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An Analysis of Difficulties Encountered by Some Seventh and Eighth Grade Pupils in the Solving of Verbal Arithmetic Problems

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AN ANALYSIS OF DIFFICULTIES ENCOUNTERED BY SOME SEVENTH AND EIGHTH GRADE PUPILS IN THE SOLVING OF VERBAL ARITHMETIC PROBLEMS

by

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TABLE OF CONTENTS

LIST OF TABLES

CHAPTER I

INTRODUCTION

When one of the foremost authorities in the field of arithmetic calls for more research to be done along that line,[1] it is ample reason for choosing that field in which to do a thesis. There seems to be an unlimited supply of topics for research work, but the one that seems to have found the most interested following is that of problem solving. Probably that can be accounted for by the innumerable factors which enter in when one considers success or non-success in relation to the solving of verbal problems. One question which constantly came to the mind of the writer was why the children who can attain a grade level above their actual grade level in problem solving have such an antipathy toward it. Why also do they claim that they are unable to solve verbal problems?

It was questions such as those mentioned which gave birth to the idea of this study. The author has attempted to find out some of the difficulties

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1 Guy T. Buswell, Methods of Studying Pupils' Thinking in Arithmetic, Supplementary Educational Monographs, No. 70, Chicago University, 1949, 55-63.

which her students were having in the solving of word problems. During the study the following objectives were kept in mind:

1. To endeavor to discover the method which a child uses to solve a verbal problem.

2. To note the improvement in the pupils'ability to solve verbal problems during the school year 1953-1954 by stressing various methods of solution.

3. To find out to what extent it is possible to teach children who have no definite method of their own for solving verbal problems, a technique which would be helpful in solving them.

4. To form an opinion as to which techniques have proved most helpful.

The experimental study was carried on in two private school in the city of Chicago. One school is a private girls' school in which the writer teaches the eighth grade arithmetic. She has also been allowed to include the seventh grade in this experimental work. Together the number of the children in the two classes is twenty-five. The other school is a private boys' school in which the writer teaches the seventh grade arithmetic, and has included the eighth grade in her experiment. They totalled twenty-four boys in these two classes.

The children in both schools come from families whose financial status for the most part puts them in the upper income brackets. A few of the children come from homes where the income is average but sufficiently large to give them a comfortable living. All of the children have traveled widely in the United States and some of them in foreign countries. This has given them a very broad general knowledge. These experiences eliminate many difficulties that come from being unable to visualize material around which a problem centers.

Since the writer of this study has taught in these two schools for the past four years, the children who took part in the experimental study were well known to her. She had taught them all previously - except those who were new to the schools this year - either arithmetic or some other core subject. As the classes are very small, the children from the beginning feel right at home and at ease which is so necessary for this type of study as Buswell says, "The first requirement is that the teacher establish good rapport with the pupil so that he will express his thoughts freely and fully as he does his computations or his verbal problem."[2] Both schools have a family atmosphere which gives the child the knowledge that he belongs and has a very important part to play in his class. There is rarely any reticence on the child's part in making known the difficulty he experiences. The children know that the teacher is willing to help them individually, and if they hesitate to ask during a class they have ample opportunity to ask in private.

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2 _Ibid._, 58.

For the past three years in both schools, the children have been grouped for arithmetic, or taken individually, as their ability warrants. The basis for this grouping is usually dependent on three things: (1) the attitude of the child toward the subject. This can be gathered chiefly from the teacher who had him in the past. She notes whether he is eager to learn. Does he want to get by with as little work as possible? Is he a plodder and willing to do all that is assigned? (2) The grade level attained on one of the reliable achievement tests. The children in both schools are tested at least twice a year, and usually as often as every quarter. (3) The child's I.Q. and Reading Grade Equivalent.

In the girls' school the arithmetic period is fifty minutes in length daily. In the boys' school the arithmetic period is sixty-five minutes in length daily. No teacher is obliged to take her entire group or groups any day. If she wants to take one child individually to clear up a difficulty, she may; while the remainder of the children with whom she is charged have sufficient work to continue on their own. The children, when not with one of the arithmetic teachers, are at a supervised study. There the one in charge reports to the arithmetic teachers when one in her group is not doing arithmetic. The cause of his not doing arithmetic is discovered and remedied.

It is not obligatory for the teachers to assign homework. However, the teachers do assign homework, if not every night, at least three nights a week.

Given the freedom from usual classroom procedures in the arithmetic department of both schools, the writer was able to carry on her study - a study which would have been more difficult to make in a more conventional school.

The schools have found this grouping very satisfactory. The dull child finds joy in going to class and understanding what he is doing. This sense of achievement is a great incentive to harder work. The average child can work to his full ability without being held back by the dull child or constantly outstripped for the honors by the very bright child. It is the author's opinion that the keenest satisfaction is found on the part of the bright child. Arithmetic is presented to him as a challenge, something that is a little beyond his reach, but which he can attain by stretching. This goal when reached gives him a sense of achievement and widens his horizons to see further goals. He loves to conquer the difficult, and this can be presented to a group of children of this type in such a way that it will be a spur to work harder. On the other hand, if the same type of work were presented to a mixed group, what would be difficult to the very intelligent child would become a nightmare to the average child and impossible to the dull, thus encouraging the last two groups to build up the feeling which so many have, "I could never do arithmetic - I never knew what I was doing." Morton expressed this idea when he said:

> Not the least important of the findings...is the fact that problem assignments should be varied to suit the greatly varying capacities of the children to be taught. What is suitable for average children is too easy for the gifted and too difficult for the dull. Providing a uniform assignment for all is likely to result in a loss of interest on the part of the brighter children because the tasks are not a fit challenge to their abilities and an equal or greater loss of interest on the part of the duller because they fail to understand what is required of them. Only by making the assignment conform to the different capacities and interests of the pupils in the class can we hope to accomplish a maximum of benefit for all.[3]

The schools have found that not all teachers share these views on the group system. The teacher who is enthusiastic about his subject has been found to be completely in favor of it, but those who find no interest in the teaching of arithmetic have the strong objection that having groups at all different levels calls for much more work on the part of the teacher. That is very true. However, they are all unanimous in admitting that the actual teaching within the classroom is easier.

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3 Robert Lee Morton, Teaching Arithmetic in the Elementary School, Vol. II, Chicago, 1938, 459.

CHAPTER II

REVIEW OF RELATED LITERATURE

Problem solving could undoubtedly claim to trace its history from the beginning of man, for with the existence of man problems came into being. However, written records as to the nature and essence of problem solving in arithmetic came before the public eye shortly after Rice's first published report in 1902. Rice's report was not dealing with problem solving but was his explanation of how to measure the progress in arithmetic scientifically.[1] This gave an impetus to others in the field of arithmetic both in computation and problem solving. Though the ball began to roll with Rice, its progress was slow. In 1927 Buswell and Judd made a survey of all the literature which had been published concerning problem solving before 1927.[2] This survey presents a very concise summary of all articles written and gives a clear picture

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1 Charles Judd and Guy T. Buswell, Summary of Arithmetic Investigations (1926) Problem Solving, Chicago, 1927.

2 Ibid.

of the experimental work which had been done on problem solving up to that time.

Among the many studies that Judd and Buswell reviewed was Hydle and Clapp's[3]. Their work was concerned with sixth through ninth grades. They studied the following elements of difficulty as it concerns problem solving: (1) objective setting, (2) size of numbers, (3) unfamiliar objects, (4) arrangement of problems within a series, (5) non-essential terms, (6) experience and visualization, (7) the problem form of statement compared with the project form of statement, and lastly, (8) symbolic terms. They found that there was a great difference in the solution of problems with and without the first five elements. With the sixth and seventh element they could not find any significant difference;with the eighth there was a significant difference for grades four and five, but not for the higher grades. They concluded that their study gave evidence that pupils' thinking is merely a matter of visualization and that if we are to train the children to think, the problems that they are given must not be too easy to visualize.

Another study that was published before 1927 was the experimental work done by Lutes . He aimed to improve the problem solving ability of the

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3 L. L. Hydle and Frank L. Clapp, Elements of Difficulty in the Interpretation of Concrete Problems in Arithmetic, University of Wisconsin Bureau of Educational Research Bulletin, No. 9, Madison, Sept. 1927.

sixth grade classes in the large public schools in one of Iowa's cities. In judging his results[4] Lutes considered that the correlations between computation and reasoning low, being 0.439 ± .035 and 0.494 ± 0.033. He found that in the groups where the emphasis was laid on computational skill there were greater results to show in both computation and reasoning. However, in the beginning of his experiment he did not define problem solving ability as ability to reason but the ability to obtain correct mathematical answers.

It would seem that it might be well to keep in mind Lutes' stress on computational practice, for, according to an article by Habel there is a great deficiency. "Examination of many articles relating to arithmetic tests administered to college freshmen has inexorably emphasized the unhappy fact that thirty to forty percent of the freshmen in most sections of the country are inferior to the average eighth grade student in computational skill."[5] This could mean that the students of the eighth grade today are better trained in computational skills. However, to have this ability last for the next five years the present eighth graders will have to have a thorough understanding

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4 Olin S. Lutes, An Evaluation of Three Techniques for Improving Ability to Solve Arithmetic Problems, University of Iowa Monographs in Education, No. 6, Iowa City, June, 1926.

5 E. A. Habel, "Deficiencies of College Freshmen in Arithmetic: Diagnosis and Remedy", School Science and Mathematics, 50: June 1950 480-481.

of the computational skills, or they will be among the freshmen whom next year's eighth graders surpass.

In 1929 Neulen did a study with three groups which had been divided on the I.Q. basis. Though there was non-overlapping on the point of I.Q. in any of these groups, there was a decided overlapping in arithmetic ability. There were some pupils in each group who solved all twenty-five problems correctly and some in each group who solved none of the three-step problems correctly. He felt that it showed that homogenous grouping according to I.Q. did not necessarily produce homogeneity for problem solving. However, he did note that the per cent of pupils in the lower I.Q. groups which solved the problems incorrectly was greater than those in the higher groups; likewise, the per cent solving the greater number of problems correctly were to be found in the two higher groups.[6]

Hanna has been much quoted for the experimental work which he carried on endeavoring to find out which of three methods was the most successful in teaching children to solve verbal problems.[7] The methods were

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6 Leon Nelson Neulen, Problem Solving in Arithmetic, The Lincoln School of Teachers College, Bureau of Publications, Columbia University, New York, 1929.

7 Paul R. Hanna, Arithmetic Problem Solving, The Lincoln School of Teachers College, Bureau of Publications, Columbia University, New York 1929.

the dependencies, conventional and the individual method. He found out at the end of a six weeks' trial that the conventional method was inferior to the other two, but that statistics showed no significant difference between the dependencies and the independent methods.

A little less than ten years later, Sheerin carried on an experiment with forty-two classes from fourth through eighth grades in an attempt to evaluate which of the three methods was best for teaching problem solving.[8] She had the children use the conventional, equation and box technique. At the conclusion of this work she found that there was no technique which was significantly superior to another, but there was a significant trend in favor of the box technique.

Another means that has been employed to help children improve in problem solving is that of estimating answers. It is thought that if one could teach a child to give a reasonable estimate of the answer to a problem before the work is begun, it would eliminate much of the random guessing so harmful to real thinking. Dickey did an experimental study with a control group and an experimental group with which he worked for fifty days, training them to estimate answers to both problems and examples. The end of the experiment there was no significant difference between the control and experimental

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8 Ethel M. Sheerin, An Evaluation of Arithmetical Problem-Solving Techniques, Unpublished Doctor's Dissertation, New York University, N.Y.

groups as to their ability to solve problems. This experiment was done with children in the sixth grade and Dickey said that he felt there might be a significant difference if the experiment was tried with older students with more mature judgment.[9]

Dieterle was in agreement with Dickey as a result of an experiment which she carried on with a fourth grade. She felt that there was no significant difference in the students' ability to solve verbal problems and noted further that those of the class with lower intelligence found it confusing to have also the estimate of the answer besides doing the problem. At the end of the experiment they got more problems wrong than previously, while the brighter children made the better estimate and attained a higher score in problem solving.[10]

Where is the advantage of estimating answers if by itself it is not considered the way to improve problem solving? It has been said by Shane and McSwain that one of the "Desired Outcomes in Problem-Solving" is "(3) Are the children able to approximate a reasonable answer before computing to

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9 John W. Dickey, "The Value of Estimating Answers to Arithmetical Problems and Examples", The Elementary School Journal XXXV, Sept. 1934, 25-31.

10 Louise E. Dieterle, The Effect of Estimating Answers in Advance to Solving Word Problems in Arithmetic on Achievement in Arithmetic in Grade 4B, Unpublished Master's Thesis, Loyola University, Chicago, 1953.

find the correct answer?"[11] It would seem, then, that if the ability to estimate the correct answer is a desirable outcome, it evolves from a real understanding of the problem. The ability to estimate answers is one of the effects of the ability to solve problems and not the foundation upon which this ability may be built.

Though there is a wide divergence of opinion as to the best method of teaching problem solving, there is considerable unity on some stumbling blocks in the way of successful problem solving. The first of these is the ability to read problems and by that is not meant simply the general skill of reading, but the ability to read and comprehend detailed, technical reading. Parker gives an example of a mathematics teacher faced with a student who would say that he didn't understand a problem. The teacher would then ask if not any of the problem was understood. Then he would tell the student to look at his book and read the first few words. The pupil would be stopped with the question, "Do you understand that?" This method would go on for each phrase. The teacher affirmed that frequently this detailed attack was all that was necessary to enable a student to understand a problem.[12] Cole backs

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11 H.G. Shane and E.T. McSwain, Evaluation and the Elementary Curriculum, New York, 1951, 198.

12 Samuel C. Parker, Types of Elementary Teaching and Learning, Chicago, 1923, 379.

up this idea by stating that the difficulty with children is that they read problems as they would read fiction. They will read a problem quickly missing many of the details essential to solving the problem.[13] This same reading difficulty is stated by Morton,[14] Lutes,[15] Haynes,[16] John,[17] Brueckner,[18] and Spencer and Brydegard.[19]

Closely allied to the reading difficulty is the hurdle of vocabulary. It is necessary for authors when writing arithmetic text books to correlate the vocabulary of the books with the standard vocabulary list for each grade. This has not always been done, and in consequence, problems were frequently incorrectly solved by children. Yet those identical problems were easily solved by the same children when the vocabulary was changed to meet their level.

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13 Luella Cole, The Elementary School Subjects, New York, 1946, 373.

14 Mprton, Teaching Arithmetic, 454.

15 Lutes, An Evaluation, 12.

16 Jessie P. Haynes, Problems of a Supervisor of Arithmetic in the Elementary Schools, 2nd Yearbook, National Council of Teachers of Mathematics, 94.

17 Lenore John, "Difficulties in Solving Problems in Arithmetic," The Elementary School Journal, XXXI, Nov. 1930.

18 Leo J. Brueckner and Foster Grossnickle, How To Make Arithmetic Meaningful, Chicago, 1947.

19 Peter Lincoln Spencer and Marguerite Brydegaard, Building Mathematical Concepts in the Elementary School, N.Y., 1952, 311-312.

There are many who claim that the difficulty of vocabulary is one of the main factors for inability to solve problems. A few of these are Hildreth[20] Bell, Coston and Gates,[21] Gesling,[22] Stevenson,[23] Buswell[24] and Foran.[25]

Questions have arisen as to whether the difficulty of the problem depends on the process involved within the problem. Becker did a study along that line and came to the following conclusions:

1. The sequences of processes are elements in determining the difficulty of concrete problems.
2. The processes aside from their sequence do not appear to be elements which determine the difficulty of concrete problems. Subtraction-division and division-subtraction include the same processes but are at opposite extremes of the scale of difficulty.
3. Those concrete problems in which the first step is addition or or subtraction are generally less difficult than those problems in which the first step is multiplication or division.
4. Concrete problems in which the first step is division are

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20 Gertrude Hildreth. Learning the Three R's, Minneapolis, 1947,

21 Elizabeth Bell, Arleta Coston and Elizabeth Gates, "Solving Your Arithmetic Problems", National Educational Association Journal, Vol. 41, Nov. 1952, 477-8.

22 P.R. Stevenson, Difficulties in Problem Solving", Journal of Educational Research, XXV, May, 1932, 253-260.

23 Guy T. Buswell, Curriculum Problems in Arithmetic, The National Council of Teachers of Mathematics, 2nd Year Book, N.Y. 1927.

24 Thomas G. Foran, "The Reading of Problems in Arithmetic", Catholic Educational Review XXXI, Dec. 1933, 601-612.

decisively more difficult than those in which the second step is division.[25]

This experiment which Becker did was carried on with eighth grade pupils.

A similar study by Berglund-Gray reached the same conclusion. The position of a process in a problem is a factor in the degree of difficulty of the problem. "The ascending order of difficulty of the prosesses when used as the first step in the solution is as follows: addition, multiplication, subtraction, division."[26]

In the past ten years Brueckner has made some interesting observations on the teaching of problem solving. He states that in many cases the verbal problems turn out to be nothing more than a disguised drill; he, as others in the past, stresses that the problems which the children are given to solve should come from direct experience. He specifies, though, that problems containing vicarious experiences will be helpful to the child if from real experience the child knows the material with which the problem deals.[27]

The necessity of experience in the situations with which the word

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25 Fredericka M. Becker, Effect of the Processes and Their Order Upon the Difficulty of Arithmetic Problems, Unpublished Master's Thesis, University of Pittsburg, 1943, 31.

26 Gunborg Berglund-Gray, "Difficulties of the Arithmetic Processes", The Elementary School Journal, XL, Nov. 1939, 198-203.

27 Brueckner and Grossnickle, How to Make, 197.

problems deal was also a conclusion drawn by Lazerte [28] after several studies in problem solving. This same conclusion was drawn by White as a result of her experiment; she also found that the more steps there are in a problem the greater is the necessity for experience in the situation. [29] Bowman expressed the idea that interest in a problem would be built up if the children were given problems which represented genuine childhood situations. [30]

One of the more recent studies which should prove a valuable aid to the teachers of arithmetic was Sutherland's work in problem patterns. [31] She examined the textbooks from third to sixth grade inclusive, of four different series, analysing all the one-step, two-step and three-step problems and categorized them according to patterns. She found thirty-eight problem patterns which were divided as follows: sixteen of division, ten of subtraction,

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28 M.E. Lazerte, The Development of Problem Solving Ability in Arithmetic, Toronto, 1933, 136.

29 Helen M. White, "Does Experience in the Situation Involved Affect the Solving of a Problem", Education, LIV, April, 1934, 451-455.

30 Herbert L. Bowman, "The Relation of Reported Preference to Performance on Problem Solving", Journal of Educational Psychology, XXIII, April, 1932, 266-276.

31 Ethel Sutherland, One-Step Problem Patterns and Their Relation to Problem Solving in Arithmetic, Teachers College, Bureau of Publications, New York, 1947.

eight of multiplication and four of addition. As it was necessary to limit the scope of her work, she omitted certain types of problems in her classification. If these also had been examined, the number would be greater. Her idea is not to have the patterns as such presented to the children, but the teacher, knowing these patterns, will be able to give the children enough problems of one pattern to facilitate recognition of them. This should be a remedy for the confusion of mind that many children feel because of different types of problems.

The above named study must not be judged to be similar with a study by McEwen where the children were trained to recognize cues in order to decide how to work the problem.[32] Some of the common cues were words as: in all, both, difference between, share equally, times, needed, products. At the end of his experimental study he concluded that cues were more used by children of the younger grades, and that within each grade the children of a low rank in problem solving achievement were more affected by the verbal cues than those with a higher rank. As a result of his study he felt that it was not advisable to teach children to solve problems by means of verbal cues for they "interfere with sound progress in quantitative thinking."[33] This

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32 Noble Ralph McEwen, The Effect of Selected Cues in Children's Solutions of Verbal Problems in Arithmetic, Unpublished Doctor's Dissertation, Durham, 1941.

33 Ibid, 171.

study differed from Sutherland's work for though she pointed out cues that could be found in the different patterns, she does not advocate drill on them as an aid to problem solving.

In reviewing the literature concerned with problem solving no studies were encountered where the experimenter worked individually with the students having them do their work orally. It is in that way that this study differs from previous works. Many points of the study are similar to earlier experiments, but the manner of procedure differs.

CHAPTER III

METHOD OF PROCEDURE OF EXPERIMENTAL STUDY

It was decided that to have a basis for comparison for the students' improvement in problem solving, an achievement test in arithmetic would be given. The investigator administered to both schools within the first week of the opening of the school year, 1953-1954, the Stanford Achievement Test, advanced form Jm, the revised 1953 edition in both reading and arithmetic. The results are listed in Table II and III.

From the results of these tests it can be seen that the majority of the students have a higher grade level in problem solving than in computation. That seemed an important factor in considering what type of word problems to choose in compiling a test which would consist of twelve verbal problems to be given to each child individually.

The problems which were chosen were selected on the basis that they presented a problem to be solved, and in all cases the computations needed were only the four fundamentals which should not be a stumbling block to the seventh and eighth graders. These problems were entirely new to them and it

TABLE I

AGE - GRADE - I.Q.

Name	Age: Sept. 1953	Grade	I.Q.[a]
Annie	13:2	8	113
Janice	13:2	8	127
Jody	13:0	8	116
Pat	12:3	8	134
May	12:11	8	104
Joanne	13:1	8	106
Mickey	14:0	8	96
Dotty	12:10	8	124
Cathy	12:1	8	112
Charlene	12:0	8	127
Eloise	13:7	8	125
Agnes	13:6	8	126
Eileen	13:0	8	109
Marie	13:0	8	101
Dot	13:4	8	111

a Otis Self-Administering Tests of Mental Ability: Form A

TABLE I

AGE - GRADE - I.Q. (Continued)

Name	Age: Sept. 1953	Grade	I.Q.
Candee	12:0	7	99
Teresa	11:4	7	107
Nonie	12:9	7	126
Penny	11:0	7	106
Jackie	12:0	7	121
Lindy	11:8	7	136
June	12:1	7	112
Lila	12:2	7	125
LIz	11:8	7	118
Susie	11:11	7	116
Dan	12:9	8	129
Paul	12:5	8	133
Gerald	13:3	8	104
Dick	12:6	8	122
Walter	12:9	8	108

TABLE I

AGE - GRADE - I.Q. (Continued)

Name	Age: Sept. 1953	Grade	I.Q.
Fred	14:4	8	111
Bud	13:4	8	95
Bill	13:9	8	101
Roy	12:9	8	118
Lester	13:11	8	99
Ted	14:3	8	95
Lloyd	13:4	8	90
Chris	12:5	7	94
Matthew	12:0	7	119
Henry	11:9	7	124
Ralph	12:9	7	109
Clark	11:4	7	117
Tony	12:2	7	126
Bob	12:3	7	129
Ed	11:1	6	132
Benedict	11:1	6	134

TABLE II

STANFORD ACHIEVEMENT TEST : READING

Name	Form Jm (Sept.) Comp.	Form Jm (Sept.) Vocab.	Form Km (May) Comp.	Form Km (May) Vocab.
Annie	10.4	10.4	8.2	10.4
Janice	12.9	12.4	12.8	12.4
Jody	10.4	10.6	10.6	11.6
Pat	10.4	12.9	11.7	12.9
May	12.8	11.6	11.7	11.9
Joanne	11.5	10.6	11.4	11.4
Mickey	8.1	8.0	6.3	8.4
Dotty	11.1	10.8	11.4	11.9
Cathy	10.8	9.3	8.5	10.6
Charlene	12.9	11.9	10.9	12.4
Eloise	12.9	12.7	12.9	12.2
Agnes	11.8	12.4	11.7	12.9
Eileen	11.5	10.4	10.9	10.6
Marie	8.1	7.6	7.9	7.7
Dot	7.2	10.4	8.5	11.9

TABLE II

STANFORD ACHIEVEMENT TEST: READING (Continued)

Name	Form Jm Comp.	(Sept.) Vocab.	Form Km Comp.	(May) Vocab.
Candee	6.2	6.3	8.2	6.5
Teresa	5.8	9.3	7.0	8.4
Nonie	9.3	8.4	8.5	8.8
Penny	9.8	10.1	9.0	9.6
Jackie	11.8	10.1	11.7	11.6
Lindy	12.8	12.2	12.5	11.9
June	8.9	11.9	10.9	11.4
Lila	12.1	11.6	12.0	12.2
Liz	9.8	10.8	12.0	11.1
Susie	10.1	10.4	11.4	10.6
Dan	10.4	11.9	11.3	11.3
Paul	12.1	12.7	11.6	12.2
Gerald	7.8	8.2	8.5	7.6
Dick	11.5	10.1	12.0	11.6
Walter	6.2	9.1	7.7	10.4

TABLE II

STANFORD ACHIEVEMENT TEST : READING (Continued)

Name	Form Jm (Sept.) Comp.	Vocab.	Form Km (May) Comp.	Vocab.
Fred	11.8	11.1	12.0	11.9
Bud	8.9	8.2	8.5	9.1
Bill	8.1	7.1	5.5	7.0
Roy	8.5	11.4	11.0	11.0
Lester	5.6	6.8	6.0	9.1
Ted	8.5	8.2	8.0	7.6
Lloyd	6.2	8.8	4.8	8.2
Chris	5.8	6.1	5.5	7.2
Matthew	8.9	10.4	7.1	7.6
Henry	9.3	9.8	8.9	9.1
Ralph	7.8	6.3	6.5	6.1
Clark	10.1	8.4	8.9	9.1
Tony	8.1	7.1	8.5	7.9
Bob	10.4	9.3	10.3	10.7
Ed	9.0	8.9	9.7	8.9
Benedict	10.4	7.7	9.7	8.7

TABLE III

STANFORD ACHIEVEMENT TEST: ARITHMETIC

Name	Form Jm (Sept.) Comp.	Reas.	Form Km (May) Comp.	Reas.
Annie	10.9	11.7	11.3	12.1
Janice	11.2	11.7	12.3	12.1
Jody	8.4	10.0	11.6	11.7
Pat	12.4	12.7	11.6	11.7
May	8.7	7.9	10.7	12.1
Joanne	9.9	9.7	11.0	10.3
Mickey	6.4	6.4	6.9	8.3
Dotty	9.0	9.7	10.7	11.1
Cathy	6.9	7.4	9.4	9.8
Charlene	8.2	8.5	8.7	11.1
Eloise	10.6	11.7	11.0	12.1
Agnes	10.9	10.8	11.3	10.0
Eileen	7.1	8.8	8.5	10.8
Marie	7.9	8.5	6.8	9.0
Dot	7.7	7.4	10.3	9.4

TABLE III

STANFORD ACHIEVEMENT TEST: ARITHMETIC (Continued)

Name	Form Jm Comp	(Sept.) Reas.	Form Km Comp.	(May) Reas.
Candee	6.8	7.7	7.7	7.1
Teresa	6.6	6.8	7.0	8.0
Nonie	6.6	6.6	6.8	7.3
Penny	8.1	6.8	9.4	9.8
Jackie	7.9	7.4	7.0	8.3
Lindy	8.4	8.5	12.0	12.7
June	9.3	9.3	11.3	11.1
Lila	8.4	8.1	8.1	9.4
Liz	7.2	7.4	9.1	9.4
Susie	7.5	8.1	7.9	7.1
Dan	10.6	10.8	9.9	12.9
Paul	12.1	12.7	12.3	12.7
Gerald	9.5	8.5	7.9	8.5
Dick	6.9	8.8	8.2	11.1
Walter	6.9	7.0	7.9	9.0

TABLE III

STANFORD ACHIEVEMENT TEST: ARITHMETIC (Continued)

Name	Form Jm Comp.	(Sept.) Reas.	Form Km Comp.	(May) Reas.
Fred	7.5	6.8	8.5	9.0
Bud	8.4	7.2	8.2	7.8
Bill	8.5	7.7	9.9	11.7
Roy	9.0	7.7	8.5	9.8
Lester	7.9	6.4	7.1	7.0
Ted	8.4	10.0	11.3	11.7
Lloyd	5.2	5.4	6.6	4.2
Chris	5.2	5.8	6.3	7.0
Matthew	8.2	6.6	7.3	8.5
Henry	6.9	6.1	7.3	8.8
Ralph	7.5	7.4	9.1	8.8
Clark	6.3	7.7	9.1	9.4
Tony	9.3	8.5	11.0	11.1
Bob	9.9	10.5	10.3	12.1
Ed	8.4	7.9	10.2	10.6
Benedict	8.6	8.8	9.2	10.0

would offer an opportunity to show how they tackled a new and strange problem. It was hoped that it would show if they had any method or if they were simply making random guesses.

If a child is given a problem as the following: "During a vacation Jim Hall worked in a store. He could buy anything in the store at a 20% discount. Find what a $1.25 box of fancy soap would have cost."[1] In spite of the wording which makes the problem, in reality it is not more than percentage computation. One can do the problem if one knows percentage; if not, one can not. The aim of this thesis was not to find out in what fundamentals the children were weak, but why they could not solve verbal problems. Also as many of the conventional type word problems easily found in many text books are often nothing more than "a disguised drill"[2] for some form of computation, it was judged better to take the problems from some other source. It was hoped that the problems which were chosen would give a clearer picture of the child's ability to think.

Both the schools open late in September, and after they had been underway for a month, the individual testing of each child was begun. The investigator felt that a clearer insight into the child's thinking could be gathered if

1 F.B. Knight, J.W. Studebaker, Gladys Tate, Study Arithmetics, Book 7, Chicago 1948, 156.

2 Brueckner and Grossnickel, How to Make, 450.

while doing the problems on the individual test, the child did his work out loud.

Another reason for this decision was that in doing the preliminary research necessary for this study it was found that many well known educators, for example, Osburn, Morton, Lutes, Hall and Wilson had done work in the field of problem solving stressing the analysis of the child's work or his answers, but apparently had not stressed having the children do their work orally.[3] The success of the study made by Bloom and Broder and their opinion that

> Until the educator knows and understands the relations between the solutions given by the students to academic problems and the thought processes which led to the solutions, he is unable to determine when and under what conditions such good habits are established. Present emphasis on accuracy of solutions undoubtedly gives a misleading picture about the nature of the student's thinking.[4]

also encouraged the author to have her students do their work orally. Buswell also praised the method of having students do their work orally as it would give insight into pupils' difficulties and thinking.[5]

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3 Hildreth, Learning, 787.

4 Benjamin S. Bloom and Lois J. Broder, Problem-Solving Processes of College Students, Illinois Supplementary Educational Monograph, No. 73, 1950, 3.

5 Guy T. Buswell, Method of Studying Pupils' Thinking in Arithmetic, Supplementary Educational Monographs, No. 70, 1949, 58.

Each problem of the test was typed separately on a slip of paper and handed to the child one at a time. He could ask any questions that did not have to do with the direct solving of the problems. One of the most current questions was in connection with problem two. "What are A and B?" The children were not sure if it was a mistake in typing or if it was something which they had not heard about.

When a child felt that he had solved a problem, he would tell the one conducting the test what his answer was, how he arrived at it and why he chose the method he did in solving the problem. All the conversation between the pupil and the one administering the test was being taken down on the wire recorder.

At first the wire recorder presented a difficulty as it was not thought that the children would speak naturally if they were aware that what they said was being taken down. As it was impossible to hide the recorder, it was left on the desk of the teacher in full view of all the pupils some two weeks before the testing, and from time to time it was put on during class, and to the amusement of the children they were allowed to listen to themselves after school. This made them lose their self-consciousness and they soon paid no attention as to whether the recorder was on or not.

As there was not a time limit, the student could spend as much time as he wanted over the problem. Some of the children took an hour and a half trying to work out the problems. This part of the thesis took the most time for the individual tests lasted from the middle of October to the end of November. While they were conducted, no special help other than what would ordinarily come up about work with word problems was given to the children.

As the testing was spread over such a long period of time, it was thought that perhaps the children who came toward the end of the period would have been told what to expect by the other children. However, there did not seem to be any evidence of it. Each child as he was given the test was asked as a favor not to mention what the problems were about to any companion. The investigator further explained to the child no mark would be given for the test and no report of it would go home, but it was being given simply for the investigator's knowledge. The children were most co-operative.

As the results of these tests are taken up in Chapter IV no more mention need be made of them here.

After the period of testing was passed, a class was organized for the purpose of problem solving. This class was held twice a week during the regular arithmetic time and it lasted for twenty minutes on each day. At that time both the seventh and eighth grades were taken together in both the boys' and the girls' schools. As the children had been told that they would be taught how to do all the problems on the individual test, one class period was devoted to each of the twelve problems. When doing the test many of the children were positive that they did not know how to do problems ten and eleven, and they particularly asked if they would be taught how to do them. These children were very eager to have the problem solving class.

At the beginning of the class the instructor would read one of the problems which had been given on the individual test. She would point out the almost general error - if there was one - as in problem one. "If two pounds of candy cost $1.20, what would 1/2 pound cost at the same rate?" The children made their mistake in reading the problem incorrectly. Instead of finding the price of a half pound, they found out the price of half the given quantity thus, finding out the price per pound.

When the instructor read the problem out loud, she asked the children what they were to find. One of the children whom she knew had done the problem incorrectly was called on to answer. The student said, "You are to find out what a half cost, so you divide the $1.20 by two." The child was told to listen carefully while the problem was read a second time and then asked again if he was sure what the problem was asking. The mistake was seen readily and the answer came, "They don't want half the price, but the price of half a pound so you would divide by four."

A minute or two of the period was spent discussing how the mistake was made from misreading the problem trying to impress on the children that that could be one source of their errors. Then similar problems were given with no particular respect as to the difficulty of the numbers, but care to have them the same general pattern. A few like these were: "Nuts sell for 4 pounds for $1.60. At that rate what is the cost of 1/4 pound?"

"Two dozen dolls can be bought wholesale for $48. What would one pay to buy only 1/2 dozen wholesale?"

"The price of ribbon is 90 cents for a bolt of three yards. At the same price what will be paid for 1/3 of a yard?"

For problem one the misreading was the only type of mistake made, but on problem two there was more than one type, but that mistake which the majority made was to divide by four rather than by five. The problem was: "Two men caught 60 trout. A caught 4 times as many as B. How many trout did B catch?" The words "4 times" caused the trouble for the first reaction was to divide 60 by 4, saying that A caught 45 fish and B caught 15. The children were asked what four times any number meant. They explained it was to multiply a number by four. Then, it was asked if the answer that B got fifteen fish was reasonable? Some said it was claiming that four times fifteen is sixty; while others saw the fallacy without seeing how to do the problem correctly. The problem was illustrated on the board showing A's four fish and B's fish and that altogether there were five. At that point the greater number of students saw that you would divide by five; but several of the duller ones could not comprehend the reason.

It was at this point that one boy interrupted to ask if he might explain the way that he saw the problem. He had worked it correctly on the individual test. His explanation was this: "I always see this type of problem like a card game. You are the dealer and the fellow you play with gets four cards while the dealer gets one, and that is one round; the next round it is the same thing. Now in each of the rounds that you deal, you have dealt out five cards. So the simple thing is to divide the total number of cards - which in this problem would be sixty fish - and you get twelve. That means you can go twelve rounds in the dealing, so the fellow who only gets one card each time will have twelve cards, and the one that got four cards each round will get four times twelve or forty-eight. Add the twelve and forty-eight together and it checks to be sixty."

Probably because all the children can play cards and do it often the explanation had an appeal to many of them and they seemed to have the idea clearly in mind. Then many problems of the same type were given. Some of these were: "May and Jean had 20 dolls together, but of that number May owned 4 times as many as Jean. How many belonged to each girl?"

"Fred and Dick pooled their supplies of marbles totaling seventy-two. Before they pooled them, Fred had six times as many as Dick. How many had Fred?"

After the original test problems were explained, on the average of five to eight other problems of the same type were given. These were not on slips of paper, but were read by the instructor. A different child was called on each time to explain how he had done the problem. For the problems where there was more than one way to work it, it would always happen that the children who had the right answer but had done it differently than the way that was being explained, would want to explain their way. They were allowed to do this.

To explain problem four, "A man drove 84 miles in 3 hours. At that rate how many hours will it take him to drive 126 miles?" It was taken as two separate problems. The first question was to ask how one would figure out how far a man could drive in one hour if in three hours he could travel eighty-four miles. Most of the children saw that it was a question of finding the average, which they linked immediately with division. Then the second problem was presented to them thus: if a man can drive twenty-eight miles in one hour, how many hours will it take him to cover a distance of 126 miles? To help the children see this, a road whose length represented 126 miles was drawn on the board and divided up into sections of twenty-eight miles each. That way they could picture the process of division. However, after that was explained a child asked if he might explain the way he saw the problem.

This child said that he could not see the problem as two problems but that it was all one. The total distance that had to be covered was 126 miles and the one thing that you know is that you are able to cover eighty-four miles in three hours; therefore you cut the 126 miles into groups of of eighty-four. It is contained only once in 126 miles; that once is equal to three hours. There is a remainder of forty-two. By putting the remainder over your divisor the fraction equals one-half. That isn't half an hour, but half of eighty-four miles. Thus it would take one-half the time it took to go the eighty-four miles, which is one and a half hours. The total length of time is four and one-half hours. As this method was shorter, it appealed to the more intelligent children of the class. The children that found it complicated could not see that the one-half remainder was equal to one and a half hours. They said that the time should simply be three and a half hours. When more problems similar to this one were given and the explanations were asked for, it was easily seen that the duller children held to the first explanation and did not venture to try the second way.

Besides calling on the children for correct explanations the children who got a problem wrong were called on to explain where they made their mistake' if they could explain it easily, then the instructor was able to see what the trouble had been. As for example in the sixth problem: "At the rate of $.35 for the first half mile and $.10 for each additb nal 1/2 mile, how much would it cost to ride 5 miles in a taxicab?" After the manner of doing the problem was explained a similar one was given. As with the others, the first problem to b8 given after the original was almost identical, as, "at the rate of $.45 for the first half mile and $.10 for each additional 1/2 mile how much would it cost to ride 6 miles in a taxicab?"

One child said that his answer was ninety-five cents and explained that it was forty-five cents for the first half mile, which left five miles at ten cents each; therefore, fifty cents and forty-five cents are ninety-five cents. As the child was reading the problem to explain his work, he stopped and said "I didn't do what they said. The problem gave the price per half mile and I took it for a mile." He stopped and thought and said, "Well, it should have been forty-five cents plus $1.10 or $1.55."

Then another child raised his difficulty, "I did the problem correctly, but I come out with the wrong answer and I can't explain why for I don't see it." He said this was what he did: "It is forty-five cents for the first half mile; that leaves five and one-half miles still to be paid for. There are eleven halves in five and one-half, so I multiplied it by ten cents and got an answer of fifty-five cents. That added to forty-five gives me $1.00." It was then explained to the child that when he multiplied eleven halves by ten cents that he took eleven halves of ten cents which was really multiplying the ten cents by eleven half miles and then dividing the answer in two. This was a common mistake.

When a child was able to explain her difficulty, the instructor noted that the child usually was able to get the very next problem. When a child could not explain how to do a problem or could not see what his difficulty was, then the instructor helped that child individually.

After a class period had been spent on each of the twelve original problems, the class time was given to problems of all different types. In each period there were always problems which were similar to the original twelve, but there were many others. There were mistakes frequently in the different types. If these mistakes were general, the problem was explained to the entire class, but if the mistake was on the part of one or two, it was cleared up individually for them.

At the close of April the problem solving class ended and the retesting of each child began. A test of twelve problems was made out which was thought to be similar to the original twelve problems. The question arose that these problems might be easier or more difficult and that there should be some basis of comparison between the problems for the last individual test with the problems used for the first individual test. It was decided that preliminary tests, one for each type of problem, consisting of five problems would be given to the freshmen in one of the large public high schools of the city. The first six tests were given to freshmen who were taking general mathematics; the other six tests were given to an algebra class. Copies of these tests are in the Appendix III. The first problem on each test was taken from the first individual test and the other four were thought to be similar to it.

After the tests were corrected the problems that rated the same number of correct answers as the first problem were chosen to be in the final test. For example in the preliminary test one; problem one had thirty-three correct answers; problem two had one correct answer; problem three had thirty-two correct answers; Problem four had thirty-four correct answers; and problem five had fourteen correct answers. It was decided that in the final test either problem three or four could be used for they seemed to be the same level of difficulty as problem one. For preliminary test six the differences in the answers were as follows: problem one had eight; problem two had none; problem three had twelve; problem four had three and problem five had seven. It seemed obvious that problems two and four were too difficult to be used while problem three was too easy. Problem five seemed close to problem one only varying by one correct answer; it was chosen for the final test. In this way all the problems for the final individual test were decided upon.

The individual re-testing took a much shorter time at the second testing. Perhaps this could be accounted for in that the child knew what to expect, hence, the slowness which characterized his encounter with an unfamiliar situation was gone.

The last step in the experiment was to administer to the children in both schools the Stanford Achievement Test, advanced form Km, the revised 1953 edition, in both reading and arithmetic. The results can be seen in the tables.

CHAPTER IV

ANALYSES OF INDIVIDUAL CASE STUDIES

Each individual is unique in creation, a masterpiece of God. That is why the investigator found that working with the child alone proved so interesting and the most attractive part of this study.

To explain the method the children used to solve word problems, it seems best for the most part to let them speak for themselves. In this chapter each explanation of a problem which is enclosed in quotation marks is the words of the child taken down by wire recorder. The name for each record is fictitious.

It was thought advisable to take the problems in order, giving for each problem any solutions which contained interesting factors whether of correct or faulty reasoning, although the latter case is more likely to be a stab-in-the-dark guessing. After the solutions for any problem from the first individual test are given, the solutions for the problem similar to it from the May test will be discussed. A general analyses of the children's solutions are shown in Table IV.

For the first problem, which read, "If two pounds of candy cost

TABLE IV

ANALYSES OF THE INDIVIDUAL SOLUTIONS

Problem	Computation	Reading	Reasoning	Correct	Didn't Try
1 a	0	24	3	18	0
1 b	1	9	2	33	0
2 a	1	0	37	7	0
2 b	2	1	21	21	0
3 a	0	0	33	11	1
3 b	0	0	19	24	2
4 a	10	0	19	14	2
4 b	0	0	17	26	2
5 a	0	0	33	9	3
5 b	3	0	14	18	10
6 a	1	0	28	16	0
6 b	2	0	16	27	0
7 a	0	6	23	11	5
7 b	3	3	10	27	2
8 a	2	0	34	9	0
8 b	1	0	21	20	3
9 a	1	0	32	10	2
9 b	0	0	30	7	8
10 a	0	0	45	0	0
10 b	7	0	23	6	9
11 a	1	0	35	9	0
11 b	0	0	23	18	4
12 a	0	0	37	8	0
12 b	0	0	4	36	5

$1.20, what would 1/2 pound cost at the same rate?" the most common mistake was dividing the total cost in half instead of finding the price of a half pound. Over half the group made this mistake, while the other solutions differed.

Lloyd's explanation was, "You change one-half to its per cent equivalent, fifty per cent, and that is your answer." He had no further explanation and the investigator could not see any reasoning behind what he had said.

After frequently re-reading the problem Bud said, "One pound is $1.20 and it says that a half pound is the same price, so the answer is $1.20.

Lester was very sure of his explanation which was, "I've got this one. There are sixteen ounces in a pound and you are given the price of two pounds and you want the price of a half pound, so that would be eight ounces. You multiply eight by five-tenths, which is your half pound. That comes out to be forty cents." Lester is always satisfied with any answer he gives if he can find some relation between words in the problem and something he knows, never thinking whether that is what is asked for or not! It seems that it is not a matter of reasoning but of recognition of terms with the lack of ability to know what to do with them.

Henry: "Well, you have two pounds and you cut that in half. That gives you a pound in each pile. Next you cut each of the piles in half, which gives four piles each of a half pound. Well, if you have four piles, divide the $1.20 by four and you have the price of a half pound, which is thirty. cents."

In the second test for the first problem only nine of the students made the mistake of dividing the total cost in half. There were, however, some varied solutions. The problem stated: "If two dozen oranges cost $1.80, what would a half dozen cost at the same rate?"

Jackie did not read the problem accurately as can be seen from her own words. "It's easy. They say at the same rate, so the answer is the same, $1.80."

Two children gave this for their explanation. "As you are to find out the price of a half dozen, that is equal to six. You divide the total cost by six and you find that thirty cents is the price for a half dozen." The reasoning for the problem was correct, but the mistake was in mixing the units with dozens which caused them to arrive at their solution.

Chris made his mistake in reading. "They want to find out how much one-twelfth of a dozen will cost. One-twelfth is equal to one orange and that would cost fifteen cents, for one dozen cost $1.80."

Bud worked his problem correctly this time but did it in a manner that was different from the others. "The price of two dozen or twenty-four oranges is $1.80. I divided by twenty-four and found out that the price of one orange is seven and one-half cents. As you want the cost of six of them, I multiplied six by seven and one-half and got forty-five cents."

All the other students did the work correctly and did it either by dividing $1.80 by four or dividing $1.80 by two, and then dividing ninety cents by two.

Very few of the students were able to work the second problem correctly. As it appears, the computation is not difficult, but the majority of the children did not understand how to arrive at the correct answer. They could tell what the problem was asking, but they did not see how to obtain their answer. The problem was: "Two men caught 60 trout. A caught 4 times as many as B. How many trout did B catch?" As was pointed out in Chapter III, the common mistake was dividing by four and saying that B caught fifteen fish. Several children were unable to figure out what A and B were. The investigator always told them that A and B were persons.

Paul did the entire test in twenty minutes. All his work is characterized by speed, and usually with a high rate of accuracy. He was sure when he knew a problem and did it immediately, and when he came to those he got wrong, he was just as sure they were wrong for he would say in giving his work, "I know this isn't right for the answer isn't reasonable, but I can't see how else to do it." He had no difficulty with problem two and his solution was: "A caught four fish and B caught one and altogether that would make five. So divide five into sixty and that is twelve which would be the number B caught. To prove my work I multiply four by twelve and that is forty-eight and add on the twelve and it comes out to be sixty. So it's right."

Susie kept puzzling over the problem and finally asked, "What is meant by B?" When that was cleared up, she worked the problem and gave the following explanation. "First I divide the number of fish that they caught into two parts, one part for each man. Therefore, one man catches thirty fish and the other catches four times that number which would be 120 fish." There was apparently no question in her mind about the reasonableness of her answer. She felt that she had done the problem correctly since her numbers fulfilled the condition of having one number four times the other.

Annie worked her problem as Dot whose solution was as follows: "First divide the sixty into two parts. Thirty is what A got; then divide the other thirty by four and the answer is seven and one-half, which is what B got." Dot did not seem to realize that she had accounted for only thirty-seven and a half fish when she gave her final answer. Nor did it seem strange to her that one could catch a half fish! As can be seen from her solution, she, too, was not clear on the idea that the number of fish which A had was four times that which B had.

Several children gave solutions which were identical with that of Mickey's. "If one had four times as many as the other then you simply multiply four by sixty and you get 240 for the answer." This explanation shows a complete lack of understanding of what was given in the problem. They miss the fact that sixty fish was the total amount caught. In their answer it is apparent that they did not realize that the number of fish which B caught would be less than those caught by A. They, too, fixed their attention on the one thought, "four times as many", but they did not see it in relation to the rest of the problem.

Janice's solution was not clearly expressed, but she did understand the problem. "B gets only one-fourth of what A gets. The only numbers in sixty which would work are forty-eight and twelve for twelve is one-fourth of forty-eight, therefore, that is how many fish that B caught. I can prove that it is right by adding the two numbers together and getting sixty." When she had finished her work, the investigator asked her how she got the forty-eight. She said, "You just know that those numbers are the ones that will work." She stopped and thought for a moment and continued, "You could get the same numbers by dividing the sixty by five, but I didn't. Does that matter?" She was given the assurance that she could solve the problems anyway she wanted and that there was no set way for doing them. Janice is a slow and very thorough worker. She spent over an hour and a half doing the test. Every problem she knew how to do, she would prove to be sure that she was right. Those that she could not do, she did not want to leave without success, but she had to stop knowing that there were some things which she did not understand about the problems.

Several children gave for their answers that A caught forty fish and B caught twenty. Some said that they could not explain it, but that they knew it was right, however, the following is Lester's solution: "If two men caught sixty trout and if A got four times as many as B, then I multiply four by sixty and get 240. Then divide that (240) by sixty and I get forty, so A caught forty and B caught twenty fish." This the investigator thinks is a perfect example of random guessing. He thought that the answer should be twenty and forty and there is apparently no connection between his explanation and what is asked for in the problem. His division is incorrect, and that is not because he couldn't do it correctly, but he felt that the answer should be forty for four would be too small, so he simply added a zero to his quotient. He could explain that he had no right to change an answer, but also he would defend what he had done on the principle that he needed forty. With the other children that arrived at the same answer the universal explanation was that they "just felt" that was the answer. One child said that forty was four times ten and when you subtract that from sixty you would get twenty.

Ralph: "I multiplied four by twelve and subtract the product from sixty and that is twelve which is what B caught." When the investigator asked him where he got the original twelve which he multiplied by four, he said, "I don't know, but I knew that was the number that would work, and it did."

When the second individual test for problem solving was given, there was a greater number of students who got the second problem correct, as can be seen from Table IV. The problem read: "Bob and Jim earned $3936 together. Jim's share of the money was three times Bob's. What did they both get?"

As on the first test several students made the mistake of dividing by three saying that since Jim's share was three times Bob's then you divide the total by three and the quotient, which is $1312, is Bob's share and Jim's is $2624, which is the difference between Bob's and the total amount.

Fred understood the problem as can be seen from his explanation, but his answer was incorrect owing to a computation mistake. "Jim's share is three-fourths of the total amount which is $3936, so his share is $2974.50. When that is subtracted from $3936, one finds Bob's share is $991.50."

A mistake in reading is evident from the solution given by Ted. "It says: 'What did they both get' and it tells you in the problem that they earned $3936, so that is the answer." Ted completely missed the fact that Jim got three times what Bob got.

The children who solved the problem correctly reasoned as Dotty, though only a few spoke of shares. Instead others used the words piles, parts or pieces. "There are four shares, so I divided four into the total amount and that was $984. That equalled Bob's share which I subtracted from the total amount and got $2952 which was Jim's share."

A few children solved the problem incorrectly in a manner similar to Dick. "I divided two into $3936 and I got $1968. Then I divided $1968 by three and got $656, which is Bob's share, and Jim's, of course, is the other $1968." No child seemed to worry over the $1312 not accounted for, nor the fact that according to their answers Jim's share was not three times as big as Bob's.

The remark made by many children as they came to problem three was, "The problems get harder as you continue the test." On the first test, problem three read, "If 3 1/2 yards of silk cost $21, what will 7 1/2 yards cost?" The difficulty in that problem is breaking down the total price so that the price of one-half yard can be determined. The greater number of children got the problem wrong for that reason, or from not knowing what to do as is shown from the following explanation.

Susie: "I have an answer that doesn't make sense. It says three and one-half yards cost twenty-one dollars and for my answer I have seventeen dollars. The investigator did not see how she got the answer, so she asked. "You subtract three and one-half from the twenty-one and it is seventeen," - she did not notice that she had made a computation mistake - "but it should be more than twenty-one for it doesn't make sense to have seven and one-half yards cost less than three and one-half." She spent more time on the problem and asked if she could do it over for she knew where her mistake was. She was told that she might. "I multiplied three and one-half by two and that was seven; so I multiplied two by twenty-one and that is forty-two dollars and that is the answer." She apparently forgot about the one-half that was not accounted for, but she was tired of working with the problem and was satisfied that this answer sounded reasonable.

Though Susie did not know how to obtain the price of the extra half yard, her answer differed greatly from the following as it was characterized by good sense.

Lester: "Three and one-half yards is worth twenty-one dollars, so I added the three and one-half and the seven and one-half and got eleven dollars. Then I added the twenty-one dollars and the eleven dollars and I have thirty-two dollars which is the total cost of seven and one-half yards of silk." It is all the same to Lester whether he adds yards to dollars or dollars to dollars, for by his way of doing the work he will come out with dollars anyway! He seems to follow the theory that if there are numbers in the problem, the answer is bound to come if you combine the numbers often enough! The interesting fact is that he assured the investigator each time that, "This is simple." "I've got this one!" and "Is this ever easy!"

However, there were others who could solve the problem, as June. "The first thing that I did was to find out that three dollars was the price for a half yard. I did that by dividing twenty-one dollars by seven halves. Then I multiplied my three dollars by the seven add one-half yards and the price was forty-five dollars."

Several children saw that they would have to find the price of a half yard, but they could not see clearly the relation of the price given for three and one-half yards to a half yard, and in consequence, they solved it in a manner similar to Jody: "I multiplied twenty-one dollars by two for that would be the price of seven yards. Then I added ten dollars and fifty cents, which is the price of a half yard, to my product, and I found that the total cost was fifty-two dollars and fifty cents." She explained that she got the ten dollars and fifty cents by dividing the twenty-one dollars in half to find the price of a half yard.

Lindy expressed her difficulty clearly! "This problem is awful!

I could do it if he only bought seven yards, but I can't find the price if he is going to buy that extra half yard." That was settled and she went to the next problem.

An explanation which was as brief as Lindy's but correct was that of Eloise. "Double twenty-one dollars and it is forty-two dollars, the price of seven yards. Divide twenty-one by seven and you have three dollars for a half yard. The total price was forty-five dollars."

Pat confused the price of a yard with that of a half and that caused her trouble. "Three and one-half and three and one-half are seven which equals forty-two dollars. Then I divide forty-two by seven and I have six dollars, the price of a half yard. I add that to the forty-two and my total cost is forty-eight dollars."

One last solution to this third problem shows inventiveness. This was given by Dan. "I can't do it the way it should be done, but I can figure out the answer. It is forty-five dollars." The investigator asked him to explain what he had done, if it wasn't the usual way. "Well, I drew three lines the same size and one line which would be just half the size of one of the others. Then I cut the three lines in half. That showed me I had seven parts to deal with and they sell for twenty-one dollars; then one part would cost three dollars. Then I drew the other lines and cut them in half, which would equal the seven and one-half yards. So when I went to count them up, I counted each half, as three, saying three, six, nine, etc. So I came to forty-five dollars."

The investigator asked him why he didn't just multiply and he explained that he could never figure out if he should multiply by fifteen halves or just by the fifteen, but that if he did it his own way he wouldn't make a mistake.

Between the first test and the last, Dan was taken for individual work in fractions, and his difficulty was cleared away. In May he solved the third problem, which was:" If 4 1/2 feet of rope cost $.45, what will 9 1/2 feet cost?" in the following way: "You find out the cost of one-half foot by dividing forty-five cents by nine and that is five cents. Next you multiply five by nineteen for there are nineteen halves in nine and one-half. That gives the total cost to be ninety-five cents."

Many of the students who solved the third problem correctly did it as Dan, but some differed.

Ted: "I first found out the price of one foot by dividing forty-five cents by nine halves. That was ten cents per foot. Then multiply ten cents' by nineteen halves and you come out with the total cost of ninety-five cents."

Paul: "I did it by proportion and the other way and they check. The answer is ninety-five cents. By proportion you say four and one-half is to nine and one-half as forty-five cents is to X. The X, which is the price, comes out to be ninety-five cents."

However, the impression that all got this problem should not be given. There were still repetitions of earlier mistakes. When the problem was incorrectly solved, it was always easy to see that the difficulty was in finding the price of the half yard.

Roy: "I can't seem to get this. First I add four and one-half and four and one-half and that equals nine feet which cost ninety cents. But now I have to find the cost of the half yard. It is one-fiftieth." The investigator stopped him to ask what it was one-fiftieth of. "It is of the forty-five cents. Oh, I don't know what it is, and I am stuck and don't know what to do next." He stopped working there. It seemed the more he worked with the numbers of the problem the more confused he got, and at the end of his work he couldn't even remember what he was looking for.

Benedict used decimals in his solution which he gave very briefly. "I divided four and five-tenths into forty-five cents and I got ten cents which is the price per foot; therefore, the answer is ninety-five cents."

The fourth problem of the test dealt with distance. It was: "A man drove 84 miles in 3 hours. At that rate how many hours will it take him to drive 126 miles ?" The difficulty which presented itself in this problem was that the children could not see how to establish a relationship between the eighty-four miles and the one hundred twenty-six. Some would start the problem correctly, but would not know how to carry it through to completion. The following is an example of that.

Cathy: " I divided three into eighty-four and that gave me twenty-eight and I got nine and one-third hours." She completely omitted work with the 126 miles which is evidence that there was little thought in her solution.

Another common error of this problem, previously pointed out in Chapter III, was that of not being able to interpret the fractions. As Agnes' work shows this, it is quoted here. "Eighty-four goes into 126 one and a half times; the one is equal to three hours and the one-half is equal to one-half hour. The total amount of time is three and one-half hours." She missed the idea that the one-half was equal to one-half of the three hours, or one and one-half hours.

Nonie made a computation mistake, but she had reasoned it correctly in spite of her feeling of insecurity. She said, "I don't get it." She re-read it again. "First I divided three into eighty-four and that went twenty-six times. So that meant he drove twenty-six miles each hour. Then twenty-six goes into 126 to find out how many hours it took him, but it comes out an uneven number, so that can't be right. I don't know what else to do so I guess this must be the answer. It took four hours and twenty-two minutes, but it seems like a funny answer to me." During the course of this experiment the investigator found that it was often characteristic of a girl to say that the answer was wrong and give as a reason that the number was uneven. One wonders if it is part of the nature of a girl to want things smooth and even. The boys never gave that reason.

Another example of this type of work was found in Penny's solution. "Well, first I multiplied and it gave me a funny answer, so I know that isn't right. Then I tried to divide, but it doesn't turn out even, so that can't be it.

I don't know what to do!" She had no need to add that last remark for she had given adequate proof!

Two of the children had no idea what to do with the fourth problem and would do nothing, saying, "I can't get this one." A higher percentage of the children made computation errors on this problem than on any previous problem.

Some solutions which gave the correct answer were as unique as some of those which were incorrect. The following solution is awkward and involves the boy in more work than is necessary.

Dick: " I added eighty-four and eighty-four and that makes 168. That is twice the amount of eighty-four, so then I subtract 126 from 168 and that gave me forty-two. I put forty-two over eighty-four and reduced it and it was equal to one-half, so that made one and one-half hours extra. Thus the answer is four and one-half hours."

The work of some children is characterized by brevity, as, for example, Eloise'. She does the greater part of the work in her head for she can see the relation of numbers clearly. Her explanation was as follows: "126 is one and a half times eighty-four, so it would take one and a half times as long, or four and one-half hours."

Pat gave the more common solution used by the children who did the work correctly. She said: "I divide three into eighty-four and get twenty-eight miles. Then I divided that into 126 miles and I get four and one-half hours."

The fourth problem on the May test was: "A ship can travel 112 knots in 4 hours. How long will it take to travel 280 knots?" The word knots had been used in many of the problems in the classes, but probably owing to absence one child did not know what the word meant. She, however, worked the probelm correctly.

Lindy: "I don't know what a knot is, but I just supposed that it must be some kind of a measure of distance, so I did the problem in just the way I would have done it had the word been miles. That is, I divided 112 into 280 and it goes in two and one-half times, therefore, that equals ten hours."

In spite of the many problems involving distance and time which had been done in the problem solving classes, it still presented the obstacle of seeing the relation between the number of knots traveled and the time that it took. Matthew is one that didn't see the relation clearly as can be judged from his solution: "I divided 112 into 280 and it went in two and one-half times. So I added the two and one-half on to the four hours and it gave the total time of six and one-half hours."

A clumsy way to do the problem is the following solution given by Marie." Divide four into 112 knots and you get twenty-eight knots per hour. I multiplied ten by twenty-eight and it comes out to be 280 and so the right answer is ten hours." The investigator asked her how she knew that she should multiply by ten. She answered that she had tried all kinds of numbers first to see which one would work, as multiplying twenty-eight by eight, by twelve, by nine and finally by ten and that was the number which gave 280

Most of the children solved their problem in the following way which is the solution given by Charlene. "I divided 112 knots by four and that was twenty-eight knots per hour. Then I divided twenty-eight into 280 knots and the time was ten hours."

Jody started her work correctly, but she lost her trend of thought in the problem. She gave the following solution: "112 knots will go into 280 two times and that was equal to eight hours. Then there was fifty-six knots left over which I put over 280 and that was one-fifth. So it took the ship two and one-fifth hours."

Most often in class when a problem was solved in an unusual way it would be done by Tony. He had his own way for almost every problem. This is his solution for the fourth problem. "Divide four into 112 and that is twenty-eight knots an hour. Then divide four into 280 and you get seventy. Add twenty-eight and twenty-eight and that is fifty-six, and it needs fourteen to make the seventy. Now fourteen is half of twenty-eight so that would be equal to two hours. Thus the total time would be ten hours."

The fifth problem ranks in difficulty with the second if one may judge by the number of correct answers obtained. One would expect a comparison as the reasoning in both problems is similar. The problem stated: "A man died leaving $1200 to be divided among his wife, his son, and his daughter. For every dollar the daughter got, the mother got $5 and the son $2. How much did the daughter get?" The most common mistake is dividing the total by seven and saying that the daughter's share is $171. Seemingly they even overlooked the cents which rightly go with the $171.

Dot spent approximately twenty minutes trying to do the problem and she filled a sheet of paper with numbers in that time. This was her explanation: "I multiplied $175 by five and then I multiplied $175 by two and I added the two products which gave a total of $1125. So the daughter got $175." The investigator asked her where she got the $175. She explained she had picked many numbers and tried multiplying them by five and then by two and adding the products, but none of them came out to be $1200, but that the closest to that was $175, so she thought that was what the daughter got. Dot did not realize that the total number of shares could be divided into $1200 to give her the daughter's share, but she did realize the relationship between the daughter's share and that of the mother and the son, though she did not know how to find it.

Eileen came out with the correct answer through long hard work. She is a very slow worker, and it took her over an hour and a half to do the test. She must have spent fifteen minutes on this problem. "It seems so simple, but I just can't seem to get it. I divide seven into $1200 and then I multiply the quotient by five and that gives me what the mother got and then I do it by two and that is what the son got, but then there is nothing left for the girl.

So that can't be right. There has to be some money left for the daughter - oh, maybe that's it! You add her share in and then you divide by eight. The daughter would get $150, which doesn't seem very fair, but if you add it to the son's and the mother's it checks, so perhaps it is right." Virginia went one step further than most of the children. She reasoned that when she had divided by seven that there was not any money left for the daughter, whereas the other children were not bothered about that.

Lindy was disturbed trying to explain her problem for she felt that it was involved, and that no one would understand her explanation. "This is most unexplainable; couldn't I just give you my answer?" The investigator said that she would like the answer, but that she would also like Lindy to explain how she did it for it didn't matter what method was used. "I didn't use any method. I pretended that I had the $1200 and I gave five dollars to one of my friends and then I gave two dollars to another friend and I kept one dollar for myself. That amounted to eight dollars. Now if I kept on doing that I would give to my friends and myself as many times as eight will go into $1200. So I divided by the eight and it means that I will do it $150 times, and the person who is only getting one dollar each time will have a sum of $150. Therefore, I think that is what the daughter got."

One of the children who did it most efficiently was Janice. She could explain it clearly, as is seen from the following: " I put eight into $1200 for if the mother got five times as much as the daughter and the son two times as much as the daughter, then it means that the daughter got one-eighth of the money and that is $150. Then I checked my work by multiplying $150 by five and by two and I added the products to $150 and the sum was $1200."

In the May test the problem was "A carnival wheel run by three girls brought in $1296. For every $5 Connie took in, Joan took in $4 and Betty $3. How much did each girl take in?" The mistake prevalent among the children was that shown in the work of Mickey. "I divide five into $1296 and I found that Connie got $259; then I divided 4 into $1296 and found that Joan had taken in $324. Then when I divided the total by three I found that Betty had taken in $432." There was no thought on the part of the children who solved the problem that way that the sum of the amounts they assigned to each girl did not equal the total amount, neither did they recognize that the one who was taking in the least came out with the greatest amount. It seems to be another example of guessing.

Ed's solution was the same as the one just given, but he went on: "This can't be right for it just doesn't make sense. If I am right I should be able to prove it and I can't." As he was about to give up, the idea came to him to divide the total by twelve and then multiply by each girl's share. He did it correctly and was quite elated to have found his error.

Several of the children had solutions similar to Penny's. "I divide the sum by twelve and I got $108. Then I multiply $108 by five and I got $540 which is Connie's; then I multiplied it by four and I got $432 and that is Joan's

So altogether they got: Connie, $540, Joan $432 and Betty $108." They evidently forgot that they had to multiply by three to find out Betty's share.

Chris' explanation showed little ability to carry through his reasoning. "I divided $1296 by twelve and I came out with $108. Since there were three girls I divided by three and came out with sixty dollars for each girl." He missed the point that the shares for the girls were not to be equal.

Many of the children solved it correctly as did Ted. "I added the five, four and three dollars for that was how much each girl got and that came out to be twelve. I divided that into the total and that came out to be $108. I multiplied that by what the girls each received to find the individual share, and that came out to be $324 for Betty, $432 for Joan and $540 for Connie."

Another solution that was correct was that of Bob's. He was the only child that did his with fractions. "Connie got five-twelfths of $1296 and that is $540; and Joan's was four-twelfths which is $432; Betty had three-twelfths and that is $324."

The sixth problem of the test seemed very simple; in the reading it was, but in the working it did not prove to be so. It read: "At the rate of $.35 for the first half mile and $10 for each additional 1/2 mile, how much would it cost to ride 5 miles in a taxicab?"

The mistake that Charlene made was evidently one of reading, judging from her own words. "I got $4.35 for my answer. I multiplied four by ten and that gave me $4.00 and then I added the thirty-five cents for the first mile and I ended with a total cost of $4.35." She had the mile confused with the half mile.

Another common mistake was that shown in the work of Cathy. "I added thirty-five cents and ten cents together for that would be the price of the first mile. As there were four more miles to be accounted for, I multiplied four by the forty-five cents and I got $1.80 for the total cost of the taxi-cab ride." She did not distinguish between the cost of the first mile and each successive mile.

It was worked correctly by Agnes who said, "There are ten halves in five miles. The first half is worth thirty-five cents and to find the cost of the other nine, I multiplied nine by ten cents and it was ninety cents. I add the ninety cents and the thirty-five cents and the total cost was $1.25."

One who did it correctly but did unnecessary work was Joanne. "As the first half mile cost thirty-five cents, I put thirty-five cents on my paper, and underneath it I listed a dime for each of the nine half miles. Then I added it all up and it came out to be $1.25".

Another mistake that was made by more than one child was forgetting that once they had added the thirty-five cents they no longer had that half mile to think about. Gerald shows that mistake in his solution. "It is thirty-five cents for the first half mile. Then it is ten cents for each additional half mile. There are five miles which means ten half miles, so ten times ten cents is one dollar and I add the thirty-five cents and the total cost of the ride is

$1.35."

The problem in the May test read, "If a car can be rented at $10 for the first hour, and $1.50 for each additional 1/2 hour, what does it cost to rent a car for 5 1/2 hours?"

Annie did most of her work without using pencil and paper. She wrote down the ten dollars and then said, "It is ten dollars for the first hour and that leaves four and one-half hours. At that rate it is three dollars an hour, so that would be three, six, nine, twelve, thirteen dollars and fifty cents and then I add on the first ten dollars and the answer is $23.50."

Gerald made a mistake in multiplying by fractions for he multiplied by nine halves instead of by nine. He said, "It is ten dollars for the first hour and then there are four and one-half hours left. That is equal to nine halves which I multiplied by $1.50 and it came out to be $6.75 and that gave me a total cost of $16.50."

Teresa's mistake could be accounted either to reading or forgetfulness. Her solution was, "You multiply four by one dollar and fifty cents and you get six dollars and fifty cents. It was ten dollars for the first hour so I added the two together and the answer is $16.50." She did not seem to notice that there was one-half hour not accounted for.

One mistake which is definitely a reading mistake was that of Candee's. She interpreted the problem to say that it was ten dollars per hour and only one dollar and fifty cents for a half hour. She gave her solution as follows: "I multiplied ten dollars by five and that is fifty dollars; I added one dollar and fifty cents and I got my total cost of $51.50".

"A woman had 30 oranges to sell at 3 for a nickel, and 30 to sell at 2 for a nickel. She sold them at 5 for a dime. How much did she lose?", was the seventh problem given on the first test. The greatest difficulty was that many of the children forgot what they had read once they had finished reading the problem. This problem was re-read more than any of the others. Several of the children simply did nothing, saying that they did not know what the problem was asking for. One example of this state of confusion was the solution of Marie's.

"I don't know what they are asking for. What are they looking for - their money? I can't explain this problem for it is all mixed up. You divide two into thirty and you get fifteen and then you divide three into thirty and you get fifteen, no, I mean ten. Add them together and that gives you twenty-five. Subtract the ten cents which is how much she sold five for and you get fifteen cents and that is how much she lost."

Another way of mis-reading the problem is shown in the solution given by Annie. "She sold them at three for a nickel and two for a nickel which is the same as saying that they were sold five for a dime and so of course she doesn't lose, but she breaks even."

The problem was correctly solved by Paul who gave the following solution. "First you find out the cost of the oranges which were sold two for a nickel and then the amount she made at selling them three for a nickel. Then find out how much she made by selling them at five for a dime. Subtract the sum of the first two costs from the third and you find out that she loses a nickel."

There were no other kinds of solutions for that problem. It seemed that those who could read it and saw clearly what it was asking for, solved it as Paul, otherwise their work was a state of confusion.

In the May test the problem read: "A man had 40 candy bars to sell at 2 for 15 cents, and 60 to sell at 3 for 10 cents. He sells them at 5 cents each. How much does he gain or lose?"

The number of children who did this problem correctly was double the number that had done the problem of the oranges correctly.

Roy: "I divided forty by two and that left twenty bars for fifteen cents each, or a total of $3.00. Then I divided sixty by three and that is twenty to sell at ten cents each or a total of $2.00. However, he sells the forty and sixty, which is 100, at five cents each, so that equals $5.00. So there was no gain or loss."

Some of the children made a computation mistake and said that the first half of the problem totaled six dollars and thus they came out with a loss of a dollar.

Cathy's solution showed a reading mistake. "I took half of forty and that is twenty which I multiplied by fifteen cents and I got $3.00. Then I took one-third of sixty and I got twenty which I multiplied by ten cents and I got $2.00, so the amount that he made was $5.00." She completely missed the last part of the problem.

Another one who missed the point of the problem was Jackie. She said, "Well, it says that he finally decided to sell them all for five cents each and as there were one hundred bars to sell, he made $5.00."

The following problem the investigator thought was going to present no difficulty unless some child would make a computation mistake, but she was quite surprised to see how few of the children could do the problem correctly. The eighth problem was, "Helen's grades in 4 tests are 82, 76, 80 and 70. What grade must she get on the 5th test to raise her average to 80?"

One of the answers that showed how little a child understood the problem was that given by Walt. He said, "You add all the marks up and divide by four and it comes out to be seventy-seven. It is a cinch to get an eighty average at that rate for all she has to get on the next test is three per cent."

Some of the children who solved the problem correctly did it as June did. "I added all the numbers together, it was 308. Then I multiplied eighty by five and it is 400. So I subtracted 308 from the 400 and I found the difference to be 92 which is the mark she will need on the next test."

Some not knowing how to do it the way just named, but who had the right idea, did it in the following manner.

Pat: "I don't know how to find the right number so I kept adding different numbers on to my original four and then dividing the sum by five. I finally found that ninety-two would work and that I would get an average of eighty."

The problem in the May test the investigator now feels was more difficult for the children than the one on the original test. It reads: "Northwestern averaged 15 points for each home game. What was her score in the fifth game, if in the other four the scores were 18, 20, 6 and 19?"

Clark was one of the students who did the problem correctly. His solution was, "I first multiplied fifteen by five and that is seventy-five. Then I added all the points the team had already made and that equaled sixty-three. Then I subtracted that from the seventy-five and I found out that she needs twelve points."

Chris did not see how to reason through the problem as can be seen from his solution. "I added all the scores together and divided by four and that is fifteen and three-fourths, so they don't need to play the fifth game for they already have an average score of fifteen."

Chris' solution was similar to all the children who did not do the problem correctly. It was a general mistake to divide by four and so the conclusion that they came to was that the fifth game need not be played, or that they could get zero in the next game. Some, however, forgot about the fifth game and said that the average was fifteen and three-fourths.

"A freight train running 20 miles an hour is 120 miles ahead of an express train running 50 miles an hour. In how many hours will the express train overtake the freight?" is the ninth problem on the first test. It ranked among the most difficult problems on the test.

Bob had his own way of doing it. "The trains are twenty miles apart so to the 120 I add twenty miles and that is 140 miles, but then I subtract fifty and it makes them only ninety miles apart; then I add twenty again and it brings it to 110 miles apart, but I can still subtract fifty miles which makes sixty miles between the two trains. I've used two hours. Well, I add twenty to sixty and it makes eighty, but when I subtract fifty and add another twenty, they are just fifty miles apart. I subtract the final fifty and that means that the trains are now together and it took four hours." That manner of doing the problem could certainly be criticized on the basis of lengthiness, but one can easily see that the boy understood the problem and was able to reason it out. Once he was taught how to do the problem so as to save himself work, he abandoned his former method.

Likewise it can be seen from the following solution that the boy was not able to reason out how to do the problem.

Fred: "I divide twenty into 120 and I got six." There was a long pause. "Well, now that I got the six I don't know what to do with it; perhaps that is the answer."

Candee's solution was another example of the inability to see the problem clearly and to be able to work through to a solution. She said, " I multiply 120 by twenty and then I multiply 120 by fifty and then I subtracted the first product from the second and that gave me an answer of thirty-six, so, it took thirty-six hours." It seems to be a case of doing the most convenient thing with the numbers.

Though Benedict's solution was not correct, it does show that he reasoned out the problem. He made his mistake in thinking the two trains were coming toward each other instead of going in the same direction. His solution was: "They would be going seventy miles an hour together, therefore, it would take them one and five-sevenths hours until they meet."

An improvement can be seen from the chart in the number of children who got the ninth problem right on the first test compared to the May test. In May's test the problem read "A French Ocean Liner leaves New York sailing 40 miles an hour. Eight hours later an American Liner leaves sailing 80 miles an hour. In how many hours will they be together?"

Though the following solution will show that Roy did not know how to solve the problem, it can be noted that he knew more about boats than the person who made up the problem! "When is this going to take place?" The investigator asked him what he meant. "There is no boat in 1954 that can go eighty miles an hour if it is an ocean liner. If there was a boat, it would take two hours. The eighty is twice as fast as the forty, so it would take two hours to catch up." Roy saw only the relation between the two speeds and none between the speeds and the distance apart.

Dan worked it out briefly and accurately as can be seen from his solution. "The American Liner picks up forty miles every hour, so simply divide that into the distance apart from the French Liner, which is 320 miles, and it is eight hours."

One could hardly call the following solution an example of reasoning; rather it shows a lack of understanding of numbers.

Liz: "I multiplied eighty by eight and it was 640. Then I subtracted forty for that would give me 600. Then I crossed out the two zeros and the time would be six hours."

One of the common mistakes was to solve the problem as Susie did. She said, "In four hours the French Liner will go 320 miles. Four times eighty is also 320 miles, so it will take four hours." She, as the others, forgot that the French Liner was not docked but moving.

The common correct solution can be seen from what Tony said. "In the first eight hours the first liner has gone 320 miles and in another eight hours it will have gone 640 miles, while the second liner will cover 640 miles in eight hours. Therefore, it will only take eight hours for them to be together."

The most difficult problem on the test was the tenth which said, "A can do a piece of work in 4 hours, and B in 6 hours. How many hours will it take them if they work together?" No child was able to solve the problem correctly. There is no need in quoting their solutions for they all solved it in a similar manner which was adding the four and six and dividing the sum by two and giving for the final answer five hours; or multiplying four by six and giving an entire day for the answer; lastly, by subtracting four from six and then adding the remainder to four and giving six hours as the answer.

The investigator found that problems similar to the tenth were very difficult to teach and that the majority of the children were not able to grasp the idea. It was not surprising to find in the second test very few correct solutions. The mistakes were the same as those mentioned above. However, many of the children simply said they could not do it. The problem was "Jim mows a lawn in 5 hours while it takes Walter 8. If they mowed the lawn together, how long would it take them?"

The few children who solved the problem correctly did it in the manner that Tony did his. "Walter will do five-fortieths of the lawn in one hour and Jim will do eight-fortieths of the lawn. If they work together they can do thirteen-fortieths in one hour, therefore, it will take them three and one-thirteenth of an hour to do the entire lawn."

It might be a matter of interest to know that the investigator gave this problem to eighty-four high school students ranging from freshman to senior year and that from that group only two were able to do the tenth problem correctly.

The universal reaction to the eleventh problem which read, "In a fort there are sixty men, and enough food to keep the 60 men for 20 days. If 20 new men come and 40 of the first go, how many days will the food last?" was that they had no idea how to solve the problem. There were many efforts but these consisted mainly in seeing what numbers were easiest to combine. None of the solutions of that type showed any thought, nor could any student give a reasonable explanation for his work.

Janice worked the problem correctly and gave for her solution:" " The fort loses forty men and now has a total of forty men. I multiplied twenty by sixty to see how much food there was for one man. Then I divided the forty men into the 1200 days and came out with thirty days."

It was not difficult to teach that type of problem and the children could easily understand the solution; therefore, they did not have any great difficulties in doing the eleventh problem in the final test. It read, " If a stable has enough oats to last 10 horses 8 days, how many days will the oats last 4 horses?"

Many of the children solved the problem in a way that was similar to Janice's solution on the first test. However, Dotty did it a different way. "Ten is two and one-half times bigger than four so I took five halves times eight and I got twenty. That is the number of days."

Another solution which was correct but differant was that of Dan's "I set up a proportion - greater to least, and then multiplied extremes and the means. That brought four X to equal eighty, so the answer is twenty days."

The twelfth problem was difficult only in the ability to read it correctly. In consequence there were just two common solutions, one was right, the other wrong. The problem read, "A man sold a motorcycle for $80, bought it back for $100, and sold it for $120. Did he gain or lose on the transaction and if he did, how much?" The majority of the children said that he broke even, having neither a gain or a loss. A few who reasoned it correctly said that he made $100.

In the May test the majority of the children got the twelfth problem correctly solved. It read, "A watch gains at the rate of 30 seconds per day of 12 hours. If the watch is set at 12 noon, how fast will it be at six P.M. the same day?"

The answer given by Bill was that of most of the children. "In six hours it will gain only fifteen seconds, since six hours is half of twelve hours."

The children who did not get the problem right said that they could not understand it, or they did not know what they were to do.

CHAPTER V

CONCLUSION

When the investigator started this experimental study, she had in mind four aims. Now that the study has been completed, it would be well to see if those aims have been realized.

The first was 'To endeavor to discover the method which a child uses to solve a verbal problem.' It is easily seen from the different solutions which were quoted, that the children did not have a definite method for attacking their problems. The children who knew how to do the problems simply stated the relationships found within the problems and did the computation work accordingly. Those who could not work the problems, did any random manipulation with the numbers that came to them. There was one incident which clearly shows this. In the space of an hour's work a boy solved a problem incorrectly one way, and because he was not doing the problems in order, he forgot that he had solved that problem and did it again. This time the solution contained no reasoning either, and the manipulation of the numbers was the opposite of the first trial.

The second aim was 'To find out to what extent it is possible to teach children, who have no definite method of their own for solving verbal problems, a technique which would be helpful in solving them.' This was carried out during the problem solving classes and in the individual contacts with the children. During the classes the conventional method, diagram method, analysis, and the estimating of answers were presented to the children as helps toward solving the problems. To all these methods the investigator found two universal reactions. The brighter children would ask why they had to do the work required by a method when they simply could do the problem. The duller children would get so involved in the method that they could not remember what they were to do with the problem. For them, method seemed only a burden added to their state of confusion. However, there were some points taken from the different methods that many children, though not all, put to use, and the investigator saw this in the last individual test. The individual improvement from the first to last test may be noted in Table V.

The first idea which some of the children adopted came from estimating answers, and it was simply judging if the answer within their work was reasonable or not. They did not bother to estimate an answer before working the problem, but afterwards, they would look over their work to see if their answer was reasonable in the light of what was stated in the problem or what was required. This idea prevented some of the children from doing random guessing and giving solutions which were completely lacking in sense.

TABLE V

MEASURE OF ACCURACY ON INDIVIDUAL PROBLEM TESTS

Name	Test	1	2	3	4	5	6	7	8	9	10	11	12
Annie	I	X	X	C	C	X	X	X	X	C	X	C	C
	II	C	C	X	C	C	C	C	X	X	C	X	C
Janice	I	C	C	C	C	C	X	C	C	C	X	C	X
	II	C	C	C	C	C	C	C	C	C	C	C	C
Jody	I	C	X	X	C	X	C	X	C	X	X	X	X
	II	C	C	C	X	C	C	C	X	X	X	C	C
Pat	I	C	X	X	C	C	C	C	C	X	X	X	C
	II	C	C	C	C	C	C	X	C	C	C	C	C
May	I	C	X	X	X	X	C	X	X	X	X	X	X
	II	C	X	X	C	X	C	X	X	X	X	X	C
Joanne	I	X	X	X	X	X	C	X	X	X	X	X	X
	II	C	X	C	C	X	C	C	X	X	X	X	X
Mickey	I	X	X	X	X	X	X	X	X	X	X	X	X
	II	C	X	X	X	X	X	X	X	X	X	X	X
Dotty	I	X	X	X	X	X	X	X	X	X	X	X	X
	II	C	C	C	X	X	C	C	X	X	X	C	C

TABLE V

MEASURE OF ACCURACY ON INDIVIDUAL PROBLEM TESTS (Cont.)

Name	Test	1	2	3	4	5	6	7	8	9	10	11	12
Cathy	I	C	X	X	X	X	X	X	X	X	X	X	X
	II	C	X	X	X	X	X	X	X	X	X	X	X
Charlene	I	X	X	X	X	X	X	X	X	C	X	X	X
	II	X	X	X	C	X	X	X	C	X	X	X	X
Eloise	I	C	X	C	C	C	X	X	X	X	X	X	X
	II	C	C	C	C	C	X	C	C	X	X	C	C
Agnes	I	C	X	X	X	X	C	X	C	X	X	X	X
	II	C	C	X	X	C	X	C	X	X	X	C	C
Marie	I	X	X	X	X	X	X	X	X	X	X	X	X
	II	C	X	C	C	X	X	X	C	X	X	X	C
Dot	I	X	X	X	X	X	C	X	X	X	X	C	C
	II	X	X	C	C	X	C	X	X	X	X	C	C
Eileen	I	X	X	X	X	C	X	X	X	X	X	X	X
	II	C	X	X	C	X	X	C	X	X	X	X	C
Candee	I	X	X	X	X	X	X	X	X	X	X	X	X
	II	X	X	X	X	X	X	X	X	X	X	X	X

TABLE V

MEASURE OF ACCURACY ON INDIVIDUAL PROBLEM TESTS (Cont.)

Name	Test	1	2	3	4	5	6	7	8	9	10	11	12
Teresa	I	X	X	X	X	X	X	X	X	X	X	X	X
	II	X	X	X	X	X	X	C	C	X	X	X	X
Nonie	I	C	X	X	X	X	C	X	X	X	X	X	C
	II	C	C	X	X	X	C	X	C	X	X	X	X
Penny	I	X	X	X	X	C	X	X	X	X	X	X	X
	II	X	X	X	X	X	X	X	X	X	X	C	C
Jackie	I	X	X	X	X	X	X	X	X	C	X	X	X
	II	X	X	X	X	X	C	X	C	X	X	X	C
Lindy	I	C	X	X	C	C	C	C	X	C	X	X	X
	II	C	C	C	C	C	C	C	C	X	X	C	C
June	I	C	X	C	X	X	C	X	C	C	X	X	X
	II	C	C	X	C	C	C	C	C	C	X	C	C
Lila	I	X	C	X	X	X	C	C	X	X	X	X	X
	II	C	C	C	C	X	X	C	C	X	X	C	C
Liz	I	X	X	C	X	X	X	X	C	X	X	X	X
	II	C	X	X	X	X	C	C	X	X	X	X	C

TABLE V

MEASURE OF ACCURACY ON INDIVIDUAL PROBLEM TESTS (Cont.)

Name	Test	1	2	3	4	5	6	7	8	9	10	11	12
Susie	I	X	X	X	X	X	X	X	X	X	X	X	X
	II	C	X	X	X	X	X	X	X	X	X	X	X
Dan	I	C	C	C	C	C	C	C	C	C	X	X	C
	II	C	C	C	C	C	C	C	C	C	X	C	C
Paul	I	X	C	C	C	C	C	C	X	C	X	X	C
	II	C	C	C	C	C	C	C	C	X	X	C	C
Gerald	I	X	X	X	X	X	X	X	X	X	X	X	X
	II	X	C	C	C	C	X	X	C	X	X	X	C
Dick	I	C	X	X	C	X	X	X	X	X	X	X	C
	II	C	X	X	X	C	C	X	X	X	X	X	C
Walter	I	X	X	X	X	X	X	C	X	X	X	C	X
	II	X	X	C	X	X	X	C	X	X	X	X	C
Fred	I	X	X	X	X	C	X	C	X	X	X	X	X
	II	C	X	C	C	X	X	C	C	X	X	C	C
Bud	I	C	X	X	X	X	X	X	X	X	X	X	X
	II	C	X	C	X	X	C	C	X	C	X	X	C

TABLE V

MEASURE OF ACCURACY ON INDIVIDUAL PROBLEM TESTS (Cont.)

Name	Test	1	2	3	4	5	6	7	8	9	10	11	12
Bill	I	C	X	X	X	X	X	X	X	X	X	X	C
	II	C	C	C	C	C	C	C	C	X	X	C	C
Roy	I	X	X	X	C	X	C	X	X	C	X	X	X
	II	C	X	X	X	X	C	C	X	X	X	X	C
Lester	I	X	X	X	X	X	X	X	X	X	X	X	X
	II	X	X	X	X	X	X	X	X	X	X	X	C
Ted	I	X	X	X	X	X	X	X	X	X	X	X	X
	II	X	X	C	C	C	X	C	X	X	C	C	C
Lloyd	I	X	X	X	X	X	X	X	X	X	X	X	X
	II	X	X	X	C	X	X	X	X	X	X	X	X
Chris	I	X	X	X	X	X	X	X	X	X	X	C	X
	II	X	X	X	X	X	C	C	X	X	X	X	C
Matthew	I	C	X	X	X	X	X	X	X	X	X	C	X
	II	C	X	X	X	X	X	X	X	X	X	X	C

TABLE V

MEASURE OF ACCURACY ON INDIVIDUAL PROBLEM TESTS (Cont.)

Name	Test	1	2	3	4	5	6	7	8	9	10	11	12
Henry	I	C	X	C	C	X	X	X	X	X	X	X	X
	II	C	C	C	X	X	C	C	X	X	X	X	C
Ralph	I	X	C	X	X	X	X	X	X	X	X	C	X
	II	C	X	X	X	X	C	X	X	X	X	X	C
Clark	I	X	X	X	X	X	X	X	X	X	X	X	X
	II	C	C	C	C	X	C	X	C	X	X	X	C
Tony	I	C	C	C	C	X	C	C	X	X	X	X	X
	II	C	C	C	C	C	C	C	C	C	C	C	C
Bob	I	C	C	C	C	X	C	C	C	C	X	X	X
	II	C	C	C	C	C	C	C	C	X	X	X	C
Ed	I	X	X	X	C	X	X	X	X	X	X	C	X
	II	C	C	C	C	C	C	C	X	X	X	C	C
Benedict	I	C	X	C	X	C	C	C	X	X	X	C	X
	II	C	C	C	C	C	X	C	C	C	C	C	C

Many children held on to the idea of diagramingthe problems, and this was especially true in problems of distance. They did not seem to find it helpful always and therefore only used it with certain types of problems. In the final test the investigator noted that in doing both problems four and nine many children drew a diagram for themselves.

Lastly, there were some children who kept the basic idea of the conventional method. Though they did not state out loud that they wanted to know what the problem asked for, this seemed implied in the answers of the children who said, "Now the problem wants you to find..."

When the investigator saw that a few children clung to one idea, and a few to another while others tried ideas entirely different, it re-enforced her own thought that each child thinks differently and what will appeal to one child will not appeal to another. For that reason it seems that no one method will prove best for solving verbal problems.

The investigator feels that in problem solving, as with any other type of mental activity, there should be freedom accompanied by guidance . Those that are completely lost and know not where to turn may lean more heavily on the guidance than others, while those who have an idea where they are going will use the guidance only where it will help them to reach their destination more surely and quickly.

The third aim set down by the investigator was 'To note the improvement in the pupils' ability to solve verbal problems during the school year 1953-1954 by stressing various methods of solutions.' The investigator deliberately chose the words "to note" instead of to prove, for she knew that the experimental study was set up in such a way that it would be difficult to give proof of the improvement of the group with which she worked. Undoubtedly it could be questioned, when the investigator says that she feels there is an improvement, on the basis that this improvement might be owing to other factors which were not held constant during the experiment. That would be granted, and for that reason the investigator is only offering the scores on the Stanford Achievement tests and the individual tests for the interest of the reader and to let him draw his own conclusions.

However, there are a few points which mean more to the writer of this study in the line of improvement than statistical, significant proof. They are:

1. The general change in attitude of both boys and girls toward problem solving. By the end of February the dread of problem solving had been replaced by an actual liking for it. This was often shown by the number of requests the investigator received for extra classes in problem solving, and from personal remarks which showed the children now found pleasure in solving problems.

2. Many had accepted the idea that problems are not just a

matter of chance where one tries something with the numbers given hoping this will be the lucky time; they came to understand that problems represent real situations and can be reasoned out, even if they, do not see how.

The last aim was 'To form an opinion as to which techniques have proved most helpful.' This the writer thinks has already been answered in explaining the results of the second aim. However, it may be added that, though no one technique seemed more helpful than others, the persistent work with problem solving proves very helpful to the children.

In conclusion the investigator may state that she has learned from this experimental study:

1. The technique of working with the child individually and having the child do his work out loud pays rich dividends. In that way one can see what is the fundamental cause of the child's failure.

2. That there is no set technique which can be taught to all the children to assure success in problem solving.

3. That stress should be laid upon presenting a variety of problems to the children rather than on any particular method to be used.

4. That both the bright and the dull children are helped by work in problem solving.

5. That the stigma attached to problem solving can be removed if the teacher herself is convinced that it can be interesting.

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APPENDIX I

FIRST INDIVIDUAL PROBLEM SOLVING TEST

1. If two pounds of candy cost $1.20, what would 1/2 pound cost at the same rate?
2. Two men caught 60 trout. A caught 4 times as many as B. How many trout did B catch?
3. If 3 1/2 yards of silk cost $21, what will 7 1/2 yards cost?
4. A man drove 84 miles in 3 hours. At that rate how many hours will it take him to drive 126 miles?
5. A man died leaving $1,200 to be divided among his wife, his son, and his daughter. For every dollar the daughter got, the mother took $5 and the son $2. How much did the daughter get?
6. At the rate of $.35 for the first half mile and $.10 for each additional 1/2 mile how much would it cost to ride 5 miles in a taxicab?
7. A woman had 30 oranges to sell at 3 for a nickel, and 30 to sell at 2 for a nickel. She sold them at 5 for a dime. How much did she lose?
8. Helen's grades in 4 tests are 82, 80, 76 and 70. What grade must she get on the 5th test to raise her average to 80?

9. A freight train running 20 miles an hour is 120 miles ahead of an express train running 50 miles an hour. In how many hours will the express overtake the freight?

10. A can do a piece of work in 4 hours, and B in 6 hours. How many hours will it take them if they work together?

11. In a fort there are 60 men, and enough food to keep the 60 men for 20 days. If 20 new men come and 40 of the first go, how many days will the food last?

12. A man sold a motorcycle for $80, and bought it back for $100 and sold it for $120. Did he gain or lose on the transaction and if he did, how much?

APPENDIX II

SECOND INDIVIDUAL PROBLEM SOLVING TEST

1. If two dozen oranges cost $1.80, what would a 1/2 dozen cost at the same rate?

2. Bob and Jim earned $3936 together. Jim's share of the money was three times Bob's. What did they both get?

3. If 4 1/2 feet of rope cost $.45, what will 9 1/2 feet cost?

4. A ship can travel 112 knots in 4 hours. How long will it take to travel 280 knots?

5. A carnival wheel run by three girls brought in $1296. For every $5 Connie took in, Joan took in $4 and Betty $3. How much did each girl take in?

6. If a car can be rented at $10 for the first hour, and $1.50 for each additional 1/2 hour, what does it cost to rent a car for 5 1/2 hours?

7. A man had 40 candy bars to sell at 2 for 15 cents, and 60 to sell at 3 for 10 cents. He sells them at 5 cents each. How much does he gain or lose?

8. Northwestern averaged 15 points for each home game. What was her score in the fifth game, if in the other four the scores were 18, 20, 6 and 19?

9. A French Ocean Liner leaves New York sailing 40 miles an hour. Eight hours later an American Liner leaves sailing 80 miles an hour. In how many hours will they be together?

10. Jim mows a lawn in 5 hours while it takes Walter 8. If they mowed the lawn together, how long would it take them?

11. If a stable has enough oats to last 10 horses 8 days, how many days will the oats last 4 horses?

12. A watch gains at the rate of 30 seconds per day of 12 hours. If the watch is set at 12 noon, how fast will it be at six P.M. the same day?

APPENDIX III

TWELVE PRELIMINARY TESTS GIVEN TO TWO HIGH SCHOOL CLASSES

Test 1

1. If two pounds of candy cost $1.20, what would 1/2 pound cost at the same rate?
2. A box of six Easter eggs, each 1/4 pound, sells at $1.89. At the same rate what is the cost of a 1 pound Easter egg?
3. If two dozen oranges cost $1.80, what would a 1/2 dozen cost at the same rate?
4. Figs sell at 5 pounds for a dollar, at that rate what will 2 pounds cost?
5. If three pounds of nuts cost $.90, what would 1/3 pound cast at the same rate?

Test 2

1. Two men caught 60 trout. A caught 4 times as many as B. How many trout did B catch?
2. Bob and Jim earned $3936 together. Jim's share of the money was three times Bob's. What did they both get?
3. Betty and Jean bought Christmas presents amounting to forty. Jean bought seven times as many as Betty. How many did Betty buy?
4. Fred and Ralph have collected match covers of 96 different types. Fred collected eleven times as many as Ralph. How many did Fred collect?
5. Steve and his cousin walk 10 miles to school if you add their walks together. Separately Steve walks 4 times as far as his cousin. How far is that?

Test 3

1. If 3 1/2 yards of silk cost $21, what will 7 1/2 yards cost?
2. A costume containing 2 1/4 yards of cotton felt cost $6.12. At the same

rate what would a costume cost which needed 4 and 3/4 yards?

3. If 4 1/2 feet of rope cost $.45, what will 9 1/2 feet cost?
4. If 5 1/4 pounds of walnuts cost $.84, what does 1 pound cost?
5. Carmels are selling at 2 1/2 pounds for $2.25. What is the cost of 6 1/2 pounds?

Test 4

1. A man drove 84 miles in 3 hours. At that rate how many hours will it take him to drive 126 miles?
2. A ship can travel 112 knots in 4 hours. How long will it take to travel 280 knots?
3. If a ship can travel 96 knots in 4 hours, how long will it take to travel 240 knots?
4. Mother uses 24 apples in baking 4 pies. How many papples will she use to bake 14 pies?
5. A fast train can travel 450 miles in 5 hours. How long will it take to cover a distance of 720 miles?

Test 5

1. A man died leaving $1200 to be divided among his wife, his son, and his daughter. For every dollar the daughter got, the mother go $5 and the son $2. How much did the daughter get?
2. May, Ann and Sue divided 420 jelly beans among themselves. For every 2 which May received, Ann received 5 and Sue 7. How many did each receive?
3. A carnival wheel run by three girls brought in $1296. For every $5 Connie took in, Joan took in $4 and Betty $3. How much did each girl take in?
4. Three basketball teams gained a total of 240 points. On every 3 points of which Team A made, Team B gained 4 points and Team C 5 points. How many points did each team make?
5. An estate of $2400 was divided among three children. For every dollar Ross got, Dick got $5 and Bill got $2. How much did Ross get?

Test 6

1. At the rate of $.35 for the first half mile and $.10 for each additional 1/2 mile, how much would it cost to ride 5 miles in a taxicab?
2. The distance between two cities is 510 miles. At the rate of $5 for the first hundred miles and $1 for each additional 25 miles, what would be the

cost to go from one city to the other ?

3. Canoes are rented at $.45 for the first hour and $.15 for each additional 1/2 hour. How much would it cost to rent a canoe for 6 hours?
4. At the rate of $.45 for the first half mile and $.20 for each additional 1/2 mile, how much would it cost to ride 10 miles in a taxicab?
5. If a car can be rented at $10 for the first hour, and $1.50 for each additional 1/2 hour, what does it cost to rent a car for 5 1/2 hours?

Test 7

1. A woman had 30 oranges to sell at 3 for a nickel , and 30 to sell at 2 for a nickel. She sold them at 5 for a dime. How much did she lose?
2. A store has a supply of 30 $.40 cartons of 7-Up, (6 bottles per carton) and 25 $.35 cartons of Coca Cola. However the grocer sells them for seven cents a bottle. Does he gain or lose and how much?
3. A man had 40 candy bars to sell at 2 for $.15, and 60 at 3 for $.10. He sells them at five cents each. How much does he gain or lose?
4. A little boy has 60 ballons to sell at 3 for five cents, and 30 to sell at 2 for five cents. He sells them at 5 for $.10. How much does he gain or lose?
5. A grocer had 75 cans of peas to sell at 3 for $.57 and 120 cans of corn to sell at 2 for $.25. How much did he lose if he sold all at 15 cents a can?

Test 8

1. Helen's grades in 4 tests are 82, 80, 76 and 70. What grade must she get on the 5th test to raise her average to 80?
2. What must the temperature be on the seventh day of the week to make an average of 87 degrees, if on the other days the temperature was: 86, 92, 90, 80, 83, 87.
3. To maintain a 95% average what mark would Fred have to get in his fifth subject if his other marks were 98, 90, 94 and 98?
4. During the weekly spelling tests in April David received 72, 82, 78 and 84 What must he get in the last test to have an average of 82?
5. Northwestern averaged 15 points for each home game. What was her score in the fifth game, if in the other four the scores were 18, 20, 6, 19?

Test 9

1. A freight train running 20 miles an hour is 120 miles ahead of an express train running 50 miles an hour. In how many hours will the express overtake the freight?

2. A French Ocean Liner leaves New York sailing 40 miles an hour. Eight hours later an American Liner leaves sailing 80 miles an hour. In how many hours will they be together?
3. Bob's motorcycle going 30 miles an hour is 130 miles ahead of Al's which is making 60 miles an hour. In what time will Al overtake Bob?
4. An airplane takes off from Chicago traveling at 110 miles an hour. Five hours later another takes off traveling 150 miles an hour. In how much time will they meet?
5. In going to Seattle Mrs. Jones has a 500 mile lead on her son. She is driving at 40 miles an hour while her son is driving at 70 miles an hour. In how many hours will they meet?

Test 10

1. A can do a piece of work in 4 hours, and B in 6 hours. How many hours will it take them if they work together?
2. It takes Emmett 5 hours to mow a lawn, while it takes Walter 8. If they mowed the lawn together, how long would it take them?
3. Jean can do the house cleaning in 3 hours while it takes Betty 7 hours. How many hours will it take them if they work together?
4. Jim can trim the shrubs in his yard in 5 hours which is the same time it takes his father. How long would it take them if they worked together?
5. Ellen can make a formal in 2 hours, while Nancy is just learning and it takes her 8 hours. How long would it take if they worked together?

Test 11

1. In a fort there are 60 men and enough food to keep the 60 men for 20 days If 20 new men come and 40 of the first go, how many days will the food last?
2. If a stable has enough oats to last 10 horses 8 days, how many days will the oats last 4 horses?
3. A store of provisions would last 2100 men for 16 days. How long will it last 2800 men?
4. If 50 Boy Scouts packed enough food to last them 10 days on a camping trip, and only 30 of the boys take the trip, how long will the food last?
5. Usually a shipload of food will last a regiment of 5500 men 30 days. As the shipload arrived 1/2 of the regiment is released. How long will the food last the remainder of the regiment?

Test 12

1. A man sold a motorcycle for $80 and bought it back for $100 and sold it

for $120. Did he gain or lose and if he did, how much?

2. A watch gains at the rate of 30 seconds per day of 12 hours. If the watch is set at 12 noon, how fast will it be at six P.M. the same day?
3. If apples cost one-fourth as much as oranges and oranges cost twice as much as bananas, how much will twenty apples cost if the price of bananas is $.30 per dozen?
4. If Mary had $.25 more than she spent today she would have $.70. How much did she spend?
5. How much more will a dozen books cost at $6 a dozen than 12 pencils at 5 cents each?

APPROVAL SHEET

The thesis submitted by Bessie H. Chambers, R.S.C.J. has been read and approved by three members of the Department of Education.

The final copies have been examined by the director of the thesis and the signature which appears below verifies the fact that any necessary changes have been incorporated, and that the thesis is now given final approval with reference to content, form, and mechanical accuracy.

The thesis is therefore accepted in partial fulfillment of the requirements for the Degree of Master of Arts.

1/20/55
Date

Arthur P. O'Mara
Signature of Adviser