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## Economic Growth with Unlimited Supplies of Foreign Labor: Theory and Some Evidence from the GCC

Tarek Coury<sup>1</sup> and Mohamed Lahouel<sup>2</sup>

**Abstract:** We develop a modified version of the standard Solow and Ramsey growth models suited for countries with high proportions of foreign workers: firms hire foreign workers who are assumed to send a proportion of their wages as remittances. The paper shows that as the (foreign) supply of labor becomes more elastic, per capita income growth along the transitional dynamics converges to zero, the effect of TFP growth on per capita growth gradually disappears and growth in overall output converges to an AK-style model of growth. The model yields several testable predictions: Empirically, we consider the case of the states comprising the Gulf Cooperation Council and show that growth experiences of these countries are consistent with the predictions of this modified growth model. The model sheds light on certain causes of the natural resource curse as they apply to these countries and helps in explaining growth experiences of countries with high proportions of foreign workers.

**Keywords:** economic growth, economic development, labor surplus, unlimited supplies of labor, dual economies, GCC, natural resource curse, Dutch Disease.

**JEL Codes:** E10, E11, E13, E22, E23, E26, F43, F23, K31, O11, O43, O53

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# 1 Introduction

The structure of labor markets has important implications for economic growth: while countries in the OECD have labor markets that are relatively closed to foreign workers and labor laws that protect domestic workers (nationals and non-nationals), developing countries such as those of the Gulf Cooperation Council<sup>3</sup> place few restrictions on the hiring of foreign workers by domestic firms. With open labor markets, labor supply elasticity increases. In particular, the market for unskilled workers may be characterized by "unlimited supplies of labor," a term coined by Sir Arthur Lewis [15]. For example, the states comprising the GCC have acquired sizable proportions of foreign workers relative to their total workforce, in a bid to diversify their economies away from dependence on oil and natural gas revenue. By 2005, foreign workers as a percentage of the total workforce ranged from 19% for Oman to 90% for the UAE.<sup>4</sup> This increase in the foreign workforce is fuelled by retained earnings from hydrocarbon revenue.<sup>5</sup> This workforce originates predominantly from countries in East Asia and the Middle East and comprises mostly unskilled workers. For the period 1980 to 2005, growth dynamics in the Gulf are consistent with the following "stylized facts":<sup>6</sup> a) growth in overall (non-hydrocarbon) output is high, relative to the OECD; b) population growth is high and mirrors growth in output and therefore growth in output per worker is close to zero; finally c) population growth has little to no impact on growth in output per worker in Barro-like [5] growth regressions. This last point is made in a companion paper by Coury and Dave [12]: using data on GDP for the period 1980-2005, they show that population growth has a negligible impact on per capita income growth, when income is measured either as overall GDP or non-hydrocarbon GDP. This is in contrast to similar regressions for OECD countries (see for example Bassanini et al. [9]) and predictions of traditional growth models.

This paper proposes a model that explains these dynamics and explores related issues. Traditional models of growth (Solow [20]) and much of neoclassical growth theory typically assume that population growth is exogenous. In contrast, population growth is driven by economic motives in addition to fertility and mortality rates when countries allow large, unfettered flows of foreign labor. As a result, population growth is endogenous to the economic environment and reflects economic outcomes. This is the case for example for the states comprising the GCC.

Initially, we modify the neoclassical growth framework to allow for a labor supply curve which is perfectly elastic (section 2). Our growth model is inspired by the work of Sir Arthur Lewis [15]. While Lewis's analysis was motivated by issues of internal sectoral migration in developing countries, we use the assumption to analyze international labor migration to the

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<sup>3</sup>GCC states consist of Bahrain (BHR), Kuwait (KWT), Oman (OMN), Qatar (QAT), Saudi Arabia (SAU), and the United Arab Emirates (UAE).

<sup>4</sup>Source: UN International Migration Report [21]. For other GCC states, foreign workers as a proportion of total workforce was 89% for Qatar, 53% for Kuwait, 34% for Bahrain, 21% for Saudi Arabia for the year 2005.

<sup>5</sup>At the end of 2009, proved reserves of oil as a proportion of world total are 19.8% for Saudi Arabia, 7.6% for Kuwait, 7.3% for the UAE, 2% for Qatar, and 0.4% for Oman. Natural gas reserves as a proportion of world total are 13.5% for Qatar, 4.2% for Saudi Arabia, 3.4% for the UAE, 1% for Kuwait, and 0.5% for Oman. Source: 2010 British Petroleum Online Database.

<sup>6</sup>See tables in Appendix C.

GCC states.<sup>7</sup> We integrate the assumption of a perfectly elastic supply of foreign labor in a standard Solow [20] model and analyze the dynamics of income per worker. The results are in marked contrast with those of the conventional Solow growth model. First, we find that capital accumulation has no impact on growth in income per worker: when reservation wages are kept fixed, capital growth is accommodated one-for-one with growth in foreign labor and resulting income per worker remains unchanged. Second, TFP growth also has no impact on income per worker; just as with capital accumulation, greater TFP growth causes labor demand to rise and existing capital per worker to fall as wages remain fixed at all levels of production. Gains in output productivity offset this fall and resulting income per worker remains unchanged. Third, income per worker grows one-for-one with growth in foreign reservation wages. If the latter is positive, domestic firms hire fewer workers which results in positive income growth.

For the case of the GCC states, this first model predicts counterfactually that income per worker converges instantaneously.<sup>8</sup> We therefore modify the basic model of section 2 to allow for a labor supply curve for foreign workers that is upward sloping. This ensures that convergence takes place; it is the result of decreasing marginal returns to capital in the reduced-form production function. In a model with exogenous saving (section 5), we find that the speed of convergence of income per worker along the transitional dynamics can be made arbitrarily low, depending on the elasticity of the supply curve. As the elasticity becomes infinite, we recover the dynamics of the model in section 2. When supply becomes inelastic, convergence in income per worker accelerates but overall steady-state output falls. In section 6, we endogenize the saving rate of nationals in a Ramsey-type framework and analyze how equilibrium saving varies with remittance rates.

The literature on so-called dual economies, characterized by a division of economic activity between a formal (or modern) sector and an informal (or agricultural) sector, dates back at least to the seminal work of Lewis [15]. Workers migrate from the rural to urban sector as wages in the urban sector are higher. The migration may cause income per worker in the urban sector to fall and stagnate before eventually increasing.<sup>9</sup> Models of migration, such as those of Braun [10] consider optimizing models of labor migration where higher domestic wages tend to attract more workers.<sup>10</sup>

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<sup>7</sup>Lewis [15] recognizes the importance of labor migration: "In such an economy [with surplus labour] employment expands in a capitalist sector as capital formation occurs. Capital formation and technical progress result not in raising wages, but in raising the share of profits in the national income. The capitalist sector cannot expand in these ways indefinitely, since capital accumulation can proceed faster than population can grow. When the surplus is exhausted, wages begin to rise above the subsistence level. The country is still, however, surrounded by other countries which have surplus labour. Accordingly as soon as its wages begin to rise, mass immigration and the export of capital operate to check the rise."

<sup>8</sup>See growth regressions in appendix A of Coury and Dave [12].

<sup>9</sup>In chapter 21, Acemoglu [2] develops a reduced-form model characterizing Lewis's surplus labor dynamics. Under plausible conditions for a developing country, barriers to migration cause per capita income in the urban sector to fall before depletion of the rural workforce causes per capita income to either rise again or remain stagnant.

<sup>10</sup>See also chapter 9 of Barro and Sala-i-Martin [8]. We discuss this in more detail in section 3.

## 2 Perfectly Elastic Labor Supply

Throughout this paper, we assume the existence of a national household that owns and operates a (representative) domestic firm hiring foreign workers. For clarity of exposition, we limit our analysis to a Cobb-Douglas specification for the production function,  $Y_t = A_t K_t^\alpha L_t^{1-\alpha}$  where  $A_t$ ,  $K_t$  and  $L_t$  denote total factor productivity, physical capital and (foreign) labor in period  $t$ , respectively. Wages are given by an exogenous stream  $\{w_{r,t}\}$ . The wage rate  $w_{r,t}$  represents a real wage expressed in terms of the domestic consumption good and is the minimum wage required to attract a perfectly elastic supply of foreign workers in the domestic economy. This exogenous wage rate is therefore also the equilibrium wage rate in period  $t$ . The firm hires workers until the marginal product of labor equals  $w_{r,t}$ :

$$\frac{\partial Y_t}{\partial L_t} = (1-\alpha) A_t K_t^\alpha L_t^{-\alpha} = w_{r,t}. \quad (1)$$

Labor demand in period  $t$  is therefore given by:

$$L_t = A_t^{\frac{1}{\alpha}} \left( \frac{1-\alpha}{w_{r,t}} \right)^{\frac{1}{\alpha}} K_t. \quad (2)$$

A perfectly elastic labor supply schedule ensures that labor growth adjusts in a way that "accommodates" capital growth one-for-one:  $\frac{\dot{L}_t}{L_t} = \frac{\dot{K}_t}{K_t}$  (this follows from equation 2 and assumes that  $A_t$  and  $w_{r,t}$  are constant). Using the labor demand equation, one obtains the following reduced-form production function:<sup>11</sup>

$$Y_t = A_t^{\frac{1}{\alpha}} \left( \frac{1-\alpha}{w_{r,t}} \right)^{\frac{1-\alpha}{\alpha}} K_t. \quad (3)$$

The capital accumulation equation is given, as in the Solow growth model, by  $\dot{K}_t = sY_t - \delta K_t$ , where  $s$  is the saving rate<sup>12</sup> and  $\delta$  is the instantaneous rate of depreciation

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<sup>11</sup>A more general production function, displaying degree 1 homogeneity in capital and labor gives a marginal product of labor that is degree 0 homogeneous. The resulting reduced-form production function is also of the AK-type.

<sup>12</sup>The saving rate  $s$  is the "instantaneous" proportion of income that the national representative household saves toward capital accumulation. Foreign workers are assumed to send all of their wage income back as remittances. The present model will be amended in section 4 to allow for domestic spending by foreign workers. The qualitative features of the results in this section are robust to changes in this assumption.

of capital. In the aggregate, this model displays AK-style endogenous growth dynamics.<sup>13</sup> One obtains the following equation describing the dynamics of the growth rate of capital per worker from the labor demand equation:

$$\frac{\dot{k}_t}{k_t} = \frac{1}{\alpha} \left( \frac{\dot{w}_{r,t}}{w_{r,t}} - \frac{\dot{A}_t}{A_t} \right). \quad (4)$$

We combine equations (2) and (3) to solve for the level of output per (foreign) worker  $y_t = \frac{Y_t}{L_t}$  in the domestic economy:

$$y_t = \frac{w_{r,t}}{1-\alpha}. \quad (5)$$

The corresponding rate of growth in output per worker is expressed as follows:

$$\frac{\dot{y}_t}{y_t} = \frac{\dot{w}_{r,t}}{w_{r,t}}. \quad (6)$$

The conclusions reached here are in contrast to those of the conventional Solow growth model. For example, this model doesn't allow for gradual convergence in output per worker, as the latter instantaneously reaches its steady-state value when exogenous wages are fixed. In the conventional model, output per worker gradually rises until decreasing marginal returns to capital per worker cause output per worker to reach a steady-state level. In addition, the Solow growth model predicts that an exogenous increase in the population growth rate causes capital per worker (and therefore output per worker) to fall permanently to a lower steady-state level. In this modified model, the causation runs in the opposite direction: a fall in wages causes the representative domestic firm to hire more workers; this in turn causes the population growth rate to increase. This endogenous change in (foreign) population would also come about because of changes in TFP or capital formation. Indeed, the dynamic version of equation (2) takes the following form:

$$\frac{\dot{L}_t}{L_t} = \frac{1}{\alpha} \left( \frac{\dot{A}_t}{A_t} - \frac{\dot{w}_{r,t}}{w_{r,t}} \right) + \frac{\dot{K}_t}{K_t}. \quad (7)$$

We solve for the capital accumulation equation to obtain  $\dot{K}_t / K_t$ . We know that  $\dot{K}_t = sY_t - \delta K_t$  and  $Y_t$  takes the reduced-form in equation (3). We get the following expression for the capital accumulation equation:

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<sup>13</sup>This point is not new. A Lewis-type [15] labor supply has been analyzed in similar contexts by Marglin [17], Dixit [13], and Braun [10], among others. It also mirrors the effects of capital accumulation in an economy with perfect capital mobility. See Barro and Sala-i-Martin [8] pp. 163.

$$\frac{\dot{K}_t}{K_t} = sA_t^{\frac{1}{\alpha}} \left( \frac{1-\alpha}{w_{r,t}} \right)^{\frac{1-\alpha}{\alpha}} - \delta. \quad (8)$$

From equations (7) and (8), we get:

$$\frac{\dot{L}_t}{L_t} = \frac{1}{\alpha} \left( \frac{\dot{A}_t}{A_t} - \frac{\dot{w}_{r,t}}{w_{r,t}} \right) + sA_t^{\frac{1}{\alpha}} \left( \frac{1-\alpha}{w_{r,t}} \right)^{\frac{1-\alpha}{\alpha}} - \delta. \quad (9)$$

This equation expresses population growth as a function of exogenous processes and parameters in the model. An (exogenous) fall in the rate of wage growth causes domestic firms to hire more workers so the existing stock of capital per worker falls. This results in a negative relationship between output per worker and population growth. But as equation (9) illustrates, other factors cause population growth to vary. For example, higher TFP growth results in higher growth rates of labor causing existing capital stock per worker to fall. The gains in productivity offset the fall in capital per worker and imply that output per worker is not affected, as seen in equation (5).<sup>14</sup>

In the absence of shocks to the growth rates of TFP or wages, the stock of foreign labor accumulates linearly with the capital stock. Population growth resulting from capital accumulation however does not impact output per worker. Using equation (3) one can compute the growth rate of overall output:

$$\frac{\dot{Y}_t}{Y_t} = \frac{1}{\alpha} \frac{\dot{A}_t}{A_t} - \frac{1-\alpha}{\alpha} \frac{\dot{w}_{r,t}}{w_{r,t}} + sA_t^{\frac{1}{\alpha}} \left( \frac{1-\alpha}{w_{r,t}} \right)^{\frac{1-\alpha}{\alpha}} - \delta. \quad (10)$$

Notice that while a fall in the growth rate of wages causes output per worker to fall, overall output rises. An increase in the growth rate of TFP causes an increase in the growth rate of overall output while output per worker is unaffected. Finally, in the absence of shocks to growth rates of TFP or wages, overall growth in output depends on the sign of the term

$sA_t^{\frac{1}{\alpha}} \left( \frac{1-\alpha}{w_{r,t}} \right)^{\frac{1-\alpha}{\alpha}} - \delta$ . If it is positive, overall output and capital accumulate. If it is negative, capital decumulates, the population shrinks but output per worker remains constant. Sufficiently low wages will ensure capital accumulation. Letting  $n_t = \dot{L}_t / L_t$ , we can express overall output growth as:

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{w}_{r,t}}{w_{r,t}} + n_t.$$

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<sup>14</sup>Testing for the effects of TFP on growth in the GCC is beyond the scope of this paper. In a carefully researched paper, Abu-Qarn and Abu-Bader [1] consider growth dynamics for a group of MENA countries. They conclude that "the analysis of the sources of growth shows that for the selected MENA countries, the role of TFP in determining economic growth is insignificant and often detrimental." While implications of our theoretical model are consistent with their empirical results, the underlying mechanism for explaining growth in these MENA countries may be very different.

Note that in addition to responding to growth rates of TFP, population growth increases with the level of TFP. As TFP increases, capital accumulation accelerates which in turn causes population growth and finally overall output growth to rise to new levels. This analysis mirrors the dynamics of an AK-style growth model.

### 3 Labor Markets and Firm Optimization

The introduction of a perfectly elastic supply of labor in an otherwise neoclassical economy alters substantially the conclusions of the Solow growth model. The model helps in explaining why GCC states have such low growth rates in per capita income despite massive oil and gas revenues.<sup>15</sup> The model however counterfactually predicts that these countries' per capita income converges "instantaneously".

In this section, we allow foreign labor supply to be upward-sloping. The assumption implies convergence because the reduced-form production function displays decreasing marginal returns to capital. As the elasticity of labor supply increases, growth rates in income per worker along the transitional dynamics tend to zero.<sup>16</sup> The dynamics in terms of overall capital and output are similar to models featuring a work/leisure tradeoff (Cass [11], Koopmanns [14]).

As before, we assume that output requires physical capital and labor and production can be described using a Cobb-Douglas function:  $Y_t = A_t K_t^\alpha L_t^{1-\alpha}$ . The firm's flow of profits is given by:

$$\pi_t = A_t K_t^\alpha L_t^{1-\alpha} - (r_t + \delta) K_t - w(L_t) \cdot L_t. \quad (11)$$

We assume that the domestic firm faces the following inverse labor supply schedule of foreign workers:

$$w(L_t) = w_{\pi} L_t^\lambda \text{ where } \lambda > 0, w_{\pi} > 0, \quad (12)$$

and where  $w(L_t)$  is the reservation wage for the marginal foreign worker when  $L_t$  workers have already been hired. As  $\lambda$  tends to zero, labor supply becomes perfectly elastic and the reservation wage rate is  $w_{\pi}$  for all foreign workers.<sup>17</sup> The firm maximizes its profits, taking the interest rate and wages as given. Optimality conditions<sup>18</sup> for this optimization problem can be expressed as follows:

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<sup>15</sup>See tables in appendix C.

<sup>16</sup>The dynamics of the limiting case are discussed in section 2.

<sup>17</sup>The elasticity of labor supply is  $\varepsilon = \frac{\partial L / L}{\partial w / w}$ . Using the functional form of inverse labor supply in equation (12),

one obtains  $\varepsilon = \frac{1}{\lambda}$ .

<sup>18</sup>First-order conditions are necessary and sufficient since the profit function is strictly concave in capital and labor for all  $\lambda \geq 0$ .



$$A_t \alpha K_t^{\alpha-1} L_t^{1-\alpha} = r_t + \delta, \quad (13)$$

$$A_t (1-\alpha) K_t^\alpha L_t^{-\alpha} = w(L_t). \quad (14)$$

Using equations (11,12,13,14) one obtains the usual zero economic profit condition:

$$\pi_t = 0. \quad (15)$$

Using equations (12,14), one can solve for the level of labor demanded:

$$L_t = A_t^{\frac{1}{\alpha+\lambda}} B_t^{\frac{1}{\alpha+\lambda}} K_t^{\frac{\alpha}{\alpha+\lambda}} \text{ where } B_t = \frac{1-\alpha}{w_{rt}}. \quad (16)$$

One can then solve for the production function in reduced-form:

$$Y_t = A_t^{\frac{1+\lambda}{\alpha+\lambda}} B_t^{\frac{1-\alpha}{\alpha+\lambda}} K_t^{\frac{1+\lambda}{\alpha+\lambda}}. \quad (17)$$

Note that as  $\lambda$  converges to 0 (infinite labor supply elasticity), we retrieve the same AK-type production function of section 2.

## 4 Capital Accumulation and Remittance Outflow

In this section, we derive the reduced-form capital accumulation model when the supply of labor is upward sloping as expressed in equation (12). In addition, we allow foreign workers to remit a proportion of their wages back to their home country. First, we derive the capital accumulation as a function of factor prices:

$$\dot{K}_t = \pi_t + r_t K_t - C_t, \quad (18)$$

or, using equation (15),

$$\dot{K}_t = r_t K_t - C_t. \quad (19)$$

Here,  $C_t$  denotes consumption by the representative national household. We modify the equation by allowing a proportion  $(1-s_f)$  of wages earned by foreigners to be spent domestically while the rest is sent back as remittances.<sup>19</sup> The capital accumulation equation then takes the following form:

$$\dot{K}_t = r_t K_t - C_t + (1-s_f) w(L_t) L_t. \quad (20)$$

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<sup>19</sup>Data on monthly remittances to labor exporting countries dating back to 2003 are available on the World Bank's Migration & Remittances Data website. Select data on worker remittances paid from GCC countries can be found in the IMF Balance of Payments Statistics Yearbook and dates back to 1990. In 2004, remittances ranged from \$1.1 billion for Bahrain to \$13.5 billion for Saudi Arabia. They do not include the unknown but substantial remittances transferred through informal means.

This equation can be written as a function of consumption and capital only, using equations (12,16) and (20):

$$\dot{K}_t = w_{rt} \left(1 - s_f\right) A_t^{\frac{1+\lambda}{\alpha+\lambda}} B_t^{\frac{1+\lambda}{\alpha+\lambda}} K_t^{\alpha \frac{1+\lambda}{\alpha+\lambda}} + r_t K_t - C_t. \quad (21)$$

From equations (13) and (16), we get:

$$r_t + \delta = \alpha A_t^{\frac{1+\lambda}{\alpha+\lambda}} B_t^{\frac{1-\alpha}{\alpha+\lambda}} K_t^{\alpha \frac{1+\lambda}{\alpha+\lambda} - 1}. \quad (22)$$

Replace equations (12,16,22) in equation (20) to obtain:

$$\dot{K}_t = \left(1 - s_f (1 - \alpha)\right) A_t^{\frac{1+\lambda}{\alpha+\lambda}} B_t^{\frac{1-\alpha}{\alpha+\lambda}} K_t^{\alpha \frac{1+\lambda}{\alpha+\lambda}} - \delta K_t - C_t. \quad (23)$$

The national household is assumed as before to save a proportion  $s$  of domestic income. We then have  $C_t = (1-s)Y_t$  so equation (23) can now be written as:

$$\dot{K}_t = \tilde{s} A_t^{\frac{1+\lambda}{\alpha+\lambda}} B_t^{\frac{1-\alpha}{\alpha+\lambda}} K_t^{\alpha \frac{1+\lambda}{\alpha+\lambda}} - \delta K_t \text{ where } B_t = \frac{1-\alpha}{w_{rt}}, \quad (24)$$

and where  $\tilde{s} = s - s_f(1-\alpha)$  is the effective national saving rate. Equation (24) is our reduced-form capital accumulation equation. It is a function of  $K_t$  and underlying parameters of the model. Although no time-series data exist on the proportion of income sent back as remittances, migrant workers send back considerable sums of money.<sup>20</sup> Assuming that their saving rate is the same as the national population, we have  $s - s_f(1-\alpha) > 0$  since  $\alpha < 1$ ; this ensures that overall capital accumulates when  $K_t$  is sufficiently low.

We analyze two cases: first, we consider the dynamics of capital per worker from equation (24) where the saving rate is exogenous. We then consider a Ramsey [18] version of this model, where the saving rate  $s$  becomes an equilibrium decision by the national household. Throughout the paper, we assume that  $s_f$  is exogenous.

## 5 Exogenous saving

### 5.1 Steady-State and Convergence

In this section, we consider the dynamics of the capital accumulation equation (24) where the national household is assumed to save a proportion  $s$  for capital accumulation while  $s_f$  denotes the proportion of remittances from wages. A steady-state  $K^* > 0$  exists whenever the effective national saving rate  $\tilde{s}$  is positive. The labor supply schedule does not

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<sup>20</sup>See also Adams [3] and Richards and Waterbury [19] chapter 15 on the social impact of labor migration and remittances.

affect the minimum proportion of domestic spending required by foreigners to ensure that a positive steady-state exists. Indeed, as more workers are hired and for all  $\lambda$ , the size of remittance outflow relative to overall output remains constant, or:

$$\frac{(1-s_f)w(L_t)L_t}{Y_t} = 1-s_f(1-\alpha).$$

The dynamics of overall output and capital in this model are similar qualitatively to the Solow growth model when  $\lambda \in (0, +\infty)$ . As  $\lambda \rightarrow 0$ , decreasing marginal returns to capital in the reduced-form production function gradually disappear and resulting dynamics converge to those of an  $AK$ -style growth model. As  $\lambda \rightarrow +\infty$  labor is supplied inelastically: the number of workers hired converges to unity for any value of  $K_t$  (equation 16) and the overall production function takes the form  $A_t K_t^\alpha$ .

## 5.2 The Golden Rule

We solve for the saving rate that maximizes steady-state consumption by the national household  $C^*$ . We know that  $C^* = (1-s)Y^*$  where  $Y^*$  is a function of  $K^*$  and  $s$ . Foreign workers are as before assumed to remit a portion  $s_f$  of their wages. Solving for  $K^*$  using equation (24) and solving for  $s$  in the equation  $\frac{\partial C^*}{\partial s} = 0$  gives the following Golden Rule saving rate:

$$s_{GR}^* = \frac{\alpha(1+\lambda) + \lambda s_f (1-\alpha)^2}{\alpha + \lambda}. \quad (25)$$

As  $\lambda \rightarrow 0$ , the Golden Rule saving rate converges to unity: low wages at different levels of production ensure that capital accumulation is "cheap". As  $\lambda \rightarrow +\infty$ ,  $s_{GR}^*$  converges to  $\alpha + s_f(1-\alpha)^2$  with an effective saving rate of  $\tilde{s}_{GR}^* = \alpha[1 - s_f(1-\alpha)]$ . The latter corresponds to the usual Golden Rule (in a closed economy Solow setting) modified for remittances from the domestic economy.

## 5.3 Transitional Dynamics

In this section, we characterize the transitional dynamics of capital per worker in our modified growth model.<sup>21</sup> We re-write the capital accumulation (24) in intensive form. Note that  $\dot{k}_t/k_t = \dot{K}_t/K_t - n_t$  where  $n_t = \dot{L}_t/L_t$  is the rate of population growth. Replacing this in equation (24) and using equation (16) gives:

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<sup>21</sup>The dynamics of this modified Solow growth model for overall capital  $K$  are similar to those of the traditional growth model with exogenous savings and are therefore omitted.

$$\frac{\dot{k}_t}{k_t} = \tilde{s}A_t k_t^{\alpha-1} - (\delta + n_t). \quad (26)$$

Unlike conventional growth models, the population growth rate itself depends on the rate of capital accumulation, among other things and can be derived from equation (16):

$$n_t = \frac{\dot{L}_t}{L_t} = \frac{1}{\alpha + \lambda} \left( \alpha \frac{\dot{K}_t}{K_t} + \frac{\dot{A}_t}{A_t} - \frac{\dot{w}_{rt}}{w_{rt}} \right). \quad (27)$$

For simplicity, we will assume that TFP and wages are constant for the remainder of section 5. Population growth can then be re-expressed in intensive form as

$$n_t = \frac{\alpha}{\lambda} \frac{\dot{k}_t}{k_t}. \quad (28)$$

Solving for  $\dot{k}_t / k_t$  in equation (26) gives

$$\frac{\dot{k}_t}{k_t} = \left( 1 + \frac{\alpha}{\lambda} \right)^{-1} (\tilde{s}A_t k_t^{\alpha-1} - \delta). \quad (29)$$

The number of workers hired varies with the parameter  $\lambda$ . As  $\lambda \rightarrow 0$ , the growth rate in the number of workers is equal to the growth rate of overall capital. In this case, growth of capital per worker and output per worker converges to zero and we retrieve the dynamics of the perfectly elastic labor supply case of section 2. As  $\lambda \rightarrow +\infty$ , the number of workers hired converges to unity and transitional dynamics of capital per worker mimic those of the Solow growth model without population growth. Because of remittance outflows, the effective saving rate is lower than  $s$ .

## 5.4 Simulations

In this section, we use various parameter specifications and consider resulting transitional dynamics for capital per worker as expressed in equation (29). We re-formulate it to highlight the importance of wages paid to foreigners. The functional form of the labor supply curve allows us to express wages per worker  $w(L_t)$  as a function of the capital-labor ratio. From equation (14) we have

$$w_t = (1 - \alpha) A k_t^\alpha. \quad (30)$$

We can now re-express equation (29) in the following way:

$$\frac{\dot{k}_t}{k_t} = \left( 1 + \frac{\alpha}{\lambda} \right)^{-1} \left[ s A k_t^{\alpha-1} - \left( \delta + s_f \frac{w_t}{k_t} \right) \right]. \quad (31)$$

In the following simulations, we use the following baseline parameters.<sup>22</sup>  $s = 0.2$ ;  
 $A = 1$ ;  $\alpha = 0.3$ ;  $\delta = 0.05$ ;  $w_r = 1$ . Define  $h(k) = \frac{sAK^{\alpha-1}}{1 + \frac{\alpha}{\lambda}}$  and  $g(k) = \frac{\delta + s_f w}{1 + \frac{\alpha}{\lambda}}$  so that:

$$\frac{\dot{k}_t}{k_t} = h(k) - g(k).$$

FIGURE 1: Convergence:  $\tilde{s} > 0$ ,  $s_f = 0.2$ ,  $\lambda = 1$

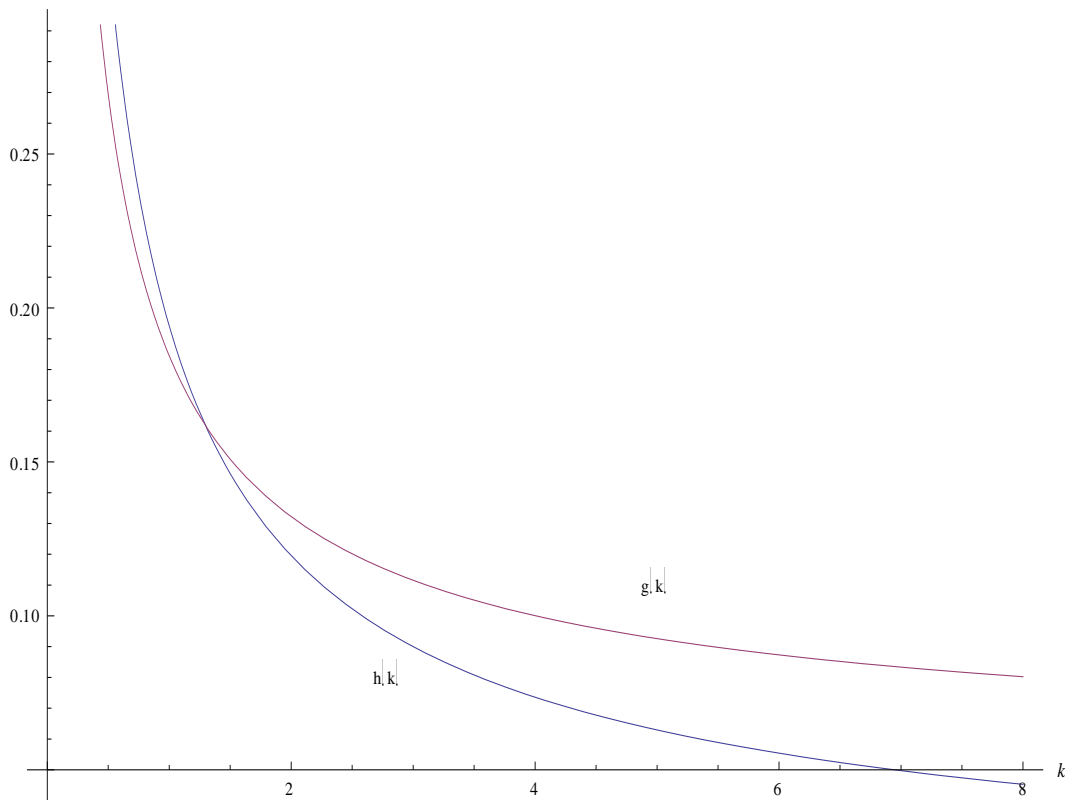
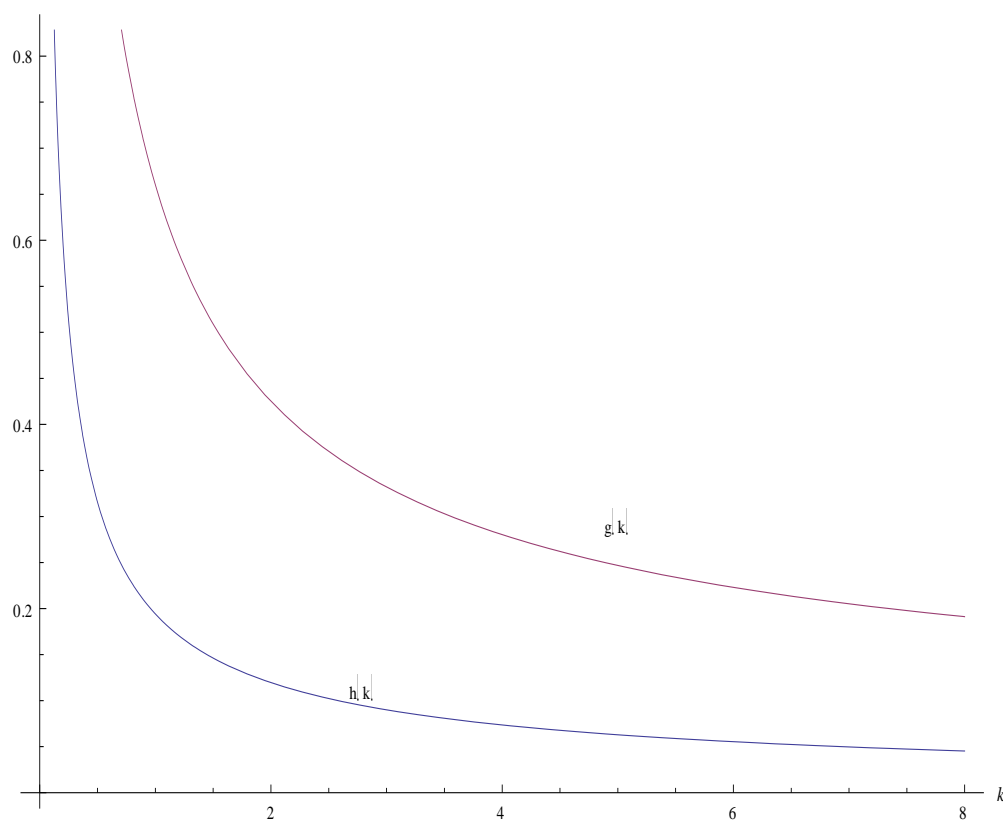


FIGURE 2: Divergence:  $\tilde{s} < 0$ ,  $s_f = 0.9$ ,  $\lambda = 1$

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<sup>22</sup>  $s, \alpha, \delta$  are standard from Barro and Sala-i-Martin [8].



In Figure 1, remittances are sufficiently low to allow for capital accumulation and convergence to a steady-state. In contrast, when remittances are high (and keeping other parameters the same), capital per worker converges to zero in the long-run: in this economy reliance on foreign workers, combined with high remittance rates, impoverish the domestic economy (Figure 2).

When  $\lambda$  is relatively large labor supply displays low elasticity; "cheap" labor initially fuels capital accumulation but as labor becomes more expensive, marginal returns to capital decrease and per capita income converges to a steady-state. Along the transitional dynamics, low wages (corresponding to low capital per worker) gradually increase as more foreign workers are hired and more capital is accumulated – these dynamics mirror those of the standard Solow growth model: high per capita income growth for low income countries gradually falling as income rises. Conversely when  $\lambda$  is low (corresponding to the case where labor supply is very elastic) capital accumulation is accommodated with rates of population growth that are almost as high as the rate of growth in overall capital. The limiting case is discussed in section 2 when  $\lambda \rightarrow 0$ . As a result, growth in capital per worker remains low along the transitional dynamics. The elasticity of the labor supply curve does not however affect the steady-state level of capital per worker.

Barro and Sala-i-Martin [8]<sup>23</sup> consider various models of labor migration. In a modified Ramsey model, they posit a foreign labor supply curve that is a function of the domestic capital

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<sup>23</sup>See chapter 9 of their textbook.

intensity. Above a threshold capital-labor ratio  $\tilde{k}$ , there is an inflow of foreign workers. If  $k < \tilde{k}$ , the growth rate of capital per worker is higher than in the Solow growth model of  $\delta + n$  (where  $n$  is population growth) because workers are flowing out of the economy; conversely, when  $k > \tilde{k}$ , the growth rate falls below  $\delta + n$ . Transitional dynamics in their model are driven by the position of the domestic economy relative to foreign economies.

Our model instead assumes that the domestic firm owned by the national household decides the number of workers entering the economy. The number of workers flowing in reflects the willingness of the domestic firm to hire workers. Unlike their model, the growth rate of the population of foreign workers in our model is not unintended.

Other strands of macroeconomics emphasize the importance of population growth on economic outcomes, prominent among them endogenous growth models. These models argue that both population growth and population levels (scale effects) may affect per capita income growth.<sup>24</sup> In our setting, we do not assume that foreign workers bring with them "human capital" – output dynamics are instead driven by labor supply elasticity.

## 6 Endogenous saving

In this section, we derive the implications of foreign labor flows when the national household chooses its saving rate endogenously. In particular, we investigate how the national saving rate adjusts to changes in the proportion  $s_f$  of wages remitted by foreign workers. We assume that the household is choosing a saving rate to maximize a discounted sum of utility flows of the type:

$$\int_0^{+\infty} u(C_t) e^{-\rho t} dt, \quad (32)$$

Where  $C_t$  is consumption by the national household,  $u(C_t)$  a twice differentiable instantaneous utility of consumption and  $\rho$  is the time preference parameter in the discounting function  $e^{-\rho t}$ . Consumption by foreign workers remains exogenous and consists of the proportion of their wages that is not sent home as remittances. The optimization problem for the household is to choose consumption to maximize objective (32) subject to the capital accumulation equation  $\dot{K}_t = r_t K_t + (1 - s_f) w(L_t) L_t - C_t$  where factor prices are taken as a given. Notice that the objective is a function of  $C_t$ , not consumption per worker since consumption  $C_t$  accrues to the national household; foreign workers receive wages but their consumption in the domestic economy is used either for capital accumulation or for consumption by the domestic household. Using Pontryagin's maximum principle, we obtain the following equation:

$$\rho - \frac{u''}{u'} \dot{C}_t = r_t. \quad (33)$$

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<sup>24</sup>Aghion and Howitt [4] provide a textbook coverage of endogenous growth.

Assuming an isoelastic utility function of the form  $u(C) = \frac{C^{1-\theta} - 1}{1-\theta}$ , we obtain:

$$\frac{\dot{C}_t}{C_t} = \frac{1}{\theta}(r_t - \rho). \quad (34)$$

Equation (22) then gives:

$$\frac{\dot{C}_t}{C_t} = \frac{1}{\theta} \left[ \alpha \frac{Y_t}{K_t} - (\rho + \delta) \right]. \quad (35)$$

Using equation (17), equation (23) can be re-stated as follows:

$$\dot{K}_t = [1 - s_f(1 - \alpha)]Y_t - \delta K_t - C_t. \quad (36)$$

Given  $K_0 > 0$ , equations (35,36) and the transversality  $\lim_{t \rightarrow +\infty} u'(C_t)e^{-\rho t} K_t = 0$  determine equilibrium dynamics of  $C_t$  and  $K_t$  over time.

## 6.1 Steady-state and Convergence

From equation (35), the expression  $\dot{C}_t = 0$  gives the steady-state level of capital accumulation, captured in the following equation:

$$\frac{Y^*}{K^*} = \frac{\rho + \delta}{\alpha}. \quad (37)$$

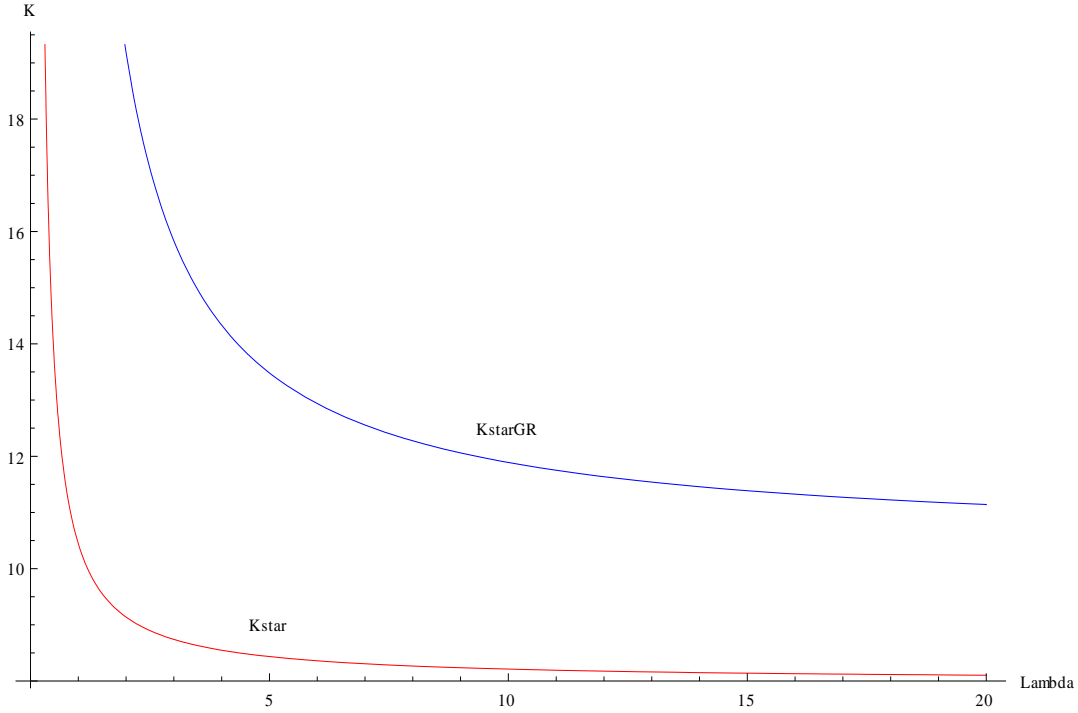
It is straightforward to show that the resulting steady-state level of capital is a decreasing function of  $\lambda$ : using equation (17), one finds that as labor elasticity decreases ( $\lambda$  becomes larger), labor becomes gradually more expensive, causing steady-state capital to fall. Using equation (16), one finds that the steady-state level of the stock of labor decreases with  $\lambda$ . Figure 3 below illustrates this with the following baseline parameter values:<sup>25</sup>  $\theta = 3$ ,  $\alpha = 0.3$ ,  $\delta = 0.05$ , and  $\rho = 0.02$  along with  $A = 1$ ,  $w_r = 1$ ,  $s_f = 0.2$ .

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<sup>25</sup>From Barro and Sala-i-Martin [8], pp. 114.



FIGURE 3:  $K^*$  and  $K_{GR}^*$  as a function of  $\lambda$



The level of accumulated capital in the steady-state is dynamically efficient and therefore below the Golden Rule level of accumulated capital.<sup>26</sup> Finally, note that the saving rate adjusts for different rates of remittances to ensure that an equilibrium with a positive level of accumulated capital exists. Let  $s^*$  denote the saving rate of the national household in the steady-state, and let  $C^*$  denote the correspond level of consumption. Then,  $C^* = (1 - s^*)Y^*$ . Using equation (36) evaluated in the steady-state, one obtains the following expression for  $s^*$ :<sup>27</sup>

$$s^* = s_f(1 - \alpha) + \frac{\alpha\delta}{\rho + \delta}$$

As  $s_f$  increases to unity (so that all wage income is used for remittances), the steady-state saving rate adjusts upward: greater capital accumulation is required to offset remittance losses. Notice however that steady-state accumulated capital (using equations 17 and 37) is not affected by the remittances, only steady-state consumption by the national household.

## 6.2 Speed of Convergence

In this section, we investigate the transitional dynamics of our modified Ramsey model

<sup>26</sup>See Appendix B for phase diagrams with various parameter specifications.

<sup>27</sup>We have  $(1 - s_f(1 - \alpha))Y^* = \delta K^* + (1 - s^*)Y^*$ . Solving gives  $s^* = (s_f(1 - \alpha)Y^* + \delta K^*)/Y^*$ . Finally use equation (37) to obtain the result.

and contrast it to the dynamics of our modified Solow model from section 2. We re-write the system of equations (35, 36) in per capita terms. Let  $n_t$  denote population growth then

$\frac{\dot{c}_t}{c_t} = \frac{\dot{C}_t}{C_t} - n_t$ . Using equation (35) and  $\frac{Y_t}{K_t} = \frac{y_t}{k_t}$ , one obtains:

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\theta} \left[ \alpha \frac{y_t}{k_t} - (\rho + \delta) \right] - n_t. \quad (38)$$

From equation (36), we have  $\frac{\dot{K}_t}{K_t} = (1 - s_f(1 - \alpha)) \frac{Y_t}{K_t} - \delta - \frac{C_t}{K_t}$ . Using  $\frac{\dot{k}_t}{k_t} = \frac{\dot{K}_t}{K_t} - n_t$  and  $\frac{C_t}{K_t} = \frac{c_t}{k_t}$ , we have:

$$\frac{\dot{k}_t}{k_t} = (1 - s_f(1 - \alpha)) \frac{y_t}{k_t} - \frac{c_t}{k_t} - (\delta + n_t), \quad (39)$$

where  $n_t$ , the population growth rate, is given by the expression  $n_t = \frac{\alpha \dot{k}_t}{\lambda k_t}$  (equation 28). Replacing this in equation (39), one obtains:

$$\frac{\dot{k}_t}{k_t} = \left( 1 + \frac{\alpha}{\lambda} \right)^{-1} \left[ (1 - s_f(1 - \alpha)) \frac{y_t}{k_t} - \frac{c_t}{k_t} - \delta \right]. \quad (40)$$

Equations (28,38,40) along with a transversality condition describe equilibrium dynamics of the variables  $c_t$  and  $k_t$ . To allow comparison with our modified Solow model, we re-write equation (40) as:

$$\frac{\dot{k}_t}{k_t} = \left( 1 + \frac{\alpha}{\lambda} \right)^{-1} \left[ \left( \frac{y_t}{k_t} - \frac{c_t}{k_t} \right) - \left( \delta + s_f(1 - \alpha) \frac{y_t}{k_t} \right) \right].$$

The last term relates to wage income spent domestically. From equation (30), the above can be re-written as:

$$\frac{\dot{k}_t}{k_t} = \left( 1 + \frac{\alpha}{\lambda} \right)^{-1} \left[ \left( \frac{y_t}{k_t} - \frac{c_t}{k_t} \right) - \left( \delta + s_f \frac{w_t}{k_t} \right) \right].$$

Transitional dynamics are similar to those observed in the modified Solow growth model of section 5 with the exception that the saving rate is endogenous. For high elasticities of labor, population grows almost as quickly as capital. As a result, growth in output per worker is lower along the transitional dynamics than in a setting where labor elasticity is lower. To illustrate

this, let  $h^*(k) = \left(1 + \frac{\alpha}{\lambda}\right)^{-1} \left(\frac{y}{k} - \frac{c}{k}\right)$  and let  $g^*(k) = \left(1 + \frac{\alpha}{\lambda}\right)^{-1} \left(\delta + s_f \frac{w_t}{k_t}\right)$ . As  $\lambda$  converges to zero, labor elasticity becomes infinite and the rate of growth of  $k$  along the transitional dynamics, which is proportional to the distance between  $h^*$  and  $k^*$ , converges to zero.

FIGURE 4: Transitional Dynamics:  $\lambda = 1$  and  $s_f = .2$

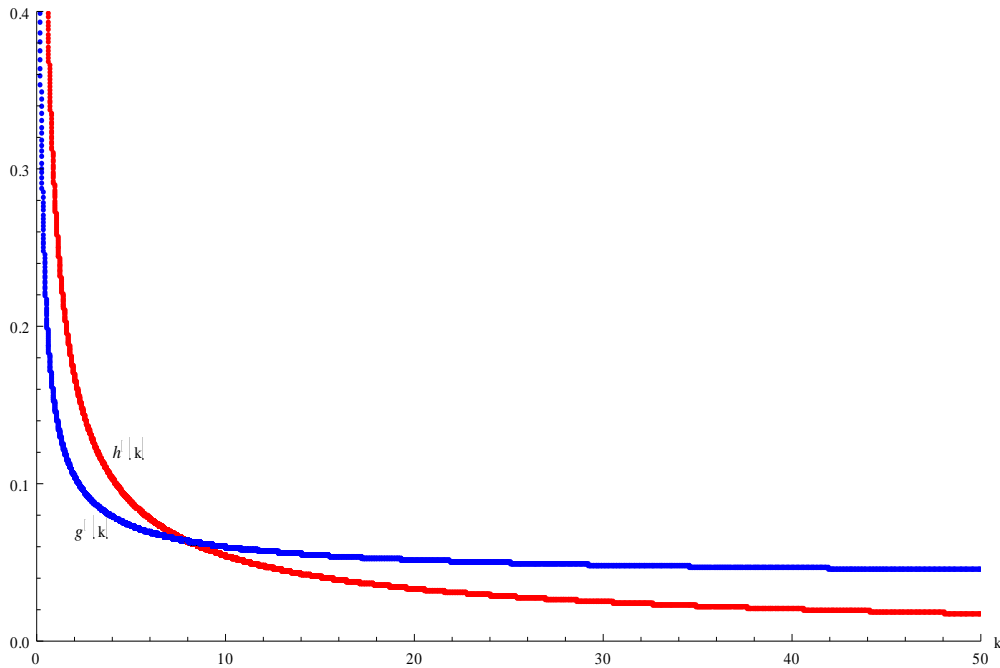
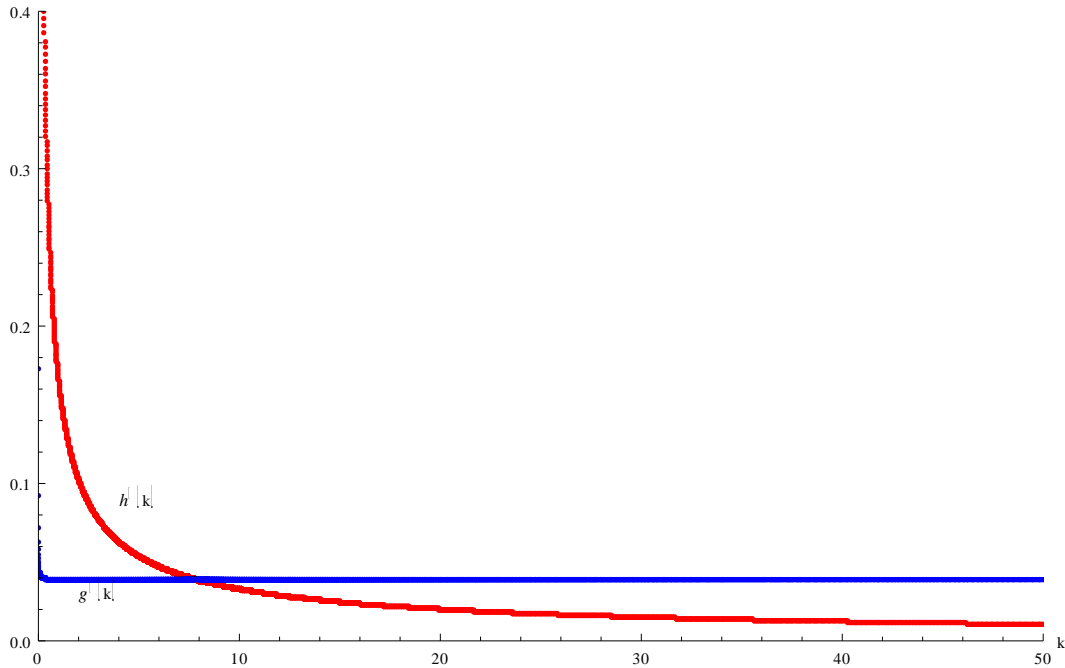


FIGURE 5: Transitional Dynamics:  $\lambda = 1$  and  $s_f = .001$



As discussed in the previous section, steady-state values of  $K$  and  $L$  do not depend on remittance rates by foreign workers. As a result, the steady-state value of capital per worker is also independent of  $s_f$ . Transitional dynamics however will be affected, as illustrated in Figures 4 and 5. With relatively high remittance rates ( $s_f = .2$ ), transitional dynamics display low growth rates of capital per worker while with much lower remittance rates ( $s_f = .001$ ), convergence occurs at a very high rate as most foreign income is re-invested toward further capital accumulation.<sup>28</sup>

## 7 Conclusion

We introduce the notion of abundant supplies of labor, present in many developing countries such as those of the GCC, in an economic growth framework: we assume that firms face an (elastic) foreign labor supply schedule, and that foreigners remit a proportion of their wage income. We find that if foreign labor is provided elastically, growth in income per worker is arbitrarily small along the transitional dynamics, while growth in overall income is high. This may provide another mechanism for the lack of "unconditional convergence" for the world (see Barro and Sala-i-Martin [6], [7]). In particular, the growth model predicts low per capita growth and high overall growth which appears to be in line with growth experiences among GCC states. In addition, the effect of TFP growth on income per worker is small when labor supply is elastic: this in turn may provide an alternative explanation for the natural resource curse, at least as this hypothesis applies to resource- and foreign labor-reliant economies such as those of the GCC.

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<sup>28</sup>We use the same parameterization as in section 6 for the other parameters.

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## 8 Appendix A – Derivation of Selected Equations

- Derivation of equation (26). From equation (24), we have

$$\dot{K}_t / K_t = \tilde{s} A_t^{\frac{1+\lambda}{\alpha+\lambda}} B_t^{\frac{1-\alpha}{\alpha+\lambda}} K_t^{\alpha \frac{1+\lambda}{\alpha+\lambda} - 1} - \delta. \quad \square \quad \text{Also, } k_t / k_t = K_t / K_t - n \quad \square \quad \text{so that}$$

$$\square \quad k_t / k_t = \tilde{s} A_t^{\frac{1+\lambda}{\alpha+\lambda}} B_t^{\frac{1-\alpha}{\alpha+\lambda}} K_t^{(\alpha-1) \frac{\lambda}{\alpha+\lambda}} - (\delta + n). \quad \text{Also, } k_t = K_t / L_t = A_t^{-\frac{1}{\alpha+\lambda}} B_t^{-\frac{1}{\alpha+\lambda}} K_t^{-\frac{\alpha}{\alpha+\lambda} + 1} \quad \text{using equation}$$

(16). Re-writing gives  $k_t = J_t K_t^{\frac{\lambda}{\alpha+\lambda}}$  where  $J_t = A_t^{-\frac{1}{\alpha+\lambda}} B_t^{-\frac{1}{\alpha+\lambda}}$ . So,  $k_t^{\alpha-1} = J_t^{\alpha-1} K_t^{(\alpha-1) \frac{\lambda}{\alpha+\lambda}}$  so

$$K_t^{(\alpha-1) \frac{\lambda}{\alpha+\lambda}} = \frac{k_t^{\alpha-1}}{J_t^{\alpha-1}}. \quad \text{Replace to obtain } \square \quad k_t / k_t = \tilde{s} A_t^{\frac{1+\lambda}{\alpha+\lambda}} B_t^{\frac{1-\alpha}{\alpha+\lambda}} \frac{k_t^{\alpha-1}}{J_t^{\alpha-1}} - (\delta + n) \quad \text{and simplify to obtain}$$

equation (26).

- Derivation of equation (33). The present value Hamiltonian for this problem is

given by  $H = u(C_t) e^{-\rho t} + \mu_t (r_t K_t + (1-s_f) w(L_t) L_t - C_t)$ . First order condition for the control

gives:  $\frac{\partial H}{\partial C_t} = 0 = u'(C_t) e^{-\rho t} - \mu_t$ . First order condition for the state gives:  $\frac{\partial H}{\partial K_t} = -\dot{\mu}_t = \mu_t r_t$ .

Given that  $u'(C_t) e^{-\rho t} = \mu_t$ , differentiate with respect to time to obtain  $u'' \dot{C}_t e^{-\rho t} - \rho u' e^{-\rho t} = \dot{\mu}_t$ .

Using the other FOC, we get  $u'' \dot{C}_t e^{-\rho t} - \rho u' e^{-\rho t} = -\mu_t r_t$ . Finally, plugging back  $u'(C_t) e^{-\rho t} = \mu_t$

into this equation, one obtains  $u'' \dot{C}_t e^{-\rho t} - \rho u' e^{-\rho t} = -u' e^{-\rho t} r_t$  or  $\rho - \frac{u''}{u} \dot{C}_t = r_t$ .

- Derivation of equation (34). Since  $u' = C_t^{-\theta}$  and  $u'' = -\theta C_t^{-\theta-1}$ , we get



$$\rho + \theta \frac{\dot{C}_t}{C_t} = r_t \quad \text{or:} \quad \frac{\dot{C}_t}{C_t} = \frac{1}{\theta} (r_t - \rho).$$

• Derivation of equation (35). From equation (22) we have:

$$\frac{\dot{C}_t}{C_t} = \frac{1}{\theta} \left[ \alpha A_t^{\frac{1+\lambda}{\alpha+\lambda}} B_t^{\frac{1-\alpha}{\alpha+\lambda}} K_t^{\alpha \frac{1+\lambda}{\alpha+\lambda} - 1} - (\rho + \delta) \right] \quad \text{or} \quad \frac{\dot{C}_t}{C_t} = \frac{1}{\theta} \left[ \alpha \frac{Y_t}{K_t} - (\rho + \delta) \right].$$

## 9 Appendix B – Simulations

In this section, we illustrate the phase diagram corresponding to the dynamics of the modified Ramsey model of section 6. We use the following baseline parameters:  $\alpha = 0.3$ ,  $\delta = 0.05$ ,  $\theta = 3$  and  $\rho = 0.02$  (from Barro and Sala-i-Martin [8], pp. 114) along with  $A = 1$ ,  $w_r = 1$ ,  $s_f = 0.2$  and  $\lambda = 1$ . We change  $\lambda$  to investigate the household's willingness to substitute current for future consumption. The blue line represents  $\dot{C} = 0$ , the black curve represents  $\dot{K} = 0$  and the red curve is the stable arm. Arrows display vector fields representing the dynamics of equations (35,36).

FIGURE B1: Phase Diagram with  $\lambda = 1$

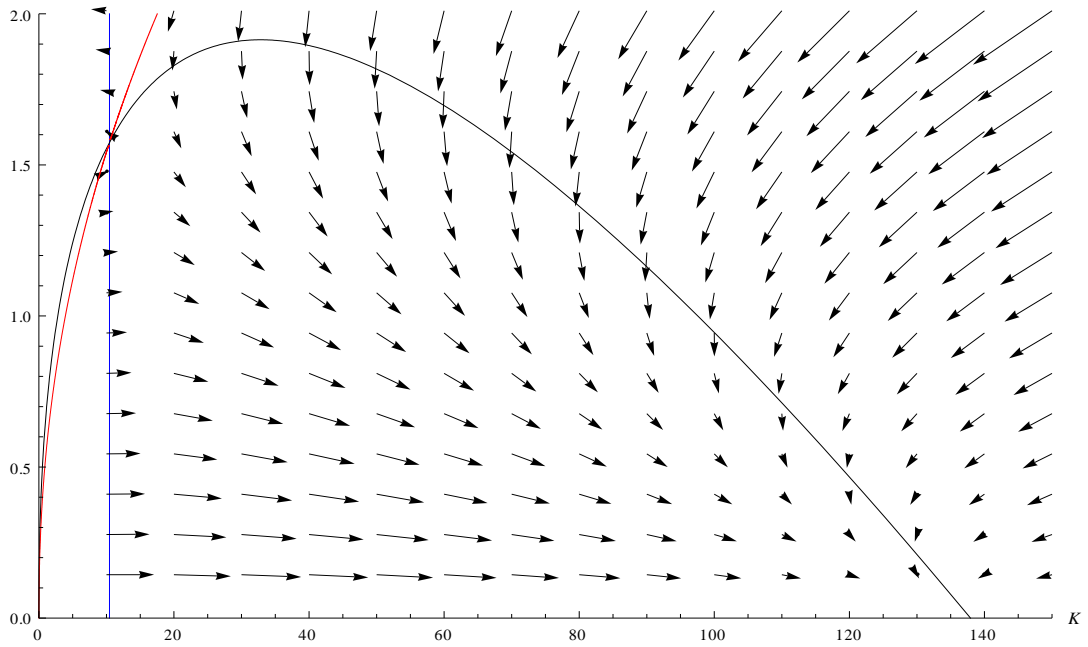
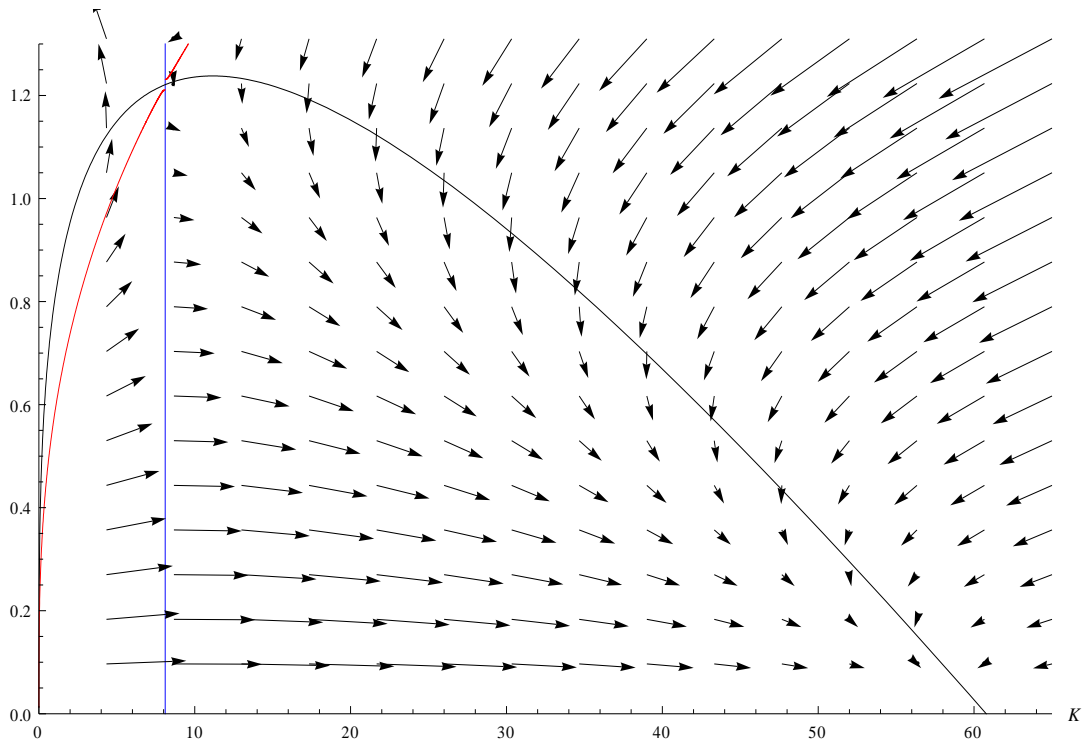


FIGURE B2: Phase Diagram with  $\lambda = 20$



## 10 Appendix C – Tables

The tables are reproduced from Coury and Dave (2009).

Table C1 — Average of y/o/y NHGDP growth in preceding 5 years<sup>29</sup>

	1985	1990	1995	2000	2005	Average
BHR	1.54%	4.28%	7.92%	3.84%	3.64%	3.96%
KWT	32.78%	16.08%	0.88%	3.51%	2.44%	5.30%
OMN	26.42%	8.87%	8.93%	3.15%	-1.01%	5.79%
QAT	46.60%	4.83%	0.66%	6.52%	9.77%	5.49%
SAU	44.22%	1.29%	5.17%	3.17%	-5.98%	3.21%
UAE	41.32%	-2.37%	10.39%	8.55%	3.36%	6.62%

Table C2 — Average of y/o/y population growth rate in preceding 5 years<sup>30</sup>

	1985	1990	1995	2000	2005	Average
BHR	4.37%	3.48%	3.52%	2.87%	2.28%	3.07%
KWT	5.89%	4.21%	-16.96%	47.80%	3.35%	2.64%
OMN	5.45%	4.62%	3.64%	2.82%	1.78%	3.37%
QAT	8.04%	8.15%	3.83%	2.22%	4.43%	5.11%
SAU	6.07%	5.80%	3.39%	2.79%	2.12%	3.71%
UAE	10.07%	6.04%	5.52%	5.72%	5.67%	6.05%

Table C3 — Average of y/o/y per cap. NHGDP growth in preceding 5 years<sup>31</sup>

	1985	1990	1995	2000	2005	Average
BHR	-2.69%	0.77%	4.25%	0.94%	1.33%	0.87%
KWT	25.26%	11.43%	46.96%	-14.54%	-0.99%	2.60%
OMN	19.88%	4.06%	5.11%	0.32%	-2.74%	2.35%
QAT	36.56%	-3.08%	-2.98%	4.21%	5.12%	0.36%
SAU	35.99%	-4.26%	1.79%	0.38%	-7.89%	-0.49%
UAE	27.78%	-7.93%	4.63%	2.68%	-2.18%	0.54%

<sup>29</sup> Source: World Bank, Energy Administration Information, British Petroleum. NHGDP is non-hydrocarbon GDP. Last column is the average year-on-year growth for 1980-2005. Last col. is the average year-on-year growth for 1980-2005.

<sup>30</sup> Source: World Bank. Last column is the average year-on-year growth for 1980-2005.

<sup>31</sup> Source: World Bank, Energy Administration Information, British Petroleum.

Table C4 — Average of y/o/y overall GDP growth in preceding 5 years<sup>32</sup>

	1985	1990	1995	2000	2005	Average
BHR	-1.36%	2.84%	7.00%	4.04%	5.59%	3.88%
KWT	-6.94%	5.66%	10.97%	1.96%	7.29%	2.76%
OMN	15.81%	6.06%	4.86%	3.32%	4.56%	6.09%
QAT	-5.55%	-2.69%	0.72%	7.74%	18.71%	3.51%
SAU	-2.60%	1.01%	4.55%	1.65%	3.69%	1.51%
UAE	-0.67%	-1.59%	5.37%	5.78%	6.18%	2.83%

Table C5 — Average of y/o/y per cap. GDP growth in preceding 5 years<sup>33</sup>

	1985	1990	1995	2000	2005	Average
BHR	-5.47%	-0.62%	3.36%	1.14%	3.24%	0.78%
KWT	-12.06%	1.40%	58.65%	-15.52%	3.81%	0.12%
OMN	9.82%	1.38%	1.17%	0.48%	2.73%	2.64%
QAT	-12.52%	-9.91%	-3.00%	5.40%	13.70%	-1.53%
SAU	-8.17%	-4.52%	1.09%	-1.10%	1.54%	-2.13%
UAE	-9.74%	-7.19%	-0.14%	0.06%	0.48%	-3.03%

Table C6 — Hydrocarbon GDP as a percentage of overall GDP<sup>34</sup>

	1980	1985	1990	1995	2000	2005
BHR	29.20%	24.44%	18.11%	12.35%	20.75%	27.65%
KWT	83.27%	54.60%	46.68%	50.52%	64.13%	68.43%
OMN	64.69%	52.03%	52.85%	40.67%	55.86%	67.20%
QAT	85.39%	59.33%	57.45%	44.26%	65.62%	76.09%
SAU	84.36%	36.47%	54.42%	41.52%	55.81%	75.93%
UAE	80.60%	51.25%	62.36%	38.27%	45.73%	50.33%

<sup>32</sup> Source: World Bank. Computed using PPP-adjusted GDP in 2005 Constant International Dollars. Last column is the average year-on-year growth for 1980-2005.

<sup>33</sup> Source: World Bank. Computed using PPP-adjusted GDP in 2005 Constant International Dollars. Last column is the average year-on-year growth for 1980-2005.

<sup>34</sup> Source: World Bank, Energy Administration Information, British Petroleum.