DESIGN AND ANALYSIS
OF
SIMULTANEOUS CONTROL CHARTING SCHEMES

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A THESIS SUBMITTED FOR THE DEGREE OF
MASTER OF SCIENCE
DEPARTMENT OF STATISTICS AND APPLIED PROBABILITY
NATIONAL UNIVERSITY OF SINGAPORE

2007

Acknowledgments

It is with the greatest respect and veneration that I express my sincere thanks to my supervisor, **Associate Professor Gan Fah Fatt**. He was present at all times whenever assistance was needed. And without his valuable advices and guidance extended to me obtaining this qualification would be not easy. Not forgetting the number of hours he patiently spent in amending my draft.

I would wish to thank National University of Singapore (NUS) for awarding me Research Scholarship which financially supports me throughout my studies in NUS. I also express my thanks to **Dr D R Weerasekera**, the Head of the Department of Statistics, University of Colombo, Sri Lanka and all the staff member, since they undertake lots of academic and administrative work, during the period when I am reading masters degree at NUS.

I express my heartiest thanks to **Yvone** and **Zhang Rong**, staff members of Department of Statistics and Applied Probability (DSAP), NUS for the help given to me in numerous ways during my stay in Singapore. Also I wish to express my sincere thanks to all other staff members of DSAP for helping me during my studies.

It is obligatory to convey my sincere thanks to all my friends, especially to **Chok Kang** and **Tsung Fei** for helping me and encouraging me throughout the course of this study. And also I have to thank **Neluka** (Amba Research, Sri Lanka), **Sanjeewa** (Ceylinco Shriram, Sri Lanka), **Darmshri** (Central Bank of Sri Lanka) for their valuable friendship and for helping me to come to NUS.

At last but not least, I wish to express my indebtedness and heartfelt gratitude to my parents, brother, sister and especially to my wife **Eisha** and daughters **Manuthi** and **Senuthi**, for their inspiration, understanding and sacrifices made throughout my studies. Without Eisha's comments, thoughts, understanding about my studies and my busy academic life, obtaining this qualification would be a dream that cannot be realized.

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Rushan A B Abeygunawardana

July 2007

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Summary

Most of the optimal design procedures for the cumulative sum (CUSUM) and exponentially weighted moving average (EWMA) charts require a shift to be specified for which the chart is optimal in detecting. Such a chart would perform well at the intended shift but it will be increasingly insensitive if the shift moves further away from the intended shift. The specification of a shift may not be practical because in reality, the shift that occurs is more likely to be random. Thus it does not make good sense to design a chart this way. We propose simultaneous schemes which do not require any specification of a shift in advance. These simultaneous schemes comprise a few CUSUM or EWMA charts including a Shewhart chart that run simultaneously. We conduct a comprehensive study of the simultaneous schemes and develop a simple design procedure for determining the chart parameters. Such a scheme is found to be sensitive in detecting a range of shifts and has better overall run length performances as compared to individual charts and other simultaneous schemes.

KEY WORDS: Average run length; Multiple charting procedures; Optimal design procedure; Shewhart chart; Statistical process control

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Part 1

Simultaneous Cumulative Sum Charting Schemes

1.1   Introduction

The Shewhart chart and the cumulative sum (CUSUM) charts are widely used for monitoring the process mean of a quality characteristic. Suppose that a quality characteristic, which is denoted by x is normally distributed with mean μ and standard deviation σ_0, $N(\mu, \sigma_0^2)$. Without loss of generality assume that $\mu = 0$. Consider taking a sample of size n from the process at each sample number t. The successive CUSUM of sample means $\bar{x}_t$ plotted against the sample number t in an upper-sided chart and a lower-sided chart can be described by

$$S_t = \max\{0, S_{t-1} + (\bar{x}_t - k)\}, \tag{1}$$

and

$$T_t = \min\{0, T_{t-1} + (\bar{x}_t + k)\}, \tag{2}$$

respectively for $t = 1, 2, ...$, where $k > 0$, $S_0 = u$, $0 < u < h$ and $T_0 = v$, $-h < v < 0$. The upper-sided chart is intended to detect an upward shift and it issues a signal at the first t for which $S_t > h$. Similarly, the lower-sided chart is intended to detect a downward shift. It issues a signal at the first t for which $T_t < -h$. A two-sided chart is obtained by running the both lower- and upper-sided charts simultaneously. The reference value k can be chosen such that it is optimal in detecting a particular shift of $\Delta \sigma_0 / \sqrt{n}$. Moustakides (1986) showed that the optimal k is given as $\Delta \frac{\sigma_0}{2\sqrt{n}}$.

A design procedure of a chart involves determining the chart parameters for a given in-control average run length (ARL). The ARL is the average number of samples taken until a signal is issued. The steady-state ARL refers to the ARL of a chart evaluated from some point in time after the monitoring statistic has reached a steady state. Here we will study the various schemes using the steady state ARL because we want to focus on assignable causes not related to a start-up situation. Most of the design procedures for the CUSUM charts (see Lucas, 1976, Crowder, 1989, Lucas and Saccucci, 1990, Gan, 1991 and Montgomery, 2001) require

a shift to be specified for which the chart is optimal in detecting. This amounts to testing a simple null hypothesis against a simple alternative hypothesis. In reality it is difficult to anticipate the size of a shift. The general practice now is to decide a shift that is deemed the most important to be detected and then implement a chart that is optimal in detecting this shift. Such a chart performs well at the intended shift, but not at other shifts. Westgard et al. (1977) suggested the use of a combined Shewhart-CUSUM scheme to improve its ability in detecting large shifts. Lucas (1982) investigated this scheme further and concluded that such a scheme is sensitive in detecting both small and large shifts as compared to a single chart.

Sparks (2000) also looked into this problem and suggested two alternative approaches. One approach was to use an adaptive CUSUM statistic that continuously adjusts the parameters h and k by one-step-ahead forecast for signaling a deviation from its target value. This approach is complicated because it requires updating of the charting parameters sequentially. Sparks concluded that the adaptive CUSUM scheme is expected to perform well provided that the mean can be estimated accurately using a one-step-ahead forecast. However, there is no satisfactory solution at the moment for one-step-ahead forecast and Sparks also did not provide any solution to this problem. The other approach was to use a simultaneous scheme which consists of $2, 3$ or 4 CUSUM charts. He suggested a simultaneous scheme with 2 CUSUM charts if we are interested in detecting shifts in the range $0.75 \leq \Delta \leq 1.25$, 3 CUSUM charts for $0.5 \leq \Delta \leq 2.0$ and 4 CUSUM charts for $0.5 \leq \Delta \leq 4.0$. Although Sparks investigated the run length performances of his schemes, he did not provide any design procedure for these schemes. Furthermore, he did not provide any justification for the number of CUSUM charts used. Neelakantan (2002) independently had proposed a 'super' CUSUM scheme consisting of nine CUSUM charts and a Shewhart chart with the intention of providing protection over a wide range of shifts. No justification was given for the number of charts used but a simple

design procedure was provided.

We propose simultaneous CUSUM schemes which do not require any specification of the shift in advance and have good performance over a range of shifts. A simultaneous CUSUM scheme comprises a few CUSUM charts including a Shewhart chart that run simultaneously. An advantage of a simultaneous scheme is that it provides protection over a range of shifts effectively. The run length of a simultaneous scheme refers to the minimum run length of any of the charts. The run length of a simultaneous scheme remains mathematically intractable, so we use simulation to study its run length distribution.

In this thesis, we conduct a comprehensive study of the simultaneous CUSUM schemes. In Section 1.2, we investigate if there is a suitable number of charts to be used in a simultaneous scheme. In Section 1.3, we do a comprehensive run length study of the various simultaneous schemes because the run length comparison given in Sparks (2000) is limited. In Section 1.4, we provide a simple design procedure for determining the chart parameters of a simultaneous CUSUM scheme. The implementation of a simultaneous scheme is illustrated in Section 1.5 with a conclusion given in Section 1.6.

1.2 Simultaneous CUSUM Control Charting Schemes

Combined Shewhart-CUSUM scheme (Lucas, 1982), simultaneous CUSUM schemes (Sparks, 2000) and 'super' CUSUM scheme (Neelakantan, 2002) are three main developments in the area of simultaneous CUSUM charting schemes. However, none of them provided any justification for the number of charts used.

In order to investigate the effect of adding more CUSUM charts in a simultaneous scheme, we first consider a CUSUM chart with $k = 0.2$ that has an in-control ARL of 1000. We then add a Shewhart chart with an in-control ARL of 1000 to the CUSUM chart to obtain a combined Shewhart-CUSUM scheme. The ARL of a scheme is computed using simulations such that the standard error of each

simulated ARL is not more than 1% of the simulated ARL. The ARL of this combined scheme is found to be 510. Then a second CUSUM chart with $k = 0.8$ that has an in-control ARL of 1000 is added to obtain a 2–CUSUM (2 CUSUM charts and a Shewhart chart) scheme. Such a scheme is found to have an ARL of about 384. This procedure is continued with more CUSUM charts added. A plot of the ARL of the simultaneous scheme against the number of charts in the scheme is displayed in Figure 1. The order of the charts added is given by $k = 0.2$, Shewhart, $k = 0.8, 0.4, 1.0, 0.6, 2.0, 0.3, 0.9, 0.5, 1.5, 0.7$ and 3.0. Figure 1 shows that the ARL of the simultaneous CUSUM scheme did not change appreciably beyond 5 charts. This suggests that using more than 5 charts may not be necessary. Figure 1 also reveals that a scheme with $3, 4$ or 5 charts including the Shewhart chart would be sufficient. As long as the simultaneous CUSUM scheme contains the CUSUM charts for small shift ($k = 0.2$), moderate shift ($k = 0.8$) and the Shewhart chart for large shift, the ARL curve shown in Figure 1 did not change appreciably when the other CUSUM charts were added in different orders. Similar results were obtained for schemes with minimum k of 0.5 and 0.75 and for the one-sided schemes.

What remains to be investigated would be the performance of these simultaneous CUSUM schemes. This is done in the next section. For each of the 2–CUSUM, 3–CUSUM and 4–CUSUM schemes, we propose 3 simultaneous schemes and these are designed to detect shift in the range $[0.4, \infty)$, $[1.0, \infty)$ and $[1.5, \infty)$ which correspond to small to very large, medium to very large and large to very large shifts respectively. Each of these schemes includes a Shewhart chart. These charts were chosen such that they are optimal in detecting selected shifts in a range specified. The reference values of the CUSUM charts used in our simultaneous schemes are given in the Table 1. We consider both one-sided and two-sided simultaneous schemes. Sparks (2000) uses $k = 0.375, 0.5$ and 0.75 for his 3-CUSUM scheme while Neelakantan (2002) uses $k = 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 1.0$ and 1.5 for her 'super'

scheme.

The ARL relationships between the simultaneous schemes and the individual CUSUM charts are given in the Figures 2–4 and 6–8 for two-sided and one-sided schemes respectively. These figures provide a better understanding of the various schemes. Although we found that the 'super' scheme contains more CUSUM charts than sufficient, we have included the ARL curves for the 'super' scheme because it is close to the limiting case. Similarly, the ARL curves for Sparks' schemes are included for a better understanding of the schemes.

1.3   Comparison of the Average Run Length Profiles

Control charts are usually compared using the ARL. In order to do a comprehensive comparison of simultaneous schemes, we consider schemes with $1, 2, 3$ or 4 CUSUM charts together with a Shewhart chart, Sparks' 3–CUSUM scheme and the 'super' scheme. In addition, 5 individual CUSUM charts optimal in detecting $\Delta = 0.4, 1.0, 1.5, 2.0$ and 2.5 and the Shewhart chart are also included for comparison. The programs for simulation were written in SAS and each ARL was simulated such that the standard error of each simulated ARL is not more than 1% of the simulated ARL. The in-control ARL was fixed at 370. Shifts of $\Delta = 0.2, 0.3, 0.4, 0.5, 0.6, 0.8, 1.0, 1.5, 2.0, 2.5$ and 4.0 are considered. Tables 2 and 3 contain the ARL profiles of the 2-sided and 1-sided charts and schemes respectively.

As expected, an individual CUSUM chart that is optimal in detecting a particular shift has the smallest ARL at that shift. As the shift moves away from this intended shift, the sensitivity of the individual CUSUM charts decreases. When a Shewhart chart is added to a CUSUM chart to form a combined CUSUM-Shewhart scheme, the scheme becomes more sensitive in detecting large shifts but it becomes less sensitive in detecting small shifts. As more CUSUM charts are added to a scheme, the scheme becomes more sensitive in detecting the corresponding intended shifts. In general, a simultaneous scheme offers a better protection over a range of

shifts. Although the individual chart can be made to be more sensitive in detecting very small shifts like $\Delta = 0.2$ and 0.3, Hawkins and Olwell (1998) pointed out that aiming at too-small a shift is potentially harmful because a certain amount of natural variability will always exist. These too-small shifts are generally due to common cause of variation and a process that is operating with only this type of variation is said to be in statistical control (Montgomery, 2005).

Consider the 4–CUSUM schemes. The scheme intended for the range $[0.4, \infty)$ is the most sensitive in detecting $\Delta \leq 1.0$ and slightly less sensitive in detecting $\Delta > 1.0$. The scheme intended for medium shift and beyond, that is $[1.0, \infty)$ is sensitive in detecting medium shift and beyond. For small shifts, this scheme is less sensitive as expected because we do not want such a scheme to be sensitive to small changes in the mean. For the scheme intended for large shift and beyond, that is $[1.5, \infty)$, the sensitivity of this scheme improves further for large shifts and become less sensitive for small and medium shifts. Similar observations can be made for 2–CUSUM and 3–CUSUM schemes and for one-sided schemes.

Sparks' 3–CUSUM scheme is intended for Δ in the range $[0.75, 1.5]$ and as such it is the least sensitive in detecting large shifts among all the simultaneous schemes. For small and medium shifts, its run length behavior is in between the simultaneous schemes intended for detecting Δ in the range $[0.4, \infty)$ and $[1.0, \infty)$. Sparks' 3–CUSUM scheme is not sensitive in detecting large shifts. The 'super' scheme and the 4–CUSUM scheme intended for detecting $[0.4, \infty)$ have very similar run length performances. This further shows that using 4 CUSUM charts in a scheme is sufficient. For a simpler scheme, quality control engineers can use a 2–CUSUM or 3–CUSUM scheme. A comparison of the adaptive CUSUM scheme with our simultaneous schemes can be found in Table A1. It is found that the simultaneous schemes are slightly more sensitive than adaptive CUSUM schemes.

1.4 Designs of Simultaneous CUSUM Schemes

Procedures for designing control charts are usually based on the ARL. We provide design procedures for both one- and two-sided CUSUM schemes with $1, 2, 3$ or 4 CUSUM charts intended for detecting shift in the range $[0.4, \infty)$, $[1.0, \infty)$ and $[1.5, \infty)$. A quality control engineer will have to decide on one of the ranges for his process. This decision is much easier than specifying a single shift to be detected as in the case of designing an individual CUSUM chart.

The following steps are recommended for the design of a one-sided or two-sided simultaneous scheme:

Step 1. Select the smallest acceptable in-control ARL of the simultaneous CUSUM scheme.

Step 2. Find the corresponding ARL of the individual component charts in the scheme based on the ARL specified in Step 1.

Step 3. Determine the chart limits of the component CUSUM charts and the Shewhart chart.

In Step 1, the choice of the ARL depends on the rate of production, frequency of sampling, size of the sample, cost etc. In order to simplify Step 2, we have determined the relationships between the ARL of the individual component charts and the ARL of the simultaneous schemes. The ARL's of the simultaneous schemes were simulated by considering the ARL of individual component charts to be 50, 100, 200, 300, 370, 400, 500, 600, 700, 800, 900, 1000, 1500, 2000, 2500, 3000, 3500 and 4000. The programs were written in SAS and each ARL was simulated using 100,000 simulations. The standard error of each simulated ARL is not more than 1% of the simulated ARL. The relationships are displayed in Figures 2–4 for two-sided schemes and Figures 6–8 for one-sided schemes. These figures can be used for determining the ARL of the individual component charts easily.

In order to simplify Step 3, the chart parameter h of a CUSUM chart was determined for $k = 0.2, 0.3, 0.375, 0.4, 0.5, 0.6, 0.7, 0.75, 0.8, 1.0, 1.25$ and 1.5 with respect to ARL of 50, 100, 150, 200, 250, 300, 350, 400, 450, 500, 600, 700, 800, 900, 1000, 1500, 2000, 2500, 3000 and 4000. These are plotted in Figure 5 for two-sided charts and Figure 9 for one-sided charts. Given a particular k and an ARL, the chart limit h can be read from Figure 5 or 9 easily. The chart limits for the one- and two-sided Shewhart chart can be obtained easily by using Figure 10. Figures 5 and 9 are developed for a process with $N(0,1)$ as the in-control distribution. However, if the underlying distribution is $N(\mu_0, \sigma_0^2)$, then the chart parameters of a CUSUM chart can be determined as $k^\star = \mu_0 + \frac{\sigma_0}{\sqrt{n}}k$ and $h^\star = \frac{\sigma_0}{\sqrt{n}}h$ where k and h are the values obtained from the figures.

1.5   Example

In this section we use 56 standard normal variates (Gan, 1991) to demonstrate the design procedures of a 4–CUSUM scheme intended for detecting shift in the range $[0.4, \infty)$ with an in-control ARL of 370. The k values of the scheme can be determined as $0.2, 0.3, 0.5$ and 1.0 from Table 1. The 3 steps for designing this scheme are as follows;

Step 1. The desired in-control ARL of the 4–CUSUM scheme is 370.

Step 2. Using Figure 2 (Scheme D), the ARL of the individual chart is determined to be 1115 using the ARL of 370 as specified in Step 1.

Step 3. Using Figure 5, the chart limits of the 4 CUSUM charts are determined as $11.97, 8.89, 5.88$, and 3.07 for $k = 0.2, 0.3, 0.5$ and 1.0 respectively. The chart limit of the Shewhart chart $h = 3.32$ can be obtained from Figure 10.

To simulate a shift of $\Delta = 3.0$ which is a large shift; we added 3.0 to the last data value in the data set. The 4–CUSUM scheme, the individual CUSUM chart with $k = 0.5$, the individual Shewhart chart and Sparks' 3–CUSUM scheme for

this data set are displayed in Figures 11–14 respectively. These figures show that only the 4–CUSUM scheme and the individual Shewhart chart signal immediately. This example demonstrates the weakness of the Sparks' scheme and the individual CUSUM charts in detecting a large shift.

1.6 Conclusions

Most of the optimal design procedures for the CUSUM chart require the specification of a shift in advance for which the chart is optimal in detecting. Such a chart would perform well at the intended shift but it will be be increasingly insensitive if the shift moves further away from the intended shift. In reality, the shift that occurs is more likely to be random, so it may not make good sense to design a chart that is optimal in detecting a particular shift only. Here, we develop simultaneous CUSUM schemes in order to provide protection over a range of shifts. Instead of using the 'super' scheme with 9 CUSUM charts (see Neelakantan 2002), our study shows that a 4–CUSUM scheme would be sufficient. One could also consider a simpler 2–CUSUM or a 3–CUSUM scheme for implementation. The component charts are chosen such that they are optimal in detecting shifts in a specified range . We have developed schemes for detecting shifts in the following ranges: $[0.4, \infty)$, $[1.0, \infty)$ and $[1.5, \infty)$. Although Sparks' had considered simultaneous schemes, he did not provide any procedure for designing his scheme. We have provided simple design procedures for designing simultaneous schemes. These procedures can also be used to design Sparks' 3–CUSUM scheme. A comprehensive comparison of simultaneous schemes shows that a simultaneous scheme indeed provides a better protection over a specified range of shift. Sparks' scheme lacks the sensitivity in detecting large shift but this is critical in many applications. A comparison between the adaptive scheme and the Sparks' 3–CUSUM scheme in Table 3 of Sparks (2000) show that these two schemes' performances are similar. Sparks concluded that the adaptive CUSUM scheme is expected to perform well provided that the mean can be es-

timated accurately using a one-step-ahead forecast but no satisfactory estimation procedure is available at the moment. Furthermore, the design and implementation of the adaptive scheme are much more complicated than the simultaneous CUSUM schemes (see Sparks, 2000). An advantage of the simultaneous CUSUM scheme over the adaptive scheme is that quality control engineers who are currently using CUSUM charts can migrate easily to simultaneous CUSUM schemes with a lower learning curve.

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Figures and Tables

for the

Simultaneous Cumulative Sum Charting Schemes

Average Run Length
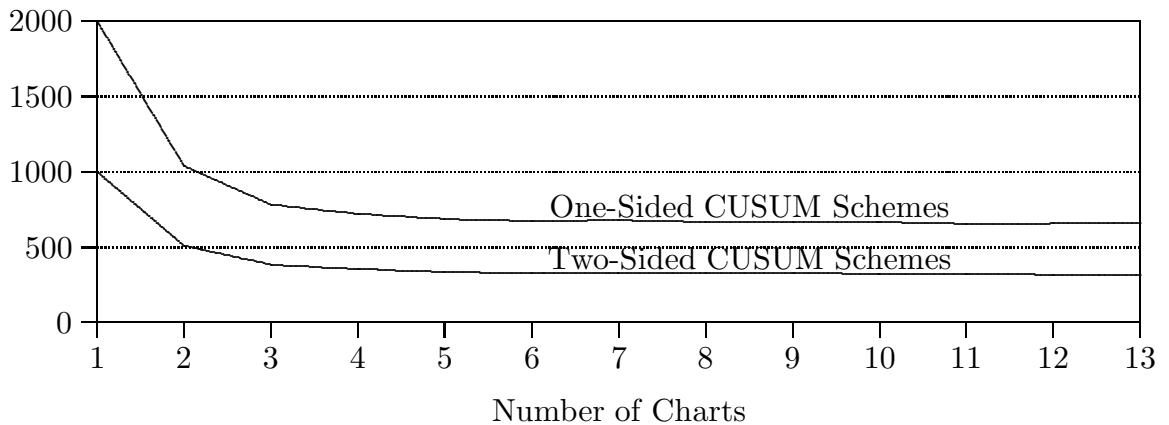
Figure 1. Steady-State In-Control ARL of Simultaneous CUSUM Schemes Designed for Detecting a Shift in the Range $[0.4,\infty)$ With Respect to the Number of Charts in a Scheme

ARL of Individual CUSUM
Scheme

Figure 2. Relationships between In-Control ARL of Two-Sided Individual CUSUM Charts and In-Control ARL of Simultaneous Schemes Designed for Detecting a Shift in the Range $[0.4, \infty)$

A : One CUSUM ($k = 0.2$) and Shewhart chart

B : Two CUSUMs ($k = 0.2, 0.5$) and Shewhart chart

C : Three CUSUMs ($k = 0.2, 0.3, 0.5$) and Shewhart chart

D : Four CUSUMs ($k = 0.2, 0.3, 0.5, 1.0$) and Shewhart chart

E : Nine CUSUMs ($k = 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 1.0, 1.5$) and Shewhart chart

ARL of Individual CUSUM   Scheme

ARL of Simultaneous CUSUM Scheme

Figure 3. Relationships between In-Control ARL of Two-Sided Individual CUSUM
Charts and In-Control ARL of Simultaneous Schemes Designed for De-
tecting a Shift in the Range $[1.0, \infty)$

A : One CUSUM ($k = 0.5$) and Shewhart chart

B : Two CUSUMs ($k = 0.5, 0.75$) and Shewhart chart

C : Three CUSUMs ($k = 0.5, 0.75, 1.0$) and Shewhart chart

D : Four CUSUMs ($k = 0.5, 0.75, 1.0, 1.25$) and Shewhart chart

E : Nine CUSUMs ($k = 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 1.0, 1.5$) and Shewhart chart

F : Sparks' three CUSUMs ($k = 0.375, 0.5, 0.75$) scheme

ARL of Individual CUSUM   Scheme

Figure 4. Relationships between In-Control ARL of Two-Sided Individual CUSUM
Charts and In-Control ARL of Simultaneous Schemes Designed for De-
tecting a Shift in the Range $[1.5, \infty)$

A : One CUSUM ($k = 0.75$) and Shewhart chart

B : Two CUSUMs ($k = 0.75, 1.0$) and Shewhart chart

C : Three CUSUMs ($k = 0.75, 1.0, 1.25$) and Shewhart chart

D : Four CUSUMs ($k = 0.75, 1.0, 1.25, 1.5$) and Shewhart chart

E : Nine CUSUMs ($k = 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 1.0, 1.5$) and Shewhart chart

Figure 5.   Combination of (k, h) Values for the ARL of the Two-Sided CUSUM Chart

ARL of Individual CUSUM   Scheme
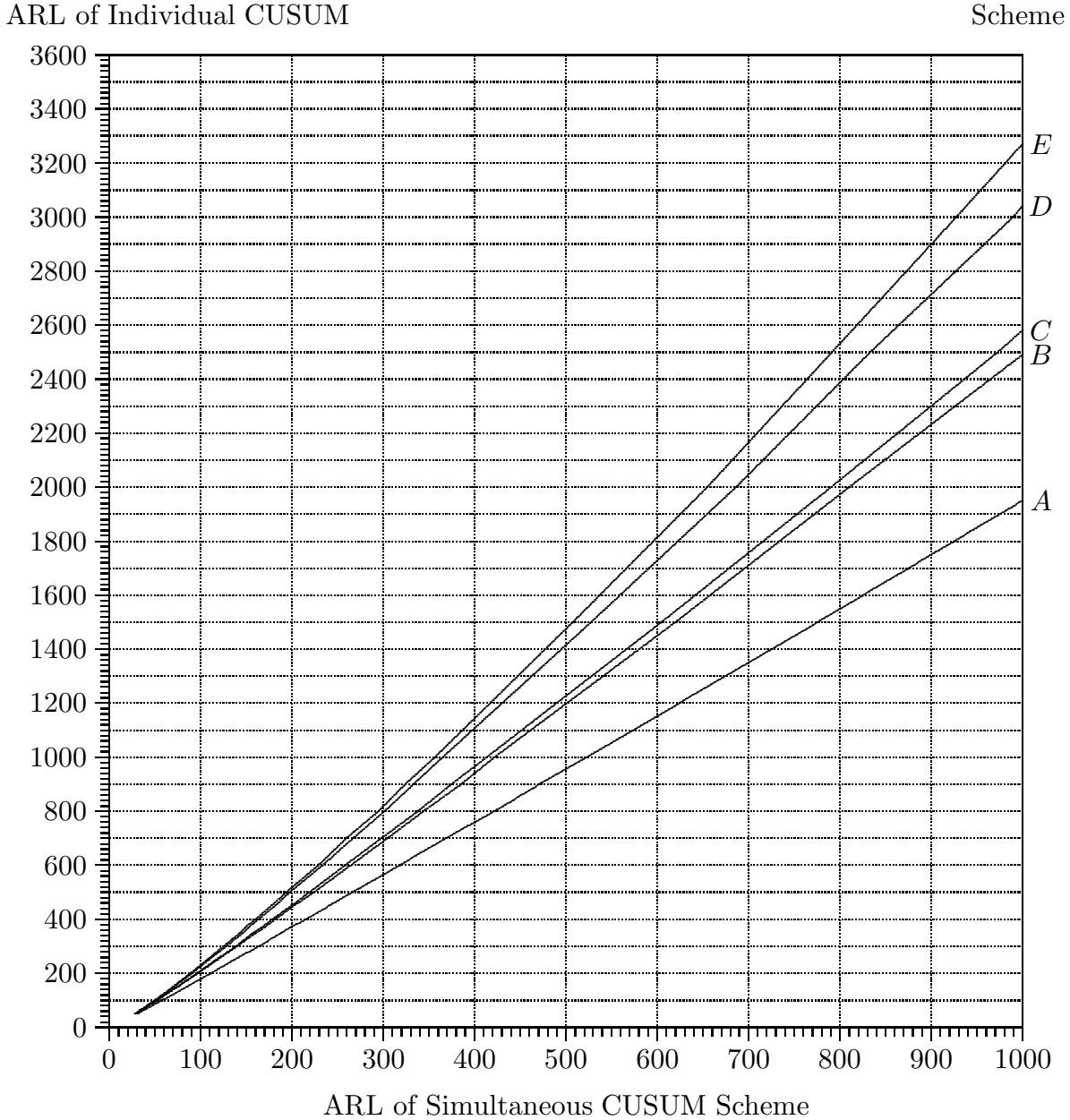
Figure 6. Relationships between In-Control ARL of One-Sided Individual CUSUM
Charts and In-Control ARL of Simultaneous Schemes Designed for De-
tecting a Shift in the Range $[0.4, \infty)$

A : One CUSUM ($k = 0.2$) and Shewhart chart

B : Two CUSUMs ($k = 0.2, 0.5$) and Shewhart chart

C : Three CUSUMs ($k = 0.2, 0.3, 0.5$) and Shewhart chart

D : Four CUSUMs ($k = 0.2, 0.3, 0.5, 1.0$) and Shewhart chart

E : Nine CUSUMs ($k = 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 1.0, 1.5$) and Shewhart chart

ARL of Individual CUSUM   Scheme
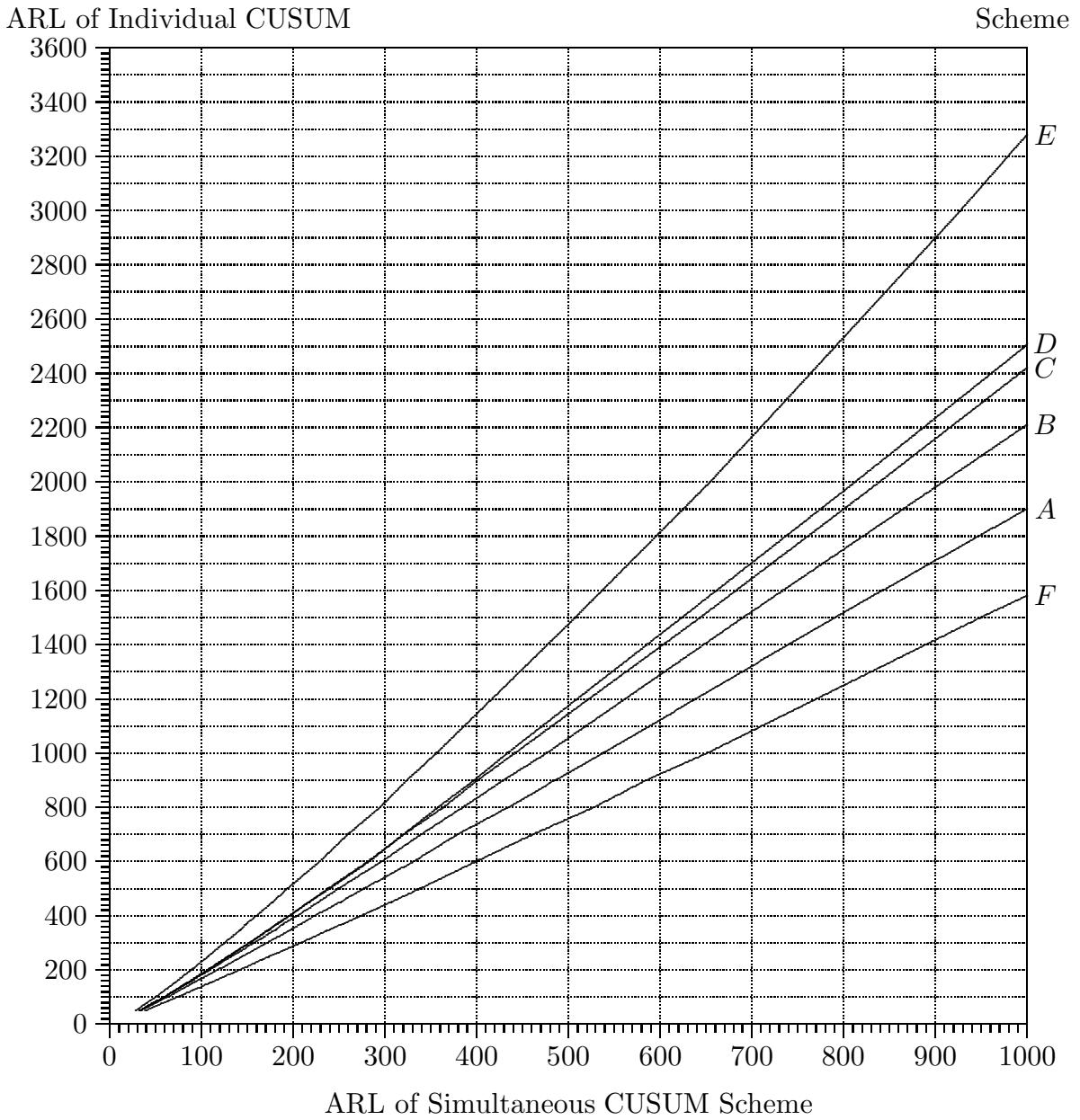
Figure 7. Relationships between In-Control ARL of One-Sided Individual CUSUM
Charts And In-Control ARL of Simultaneous Schemes Designed for De-
tecting a Shift in the Range $[1.0, \infty)$

A : One CUSUM ($k = 0.5$) and Shewhart chart

B : Two CUSUMs ($k = 0.5, 0.75$) and Shewhart chart

C : Three CUSUMs ($k = 0.5, 0.75, 1.0$) and Shewhart chart

D : Four CUSUMs ($k = 0.5, 0.75, 1.0, 1.25$) and Shewhart chart

E : Nine CUSUMs ($k = 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 1.0, 1.5$) and Shewhart chart

F : Sparks' three CUSUMs ($k = 0.375, 0.5, 0.75$) scheme

ARL of Individual CUSUM   Scheme
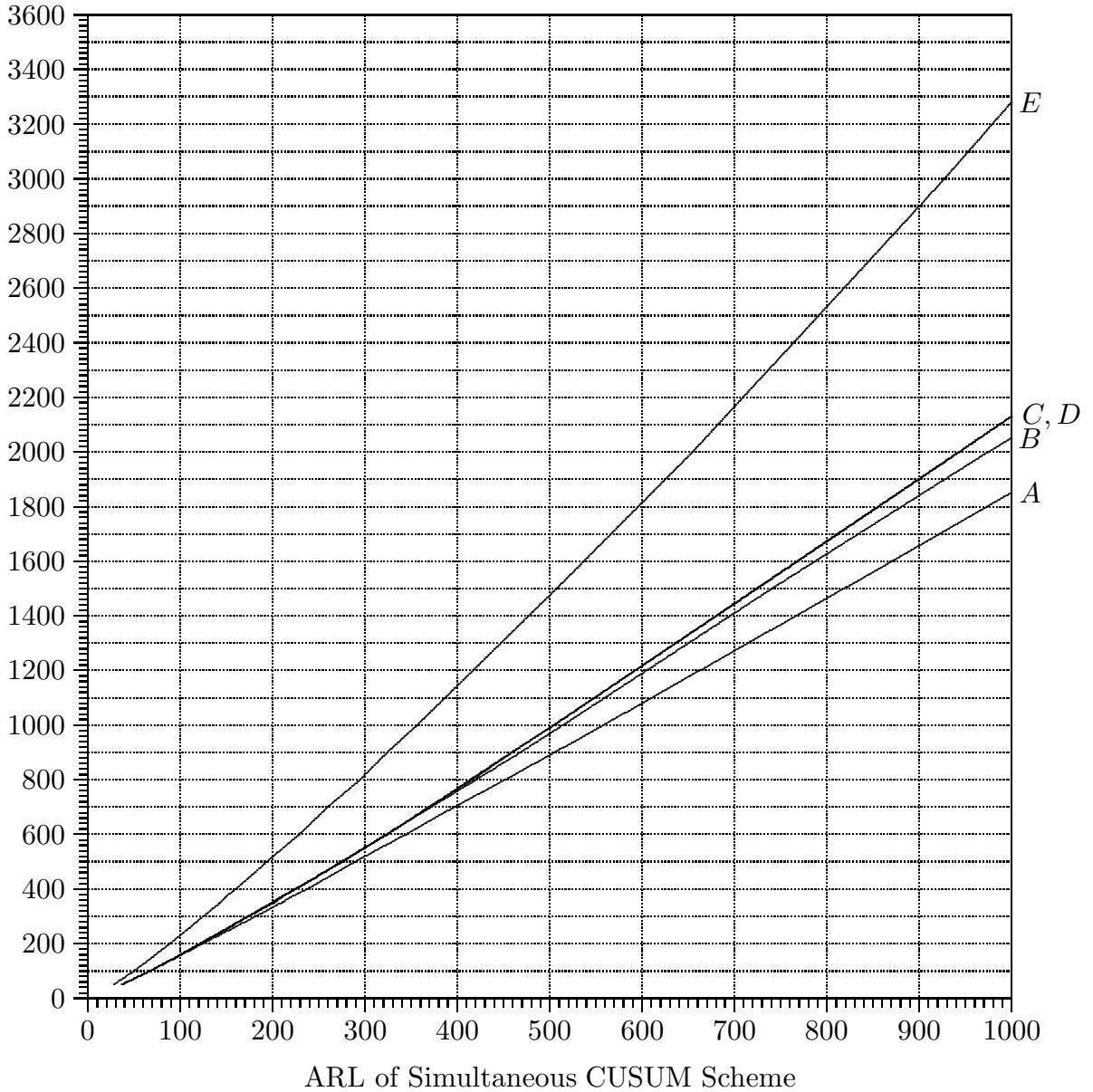
Figure 8. Relationships between In-Control ARL of One-Sided Individual CUSUM
          Charts And In-Control ARL of Simultaneous Schemes Designed for De-
          tecting a Shift in the Range $[1.5, \infty)$

A : One CUSUM ($k = 0.75$) and Shewhart chart

B : Two CUSUMs ($k = 0.75, 1.0$) and Shewhart chart

C : Three CUSUMs ($k = 0.75, 1.0, 1.25$) and Shewhart chart

D : Four CUSUMs ($k = 0.75, 1.0, 1.25, 1.5$) and Shewhart chart

E : Nine CUSUMs ($k = 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 1.0, 1.5$) and Shewhart chart
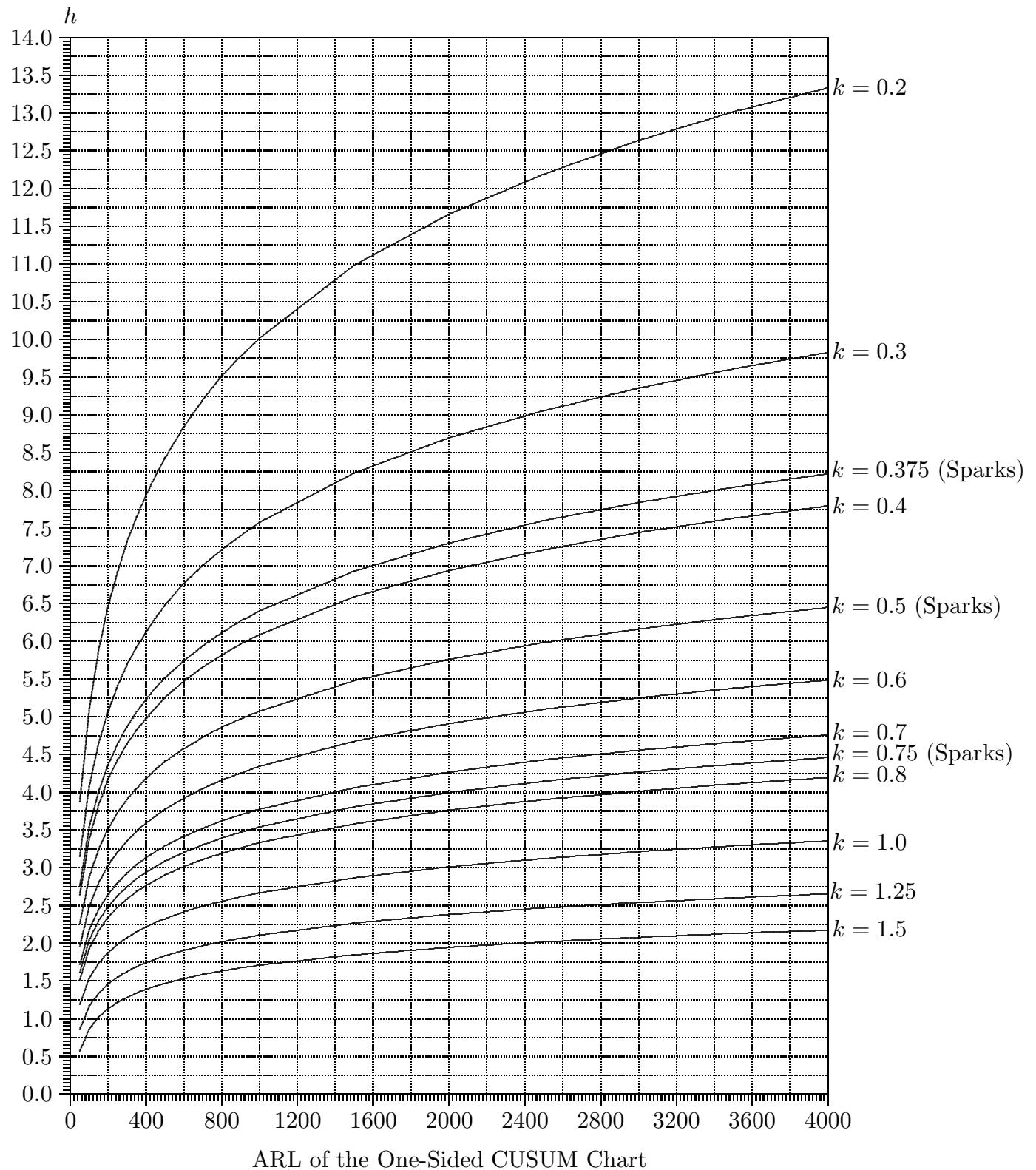
Figure 9. Combination of (k, h) Values for the ARL of the One-Sided CUSUM
         Chart
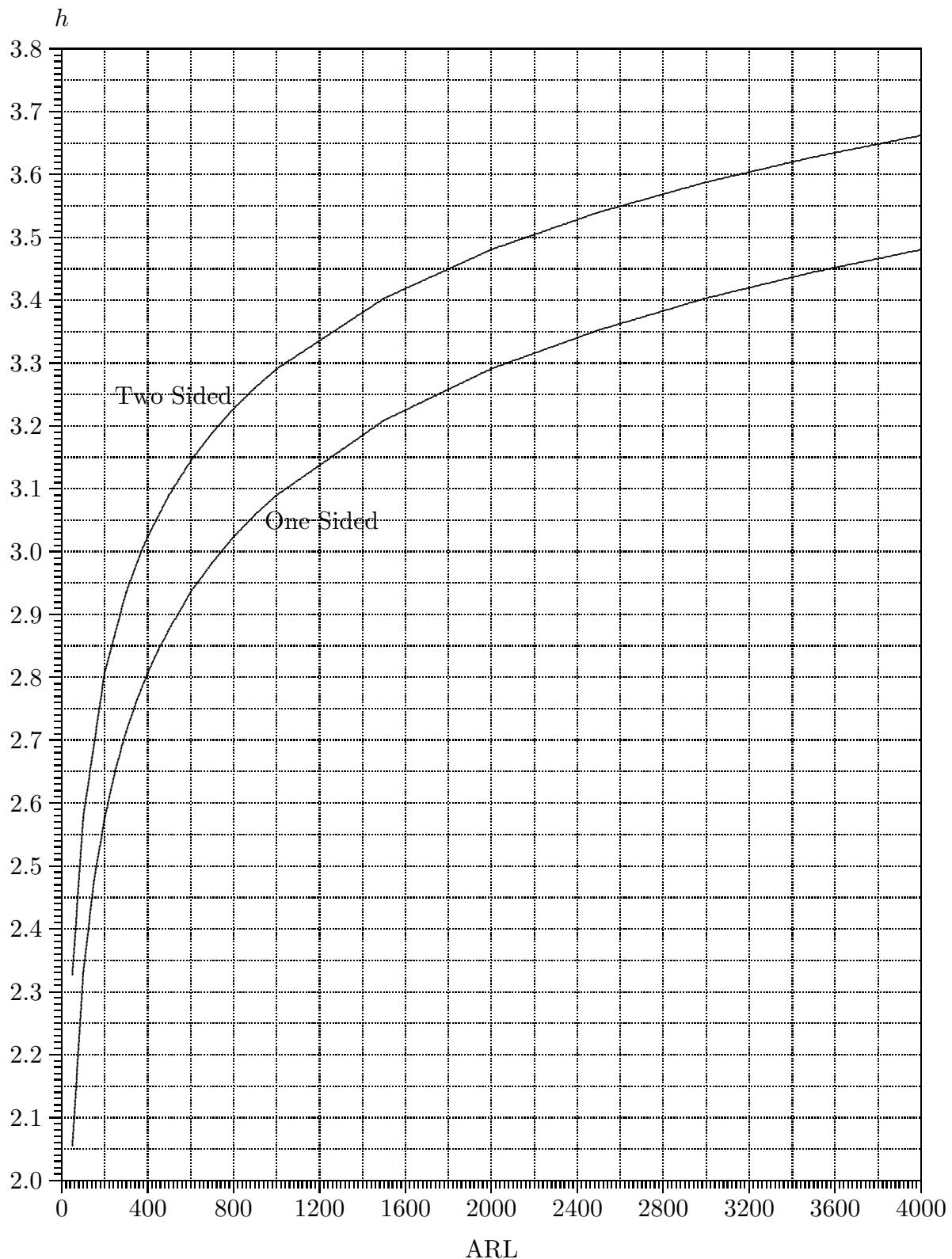
Figure 10. Chart Limits for the ARL of the Shewhart Chart
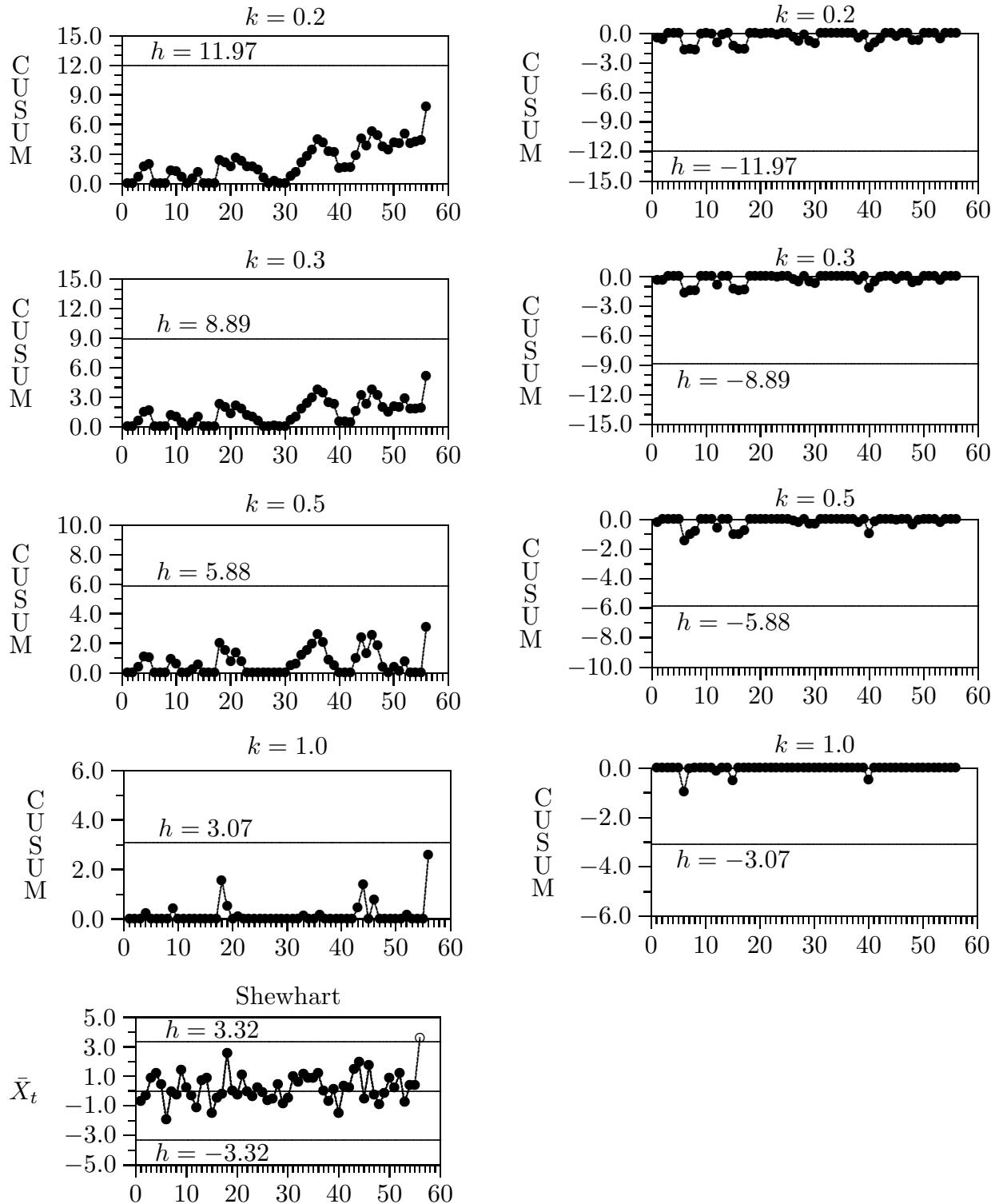
Figure 11. A 4-CUSUM Simultaneous Scheme for a Data Set with 3.0 Added to
the Last Data Value
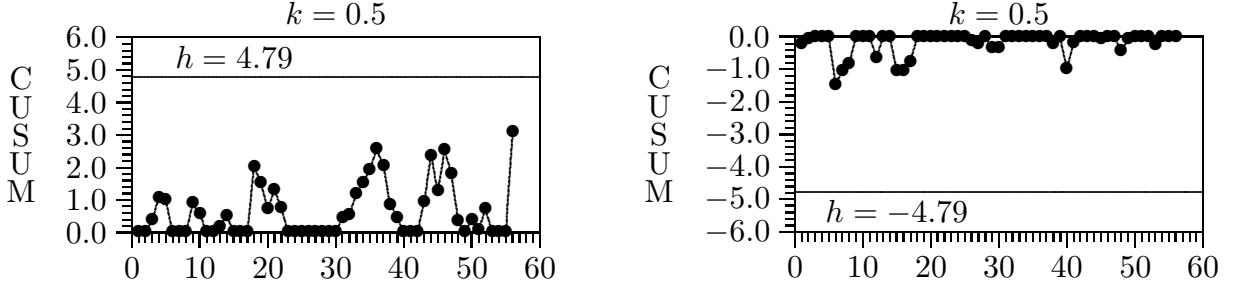
Figure 12. Two-Sided Individual CUSUM Chart for a Data Set with 3.0 Added to the Last Data Value
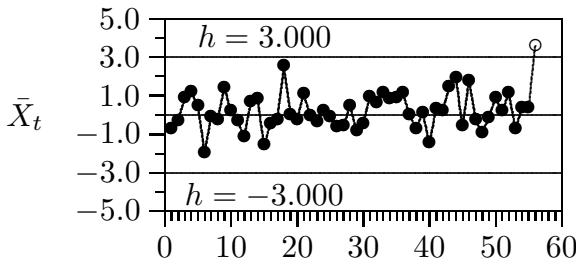
Figure 13. The Shewhart Chart for a Data Set with 3.0 Added to the Last Data Value
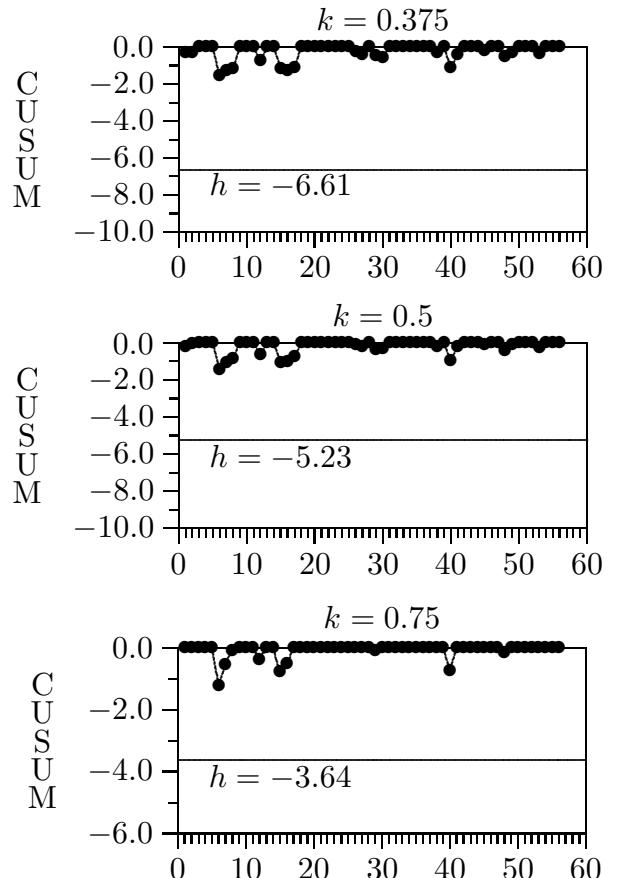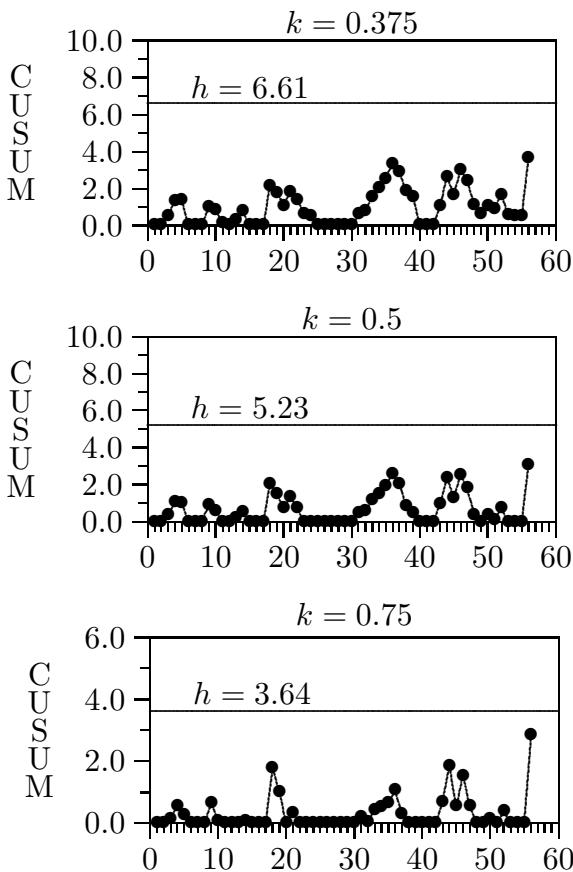
Figure 14. Sparks' 3-CUSUM Scheme for a Data Set with 3.0 Added to the Last Data Value

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Table 1. Reference Values (k) for which the CUSUM Charts are Optimal in Detecting, in Simultaneous CUSUM Scheme

Range of Shift	Combined CUSUM and Shewhart	Two CUSUMs and Shewhart	Three CUSUMs and Shewhart	Four CUSUMs and Shewhart
Small to very Large $[0.4 \leq \Delta < \infty)$	0.2	0.2, 0.5	0.2, 0.3, 0.5	0.2, 0.3, 0.5, 1.0
Medium to very Large $[1.0 \leq \Delta < \infty)$	0.5	0.5, 0.75	0.5, 0.75, 1.0	0.5, 0.75, 1.0, 1.25
Large to very Large $[1.5 \leq \Delta < \infty)$	0.75	0.75, 1.0	0.75, 1.0, 1.25	0.75, 1.0, 1.25, 1.5

Table 2. Steady State ARL Profiles of the Two-Sided CUSUM Charts, Shewhart Chart and Simultaneous CUSUM Schemes

	Single CUSUM Chart					Shewhart Chart	Combined CUSUM and Shewhart			Two CUSUMs and Shewhart			Three CUSUMs and Shewhart			Four CUSUMs and Shewhart			Sparks' Scheme	Nine CUSUMs and Shewhart*
Intended shift	0.4	1.0	1.5	2.0	2.5	∞	[0.4,∞)	[1.0,∞)	[1.5,∞)	[0.4,∞)	[1.0,∞)	[1.5,∞)	[0.4,∞)	[1.0,∞)	[1.5,∞)	[0.4,∞)	[1.0,∞)	[1.5,∞)	[0.75,1.5]	[0.4,∞)
k																3.320	3.264	3.214		
													3.274	3.254	3.215	1.0	1.25	1.5		
										3.267	3.230	3.208	0.5	1.0	1.25	0.5	1.0	1.25		0.75
							3.198	3.189	3.177	0.5	0.75	1.0	0.3	0.75	1.0	0.3	0.75	1.0		0.5
	0.2	0.5	0.75	1.0	1.25	3.000	0.2	0.5	0.75	0.2	0.5	0.75	0.2	0.5	0.75	0.2	0.5	0.75		0.375
h																3.320	3.264	3.214		
													3.274	3.254	3.215	3.066	2.343	1.853		
										3.267	3.230	3.208	5.714	2.948	2.274	5.879	2.965	2.273		3.636
							3.198	3.189	3.177	5.679	3.859	2.863	8.623	3.917	2.877	8.890	3.940	2.876		5.226
	9.412	4.794	3.345	2.519	1.987	3.000	10.940	5.415	3.737	11.500	5.554	3.807	11.580	5.642	3.825	11.970	5.676	3.824	3.636	6.606
Δ												ARL								
0.00	370	370	370	370	370	370	370	370	370	370	370	370	370	370	370	370	370	370	370	370
0.20	100	163	206	238	265	310	116	187	228	119	190	231	120	193	230	123	194	230	148	124
0.30	54.5	89.5	125	159	191	255	63.1	106	146	64.5	108	148	64.4	110	148	66.2	111	147	79.3	66.4
0.40	35.0	53.2	77.2	105	132	200	40.4	62.1	91.2	41.0	63.2	93.0	40.7	64.3	93.2	41.9	65.0	92.5	47.0	41.9
0.50	25.4	33.8	49.1	68.6	90.8	156	29.1	39.1	57.7	29.0	39.8	58.7	28.8	40.3	58.9	29.5	40.7	58.7	30.7	29.5
0.60	19.8	23.5	32.9	46.5	63.2	120	22.6	26.6	38.1	22.1	27.0	38.8	21.9	27.4	39.0	22.3	27.7	38.8	21.8	22.2
0.80	13.6	13.6	17.0	23.4	32.0	72.1	15.4	15.0	19.2	14.3	15.2	19.5	14.2	15.3	19.5	14.3	15.4	19.4	13.3	14.3
1.00	10.4	9.21	10.5	13.4	17.7	44.0	11.6	10.1	11.5	10.2	10.1	11.6	10.1	10.2	11.6	10.1	10.2	11.6	9.33	10.1
1.50	6.59	5.07	4.93	5.29	6.18	15.0	6.93	5.40	5.23	5.57	5.21	5.24	5.59	5.19	5.25	5.38	5.20	5.24	5.06	5.32
2.00	4.88	3.54	3.21	3.16	3.30	6.32	4.59	3.58	3.30	3.71	3.36	3.25	3.73	3.31	3.24	3.43	3.30	3.25	3.37	3.40
2.50	3.91	2.76	2.42	2.27	2.25	3.25	3.12	2.57	2.36	2.68	2.42	2.31	2.69	2.35	2.29	2.42	2.34	2.29	2.56	2.41
4.00	2.55	1.80	1.47	1.29	1.21	1.19	1.26	1.24	1.22	1.27	1.23	1.22	1.27	1.23	1.21	1.25	1.23	1.22	1.57	1.25

* Chart parameters of nine CUSUMs are (0.2,12.070), (0.3,8.953), (0.4,7.133), (0.5 ,5.921), (0.6,5.0421), (0.7,4.379), (0.8,3.858), (1.0,3.089), (1.5,1.996) and chart limit for the Shewhart chart is 3.335

Table 3. Steady State ARL Profiles of the One-Sided CUSUM Charts, Shewhart Chart and Simultaneous CUSUM Schemes

	Single CUSUM Chart					Shewhart Chart	Combined CUSUM and Shewhart			Two CUSUMs and Shewhart			Three CUSUMs and Shewhart			Four CUSUMs and Shewhart			Sparks' Scheme	Nine CUSUMs and Shewhart*
Intended shift	0.4	1.0	1.5	2.0	2.5	∞	[0.4,∞)	[1.0,∞)	[1.5,∞)	[0.4,∞)	[1.0,∞)	[1.5,∞)	[0.4,∞)	[1.0,∞)	[1.5,∞)	[0.4,∞)	[1.0,∞)	[1.5,∞)	[0.75,1.5]	[0.4,∞)
k	0.2	0.5	0.75	1.0	1.25		2.982 0.2	2.971 0.5	2.960 0.75	3.045 0.5 0.2	3.010 0.75 0.5	2.981 1.0 0.75	3.055 0.5 0.3 0.2	3.031 1.0 0.75 0.5	2.987 1.25 1.0 0.75	3.093 1.0 0.5 0.3 0.2	3.034 1.25 1.0 0.75 0.5	2.987 1.50 1.25 1.0 0.75	0.75 0.5 0.375	
h	7.767	4.104	2.888	2.178	1.359	2.781	2.982 9.199	2.971 4.694	2.960 3.256	3.045 4.932 9.685	3.010 3.368 4.822	2.981 2.491 3.304	3.055 4.960 7.394 9.750	3.031 2.571 3.412 4.884	2.987 1.968 2.497 3.314	3.093 2.672 5.095 7.596 10.501	3.034 2.033 2.572 3.422 4.899	2.987 1.591 1.971 2.496 3.313	3.150 4.498 5.628	
Δ										ARL										
0.00	370	370	370	370	370	370	370	370	370	370	370	370	370	370	370	370	370	370	370	370
0.20	71.8	105	125	153	172	203	84.6	123	148	86.1	125	149	87.2	126	150	89.1	127	149	99.3	88.8
0.30	41.9	61.5	78.9	100	119	153	48.9	72.8	94.4	49.5	74.0	95.0	49.6	75.4	95.6	50.7	75.6	95.2	57.1	50.9
0.40	27.9	39.0	51.1	67.9	83.5	116	32.3	45.4	61.0	32.6	46.4	62.2	32.5	46.9	62.2	33.1	47.2	62.2	35.7	33.3
0.50	20.5	26.2	34.6	46.9	59.3	88.6	23.5	30.3	41.1	23.4	30.7	41.5	23.4	30.9	41.9	23.8	31.4	41.6	24.6	23.8
0.60	16.0	19.0	24.5	33.0	42.6	68.8	18.3	21.4	28.5	18.0	21.8	28.9	17.9	22.0	28.8	18.2	22.1	29.0	17.9	18.1
0.80	11.2	11.3	13.6	18.0	23.3	42.0	12.6	12.7	15.5	11.9	12.7	15.6	11.8	12.8	15.7	12.0	12.8	15.6	11.2	11.9
1.00	8.54	7.89	8.77	11.0	14.0	26.7	9.49	8.63	9.70	8.58	8.65	9.78	8.52	8.67	9.80	8.55	8.68	9.79	7.96	8.48
1.50	5.45	4.41	4.28	4.65	5.32	10.0	5.64	4.66	4.58	4.78	4.54	4.57	4.78	4.53	4.57	4.66	4.52	4.58	4.40	4.62
2.00	4.04	3.10	2.83	2.82	2.96	4.61	3.74	3.10	2.91	3.20	2.96	2.88	3.21	2.92	2.88	3.01	2.91	2.88	2.98	2.99
2.50	3.25	2.43	2.15	2.04	2.02	2.56	2.57	2.23	2.09	2.32	2.14	2.06	2.32	2.10	2.06	2.15	2.10	2.06	2.29	2.15
4.00	2.14	1.58	1.29	1.19	1.14	1.13	1.18	1.16	1.15	1.18	1.16	1.15	1.18	1.16	1.15	1.18	1.16	1.15	1.39	1.18

* Chart parameters of nine CUSUMs are (0.2,10.135), (0.3,7.637), (0.4,6.146), (0.5 ,5.125), (0.6,4.373), (0.7,3.805), (0.8,3.353), (1.0,2.686), (1.5,1.723) and chart limit for the Shewhart chart is 3.103

Part 2

Simultaneous Exponentially Weighted Moving Average Charting Schemes

2.1   Introduction

The exponentially weighted moving average (EWMA) chart is a good alternative to the cumulative sum (CUSUM) chart. Roberts (1959) introduced the EWMA chart and extensive research that followed has shown that the performance of the EWMA chart is almost good as the CUSUM chart. In practice, the EWMA chart is easier to understand. Suppose that a quality characteristic which is denoted by x is normally distributed with mean μ and standard deviation σ_0. Consider taking a sample of size n from the process at each sample. The successive EWMA (Q_t) of sample means $\bar{x}_t$ plotted against the sample number t can be expressed as

$$Q_t = (1 - \lambda)Q_{t-1} + \lambda \bar{x}_t, \tag{1}$$

for $t = 1, 2, ...$, where λ which is a fixed smoothing constant such that $0 < \lambda \leq 1$. The quantity λ can be chosen such that the chart is optimal in detecting a particular shift of $\Delta\sigma_0/\sqrt{n}$. The EWMA assigns the largest weight to the current sample mean and the weights for the past samples decrease exponentially towards the past. Shewhart chart is a special case of the EWMA chart with $\lambda = 1$. The starting value Q_0 is usually set to the target value μ_0. The time-varying upper and lower control limits of EWMA chart can be expressed as

$$UCL(t) = \mu_0 + L\frac{\sigma_0}{\sqrt{n}}\sqrt{\frac{\lambda}{2 - \lambda}[1 - (1 - \lambda)^{2t}]}, \tag{2}$$

$$LCL(t) = \mu_0 - L\frac{\sigma_0}{\sqrt{n}}\sqrt{\frac{\lambda}{2 - \lambda}[1 - (1 - \lambda)^{2t}]}, \tag{3}$$

where L is a suitably chosen constant. The EWMA chart will issue an out-of control signal when Q_t falls outside the limits. As the sample number t increases, note that the term $1 - (1 - \lambda)^{2t}$ approaches to 1. This means that after the EWMA chart has been running for a sufficiently long period, the time-varying limits will converge to the asymptotic limits as given in equations (4) and (5).

$$UCL = \mu_0 + L\frac{\sigma_0}{\sqrt{n}}\sqrt{\frac{\lambda}{2 - \lambda}}, \tag{4}$$

$$LCL = \mu_0 - L\frac{\sigma_0}{\sqrt{n}}\sqrt{\frac{\lambda}{2-\lambda}}, \tag{5}$$

The rate of convergence depends on λ, with convergence being much slower for a smaller λ. The time-varying and asymptotic limits can also be expressed as

$$UCL(t) = \mu_0 + h\sqrt{1 - (1-\lambda)^{2t}}, \tag{6}$$

$$LCL(t) = \mu_0 - h\sqrt{1 - (1-\lambda)^{2t}}, \tag{7}$$

and

$$UCL = \mu_0 + h, \tag{8}$$

$$LCL = \mu_0 - h, \tag{9}$$

where

$$h = L\sqrt{\frac{\lambda}{2-\lambda}}, \tag{10}$$

corresponding to the case $N(0,1)$. It was shown by Montgomery (2005) and Steiner (1999) that the EWMA chart with time-varying limits is more sensitive to start-up quality problems than that with asymptotic limits. Thus, we also recommend EWMA charts with time-varying limits given in equations (6) and (7).

The average run length (ARL) is the average number of samples taken until a signal is given. ARL is an important measure of performance of a chart and it is desired to be a large when the process is in-control and small when the process is out-of-control. The steady-state ARL refers to the ARL of a chart evaluated from some point in time after the monitoring statistic has reached a steady state. Here we will study the various schemes using the steady state ARL. Design procedures of EWMA charts are usually based on the run length properties. It involves determining the chart parameters for a given in-control ARL.

Most of the design procedures of EWMA charts (see Crowder, 1989, Gan, 1998, Steiner, 1999 and Montgomery, 2005) require a shift to be specified for which the

chart is optimal in detecting. This amounts to testing a simple null hypothesis against a simple alternative hypothesis. In a quality control setting, we should be testing a simple null hypothesis against a composite alternative hypothesis. In reality it is difficult to anticipate the size of a shift. The general practice now is to decide a shift that is deemed the most important to be detected and then implement a chart that is optimal in detecting this shift. Such a chart performs well at the intended shift, but not at other shifts. Wesgard et al. (1977) suggested the use of the simultaneous scheme using the cumulative sum (CUSUM) charts. Thereafter, Lucas and Saccucci (1990) suggested the use of a combined EWMA-Shewhart scheme to improve the ability of the EWMA chart in detecting large shifts. They have found that such a scheme is sensitive in detecting both small and large shifts as compared to a single EWMA chart.

Neelakantan (2002) proposed a 'super' EWMA scheme consisting of nine EWMA charts and a Shewhart chart with the intention of providing protection over a wide range of shifts. No justification was given for the number of charts used but a simple design procedure was provided. An adaptive EWMA scheme that weights the past observations using a suitable function of the current error, was proposed by Capizzi and Masarotto (2003). This scheme is complicated because λ depends on some complicated function.

We propose simultaneous EWMA schemes which do not require any specification of the shift in advance and have good performance over a range of shifts. A simultaneous EWMA scheme comprises a few EWMA charts including a Shewhart chart that run simultaneously. The run length of a simultaneous scheme refers to minimum run length of the charts. The run length of a simultaneous scheme remains mathematically intractable, so we use simulation to study its run length distribution. In this thesis, we conduct a comprehensive study of the simultaneous EWMA schemes. In the next section, we investigate to find out if there is a suitable

number of charts to be used in a simultaneous scheme. We then do a comprehensive run length study of the various simultaneous schemes. We also provide a simple design procedure for determining the chart parameters of a simultaneous EWMA scheme. The implementation of a simultaneous EWMA scheme is illustrated and a conclusion is given.

2.2   Simultaneous EWMA Control Charting Schemes

Combined EWMA-Shewhart scheme (Lucas and Saccucci, 1990) and 'super' EWMA scheme (Neelakantan, 2002) are two main developments in the area of simultaneous EWMA charting schemes. However, all of them did not provide any justification for the number of charts used. In order to investigate the effect of adding more EWMA charts to a simultaneous scheme, we first consider a EWMA chart with $\lambda = 0.032$ (optimal in detecting a shift of $0.4\sigma_0/\sqrt{n}$) that has an in-control ARL of 1000. We then add a Shewhart chart with an in-control ARL of 1000 to the EWMA chart to obtained a combined EWMA-Shewhart scheme. The ARL of a scheme is computed using simulations such that the standard error of the simulated ARL is not more than 1% of the simulated ARL. The ARL of this combined scheme is found to be about 511. Then the second EWMA chart with $\lambda = 0.224$ (optimal in detecting a shift of $1.6\sigma_0/\sqrt{n}$) that has an in-control ARL of 1000 is added to obtain a 2–EWMA (2 EWMA charts and a Shewhart chart) scheme. Such a scheme is found to have an ARL of about 389. This procedure is continued by adding more EWMA charts to the scheme.

A plot of the ARL of the simultaneous scheme against the number of charts in the scheme is displayed in Figure 1. The order of the charts added is given by $\lambda = 0.032$, Shewhart, $\lambda = 0.224, 0.084, 0.307, 0.149, 0.750, 0.056, 0.265, 0.116, 0.545$ and 0.186 which are optimal in detecting $0.4, \infty, 1.6, 0.8, 2.0, 1.2, 4.0, 0.6, 1.8, 1.0, 3.0,$ and 1.4 units of $\sigma_0/\sqrt{n}$. Figure 1 shows that the ARL of the simultaneous EWMA scheme did not change appreciably beyond 5 charts. This suggests that using more

than 5 charts may not be necessary. Figure 1 also suggests that a 2–EWMA, 3–EWMA or 4–EWMA scheme would be sufficient. As long as the simultaneous EWMA scheme contains the optimal EWMA charts in detecting small shift ($\Delta = 0.4$), moderate shift ($\Delta = 1.6$) and the Shewhart chart for large shift, the ARL curve shown in Figure 1 did not change appreciably when the other EWMA charts were added in different orders. Similar results were obtained for the schemes intended in detecting shift in the ranges $[1.0, \infty)$ and $[1.5, \infty)$.

What remains to be investigated would be the performance of these simultaneous EWMA schemes. This is done in the next section. For each of the 2–EWMA, 3–EWMA and 4–EWMA schemes we propose 3 simultaneous schemes and these are designed to detect shift in the range $[0.4, \infty)$, $[1.0, \infty)$ and $[1.5, \infty)$ which corresponds to small to very large, medium to very large and large to very large shifts respectively. Each of these schemes includes a Shewhart chart. These EWMA charts were chosen such that they are optimal in detecting selected shifts in a range specified. The shifts being considered in our schemes for which the EWMA charts are optimal in detecting are given in the Table 1. Neelakantan (2002) used λ's for her 'super' scheme that are optimal in detecting $\Delta = 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6, 2.0$ and 3.0. Although the 'super' scheme gives better protection over the range of shift, our finding shows that a smaller number of charts can give similar protection. The ARL relationships between a simultaneous EWMA schemes and the individual EWMA charts are given in Figures 2–4. Although we found that the 'super' scheme contains more EWMA charts than sufficient, we have included the ARL curves for the 'super' scheme because it represents an approximation to the limiting case. These figures show the differences among the combined EWMA-Shewhart, 2–EWMA, 3–EWMA, 4–EWMA and the 'super' scheme.

2.3   Comparison of the Average Run Length Profiles

Control charts are usually compared using the ARL. In order to do a comprehensive comparison of simultaneous EWMA schemes, we consider schemes with $1, 2, 3$ or 4 EWMA charts together with a Shewhart chart, 4–CUSUM simultaneous scheme (see Part 1 of this thesis) and the 'super' scheme. In addition, the Shewhart chart and 5 individual EWMA charts which are optimal in detecting $\Delta = 0.4$, $1.0, 1.5, 2.0$ and 2.5 are also included for comparison. The programs for simulation were written in SAS and each ARL was simulated such that the simulated ARL is not more than 1% of the simulated ARL. The in-control ARL was fixed at 370. Shifts of $\Delta = 0.2, 0.3, 0.4, 0.5, 0.6, 0.8, 1.0, 1.5, 2.0, 2.5$ and 4.0 are considered. Table 2 shows the ARL profiles of these charts and schemes.

As expected, an individual EWMA chart that is optimal in detecting a particular shift has the smallest ARL at that shift. As the shift moves away from this intended shift, the sensitivity of the individual EWMA charts decreases. When a Shewhart chart is added to a EWMA chart to form a combined EWMA-Shewhart scheme, the scheme becomes more sensitive in detecting large shifts but it becomes less sensitive in detecting small shifts. As more EWMA charts are added to a scheme, the scheme becomes more sensitive at detecting the corresponding intended shifts. In general, a simultaneous scheme offers a better protection over a range of shifts. Although the individual chart can be made to be more sensitive in detecting very small shifts like $\Delta = 0.2$ and 0.3, Hawkins and Olwell (1998) pointed out that aiming at too-small shift is potentially harmful because a certain amount of natural variability will always exist. These too-small shifts are generally due to common cause of variation and a process that is operating with only this type of variation is said to be in statistical control (Montgomery, 2005).

If we compare simultaneous EWMA schemes intended for detecting a shift in the range $[0.4, \infty)$, the 4–EWMA scheme is seem more sensitive in detecting $\Delta \geq 0.8$

and slightly less sensitive in detecting $\Delta < 0.8$. The schemes intended for detecting a shift in the range $[1.0, \infty)$ are more sensitive in detecting $\Delta \geq 1.0$, but for small shifts, these schemes are less sensitive as expected. For the schemes intended for detecting a shift in the range $[1.5, \infty)$, the sensitivity of these schemes improve further for large shifts and become less sensitive for small and medium shifts.

The 'super' scheme and the 4–EWMA scheme intended for detecting $[0.4, \infty)$ have very similar run length performances. This further shows that using 4 EWMA charts in a scheme is sufficient. For a simpler scheme, quality control engineers could consider using a 2–EWMA or 3–EWMA scheme. It is found that the 4–EWMA scheme intended for detecting a shift in the range $[0.4, \infty)$ is more sensitive in detecting $\Delta \leq 0.8$ and less sensitive in detecting $\Delta > 0.8$ than the corresponding simultaneous 4–CUSUM scheme. Simultaneous 4–CUSUM schemes intended for detecting $[1.0, \infty)$ and $[1.5, \infty)$ are found to be slightly more sensitive than the corresponding 4–EWMA schemes in the intended range of shifts.

A comparison of the adaptive EWMA scheme with our simultaneous schemes can be found in Table A2. It is found that the simultaneous schemes intended for detecting a shift in the range $[1.0, \infty)$ are slighly more sensitive in the intended range of shifts than the corresponding adaptive EWMA schemes with $\mu_1 = 1.0$ and $\mu_2 = 6.0$.

2.4 Designs of Simultaneous EWMA Schemes

Procedures for designing control charts are usually based on the ARL. We provide design procedures for the two-sided EWMA schemes with $1, 2, 3$ or 4 EWMA charts intended for detecting shift in the range $[0.4, \infty)$, $[1.0, \infty)$ and $[1.5, \infty)$. The shifts (Δ's) for which the component EWMA charts optimal in detecting can be determined from Table 1. A quality control engineer will have to decide on one of the ranges for his process. This decision is more logical than specifying a single shift to be detected as in the case of designing an individual EWMA chart.

The following steps are recommended for the design simultaneous EWMA scheme:

Step 1. Select the smallest acceptable in-control ARL of the simultaneous EWMA scheme.

Step 2. Find the corresponding ARL of the individual component charts in the scheme based on the ARL specified in Step 1.

Step 3. Determine the values of λ of the component EWMA charts for the ARL found in Step 2 and the Δ's for which the component EWMA charts are optimal in detecting.

Step 4. Determine the chart limits h of the component EWMA charts for the ARL found in Step 2 and values of λ obtained in Step 3. And also determine the chart limit for the Shewhart chart for the ARL found in Step 2.

In Step 1, the choice of the ARL depends on the rate of production, frequency of sampling, size of the sample, cost etc. In order to simplify Step 2, we have determined the relationship between the ARL of the individual component charts and the ARL of the simultaneous scheme. The ARL's of the simultaneous schemes were simulated by considering the ARL of individual component charts to be 50, 100, 200, 300, 370, 400, 500, 600, 700, 800, 900, 1000, 1500, 2000, 2500, 3000, 3500 and 4000. The programs for simulation were written in SAS and each ARL was simulated using such that the standard error of each simulated ARL is not more than 1% of the simulated ARL. The relationships are displayed in Figures 2–4. These figures can be used for determining the ARL of the individual component charts easily.

In order to simplify Step 3, the optimal λ of a EWMA chart was found by simulation for $\Delta = 0.4$, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6, 2.0, 2.5 and 3.0 with respect to ARL of 50, 100, 150, 200, 250, 300, 350, 400, 450, 500, 600,700, 800, 900, 1000, 1500, 2000, 2500, 3000 and 4000. These are plotted in Figures 5 and 6. For the

Step 4, the chart limit h of EWMA chart was determined for different values of λ. For a specific λ and an ARL, the chart limit h can be read from Figures 7–9 easily. The chart limits for the two-sided Shewhart chart can be obtained easily by using Figure 10. Figures 7–9 are developed for a process with $N(0, 1)$ as the in-control distribution. However, if the underlying distribution is $N(\mu_0, \sigma_0^2)$, then the chart limits of an EWMA chart can be determined using equations (2) to (5) and $h = L\sqrt{\frac{\lambda}{2-\lambda}}$.

2.5   Example

In this section we use 25 sample of measurements of inside diameter of forged piston rings used in an automobile engine (Montgomery, 2005) to demonstrate the design procedure of a 4–EWMA scheme. The in-control mean and standard deviation are estimated to be $\mu_0 = 74.001$ and $\sigma_0 = 0.0094$ respectively. Suppose we want a scheme intended for detecting a shift in the range $[0.4, \infty)$ with an in-control ARL of 370. The shifts (Δ's) for which the component EWMA charts optimal in detecting, can be determined as $0.4, 0.6, 1.0$ and 2.0 from Table 1. The 4 steps for designing this scheme are as follows;

Step 1. The desired in-control ARL of the 4–EWMA scheme is 370.

Step 2. Using Figure 2 (Scheme D), the ARL of the individual chart is determined to be 1070 using the ARL of 370 as specified in Step 1.

Step 3. Using Figures 5 and 6 and ARL of 1070, the values of λ of the 4 componenet EWMA charts are determined as $\lambda = 0.031, 0.057, 0.119,$ and 0.312 for $\Delta = 0.4, 0.6, 1.0$ and 2.0 respectively.

Step 4. Using Figures 7 and 8, the chart limits of the 4 EWMA charts are determined as $h = 0.345, 0.510, 0.780,$ and 1.390 for the values of λ's obtained in Step 3 and ARL specified in Step 1. The chart limit of the Shewhart chart $h = 3.31$ can be obtained from Figure 10.

The time-varying and asymptotic limits chart limits of each of the component charts can then be calculated using equations (2) to (5) and $h = L\sqrt{\frac{\lambda}{2-\lambda}}$ as

$$UCL(t) = \mu_0 + h\frac{\sigma_0}{\sqrt{n}}\sqrt{1 - (1 - \lambda)^{2t}}, \qquad (11)$$

$$LCL(t) = \mu_0 - h\frac{\sigma_0}{\sqrt{n}}\sqrt{1 - (1 - \lambda)^{2t}}, \qquad (12)$$

and

$$UCL = \mu_0 + h\frac{\sigma_0}{\sqrt{n}}, \qquad (13)$$

$$LCL = \mu_0 - h\frac{\sigma_0}{\sqrt{n}}, \qquad (14)$$

To simulate a shift of $\Delta = 1.25$ which is a moderate shift; we added $1.25\sigma_0/\sqrt{n}$ to each of the measurements in the last 10 samples. The individual EWMA chart with $\lambda = 0.142$ which is optimal at detecting $\Delta = 1.0$, the combined EWMA-Shewhart scheme (EWMA chart with $\lambda = 0.126$ which is optimal at detecting $\Delta = 1.0$) and the 4–EWMA scheme for this data set are displayed in Figure 11. It shows that both the individual EWMA chart and the combined EWAM-Shewhart scheme signal at sample number 22, while the 4–EWMA scheme signals earlier (EWMA chart with $\lambda = 0.312$ which is optimal at detecting $\Delta = 1.0$) at sample number 20.

To simulate a big shift, we added $2.4\sigma_0/\sqrt{n}$ to each of the measurements in the last 2 samples. The various charting schemes for this data set are displayed in Figure 12. The individual EWMA chart was not able to detect this shift. Both the Shwehart charts of the combined EWMA-Shewhart scheme and the simultaneous 4–EWMA scheme detected this shift at sample number 24 when the shift occurs. These 2 data sets illustrates clearly the ability of the simultaneous EWMA scheme in detecting a shift of any magnitude in a specified range.

2.6   Conclusions

Most of the optimal design procedures for the EWMA chart require the specification of a shift in advance for which the chart is optimal in detecting. Such a chart would perform well at the intended shift but it will be increasingly insensitive if the shift moves further away from the intended shift. In reality, the shift that occurs is more likely to be random, so it may not make good sense to design a chart that is optimal in detecting a particular shift only. Here, we develop simultaneous EWMA schemes in order to provide protection to a range of shifts. We have developed schemes in detecting shifts in the ranges: $[0.4, \infty)$, $[1.0, \infty)$ and $[1.5, \infty)$. The component charts are chosen such that they are optimal in detecting shifts in a specified range. Instead of using the 'super' scheme with 9 EWMA charts (see Neelakantan 2002), our study shows that a 4–EWMA scheme would be sufficient. One could also consider a simpler 2–EWMA or a 3–EWMA scheme for implementation with less sensitivity over the intended range of shifts. We have provided a simple design procedure for determining the chart parameters of a simultaneous scheme. A comprehensive comparison shows that a simultaneous EWMA scheme indeed provides a better protection over a specified range of shift. Simultaneous schemes with CUSUM charts were also developed and details can be found in the first part of this thesis. It is found that simultaneous CUSUM and simultaneous EWMA schemes have similar ARL profiles. An advantage of the simultaneous EWMA scheme is that quality control engineers who are currently using individual EWMA charts can migrate easily to simultaneous EWMA schemes with a lower learning curve.

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Figures and Tables

for the

Simultaneous Exponentially Weighted Moving Average Charting Schemes

Average Run Length

Figure 1. Steady-State In-Control ARL of Simultaneous EWMA Schemes Designed for Detecting a Shift in the Range [0.4,∞) With Respect to the Number of Charts in a Scheme
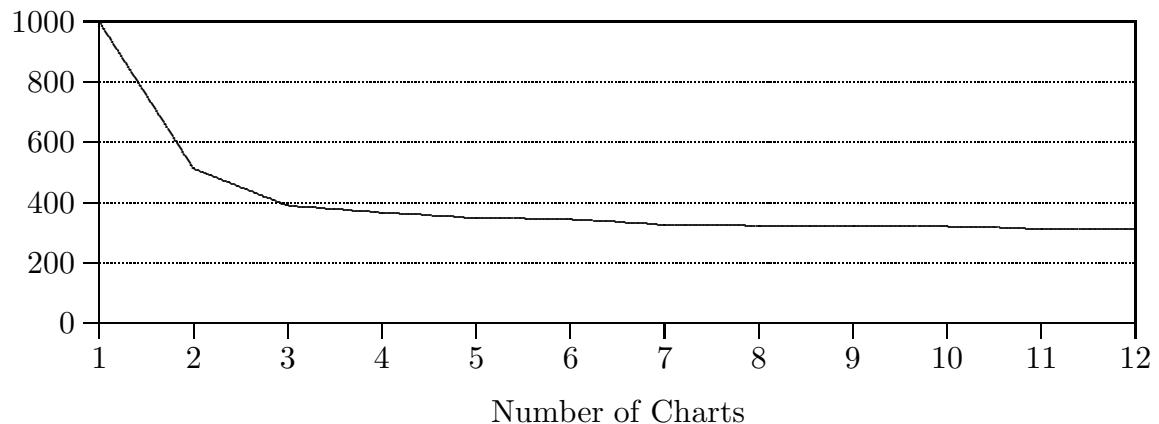
ARL of Individual EWMA   Scheme
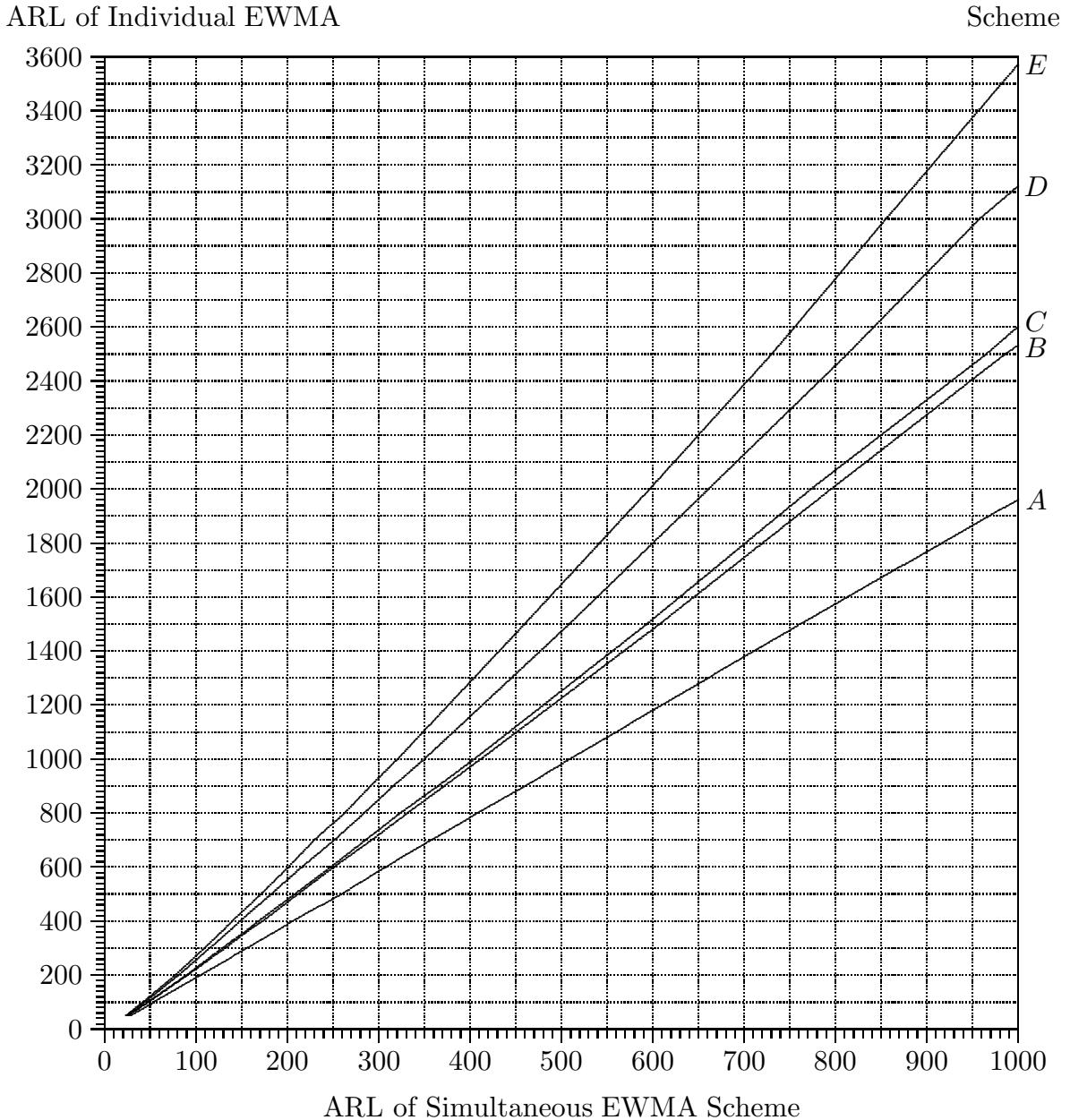
Figure 2. Relationships Between In-Control ARL of a Simultaneous Scheme De-
           signed for Detecting a Shift in the Range $[0.4, \infty)$ and In-Control ARL
           of Individual EWMA Charts

A : One EWMA ($\Delta = 0.4$) and Shewhart chart

B : Two EWMAs ($\Delta = 0.4$, 1.0) and Shewhart chart

C : Three EWMAs ($\Delta = 0.4$, 0.6, 1.0) and Shewhart chart

D : Four EWMAs ($\Delta = 0.4$, 0.6, 1.0, 2.0) and Shewhart chart

E : Nine EWMAs ($\Delta = 0.4$, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6, 2.0, 3.0) and Shewhart chart

ARL of Individual EWMA
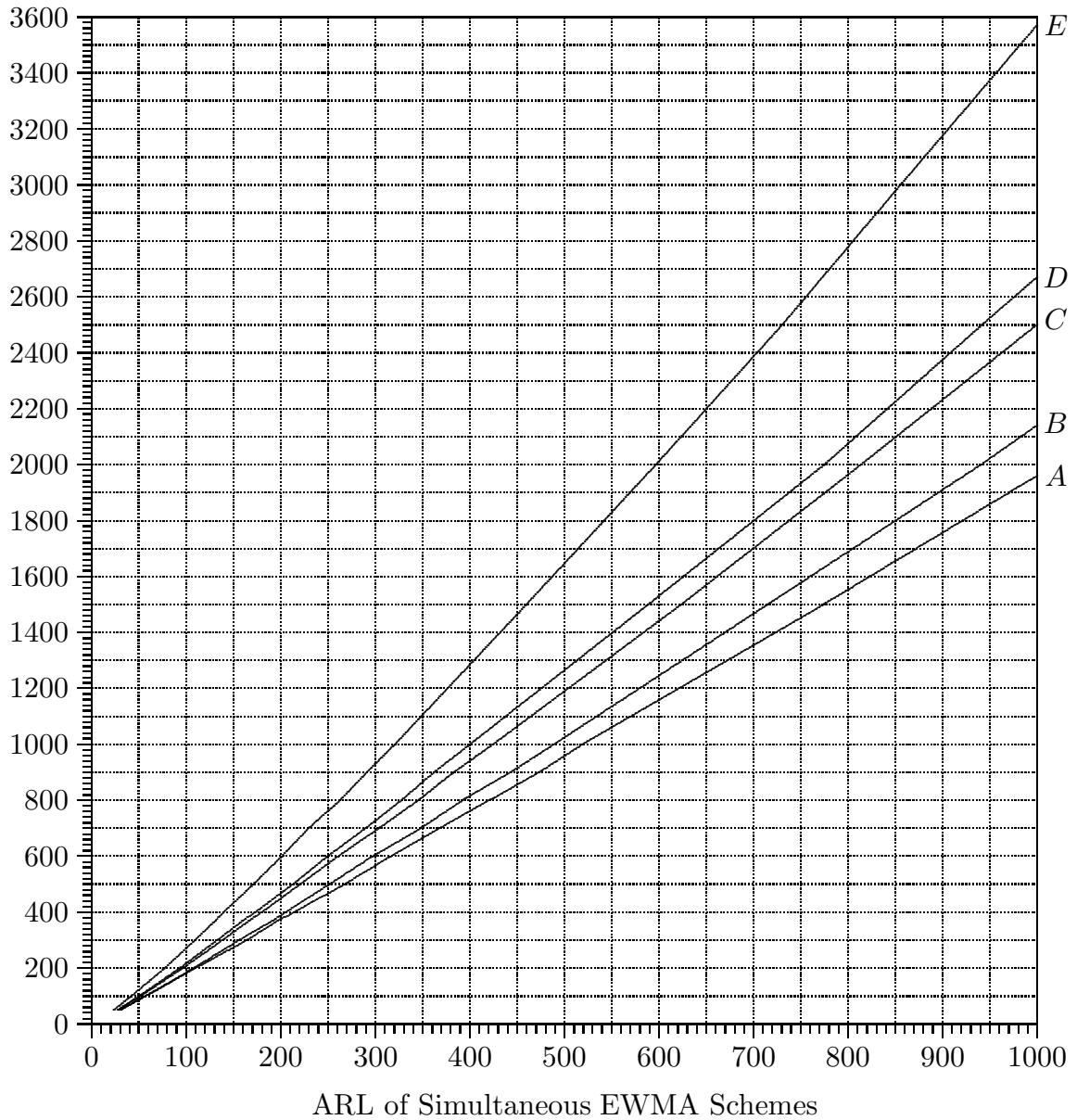
Scheme

Figure 3. Relationships Between In-Control ARL of a Simultaneous Scheme Designed for Detecting a Shift in the Range $[1.0, \infty)$ and In-Control ARL of Individual EWMA Charts

A : One EWMA ($\Delta = 1.0$) and Shewhart chart

B : Two EWMAs ($\Delta = 1.0, 1.5$) and Shewhart chart

C : Three EWMAs ($\Delta = 1.0, 1.5, 2.0$) and Shewhart chart

D : Four EWMAs ($\Delta = 1.0, 1.5, 2.0, 2.5$) and Shewhart chart

E : Nine EWMAs ($\Delta = 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6, 2.0, 3.0$) and Shewhart chart

ARL of Individual EWMA   Scheme
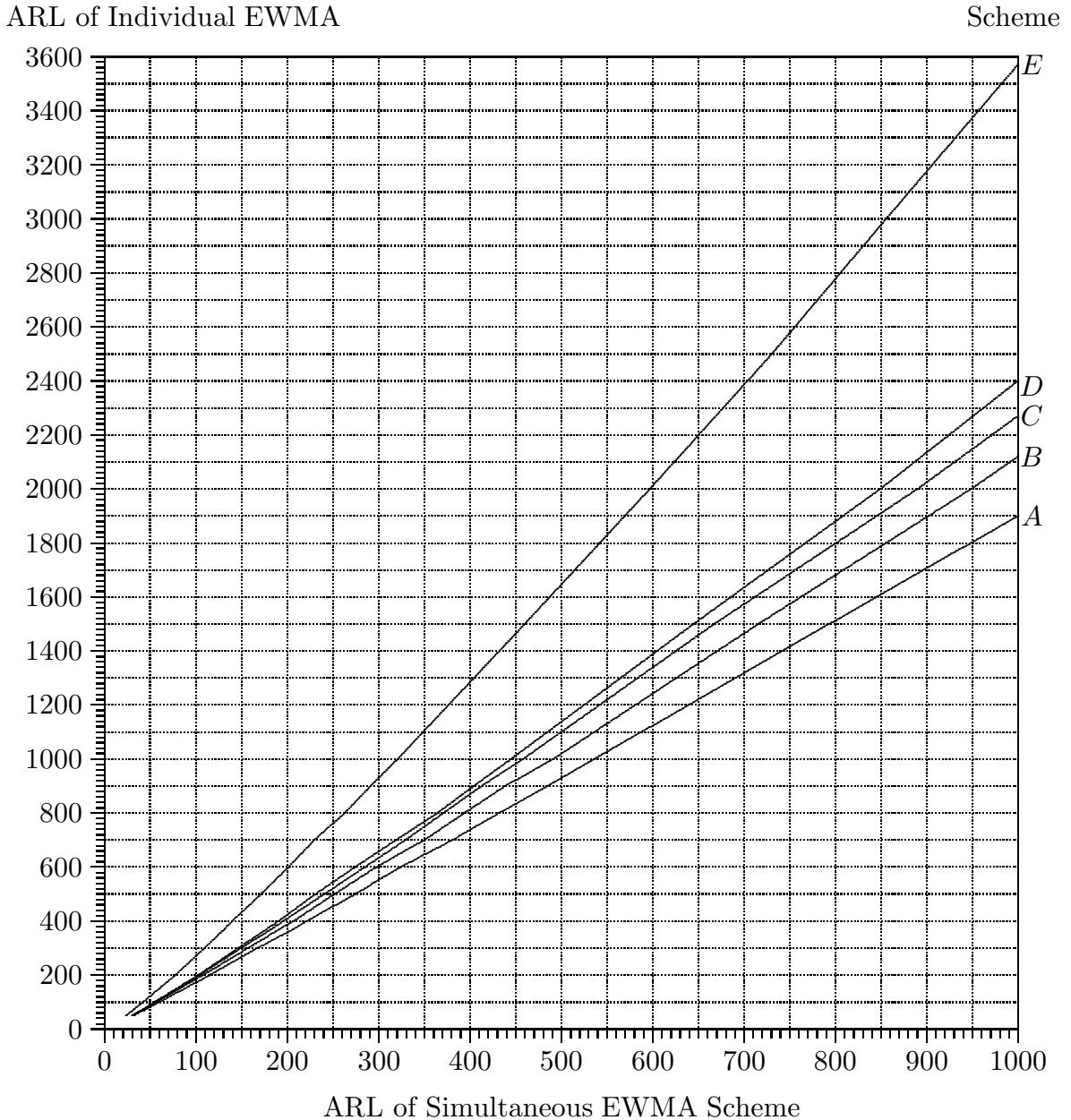
Figure 4. Relationships Between In-Control ARL of a Simultaneous Scheme De-
           signed for Detecting a Shift in the Range $[1.5, \infty)$ and In-Control ARL
           of Individual EWMA Charts

A : One EWMA ($\Delta = 1.5$) and Shewhart chart

B : Two EWMAs ($\Delta = 1.5$, 2.0) and Shewhart chart

C : Three EWMAs ($\Delta = 1.5$, 2.0, 2.5) and Shewhart chart

D : Four EWMAs ($\Delta = 1.5$, 2.0, 2.5, 3.0) and Shewhart chart

E : Nine EWMAs ($\Delta = 0.4$, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6, 2.0, 3.0) and Shewhart chart
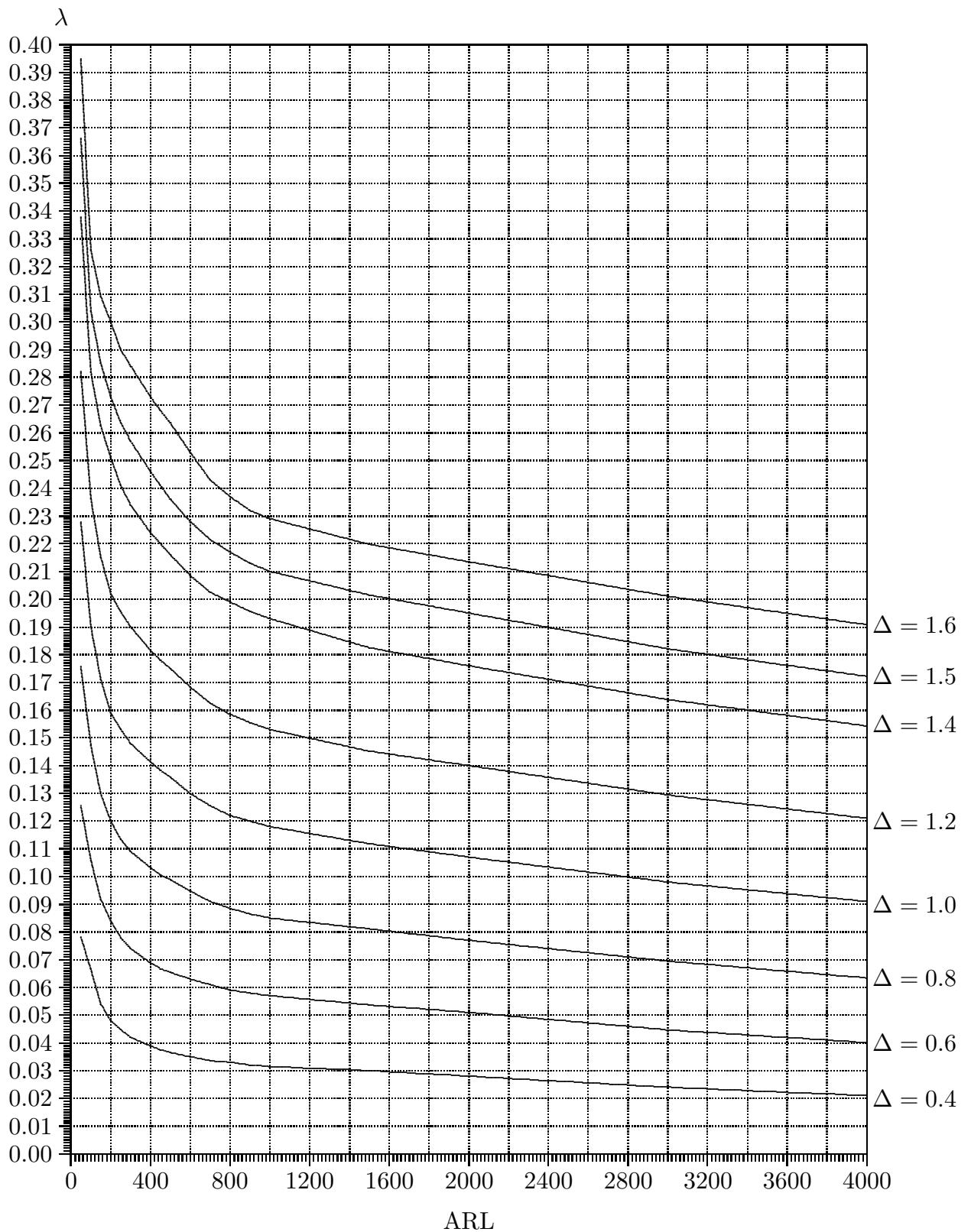
Figure 5. Combinations of λ and ARL Values of Individual EWMA Charts for $\Delta = 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.5$ and 1.6
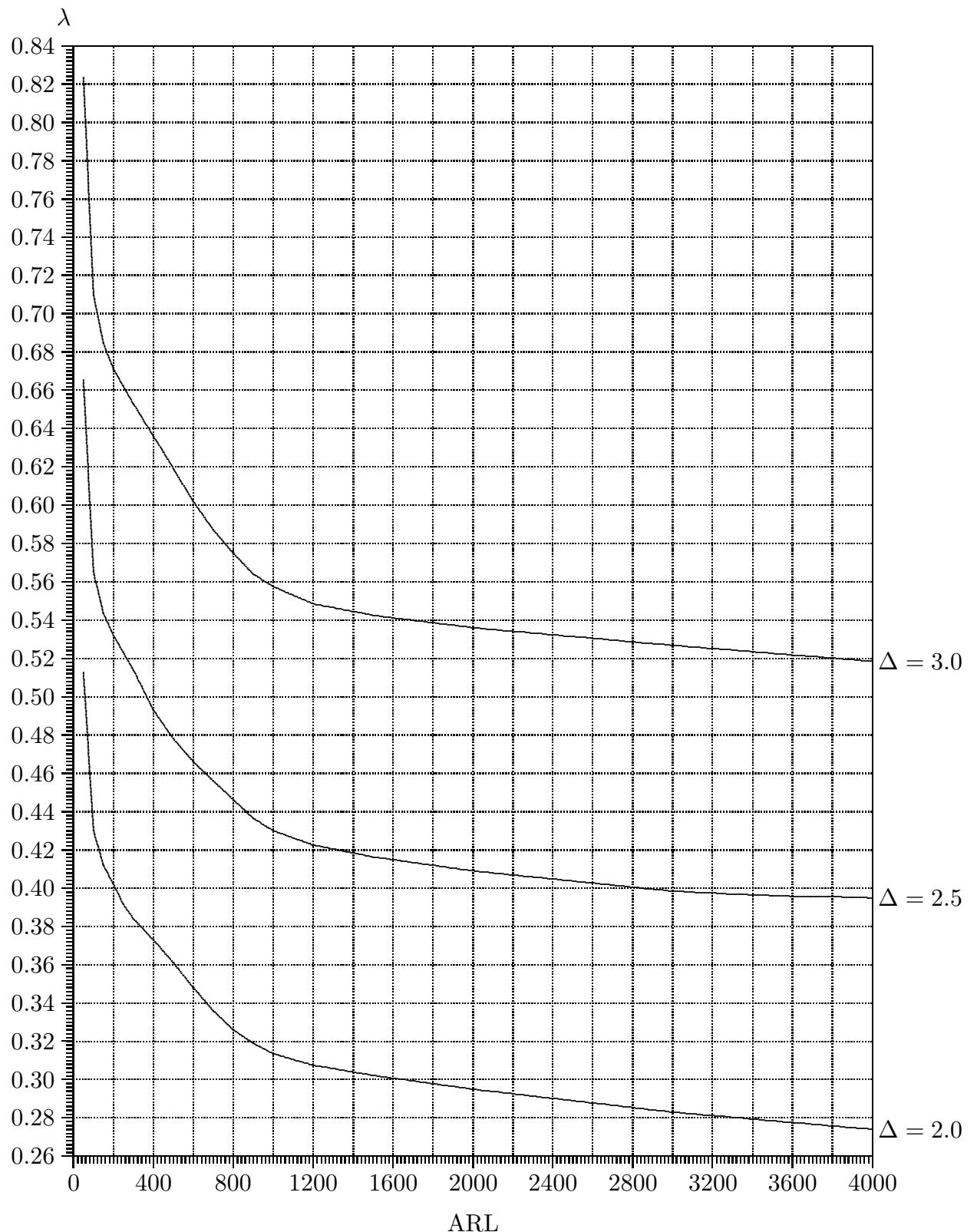
Figure 6. Combinations of λ and ARL Values of Individual EWMA Charts for
$\Delta = 2.0, 2.5$ and 3.0
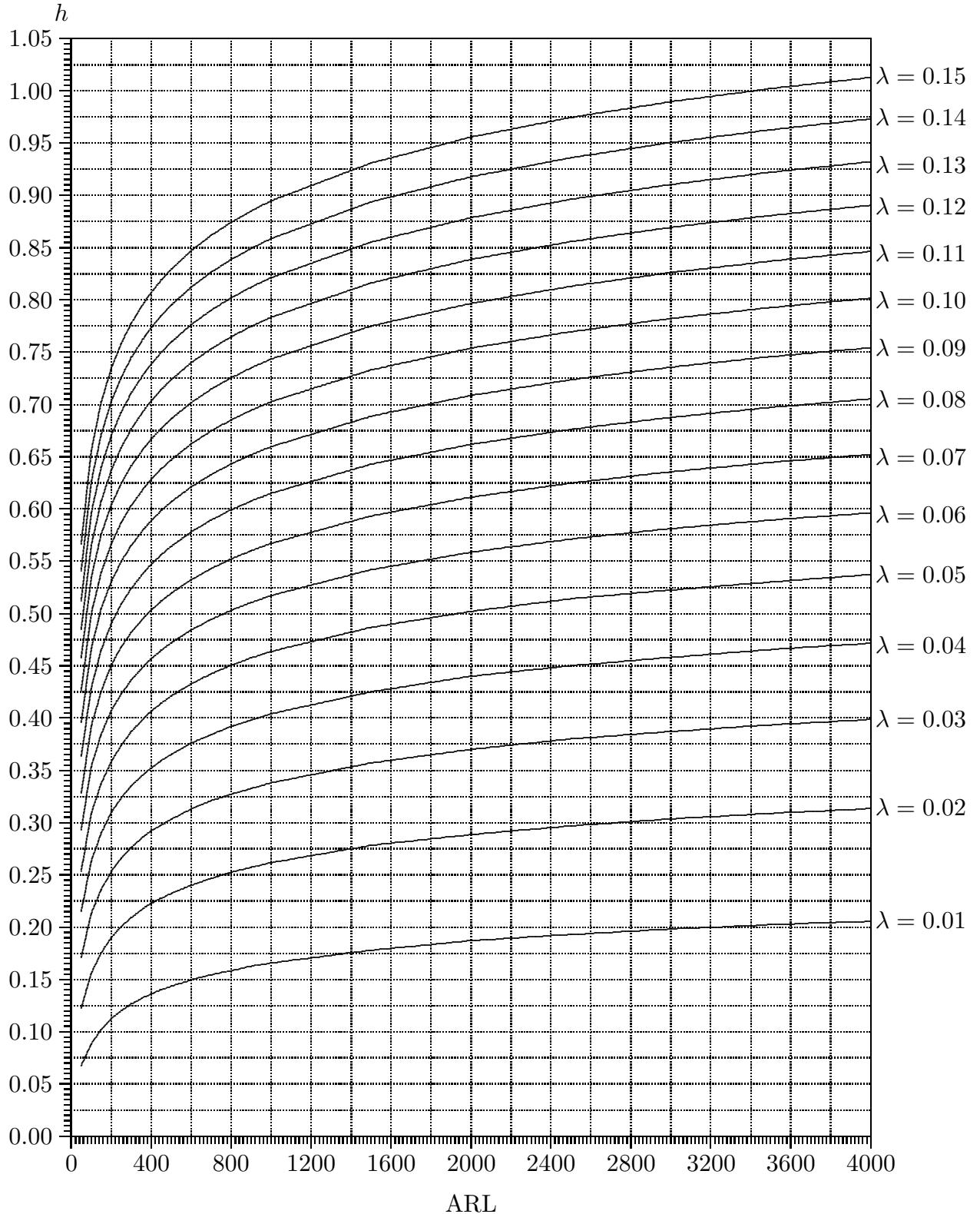
Figure 7. Combinations of h and ARL Values of Individual EWMA Charts for
$\lambda = 0.01, 0.02, 0.03, ..., 0.15$
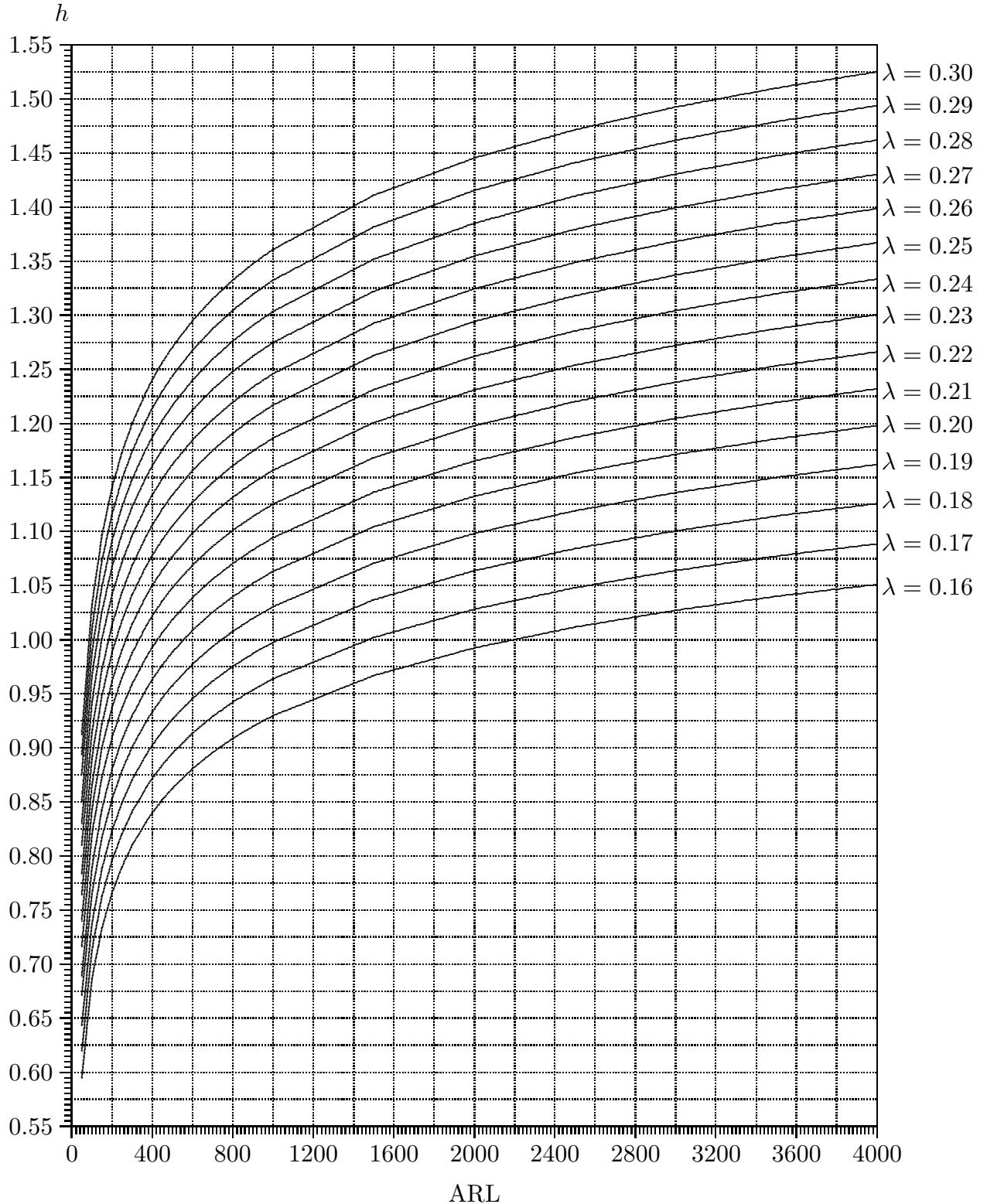
Figure 8. Combinations of h and ARL Values of Individual EWMA Charts for $\lambda = 0.16, 0.17, 0.18, ..., 0.30$
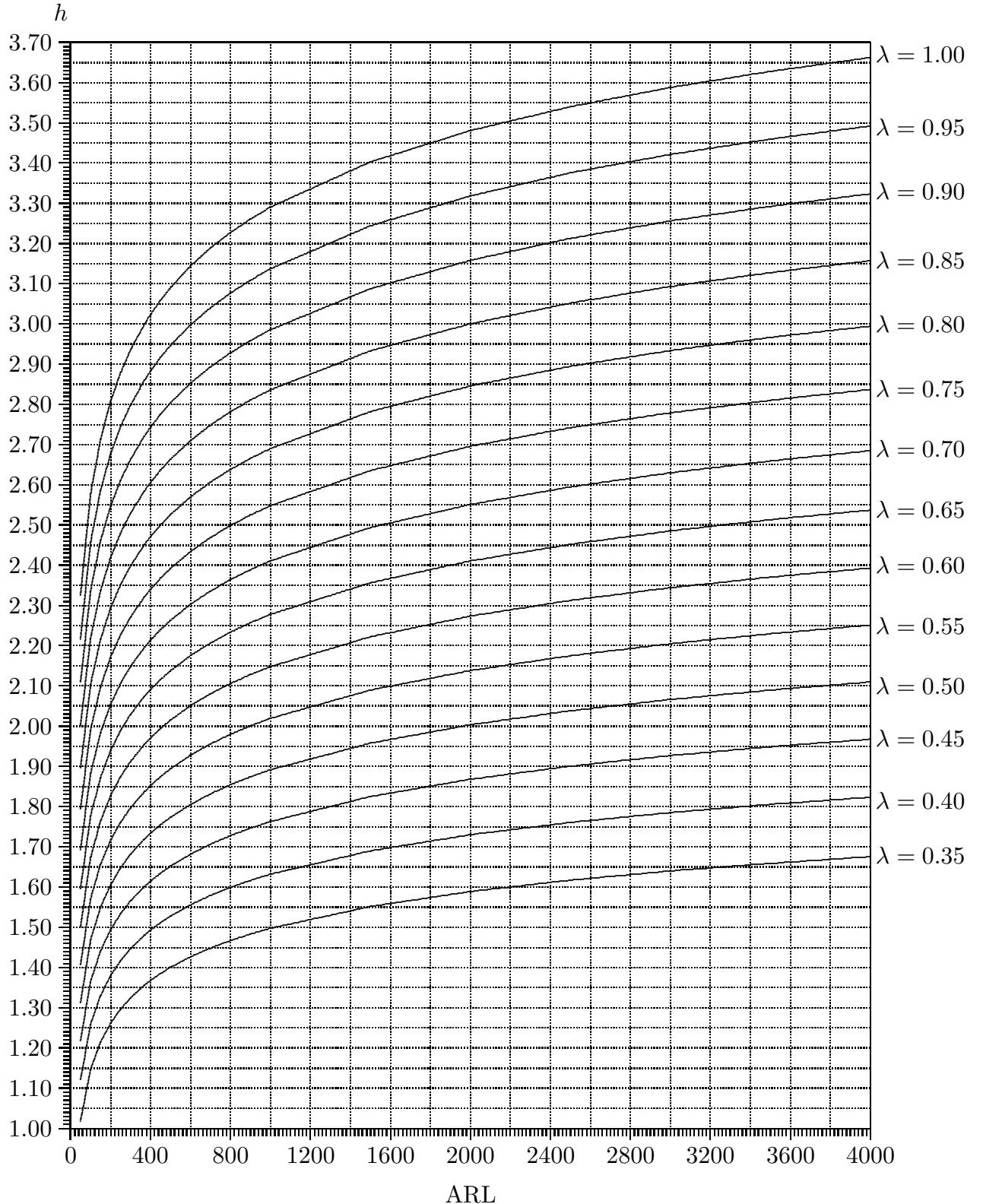
Figure 9. Combinations of h and ARL Values of Individual EWMA Charts for $\lambda = 0.35, 0.40, 0.45, ..., 1.00$
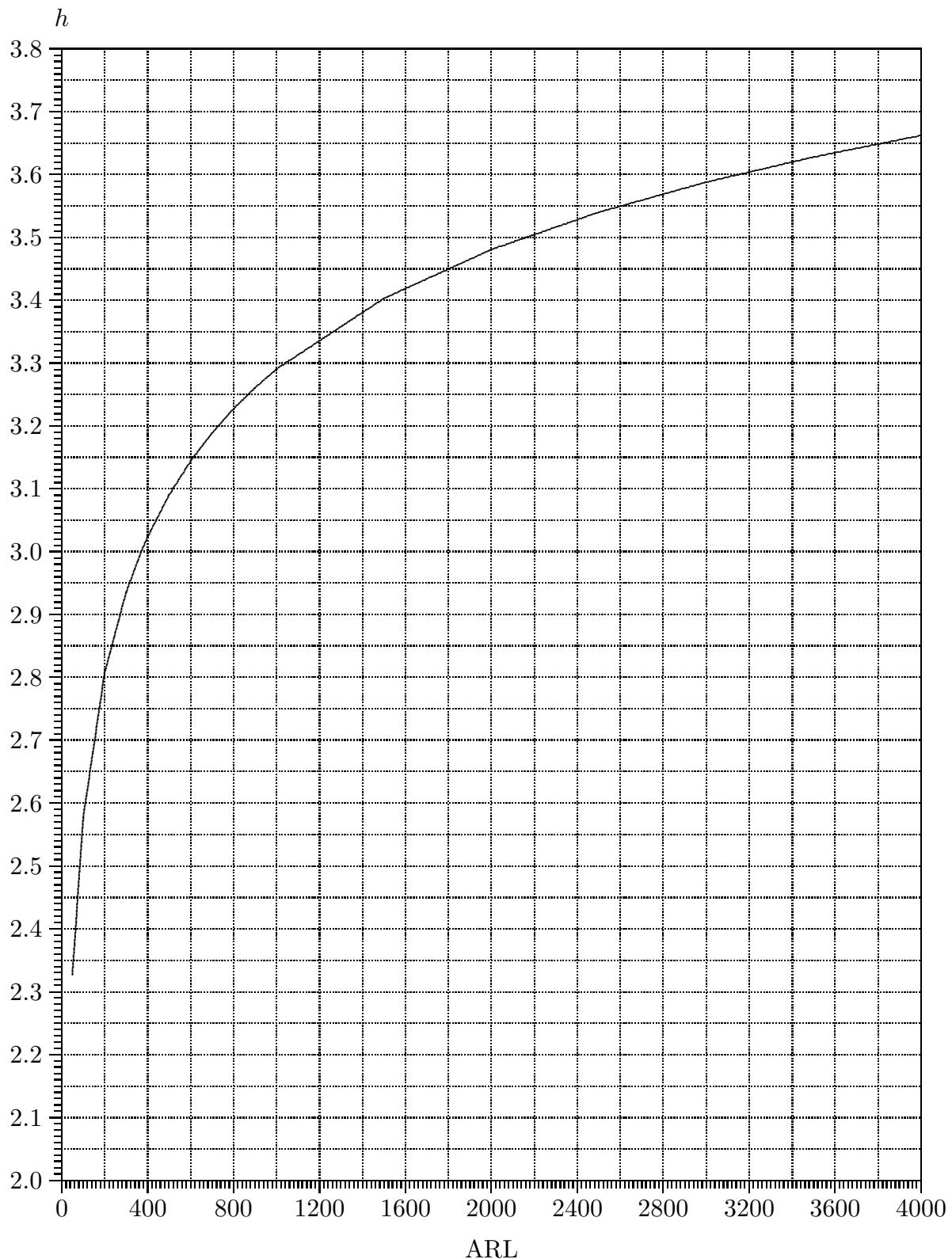
Figure 10. Chart Limits of the Shewhart Chart for ARL from 50 to 4000

Individual EWMA Control Chart
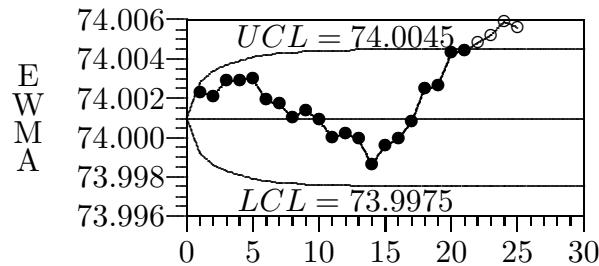
$\lambda = 0.142, \Delta = 1.0$

Combined EWMA-Shewhart Scheme

$\lambda = 0.126, \Delta = 1.0$

Shewhart

Simultaneous 4–EWMA Scheme

$\lambda = 0.031, \Delta = 0.4$

$\lambda = 0.057, \Delta = 0.6$

$\lambda = 0.119, \Delta = 1.0$

$\lambda = 0.312, \Delta = 2.0$

Shewhart

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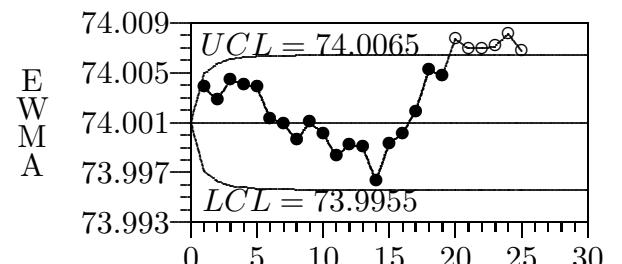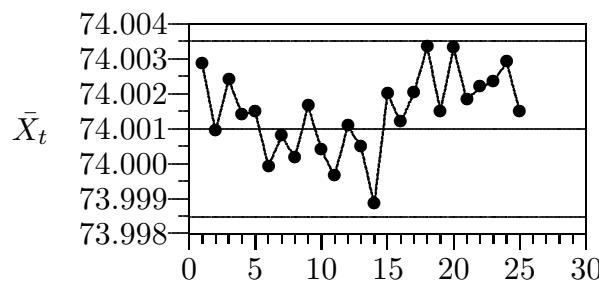
Figure 11. Control Charting Schemes for the Piston Ring Data Set when $1.25\sigma_0/\sqrt{n}$ was Added to Each of the Measurements of the Last 10 Samples

Individual EWMA Control Chart
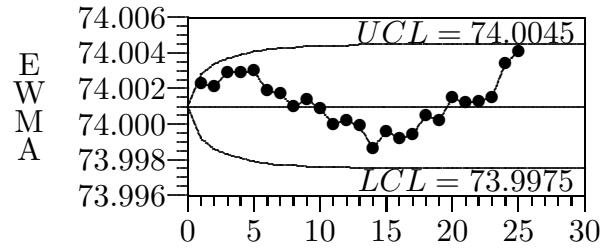
$\lambda = 0.142, \Delta = 1.0$

Combined EWMA-Shewhart Scheme

$\lambda = 0.126, \Delta = 1.0$
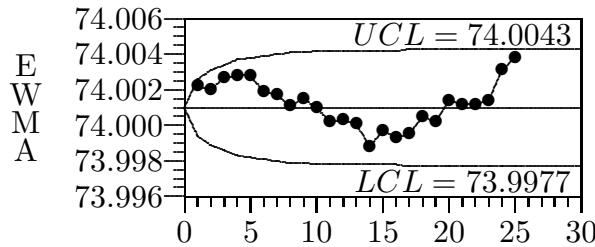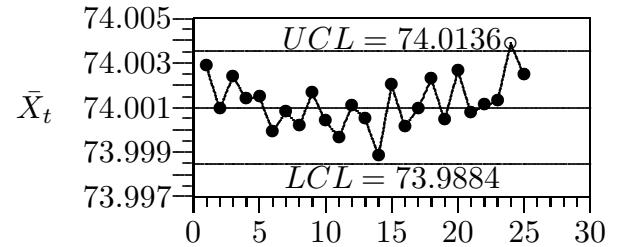
Shewhart

Simultaneous 4–EWMA Scheme

$\lambda = 0.031, \Delta = 0.4$
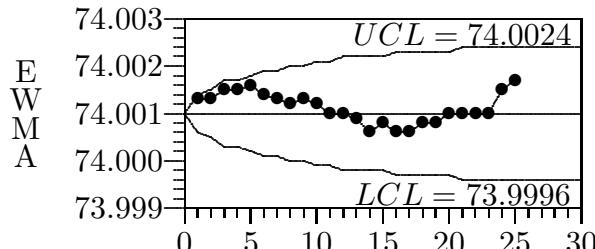
$\lambda = 0.057, \Delta = 0.6$
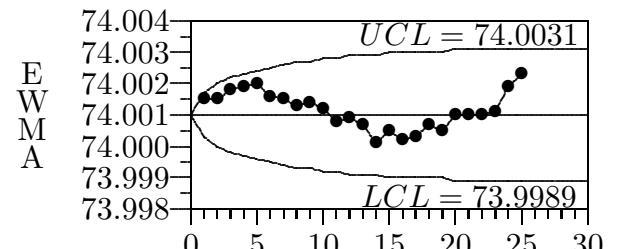
$\lambda = 0.119, \Delta = 1.0$
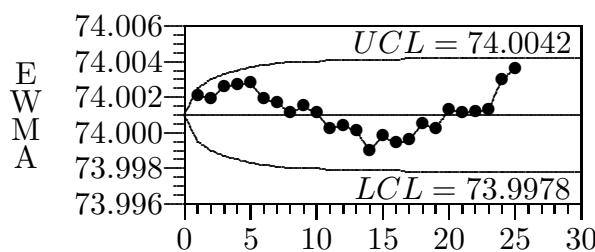
$\lambda = 0.312, \Delta = 2.0$
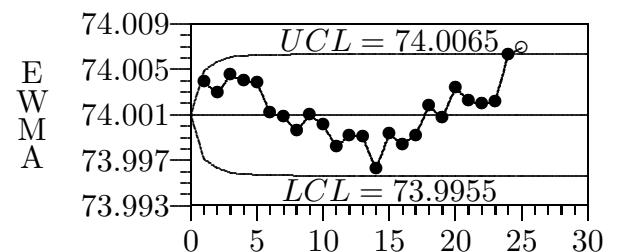
Shewhart
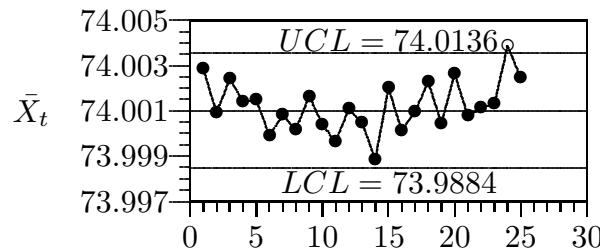
Figure 12. Control Charting Schemes for the Piston Ring Data Set when $2.4\sigma_0/\sqrt{n}$ was Added to Each of the Measurements of the Last 2 Samples

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Table 1. Shifts for which the EWMA Charts are Optimal in Detecting, in a Simultaneous EWMA Scheme

Range of Shift	Combined EWMA and Shewhart	Two EWMAs and Shewhart	Three EWMAs and Shewhart	Four EWMAs and Shewhart
Small to very Large $[0.4 \le \Delta < \infty)$	0.4	0.4, 1.0	0.4, 0.6, 1.0	0.4, 0.6, 1.0, 2.0
Medium to very Large $[1.0 \le \Delta < \infty)$	1.0	1.0, 1.5	1.0, 1.0, 2.0	1.0, 1.5, 2.0, 2.5
Large to very Large $[1.5 \le \Delta < \infty)$	1.5	1.5, 2.0	1.5, 2.0, 2.5	1.5, 2.0, 2.5, 3.0

Table 2. Steady State ARL Profiles of the Two-Sided EWMA Charts, Shewhart Chart, Simultaneous EWMA Schemes and 4–CUSUM scheme

	Single EWMA Chart					Shewhart Chart	Combined EWMA and Shewhart			Two EWMAs and Shewhart			Three EWMAs and Shewhart			Four EWMAs and Shewhart			Four CUSUMs and Shewhart+			Nine EWMAs and Shewhart*
Intended shift	0.4	1.0	1.5	2.0	2.5	∞	[0.4,∞)	[1.0,∞)	[1.5,∞)	[0.4,∞)	[1.0,∞)	[1.5,∞)	[0.4,∞)	[1.0,∞)	[1.5,∞)	[0.4,∞)	[1.0,∞)	[1.5,∞)	[0.4,∞)	[1.0,∞)	[1.5,∞)	[0.4,∞)
λ																1.000	1.000	1.000				
													1.000	1.000	1.000	0.306	0.431	0.559				
										1.000	1.000	1.000	0.118	0.318	0.431	0.119	0.317	0.441				
							1.000	1.000	1.000	0.125	0.220	0.323	0.057	0.213	0.324	0.054	0.211	0.322				
	0.039	0.142	0.251	0.375	0.498	1.000	0.031	0.120	0.220	0.031	0.119	0.220	0.031	0.118	0.222	0.029	0.122	0.212				
h																3.308	3.263	3.236	3.320	3.264	3.214	
													3.266	3.250	3.225	1.386	1.698	2.011	3.066	2.343	1.853	
										3.259	3.208	3.207	0.768	1.395	1.670	0.784	1.395	1.712	5.879	2.965	2.273	
							3.198	3.190	3.182	0.793	1.094	1.337	0.496	1.089	1.396	0.489	1.086	1.397	8.890	3.940	2.876	
	0.342	0.772	1.097	1.419	1.714	3.000	0.329	0.753	1.084	0.340	0.756	1.094	0.341	0.762	1.106	0.334	0.783	1.080	11.970	5.676	3.824	
Δ									ARL													
0.00	370	370	370	370	370	370	370	370	370	370	370	370	370	370	370	370	370	370	370	370	370	370
0.20	93.5	139	178	211	236	307	107	163	204	112	165	206	112	171	209	115	175	210	123	194	230	118
0.30	53.2	76.3	103	132	158	252	61.1	90.0	123	62.3	91.3	127	62.0	94.5	128	63.2	98.1	128	66.2	111	147	65.4
0.40	35.3	46.0	63.2	83.9	105	200	40.5	53.7	75.0	40.4	54.4	76.8	40.1	56.4	78.7	40.9	58.6	78.2	41.9	65.0	92.5	41.8
0.50	26.1	30.5	40.8	55.1	70.7	154	29.9	35.1	47.9	29.0	35.9	49.3	28.8	36.5	50.6	29.1	37.8	49.7	29.5	40.7	58.7	29.7
0.60	20.6	22.0	28.0	37.8	48.9	119	23.6	24.9	32.6	22.2	25.2	33.4	21.9	25.5	34.0	22.2	26.3	33.6	22.3	27.7	38.8	22.4
0.80	14.5	13.4	15.4	19.9	25.6	71.4	16.4	14.9	17.5	14.5	14.9	17.9	14.4	15.0	18.0	14.3	15.2	17.9	14.3	15.4	19.4	14.4
1.00	11.2	9.39	10.1	12.0	15.0	44.0	12.5	10.4	11.1	10.4	10.3	11.2	10.4	10.2	11.3	10.2	10.2	11.2	10.1	10.2	11.6	10.2
1.50	7.15	5.37	5.09	5.31	5.88	15.0	7.39	5.69	5.43	5.83	5.42	5.36	5.88	5.32	5.32	5.51	5.29	5.33	5.38	5.20	5.24	5.44
2.00	5.30	3.80	3.41	3.29	3.36	6.30	4.82	3.79	3.49	3.88	3.49	3.38	3.94	3.44	3.33	3.55	3.39	3.33	3.43	3.30	3.25	3.48
2.50	4.26	2.99	2.61	2.42	2.35	3.25	3.20	2.69	2.48	2.77	2.49	2.40	2.80	2.45	2.36	2.52	2.41	2.36	2.42	2.34	2.29	2.46
4.00	2.77	1.93	1.64	1.46	1.33	1.19	1.26	1.24	1.22	1.26	1.24	1.22	1.27	1.24	1.22	1.26	1.23	1.22	1.25	1.23	1.22	1.25

+ Reference values of 4–CUSUM scheme for [0.4,∞) are (0.2, 0.3, 0.5, 1.0) for [1.0,∞) are (0.5, 0.75, 1.0, 1.25) and for [1.5,∞) are (0.75, 1.0, 1.25, 1.5).

* Chart parameters (λ, h) of nine EWMAs for $\Delta = 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6, 2.0,$ and 3.0 are given as (0.031, 0.352), (0.055, 0.500), (0.083, 0.640), (0.114, 0.772), (0.148, 0.902), (0.184, 1.027), (0.223, 1.151), (0.305, 1.395), (0.543, 2.031) and chart limit for the Shewhart chart is 3.336

Glossary

Adaptive CUSUM Scheme

A scheme which continuously adjusts the parameters by predicting the one-step-ahead forecast of future shift.

Adaptive EWMA Scheme

A scheme which weights the past observations using a suitable function of the current error.

Assignable Causes of Quality Variation

Variability in a quality characteristic due to improperly adjusted or controlled machines, operator errors, defective raw materials etc.

Average Run Length (ARL)

The average number of samples taken until a signal is issued.

Chance Cause of Quality Variation

A certain amount of inherent or natural variability which always exists in any production process regardless of how well it is designed or carefully maintained.

Combined Scheme

A scheme which comprises a CUSUM chart or a EWMA chart and a Shewhart chart.

Delta (Δ)

Size of the observed shift of a process parameter.

Design Procedure of Control Chart

Finding the chart parameters of a control chart given the desired in-control ARL and the shift to be detected quickly.

In-Control

A process that is operating only with common causes of variation.

Intended or Specified Shift

A shift for which a quick detection is desired.

One-Sided and Two-Sided Simultaneous Schemes

A simultaneous scheme comprises one-sided charts is known as a one-sided scheme. A simultaneous scheme comprises two-sided charts is known as a two-sided scheme.

Optimal Control Chart for a Specified Shift

A control chart which has the smallest ARL in detecting a specified shift amung all charts with the same in-control ARL.

Out of Control

A process that is operating with assignable causes of variation is said to be out of control.

Quality Characteristic

A variable which measure the quality of a product.

Run Length of a Simultaneous Scheme

The minimum run length of all the charts in a scheme.

Shift

A change in the process parameter.

'Signal' of a Control Chart

An indication of a possible occurrence of a assignable cause of variation in a production process.

Simultaneous Charting Scheme

A scheme comprises of two or more control charts.

Target Value

A desired process parameter.

Upper-Control Limit (UCL) and Lower-Control Limit (LCL)

Limits chosen on a control chart such that a signal will be issued if a point plotted on the chart is beyond one of these limits.

Appendix 1

Additional Figures and Tables

Average Run Length

Figure A1. Steady-State In-Control ARL of a Simultaneous Scheme With Respect to the Number of Charts in a Scheme

When the Shift of 0.4 was Added to the Process

Average Run Length

Number of Charts in a Scheme
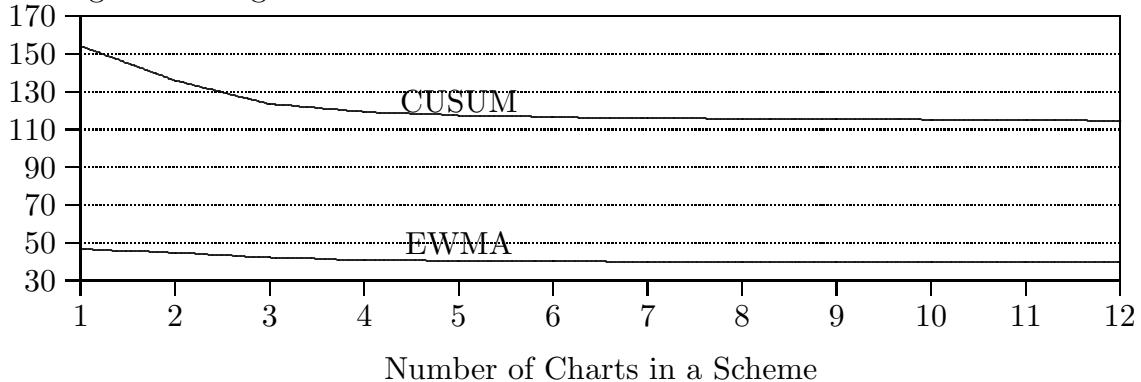
When the Shift of 1.0 was Added to the Process

Average Run Length

Number of Charts in a Scheme
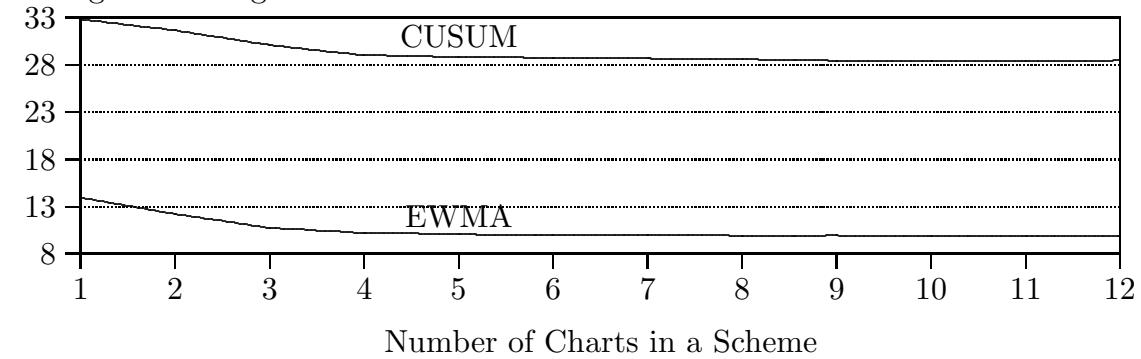
When the Shift of 1.5 was Added to the Process

Average Run Length

Number of Charts in a Scheme
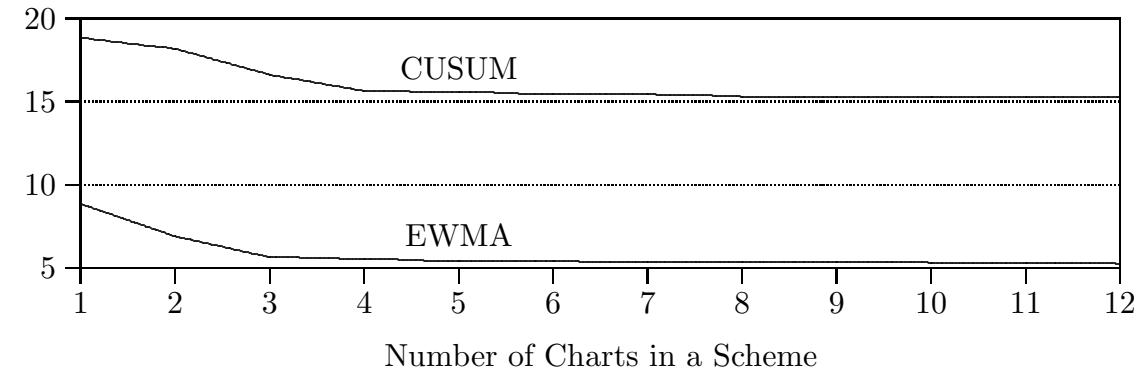
Figure A2. Steady-State Out-of-Control ARL of a Simultaneous Scheme With Respect to the Number of Charts in a Scheme

ARL of Individual Chart
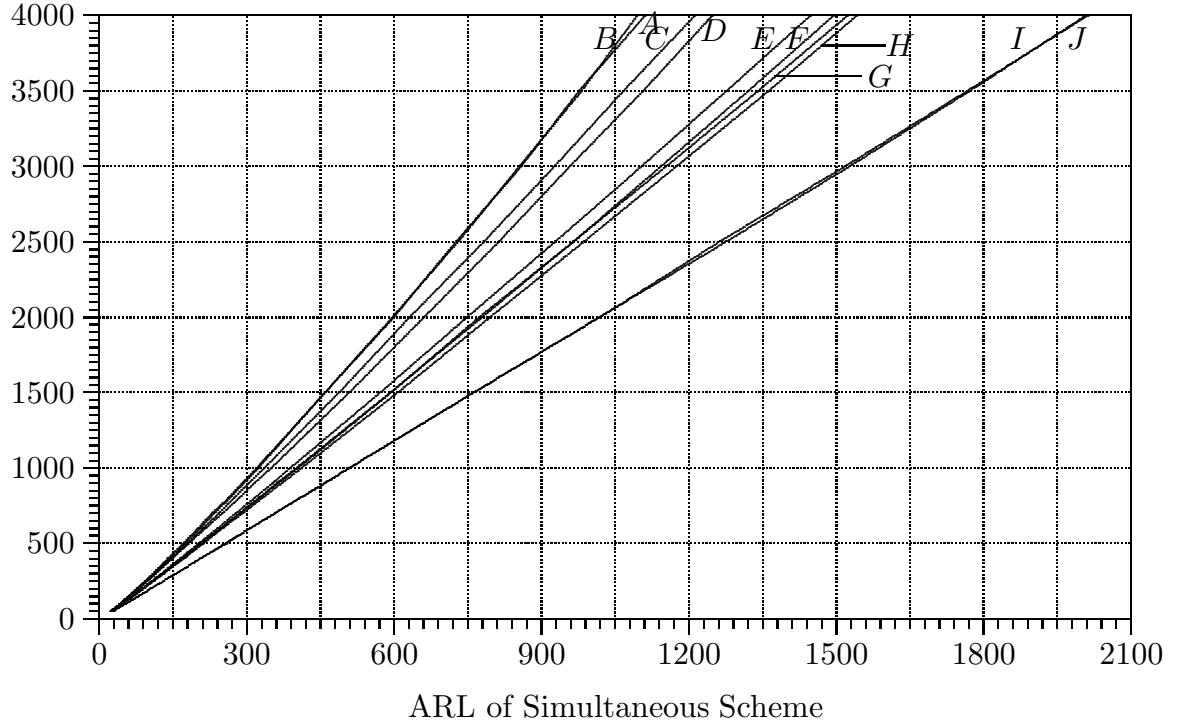
Figure A3. Relationships Between In-Control ARL of a Simultaneous Scheme Designed for Detecting a Shift in the Range $[0.4, \infty)$ and In-Control ARL of Individual Charts

A : Nine CUSUMs ($k = 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 1.0, 1.5$) and Shewhart chart

B : Nine EWMAs ($\Delta = 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6, 2.0, 3.0$) and Shewhart chart

C : Four CUSUMs ($k = 0.2, 0.3, 0.5, 1.0$) and Shewhart chart

D : Four EWMAs ($\Delta = 0.4, 0.6, 1.0, 2.0$) and Shewhart chart

E : Three CUSUMs ($k = 0.2, 0.3, 0.5$) and Shewhart chart

F : Three EWMAs ($\Delta = 0.4, 0.6, 1.0$) and Shewhart chart

G : Two CUSUMs ($k = 0.2, 0.5$) and Shewhart chart

H : Two EWMAs ($\Delta = 0.4, 1.0$) and Shewhart chart

I : One CUSUM ($k = 0.2$) and Shewhart chart

J : One EWMA ($\Delta = 0.4$) and Shewhart chart

ARL of Individual Chart
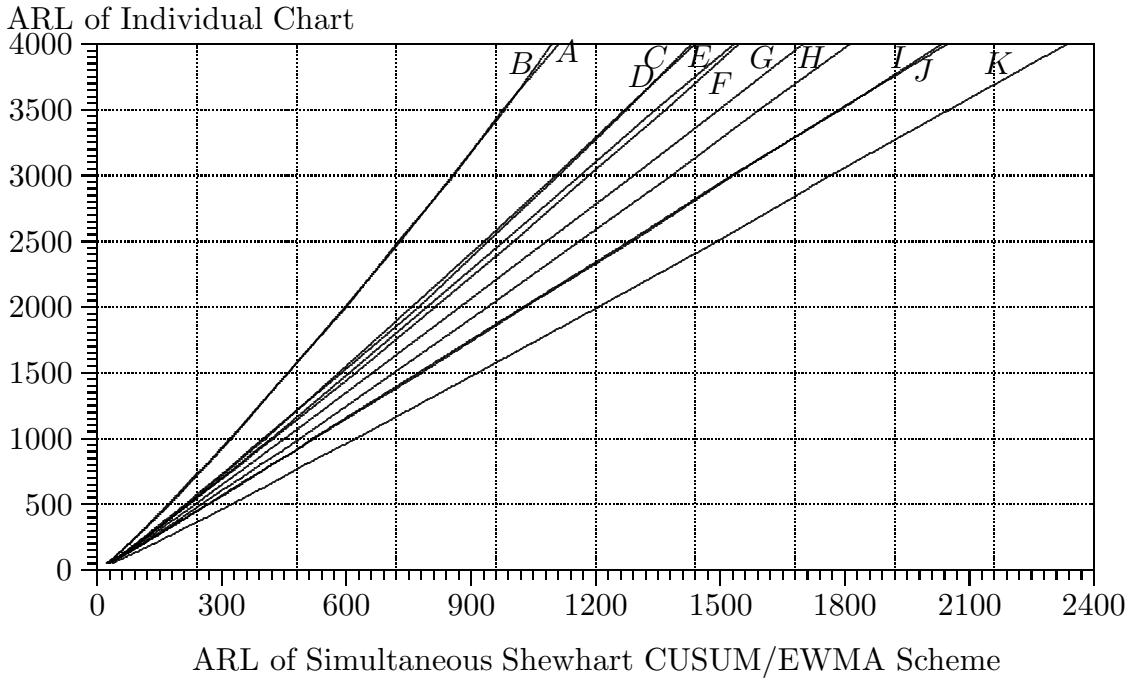
Figure A4. Relationships Between In-Control ARL of a Simultaneous Scheme Designed for Detecting a Shift in the Range $[1.0, \infty)$ and In-Control ARL of Individual Charts

A : Nine CUSUMs ($k = 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 1.0, 1.5$) and Shewhart chart

B : Nine EWMAs ($\Delta = 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6, 2.0, 3.0$) and Shewhart chart

C : Four CUSUMs ($k = 0.5, 0.75, 1.0, 1.25$) and Shewhart chart

D : Four EWMAs ($\Delta = 1.0, 1.5, 2.0, 2.5$) and Shewhart chart

E : Three CUSUMs ($k = 0.5, 0.75, 1.0$) and Shewhart chart

F : Three EWMAs ($\Delta = 1.0, 1.5, 2.0$) and Shewhart chart

G : Two CUSUMs ($k = 0.5, 0.75$) and Shewhart chart

H : Two EWMAs ($\Delta = 1.0, 1.5$) and Shewhart chart

I : One CUSUM ($k = 0.5$) and Shewhart chart

J : One EWMA ($\Delta = 1.0$) and Shewhart chart

K : Sparks' Three CUSUMs ($k = 0.375, 0.5, 0.75$)

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ARL of Individual Chart
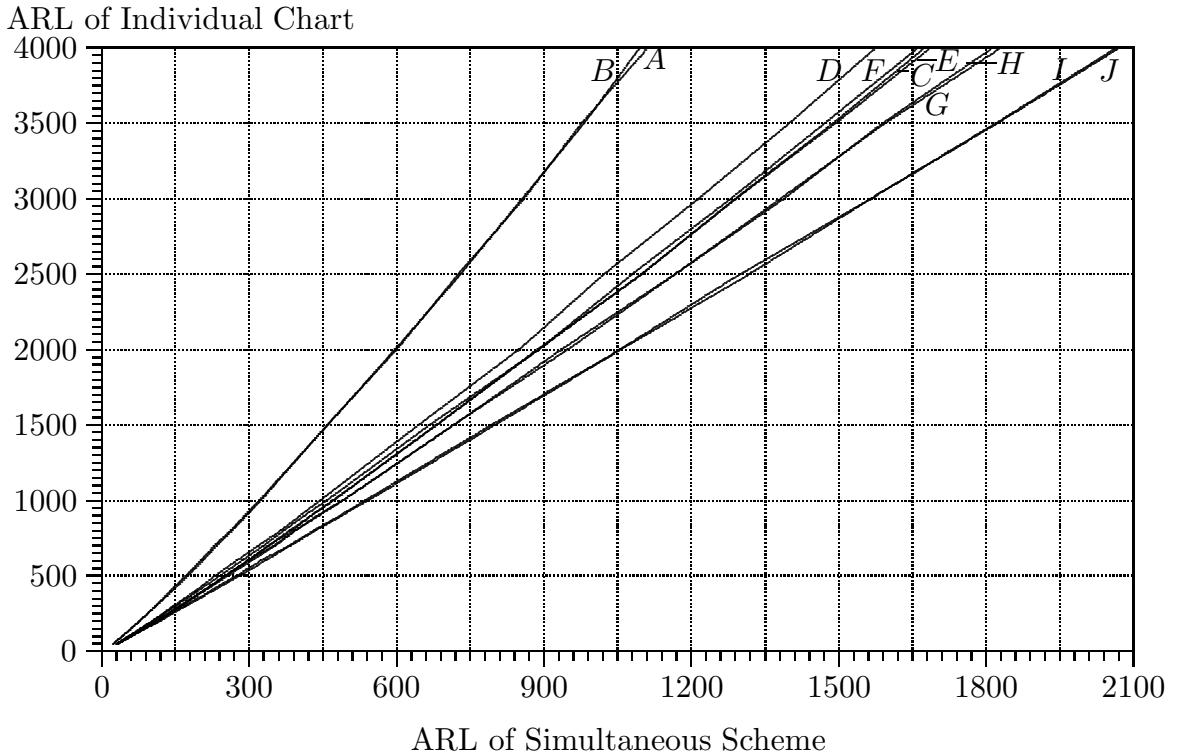
Figure A5. Relationships Between In-Control ARL of a Simultaneous Scheme De-
signed for Detecting a Shift in the Range [1.5,∞) and In-Control ARL
of Individual Charts

A : Nine CUSUMs ($k = 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 1.0, 1.5$) and Shewhart chart

B : Nine EWMAs ($\Delta = 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6, 2.0, 3.0$) and Shewhart chart

C : Four CUSUMs ($k = 0.75, 1.0, 1.25, 1.5$) and Shewhart chart

D : Four EWMAs ($\Delta = 1.5, 2.0, 2.5, 3.0$) and Shewhart chart

E : Three CUSUMs ($k = 0.75, 1.0, 1.25$) and Shewhart chart

F : Three EWMAs ($\Delta = 1.5, 2.0, 2.5$) and Shewhart chart

G : Two CUSUMs ($k = 0.75, 1.0$) and Shewhart chart

H : Two EWMAs ($\Delta = 1.5, 2.0$) and Shewhart chart

I : One CUSUMs ($k = 0.75$) and Shewhart chart

J : One EWMAs ($\Delta = 1.5$) and Shewhart chart

Table A1. Steady State ARL Profiles of the Individual CUSUM Charts, Shewhart
Chart, Simultaneous CUSUM Schemes, Sparks' 3–CUSUM Scheme and
Adaptive CUSUM Schemes

					Δ				
Intended Shift	0.00	0.25	0.50	0.75	1.00	1.50	2.00	2.50	3.00
Individual CUSUM Charts									
0.4	390	73.5	25.9	15.0	10.6	6.68	4.92	3.96	3.33
1.0	390	124	34.4	15.4	9.32	5.12	3.57	2.79	2.31
1.5	390	168	50.6	20.1	10.6	4.97	3.24	2.44	2.00
2.0	390	205	70.6	27.8	13.5	5.34	3.18	2.29	1.82
2.5	390	236	93.8	38.6	18.1	6.23	3.33	2.27	1.73
3.0	390	262	117	51.3	24.4	7.07	3.69	2.33	1.72
Shewhart Chart									
∞	390	297	163	85.0	45.4	15.0	6.48	3.31	2.02
Combined CUSUM-Shewhart Scheme									
[0.4,∞)	390	85.3	29.6	17.0	11.8	7.02	4.65	3.16	2.18
[1.0,∞)	390	146	39.7	17.2	10.2	5.46	3.61	2.60	1.95
[1.5,∞)	390	191	59.2	22.8	11.7	5.31	3.33	2.38	1.81
Simultaneous 2–CUSUM Scheme									
[0.4,∞)	390	87.7	29.6	16.0	10.3	5.63	3.76	2.71	2.02
[1.0,∞)	390	149	40.5	17.4	10.2	5.27	3.41	2.44	1.86
[1.5,∞)	390	195	60.1	23.1	11.8	5.31	3.29	2.33	1.77
Simultaneous 3–CUSUM Scheme									
[0.4,∞)	390	87.6	29.3	15.8	10.2	5.66	3.78	2.73	2.03
[1.0,∞)	390	151	41.5	17.6	10.3	5.27	3.34	2.37	1.81
[1.5,∞)	390	195	60.8	23.1	11.8	5.30	3.27	2.31	1.77
Simultaneous 4–CUSUM Scheme									
[0.4,∞)	390	90.5	29.8	16.0	10.3	5.45	3.46	2.44	1.86
[1.0,∞)	390	153	41.7	17.6	10.3	5.27	3.33	2.35	1.81
[1.5,∞)	390	195	60.8	23.3	11.8	5.29	3.28	2.31	1.76
Super CUSUM Scheme									
[0.4,∞)	390	90.5	30.0	16.0	10.2	5.37	3.42	2.43	1.85
Sparks' 3–CUSUM Schemes									
Scheme 1	390	110	31.0	14.9	9.38	5.08	3.41	2.58	2.11
Adaptive CUSUM Schemes*									
Scheme 1	390	117	32.4	15.1	9.29	5.07	3.50	2.71	2.26
Scheme 2	390	108	30.9	15.0	9.34	5.10	3.53	2.73	2.28
Scheme 3	390	102	30.6	15.1	9.35	5.12	3.54	2.74	2.30

* ARL velues extracted form the Table 3 of Sparks (2000).

Table A2. Steady State ARL Profiles of the Individual EWMA Charts, Shewhart Chart, Simultaneous EWMA Schemes and Adaptive EWMA scheme

Intended Shift	0.00	0.25	0.50	0.75	1.00	Δ 1.50	2.00	2.50	3.00	3.50	4.00	5.00	6.00
Individual EWMA Charts													
1.0	500	121	34.0	16.1	10.0	5.67	4.00	3.13	2.62	2.26	2.01	1.68	1.45
1.5	500	165	46.3	19.4	10.8	5.39	3.60	2.75	2.25	1.94	1.72	1.41	1.19
2.0	500	211	64.8	25.8	13.1	5.59	3.46	2.53	2.03	1.73	1.51	1.21	1.06
2.5	500	246	84.5	33.8	16.5	6.24	3.54	2.46	1.92	1.60	1.38	1.12	1.02
Shewhart Chart													
∞	500	375	202	103	54.4	17.9	7.25	3.58	2.15	1.52	1.22	1.03	1.00
Combined EWMA-Shewhart Scheme													
[0.4,∞)	500	90.2	32.4	18.9	13.1	7.81	5.15	3.43	2.31	1.66	1.30	1.04	1.00
[1.0,∞)	500	148	39.6	18.1	11.1	6.04	4.00	2.84	2.08	1.58	1.28	1.04	1.00
[1.5,∞)	500	199	55.2	22.1	12.0	5.73	3.66	2.61	1.96	1.53	1.26	1.04	1.00
Simultaneous 2–EWMA Scheme													
[0.4,∞)	500	93.5	31.6	17.1	11.2	6.23	4.15	2.96	2.16	1.64	1.31	1.05	1.00
[1.0,∞)	500	150	40.0	18.2	11.0	5.66	3.69	2.64	1.98	1.54	1.27	1.04	1.00
[1.5,∞)	500	204	56.8	22.4	12.1	5.66	3.56	2.52	1.90	1.51	1.26	1.04	1.00
Simultaneous 3–EWMA Scheme													
[0.4,∞)	500	92.8	31.1	16.9	11.1	6.25	4.18	2.98	2.17	1.64	1.31	1.05	1.00
[1.0,∞)	500	155	41.3	18.3	10.9	5.65	3.62	2.58	1.94	1.53	1.27	1.05	1.00
[1.5,∞)	500	205	57.5	22.7	12.1	5.64	3.52	2.50	1.88	1.50	1.25	1.04	1.00
Simultaneous 4–EWMA Scheme													
[0.4,∞)	500	96.8	31.9	17.1	10.9	5.83	3.74	2.65	2.00	1.56	1.30	1.05	1.00
[1.0,∞)	500	156	41.4	18.3	10.9	5.61	3.57	2.52	1.90	1.51	1.26	1.05	1.00
[1.5,∞)	500	207	58.2	22.8	12.1	5.64	3.51	2.46	1.86	1.48	1.25	1.04	1.00
Super EWMA Scheme													
[0.4,∞)	500	98.2	32.2	17.1	10.9	5.76	3.66	2.58	1.94	1.54	1.29	1.05	1.00
Adaptive EWMA Schemes*													
Schemes based on $\phi_{hu}(.)$													
[0.25,4]	500	98.5	40.9	25.0	17.6	10.1	6.08	3.66	2.29	1.60	1.26	1.04	1.00
[0.50,4]	500	115	36.4	19.9	13.4	7.71	4.93	3.24	2.19	1.58	1.26	1.04	1.00
[1.00,4]	500	168	44.9	19.6	11.6	6.13	3.98	2.78	2.03	1.55	1.26	1.04	1.00
[0.25,5]	500	77.3	33.0	20.7	15.0	9.39	6.44	4.43	2.98	2.04	1.49	1.08	1.01
[0.50,5]	500	86.0	29.7	17.0	11.8	7.20	5.01	3.62	2.63	1.92	1.47	1.08	1.01
[1.00,5]	500	131	36.3	16.9	10.4	5.74	3.92	2.92	2.25	1.76	1.42	1.08	1.01
[0.25,6]	500	74.0	30.9	19.3	14.0	9.07	6.69	5.19	4.04	3.08	2.28	1.33	1.05
[0.50,6]	500	82.8	28.6	16.4	11.4	7.12	5.19	4.09	3.24	2.58	2.04	1.32	1.05
[1.00,6]	500	120	34.0	16.2	10.2	5.75	4.04	3.13	2.54	2.09	1.74	1.27	1.05
Schemes based on $\phi_{bs}(.)$													
[0.25,4]	500	135	42.7	22.0	13.9	7.12	4.25	2.80	2.01	1.55	1.28	1.05	1.00
[0.50,4]	500	139	41.2	20.3	12.7	6.59	4.05	2.73	1.99	1.55	1.28	1.05	1.00
[1.00,4]	500	164	45.5	20.0	11.7	5.94	3.74	2.60	1.94	1.53	1.28	1.05	1.00
[0.25,5]	500	99.8	35.9	20.2	13.4	7.27	4.50	3.03	2.19	1.69	1.38	1.08	1.01
[0.50,5]	500	106	33.7	18.0	11.8	6.50	4.16	2.89	2.14	1.67	1.37	1.08	1.01
[1.00,5]	500	148	40.9	18.2	10.8	5.62	3.66	2.65	2.03	1.63	1.36	1.08	1.01
[0.25,6]	500	78.0	31.6	19.1	13.4	7.95	5.32	3.80	2.84	2.22	1.79	1.27	1.05
[0.50,6]	500	86.0	29.3	16.5	11.3	6.70	4.61	3.41	2.64	2.12	1.75	1.27	1.05
[1.00,6]	500	123	34.6	16.4	10.2	5.67	3.90	2.96	2.38	1.98	1.69	1.27	1.05
Schemes based on $\phi_{cub}(.)$													
[0.25,4]	500	97.0	41.5	25.7	18.2	10.5	6.36	3.80	2.35	1.62	1.27	1.04	1.00
[0.50,4]	500	113	35.8	19.5	13.0	7.38	4.67	3.08	2.12	1.57	1.27	1.04	1.00
[1.00,4]	500	156	42.2	19.0	11.5	6.06	3.89	2.70	1.98	1.54	1.27	1.05	1.00
[0.25,5]	500	77.3	32.9	20.5	14.9	9.33	6.41	4.43	2.99	2.04	1.50	1.08	1.01
[0.50,5]	500	86.0	29.8	17.0	11.8	7.20	4.99	3.59	2.59	1.90	1.46	1.08	1.01
[1.00,5]	500	128	35.8	16.8	10.4	5.73	3.88	2.84	2.17	1.71	1.39	1.08	1.01
[0.25,6]	500	74.0	31.3	19.6	14.3	9.29	6.86	5.33	4.15	3.15	2.32	1.34	1.05
[0.50,6]	500	82.4	28.6	16.4	11.4	7.14	5.19	4.03	3.19	2.52	1.98	1.30	1.05
[1.00,6]	500	119	33.8	16.2	10.2	5.77	4.06	3.15	2.56	2.10	1.75	1.27	1.05

* ARL velues extracted form the Table 7 of Capizzi and Masarotto (2003) and the values in [,] represent the values of μ_1 and μ_2 respectively.

Table A3. Steady State ARL Profiles of the Simultaneous CUSUM and Simultaneous EWMA Schemes Designed for Detecting Shifts in the Range $[0.4, \infty)$

	Combined CUSUM	Combined EWMA	2–CUSUM	2–EWMA	3–CUSUM	3–EWMA	4–CUSUM	4–EWMA	Super* CUSUM	Super# EWMA
							3.320	1.000		
λ					3.274	1.000	1.0	0.306		
or			3.267	1.000	0.5	0.118	0.5	0.119		
k	3.198	1.000	0.5	0.125	0.3	0.057	0.3	0.054		
	0.2	0.031	0.2	0.031	0.2	0.031	0.2	0.029		
							3.320	3.308		
					3.274	3.266	3.066	1.386		
h			3.267	3.259	5.714	0.768	5.879	0.784		
	3.198	3.198	5.679	0.793	8.623	0.496	8.890	0.489		
	10.940	0.329	11.500	0.340	11.580	0.341	11.970	0.334		
Δ					ARL					
0.00	370	370	370	370	370	370	370	370	370	370
0.20	116	107	119	112	120	112	123	115	124	118
0.30	63.1	61.1	64.5	62.3	64.4	62.0	66.2	63.2	66.4	65.4
0.40	40.4	40.5	41.0	40.4	40.7	40.1	41.9	40.9	41.9	41.8
0.50	29.1	29.9	29.0	29.0	28.8	28.8	29.5	29.1	29.5	29.7
0.60	22.6	23.6	22.1	22.2	21.9	21.9	22.3	22.2	22.2	22.4
0.80	15.4	16.4	14.3	14.5	14.2	14.4	14.3	14.3	14.3	14.4
1.00	11.6	12.5	10.2	10.4	10.1	10.4	10.1	10.2	10.1	10.2
1.50	6.93	7.39	5.57	5.83	5.59	5.88	5.38	5.51	5.32	5.44
2.00	4.59	4.82	3.71	3.88	3.73	3.94	3.43	3.55	3.40	3.48
2.50	3.12	3.20	2.68	2.77	2.69	2.80	2.42	2.52	2.41	2.46
4.00	1.26	1.26	1.27	1.26	1.27	1.27	1.25	1.26	1.25	1.25

* Chart parameters (k, h) of nine CUSUMs are (0.2,12.070), (0.3,8.953), (0.4,7.133), (0.5 ,5.921), (0.6,5.0421), (0.7,4.379), (0.8,3.858), (1.0,3.089), (1.5,1.996) and chart limit for the Shewhart chart is 3.335.

Chart parameters (λ, h) of nine EWMAs for $\Delta = 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6, 2.0$, and 3.0 are (0.031, 0.352), (0.055, 0.500), (0.083, 0.640), (0.114, 0.772), (0.148, 0.902), (0.184, 1.027), (0.223, 1.151), (0.305, 1.395), (0.543, 2.031) and chart limit for the Shewhart chart is 3.335.

Table A4. Steady State ARL Profiles of the Simultaneous CUSUM and Simultaneous EWMA Schemes Designed for Detecting Shifts in the Range $[1.0,\infty)$

|	Combined CUSUM	Combined EWMA	2–CUSUM	2–EWMA	3–CUSUM	3–EWMA	4–CUSUM	4–EWMA	Sparks' 3–CUSUM	Super* EWMA	Super# EWMA
							3.264	1.000			
λ					3.254	1.000	1.25	0.431			
or			3.230	1.000	1.0	0.318	1.0	0.317	0.75		
k	1.389	1.000	0.75	0.220	0.75	0.213	0.75	0.211	0.5		
	0.5	0.120	0.5	0.119	0.5	0.118	0.5	0.122	0.086		
							3.264	3.263			
					3.254	3.250	2.343	1.698			
h			3.230	3.208	2.948	1.395	2.965	1.395	3.636		
	3.189	3.190	3.859	1.094	3.917	1.089	3.940	1.086	5.226		
	5.415	0.754	5.554	0.756	5.642	0.762	5.676	0.783	6.606		
Δ					ARL						
0.00	370	370	370	370	370	370	370	370	370	370	370
0.20	187	163	190	158	193	171	194	175	148	124	118
0.30	106	90.0	108	89.3	110	94.5	111	98.1	79.3	66.4	65.4
0.40	62.1	53.7	63.2	53.4	64.3	56.4	65.0	58.6	47.0	41.9	41.8
0.50	39.1	35.1	39.8	34.9	40.3	36.5	40.7	37.8	30.7	29.5	29.7
0.60	26.6	24.9	27.0	24.6	27.4	25.5	27.7	26.3	21.8	22.2	22.4
0.80	15.0	14.5	15.2	14.7	15.3	15.0	15.4	15.2	13.3	14.3	14.4
1.00	10.1	10.4	10.1	10.0	10.2	10.2	10.2	10.2	9.33	10.1	10.2
1.50	5.40	5.69	5.21	5.32	5.19	5.32	5.20	5.29	5.06	5.32	5.44
2.00	3.58	3.79	3.36	3.49	3.31	3.44	3.30	3.39	3.37	3.40	3.48
2.50	2.57	2.69	2.42	2.49	2.35	2.45	2.34	2.41	2.56	2.41	2.46
4.00	1.24	1.24	1.23	1.23	1.23	1.24	1.23	1.23	1.57	1.25	1.25

* Chart parameters (k,h) of nine CUSUMs are (0.2,12.070), (0.3,8.953), (0.4,7.133), (0.5 ,5.921), (0.6,5.0421), (0.7,4.379), (0.8,3.858), (1.0,3.089), (1.5,1.996) and chart limit for the Shewhart chart is 3.335.

Chart parameters (λ, h) of nine EWMAs for $\Delta = 0.4$, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6, 2.0, and 3.0 are (0.031, 0.352), (0.055, 0.500), (0.083, 0.640), (0.114, 0.772), (0.148, 0.902), (0.184, 1.027), (0.223, 1.151), (0.305, 1.395), (0.543, 2.031) and chart limit for the Shewhart chart is 3.335.

Table A5. Steady State ARL Profiles of the Simultaneous CUSUM and Simultaneous EWMA Schemes Designed for Detecting Shifts in the Range [1.5,∞)

	Combined CUSUM	Combined EWMA	2–CUSUM	2–EWMA	3–CUSUM	3–EWMA	4–CUSUM	4–EWMA	Super* CUSUM	Super# EWMA
							3.214	1.000		
λ					3.215	1.000	1.5	0.559		
or			3.208	1.000	1.25	0.431	1.25	0.441		
k	3.177	1.000	1.0	0.323	1.0	0.324	1.0	0.322		
	0.75	0.220	0.75	0.220	0.75	0.222	0.75	0.212		
							3.214			
					3.215	3.225	1.853	2.011		
h			3.208	3.207	2.274	1.670	2.273	1.712		
	3.177	3.182	2.863	1.337	2.877	1.396	2.876	1.397		
	3.737	1.084	3.807	1.094	3.825	1.106	3.824	1.080		
Δ					ARL					
0.00	370	370	370	370	370	370	370	370	370	370
0.20	228	204	231	206	230	209	230	210	124	118
0.30	146	123	148	127	148	128	147	128	66.4	65.4
0.40	91.2	75.0	93.0	76.8	93.2	78.7	92.5	78.2	41.9	41.8
0.50	57.7	47.9	58.7	49.3	58.9	50.6	58.7	49.7	29.5	29.7
0.60	38.1	32.6	38.8	33.4	39.0	34.0	38.8	33.6	22.2	22.4
0.80	19.2	17.5	19.5	17.9	19.5	18.0	19.4	17.9	14.3	14.4
1.00	11.5	11.1	11.6	11.2	11.6	11.3	11.6	11.2	10.1	10.2
1.50	5.23	5.43	5.24	5.36	5.25	5.32	5.24	5.33	5.32	5.44
2.00	3.30	3.49	3.25	3.38	3.24	3.33	3.25	3.33	3.40	3.48
2.50	2.36	2.48	2.31	2.40	2.29	2.36	2.29	2.36	2.41	2.46
4.00	1.22	1.22	1.22	1.22	1.21	1.22	1.22	1.22	1.25	1.25

* Chart parameters (k, h) of nine CUSUMs are (0.2,12.070), (0.3,8.953), (0.4,7.133), (0.5,5.921), (0.6,5.0421), (0.7,4.379), (0.8,3.858), (1.0,3.089), (1.5,1.996) and chart limit for the Shewhart chart is 3.335.

Chart parameters (λ, h) of nine EWMAs for Δ = 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6, 2.0, and 3.0 are (0.031, 0.352), (0.055, 0.500), (0.083, 0.640), (0.114, 0.772), (0.148, 0.902), (0.184, 1.027), (0.223, 1.151), (0.305, 1.395), (0.543, 2.031) and chart limit for the Shewhart chart is 3.335.

Appendix 2

SAS Programs for Parts 1 and Part 2

SAS Programs for Simultaneous CUSUM Charting Schemes

SAS Programs for Simultaneous EWMA Charting Schemes

SAS Programs for Shewhart Charts

Program 1:

```
*********************************************************************
*                         Two Sided Charts                         *
*            SAS Program to Find the Number of CUSUM Charts in a    *
*            Simultaneous Scheme Together With the Shewhart Chart.  *
*                                                                   *
*      1. Simultaneous CUSUM Schemes with Steady State Limits.      *
*      2. ARL of Each CUSUM Chart and the Shewhart Chart is 1000.   *
*      3. ARL of Each Simultaneous CUSUM Scheme will be Calculated. *
*      4. The Order of Charts Added to a Scheme is as Follows;      *
*            Delta = 0.4, Shewhart, 1.6, 0.8, 2.0, 1.2, 4.0,        *
*                    0.6, 1.8, 1.0, 3.0 1.4 And 6.0                 *
*********************************************************************;
data;
seed=499862659;
delta=0.0;
n=4;
mu=0.0;
sigma=2.0;
oneml=(1-lambda);
shift=delta*sigma/n**0.5;
numrun=100000;

* h values for the different reference (rk) values and delta's with
individual ARL 1000;
rk1=0.2;          h1=11.6919;        *optimal for detecting delta=0.4;
*rk2=0.3;         *h2=8.6977;        *optimal for detecting delta=0.6;
*rk3=0.4;         *h3=6.9393;        *optimal for detecting delta=0.8;
*rk4=0.5;         *h4=5.7648;        *optimal for detecting delta=1.0;
*rk5=0.6;         *h5=4.9117;        *optimal for detecting delta=1.2;
*rk6=0.7;         *h6=4.2667;        *optimal for detecting delta=1.4;
*rk7=0.8;         *h7=3.7595;        *optimal for detecting delta=1.6;
*rk8=0.9;         *h8=3.3484;        *optimal for detecting delta=1.8;
*rk9=1.0;         *h9=3.0101;        *optimal for detecting delta=2.0;
*rk10=1.5;        *h10=1.9425;       *optimal for detecting delta=3.0;
*rk11=2.0;        *h11=1.3171;       *optimal for detecting delta=4.0;
*rk12=3.0;        *h12=0.2907;       *optimal for detecting delta=6.0;
*lambda=1.0;      *h13=3.2905;       *Shewhart chart;

*Upper and Lower control Limits;
lcl1=-h1;
ucl1=h1;
*lcl2=-h2;
*ucl2=h2;
*lcl3=-h3;
*ucl3=h3;
*lcl4=-h4;
*ucl4=h4;
*lcl5=-h5;
*ucl5=h5;
*lcl6=-h6;
*ucl6=h6;
*lcl7=-h7;
*ucl7=h7;
*lcl8=-h8;
*ucl8=h8;
*lcl9=-h9;
*ucl9=h9;
*lcl10=-h10;
*ucl10=h10;
*lcl11=-h11;
```

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```
*ucl11=h11;
*lcl12=-h12;
*ucl12=h12;
*lcl13=-h13;
*ucl13=h13;


*Variable to count the number of signals;
signal1=0;
*signal2=0;
*signal3=0;
*signal4=0;
*signal5=0;
*signal6=0;
*signal7=0;
*signal8=0;
*signal9=0;
*signal10=0;
*signal11=0;
*signal12=0;
*signal13=0;


*Simulations;
do i=1 to numrun;
restart:
*Upper CUSUM statistic;
      cup1=0.0;
      *cup2=0.0;
      *cup3=0.0;
      *cup4=0.0;
      *cup5=0.0;
      *cup6=0.0;
      *cup7=0.0;
      *cup8=0.0;
      *cup9=0.0;
      *cup10=0.0;
      *cup11=0.0;
      *cup12=0.0;

*Lower CUSUM statistic;
      cdown1=0.0;
      *cdown2=0.0;
      *cdown3=0.0;
      *cdown4=0.0;
      *cdown5=0.0;
      *cdown6=0.0;
      *cdown7=0.0;
      *cdown8=0.0;
      *cdown9=0.0;
      *cdown10=0.0;
      *cdown11=0.0;
      *cdown12=0.0;
      m2=0;
      runlength=0;
*For Steady State;
      do j=1 to 100;
              x1=mu+rannor(seed)*sigma;
              x2=mu+rannor(seed)*sigma;
              x3=mu+rannor(seed)*sigma;
              x4=mu+rannor(seed)*sigma;
              xbar=(x1+x2+x3+x4)/n;
              Zt=xbar;
```

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```
*upper sided cusum;
            cup1=(Zt-rk1)+cup1;
            *cup2=(Zt-rk2)+cup2;
            *cup3=(Zt-rk3)+cup3;
            *cup4=(Zt-rk4)+cup4;
            *cup5=(Zt-rk5)+cup5;
            *cup6=(Zt-rk6)+cup6;
            *cup7=(Zt-rk7)+cup7;
            *cup8=(Zt-rk8)+cup8;
            *cup9=(Zt-rk9)+cup9;
            *cup10=(Zt-rk10)+cup10;
            *cup11=(Zt-rk11)+cup11;
            *cup12=(Zt-rk12)+cup12;

*lower sided cusum;
            cdown1=(Zt+rk1)+cdown1;
            *cdown2=(Zt+rk2)+cdown2;
            *cdown3=(Zt+rk3)+cdown3;
            *cdown4=(Zt+rk4)+cdown4;
            *cdown5=(Zt+rk5)+cdown5;
            *cdown6=(Zt+rk6)+cdown6;
            *cdown7=(Zt+rk7)+cdown7;
            *cdown8=(Zt+rk8)+cdown8;
            *cdown9=(Zt+rk9)+cdown9;
            *cdown10=(Zt+rk10)+cdown10;
            *cdown11=(Zt+rk11)+cdown11;
            *cdown12=(Zt+rk12)+cdown12;

*check upper sided cusum;
            if cup1<0 then cup1=0;
            *if cup2<0 then cup2=0;
            *if cup3<0 then cup3=0;
            *if cup4<0 then cup4=0;
            *if cup5<0 then cup5=0;
            *if cup6<0 then cup6=0;
            *if cup7<0 then cup7=0;
            *if cup8<0 then cup8=0;
            *if cup9<0 then cup9=0;
            *if cup10<0 then cup10=0;
            *if cup11<0 then cup11=0;
            *if cup12<0 then cup12=0;

*check lower sided cusum;
            if cdown1>0 then cdown1=0;
            *if cdown2>0 then cdown2=0;
            *if cdown3>0 then cdown3=0;
            *if cdown4>0 then cdown4=0;
            *if cdown5>0 then cdown5=0;
            *if cdown6>0 then cdown6=0;
            *if cdown7>0 then cdown7=0;
            *if cdown8>0 then cdown8=0;
            *if cdown9>0 then cdown9=0;
            *if cdown10>0 then cdown10=0;
            *if cdown11>0 then cdown11=0;
            *if cdown12>0 then cdown12=0;

            *m2=oneml*m2 + lambda*xbar;
        end;
        if
            cup1>ucl1 or cdown1<lcl1
        then goto restart;
```

```
        *
        or cup2>ucl2 or cdown2<lcl2
        or cup3>ucl3 or cdown3<lcl3
        or cup4>ucl4 or cdown4<lcl4
        or cup5>ucl5 or cdown5<lcl5
        or cup6>ucl6 or cdown6<lcl6
        or cup7>ucl7 or cdown7<lcl7
        or cup8>ucl8 or cdown8<lcl8
        or cup9>ucl9 or cdown9<lcl9
        or cup10>ucl10 or cdown10<lcl10
        or cup11>ucl11 or cdown11<lcl11
        or cup12>ucl12 or cdown12<lcl12
        or m2<lcl13 or m2>ucl13
        ;

        nextsamp:
        runlength=runlength+1;
        x1=(mu+shift)+rannor(seed)*sigma;
        x2=(mu+shift)+rannor(seed)*sigma;
        x3=(mu+shift)+rannor(seed)*sigma;
        x4=(mu+shift)+rannor(seed)*sigma;
        xbar=(x1+x2+x3+x4)/n;
        Zt=xbar;

*upper sided cusum;
        cup1=(Zt-rk1)+cup1;
        *cup2=(Zt-rk2)+cup2;
        *cup3=(Zt-rk3)+cup3;
        *cup4=(Zt-rk4)+cup4;
        *cup5=(Zt-rk5)+cup5;
        *cup6=(Zt-rk6)+cup6;
        *cup7=(Zt-rk7)+cup7;
        *cup8=(Zt-rk8)+cup8;
        *cup9=(Zt-rk9)+cup9;
        *cup10=(Zt-rk10)+cup10;
        *cup11=(Zt-rk11)+cup11;
        *cup12=(Zt-rk12)+cup12;

*lower sided cusum;
        cdown1=(Zt+rk1)+cdown1;
        *cdown2=(Zt+rk2)+cdown2;
        *cdown3=(Zt+rk3)+cdown3;
        *cdown4=(Zt+rk4)+cdown4;
        *cdown5=(Zt+rk5)+cdown5;
        *cdown6=(Zt+rk6)+cdown6;
        *cdown7=(Zt+rk7)+cdown7;
        *cdown8=(Zt+rk8)+cdown8;
        *cdown9=(Zt+rk9)+cdown9;
        *cdown10=(Zt+rk10)+cdown10;
        *cdown11=(Zt+rk11)+cdown11;
        *cdown12=(Zt+rk12)+cdown12;

        if cup1<0 then cup1=0;
        *if cup2<0 then cup2=0;
        *if cup3<0 then cup3=0;
        *if cup4<0 then cup4=0;
        *if cup5<0 then cup5=0;
        *if cup6<0 then cup6=0;
        *if cup7<0 then cup7=0;
        *if cup8<0 then cup8=0;
        *if cup9<0 then cup9=0;
```

```
*if cup10<0 then cup10=0;
*if cup11<0 then cup11=0;
*if cup12<0 then cup12=0;

if cdown1>0 then cdown1=0;
*if cdown2>0 then cdown2=0;
*if cdown3>0 then cdown3=0;
*if cdown4>0 then cdown4=0;
*if cdown5>0 then cdown5=0;
*if cdown6>0 then cdown6=0;
*if cdown7>0 then cdown7=0;
*if cdown8>0 then cdown8=0;
*if cdown9>0 then cdown9=0;
*if cdown10>0 then cdown10=0;
*if cdown11>0 then cdown11=0;
*if cdown12>0 then cdown12=0;

*m2=oneml*m2 + lambda*xbar;
nsignal=0;
If cdown1<lcl1 or cup1>ucl1 then do;
                    signal1=signal1+1;
                    nsignal=nsignal+1;
                        end;
*If cdown2<lcl2 or cup2>ucl2 then do;
*                   signal2=signal2+1;
*                   nsignal=nsignal+1;
*                       end;
*If cdown3<lcl3 or cup3>ucl3 then do;
*                   signal3=signal3+1;
*                   nsignal=nsignal+1;
*                       end;
*If cdown4<lcl4 or cup4>ucl4 then do;
*                   signal4=signal4+1;
*                   nsignal=nsignal+1;
*                       end;
*If cdown5<lcl5 or cup5>ucl5 then do;
*                   signal5=signal5+1;
*                   nsignal=nsignal+1;
*                       end;
*If cdown6<lcl6 or cup6>ucl6 then do;
*                   signal6=signal6+1;
*                   nsignal=nsignal+1;
*                       end;
*If cdown7<lcl7 or cup7>ucl7 then do;
*                   signal7=signal7+1;
*                   nsignal=nsignal+1;
*                       end;
*If cdown8<lcl8 or cup8>ucl8 then do;
*                   signal8=signal8+1;
*                   nsignal=nsignal+1;
*                       end;
*If cdown9<lcl9 or cup9>ucl9 then do;
*                   signal9=signal9+1;
*                   nsignal=nsignal+1;
*                       end;
*If cdown10<lcl10 or cup10>ucl10 then do;
*                   signal10=signal10+1;
*                   nsignal=nsignal+1;
*                       end;
*If cdown11<lcl11 or cup11>ucl11 then do;
*                   signal11=signal11+1;
```

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```
*                              nsignal=nsignal+1;
*                                  end;
*If cdown12<lcl12 or cup12>ucl12 then do;
*                              signal12=signal12+1;
*                              nsignal=nsignal+1;
*                                  end;
*If m2<lcl13 or m2>ucl13 then do;
*                              signal13=signal13+1;
*                              nsignal=nsignal+1;
*                                  end;
        If nsignal>0 then goto endchart;
        else goto nextsamp;
        endchart:
             output;
end;


proc means n mean stderr;
   var runlength;
run;



Program 2:
*********************************************************************
*                       One Sided Charts                           *
*         SAS Program to Find the Number of CUSUM Charts in a       *
*         Simultaneous Scheme Together With the Shewhart Chart.     *
*                                                                   *
*      1. Simultaneous CUSUM Schemes with Steady State Limits.      *
*      2. ARL of Each CUSUM Chart and the Shewhart Chart is 1000.   *
*      3. ARL of Each Simultaneous CUSUM Scheme will be Calculated. *
*      4. The Order of Charts Added to a Scheme is as Follows;      *
*           Delta = 0.4, Shewhart, 1.6, 0.8, 2.0, 1.2, 4.0,         *
*                   0.6, 1.8, 1.0, 3.0 1.4 And 6.0                  *
*********************************************************************;
data;
seed=499862659;
delta=0.0;
n=4;
mu=0.0;
sigma=2.0;
oneml=(1-lambda);
shift=delta*sigma/n**0.5;
numrun=100000;

* h values for the different reference (rk) values and delta's with
individual ARL 1000;
rk1=0.2;        h1=10.01760;        *optimal for detecting delta=0.4;
*rk2=0.3;       *h2=7.57635;        *optimal for detecting delta=0.6;
*rk3=0.4;       *h3=6.08775;        *optimal for detecting delta=0.8;
*rk4=0.5;       *h4=5.07383;        *optimal for detecting delta=1.0;
*rk5=0.6;       *h5=4.34385;        *optimal for detecting delta=1.2;
*rk6=0.7;       *h6=3.77455;        *optimal for detecting delta=1.4;
*rk7=0.8;       *h7=3.33300;        *optimal for detecting delta=1.6;
*rk8=0.9;       *h8=2.96408;        *optimal for detecting delta=1.8;
*rk9=1.0;       *h9=2.66657;        *optimal for detecting delta=2.0;
*rk10=1.5;      *h10=1.70807;       *optimal for detecting delta=3.0;
*rk11=2.0;      *h11=1.11104;       *optimal for detecting delta=4.0;
*rk12=3.0;      *h12=0.08977;       *optimal for detecting delta=6.0;
*lambda=1.0;    *h13=3.08969;       *Shewhart chart;
```

```
*Upper control Limits;
ucl1=h1;
*ucl2=h2;
*ucl3=h3;
*ucl4=h4;
*ucl5=h5;
*ucl6=h6;
*ucl7=h7;
*ucl8=h8;
*ucl9=h9;
*ucl10=h10;
*ucl11=h11;
*ucl12=h12;
*ucl13=h13;

*Variable to count the number of signals;
signal1=0;
*signal2=0;
*signal3=0;
*signal4=0;
*signal5=0;
*signal6=0;
*signal7=0;
*signal8=0;
*signal9=0;
*signal10=0;
*signal11=0;
*signal12=0;
*signal13=0;

*Simulations;
do i=1 to numrun;
restart:
*Upper CUSUM statistic;
      cup1=0.0;
      *cup2=0.0;
      *cup3=0.0;
      *cup4=0.0;
      *cup5=0.0;
      *cup6=0.0;
      *cup7=0.0;
      *cup8=0.0;
      *cup9=0.0;
      *cup10=0.0;
      *cup11=0.0;
      *cup12=0.0;

      m2=0;
      runlength=0;

      *For Steady State;
      do j=1 to 100;
            x1=mu+rannor(seed)*sigma;
            x2=mu+rannor(seed)*sigma;
            x3=mu+rannor(seed)*sigma;
            x4=mu+rannor(seed)*sigma;
            xbar=(x1+x2+x3+x4)/n;
            Zt=xbar;
*upper sided cusum;
            cup1=(Zt-rk1)+cup1;
            *cup2=(Zt-rk2)+cup2;
```

```
            *cup3=(Zt-rk3)+cup3;
            *cup4=(Zt-rk4)+cup4;
            *cup5=(Zt-rk5)+cup5;
            *cup6=(Zt-rk6)+cup6;
            *cup7=(Zt-rk7)+cup7;
            *cup8=(Zt-rk8)+cup8;
            *cup9=(Zt-rk9)+cup9;
            *cup10=(Zt-rk10)+cup10;
            *cup11=(Zt-rk11)+cup11;
            *cup12=(Zt-rk12)+cup12;
*check upper sided cusum;
            if cup1<0 then cup1=0;
            *if cup2<0 then cup2=0;
            *if cup3<0 then cup3=0;
            *if cup4<0 then cup4=0;
            *if cup5<0 then cup5=0;
            *if cup6<0 then cup6=0;
            *if cup7<0 then cup7=0;
            *if cup8<0 then cup8=0;
            *if cup9<0 then cup9=0;
            *if cup10<0 then cup10=0;
            *if cup11<0 then cup11=0;
            *if cup12<0 then cup12=0;

            *m2=oneml*m2 + lambda*xbar;
        end;

        if
            cup1>ucl1
        then goto restart;
        *
        or cup2>ucl2
        or cup3>ucl3
        or cup4>ucl4
        or cup5>ucl5
        or cup6>ucl6
        or cup7>ucl7
        or cup8>ucl8
        or cup9>ucl9
        or cup10>ucl10
        or cup11>ucl11
        or cup12>ucl12
        or m2>ucl13
        ;

        nextsamp:
        runlength=runlength+1;
        x1=(mu+shift)+rannor(seed)*sigma;
        x2=(mu+shift)+rannor(seed)*sigma;
        x3=(mu+shift)+rannor(seed)*sigma;
        x4=(mu+shift)+rannor(seed)*sigma;
        xbar=(x1+x2+x3+x4)/n;
        Zt=xbar;

*upper sided cusum;
        cup1=(Zt-rk1)+cup1;
        *cup2=(Zt-rk2)+cup2;
        *cup3=(Zt-rk3)+cup3;
        *cup4=(Zt-rk4)+cup4;
        *cup5=(Zt-rk5)+cup5;
        *cup6=(Zt-rk6)+cup6;
```

```
*cup7=(Zt-rk7)+cup7;
*cup8=(Zt-rk8)+cup8;
*cup9=(Zt-rk9)+cup9;
*cup10=(Zt-rk10)+cup10;
*cup11=(Zt-rk11)+cup11;
*cup12=(Zt-rk12)+cup12;
if cup1<0 then cup1=0;
*if cup2<0 then cup2=0;
*if cup3<0 then cup3=0;
*if cup4<0 then cup4=0;
*if cup5<0 then cup5=0;
*if cup6<0 then cup6=0;
*if cup7<0 then cup7=0;
*if cup8<0 then cup8=0;
*if cup9<0 then cup9=0;
*if cup10<0 then cup10=0;
*if cup11<0 then cup11=0;
*if cup12<0 then cup12=0;

*m2=oneml*m2 + lambda*xbar;
nsignal=0;
If cup1>ucl1 then do;
                          signal1=signal1+1;
                          nsignal=nsignal+1;
                  end;
*If cup2>ucl2 then do;
*                         signal2=signal2+1;
*                         nsignal=nsignal+1;
*                 end;
*If cup3>ucl3 then do;
*                         signal3=signal3+1;
*                         nsignal=nsignal+1;
*                 end;
*If cup4>ucl4 then do;
*                         signal4=signal4+1;
*                         nsignal=nsignal+1;
*                 end;
*If cup5>ucl5 then do;
*                         signal5=signal5+1;
*                         nsignal=nsignal+1;
*                 end;
*If cup6>ucl6 then do;
*                         signal6=signal6+1;
*                         nsignal=nsignal+1;
*                 end;
*If cup7>ucl7 then do;
*                         signal7=signal7+1;
*                         nsignal=nsignal+1;
*                 end;
*If cup8>ucl8 then do;
*                         signal8=signal8+1;
*                         nsignal=nsignal+1;
*                 end;
*If cup9>ucl9 then do;
*                         signal9=signal9+1;
*                         nsignal=nsignal+1;
*                 end;
*If cup10>ucl10 then do;
*                         signal10=signal10+1;
*                         nsignal=nsignal+1;
*                 end;
```

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```
        *If cup11>ucl11 then do;
        *                       signal11=signal11+1;
        *                       nsignal=nsignal+1;
        *                end;
        *If cup12>ucl12 then do;
        *                       signal12=signal12+1;
        *                       nsignal=nsignal+1;
        *                end;
        *If m2<lcl13 or m2>ucl13 then do;
        *                       signal13=signal13+1;
        *                       nsignal=nsignal+1;
        *                end;

        If nsignal>0 then goto endchart;
        else goto nextsamp;
        endchart:
                output;
end;
proc means n mean stderr;
   var runlength;
run;
```

Program 3:
```
********************************************************************
*                       Two-Sided Charts                          *
*       SAS Program to Find the ARL Profile of Simultaneous        *
*                       CUSUM Schemes                             *
*                                                                  *
*       1. Find the ARL of Simultaneous CUSUM Scheme              *
*       2. Firstly, Find the ARL of Component Individual Chart     *
*          to Get the ARL of Simultaneous CUSUM Scheme as 370     *
*       3. Then Find the Parameters of These Individual Charts     *
*       4. Then Run These Charts Simultaneously                   *
********************************************************************;
data;
delta=0.0;
mu=0.0;
sigma=2.0;
oneml=(1-lambda);
shift=delta*sigma/n**0.5;
n=4;
numrun=100000;
seed=499852037;

* Two-sided individual CUSUM charts to get ARL=370;
        *rk1=0.2;   *h1=9.4119;         *optimal for detecting delta=0.4;
        *rk1=0.25;  *h1=8.1047;         *optimal for detecting delta=0.5;
        *rk1=0.3;   *h1=7.1335;         *optimal for detecting delta=0.6;
        *rk1=0.4;   *h1=5.7435;         *optimal for detecting delta=0.8;
        *rk1=0.5;   *h1=4.7935;         *optimal for detecting delta=1.0;
        *rk1=0.6;   *h1=4.0974;         *optimal for detecting delta=1.2;
        *rk1=0.7;   *h1=3.5629;         *optimal for detecting delta=1.4;
        *rk1=0.75;  *h1=3.3454;         *optimal for detecting delta=1.5;
        *rk1=0.8;   *h1=3.1435;         *optimal for detecting delta=0.8;
        *rk1=1.0;   *h1=2.5194;         *optimal for detecting delta=2.0;
        *rk1=1.25;  *h1=1.9872;         *optimal for detecting delta=2.5;
        *rk1=1.5;   *h1=1.6035;         *optimal for detecting delta=3.0;
        *rk1=2.0;   *h1=1.0167;         *optimal for detecting delta=4.0;

* Combined Shewhart-CUSUM Schemes with two-sided individual charts;
```

```
        *To get ARL=370 for combined Shewhart-CUSUM scheme in detecting
        [0.4,infinity)
         ARL of component individual chart ARL= 721;
        *rk1=0.2;    *h1=10.940;         *optimal for detecting delta=0.4;
        *lambda=1.0;*h10=3.198;          *Shewhart chart;


        *To get ARL=370 for combined Shewhart-CUSUM scheme in detecting
        [1.0, infinity)
         ARL of component individual chart ARL= 701;
        *rk1=0.5;    *h1=5.4150;         *optimal for detecting delta=1.0;
        *lambda=1.0;*h10=3.1891;         *Shewhart chart;


        *To get ARL=370 for combined Shewhart-CUSUM scheme in detecting
        [1.5, infinity)
         ARL of component individual chart ARL= 675;
        *rk1=0.75;   *h1=3.7370;         *optimal for detecting delta=1.5;
        *lambda=1.0;*h10=3.1775;         *Shewhart chart;


* 2-CUSUM Schemes with two-sided individual charts;
        *To get ARL=370 for 2-CUSUM scheme in detecting [0.4, infinity)
         ARL of component individual chart ARL= 915;
        *rk1=0.2;    *h1=11.496;         *optimal for detecting delta=0.4;
        *rk2=0.5;    *h2=5.6790;         *optimal for detecting delta=1.0;
        *lambda=1.0;*h10=3.2668;         *Shewhart Chart;


        *To get ARL=370 for 2-CUSUM scheme in detecting [1.0, infinity)
         ARL of component individual chart ARL= 805;
        *rk1=0.5;    *h1=5.5541;         *optimal for detecting delta=1.0;
        *rk2=0.75;   *h2=3.8592;         *optimal for detecting delta=1.5;
        *lambda=1.0;*h10=3.2299;         *Shewhart chart;


        *To get ARL=370 for 2-CUSUM scheme in detecting [1.5, infinity)
         ARL of component individual chart ARL= 745;
        *rk1=0.75;   *h1=3.8078;         *optimal for detecting delta=1.5;
        *rk2=1.0;    *h2=2.8634;         *optimal for detecting delta=2.0;
        *lambda=1.0;*h10=3.2075;         *Shewhart chart;


* 3-CUSUM Schemes with two-sided individual charts;
        *To get ARL=370 for 3-CUSUM scheme in detecting [0.4, infinity)
         ARL of component individual chart ARL= 945;
        *rk1=0.2;    *h1=11.5751;        *optimal for detecting delta=0.4;
        *rk2=0.3;    *h2=8.6232;         *optimal for detecting delta=0.6;
        *rk3=0.5;    *h3=5.7142;         *optimal for detecting delta=1.0;
        *lambda=1.0;*h10=3.27417;        *Shewhart chart;


        *To get ARL=370 for 3-CUSUM scheme in detecting [1.0, infinity)
         ARL of component individual chart ARL= 880;
        *rk1=0.5;    *h1=5.6418;         *optimal for detecting delta=1.0;
        *rk2=0.75;   *h2=3.9171;         *optimal for detecting delta=1.5;
        *rk3=1.0;    *h3=2.9478;         *optimal for detecting delta=2.0;
        *lambda=1.0;*h10=3.2543;         *Shewhart chart;


        *To get ARL=370 for 3-CUSUM scheme in detecting [1.5, infinity)
         ARL of component individual chart ARL= 765;
        *rk1=0.75;   *h1=3.8254;         *optimal for detecting delta=1.5;
        *rk2=1.0;    *h2=2.8768;         *optimal for detecting delta=2.0;
        *rk3=1.25;   *h3=2.2742;         *optimal for detecting delta=2.5;
        *lambda=1.0;*h10=3.2151;         *Shewhart chart;


* 4-CUSUM Schemes with two-sided individual charts;
        *To get ARL=370 for 4-CUSUM scheme in detecting [0.4, infinity)
```

```
        ARL of component individual chart ARL= 1115;
     *rk1=0.2;   *h1=11.9695;       *optimal for detecting delta=0.4;
     *rk2=0.3;   *h2=8.8895;        *optimal for detecting delta=0.6;
     *rk3=0.5;   *h3=5.8790;        *optimal for detecting delta=1.0;
     *rk4=1.0;   *h4=3.0656;        *optimal for detecting delta=2.0;
     *lambda=1.0;*h10=3.3203;       *Shewhart chart;

     *To get ARL=370 for 4-CUSUM scheme in detecting [1.0, infinity)
      ARL of component individual chart ARL=910;
     *rk1=0.5;   *h1=5.6755;        *optimal for detecting delta=0.4;
     *rk2=0.75;  *h2=3.9399;        *optimal for detecting delta=0.6;
     *rk3=1.0;   *h3=2.9650;        *optimal for detecting delta=1.0;
     *rk4=1.25;  *h4=2.3431;        *optimal for detecting delta=2.0;
     *lambda=1.0;*h10=3.2635;       *Shewhart chart;

     *To get ARL=370 for 4-CUSUM scheme in detecting [1.5, infinity)
      ARL of component individual chart ARL= 763;
     *rk1=0.75;  *h1=3.8238;        *optimal for detecting delta=1.5;
     *rk2=1.0;   *h2=2.87555;       *optimal for detecting delta=2.0;
     *rk3=1.25;  *h3=2.2732;        *optimal for detecting delta=2.5;
     *rk4=1.5;   *h4=1.85285;       *optimal for detecting delta=3.0;
     *lambda=1.0;*h10=3.2143;*      Shewhart chart;*ARL=763.04;

* Sparks Simultaneous CUSUM Schemes with two-sided individual charts;
     *To get ARL=370 for Sparks 2-CUSUM scheme in detecting [0.75,
     1.25]
      ARL of component individual chart ARL= 490
     *rk1=0.4;   *h1=6.0764;        *optimal for detecting delta=0.8;
     *rk2=0.6;   *h2=4.3252;        *optimal for detecting delta=1.2;

     *To get ARL=370 for Sparks 3-CUSUM scheme in detecting [0.5, 2.0]
      ARL of component individual chart ARL= 578;
     *rk1=0.375; *h1=6.6069;        *optimal for detecting delta=0.75;
     *rk2=0.5;   *h2=5.2261;        *optimal for detecting delta=1.0;
     *rk3=0.75;  *h3=3.6363;        *optimal for detecting delta=1.5;

     *To get ARL=370 for Sparks 4-CUSUM scheme in detecting [0.5, 2.0]
      ARL of component individual chart ARL= 880;
     *rk1=0.3;   *h1=8.5093;        *optimal for detecting delta=0.6;
     *rk2=0.5;   *h2=5.6375;        *optimal for detecting delta=1.0;
     *rk3=1.0;   *h3=2.94458;       *optimal for detecting delta=2.0;
     *rk4=1.5;   *h4=1.89928;       *optimal for detecting delta=3.0;

*Upper and Lower control limits of CUSUM charts;
     *lcl1=-h1;
     *ucl1=h1;
     *lcl2=-h2;
     *ucl2=h2;
     *lcl3=-h3;
     *ucl3=h3;
     *lcl4=-h4;
     *ucl4=h4;

*Chart limits of Shewhart chart;
     *lcl10=mu-h10;
     *ucl10=mu+h10;

*Variable to count signals;
     *signal1=0;
     *signal2=0;
     *signal3=0;
```

```
        *signal4=0;
        *signal10=0;

*simulation;
do i=1 to numrun;
restart:
*Upper sided CUSUM statistic;
        *cup1=0.0;
        *cup2=0.0;
        *cup3=0.0;
        *cup4=0.0;
*Lower sided CUSUM statistic;
        *cdown1=0.0;
        *cdown2=0.0;
        *cdown3=0.0;
        *cdown4=0.0;

        m2=0;
        runlength=0;
*To become steady state;
        do j=1 to 100;
                x1=mu+rannor(seed)*sigma;
                x2=mu+rannor(seed)*sigma;
                x3=mu+rannor(seed)*sigma;
                x4=mu+rannor(seed)*sigma;
                xbar=(x1+x2+x3+x4)/n;
                Zt=xbar;

*upper sided cusum;
                *cup1=(Zt-rk1)+cup1;
                *cup2=(Zt-rk2)+cup2;
                *cup3=(Zt-rk3)+cup3;
                *cup4=(Zt-rk4)+cup4;

*lower sided cusum;
                *cdown1=(Zt+rk1)+cdown1;
                *cdown2=(Zt+rk2)+cdown2;
                *cdown3=(Zt+rk3)+cdown3;
                *cdown4=(Zt+rk4)+cdown4;

*Reset the upper chart limits;
                *if cup1<0 then cup1=0;
                *if cup2<0 then cup2=0;
                *if cup3<0 then cup3=0;
                *if cup4<0 then cup4=0;

*Reset the lower chart limits;
                *if cdown1>0 then cdown1=0;
                *if cdown2>0 then cdown2=0;
                *if cdown3>0 then cdown3=0;
                *if cdown4>0 then cdown4=0;

*For Shewhart chart;
                *m2=oneml*m2 + lambda*xbar;
        end;

* have to select the decision criteria form the following list and add in
between if condition below;
        if

        then goto restart;
```

```
        *
        cup1>ucl1 or cdown1<lcl1
        or cup2>ucl2 or cdown2<lcl2
        or cup3>ucl3 or cdown3<lcl3
        or cup4>ucl4 or cdown4<lcl4
        or m2<lcl10 or m2>ucl10
        ;
        nextsamp:
        runlength=runlength+1;
        x1=(mu+shift)+rannor(seed)*sigma;
        x2=(mu+shift)+rannor(seed)*sigma;
        x3=(mu+shift)+rannor(seed)*sigma;
        x4=(mu+shift)+rannor(seed)*sigma;
        xbar=(x1+x2+x3+x4)/n;
        Zt=xbar;

*upper sided cusum;
        *cup1=(Zt-rk1)+cup1;
        *cup2=(Zt-rk2)+cup2;
        *cup3=(Zt-rk3)+cup3;
        *cup4=(Zt-rk4)+cup4;

*lower sided cusum;
        *cdown1=(Zt+rk1)+cdown1;
        *cdown2=(Zt+rk2)+cdown2;
        *cdown3=(Zt+rk3)+cdown3;
        *cdown4=(Zt+rk4)+cdown4;

*Reset the upper chart limits;
        *if cup1<0 then cup1=0;
        *if cup2<0 then cup2=0;
        *if cup3<0 then cup3=0;
        *if cup4<0 then cup4=0;

*Reset the lower chart limits;
        *if cdown1>0 then cdown1=0;
        *if cdown2>0 then cdown2=0;
        *if cdown3>0 then cdown3=0;
        *if cdown4>0 then cdown4=0;

*For Shewhart chart;
        *m2=oneml*m2 + lambda*xbar;
        nsignal=0;
        *If cdown1<lcl1 or cup1>ucl1 then do;
        *                                 signal1=signal1+1;
        *                                 nsignal=nsignal+1;
        *                                       end;
        *If cdown2<lcl2 or cup2>ucl2 then do;
        *                                 signal2=signal2+1;
        *                                 nsignal=nsignal+1;
        *                                       end;
        *If cdown3<lcl3 or cup3>ucl3 then do;
        *                                 signal3=signal3+1;
        *                                 nsignal=nsignal+1;
        *                                       end;
        *If cdown4<lcl4 or cup4>ucl4 then do;
        *                                 signal4=signal4+1;
        *                                 nsignal=nsignal+1;
        *                                       end;
        *If m2<lcl10 or m2>ucl10 then do;
        *                                 signal10=signal10+1;
```

```
*                                  nsignal=nsignal+1;
*                                 end;
      If nsignal>0 then goto endchart;
      else goto nextsamp;
      endchart:
            output;
end;
proc means n mean stderr;
  var runlength;
run;
```

Program 4:

```
*********************************************************************
*                      One-Sided Charts                            *
*      SAS Program to Find the ARL Profile of Simultaneous          *
*                        CUSUM Schemes                              *
*                                                                   *
*      1. Find the ARL of Simultaneous CUSUM Scheme                 *
*      2. Firstly, Find the ARL of Component Individual Chart       *
*         to Get the ARL of Simultaneous CUSUM Scheme as 370        *
*      3. Then Find the Parameters of These Individual Charts       *
*      4. Then Run These Charts Simultaneously                      *
*********************************************************************;
data;
delta=0.0;
numrun=100000;
n=4;
mu=0.0;
sigma=2.0;
oneml=(1-lambda);
shift=delta*sigma/n**0.5;
seed=498188191;
* One-sided individual CUSUM charts to get ARL=370;
      *rk1=0.2;   *h1=7.76721;       *optimal for detecting delta=0.4;
      *rk2=0.3;   *h2=6.00099;       *optimal for detecting delta=0.6;
      *rk3=0.4;   *h3=4.88652;       *optimal for detecting delta=0.8;
      *rk4=0.5;   *h4=4.10411;       *optimal for detecting delta=1.0;
      *rk8=1.0;   *h8=2.17750;       *optimal for detecting delta=2.0;
      *rk9=1.5;   *h9=1.35904;       *optimal for detecting delta=3.0;
      *lambda=1.0;*h10=2.78077;      *Shewhart chart;
      *rk11=0.375;*h11=5.13154;      *optimal for detecting delta=0.75;
      *rk12=0.75; *h12=2.88797;      *optimal for detecting delta=1.5;
      *rk13=1.25; *h13=1.70984;      *optimal for detecting delta=2.5;

* Combined Shewhart-CUSUM Schemes with one-sided individual charts;
      *To get ARL=370 for combined Shewhart-CUSUM scheme in detecting
      [0.4, infinity)
      *ARL of component individual chart ARL= 700.00;
      *rk1=0.2;   *h1=9.19925;       *optimal for detecting delta=0.4;
      *lambda=1.0;*h10=2.98239;      *Shewhart chart;

      *To get ARL=370 for combined Shewhart-CUSUM scheme in detecting
      [1.0, infinity)
       ARL of component individual chart ARL= 675.00;
      *rk1=0.5;   *h1=4.69405;       *optimal for detecting delta=1.0;
      *lambda=1.0;*h10=2.97119;      *Shewhart chart;

      *To get ARL=370 for combined Shewhart-CUSUM scheme in detecting
      [1.5, infinity)
```

```
        ARL of component individual chart ARL= 650.00;
     *rk1=0.75;  *h1=3.25613;       *optimal for detecting delta=1.5;
     *lambda=1.0;*h10=2.95975;      *Shewhart chart;


* 2-CUSUM Schemes with two-sided individual charts;
     *To get ARL=370 for 2-CUSUM scheme in detecting [0.4, infinity)
      ARL of component individual chart ARL= 865;
     *rk1=0.2;   *h1=9.68529;       *optimal for detecting delta=0.4;
     *rk4=0.5;   *h4=4.93175;       *optimal for detecting delta=1.0;
     *lambda=1.0;*h10=3.04540;      *Shewhart chart;


     *To get ARL=370 for 2-CUSUM scheme in detecting [1.0, infinity)
      ARL of component individual chart ARL= 770;
     *rk1=0.5;   *h1=4.82212;       *optimal for detecting delta=1.0;
     *rk4=0.75;  *h4=3.36820;       *optimal for detecting delta=1.5;
     *lambda=1.0;*h10=3.01;         *Shewhart chart;


     *To get ARL=370 for 2-CUSUM scheme in detecting [1.5, infinity)
      ARL of component individual chart ARL= 700;
     *rk1=0.75;  *h1=3.30445;       *optimal for detecting delta=1.5;
     *rk4=1.0;   *h4=2.49102;       *optimal for detecting delta=2.0;
     *lambda=1.0;*h10=2.98088;      *Shewhart chart;


* 3-CUSUM Schemes with two-sided individual charts;
     *To get ARL=370 for 3-CUSUM scheme in detecting [0.4, infinity)
      ARL of component individual chart ARL= 890;
     *rk1=0.2;   *h1=9.75042;       *optimal for detecting delta=0.4;
     *rk2=0.3;   *h2=7.39426;       *optimal for detecting delta=0.6;
     *rk4=0.5;   *h4=4.96008;       *optimal for detecting delta=1.0;
     *lambda=1.0;*h10=3.05495;      *Shewhart chart;


     *To get ARL=370 for 3-CUSUM scheme in detecting [1.0, infinity)
      ARL of component individual chart ARL= 825;
     *rk1=0.5;   *h1=4.88410;       *optimal for detecting delta=1.0;
     *rk2=0.75;  *h2=3.41215;       *optimal for detecting delta=1.5;
     *rk4=1.0;   *h4=2.57118;       *optimal for detecting delta=2.0;
     *lambda=1.0;*h10=3.03130;      *Shewhart chart;


     *To get ARL=370 for 3-CUSUM scheme in detecting [1.5, infinity)
      ARL of component individual chart ARL= 710;
     *rk1=0.75;  *h1=3.31355;       *optimal for detecting delta=1.5;
     *rk2=1.0;   *h2=2.49660;       *optimal for detecting delta=2.0;
     *rk4=1.25;  *h4=1.96828;       *optimal for detecting delta=2.5;
     *lambda=1.0;*h10=2.98732;      *Shewhart chart;


* 4-CUSUM Schemes with two-sided individual charts;
     *To get ARL=370 for 4-CUSUM scheme in detecting [0.4, infinity)
      ARL of component individual chart ARL= 1015;
     *rk1=0.2;   *h1=10.0615;       *optimal for detecting delta=0.4;
     *rk2=0.3;   *h2=7.60465;       *optimal for detecting delta=0.6;
     *rk4=0.5;   *h4=5.0993;        *optimal for detecting delta=1.0;
     *rk8=1.0;   *h8=2.67452;       *optimal for detecting delta=2.0;
     *lambda=1.0;*h10=3.09469;      *Shewhart chart;


     *To get ARL=370 for 4-CUSUM scheme in detecting [1.0, infinity)
      ARL of component individual chart ARL= 830;
     *rk1=0.5;   *h1=4.89890;       *optimal for detecting delta=1.0;
     *rk2=0.75;  *h2=3.42190;       *optimal for detecting delta=1.5;
     *rk4=1.0;   *h4=2.57235;       *optimal for detecting delta=2.0;
     *rk8=1.25;  *h8=2.03293;       *optimal for detecting delta=2.5;
     *lambda=1.0;*h10=3.03411;      *Shewhart chart;
```

```
      *To get ARL=370 for 4-CUSUM scheme in detecting [1.5, infinity)
       ARL of component individual chart ARL= 710;
      *rk1=0.75;  *h1=3.31329;      *optimal for detecting delta=1.0;
      *rk2=1.0;   *h2=2.49583;      *optimal for detecting delta=1.5;
      *rk4=1.25;  *h4=1.97064;      *optimal for detecting delta=2.0;
      *rk8=1.5;   *h8=1.59091;      *optimal for detecting delta=2.5;
      *lambda=1.0;*h10=2.98680;     *Shewhart chart;


* Sparks Simultaneous CUSUM Schemes with two-sided individual charts;
      *To get ARL=370 for Sparks 2-CUSUM scheme in detecting [0.75, 1.25]
       ARL of component individual chart ARL= 480;
      *rk1=0.4;   *h1=5.19695;      *optimal for detecting delta=0.8;
      *rk2=0.6;   *h2=3.73475;      *optimal for detecting delta=1.2;


      *To get ARL=370 for Sparks 3-CUSUM scheme in detecting [0.5, 2.0]
       ARL of component individual chart ARL= 550;
      *rk1=0.375; *h1=5.62847;      *optimal for detecting delta=0.75;
      *rk2=0.5;   *h2=4.49802;      *optimal for detecting delta=1.0;
      *rk3=0.75;  *h3=3.15022;      *optimal for detecting delta=1.5;


      *To get ARL=370 for Sparks 4-CUSUM scheme in detecting [0.5, 2.0]
       ARL of component individual chart ARL= 825;
      *rk1=0.3;   *h1=7.26579;      *optimal for detecting delta=0.6;
      *rk2=0.5;   *h2=4.89262;      *optimal for detecting delta=1.0;
      *rk3=1.0;   *h3=2.57096;      *optimal for detecting delta=2.0;
      *rk4=1.5;   *h4=1.64197;      *optimal for detecting delta=3.0;


* For 10 CUSUMs.  Individual ARL 1045;
      *rk1=0.2;   *h1=10.13429;     *optimal for detecting delta=0.4;
      *rk2=0.3;   *h2=7.63650;      *optimal for detecting delta=0.6;
      *rk3=0.4;   *h3=6.1455;       *optimal for detecting delta=0.8;
      *rk4=0.5;   *h4=5.12522;      *optimal for detecting delta=1.0;
      *rk5=0.6;   *h5=4.37346;      *optimal for detecting delta=1.2;
      *rk6=0.7;   *h6=3.80545;      *optimal for detecting delta=1.4;
      *rk7=0.8;   *h7=3.35324;      *optimal for detecting delta=1.6;
      *rk8=1.0;   *h8=2.68617;      *optimal for detecting delta=2.0;
      *rk9=1.5;   *h9=1.72282;      *optimal for detecting delta=3.0;
      *lambda=1.0;*h10=3.10265;     *Shewhart chart;

*ucl1=h1;
*ucl2=h2;
*ucl3=h3;
*ucl4=h4;
*ucl5=h5;
*ucl6=h6;
*ucl7=h7;
*ucl8=h8;
*ucl9=h9;
*ucl10=mu+h10;
*ucl11=h11;
*ucl12=h12;
*ucl13=h13;
*signal1=0;
*signal2=0;
*signal3=0;
*signal4=0;
*signal5=0;
*signal6=0;
*signal7=0;
*signal8=0;
```

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```
*signal9=0;
*signal10=0;
*signal11=0;
*signal12=0;
*signal13=0;

do i=1 to numrun;
restart:
      *cup1=0.0;
      *cup2=0.0;
      *cup3=0.0;
      *cup4=0.0;
      *cup5=0.0;
      *cup6=0.0;
      *cup7=0.0;
      *cup8=0.0;
      *cup9=0;
      *cup11=0.0;
      *cup12=0.0;
      *cup13=0.0;

      m2=0;
      runlength=0;
      do j=1 to 100;
            x1=mu+rannor(seed)*sigma;
            x2=mu+rannor(seed)*sigma;
            x3=mu+rannor(seed)*sigma;
            x4=mu+rannor(seed)*sigma;
            xbar=(x1+x2+x3+x4)/n;
            Zt=xbar;
*upper sided cusum;
            *cup1=(Zt-rk1)+cup1;
            *cup2=(Zt-rk2)+cup2;
            *cup3=(Zt-rk3)+cup3;
            *cup4=(Zt-rk4)+cup4;
            *cup5=(Zt-rk5)+cup5;
            *cup6=(Zt-rk6)+cup6;
            *cup7=(Zt-rk7)+cup7;
            *cup8=(Zt-rk8)+cup8;
            *cup9=(Zt-rk9)+cup9;
            *cup11=(Zt-rk11)+cup11;
            *cup12=(Zt-rk12)+cup12;
            *cup13=(Zt-rk13)+cup13;

            *if cup1<0 then cup1=0;
            *if cup2<0 then cup2=0;
            *if cup3<0 then cup3=0;
            *if cup4<0 then cup4=0;
            *if cup5<0 then cup5=0;
            *if cup6<0 then cup6=0;
            *if cup7<0 then cup7=0;
            *if cup8<0 then cup8=0;
            *if cup9<0 then cup9=0;
            *if cup11<0 then cup11=0;
            *if cup12<0 then cup12=0;
            *if cup13<0 then cup13=0;

            *m2=oneml*m2 + lambda*xbar;
      end;
      if
```

```
        then goto restart;
*
        cup1>ucl1
        or cup2>ucl2
        or cup3>ucl3
        or cup4>ucl4
        or cup5>ucl5
        or cup6>ucl6
        or cup7>ucl7
        or cup8>ucl8
        or cup9>ucl9
        or m2>ucl10
        or cup11>ucl11
        or cup12>ucl12
        or cup13>ucl13
;
        nextsamp:
        runlength=runlength+1;
        x1=(mu+shift)+rannor(seed)*sigma;
        x2=(mu+shift)+rannor(seed)*sigma;
        x3=(mu+shift)+rannor(seed)*sigma;
        x4=(mu+shift)+rannor(seed)*sigma;
        xbar=(x1+x2+x3+x4)/n;
        Zt=xbar;

*upper sided cusum;
        *cup1=(Zt-rk1)+cup1;
        *cup2=(Zt-rk2)+cup2;
        *cup3=(Zt-rk3)+cup3;
        *cup4=(Zt-rk4)+cup4;
        *cup5=(Zt-rk5)+cup5;
        *cup6=(Zt-rk6)+cup6;
        *cup7=(Zt-rk7)+cup7;
        *cup8=(Zt-rk8)+cup8;
        *cup9=(Zt-rk9)+cup9;
        *cup11=(Zt-rk11)+cup11;
        *cup12=(Zt-rk12)+cup12;
        *cup13=(Zt-rk13)+cup13;

        *if cup1<0 then cup1=0;
        *if cup2<0 then cup2=0;
        *if cup3<0 then cup3=0;
        *if cup4<0 then cup4=0;
        *if cup5<0 then cup5=0;
        *if cup6<0 then cup6=0;
        *if cup7<0 then cup7=0;
        *if cup8<0 then cup8=0;
        *if cup9<0 then cup9=0;
        *if cup11<0 then cup11=0;
        *if cup12<0 then cup12=0;
        *if cup13<0 then cup13=0;

        *m2=oneml*m2 + lambda*xbar;
        nsignal=0;
        *If cup1>ucl1 then do;
        *          signal1=signal1+1;
        *          nsignal=nsignal+1;
        *              end;
        *If cup2>ucl2 then do;
        *          signal2=signal2+1;
        *          nsignal=nsignal+1;
```

```
*                   end;
*If cup3>ucl3 then do;
*           signal3=signal3+1;
*           nsignal=nsignal+1;
*                   end;
*If cup4>ucl4 then do;
*           signal4=signal4+1;
*           nsignal=nsignal+1;
*                   end;
*If cup5>ucl5 then do;
*           signal5=signal5+1;
*           nsignal=nsignal+1;
*                   end;
*If cup6>ucl6 then do;
*           signal6=signal6+1;
*           nsignal=nsignal+1;
*                   end;
*If cup7>ucl7 then do;
*           signal7=signal7+1;
*           nsignal=nsignal+1;
*                   end;
*If cup8>ucl8 then do;
*           signal8=signal8+1;
*           nsignal=nsignal+1;
*                   end;
*If cup9>ucl9 then do;
*           signal9=signal9+1;
*           nsignal=nsignal+1;
*                   end;
*If m2>ucl10 then do;
*           signal10=signal10+1;
*           nsignal=nsignal+1;
*                   end;
*If cup11>ucl11 then do;
*           signal11=signal11+1;
*           nsignal=nsignal+1;
*                   end;
*If cup12>ucl12 then do;
*           signal12=signal12+1;
*           nsignal=nsignal+1;
*                   end;
*If cup13>ucl13 then do;
*           signal13=signal13+1;
*           nsignal=nsignal+1;
*                   end;
If nsignal>0 then goto endchart;
else goto nextsamp;
endchart:
     output;
end;
proc means n mean stderr;
  var runlength;
run;
```

Program 5:
```
******************************************************************
*                    Two-Sided Charts                           *
*     SAS Program to Find the ARL Relationship Between the       *
*     Individual CUSUM and the Simultaneous CUSUM Schemes.       *
```

```
*                                                                *
*       1. Simultaneous CUSUM Schemes with Steady State Limits.  *
*       2. ARL of Each Simultaneous CUSUMA Scheme is Calculated.  *
*                                                                *
*****************************************************************;
data;

*Parameters (rk,h) for Individual ARL=50;
*rk1=0.2;          *h1=5.3555;          *optimal for detecting delta=0.4;
*rk2=0.3;          *h2=4.1868;          *optimal for detecting delta=0.6;
*rk3=0.4;          *h3=3.4539;          *optimal for detecting delta=0.8;
*rk4=0.5;          *h4=2.9065;          *optimal for detecting delta=1.0;
*rk5=0.6;          *h5=2.5204;          *optimal for detecting delta=1.2;
*rk6=0.7;          *h6=2.2016;          *optimal for detecting delta=1.4;
*rk7=0.8;          *h7=1.9275;          *optimal for detecting delta=1.6;
*rk8=1.0;          *h8=1.5442;          *optimal for detecting delta=2.0;
*rk9=1.5;          *h9=0.8535;          *optimal for detecting delta=3.0;
*lambda=1.0;       *h10=2.3264;         *Shewhart chart;
*rk11=0.75;        *h11=2.0592;         *optimal for detecting delta=1.5;
*rk12=1.25;        *h12=1.1742;         *optimal for detecting delta=2.5;
*rk13=0.375;       *h13=3.6039;         *optimal for detecting delta=0.75;


*Parameters (rk,h) for Individual ARL=100;
*rk1=0.2;          *h1=6.6706;          *optimal for detecting delta=0.4;
*rk2=0.3;          *h2=5.1513;          *optimal for detecting delta=0.6;
*rk3=0.4;          *h3=4.2118;          *optimal for detecting delta=0.8;
*rk4=0.5;          *h4=3.5451;          *optimal for detecting delta=1.0;
*rk5=0.6;          *h5=3.0463;          *optimal for detecting delta=1.2;
*rk6=0.7;          *h6=2.6588;          *optimal for detecting delta=1.4;
*rk7=0.8;          *h7=2.3503;          *optimal for detecting delta=1.6;
*rk8=1.0;          *h8=1.8801;          *optimal for detecting delta=2.0;
*rk9=1.5;          *h9=1.1311;          *optimal for detecting delta=3.0;
*lambda=1.0;       *h10=2.5758;         *Shewhart chart;
*rk11=0.75;        *h11=2.4948;         *optimal for detecting delta=1.5;
*rk12=1.25;        *h12=1.4625;         *optimal for detecting delta=2.5;
*rk13=0.375;       *h13=4.4096;         *optimal for detecting delta=0.75;


*Parameters (rk,h) for Individual ARL=200;
*rk1=0.2;          *h1=8.0678;          *optimal for detecting delta=0.4;
*rk2=0.3;          *h2=6.1976;          *optimal for detecting delta=0.6;
*rk3=0.4;          *h3=5.0240;          *optimal for detecting delta=0.8;
*rk4=0.5;          *h4=4.2090;          *optimal for detecting delta=1.0;
*rk5=0.6;          *h5=3.5958;          *optimal for detecting delta=1.2;
*rk6=0.7;          *h6=3.1339;          *optimal for detecting delta=1.4;
*rk7=0.8;          *h7=2.7666;          *optimal for detecting delta=1.6;
*rk8=1.0;          *h8=2.2169;          *optimal for detecting delta=2.0;
*rk9=1.5;          *h9=1.3855;          *optimal for detecting delta=3.0;
*lambda=1.0;       *h10=2.8070;         *Shewhart chart;
*rk11=0.75;        *h11=2.9471;         *optimal for detecting delta=1.5;
*rk12=1.25;        *h12=1.7428;         *optimal for detecting delta=2.5;
*rk13=0.375;       *h13=5.2543;         *optimal for detecting delta=0.75;


*Parameters (rk,h) for Individual ARL=300;
*rk1=0.2;          *h1=8.9300;          *optimal for detecting delta=0.4;
*rk2=0.3;          *h2=6.8023;          *optimal for detecting delta=0.6;
*rk3=0.4;          *h3=5.4926;          *optimal for detecting delta=0.8;
*rk4=0.5;          *h4=4.5931;          *optimal for detecting delta=1.0;
*rk5=0.6;          *h5=3.9237;          *optimal for detecting delta=1.2;
*rk6=0.7;          *h6=3.4168;          *optimal for detecting delta=1.4;
*rk7=0.8;          *h7=3.0144;          *optimal for detecting delta=1.6;
*rk8=1.0;          *h8=2.4151;          *optimal for detecting delta=2.0;
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```
*rk9=1.5;             *h9=1.5302;        *optimal for detecting delta=3.0;
*lambda=1.0;          *h10=2.9352;       *Shewhart chart;
*rk11=0.75;           *h11=3.2089;       *optimal for detecting delta=1.5;
*rk12=1.25;           *h12=1.9043;       *optimal for detecting delta=2.5;
*rk13=0.375;          *h13=5.7610;       *optimal for detecting delta=0.75;


*Parameters (rk,h) for Individual ARL=370;
*rk1=0.2;             *h1=9.3923;        *optimal for detecting delta=0.4;
*rk2=0.3;             *h2=7.1234;        *optimal for detecting delta=0.6;
*rk3=0.4;             *h3=5.7397;        *optimal for detecting delta=0.8;
*rk4=0.5;             *h4=4.7944;        *optimal for detecting delta=1.0;
*rk5=0.6;             *h5=4.0945;        *optimal for detecting delta=1.2;
*rk6=0.7;             *h6=3.5640;        *optimal for detecting delta=1.4;
*rk7=0.8;             *h7=3.1434;        *optimal for detecting delta=1.6;
*rk8=1.0;             *h8=2.5180;        *optimal for detecting delta=2.0;
*rk9=1.5;             *h9=1.6036;        *optimal for detecting delta=3.0;
*lambda=1.0;          *h10=2.9997;       *Shewhart chart;
*rk11=0.75;           *h11=3.3474;       *optimal for detecting delta=1.5;
*rk12=1.25;           *h12=1.9873;       *optimal for detecting delta=2.5;
*rk13=0.375;          *h13=6.0264;       *optimal for detecting delta=0.75;


*Parameters (rk,h) for Individual ARL=400;
*rk1=0.2;             *h1=9.5667;        *optimal for detecting delta=0.4;
*rk2=0.3;             *h2=7.2439;        *optimal for detecting delta=0.6;
*rk3=0.4;             *h3=5.8322;        *optimal for detecting delta=0.8;
*rk4=0.5;             *h4=4.8696;        *optimal for detecting delta=1.0;
*rk5=0.6;             *h5=4.1582;        *optimal for detecting delta=1.2;
*rk6=0.7;             *h6=3.6188;        *optimal for detecting delta=1.4;
*rk7=0.8;             *h7=3.1915;        *optimal for detecting delta=1.6;
*rk8=1.0;             *h8=2.5564;        *optimal for detecting delta=2.0;
*rk9=1.5;             *h9=1.6307;        *optimal for detecting delta=3.0;
*lambda=1.0;          *h10=3.0233;       *Shewhart chart;
*rk11=0.75;           *h11=3.3976;       *optimal for detecting delta=1.5;
*rk12=1.25;           *h12=2.0181;       *optimal for detecting delta=2.5;
*rk13=0.375;          *h13=6.1343;       *optimal for detecting delta=0.75;


*Parameters (rk,h) for Individual ARL=500;
*rk1=0.2;             *h1=10.0723;       *optimal for detecting delta=0.4;
*rk2=0.3;             *h2=7.5923;        *optimal for detecting delta=0.6;
*rk3=0.4;             *h3=6.0987;        *optimal for detecting delta=0.8;
*rk4=0.5;             *h4=5.0859;        *optimal for detecting delta=1.0;
*rk5=0.6;             *h5=4.3409;        *optimal for detecting delta=1.2;
*rk6=0.7;             *h6=3.7761;        *optimal for detecting delta=1.4;
*rk7=0.8;             *h7=3.3294;        *optimal for detecting delta=1.6;
*rk8=1.0;             *h8=2.6664;        *optimal for detecting delta=2.0;
*rk9=1.5;             *h9=1.7077;        *optimal for detecting delta=3.0;
*lambda=1.0;          *h10=3.0902;       *Shewhart chart;
*rk11=0.75;           *h11=3.5455;       *optimal for detecting delta=1.5;
*rk12=1.25;           *h12=2.1061;       *optimal for detecting delta=2.5;
*rk13=0.375;          *h13=6.4209;       *optimal for detecting delta=0.75;


*Parameters (rk,h) for Individual ARL=600;
*rk1=0.2;             *h1=10.4917;       *optimal for detecting delta=0.4;
*rk2=0.3;             *h2=7.8800;        *optimal for detecting delta=0.6;
*rk3=0.4;             *h3=6.3182;        *optimal for detecting delta=0.8;
*rk4=0.5;             *h4=5.2636;        *optimal for detecting delta=1.0;
*rk5=0.6;             *h5=4.4906;        *optimal for detecting delta=1.2;
*rk6=0.7;             *h6=3.9049;        *optimal for detecting delta=1.4;
*rk7=0.8;             *h7=3.4423;        *optimal for detecting delta=1.6;
*rk8=1.0;             *h8=2.7565;        *optimal for detecting delta=2.0;
*rk9=1.5;             *h9=1.7701;        *optimal for detecting delta=3.0;
```

```
*lambda=1.0;        *h10=3.1440;        *Shewhart chart;
*rk11=0.75;         *h11=3.6661;        *optimal for detecting delta=1.5;
*rk12=1.25;         *h12=2.1778;        *optimal for detecting delta=2.5;
*rk13=0.375;        *h13=6.6545;        *optimal for detecting delta=0.75;


*Parameters (rk,h) for Individual ARL=700;
*rk1=0.2;           *h1=10.8504;        *optimal for detecting delta=0.4;
*rk2=0.3;           *h2=8.1251;         *optimal for detecting delta=0.6;
*rk3=0.4;           *h3=6.5047;         *optimal for detecting delta=0.8;
*rk4=0.5;           *h4=5.4144;         *optimal for detecting delta=1.0;
*rk5=0.6;           *h5=4.6174;         *optimal for detecting delta=1.2;
*rk6=0.7;           *h6=4.0139;         *optimal for detecting delta=1.4;
*rk7=0.8;           *h7=3.5379;         *optimal for detecting delta=1.6;
*rk8=1.0;           *h8=2.8329;         *optimal for detecting delta=2.0;
*rk9=1.5;           *h9=1.8224;         *optimal for detecting delta=3.0;
*lambda=1.0;        *h10=3.1888;        *Shewhart chart;
*rk11=0.75;         *h11=3.7672;        *optimal for detecting delta=1.5;
*rk12=1.25;         *h12=2.2385;        *optimal for detecting delta=2.5;
*rk13=0.375;        *h13=6.8525;        *optimal for detecting delta=0.75;


*Parameters (rk,h) for Individual ARL=800;
*rk1=0.2;           *h1=11.1637;        *optimal for detecting delta=0.4;
*rk2=0.3;           *h2=8.3386;         *optimal for detecting delta=0.6;
*rk3=0.4;           *h3=6.6670;         *optimal for detecting delta=0.8;
*rk4=0.5;           *h4=5.54534;        *optimal for detecting delta=1.0;
*rk5=0.6;           *h5=4.7275;         *optimal for detecting delta=1.2;
*rk6=0.7;           *h6=4.1085;         *optimal for detecting delta=1.4;
*rk7=0.8;           *h7=3.6209;         *optimal for detecting delta=1.6;
*rk8=1.0;           *h8=2.8992;         *optimal for detecting delta=2.0;
*rk9=1.5;           *h9=1.8675;         *optimal for detecting delta=3.0;
*lambda=1.0;        *h10=3.2272;        *Shewhart chart;
*rk11=0.75;         *h11=3.8561;        *optimal for detecting delta=1.5;
*rk12=1.25;         *h12=2.2910;        *optimal for detecting delta=2.5;
*rk13=0.375;        *h13=7.0300;        *optimal for detecting delta=0.75;


*Parameters (rk,h) for Individual ARL=900;
*rk1=0.2;           *h1=11.4418;        *optimal for detecting delta=0.4;
*rk2=0.3;           *h2=8.5278;         *optimal for detecting delta=0.6;
*rk3=0.4;           *h3=6.8105;         *optimal for detecting delta=0.8;
*rk4=0.5;           *h4=5.6611;         *optimal for detecting delta=1.0;
*rk5=0.6;           *h5=4.8247;         *optimal for detecting delta=1.2;
*rk6=0.7;           *h6=4.1920;         *optimal for detecting delta=1.4;
*rk7=0.8;           *h7=3.6940;         *optimal for detecting delta=1.6;
*rk8=1.0;           *h8=2.9577;         *optimal for detecting delta=2.0;
*rk9=1.5;           *h9=1.9072;         *optimal for detecting delta=3.0;
*lambda=1.0;        *h10=3.2608;        *Shewhart chart;
*rk11=0.75;         *h11=3.9347;        *optimal for detecting delta=1.5;
*rk12=1.25;         *h12=2.3373;        *optimal for detecting delta=2.5;
*rk13=0.375;        *h13=7.1795;        *optimal for detecting delta=0.75;


*Parameters (rk,h) for Individual ARL=1000;
*rk1=0.2;           *h1=11.6919;        *optimal for detecting delta=0.4;
*rk2=0.3;           *h2=8.6977;         *optimal for detecting delta=0.6;
*rk3=0.4;           *h3=6.9393;         *optimal for detecting delta=0.8;
*rk4=0.5;           *h4=5.7648;         *optimal for detecting delta=1.0;
*rk5=0.6;           *h5=4.9117;         *optimal for detecting delta=1.2;
*rk6=0.7;           *h6=4.2667;         *optimal for detecting delta=1.4;
*rk7=0.8;           *h7=3.7595;         *optimal for detecting delta=1.6;
*rk8=1.0;           *h8=3.0101;         *optimal for detecting delta=2.0;
*rk9=1.5;           *h9=1.9425;         *optimal for detecting delta=3.0;
*lambda=1.0;        *h10=3.2905;        *Shewhart chart;
```

```
*rk11=0.75;          *h11=4.0058;          *optimal for detecting delta=1.5;
*rk12=1.25;          *h12=2.3788;          *optimal for detecting delta=2.5;
*rk13=0.375;         *h13=7.3165;          *optimal for detecting delta=0.75;


*Parameters (rk,h) for Individual ARL=1500;
*rk1=0.2;            *h1=12.6639;          *optimal for detecting delta=0.4;
*rk2=0.3;            *h2=9.3556;           *optimal for detecting delta=0.6;
*rk3=0.4;            *h3=7.4369;           *optimal for detecting delta=0.8;
*rk4=0.5;            *h4=6.1653;           *optimal for detecting delta=1.0;
*rk5=0.6;            *h5=5.2472;           *optimal for detecting delta=1.2;
*rk6=0.7;            *h6=4.5548;           *optimal for detecting delta=1.4;
*rk7=0.8;            *h7=4.0119;           *optimal for detecting delta=1.6;
*rk8=1.0;            *h8=3.2122;           *optimal for detecting delta=2.0;
*rk9=1.5;            *h9=2.0775;           *optimal for detecting delta=3.0;
*lambda=1.0;         *h10=3.4029;          *Shewhart chart;
*rk11=0.75;          *h11=4.2725;          *optimal for detecting delta=1.5;
*rk12=1.25;          *h12=2.5385;          *optimal for detecting delta=2.5;
*rk13=0.375;         *h13=7.8478;          *optimal for detecting delta=0.75;


*Parameters (rk,h) for Individual ARL=2000;
*rk1=0.2;            *h1=13.3614;          *optimal for detecting delta=0.4;
*rk2=0.3;            *h2=9.8259;           *optimal for detecting delta=0.6;
*rk3=0.4;            *h3=7.7919;           *optimal for detecting delta=0.8;
*rk4=0.5;            *h4=6.4505;           *optimal for detecting delta=1.0;
*rk5=0.6;            *h5=5.4857;           *optimal for detecting delta=1.2;
*rk6=0.7;            *h6=4.7594;           *optimal for detecting delta=1.4;
*rk7=0.8;            *h7=4.1913;           *optimal for detecting delta=1.6;
*rk8=1.0;            *h8=3.3558;           *optimal for detecting delta=2.0;
*rk9=1.5;            *h9=2.1725;           *optimal for detecting delta=3.0;
*lambda=1.0;         *h10=3.4808;          *Shewhart chart;
*rk11=0.75;          *h11=4.4631;          *optimal for detecting delta=1.5;
*rk12=1.25;          *h12=2.6523;          *optimal for detecting delta=2.5;
*rk13=0.375;         *h13=8.2242;          *optimal for detecting delta=0.75;


*Parameters (rk,h) for Individual ARL=2500;
*rk1=0.2;            *h1=13.9058;          *optimal for detecting delta=0.4;
*rk2=0.3;            *h2=10.1922;          *optimal for detecting delta=0.6;
*rk3=0.4;            *h3=8.0679;           *optimal for detecting delta=0.8;
*rk4=0.5;            *h4=6.6720;           *optimal for detecting delta=1.0;
*rk5=0.6;            *h5=5.6708;           *optimal for detecting delta=1.2;
*rk6=0.7;            *h6=4.9183;           *optimal for detecting delta=1.4;
*rk7=0.8;            *h7=4.3303;           *optimal for detecting delta=1.6;
*rk8=1.0;            *h8=3.4673;           *optimal for detecting delta=2.0;
*rk9=1.5;            *h9=2.2460;           *optimal for detecting delta=3.0;
*lambda=1.0;         *h10=3.5401;          *Shewhart chart;
*rk11=0.75;          *h11=4.6099;          *optimal for detecting delta=1.5;
*rk12=1.25;          *h12=2.7403;          *optimal for detecting delta=2.5;
*rk13=0.375;         *h13=8.5168;          *optimal for detecting delta=0.75;


*Parameters (rk,h) for Individual ARL=2500;
*rk1=0.2;            *h1=14.3523;          *optimal for detecting delta=0.4;
*rk2=0.3;            *h2=10.4922;          *optimal for detecting delta=0.6;
*rk3=0.4;            *h3=8.2938;           *optimal for detecting delta=0.8;
*rk4=0.5;            *h4=6.8533;           *optimal for detecting delta=1.0;
*rk5=0.6;            *h5=5.8223;           *optimal for detecting delta=1.2;
*rk6=0.7;            *h6=5.0482;           *optimal for detecting delta=1.4;
*rk7=0.8;            *h7=4.4440;           *optimal for detecting delta=1.6;
*rk8=1.0;            *h8=3.5585;           *optimal for detecting delta=2.0;
*rk9=1.5;            *h9=2.3059;           *optimal for detecting delta=3.0;
*lambda=1.0;         *h10=3.5879;          *Shewhart chart;
*rk11=0.75;          *h11=4.7295;          *optimal for detecting delta=1.5;
```

```
*rk12=1.25;        *h12=2.8125;        *optimal for detecting delta=2.5;
*rk13=0.375;       *h13=8.7553;        *optimal for detecting delta=0.75;


*Parameters (rk,h) for Individual ARL=3500;
*rk1=0.2;          *h1=14.7303;        *optimal for detecting delta=0.4;
*rk2=0.3;          *h2=10.7460;        *optimal for detecting delta=0.6;
*rk3=0.4;          *h3=8.4851;         *optimal for detecting delta=0.8;
*rk4=0.5;          *h4=7.0067;         *optimal for detecting delta=1.0;
*rk5=0.6;          *h5=5.9504;         *optimal for detecting delta=1.2;
*rk6=0.7;          *h6=5.1580;         *optimal for detecting delta=1.4;
*rk7=0.8;          *h7=4.5401;         *optimal for detecting delta=1.6;
*rk8=1.0;          *h8=3.6356;         *optimal for detecting delta=2.0;
*rk9=1.5;          *h9=2.3564;         *optimal for detecting delta=3.0;
*lambda=1.0;       *h10=3.6279;        *Shewhart chart;
*rk11=0.75;        *h11=4.8324;        *optimal for detecting delta=1.5;
*rk12=1.25;        *h12=2.8724;        *optimal for detecting delta=2.5;
*rk13=0.375;       *h13=8.9620;        *optimal for detecting delta=0.75;


*Parameters (rk,h) for Individual ARL=4000;
*rk1=0.2;          *h1=15.0584;        *optimal for detecting delta=0.4;
*rk2=0.3;          *h2=10.9665;        *optimal for detecting delta=0.6;
*rk3=0.4;          *h3=8.6510;         *optimal for detecting delta=0.8;
*rk4=0.5;          *h4=7.1396;         *optimal for detecting delta=1.0;
*rk5=0.6;          *h5=6.0614;         *optimal for detecting delta=1.2;
*rk6=0.7;          *h6=5.2532;         *optimal for detecting delta=1.4;
*rk7=0.8;          *h7=4.6234;         *optimal for detecting delta=1.6;
*rk8=1.0;          *h8=3.7024;         *optimal for detecting delta=2.0;
*rk9=1.5;          *h9=2.4002;         *optimal for detecting delta=3.0;
*lambda=1.0;       *h10=3.6623;        *Shewhart chart;
*rk11=0.75;        *h11=4.9225;        *optimal for detecting delta=1.5;
*rk12=1.25;        *h12=2.9257;        *optimal for detecting delta=2.5;
*rk13=0.375;       *h13=9.1390;        *optimal for detecting delta=0.75;

numrun=100000;
seed=499624291;
n=4;
delta=0.0;
mu=0.0;
sigma=2.0;
oneml=(1-lambda);
shift=delta*sigma/n**0.5;

*lcl1=-h1;
*ucl1=h1;
*lcl2=-h2;
*ucl2=h2;
*lcl3=-h3;
*ucl3=h3;
*lcl4=-h4;
*ucl4=h4;
*lcl5=-h5;
*ucl5=h5;
*lcl6=-h6;
*ucl6=h6;
*lcl7=-h7;
*ucl7=h7;
*lcl8=-h8;
*ucl8=h8;
*lcl9=-h9;
*ucl9=h9;
*lcl10=mu-h10;
```

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```
*ucl10=mu+h10;
*lcl11=-h11;
*ucl11=h11;
*lcl12=-h12;
*ucl12=h12;
*lcl13=-h13;
*ucl13=h13;


*signal1=0;
*signal2=0;
*signal3=0;
*signal4=0;
*signal5=0;
*signal6=0;
*signal7=0;
*signal8=0;
*signal9=0;
*signal10=0;
*signal11=0;
*signal12=0;
*signal13=0;

do i=1 to numrun;
restart:
      *cup1=0.0;
      *cup2=0.0;
      *cup3=0.0;
      *cup4=0.0;
      *cup5=0.0;
      *cup6=0.0;
      *cup7=0.0;
      *cup8=0.0;
      *cup9=0;
      *cup11=0.0;
      *cup12=0.0;
      *cup13=0.0;

      *cdown1=0.0;
      *cdown2=0.0;
      *cdown3=0.0;
      *cdown4=0.0;
      *cdown5=0.0;
      *cdown6=0.0;
      *cdown7=0.0;
      *cdown8=0.0;
      *cdown9=0.0;
      *cdown11=0.0;
      *cdown12=0.0;
      *cdown13=0.0;

      m2=0;
      runlength=0;

      do j=1 to 100;
            x1=mu+rannor(seed)*sigma;
            x2=mu+rannor(seed)*sigma;
            x3=mu+rannor(seed)*sigma;
            x4=mu+rannor(seed)*sigma;
            xbar=(x1+x2+x3+x4)/n;
            Zt=xbar;
```

```
*upper sided cusum;
            *cup1=(Zt-rk1)+cup1;
            *cup2=(Zt-rk2)+cup2;
            *cup3=(Zt-rk3)+cup3;
            *cup4=(Zt-rk4)+cup4;
            *cup5=(Zt-rk5)+cup5;
            *cup6=(Zt-rk6)+cup6;
            *cup7=(Zt-rk7)+cup7;
            *cup8=(Zt-rk8)+cup8;
            *cup9=(Zt-rk9)+cup9;
            *cup11=(Zt-rk11)+cup11;
            *cup12=(Zt-rk12)+cup12;
            *cup13=(Zt-rk13)+cup13;

*lower sided cusum;
            *cdown1=(Zt+rk1)+cdown1;
            *cdown2=(Zt+rk2)+cdown2;
            *cdown3=(Zt+rk3)+cdown3;
            *cdown4=(Zt+rk4)+cdown4;
            *cdown5=(Zt+rk5)+cdown5;
            *cdown6=(Zt+rk6)+cdown6;
            *cdown7=(Zt+rk7)+cdown7;
            *cdown8=(Zt+rk8)+cdown8;
            *cdown9=(Zt+rk9)+cdown9;
            *cdown11=(Zt+rk11)+cdown11;
            *cdown12=(Zt+rk12)+cdown12;
            *cdown13=(Zt+rk13)+cdown13;

            *if cup1<0 then cup1=0;
            *if cup2<0 then cup2=0;
            *if cup3<0 then cup3=0;
            *if cup4<0 then cup4=0;
            *if cup5<0 then cup5=0;
            *if cup6<0 then cup6=0;
            *if cup7<0 then cup7=0;
            *if cup8<0 then cup8=0;
            *if cup9<0 then cup9=0;
            *if cup11<0 then cup11=0;
            *if cup12<0 then cup12=0;
            *if cup13<0 then cup13=0;

            *if cdown1>0 then cdown1=0;
            *if cdown2>0 then cdown2=0;
            *if cdown3>0 then cdown3=0;
            *if cdown4>0 then cdown4=0;
            *if cdown5>0 then cdown5=0;
            *if cdown6>0 then cdown6=0;
            *if cdown7>0 then cdown7=0;
            *if cdown8>0 then cdown8=0;
            *if cdown9>0 then cdown9=0;
            *if cdown11>0 then cdown11=0;
            *if cdown12>0 then cdown12=0;
            *if cdown13>0 then cdown13=0;

            *m2=oneml*m2 + lambda*xbar;
        end;

        if

        then goto restart;
```

```
*
        cup1>ucl1 or cdown1<lcl1
        or cup2>ucl2 or cdown2<lcl2
        or cup3>ucl3 or cdown3<lcl3
        or cup4>ucl4 or cdown4<lcl4
        or cup5>ucl5 or cdown5<lcl5
        or cup6>ucl6 or cdown6<lcl6
        or cup7>ucl7 or cdown7<lcl7
        or cup8>ucl8 or cdown8<lcl8
        or cup9>ucl9 or cdown9<lcl9
        or cup11>ucl11 or cdown11<lcl11
        or cup12>ucl12 or cdown12<lcl12
        or cup13>ucl13 or cdown13<lcl13
        or m2<lcl10 or m2>ucl10

;
        nextsamp:
        runlength=runlength+1;
        x1=(mu+shift)+rannor(seed)*sigma;
        x2=(mu+shift)+rannor(seed)*sigma;
        x3=(mu+shift)+rannor(seed)*sigma;
        x4=(mu+shift)+rannor(seed)*sigma;
        xbar=(x1+x2+x3+x4)/n;
        Zt=xbar;

*upper sided cusum;
        *cup1=(Zt-rk1)+cup1;
        *cup2=(Zt-rk2)+cup2;
        *cup3=(Zt-rk3)+cup3;
        *cup4=(Zt-rk4)+cup4;
        *cup5=(Zt-rk5)+cup5;
        *cup6=(Zt-rk6)+cup6;
        *cup7=(Zt-rk7)+cup7;
        *cup8=(Zt-rk8)+cup8;
        *cup9=(Zt-rk9)+cup9;
        *cup11=(Zt-rk11)+cup11;
        *cup12=(Zt-rk12)+cup12;
        *cup13=(Zt-rk13)+cup13;

*lower sided cusum;
        *cdown1=(Zt+rk1)+cdown1;
        *cdown2=(Zt+rk2)+cdown2;
        *cdown3=(Zt+rk3)+cdown3;
        *cdown4=(Zt+rk4)+cdown4;
        *cdown5=(Zt+rk5)+cdown5;
        *cdown6=(Zt+rk6)+cdown6;
        *cdown7=(Zt+rk7)+cdown7;
        *cdown8=(Zt+rk8)+cdown8;
        *cdown9=(Zt+rk9)+cdown9;
        *cdown11=(Zt+rk11)+cdown11;
        *cdown12=(Zt+rk12)+cdown12;
        *cdown13=(Zt+rk13)+cdown13;

        *if cup1<0 then cup1=0;
        *if cup2<0 then cup2=0;
        *if cup3<0 then cup3=0;
        *if cup4<0 then cup4=0;
        *if cup5<0 then cup5=0;
        *if cup6<0 then cup6=0;
        *if cup7<0 then cup7=0;
        *if cup8<0 then cup8=0;
```

```
        *if cup9<0 then cup9=0;
        *if cup11<0 then cup11=0;
        *if cup12<0 then cup12=0;
        *if cup13<0 then cup13=0;

        *if cdown1>0 then cdown1=0;
        *if cdown2>0 then cdown2=0;
        *if cdown3>0 then cdown3=0;
        *if cdown4>0 then cdown4=0;
        *if cdown5>0 then cdown5=0;
        *if cdown6>0 then cdown6=0;
        *if cdown7>0 then cdown7=0;
        *if cdown8>0 then cdown8=0;
        *if cdown9>0 then cdown9=0;
        *if cdown11>0 then cdown11=0;
        *if cdown12>0 then cdown12=0;
        *if cdown13>0 then cdown13=0;

        *m2=oneml*m2 + lambda*xbar;
        nsignal=0;
        *If cdown1<lcl1 or cup1>ucl1 then do;
        *                            signal1=signal1+1;
        *                            nsignal=nsignal+1;
        *                                    end;
        *If cdown2<lcl2 or cup2>ucl2 then do;
        *                            signal2=signal2+1;
        *                            nsignal=nsignal+1;
        *                                    end;
        *If cdown3<lcl3 or cup3>ucl3 then do;
        *                            signal3=signal3+1;
        *                            nsignal=nsignal+1;
        *                                    end;
        *If cdown4<lcl4 or cup4>ucl4 then do;
        *                            signal4=signal4+1;
        *                            nsignal=nsignal+1;
        *                                    end;
        *If cdown5<lcl5 or cup5>ucl5 then do;
        *                            signal5=signal5+1;
        *                            nsignal=nsignal+1;
        *                                    end;
        *If cdown6<lcl6 or cup6>ucl6 then do;
        *                            signal6=signal6+1;
        *                            nsignal=nsignal+1;
        *                                    end;
        *If cdown7<lcl7 or cup7>ucl7 then do;
        *                            signal7=signal7+1;
        *                            nsignal=nsignal+1;
        *                                    end;
        *If cdown8<lcl8 or cup8>ucl8 then do;
        *                            signal8=signal8+1;
        *                            nsignal=nsignal+1;
        *                                    end;
        *If cdown9<lcl9 or cup9>ucl9 then do;
        *                            signal9=signal9+1;
        *                            nsignal=nsignal+1;
        *                                    end;
        *If m2<lcl10 or m2>ucl10 then do;
        *                            signal10=signal10+1;
        *                            nsignal=nsignal+1;
        *                                    end;
        *If cdown11<lcl11 or cup11>ucl11 then do;
```

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```
*                                           signal11=signal11+1;
*                                           nsignal=nsignal+1;
*                                                  end;
*If cdown12<lcl12 or cup12>ucl12 then do;
*                                           signal12=signal12+1;
*                                           nsignal=nsignal+1;
*                                                  end;
*If cdown13<lcl13 or cup13>ucl13 then do;
*                                           signal13=signal13+1;
*                                           nsignal=nsignal+1;
*                                                  end;
If nsignal>0 then goto endchart;
else goto nextsamp;
endchart:
       output;
end;


proc means n mean stderr;
  var runlength;
run;
```


Program 6:

```
***********************************************************************
*                          One-Sided Charts                          *
*      SAS Program to Find the ARL Relationship Between the           *
*      Individual CUSUM and the Simultaneous CUSUM Schemes.           *
*                                                                     *
*      1. Simultaneous CUSUM Schemes with Steady State Limits.        *
*      2. ARL of Each Simultaneous CUSUMA Scheme is Calculated.       *
*                                                                     *
***********************************************************************;
data;

*Parameters (rk,h) for Individual ARL=50;
*rk1=0.2;        *h1=3.87717;       *optimal for detecting delta=0.4;
*rk2=0.3;        *h2=3.14950;       *optimal for detecting delta=0.6;
*rk3=0.4;        *h3=2.63750;       *optimal for detecting delta=0.8;
*rk4=0.5;        *h4=2.26000;       *optimal for detecting delta=1.0;
*rk5=0.6;        *h5=1.95800;       *optimal for detecting delta=1.2;
*rk6=0.7;        *h6=1.71622;       *optimal for detecting delta=1.4;
*rk7=0.8;        *h7=1.51345;       *optimal for detecting delta=1.6;
*rk8=1.0;        *h8=1.18385;       *optimal for detecting delta=2.0;
*rk9=1.5;        *h9=0.57111;       *optimal for detecting delta=3.0;
*lambda=1.0;     *h10=2.05425;      *Shewhart chart;
*rk11=0.375;     *h11=2.75015;      *optimal for detecting delta=0.75;
*rk12=0.75;      *h12=1.61150;      *optimal for detecting delta=1.5;
*rk13=1.25;      *h13=0.85638;      *optimal for detecting delta=2.5;


*Parameters (rk,h) for Individual ARL=100;
*rk1=0.2;        *h1=5.10380;       *optimal for detecting delta=0.4;
*rk2=0.3;        *h2=4.08500;       *optimal for detecting delta=0.6;
*rk3=0.4;        *h3=3.38373;       *optimal for detecting delta=0.8;
*rk4=0.5;        *h4=2.87595;       *optimal for detecting delta=1.0;
*rk5=0.6;        *h5=2.48875;       *optimal for detecting delta=1.2;
*rk6=0.7;        *h6=2.17722;       *optimal for detecting delta=1.4;
*rk7=0.8;        *h7=1.92722;       *optimal for detecting delta=1.6;
*rk8=1.0;        *h8=1.53871;       *optimal for detecting delta=2.0;
*rk9=1.5;        *h9=0.86157;       *optimal for detecting delta=3.0;
*lambda=1.0;     *h10=2.32708;      *Shewhart chart;
```

```
*rk11=0.375;        *h11=3.54401;        *optimal for detecting delta=0.75;
*rk12=0.75;         *h12=2.04250;        *optimal for detecting delta=1.5;
*rk13=1.25;         *h13=1.16650;        *optimal for detecting delta=2.5;


*Parameters (rk,h) for Individual ARL=150;
*rk1=0.2;           *h1=5.89445;         *optimal for detecting delta=0.4;
*rk2=0.3;           *h2=4.65879;         *optimal for detecting delta=0.6;
*rk3=0.4;           *h3=3.83635;         *optimal for detecting delta=0.8;
*rk4=0.5;           *h4=3.25068;         *optimal for detecting delta=1.0;
*rk5=0.6;           *h5=2.80208;         *optimal for detecting delta=1.2;
*rk6=0.7;           *h6=2.45332;         *optimal for detecting delta=1.4;
*rk7=0.8;           *h7=2.16935;         *optimal for detecting delta=1.6;
*rk8=1.0;           *h8=1.73621;         *optimal for detecting delta=2.0;
*rk9=1.5;           *h9=1.02130;         *optimal for detecting delta=3.0;
*lambda=1.0;        *h10=2.47483;        *Shewhart chart;
*rk11=0.375;        *h11=4.01789;        *optimal for detecting delta=0.75;
*rk12=0.75;         *h12=2.30228;        *optimal for detecting delta=1.5;
*rk13=1.25;         *h13=1.33828;        *optimal for detecting delta=2.5;


*Parameters (rk,h) for Individual ARL=200;
*rk1=0.2;           *h1=6.47450;         *optimal for detecting delta=0.4;
*rk2=0.3;           *h2=5.07175;         *optimal for detecting delta=0.6;
*rk3=0.4;           *h3=4.17599;         *optimal for detecting delta=0.8;
*rk4=0.5;           *h4=3.52075;         *optimal for detecting delta=1.0;
*rk5=0.6;           *h5=3.03408;         *optimal for detecting delta=1.2;
*rk6=0.7;           *h6=2.64508;         *optimal for detecting delta=1.4;
*rk7=0.8;           *h7=2.34304;         *optimal for detecting delta=1.6;
*rk8=1.0;           *h8=1.87561;         *optimal for detecting delta=2.0;
*rk9=1.5;           *h9=1.13269;         *optimal for detecting delta=3.0;
*lambda=1.0;        *h10=2.57553;        *Shewhart chart;
*rk11=0.375;        *h11=4.36693;        *optimal for detecting delta=0.75;
*rk12=0.75;         *h12=2.48315;        *optimal for detecting delta=1.5;
*rk13=1.25;         *h13=1.46111;        *optimal for detecting delta=2.5;


*Parameters (rk,h) for Individual ARL=250;
*rk1=0.2;           *h1=6.92055;         *optimal for detecting delta=0.4;
*rk2=0.3;           *h2=5.41295;         *optimal for detecting delta=0.6;
*rk3=0.4;           *h3=4.43145;         *optimal for detecting delta=0.8;
*rk4=0.5;           *h4=3.73014;         *optimal for detecting delta=1.0;
*rk5=0.6;           *h5=3.20850;         *optimal for detecting delta=1.2;
*rk6=0.7;           *h6=2.80175;         *optimal for detecting delta=1.4;
*rk7=0.8;           *h7=2.47715;         *optimal for detecting delta=1.6;
*rk8=1.0;           *h8=1.98305;         *optimal for detecting delta=2.0;
*rk9=1.5;           *h9=1.21537;         *optimal for detecting delta=3.0;
*lambda=1.0;        *h10=2.65225;        *Shewhart chart;
*rk11=0.375;        *h11=4.64080;        *optimal for detecting delta=0.75;
*rk12=0.75;         *h12=2.62993;        *optimal for detecting delta=1.5;
*rk13=1.25;         *h13=1.54918;        *optimal for detecting delta=2.5;


*Parameters (rk,h) for Individual ARL=300;
*rk1=0.2;           *h1=7.32451;         *optimal for detecting delta=0.4;
*rk2=0.3;           *h2=5.69457;         *optimal for detecting delta=0.6;
*rk3=0.4;           *h3=4.64350;         *optimal for detecting delta=0.8;
*rk4=0.5;           *h4=3.90925;         *optimal for detecting delta=1.0;
*rk5=0.6;           *h5=3.35750;         *optimal for detecting delta=1.2;
*rk6=0.7;           *h6=2.92715;         *optimal for detecting delta=1.4;
*rk7=0.8;           *h7=2.58600;         *optimal for detecting delta=1.6;
*rk8=1.0;           *h8=2.07409;         *optimal for detecting delta=2.0;
*rk9=1.5;           *h9=1.28128;         *optimal for detecting delta=3.0;
*lambda=1.0;        *h10=2.71366;        *Shewhart chart;
*rk11=0.375;        *h11=4.86925;        *optimal for detecting delta=0.75;
```

```
*rk12=0.75;          *h12=2.74849;        *optimal for detecting delta=1.5;
*rk13=1.25;          *h13=1.62484;        *optimal for detecting delta=2.5;


*Parameters (rk,h) for Individual ARL=350;
*rk1=0.2;            *h1=7.64542;         *optimal for detecting delta=0.4;
*rk2=0.3;            *h2=5.92733;         *optimal for detecting delta=0.6;
*rk3=0.4;            *h3=4.82320;         *optimal for detecting delta=0.8;
*rk4=0.5;            *h4=4.05222;         *optimal for detecting delta=1.0;
*rk5=0.6;            *h5=3.48265;         *optimal for detecting delta=1.2;
*rk6=0.7;            *h6=3.04300;         *optimal for detecting delta=1.4;
*rk7=0.8;            *h7=2.68221;         *optimal for detecting delta=1.6;
*rk8=1.0;            *h8=2.14915;         *optimal for detecting delta=2.0;
*rk9=1.5;            *h9=1.33851;         *optimal for detecting delta=3.0;
*lambda=1.0;         *h10=2.76359;        *Shewhart chart;
*rk11=0.375;         *h11=5.06410;        *optimal for detecting delta=0.75;
*rk12=0.75;          *h12=2.85298;        *optimal for detecting delta=1.5;
*rk13=1.25;          *h13=1.68782;        *optimal for detecting delta=2.5;


*Parameters (rk,h) for Individual ARL=370;
*rk1=0.2;            *h1=7.76721;         *optimal for detecting delta=0.4;
*rk2=0.3;            *h2=6.00099;         *optimal for detecting delta=0.6;
*rk3=0.4;            *h3=4.88652;         *optimal for detecting delta=0.8;
*rk4=0.5;            *h4=4.10411;         *optimal for detecting delta=1.0;
*rk5=0.6;            *h5=3.52899;         *optimal for detecting delta=1.2;
*rk6=0.7;            *h6=3.07561;         *optimal for detecting delta=1.4;
*rk7=0.8;            *h7=2.71453;         *optimal for detecting delta=1.6;
*rk8=1.0;            *h8=2.17750;         *optimal for detecting delta=2.0;
*rk9=1.5;            *h9=1.35904;         *optimal for detecting delta=3.0;
*lambda=1.0;         *h10=2.78077;        *Shewhart chart;
*rk11=0.375;         *h11=5.13154;        *optimal for detecting delta=0.75;
*rk12=0.75;          *h12=2.88797;        *optimal for detecting delta=1.5;
*rk13=1.25;          *h13=1.70984;        *optimal for detecting delta=2.5;


*Parameters (rk,h) for Individual ARL=400;
*rk1=0.2;            *h1=7.93856;         *optimal for detecting delta=0.4;
*rk2=0.3;            *h2=6.12335;         *optimal for detecting delta=0.6;
*rk3=0.4;            *h3=4.97521;         *optimal for detecting delta=0.8;
*rk4=0.5;            *h4=4.18115;         *optimal for detecting delta=1.0;
*rk5=0.6;            *h5=3.58958;         *optimal for detecting delta=1.2;
*rk6=0.7;            *h6=3.13519;         *optimal for detecting delta=1.4;
*rk7=0.8;            *h7=2.76360;         *optimal for detecting delta=1.6;
*rk8=1.0;            *h8=2.21573;         *optimal for detecting delta=2.0;
*rk9=1.5;            *h9=1.38779;         *optimal for detecting delta=3.0;
*lambda=1.0;         *h10=2.80691;        *Shewhart chart;
*rk11=0.375;         *h11=5.22862;        *optimal for detecting delta=0.75;
*rk12=0.75;          *h12=2.93875;        *optimal for detecting delta=1.5;
*rk13=1.25;          *h13=1.74075;        *optimal for detecting delta=2.5;


*Parameters (rk,h) for Individual ARL=450;
*rk1=0.2;            *h1=8.19390;         *optimal for detecting delta=0.4;
*rk2=0.3;            *h2=6.31099;         *optimal for detecting delta=0.6;
*rk3=0.4;            *h3=5.12021;         *optimal for detecting delta=0.8;
*rk4=0.5;            *h4=4.30163;         *optimal for detecting delta=1.0;
*rk5=0.6;            *h5=3.68325;         *optimal for detecting delta=1.2;
*rk6=0.7;            *h6=3.21523;         *optimal for detecting delta=1.4;
*rk7=0.8;            *h7=2.83433;         *optimal for detecting delta=1.6;
*rk8=1.0;            *h8=2.27508;         *optimal for detecting delta=2.0;
*rk9=1.5;            *h9=1.42861;         *optimal for detecting delta=3.0;
*lambda=1.0;         *h10=2.84380;        *Shewhart chart;
*rk11=0.375;         *h11=5.38250;        *optimal for detecting delta=0.75;
*rk12=0.75;          *h12=3.01455;        *optimal for detecting delta=1.5;
```

```
*rk13=1.25;          *h13=1.78862;       *optimal for detecting delta=2.5;


*Parameters (rk,h) for Individual ARL=500;
*rk1=0.2;            *h1=8.42935;        *optimal for detecting delta=0.4;
*rk2=0.3;            *h2=6.47801;        *optimal for detecting delta=0.6;
*rk3=0.4;            *h3=5.25515;        *optimal for detecting delta=0.8;
*rk4=0.5;            *h4=4.40605;        *optimal for detecting delta=1.0;
*rk5=0.6;            *h5=3.77347;        *optimal for detecting delta=1.2;
*rk6=0.7;            *h6=3.29017;        *optimal for detecting delta=1.4;
*rk7=0.8;            *h7=2.90253;        *optimal for detecting delta=1.6;
*rk8=1.0;            *h8=2.32286;        *optimal for detecting delta=2.0;
*rk9=1.5;            *h9=1.46718;        *optimal for detecting delta=3.0;
*lambda=1.0;         *h10=2.87603;       *Shewhart chart;
*rk11=0.375;         *h11=5.51488;       *optimal for detecting delta=0.75;
*rk12=0.75;          *h12=3.08575;       *optimal for detecting delta=1.5;
*rk13=1.25;          *h13=1.83162;       *optimal for detecting delta=2.5;


*Parameters (rk,h) for Individual ARL=600;
*rk1=0.2;            *h1=8.84539;        *optimal for detecting delta=0.4;
*rk2=0.3;            *h2=6.76955;        *optimal for detecting delta=0.6;
*rk3=0.4;            *h3=5.47268;        *optimal for detecting delta=0.8;
*rk4=0.5;            *h4=4.57800;        *optimal for detecting delta=1.0;
*rk5=0.6;            *h5=3.92008;        *optimal for detecting delta=1.2;
*rk6=0.7;            *h6=3.41359;        *optimal for detecting delta=1.4;
*rk7=0.8;            *h7=3.01583;        *optimal for detecting delta=1.6;
*rk8=1.0;            *h8=2.41445;        *optimal for detecting delta=2.0;
*rk9=1.5;            *h9=1.52967;        *optimal for detecting delta=3.0;
*lambda=1.0;         *h10=2.93573;       *Shewhart chart;
*rk11=0.375;         *h11=5.74282;       *optimal for detecting delta=0.75;
*rk12=0.75;          *h12=3.20353;       *optimal for detecting delta=1.5;
*rk13=1.25;          *h13=1.90332;       *optimal for detecting delta=2.5;


*Parameters (rk,h) for Individual ARL=700;
*rk1=0.2;            *h1=9.20110;        *optimal for detecting delta=0.4;
*rk2=0.3;            *h2=7.00641;        *optimal for detecting delta=0.6;
*rk3=0.4;            *h3=5.65801;        *optimal for detecting delta=0.8;
*rk4=0.5;            *h4=4.72636;        *optimal for detecting delta=1.0;
*rk5=0.6;            *h5=4.04813;        *optimal for detecting delta=1.2;
*rk6=0.7;            *h6=3.52214;        *optimal for detecting delta=1.4;
*rk7=0.8;            *h7=3.10878;        *optimal for detecting delta=1.6;
*rk8=1.0;            *h8=2.48886;        *optimal for detecting delta=2.0;
*rk9=1.5;            *h9=1.58483;        *optimal for detecting delta=3.0;
*lambda=1.0;         *h10=2.98185;       *Shewhart chart;
*rk11=0.375;         *h11=5.93873;       *optimal for detecting delta=0.75;
*rk12=0.75;          *h12=3.30205;       *optimal for detecting delta=1.5;
*rk13=1.25;          *h13=1.96509;       *optimal for detecting delta=2.5;


*Parameters (rk,h) for Individual ARL=800;
*rk1=0.2;            *h1=9.51679;        *optimal for detecting delta=0.4;
*rk2=0.3;            *h2=7.21335;        *optimal for detecting delta=0.6;
*rk3=0.4;            *h3=5.81561;        *optimal for detecting delta=0.8;
*rk4=0.5;            *h4=4.86254;        *optimal for detecting delta=1.0;
*rk5=0.6;            *h5=4.16065;        *optimal for detecting delta=1.2;
*rk6=0.7;            *h6=3.62128;        *optimal for detecting delta=1.4;
*rk7=0.8;            *h7=3.18981;        *optimal for detecting delta=1.6;
*rk8=1.0;            *h8=2.55503;        *optimal for detecting delta=2.0;
*rk9=1.5;            *h9=1.63177;        *optimal for detecting delta=3.0;
*lambda=1.0;         *h10=3.02320;       *Shewhart chart;
*rk11=0.375;         *h11=6.11313;       *optimal for detecting delta=0.75;
*rk12=0.75;          *h12=3.39185;       *optimal for detecting delta=1.5;
*rk13=1.25;          *h13=2.01719;       *optimal for detecting delta=2.5;
```

```
*Parameters (rk,h) for Individual ARL=900;
*rk1=0.2;          *h1=9.77741;        *optimal for detecting delta=0.4;
*rk2=0.3;          *h2=7.39930;        *optimal for detecting delta=0.6;
*rk3=0.4;          *h3=5.96629;        *optimal for detecting delta=0.8;
*rk4=0.5;          *h4=4.97115;        *optimal for detecting delta=1.0;
*rk5=0.6;          *h5=4.25200;        *optimal for detecting delta=1.2;
*rk6=0.7;          *h6=3.70041;        *optimal for detecting delta=1.4;
*rk7=0.8;          *h7=3.26282;        *optimal for detecting delta=1.6;
*rk8=1.0;          *h8=2.61288;        *optimal for detecting delta=2.0;
*rk9=1.5;          *h9=1.67102;        *optimal for detecting delta=3.0;
*lambda=1.0;       *h10=3.05867;       *Shewhart chart;
*rk11=0.375;       *h11=6.26855;       *optimal for detecting delta=0.75;
*rk12=0.75;        *h12=3.46941;       *optimal for detecting delta=1.5;
*rk13=1.25;        *h13=2.06280;       *optimal for detecting delta=2.5;


*Parameters (rk,h) for Individual ARL=1000;
*rk1=0.2;          *h1=10.01760;       *optimal for detecting delta=0.4;
*rk2=0.3;          *h2=7.57635;        *optimal for detecting delta=0.6;
*rk3=0.4;          *h3=6.08775;        *optimal for detecting delta=0.8;
*rk4=0.5;          *h4=5.07383;        *optimal for detecting delta=1.0;
*rk5=0.6;          *h5=4.34385;        *optimal for detecting delta=1.2;
*rk6=0.7;          *h6=3.77455;        *optimal for detecting delta=1.4;
*rk7=0.8;          *h7=3.33300;        *optimal for detecting delta=1.6;
*rk8=1.0;          *h8=2.66657;        *optimal for detecting delta=2.0;
*rk9=1.5;          *h9=1.70807;        *optimal for detecting delta=3.0;
*lambda=1.0;       *h10=3.08969;       *Shewhart chart;
*rk11=0.375;       *h11=6.40310;       *optimal for detecting delta=0.75;
*rk12=0.75;        *h12=3.53958;       *optimal for detecting delta=1.5;
*rk13=1.25;        *h13=2.10614;       *optimal for detecting delta=2.5;


*Parameters (rk,h) for Individual ARL=1500;
*rk1=0.2;          *h1=10.98375;       *optimal for detecting delta=0.4;
*rk2=0.3;          *h2=8.23050;        *optimal for detecting delta=0.6;
*rk3=0.4;          *h3=6.58942;        *optimal for detecting delta=0.8;
*rk4=0.5;          *h4=5.47805;        *optimal for detecting delta=1.0;
*rk5=0.6;          *h5=4.67498;        *optimal for detecting delta=1.2;
*rk6=0.7;          *h6=4.05885;        *optimal for detecting delta=1.4;
*rk7=0.8;          *h7=3.57940;        *optimal for detecting delta=1.6;
*rk8=1.0;          *h8=2.86540;        *optimal for detecting delta=2.0;
*rk9=1.5;          *h9=1.84542;        *optimal for detecting delta=3.0;
*lambda=1.0;       *h10=3.20879;       *Shewhart chart;
*rk11=0.375;       *h11=6.92935;       *optimal for detecting delta=0.75;
*rk12=0.75;        *h12=3.80658;       *optimal for detecting delta=1.5;
*rk13=1.25;        *h13=2.26510;       *optimal for detecting delta=2.5;


*Parameters (rk,h) for Individual ARL=2000;
*rk1=0.2;          *h1=11.65871;       *optimal for detecting delta=0.4;
*rk2=0.3;          *h2=8.69551;        *optimal for detecting delta=0.6;
*rk3=0.4;          *h3=6.93700;        *optimal for detecting delta=0.8;
*rk4=0.5;          *h4=5.76034;        *optimal for detecting delta=1.0;
*rk5=0.6;          *h5=4.90900;        *optimal for detecting delta=1.2;
*rk6=0.7;          *h6=4.26521;        *optimal for detecting delta=1.4;
*rk7=0.8;          *h7=3.76295;        *optimal for detecting delta=1.6;
*rk8=1.0;          *h8=3.01152;        *optimal for detecting delta=2.0;
*rk9=1.5;          *h9=1.94297;        *optimal for detecting delta=3.0;
*lambda=1.0;       *h10=3.29113;       *Shewhart chart;
*rk11=0.375;       *h11=7.30048;       *optimal for detecting delta=0.75;
*rk12=0.75;        *h12=3.99522;       *optimal for detecting delta=1.5;
*rk13=1.25;        *h13=2.37947;       *optimal for detecting delta=2.5;
```

```
*Parameters (rk,h) for Individual ARL=2500;
*rk1=0.2;        *h1=12.18786;     *optimal for detecting delta=0.4;
*rk2=0.3;        *h2=9.05526;      *optimal for detecting delta=0.6;
*rk3=0.4;        *h3=7.20958;      *optimal for detecting delta=0.8;
*rk4=0.5;        *h4=5.98423;      *optimal for detecting delta=1.0;
*rk5=0.6;        *h5=5.09765;      *optimal for detecting delta=1.2;
*rk6=0.7;        *h6=4.42932;      *optimal for detecting delta=1.4;
*rk7=0.8;        *h7=3.90130;      *optimal for detecting delta=1.6;
*rk8=1.0;        *h8=3.12008;      *optimal for detecting delta=2.0;
*rk9=1.5;        *h9=2.01664;      *optimal for detecting delta=3.0;
*lambda=1.0;     *h10=3.35259;     *Shewhart chart;
*rk11=0.375;     *h11=7.59791;     *optimal for detecting delta=0.75;
*rk12=0.75;      *h12=4.14420;     *optimal for detecting delta=1.5;
*rk13=1.25;      *h13=2.46664;     *optimal for detecting delta=2.5;


*Parameters (rk,h) for Individual ARL=3000;
*rk1=0.2;        *h1=12.63480;     *optimal for detecting delta=0.4;
*rk2=0.3;        *h2=9.35319;      *optimal for detecting delta=0.6;
*rk3=0.4;        *h3=7.44052;      *optimal for detecting delta=0.8;
*rk4=0.5;        *h4=6.16128;      *optimal for detecting delta=1.0;
*rk5=0.6;        *h5=5.24815;      *optimal for detecting delta=1.2;
*rk6=0.7;        *h6=4.55542;      *optimal for detecting delta=1.4;
*rk7=0.8;        *h7=4.01062;      *optimal for detecting delta=1.6;
*rk8=1.0;        *h8=3.21163;      *optimal for detecting delta=2.0;
*rk9=1.5;        *h9=2.07741;      *optimal for detecting delta=3.0;
*lambda=1.0;     *h10=3.40334;     *Shewhart chart;
*rk11=0.375;     *h11=7.83735;     *optimal for detecting delta=0.75;
*rk12=0.75;      *h12=4.26742;     *optimal for detecting delta=1.5;
*rk13=1.25;      *h13=2.53832;         *optimal for detecting delta=2.5;


*Parameters (rk,h) for Individual ARL=3500;
*rk1=0.2;        *h1=13.01356;     *optimal for detecting delta=0.4;
*rk2=0.3;        *h2=9.60850;      *optimal for detecting delta=0.6;
*rk3=0.4;        *h3=7.62684;      *optimal for detecting delta=0.8;
*rk4=0.5;        *h4=6.31508;      *optimal for detecting delta=1.0;
*rk5=0.6;        *h5=5.37810;      *optimal for detecting delta=1.2;
*rk6=0.7;        *h6=4.66315;      *optimal for detecting delta=1.4;
*rk7=0.8;        *h7=4.11117;      *optimal for detecting delta=1.6;
*rk8=1.0;        *h8=3.28862;      *optimal for detecting delta=2.0;
*rk9=1.5;        *h9=2.12839;      *optimal for detecting delta=3.0;
*lambda=1.0;     *h10=3.44485;     *Shewhart chart;
*rk11=0.375;     *h11=8.03908;     *optimal for detecting delta=0.75;
*rk12=0.75;      *h12=4.36960;     *optimal for detecting delta=1.5;
*rk13=1.25;      *h13=2.59888;     *optimal for detecting delta=2.5;


*Parameters (rk,h) for Individual ARL=4000;
*rk1=0.2;        *h1=13.33031;     *optimal for detecting delta=0.4;
*rk2=0.3;        *h2=9.82512;      *optimal for detecting delta=0.6;
*rk3=0.4;        *h3=7.79249;      *optimal for detecting delta=0.8;
*rk4=0.5;        *h4=6.44660;      *optimal for detecting delta=1.0;
*rk5=0.6;        *h5=5.48548;      *optimal for detecting delta=1.2;
*rk6=0.7;        *h6=4.75766;      *optimal for detecting delta=1.4;
*rk7=0.8;        *h7=4.19180;      *optimal for detecting delta=1.6;
*rk8=1.0;        *h8=3.35613;      *optimal for detecting delta=2.0;
*rk9=1.5;        *h9=2.17222;      *optimal for detecting delta=3.0;
*lambda=1.0;     *h10=3.48054;     *Shewhart chart;
*rk11=0.375;     *h11=8.21962;     *optimal for detecting delta=0.75;
*rk12=0.75;      *h12=4.46003;     *optimal for detecting delta=1.5;
*rk13=1.25;      *h13=2.65322;     *optimal for detecting delta=2.5;


delta=0.0;
```

```
numrun=100000;
n=4;
mu=0.0;
sigma=2.0;
oneml=(1-lambda);
shift=delta*sigma/n**0.5;
seed=499632567;

*ucl1=h1;
*ucl2=h2;
*ucl3=h3;
*ucl4=h4;
*ucl5=h5;
*ucl6=h6;
*ucl7=h7;
*ucl8=h8;
*ucl9=h9;
*ucl10=h10;
*ucl11=h11;
*ucl12=h12;
*ucl13=h13;

*signal1=0;
*signal2=0;
*signal3=0;
*signal4=0;
*signal5=0;
*signal6=0;
*signal7=0;
*signal8=0;
*signal9=0;
*signal10=0;
*signal11=0;
*signal12=0;
*signal13=0;

do i=1 to numrun;
restart:
      *cup1=0.0;
      *cup2=0.0;
      *cup3=0.0;
      *cup4=0.0;
      *cup5=0.0;
      *cup6=0.0;
      *cup7=0.0;
      *cup8=0.0;
      *cup9=0;
      *cup11=0.0;
      *cup12=0.0;
      *cup13=0.0;

      m2=0;
      runlength=0;

      do j=1 to 100;
            x1=mu+rannor(seed)*sigma;
            x2=mu+rannor(seed)*sigma;
            x3=mu+rannor(seed)*sigma;
            x4=mu+rannor(seed)*sigma;
            xbar=(x1+x2+x3+x4)/n;
            Zt=xbar;
```

```
*upper sided cusum;
            *cup1=(Zt-rk1)+cup1;
            *cup2=(Zt-rk2)+cup2;
            *cup3=(Zt-rk3)+cup3;
            *cup4=(Zt-rk4)+cup4;
            *cup5=(Zt-rk5)+cup5;
            *cup6=(Zt-rk6)+cup6;
            *cup7=(Zt-rk7)+cup7;
            *cup8=(Zt-rk8)+cup8;
            *cup9=(Zt-rk9)+cup9;
            *cup11=(Zt-rk11)+cup11;
            *cup12=(Zt-rk12)+cup12;
            *cup13=(Zt-rk13)+cup13;

            *if cup1<0 then cup1=0;
            *if cup2<0 then cup2=0;
            *if cup3<0 then cup3=0;
            *if cup4<0 then cup4=0;
            *if cup5<0 then cup5=0;
            *if cup6<0 then cup6=0;
            *if cup7<0 then cup7=0;
            *if cup8<0 then cup8=0;
            *if cup9<0 then cup9=0;
            *if cup11<0 then cup11=0;
            *if cup12<0 then cup12=0;
            *if cup13<0 then cup13=0;

            *m2=oneml*m2 + lambda*xbar;
        end;

        if

        then goto restart;

*
        cup1>ucl1
        or cup2>ucl2
        or cup3>ucl3
        or cup4>ucl4
        or cup5>ucl5
        or cup6>ucl6
        or cup7>ucl7
        or cup8>ucl8
        or cup9>ucl9
        or cup11>ucl11
        or cup12>ucl12
        or cup13>ucl13
        or m2>ucl10
;
        nextsamp:
        runlength=runlength+1;
        x1=(mu+shift)+rannor(seed)*sigma;
        x2=(mu+shift)+rannor(seed)*sigma;
        x3=(mu+shift)+rannor(seed)*sigma;
        x4=(mu+shift)+rannor(seed)*sigma;
        xbar=(x1+x2+x3+x4)/n;
        Zt=xbar;

*upper sided cusum;
        *cup1=(Zt-rk1)+cup1;
```

```
*cup2=(Zt-rk2)+cup2;
*cup3=(Zt-rk3)+cup3;
*cup4=(Zt-rk4)+cup4;
*cup5=(Zt-rk5)+cup5;
*cup6=(Zt-rk6)+cup6;
*cup7=(Zt-rk7)+cup7;
*cup8=(Zt-rk8)+cup8;
*cup9=(Zt-rk9)+cup9;
*cup11=(Zt-rk11)+cup11;
*cup12=(Zt-rk12)+cup12;
*cup13=(Zt-rk13)+cup13;

*if cup1<0 then cup1=0;
*if cup2<0 then cup2=0;
*if cup3<0 then cup3=0;
*if cup4<0 then cup4=0;
*if cup5<0 then cup5=0;
*if cup6<0 then cup6=0;
*if cup7<0 then cup7=0;
*if cup8<0 then cup8=0;
*if cup9<0 then cup9=0;
*if cup11<0 then cup11=0;
*if cup12<0 then cup12=0;
*if cup13<0 then cup13=0;

*m2=oneml*m2 + lambda*xbar;
nsignal=0;
*If cup1>ucl1 then do;
*          signal1=signal1+1;
*          nsignal=nsignal+1;
*               end;
*If cup2>ucl2 then do;
*          signal2=signal2+1;
*          nsignal=nsignal+1;
*               end;
*If cup3>ucl3 then do;
*          signal3=signal3+1;
*          nsignal=nsignal+1;
*               end;
*If cup4>ucl4 then do;
*          signal4=signal4+1;
*          nsignal=nsignal+1;
*               end;
*If cup5>ucl5 then do;
*          signal5=signal5+1;
*          nsignal=nsignal+1;
*               end;
*If cup6>ucl6 then do;
*          signal6=signal6+1;
*          nsignal=nsignal+1;
*               end;
*If cup7>ucl7 then do;
*          signal7=signal7+1;
*          nsignal=nsignal+1;
*               end;
*If cup8>ucl8 then do;
*          signal8=signal8+1;
*          nsignal=nsignal+1;
*               end;
*If cup9>ucl9 then do;
*          signal9=signal9+1;
```

```
*            nsignal=nsignal+1;
*                end;
*If m2>ucl10 then do;
*            signal10=signal10+1;
*            nsignal=nsignal+1;
*                end;
*If cup11>ucl11 then do;
*            signal11=signal11+1;
*            nsignal=nsignal+1;
*                end;
*If cup12>ucl12 then do;
*            signal12=signal12+1;
*            nsignal=nsignal+1;
*                end;
*If cup13>ucl13 then do;
*            signal13=signal13+1;
*            nsignal=nsignal+1;
*                end;
If nsignal>0 then goto endchart;
else goto nextsamp;
endchart:
        output;
end;


proc means n mean stderr;
  var runlength;
run;




Program 7:
**********************************************************************
*                    Two Sided Charts                               *
*    SAS Program to Find the h for Different "Reference k" (rk)     *
*         Values of CUSUM Chart                                      *
*    1. Calculate the 'h' Values by Considering Steady State        *
*    2. Firstly Select the 'rk' which Need to Find 'h' Value        *
*    3. Change 'h' Value (By-Section Algorithm) to Get Desired      *
*       ARL for These 'h' and 'Lambda'                              *
*    4. This Program Can be Used to find the ARL for                *
*       Given 'rk' and 'h'.                                          *
**********************************************************************;
data;
rk=0.2;
h=9.53253;

n=4;
delta=0.00;
mu=0.0;
sigma=2.0;
shift=delta*sigma/n**0.5;
numrun=100000;
seed=498643679;

*Upper and Lower chart limits;
    lcl=-h;
    ucl=h;

*Simulations;
do i=1 to numrun;
restart:
```

```
        cup=0.0;
        cdown=0.0;
        runlength=0;

*For steady state;
        do j=1 to 100;
                x1=mu+rannor(seed)*sigma;
                x2=mu+rannor(seed)*sigma;
                x3=mu+rannor(seed)*sigma;
                x4=mu+rannor(seed)*sigma;
                xbar=(x1+x2+x3+x4)/n;
                Zt=xbar;

*upper sided cusum;
                cup=(Zt-rk)+cup;
*lower sided cusum;
                cdown=(Zt+rk)+cdown;

*Re-set the parameters;
                if cup<0 then cup=0;
                if cdown>0 then cdown=0;
        end;

*Decision critera;
        if
                cup>ucl or cdown<lcl
        then goto restart;


*After steady state;
        nextsamp:
        runlength=runlength+1;
        x1=(mu+shift)+rannor(seed)*sigma;
        x2=(mu+shift)+rannor(seed)*sigma;
        x3=(mu+shift)+rannor(seed)*sigma;
        x4=(mu+shift)+rannor(seed)*sigma;
        xbar=(x1+x2+x3+x4)/n;
        Zt=xbar;

*upper sided cusum;
        cup=(Zt-rk)+cup;

*lower sided cusum;
        cdown=(Zt+rk)+cdown;

*Re-set the parameters;
        if cup<0 then cup=0;
        if cdown>0 then cdown=0;

*Decision critera;
        if
                cup>ucl or cdown<lcl
        then goto endchart;
        else goto nextsamp;
        endchart:
                output;
end;

proc means n mean stderr;
   var runlength;
run;
```

Program 8:

```
********************************************************************
*                        One-Sided Charts                        *
*      SAS Program to Find the h for Different "Reference k" (rk) *
*            Values of CUSUM Chart                                *
*      1. Calculate the 'h' Values by Considering Steady State   *
*      2. Firstly Select the 'rk' which Need to Find 'h' Value   *
*      3. Change 'h' Value (By-Section Algorithm) to Get Desired *
*         ARL for These 'h' and 'Lambda'                          *
*      4. This Program Can be Used to find the ARL for           *
*         Given 'rk' and 'h'.                                     *
********************************************************************;
data;
rk1=0.4;
h1=5.19695;

delta=0.0;
n=4;
mu=0.0;
sigma=2.0;
oneml=(1-lambda);
shift=delta*sigma/n**0.5;
numrun=100000;
seed=499875789;

ucl1=h1;
signal1=0;

do i=1 to numrun;
restart:
      cup1=0.0;

runlength=0;

      do j=1 to 100;
            x1=mu+rannor(seed)*sigma;
            x2=mu+rannor(seed)*sigma;
            x3=mu+rannor(seed)*sigma;
            x4=mu+rannor(seed)*sigma;
            xbar=(x1+x2+x3+x4)/n;
            Zt=xbar;

*upper sided cusum;
            cup1=(Zt-rk1)+cup1;

            if cup1<0 then cup1=0;

      end;

      if
            cup1>ucl1

      then goto restart;

nextsamp:
      runlength=runlength+1;
      x1=(mu+shif)+rannor(seed)*sigma;
      x2=(mu+shif)+rannor(seed)*sigma;
      x3=(mu+shif)+rannor(seed)*sigma;
      x4=(mu+shif)+rannor(seed)*sigma;
      xbar=(x1+x2+x3+x4)/n;
```

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```
        Zt=xbar;

*upper sided cusum;

            cup1=(Zt-rk1)+cup1;

            if cup1<0 then cup1=0;

        nsignal=0;

*one sided CUSUM;
        If cup1>ucl1 then do;
                    signal1=signal1+1;
                    nsignal=nsignal+1;
                        end;

        If nsignal>0 then goto endchart;
        else goto nextsamp;
        endchart:
            output;
end;
proc means n mean stderr;
  var runlength;
run;
```

Program 9:
```
********************************************************************
*           SAS Program to Find the Number of EWMA Charts          *
*                                                                  *
*     1. Simultaneous EWMA Schemes with Steady State Limits.       *
*     2. ARL of Each EWMA Chart and the Shewhart Chart is 1000.    *
*     3. ARL of Each Simultaneous EWMA Scheme is Calculated.       *
*     4. The Order of Charts Added to a Scheme is as Follows;      *
*     5. Delta = 0.4, Shewhart, 1.6, 0.8, 2.0, 1.2, 4.0,          *
*                0.6, 1.8, 1.0, 3.0 And 1.4.                      *
********************************************************************;
data ARL;
delta=0.0;
mu=0.0;
sigma=2.0;
n=4;
shift=delta*sigma/n**0.5;
numrun=100;
seed=495413117;

*Parameters (h,lambda) of the EWMA and Shewhart chart with individual
ARL=1000;
     h1=0.3515;        lambda1=0.032;        *delta = 0.4;
     *h2=0.4965;       *lambda2=0.056;       *delta = 0.6;
     *h3=0.6320;       *lambda3=0.084;       *delta = 0.8;
     *h4=0.7670;       *lambda4=0.116;       *delta = 1.0;
     *h5=0.8920;       *lambda5=0.149;       *delta = 1.2;
     *h6=1.0180;       *lambda6=0.186;       *delta = 1.4;
     *h7=1.1380;       *lambda7=0.224;       *delta = 1.6;
     *h8=1.2500;       *lambda8=0.265;       *delta = 1.8;
     *h9=1.3800;       *lambda9=0.307;       *delta = 2.0;
     *h10=2.0100;      *lambda10=0.545;      *delta = 3.0;
     *h11=2.5480;      *lambda11=0.75;       *delta = 4.0;
     *h12=3.2905;      *lambda12=1.00;       *Shewhart;
```

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```
*Weights for the past samples;
      oneml1=(1-lambda1);
      *oneml2=(1-lambda2);
      *oneml3=(1-lambda3);
      *oneml4=(1-lambda4);
      *oneml5=(1-lambda5);
      *oneml6=(1-lambda6);
      *oneml7=(1-lambda7);
      *oneml8=(1-lambda8);
      *oneml9=(1-lambda9);
      *oneml10=(1-lambda10);
      *oneml11=(1-lambda11);
      *oneml12=(1-lambda12);

* upper and lower chart Limits;
      lcl1=mu-h1;
      ucl1=mu+h1;
      *lcl2=mu-h2;
      *ucl2=mu+h2;
      *lcl3=mu-h3;
      *ucl3=mu+h3;
      *lcl4=mu-h4;
      *ucl4=mu+h4;
      *lcl5=mu-h5;
      *ucl5=mu+h5;
      *lcl6=mu-h6;
      *ucl6=mu+h6;
      *lcl7=mu-h7;
      *ucl7=mu+h7;
      *lcl8=mu-h8;
      *ucl8=mu+h8;
      *lcl9=mu-h9;
      *ucl9=mu+h9;
      *lcl10=mu-h10;
      *ucl10=mu+h10;
      *lcl11=mu-h11;
      *ucl11=mu+h11;
      *lcl12=mu-h12;
      *ucl12=mu+h12;

*variable to count the number of signals;
      signal1=0;
      *signal2=0;
      *signal3=0;
      *signal4=0;
      *signal5=0;
      *signal6=0;
      *signal7=0;
      *signal8=0;
      *signal9=0;
      *signal10=0;
      *signal11=0;
      *signal12=0;

*Starting the simulations;
do i=1 to numrun;
      restart:
      m1=0.0;
      *m2=0.0;
      *m3=0.0;
      *m4=0.0;
```

```
        *m5=0.0;
        *m6=0.0;
        *m7=0.0;
        *m8=0.0;
        *m9=0.0;
        *m10=0.0;
        *m11=0.0;
        *m12=0.0;
*To reach the steady state;
        runlength=0;
        do j=1 to 300;
                x1=mu+rannor(seed)*sigma;
                x2=mu+rannor(seed)*sigma;
                x3=mu+rannor(seed)*sigma;
                x4=mu+rannor(seed)*sigma;
                xbar=(x1+x2+x3+x4)/n;

*calculate the ewma statistic;
                m1=oneml1*m1 + lambda1*xbar;
                *m2=oneml2*m2 + lambda2*xbar;
                *m3=oneml3*m3 + lambda3*xbar;
                *m4=oneml4*m4 + lambda4*xbar;
                *m5=oneml5*m5 + lambda5*xbar;
                *m6=oneml6*m6 + lambda6*xbar;
                *m7=oneml7*m7 + lambda7*xbar;
                *m8=oneml8*m8 + lambda8*xbar;
                *m9=oneml9*m9 + lambda9*xbar;
                *m10=oneml10*m10+lambda10*xbar;
                *m11=oneml11*m11+lambda11*xbar;
                *m12=oneml12*m12+lambda12*xbar;
        end;

* have to select the decision criteria form the following list and add in
between if condition below;
        If
                m1<lcl1 or m1>ucl1
        then goto restart;

        *
        m1<lcl1 or m1>ucl1
        or m2<lcl2 or m2>ucl2
        or m3<lcl3 or m3>ucl3
        or m4<lcl4 or m4>ucl4
        or m5<lcl5 or m5>ucl5
        or m6<lcl6 or m6>ucl6
        or m7<lcl7 or m7>ucl7
        or m8<lcl8 or m8>ucl8
        or m9<lcl9 or m9>ucl9
        or m10<lcl10 or m10>ucl10
        or m11<lcl11 or m11>ucl11
        or m12<lcl12 or m12>ucl12
        ;

*After the steady state;
        nextsamp:
        runlength=runlength+1;
        x1=(mu+shift)+rannor(seed)*sigma;
        x2=(mu+shift)+rannor(seed)*sigma;
        x3=(mu+shift)+rannor(seed)*sigma;
        x4=(mu+shift)+rannor(seed)*sigma;
        xbar=(x1+x2+x3+x4)/n;
```

```
*calculate the ewma statistic;
      m1=oneml1*m1 + lambda1*xbar;
      *m2=oneml2*m2 + lambda2*xbar;
      *m3=oneml3*m3 + lambda3*xbar;
      *m4=oneml4*m4 + lambda4*xbar;
      *m5=oneml5*m5 + lambda5*xbar;
      *m6=oneml6*m6 + lambda6*xbar;
      *m7=oneml7*m7 + lambda7*xbar;
      *m8=oneml8*m8 + lambda8*xbar;
      *m9=oneml9*m9 + lambda9*xbar;
      *m10=oneml10*m10+lambda10*xbar;
      *m11=oneml11*m11+lambda11*xbar;
      *m12=oneml12*m12+lambda12*xbar;


*Variable to count the number of signals;
      nsignal=0;


*Decide whether there is a signal or not;
      If m1<lcl1 or m1>ucl1 then do;
                            signal1=signal1+1;
                            nsignal=nsignal+1;
                                  end;
      *If m2<lcl2 or m2>ucl2 then do;
      *                     signal2=signal2+1;
      *                     nsignal=nsignal+1;
      *                           end;
      *If m3<lcl3 or m3>ucl3 then do;
      *                     signal3=signal3+1;
      *                     nsignal=nsignal+1;
      *                           end;
      *If m4<lcl4 or m4>ucl4 then do;
      *                     signal4=signal4+1;
      *                     nsignal=nsignal+1;
      *                           end;
      *If m5<lcl5 or m5>ucl5 then do;
      *                     signal5=signal5+1;
      *                     nsignal=nsignal+1;
      *                           end;
      *If m6<lcl6 or m6>ucl6 then do;
      *                     signal6=signal6+1;
      *                     nsignal=nsignal+1;
      *                           end;
      *If m7<lcl7 or m7>ucl7 then do;
      *                     signal7=signal7+1;
      *                     nsignal=nsignal+1;
      *                           end;
      *If m8<lcl8 or m8>ucl8 then do;
      *                     signal8=signal8+1;
      *                     nsignal=nsignal+1;
      *                           end;
      *If m9<lcl9 or m9>ucl9 then do;
      *                     signal9=signal9+1;
      *                     nsignal=nsignal+1;
      *                           end;
      *If m10<lcl10 or m10>ucl10 then do;
      *                     signal10=signal10+1;
      *                     nsignal=nsignal+1;
      *                           end;
      *
      *If m11<lcl11 or m11>ucl11 then do;
      *                     signal11=signal11+1;
```

```
*                          nsignal=nsignal+1;
*                              end;
*If m12<lcl12 or m12>ucl12 then do;
*                          signal12=signal12+1;
*                          nsignal=nsignal+1;
*                              end;

        If nsignal>0 then goto endchart;
        else goto nextsamp;

        endchart:
                output;
end;
proc means data=ARL N Mean Stderr;
 var runlength;
run;
```

Program 10:

```
************************************************************************
*          SAS Program to Find the ARL Profile of                     *
*               Simultaneous EWMA Schemes.                            *
*                                                                     *
*     1. Simultaneous EWMA Schemes with Steady State Limits.          *
*     2. ARL of Each Simultaneous EWMA Scheme is Calculated.          *
*     3. Parameters (h, Lambda) of the Individual EWMA Chart          *
*        with Different ARL's in Different Schemes to Give ARL of      *
*        Simultaneous EWMA Scheme as 370                              *
*                                                                     *
************************************************************************;
data ARL;
delta=0.00;
n=4;
mu=0.0;
sigma=2.0;
shift=delta*sigma/n**0.5;
numrun=100000;
seed=499563251;

*Individual EWMA Charts;
      *Parameters (h, lambda) of the individual EWMA chart to give
      ARL=370;
            *h1=0.342000;    *lambda1=0.0390;        *delta =0.4;
            *h2=0.497392;    *lambda2=0.0700;        *delta =0.6;
            *h3=0.600720;    *lambda3=0.0950;        *delta =0.75;
            *h4=0.637000;    *lambda4=0.1040;        *delta =0.8;
            *h5=0.772000;    *lambda5=0.1420;        *delta =1.0;
            *h6=0.903500;    *lambda6=0.1830;        *delta =1.2;
            *h7=1.032500;    *lambda7=0.2270;        *delta =1.4;
            *h8=1.097480;    *lambda8=0.2505;        *delta =1.5;
            *h9=1.150609;    *lambda9=0.2700;        *delta =1.6;
            *h10=1.419000;   *lambda10=0.3750;       *delta =2.0;
            *h11=1.713450;   *lambda11=0.4980;       *delta =2.5;
            *h12=2.100000;   *lambda12=0.6600;       *delta =3.0;
            *h13=2.999670;   *lambda13=1.00;         *Shewhart;

*Combined EWMA-Shewhart schemes;
      *Combined EWMA-Shewhart scheme in detecting [0.4,infinity);
      *ARL for Individual EWMA = 720.00;
            h1=0.328520;     lambda1=0.031;          *delta =0.4;
            h13=3.19775;     lambda13=1.00;          *Shewhart;
```

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```
      *Combined EWMA-Shewhart scheme in detecting [1.0,infinity);
      *ARL for Individual EWMA = 705.00;
           *h5=0.753770;      *lambda5=0.120;          *delta =1.0;
           *h13=3.19036;      *lambda13=1.00;          *Shewhart;


      *Combined EWMA-Shewhart scheme in detecting [1.5,infinity);
      *ARL for Individual EWMA = 685.00;
           *h8=1.084420;      *lambda8=0.220;          *delta =1.5;
           *h13=3.18221;      *lambda13=1.00;          *Shewhart;


*2-EWMA schemes;
      *2-EWMA scheme in detecting [0.4,infinity);
      *ARL for Individual EWMA = 898.00;
           *h1=0.33952;       *lambda1=0.031;          *delta =0.4;
           *h5=0.79280;       *lambda5=0.125;          *delta =1.0;
           *h13=3.2589;       *lambda13=1.00;          *Shewhart;


      *2-EWMA scheme in detecting [1.0,infinity);
      *ARL for Individual EWMA = 752.00;
           *h5=0.75552;       *lambda5=0.119;          *delta =1.0;
           *h8=1.09425;       *lambda8=0.220;          *delta =1.5;
           *h13=3.20838;      *lambda13=1.0;           *Shewhart;


      *2-EWMA scheme in detecting [1.5,infinity);
      *ARL for Individual EWMA = 749.00;
           *h8=1.09391;       *lambda8=0.220;          *delta =1.5;
           *h10=1.38667;      *lambda10=0.323;         *delta =2.0;
           *h13=3.20735;      *lambda13=1.0;           *Shewhart;


*3-EWMA schemes;
      *3-EWMA scheme in detecting [0.4,infinity);
      *ARL for Individual EWMA = 916.00;
           *h1=0.34058;       *lambda1=0.031;          *delta =0.4;
           *h2=0.49584;       *lambda2=0.057;          *delta =0.6;
           *h5=0.76763;       *lambda5=0.118;          *delta =1.0;
           *h13=3.2657;       *lambda13=1.00;          *Shewhart;


      *3-EWMA scheme in detecting [1.0,infinity);
      *ARL for Individual EWMA = 865.00;
           *h5=0.76239;       *lambda5=0.118;          *delta =1.0;
           *h8=1.08871;       *lambda8=0.213;          *delta =1.5;
           *h10=1.39509;      *lambda10=0.318;         *delta =2.0;
           *h13=3.24955;      *lambda13=1.00;          *Shewhart;


      *3-EWMA scheme in detecting [1.5,infinity);
      *ARL for Individual EWMA = 793.00;
           *h8=1.10555;       *lambda8=0.222;          *delta =1.5;
           *h10=1.39586;      *lambda10=0.324;         *delta =2.0;
           *h11=1.66967;      *lambda11=0.431;         *delta =2.5;
           *h13=3.22453;      *lambda13=1.00;          *Shewhart;


*4-EWMA schemes;
      *4-EWMA scheme in detecting [0.4,infinity);
      *ARL for Individual EWMA = 1068.00;
           *h1=0.33375;       *lambda1=0.029;          *delta =0.4;
           *h2=0.48869;       *lambda2=0.054;          *delta =0.6;
           *h5=0.78373;       *lambda5=0.119;          *delta =1.0;
           *h10=1.38554;      *lambda10=0.306;         *delta =2.0;
           *h13=3.30790;      *lambda13=1.00;          *Shewhart;


      *4-EWMA scheme in detecting [1.0,infinity);
```

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```
      *ARL for Individual EWMA = 908.00;
          *h5=0.78345;        *lambda5=0.122;        *delta =1.0;
          *h8=1.08564;        *lambda8=0.211;        *delta =1.5;
          *h10=1.39530;       *lambda10=0.317;       *delta =2.0;
          *h11=1.69779;       *lambda11=0.431;       *delta =2.5;
          *h13=3.26320;       *lambda13=1.00;        *Shewhart;

      *4-EWMA scheme in detecting [1.5,infinity);
      *ARL for Individual EWMA = 826.00;
          *h8=1.08022;        *lambda8=0.212;        *delta =1.5;
          *h10=1.39672;       *lambda10=0.322;       *delta =2.0;
          *h11=1.71165;       *lambda11=0.441;       *delta =2.5;
          *h12=2.01067;       *lambda12=0.559;       *delta =3.0;
          *h13=3.23630;       *lambda13=1.0;         *Shewhart;


*Super-EWMA scheme;
      *Super-EWMA scheme in detecting [0.4,infinity);
      *ARL for Individual EWMA = 1176.00;
          *h1=0.3522;         *lambda1=0.031;        *delta =0.4;
          *h2=0.5000;         *lambda2=0.055;        *delta =0.6;
          *h4=0.6400;         *lambda4=0.083;        *delta =0.8;
          *h5=0.7720;         *lambda5=0.114;        *delta =1.0;
          *h6=0.9020;         *lambda6=0.148;        *delta =1.2;
          *h7=1.0270;         *lambda7=0.184;        *delta =1.4;
          *h9=1.1510;         *lambda9=0.223;        *delta =1.6;
          *h10=1.3950;        *lambda10=0.305;       *delta =2.0;
          *h12=2.03100;       *lambda12=0.543;       *delta =3.0;
          *h13=3.33591;       *lambda13=1.00;        *Shewhart;


*Weights for past samples;
      oneml1=(1-lambda1);
      *oneml2=(1-lambda2);
      *oneml3=(1-lambda3);
      *oneml4=(1-lambda4);
      *oneml5=(1-lambda5);
      *oneml6=(1-lambda6);
      *oneml7=(1-lambda7);
      *oneml8=(1-lambda8);
      *oneml9=(1-lambda9);
      *oneml10=(1-lambda10);
      *oneml11=(1-lambda11);
      *oneml12=(1-lambda12);
      oneml13=(1-lambda13);


*Upper and lower control limits;
      lcl1=mu-h1;
      ucl1=mu+h1;
      *lcl2=mu-h2;
      *ucl2=mu+h2;
      *lcl3=mu-h3;
      *ucl3=mu+h3;
      *lcl4=mu-h4;
      *ucl4=mu+h4;
      *lcl5=mu-h5;
      *ucl5=mu+h5;
      *lcl6=mu-h6;
      *ucl6=mu+h6;
      *lcl7=mu-h7;
      *ucl7=mu+h7;
      *lcl8=mu-h8;
      *ucl8=mu+h8;
```

```
      *lcl9=mu-h9;
      *ucl9=mu+h9;
      *lcl10=mu-h10;
      *ucl10=mu+h10;
      *lcl11=mu-h11;
      *ucl11=mu+h11;
      *lcl12=mu-h12;
      *ucl12=mu+h12;
      lcl13=mu-h13;
      ucl13=mu+h13;

*variable to count the number of signals;
      signal1=0;
      *signal2=0;
      *signal3=0;
      *signal4=0;
      *signal5=0;
      *signal6=0;
      *signal7=0;
      *signal8=0;
      *signal9=0;
      *signal10=0;
      *signal11=0;
      *signal12=0;
      signal13=0;

*Simulations;
do i=1 to numrun;
      restart:
      m1=0.0;
      *m2=0.0;
      *m3=0.0;
      *m4=0.0;
      *m5=0.0;
      *m6=0.0;
      *m7=0.0;
      *m8=0.0;
      *m9=0.0;
      *m10=0.0;
      *m11=0.0;
      *m12=0.0;
      m13=0.0;

*For steady state;
      runlength=0;
      do j=1 to 100;
            x1=mu+rannor(seed)*sigma;
            x2=mu+rannor(seed)*sigma;
            x3=mu+rannor(seed)*sigma;
            x4=mu+rannor(seed)*sigma;
            xbar=(x1+x2+x3+x4)/n;

            m1=oneml1*m1 + lambda1*xbar;
            *m2=oneml2*m2 + lambda2*xbar;
            *m3=oneml3*m3 + lambda3*xbar;
            *m4=oneml4*m4 + lambda4*xbar;
            *m5=oneml5*m5 + lambda5*xbar;
            *m6=oneml6*m6 + lambda6*xbar;
            *m7=oneml7*m7 + lambda7*xbar;
            *m8=oneml8*m8 + lambda8*xbar;
            *m9=oneml9*m9 + lambda9*xbar;
```

```
             *m10=oneml10*m10+lambda10*xbar;
             *m11=oneml11*m11+lambda11*xbar;
             *m12=oneml12*m12+lambda12*xbar;
             m13=oneml13*m13+lambda13*xbar;
        end;


* have to select the decision criteria form the following list and add in
between if condition below;
        If
             m1<lcl1 or m1>ucl1
             or m13<lcl13 or m13>ucl13
        then goto restart;


        *
        m1<lcl1 or m1>ucl1
        or m2<lcl2 or m2>ucl2
        or m3<lcl3 or m3>ucl3
        or m4<lcl4 or m4>ucl4
        or m5<lcl5 or m5>ucl5
        or m6<lcl6 or m6>ucl6
        or m7<lcl7 or m7>ucl7
        or m8<lcl8 or m8>ucl8
        or m9<lcl9 or m9>ucl9
        or m10<lcl10 or m10>ucl10
        or m11<lcl11 or m11>ucl11
        or m12<lcl12 or m12>ucl12
        or m13<lcl13 or m13>ucl13
        ;
*After the steady state;
        nextsamp:
        runlength=runlength+1;
        x1=(mu+shift)+rannor(seed)*sigma;
        x2=(mu+shift)+rannor(seed)*sigma;
        x3=(mu+shift)+rannor(seed)*sigma;
        x4=(mu+shift)+rannor(seed)*sigma;
        xbar=(x1+x2+x3+x4)/n;

*calculate the ewma statistic;
        m1=oneml1*m1 + lambda1*xbar;
        *m2=oneml2*m2 + lambda2*xbar;
        *m3=oneml3*m3 + lambda3*xbar;
        *m4=oneml4*m4 + lambda4*xbar;
        *m5=oneml5*m5 + lambda5*xbar;
        *m6=oneml6*m6 + lambda6*xbar;
        *m7=oneml7*m7 + lambda7*xbar;
        *m8=oneml8*m8 + lambda8*xbar;
        *m9=oneml9*m9 + lambda9*xbar;
        *m10=oneml10*m10+lambda10*xbar;
        *m11=oneml11*m11+lambda11*xbar;
        *m12=oneml12*m12+lambda12*xbar;
        m13=oneml13*m13+lambda13*xbar;

*Variable to count the number of signals;
        nsignal=0;

*Decide whether there is a signal or not;
        If m1<lcl1 or m1>ucl1 then do;
                        signal1=signal1+1;
                        nsignal=nsignal+1;
                             end;
        *If m2<lcl2 or m2>ucl2 then do;
```

```
*                         signal2=signal2+1;
*                         nsignal=nsignal+1;
*                              end;
*If m3<lcl3 or m3>ucl3 then do;
*                         signal3=signal3+1;
*                         nsignal=nsignal+1;
*                              end;
*If m4<lcl4 or m4>ucl4 then do;
*                         signal4=signal4+1;
*                         nsignal=nsignal+1;
*                              end;
*If m5<lcl5 or m5>ucl5 then do;
*                         signal5=signal5+1;
*                         nsignal=nsignal+1;
*                              end;
*If m6<lcl6 or m6>ucl6 then do;
*                         signal6=signal6+1;
*                         nsignal=nsignal+1;
*                              end;
*If m7<lcl7 or m7>ucl7 then do;
*                         signal7=signal7+1;
*                         nsignal=nsignal+1;
*                              end;
*If m8<lcl8 or m8>ucl8 then do;
*                         signal8=signal8+1;
*                         nsignal=nsignal+1;
*                              end;
*If m9<lcl9 or m9>ucl9 then do;
*                         signal9=signal9+1;
*                         nsignal=nsignal+1;
*                              end;
*If m10<lcl10 or m10>ucl10 then do;
*                         signal10=signal10+1;
*                         nsignal=nsignal+1;
*                              end;
*If m11<lcl11 or m11>ucl11 then do;
*                         signal11=signal11+1;
*                         nsignal=nsignal+1;
*                              end;
*If m12<lcl12 or m12>ucl12 then do;
*                         signal12=signal12+1;
*                         nsignal=nsignal+1;
*                              end;
If m13<lcl13 or m13>ucl13 then do;
                         signal13=signal13+1;
                         nsignal=nsignal+1;
                              end;

        If nsignal>0 then goto endchart;
        else goto nextsamp;

        endchart:
              output;
end;
proc means data=ARL N Mean Stderr;
 var runlength;
run;
```

Program 11:

```
************************************************************************
*       SAS Program to Find the ARL Relationship Between the          *
*             Individual EWMA and Simultaneous EWMA Schemes.          *
*                                                                     *
*       1. Simultaneous EWMA Schemes with Steady State Limits.        *
*       2. ARL of Each Simultaneous EWMA Scheme is Calculated.        *
************************************************************************;
data ARL;
n=4;
delta=0.0;
mu=0.0;
sigma=2.0;
shift=delta*sigma/n**0.5;
numrun=100000;
seed=489563627;


*Parameters (h,lambda) for Individual ARL=50;
*h1=0.360;           *lambda1=0.078;          *delta=0.4;
*h2=0.505;           *lambda2=0.126;          *delta=0.6;
*h3=0.634;           *lambda3=0.176;          *delta=0.8;
*h4=0.760;           *lambda4=0.228;          *delta=1.0;
*h5=0.880;           *lambda5=0.282;          *delta=1.2;
*h6=0.995;           *lambda6=0.338;          *delta=1.4;
*h7=1.110;           *lambda7=0.395;          *delta=1.6;
*h8=1.340;           *lambda8=0.513;          *delta=2.0;
*h9=1.950;           *lambda9=0.823;          *delta=3.0;
*h10=2.326;          *lambda10=1.00;          *Shewhart;
*h11=1.052;          *lambda11=0.366;         *delta=1.5;
*h12=1.628;          *lambda12=0.665;         *delta=2.5;
*h13=0.601;          *lambda13=0.163;         *delta=0.75;


*Parameters (h,lambda) for Individual ARL=100;
*h1=0.379;           *lambda1=0.067;          *delta=0.4;
*h2=0.519;           *lambda2=0.106;          *delta=0.6;
*h3=0.645;           *lambda3=0.147;          *delta=0.8;
*h4=0.765;           *lambda4=0.190;          *delta=1.0;
*h5=0.875;           *lambda5=0.234;          *delta=1.2;
*h6=0.988;           *lambda6=0.280;          *delta=1.4;
*h7=1.094;           *lambda7=0.326;          *delta=1.6;
*h8=1.300;           *lambda8=0.421;          *delta=2.0;
*h9=1.840;           *lambda9=0.680;          *delta=3.0;
*h10=2.576;          *lambda10=1.00;          *Shewhart;
*h11=1.038;          *lambda11=0.303;         *delta=1.5;
*h12=1.595;          *lambda12=0.562;         *delta=2.5;
*h13=0.615;          *lambda13=0.137;         *delta=0.75;


*Parameters (h,lambda) for Individual ARL=200;
*h1=0.378;           *lambda1=0.054;          *delta=0.4;
*h2=0.530;           *lambda2=0.090;          *delta=0.6;
*h3=0.653;           *lambda3=0.125;          *delta=0.8;
*h4=0.785;           *lambda4=0.167;          *delta=1.0;
*h5=0.909;           *lambda5=0.210;          *delta=1.2;
*h6=1.030;           *lambda6=0.256;          *delta=1.4;
*h7=1.149;           *lambda7=0.303;          *delta=1.6;
*h8=1.385;           *lambda8=0.403;          *delta=2.0;
*h9=2.010;           *lambda9=0.680;          *delta=3.0;
*h10=2.807;          *lambda10=1.00;          *Shewhart;
*h11=1.074;          *lambda11=0.273;         *delta=1.5;
*h12=1.679;          *lambda12=0.532;         *delta=2.5;
*h13=0.599;          *lambda13=0.109;         *delta=0.75;
```

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```
*Parameters (h,lambda) for Individual ARL=300;
*h1=0.351;         *lambda1=0.043;        *delta=0.4;
*h2=0.502;         *lambda2=0.075;        *delta=0.6;
*h3=0.645;         *lambda3=0.111;        *delta=0.8;
*h4=0.778;         *lambda4=0.150;        *delta=1.0;
*h5=0.906;         *lambda5=0.192;        *delta=1.2;
*h6=1.034;         *lambda6=0.237;        *delta=1.4;
*h7=1.158;         *lambda7=0.284;        *delta=1.6;
*h8=1.410;         *lambda8=0.385;        *delta=2.0;
*h9=2.080;         *lambda9=0.670;        *delta=3.0;
*h10=2.935;        *lambda10=1.00;        *Shewhart;
*h11=1.088;        *lambda11=0.258;       *delta=1.5;
*h12=1.713;        *lambda12=0.515;       *delta=2.5;
*h13=0.603;        *lambda13=0.100;       *delta=0.75;


*Parameters (h,lambda) for Individual ARL=370;
*h1=0.342;         *lambda1=0.039;        *delta=0.4;
*h2=0.497;         *lambda2=0.070;        *delta=0.6;
*h3=0.637;         *lambda3=0.104;        *delta=0.8;
*h4=0.772;         *lambda4=0.142;        *delta=1.0;
*h5=0.904;         *lambda5=0.183;        *delta=1.2;
*h6=1.033;         *lambda6=0.227;        *delta=1.4;
*h7=1.151;         *lambda7=0.270;        *delta=1.6;
*h8=1.419;         *lambda8=0.375;        *delta=2.0;
*h9=2.100;         *lambda9=0.660;        *delta=3.0;
*h10=2.999;        *lambda10=1.00;        *Shewhart;
*h11=1.098;        *lambda11=0.251;       *delta=1.5;
*h12=1.714;        *lambda12=0.498;       *delta=2.5;
*h13=0.601;        *lambda13=0.095;       *delta=0.75;


*Parameters (h,lambda) for Individual ARL=400;
*h1=0.347;         *lambda1=0.039;        *delta=0.4;
*h2=0.499;         *lambda2=0.069;        *delta=0.6;
*h3=0.640;         *lambda3=0.103;        *delta=0.8;
*h4=0.777;         *lambda4=0.141;        *delta=1.0;
*h5=0.912;         *lambda5=0.183;        *delta=1.2;
*h6=1.042;         *lambda6=0.227;        *delta=1.4;
*h7=1.161;         *lambda7=0.270;        *delta=1.6;
*h8=1.430;         *lambda8=0.375;        *delta=2.0;
*h9=2.115;         *lambda9=0.660;        *delta=3.0;
*h10=3.023;        *lambda10=1.00;        *Shewhart;
*h11=1.099;        *lambda11=0.247;       *delta=1.5;
*h12=1.713;        *lambda12=0.493;       *delta=2.5;
*h13=0.604;        *lambda13=0.094;       *delta=0.75;


*Parameters (h,lambda) for Individual ARL=500;
*h1=0.342;         *lambda1=0.036;        *delta=0.4;
*h2=0.496;         *lambda2=0.065;        *delta=0.6;
*h3=0.644;         *lambda3=0.099;        *delta=0.8;
*h4=0.781;         *lambda4=0.136;        *delta=1.0;
*h5=0.918;         *lambda5=0.177;        *delta=1.2;
*h6=1.052;         *lambda6=0.221;        *delta=1.4;
*h7=1.181;         *lambda7=0.267;        *delta=1.6;
*h8=1.447;         *lambda8=0.368;        *delta=2.0;
*h9=2.160;         *lambda9=0.658;        *delta=3.0;
*h10=3.090;        *lambda10=1.00;        *Shewhart;
*h11=1.094;        *lambda11=0.236;       *delta=1.5;
*h12=1.719;        *lambda12=0.478;       *delta=2.5;
*h13=0.606;        *lambda13=0.090;       *delta=0.75;


*Parameters (h,lambda) for Individual ARL=600;
```

```
*h1=0.345;          *lambda1=0.035;          *delta=0.4;
*h2=0.499;          *lambda2=0.063;          *delta=0.6;
*h3=0.642;          *lambda3=0.095;          *delta=0.8;
*h4=0.776;          *lambda4=0.130;          *delta=1.0;
*h5=0.907;          *lambda5=0.168;          *delta=1.2;
*h6=1.035;          *lambda6=0.209;          *delta=1.4;
*h7=1.165;          *lambda7=0.253;          *delta=1.6;
*h8=1.419;          *lambda8=0.347;          *delta=2.0;
*h9=2.094;          *lambda9=0.616;          *delta=3.0;
*h10=3.144;         *lambda10=1.00;          *Shewhart;
*h11=1.089;         *lambda11=0.227;         *delta=1.5;
*h12=1.719;         *lambda12=0.466;         *delta=2.5;
*h13=0.605;         *lambda13=0.086;         *delta=0.75;


*Parameters (h,lambda) for Individual ARL=700;
*h1=0.347;          *lambda1=0.034;          *delta=0.4;
*h2=0.494;          *lambda2=0.060;          *delta=0.6;
*h3=0.633;          *lambda3=0.090;          *delta=0.8;
*h4=0.768;          *lambda4=0.124;          *delta=1.0;
*h5=0.899;          *lambda5=0.161;          *delta=1.2;
*h6=1.025;          *lambda6=0.200;          *delta=1.4;
*h7=1.152;          *lambda7=0.242;          *delta=1.6;
*h8=1.406;          *lambda8=0.333;          *delta=2.0;
*h9=2.068;          *lambda9=0.594;          *delta=3.0;
*h10=3.189;         *lambda10=1.00;          *Shewhart;
*h11=1.091;         *lambda11=0.221;         *delta=1.5;
*h12=1.708;         *lambda12=0.451;         *delta=2.5;
*h13=0.605;         *lambda13=0.083;         *delta=0.75;


*Parameters (h,lambda) for Individual ARL=800;
*h1=0.347;          *lambda1=0.033;          *delta=0.4;
*h2=0.498;          *lambda2=0.059;          *delta=0.6;
*h3=0.639;          *lambda3=0.089;          *delta=0.8;
*h4=0.772;          *lambda4=0.122;          *delta=1.0;
*h5=0.906;          *lambda5=0.159;          *delta=1.2;
*h6=1.033;          *lambda6=0.198;          *delta=1.4;
*h7=1.152;          *lambda7=0.237;          *delta=1.6;
*h8=1.390;          *lambda8=0.321;          *delta=2.0;
*h9=2.005;          *lambda9=0.559;          *delta=3.0;
*h10=3.227;         *lambda10=1.00;          *Shewhart;
*h11=1.092;         *lambda11=0.217;         *delta=1.5;
*h12=1.707;         *lambda12=0.442;         *delta=2.5;
*h13=0.603;         *lambda13=0.081;         *delta=0.75;


*Parameters (h,lambda) for Individual ARL=900;
*h1=0.353;          *lambda1=0.033;          *delta=0.4;
*h2=0.500;          *lambda2=0.058;          *delta=0.6;
*h3=0.638;          *lambda3=0.087;          *delta=0.8;
*h4=0.775;          *lambda4=0.120;          *delta=1.0;
*h5=0.910;          *lambda5=0.157;          *delta=1.2;
*h6=1.039;          *lambda6=0.196;          *delta=1.4;
*h7=1.160;          *lambda7=0.235;          *delta=1.6;
*h8=1.396;          *lambda8=0.318;          *delta=2.0;
*h9=2.015;          *lambda9=0.554;          *delta=3.0;
*h10=3.261;         *lambda10=1.00;          *Shewhart;
*h11=1.099;         *lambda11=0.215;         *delta=1.5;
*h12=1.702;         *lambda12=0.435;         *delta=2.5;
*h13=0.603;         *lambda13=0.079;         *delta=0.75;


*Parameters (h,lambda) for Individual ARL=1000;
*h1=0.352;          *lambda1=0.032;          *delta=0.4;
```

```
*h2=0.497;          *lambda2=0.056;          *delta=0.6;
*h3=0.632;          *lambda3=0.084;          *delta=0.8;
*h4=0.767;          *lambda4=0.116;          *delta=1.0;
*h5=0.892;          *lambda5=0.149;          *delta=1.2;
*h6=1.018;          *lambda6=0.186;          *delta=1.4;
*h7=1.138;          *lambda7=0.224;          *delta=1.6;
*h8=1.380;          *lambda8=0.307;          *delta=2.0;
*h9=2.010;          *lambda9=0.545;          *delta=3.0;
*h10=3.291;         *lambda10=1.00;          *Shewhart;
*h11=1.102;         *lambda11=0.212;         *delta=1.5;
*h12=1.716;         *lambda12=0.432;         *delta=2.5;
*h13=0.600;         *lambda13=0.077;         *delta=0.75;


*Parameters (h,lambda) for Individual ARL=1500;
*h1=0.357;          *lambda1=0.030;          *delta=0.4;
*h2=0.509;          *lambda2=0.054;          *delta=0.6;
*h3=0.647;          *lambda3=0.081;          *delta=0.8;
*h4=0.784;          *lambda4=0.112;          *delta=1.0;
*h5=0.912;          *lambda5=0.145;          *delta=1.2;
*h6=1.040;          *lambda6=0.181;          *delta=1.4;
*h7=1.166;          *lambda7=0.219;          *delta=1.6;
*h8=1.417;          *lambda8=0.302;          *delta=2.0;
*h9=2.068;          *lambda9=0.541;          *delta=3.0;
*h10=3.403;         *lambda10=1.00;          *Shewhart;
*h11=1.104;         *lambda11=0.200;         *delta=1.5;
*h12=1.735;         *lambda12=0.416;         *delta=2.5;
*h13=0.614;         *lambda13=0.074;         *delta=0.75;


*Parameters (h,lambda) for Individual ARL=2000;
*h1=0.355;          *lambda1=0.028;          *delta=0.4;
*h2=0.513;          *lambda2=0.052;          *delta=0.6;
*h3=0.656;          *lambda3=0.079;          *delta=0.8;
*h4=0.792;          *lambda4=0.109;          *delta=1.0;
*h5=0.925;          *lambda5=0.142;          *delta=1.2;
*h6=1.058;          *lambda6=0.178;          *delta=1.4;
*h7=1.185;          *lambda7=0.216;          *delta=1.6;
*h8=1.443;          *lambda8=0.299;          *delta=2.0;
*h9=2.110;          *lambda9=0.539;          *delta=3.0;
*h10=3.481;         *lambda10=1.00;          *Shewhart;
*h11=1.120;         *lambda11=0.196;         *delta=1.5;
*h12=1.791;         *lambda12=0.409;         *delta=2.5;
*h13=0.619;         *lambda13=0.072;         *delta=0.75;


*Parameters (h,lambda) for Individual ARL=2500;
*h1=0.349;          *lambda1=0.026;          *delta=0.4;
*h2=0.507;          *lambda2=0.049;          *delta=0.6;
*h3=0.646;          *lambda3=0.074;          *delta=0.8;
*h4=0.787;          *lambda4=0.104;          *delta=1.0;
*h5=0.920;          *lambda5=0.136;          *delta=1.2;
*h6=1.052;          *lambda6=0.171;          *delta=1.4;
*h7=1.185;          *lambda7=0.209;          *delta=1.6;
*h8=1.445;          *lambda8=0.291;          *delta=2.0;
*h9=2.130;          *lambda9=0.533;          *delta=3.0;
*h10=3.540;         *lambda10=1.00;          *Shewhart;
*h11=1.112;         *lambda11=0.188;         *delta=1.5;
*h12=1.773;         *lambda12=0.406;         *delta=2.5;
*h13=0.615;         *lambda13=0.068;         *delta=0.75;


*Parameters (h,lambda) for Individual ARL=3000;
*h1=0.339;          *lambda1=0.024;          *delta=0.4;
*h2=0.492;          *lambda2=0.045;          *delta=0.6;
```

```
*h3=0.630;           *lambda3=0.069;          *delta=0.8;
*h4=0.773;           *lambda4=0.098;          *delta=1.0;
*h5=0.906;           *lambda5=0.129;          *delta=1.2;
*h6=1.042;           *lambda6=0.164;          *delta=1.4;
*h7=1.175;           *lambda7=0.201;          *delta=1.6;
*h8=1.440;           *lambda8=0.283;          *delta=2.0;
*h9=2.142;           *lambda9=0.527;          *delta=3.0;
*h10=3.588;          *lambda10=1.00;          *Shewhart;
*h11=1.108;          *lambda11=0.182;         *delta=1.5;
*h12=1.783;          *lambda12=0.398;         *delta=2.5;
*h13=0.598;          *lambda13=0.063;         *delta=0.75;


*Parameters (h,lambda) for Individual ARL=3500;
*h1=0.336;           *lambda1=0.023;          *delta=0.4;
*h2=0.486;           *lambda2=0.043;          *delta=0.6;
*h3=0.623;           *lambda3=0.066;          *delta=0.8;
*h4=0.765;           *lambda4=0.094;          *delta=1.0;
*h5=0.901;           *lambda5=0.125;          *delta=1.2;
*h6=1.037;           *lambda6=0.159;          *delta=1.4;
*h7=1.171;           *lambda7=0.196;          *delta=1.6;
*h8=1.441;           *lambda8=0.278;          *delta=2.0;
*h9=2.155;           *lambda9=0.523;          *delta=3.0;
*h10=3.628;          *lambda10=1.00;          *Shewhart;
*h11=1.093;          *lambda11=0.174;         *delta=1.5;
*h12=1.794;          *lambda12=0.396;         *delta=2.5;
*h13=0.590;          *lambda13=0.060;         *delta=0.75;


*Parameters (h,lambda) for Individual ARL=4000;
*h1=0.323;           *lambda1=0.021;          *delta=0.4;
*h2=0.472;           *lambda2=0.040;          *delta=0.6;
*h3=0.615;           *lambda3=0.063;          *delta=0.8;
*h4=0.754;           *lambda4=0.090;          *delta=1.0;
*h5=0.895;           *lambda5=0.121;          *delta=1.2;
*h6=1.029;           *lambda6=0.154;          *delta=1.4;
*h7=1.166;           *lambda7=0.191;          *delta=1.6;
*h8=1.437;           *lambda8=0.272;          *delta=2.0;
*h9=2.160;           *lambda9=0.518;          *delta=3.0;
*h10=3.662;          *lambda10=1.00;          *Shewhart;
*h11=1.097;          *lambda11=0.172;         *delta=1.5;
*h12=1.809;          *lambda12=0.395;         *delta=2.5;
*h13=0.581;          *lambda13=0.057;         *delta=0.75;


*Weights for the past observation;
      *oneml1=(1-lambda1);
      *oneml2=(1-lambda2);
      *oneml3=(1-lambda3);
      *oneml4=(1-lambda4);
      *oneml5=(1-lambda5);
      *oneml6=(1-lambda6);
      *oneml7=(1-lambda7);
      *oneml8=(1-lambda8);
      *oneml9=(1-lambda9);
      *oneml10=(1-lambda10);
      *oneml11=(1-lambda11);
      *oneml12=(1-lambda12);
      *oneml13=(1-lambda13);


*Upper and Lower control limits;
      *lcl1=mu-h1;
      *ucl1=mu+h1;
      *lcl2=mu-h2;
```

```
      *ucl2=mu+h2;
      *lcl3=mu-h3;
      *ucl3=mu+h3;
      *lcl4=mu-h4;
      *ucl4=mu+h4;
      *lcl5=mu-h5;
      *ucl5=mu+h5;
      *lcl6=mu-h6;
      *ucl6=mu+h6;
      *lcl7=mu-h7;
      *ucl7=mu+h7;
      *lcl8=mu-h8;
      *ucl8=mu+h8;
      *lcl9=mu-h9;
      *ucl9=mu+h9;
      *lcl10=mu-h10;
      *ucl10=mu+h10;
      *lcl11=mu-h11;
      *ucl11=mu+h11;
      *lcl12=mu-h12;
      *ucl12=mu+h12;
      *lcl13=mu-h13;
      *ucl13=mu+h13;

*Variable to count number of signals;
      *signal1=0;
      *signal2=0;
      *signal3=0;
      *signal4=0;
      *signal5=0;
      *signal6=0;
      *signal7=0;
      *signal8=0;
      *signal9=0;
      *signal10=0;
      *signal11=0;
      *signal12=0;
      *signal13=0;

*Simulations;
do i=1 to numrun;
      restart:

      *m1=0.0;
      *m2=0.0;
      *m3=0.0;
      *m4=0.0;
      *m5=0.0;
      *m6=0.0;
      *m7=0.0;
      *m8=0.0;
      *m9=0.0;
      *m10=0.0;
      *m11=0.0;
      *m12=0.0;
      *m13=0.0;

      *For steady state;
      runlength=0;
      do j=1 to 100;
            x1=mu+rannor(seed)*sigma;
```

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```
            x2=mu+rannor(seed)*sigma;
            x3=mu+rannor(seed)*sigma;
            x4=mu+rannor(seed)*sigma;
            xbar=(x1+x2+x3+x4)/n;

*calculate the ewma statistic;
            *m1=oneml1*m1 + lambda1*xbar;
            *m2=oneml2*m2 + lambda2*xbar;
            *m3=oneml3*m3 + lambda3*xbar;
            *m4=oneml4*m4 + lambda4*xbar;
            *m5=oneml5*m5 + lambda5*xbar;
            *m6=oneml6*m6 + lambda6*xbar;
            *m7=oneml7*m7 + lambda7*xbar;
            *m8=oneml8*m8 + lambda8*xbar;
            *m9=oneml9*m9 + lambda9*xbar;
            *m10=oneml10*m10+lambda10*xbar;
            *m11=oneml11*m11+lambda11*xbar;
            *m12=oneml12*m12+lambda12*xbar;
            *m13=oneml13*m13+lambda13*xbar;
        end;

* have to select the decision criteria form the following list and add in
between if condition below;
        If

        then goto restart;

*
        m1<lcl1 or m1>ucl1
        or m2<lcl2 or m2>ucl2
        or m3<lcl3 or m3>ucl3
        or m4<lcl4 or m4>ucl4
        or m5<lcl5 or m5>ucl5
        or m6<lcl6 or m6>ucl6
        or m7<lcl7 or m7>ucl7
        or m8<lcl8 or m8>ucl8
        or m9<lcl9 or m9>ucl9
        or m10<lcl10 or m10>ucl10
        or m11<lcl11 or m11>ucl11
        or m12<lcl12 or m12>ucl12
        or m13<lcl13 or m13>ucl13

;
*After the steady state;
        nextsamp:
        runlength=runlength+1;
        x1=(mu+shift)+rannor(seed)*sigma;
        x2=(mu+shift)+rannor(seed)*sigma;
        x3=(mu+shift)+rannor(seed)*sigma;
        x4=(mu+shift)+rannor(seed)*sigma;
        xbar=(x1+x2+x3+x4)/n;

*calculate the ewma statistic;
        *m1=oneml1*m1 + lambda1*xbar;
        *m2=oneml2*m2 + lambda2*xbar;
        *m3=oneml3*m3 + lambda3*xbar;
        *m4=oneml4*m4 + lambda4*xbar;
        *m5=oneml5*m5 + lambda5*xbar;
        *m6=oneml6*m6 + lambda6*xbar;
        *m7=oneml7*m7 + lambda7*xbar;
        *m8=oneml8*m8 + lambda8*xbar;
```

```
        *m9=oneml9*m9 + lambda9*xbar;
        *m10=oneml10*m10+lambda10*xbar;
        *m11=oneml11*m11+lambda11*xbar;
        *m12=oneml12*m12+lambda12*xbar;
        *m13=oneml13*m13+lambda13*xbar;


*Variable to count the number of signals;
        nsignal=0;
*Decide whether there is a signal or not;
        *If m1<lcl1 or m1>ucl1 then do;
        *               signal1=signal1+1;
        *               nsignal=nsignal+1;
        *                   end;
        *If m2<lcl2 or m2>ucl2 then do;
        *               signal2=signal2+1;
        *               nsignal=nsignal+1;
        *                   end;
        *If m3<lcl3 or m3>ucl3 then do;
        *               signal3=signal3+1;
        *               nsignal=nsignal+1;
        *                   end;
        *If m4<lcl4 or m4>ucl4 then do;
        *               signal4=signal4+1;
        *               nsignal=nsignal+1;
        *                   end;
        *If m5<lcl5 or m5>ucl5 then do;
        *               signal5=signal5+1;
        *               nsignal=nsignal+1;
        *                   end;
        *If m6<lcl6 or m6>ucl6 then do;
        *               signal6=signal6+1;
        *               nsignal=nsignal+1;
        *                   end;
        *If m7<lcl7 or m7>ucl7 then do;
        *               signal7=signal7+1;
        *               nsignal=nsignal+1;
        *                   end;
        *If m8<lcl8 or m8>ucl8 then do;
        *               signal8=signal8+1;
        *               nsignal=nsignal+1;
        *                   end;
        *If m9<lcl9 or m9>ucl9 then do;
        *               signal9=signal9+1;
        *               nsignal=nsignal+1;
        *                   end;
        *If m10<lcl10 or m10>ucl10 then do;
        *               signal10=signal10+1;
        *               nsignal=nsignal+1;
        *                   end;
        *If m11<lcl11 or m11>ucl11 then do;
        *               signal11=signal11+1;
        *               nsignal=nsignal+1;
        *                   end;
        *If m12<lcl12 or m12>ucl12 then do;
        *               signal12=signal12+1;
        *               nsignal=nsignal+1;
        *                   end;
        *If m13<lcl13 or m13>ucl13 then do;
        *               signal13=signal13+1;
        *               nsignal=nsignal+1;
        *                   end;
```

```
        If nsignal>0 then goto endchart;
        else goto nextsamp;

        endchart:
              output;
end;
proc means data=ARL N Mean Stderr;
 var runlength;
run;
```

Program 12:

```
**********************************************************************
*      SAS Program to Find the h for Different Lambda Values    *
*                                                               *
*      1. Calculate the 'h' Values by Considering Steady State  *
*      2. First Select the Lambda which Need to Find h Value    *
*      3. Change h Value (By-Section Algorithm) to Get Desired  *
*         ARL for these h and Lambda                            *
*                                                               *
**********************************************************************;
data;
h=1.104475;
lambda=0.223;

numrun=100000;
n=4;
mu=0.0;
sigma=2.0;
delta=0.0;
shift=delta*sigma/n**0.5;
seed= 499920617;

*Weights for past samples;
oneml=(1-lambda);

*Upper and Lower limite;
lcl=mu-h;
ucl=mu+h;


*Simulations;
do i=1 to numrun;
restart:
      m=0.0;

*For steady state;
      runlength=0;
      do j=1 to 100;
              x1=mu+rannor(seed)*sigma;
              x2=mu+rannor(seed)*sigma;
              x3=mu+rannor(seed)*sigma;
              x4=mu+rannor(seed)*sigma;
              xbar=(x1+x2+x3+x4)/n;

*Calculate ewma statistic;
              m=oneml*m + lambda*xbar;
      end;

*Decision critera;
```

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```
        If
                m<lcl or m>ucl
        then goto restart;


*After the steady state;
nextsamp:
        runlength=runlength+1;
        x1=(mu+delta)+rannor(seed)*sigma;
        x2=(mu+delta)+rannor(seed)*sigma;
        x3=(mu+delta)+rannor(seed)*sigma;
        x4=(mu+delta)+rannor(seed)*sigma;
        xbar=(x1+x2+x3+x4)/n;


*Calculate ewma statistic;
        m=oneml*m + lambda*xbar;

*Decide whether there is a signal or not;
        If
                m<lcl or m>ucl
        then goto endchart;
                        else goto nextsamp;
        endchart:
                output;

end;

proc means N Mean Stderr;
        var runlength;
run;



Program 13:
******************************************************************
*       SAS Program to Find the h for Two-Sided Shewhart Chart   *
*                                                                *
*       1. Calculate the 'h' Values by Considering Steady State  *
*       2. Use EWMA Statistic with Lambda=1.0                    *
*       3. Change h Value (By-Section Algorithm) to Get Desired ARL *
*                                                                *
******************************************************************;
data ARL;

h10=3.2615;

n=4;
k=0.0;
mu=0.0;
sigma=2.0;
delta=k*sigma/n**0.5;
numrun=100000;
lambda10=1.0;
seed=499869757;

lcl10=mu-h10;
ucl10=mu+h10;

signal10=0;
```

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```
oneml10=(1-lambda10);

do i=1 to numrun;
      restart:

      m10=0.0;
      runlength=0;
      do j=1 to 100;
            x1=mu+rannor(seed)*sigma;
            x2=mu+rannor(seed)*sigma;
            x3=mu+rannor(seed)*sigma;
            x4=mu+rannor(seed)*sigma;
            xbar=(x1+x2+x3+x4)/n;

      m10=oneml10*m10+lambda10*xbar;
      end;

*Decision conditions;
      If
      m10<lcl10 or m10>ucl10
      then goto restart;

*After reached to the steady state;
      nextsamp:
      runlength=runlength+1;
      x1=(mu+delta)+rannor(seed)*sigma;
      x2=(mu+delta)+rannor(seed)*sigma;
      x3=(mu+delta)+rannor(seed)*sigma;
      x4=(mu+delta)+rannor(seed)*sigma;
      xbar=(x1+x2+x3+x4)/n;

      m10=oneml10*m10+lambda10*xbar;
      nsignal=0;

*Decision conditions;
      If m10<lcl10 or m10>ucl10 then do;
                        signal10=signal10+1;
                        nsignal=nsignal+1;
                              end;
      If nsignal>0 then goto endchart;
      else goto nextsamp;

      endchart:
            output;
end;
proc means data=ARL N Mean Stderr;
 var runlength;
run;
```

Program 14:

```
****************************************************************
*     SAS Program to Find the h for One-Sided Shewhart Chart   *
*                                                              *
*     1. Calculate the 'h' Values by Considering Steady State  *
*     2. Use EWMA Statistic with Lambda=1.0                    *
*     3. Change h Value (By-Section Algorithm) to Get Desired ARL *
****************************************************************;
data;
h10=3.054;
```

```
lambda=1.0;

delta=0.0;
n=4;
mu=0.0;
sigma=2.0;
oneml=(1-lambda);
shift=delta*sigma/n**0.5;
numrun=100000;
seed=499901405;


ucl10=mu+h10;

signal10=0;
do i=1 to numrun;
restart:
      cup1=0.0;
      m2=0;
      runlength=0;
      do j=1 to 100;
            x1=mu+rannor(seed)*sigma;
            x2=mu+rannor(seed)*sigma;
            x3=mu+rannor(seed)*sigma;
            x4=mu+rannor(seed)*sigma;
            xbar=(x1+x2+x3+x4)/n;
            Zt=xbar;
      m2=oneml*m2 + lambda*xbar;
      end;
      if
            m2>ucl10
      then goto restart;
nextsamp:
      runlength=runlength+1;
      x1=(mu+shift)+rannor(seed)*sigma;
      x2=(mu+shift)+rannor(seed)*sigma;
      x3=(mu+shift)+rannor(seed)*sigma;
      x4=(mu+shift)+rannor(seed)*sigma;
      xbar=(x1+x2+x3+x4)/n;
      Zt=xbar;
      m2=oneml*m2 + lambda*xbar;
      nsignal=0;
If m2>ucl10 then do;
                        signal10=signal10+1;
                        nsignal=nsignal+1;
                  end;
If nsignal>0 then goto endchart;
      else goto nextsamp;
      endchart:
            output;
end;
proc means n mean stderr;
  var runlength;
run;
```