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Enabling Greater Access to Home Meal Delivery

Maciek Nowak
Loyola University Chicago, mnowak4@luc.edu

Leo Gala
Rochester Institute of Technology

Mike Hewitt
Rochester Institute of Technology

Enabling Greater Access to Home Meal Delivery

Leo Gala, Mike Hewitt [1]

Department of Industrial and Systems Engineering
Rochester Institute of Technology
Rochester, NY 14534, U.S.A, Email: mrheie@rit.edu

Maciek Nowak

Information Systems and Operations Management
Graduate School of Business
Loyola University Chicago

Abstract

Non-profit organizations like the Meals On Wheels (MOW) association of America prepare and deliver meals, typically daily, to approximately one million homebound citizens in the United States alone. However, many MOW agencies are facing a steadily increasing number of clients requesting meal service without an increase in resources (either financial or human). One strategy for accommodating these requests is to deliver multiple (frozen) meals at a time and thus make fewer deliveries. However, many of the stakeholders (funders, volunteers, meal recipients) value the relationships that are developed by having a client receive daily deliveries from the same volunteer. Further, meal recipients may be concerned with the quality of food delivered through a frozen meal. In this paper, we develop a method for introducing consolidation into home meal delivery while minimizing operational disruptions and maintaining client satisfaction. With an extensive computational study, the savings associated with various levels and types of disruptions are detailed.

Keywords: OR in health services, transportation, integer programming, heuristics.

1. Introduction

On a daily basis, non-profit organizations like the Meals On Wheels Association of America (MOW) deliver approximately one million meals throughout communities

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[1]corresponding author; email: mrheie@rit.edu, phone: +1 585 475 5571, fax: +1 585 475 2520

in the United States. Within each of these communities, many individuals aged 60 and older rely on government funded programs like MOW to meet their dietary needs for sustaining a healthy lifestyle. In addition to the aging population, MOW serves individuals who are incapable of sustaining themselves due to medical limitations. The assistance MOW provides enables their clients to remain comfortable in their homes instead of requiring them to relocate to subsidized housing or nursing homes, either at personal or government expense. Individuals who wish to receive free or reduced-cost meal assistance must qualify, where the qualification process also determines the number of meals the individual should receive in a week. To deliver meals, MOW relies on a workforce comprised of both professionals and volunteers from each community, and routes between 800,000 and 1.2 million volunteers annually in the United States [17].

MOW is seeing is a steadily increasing demand for meals. From 1980 to 2002, the demand for meals in the United States increased by 290%. While contributions from private organizations and cost reduction efforts have helped MOW to increase the number of meals it delivers, their capabilities have not grown at the same rate as demand. As a result, MOW agencies are often forced to put some or all of a clients deliveries on a wait-list, although this is clearly undesirable. Similarly, MOW anticipates a significant increase in the number of seniors in America that face the threat of hunger (8.3. million in 2010, 9.5 million projected in 2025), and has stated that its vision is to end senior hunger by 2020 [18].

One strategy for reducing costs and increasing capacity that some MOW agencies have undertaken (while there is a national association, agencies are locally owned and operated) is to deliver frozen meals. An official from New York City stated, "for us it's really been about creating a more efficient system and not having anyone on a waiting list." That same official mentioned that by delivering frozen meals twice a week to 40% of its recipients they were able to reduce the number of professional drivers they paid from seventeen to three [14]. Similarly, delivering frozen meals has enabled MOW agencies to serve recipients in rural areas that, given limited driver resources, were too costly to reach on a daily basis [12]. By using frozen meals, MOW agencies find the benefits of consolidating deliveries that many other transportation providers have already discovered. However, while frozen meals allow for considerable cost savings, there are several operational challenges.

While the primary purpose of a MOW agency is to provide nutritious meals to those who can not provide one for themselves, many agencies and their supporters (both volunteers and funders) also see themselves as providing human interactions and relationships to those who may otherwise have little human contact [15]. As an example, many MOW agencies like to invoke the slogan "more than just a meal."

Thus, while there are some clear efficiencies that can be gained by delivering frozen meals, some funding agencies that support MOW programs are reticent to support their delivery as they fear doing so will diminish the health monitoring capabilities of Meals On Wheels. Further, many have claimed that the quality of the frozen meals is simply inferior to the hot meals [8].

Like major parcel delivery companies such as FedEx and UPS, MOW faces the problem of making home deliveries in large sized communities, containing demand at varied locations, and with delivery routes limited by vehicle capacity. However, while industry leaders in parcel delivery have the engineering and financial resources to invest in technology that handles these routing scenarios and to answer strategic what-if questions regarding how deliveries are made, many community based MOW agencies do not. Creating a good set of routes that are repeated on a daily basis can be difficult enough without an advanced routing algorithm. Consolidating deliveries on those routes may require the creation of a unique route for each day of service, putting an even greater strain on the MOW agency's limited operational resources.

The contribution of this paper is a methodology that MOW agencies can use to effectively introduce frozen meal delivery while limiting operational disruption. While this work is presented in the context of its inspiration, Meals On Wheels, the methodology and results presented are pertinent to any organization that wishes to quantify the tradeoffs between consolidating deliveries and maintaining high levels of customer service, such as a parcel delivery or LTL carrier. In particular, the approach presented here is configurable to the operational realities faced and quality metrics valued by the MOW agencies. The focus is on a setting where clients currently receive hot meals five days a week and the agency has an established set of delivery routes that are executed daily. Based on those routes, this approach can maintain consistency with historical operations by ensuring that a client remains on the same route as before, and that clients are seen in the same order.

Introducing consolidation into daily delivery routes is reminiscent of vendor-managed inventory or the Inventory Routing Problem [1, 5]. However, we consider constraints that have not been included in the inventory routing problem. For example, this approach can support fixed routes [4, 9] by serving the clients in the same order on every route. Such a restriction can have many operational advantages, including making it easier to synchronize the meal preparation and vehicle loading process. This problem is inherently periodic and thus shares characteristics with the Periodic Vehicle Routing Problem (PVRP) [11]. However, like the work on the PVRP with Service Choice [10], the number of times an individual is visited is not an input to the model but a decision variable.

Some of the earliest work on the home meal delivery problem was presented in [3],

which used space-filling curves as the basis of a system that could be implemented on two rolodex cards. While the approach presented here is fundamentally different, the emphasis on operational ease is shared, as the focus is on minimizing the operational disruptions associated with introducing consolidation into delivery operations. This approach can also enforce various quality of service-type metrics. For an agency that is concerned with the quality disparity between frozen and hot meals, this approach can ensure that each client receives at least a minimum number of hot meals. For an agency that wishes to maintain the relationships their drivers have with clients this approach can enforce a maximum number of days between visits and that a client is always visited by the same driver. Quality-of-service metrics were also studied in the context of inventory routing in [7].

To determine the potential efficiencies associated with delivering frozen meals, the problem of introducing consolidation is formulated as a mixed integer program (MIP). Different variants of this MIP are considered, based on the constraints to be enforced, e.g. fixed routes, a maximum number of days between deliveries to a client, etc. A characteristic of the problem is exploited to limit the solution space, allowing for shorter computational times. Transportation costs incurred by executing daily routes with and without consolidation are compared in an extensive computational analysis that details the constraints that most impact transportation costs. It is found that there is virtually no impact on the savings from consolidation when operational disruptions are prohibited. The savings decrease as quality of service is more strictly enforced, but even when consolidating just two deliveries into one for each customer, transportation costs are reduced by 10%. For MOW agencies that are operating at such tight margins, those savings can have a very significant impact. Finally, it is shown that through consolidation, more space may be created on the daily routes to service additional clients while still cutting transportation costs.

The remainder of this paper is organized as follows. In Section 2, the problem of introducing consolidation is presented along with several constraints used to limit operational disruption and client dissatisfaction. In Section 3 the heuristic used in this study is developed. Section 4 contains the results of an extensive computational study of the benefits associated with consolidation under various settings. Finally, Section 5 provides managerial insights based on the results.

2. Home Meal Consolidation and Delivery Problem

We next describe the problem of introducing consolidation into routes that are executed by a set of delivery vehicles, $v \in V$, every day, t, over a planning horizon, T, and how this problem is modeled as a mixed integer program (MIP). One week

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consisting of five days is used for this problem (thus $|T| = 5$). In this work, it is assumed that the solution for a week can be used repeatedly to provide service over a longer period, such that frozen meals left at the end of one week may be used in the following week. Each day the set of delivery vehicles begin at a common depot, D, visiting a set of clients, I, delivering a single meal to each, and returning to the depot. To preserve the quality of the meals delivered, they are stored in a container which can hold at most K meals. In the computational study in Section 4, K is the same for both frozen and hot meals. The cost of traveling from client i to client j is denoted by c_{ij}.

By delivering frozen meals and, in particular, multiple meals on a single day, the delivery route may skip a client on certain days. Thus, the decision variable $y_i^{vt} \in \{0, 1\}$ is defined to denote whether or not vehicle v visits client $i \in I$ on day $t \in T$, $q_{ci}^{vt} \in \mathbb{Z}$ to represent the number of cold meals delivered by vehicle v, and $q_{hi}^{vt} \in \{0, 1\}$ the number of hot meals delivered by vehicle v. Because hot and frozen meals can not be in the same meal container (and each vehicle carries at most one container), delivering a hot meal to one client on day t precludes delivering frozen meals to any client that day by that vehicle. Thus, the variable $r^{vt} \in \{0, 1\}$ is defined to indicate whether or not vehicle v delivers hot meals on day t. The decision variable $x_{ij}^{vt} \in \{0, 1\}$ is defined to denote whether vehicle v travels from location $i \in D \cup I$ to location $j \in D \cup I$ on day $t \in T$. Finally, define $z_i^v \in \{0, 1\}$ to represent whether client $i \in I$ is served by vehicle $v \in V$. The baseline home meal consolidation and delivery problem (HMCD) problem is then to:

$$\text{minimize} \sum_{v \in V} \sum_{t \in T} \sum_{i \in I \cup D} \sum_{j \in I \cup D} c_{ij} x_{ij}^{vt}$$

subject to

$$\sum_{v \in V} z_i^v = 1 \quad \forall i \in I, \tag{1}$$

$$y_i^{vt} \leq z_i^v \quad \forall i \in I, v \in V, t \in T, \tag{2}$$

$$\sum_{v \in V} \sum_{t \in T} q_{ic}^{vt} + q_{hi}^{vt} = |T| \quad \forall i \in I, \tag{3}$$

$$q_{ic}^{vt} \leq |T| y_i^{vt} \quad \forall i \in I, v \in V, t \in T,$$
$$q_{ih}^{vt} \leq y_i^{vt} \quad \forall i \in I, v \in V, t \in T, \tag{4}$$

$$r^{vt} \leq q_{ih}^{vt} \leq r^{vt} \quad \forall i \in I, v \in V, t \in T, \tag{5}$$

$$q_{ic}^{vt} \leq |T| - |T| r^{vt} \quad \forall i \in I, v \in V, t \in T, \tag{6}$$

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$$\sum_{i \in I} q_{ic}^{vt} + q_{ih}^{vt} \leq K \quad \forall v \in V, t \in T, \tag{7}$$

$$\sum_{j \in I} x_{ij}^{vt} = y_i^{vt} \quad \forall i \in I, v \in V, t \in T, \tag{8}$$

$$\sum_{j \in I} x_{ji}^{vt} = y_i^{vt} \quad \forall i \in I, v \in V, t \in T, \tag{9}$$

$$\sum_{j \in I \cup D} x_{ij}^{vt} = \sum_{j \in I \cup D} x_{ji}^{vt} \quad \forall i \in I, v \in V, t \in T, \tag{10}$$

$$\sum_{i \in S} \sum_{j \in S} x_{ij}^{vt} \leq |S| - 1, \quad \forall S \subseteq I \cup D, |D| \geq 2, \forall v \in V, t \in T. \tag{11}$$

The objective is to minimize the total cost associated with travel. Constraints (1) and (2) ensure that each client is seen by exactly one delivery person during the week. To help foster relationships between clients and delivery persons, as was shown to be beneficial in [19], when a client is moved between vehicles on one day, she is moved to that vehicle on all days, ensuring that each client is seen by at most one delivery person each week. Constraints (3) ensure that each client receives a meal for every day of the time horizon, whereas constraints (4) ensure that a client is not delivered a hot or cold meal on a day unless the delivery vehicle visits them on that day. Note that an individual can receive at most $|T|$ frozen meals and at most one hot meal in one delivery. Constraints (5) and (6) together ensure that if one client receives a hot meal on day t then no other client receives a frozen meal on that day. Note constraints (5) also ensure that when hot meals are delivered on a day, every client receives a hot meal that day. Constraints (7) ensure that vehicle capacity for meals is not exceeded. Constraints (8, 9, 10, and 11) are standard routing constraints.

While the HMCD ensures that each client receives five meals a week, it requires little else. From the delivery agencies perspective, it may yield delivery routes that vary significantly by day, which could complicate administrative tasks. From the client's perspective, the use of frozen meals limits the number of deliveries and the time for interaction between client and volunteer, harming this relationship. Also, some clients only receive frozen meals, which they may enjoy less than hot meals. Thus, we next discuss constraints that can be added to the model to mitigate negative side effects associated with delivering frozen meals. Then, in Section 4 an extensive computational study is reported regarding how different combinations of these constraints impact costs.

2.1 Minimizing Operational Disruption

The modifications to the model discussed in this section are meant to minimize the degree to which consolidation can change the daily operations of the Meals On Wheels agency. This can be done by ensuring that the new set of routes for the delivery vehicles follow a pre-defined order throughout the week, by fixing the assignment of clients to drivers, or by maintaining consistency with respect to the order clients are visited in the current daily routes.

Fixed Route: Fixed routes, or a set of routes that visit locations in the same order every day, often offer advantages over routes that vary by day. For home meal delivery, a fixed route can make it easier to synchronize the meal preparation and vehicle loading process. Similarly, while some routes are executed by volunteers, some are done by professional drivers, in which case the same driver executes the route every day. In this case, even though some clients may be skipped on some days, ensuring that they are always visited in the same order can make executing the delivery route easier. Thus, to ensure the delivery routes follow a fixed ordering of clients, we define the variable $o_{ij}^v \in \{0,1\}$ to indicate whether client i is visited before client j by delivery vehicle v on days they are both visited and add the following constraints to the HMCD:

$$o_{ij}^v + o_{ji}^v = 1 \quad \forall i,j \in I, v \in V, \tag{12}$$

$$o_{ij}^v + o_{jk}^v + o_{ki}^v \leq 2 \quad \forall i,j,k \in I, v \in V, \tag{13}$$

$$x_{ij}^{vt} \leq o_{ij}^v \quad \forall i,j \in I, v \in V, t \in T. \tag{14}$$

Constraints (12) and (13) are standard ordering constraints, with the first ensuring that either i precedes j or j precedes i, and the second ensuring for a triplet i,j,k the "ordering" $i \to j \to k \to i$ is not allowed. Finally, constraints (14) ensure that daily travel corresponds to the ordering prescribed for the week. These are referred to as the *Fixed Route* constraints.

Client Consistency: The model focuses on introducing consolidation into routes that are already executed on a daily basis by the same MOW provider, and, typically, in the same order on every day. However, the HMCD, even with the fixed route constraints above, may change the vehicle, and therefore the driver, that services a client. A driver may have developed a bond with his or her clients and disrupting that bond in the interest of operational efficiency may be discouraged. Therefore, it may be of interest to keep the same clients assigned to the same vehicles as on the

currently established routes. Consistency with clients is maintained by adding the constraint:

$$y_i^{vt} = 0 \quad \forall t \in T, v \in V, i \in I \text{ if } i \text{ is not currently visited by } v. \tag{15}$$

This is referred to as the *Client Consistency* constraint. When these constraints are added to the HMCD, it separates into a single-vehicle variant of the problem for each vehicle.

Order Consistency: The HMCD may also change the order in which clients are visited on currently established routes, even with the fixed route constraints above. As with the developed client relationship, a driver may have an established route that she is familiar with and disrupting that route may add some intangible operating costs. Therefore, it may be of interest to keep the order in which clients are visited the same as on the currently established routes. Consistency with order of clients is maintained by adding the constraint:

$$x_{ij}^{vt} = 0 \quad \forall t \in T, v \in V, i, j \in I \text{ if } j \text{ visited before } i \text{ in current route.} \tag{16}$$

This is referred to as the *Order Consistency* constraint.

2.2 Minimizing Client Dissatisfaction

While consolidation has been shown to yield tremendous savings in many settings, Meals On Wheels is primarily focused on the wellness and betterment of its clients. Thus, the client experience within a meal delivery program that delivers frozen meals is of utmost importance. The human interactions and relationships offered by Meals On Wheels may be maintained by limiting the number of days that can elapse between visits. Also, a minimum may be placed on the number of hot meals that each client receives as most prefer these to frozen meals.

Delivery Frequency: The y_i^t variables may be used to derive a constraint ensuring that a client must be seen at least once within a specific span of days, ensuring that there are never too many days between visits. For example, to allow no more than two days between visits (recall that $|T| = 5$), the following constraints are added to

the HMCD:

$$\sum_{v \in V} y_i^{v1} + \sum_{v \in V} y_i^{v2} + \sum_{v \in V} y_i^{v3} \geq 1 \quad \forall i \in I,$$

$$\sum_{v \in V} y_i^{v2} + \sum_{v \in V} y_i^{v3} + \sum_{v \in V} y_i^{v4} \geq 1 \quad \forall i \in I,$$

$$\sum_{v \in V} y_i^{v3} + \sum_{v \in V} y_i^{v4} + \sum_{v \in V} y_i^{v5} \geq 1 \quad \forall i \in I,$$

$$\sum_{v \in V} y_i^{v4} + \sum_{v \in V} y_i^{v5} + \sum_{v \in V} y_i^{v1} \geq 1 \quad \forall i \in I,$$

$$\sum_{v \in V} y_i^{v5} + \sum_{v \in V} y_i^{v1} + \sum_{v \in V} y_i^{v2} \geq 1 \quad \forall i \in I. \tag{17}$$

These types of constraints are referred to as *Delivery Frequency* constraints. Note that because each y_i^{vt} is binary, the valid inequality $y_i^{v1} + y_i^{v2} + y_i^{v3} + y_i^{v4} + y_i^{v5} \geq 2$ may be derived from these constraints with a rounding argument.

<u>*Hot Meal Delivery:*</u> To ensure that at least H hot meals are delivered to each individual, the following constraint is added to the HMCD:

$$\sum_{t \in T} q_{ih}^{vt} \geq H \quad \forall i \in I, v \in V. \tag{18}$$

This is referred to as the *Hot Meal* constraint.

Some analysis of these constraints can be useful in restricting the solution space of the HMCD. By guaranteeing a minimum number of hot meals and a maximum time between visits, the scheduling options are strictly limited. Specifically, a lemma regarding the schedule follows:

Lemma 2.1. *Given a time horizon of five days ($|T| = 5$), if at least two hot meals must be delivered to each individual ($H = 2$) and at most two days are allowed between visits, then there is an optimal solution to HMCD where hot meals are delivered on days two and four (Tuesday and Thursday).*

Proof Consider a solution $(\bar{q}_c, \bar{q}_h, \bar{x}, \bar{y}, \bar{r})$ to the HMCD where at least two hot meals are delivered and at most two days pass between visits. Suppose hot meals are delivered on days t', t''. For vehicle v, set the value of all variables associated with day two equal to the values of the variables associated with t' and day four variables

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equal to the values of the t'' variables, while similarly setting the variables associated with t' equal to the day two variables and t'' equal to day four (i.e. set $x_{ij}^{v2} = \bar{x}_{ij}^{vt'}, x_{ij}^{vt'} = \bar{x}_{ij}^{v2}, q_{ic}^{v2} = \bar{q}_{ic}^{vt'}, q_{ic}^{vt'} = \bar{q}_{ic}^{v2}, q_{ih}^{v2} = \bar{q}_{ih}^{vt'}, q_{ih}^{vt'} = \bar{q}_{ih}^{v2}, r^{v2} = \bar{r}^{vt'}, r^{vt'} = \bar{r}^{v2}, y_i^{v2} = \bar{y}_i^{vt'}, y_i^{vt'} = \bar{y}_i^{v2}$ and similarly for days four and t''). This may be repeated for all vehicles $v \in V$. Changing the day on which a route is executed does not have an effect on the cost of that route, so the cost of this new solution is the same as the original solution. This new solution is still feasible because the number of hot meals delivered to each client during the week remains the same. Also, by ensuring everyone is delivered to on days two and four, there can be at most two days between visits (days five and one if an individual does not receive a delivery on those days). Thus, any optimal solution to the HMCD can be translated to a solution where hot meals are delivered on days two and four, and because the cost of that new solution is the same, it is also optimal. $\square$

For instances of the HMCD where the premise of Lemma 2.1 applies, we can fix $r^{v2} = r^{v4} = 1, r^{v1} = r^{v3} = r^{v5} = 0$ for all $v \in V$, which in turn reduces the problem to one where decisions only need to be made for days one, three, and five. Similarly, one can argue that if three hot meals must be delivered to each individual, then there is an optimal solution where they are delivered on days one, three, and five. While this lemma has algorithmic implications in that instances of the HMCD may be solved in much less time by fixing r^{vt} variables, it also has practical implications. Specifically, this lemma implies that an agency only needs to prepare hot meals on specific days and thus resource requirements for meal preparation may be re-allocated.

3. Introducing consolidation into daily delivery routes

One of two algorithms is used to introduce consolidation into daily delivery routes, with the choice of algorithm dependent on whether *Client Consistency* is enforced. Each algorithm is based on solving restrictions of the HMCD, and thus can be used (unchanged) when the model is extended to include other constraints such as the *Fixed Route* or *Delivery Frequency* constraints. In describing these algorithms, the constraints described in Sections 2.1 and 2.2 are collectively referred to as *Operational Considerations*. As alluded to above, when Lemma 2.1 applies, restrictions of the HMCD are further constrained by fixing the appropriate r^{vt} variables to 1 or 0.

With *Client Consistency*: As noted previously, when *Client Consistency* is enforced, such that the constraints

$$y_i^{vt} = 0 \quad \forall t \in T, v \in V, i \in I \text{ if } i \text{ is not currently visited by } v$$

are added to the HMCD, then the problem separates into a single-vehicle variant of the problem for each vehicle. In this case, consolidation is achieved by solving $|V|$ restrictions of the HMCD, one for each vehicle, which we do to (near-) optimality with a commercial integer programming software package. Specifically, Algorithm 1 is executed.

Algorithm 1 Introducing consolidation while maintaining driver-client assignments

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Require: Set of *Operational Considerations* to enforce
 1: Given the pre-existing daily routes used by the MOW provider, create initial solution where these routes are executed every day of the time horizon
 2: **for all** $v \in V$ **do**
 3:    Create instance of HMCD where the set of clients, I, corresponds to those visited by v in daily routes
 4:    Add to HMCD constraints that model *Operational Considerations*
 5:    Solve resulting instance of HMCD
 6: **end for**

<u>Without *Client Consistency*</u>: When *Client Consistency* is not enforced, the HMCD does not separate by vehicle, leaving an optimization problem that, for realistically-sized instances, is difficult to solve to near-optimality with off-the-shelf software. As a result, to introduce consolidation into daily delivery routes, a heuristic is used that repeatedly solves restricted versions of the HMCD in which certain values are fixed during execution of the heuristic. These restricted versions are created by first partitioning the set of vehicles into subsets, $\bar{V}_1, \bar{V}_2, \ldots, \bar{V}_k \subseteq V$, and then, the clients seen by vehicles in a subset $\bar{V}_i$, $I(\bar{V}_i)$, are further partitioned into subsets, $\bar{I}_1(\bar{V}_i), \ldots, \bar{I}_k(\bar{V}_i) \subseteq I(\bar{V})$. Given these subsets, the following variables of the HMCD are fixed to their values in the current solution:

- $z_i^v, y_i^{vt}, x_{ij}^{vt}, \quad \forall v \in V \setminus \bar{V}$

- $z_i^v, y_i^{vt}, \quad \forall v \in \bar{V}, i \in I(\bar{V}) \setminus \bar{I}(\bar{V})$.

That is, all but the delivery quantity variables are fixed for a subset of vehicles. Then, for the remainder of the vehicles, a subset of the client-assignment variables are fixed (but none of the routing variables are fixed). This is done in the context of executing Algorithm 2

The *k-means* clustering algorithm [16] is used to partition the vehicles and clients into the subsets. The purpose of k-means clustering is to partition n data points into k clusters such that each data point is in the cluster whose mean is closest. While

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Algorithm 2 Introducing consolidation while allowing driver-client assignments to change

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Require: Set of *Operational Considerations* to enforce

 1: Given the pre-existing daily routes used by the MOW provider, create initial solution where these routes are executed every day of the time horizon
 2: **while** not stop **do**
 3:     Partition vehicles into groups $\bar{V}_1, \bar{V}_2, \ldots, \bar{V}_k$
 4:     **for all** vehicle groups $\bar{V}_i$ **do**
 5:         Partition clients $I(\bar{V}_i)$ delivered to by some $v \in \bar{V}_i$ into groups $\bar{I}_1(\bar{V}_i), \ldots, \bar{I}_k(\bar{V}_i)$
 6:         **for all** client groups $I_j(\bar{V}_i)$ **do**
 7:             Formulate HMCD with constraints that model *Operational Considerations*
 8:             Fix variables $z_i^v, y_i^{vt}, x_{ij}^{vt}, \quad \forall v \in V \setminus \bar{V}$ and then $z_i^v, y_i^{vt}, \quad \forall v \in \bar{V}, i \in I(\bar{V}) \setminus \bar{I}(\bar{V})$
 9:             Solve resulting instance to get new solution
10:             **if** improved solution is found **then**
11:                 Set current solution to improved solution
12:             **end if**
13:         **end for**
14:     **end for**
15: **end while**

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determining the optimal set of k clusters for a fixed k is NP-Hard, many computationally effective and efficient heuristics have been developed. In particular, we use the heuristic k-means++ [2] which has been shown to have approximation guarantees. To use the k-means++ algorithm to partition a set of objects into groups each object must be represented by a data point. Once this representation of each object is determined, the k-means++ algorithm may be executed to cluster the data points into groups. The representations for vehicles and clients is discussed next.

Partitioning vehicles: To partition the vehicles into groups a $2|T|$-dimensional data point is created for each vehicle containing the average (x, y) coordinates for that vehicle for each day over the problem horizon. Assume the coordinates (x^i, y^i) are given for each client and that vehicle v sees m_t clients on day t. The average coordinates for each day are calculated as $avg_x^t = \sum_{i \text{ seen on } t} x^i/m_t$ and $avg_y^t = \sum_{i \text{ seen on } t} y^i/m_t$. The data point for vehicle v is then $[avg_x^1 \quad avg_y^1 \quad \dots avg_x^{|T|} \quad avg_y^{|T|}]$. As an example, consider a two day planning horizon $(T = 2)$ and a vehicle that on day one visits clients at locations $(20, 40), (10, 90)$, and $(0, -10)$ and on day two visits clients at locations $(10, 10)$, and $(50, 50)$. Then, the data point for that vehicle would be $[10 \quad 30 \quad 30 \quad 30]$.

Partitioning clients: To partition the clients into groups, each client is represented by a 2-dimensional data point that contains the (x, y) coordinates for that client.

While the k-means++ algorithm is the primary mechanism for partitioning vehicles and clients into groups, to introduce diversity into the search, the heuristic periodically partitions them randomly.

4. Computational Analysis

For these experiments, eight of the Christofides instances are modified from [6], specifically, instances one through four and six through nine. There are then two instances each with 50, 75, 100 and 150 customers. The instances are modified so that each customer has a demand of one unit per day (in our context, a meal) and each vehicle has a capacity of eight. The number of vehicles is adjusted so that the utilization of each vehicle is roughly 90%, with 7, 10, 13 and 20 vehicles for the problems in increasing size. As the algorithm indicates, the initial solution is the pre-existing routes daily routes used by the provider executed on each day of the problem horizon, such that each client receives a hot meal on each day. Given that pre-existing routes do not exist for the modified Christofides instances, a heuristic presented in [13] is used to create these daily routes with costs that are within 5% of the best known solution for each of the eight original Christofides instances. Note

that most MOW providers do not utilize advanced routing techniques such as this heuristic, and thus their current daily routes may not be as cost efficient.

After the heuristic establishes the baseline daily routes providing service to every client on each day, either Algorithm 1 or 2 is applied, depending on which is appropriate under the various conditions described in Section 2 and comparisons may be made with the baseline routes. When solving integer programs in the context of executing either Algorithm 1 or 2, Gurobi 4.6 is used with the optimality tolerance parameter set to 1%. When solving instances of HMCD in the course of executing Algorithm 1, solution time is limited to ten minutes. When solving restrictions of HMCD in the course of executing Algorithm 2, solution time is limited to one minute. Finally, the complete runtime for Algorithm 2 is limited to one hour.

Two questions are explored in this computational analysis. The first is, given a set of clients, I, that are currently being served, how much money can be saved by delivering frozen meals to the clients in I, as compared to delivering a hot meal every day to each of those clients? The second is, given a set of clients, I, that are currently being served and a set of clients, U, that are currently unserved, by delivering frozen meals how many clients in U can be served a meal while still saving money, as compared to delivering a hot meal every day to each client in I? These questions are answered in the next two subsections for the different settings discussed in Sections 2.1 and 2.2.

4.1 Savings through consolidation

Table 1 presents several metrics for instances with consolidation that have no additional constraints and instances with the constraints used for minimizing operational disruption. These metrics include the averages of the savings in travel cost compared to the baseline case where only hot meals are delivered, and the number of hot meals and visits a client receives in a five day week. As might be expected, using frozen meals to allow for load consolidation leads to considerable cost savings. However, these results show that cost savings may be found without significantly altering current operations. When implementing the constraints to minimize disruption from current operations, the cost savings are virtually equivalent, as are the other metrics of interest.

These findings indicate that the cost savings related to consolidation are not a result of major changes in the routes used to deliver service. The heuristic used produces base daily routes that deliver hot meals every day that are within 5% of the best known solution, with little room for improvement from swapping clients between routes or reordering them within a route. Consolidation of deliveries results in the elimination of clients from routes on certain days and the routes are robust

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enough that removing stops does not open up savings opportunities associated with altering the route in other ways. Thus, the order of clients served and the assignment of clients to drivers may be maintained while travel costs are reduced by skipping multiple stops. If implemented in a real world setting with lower quality routes, this algorithm may find more opportunities to improve the solution through route adjustments. However, these results show that consolidation can lead to savings even without making those adjustments. From a managerial perspective, this is important as it indicates that operational costs may be reduced with minimal change management required for the pool of drivers.

Constraint	Cost savings	Hot meals per client	Visits per client
No operational constraints	21.21%	1.313	3.061
Fixed route	21.14%	1.316	3.056
Client consistency	21.21%	1.309	3.035
Order consistency	21.04%	1.308	3.036
All constraints	21.03%	1.307	3.036

Table 1: Evaluation of constraints minimizing operation disruption

While the impact on drivers may be minimized, consolidation leads to clients receiving more frozen meals and fewer visits. Table 2 presents the results when constraints are placed on the amount that a client may be inconvenienced through consolidation, attempting to limit dissatisfaction with the MOW service. Tests were not run with certain combinations of minimum hot meals and maximum days between visits as they were not feasible (e.g., four frozen meals must be delivered the day before there are three days between visits). These constraints have a much greater impact on the savings associated with consolidation than the operational constraints. As might be expected, the savings decrease as more hot meals are required per week and less consolidation is allowed. The minimum number of hot meals allowed by the constraints are delivered to the clients for each constraint setting. The savings also increased as more days were allowed between visits. However, a 10% cost savings still results when a minimum of three hot meals are required. Allowing two frozen meals per customer is equivalent to removing one day of service to each customer. While this is not a drastic adjustment to the MOW operations, with such tight margins, even a savings of 10% can be significant.

To see the effect of fixing r^{vt} variables when Lemma 2.1 holds, Algorithm 2 was run twice for each of the instances, once with the variables fixed and once without. This was done for the setting with a maximum of two days between service, a minimum

Maximum days between service	Minimum number of hot meals	Cost savings	Hot meals per client	Visits per client
1	0	22.6%	0.03	3.03
	1	20.8%	1.00	3.09
	2	11.4%	2.03	4.00
	3	10.2%	3.00	4.01
2	0	31.3%	0.01	2.09
	1	22.0%	1.02	3.01
	2	19.6%	2.00	3.12
3	0	32.7%	0.02	2.03
	1	30.7%	1.00	2.07

Table 2: Evaluation of constraints minimizing client dissatisfaction

of two hot meals per client, and no additional operational constraints. For each execution and instance, Algorithm 2 found the same solution. However, with the r^{vt} variables fixed, Algorithm 2 was able to find that solution an average of almost eight minutes faster than when the variables were not fixed. Similarly, fixing the r^{vt} variables enabled Gurobi to solve instances of the HMCD in much less time, as evidenced by the fact that Algorithm 2 was able to execute 48% more iterations in one hour when doing so.

4.2 Expanding the client set

For these experiments, initial daily routes are again generated for each instance. Then, assuming $|I|$ clients are in an instance, $0.1 * |I|$ new clients are randomly distributed across the region spanned by the original $|I|$ clients. All of these new clients are placed on one route for a "dummy" vehicle, wherein the transportation costs for this route are multiplied by a large number. Algorithm 2 is then executed on this instance, with the dummy vehicle included in each vehicle subset and each client subset containing all of the clients served by this vehicle. Thus, when solving restrictions of the HMCD in the course of executing Algorithm 2, clients may be moved from one real vehicle to another, as well as from the dummy vehicle to a real vehicle. By modeling a "waiting list" of clients with this dummy vehicle, it is possible that a client only receives one or two meals over the time horizon. This is in line with how agencies operate, as some clients may be assigned delivery on fewer than five days a week if that is what the current routes will allow.

Table 3 presents several metrics to evaluate the addition of new clients when using consolidation. These metrics are presented for the four problem sizes, defined by the number of vehicles used to deliver the meals. The first values show the increase in

the total number of meals served when the new clients are added, as well as the average number of additional meals that each vehicle delivers. As the problem size increases the total number of additional meals served increases, while the efficiency of the vehicles decreases. The heuristic is effective with the larger problems, but as problem size increases it becomes difficult to find available capacity for as many new clients on the routes.

Number of vehicles	Initial meals served	Meals served with additional clients	Additional meals per vehicle
7	250	268	2.59
10	375	398	2.66
13	500	520	2.33
20	750	787	1.90

Table 3: Additional meals served with increased client base

Table 4 presents the savings with consolidation over the baseline routes that deliver all hot meals, comparing the instances with and without the additional clients. Despite the increase in routing costs from serving additional clients, consolidation still leads to considerable savings, while also allowing for the delivery of more meals. The savings with additional clients increase with problem size as a smaller percentage of the new clients may be inserted into the routes, leading to less of an increase in routing costs. Conversely, the savings without additional clients decreases as the problem size increases. Due to the larger problem size and solution space, the heuristic cannot explore as many of the available opportunities for consolidation.

Number of vehicles	Savings without additional clients	Savings with additional clients	Baseline cost per meal	Cost per meal without additional clients	Cost per meal with additional clients
7	20.1%	9.8%	11.68	9.33	9.83
10	19.4%	11.6%	11.14	8.99	9.28
13	18.0%	14.9%	10.80	8.85	8.84
20	16.4%	14.0%	9.92	8.30	8.13

Table 4: Impact on consolidation savings with an increased client base

An analysis of the costs per meal for the three problem scenarios indicates not only the considerable savings per meal from consolidation, but the reduction in cost as the problem size increases. Further, while the cost per meal is initially greater

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with the new clients for the smallest problem size, it drops slightly below the cost without the new clients for the problems with 13 and 20 vehicles (or 100 and 150 customers). In combination with the results in Table 3, this indicates that while the heuristic can not find as much available capacity with the larger problem sizes, efficiency is maintained with routing those additional clients that can be inserted.

5. Conclusion

In this paper a model and solution methodology are presented that will enable home delivery companies to balance the efficiencies gained from consolidation with customer satisfaction and ease of operation metrics. The results provide several insights that may be beneficial to a MOW agency manager, or any transportation provider interested in consolidating loads while minimizing operational disruption.

- The order in which clients are serviced and the assignment of drivers to clients do not have to be altered in order to reduce costs through consolidation. This may have a greater impact on other transportation providers focused more on limiting operational disruption and less on customer service. However, while MOW agencies may be more client-centric, a methodology that does not impact their service providers is likely to be viewed favorably.

- While savings from consolidation decrease as customer service requirements increase, a non-negligible benefit may be found even with a minimal amount of consolidation. As described earlier, many MOW agencies are struggling to remain operational with limited funding and even a small cost savings may be what allows an agency to stay in business.

- Consolidation can lead to a reduction in transportation cost, while also allowing for previously unserved clients to receive meals. Clients may be unhappy with frozen meals; however, it may be easier to convince them of the benefit by indicating how many additional people the program can aid when some hot meals are replaced.

- The lemma implies that when certain customer service limits are in place, an agency only needs to prepare hot meals on specific days. This allows for operational planning to be simplified and for resource requirements for meal preparation to be re-allocated.

As more people require the service provided by MOW and funding becomes harder to find, the cost savings found through the use of frozen meals will be difficult to

resist, despite some dissatisfaction on the part of clients. However, the methodology presented in this paper may be used to make the transition easier by limiting disruption to both clients and service providers.

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