MONEY MANAGEMENT

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ABSTRACT

This thesis is composed of two different sections. In the first section, the effects of the choice of inputs into a neural network model for the prediction of foreign exchange rates are examined. Fundamental indicators such as interest rates and gross domestic products, and technical indicators, such as moving averages and support and resistance levels, are fed into the neural networks to see if any relationship may be captured and improve the predictive capabilities of the model. In the second section, a comparison of different trading strategies and their resulting profitability when applied on a stock market with mean-reverting properties is made. The focus is on two main strategies, dollar cost averaging and value averaging. Dollar cost averaging is an investment strategy which reduces the investment risk through the systematic purchase of securities at predetermined intervals and set amounts. Value averaging is a strategy in which an investor adjusts the amount invested to meet a prescribed target. Results indicate that value averaging does have higher expected investment returns in a mean-reverting financial market when considering the cash flow stream of the investment. However, when a side-fund which provides loans and deposits is introduced into the cash flow stream, value averaging fails to outperform the market. Dollar cost averaging on the other hand does not provide superior performance to a random investing technique.

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF ABBREVIATIONS

BH	Buy and Hold
DCA	Dollar Cost Averaging
VA	Value Averaging
RI	Random Investing
IRR	Internal Rate(s) of Return
NPV	Net Present Value
NFV	Net Future Value
RMSE	Root Mean Squared Error
S&P 500	Standard & Poor’s 500
DJIA	Dow Jones Industrial Average
STI	Straits Times Index
P.A.	Per Annum

CHAPTER 1 INTRODUCTION

This thesis addresses two different aspects of investment using modern engineering methods. First, it studies the effectiveness of a non-linear model called artificial neural network in the prediction of daily foreign exchange rates. Second, it compares the performance of two different investment strategies, dollar cost averaging and value averaging, in a financial market with mean-reverting characteristics. In chapters 2 to 4 the artificial neural network is discussed along with the empirical findings of the experiments. The following chapter 5 presents mean-reversion and how it is modelled. Finally, chapter 6 reports the simulation results for the two different investment strategies when used on a financial market with mean-reverting characteristics.

1.1 Forecasting Exchange Rates with ANN

The amount of international trade has experienced unprecedented growth over the past few decades. This increase in global operations and interactions has propelled the foreign exchange market to be the largest and most liquid of the financial markets. It has also become a crucial factor for the success of many international businesses and fund managers who deal with currency risk on a daily basis.

The foreign exchange market also sees direct intervention from governments as exchange rates affect economics and politics. The complex interaction of these varied factors from both the private and public sector and on a macro and micro economic levels makes exchange rate prediction one of the most challenging amongst time series forecasting. Yet, the financial benefits of predicting this volatile and noisy market have driven academics and practitioners to predict exchange rates using numerous techniques.

Amongst these methods, artificial neural networks (ANNs) which are function approximators in system modeling have been used as a potential alternative for time series analysis. As a multivariate model, it is able to use a greater range of information instead of being limited to pure time-delayed data. In the case of foreign exchange prediction where multiple factors such as fundamental and technical indicators interact, it would be ideal to include them as predictors in the model. Fundamental inputs include the macroeconomic indicators such as consumer price index, Gross Domestic Product (GDP), trade surplus and interest rates. Technical inputs include, moving averages, supports and resistance levels. Individual forecast results from various sources could also be used as inputs.

Furthermore, ANNs are a non-linear, non-parametric model which are data driven. This allows the entire set of data to be utilized without having to place parametric modeling assumptions. These two advantages of ANNs over other linear and nonlinear approaches have led to an increase in research on the applications of ANNs in the prediction of exchange rates.

For a good introduction to forecasting foreign exchange rates using ANN, interested readers may refer to a survey of this research area done by Huang et al. [26]. They compare the different methods used by researchers along the forecasting process from input selection to data pre-processing methods and model selection. They highlight the advantages and disadvantages of the various ideas put forward.

Another comparison of predictive performances of ANN on exchange rates was done by Yu L. et al. [7] and focuses on a quantitative analysis of the different performance metrics and empirical findings of different researchers. They conclude that their methodology produces significantly better results when using a principal component analysis of the different performances. This method of analysis was used to overcome the variety of performance metrics proposed by different researchers.

Yao et al. [6] was one of the earlier examples of using ANN in forecasting foreign exchange rates, they found the forecasting results very promising for most currencies except the yen. They used simple technical indicators like the moving average as inputs to the network. They also highlighted the lack of an automatic facility to model construction which could alleviate the time consuming trial and error method. In fact, this problem was addressed by Refenes et al. [27] around the same time that

Yao et al. first finished their paper. They proposed and adopted a number of methods to deal with variable selection and testing model misspecification.

A variety of different architectures of ANNs were put forward and compared. Karmruzzman and Sarker [8] showed that the Scaled Conjugate Gradient and Backpropagation with Baysian Regularization models were comparable in performance when forecasting six foreign currencies against the Australian dollar. Qi and Zhang [5] expose several problems in using information-based in-sample model selection criteria in selecting the best model architecture. They conclude that there is no apparent connection between in-sample model fit and out-of-sample forecasting performance.

This is further supported by Panda and Narasimhan [9] who used a single hidden layer feedforward ANN to make daily predictions of Indian rupee/US dollar exchange rates. They conclude that ANN give better in-sample forecasts than linear autoregressive and random walk models. However, the out-of-sample results for the ANN are mixed and do not outperform the linear autoregressive model consistently.

Using monetary fundamentals as inputs, Qi and Wu [28] find that an ANN with market fundamentals as input cannot beat the random walk in out-of-sample forecast accuracy.

1.2 Mean Reversion and Money Management

Dollar cost averaging has been touted by many professional financial advisers as a superior investment technique. The investor with a sum of money to invest does not invest the entire sum immediately. Instead, at equally scheduled intervals through time, a fixed amount of the capital will be invested. In this way, the investor will purchase more shares when prices are low and less shares when prices are high.

Value averaging amplifies the benefits of dollar cost averaging. If buying fewer shares when prices are high is a good idea, then one should take the opportunity to sell some shares as well. This technique requires the investment to grow by a predefined amount each period. The amount of money needed to bring the investment up to the target level is added each period. If the value of the investment is above the target level, we bring the investment back down to the target level by selling shares.

These trading rules and their resulting profitability rely on the properties of the financial markets. The random walk description of markets has recently come under attack as such a process may diverge over time, resulting in infinite profits or losses. There is no longer an acceptable model which can be used to prove the effectiveness of these rules. However, mean reversion behavior exhibited by security prices has recently been recognized by theorists. In real world financial markets, arbitrage opportunities do arise, generating trading activity aimed at exploiting mispricing. This contributes to drive the asset prices toward their theoretically fair or equilibrium values. Mean reversion is the best way to capture this effect.

Dollar cost averaging has been around for decades, from Sharpe's classic text in 1978 to current popular publications like Malkiel's *Random Walk down Wall Street* [22], which is a compilation of academic theories on investment explained for the average man in the street. It has a section which claims that, "This technique is controversial, but it does help you avoid the risk of putting all of your money in the stock or bond market at the wrong time." No explanation is given for the controversial aspect of this technique.

Johnson and Krueger [13] have shown that dollar cost averaging falls short of a buy and hold strategy, which involves a lump sum investment up front. This is when the two techniques are used on two decades of historical closings of the Standard & Poor's 500 and Dow Jones Composite Index. Only the NASDAQ showed contrary results. They used two different metrics of performance, the dollar value of investments at the terminal date and the compounded annual returns.

Marshall and Baldwin [14] did a statistical comparison of dollar cost averaging and random investing techniques. They used the internal rate of return to an investor from simulated market scenarios under both dollar cost averaging and random investing techniques as a base of comparison. They found that there is no statistical difference in the internal rates of return achieved by each technique. Later on, Marshall [15] compares dollar cost averaging to an alternative investment strategy, value averaging. He prevents extensive evidence that value averaging does actually provide a performance advantage over dollar cost averaging and random investment techniques without incurring additional risk. He also confirms the earlier work of Marshall and Baldwin [14]. However, Marshall was unable to claim that there was a statistical difference for market scenarios with low variability and a short investment time horizon.

Marshall [15] uses a random walk hypothesis and does not implement the use of a side fund in his simulations. The random walk hypothesis is now generally rejected as an adequate description of stock price behavior. Poterba and Summers [16] presented evidence of mean reversion in stock price behavior. They presented an auto-regressive (1) model and their results suggested that stock returns show positive serial correlation over short periods and negative correlation over longer intervals. The data sets did not permit the rejection of the random-walk hypothesis at high significance levels but the sets together supported the case against the adequacy of the hypothesis.

Hillebrand [25] reviews the different mean reverting models proposed and highlights the difference between the two possible mean reversion models, mean reversion in returns and mean reversion in volatility. The latter complies with the hypothesis of efficient markets while the other does not. Using daily data of the Dow Jones Industrial Average and Standard & Poor's 500 index, he showed that mean reversion in returns is a transient but recurring phenomenon. Hence confirming Poterba and Summers' [16] work demonstrating that mean reversion in prices and returns does exist and it is impossible to tell the null hypothesis of a random walk apart from a mean reverting trend.

Brennan, Li and Torous [17] found that 'rational' individual investors with a well-defined von Neumann-Morgenstern utility function benefited from dollar cost averaging when purchasing individual stocks to add to an existing portfolio. The same was found for the purchase of a single stock. In both cases, dollar cost averaging was compared with buy and hold and measured with the Marginal Value Ratio, the ratio between the expected marginal utilities per dollar for each strategy. They were able to reproduce the same benefits when simulated with a mean reverting model based on the one suggested by Poterba and Summers [16]. The parameters of this modified model were chosen by trial and error to generate a pattern of positive serial correlation for short horizons followed by negative correlations.

1.3 Focus and Contributions

The goal of the first section of this thesis is to add to the existing literature by examining the choice of input variables into a single hidden layer back propagation neural network. The architecture will be fixed while varying only the number of hidden neurons in the hidden layer. First, the effects on the different methods of data pre-processing are examined. Second, the use of fundamental economic data is introduced to analyze the impact it has on the predictive capability of the ANN.

This second part of this thesis aims to contribute to the literature comparing the benefits of investment strategies dollar cost averaging and value averaging. It will add onto prior research in three ways. Firstly, these strategies will be compared against the buy and hold strategy. To make value averaging comparable to the buy and hold strategy, a modified version of value averaging is used. Secondly, these strategies will be simulated on a financial environment exhibiting mean reverting behavior. Finally, we examine the effects of including a side money market fund which allows making loans and deposits at a fixed interest rate. The performance of the different investment strategies will be measured by the internal rate of return found using Monte Carlo simulations.

CHAPTER 2 ARTIFICIAL NEURAL NETWORKS

The idea behind an artificial neural network (ANN) is to replicate the way that the human brain processes information. Similar to the brain, the ANN consists of interconnected units called neurons which are commonly grouped into layers. Each of these connections between the neurons has a weight value associated to it. It is through the tuning of these weights that the ANN is trained to perform its task.

As an input-output model, one of the tasks which an ANN is capable of doing is the modelling of nonlinear relationships. This ability to extract complex nonlinear interactions between inputs is attributed to its massive parallelism and multiple layers of neurons. The most commonly used ANN is the feed-forward Multi Layer Perceptron (MLP) network which is trained via back-propagation. The popularity of this particular ANN is due to the extensive mathematical documentation by Rumelhart et al. [1] on the MLP and the back-propagation algorithm.

2.1 Architecture

A MLP network consists of at least one input layer represented by the vector $\boldsymbol{X} = (x_1, x_2, \ldots, x_n)'$ and one output layer $\boldsymbol{Y} = (y_1, y_2, \ldots, y_m)'$ where n and m are the numbers of inputs and outputs. In between these two layers are k hidden layers each with their own number of hidden neurons.

The feed-forward property means the connections between the neurons are only allowed to move forward. Neurons in the same layer are not permitted to be connected nor are feed-back connections possible. Each neuron in one layer is connected to every single neuron of the following layer. As the signal is passed forward, the real-valued weight of each connection will modify the signal while the receiving neuron will sum up all the signals it receives and add a bias term before transferring it through its transfer function φ and relaying it on.

This transfer function is also known as an activation function and is necessarily continuous and differentiable. The common transfer functions used in ANN neurons are the sigmoidal-type functions like the logistic and hyperbolic tangent functions. Other known functions are the radial basis function and the polynomial function.

Hornik et al. [2] have shown that a typical back-propagation ANN with one hidden layer is able to approximate any function if given sufficient free parameters. With this in mind, the architecture of the ANN used in the simulations is fixed to one layer. Considering that the number of neurons in both the input and output layers depends on the input-output model, the only variable is the number of neurons in the single hidden layer. In other studies, either an evolutionary approach or a trial and error method was used to find the optimal solution. For this study, the focus is on the effect of the types of input hence the number of hidden neurons is varied within a fixed range.

There is only one output neuron which gives the predicted exchange rate. The use of a single output is to prevent a situation where multiple outputs lead to conflicting weights and biased results.

For a three-layered feed-forward MLP network with n inputs and h hidden neurons, the neurons in the input layer do not have transfer functions and are used to distribute the input signals to every neuron in the hidden layer. The output from the hidden layer is noted by the vector $Z = (z_1, z_2, \ldots, z_h)'$. The bias terms which are always equal to one are noted as x_0 and z_0. The weight associated with each connection from input neuron i to hidden neuron j is β_{ij}^1 while the weight of the connection from hidden neuron i to the single output neuron is β_i^2. Thus, the outputs from the hidden neurons and output neuron are

$$z_j = \varphi^1\left(\sum_{i=0}^{h} \beta_{ij}^1 x_i\right), \quad j = 1,2,\ldots,h.$$

$$y = \varphi^2\left(\sum_{i=0}^{h} \beta_i^2 z_i\right).$$

The common choice of transfer function for an ANN with predictive out-of-sample tasks is a log-sigmoidal transfer function for the hidden layer and a linear transfer function in the output layer.

$$\varphi^1(x) = \frac{1}{(1+e^{-x})},$$

$$\varphi^2(x) = x.$$

This enables the ANN to extrapolate out of the range of its training data which is possible in the context of predicting foreign exchange rates.

2.2 Training

The ANN is trained with a training set of the form

$$G = \{(X_1, d_1), (X_2, d_2), \cdots, (X_p, d_p)\},$$

where d_p is the desired output from the single output neuron when the input to the ANN is X_p and p is the total number of pairs in the training set. The aim of this training is to minimize the sum squared error E at the output layer over all the training data by adjusting the weights systematically.

$$E = \frac{1}{2}\sum_{l=1}^{p}(d_l - y_l)^2 .$$

This cost function is dependent only on the weights β_{ij}^1 and β_i^2. The standard back-propagation algorithm by Rumelhart et al. [1] minimizes the cost function using the steepest gradient descent technique to approximate the change required to each weight by

$$\Delta\beta_{ij}^k = -\alpha\frac{\partial E}{\partial\beta_{ij}^k},$$

where α is the learning rate. This is the most important parameter which determines how fast the cost function converges. The optimum value of the learning rate depends on the error surface which is often too complex to calculate and is often found through experimentation.

To improve on the speed of training and convergence, the Levenberg-Marquardt algorithm which is an approximation to Newton's method is introduced. This method requires the calculation of the Jacobian matrix J of the partial derivatives of the network errors with respect to the weights [3]. The matrix for the weight updates becomes

$$\Delta\beta = \left(J^T J + \mu I\right)^{-1} J^T e ,$$

where μ is a parameter multiplied by some factor σ whenever a step would result in an increased E and is divided by σ when there is a decreased E. A large value of μ makes the algorithm into the steepest descent while a small μ the Gauss-Newton is obtained.

Each of the iterations in this algorithm consists of two passes. First the input vector $\boldsymbol{X}$ from a pair in training set $\boldsymbol{T}$ is applied to the ANN to produce output y. Second the output is compared with the desired output d for the pair and the error E is propagated backwards and the weights adjusted accordingly. If the weights are adjusted at each of the iterations then this is known as the online mode. On the other hand, if all the errors are calculated for every pair in the training set, which is also known as an epoch, before updating the weights, it is known as offline or batch mode. This is the mode preferred by other researchers as it gives a better approximation of the gradient at each weight update. Hence, the batch mode is used in the training algorithms in this study.

The initial values of the weights are generated with Nguyen and Widrow's [4] method. The combination of these methods of initialisation and estimation may still encounter the problem of local minimum, hence each network simulation is run 10 times based on 10 different initial parameter values and the one with the least sum of square errors is used for the out-of sample prediction and evaluation. This best represents the actual use of an ANN for prediction where the user has no knowledge of the future and will train the ANN till it has the least error before using it. However, it should be noted that Qi and Zhang [5] concluded that the in-sample model selection criteria does not provide a reliable guide to out-of sample prediction performance. Nevertheless, the lack of more appropriate methods leaves this as the most realistic choice.

2.3 Validation

A well trained ANN is able to generalize and give good results for independent input-output pairs not in the training set. If the performance for the training set is much better than the independent data set, it is highly probable that the network has overfitted by fitting the noise found in the training data. A solution to this problem is the use of a validation set which monitors the performance of the network after each update of the weights and stops the training when the performance gets worse. For this study, the training was stopped when the performance for the validation set got worse for 5 consecutive updates.

According to the work of Yao and Tan [6], the size of the training, validation and testing sets should be 70%, 20% and 10% of the collected data respectively. This recommendation comes as a result of the researchers' experience. The division of the collected data is done in a sequential fashion with the training set comprising of the oldest data and the test set has the latest data.

2.4 Performance Measure

This study is focused on the out-of-sample predictions of a trained ANN. The ANN is first trained with the training set until the training is stopped when the validation set performance deteriorates or the maximum number of training epochs is reached. With this ANN, predictions are made with the *k* input-output pairs in the test set and the network's performance is measured by the root mean square error (RMSE) and the directional accuracy (DA) which is the percentage of correct predictions in terms of direction changes. These are widely used performance metrics and were the two main metrics used by Yu et al. [7] in their comprehensive comparison analysis model of fifteen studies which applied ANN to exchange rate prediction.

Other common performance metrics include the trend accuracy (TA), the mean absolute percentage error (MAPE), the mean absolute error (MAE) and the goodness of fit (R-value).

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(y_i - \hat{y}_i\right)^2} \quad ,$$

$$DA = \frac{1}{N}\sum_{i=1}^{N} a_i \text{ where } a_i = \begin{cases} 1 \; if \; (y_{i+1} - y_i)\left(\hat{y}_{i+1} - y_i\right) > 0 \\ 0 \quad otherwise \end{cases} \quad ,$$

$$TA = \frac{1}{N}\sum_{i=1}^{N} a_i \text{ where } a_i = \begin{cases} 1 \; if \; (y_{i+1} - y_i)\left(\hat{y}_{i+1} - \hat{y}_i\right) > 0 \\ 0 \quad otherwise \end{cases} \quad .$$

$$MAPE = \frac{1}{N}\sum_{i=1}^{N}\left|\frac{\left(y_i - \hat{y}_i\right)}{y_i}\right| \quad ,$$

$$MAE = \frac{1}{N}\sum_{i=1}^{N}\left|\left(y_i - \hat{y}_i\right)\right| \quad ,$$

$$R^2 = 1 - \left(\frac{\sum_{i=1}^{N}\left(y_i - \hat{y}_i\right)^2}{\sum_{i=1}^{N}\left(y_i - \bar{y}_i\right)^2}\right) \quad .$$

The RMSE is a relevant measurement of performance only when the aim of the predictions is to minimize the size of the squared errors without taking into consideration the direction of the errors. However in the financial context, it is essential that the predictions are in the correct direction to determine the course of action to take, whether the trader goes long or short. Hence, this metric has to be used together with DA to ensure profitability in the predictions made by the ANN.

2.5 Simulation Environment and Verification

The simulations were run using the Matlab 7.2 software package implemented on Windows XP.

To ensure that the neural network designed in Matlab was performing correctly, the network was used to replicate the results of other researchers and their experiments. The principle considerations when choosing which experiments to replicate were:

a) *Detailed source and period of training and testing data:* The paper has to be very specific about the source of its data and the choice of inputs to the neural network. Furthermore, the data has to be readily available online.

b) *Training algorithm:* The performance of a network is dependent on the parameters used in a training algorithm. Thus, the parameters have to be explicitly stated in the paper.

c) *Network architecture:* The choices of transfer function and number of hidden neurons have to be detailed to ensure that the results are reproducible.

After searching for research papers with such detailed discussions on the development and design of the ANN, three papers were short listed, Kamruzzaman & Sarker [8], Yao & Tan [6] and Panda & Narasimhan [9].

a) *Kamruzzaman & Sarker:* The paper chose its weekly data from the Reserve Bank of Australia and used daily simple moving averages as inputs to the network. The back-propagation training algorithm did not use a validation set and terminated its training between 5000 and 10000 iterations. There were no details as to how the maximum number of iterations was chosen. The performance results of these authors were not reproducible in the Matlab environment developed for this study.

b) *Yao & Tan:* The training and testing periods used for the evaluation of the ANN were clearly specified. However, the source of data from which they obtained the Friday closing of the Singapore Exchange's foreign exchange rates was not stated. Data from another source was used for the same time period and comparison of the statistics of the observations used was made. There was a slight difference in the statistics yet the out-of-sample forecasting results for the specified model architecture (5-3-1) were not reproducible with Yao's performance being much better. Despite getting in contact with Yao through e-mail, no further details were given regarding the development or initial parameters of the ANN.

c) *Panda & Narasimhan:* This article studies the daily spot rates of the Indian rupee/US dollar exchange rate. The data set is from the Pacific FX database which is readily accessible online. The authors detailed the training algorithm and specified the various network architectures used. They had chosen to run the simulations on the Matlab 6.1 software package. The training performance of both the in-sample and out-sample forecasts were all replicable.

This short exercise in verifying the development and simulation environment for the ANN has demonstrated the difficulties in replicating the results of previous studies due to the sensitive nature of ANN to its training and initial parameters. It has also verified that the programming of the simulation environment is at least consistent with other researchers.

CHAPTER 3 DATA PRE-PROCESSING

The foreign exchange markets are considered as highly liquid markets with over three trillion U.S. dollars in them. The markets are open 24 hours and traded on three main exchanges each on their own continent and time zone. This makes the already volatile market have different characteristics on different exchanges. To improve the quality of the raw data, which in this case is the daily closing exchange rate, the data has to be pre-processed.

Data pre-processing is an important process of developing an ANN so as to ensure that the essential features of the data may be extracted. In our study, it acts as a filter which cancels out the noise through the use of moving averages or logarithmic returns. Another important step is to determine the effect that the number of lags will have on the performance of the network. Lagged data refers to older data in a given time series. If the ANN has an input of the closing rate c up to 2 lags, then it would use c_T, c_{T-1} and c_{T-2} to predict c_{T+1} at its output. Too many redundant inputs will slow down the training duration and introduce additional degrees of freedom which could lead to overfitting.

3.1 Data Sets

The data used in this study is the daily foreign exchange rate of the U.S. dollar against the other three core currencies in the global economy today, the Japanese Yen, the British pound and the European Union Euro.

For this section where the different methods of data pre-processing are examined, only the daily exchange rates are required and the sample data set is taken from Pacific FX database which is maintained by the Sauder School of Business at the University of British Columbia. This database may be accessed online at http://fx.sauder.ubc.ca/data.html. Two different data sets were used. The test data sets were fixed with 252 observations each starting from May 1, 2003 to April 30, 2004 and Dec 29, 2005 to Dec 29, 2006. The training data was chosen to start from both Jan 8, 1993 and Aug 25, 1995. This provides 2611 observations immediately before the evaluation periods.

The earlier test period was chosen to make the results of this study comparable to Yu et al. [7] who concluded using a comprehensive comparison analysis model that they had the best prediction ANN compared to earlier research. To ensure that the performance of the network is not unique to a particular period, two different time periods were used in the simulations.

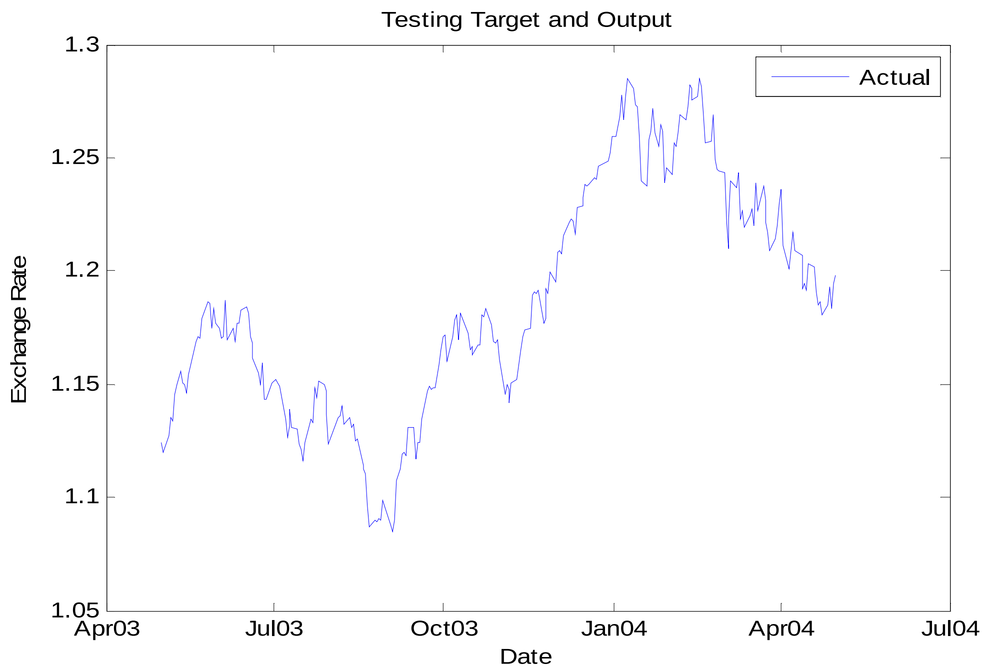

Figure 3-1 Historical EUR/USD Exchange Rate for Scenario A

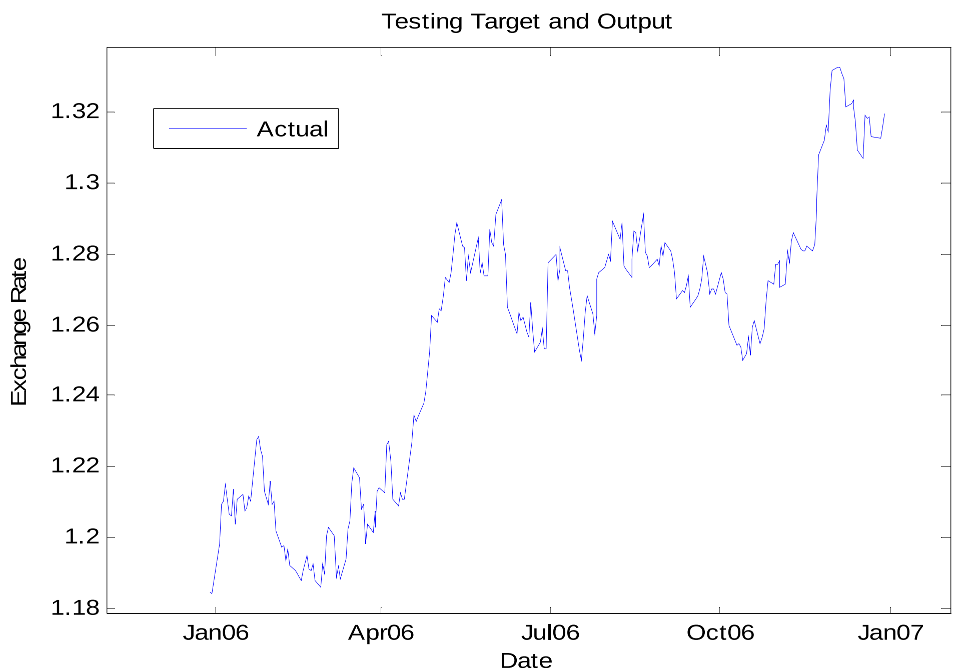

Figure 3-2 Historical EUR/USD Exchange Rate for Scenario B

3.2 Data Division and Normalization

Using the rule of thumb by Yao and Tan [6] mentioned earlier as guidance, the training set was further split into a training and validation set with the validation set making up 20% of the entire historical data set.

Data normalization is one of the most important steps in the use of an ANN. Different inputs could have different ranges and to ensure that none of the transfer functions in the neurons becomes saturated due to a large input, the input data has to be normalized. The data set into each input is normalized using the maximum and minimum of each set and it is done independently of the other sets. Whereas the range of the normalization is kept fixed as transfer functions in the hidden layer are all the same. As the ANN in this study uses the log-sigmoidal transfer function with an output range of 0 to 1, both the input and target data should be normalized to (0, 1). This may be done linearly or using the logistic function. For this study, the training data is normalized linearly.

Apart form the obvious methods of normalization, if the data is processed from raw daily closing rates to log-returns, the data will be reduced to the (-1, 1) range which is an acceptable range when using the log-sigmoidal transfer function. This alternative method of data pre-processing is further examined via simulations.

3.3 Experiment Design

In these series of simulations, the ANN will be used for one-step-ahead prediction of daily exchange rates. Despite having the ANN architecture fixed to a three-layer feed-forward network and specifying the transfer functions used in each layer, there are still other parameters which affect the performance of an ANN. The other choices of the number of inputs n and the number of hidden neurons h in the middle layer will also affect the network by changing the total number of parameters q as $q = h(2+n)+1$. Having too many free parameters with respect to the number of inputs and observations could result in over-fitting. On the other hand, if there were too few hidden neurons compared to the number of inputs, an under-fitted network would be obtained. There are no fixed rules regarding such choices but there exist some guidelines like Widrow's rule of thumb and Baum and Haussler's result for valid generalization, both of which are briefly discussed in Haykin [29].

In this case, as there is only a single type of input, the number of inputs would vary only by the choice of the number of lagged data to be used. In order to avoid a model selection bias, both the number of lagged data and the number of hidden neurons will vary. The input data will have lags from 2 to 5 for each method of data pre-processing while the number of hidden neurons will vary from 4 to 10. The different performances will be compared to ascertain the advantages of each pre-processing method.

3.4 Results and Discussion

The results of the simulations prediction are shown in the Appendix. The different tables report the results on the one-step-ahead predictive performance for each of the exchange pairs. The discussion will focus on the EUR/USD exchange pair in both scenarios and then cover briefly the other two exchange pairs.

3.4.1 Pure Time Delayed Closing Rates

The essential performance measurements of the predictive results for pure time delayed inputs are shown in the Table 3-1. Fig. 3-1 and 3-2 are plots showing the actual historical time series of the test data for the EUR/USD exchange rate for the two scenarios.

Table 3-1 Using Pure Time Delayed Rates as inputs for EUR/USD

No. of Lags	2	3	4	5	2	3	4	5
Model	RMSE				DA			
Scenario A								
n=4	0.00833	0.00830	0.00838	0.00850	0.48413	0.48810	0.48413	0.49603
n=5	0.00829	0.00841	0.00837	0.00831	0.50000	0.47619	0.45635	0.48016
n=6	0.00840	0.00837	0.00835	0.00833	0.40476	0.47222	0.44841	0.49603
n=7	0.00838	0.00846	0.00845	0.00842	0.43254	0.49603	0.45635	0.49603
n=8	0.00836	0.00836	0.00844	0.00844	0.47222	0.46429	0.48810	0.45635
n=9	0.00835	0.00848	0.00860	0.00847	0.44444	0.48810	0.50000	0.46825
n=10	0.00834	0.00836	0.00852	0.00841	0.47222	0.50000	0.45635	0.48413
Scenario B								
n=4	0.00590	0.00594	0.00591	0.00593	0.50794	0.52778	0.50397	0.51190
n=5	0.00592	0.00590	0.00591	0.00591	0.51587	0.49603	0.50000	0.50794
n=6	0.00594	0.00593	0.00592	0.00592	0.49603	0.54762	0.54365	0.51190
n=7	0.00597	0.00591	0.00594	0.00595	0.49603	0.48810	0.53571	0.54365
n=8	0.00594	0.00595	0.00593	0.00594	0.54762	0.50794	0.51190	0.54762
n=9	0.00594	0.00596	0.00588	0.00596	0.49206	0.54762	0.49603	0.53571
n=10	0.00595	0.00592	0.00594	0.00597	0.55556	0.52381	0.52778	0.51190

The initial impression from both the performance metrics and the plots is very good. The RMSE value is low and the predicted plot fits the actual time series well. The DA on the other hand is poor with values around 50% which imply that the ANN does not have a market-timing ability when using pure time delayed closing rates. This is the case regardless of the number of lagged data at the input and the number of hidden neurons.

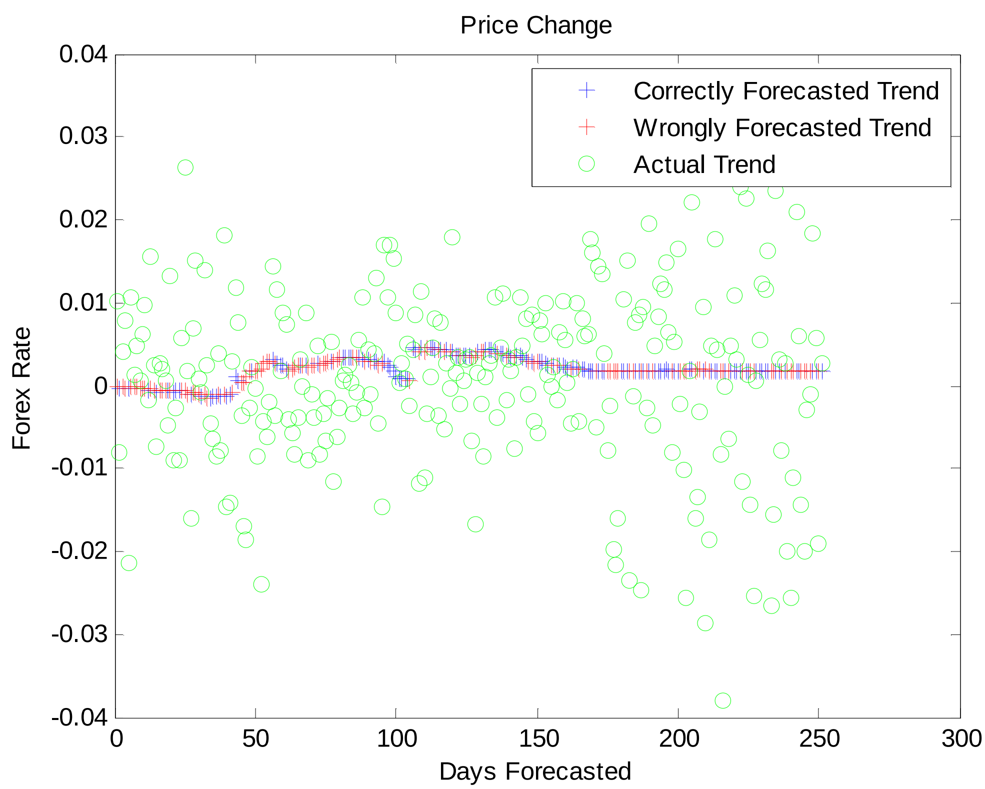

Figure 3-3 Price Change Prediction Performance for EUR/USD Scenario A

Using the trend analysis, it may be seen that the network has been trained to output today's exchange rate as the predicted rate for tomorrow. Fig. 3-3 shows the rate change between tomorrow's predicted rate and today's rate $\hat{y}_{T+1} - y_T$ while Fig. 3-4 shows the trend change between tomorrow's predicted rate and today's predicted rate $\hat{y}_{T+1} - \hat{y}_T$. The rate change is close to zero as tomorrow's predicted rate is today's actual rate multiplied by a weighting close to unity. The trend change has the same magnitude as the actual changes but is delayed by on period. This demonstrates that the ANN was not able to extract any relationships between the time delayed closing exchange rates.

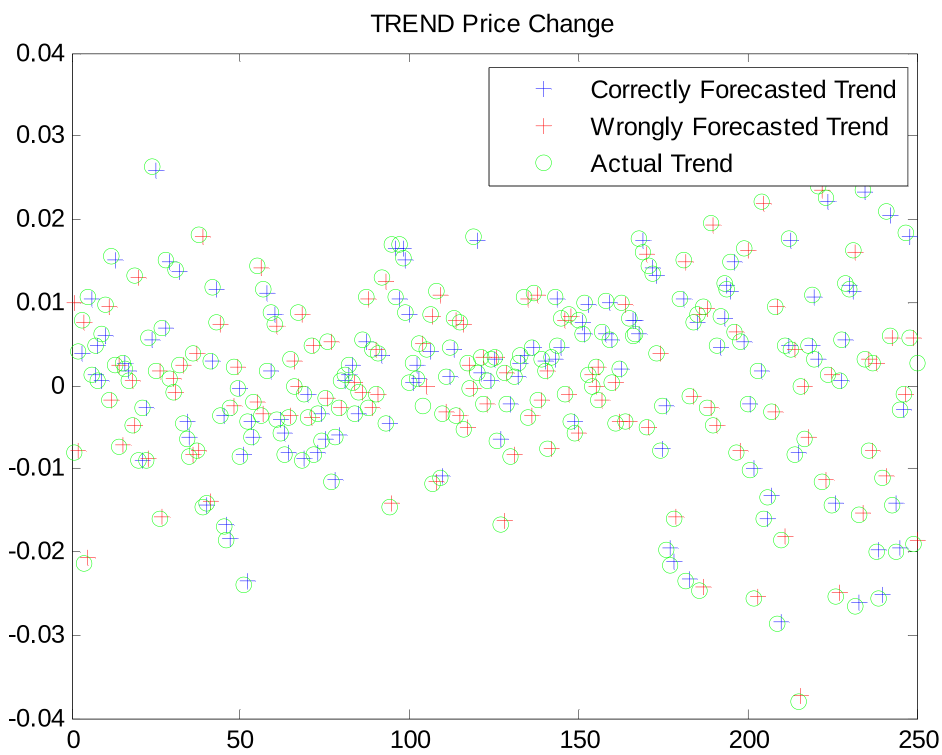

Figure 3-4 Trend Change Prediction Performance for EUR/USD Scenario A

These results are similar to those for the USD/JPY and the GBP/USD exchange pairs. Their detailed results may be found in the Appendix.

3.4.2 Moving Averages

Moving averages are one of the most common technical analysis indicator used by traders. In this study, the simple moving average M^d is used,

$$M^d = \frac{\left(y_T + y_{T-1} + \ldots + y_{T-d+1}\right)}{d}.$$

This represents the *d* trading days' moving average. These indicators act as a filter to the noise present in the daily closing rates.

Other researchers like Yao & Tan [6] and Kamruzzaman & Sarker [8] used the indicators $M^5{}_T$, $M^{10}{}_T$, $M^{20}{}_T$, $M^{60}{}_T$, $M^{120}{}_T$ and y_T as inputs to the ANN to predict y_{T+1}. However, they used weekly data instead of daily. With this setup, they were able to achieve DA above 65%.

When using the 5, 10, 20, 60 and 120 day moving averages as inputs, the same results as shown in Table 3-2 below were not achieved in our simulations. The number of hidden neurons was varied between 4 and 6 to check for any model selection bias. One of the possible reasons is that the weekly data contains less noise and moves much smoother than daily data.

Table 3-2 Using Moving Averages as inputs for EUR/USD

Model	RMSE	DA
Scenario A		
n=4	0.00834	0.45635
n=5	0.00834	0.51984
n=6	0.00834	0.51984
Scenario B		
n=4	0.00595	0.50794
n=5	0.00598	0.48016
n=6	0.00496	0.73810

Our study on the other hand uses q time delayed d days moving average indicators (M^d_T, M^d_{T-1}, …, M^d_{T-q}) together with today's rate y_T to predict y_{T+1}. The number of lagged or time delayed data q is varied from 0 to 3 while d is taken as 5 or 10.

The results of these sets of simulations are shown in Table 3-3. Similar to the results seen earlier when pure time delayed closing rates were used as inputs; the ANN has learnt the trend and outputs it with a one period delay. Their performance is unaffected by the number of lagged data used. It is also the same regardless of the number of hidden neurons in the network. As before, the same results are obtained for the other two exchange pairs. This leads to the conclusion that lagged 5 day or 10 day moving averages when used separately does not have any relationship with the closing rate which allows them to be used in time series prediction.

Table 3-3 Using Lagged 5-day Moving Average as inputs for EUR/USD

No. of Lags	2	3	4	5	2	3	4	5
Model	RMSE				DA (%)			
Scenario A								
n=4	0.00835	0.00833	0.00832	0.00830	0.45238	0.51190	0.49206	0.50397
n=5	0.00834	0.00837	0.00840	0.00839	0.48016	0.46825	0.45238	0.46032
n=6	0.00838	0.00835	0.00837	0.00838	0.49206	0.47222	0.46825	0.46429
n=7	0.00833	0.00837	0.00833	0.00845	0.50794	0.48413	0.49206	0.47619
n=8	0.00835	0.00836	0.00842	0.00845	0.46429	0.46429	0.46429	0.45635
n=9	0.00834	0.00836	0.00841	0.00843	0.47619	0.46032	0.48810	0.42460
n=10	0.00837	0.00835	0.00839	0.00838	0.45238	0.49206	0.47222	0.42857
Scenario B								
n=4	0.00591	0.00593	0.00594	0.00592	0.49206	0.48413	0.49206	0.46825
n=5	0.00590	0.00594	0.00597	0.00597	0.49206	0.48810	0.48810	0.47222
n=6	0.00595	0.00595	0.00597	0.00594	0.51984	0.50397	0.50397	0.52778
n=7	0.00599	0.00597	0.00601	0.00604	0.52381	0.53571	0.49603	0.51984
n=8	0.00590	0.00598	0.00595	0.00597	0.51587	0.51587	0.48413	0.48016
n=9	0.00594	0.00592	0.00601	0.00593	0.56746	0.50397	0.47222	0.46032
n=10	0.00598	0.00601	0.00601	0.00600	0.48810	0.49603	0.50397	0.51190

Table 3-4 Using Lagged 10-day Moving Average as inputs for EUR/USD

No. of Lags	2	3	4	5	2	3	4	5
Model	RMSE				DA (%)			
Scenario A								
n=4	0.00833	0.00835	0.00835	0.00834	0.47222	0.50794	0.47619	0.48016
n=5	0.00833	0.00833	0.00836	0.00828	0.44841	0.48810	0.45238	0.46429
n=6	0.00836	0.00837	0.00832	0.00836	0.46032	0.46825	0.50794	0.48810
n=7	0.00835	0.00835	0.00833	0.00840	0.48016	0.50794	0.49206	0.45635
n=8	0.00832	0.00833	0.00835	0.00837	0.48810	0.48016	0.48016	0.49603
n=9	0.00838	0.00838	0.00838	0.00837	0.43651	0.46825	0.47619	0.46032
n=10	0.00834	0.00834	0.00836	0.00835	0.47222	0.50397	0.49206	0.48016
Scenario B								
n=4	0.00591	0.00592	0.00591	0.00589	0.49206	0.48016	0.48016	0.52381
n=5	0.00591	0.00590	0.00594	0.00591	0.49206	0.47222	0.47619	0.51190
n=6	0.00590	0.00593	0.00593	0.00594	0.55556	0.50397	0.48810	0.51587
n=7	0.00600	0.00593	0.00600	0.00593	0.54365	0.50794	0.48810	0.50794
n=8	0.00595	0.00596	0.00595	0.00593	0.53571	0.51587	0.51587	0.51190
n=9	0.00592	0.00601	0.00595	0.00590	0.52381	0.52381	0.46429	0.56746
n=10	0.00611	0.00599	0.00594	0.00597	0.55556	0.51984	0.53571	0.53571

3.4.3 Log-Returns

It is common for research in the finance field to use logarithmic returns when analyzing a price series instead of the price series itself. This is because the use of log-returns enables us to get past the problem of non-stationarity and outlier effects. In so doing, the prediction bias may be removed.

The log-return *r* at day *T* is

$$r_T = \log\left(\frac{y_T}{y_{T-1}}\right).$$

Using 0 to 3 lagged log-returns (r_T, r_{T-1}, r_{T-2}, r_{T-3}) as inputs, the ANN is trained to predict the following day's log-return r_{T+1}. To avoid model selection bias, the number of hidden neurons is varied from 4 to 10.

As discussed earlier, converting the raw closing rate series data into log-returns is a form of normalization to an appropriate range for input into an ANN. This method of normalization is compared against using the maximum and minimums of the training data to normalize linearly into the (0, 1) range.

Table 3-5 and 3-6 compares the performance of these two different methods of data pre-processing during the test data set. As compared to the previous two methods of using lagged closing rates and moving averages, the DA performance improves greatly when lagged log-returns are used as inputs to predict the following day's log-returns. The average DA for these networks is approximately 70%. When looking at the R-value of the returns performance, a surprisingly low average of 0.58 is found. This indicates a poor fit of the predicted out of sample performance.

Both methods of normalization produce approximately the same results. The choice of the number of hidden neurons and the number of lagged log-returns data to be used as inputs did not have a major impact on the results. Once again, the same trends are seen for the other exchange pairs.

Table 3-5 Using Log-Returns to Predict Log-Returns without normalization

No. of Lags	0	1	2	3	0	1	2	3
Model	RMSE				DA (%)			
Scenario A								
n=4	0.00698	0.00697	0.00696	0.00704	0.74206	0.74603	0.74603	0.74603
n=5	0.00698	0.00700	0.00703	0.00704	0.74603	0.74206	0.75000	0.75397
n=6	0.00698	0.00702	0.00705	0.00708	0.74206	0.74206	0.75397	0.73810
n=7	0.00699	0.00710	0.00701	0.00709	0.74603	0.74206	0.74603	0.74206
n=8	0.00697	0.00705	0.00697	0.00720	0.74206	0.74603	0.73810	0.74603
n=9	0.00696	0.00700	0.00701	0.00735	0.74603	0.74206	0.74206	0.73016
n=10	0.00711	0.00704	0.00716	0.00714	0.73810	0.74603	0.74603	0.73413
Scenario B								
n=4	0.00467	0.00470	0.00468	0.00473	0.77381	0.76984	0.77381	0.76984
n=5	0.00467	0.00470	0.00468	0.00474	0.77778	0.77381	0.77381	0.75794
n=6	0.00467	0.00469	0.00468	0.00468	0.76587	0.77381	0.76587	0.77778
n=7	0.00468	0.00469	0.00470	0.00472	0.77381	0.77381	0.75794	0.76587
n=8	0.00466	0.00469	0.00470	0.00469	0.76984	0.76587	0.76587	0.76984
n=9	0.00466	0.00471	0.00471	0.00471	0.76587	0.76984	0.77381	0.76190
n=10	0.00466	0.00468	0.00470	0.00473	0.76587	0.75794	0.76587	0.76984

Table 3-6 Using Log-Returns to Predict Log-Returns with linear normalization (0, 1)

No. of Lags	2	3	4	5	2	3	4	5
Model	RMSE				DA (%)			
Scenario A								
n=4	0.00698	0.00699	0.00697	0.00696	0.74206	0.74206	0.74603	0.75000
n=5	0.00699	0.00700	0.00693	0.00701	0.75000	0.74603	0.74603	0.75397
n=6	0.00697	0.00702	0.00691	0.00714	0.74206	0.75000	0.74206	0.74206
n=7	0.00699	0.00702	0.00696	0.00704	0.74603	0.74603	0.75000	0.73810
n=8	0.00699	0.00697	0.00701	0.00706	0.75000	0.74206	0.75000	0.75000
n=9	0.00698	0.00699	0.00703	0.00704	0.74603	0.75000	0.74603	0.76190
n=10	0.00700	0.00710	0.00712	0.00699	0.74603	0.74206	0.75000	0.75000
Scenario B								
n=4	0.00468	0.00470	0.00469	0.00472	0.77778	0.77778	0.77381	0.77381
n=5	0.00467	0.00470	0.00470	0.00471	0.77778	0.77381	0.78571	0.77778
n=6	0.00466	0.00469	0.00470	0.00473	0.76984	0.77778	0.77381	0.76587
n=7	0.00467	0.00469	0.00471	0.00470	0.77778	0.77381	0.76587	0.77381
n=8	0.00467	0.00469	0.00468	0.00470	0.76984	0.77778	0.77381	0.77778
n=9	0.00465	0.00468	0.00466	0.00474	0.76587	0.77778	0.76984	0.76984
n=10	0.00465	0.00470	0.00470	0.00469	0.76587	0.77778	0.77778	0.77778

3.4.4 Rates Prediction with Returns

The ability to predict log-returns in the correct direction will only be useful if this may be translated into the more difficult prediction of exchange rates. Two different methods are examined. First, the predicted log-return is taken from the ANN, then its exponential is used together with today’s exchange rate to calculate tomorrow’s estimated rate. Second, today’s exchange rate is included as one of the inputs into the ANN to check if there exists a non-linear relationship which may be extracted.

These two simulations will use the linear normalization method. Since the performance of the returns prediction earlier on are relatively insensitive to the number of lagged inputs and hidden neurons used, the simulations were ran varying the lagged log-returns as inputs from 0 – 3 and with the ANN architecture of varying at a reduced scale between 5 - 7 hidden neurons to prevent model selection bias. The results of the simulations are presented in Table 3-7 and Table 3-8.

These results are in line with the paper written by Panda & Narasimhan [9] where both the out and in sample performance of neural networks was tested on the INR/USD exchange rate pair using log-returns as inputs. In this study, it was found that the performance of the ANN was much poorer when the predicted log-returns were converted back to the price series.

Table 3-7 Returns Added Back on Price for EUR/USD

No. of Lags	0	1	2	3	0	1	2	3
Model	RMSE				DA			
Scenario A								
n=5	0.0083	0.0083	0.0084	0.0084	0.4802	0.5595	0.5159	0.5238
n=6	0.0083	0.0083	0.0083	0.0084	0.5317	0.5317	0.5357	0.5397
n=7	0.0083	0.0084	0.0083	0.0085	0.5000	0.4762	0.4683	0.5000
Scenario B								
n=5	0.0059	0.0059	0.0059	0.0059	0.5040	0.5437	0.5119	0.4683
n=6	0.0058	0.0059	0.0059	0.0059	0.5238	0.5119	0.5159	0.4921
n=7	0.0059	0.0059	0.0059	0.0059	0.5397	0.5079	0.5159	0.4643

When comparing these two methods of incorporating the predicted returns series to estimate the exchange rate, neither method outperforms the other consistently. They are both sensitive to the choice of the number of inputs and the number of hidden neurons without any clear relationship as to how these parameters affect their performance. None of the performance metrics are significant enough for use in actual trading and are similar to the results found earlier when using the pure time delayed closing rates and moving averages.

Table 3-8 Using Returns and Price as Input for EUR/USD

No. of Lags	0	1	2	3	0	1	2	3
Model	RMSE				DA			
Scenario A								
n=5	0.0083	0.0084	0.0083	0.0083	0.4722	0.5000	0.4643	0.4921
n=6	0.0084	0.0083	0.0083	0.0084	0.4881	0.5040	0.4802	0.4881
n=7	0.0084	0.0084	0.0084	0.0084	0.4603	0.4762	0.5159	0.4960
Scenario B								
n=5	0.0059	0.0059	0.0059	0.0059	0.4921	0.5119	0.5079	0.5079
n=6	0.0059	0.0059	0.0059	0.0060	0.4921	0.4722	0.5159	0.4881
n=7	0.0059	0.0059	0.0059	0.0059	0.5000	0.4881	0.5198	0.4841

CHAPTER 4 USE OF FUNDAMENTAL DATA

The foreign exchange rate is linked to the fiscal standings of the countries, their trade relations, key interest rates and inflations rate. There are other factors and economic data which do affect the minor fluctuations but these generally do not influence the long term trend. Economic data which are known indicators of trade relations and the level of inflation are the Gross Domestic Product (GDP), quarterly export and import numbers and the Consumer Price Index (CPI). Together with interest rates, these sets of figures were chosen by Yu et al. [7] as inputs into their ANN for three reasons: (1) these variables represent fundamental features which an ANN should detect in order to obtain correct outputs; (2) they should have variation which leads to generalization and not memorization; and (3) they should not have a case where identical inputs give different outputs. The second reason provided does not justify the choice of such data as Yu et al. later go on to point out that these macroeconomic figures are not available on a daily basis and are input as constants over the given quarter or time period when it is left unchanged. They consider these "explanatory variables as dumb variables to adjust the neural network forecasting model".

Financial markets have been hypothesized to be leading indicators of the economic or business cycles. They are bearish before a recession and bullish before economies start expanding again. This is because the prices of financial securities or exchange rates in this case are determined not only by arbitrage free formulas but also with investor's sentiments and expectations. Of course the move will not necessarily be in the correct direction all the time and even then the market is quick to correct itself. Whether this is done efficiently or not is still up for debate among the academics.

If the financial markets are in fact leading indicators, then the introduction of coincident or lagging indicators such as the GDP and CPI should not contain information or relationships which may improve the performance of the ANN. An alternative to this is to use measures of market sentiment or expectations as inputs into an ANN to see if it is able to extract a relationship between this data and the exchange rates to predict future rates.

In this chapter, the influence of the fundamental economic figures on the performance of the ANN will be examined. On top of that, a novel approach is used to check the effects of using a leading indicator of exchange rates to the predictive power of an ANN. This approach helps to overcome the problem of insufficient data and uses a future closing rate as a measure of market sentiment. The limit at which this measure loses its predictive properties is also investigated.

To avoid any confusion, the single output of the ANN is known as the *predicted* exchange rate while the input which serves as the future price is known as the *forecasted* exchange rate.

4.1 Perfect Future price

In the previous chapter, simulations of ANN using various pre-processing methods were examined. Unfortunately, none of the methods showed any contribution to an improved predictive performance of the ANN. Hence in the following simulations of this chapter, only pure time delayed exchange rates will be used as inputs along with the market future price. The imaginary experimental indicator which was chosen is the weekly closing exchange rate. This is the closing rate of the last trading day of each week. As the quotes on the Pacific Sauder database are noon spot exchange rates from the Bank of Canada at around 3pm Eastern Time, the Canadian holiday schedule applies and if there is no trading on a Friday, then the latest trading day's close will be taken.

It was demonstrated earlier that the number of hidden neurons did not have a significant impact on the performance of the ANN when they were in the range of 4 to 6. To save time, the simulations in this chapter were run on an ANN with 4 to 6 hidden neurons and pure time delayed exchange rates from 2 to 4 lags. Further simulations were run to eliminate the possibility of model selection bias but their results were left out to save space as they were inconsequential.

4.1.1 Different Time Frames

The future prices have a weekly frequency while the time series which the ANN is predicting has a daily frequency. There are two possible approaches to overcome this issue of different time frames. First, the weekly future prices may be kept constant and used as daily inputs. Second, the future price and this week's closing price may be used to interpolate 5 data points to represent a daily future price series as shown in the figure below.

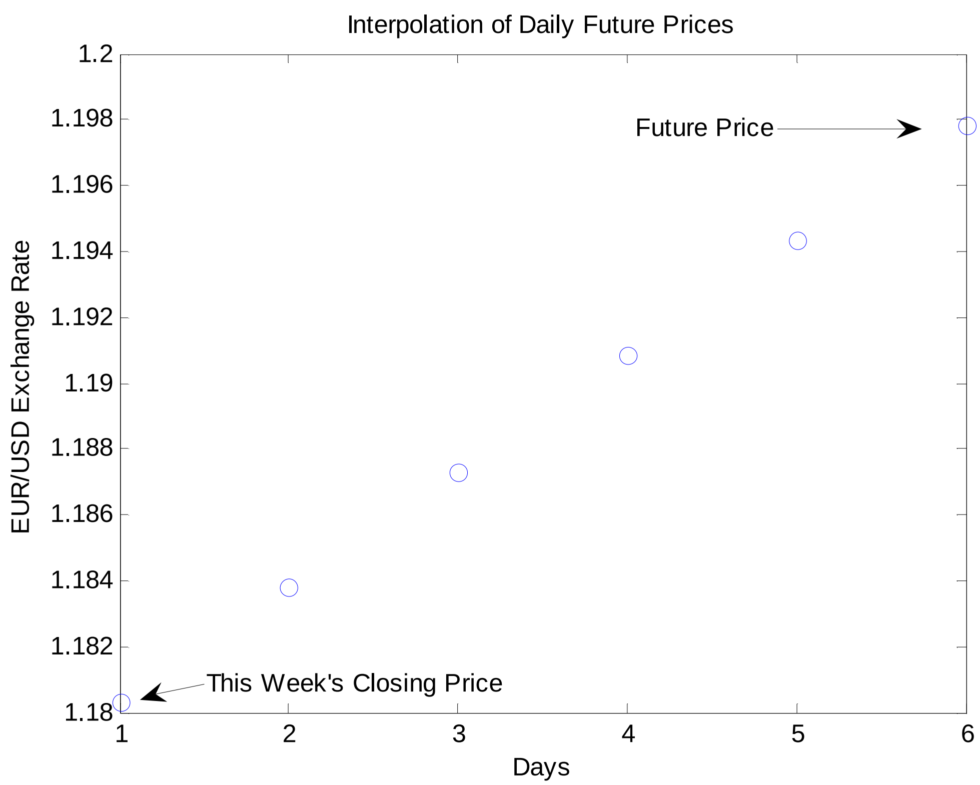

Figure 4-1 Interpolations of Daily Future Prices

4.1.2 Interpolated Future Price

The results of the simulations for the EUR/USD and the two different periods are shown in Table 4-1. It is evident that the use of the interpolated future weekly closing rate as an input to the ANN is able to improve its predictive performance. This is seen from the reduced RMSE and the higher DA when compared to the performance of an ANN without the use of *forecasted* (No FC) or future prices as inputs. Using the Pesaran-Timmermann test for significance of directional accuracy, a p-value close to 0 is obtained for all of the simulations, which indicates the predicted exchange rates have market-timing ability even higher than a 99% level of significance.

Table 4-1 Using Interpolated Future Price as Inputs for the EUR/USD

No. of Lags	2	3	4	NO FC	2	3	4	NO FC
Model	RMSE				DA			
Scenario A								
n=4	0.0065	0.0065	0.0065	0.0083	0.7063	0.6905	0.6984	0.5119
n=5	0.0064	0.0064	0.0064	0.0083	0.7024	0.7063	0.7024	0.5159
n=6	0.0065	0.0065	0.0064	0.0084	0.7103	0.7103	0.7143	0.5040
Scenario B								
n=4	0.0044	0.0045	0.0045	0.0060	0.7421	0.7421	0.7421	0.5119
n=5	0.0044	0.0044	0.0045	0.0060	0.7381	0.7421	0.7421	0.4881
n=6	0.0045	0.0045	0.0045	0.0059	0.7500	0.7460	0.7421	0.4881

4.1.3 Constant Future Price

The results of the simulations using constant future prices as inputs for the EUR/USD pair and the two different periods are shown in Table 4-2. It is evident that the use of the constant future weekly closing rate as an input to the ANN is able to improve its predictive performance. This is seen from the reduced RMSE and the higher DA when compared to the performance of an ANN without the use of *forecasted* (No FC) or future prices as inputs. Using the Pesaran-Timmermann test for significance of directional accuracy, a p-value close to 0 is obtained for all of the simulations, which indicates the predicted exchange rates have market-timing ability even higher than a 99% level of significance.

Table 4-2 Using Constant Perfect Future Prices as Inputs for EUR/USD

No. of Lags	2	3	4	NO FC	2	3	4	NO FC
Model	RMSE				DA (%)			
Scenario A								
n=4	0.0068	0.0068	0.0069	0.0083	0.7302	0.7421	0.7262	0.5119
n=5	0.0067	0.0068	0.0069	0.0083	0.7341	0.7262	0.7341	0.5159
n=6	0.0068	0.0068	0.0068	0.0084	0.7341	0.7302	0.7262	0.5040
Scenario B								
n=4	0.0049	0.0049	0.0049	0.0060	0.7302	0.7302	0.7302	0.5119
n=5	0.0049	0.0049	0.0049	0.0060	0.7222	0.7341	0.7302	0.4881
n=6	0.0050	0.0050	0.0049	0.0059	0.7381	0.7341	0.7222	0.4881

The use of the interpolated future price and the constant future price does not have a significant difference when compared against each other.

4.1.4 Step Analysis

To further understand the impact of using the perfect future exchange rate as an input, an analysis of the degree of errors classified by the number of steps away from the end of the week was made. If the last trading day of the week is a Thursday then the predicted rate for the Monday at the beginning of the same week is considered to be three steps away. Table 4-3 shows the breakdown of how many days of the 252 test days fall into each category of steps for an ANN with 4 hidden neurons. As expected, there are fewer days which are 5 steps or trading days away from the end of the week.

Table 4-3 Step Analysis When Using Perfect Future Prices as inputs for EUR/USD

Steps	EUR/USD	GBP/USD	USD/JPY	EUR/USD	GBP/USD	USD/JPY
	RMSE			DA		
Scenario A						
s=1	0.0064	0.0302	0.4046	0.9434	0.6792	0.8868
s=2	0.0068	0.0332	0.4016	0.5094	0.7358	0.6415
s=3	0.0065	0.0330	0.5451	0.6731	0.6346	0.7500
s=4	0.0078	0.0364	0.4808	0.7843	0.5490	0.7255
s=5	0.0066	0.0370	0.7098	0.7209	0.4884	0.5349
Scenario B						
s=1	0.0044	0.0087	0.4611	0.9434	0.8113	0.9245
s=2	0.0041	0.0071	0.3730	0.7736	0.6792	0.8113
s=3	0.0045	0.0072	0.4529	0.6731	0.7115	0.7500
s=4	0.0050	0.0086	0.5753	0.6863	0.6471	0.6078
s=5	0.0064	0.0080	0.5772	0.5581	0.6279	0.6047

The RMSE and DA values given above are with respect to the predictions for that category of steps. This breakdown demonstrates that the predictions made 1 step away from the last trading day of the week were consistently the most accurate in terms of direction except for scenario A of the GBP/USD pair. On the other hand, the RMSE does not have a consistent trend. The two amongst the five steps with the lowest RMSE values were found amongst steps 1, 2 or 3. The overall results come as no surprise and leads to the conclusion that using a perfect future price does help the predictive capabilities of an ANN.

4.2 Noisy Future price

Indicators ideally have a perfect correlation with the underlying property which they track. However, sentiment or market expectation is impossible to quantify unless a thorough survey of all market participants can be made. The closest thing to a survey of every investor's sentiment is the change in prices as buyers and sellers come to a consensus on the value of the security. Even then, no one can be certain if the prices readily reflect the crowd's expectation.

The effects of such uncertainties are examined in detail with the aim of finding the limit at which the *forecasted* price will cease to aid the ANN in improving its predictive performance. These uncertainties are modelled as noise into the simulations introducing into the testing set's *forecasted* price inputs, leaving the *forecasted* price series in the training set perfect. The level of white noise with zero mean and unit variance to be added varies from 1.0% to 5.5% of the mean of the perfect *forecasted* price series.

Due to the stochastic nature of introducing this white Gaussian noise, the level of noise effectively added onto the *forecasted* series is measured by the level of correlation between the noisy and perfect series. These simulations are run 1000 times at each 0.5% interval and the mean of the correlations noted. The mean of the performance metrics, RMSE and DA, will also be taken to be representative of the predictive capabilities of the ANN with that level of noise.

In this experiment, the architecture of the ANN is fixed with 4 hidden neurons. The pure time delayed exchange rates as inputs will also be fixed at 4 lags. These rates are input along with the noisy *forecasted* rates for a one-step prediction of the following day's exchange rate.

The results of the simulations for the EUR/USD pair in both scenarios highlight that the accuracy of the future price does have an effect on the predictive performance of the network. As the level of noise decreases, as indicated by an increasing correlation between the noisy and the perfect *forecasted* input in the test period, the directional accuracy of the trained ANN improves. The size of the RMSE also decreases significantly. A similar result is also found for the other two currency pairs and for both scenarios. The graphs are found in the appendix.

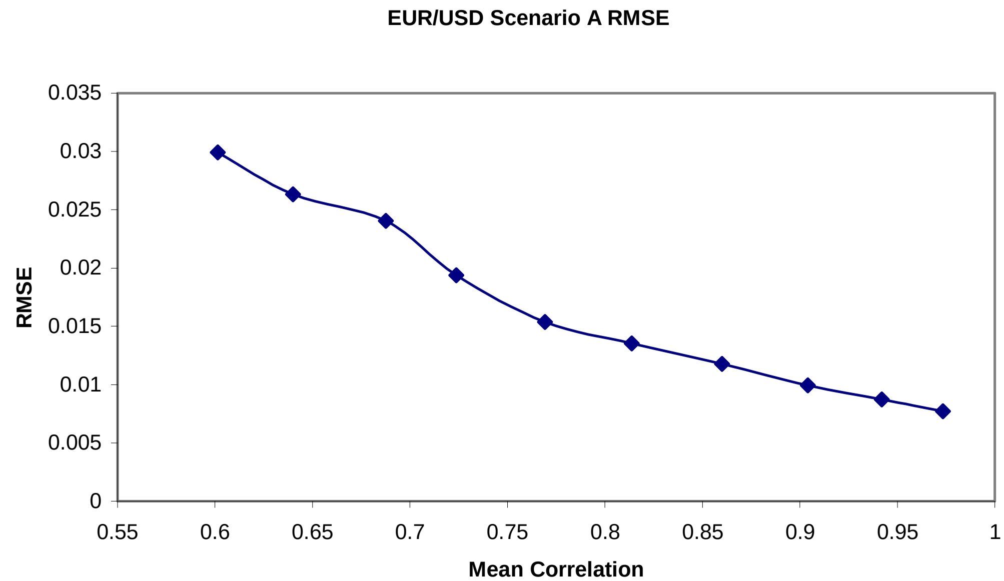

Figure 4-2 Noisy Future Prices against RMSE for EUR/USD Scenario A

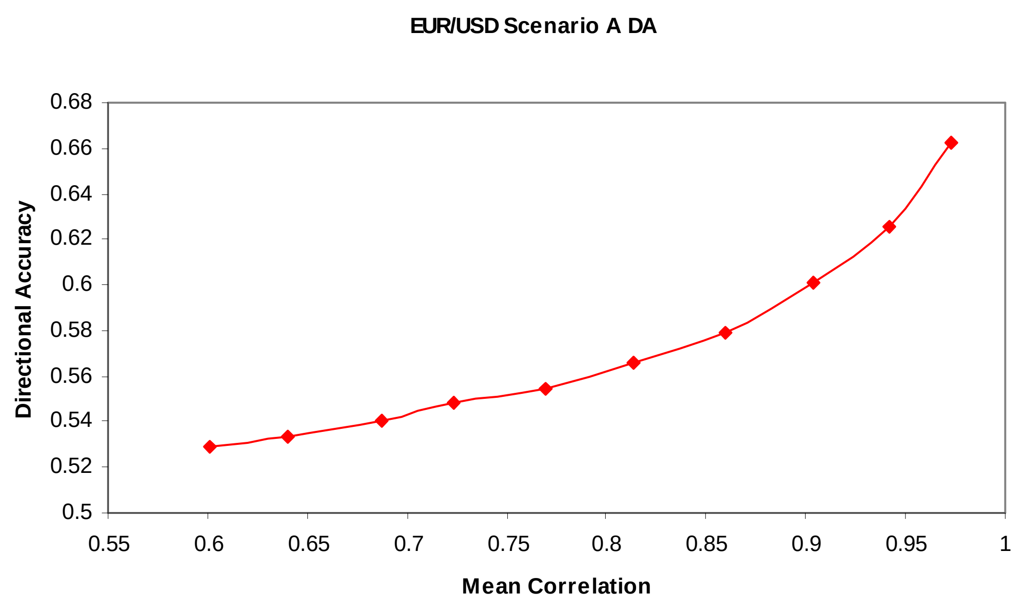

Figure 4-3 Noisy Future Prices against DA for EUR/USD Scenario A

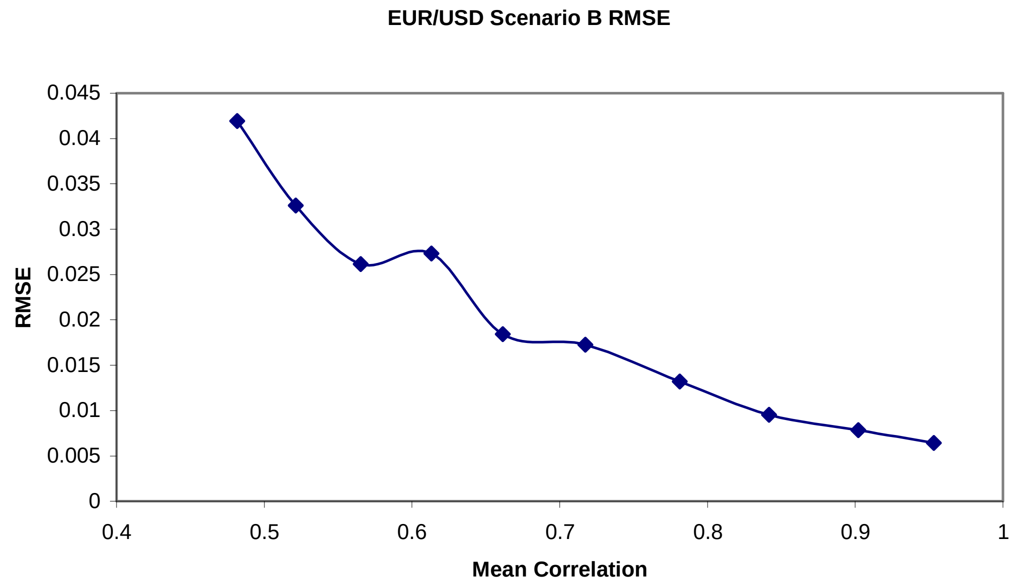

Figure 4-4 Noisy Future Prices against RMSE for EUR/USD Scenario B

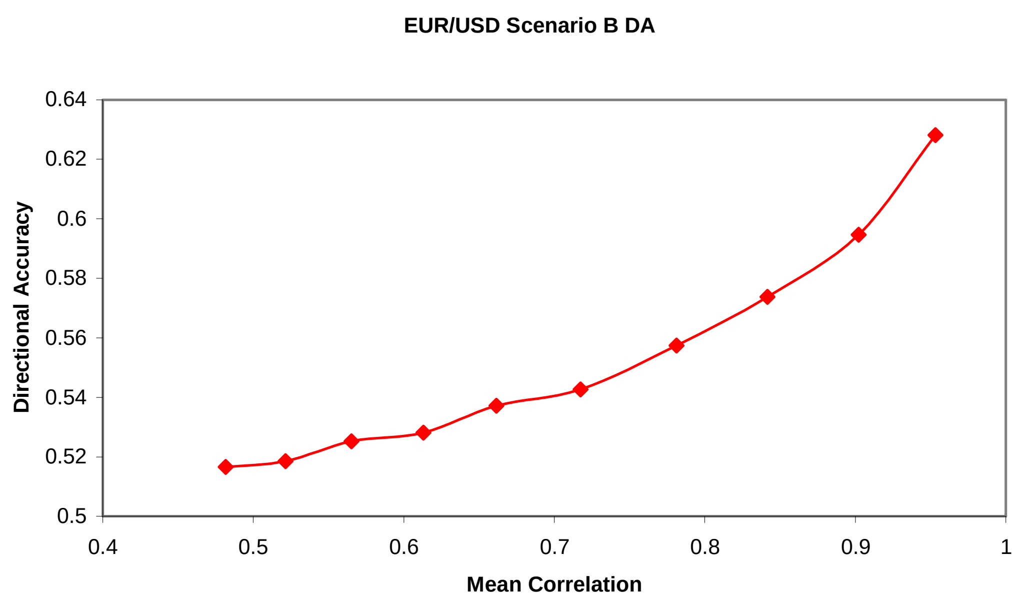

Figure 4-5 Noisy Future Prices against DA for EUR/USD Scenario B

4.3 Fundamental Data

Economic data like the interest rates, Gross Domestic Product (GDP), quarterly trade balance numbers and the Consumer Price Index (CPI) have traditionally been the fundamental driving forces for exchange rate trends. Detailed explanations of how such figures reflect the current productivity and trade relations of countries are found in most entry level economic textbooks. In brief, inflation is measured by changes in the Consumer Price Index. When inflation increases, then it implies that the domestic price of consumer goods has run up as compared to its neighbours. Exports would become too expensive while imports will look increasingly cheaper. The currency would thus become uncompetitive and would result in a balance of payment crisis and a trade account deficit. Foreigners would lose confidence in the currency and sell it off to repatriate funds.

With such well established qualitative relationships in economic literature, it could be possible that a more quantitative relationship may be extracted from this set of economic figures. This was examined by Yu et al. [7] who had outstanding results using these fundamentals along with lagged exchange rates when training an ANN to predict future rates. They concluded that they had outstanding results after doing a comprehensive principal component analysis of past published research in the same field.

The research study included the EUR/USD pair even before the actual inception of the European Union (EU). There were no details as to how they obtained the economic figures of the Euro area for the early periods. They could possibly have blended the individual economic data and exchange rates of each EU country to achieve a representative figure. However, they later conclude these “explanatory variables as dumb variables to adjust the neural network forecasting model”. It should also be noted that Yu et al. did introduce a modified cost function whilst training their ANN. Perhaps it was this adjustment which led to the improved performance.

Similar to their work, a study on the relationship between this set of four economic figures and their ability to improve the predictive performance of an ANN is made. As there are two countries for each exchange rate pair, a total of eight new inputs are introduced into the ANN. These figures are updated on a quarterly basis and are kept constant throughout the quarter. The source of such information is the International Financial Statistics database maintained by the International Monetary Fund.

The number of hidden neurons of the ANN is varied from 4 to 6 to prevent any model selection bias while the pure time delayed exchange rates used as inputs are fixed at 4 lags. To form a comparison, a perfect future price such as the *forecasted* exchange rate used earlier is added on as an input.

First, the influence of each of these fundamental indicators is tested independently by using only a pair of indicators, one from each country, as inputs into the ANN. Second, all the indicators will be used as inputs at the same time to examine if they improve the predictive capabilities of the ANN.

4.3.1 Interest Rates

The foreign exchange rate between two currencies is related to the interest rates in the two countries. If the interest rate of a foreign currency relative to the home country goes up, the home currency weakens. In other words, it takes more of the home currency to buy the same amount of foreign currency.

When interest rates in a country rise, investments held in that country's currency will earn a higher rate of return. Therefore, money and investments will tend to flow into that country. This in turn drives up the value of its currency. The reverse is true when a country's interest rate falls.

From the results in Table 4-4, the use of interest rates or the difference between the rates does not improve the performance of the ANN consistently.

Table 4-4 Using Individual Interest Rates or Their Difference as Inputs for GBP/USD

Forecast Model	Individual	Difference	Individual	Difference
	RMSE		DA	
Scenario A				
n=4	0.0199	0.0162	0.4722	0.4365
n=5	0.0144	0.0646	0.5079	0.4444
n=6	0.0185	0.0173	0.4325	0.4405
Scenario B				
n=4	0.0236	0.0112	0.4802	0.4484
n=5	0.0148	0.0132	0.4722	0.4643
n=6	0.0124	0.0123	0.5437	0.4921

4.3.2 Gross Domestic Product

The Gross Domestic Product (GDP) is the measure of average economic activity and it is the broadest measure available. GDP growth is widely considered as the primary indicator of the strength of economic activity in a country. GDP is a representation of the total value of a country's production within the period and is made up of the purchases of domestically produced goods and services by individuals, businesses, foreigners and the government. The GDP is reported on a quarterly basis.

As GDP reports are often subject to substantial quarter-to-quarter volatility and revisions, it is preferable to follow the indicator on a year-to-year basis. It can be valuable to follow the trend rate of growth in each of the major categories of GDP to determine the strengths and weaknesses in the economy.

A high GDP figure is often associated with the expectations of higher interest rates, which is frequently positive, at least in the short term, for the currency involved, unless expectations of increased inflation pressure is concurrently undermining confidence in the currency.

Table 4-5 Using Individual GDP or Their Difference as Inputs for GBP/USD

Forecast Model	Individual	Difference	Individual	Difference
	RMSE		DA	
Scenario A				
n=4	0.0298	0.0298	0.4484	0.4722
n=5	0.0227	0.0372	0.4444	0.4365
n=6	0.0471	0.0381	0.4524	0.4325
Scenario B				
n=4	0.0100	0.0229	0.4683	0.5556
n=5	0.0562	0.0161	0.5278	0.5516
n=6	0.0112	0.0260	0.4921	0.5238

From the results in Table 4-5, the use of GDP or the difference between the indicators does not improve the performance of the ANN consistently.

4.3.3 Consumer Price Index

The Consumer Price Index (CPI) is a measure of the aggregate level of prices of a fixed basket of goods and services. The changes in the CPI are considered as an inflation indicator.

Reported on a monthly basis, the CPI is a primary inflation indicator because consumer spending makes up approximately two-thirds of a country's economic activity. There is also the core CPI which is followed. This excludes the price of items that are generally much more volatile than the rest of the CPI and can obscure the more important underlying trend. Core CPI excludes items like food and energy.

Rising consumer price inflation would normally lead to higher short term interest rates and may therefore strengthen a currency in the near term. Despite the short term benefits, an inflation problem will eventually undermine confidence in the currency and depreciation will follow.

Table 4-6 Using Individual CPI or Their Difference as Inputs for GBP/USD

Forecast Model	Individual	Difference	Individual	Difference
	RMSE		DA	
Scenario A				
n=4	0.0173	0.0231	0.4246	0.4444
n=5	0.0203	0.0156	0.4325	0.4802
n=6	0.0182	0.0208	0.5278	0.4603
Scenario B				
n=4	0.0124	0.0126	0.4960	0.4683
n=5	0.0149	0.0251	0.5476	0.4722
n=6	0.0169	0.0213	0.4444	0.4802

From the results in Table 4-6, the use of CPI or the difference between the indices does not improve the performance of the ANN consistently.

4.3.4 Trade Balance

The trade balance is the difference between the dollar amount of imports and exports of goods and services. The amount of trade balance and the changes in amount exported and imported are tracked closely by foreign exchange markets. Trade balance is considered as a major indicator of foreign exchange rate trends. This is because measures of imports and exports are indicators of the economic activity in the country.

The trend growth rates for exports and imports independently are often of more interest. Changes in export activities reflect the competitive position of the country and also the strength of economic activity abroad. On the other hand, changes in imports reflect the amount of economic activity in the country. A country with a trade balance deficit will have a weaker currency due to the continued commercial selling of the currency.

Table 4-7 Using Individual Trade Balance or Their Difference As Inputs for GBP/USD

Forecast Model	Individual	Difference	Individual	Difference
	RMSE		DA	
Scenario A				
n=4	0.0225	0.0187	0.4167	0.4325
n=5	0.0245	0.0188	0.4325	0.4405
n=6	0.0990	0.0180	0.4325	0.4563
Scenario B				
n=4	0.0317	0.0185	0.4762	0.5238
n=5	0.0167	0.0214	0.4960	0.5635
n=6	0.0178	0.0281	0.5635	0.5278

From the results in Table 4-7, the use of trade balance or the difference between the amounts of trade does not improve the performance of the ANN consistently.

4.3.5 Combined Input

On their own, the different fundamental indicators may not make much sense. Perhaps when they are combined, they will be able to help predict changes in the foreign exchange rates.

Table 4-8 Using Fundamental Data as inputs for GBP/USD

Forecast Model	Without	With	Without	With
	RMSE		DA	
Scenario A				
n=4	0.0114	0.0264	0.5714	0.5317
n=5	0.0287	0.0131	0.4444	0.6349
n=6	0.0126	0.0239	0.5198	0.4643
Scenario B				
n=4	0.0613	0.0085	0.5079	0.6984
n=5	0.0157	0.0084	0.5595	0.7103
n=6	0.1119	0.0134	0.4722	0.6349

From the results in Table 4-8, the use of fundamental data does not improve the performance of the ANN consistently. Furthermore, the performance of the ANN has decreased significantly when fundamental data has been introduced as inputs in training the network. This could be attributed to the fact that the fundamental data is constant throughout the quarter.

4.3.6 Summary

The use of fundamental data independently or combined did not have a significant impact on the performance of the ANN. In the combined case, the introduction of fundamental data even decreased the performance of the ANN in one of the scenarios of the GBP/USD exchange pair. A problem with the use of fundamental data is the slow rate at which it is updated. An analysis of the exchange rates could be done at a quarterly rate which is inline with the rate at which the fundamental data is updated; however, the amount of data available may not be optimum to train the

ANN. Perhaps, with economic indicators which are more frequently updated, there may be a greater contribution to the predictive performance of the ANN. This was also noted by Yu et al. [7] who remarked, "…, these explanatory variables are used as dumb variables to adjust the neural network forecasting model." The better performance of their model could stem from the different error function which they used. Their error function included a function to depict the directional error.

CHAPTER 5 MEAN REVERSION

This idea was motivated by Poterba and Summers [16] which discussed the evidence of mean reversion in stock returns. Mean reversion is seen when the stock price diverges from the fundamental value of the company; allowing speculators to eliminate the difference and force the stock price back to its fundamental value. To model this behavior, a mean reverting process was studied by Dixit and Pindyck [24] and is known as a Geometric Ornstein-Uhlenbeck or Dixit and Pindyck model. This model also appears in Metcalf and Hassett [18] and Hillebrand [25].

Evaluating the values of the parameters in the model using an economic analysis would be ideal. Alternatively, we may use a data based approach to calibrate the parameters of the model. The aim is to contribute to the literature on modeling the mean reverting behavior by analyzing two methods of parameter estimation, Least Squares Estimation (LSE) and Maximum Likelihood Estimation (MLE).

Sections 1 to 3 present a review of the necessary mathematical tools and information required to have a clear understanding of how they can be used to advance this project. Then Section 4 provides an overview of the theoretical arguments that motivate mean reverting behavior in the stock markets will be presented. Next, the selected model will be elaborated on and a demonstration of the properties of the parameters in the model. Section 5 focuses on the two methods of parameter estimation, least squares estimation and maximum likelihood estimation. Using Monte Carlo methods, the root mean square error, which is a measure of the accuracy of the estimation, is calculated. Finally, using monthly data for the Dow Jones Industrial Average and the Singapore Straits Times Index, the accuracy of both methods is analyzed.

5.1 Stock Market Indices

A stock market index is a listing of stocks and a statistic which reflects the composite value of its underlying components. It is used as a tool to represent the characteristics of its component stocks and the general performance of the stock market. These stocks have some commonality such as being traded on the same stock market exchange, belonging to the same industry, or having similar market capitalizations. Many indices compiled by news or financial services firms are used to benchmark the performance of portfolios such as mutual funds. Two of the most well known and tracked indices are the Dow Jones Industrial Average and the Standard & Poor's 500.

5.1.1 Dow Jones Industrial Average

The Dow Jones Industrial Average (DJIA) is the oldest continuing stock market index in the United States of America. It consists of 30 of the largest and most widely held public companies in the United States. These companies are listed on either the New York Stock Exchange (NYSE) or the National Association of Securities Dealers Automated Quotations (NASDAQ). Some of the more well known stocks found in the Dow Jones Industrial Average are:

- Coca-Cola Co. (NYSE: KO)
- Intel Corp. (NASDAQ: INTC)
- McDonald's Corp. (NYSE: MCD)
- Microsoft Corp. (NASDAQ: MSFT)

The Dow Jones Industrial Average is criticized for being a price-weighted average. This means that higher-priced stocks have more influence over the average than the other lower-priced stocks. This can produce misleading results, as a $1 increase in a lower-priced stock will be negated by a $1 decrease in a much higher-priced stock, even though the first stock experienced a larger percentage change. Furthermore, the small sample size in the average has brought on additional criticism to the accuracy of the index as a reflection of market conditions. Despite these flaws, the Dow Jones Industrial Average is widely used as an indicator of overall market performance. Many critics of the Dow Jones Industrial Average choose the float-adjusted market-value weighted Standard & Poor's 500 as a better indicator of the overall economic conditions.

5.1.2 Standard & Poor's 500

The Standard & Poor's 500 (S&P 500) is an index containing the stocks of 500 large market capitalization corporations which are North American. Similar to the Dow Jones Industrial Average, all of the stocks in the index are those of large publicly held companies and trade on major US stock exchanges such as the New York Stock Exchange and NASDAQ. After the Dow Jones Industrial Average, the S&P 500 is the most widely watched index of large-cap US stocks.

Most of the companies found in the Dow Jones Industrial Average are found in the S&P 500 as well. The index was previously market-value weighted. This means that companies whose total market valuation is larger will have a greater effect on the index than companies whose market valuation is smaller. Just last year, the index was converted to float weighted, where only shares which Standard & Poor’s determines are available for public trading ("float") are counted when determining the market valuation.

5.2 Lognormal Prices

5.2.1 Lognormal Random Variables

A random variable z is lognormal if the random variable $ln\ z$ is normally distributed. The opposite is also true, if y is normal, then $z = e^x$ is lognormal. This implies that the probability density function for z has the form

$$p(\zeta) = \frac{1}{\sqrt{2\pi}\sigma\zeta} e^{-\frac{1}{2\sigma^2}(\ln\zeta - \nu)^2}.$$

This distribution has the following properties:

$$E(z) = e^{\left(\nu+\sigma^2/2\right)} ,$$

$$E(\ln z) = \nu ,$$

$$\text{var}(z) = e^{\left(2\nu+\sigma^2\right)}\left(e^{\sigma^2} - 1\right),$$

$$\text{var}(\ln z) = \sigma^2 .$$

Following the summation result for jointly normal random variables, products and power of jointly lognormal variables are also lognormal. If u and v are lognormal, then $z = u^{\alpha}v^{\beta}$ is also lognormal. The general shape of the density function of a lognormal random variable is shown in Figure 5-1. The function is always positive and slightly skewed in the positive direction. It is this property which gives an idea as to how the value of σ influences the lognormal distribution. If σ is increased, the distribution will spread out. As this can spread upwards but cannot spread below zero, the mean value increases as well.

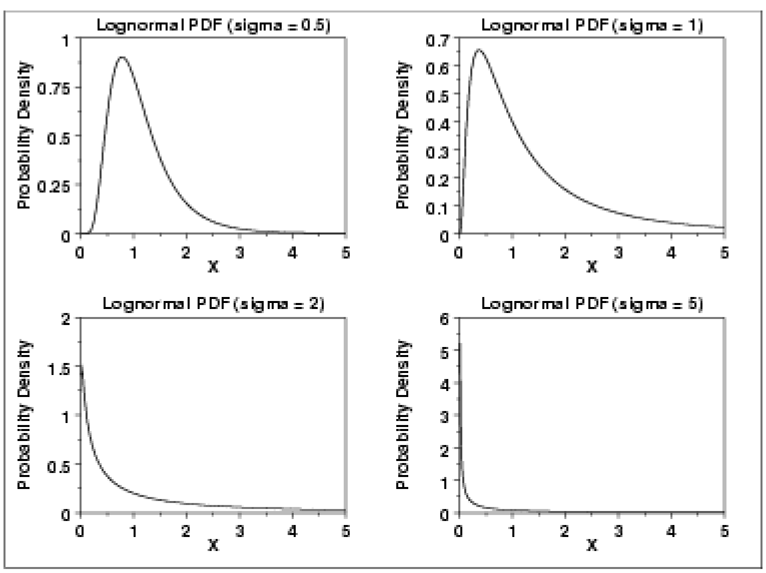

Figure 5-1 Lognormal probability density function for four values of σ [23]

5.2.2 Real Stock Returns Distributions

Let X_t stand for the price of an investment at time t. If the investment is sold at sale day $t > 0$, then the return of the investment $R_t = X_t / X_0$. Based on an analysis of past stock price records, the logarithm of stock returns are fairly close to lognormal. To verify this, a period length is chosen and the many logarithms of returns, $\log R_t = \log X_t - \log X_{t-1}$, are recorded. As the historical data which were used were the monthly closing levels of the Dow Jones Industrial Average and the Standard & Poor's 500, the monthly changes in the logarithm of the closing levels were calculated. The Dow Jones Industrial Average data runs from 1st March 1929 to the 1st March 2006 while the Standard & Poor's 500 data is from the 3rd March 1950 to 1st March 2006.

A histogram is then constructed and compared with that of a normal distribution of the same variance as seen in Figure 5-2 and Figure 5-3. The distribution observed is fairly similar to normal. A difference between the distributions is the "fat tails" at the positive and negative large values where the distribution is larger. This indicates that large changes in price levels occur more frequently than would be predicted by a normal distribution.

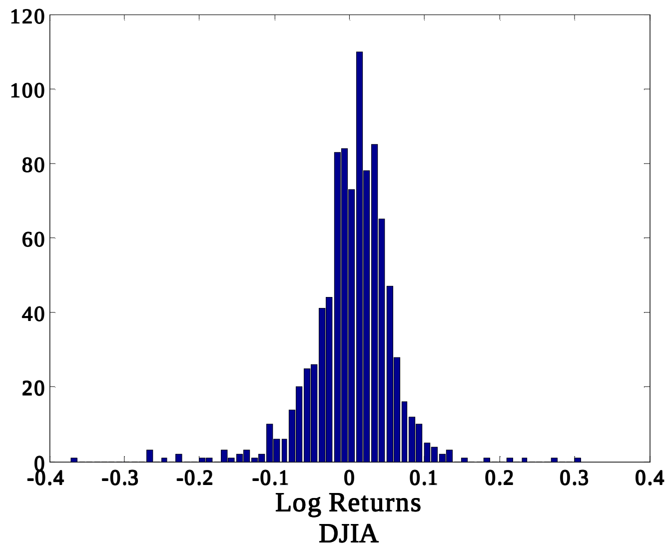

Figure 5-2 Log returns of the DJIA which show a normal distribution

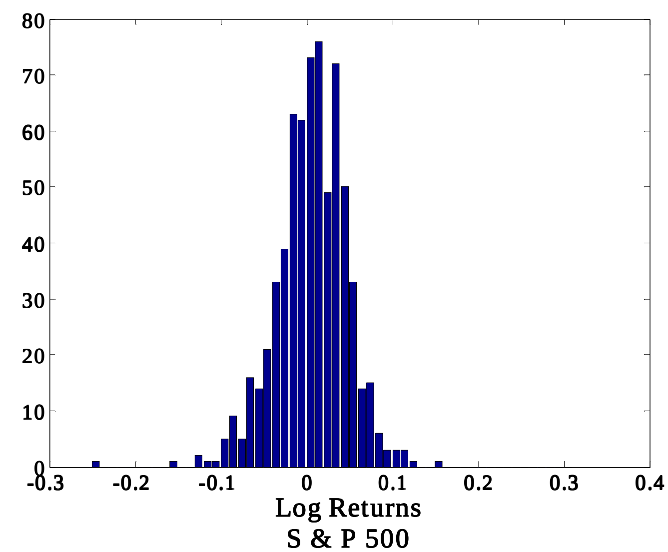

Figure 5-3 Log returns of the S&P 500 which show a normal distribution

5.3 Data and Analysis of Statistics

The forecasting of future prices is a procedure which requires the use of the expected returns and variances of these returns. It is necessary to assign values to the different parameters of the model such that the output of the forecasting model will be realistic. These parameters may be estimated from the historical data of the securities returns. If we are searching for the expected monthly rate of return for the stock market, we may average the monthly rates of return of the Dow Jones Industrial Average or Standard & Poor's 500 over a sufficiently long period of time. This method may give a fairly accurate estimate of the actual mean monthly rate of return.

There are many simple methods used for estimation from historical returns data. This is a convenient method since the sources of data are easily available from financial service organizations. It is often the case that these organizations will provide the parameter estimates along with their data. However, the methods used may differ and it is essential to understand how the estimates were made and the reliability of these methods. Not all parameters may be estimated reliably. One such parameter which has an unreliable estimation is the expected returns. This unreliability stems from a basic limitation from the estimation process. It is often referred to as the "blur of history". Other parameter estimates are more reliable, such as the variance and covariance.

5.3.1 Effects of Period-Length

Given that an investment has an annual return of $1 + r_a$. This implies that the yearly return can be considered as the result of 12 monthly returns. If these monthly returns are small, we are able to approximate the yearly returns as the sum of the 12 individual monthly returns by keeping the first-order terms of the expanded equation.

$$1 + r_a = (1 + r_1)(1 + r_2)\ldots(1 + r_{12}),$$

$$1 + r_a \approx 1 + r_1 + r_2 + \ldots + r_{12}.$$

This approximation fails to capture the effect of compounded interest but will be sufficient to demonstrate the effects of period-length on the estimation of the expected returns and variance.

Suppose that the monthly returns are mutually uncorrelated and statistically similar with mean α and standard deviation σ. With the earlier approximation and the fact that the returns are uncorrelated, we obtain the following equations for the monthly statistical properties in yearly terms:

$$\alpha = \frac{1}{12}\overline{r_a},$$
$$\sigma = \frac{1}{\sqrt{12}}\overline{\sigma_a}.$$

This case may be generalized to any period length as long as the same assumptions that the periodic returns are uncorrelated and have similar statistical properties.

Observe that the expected rate of return over a period increases linearly with the length of the period. However, the standard deviation increases as the square root of the length of the period. This implies that the ratio of the standard deviation to the expected rate of return increases dramatically as the period length is made shorter. As the period length tends to zero, the ratio will approach infinity. Hence, the rates of return for small periods have significantly higher standard deviations compared to their expected values.

5.3.2 Mean Blur

Suppose that we wish to estimate the mean value of returns using historical data. If we have *n* samples of periodic returns, the best estimate of the mean rate of return would be the average of the samples,

$$\hat{r} = \frac{1}{n} \sum_{i=1}^{n} r_i \ .$$

This estimate is itself a random variable with probabilistic properties such as a mean and variance. The expected value of the estimate is the true value $\bar{r}$ since

$$E\left(\hat{r}\right) = E\left(\frac{1}{n}\sum_{i=1}^{n} r_i\right) = \bar{r} \ .$$

To determine the accuracy and reliability of this estimation, we have to consider the variance and standard deviation of the estimate,

$$\sigma_{\hat{r}}^2 = E\left[\left(\hat{r} - \bar{r}\right)^2\right] = E\left[\left(\frac{1}{n}\sum_{i=1}^{n}\left(r_i - \bar{r}\right)\right)\right]^2 = \frac{1}{n}\sigma^2 \ ,$$

$$\sigma_{\hat{r}} = \frac{\sigma}{\sqrt{n}} \ .$$

This is the expression which governs the error in the estimate of the mean rate of return.

To better understand the implications of this expression, some fixed values are put into them. Suppose the returns to be estimated are monthly returns with mean of 1% and standard deviation of 4.5%. If the investor has 12 months of data to use for his estimation of the mean, he will have a standard deviation

$$\sigma_{\hat{r}} = \frac{4.5\%}{\sqrt{12}} = 1.30\% \ .$$

This implies that the standard deviation is larger than the estimated mean itself. The investor will be able to reduce the deviation by a factor of 2 if he uses 4 years or 48 months of data. However, to get a reasonable estimate, the standard deviation should not be more than one-tenth of the actual mean value. This would require $n = (45)^2 = 2025$ or about 168 years of monthly data. Furthermore, the mean value would not be a constant throughout this period, thus worsening the estimation problem.

It is basically impossible to accurately estimate the mean rate of return using historical data and the accuracy may not be improved by varying the length of the period. If the investor chooses a short period length, he will have more samples to work with but each sample is worse in terms of the ratio of the standard deviation to the mean value. On the other hand, if he uses longer period lengths, he will have less data available but each sample will be more reliable.

5.4 Modeling

5.4.1 Mean Reversion in the Market

The random walk description of capital markets has recently been challenged as such a process may diverge over time, resulting in unbounded profits or losses. However, mean reversion behavior exhibited by security prices has recently been recognized by theorists. In financial markets, arbitrage opportunities arise and generate trading activities which exploit the mispricing. This contributes to drive the prices of the securities towards their theoretically fair or equilibrium values. Mean reversion is the best way to capture this effect.

There are numerous ways of modeling the fair value of a share, with prices deviating from this value to the extent that the assumptions in the particular model differ.

We may assume that various capital markets are efficient and free of arbitrage opportunities. In such an ideal situation, the mean value of a share would be the book value of the tangible and intangible assets. If the price were higher, an arbitrageur could create a new company, purchase the exact same assets and then sell the shares for a premium. A higher price would imply that there exist incentives for new firms to compete in that market or for the existing firms to expand. This will continue until profits decline to zero as an increase in supply will damp the price increase. If the price were lower, then the high-cost companies will exit the market whereas the other companies would stop replacement of assets and would issue excess capital back to their shareholders through dividends until the overall supply in the market was sufficiently reduced. This will soften the price fall.

5.4.2 The Model

We begin by defining a Geometric Brownian Motion (GBM) model which was previously accepted by academic theorists. Subsequently, we will review the different stochastic differential equations which may be used for modeling prices which have a mean reverting property. A price (P_t) following GBM can be characterized by the following equation:

$$dP_t = \alpha P_t dt + \sigma P_t dz ,$$

where dz is a Weiner Process with zero mean and unit variance. The parameter α measures the trend in the price series while σ is a measure of volatility.

The Geometric Mean Reversion (GMR) model [20] follows this process:

$$dP_t = \left(\alpha + \lambda\left(P_0 e^{\alpha t} - P_t\right)\right)P_t dt + \sigma P_t dz ,$$

where λ is a parameter which measures the speed of reversion if it is positive or aversion if it is negative. P_0 , a constant, is the long-run equilibrium price which the prices tend to revert to. The α term is the rate at which the equilibrium price rises exponentially. The difference between the GBM and GMR is the P_0 which attracts the price in its direction. The further away from P_0, the higher the tendency of reversion.

If we take α to be zero, we find a process which is known as Geometric Ornstein-Uhlenbeck (OU) or Dixit & Pindyck model:

$$dP_t = \lambda(P_0 - P_t)P_t dt + \sigma P_t dz \, .$$

The model gets its name as it was first studied by Dixit & Pindyck in financial literature but was introduced by G.E. Uhlenbeck and L.S. Ornstein in a physics review. This model appeared in Metcalf & Hassets' [18]. Their research showed that a cumulative investment strategy would yield the same returns when applied in either a mean reverting or random walk environment.

Another model is the Arithmetic Mean Reversion process for the logarithm of the stochastic variable. We find that this is a simpler model to work with for simulations and parameters estimation. We let the $x = ln(P_t)$ and consider that it follows the arithmetic Ornstein-Uhlenbeck process towards an equilibrium price which is a Vasicek-type model,

$$dx = \alpha dt + \lambda(x_0 + \alpha t - x_t)dt + \sigma \, dz \, . \quad (1)$$

It is an accepted theoretical model that real stock prices are actually close to a lognormal distribution. Hence the variable P_t has a lognormal distribution and variable x_t has normal distribution. To calculate the expected value, it is noted that the expectation of the Itô integral goes to zero and a solution of (1) is:

$$x(t) = x_0 + \alpha t .$$

$$E[x] = x_0 + \alpha t .$$

To calculate the variance, we take into consideration the stochastic integral format for the process where:

$$x(t) = x_0 + \alpha t + \int_0^t \sigma e^{\lambda(\tau - t)} dz(\tau) ,$$

$$Var[x(t)] = \sigma^2 e^{-2\lambda t} E[\int_0^t e^{2\lambda\tau} dz(\tau)],$$

$$Var[x(t)] = \frac{\sigma^2}{2\lambda} e^{-2\lambda t} (e^{2\lambda t} - 1),$$

$$Var[x(t)] = \frac{\sigma^2}{2\lambda} (1 - e^{-2\lambda t}) . \tag{2}$$

For a long time horizon the variance of this process tends towards $\frac{\sigma^2}{2\lambda}$. Unlike the GBM the variance is bounded and does not grow to infinity.

A continuous-time process may be simulated in its standard form or using the log form. These two methods are not exactly equivalent but it can be shown that their differences tend to be negligible in the long run [19].

5.4.3 Examples of Model Properties

Figures 5-4 to 5-6 demonstrate the properties of the model and the effect that each parameter has on the model.

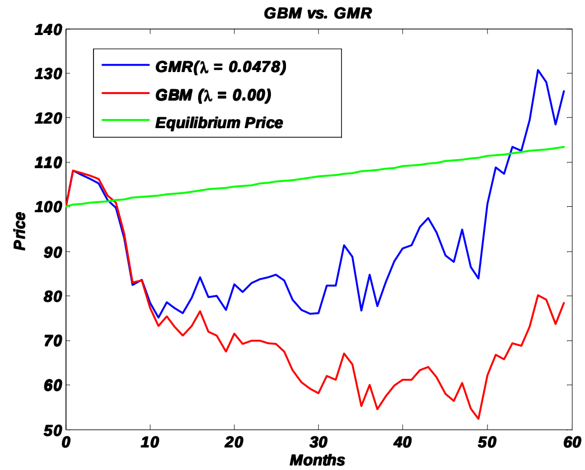

Figure 5-4 Difference between a GMR model and a GBM

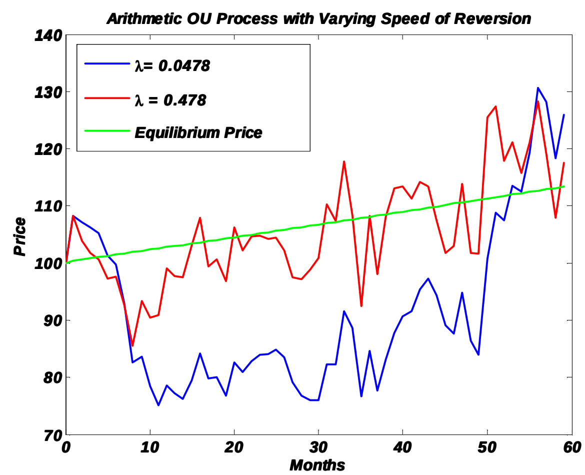

Figure 5-5 Demonstrates the property of parameter λ, speed of mean reversion

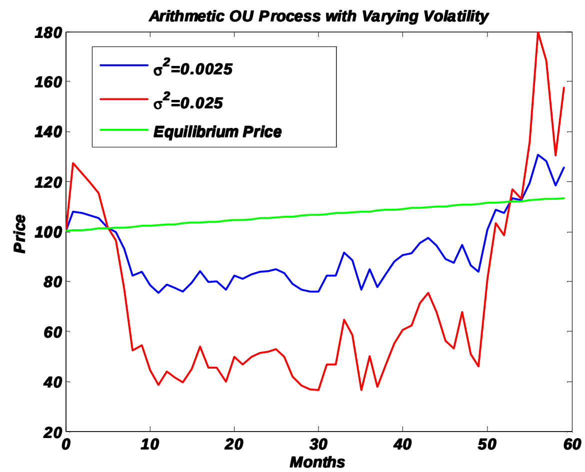

Figure 5-6 Demonstrates the effect of varying σ, volatility

5.5 Parameter Estimation

5.5.1 Maximum Likelihood

From model (1) we are able to analyze the conditional distribution of the log-returns $x_{t+1} - x_t$:

$$(x_{t+1} - x_t) \sim N\left(\alpha + \lambda(x_0 + \alpha t - x_t), \sigma^2\right).$$

The parameters of the model, λ, σ and α may be estimated by the Maximum Likelihood Estimation given the data of the time series through date t. For the sample, x_0, x_1, .., x_{T-1}, of size T, the log of the likelihood to be maximized is:

$$\ln L(\lambda, \sigma, \alpha, x_t) = -\frac{T-1}{2}\ln\sigma^2 - \frac{1}{2\sigma^2}\sum_{t=0}^{T-1}\left(x_{t+1} - x_t - \alpha - \lambda(x_0 + \alpha t - x_t)\right)^2.$$

We use the ‘fminunc’ function found in MATLAB which uses a quasi-Newton method with line search.

5.5.2 Least Squares

To estimate the parameters by Least Squares Estimation, we write (1) in the following form:

$$x_t - x_{t-1} = \alpha + \lambda(x_0 + \alpha t - x_t) + \varepsilon_t \,.$$

This represents a nonlinear least squares regression equation where the parameters, λ and α can be chosen to minimize $\Sigma\varepsilon_t^2$. From (2) we find that the standard deviation of the regression, $\sigma_\varepsilon = \sigma$. Using the ‘lsqnonlin’ function in MATLAB we are able to solve the nonlinear regression.

5.5.3 Monte Carlo Simulations

This section examines the accuracy of both the maximum likelihood and least squares estimators using Monte Carlo experimentation. In each experiment the results reported are based on 5000 replications. Random normal errors are generated using the ‘randn’ function with the seed being a function of the computer time.

For each replication, a log-price series was created and we then estimated the parameters using the two methods. Using the estimated parameters, we create the estimated log-price series without any noise (σ = 0) and compare it with the original log-price series to obtain the Root Mean Squared Error (RMSE).

5.5.4 Results and Analysis

Comparing the two methods, we find that both methods estimate σ accurately and without bias. However, α and λ are estimated with a degree of bias. When comparing the estimated price series and using the RMSE as a measure of accuracy, we find that MLE gives us a more accurate replication of the original log-price series. MLE is also more accurate when estimating the speed of reversion parameter, λ for a smaller number of samples.

Table 5-1 Simulation Results: Least Squares Estimation

Parameter	True	T=60		T=120	
		Mean	RMSE	Mean	RMSE
λ	0.0478	0.0982	0.0876	0.0569	0.0396
σ	0.0025	0.0024	0.0005	0.0025	0.0003
α	0.0021	0.0001	0.0025	-0.0001	0.0023

Table 5-2 Simulation Results: Maximum likelihood Estimation

Parameter	True	T=60		T=120	
		Mean	RMSE	Mean	RMSE
λ	0.0478	0.0372	0.0664	0.0188	0.0452
σ	0.0025	0.0025	0.0005	0.0025	0.0003
α	0.0021	0.0004	0.0024	0.0004	0.0020

Table 5-3 Simulation Results: Mean RMSE

T=60		T=120	
LSE	MLE	LSE	MLE
0.1574	0.1516	0.2117	0.1955

5.6 Application to the DJIA & STI

We apply the two methods to historical market data and evaluate the accuracy of their forecasts. The data used are monthly closings of the Dow Jones Industrial Average ranging from June 1933 to March 2006 which has 874 observations. The monthly closings of the Straits Times Index from December 1987 to Jan 2007 covering 230 observations were also used. Both sets of data were obtained from Yahoo.

We estimated the parameters on a rolling window of 60 points length which was moved forward by one point every step. With the estimated parameters we create a log-price series without noise (σ = 0) and calculate the RMSE between the estimated series and the historical market data.

5.7 Summary

This section has provided an introduction to mean reverting models which are analyzed in terms of model specification and estimation. In the case of estimation methods, we demonstrate the advantages of Maximum Likelihood Estimation compared to Least Square Estimation. Using Root Mean Squared Error as a measure of accuracy, we find that the method of Maximum Likelihood estimation provides a better forecast of the log-price series compared to Least Square Estimation. Furthermore, it gives a more accurate estimate of the model parameters.

Table 5-4 Mean RMSE When Applied To Historical Market Data

DJIA		*STI*	
LSE	MLE	LSE	MLE
0.2116	0.2156	0.2803	0.2506

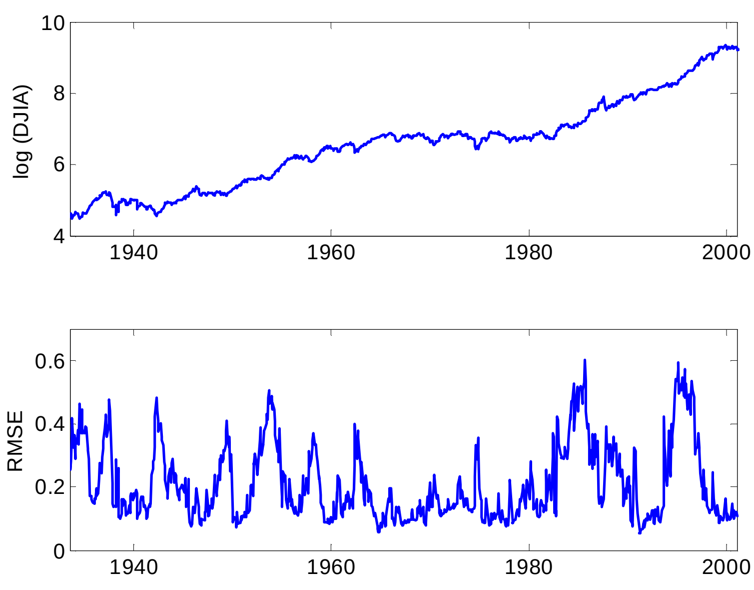

Figure 5-7 Top: Log-price series of the DJIA. Bottom: RMSE of log-price series created with estimated parameters, λ and α, with $\sigma = 0$. Parameters estimated using LSE and window of 60 points.

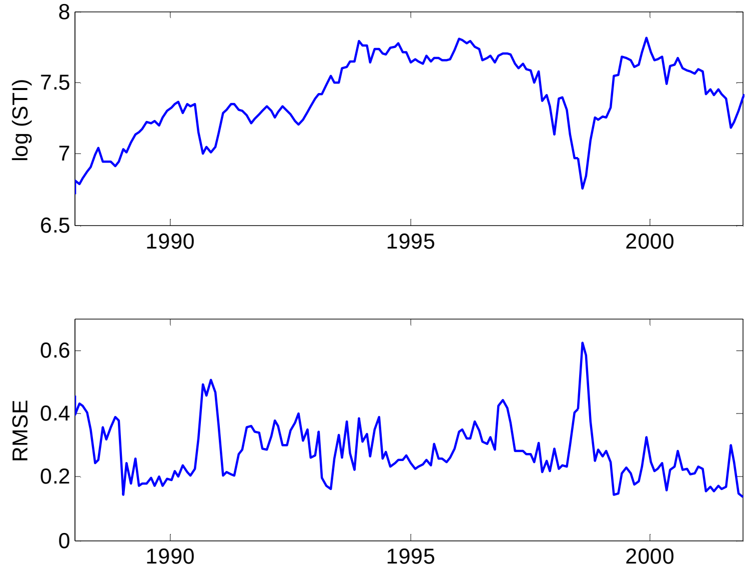

Figure 5-8 Top: Log-price series of the STI. Bottom: RMSE of log-price series created with estimated parameters, λ and α, with σ = 0. Parameters estimated using LSE and window of 60 points.

CHAPTER 6 MONEY MANAGEMENT RULES

Dollar cost averaging has been touted by many professional financial advisers as a superior investment technique. The investor with a sum of money to invest does not invest the entire sum immediately. Instead, at equally scheduled intervals through time, a fixed amount of the capital will be invested. In this way, the investor will purchase more shares when prices are low and less shares when prices are high.

Value averaging amplifies the benefits of dollar cost averaging. If buying fewer shares when prices are high is a good idea, then one should take the opportunity to sell some shares as well. This technique requires the investment to grow by a predefined amount each period. The amount of money needed to bring the investment up to the target level is added each period. If the value of the investment is above the target level, we bring the investment back down to the target level by selling shares.

These investment strategies and their resulting profitability rely on the properties of the financial markets. The random walk description of markets has recently come under attack as such a process may diverge over time, resulting in infinite profits or losses. There is no longer an acceptable model which can be used to prove the effectiveness of these rules. However, mean reversion behavior exhibited by security prices has recently been recognized by theorists. In real world financial markets, arbitrage opportunities do arise, generating trading activity aimed at exploiting mispricing. This contributes to drive the asset prices toward their theoretically fair or equilibrium values. Mean reversion is the best way to capture this effect.

Sections 1 to 3 present a review of the necessary mathematical tools and information required to have a clear understanding of how they can be used to advance this project. Then Section 4 analyzes the performance of these strategies on historical data. Finally, Sections 5 to 10 analyze the performance of these strategies in a simulated financial market with mean reverting characteristics.

6.1 Different Investment Strategies

6.1.1 Buy and Hold

Buy and Hold (BH) is the simplest strategy which only has a rule for buying and none for selling. The entire capital is invested from the on start and kept till the end of the investment period. The return from this strategy best represents the market return.

Edleson [12] remarked that comparing dollar cost averaging and value averaging against the buy and hold strategy would not be a fair comparison. The risk characteristics of a single lump-sum investment are totally different from a gradual investment over time. However, the average investor will not necessarily be limiting himself to accumulative investment strategies and will be open to other trading strategies which provide higher returns.

6.1.2 Dollar Cost Averaging

Dollar cost averaging (DCA) is an investment strategy which reduces the investment risk through the systematic purchase of securities at predetermined intervals and set amounts. Many investors already practice this strategy out of necessity without realizing it. They have a monthly budget and their investments are made on a monthly basis. Instead of investing assets in a lump sum, the investor works his way into a position by slowly buying smaller fixed amounts over a longer period of time. This spreads the cost out over the investment period, protecting the investor against changes in market price. The following steps are important in realizing this plan:

1) The investor needs to decide exactly how much money to invest each period.
2) The investor needs to select an investment that he wants to hold for the long term, preferably five to ten years or longer.
3) The investor needs to choose a regular interval at which to invest. Weekly, monthly or quarterly are suitable choices.

Dollar cost averaging works better than the periodic purchase of a constant number of shares [12]. The rationale is that market volatility should work in the investor's favor, because he will automatically be purchasing more shares when the price is low and fewer shares when the price is high. This strategy lacks a sell rule and does not profit from high prices. A numerical example of dollar cost averaging is shown in Table 6-1.

Table 6-1 Example of Dollar Cost Averaging

Period	Market Price	Amount Invested	Shares Bought	Shares Owned	Total Value
1	$50	$1000	20	20	$1000
2	$25	$1000	40	60	$1500
3	$20	$1000	50	110	$2200
4	$40	$1000	25	135	$5400
5	$50	$1000	20	155	$7750
Average Market Price:		$37.00	IRR: 22.07%		
Average Price Paid:		$32.26			

6.1.3 Value Averaging

Value averaging (VA) is a strategy in which a person adjusts the amount invested to meet a prescribed target in the future. The investor then buys or sells units of the investment such that his total investment has the target value at each revaluation point. The additional sell rule allows value averaging to profit when prices are high.

The first step in value averaging is to decide how much and how often you intend to invest. With the funds available and the duration of each period, the investor is able to determine realistic target asset values. The target asset values are defined implicitly by requiring the value of the investment to grow by a predefined amount each period. At the start of each period, the net asset value of the investment is evaluated and compared to the target asset value. The net asset value is then readjusted by buying or selling the riskier security in order to meet the target value.

For a clearer understanding, a detailed example will now be presented. Before commencing on this strategy, there are certain values to be predefined. Firstly, the investor fixes the time of investment to be from $t=0$ to $t=T$. Secondly, he determines the amount of funds which will be available at the start of each period, F_t for $t=0$ to $t=T-1$ with him divesting at $t=T$. Lastly, he defines his target values V_t. Given a target rate of return, $g\%$ per period plus the fund available for the period, $V_{t+1} = ((1+g)*V_t) + F_{t+1}$ where $V_0 = F_0$.

This strategy may be seen as an investment into two different financial products. The investor invests a constant amount into this two product portfolio. He is then able to assess his returns on the total portfolio and the returns from each of the individual products. In this example, the two products are a risky security and a risk-free side-fund such as an interest paying bank account.

Deposits into this bank account enjoy a fixed interest rate of $i\%$ per period. This bank account allows the investor to make loans as well. For convenience, the interest rate on the loans will be kept at *i%* per period. The total balance in the side-fund is noted as B_t and the cash flow in and out of the side-fund as S_t, where a deposit is considered negative and a withdrawal or loan is positive. Hence, the total balance for the following periods may be calculated as $B_{t+1} = ((1+i)*B_t) - S_{t+1}$.

To simulate the risky security, a price series P_t for time $t=0$ to $t=T$ is generated. At the start, the funds are directed to the risky security, and the total amount of security owned is noted as A_t. The cash flow into the risky security is noted as R_t where buying the security is negative. Hence, $R_0 = -F_0$ and $R_0 = F_0/P_0$. To simplify this example, the purchases of fractional shares are permitted.

The key feature of value averaging is the revaluation of the net risky assets ($N_t = P_t*A_{t-1}$) at the beginning of each period and comparing it to the target value V_t. If N_t is above V_t, then the risky security is sold to bring down the net asset value to the target level with the proceeds being deposited into the bank account. All the unused funds available will then be deposited into the bank account.

If N_t is below V_t, then more of the risky security is bought. The amount of money needed to purchase the security is first taken from the fund available at each is insufficient; the investor uses the cash available in the bank account. Loans from the bank will be made if the balance in the bank account is not enough as well. The loans will be paid back with interest at the start of the following period.

At the end of the investment horizon, the investor divests from the risky security and the risk free side-fund, $R_T = P_T*A_{T-1}$ and $S_T = B_{T-1}*(1+i)$. To analyze the effectiveness of this strategy, the internal rates of return for the following cash flows, R_t, $(R_t + S_t)$ are calculated.

Table 6-2 Example of Value Cost Averaging Assuming A 10% Return & Bank Interest Rate of 2%

		Risky Security						Risk-free Security		
	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	(ix)	(x)
Time	Cash Flow of total investment *R+S*	Price of risky security P	Shares Outstanding *A*	Net Risky Assets *N*	Target Value *V*	Cash Flow out of risky security, *R*	New Shares Bought / (Sold) *D*	Cash Flow out of Bank, *S*	Bank Balance, *B*	Net Asset Value *G*
t	R_t+S_t	P_t	D_t+A_{t-1}	P_t*A_{t-1}	$1.1*V_{t-1}+F_t$	V_t-N_t	R_t/P_t	$(R_t+S_t)-R_t$	$1.02*B_{t-1}-S_t$	B_t+V_t
0	$(400)	$10	40.0	$0.0	$400.0	$(400.0)	40.0	$0.0	$0.0	$0.0
1	$(400)	$5	168.0	$200.0	$840.0	$(640.0)	128.0	$240.0	$(240.0)	$(40.0)
2	$(400)	$3	441.3	$504.0	$1324.0	$(820.0)	273.3	$420.0	$(664.8)	$(160.8)
3	$(400)	$6	309.4	$2648.0	$1856.4	$791.6	(131.9)	$(1191.6)	$513.5	$3161.5
4	$(400)	$5	488.4	$1547.0	$2442.0	$(895.0)	179.0	$495.0	$28.7	$1575.7
5	$4913.4	$10	0.0	$4884.1	$2686.2	$4884.1	(488.4)	$29.3	$0.0	$4913.4

CF IRR: 2.0%
CS IRR: 29.6%
(CF+CS) IRR: 31.6%
Return of Risky Security: 0.0%

In Table 6-2 is an example where the investor has funds (F_t) of $400 per period with his target return set at 10% per period. The interest rate on both loans and deposits of the bank account available to him is 2% per period. Considering time $t=2$, the target value of the investor V_2 is 10% more than his previous target plus the additional funds for that period, ($840 * 110%) + $400 = $1324. However N_2, the net risky asset owned at the beginning of the period is only, 168 * $3 = $504. Hence he has to make up for the difference by investing more in the risky security, R_2, $1324 - $504 = $820. This amount is made up of the additional fund for this period, $400 and from the bank account, S_2 = $420. Such a withdrawal from the bank account requires the investor to make a loan and leaves behind $(664.8) as the balance.

6.1.4 Modified Value Averaging

In a bid to make the accumulative investment method of value averaging comparable to the buy and hold strategy, a modified method of value averaging was introduced. To remove the "accumulative" property of value averaging, no additional funds will be provided after the initial investment. The target value will still grow at the targeted rate of return. Any additional investments to meet the target will be made with money borrowed from the side-fund at the stipulated interest rate while proceeds from sales will be deposited into the side-fund to earn interest.

6.1.5 Random Investing

Similar to Marshall [15], a random investing (RI) strategy is introduced as a basis for comparison. This is a strategy which approximates a normal investment pattern common among investors. At regular intervals, the investor is equally likely to invest 50% of his available capital while the extra capital will be saved, or 150% of his available capital, where borrowing is required. A uniform distribution between 0% and 200% is used to represent the percentage of the capital to be invested. The expected investment value of this strategy is the same as the dollar cost averaging method. This keeps our comparison fair by investing the same amount for both methods.

There are two possible methods to analyze the performance of this strategy. This is similar to the value averaging case where there are two streams of cash flows to be considered, the combined cash flow, $(R_t + S_t)$, of both the investment into the risky security and the deposit into the side-fund or the investment into the risky security alone, R_t.

6.2 Criterion for Investment Evaluation

The aim of an investor is to select the best possible cash flow stream to meet his investment targets. In order to evaluate and differentiate the different investment options in a fair and logical way, the investor has several different criteria to use. The two most important methods are those based on the net present value and the internal rate of return. Both these methods involve the time value of money which states that the present value of money is less than the face value of that amount in the future. This allows us to make comparisons between cash flows spent or received at different time intervals.

The two methods have both attractive features and limitations. Hence the choice of criteria depends on the situation presented to the investor. The investment could be a one-time opportunity and cannot be repeated, in which case, net present value would be the appropriate criterion. On the other hand, the investment cannot be repeated or it could be an investment which may be repeatedly reinvested. In this situation, the internal rate of return would be the ideal criterion.

6.2.1 Net Present Value

Money invested today leads to an increased value in the future as a result of interest. This concept may be reversed to calculate the value to be assigned to money at the present moment that is to be received in the future. The process of evaluating an equivalent present value of a future cash flow is known as discounting. The present value of future money is less than the face value of that amount because money in the present is more useful and the future cash flow bears with it a risk of default. Hence, the future value has to be discounted to obtain the present value. The factor by which the future value has to be discounted is referred to as the discount factor. This is an example of a future value, *FV* to be received in *n*-period discounted by a 1-period discount factor, *r*,

$$PV = \frac{FV}{(1+r)^n}.$$

The present value, *PV* is dependent on the choice of the discount factor.

This concept may be extended to cash flow streams over multiple periods. Assume that we have a cash flow (x_0, x_1, x_2, …, x_n) and that the cash flow occurs at the end of each period. The net present value (*NPV*) of this cash flow can be calculated if we consider each stream individually. The present value of x_0 would be that value itself and the present value of x_1 is discounted for one period by $1/(1+r)$ while x_2 is discounted by two periods by $1/(1+r)^2$. This continues on for the rest of the cash flow stream to give,

$$NPV = x_0 + \frac{x_1}{1+r} + \frac{x_2}{(1+r)^2} + \ldots + \frac{x_n}{(1+r)^n} .$$

This may be regarded as the equivalent present payment amount from the entire stream.

Net present value is used to rank different cash flow streams resulting from different investment schemes. It evaluates the present values of the investments and the higher the present value, the better the investment. An investment which gives a negative net present value should not be considered.

6.2.2 Internal Rate of Return

Internal Rate of Return (IRR) is a useful number to know when we are evaluating an investment. Often used in capital budgeting, it is the interest rate, or the discount rate, that makes the net present value of all cash flows equal to zero. The internal rate of return is the true interest yield expected from an investment expressed as a percentage. It is often referred to as a break-even rate of return because it shows the discount rate below which an investment results in a positive net present value and above which an investment results in a negative net present value. Suppose we have a stream of cash flows as, (x_0, x_1, x_2, …, x_n). Then the internal rate of return of this stream is a number r satisfying the equation

$$0 = x_0 + \frac{x_1}{1+r} + \frac{x_2}{(1+r)^2} + \cdots + \frac{x_n}{(1+r)^n}. \tag{3}$$

When used as a performance criterion, the higher the internal rate of return, the more profitable an investment. However, the investment should only be considered if it has an internal rate of return higher than the current bank or treasury interest rate. If it does not have a higher internal rate of return, it would be wiser to and more profitable to invest in risk-free treasury bonds or deposit the available capital in a bank account.

6.2.3 Multiple Internal Rates of Return

One of the shortcomings that accompanies internal rate of return analysis is that if an investment has at least three periods of cash flows and if one internal rate of return can be computed, then there is likely to be at least one additional internal rate of return solution. This implies that the solution to (3) has repeated roots. For more common investment situations in which all periods following the initial outlay involve inflows, one of the two internal rates of return can be ignored because its magnitude is in contradiction to the profitable nature of the cash flows. In fact, the value of the errant internal rate of return will be less than -100%, which indicates a loss of more than the amount invested.

Unfortunately, it is not always possible to ignore one of two internal rates of returns, because both solutions may be of a magnitude that appears consistent with the investment cash flows. This situation can occur when there is a reversal in the signs of the cash flows, such that an initial investment is followed by at least one cash inflow and, subsequently, at least one negative flow. This is exactly the case when the investor is executing the value averaging strategy. The investor will be either buying or selling at each period, causing reversals in the signs of the cash flows. When solving the internal rate of return of such a cash flow, there will often be multiple internal rates of return. To select the relevant internal rate of return, a method suggested by Colwell et al. [21] was modified and used.

The method put forward by Colwell et al. addresses all cases of dual internal rates of return regardless of the magnitudes of the results. This method developed has great intuitive appeal, in that a rate is rejected if it moves in the "wrong" direction when the final cash flow changes. A rate is ignored, for example, if its value falls when computed under new assumptions that show a lower negative cash flow or a higher positive cash flow in any specified period. This technique treats a rate of return measure as irrelevant if it falls as the investment becomes more profitable or rises when the investment becomes a greater loss. This method has theoretical appeal because it is applicable to all situations in which there are two solutions to the internal rate of return equation, regardless of whether one of the rates is clearly impossible because of its magnitude.

As an example, assume that there is an investment with three cash flows. The initial cash flow relating to this investment would be a negative amount and one would normally expect subsequent cash flows to be positive, although one or more could be negative. The net present value of this investment with three cash flows is calculated with

$$NPV = x_0 + \frac{x_1}{1+r} + \frac{x_2}{(1+r)^2}. \tag{4}$$

In equation (4), the subscripted x_t represents the periodic cash flows. The initial cash flow x_0 is assumed to occur at the present; hence it is discounted for zero periods. Furthermore it has a negative value as it represents the initial investment. Variable r represents the rate by which expected cash flows are discounted in computing present values. This rate is the return that the investor would expect to earn on other available investments that would impose similar risks.

We multiply each side of equation (4) by $(1 + r)^2$ and can present the result in a form that is easier to study by replacing $(1 + r)$ by y, so that the equation appears as

$$NPV(y^2) = x_0(y^2) + x_1(y) + x_2.$$

This equation could be called net future value (*NFV*), in that it represents the future equivalent of a set of cash flows, just as net present value represents the present equivalent of a cash flow series. In our example, the equation takes the form of the well known quadratic function, the graph of which is a parabola. Figure 6-1 shows a graph of this parabolic function based on an assumption that x_1 is positive, while x_2, like x_0, has a negative value which simulates a cash flow reversal.

The roots of the function are shown graphically as points where the parabola crosses the horizontal axis. At each such point, the function's value equals zero:

$$NFV = 0 = x_0\left(y^2\right) + x_1\left(y\right) + x_2 . \tag{5}$$

We can find the dual internal rates of return, which are the r values for which NFV = 0, by subtracting 1 from each of these roots; recall that $y = 1 + r$. The two internal rates of return are shown in Figure 6-1 as r_1 and r_2. It is here that the investor has to select the one that is relevant. A computationally easy cash flow change to consider would be an increase or decrease in x_2. Note that in equation (5), x_2 is the only cash flow not multiplied by y or y^2. Graphically, a change in x_2 merely shifts the parabola up or down. Suppose that the investor revises the cash flow projections such that x_2 would rise or become less negative. It should be obvious that the investor would be better off with a higher cash flow in that final period. A graphical representation in Figure 6-1 (right) shows the parabola shifting upward. Note that the smaller root at the left would decrease, while the greater root at the right, r_2, would take the higher r_2' value shown.

On the other hand, if the investor had revised the cash flow showing a decrease in x_2 which would represent a loss for the investor, relative to the situation depicted in

Figure 6-1. As shown in Figure 6-2, the parabola shifts downward; the greater root falls from r_2 to r_2', while the smaller root rises from r_1 to r_1'. The root on the left moves in the wrong direction whenever we change an assumption regarding the cash flows. Therefore, this root must be irrelevant.

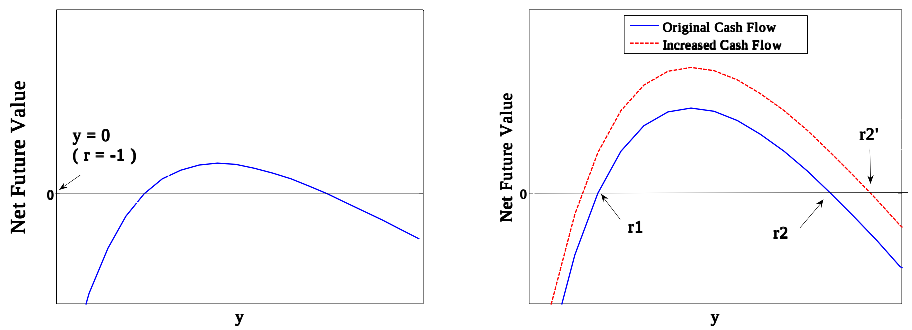

Figure 6-1 (Left) Net Future Value curve where there are three cash flows. Initial and final cash flows are negative while the second cash flow is positive. (Right) Net Future Value curves of modified cash flow where the final cash flow is increased and the origin

As an example, suppose that the following cash flow stream, (-1, 5, -6), is given. The NPV curve as seen in Figure 6-2 is obtained. The values of r which solve (4) for this cash flow stream are 100% and 200%. Graphically, increasing or decreasing the initial cash flow x_0 translates into moving the quadratic curve upwards or downwards. Intuitively, if the cash flow x_0 is increased, the relevant rate of return should increase as well. This leads us to conclude that 200% is the relevant rate.

In this example where there are two internal rates of return, it has been demonstrated that the internal rates of return associated with the root of the function where the net future value is increasing and the gradient is positive should be rejected as an indicator of investment returns. This method suggested by Colwell et al is equally applicable in the more common investment situation where there are no cash flow reversals, in which one of the two rates has a value less than –100%.

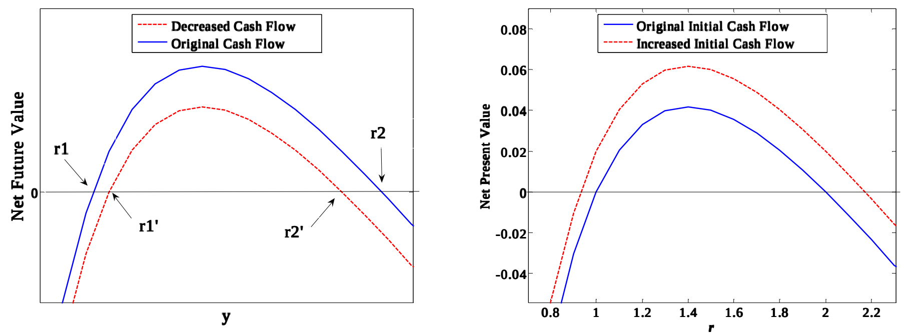

Figure 6-2 (Left) Net Future Value curves of modified cash flow where the final cash flow is decreased and the original cash flow. Root r2 moves in the correct direction to r2' when the final cash flow is increased. (Right) Net Present Value curve of the cash flow stream, (-1, 5, -6) which demonstrates multiple internal rates of return. When the initial cash flow is increased, the rate at 2 moves in the positive direction while the rate at 1 decreases. Intuitively, the rate at 2 is the relevant internal rate of return.

In general, an internal rate of return regardless of its magnitude is a relevant measure of investment return if a marginal increase in any investment cash flow results in an increase in the rate's calculated value, whereas an IRR is irrelevant if a marginal increase in a cash flow causes a decrease in the rate's value. Problems can arise in the use of this rule if there are more than two roots to the net future value function.

If there are more than two roots to the net future value function, we will first implement the above mentioned method to eliminate the irrelevant internal rates of return. From the remaining relevant internal rates of return, we will select the maximum internal rate of return.

6.2.4 Choice of Criteria

The two methods of evaluation presented each has its advantages and limitations. Net present value is easy to calculate and it does not have the problems such as the multiple roots of the internal rate of return equation as highlighted above. Furthermore, net present value analysis allows us to break it down into its component pieces for a deeper evaluation. On the other hand, internal rate of return has the advantage of depending only on the cash flow stream and not on the discount rate which does not have a standard definition.

It is possible that two methods of analysis give contradicting recommendations hence it is important to choose the right criteria for evaluation. The choice of which of the two criteria is the most appropriate for investment evaluation depends on the scenario presented to the investor. If the investment may be made repeatedly but scaled in size, the investment option which gives the largest internal rate of return should be selected as it allows the greatest growth of capital. However, if this investment opportunity is a one-off event and cannot be repeated, the net present value would be the more appropriate criterion since it compares the investment with other options offering the prevailing interest rate.

In our research, the trading strategies being evaluated present an opportunity for continual reinvestment throughout the investment horizon. Hence, we will be using the internal rate of return as a criterion for investment evaluation.

6.3 Performance Measures

The criteria of performance will be the internal rate of return. Monthly cash flows are used to calculate the internal rates of return; hence a monthly internal rate of return is obtained. A higher overall internal rate of return indicates a better performance.

For each of the strategies which involve the use of a side fund, two different streams of cash flows will be considered in the assessment of the strategies. Firstly, the single cash flow stream only from purchasing and selling of the risky security is considered. This method of analysis is noted as VA (Stock only). Secondly, the combined cash flow stream from the overall investment into both the risky security and the side fund is evaluated. This method is noted as VA (Combined cash flow).

6.4 Historical Performance

To analyze the effectiveness of these strategies, they will be executed on the historical monthly closings of the Dow Jones Industrial Average and the Standard & Poor's 500. The Dow Jones Industrial Average data runs from 1st March 1929 to the 1st March 2006 while the Standard & Poor's 500 data is from the 3rd March 1950 to 1st March 2006.

The following investment parameters will be assumed:

- $1000 will be available for investment at the start of each month. This is available to all the strategies except the modified value averaging and the buy and hold strategies.
- Value averaging will be executed with three different target rates of return, 0%, 5% and 10% per annum.
- Both the buy and hold and the modified value averaging strategies will make one initial investment of $1000. Any subsequent funds needed by the modified value averaging strategy will be obtained via a loan from the side fund.
- The side fund will have a constant interest rate of 2% for both the deposits and loans. Any loans made will have to be paid back in the following investment period.
- Investments will be made at the start of every month. The investment portfolio is valued on the last day of the investment horizon.
- Random investing will not be considered.

6.4.1 Short-term Performance – Dow Jones Industrial Average

The short-term performances of the different strategies were compared for each year of actual stock market history for the Dow Jones Industrial Average. Each yearly period starts and ends in the month of March.

Throughout the seventy six periods, when compared to the buy and hold strategy, dollar cost averaging had a higher return 40 times and was beaten by the buy and hold strategy 36 times. When dollar cost averaging out performed the buy and hold strategy, it was on average 0.52% better and had as much as 1.94% higher returns. When dollar cost averaging performed worse than the buy and hold strategy, it averaged 0.55% lower and was no lower to the buy and hold strategy than by 3.19%.

Looking at value averaging (combined cash flow), and comparing among the three target rate of returns, it performed the best 47 times (61.8%) when it set the target returns at 10%. These results were then compared with the buy and hold strategy. Value averaging was able to outperform the buy and hold strategy 38 times and averaged 0.56% higher. On the other hand, it lost 38 times and averaged lower by 0.58%.

As for value averaging (Stock only), when only the cash flow stream into the risky security is considered in the calculation of returns, the choice of target returns for value averaging was much less significant. Having the target return at 10% gave a better performance only 39 times while having a 0% target return performed best 37 times.

A comparison of the average internal rate of return throughout these seventy six samples (Table 6-3) shows these trends more clearly. The choice of target returns in the value averaging strategy has an effect when considering both cash flow streams in the returns analysis but is less important when considering only the cash flow stream into the risky security.

Table 6-3 Mean IRR for Short-term Performance in the DJIA

BH	DCA	VA (Combined Cash Flow)			VA (Stock Only)		
		0%	5%	10%	0%	5%	10%
0.40440	0.41884	0.38977	0.39092	0.39196	0.50098	0.50092	0.50087

Comparing the modified value averaging strategy with the buy and hold strategy (Table 6-4), the opposite trend is observed. Modified value averaging, while considering a combined cash flow, performs best when the target return is set at 0%. In this case it is unable to beat the buy and hold strategy regardless of the target return chosen. When evaluating the cash flow from the risky security alone, the target return still has a smaller influence and is able to outperform the buy and hold method consistently.

Table 6-4 Mean IRR for Short-term Performance of Modified VA in the DJIA

BH	VA (Combined Cash Flow)			VA (Stock Only)		
	0%	5%	10%	0%	5%	10%
0.40440	0.32520	0.32184	0.31793	0.56844	0.56641	0.56449

6.4.2 Long-term Performance – Dow Jones Industrial Average

The long-term performance was analyzed using all possible five year periods of actual stock market history. Each period starts and ends in the month of March.

Comparing the seventy two five year periods, dollar cost averaging had a higher return 41 times and was beaten by the buy and hold strategy in 31 investment periods. When dollar cost averaging outperformed the buy and hold strategy, it averaged 0.32% higher when it won and provided as much as 1.93% higher returns when it won (1930 – 1934). When dollar cost averaging performed worse than the buy and hold strategy, it averaged 0.27% lower and was no lower than 1.59%.

Analyzing value averaging while taking into consideration both cash flow streams into the risky security and the side fund, it is found that among the three target rate of returns, value averaging performed the best 56 times (77.8%) when it set the target returns at 10%. This case is then compared with the buy and hold strategy. Value averaging was able to outperform the buy and hold strategy 42 times and averaged 0.37% higher. On the other hand, it lost 30 times and averaged lower by 0.26%.

When only the cash flow stream into the risky security is considered in the calculation of returns, the choice of target returns of value averaging played less of a role. Having the target return at 10% gave a better performance only 38 times while having a 0% target return performed best 34 times.

Using the average internal rate of return throughout these seventy two samples (Table 6-5), the above mentioned trends become more obvious. The choice of target returns in the value averaging strategy has an effect when considering both cash flow streams but is less important when considering only the cash flow stream into the risky security. Value averaging is only able to out perform dollar cost averaging if it sets the target return at 5% and above.

Table 6-5 Mean IRR for Long-term Performance in the DJIA

BH	DCA	VA (Combined Cash Flow)			VA (Stock Only)		
		0%	5%	10%	0%	5%	10%
0.49162	0.55816	0.54604	0.57102	0.59625	0.63044	0.62958	0.62871

This is similar to the case when evaluating the modified value averaging strategy. When the combined cash flow is taken into account, having the target return at 10% allowed the best performance 60 times. However, when only the cash flow into the risky security is considered, the choice of target returns does not necessarily lead to better performance.

Comparing the modified value averaging strategy with the buy and hold strategy (Table 6-6), the same trends are observed. For the modified value averaging strategy to out perform the market, the target return has to be set at 5% and above. If only the cash flow into the risky security is considered, the modified value averaging method consistently out performs the buy and hold strategy.

Table 6-6 Mean IRR for Long-term Performance of Modified VA in the DJIA

BH	VA (Combined Cash Flow)			VA (Stock Only)		
	0%	5%	10%	0%	5%	10%
0.49162	0.47266	0.50939	0.54769	0.65471	0.65035	0.64604

6.4.3 Short-term Performance – Standard & Poor's 500

The short-term performances of the different strategies were compared for each year of actual stock market history for the Standard & Poor's 500. Each yearly period starts and ends in the month of March.

Throughout the fifty five years, when compared to the buy and hold strategy, dollar cost averaging had a higher return 24 times and was beaten by the buy and hold strategy on 31 times. When dollar cost averaging outperformed the buy and hold strategy, it was on average 0.65% better and had as much as 2.09% higher returns.

When dollar cost averaging performed worse than the buy and hold strategy, it averaged 0.34% lower and was no lower to the buy and hold strategy than by 0.84%.

It is found that among the three target rate of returns for value averaging (combined cash flow), the strategy performed the best for 35 times when it set the target returns at 10%. The results of this is then compared with the buy and hold strategy. Value averaging was able to outperform the buy and hold strategy 25 times and averaged 0.64% higher. On the other hand, it lost 30 times and averaged lower by 0.36%.

When only the cash flow stream into the risky security is considered in the calculation of returns, the choice of target returns for value averaging was much less significant. Having the target return at 10% gave a better performance only 30 times while having a 0% target return performed best 25 times.

A comparison of the average internal rate of return throughout these fifty five samples (Table 6-7) shows these trends more clearly. The choice of target returns in the value averaging strategy has an effect when considering both cash flow streams in the returns analysis but is less important when considering only the cash flow stream into the risky security.

Table 6-7 Mean IRR for Short-term Performance in the S&P500

BH	DCA	VA (Combined Cash Flow)			VA (Stock Only)		
		0%	5%	10%	0%	5%	10%
0.64962	0.74287	0.73102	0.73921	0.74718	0.78946	0.79041	0.7913

The modified value averaging strategy gives results which are similar to the normal case. When the overall cash flow is taken into account, having the target return at 10% gave the best performance 35 times. However, when only the cash flow into the risky security is considered, the choice of target returns does not necessarily lead to a better performance.

Comparing the modified value averaging strategy with the buy and hold strategy (Table 6-8), the same trends are observed. For the modified value averaging strategy to out perform the market, the target return has to be set at 10% and above. If only the cash flow into the risky security is considered, the modified value averaging method consistently out performs the buy and hold strategy.

Table 6-8 Mean IRR for Short-term Performance of Modified VA in the S&P500

BH	VA (Combined Cash Flow)			VA (Stock Only)		
	0%	5%	10%	0%	5%	10%
0.64962	0.63115	0.64193	0.65246	0.72807	0.72951	0.73089

6.4.4 Long-term performance – Standard & Poor's 500

The long-term performance was analyzed using all possible five year periods of actual stock market history. Each period starts and ends in the month of March.

Of the fifty one possible five year periods, dollar cost averaging had a higher return 30 times and was beaten by the buy and hold strategy on 21 times. When dollar cost averaging out performed the buy and hold strategy, it averaged 0.19% higher when it won and provided as much as 0.61% higher returns when it won. When dollar cost averaging performed worse than the buy and hold strategy, it averaged 0.23% lower and was no lower than 0.73%.

Analyzing value averaging while taking into consideration a combined cash flow stream of the risky security and the side fund, it is found that among the three target rate of returns, value averaging performed the best 42 times when it set the target returns at 10%. This is then compared with the buy and hold strategy. Value averaging was able to outperform the buy and hold strategy 29 times and averaged 0.23% higher. On the other hand, it lost 22 times and averaged lower by 0.23%.

When only the cash flow stream into the risky security is considered in the calculation of returns, the choice of target returns of value averaging played less of a role. Having the target return at 10% gave a better performance only 25 times while having a 0% target return performed best 26 times.

Using the average internal rate of return throughout these seventy two samples (Table 6-9), the above mentioned trends become more obvious. The choice of target returns in the value averaging strategy has an effect when considering both cash flow streams but is less important when considering only the cash flow stream into the risky security.

Table 6-9 Mean IRR for Long-term Performance in the S&P500

BH	DCA	VA (Combined Cash Flow)			VA (Stock Only)		
		0%	5%	10%	0%	5%	10%
0.63448	0.65035	0.61026	0.63966	0.66981	0.70826	0.70779	0.70737

This is similar to the case when evaluating the modified value averaging strategy. When the overall cash flow is taken into account, having the target return at 10% allowed the best performance 44 times (86.2%). However, when only the cash flow into the risky security is considered, the choice of target returns does not necessarily lead to better performance.

Comparing the modified value averaging strategy with the buy and hold strategy (Table 6-10), the same trends are observed. For the modified value averaging strategy to out perform the market, the target return has to be set at 10% and above. If only the cash flow into the risky security is considered, the modified value averaging method consistently outperforms the buy and hold strategy.

Table 6-10 Mean IRR for Long-term Performance of Modified VA in the S&P500

BH	VA (Combined Cash Flow)			VA (Stock Only)		
	0%	5%	10%	0%	5%	10%
0.63448	0.58012	0.62195	0.6656	0.72505	0.72242	0.71994

6.4.5 Summary

The results above emphasize the importance of the method used to analyze the internal rate of return from value averaging. If only the cash flow stream into the risky security is considered, then value averaging will be able to outperform both the buy and hold and dollar cost averaging consistently. If the combined cash flow stream is to be analyzed, then the choice of the target returns when applying the value averaging strategy becomes an important factor. For this strategy to outperform dollar cost averaging and buy and hold, the target return has to be set above a certain threshold of about 10%.

The length of the investment horizon, short-term or long-term, does not affect both dollar cost averaging and value averaging. The long-term investment over five years did better for the historical Dow Jones Industrial Average data while the short-term investment over a year performed better for the historical Standard & Poor’s 500.

Using the mean internal rates of return as our metric for analyses has certain drawbacks as mentioned earlier in section 2.4 such as the historical mean blur. Despite a higher mean internal rate of return, the number of times a strategy out performs another may not be significantly greater. Further studies could be made into the conditions which favor one strategy over another.

6.5 Monte Carlo Simulation Methodology

The simulations were done using Matlab. To allow the results to be comparable with previous works, Marshall's [15] methodology of analyzing the performance of the investment strategies was followed. The investment return is determined by the internal rate of return of each cash flow with the Monte Carlo method. 20000 simulations of investments are used to calculate the mean return and standard deviation of the internal rate of return in each of the trading strategies.

The investments will be made on a monthly basis which is a close representation of the common investor who sets aside some of his monthly income for investment. The different investment strategies were applied to a simulated stock market with a mean reverting characteristic. Monthly market returns were generated using the model defined by (1) in Chapter 7. To ensure that the model best replicates the real world, the choice of the parameters of the model, λ, σ, and α is important. They were selected to give annual returns from 5% to 15% with the monthly returns having volatility ranging from 8% to 20%. The base values of the parameters were set as follows: λ = *0.001*, σ = *0.3281* while α was varied to give the different market returns.

The model parameters were varied around these base values to check their influence on the performance of any of the strategies. Other variables such as the duration of the investment period, amount invested and the risk-free side fund's interest rates were also varied.

6.5.1 Investment Period

The length of investment has to be long enough to ensure that the mean reverting characteristic will be shown. A period of 5 years which is consistent with Marshall [15] was chosen.

6.5.2 Amount invested

For dollar cost averaging, a constant dollar amount of $1000 is invested each period.

For value averaging, the same amount of $1000 was given at the start of each period. As for the target return of value averaging, the target value of the portfolio was grown at three different rates. The growth rates were set to be equal to, 5% over and 5% under the growth rate of the market. If the investor does not have enough funds to reach the target value, he will have to borrow the additional funds from the bank at the stipulated interest rate and repay it the following month. Likewise, the proceeds from the sale of his shares are deposited into the bank and enjoy the interest earned.

Random investing includes a uniform chance that he invests from 0% of his capital to 200% of his capital. If he has to invest more than the capital available to him, he will have to borrow the additional funds as in the value averaging case. Any capital not invested will be deposited with the bank and interest will be earned on the bank balance.

For the buy and hold strategy, $1000 was invested from the on start. In this case, the internal rate of return is independent of the amount invested. The return from this strategy is taken to represent the market's return.

6.5.3 Interest Rates

The cost of borrowing should ideally be greater than the cost of lending. However, for simplicity, both interest rates have been taken to be constant and equal. Simulations were run with the interest rates at 0%, 2% and 5% per annum.

6.6 Monte Carlo Simulations Results and Analyses

In Section 5, the framework for the Monte Carlo simulations to obtain the mean internal rates of return of the different strategies was defined. This section analyses the results from the simulations from two aspects, the expected returns and the inherent risk involved. As the different parameters of the simulation are varied to test their effects on the strategies, the analyses will take this two pronged approach.

Finally, the modified value averaging method will be compared to buy and hold strategy.

It should be noted that these results are valid only locally around the model parameters which were used.

Throughout the various sets of simulations, certain trends were present regardless of the underlying market conditions.

6.6.1 Investment Period and Amount Invested

It was found that the length of the investment period and the amount invested did not have any significant influence on the results. However, increasing the length of the investment period reduced the risk involved in the strategies.

6.6.2 Random Investing and Dollar Cost Averaging

The first noticeable result was that random investing gave approximately the same internal rate of return when analyzed with a single stream cash flow into the risky asset or with a combined cash flow stream into both the risky asset and the side fund. This is a reasonable result as the investor was equally likely to deposit cash as he was to borrow it. In the long run, the amount of his deposits would have been equal to the amount of loans.

Similar to Marshall [15], the random investing strategy gives very similar returns to dollar cost averaging. Unfortunately, both of these strategies fail to beat the buy and hold strategy regardless of the market conditions.

Dollar cost averaging however has a slight advantage of random investing as it has less risk. The degree of this advantage is greater in an environment with higher volatility.

6.6.3 Value Averaging

Value averaging is the most superior strategy when only the single cash flow stream into the risky asset is considered. It consistently outperforms the other strategies in most situations. The only time it fails to do so is when volatility is reduced extremely. In such an environment, it is still able to match up to dollar cost averaging but loses to the buy and hold strategy. This is to be expected as the success of this strategy depends on the volatility of the market.

When analyzing value averaging in this manner, the target return set by the investor affects the returns inversely (Table 6-11). If the target return is set above the actual market return, his return will be lowered. If he sets his target return below the actual market return, the return from the strategy increases. This trend is less obvious in a low volatility environment and becomes more distinct as the volatility increases.

On the other hand, when analyzing value averaging with a combined cash flow stream of both the risky asset and the side fund, the opposite is true. Setting the target return higher would result in a higher return. This makes sense as the interest earned by a deposit into the side fund is lower than the return from an investment into the financial market. However this is not true all the time as in a bearish market, the financial market would not be able to give the kind of returns a risk-free side fund.

The importance of the choice of target return is highlighted when comparing value averaging (combined cash flow) with dollar cost averaging. Value averaging will only be able to out perform dollar cost averaging if the target return is set higher than the market return.

Table 6-11 Mean IRR for Market Return of 5% with $\Sigma = 0.3281$ and $\Lambda = 0.001$

BH	DCA	VA (Combined Cash Flow)			VA (Stock Only)		
		0%	5%	10%	0%	5%	10%
0.456	0.426	0.409	0.421	0.431	0.488	0.486	0.485

The risks involved with value averaging naturally increased with a higher target rate of return. However, the level of risk increased faster for the combined cash flow analysis.

Table 6-12 Standard Deviation of IRR for Market Return of 5% with Σ = 0.3281 and Λ = 0.001

BH	DCA	VA (Combined Cash Flow)			VA (Stock Only)		
		0%	5%	10%	0%	5%	10%
0.564	0.650	0.598	0.660	0.736	0.649	0.656	0.663

6.7 Interest Rates

The interest rates charged on loans taken from the side fund and paid on deposits were varied at 3 values, 0%, 2% and 5% per annum.

6.7.1 Mean Internal Rates of Return Analysis

The interest rate has no effect on the internal rates of return for strategies which do not borrow or deposit cash with the side fund. Hence, the buy and hold and dollar cost averaging returns were unaffected.

Similarly, for the value averaging and random investing strategies, the internal rates of return when analyzing the cash flow stream into only the risky asset, was not affected by the interest rates.

Interestingly, the interest rate has no effect on the random investing strategy even when a combined cash flow stream is considered. Approximately the same internal rates of return are obtained in both of the cases, a combined cash flow and a risky asset only cash flow. This may be explained by the fact that the expected amount to be invested at every period is 100% of the available fund. The investor is equally likely to be borrowing cash as he is to be depositing cash. Thus, the interest rate charged on loans and given on deposits will not have an impact on his returns.

For the case of value averaging where a combined cash flow is considered, the effects of the interest rate will depend on the overall market return and the target return set for the strategy. Intuitively, when the target return is set higher than the market return, there is a higher probability that the investor will be borrowing more cash from the side fund to attain his target value. In the converse situation where the target return is set to be lower than the market return, more cash will be deposited into the side fund as the higher than expected prices will channel the investor to sell of some of his risky assets in order to lower his invested value to his target value. With larger deposits in the side fund, a higher interest rate should lead to higher returns. Evidence of this is seen from the results shown in Table 6-13 when the market return is 5% per annum.

However, when the market return is much higher than the interest rate of the side fund, it would be wiser to have more invested into the market. A higher target return results in a greater return. This is seen in Table 6-13 for a market which returns 10%.

Table 6-13 Mean IRR of VA(Combined Cash Flow) from Varying Interest Rates & Market Returns

Market Return p.a.		5%			10%		
VA Target Returns p.a.		0%	5%	10%	5%	10%	15%
	0%	0.396	0.419	0.444	0.760	0.810	0.866
Interest Rate	2%	0.409	0.421	0.431	0.777	0.818	0.863
	5%	0.427	0.420	0.408	0.795	0.819	0.843

6.7.2 Investment Risk Analysis

Risk in this sense is the standard deviation of the internal rates of return. It is observed that the higher the investor sets his target returns, the greater the risk. When the market has higher returns, the risk involved in the value averaging strategy declines.

Table 6-14 Standard Deviation of IRR of VA(Combined Cash Flow) from Varying Interest Rates & Market Returns

Market Return p.a.		5%			10%		
VA Target Returns p.a.		0%	5%	10%	5%	10%	15%
	0%	0.579	0.636	0.705	0.559	0.608	0.665
Interest Rate	2%	0.598	0.660	0.736	0.548	0.597	0.654
	5%	0.603	0.669	0.752	0.569	0.626	0.695

6.8 Volatility

In this set of simulations, the volatility of the market was varied to test its effects on the strategies. Volatility was manipulated by changing the value of σ in the mean reverting model parameters. During these simulations, the interest rate of the side fund was kept at 2% per annum and the rate of mean reversion, λ = 0.001.

6.8.1 Mean Internal Rates of Return Analysis

Both dollar cost averaging and value averaging have been touted to benefit from increased market volatility. This trend was noted when analyzing the returns for dollar cost averaging and value averaging when considering the return from the cash flow of the risky asset alone. Dollar cost averaging was unable to outperform the buy and hold strategy regardless of the volatility levels whereas value averaging was able to beat it consistently independently of the target value selected by the investor. However, value averaging failed to outperform buy and hold when the volatility was reduced below a certain threshold. These results are in line with previous findings [12].

Table 6-15 Mean IRR of Strategies from Varying Volatility & Market Return of 5%

Volatility		BH	DCA	VA (Combined Cash Flow)		
				0%	5%	10%
Model Parameter σ	0.033	0.420	0.414	0.395	0.414	0.434
	0.082	0.429	0.416	0.397	0.415	0.436
	0.164	0.411	0.407	0.390	0.406	0.424
	0.328	0.456	0.426	0.409	0.421	0.431
	0.492	0.512	0.452	0.430	0.426	0.368
	0.656	0.361	0.348	0.031	-0.316	-0.951
	0.984	-0.261	-0.249	-5.640	-7.948	-10.976

The focus of these simulations is on the returns from value averaging when considering a combined cash flow with the inclusion of an interest bearing side fund. In this situation, value averaging was only superior to the buy and hold strategy when market volatility was low (σ approximately less than 0.328) and the target return set by the investor was above the market return by 5%. A set of results for a simulation done with the model giving a market return of 5% is shown in Table 6-15.

In the event of an extremely volatile market, all strategies suffer heavy losses. The higher the investor set his target return for the value averaging strategy, the bigger the loss.

6.8.2 Investment Risk Analysis

The same observations are made on the level of risk for both a market return of 5% and 10%.

When analyzing value averaging with a combined cash flow stream, the risk incurred by this strategy in a volatile market is very great. It increases at a much faster rate compared to dollar cost averaging when volatility is increased. Unlike an analysis where only the cash flow into the risky security is considered, there is a cost involved in borrowing extra funds in order for the investor to meet his target value. In an extremely volatile market, such loans can take a toll on the investor as he may not be able to extend his loans without limit.

Dollar cost averaging and the buy and hold strategy are not exposed to this risk and are better suited for a high volatility environment.

Table 6-16 Standard Deviation of Strategies from Varying Volatility & Market Return of 5%

Volatility		BH	DCA	VA (Combined Cash Flow)		
				0%	5%	10%
Model Parameter Σ	0.328	0.564	0.650	0.598	0.660	0.736
	0.492	0.837	0.974	0.950	1.110	2.675
	0.656	1.120	1.310	7.426	10.374	15.247
	0.984	1.654	1.950	32.357	38.451	45.122

6.9 Rate of Mean Reversion

As seen in equation (2) of Chapter 5, the rate of mean reversion affects the simulated market price series by changing the variance of the prices. A higher rate of mean reversion will result in a smaller variance of the prices. This property is less evident at the start of the price series and becomes more obvious with time.

In this set of simulations, the rate of mean reversion, λ, was varied from 0.003 to 0.0003. The volatility, σ, and the interest rate were kept constant at 0.3281 and 2% per annum respectively throughout all the simulations.

6.9.1 Mean Internal Rates of Return Analysis

Varying the rate of mean reversion did not produce any distinct trends. No significant benefits were brought to the strategies. However, both increasing and decreasing the rate of mean reversion to the extremes resulted in a slight decrease in returns from all strategies.

Dollar cost averaging and value averaging when analyzed with a combined cash flow were both unable to outperform the buy and hold strategy consistently. Though value averaging did manage to beat the buy and hold strategy under certain conditions. However, they did not occur consistently in both the market with returns of 5% and 10% and nothing conclusive could be drawn from the results.

Table 6-17 Mean IRR of Strategies from Varying Rate of Mean Reversion for Market Return of 5%

Rate of Mean Reversion		BH	DCA	VA (Combined Cash Flow)		
				0%	5%	10%
	0.0003	0.457	0.419	0.404	0.415	0.425
Model	0.0005	0.421	0.420	0.403	0.414	0.423
Parameter	0.0010	0.456	0.426	0.409	0.421	0.431
λ	0.0020	0.470	0.429	0.413	0.425	0.436
	0.0030	0.459	0.415	0.401	0.411	0.421

6.9.2 Investment Risk Analysis

The mean reversion parameter did not have any significant influence on the risks associated with the strategies.

6.10 Modified Value Averaging

The concept of the modified value averaging strategy was to make value averaging, a cumulative investment technique, comparable with the buy and hold strategy. This method would highlight the effects of borrowing additional funds in order to meet targets. Thus, any analysis should be made with a combined cash flow stream.

In this case, the modified value averaging strategy was unable to beat the buy and hold strategy except when volatility was low and the target return was set at 5% above the market return. When volatility increased, the risk involved in this strategy grew at an increasing rate and its performance was poorer than the buy and hold.

Table 6-18 Mean IRR of Modified Value Averaging From Varying Rate of Volatility for Market Return of 5%

Volatility		BH	VA (Combined Cash Flow)		
			0%	5%	10%
Model	0.164	0.423	0.398	0.422	0.450
Parameter	0.328	0.461	0.436	0.423	0.263
σ	0.492	0.498	0.127	-0.469	-1.820

CHAPTER 7 CONCLUSION

7.1 Foreign Exchange Rate Prediction with ANN

Forecasting of foreign exchange rates is a difficult task which has been a challenge in modern time series prediction. The goal of the first section in this thesis has been to examine the different inputs which may be used in constructing an ANN for the purpose of predicting exchange rates. The performance of these ANN was based on the RMSE and DA. Data pre-processing methods were examined and it was found that using simple moving averages, returns or log-returns did not help in the out-of-sample performance of the ANN.

Economic fundamentals are important in exchange rates movements but their underlying relationships were not captured by the ANN. This was shown when they did not improve the networks predictive performance despite their use as inputs. This could be a result of the frequency of the economic fundamentals which are updated only quarterly.

Further studies may be made into the use of more frequently updated indicators of the foreign exchange market like technical indicators. The performance of the networks in this experiment pale in comparison to Yu's [7] results. Perhaps, more studies may be made into the effect of changing the cost function when training the ANN. Currently, there is no standardized procedure in data and ANN architecture selection, further research may be done to develop a system which will eliminate the trial and error process.

7.2 Money Management in a Mean-Reverting Environment

The second part of the thesis evaluates the performance of dollar cost averaging and value averaging in a mean reverting environment. The evaluation of the effectiveness of value averaging is very much dependent on the method of analysis. A prudent investor would include the cash flows in and out of his risk-free side fund into his analysis. In this case, the internal rate of return from value averaging would only be better than dollar cost averaging if the investor is able to set his target return higher than the market return. If the investor fails to set his target return for value averaging above the market, value averaging and dollar cost averaging will be unable to outperform the buy and hold strategy.

On the other hand, if the investor chooses to ignore his side fund as part of his cash flow stream, value averaging will undoubtedly be the superior strategy, consistently beating the market and dollar cost averaging except during periods of lower volatility.

These results are valid only locally around the model parameters which were used.

Further studies may be made into the characteristics of the stock market which contribute to the performance of the different strategies.

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APPENDIX A: RESULTS OF EXCHANGE RATE PAIRS

CHAPTER 3

Using Pure Time Delayed Rates as inputs for GBP/USD

No. of Lags	2	3	4	5	2	3	4	5
Model	RMSE				DA			
Scenario A								
n=4	0.10406	0.02380	0.07310	0.01489	0.49206	0.45238	0.49206	0.46032
n=5	0.17864	0.01805	0.02554	0.02094	0.47222	0.43651	0.45635	0.45635
n=6	0.02836	0.03648	0.04114	0.03461	0.47222	0.44841	0.46429	0.44444
n=7	0.16924	0.05244	0.05663	0.01242	0.44048	0.45635	0.49206	0.48016
n=8	0.21013	0.06235	0.03323	0.05766	0.45238	0.45635	0.46032	0.44444
n=9	0.23660	0.07941	0.03620	0.03829	0.46032	0.51587	0.44841	0.46032
n=10	0.20823	0.04117	0.09045	0.02393	0.50397	0.44841	0.51587	0.45635
Scenario B								
n=4	0.01277	0.01338	0.01165	0.01146	0.53175	0.47222	0.53175	0.48016
n=5	0.02939	0.01479	0.01103	0.01009	0.48810	0.51984	0.48016	0.50794
n=6	0.01527	0.02591	0.01601	0.02406	0.53175	0.49206	0.51587	0.48810
n=7	0.03416	0.02036	0.01039	0.01150	0.47619	0.47222	0.48413	0.50794
n=8	0.02222	0.01263	0.01086	0.01310	0.54365	0.53175	0.49603	0.44048
n=9	0.02857	0.01270	0.01467	0.01056	0.53571	0.42460	0.48413	0.51587
n=10	0.03503	0.01888	0.02007	0.01305	0.48016	0.50794	0.50000	0.48413

Using Pure Time Delayed Rates as inputs for USD/JPY

No. of Lags	2	3	4	5	2	3	4	5
Model	RMSE				DA			
Scenario A								
n=4	0.62167	0.62412	0.62327	0.62558	0.52381	0.49603	0.54762	0.48810
n=5	0.62408	0.62163	0.62610	0.62378	0.52778	0.53175	0.51984	0.51190
n=6	0.62396	0.62267	0.62730	0.62537	0.55556	0.55556	0.49603	0.49603
n=7	0.62412	0.62512	0.63053	0.62701	0.53968	0.53968	0.49603	0.50397
n=8	0.62588	0.62524	0.62752	0.63870	0.49206	0.48413	0.48810	0.48413
n=9	0.62260	0.62329	0.62811	0.62629	0.56349	0.54365	0.51587	0.49206
n=10	0.62440	0.62414	0.62837	0.62749	0.50794	0.51587	0.50000	0.50397
Scenario B								
n=4	0.58816	0.59311	0.59384	0.59791	0.54365	0.53175	0.47619	0.48413
n=5	0.59829	0.59422	0.59684	0.59367	0.53175	0.53968	0.51190	0.53968
n=6	0.59180	0.58909	0.59545	0.59591	0.52381	0.55556	0.58730	0.51190
n=7	0.59554	0.58542	0.59298	0.59450	0.53968	0.60714	0.47222	0.53571
n=8	0.59375	0.59609	0.59565	0.59607	0.53968	0.52381	0.51984	0.54762
n=9	0.59583	0.59072	0.59532	0.59857	0.53571	0.53571	0.58730	0.50000
n=10	0.59441	0.59205	0.59583	0.59745	0.53968	0.53968	0.54762	0.45238

Using Moving Averages as inputs for GBP/USD

Model	RMSE	DA (%)
Scenario A		
n=4	0.02564	0.48016
n=5	0.02639	0.48413
n=6	0.03214	0.59921
Scenario B		
n=4	0.01861	0.48016
n=5	0.01074	0.47222
n=6	0.01088	0.61508

Using Moving Averages as inputs for USD/JPY

Model	RMSE	DA (%)
Scenario A		
n=4	0.62724	0.44841
n=5	0.62866	0.48810
n=6	0.51227	0.71032
Scenario B		
n=4	0.60442	0.54762
n=5	0.59522	0.47619
n=6	0.59593	0.46825

Using Lagged 5-day Moving Averages as inputs for GBP/USD

No. of Lags	2	3	4	5	2	3	4	5
Model	RMSE				DA			
Scenario A								
n=4	0.01663	0.04341	0.03837	0.01985	0.46429	0.47222	0.44444	0.45635
n=5	0.05941	0.04231	0.03111	0.02992	0.46032	0.48413	0.48016	0.52381
n=6	0.08303	0.22518	0.01798	0.08132	0.46032	0.46825	0.45238	0.48016
n=7	0.06529	0.04689	0.01883	0.25496	0.42857	0.52381	0.49603	0.52778
n=8	0.21051	0.01709	0.04502	0.01264	0.48016	0.44444	0.47222	0.44048
n=9	0.28809	0.02783	0.07343	0.04862	0.46429	0.49206	0.50397	0.44841
n=10	0.38726	0.01955	0.01677	0.02121	0.45238	0.47222	0.47619	0.44444
Scenario B								
n=4	0.01965	0.01714	0.01188	0.01148	0.47222	0.44841	0.53175	0.43651
n=5	0.01323	0.01972	0.01698	0.01337	0.48413	0.50397	0.50397	0.45238
n=6	0.01686	0.01817	0.01472	0.01949	0.47222	0.46825	0.52381	0.47222
n=7	0.06611	0.01529	0.01797	0.02672	0.52778	0.49603	0.50397	0.47222
n=8	0.04249	0.01756	0.02800	0.01542	0.47222	0.45635	0.51190	0.48810
n=9	0.02546	0.01753	0.01447	0.01236	0.46429	0.44048	0.47619	0.55952
n=10	0.06541	0.04019	0.03157	0.01378	0.49603	0.49206	0.52778	0.48810

Using Lagged 5-day Moving Averages as inputs for USD/JPY

No. of Lags	2	3	4	5	2	3	4	5
Model	RMSE				DA			
Scenario A								
n=4	0.62930	0.62753	0.63086	0.62681	0.48810	0.51984	0.48016	0.48810
n=5	0.62794	0.62692	0.62425	0.63370	0.48016	0.47619	0.50000	0.49603
n=6	0.62523	0.62875	0.63259	0.63224	0.53968	0.48016	0.48810	0.48810
n=7	0.62589	0.62653	0.63505	0.63574	0.52778	0.55159	0.46032	0.49603
n=8	0.62472	0.63504	0.63369	0.62821	0.51984	0.51587	0.49603	0.47222
n=9	0.62455	0.62932	0.63082	0.63889	0.49206	0.51587	0.47619	0.44444
n=10	0.63206	0.62968	0.63465	0.63610	0.53175	0.52381	0.46032	0.48016
Scenario B								
n=4	0.59505	0.59248	0.59550	0.59492	0.54762	0.54365	0.53571	0.53968
n=5	0.59790	0.59221	0.59214	0.59869	0.53571	0.52778	0.52778	0.51587
n=6	0.59838	0.59464	0.60535	0.59446	0.53175	0.53968	0.53968	0.53571
n=7	0.59636	0.59442	0.59792	0.59783	0.53968	0.54762	0.52778	0.52381
n=8	0.59328	0.60205	0.59666	0.59545	0.54365	0.53175	0.54365	0.53968
n=9	0.59333	0.59546	0.59633	0.60786	0.53968	0.53571	0.53968	0.54762
n=10	0.59368	0.59545	0.59260	0.60341	0.53571	0.52778	0.53968	0.54762

Using Lagged 10-day Moving Averages as inputs for GBP/USD

No. of Lags	2	3	4	5	2	3	4	5
Model	RMSE				DA			
Scenario A								
n=4	0.01260	0.02886	0.01552	0.03916	0.47222	0.44841	0.47222	0.43254
n=5	0.09810	0.01973	0.02200	0.02602	0.48810	0.48413	0.45635	0.42857
n=6	0.07055	0.08712	0.04577	0.06827	0.45635	0.50397	0.45635	0.45635
n=7	0.12673	0.05177	0.01166	0.01327	0.49206	0.46429	0.46032	0.48413
n=8	0.22816	0.03756	0.20826	0.01776	0.47619	0.45635	0.51587	0.46429
n=9	0.06682	0.04546	0.03521	0.06003	0.46825	0.49206	0.47619	0.48413
n=10	0.30705	0.12766	0.06027	0.03123	0.45238	0.51984	0.49206	0.44841
Scenario B								
n=4	0.01690	0.01053	0.01539	0.01054	0.45238	0.51984	0.50000	0.51587
n=5	0.02316	0.01445	0.01131	0.01417	0.50397	0.44841	0.45238	0.47222
n=6	0.04589	0.01786	0.02587	0.01493	0.48413	0.45635	0.46429	0.48016
n=7	0.02594	0.01194	0.01069	0.02030	0.50794	0.48413	0.50794	0.48016
n=8	0.03453	0.01519	0.01566	0.01435	0.48810	0.51984	0.51190	0.47222
n=9	0.03639	0.02885	0.01405	0.01679	0.48413	0.53175	0.48016	0.49603
n=10	0.02462	0.02414	0.01969	0.01987	0.44048	0.48810	0.47619	0.46825

Using Lagged 10-day Moving Averages as inputs for USD/JPY

No. of Lags	2	3	4	5	2	3	4	5
Model	RMSE				DA			
Scenario A								
n=4	0.63178	0.63035	0.62899	0.62814	0.50397	0.50794	0.50000	0.48016
n=5	0.62832	0.63152	0.62692	0.63769	0.45635	0.48413	0.49603	0.48016
n=6	0.63123	0.63241	0.63034	0.62767	0.47619	0.48413	0.50000	0.47619
n=7	0.63049	0.63347	0.62945	0.63131	0.46825	0.48810	0.48016	0.45635
n=8	0.63063	0.62802	0.63328	0.63102	0.49206	0.47619	0.48016	0.49603
n=9	0.63179	0.62704	0.63465	0.64145	0.50000	0.50000	0.45238	0.47619
n=10	0.62828	0.63145	0.63035	0.63225	0.51190	0.47222	0.45238	0.46825
Scenario B								
n=4	0.59535	0.59295	0.59422	0.59753	0.53571	0.55159	0.54365	0.50397
n=5	0.59272	0.59529	0.59167	0.59314	0.53968	0.53968	0.53968	0.55159
n=6	0.59126	0.59750	0.59423	0.59571	0.53968	0.53968	0.52778	0.54762
n=7	0.59566	0.59860	0.60032	0.60466	0.54762	0.51984	0.51984	0.52778
n=8	0.60024	0.60001	0.59640	0.60134	0.54365	0.53968	0.52381	0.53968
n=9	0.60381	0.59907	0.60369	0.60497	0.51587	0.53571	0.52381	0.54365
n=10	0.60181	0.60826	0.60213	0.59986	0.53571	0.51190	0.53571	0.53968

Using Log-Returns to predict Log-Returns without Normalization GBP/USD

No. of Lags	2	3	4	5	2	3	4	5
Model	RMSE				DA (%)			
Scenario A								
n=4	0.00624	0.00633	0.00630	0.00621	0.71825	0.71825	0.71032	0.72222
n=5	0.00623	0.00650	0.00634	0.00625	0.72222	0.69841	0.71429	0.71825
n=6	0.00623	0.00640	0.00624	0.00619	0.71825	0.71429	0.71429	0.71825
n=7	0.00622	0.00642	0.00636	0.00625	0.71825	0.71825	0.71825	0.71825
n=8	0.00622	0.00643	0.00634	0.00623	0.71825	0.71429	0.71429	0.72222
n=9	0.00622	0.00636	0.00636	0.00634	0.71825	0.71429	0.71429	0.70635
n=10	0.00624	0.00653	0.00655	0.00629	0.71825	0.71032	0.70238	0.70635
Scenario B								
n=4	0.00489	0.00489	0.00491	0.00492	0.75000	0.75794	0.75397	0.73413
n=5	0.00489	0.00490	0.00490	0.00491	0.75397	0.75397	0.75794	0.74206
n=6	0.00489	0.00489	0.00491	0.00497	0.76190	0.75794	0.74603	0.74603
n=7	0.00489	0.00490	0.00500	0.00493	0.75397	0.75794	0.73413	0.74603
n=8	0.00489	0.00489	0.00490	0.00492	0.75794	0.75794	0.75794	0.74206
n=9	0.00490	0.00490	0.00492	0.00493	0.75000	0.75794	0.75397	0.75000
n=10	0.00490	0.00492	0.00494	0.00490	0.75000	0.76190	0.75000	0.75397

Using Log-Returns to predict Log-Returns without Normalization USD/JPY

No. of Lags	2	3	4	5	2	3	4	5
Model	RMSE				DA (%)			
Scenario A								
n=4	0.00558	0.00560	0.00559	0.00561	0.71429	0.71825	0.71429	0.71825
n=5	0.00558	0.00560	0.00563	0.00564	0.71825	0.71429	0.70635	0.70238
n=6	0.00558	0.00559	0.00557	0.00558	0.71825	0.72222	0.71032	0.71825
n=7	0.00558	0.00558	0.00564	0.00561	0.71429	0.71825	0.70238	0.70238
n=8	0.00558	0.00559	0.00561	0.00561	0.71032	0.71429	0.69841	0.71429
n=9	0.00560	0.00558	0.00564	0.00561	0.71429	0.72222	0.70238	0.71429
n=10	0.00558	0.00555	0.00559	0.00563	0.70635	0.71032	0.69841	0.71825
Scenario B								
n=4	0.00509	0.00509	0.00514	0.00515	0.71032	0.71429	0.71032	0.71429
n=5	0.00509	0.00510	0.00518	0.00512	0.71429	0.71429	0.71825	0.72222
n=6	0.00509	0.00510	0.00510	0.00515	0.71032	0.71429	0.71825	0.72222
n=7	0.00509	0.00509	0.00513	0.00514	0.71032	0.71429	0.72222	0.71825
n=8	0.00509	0.00509	0.00515	0.00515	0.71032	0.71032	0.71429	0.72619
n=9	0.00510	0.00509	0.00514	0.00515	0.71429	0.71032	0.72222	0.71429
n=10	0.00508	0.00508	0.00516	0.00516	0.71032	0.71032	0.71429	0.72619

Using Log-Returns to predict Log-Returns with Linear Normalization to (0,1) GBP/USD

No. of Lags	0	1	2	3	0	1	2	3
Model	RMSE				DA			
Scenario A								
n=4	0.00621	0.00653	0.00639	0.00658	0.72222	0.70635	0.72619	0.72222
n=5	0.00626	0.00647	0.00639	0.00696	0.71825	0.70635	0.71429	0.71429
n=6	0.00623	0.00647	0.00646	0.00635	0.71825	0.70635	0.71429	0.71032
n=7	0.00624	0.00655	0.00835	0.00628	0.71825	0.70635	0.72222	0.71825
n=8	0.00623	0.00642	0.00651	0.00630	0.71825	0.72222	0.71825	0.70635
n=9	0.00625	0.00653	0.00656	0.00681	0.71825	0.71032	0.71429	0.71429
n=10	0.00623	0.00655	0.00636	0.00636	0.71825	0.71429	0.71032	0.70635
Scenario B								
n=4	0.00490	0.00498	0.00493	0.00491	0.75397	0.75397	0.74206	0.75794
n=5	0.00490	0.00488	0.00491	0.00492	0.75000	0.75397	0.75000	0.74206
n=6	0.00490	0.00494	0.00489	0.00498	0.75000	0.75000	0.76984	0.74206
n=7	0.00490	0.00492	0.00491	0.00501	0.75000	0.75000	0.75794	0.74206
n=8	0.00489	0.00497	0.00494	0.00495	0.75397	0.75397	0.76984	0.74603
n=9	0.00490	0.00492	0.00493	0.00500	0.75000	0.75397	0.76190	0.76190
n=10	0.00490	0.00488	0.00491	0.00500	0.75000	0.76190	0.75794	0.76984

Using Log-Returns to predict Log-Returns with Linear Normalization to (0,1) USD/JPY

No. of Lags	2	3	4	5	2	3	4	5
Model	RMSE				DA (%)			
Scenario A								
n=4	0.00558	0.00560	0.00559	0.00561	0.71429	0.71825	0.71429	0.71825
n=5	0.00558	0.00560	0.00563	0.00564	0.71825	0.71429	0.70635	0.70238
n=6	0.00558	0.00559	0.00557	0.00558	0.71825	0.72222	0.71032	0.71825
n=7	0.00558	0.00558	0.00564	0.00561	0.71429	0.71825	0.70238	0.70238
n=8	0.00558	0.00559	0.00561	0.00561	0.71032	0.71429	0.69841	0.71429
n=9	0.00560	0.00558	0.00564	0.00561	0.71429	0.72222	0.70238	0.71429
n=10	0.00558	0.00555	0.00559	0.00563	0.70635	0.71032	0.69841	0.71825
Scenario B								
n=4	0.00509	0.00509	0.00514	0.00515	0.71032	0.71429	0.71032	0.71429
n=5	0.00509	0.00510	0.00518	0.00512	0.71429	0.71429	0.71825	0.72222
n=6	0.00509	0.00510	0.00510	0.00515	0.71032	0.71429	0.71825	0.72222
n=7	0.00509	0.00509	0.00513	0.00514	0.71032	0.71429	0.72222	0.71825
n=8	0.00509	0.00509	0.00515	0.00515	0.71032	0.71032	0.71429	0.72619
n=9	0.00510	0.00509	0.00514	0.00515	0.71429	0.71032	0.72222	0.71429
n=10	0.00508	0.00508	0.00516	0.00516	0.71032	0.71032	0.71429	0.72619

Returns Added Back on Price for GBP/USD

No. of Lags	0	1	2	3	0	1	2	3
Model	RMSE				DA			
Scenario A								
n=5	0.0110	0.0115	0.0112	0.0112	0.5516	0.5119	0.5040	0.5000
n=6	0.0109	0.0117	0.0112	0.0112	0.5437	0.5238	0.5238	0.4881
n=7	0.0109	0.0118	0.0113	0.0109	0.5595	0.4405	0.4802	0.4881
Scenario B								
n=5	0.0090	0.0090	0.0091	0.0091	0.5437	0.5278	0.4841	0.5159
n=6	0.0090	0.0090	0.0090	0.0090	0.5079	0.5119	0.5079	0.5238
n=7	0.0090	0.0091	0.0091	0.0092	0.5198	0.5079	0.5238	0.5397

Returns Added Back on Price for USD/JPY

No. of Lags	0	1	2	3	0	1	2	3
Model	RMSE				DA			
Scenario A								
n=5	0.6242	0.6270	0.6248	0.6271	0.5357	0.4365	0.4960	0.4802
n=6	0.6234	0.6269	0.6267	0.6257	0.5238	0.4921	0.5238	0.5238
n=7	0.6239	0.6187	0.6261	0.6246	0.5317	0.5516	0.5317	0.4921
Scenario B								
n=5	0.5907	0.5933	0.5959	0.5949	0.5397	0.5556	0.4802	0.5119
n=6	0.5904	0.5901	0.5951	0.5965	0.5238	0.5198	0.5040	0.4960
n=7	0.5903	0.5923	0.5949	0.5961	0.5000	0.5278	0.5119	0.5159

Using Returns and Price as Input for GBP/USD

No. of Lags	0	1	2	3	0	1	2	3
Model	RMSE				DA			
Scenario A								
n=5	0.0904	0.0164	0.0160	0.0281	0.4444	0.4246	0.4921	0.4524
n=6	0.0688	0.0218	0.0372	0.0127	0.4643	0.4167	0.4643	0.4286
n=7	0.0379	0.0185	0.0140	0.0180	0.4722	0.4683	0.4722	0.4365
Scenario B								
n=5	0.0323	0.0112	0.0135	0.0097	0.4841	0.4722	0.4802	0.4365
n=6	0.0126	0.0098	0.0188	0.0159	0.4524	0.4325	0.4444	0.4563
n=7	0.0184	0.0109	0.0156	0.0194	0.5595	0.4603	0.4722	0.4603

Using Returns and Price as Input for USD/JPY

No. of Lags	0	1	2	3	0	1	2	3
Model	RMSE				DA			
Scenario A								
n=5	0.6288	0.6203	0.6280	0.6246	0.5357	0.5357	0.4603	0.5516
n=6	0.6247	0.6224	0.6231	0.6286	0.5278	0.5595	0.5159	0.4365
n=7	0.6294	0.6209	0.6247	0.6249	0.5476	0.5278	0.5040	0.5159
Scenario B								
n=5	0.5916	0.5909	0.5972	0.5981	0.4683	0.4841	0.5000	0.5000
n=6	0.5915	0.5881	0.5951	0.6002	0.4960	0.4722	0.4722	0.4722
n=7	0.5914	0.5934	0.5953	0.5953	0.5437	0.5119	0.5278	0.5040

<u>**CHAPTER 4**</u>

Using Interpolated Future Price as Inputs for the GBP/USD

No. of Lags	2	3	4	NO FC	2	3	4	NO FC
Model	RMSE				DA (%)			
Scenario A								
n=4	0.0316	0.0092	0.0339	0.0176	0.6310	0.7262	0.6230	0.4167
n=5	0.0189	0.0345	0.0239	0.0282	0.6230	0.6508	0.6032	0.4762
n=6	0.0321	0.0170	0.0154	0.0405	0.5992	0.6310	0.7222	0.5000
Scenario B								
n=4	0.0077	0.0075	0.0080	0.0140	0.7183	0.7540	0.6984	0.4325
n=5	0.0138	0.0083	0.0098	0.0133	0.6230	0.6825	0.6270	0.4841
n=6	0.0109	0.0145	0.0075	0.0152	0.6151	0.6667	0.7103	0.4881

Using Interpolated Future Price as Inputs for the USD/JPY

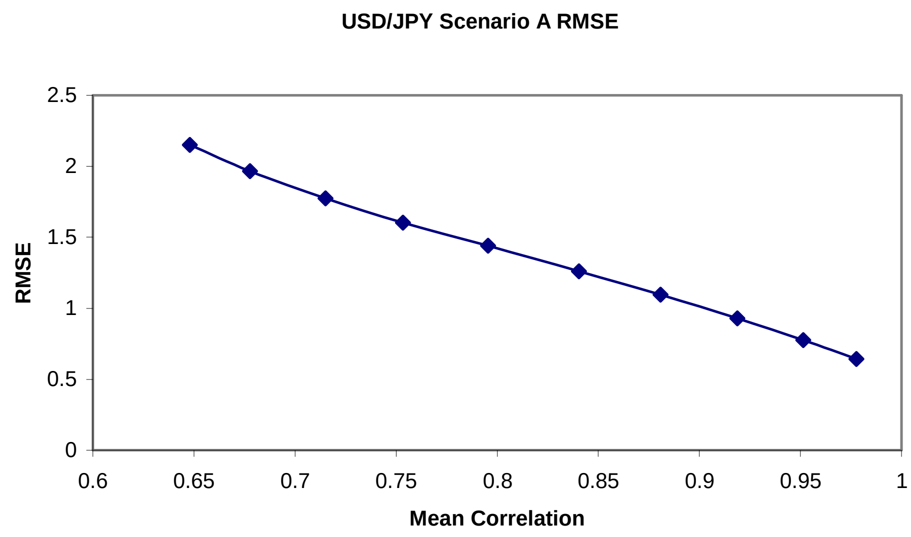
No. of Lags	2	3	4	NO FC	2	3	4	NO FC
Model	RMSE				DA (%)			
Scenario A								
n=4	0.5134	0.5150	0.5123	0.6336	0.7024	0.6786	0.7143	0.4841
n=5	0.5120	0.5122	0.5129	0.6341	0.7143	0.7222	0.7183	0.4603
n=6	0.5123	0.5107	0.5108	0.6307	0.7103	0.7222	0.7143	0.4921
Scenario B								
n=4	0.4822	0.4923	0.4901	0.6036	0.7579	0.7421	0.7460	0.5079
n=5	0.4854	0.4893	0.4938	0.5944	0.7540	0.7460	0.7341	0.4960
n=6	0.4865	0.4906	0.4875	0.5959	0.7540	0.7500	0.7460	0.4683

Using Constant Future Price as Inputs for the GBP/USD

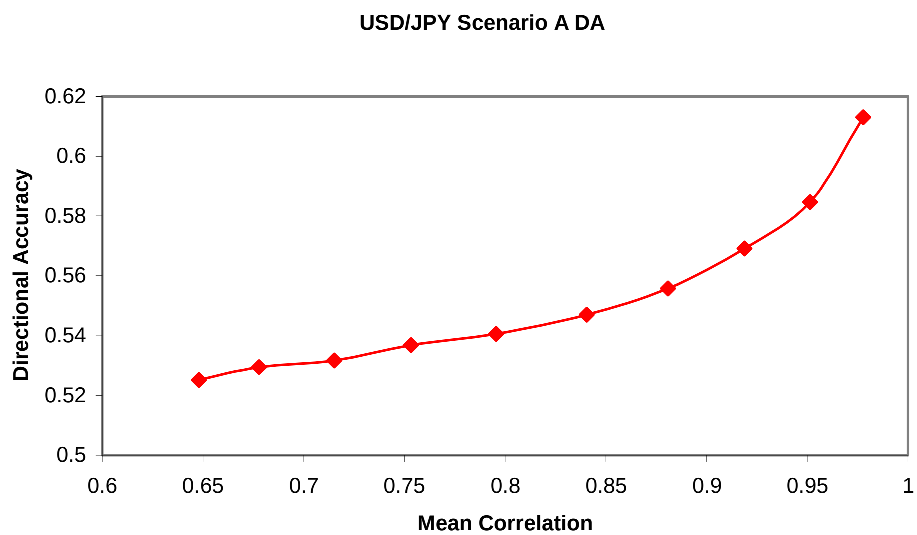
No. of Lags	2	3	4	NO FC	2	3	4	NO FC
Model	RMSE				DA (%)			
Scenario A								
n=4	0.0786	0.0594	0.0324	0.0176	0.6071	0.6468	0.6032	0.4167
n=5	0.0281	0.0322	0.0269	0.0282	0.5992	0.5952	0.6032	0.4762
n=6	0.0405	0.0286	0.0241	0.0405	0.5992	0.6032	0.5992	0.5000
Scenario B								
n=4	0.0080	0.0109	0.0091	0.0140	0.6905	0.5913	0.6429	0.4325
n=5	0.0082	0.0078	0.0084	0.0133	0.7143	0.6667	0.6548	0.4841
n=6	0.0136	0.0074	0.0081	0.0152	0.6786	0.7421	0.7222	0.4881

Using Constant Future Price as Inputs for the USD/JPY

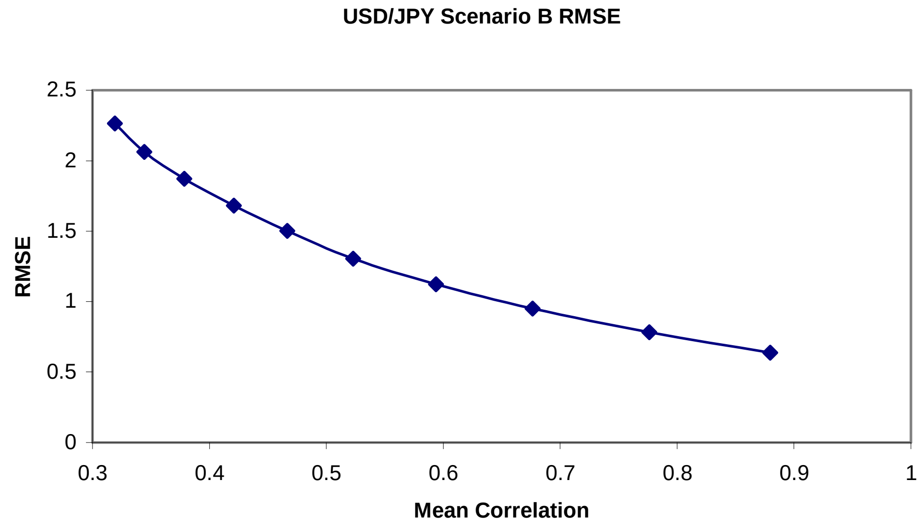
No. of Lags	2	3	4	NO FC	2	3	4	NO FC
Model	RMSE				DA (%)			
Scenario A								
n=4	0.4684	0.4722	0.4699	0.6336	0.7143	0.7143	0.7183	0.4841
n=5	0.4721	0.4716	0.4700	0.6341	0.7222	0.7222	0.7024	0.4603
n=6	0.4722	0.4722	0.4680	0.6307	0.7183	0.7183	0.6984	0.4921
Scenario B								
n=4	0.4423	0.4469	0.4421	0.6036	0.7381	0.7222	0.7381	0.5079
n=5	0.4404	0.4468	0.4477	0.5944	0.7421	0.7381	0.7222	0.4960
n=6	0.4412	0.4416	0.4450	0.5959	0.7540	0.7540	0.7381	0.4683



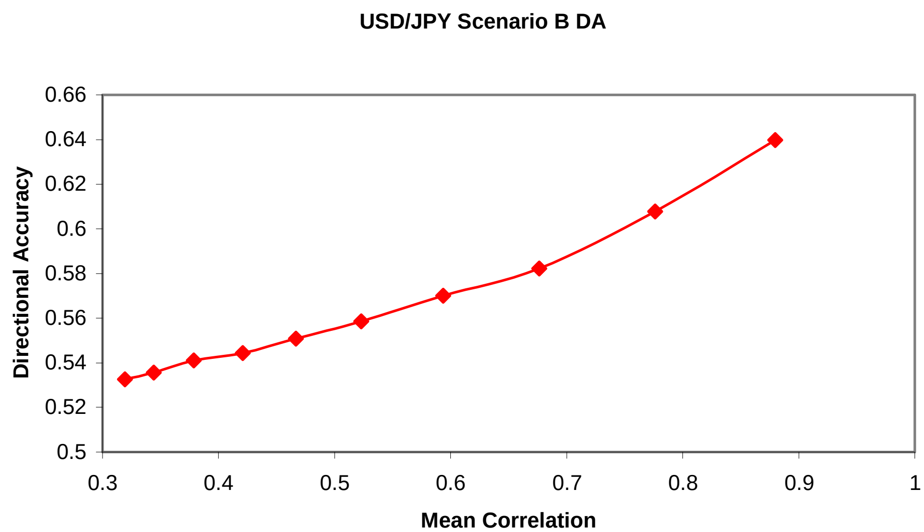
Noisy Future Prices against RMSE for USD/JPY Scenario A



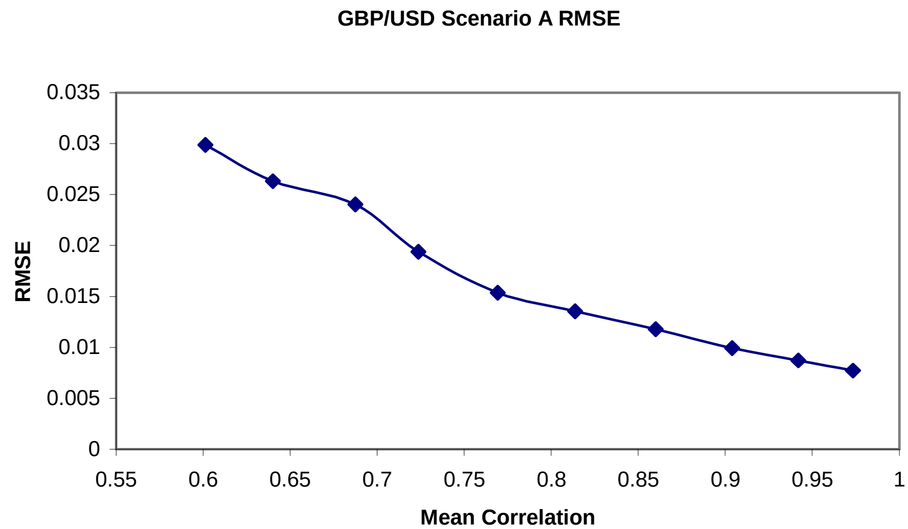
Noisy Future Prices against DA for USD/JPY Scenario A



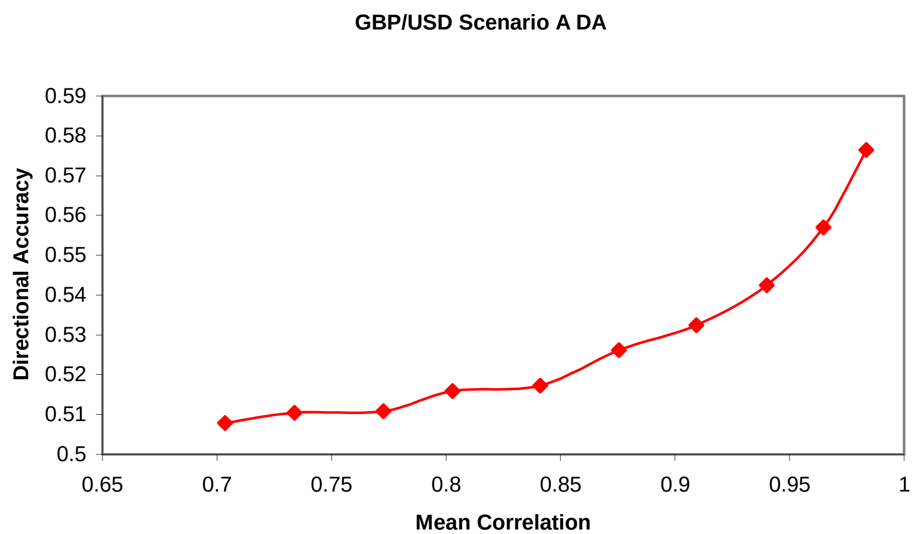
Noisy Future Prices against RMSE for USD/JPY Scenario B



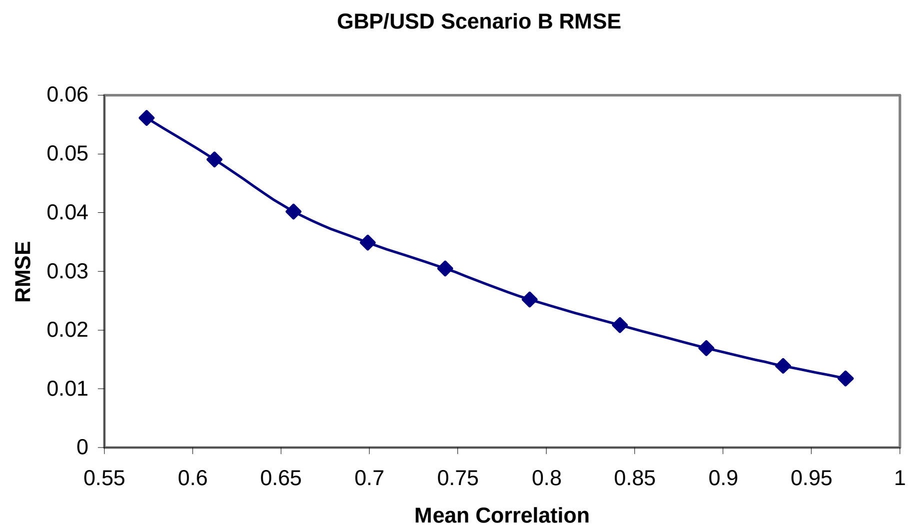
Noisy Future Prices against DA for USD/JPY Scenario B



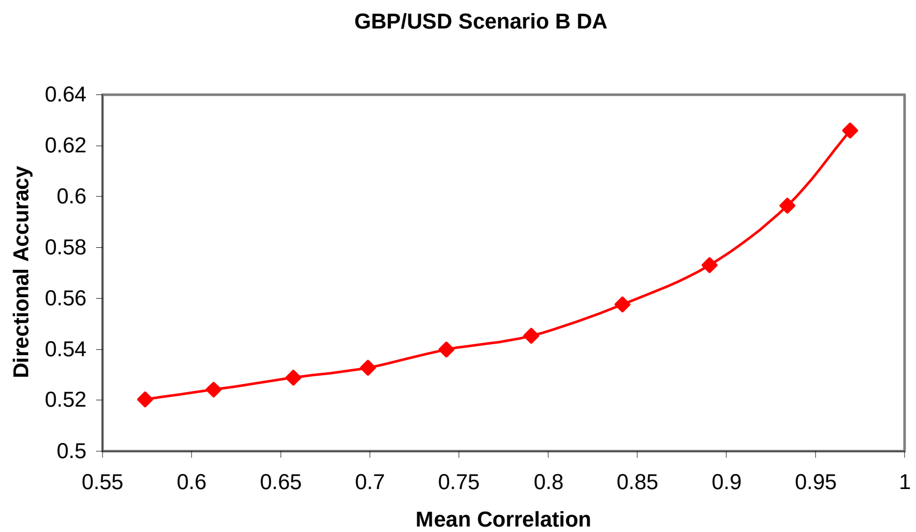
Noisy Future Prices against RMSE for GBP/USD Scenario A



Noisy Future Prices against DA for GBP/USD Scenario A



Noisy Future Prices against RMSE for GBP/USD Scenario B



Noisy Future Prices against DA for GBP/USD Scenario B

Using Individual Interest Rates or Their Difference as Inputs for GBP/USD

Forecast Model	Individual RMSE	Difference RMSE	Individual DA	Difference DA
Scenario A				
n=4	0.6457	0.6260	0.5079	0.5317
n=5	1.5312	0.6236	0.4881	0.5040
n=6	0.8178	0.6271	0.4921	0.5159
Scenario B				
n=4	0.6487	0.5965	0.4365	0.4921
n=5	0.6294	0.5979	0.4603	0.4683
n=6	0.6246	0.6058	0.4365	0.4841

Using Individual GDP or Their Difference as Inputs for USD/JPY

Forecast Model	Individual RMSE	Difference RMSE	Individual DA	Difference DA
Scenario A				
n=4	0.6850	0.6483	0.5595	0.5635
n=5	0.6331	0.6466	0.5040	0.4921
n=6	0.6699	0.6448	0.5595	0.4325
Scenario B				
n=4	0.6411	0.5983	0.4722	0.4444
n=5	0.5935	0.5953	0.5397	0.5159
n=6	0.5905	0.5906	0.5159	0.5476

Using Individual CPI or Their Difference as Inputs for USD/JPY

Forecast Model	Individual RMSE	Difference RMSE	Individual DA	Difference DA
Scenario A				
n=4	1.0959	0.6329	0.4405	0.5675
n=5	0.6676	0.6387	0.4484	0.4484
n=6	0.6424	0.6556	0.4444	0.4683
Scenario B				
n=4	0.6039	0.5912	0.5000	0.5278
n=5	0.8152	0.5966	0.4603	0.4841
n=6	0.5914	0.6011	0.5754	0.5357

Using Individual Trade Balance or Their Difference as Inputs for USD/JPY

Forecast Model	Individual RMSE	Difference RMSE	Individual DA	Difference DA
Scenario A				
n=4	0.6517	0.6476	0.5357	0.5595
n=5	4.5463	0.6381	0.4643	0.5675
n=6	0.6347	0.6226	0.5873	0.5397
Scenario B				
n=4	0.6399	0.5984	0.5397	0.4524
n=5	0.7663	0.5940	0.5397	0.4762
n=6	7.7220	0.6423	0.5317	0.4405

Using Fundamental Data as inputs for USD/JPY

Forecast Model	Without RMSE	With RMSE	Without DA	With DA
Scenario A				
n=4	1.4512	0.5793	0.4405	0.6667
n=5	0.6561	0.5269	0.5437	0.6627
n=6	0.6458	0.6420	0.5516	0.6151
Scenario B				
n=4	1.7338	0.4853	0.5397	0.7302
n=5	0.8054	0.5487	0.5397	0.6786
n=6	3.3403	2.1168	0.4603	0.4643