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The Effects of Certain Factors upon Third Grade Children's Ability to Solve Written Verbal Arithmetic Problems

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THE EFFECTS OF CERTAIN FACTORS UPON THIRD GRADE CHILDREN'S ABILITY TO SOLVE WRITTEN VERBAL ARITHMETIC PROBLEMS

by

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VITA

The author, Susan Cesaro Ireland, is the daughter of Frank Cesaro and Angela (Montano) Cesaro and the wife of Dennis Ireland, O.D. Susan was born November 4, 1950, in Chicago, Illinois.

Susan's elementary education was obtained in parochial schools of Chicago and Evergreen Park, Illinois. Her secondary education was completed at Mother McAuley High School, Chicago, Illinois, in June 1968.

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In 1973, Susan was granted an assistantship in special education at the National College of Education, Evanston, Illinois. She completed the Master of Science in Education program there in August, 1974, with a major in educational therapy.

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Following the doctoral residency, Susan served for two years as an educational therapist in the Department for Communicative Disorders of St. Joseph Hospital, Chicago, Illinois. She currently is serving as master teacher for the Eisenhower Cooperative Social and Emotional Disorders Alternate Program.

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CHAPTER I

INTRODUCTION

A basic purpose for fostering a knowledge of mathematics in a student is that he will be able to use this knowledge as a foundation for solving quantitative problems that he will encounter in later life. Linville observed that instruction in elementary school mathematics would be of little avail unless the students could use in the solution of written verbal arithmetic problems what had been learned as a result of that instruction.[1] Sinner concurred by noting that a good arithmetic program must stress the solving of written verbal arithmetic problems by including them as an important component of the mathematics curriculum.[2]

Although the importance of being able to solve written verbal arithmetic problems is apparent to those involved in the instruction of children, the literature abounds with evidence that many children, even though they may demonstrate

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[1] William Jerome Linville, "The Effects of Syntax and Vocabulary upon the Difficulty of Verbal Arithmetic Problems with Fourth Grade Students," (unpublished Ed.D. dissertation, Dept. of Education, Indiana University, June, 1969), p. 1.

[2] Clarice Sinner, "The Problem of Problem Solving," The Arithmetic Teacher, VI (April, 1959), p. 158.

skill in solving strictly computational problems, experience considerable difficulty in the solution of written verbal arithmetic problems.

Grossnickle implied that a student would have difficulty in solving a written verbal arithmetic problem if he did not possess the skills necessary to comprehend it. He noted that if a student could read a verbal arithmetic problem intelligently, he would be able to identify the one or more essential elements for solving the problem.[3]

Blecha, commenting on the same issue, suggested that part of the reason that students had difficulty with written verbal arithmetic problems was because the problems had to be read before they could be solved. He added that a child's ability to cope with written verbal arithmetic problems was determined primarily by his mental capacity and by the depth of his background in each of the following: meanings, understandings, concepts, and skills. Blecha noted that the depth of the child's background in each of these four areas could be influenced by teachers and, therefore, deserved special attention. He concluded that written verbal arithmetic problems did not constitute a separate isolable division of the arithmetic program but rather were an integral part of it.[4]

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[3]Foster E. Grossnickle, "Verbal Problem Solving," The Arithmetic Teacher, XI (January, 1964), p. 14.

[4]Milo K. Blecha, "Helping Children Understand Verbal Problems," The Arithmetic Teacher, VI (March, 1959, p. 106.

Smith, agreeing with Grossnickle[5] and Blecha,[6] stated that a student's first task in solving a written verbal arithmetic problem was one of reading. He added that if a child could not read, an evaluation of his skill in solving verbal arithmetic problems could not be made unless the problems were read to him.[7]

Riley and Pachtman indicated that written verbal arithmetic problems were frequently difficult for students to solve. They noted that the concepts and relationships in these problems were often not readily apparent and that the direct application of basic reading skills did not necessarily lead to understanding these concepts and relationships. The authors concluded, as did Blecha,[8] that it was necessary to offer students specific guidance in reading and understanding written verbal arithmetic problems.[9]

Since the development of skill in solving written verbal arithmetic problems, which has been viewed as an inte-

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[5]Grossnickle, loc. cit.

[6]Blecha, loc. cit.

[7]Frank Smith, "The Readability of Sixth Grade Word Problems," School Science and Mathematics, LXXI (June, 1961,) P. 559.

[8]Blecha, loc. cit.

[9]James D. Riley and Andrew B. Pachtman, "Reading Mathematical Word Problems: Telling Them What to Do Is Not Telling Them How to Do It," Journal of Reading, XXI (March, 1978), p. 531.

gral component of the mathematics curriculum, has proven to be difficult for many students, further study to determine those factors most closely associated with the successful solution of written verbal arithmetic problems seems justified.

Definition of terms

A verbal arithmetic problem refers to a situation which is described in words involving a quantitative question when the computational process or processes necessary to obtain a solution are not indicated to the problem solver.

A written verbal arithmetic problem is a verbal arithmetic problem which has to be read before it can be solved.

The informational component of a verbal arithmetic problem refers to that portion of a verbal arithmetic problem which includes the information necessary to solve it.

The question component of a verbal arithmetic prolem refers to that portion of a verbal arithmetic problem that requires the problem solver for a response based on the data contained in the informational component of the problem.

Purpose of the study

There are several purposes for this study. One purpose is to examine whether varying the syntax of the informational component of a written verbal arithmetic problem from two simple sentences to a compound sentence changes third grade students' ability to perform the computational process or processes required. Two informational components reflecting this change are as follows:

> The dancer earned 324 dollars last week. The gardener earned 276 dollars last week.
>
> The dancer earned 324 dollars last week, and the gardener earned 276 dollars last week.

A further purpose is to study whether changing the structure of the question component of a written verbal arithmetic problem from a form which repeats the quantified noun cited in the informational component of the problem to a form which deletes the quantified noun cited in the informational component of the problem affects third grade children's ability to respond to the computational process or processes required. An example illustrating this change is as follows:

> How many more dollars did the dancer earn than the gardener?
>
> How many more did the dancer earn than the gardener?

Additionally, this study seeks to determine whether differences exist between boys and girls and between children in Catholic schools and in public schools in terms of ability to correctly respond to written verbal arithmetic problems. Also of concern is to examine whether there are any interactions among the above variables.

Limitations of the study

The following are considered to be limitations of this study:

1. Although the four tests designed by the author were randomly assigned to subjects, the classes used represented intact groups.

2. Since the different schools that were used to obtain the necessary data for this project did not all participate in the same standardized testing program, the grade equivalent scores from three different achievement tests are represented in the achievement scores of the subjects of this study.

3. There was no control by the investigator over the reliability and validity of each of the standardized achievement tests.

4. The tests of this study were administered by each of the participating teachers to his own class following the receiving of verbal and written instructions from the researcher.

CHAPTER II

REVIEW OF RELATED LITERATURE

Verbal arithmetic problems have been an area of serious concern for educators and students alike. Many teachers have witnessed students who have been typically successful in solving strictly computational problems fail to accurately solve verbal arithmetic problems. In reaction to this situation, the professional literature abounds with information relating to the study of problems of this type. A distinguishing factor of verbal arithmetic problems is that they are presented in story format generally consisting of one or more sentences per problem. One can question, therefore, whether the structures and the contents of the sentences in verbal arithmetic problems affect children's ability to correctly respond to them.

Responding to the sentential nature of verbal arithmetic problems and to the burden that they have traditionally placed upon students, this review of the literature proceeds as follows: (1) a summary of those factors studied by various researchers to determine the correlates of sentential complexity, (2) a discussion of the factors considered by educators to contribute to the distinct nature of verbal arithmetic problems, and (3) a summary of the research studies conducted by various investigators to determine the factors most closely associated with the successful solution of verbal arithmetic problems.

Factors studied as possible correlates of sentential complexity

It is evident that the required reading becomes increasingly more difficult as one progresses through school. The factors studied by various researchers to determine the correlates of this increased difficulty are discussed below.

In his first book, Flesch cited three factors as contributing to the difficulty of written material. These factors included the number of affixes, the number of personal references, and the number of words per sentence in the text. When each of these factors was tabulated in representative reading samples from a text, a measure of complexity was obtained which ranged from very easy to very difficult. The formula read as follows: "difficulty score" = ((0.1338 x average number of words per sentence) + (0.0645 x average number of affixes per 100 words)) - ((0.0659 x average number of personal references per 100 words) = (0.75)). A resultant score was apt to be a figure between zero and seven. A "difficulty score" of zero implied that the material was very easy to read, and a "difficulty score" of seven indicated that it was very difficult to read.[1]

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[1]Rudolf Flesch, The Art of Plain Talk (New York: Harper and Row, Publishers, 1946), p. 58.

In a later publication, Flesch presented another formula to estimate the readability of written material. The variables included in this formula were the average sentence length in words and the average number of syllables per 100 words computed from several textual samples. This second formula read as follows: "reading ease" score = 206.835 - ((1.015 x average number of words per sentence) + (0.846 x average number of syllables per 100 words)). A score derived from this formula typically had a value between zero and 100. A "reading ease" score of zero indicated that the material was practically unreadable, and a "reading ease" score of 100 suggested that the material could be easily read by any literate person.[2]

In a third publication, Flesch studied the effect of level of abstraction upon the comprehensibility of written material. The level of abstraction of each passage was obtained by tabulating the number of definite words contained in it. Statistical analysis revealed a negative correlation of -0.554 between the number of definite words in a test passage and the average grade level of children who had correctly answered one-half of the test questions. These results suggested that the level of abstraction of a passage was directly related to its level of difficulty.[3]

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[2]Rudolf Flesch, The Art of Readable Writing (New York: Harper and Row, Publishers, p. 216.

[3]Rudolf Flesch, "Measuring the Level of Abstraction," Journal of Applied Psychology, XXXIV (1950), pp. 384-90.

In a study using ninety undergraduates from Johns Hopkins University, Coleman studied the effect of sentence length upon sentential comprehensibility. By slight alterations, difficult passages from a college level text were matched for number of words, sentences, syllables, prepositions, and direct words. The number of sentences in each of the original selections was ten with an average of 23.2 words per sentence. Each passage was rewritten in two other versions. One version consisted of six sentences with an average of 38.7 words per sentence. The other version consisted of fifteen sentences with an average of 15.4 words per sentence. Except for punctuation marks, little varied between the three versions of a passage. The readability level of each version of each passage was determined by cloze tests.

Using an analysis of variance with the number of correct responses on a cloze test as the dependent variable, the overall research hypothesis that shortening sentences would make them more comprehensible was supported at the .05 level of significance thereby concurring with the conclusion of Flesch[4,5] that the reading difficulty of written material was related to the average length of the sentences contained in the material. Based on twenty-six sentences, each of

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[4]Flesch, The Art of Plain Talk, p. 58.

[5]Flesch, The Art of Readable Writing, p. 216.

which had a two-sentence counterpart, the data further implied that two short sentences were typically more comprehensible than one long sentence when other factors were essentially the same, as indicated by a .02 level of significance using the Wilcoxon Matched-Pairs Signed-Rank Test.

In a more extensive statistical analysis of the same data, Coleman divided the twenty-six long sentences into two categories. The first category included ten complex sentences. The subordinate clause in each complex sentence was raised to a full sentence; consequently, two simple sentences were formed from each complex sentence. Each simple-sentence pair was compared in readability to its complex-sentence counterpart. The data suggested that with other factors being held relatively constant, two simple sentences were generally more understandable than one complex sentence, as indicated by a .07 level of significance using the Wilcoxon Matched-Pairs Signed-Rank Test.

The second category consisted of sixteen compound sentences. These sixteen sentences were divided into two categories. The first category consisted of six sentences. Each of the six sentences contained two independent clauses joined by the coordinating conjunction "and." The two independent clauses in each of these compound sentences were divided to form two simple sentences. The compound sentences were compared in readability to the sample sentences. The results, as evaluated by the Wilcoxon Matched-Pairs

Signed-Rank Test, failed to support the hypothesis that a compound sentence containing the coordinating conjunction "and" differed in terms of comprehensibility from the two simple sentences derived from it. The second category consisted of ten compound sentences. Each of the ten sentences contained two independent clauses joined by a coordinating conjunction other than "and." Again the two independent clauses in each of the compound sentences were divided to form two simple sentences. The finding of the Wilcoxon Matched-Pairs Signed-Rank Test, as evaluated at the .025 level of significance, supported the hypothesis that with other factors being held constant, two simple sentences were more comprehensible than one compound sentence containing a coordinating conjunction other than "and."[6]

In a later study, also using students from Johns Hopkins University, Coleman studied the effects of several grammatical transformations upon comprehension. The project consisted of four separate experiments. In the first two experiments, long prose passages were simplified by applying three transformations to each. The transformations included changing passive verbs to active verbs, changing nominalizations using abstract nouns to their active-verb derivatives, and changing adjectivalizations to their ad-

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[6]E. B. Coleman, "Improving Comprehensibility by Shortening Sentences," Journal of Applied Psychology, XLVI (April, 1962), pp. 131-34.

jectival or adverbial forms. Although there was no significant difference in the mean number of words read per simplified passage as compared to the mean number of words read per original passage during a twelve minute period, it was determined that more students had answered more multiple-choice questions correctly in the simplified passages as compared to the original passages, as measured at the .005 level of significance by the Binomial Test. In addition, utilizing four scoring systems measuring different aspects of recall, the results indicated that the simplified passages had been more accurately recalled than the original passages; however, although tending to favor the simplified passages, one of the comparisons failed to reach significance. Each of these recall analyses utilized the Wilcoxon Matched-Pairs Signed-Rank Test.

The third and fourth experiments, the latter of which was distinct from the first three experiments since the subjects were students at Sul Ross College and not at Johns Hopkins University, compared nominalized sentences to their active-verb transformations. Using the Wilcoxon Matched-Pairs Signed-Rank Test, the simplified active-verb sentences were found to have been more accurately recalled than the original nominalized sentences. In addition, using the same statistical test as before and a significance level of .01, the simplified active-verb transformations were found to have been more promptly recalled than the original nominalized sentences. Although the results from multiple-choice tests failed to reach significance, the difference favored the simplified active-verb sentences.[7]

Using twenty students from the Massachusetts Institute of Technology, Fodor and Garrett studied the effect of the presence of relative pronouns versus their absence upon the complexity of sentences. Each student's scores were based upon the speed and accuracy with which he was able to paraphrase the two different types of sentences after they had been orally presented. The results, which were evaluated at a .05 level of significance using the Wilcoxon Matched-Pairs Signed-Rank Test, indicated that those sentences with relative pronouns had been significantly better understood than matching sentences with deleted relative pronouns. The authors stated that the complexity of a sentence was not only a function of the transformational distance from its base structure to its surface structure but also of the degree to which the elements in the surface structure provided clues to the relations of elements in the deep structure.[8]

Bormuth, Manning, Carr, and Pearson studied the ability of fourth grade children to comprehend varying types

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[7] E. B. Coleman, "The Comprehensibility of Several Grammatical Transformations," Journal of Applied Psychology, IIL (June, 1964), pp. 186-90.

[8] J. A. Fodor and M. Garrett, "Sentential Complexity," Perception and Psychophysics, II, No. 7 (1967), pp. 290-91.

of syntactic structures. They also studied the ability of the children to respond to different types of comprehension questions. Included in the study were 240 children from three semi-rural schools. The children's reading comprehension of each of twenty-five within-sentence structures, fourteen anaphoric structures, and sixteen intersentence structures was assessed through their responses to different types of wh-questions.

For each of the major categories, an analysis of variance based on comprehension scores indicated that the different tested structures had not been equally understood by the children. The level of significance for each comparison was .01. In addition, the format of questions was determined to have significantly affected students' ability to respond only with respect to the within-sentence structures. Finally, the data indicated that the students had found the intersentence structures more difficult to understand than either the within-sentence structures or the anaphoric structures. The authors stated that the most surprising finding of their study was that many of the students had been unable to demonstrate a comprehension of the most basic syntactic structures used in our language.[9]

Lesgold investigated the ability of third and fourth

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[9]John R. Bormuth et al., "Children's Comprehension of Between- and Within-Sentence Syntactic Structures," Journal of Educational Psychology, LVI (October, 1970), pp. 349-57.

grade students to comprehend fourteen different anaphoric structures. Forty students from a campus lab school and forty students from an urban public school were included in the sample population. An analysis of variance indicated that there was a significant difference among the anaphoric forms in terms of the wh-comprehension scores obtained from them, as determined at a .0001 level of significance. This finding supported Bormuth's conclusion[10] that different anaphoric forms were not equally understood by children. Based on his results, Lesgold postulated that the children's poor comprehension of several of the anaphoric structures was attributable to the fact that they had not known the interpretation rules required to understand the structures in certain semantic contexts and also to the fact that they had not developed the cognitive skills necessary for applying these rules.[11]

Richek studied the effects of paraphrase alterations of anaphoric forms upon reading comprehension. Sentences containing equivalent anaphoric forms were prepared in three paraphrase alterations: noun, pronoun, and null. In each sentence containing a noun anaphoric form, the noun antecedent was referred to by the repetition of the noun. Each sentence containing a pronoun anaphoric form included a

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[10]Ibid, p. 356.

[11]Alan M. Lesgold, "Variability in Children's Comprehension of Syntactic Structures," Journal of Educational Psychology, LXVI, No. 3 (1974), pp. 333-38.

pronoun to refer back to the noun antecedent. In addition, each sentence containing a null anaphoric form did not include a specific word to refer back to the noun antecedent; however, the reference was implied. Four complexity variables and one variable dictated by the experimental design were also used in the study. One complexity variable concerned whether a sentence had zero or two embedded kernels. A second variable was defined by the number of words between the noun antecedent and the anaphoric form in a sentence. If the number of words totalled between ten and twelve, the length was considered short. If the number of words was between sixteen and eighteen, the length was considered long. A third variable identified whether the two independent clauses in a sentence were parallel or switched in construction. The last complexity variable indicated whether the anaphoric form in a sentence was the subject or the non-subject reference. Richek referred to this last variable as the "node questioned." In addition to the complexity variables, a sentence frame variable was also included. This factor was nested within all of the above variables with the exception of anaphoric form. Sentences using the same sentence frame were controlled for number and type of clauses; however, the linguistic contexts were varied. Each of 220 third grade students took either the long or the short sentences test; therefore, subjects were nested within length. After a subject had read a passage containing a test sentence, he was asked to identify an appropriate antecedent. An answer was scored as correct of the appropriate antecedent has been identified in noun form.

The results, which were analyzed by two separate analyses of variance, revealed that the paraphrase alterations involving the three anaphoric forms had significantly affected difficulty at a .05 significance level with the noun forms having been easiest to understand and the pronoun forms having been next most comprehensible. In addition, the node questioned was determined to have significantly affected comprehension at a .05 significance level with the non-subject nodes having been more difficult to produce. The data also indicated that the sentence frame variable had significantly contributed to difficulty, as determined at a .01 level of significance. Although number of kernels, length, and parallel versus switched construction did not exhibit significant main effects, they did demonstrate a three-way interaction which was significant at the .05 level. A two-way interaction significant at a .01 significance level was observed between sentence frame and paraphrase alteration, and a two-way interaction significant at a .05 significance level was noted between subjects and sentence frame. Finally, the performance scores of the subjects differed significantly, as indicated by a .01

significance level.[12]

Pearson conducted three separate experiments which suggested that grammatical complexity was often an aid to comprehension and recall. The first phase of the first experiment involved sixty-four third and fourth grade students. Eight different surface forms were generated by crossing both of the levels of one of three variables with both of the levels of each of the remaining two variables. The three variables were cue, order, and sentence. A cue, such as the word "because," was either present or absent in each structure. The order of each structure was either cause-effect or effect-cause. In addition, there was either one or two sentences for each structure. A response to a wh-question was scored as correct if it contained the major lexical elements in the cue-present, cause-effect order, one-sentence structure. A response was scored as subordinate if it was introduced by the word "because" or a reasonable semantic substitute.

In terms of correct responses, nearly every subject responded correctly to every form. Different results were obtained, however, when the dependent variable was the number of subordinate responses generated by a surface structure. Using an analysis of variance and setting the significance level at .01 for each comparison and interaction, the

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[12]Margaret Ann Richek, "Reading Comprehension of Anaphoric Forms in Varying Linguistic Contexts," <u>Reading Research Quarterly</u>, XII, No. 2 (1976-77), pp. 145-65.

following information was obtained. There was a significantly higher total of subordinate responses for the cue-present condition as compared to the cue-absent condition. In addition, significantly more subordinate responses had been given for the one-sentence versus the two-sentence forms. Only the cue x order interaction and the cue x order x sentence interaction were significant. Data for the cue x order interaction indicated that for the cue-present condition, sentences based on the effect-cause order had yielded more subordinate responses; however, for the cue-absent condition, sentences based on the cause-effect order had yielded more subordinate responses. Despite the cue x order x sentence interaction, the sentence effect was found to be in the same direction across all cue x order conditions.

In the second and third phases of the first experiment, four surface structure forms were generated for each item by applying successive transformations on the deep structure representation of a sentence containing two embedded sentences which dealt with adjectival relations. A question beginning with the word "which" was used to test each structure in the second section, and a question beginning with the word "who" was used for each structure in the third section. Based on several analyses of variance, the author concluded that the more cohesive, i.e., the more embedded forms, had yielded generally better and more stable comprehension and also that the students' response outputs had favored more cohesive adjectival responses as compared to less cohesive clausal responses.

In Pearson's second experiment, twenty-four fourth grade students were given a question for each sample item and asked to choose which of four different forms they considered to include the best, easiest, and clearest information necessary to answer the question. A clear trend was demonstrated by the students to select the more cohesive or heavily embedded forms as preferrable to the less cohesive forms.

In the third experiment conducted by Pearson, eight fourth grade students were asked to read the same clausal items that had been used in the first phase of the first experiment. The students were instructed to try to remember each sentence because they would be asked to recall it later. The data indicated that the students had tended to store each causal relation as a unified, subordinated chunk rather than in discrete units.

Pearson observed that the data from his three experiments implied that comprehension consisted of synthesizing atomistic propositions into larger conceptual units rather than of analyzing complex units into atomistic propositions. In addition, Pearson postulated that if the surface form of a statement was highly synthesized, comprehension was aided. However, if the surface structure was broken down to more closely resemble its deep structure form, comprehension was impeded.[13]

Hansell studied the effects of simplifying the syntax and the vocabulary in a passage upon the reading rates and cloze scores of children. The effects of the transformations upon the students' ratings of comprehensibility and enjoyability of a passage were also studied. The study included 216 eighth grade students. Each student was classified as high, middle, or low ability depending upon his reading achievement level. The reading passages were chosen from six texts, and each text was rated as good or poor based on its contribution to modern literature. Each passage was presented in three forms. One form was the original form as written in the text. Another form involved a syntactic simplification in which fewer embeddings, conjunctions, passives, negatives, questions, and imperative transformations were included. A third form contained simplified vocabulary.

Based on four separate analyses of variance, each of which summed variation of the dependent variable across passages, it was determined that the readings from significant modern literature had been significantly more difficult to understand than those from subliterture, that sim-

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[13] P. David Pearson, "The Effects of Grammatical Complexity on Children's Comprehension, Recall, and Conception of Certain Semantic Relations," Reading Research Quarterly, X, No. 2 (1974-75), pp. 168-87.

plifying good passages had made them much easier to understand, and that high ability students had generally preferred the unsimplified, original passages. The analyses also revealed that middle ability students and low ability students had significantly preferred the simplified vocabulary readings and the syntactically simplified readings respectively and that the original readings had been most rapidly read by the high ability students, however, most slowly read by the low ability students. An additional analysis examined the cloze scores based on each form of each passage. The results suggested that the effect of simplifying the vocabulary and the syntax of a passage was dependent upon the individual passage under consideration and was not a simple function of the original style in which the passage was presented.[14]

There is evidence to suggest, as supported by these studies, that both the syntactic structure and the content of a sentence contribute to its complexity. Consequently, one should be concerned with both the syntactic structures and the contents of language-based materials presented to students. It appears probable, for example, that a student may not possess the experiential background or grammatical skill required to understand and respond to a passage in

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[14]T. Stevenson Hansell, "Readability: Syntactic Transformations and Generative Semantics," Journal of Reading, XIX (April, 1976), pp. 560-61.

the form initially presented to him; however, he could interpret and respond to it if it were presented in an altered form.

The distinct nature of verbal arithmetic problems

The solution of verbal arithmetic problems presented in written form requires a unique blend of decoding skills, reading comprehension, and mathematical proficiency. Since they have frequently placed typically successful students under duress and have been a serious area of failure for less able students, the consideration of verbal arithmetic problems has been a favored subject in professional literature.

Buswell observed that students frequently attacked written verbal arithmetic problems as though they were unnatural situations encountered only in a schoolroom. He suggested that students be induced to solve these problems with the same kind of straightforward thinking that they generally used outside of school. He added that in order to do this, a student should be presented with problems that made sense to him and that were within the scope of his experience. Realizing that the authors of textbooks may find it difficult to present problems that are equally genuine to all pupils, he noted that teachers might occasionally have to explain the settings of problems to children or substitute new social settings but retain the same numbers and mathematical processes required. Buswell also observed that teachers could help students to solve written verbal arithmetic problems by teaching them the specialized reading skills required. Finally, he suggested that students be encouraged to think clearly about the process or processes required to solve a problem, estimate the approximate answer, and verify their results.[15]

Blecha indicated that children frequently had more difficulty solving written verbal arithmetic problems than other mathematical problems since they had to read each problem before they could decide on the process or processes necessary to solve it. Blecha further suggested that a child's ability to solve a written verbal arithmetic problem was determined by his mental capacity as well as by his ability to use the specialized reading skills of locating information, reading for details, organizing factual data, remembering what has been read, and understanding technical vocabulary. Concurring with Buswell,[16] Blecha contended that these skills were not a by-product of the regular reading program and, therefore, had to be developed through direct instruction.[17]

Sinner also noted that written verbal arithmetic

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[15]G. T. Buswell, "Solving Problems in Arithmetic," Education, LXXIX (January, 1959), pp. 287-88.

[16]Ibid.

[17]Blecha, loc. cit.

problems presented a challenge to students because there was no set formula that could guarantee success in the solution of the problems. He added that these problems as used were frequently not real to students since they did not concern actual problems that arose in the home, school, or community. Sinner observed that it was the responsibility of each arithmetic teacher to encourage children to read written verbal arithmetic problems carefully and slowly, to assist them in understanding the problems, and to provide problems that were relevant to the students.[18]

The importance of reading in the solution of written verbal arithmetic problems was also cited by Smith:

> When children are given a series of written problems to solve the first test is one of reading. If they pass this test, then their problem-solving ability can be evaluated. But for the child who cannot read, no evaluation of his ability to solve problems can be made unless the statements are read to him. As a result, children often receive low marks or poor evaluations in mathematics because of their poor reading abilities. It is also possible that achievement test scores in mathematics sometimes reflect a child's limitations in reading rather than his mathematical performance.[19]

Grossnickle proposed six steps for solving a written verbal arithmetic problem. These steps included the identification of the problems question, recognition of the operation to use, writing of the mathematical sentence to

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[18] Sinner, loc. cit. pp. 158-59.

[19] Smith, loc. cit.

express the relationship between the numbers given, finding the number which will make the sentence true, checking the solution obtained by evaluating the equation, and labeling the answer.[20] The importance of an intelligent reading of a written verbal arithmetic problems was cited by Grossnickle as a key component in identifying the problem question:

> The pupil's ability to identify the problem question is closely related to his ability to read the problem intelligently. If a pupil can read a problem intelligently, he can identify one or more of the elements which are essential in solving the problem.[21]

Maffer indicated that a written verbal arithmetic problem, in contrast to a strictly computational type of problem, required an analytical reading before it could be solved. He proposed a five-step process that students could follow to assist them in solving written verbal arithmetic problems. The steps included previewing, questioning, reading, reflecting, and rewriting.[22]

Nesher and Katriel proposed that the understanding of a verbal arithmetic problem required a recognition of the unique semantic dependencies or relations among the strings of the text of the problem. They added that the actual identity of each object in a specific verbal arith-

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[20]Grossnickle, loc. cit., pp. 14-17.

[21]Ibid., p. 14.

[22]Anthony C. Maffer, "Reading Analysis in Mathematics," Journal of Reading, XVI (April, 1973), pp. 548-49.

metic problem was far less relevant to the solution of the problem than an understanding of the semantic class to which it belonged. The authors also noted that verbal arithmetic problems were further distinguised from non-mathematical writings in that they contained significantly more numbers.[23]

Bartel recognized a significant problem concerning verbal arithmetic problems as presented in mathematics programs:

> . . . . as arithmetic difficulty increases, so do vocabulary, syntatic difficulty, and length and structure of the problem. The problem is that many children do not progress evenly in their ability to handle more difficult words, longer sentences, and problems involving more complex arithmetic processes.[24]

In a later publication, Bartel noted that in no area of mathematical performance was there more difficulty than in the solving of written verbal arithmetic problems. She offered several possible explanations for this situation including lack of practice and inadequate development of each of the following underlying capabilities: ability to perform required computations, ability to read with understanding, ability to estimate answers, acquisition of pre-

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[23]Perla Nesher and Tamar Katriel, "A Semantic Analysis of Addition and Subtraction Word Problems in Arithmetic," Educational Studies in Mathematics, VIII (October, 1977), pp. 252-53.

[24]Nettie R. Bartel, "Problems in Arithmetic Achievement," Teaching Children with Learning and Behavior Problems, eds. Donald P. Hammill and Nettie R. Bartel (Boston: Allyn and Bacon, Inc., 1975), p. 63.

requisite concepts and cognitive structures, and ability to organize problems.[25]

Bartel further proposed, as did Grossnickle[26] and Maffer,[27] a series of steps which students could follow to assist them in solving written verbal arithmetic problems. The steps which were to be carried out in sequence included previewing the problem to identify unknown words, words with unusual usages, and cue words; rereading the problem to determine what has been given and what has been asked; deciding what operation or operations need to be performed; writing the mathematical sentence or sentences; performing the required operation or operations; checking the answer; and stating the result.[28]

The challenge presented to students by written verbal arithmetic problems was also recognized by Riley and Pachtman:

> Mathematical word problems constitute a new area of difficulty for the student. Unlike the language of narrative material, the language of word problems is compact. Mathematical concepts and relationships are often 'hidden' or assumed and therefore not readily apparent to the student. Direct application of basic reading skills, such as the use of context clues or structural analysis, does not necessarily lead to un-

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[25]Nettie R. Bartel, "Problems in Arithmetic Achievement," <u>Teaching Children with Learning and Behavior Problems</u>, eds. Donald P. Hammill and Nettie R. Bartel (2d ed.; Boston: Allyn and Bacon, Inc., 1978), pp. 138-39.

[26]Grossnickle, <u>loc. cit</u>., pp. 14-17.

[27]Maffer, <u>loc. cit</u>.

[28]Bartel, <u>Teaching Children with Learning and Behavior Problems</u>, 2d ed., pp. 142-43.

derstanding the mathematical concepts. Therefore, in order for the student to overcome the difficulties of reading and understanding word problems, specific guidelines seem appropriate. The student must be able to sift out the important information and also to perceive the relationships between concepts that lead to understanding.[29]

These writings imply that verbal arithmetic problems offer the students a unique challenge that is not present in strictly computational problems. The challenge is based on the fact that verbal arithmetic problems are language based and, therefore, require interpretation skills that are not needed in solving entirely computational problems. In the case of written verbal arithmetic problems the challenge is compounded by the fact that the problems must be read before they can be solved.

Research studies of the factors associated with success in the solution of verbal arithmetic problems

The concern over verbal arithmetic problems is also evident by the large number of research studies which have been conducted to determine those factors associated with success in their solution.

Kramer studied the effects of four factors upon sixth grade students' success in the solution of written verbal arithmetic problems. She divided each of the four factors into two levels, and she prepared problems for each of the two levels of each of the four factors. The first factor, the interest factor, consisted of one level that in-

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[29]Riley and Pachtman, loc. cit.

cluded problems which reflected the interests and activities of the children and another level that included problems with traditional, relatively uninteresting content. The second factor concerned sentence form. Half of the problems were of the proverbial type in which the facts and requirements for each problem were given within the confines of a single complex-interrogative sentence. The other problems were of the declarative type. Each problem of this type was introduced by a declarative sentence with the factual material incident to the problem being given through the medium of a compound- or a complex-declarative sentence, or in some instances, by two or more declarative sentences. The question was asked by a distinct interrogative sentence or by an imperative sentence. The third factor of the investigation involved the use of details in setting forth the problem situation. One-half of the problems were briefly stated without details. The details of ordinary discourse were employed in the other half of the problems. The fourth factor concerned whether the vocabulary in a problem was relatively familiar or unfamiliar to the average sixth grade student.

The results indicated that the interesting problems had not produced notably keener or more successful arithmetic thinking than the uninteresting problems. The results also showed that in the matter of sentence form, slightly more proverbial type problems had been answered correctly than declarative type problems. Analysis of the data further revealed that children had done consistently better on briefly stated problems without irrelevant details as compared to those stated in the style of ordinary discourse. In addition, those problems with familiar vocabulary were found to have been answered more accurately than those with relatively unfamiliar vocabulary. In this research, she also studied the correlation of each of the following with achievement in solving verbal arithmetic problems: intelligence, chronological age, and computational ability. The correlation between intelligence and ability to solve verbal arithmetic problems was .386. The correlation between chronological age and achievement in solving verbal arithmetic problems was -.199. In the case of computational ability and ability to solve verbal arithmetic problems, the correlation was .598. Finally, the difference between sexes in terms of achievement in solving the verbal arithmetic problems of this study was minimal. Kramer observed that the children had frequently responded to a verbal cue rather than to the total situation and essential elements or facts given in the statement of a problem. Many of the errors were made, therefore, because the children had done little reflective thinking and had not verified their solutions.[30]

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[30]Grace Amanda Kramer, The Effect of Certain Factors in the Verbal Arithmetic Problem upon Children's Success in the Solution ("Johns Hopkins University Studies in Education," Baltimore: The Johns Hopkins Press, 1933), pp. 7-71.

Hansen administered tests consisting of verbal arithmetic problems to 681 sixth grade students in ten communities. Based on their test results, the upper twenty-seven percent were designated as superior achievers, and the lower twenty-seven percent as inferior achievers. Empolying other test results, the skills of the two groups were compared on the arithmetic factors of fundamental operations, quantitative relationships, arithmetic vocabulary, estimating answers to problems, estimating answers in fundamental operations, problem analysis, thinking abstractly with numbers, and number series. The performances of the two groups were further compared on the mental factors of general reasoning ability, noting differences, noting likenesses, non-language factors, analogies, delayed memory span, immediate memory span, memory, spatial imagery, spatial relationships, and inference. Finally, the achievement levels of the groups were compared on the reading factors of general vocabulary, speed in reading to note details, general language ability, speed in reading to predict outcomes, comprehension in reading to note details, and comprehension in reading to predict outcomes.

With the effect of chronological age and the effect of mental age statistically controlled by the use of the Johnson-Neyman Technique, the scores of the superior group were found to be significantly higher than those of the inferior group for all factors with the exceptions of speed in reading to note details, comprehension in reading to predict outcomes, and comprehension in reading to note details. All comparisons attained significance at the .05 level, and several were significant at the .01 level. The only significant difference in favor or the inferior group was for speed in reading to predict outcomes. The author concluded that the factors which were most closely associated with successful performance in solving verbal arithmetic problems were those related to numbers and reasoning and that the factors which were least closely associated were those related to vocabulary and reading.[31]

Using test results and chronological ages, Treacy collected eighteen items of information on each of 244 seventh grade pupils in two Milwaukee junior high schools. The criterion for problem-solving ability was the average performance on two standardized tests. To make the units of measurement equivalent, all scores were turned into T-scores. The students that had attained the eighty highest averaged T-scores were designated as high achievers, and the students that had attained the eighty lowest averaged T-scores were designated as poor achievers. The good and the poor achievers were then compared on each of fifteen reading skills.

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[31] Carl W. Hansen, "Factors Associated with Successful Achievement in Problem Solving in Sixth Grade Arithmetic," Journal of Educational Research, XXXVIII (October, 1944), pp. 111-18.

With the effect of mental age and the effect of chronological age statistically controlled by the use of the Johnson-Neyman Technique, good achievers were found to be better than poor achievers, as indicated by a .01 significance level, in quantitative relationships, perception of relationships, vocabulary in context, and integration of dispersed ideas. Good achievers were also superior to poor achievers, as determined at the .05 significance level, in arithmetic vocabulary, knowledge of isolated words, retentions of clearly stated details, drawing of inferences from context, and general reading ability. Finally, no significant differences between the groups were found in prediction of outcomes, understanding of precise directions, rate of comprehension, grasp of central thought, general information, and interpretation of content.[32]

Corle administered an eight problem verbal arithmetic problem test to each of seventy-four sixth grade students. Each student was interviewed singly, and a tape recorder was used to record the student's oral reading of the problems and responses to questions posed by the interviewer. Several factors were identified for each of the problems. The factors included the correctness of the solution, the accuracy of the student's concept of the prob-

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[32]John P. Treacy, "The Relationship of Reading Skills to the Ability to Solve Arithmetic Problems," Journal of Educational Research, XXXVIII (October, 1944), pp. 89-93.

lem situation, the type of reasoning employed to solve the problem, the level of the student's comprehension of the vocabulary used in the problem, and the fluency rating of the oral reading of the problem.

Data analysis using Chi-Square revealed a significant relation, at a .01 level of significance, between accuracy in problem solving and each of the following: good concept formation, computational reasoning, high level of confidence in problem-solving ability, and good vocabulary interpretation. No significant relationship was observed between oral reading fluency and problem-solving ability.[33] Although they did not limit themselves to studying the same variables, Hansen,[34] Treacy,[35] and Corle[36] all found that skill in identifying quantitative relationships as well as a good arithmetical vocabulary were conducive to successful problem solving.

Balow conducted a study to determine if level of general reading ability was associated with problem-solving ability, if level of computational skill was associated with problem-solving ability, and if a high level of ability in one of these areas would compensate for a low level of

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[33]Clyde G. Corle, "Thought Processes in Grade Six Problems," The Arithmetic Teacher, V (October, 1958), pp. 193-201.

[34]Hansen, loc. cit., p. 113.

[35]Treacy, loc. cit., p. 92.

[36]Corle, loc. cit., p. 201.

ability in the other. Although 1400 sixth grade students had participated in the testing program designed to obtain the necessary data, the scores obtained from only 368 randomly chosen students were used in analyzing the data. Controlling for the effect of intelligence, an analysis of covariance revealed a significant direct relationship between level of general reading ability and problem-solving ability, as determined by a .05 level of significance, and a significant direct relationship between level of computational skill and problem-solving ability, as indicated by a .01 significance level. The interaction between the two independent variables, level of general reading ability and computational skill, was not significant.[37] The direct relationship found by Balow[38] between level of general reading ability and problem-solving ability was also observed by Treacy.[39] In addition, the direct relationship identified by Balow[40] between level of computational skill and problem-solving ability was also observed by Hansen.[41]

Faulk and Landry investigated the effect of a five-

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[37]Irving H. Balow, "Reading and Computation Ability a Determinants of Problem Solving," <u>The Arithmetic Teacher</u>, XI (January, 1964), pp. 18-21.

[38]<u>Ibid.</u>, p. 21.

[39]Treacy, <u>loc. cit</u>.

[40]Balow, <u>loc. cit</u>.

[41]Hansen, <u>loc. cit</u>.

step method of problem solving upon the successful solution of verbal arithmetic problems. The steps, which were to be completed in sequential order, consisted of several minutes of studying arithmetic vocabulary at the beginning of each arithmetic class period, discussing the situation presented by a particular problem, drawing a simple diagram of the problem, estimating an answer to the problem, and calculating an exact answer to the problem. The last four steps of the method were to be carried out for each new problem presented. Seventy-four sixth grade students formed the experimental group and another seventy-four formed the control group. The students were paired according to sex, age, I.Q., and arithmetic reasoning achievement. The students in the control group were instructed in adherence with the directions in the teacher's guide of their arithmetic series. Specific instructional techniques which had proven successful in the past were also employed. The students in the experimental group were instructed according to the five-step method designed by the authors.

After five months of participating in the study, the mean gain per child in ability to solve verbal arithmetic problems for the experimental group was 9.6 months, and the mean gain per child for the control group was 7.2 months. The 2.4 month difference between the two groups in terms of mean gain per child in problem-solving ability was significant at the .01 significance level.[42]

Cullen studied the effect of practice in reading verbal arithmetic problems upon each of the following: total arithmetic achievement, arithmetic reasoning achievement, and total reading achievement of third grade children. Forty-two students participated in the study, and the experimental group and the control group each had twenty-one subjects. Prior to the initiation of the treatment, the two groups had not significantly differed in terms of mental age, total reading achievement, arithmetic reasoning achievement, or total arithmetic achievement.

Throughout the four month experimental period, verbal arithmetic problems were discussed with the experimental group children approximately three times a week for about thirty-five minutes each time. During these sessions, the students in the experimental group were taught various skills to assist them in extending their understanding of verbal arithmetic problems. The skills emphasized to expand this understanding were the following: (1) the ability to do thorough reading, (2) the ability to do associational reading, (3) the ability to skim material, (4) the ability to read and comprehend numbers, (5) the ability to understand arithmetical vocabulary, (6) the ability to summarize

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[42]Charles J. Faulk and Thomas R. Landry, "An Approach to Problem Solving," The Arithmetic Teacher, VIII (April, 1961), pp. 157-60.

material, and (7) the ability to evaluate material. The children in the experimental group were also given assistance in improving their skills in calculation and estimation. Finally, in order to develop proficiency in reading verbal arithmetic problems, practice sessions were provided for the experimental group in the following areas of problem solving: (1) identifying the true and false elements in a problem, (2) identifying a missing element of information needed to solve a problem, (3) identifying irrelevant details in a problem, (4) working backwards from the solution of a problem to the method of solution, and (5) determining the type of calculation implied from the language of a problem.

Upon completion of the treatment application, a t-test revealed a significant difference in favor of the experimental group in total arithmetic achievement, as determined by a .02 significance level. A separate t-test revealed a significant difference in favor of the experimental group in total reading achievement, as determind by a .01 significance level. No significant difference was evidenced between the two groups in arithmetic reasoning achievement.[43]

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[43]Mary T. Cullen, "The Effect of Practice in the Reading of Arithmetic Problems upon the Achievement in Arithmetic of Third Grade Pupils" (unpublished Master's thesis, Dept. of Education, Cardinal Stritch College, 1963) pp. 3-32.

Linville investigated whether the level of vocabulary and/or the level of syntax used in verbal arithmetic problems were factors which contributed to the degree of difficulty of the problems when the computational operations were held constant. The study included four written tests each consisting of ten verbal arithmetic problems. In addition, two levels of syntax and two levels of vocabulary were distinguished. The vocabulary of each item, excluding the question itself, was considered to be relatively easy or relatively difficult for the average fourth grade student. The syntax used in each problem was also considered to be relatively easy or relative difficulty. A relatively easy item consisted of two simple sentences, exclusive of the question. A relatively difficult item contained a subordinating clause. All of the ten items of a test contained the same level of syntax and the same level of vocabulary. Finally, each of 348 fourth grade students was randomly assigned to take one of the forms of the test.

Scheffe Tests of Contrast revealed that the easier vocabulary test item scores were significantly higher, as indicated by a .01 significance, than the more difficult vocabulary test item scores across difficulty levels of syntax. This apparent influence of the vocabulary of verbal arithmetic problems upon their level of difficulty was also identified by Kramer.[44] Linville also found that the easier

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[44]Kramer, op. cit. p. 60.

syntax test item scores were significantly higher at a .05 significance level than the more difficult syntax test item scores across difficulty levels of vocabulary. An analysis of variance further showed that there was no significant difference between the boys and the girls in terms of verbal arithmetic problem test scores. A corresponding finding was found by Kramer.[45] There was a significant difference, however, found by Linville between the children of high intelligence and the children of low intelligence in terms of verbal arithmetic problem test scores favoring the children of high intelligence, as determined at a .01 significance level. Supporting the finding of Balow[46] and Treacy,[47] the analysis of variance also revealed a significant difference in the verbal arithmetic problem test scores between the children of low reading achievement and the children of high reading achievement favoring the children of high reading achievement. This difference was also significant at the .01 significance level. Finally, all interactions among the variables of level of vocabulary, level of syntax, sex, level of intelligence, and level of reading achievement were not significant.[48]

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[45] Kramer, _op. cit_. p. 46.

[46] Balow, _loc. cit_.

[47] Treacy, _loc. cit_.

[48] Linville, _op. cit_. pp. 26-38.

The findings of these researchers indicate that successful solution of verbal arithmetic problems is dependent upon certain factors within the students as well as factors within the problems themselves. Many of these factors, moreover, could be manipulated by teachers in order to make these problems more solvable by students. A teacher, for example, in preparing verbal arithmetic problems to be solved by his students could reduce the lengths of the sentences in the problems and concern himself with sentential structures and vocabulary. The teacher could attempt to develop proficient skill in solving written verbal arithmetic problems by teaching his students functional reading skills as well as providing them with direct training, including vocabulary study, in solving this type of problem.

Summary

This review has provided evidence which indicates that certain sentential factors, including specific syntactic structures and contents affect sentential complexity. The unique challenge presented to students by verbal arithmetic problems has also been considered. Finally, several factors which appear to be related to success in solving verbal arithmetic problems were also cited.

CHAPTER III

THE METHOD

One major purpose of this study is to investigate two types of syntax of the informational components of written verbal arithmetic problems and two types of structure of the question components of written verbal arithmetic problems as they relate to the degree of difficulty third grade children experience in solving these problems. Also of primary concern to this study is to examine whether differences exist between boys and girls and between children in public schools and children in Catholic schools in terms of ability to solve written verbal arithmetic problems. Finally, this study seeks to investigate whether there are any interactions among the experimental variables cited above.

Preparation of materials

Since no standardized instruments were available which could examine the hypotheses of this study, a series of four tests was prepared by the investigator. Each of the four preliminary versions of the tests consists of twenty-four written verbal arithmetic problems. The vocabulary used in the written verbal arithmetic problems was chosen from the first- and the second-grade words of the Harris-Jacobson Readability Word List.[1] In addition, each of the four tests uses a distinct combination of the type of syntax of the informational components and the type of structure of the question components of its problems.

The informational component of each written verbal arithmetic problem in the preliminary version of Test I consists of two simple sentences. The question component of each item in this test repeats the quantified noun cited in the informational component of the problem. An example follows:

> The dancer earned 324 dollars last week. The gardener earned 276 dollars last week. How many more dollars did the dancer earn than the gardener?

In the preliminary version of Test II, the informational components of the items are identical to those in the preliminary version of Test I with the exceptions that the two independent clauses of each item are joined by a coordinating conjunction, there is a period rather than a comma between the two independent clauses, and the first letter of the second clause is lower case. The informational component of each item, therefore, is in the form of a compound sentence. The question components of the items in this test are identical to those in the preliminary version

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[1]Albert J. Harris and Milton D. Jacobson, <u>Basic Elementary Reading Vocabularies</u> (New York: The MacMillin Publishing Co., 1972), cited in Albert J. Harris and Edward R. Sipay, <u>How to Increase Reading Ability</u> (6th ed.; New York: David McKay Company, Inc., 1975), pp. 666-75.

of Test I. An example of a problem of this type is as follows:

> The dancer earned 324 dollars last week, and the gardener earned 276 dollars last week. How many more dollars did the dancer earn than the gardener?

The informational components of the items in the preliminary version of Test III are identical to those in the preliminary version of Test I. The question components of the items in the preliminary version of Test III are identical to those in the preliminary versions of Tests I and II with the exception that for each problem the quantified noun cited in the informational component of the problem is not repeated in the question component of the problem. An example follows:

> The dancer earned 324 dollars last week. The gardener earned 276 dollars last week. How many more did the dancer earn than the gardener?

The informational components of the problems in the preliminary version of Test IV are identical to those in the preliminary version of Test II. The question components of the problems in the preliminary version of Test IV are identical to those in the preliminary version of Test III. An example is as follows:

> The dancer earned 324 dollars last week, and the gardener earned 276 dollars last week. How many more did the dancer earn than the gardener?

The twenty-four written verbal arithmetic problems included in the preliminary version of each of the four tests consists of twelve addition problems and twelve subtraction problems. Each of six addition problems requires the addition of two two-digit numbers. Each of the remaining six addition problems entails the addition of two three-digit numbers. Each of six subtraction problems requires the subtraction of one two-digit number from another two-digit number. Each of the remaining six subtraction problems requires the subtraction of one three-digit number from another three-digit number.

The ninety-six written verbal arithmetic problems included in the preliminary versions of the four tests are based on a core of twenty-four computational problems which are included in each of the four tests. This implies that although the combination of the type of syntax of the informational components and the type of structure of the question components of the written verbal arithmetic problems in any given preliminary version of a test are distinct from the combination used in the preliminary version of any other test, the numbers used in the problems and the operations required to solve them remain the same across tests. In order to determine the position of an item within the preliminary version of a test, the twenty-four computational problems were randomized according to a table of random digits.[2] The number obtained from the table for each of the computational problems then became the position that

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[2]Audrey Haber and Richard Runyan, General Statistics (2d ed., Reading, Massachusetts: Addison-Wesley Publishing Company, 1973), pp. 367-70.

it occupies in all of the preliminary versions of the tests.

A pilot study was conducted using two classes of third grade children in two Catholic schools located in Chicago. Each of the two teachers whose class was included in the pilot testing program received an equal number of each of the preliminary versions of the tests plus a copy of the test directions. The test directions indicated that the copies of the tests were to be distributed randomly among the students in a class. As a result, the preliminary versions of Tests I, II, III, and IV were taken by seventeen, twelve, twelve, and fifteen students, respectively. In addition, the test directions included all the other information necessary to the administration of the tests. Following the administration of the preliminary versions of the tests, the students' test papers were scored by the investigator with the number of correct responses given by a student taken as the dependent variable.

The purposes of the pilot study were several. One purpose was that of determining the approximate time needed for the administration of the tests. The pilot also served to determine the adequacy of the test directions so that they could be standardized prior to the major study. In addition, the pilot study was used to gather data concerning the difficulty level of each of the twenty-four core problems. The pilot was also used for a reliability check.

A difficulty level was computed for each of the twenty-four core problems by summing the number of correct responses for the item number across all four tests and dividing by the number of possible correct responses. Using .50 as an optimal level of difficulty, as cited by Sax,[3] those four items that deviated the most from this optimal difficulty level were rejected as possible items for the final versions of the tests. Table I presents the difficulty level of each of the twenty-four core problems in the preliminary versions of the tests.

Based on the difficulty levels, it was decided that the items numbered two, six, ten, and twenty in the preliminary versions of the tests would not be included in the final versions of the tests. As a result, a total of twenty possible items remained for the final versions of the tests.

Again summing scores for all items of the same number across all four tests, an internal reliability was estimated by employing the Kuder Richardson Formula 20 [4] and including only those twenty items that had not been rejected as possible items for the final versions of the tests.[4] The resultant calculations yielded a reliability

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[3]Gilbert Sax, Principles of Educational Measurement and Evaluation (Belmont, California: Wadsforth Publishing Company, Inc., 1974), pp. 239-40.

[4]Ibid, p. 181.

TABLE 1.-- Difficulty levels of the core problems in the preliminary versions of Tests I, II, III, and IV

Item	Difficulty Level
1	.464
2	.821 *
3	.554
4	.321
5	.768
6	.196 *
7	.786
8	.589
9	.589
10	.857 *
11	.411
12	.679
13	.679
14	.750
15	.500
16	.607
17	.750
18	.679
19	.446
20	.160 *
21	.464
22	.482
23	.392
24	.607

* -- Rejected items

estimate of .83. Due to the high value of this obtained reliability coefficient, the tests were considered to be in their final forms.

Subjects

The major study included 312 students enrolled in fourteen third grade classes. Seven classes were in three public schools and seven were in four Catholic schools. The schools were chosen to represent different sections of the Chicago metropolitan area. One hundred seventy students were boys, and eighty-six of these boys were enrolled in public schools and eighty-four were enrolled in Catholic schools. One hundred forty-two students were girls, and seventy of the girls were enrolled in public schools and seventy-two were enrolled in Catholic schools. Finally, those students that had participated in the pilot testing were not included in the major study.

Collection of the data

The administration of the final versions of the tests was carried out during the week of May 14, 1979. Each teacher whose class was included in the major testing program received an equal number of copies of each of the four tests plus a copy of the test directions. The test directions indicated that the copies of the tests were to be distributed randomly among the students in a class. As a result, the final versions of Tests I, II, III, and IV were taken by seventy-seven, seventy-six, seventy-five, and eighty-four students, respectively. In addition, the directions included all the other information necessary to the administration of the tests. Following the administration of the tests, the students' papers were scored by the investigator with the number of correct responses for each taken as the dependent variable.

Experimental hypotheses and statistical design

The following null hypotheses were examined in this study:

1. There is no difference in terms of level of difficulty between written verbal arithmetic problems in which the informational components of each problem consists of two simple sentences and written verbal arithmetic problems in which the informational component of each problem consists of a compound sentence.

2. There is no difference in terms of level of difficulty between written verbal arithmetic problems in which the question component of each problem repeats the quantified noun cited in the informational component of the problem and written verbal arithmetic problems in which the question component of each problem deletes the quantified noun cited in the informational component of the problem.

3. There is no difference between third grade boys and third grade girls in ability to solve the types of written verbal arithmetic problems represented by the four

tests of this study.

4. There is no difference between third grade children in Catholic schools and third grade children in public schools in ability to solve the types of written verbal arithmetic problems represented by the four tests of this study.

5. There are no interactions among the following independent variables of this study: type of syntax of the informational components of written verbal arithmetic problems (simple sentences versus compound sentences), type of structure of the question components of written verbal arthimetic problems (quantified nouns repeated versus quantified nouns deleted), type of school (public versus Catholic), and sex.

In order to test the hypotheses of this study, a 2 x 2 x 2 x 2 analysis of covariance was employed. The covariates used were three: (1) grade equivalent score on a vocabulary achievement test, (2) grade equivalent score on a reading comprehension achievement test, and (3) grade equivalent score for overall mathematics achievement. The independent variables were sex, type of school (Catholic versus public), type of syntax of the informational component of each problem on a test (two simple sentences versus one compound sentence), and type of structure of the question component of each problem on a test (repetition versus deletion of the quantified noun cited in the informational component of the problem). In addition, a check was made on the assumption of homogenous regression coefficients.

The seven schools that are represented in this study did not all participate in the same standardized achievement testing program; however, all schools administered their achievement tests during the spring of 1979. The Iowa Tests of Basic Skills were administered to six classes of children in three Catholic schools. The Comprehensive Tests of Basic Skills were administered to five classes of children in two public schools. Finally, the Stanford Achievement Test was administered to two classes in a public school and one class in a Catholic school.

Depending on his school's standardized testing program, a child's achievement level in vocabulary was based either on his Vocabulary grade equivalent score on the Stanford Achievement Test, his Reading Vocabulary grade equivalent score on the Comprehensive Tests of Basic Skills, or on his Test V: Vocabulary grade equivalent score on the Iowa Tests of Basic Skills. His achievement level in reading comprehension of passages was based either on his Reading Comprehension grade equivalent score on the Stanford Achievement Test, his Reading Comprehension: Passages grade equivalent score on the Comprehensive Tests of Basis Skills, or on his Test R: Reading grade equivalent score on the Iowa Tests of Basic Skills. His achievement level in arithmetic was based either on his Total Math grade equivalent score on the Stanford Achievement Test, his Total Mathematics grade equivalent score on the Comprehensive Tests of Basic Skills, or on his Test M: Total Mathematics Skills grade equivalent score on the Iowa Tests of Basic Skills.

In addition to the analysis of covariance, an analysis of variance was conducted which examined whether there was a difference in terms of level of difficulty between two-digit written verbal arithmetic problems and three-digit written verbal arithmetic problems or between written verbal arithmetic problems involving addition and those involving subtraction. In addition, this analysis was used to determine whether there were any interactions among the following independent variables: the type of syntax of the informational components of written verbal arithmetic problem (simple sentences versus compound sentences), the type of structure of the question components of written verbal arithmetic problems (quantified nouns cited versus quantified nouns deleted), arithmetic achievement level (low versus high), the operation required in problems (addition versus subtraction), and the number of digits of each numeral of problems (two versus three). In order to perform this analysis, repeated measures were taken on the digit factor and on the operation factor. Since the final version of each test consisted of four two-digit addition problems, six two-digit subtraction problems, six three-digit addition problems, and four three-digit subtraction problems, students' scores were weighted to provide equal representation. A student's achievement level in mathematics was considered to be low if his grade level score was 3.8 or below. His achievement level in arithmetic was considered to be high if his grade equivalent score was 3.9 or above. Finally, Duncan's New Multiple Range Test[5] was employed to assess the nature of an observed interaction between the informational component, question component, and arithmetic achievement level variables.

Also determined from the data was the following information: (1) the mean and standard deviation obtained on each test, (2) the percent and number of students responding correctly to each item on each test, (3) the discrimination index, the point biserial correlation coefficient between the scores on a test item and the scores on the other test items, of each of the twenty core problems, (4) the Kuder-Richardson Formula 20 reliability coefficient based on considering all four tests as one test with twenty problems, and (5) the correlation of total score on a written verbal arithmetic problem test with each of the following: achievement grade level in vocabulary, achievement grade level in reading comprehension, and achievement grade level in arithmetic.

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[5]Roger E. Kirk, Experimental Design: Procedures for the Behavioral Sciences (Belmont, California: Brooks/Cole Publishing Company, 1968), pp. 93-94.

CHAPTER IV

RESULTS

Since no standardized instruments were available which could examine the hypotheses of this study, four tests, each consisting of twenty written verbal arithmetic problems, were prepared by the examiner. These tests were administered to 312 third grade students attending schools in the Chicago metropolitan area.

The mean number of correct responses and the standard deviation obtained on each test were computed and are presented in Table 2:

TABLE 2.--Mean and standard deviation obtained on each of the four tests of written verbal arithmetic problems

Test	Mean	Standard Deviation
I	14.000	4.036
II	14.040	4.810
III	14.107	4.961
IV	14.119	3.983

The number and percent of students responding correctly to each item of each test were also attained. These findings are reported in Table 3:

TABLE 3.--Number and percent of students responding correctly to each item on each test

Item	Number of Digits in Each Numeral of Problems	Operation Required	Test I		Test II		Test III		Test IV	
			#	%	#	%	#	%	#	%
1	3	Subtraction	39	50.6	42	55.3	42	56.0	41	48.8
2	2	Subtraction	39	50.6	48	63.2	44	58.7	49	58.3
3	3	Subtraction	32	41.6	35	46.1	38	50.7	42	50.0
4	3	Addition	58	75.3	57	75.0	59	78.7	70	80.3
5	2	Addition	62	80.5	58	76.3	61	81.3	71	84.5
6	3	Addition	55	71.4	56	73.7	50	66.7	63	75.0
7	2	Subtraction	70	90.9	61	80.3	66	88.0	68	81.0
8	2	Subtraction	45	58.4	51	67.1	52	69.3	52	61.9
9	3	Addition	65	84.4	54	71.1	61	81.3	67	79.8
10	3	Addition	66	85.7	65	85.5	57	76.0	69	82.1
11	2	Subtraction	70	90.9	64	84.2	62	82.7	73	86.7
12	2	Subtraction	37	48.1	38	50.0	35	46.7	44	52.4

TABLE 3--Continued

Item	Number of Digits in Each Numeral of Problems	Operation Required	Test I		Test II		Test III		Test IV	
			#	%	#	%	#	%	#	%
13	3	Addition	52	67.5	54	71.1	59	78.7	66	78.6
14	2	Addition	73	94.8	69	90.8	65	86.7	74	88.1
15	2	Addition	60	77.9	56	73.7	51	81.3	68	81.0
16	3	Subtraction	44	57.1	50	65.8	42	56.0	54	64.3
17	2	Subtraction	45	58.4	42	55.3	46	61.3	42	50.0
18	3	Addition	55	71.4	61	80.3	58	77.3	66	78.6
19	3	Subtaction	45	58.4	44	57.9	38	50.7	35	41.7
20	2	Addition	66	85.7	62	81.6	61	81.3	72	85.7

The discrimination index, the point biserial correlation coefficient between the scores on a test item and the scores on the other test items, of each of the twenty core problems was also computed from the data. The discrimination index of each core problem was obtained by pooling all scores attained on the problem across all four tests. The discrimination indexes of the twenty problems are presented in Table 4.

In addition, a Kuder Richardson Formula 20 reliability coefficeint, based on considering all four tests as one test with twenty problems, was tabulated and found to equal 0.846.

In order to test the hypotheses of this study, an analysis of covariance was used which controlled for the effects of vocabulary skill, reading comprehension skill, and arithmetical proficiency. The independent variables of this study consisted of the type of syntax of the informational components of the written verbal arithmetic problems of a test (simple sentences versus compound sentences), the type of structure of the question components of the written verbal arithmetic problems of a test (quantified nouns repeated versus quantified nouns deleted), type of school (public versus Catholic), and sex. The covariates were grade equivalent score on a vocabulary achievement test, grade equivalent score on a reading comprehension test, and total grade equivalent score for mathematics achievement.

TABLE 4.-- The discrimination index of each of the twenty core problems

Number Given to Core Problem in the Test	Discrimination Index
1	0.657
2	0.519
3	0.527
4	0.531
5	0.454
6	0.446
7	0.411
8	0.626
9	0.427
10	0.524
11	0.464
12	0.524
13	0.404
14	0.419
15	0.505
16	0.601
17	0.673
18	0.278
19	0.644
20	0.438

The dependent variable was the total number of correct responses on a twenty item test of written verbal arithmetic problems. The results of this analysis are presented in Table 5:

TABLE 5.--Analysis of covariance of total scores obtained on tests of written verbal arithmetic problems

Source of Variation	Sum of Squares	df	Mean Square	F	Level of Significance
Vocabulary skill	7.227	1	7.227	0.609	not significant (ns)
Reading skill	2.370	1	2.370	0.200	ns
Arithmetical skill	1330.069	1	1330.069	112.073	$p < .01$
Syntax of Informational components (A)	0.921	1	0.921	0.078	ns
Structure of question components (B)	8.045	1	8.045	0.678	ns
Sex (C)	0.476	1	0.476	0.040	ns
Type of school (D)	6.460	1	6.460	0.544	ns
A x B	17.987	1	17.987	1.516	ns
A x C	2.112	1	2.112	0.178	ns
A x D	9.407	1	9.407	0.793	ns
B x C	0.152	1	0.152	0.013	ns
B x D	16.214	1	16.214	1.366	ns

TABLE 5.--Continued

Source of Variation	Sum of Squares	df	Mean Square	F	Level of Significance
C x D	6.793	1	6.793	0.572	ns
A x B x C	8.292	1	8.292	0.740	ns
A x B x D	22.417	1	22.417	1.889	ns
A x C x D	21.581	1	21.581	1.818	ns
B x C x D	23.265	1	23.265	1.969	ns
A x B x C x D	17.242	1	17.242	1.453	ns
Error	3477.295	293	11.868		
Total		311			

Based on this analysis it was determined that the only significant covariate effect was attributable to the level of mathematical proficiency variable ($F = 112.073$; $p < .01$). The Pearson correlation of total score on a written verbal arithmetic problem test with achievement level in arithmetic was calculated and found to be 0.636. The correlation of total score with achievement level on a reading comprehension test was 0.414, and the correlation of total score with achievement level on a vocabulary test was 0.374. Since achievement level on a reading comprehension test and achievement level on a vocabulary test both correlated highly with arithmetic achievement (.654 and .550, respectively), they did not contribute significantly to the variance of the dependent variable. In addition, since corresponding F-ratios failed to reach significance at the .05 level, all of the following hypotheses of this study were accepted:

1. There is no difference in terms of level of difficulty between written verbal arithmetic problems in which the informational component of each problem consists of two simple sentences and written verbal arithmetic problems in which the informational component of each problem consists of a compound sentence.

2. There is no difference in terms of level of difficulty between written verbal arithmetic problems in which the question component of each problem repeats the quantified nouns cited in the informational component of the problem and written verbal arithmetic problems in which the question component of each problems deletes the quantified noun cited in the informational component of the problem.

3. There is no difference between third grade girls and third grade boys in ability to solve the types of written verbal arithmetic problems represented by the tests of this study.

4. There is no difference between children in Catholic schools and children in public schools in ability to solve the types of written verbal arithmetic problems represented by the four tests of this study.

5. There are no interactions among the above cited variables.

Since the analysis of covariance procedure requires that the various comparison groups have a common slope, a test for overall inequality of slopes was conducted. There was no overall difference of slopes between cells. A significant type of syntax of informational components by type of structure of question components unequal slope effect was found, however, for both the reading comprehension achievement variable and the arithmetic achievement variable. Since overall grade equivalent scores in arithmetic and grade equivalent scores in reading comprehension were highly correlated (.654) and since the analysis of covariance indicated that math was producing the major effect, the sample was split at the median total grade equivalent score in arithmetic (3.85) and math achievement level (low versus high) was incorporated as a factor in the following analysis.

In the second major analysis of the data, a 2 x 2 x 2 x 2 x 2 analysis of variance with repeated measures on the last two variables was conducted. The independent variables were the type of syntax of the information components of written verbal arithmetic problems (simple sentences versus compound sentences), the type of structure of the question components of written verbal arithmetic problems (quantified nouns repeated versus quantified nouns deleted), and arithmetic achievement level (low versus high). The repeated measures factors were the number of digits in each numeral of problems (two versus three) and the operation required to solve problems (addition versus subtraction). Since the final version of each test consisted of four two-digit addition problems, six two-digit subtraction problems, six three-digit addition problems, and four three-digit subtraction problems, students' scores on each problems type were weighted to assure equal representation. The weighting was accomplished by multiplying by six the scores based on each problem type that consisted of only four problems and multiplying by four the scores based on each problem type that consisted of six problems. As a result, a student's total weighted score could be a maximum of ninety-six. The results of this analysis are presented in Table 6:

TABLE 6.--Analysis of variance on weighted scores broken down by operation and digits

Source of Variation	Sum of Squares	df	Mean Square	F	Level of Significance
Between Ss		311			
Syntax of informational components (A)	7.704	1	7.704	0.098	not significant (ns)
Structure of question components (B)	208.765	1	208.765	2.648	ns

TABLE 6.--Continued

Source of Variation	Sum of Squares	df	Mean Square	F	Level of Significance
Arithmetic Achievement (C)	11318.317	1	11318.317	143.588	p < .01
A x B	32.837	1	32.837	0.417	ns
A x C	115.465	1	115.465	1.465	ns
B x C	15.733	1	15.733	0.200	ns
A x B x C	653.599	1	653.599	8.292	p < .01
Error	23962.719	304	78.825		
Operation (D)	7454.010	1	7454.010	154.628	p < .01
A x D	4.553	1	4.553	0.094	ns
B x D	0.816	1	0.816	0.017	ns
C x D	1246.322	1	1246.322	25.854	p < .01
A x B x D	71.192	1	71.192	1.477	ns
A x C x D	34.419	1	34.419	0.714	ns
B x C x D	0.198	1	0.198	0.004	ns
A x B x C x D	70.897	1	70.897	1.471	ns
Error	14654.482	304	48.206		
Digits (E)	1684.740	1	1684.740	91.517	p < .01
A x E	22.089	1	22.089	1.200	ns
B x E	0.036	1	0.036	0.002	ns
C x E	32.273	1	32.273	1.753	ns
A x B x E	17.324	1	17.324	0.941	ns

TABLE 6.--Continued

Source of Variation	Sum of Squares	df	Mean Square	F	Level of Significance
A x C x E	17.324	1	17.324	0.941	$p < .05$
B x C x E	19.850	1	19.850	1.078	ns
A x B x C x E	1.038	1	1.038	0.056	ns
Error	5596.190	304	18.409		
D x E	221.542	1	221.542	13.971	$p < .01$
A x D x E	0.418	1	0.418	0.026	ns
B x D x E	6.345	1	6.345	0.400	ns
C x D x E	1.592	1	1.592	0.100	ns
A x B x D x E	0.100	1	0.100	0.006	ns
A x C x D x E	20.324	1	20.324	1.282	ns
B x C x D x E	17.127	1	17.127	1.080	ns
A x B x C x D x E	4.413	1	4.413	0.278	ns
Error	4820.417	304	15.857		

This analysis revealed three significant main effects. One main effect suggested that children with high overall achievement levels in arithmetic could solve written verbal arithmetic problems better than children with low overall achievement levels in arithmetic ($F = 143.588$; $p < .01$). The mean weighted scores attained by the high and the low math achievers on the written verbal arithmetic problem tests were 77.266 and 55.405, respectively. A second main effect suggested that written verbal arithmetic problems involving addition were significantly easier to solve than written verbal arithmetic problems involving subtraction ($F = 154.628$; $p < .01$). The mean weighted scores on the addition problems and the subtraction problems were 28.841 and 28.705, respectively. The other main effect indicated that two-digit written verbal arithmetic problems were easier to solve than three-digit ones ($F = 91.517$; $p < .01$). The mean weighted scores on the two-digit problems and the three-digit problems were 35.197 and 31.269, respectively. In addition, four significant interactions were observed. All other effects examined in this analysis failed to reach significance at the .05 level.

A significant two-way interaction was observed between achievement level in arithmetic and the operation required in written verbal arithmetic and the operation required in written verbal arithmetic problems ($F = 25.854$; $p < .01$). The mean weighted scores of the high math group on the addition problems and the subtraction problems were 42.48 and 36.79, respectively. The mean weighted scores of the low math group on the addition problems and the subtraction problems were 34.58 and 20.82, respectively. A representation of the interaction is presented in Figure 1. As is evident from the figure, the subtraction problems were harder to solve in comparison to addition problems for the low math group than they were for the high math group

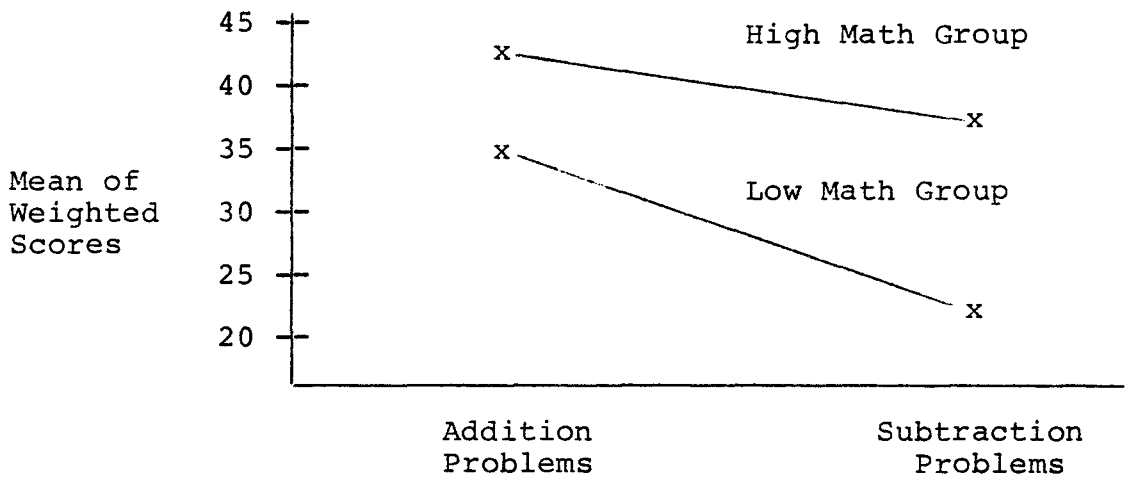


Fig. 1.--Diagram of the two-way interaction between achievement level in math and the operation required in written verbal arithmetic problems

A significant two-way interaction was also observed between the operation required in written verbal arithmetic problems and the number of digits in each numeral of problems ($F = 13.971$; $p < .01$). The mean weighted scores attained on the two-digit addition problems and the two-digit subtraction problems were 19.98 and 15.94, respectively. In addition, the mean weighted scores attained on the three-digit addition problems and the three-digit subtraction problems were 18.50 and 12.77, respectively. The interaction is represented in Figure 2 which shows that three-digit subtraction problems were harder to solve relative to the three-digit addition problems than the two-digit subtraction problems were relative to the two-digit addition problems.

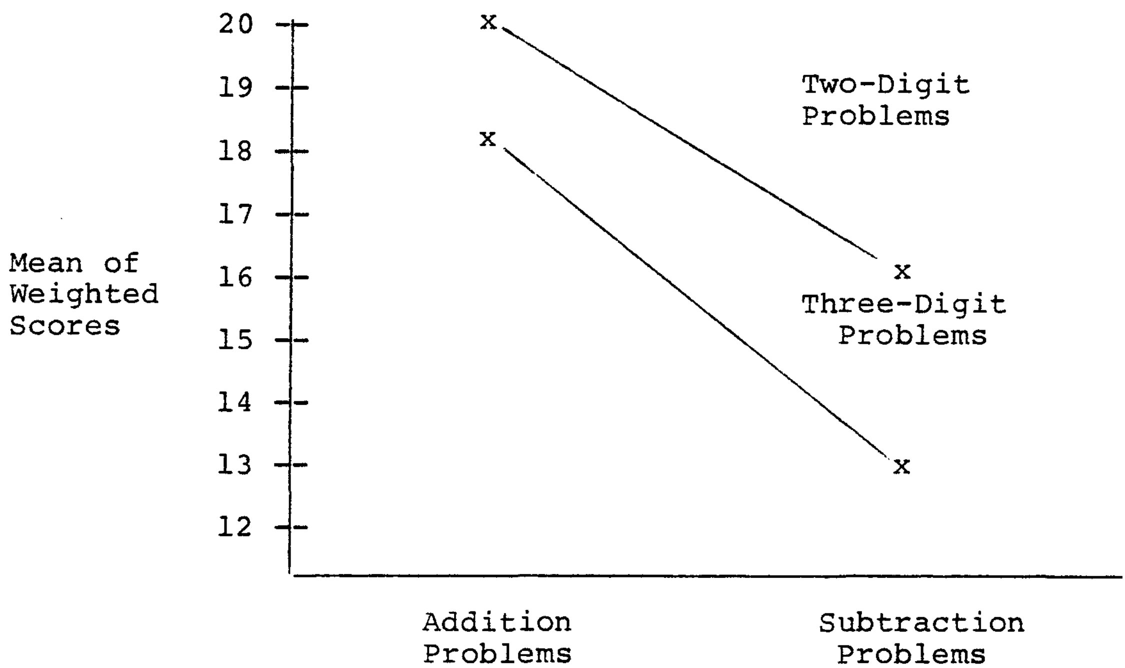


Fig. 2.--Diagram of the two-way interaction between the operation required in written verbal arithmetic problems and the number of digits in each numeral of problems

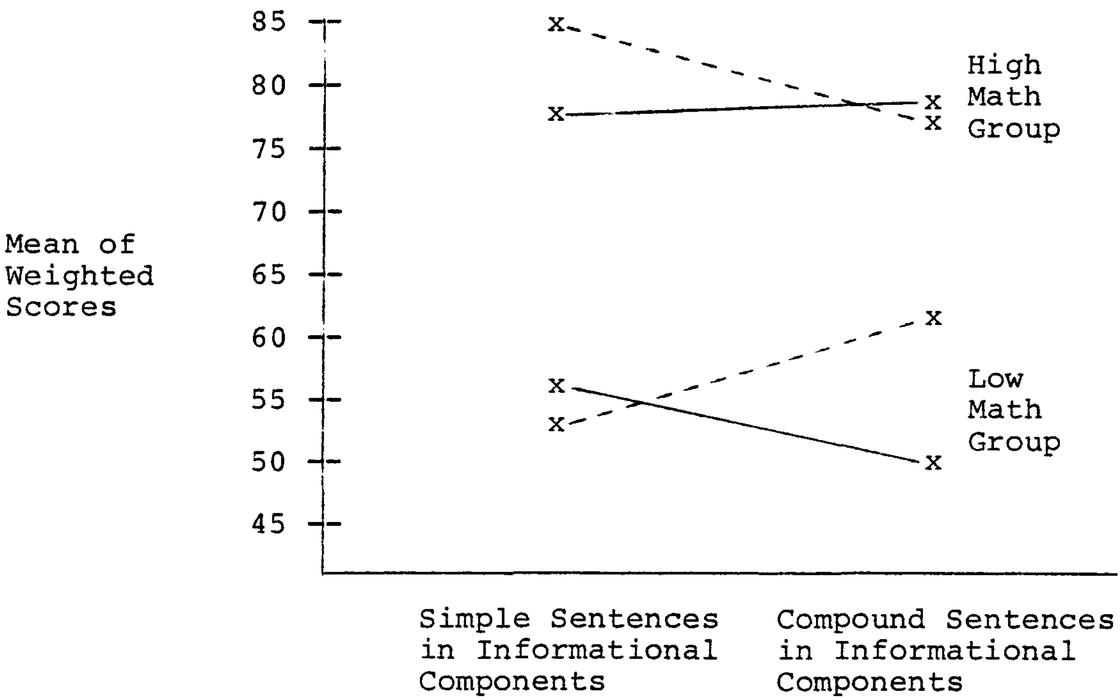
The analysis of variance also revealed a significant three-way interaction between the syntax of the informational components of written verbal arithmetic problems variable, the structure of the question components of written verbal arithmetic problems variable, and the arithmetic achievement level variable ($F = 8.292$; $p < .01$). In order to determine the precise nature of this interaction, Duncan's New Multiple Range Test was used to examine the differences between the eight cells. The results of the Duncan's analysis indicated that there were no significant differences among the high achievement groups; however, their scores were significantly higher than those of the low achievement groups. Among the four low achievement groups, the group that had been expected to attain the lowest scores since they were to respond to items that had relatively longer sentences in the informational components (compound sentences as compared to simple sentences) and which were less explicit in the question components (quantified nouns deleted as compared to quantified nouns repeated) actually scored significantly higher than the other groups with the exception of the group that had been expected to attain the highest scores (simple sentences in the informational components and quantified nouns repeated in the question components) which had the second highest scores. The group responding to problems that contained compound sentences in the informational components and that deleted the quantified nouns cited in the informational components, as predicted, had the lowest scores among the high achievement groups, even though not significantly so. Table 7 and Figure 3 present the means of the eight groups and an illustration of the interaction, respectively.

A second significant three-way interaction was observed in this analysis ($F = 4.003$; $p < .05$). This involved an interaction between the syntax of the informational components of written verbal arithmetic problems, achievement level in math, and the number of digits in each numeral of problems. The mean of each group is presented in Table 8 and a representation of the interaction is given in Figure 4. As can be seen, three-digit problems

TABLE 7.--Group means of weighted scores based on the three-way interaction between the type of syntax of the informational components of written verbal arithmetic problems, the type of structure of the question components of the problems, and achievement level in arithmetic

Structure of Question Components	Low Math Group		High Math Group	
	Syntax of Informational Components		Syntax of Informational Components	
	Simple Sentences	Compound Sentences	Simple Sentences	Compound Sentences
Quantified nouns cited in informational components are repeated	55.79	49.53	77.79	78.56
Quantified nouns cited in informational components are deleted	52.83	60.86	84.82	76.46

including compound sentences in the informational components were actually easier than the three-digit problems including simple sentences for the low math group. For every other combination, however, problems including compound sentences in the informational components were harder than problems containing simple sentences in the informational components.



——————— Quantified nouns repeated in the question components

- - - - - Quantified nouns deleted in the question components

Fig. 3.--Diagram of the three-way interaction between the type of syntax of the informational components of written verbal arithmetic problems, the type of structure of the question components of the problems, and achievement level in arithmetic

TABLE 8.--Group means of weighted scores based on the three-way interaction between achievement level in math, the syntax of the informational components of written verbal arithmetic problems, and the number of digits in each numeral or problems

Number of Digits in Each Numeral of Problems	Low Math Group		High Math Group	
	Syntax of Informational Components		Syntax of Informational Components	
	Simple Sentences	Compound Sentences	Simple Sentences	Compound Sentences
Two	30.51	30.20	42.30	41.01
Three	23.75	26.35	38.77	36.64

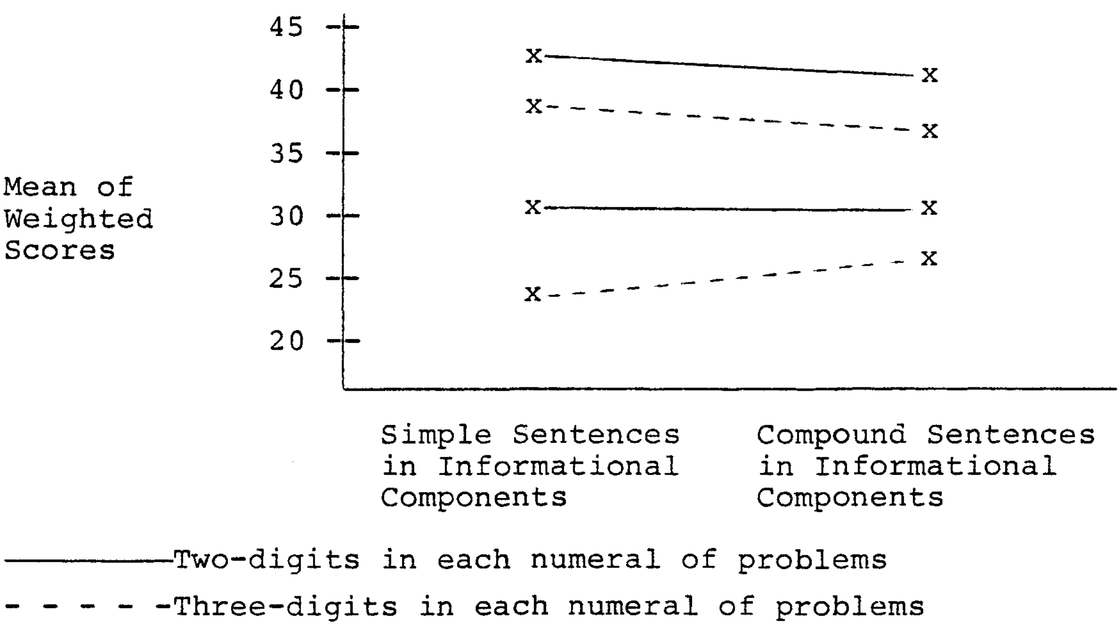


Fig. 4.--Diagram of the three-way interaction between achievement level in math, the syntax of the informational components of written verbal arithmetic problems, and the number of digits in each numeral of problems

CHAPTER V

DISCUSSION

One major purpose of current approaches to the teaching of elementary school mathematics is to foster in children the ability to think mathematically.[1] As a result much time is being spent in elementary school classrooms studying the basic underlying principles of mathematics in order that children will be able to apply these learnings to situations that will arise in their later lives.[2] The avenue that is frequently used to prepare students for possible mathematically based problem situations is the verbal arithmetic problem. Although verbal arithmetic problems have been included in mathematics texts for many years, the literature indicates that children still experience a considerable degree of difficulty solving these problems even though they may demonstrate adequate performance on strictly computational tasks. This study was designed to investigate some possible correlates of the successful solution of verbal arithmetic problems.

Review and interpretation of findings

Using third grade students and controlling for

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[1]Linville, *op. cit.*, p. 40.

[2]*Ibid.*, p. 1.

the effects of vocabulary skill, reading comprehension ability, and arithmetical proficiency, the author compared scores on tests of written verbal arithmetic problems in which the informational component of each problem consisted of two simple sentences with scores on tests in which the informational component of each problem consisted of a compound sentence. Also compared were the scores on tests in which the quantified noun cited in the informational component of each problem was repeated in the question component of the problem with the scores on tests in which the quantified noun cited in the informational component of each problem was deleted in the question component of the problem. In both of these comparisons, no significant difference was found. In addition, the scores obtained by the girls were compared to those obtained by the boys, and the scores obtained by children in Catholic schools were compared to those obtained by children in public schools. Again, no significant difference was found by either comparison. All tested interaction effects were also not significant. Of the three covariates, grade equivalent score on a vocabulary test, grade equivalent score on a reading comprehension test, and overall grade equivalent score in arithmetic, only the latter variable was found to contribute significantly to the variance of the dependent variable. One possible explanation for the repeated instances of no significance is that since all the words in the test problems were at the second grade level or below and, consequently, were so easily read by most of the students, the effects of the independent variables were nullified. The influence of the vocabulary used in written verbal arithmetic problems upon their level of difficulty has been studied by Linville who found that problems containing many easy vocabulary words were significantly easier to solve than problems containing many difficult vocabulary words.[3] It may be possible, therefore, that the level of vocabulary of problems can be simplified to such a degree that some of the other variables which normally affect the difficulty of written verbal arithmetic problems will no longer be operational.

A further analysis of the data involved the following variables: the type of syntax of the informational components of written verbal arithmetic problems (simple sentences versus compound sentences), the type of structure of the question components of written verbal arithmetic problems (quantified nouns repeated versus quantified nouns deleted), achievement level in mathematics (low versus high), the number of digits in each numeral or problems (two versus three), and the operation required in problems (addition versus subtraction). The analysis revealed three significant main effects and four significant interaction effects.

As expected, one main effect revealed that high math achievers were able to solve written verbal arithmetic problems significantly better than low math achievers. In addi-

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[3] _Ibid._, p. 38.

tion, two-digit written verbal arithmetic problems and written verbal arithmetic problems involving addition were found to be significantly easier to solve than three-digit written verbal arithmetic problems and written verbal arithmetic problems involving subtraction, respectively. These latter findings were also anticipated and coincide with the presentation sequence found in most elementary school mathematics curriculums.

A two-way interaction was observed between achievement level in math and the operation required in written verbal arithmetic problems. This interaction suggested that students achieving low in math were more challenged by written verbal arithmetic problems involving subtraction as compared to those involving addition than were students achieving high in math. One possible implication of this finding is that as the mathematics curriculum increases in difficulty, students achieving low in mathematics are disproportionately challenged relative to those achieving high in math.

A two-way interaction was also observed between the number of digits in each numberal of problems and the operation required in problems. This interaction suggested that the increased difficulty in solving written verbal arithmetic problems involving subtraction relative to those involving addition and the increased difficulty in solving three-digit written verbal arithmetic problems relative to two-digit ones were not additive when both of these factors, three digits in each numeral of problems and subtraction required to solve problems, are operating in the same problems. In fact, three-digit written verbal arithmetic problems involving subtraction were found to be more difficult relative to two-digit ones involving subtraction than three-digit written verbal arithmetic problems involving addition were relative to two-digit ones involving addition. Therefore, the possibility exists that as the number of concepts included in problems increases, students may become disproportionately more challenged by the new problems relative to more simple problems.

In addition to the two two-way interaction, a three-way interaction was observed between the type of syntax of the informational components of written verbal arithmetic problems, the type of structure of the question components of written verbal arithmetic problems, and achievement level in arithmetic. For problems in which the quantified nouns cited in the informational components of the problems were deleted in the question components of the problems, an analysis of group means revealed that both the high and the low math ability students exhibited approximately an eight point mean weighted score difference between problems including simple sentences in the informational components and those including compound sentences in the informational components. For the group achieving low in math, those problems including compound sentences in the informational components had been answered more accurately than those including simple sentences in the informational components. This situation was reversed for the high ability students. It appears that for low math ability students, when problems are less explicit, as they are when the quantified nouns cited in the informational components of problems are deleted in the question components of the problems, the students will respond poorly if the problems are presented in a fragmentary rather than a cohesive form, for example, simple sentences in the informational components of the problems rather than compound sentences. One may question, therefore, whether the combination of lack of explicitness and fragmentation in the wording of written verbal arithmetic problems can make the problems particularly difficult for low ability students. When the quantified nouns cited in the informational components of problems were deleted in the question components of the problems, high ability students in contrast to low ability students, did not appear to be more challenged if the informational components of the problems were in the form of simple sentences as compared to compound sentences. Rather, they seemed to be more challenged if the sentences in the informational components were long (compound sentences versus simple sentences).

For the problems in which the quantified nouns cited in the informational components of the problems were repeated in the question components of the problems, the high math ability group evidenced less than a one point difference between means of weighted scores in favor of those problems including compound sentences in the informational components, whereas, the low math group evidenced more than a six point difference in favor of problems including simple sentences in the informational components. One may conjecture that for high math ability students, when problems are very explicit, the effect of sentence length upon ease of solution is nullified; however, for low math ability students, as problems become more explicit, the students become increasingly sensitive to and challenged by long sentences.

The final observed interaction involved the number of digits in each numeral of problems, achievement level in math, and the type of syntax of the informational components of written verbal arithmetic problems. An analysis of weighted group means suggested that for high math ability students, the difference in terms of difficulty between two-digit and three-digit written verbal arithmetic problems was essentially the same for those problems including simple sentences in the informational components and those including compound sentences. However, for low math ability students, three-digit written verbal arithmetic problems were notably more difficult to solve relative to two-digit ones when the informational components of the problems were in the form of simple sentences. In order to explain this finding, one may offer the supposition that low ability students can become overloaded by a large number of units of information. If, therefore, one assumes that each numerical symbol in a problem accounts for a unit of information and each sentence also accounts for a unit of information, three-digit written verbal arithmetic problems would account for more units of information than any of the other possible combinations represented by problems of this study and would, if the proposed supposition holds true, account for the increased difficulty experienced by low ability students on three-digit written verbal arithmetic problems including simple sentences in the informational components.

Educational implications

Considering the findings of this study, several implications related to the teaching of elementary school mathematics can be offered. Those teachers involved in elementary school arithmetic instruction should be aware of the increased burden placed on students by subtraction problems relative to addition problems and by problems containing a large number of digits as compared to those containing fewer digits and should attempt to alleviate the corresponding problems of their students through their daily classroom mathematics activities. Also, the difficulty experienced by children of low overall arithmetical ability in solving written verbal arithmetic problems and particularly those problems involving subtraction should be considered and responded to by the mathematics curriculm. In addition, it is essential that mathematics teachers realize that the poor performance demonstrated by some of their students in solving written verbal arithmetic problems may be more a reflection of factors within the statement of the problems than of the children's true problem-solving ability.

The results of this study also provided evidence that high and low achievers in arithmetic respond differently to various phrasings of written verbal arithmetic problems. Consequently, a teacher should consider a child's aptitude in arithmetic prior to designing for him an instructional program to develop skill in solving written verbal arithmetic problems. This suggestion is consistent with the Aptitude Treatment Interaction approach proposed by Cronbach and Snow[4] which states that the most effective learning takes place when the teaching process is adapted to the learning style of the individual students.

It is the responsibility of those who are involved in the production of elementary school mathematics texts to be cognizant of those components of the mathematics curriculum which may be troublesome for many students. They should also make certain that adequate presentation and review space is included in their texts to foster mastery in

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[4]Lee J. Cronbach and Richard E. Snow, Individual Differences in Learning Ability as a Function of Instructional Variables, ERIC Document No. 029001 (Palo Alto, California: Stanford Center for Research and Development in Teaching, 1969), cited by J. Galen Saylor and William M. Alexander, Planning Curriculium for Schools (New York: Holt, Rinehart, and Winston, Inc., 1974), p. 277.

these difficult areas. This implies that arithmetic textbook publishers and authors should keep abreast of current research in the field and should properly field test their materials prior to marketing them.

Those involved in the selection of the elementary school arithmetic texts to be used in schools should also be familiar with current research and should review a wide variety of materials with the needs of their students in mind prior to making a final selection.

Recommendations for further research

The following are offered as suggestions for further research in the area of written verbal arithmetic problems:

1. Reexamination of the effects of the variables used in this study upon the successful completion of written verbal arithmetic problems, however, with the level of vocabulary used in the test problems corresponding to the grade placement of the children to be tested.

2. Further study of other factors within the statement of written verbal arithmetic problems which may contribute to their difficulty.

3. Examination of those reading skills and other student aptitudes which may be most closely associated with the successful solution of written verbal arithmetic problems.

4. Examination of the effect of socioeconomic status upon the successful solution of written verbal arithmetic problems.

5. Investigation of specific instructional strategies which may assist students in correctly solving written verbal arithmetic problems with the emphasis placed on matching the instructional program to individual student needs.

SUMMARY

Since verbal arithmetic problems have proven to be a particulary challenging component of the mathematics curriculum, the investigator designed this study to examine several specific factors within the statement of the problems which may contribute to their difficulty.

To gather the required data, four tests, each consisting of twenty two- and three-digit addition and subtraction written verbal arithmetic problems were designed. The vocabulary used in the tests was controlled by selecting for inclusion words at the second grade level or below. In Tests I and III, the informational component of each problem consists of two simple sentences. An example follows: "A girl has 186 crayons. A boy has 214 crayons." In Tests II and IV, the informational component of each problem is in the form of a compound sentence. An example is as follows: "A girl has 186 crayons, and a boy has 214 crayons." In Tests I and II, the quantified noun cited in the informational component of each problem is repeated in its question component. Following is an example: "How many crayons do they have in all?" Finally, in Tests III and IV, the quantified noun cited in the informational component of each problem is deleted in its question component. An example follows: "How many do they have in all?"

The four tests were administered to 312 third grade students in seven Catholic and public schools located in the Chicago metropolitan area.

Using an analysis of covariance with the number of correct responses on a test being the dependent variable and vocabulary skill, reading comprehension ability, and mathematical proficiency, as determined from standardized testing, taken as the covariates, no significant difference was found between the tests scores based on problems including simple sentences in the informational components and the test scores based on problems including compound sentences in the informational components. Also, the inclusion versus the deletion in the question components of problems of the quantified nouns cited in the informational components of the problems did not significantly affect difficulty. The test scores of the girls did not significantly differ from those of the boys, and the tests scores of the children in Catholic schools did not significantly differ from those of the children in public schools. In addition, all possible interaction effects were insignificant.

A further analysis of the data was conducted to determine whether there was a difference in terms of ability to solve written verbal arithmetic problems between children of high overall arithmetical ability and those of low overall arithmetical ability. This analysis of variance was also used to examine whether two-digit written verbal arithmetic problems and written verbal arithmetic problems involving the operation of addition differed in terms of difficulty from three-digit written verbal arithmetic problems and written verbal arithmetic problems involving the operation of subtraction, respectively. The presence of any interaction effects between the above variables or between the above variables and the experimental variables of the type of syntax of the informational components of the written verbal arithmetic problems on a test (simple sentences versus compound sentences) and the type of structure of the question components of the written verbal arithmetic problems on a test (quantified nouns repeated versus quantified nouns deleted) was also of concern.

The results indicated that high achievers in arithmetic were able to solve written verbal arithmetic problems better than low achievers in arithmetic. Also, three-digit written verbal arithmetic problems and written verbal arithmetic problems involving subtraction were found to be more difficult to solve than two-digit ones and those involving addition, respectively. In addition to the significant main effects, two significant two-way interactions (operation x digits and level of overall mathematical ability x operation) and two significant three-way interactions (syntax of informational components x structure of question components x level of overall arithmetical ability and syntax of informational components x level of overall arithmetical ability x digits) were identified.

The author recommends that additional research be undertaken in the area of verbal arithmetic problems and that special attention be directed toward the unique difficulties experienced by children in solving these problems.

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APPENDIX A

PRELIMINARY VERSION OF TEST I

____
Name

1. The dancer earned 324 dollars last week. The gardener earned 276 dollars last week. How many more dollars did the dancer earn than the gardener?

2. There are 30 garages on my block. There are 46 garages on your block. How many garages are there on both blocks?

3. A dress costs 70 dollars. You have only 49 dollars. How many more dollars do you need?

4. The store has 211 shovels today. The store will sell 165 shovels tomorrow. How many shovels will the store have then?

5. The baker sold 144 cupcakes to the teachers. He sold 208 cupcakes to the mothers. How many cupcakes did the baker sell in all?

6. The policeman swam 704 yards. The fireman swam 579 yards. How many more yards did the policeman swim than the fireman?

7. There are 68 cars in the parking lot. There are 26 cars in the street. How many cars are there in all?

8. The city has 594 trucks. The city needs 376 more. How many trucks does the city need in all?

9. You have only 98 pennies in your bank. You will give 37 pennies to your brother. How many pennies will you have then?

10. There are 28 animals in the brown cage. There are 23 animals in the black cage. How many animals are there in both cages?

11. There are 32 children in the first grade. There are 29 children in the third grade. How many more children are there in the first grade than in the third grade?

12. My mother has 285 radishes in her garden. My aunt has 392 radishes in her garden. How many radishes are there in all?

13. A girl has 186 crayons. A boy has 214 crayons. How many crayons do they have in all?

14. There are 77 balls in the box. The woman will take 35 balls from the box. How many balls will be left in the box?

15. The doctor mailed 41 letters. The postman mailed 53 letters. How many more letters did the postman mail than the doctor?

16. There are 682 words in my reading book. There are 743 words in your reading book. How many words are there in both books?

17. The little girl has 21 dolls. She wants 13 more dolls. How many dolls does she want to have in all?

18. The girls ate 37 apples at the picnic. The boys ate 39 apples at the picnic. How many apples in all were eaten at the picnic?

19. There are 846 blueberries on the tree. A girl will pick 493 blueberries from the tree. How many blueberries will be left on the tree?

20. The banker had 203 friends. The painter had only 176 friends. How many more friends did the banker have than the painter?

21. I have 75 fish. You have only 49 fish. How many more fish do I have than you?

22. A cowboy bought 453 horses today. He will buy 226 horses tomorrow. How many horses will the cowboy have then?

23. There are 781 houses in my town. There are 517 houses in your town. How many more houses are there in my town than in your town?

24. The man told 37 jokes. The woman told 65 jokes. How many jokes were told in all?

PRELIMINARY VERSION OF TEST II

____
Name

1. The dancer earned 324 dollars last week, and the gardener earned 276 dollars last week. How many more dollars did the dancer earn than the gardener?

2. There are 30 garages on my block, and there are 46 garages on your block. How many garages are there on both blocks?

3. A dress costs 70 dollars, but you have only 49 dollars. How many more dollars do you need?

4. The store has 211 shovels today, but the store will sell 165 shovels tomorrow. How many shovels will the store have then?

5. The baker sold 144 cupcakes to the teachers, and he sold 208 cupcakes to the mothers. How many cupcakes did the baker sell in all?

6. The policeman swam 794 yards, and the fireman swam 579 yards. How many more yards did the policeman swim than the fireman?

7. There are 68 cars in the parking lot, and there are 26 cars in the street. How many cars are there in all?

8. The city has 594 trucks, yet the city needs 376 more trucks. How many trucks does the city need in all?

9. You have only 98 pennies in your bank, yet you will give 37 pennies to your brother. How many pennies will you have then?

10. There are 28 animals in the brown cage, and there are 23 animals in the black cage. How many animals are there in both cages?

11. There are 32 children in the first grade, and there are 29 children in the third grade. How many more children are there in the first grade than in the third grade?

12. My mother has 285 radishes in her garden, and my aunt has 392 radishes in her garden. How many radishes are there in all?

13. A girl has 186 crayons, and a boy has 214 crayons. How many crayons do they have in all?

14. There are 77 balls in the box, but the woman will take 35 balls from the box. How many balls will be left in the box?

15. The doctor mailed 41 letters, and the postman mailed 53 letters. How many more letters did the postman mail than the doctor?

16. There are 682 words in my reading book, and there are 743 words in your reading book. How many words are there in both books?

17. The little girl has 21 dolls, yet she wants 13 more dolls. How many dolls does she want to have in all?

18. The girls ate 37 apples at the picnic, and the boys ate 49 apples at the picnic. How many apples in all were eaten at the picnic?

19. There are 846 blueberries on the tree, but a girl will pick 493 blueberries from the tree. How many blueberries will be left on the tree?

20. The banker had 203 friends, but the painter had only 176 friends. How many more friends did the banker have than the painter?

21. I have 75 fish, but you have only 49 fish. How many more fish do I have than you?

22. A cowboy bought 453 horses today, and he will buy 226 horses tomorrow. How many horses will the cowboy have then?

23. There are 781 houses in my town, and there are 517 houses in your town. How many more houses are there in my town than in your town?

24. The man told 37 jokes, and the woman told 65 jokes. How many jokes were told in all?

PRELIMINARY VERSION OF TEST III

____
Name

1. The dancer earned 324 dollars last week. The gardener earned 276 dollars last week. How many more did the dancer earn than the gardener?

2. There are 30 garages on my block. There are 46 garages on your block. How many are there on both blocks?

3. A dress costs 70 dollars. You have only 49 dollars. How many more do you need?

4. The store has 211 shovels today. The store will sell 165 shovels tomorrow. How many will the store have then?

5. The baker sold 144 cupcakes to the teachers. He sold 208 cupcakes to the mothers. How many did the baker sell in all?

6. The policeman swam 704 yards. The fireman swam 579 yards. How many more did the policeman swim than the fireman?

7. There are 68 cars in the parking lot. There are 26 cars in the street. How many are there in all?

8. The city has 594 trucks. The city needs 376 more trucks. How many does the city need in all?

9. You have only 98 pennies in your bank. You will give 37 pennies to your brother. How many will you have then?

10. There are 28 animals in the brown cage. There are 23 animals in the black cage. How many are there in both cages?

11. There are 32 children in the first grade. There are 29 children in the third grade. How many more are there in the first grade than in the third grade?

12. My mother has 285 radishes in her garden. My aunt has 392 radishes in her garden. How many are there in all?

13. A girl has 186 crayons. A boy has 214 crayons. How many do they have in all?

14. There are 77 balls in the box. The woman will take 35 balls from the box. How many will be left in the box?

15. The doctor mailed 41 letters. The postman mailed 53 letters. How many more did the postman mail than the doctor?

16. There are 682 words in my reading book. There are 743 words in your reading book. How many are there in both books?

17. The little girl has 21 dolls. She wants 13 more dolls. How many does she want to have in all?

18. The girls ate 37 apples at the picnic. The boys ate 49 apples at the picnic. How many in all were eaten at the picnic?

19. There are 846 blueberries on the tree. A girl will pick 493 blueberries from the tree. How many will be left on the tree?

20. The banker has 203 friends. The painter had only 176 friends. How many more did the banker have than the painter?

21. I have 75 fish. You have only 49 fish. How many more do I have than you?

22. A cowboy bought 453 horses today. He will buy 226 horses tomorrow. How many will the cowboy have then?

23. There are 781 houses in my town. There are 517 houses in your town. How many more are there in my town than in your town?

24. The man told 37 jokes. The woman told 65 jokes. How many were told in all?

PRELIMINARY VERSION OF TEST IV

____
Name

1. The dancer earned 324 dollars last week, and the gardener earned 276 dollars last week. How many more did the dancer earn than the gardener?

2. There are 30 garages on my block, and there are 46 garages on your block. How many are there on both blocks?

3. A dress costs 70 dollars, but you have only 49 dollars. How many more do you need?

4. The store has 211 shovels today, but the store will sell 165 shovels tomorrow. How many will the store have then?

5. The baker sold 144 cupcakes to the teachers, and he sold 208 cupcakes to the mothers. How many did the baker sell in all?

6. The policeman swam 704 yards, and the fireman swam 579 yards. How many more did the policeman swim than the fireman?

7. There are 68 cars in the parking lot, and there are 26 cars in the street. How many are there in all?

8. The city has 594 trucks, yet the city needs 376 more trucks. How many does the city need in all?

9. You have only 98 pennies in your bank, yet you will give 37 pennies to your brother. How many will you have then?

10. There are 28 animals in the black cage, and there are 23 animals in the brown cage. How many are there in both cages?

11. There are 32 children in the first grade, and there are 29 children in the third grade. How many more are there in the first grade than in the third grade?

12. My mother has 285 radishes in her garden, and my aunt has 392 radishes in her garden. How many are there in all?

13. A girl has 186 crayons, and a boy has 214 crayons. How many do they have in all?

14. There are 77 balls in the box, but the woman will take 35 balls from the box. How many will be left in the box?

15. The doctor mailed 41 letters, and the postman mailed 53 letters. How many more did the postman mail than the doctor?

16. There are 682 words in my reading book, and there are 743 words in your reading book. How many are there in both books?

17. The little girl has 21 dolls, yet she wants 13 more dolls. How many does she want to have in all?

18. The girls ate 37 apples at the picnic, and the boys ate 49 apples at the picnic. How many in all were eaten at the picnic?

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21. I have 75 fish, but you have only 49 fish. How many more do I have than you?

22. A cowboy bought 453 horses today, and he will buy 226 horses tomorrow. How many will the cowboy have then?

23. There are 781 houses in my town, and there are 517 houses in your town. How many more are there in my town than in your town?

24. The man told 37 jokes, and the woman told 65 jokes. How many were told in all?

APPENDIX B

TEST I

____
Name

1. The dancer earned 324 dollars last week. The gardener earned 276 dollars last week. How many more dollars did the dancer earn than the gardener?

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4. The baker sold 144 cupcakes to the teachers. He sold 208 cupcakes to the mothers. How many cupcakes did the baker sell in all?

5. There are 68 cars in the parking lot. There are 26 cars in the street. How many cars are there in all?

6. The city has 594 trucks. The city needs 376 more trucks. How many trucks does the city need in all?

7. You have only 98 pennies in your bank. You will give 37 pennies to your brother. How many pennies will you have then?

8. There are 32 children in the first grade. There are 29 children in the third grade. How many more children are there in the first grade than in the third grade?

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17. I have 75 fish. You have only 49 fish. How many more fish do I have than you?

18. A cowboy bought 453 horses today. He will buy 226 horses tomorrow. How many horses will the cowboy have then?

19. There are 781 houses in my town. There are 517 houses in your town. How many more houses are there in my town than in your town?

20. The man told 37 jokes. The woman told 65 jokes. How many jokes were told in all?

TEST II

____
Name

1. The dancer earned 324 dollars last week, and the gardener earned 276 dollars last week. How many more dollars did the dancer earn than the gardener?

2. A dress costs 70 dollars, but you have only 49 dollars. How many more dollars do you need?

3. The store has 211 shovels today, but the store will sell 165 shovels tomorrow. How many shovels will the store have then?

4. The baker sold 144 cupcakes to the teachers, and he sold 208 cupcakes to the mothers. How many cupcakes did the baker sell in all?

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17. I have 75 fish, but you have only 49 fish. How many more fish do I have than you?

18. A cowboy bought 453 horses today, and he will buy 226 horses tomorrow. How many horses will the cowboy have then?

19. There are 781 houses in my town, and there are 517 houses in your town. How many more houses are there in my town than in your town?

20. The man told 37 jokes, and the woman told 65 jokes. How many jokes were told in all?

TEST III

____
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6. The city has 594 trucks. The city needs 376 more trucks. How many does the city need in all?

7. You have only 98 pennies in your bank. You will give 37 pennies to your brother. How many will you have then?

8. There are 32 children in the first grade. There are 29 children in the third grade. How many more are there in the first grade than in the third grade?

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17. I have 75 fish. You have only 49 fish. How many more do I have than you?

18. A cowboy bought 453 horses today. He will buy 226 horses tomorrow. How many will the cowboy have then?

19. There are 781 houses in my town. There are 517 houses in your town. How many more are there in my town than in your town?

20. The man told 37 jokes. The woman told 65 jokes. How many were told in all?

TEST IV

____

Name

1. The dancer earned 324 dollars last week, and the gardener earned 276 dollars last week. How many more did the dancer earn than the gardener?

2. A dress costs 70 dollars, but you have only 49 dollars. How many more do you need?

3. The store has 211 shovels today, but the store will sell 165 shovels tomorrow. How many will the store have then?

4. The baker sold 144 cupcakes to the teachers, and he sold 208 cupcakes to the mothers. How many did the baker sell in all?

5. There are 68 cars in the parking lot, and there are 26 cars in the street. How many are there in all?

6. The city has 594 trucks, yet the city needs 376 more trucks. How many does the city need in all?

7. You have only 98 pennies in your bank, yet you will give 37 pennies to your brother. How many will you have then?

8. There are 32 children in the first grade, and there are 29 children in the third grade. How many more are there in the first grade than in the third grade?

9. My mother has 285 radishes in her garden, and my aunt has 392 radishes in her garden. How many are there in all?

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11. There are 77 balls in the box, but the woman will take 35 balls from the box. How many will be left in the box?

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16. There are 846 blueberries on the tree, but a girl will pick 493 blueberries from the tree. How many will be left on the tree?

17. I have 75 fish, but you have only 49 fish. How many more do I have than you?

18. A cowboy bought 453 horses today, and he will buy 226 horses tomorrow. How many will the cowboy have then?

19. There are 781 houses in my town, and there are 517 houses in your town. How many more are there in my town than in your town?

20. The man told 37 jokes, and the woman told 65 jokes. How many were told in all?

APPENDIX C

TEST DIRECTIONS

Dear Teacher:

You have been given an equal number of copies of each of four arithmetic tests. Please distribute these copies among your students. Each student should receive one copy of only one test. Once the test copies have been passed out, please have each child place his name on the line provided on the first page of his test copy. After this has been completed, read the following directions to the class:

> There are twenty arithmetic problems for you to solve. You are to solve each of the problems as best as you can. You will not receive a grade for your work. We are just trying to find out what kinds of problems third grade girls and boys do best. You can use the space under each problem to solve the problem. Please circle your answer to the problem. Raise your hand when you are finished, and I will collect your papers. Don't forget to circle your answer to each problem.

After the students have finished, please bring all papers to your principal's office.

Please accept my appreciation for your cooperation and participation in this project.

Sincerely,

Susan Ireland

APPROVAL

The dissertation submitted by Susan C. Ireland has been read and approved by the following Committee:

Dr. Robert C. Cienkus, Chairman
Associate Professor, Curriculum and Instruction

Dr. Todd Hoover
Assitant Professor, Curriculum and Instruction

Dr. Lois Lackner
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The final copies have been examined by the director of the dissertation and the signature which appears below verifies the fact that any necessary changes have been incorporated and that the dissertation is now given final approval by the Committee with reference to content and form.

The dissertation is therefore accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

December 13, 1979
Date

Robert C. Cienkus
Director's Signature