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Preliminary Draft

Risk Analysis of The Government Domestic Debt Stock In Turkey: Cost-At-Risk Approach

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Abstract

In this study, stochastic simulation based risk analysis is applied to the government domestic debt stock in Turkey with the motivation to identify the cost and risk characteristics of alternative debt financing strategies. Future path of interest rates is simulated by using the yield curve framework in Diebold and Li (2002), which is founded on the Nelson-Siegel model. Yield curve simulation is based on the estimated term structure of interest rates for the period June 2001-July 2004. Simulated yield curves are generally upward sloped and concave. Contrary to the common observation, long-term yields are more volatile compared to short-term yields. Under each financing strategy, debt is rolled over on top of simulated term structure of interest rates. Alternative financing strategies are compared with respect to absolute Cost-at-Risk, relative Cost-at-Risk and relative risk measures computed from the simulated cost distributions. Results of the risk analysis are influenced by the characteristics of the simulated term structure of interest rates and the additional yield imposed on the coupon bonds, which is assumed to reflect risk perception of investors for increased maturity.

JEL Classification: H63, E43

Keywords: Government Debt, Stochastic Simulation, Cost-at-Risk

I. Introduction

Sustainable public debt stock along with sound fiscal and monetary policies is the premises of a stable economy. High and poorly structured debt raises concerns related to the sustainability of the debt stock and coupled with international capital mobility damages financial stability. Poorly structured debt in terms of maturity mix and currency denomination has been important factors in inducing and propagating economic crises (IMF and World Bank, 2002). Specifically, short maturity and floating rate debt expose government budget to changing financial market conditions when this debt has to be rolled over. Refinancing problem faced by the government is exacerbated when the debt to be refinanced is in foreign currency. In these instances, inability of the government to borrow in terms of foreign currency either stemming from non-adequate foreign exchange in the economy or reluctance of investors to lend in foreign currency can exert upward pressure on the exchange rate. Sound debt structures on the other hand, alleviate the risk perception related to the sustainability of the debt stock by reducing its exposure to the interest rate and the exchange rate movements. The risk posed by the government debt, brings forward the importance of sound debt management[1]. Sovereign debt management, and the configuration of the optimal debt structure in that context, is not just the concern of the highly indebted countries with debt sustainability problems. One of the reasons for establishing debt management framework is the welfare loss that result from increase in the cost of debt. Increase in interest expenditures results in welfare losses as the tax rates are adjusted to finance the gap in the government budget. Welfare loss is aggravated further if the cost of foreign debt stock is increased, since in that case resources of the economy are transferred abroad. Another reason for establishing debt management framework is to maintain the reputation of the government in international financial markets. Governments, which are debtors in international financial markets, are expected to have transparent and accountable debt management practices including a risk management policy (Storkey, 2001).

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[1] Public debt management is defined as a process of establishing and executing a strategy for managing the government's debt in order to raise required amount of funding, achieve its risk and cost objective, and to meet any other sovereign debt management goals the government may have set, such as developing and maintaining an efficient market for government securities (IMF and World Bank Guidelines, pp. 1)

Optimal structure of public debt in terms of denomination, maturity structure and indexation features has been an issue of both academic and practitioner research. However, these lines of work consider the issue from different perspectives. In the theoretical literature, under the optimal taxation approach, the objective of the government is to minimize welfare losses resulting from distortionary taxation. Hence the government is motivated to smooth tax rates over time. Under this framework, risk is the budgetary risk, more specifically it is the risk of having to change taxes in response to the shocks hitting the government budget. On the other hand, under the public debt management practice, objective of the government is to minimize the financial cost of servicing debt with due regard to risk, which is the potential variation in the financial cost. Hence, the scope of risk concept under debt management practice is limited to the debt servicing cost. However in recent years, there is a tendency in debt management practice, towards measuring the risk under a budgetary framework, acknowledging that macroeconomic shocks not only affect debt costs but also other components of the government's budget.

Well-established debt management framework enables the government to identify cost and risk tradeoffs associated with different debt structures. One of the trade-offs is related to the maturity of the debt stock. Short-term debt is perceived to be less costly although more risky compared to long-term debt. This inference is based on the frequent refinancing necessity associated with short-term borrowing and the general observation that on the average yield curves are positively sloped and short-term rates are more volatile compared to long-term rates (Bolder, 2003; Diebold and Li, 2002). Another trade-off is related to the denomination of the debt stock. Preference of foreign currency debt against TL denominated debt also entails a tradeoff, as foreign currency denominated/ indexed debt reduces the cost of borrowing, however increases the exposure of debt stock to depreciation of the exchange rate.

Given the tradeoffs in borrowing policy, in recent years there is increased focus of countries on managing financial risk inherent in the government debt portfolio. Country surveys point out to the high level of awareness of the importance of risk management of public debt and growing consensus for the appropriate techniques for managing risk (IMF and World Bank Guidelines, 2002). This tendency in the field has rendered identification of cost and risks associated with alternative financing strategies and corresponding debt structures an important component of public debt management. Parallel to this trend in the field, also in Turkey, sovereign debt management framework has been adopted[2]. Among basic principles of the debt management framework established in 2002 is the fulfillment of financing requirements at the lowest possible cost in the medium and long term with regard to the determined levels of risk.

This paper quantitatively analyzes the cost and the risk to the Turkish government of alternative domestic borrowing strategies. In Turkey lowering debt cost is a major issue on account of the considerable share of interest expenditure within the government expenditures. At present, given that the level and the vulnerable structure of the debt stock are factors contributing to the high real interest rates in Turkey, the objective of the debt management strategy is straight forward; increasing the maturity of the debt stock and reducing the share of FX denominated debt, thereby reducing the susceptibility to interest rate and the exchange rate movements. Even though the direction of the policy is unambiguous, quantifying the effect of alternative borrowing strategies is worth the effort. Moreover, in the Turkish case, the importance of establishing risk management tools will become important in the upcoming period as the economy moves into a phase of relatively stable economic environment, in which context the feasibility of active debt management will increase, as the debt sustainability will seize to be a concern.

In this paper, comparative cost and risk analysis of financing strategies is based on stochastic simulation methodology. Cost is defined simply as the interest expenditure and risk is the likelihood that interest expenditure will be over a maximum amount specified with some probability level. Scope of the risk analysis is limited to the market risk associated with fluctuations in the interest rate, i.e. interest rate risk. Borrowing strategies comprise discounted securities of various maturities and coupon bonds denominated in domestic currency. Risk measures are computed from simulated cost distributions obtained from a stochastic simulation model, which has five-year horizon, covering the years 2005-2009.

Simulation based risk models used in sovereign debt management practice are risk measures adopted from the financial and the corporate sector. Most commonly used

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[2] New debt management framework has been established with the enforcement of law on the Regulation of Public Financing and Debt Management in March 28, 2002. and the Regulation on the Principles and Procedures for the Coordination and Execution of Debt and Risk Management in September 1, 2002.

risk measure used in these sectors is Value-at-Risk (VaR), which expresses the maximum decline, with a given probability, in the market value of a portfolio over a given period. Since bulk of debt is left outstanding until maturity, in majority of the debt management practices debt cost is not computed from the market value of the debt portfolio. Instead, only realized cost is taken into consideration, whereas unrealized mark-to market[3] costs- changes in the market value of the debt stock resulting from the movements in the market prices- are not considered. Thus, instead of VaR, a similar measure used in sovereign debt management is Cost-at-Risk[4], which is based on debt costs rather than market value. Once the statistical distribution of the debt costs is obtained, CaR is the maximum cost that could occur with some probability in a particular time period. For example with 99 percent probability, CaR is the 99th quartile value of the cost distribution (Figure 1.1).

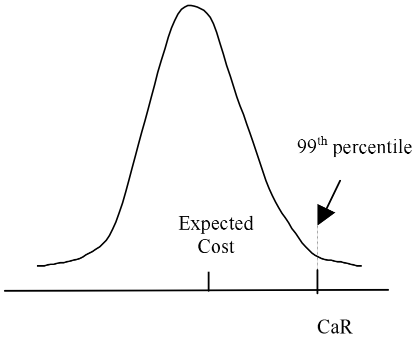

Figure 1.1 Illustration of Cost-at-Risk

Empirical studies of both the debt management offices[5] and other researchers that measure risk from cost distributions are motivated to compare cost and risk characteristics of various debt management strategies with the perspective of selecting the optimal debt strategy amongst the alternatives, although not necessarily for the

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[3] Realized mark-to-market costs are cost arising from debt buybacks and swaps.

[4] An exception to this practice is New Zealand Debt Management Office(NZDMO). NZDMO manages market risk associated with tactical trading through the use of VaR measure (IMF and World Bank Guidelines, 2002).

[5] Methods used to measure the market risk of sovereign debt portfolio differ across countries. Most use deterministic scenarios and few use stochastic models. Countries that use stochastic simulation models comprise Brazil, Colombia, Denmark, New Zealand, Canada, Italy, Portugal and Sweden. Recently Turkish Treasury, authority of managing sovereign debt, has also adopted a risk measure based on a stochastic simulation model. Debt management offices that have initially applied this method are New Zealand, Denmark and Sweden.

purpose of making a policy recommendation. In practice either a goal is set, such as minimizing cost and risk of the debt, the optimal strategy is then chosen via optimization or alternatively cost and risk characteristics of various strategies are computed through simulations and the results are compared. In the first type of analysis, optimal strategy is the model outcome, that is the problem faced by the government is formulated as stochastic dynamic optimization problem where given the outstanding debt stock and the simulated paths for the variables effecting it, the role of model is to find the borrowing policy (control variable) that minimizes the objective function that is the cost or the risk of debt (Grill and Östberg, 2003, Cannata et al., 2004, Bolder, 2003).[6]

In the latter type of analysis, stochastic simulation constitutes the first stage, which provides set of borrowing strategies each with cost and risk characteristics. Further stage is the selection of the optimal strategy amongst them. This task can be accomplished under judgmental or mathematical optimization framework on account of the other objectives of the government along with cost and risk. Analysis in this paper is limited to the first stage of the latter type of analysis.

Stochastic simulation model in this paper consists of two parts; First part comprises the stochastic model, using which term structure of interest rates is modeled and simulated. Second part contains strategy simulation, in which for each borrowing strategy debt is rolled over on top of the simulated economic environment. Cost and risk measures are then computed from a simulated cost distribution. Cost is the median of the distribution whereas risk is measured as deviation from the median with a given probability.

Analysis carried out in this paper is among the first studies attempting to measure market risk associated with the government debt in Turkey, under a stochastic simulation framework.[7] This paper does not aim to make a policy recommendation on the optimal debt strategy; rather tries to provide a tool that could serve the decision

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6 In Bolder (2003) governments borrowing decision is also conceptualized as an optimal control problem in a stochastic setting, where the government is trying to optimally select the composition of its debt portfolio to minimize expected debt cost subject to risk and liquidity constraints. However due to practical complexities regarding the use of dynamic programming technique, analyses rely on simulation of alternative debt management strategies.

[7] However, analysis is partial, in the sense that among the market risk that debt stock is exposed to, only the interest rate risk associated with domestic currency (Turkish lira) denominated portion of the debt stock is evaluated. Recently, Risk Management Unit within Turkish Treasury has adopted a new framework for risk analysis where market risk is measured on the basis of a stochastic model.

process of selecting an optimal debt strategy. Optimal debt strategy, along with the objective of cost minimization with due regard to risk, is exercised on account of other qualitative factors some of which include developing and maintaining an efficient market for government securities, maintaining sufficient level of liquidity in the secondary markets and broadening the investor base by diversifying the stock of debt among different maturities and debt instruments.

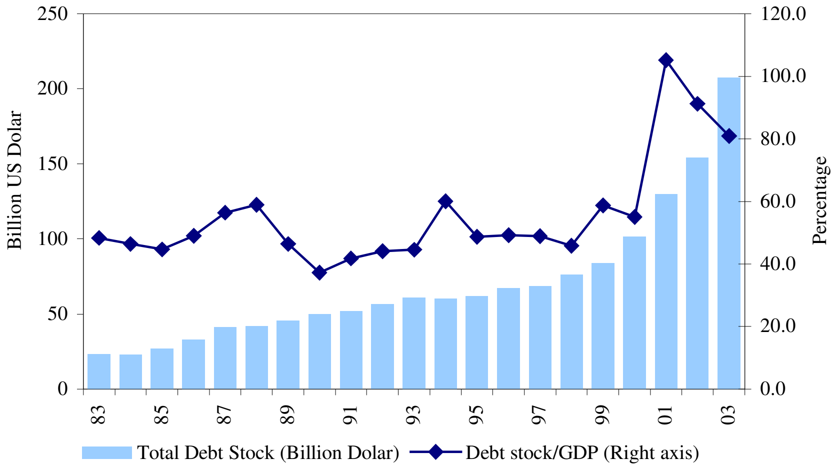
The study is structured as follows: In section 2 structure of the public debt stock in Turkey is analyzed. Section 3 and Section 4 explain stochastic and strategy simulation modules of the analysis. Section 5 contains the empirics of term structure estimation and simulation framework. Section 6 presents illustrative results of the risk analysis of the domestic currency denominated portion of the government debt stock in Turkey and Chapter 6 concludes.

II. The Structure of the Sovereign Debt Stock in Turkey

In this section, evolution of the total public sector debt stock during 1983-2004 and the present structure of the debt stock in terms of maturity and currency composition is analyzed. Gross debt stock of the public sector in Turkey increased from 48.3 percent of GDP in 1983 to 81.0 percent in 2003 (Figure 2.1). Decomposition of the change in the debt stock reveals several important points (Figure 2.2). Interest expenditure during the period has risen sharply as a result of the increased borrowing from the markets and ensuing high level of interest rates. Interest expenditure, which constituted 2 percent of GDP in 1983 increased to 17.1 percent of GDP in 2002. Another pronounced remark is the impact of the exchange rate on the evolution of the debt stock during the period 1983-2002. Excluding the years from 1997 to 2004, increase in the debt stock caused by the adjustments in the value of foreign debt expressed in domestic currency has been greater than the interest expenditure.

In the period 1983-1993, primary balance of the government was in deficit indicating lack of fiscal discipline. Although primary budget was in surplus position following 1993 until 1997, the magnitude of the primary surplus remained low compared to the heightening interest expenditures. Under a transparent fiscal framework increase in the debt stock should be explained by the abovementioned factors, namely, primary balance, interest expenditure and revaluation of the foreign currency denominated portion of the debt stock. In the Turkish case however, for several years we observe considerable amount of residual increases in the debt stock that cannot be explained by the exchange rate movements or the budget deficits. In major part, source of these increases is the securities issued to cover the off budget expenditure of the central government or the other public institutions. Substantial amount of residual increase was observed in the year 2001, prior to which, central government total debt stock to GDP ratio was around 50 percent. Hike in the debt stock mainly resulted from the increase in the non-cash debt[8], which was issued within the context of banking sector operation. In May 2001, under the framework of this operation, specially designed, non-cash debt instruments were issued to the state banks and to the private banks under Saving Depository Insurance Fund (SDIF) to strengthen their capital structure. Besides the restructuring of the capital structure, additional debt was issued to the state banks to cover the incurred duty losses. The source of duty losses dates back to 1980's, beginning from which off-budget government expenditures were carried out by the use of state banks. Substantial increase in the debt stock in 2001, for the aforementioned reasons has underlined the importance of fiscal discipline and the need to establish a control over contingent liabilities.

Figure 2.1. Gross Debt of the Public sector (1983-2003)



Source: State Planning Organization(SPO), Treasury

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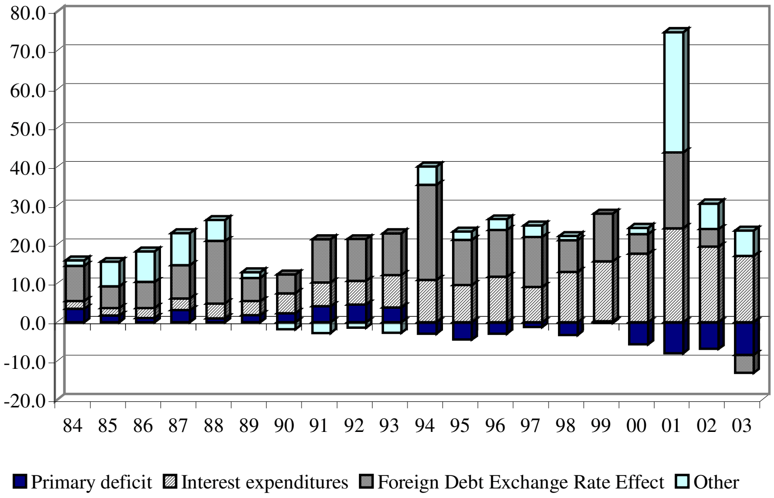
[8] Non-cash securities are debt instruments against which no cash is received.

Figure 2.2. Decomposition of the Change in the Gross Total Public Debt Stock (Share in GDP)



Source: SPO, Treasury.

Despite the declining trend in the debt stock to GDP ratio, following the substantial increase in the year 2001, public debt sustainability is still a concern in Turkey. Along with its high level, the structure of the debt stock in terms of maturity and currency composition raise concerns. In the remaining of this section, structure of the central government debt stock[9], for the recent period is analyzed based on the information provided in Table I.1 (Appendix I). Accordingly, main weaknesses of the central government debt are the significant share of FX-linked debt and the short-maturity of the domestic debt stock. External debt stock of the central government, which constitutes 30 percent of the total central government debt stock as of the year 2004, mainly consists of program credits from international organizations and Eurobond issues. Its share has increased in 2002 due to IMF credits and decreased in the following period as a consequence of the preference of the debt management authorities for domestic borrowing over foreign borrowing. As of 2004, foreign debt and the FX-linked portion of the domestic debt stock taken together constitute 44% of the total debt.

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[9] Structure of the government debt stock is analyzed on the basis of the central government debt stock. Major segment of the public sector debt stock is held by the central government, thus sensitivity of the debt stock with respect to the movements in the interest rate and the exchange rate can be captured by the evaluation of the central government debt stock.

In terms of its maturity external debt raise no concerns; as of 2004 duration of the foreign bond stock was 4.3 years. The increase in the FX-linked domestic debt observed in 2001 was the consequence of the debt issued within the framework of the banking sector operation and the IMF credits that were used under the standby program. An additional factor was the loss in confidence in the Turkish lira, which directed the government to borrow from the markets in FX denominated/indexed securities.

Domestic debt stock comprises securities denominated in foreign currency, fixed rate securities- mainly discounted bonds- and securities indexed to the short-term interest rate, the inflation and the exchange rate. As mentioned previously, weak spot of the domestic debt stock has been its short maturity. In year 2001, maturity of the overall debt stock has increased as the non-cash securities with relatively high maturity were issued under the framework of the aforementioned banking sector operation. In the period following 2001, maturity of the domestic debt declined, approaching the maturity of cash debt stock, as the share and remaining maturity of non-cash debt stock decreased. This trend will continue in the coming years and the maturity of cash borrowing will determine the maturity of the domestic debt stock. It should be emphasized that, non-cash borrowing takes place under special circumstances and the regular means of financing debt service is via cash borrowing from the markets. Therefore, although maturity of the total domestic debt stock reflects the risks faced by the government, the maturity of cash debt stock and cash borrowing are better indicators for evaluating the marginal improvements in the maturity structure. Cash borrowing is carried out mainly in TL denominated discounted securities in the maturity range of 3 to 20 months and two year floating rate securities with quarterly coupon payments. Maturity of cash borrowing has been increasing as of 2002. Although the maturity of borrowing has been extending it did not suffice to extend the maturity of cash debt stock. In the 2001-2004 period, significant steps have been taken towards achieving economic stability, consequently this period enabled improvement in terms of effective execution of the borrowing policy. The improvement in the borrowing maturity is notable than the figures implied by the maturity of cash borrowing due to the change in the borrowing composition. Maturity in discounted auctions, which constitute significant portion of cash borrowing, has lengthened significantly. Despite this improvement, as the composition of cash borrowing shifted towards discounted securities, whose maturities are shorter than those with floating rate bonds, the extension in the borrowing maturity remained limited. Despite the progress witnessed in the recent period, existing maturity of the domestic debt stock is short. Duration of the TL-denominated cash debt stock, which increased to 6.6 percent in 2004, highlights the seriousness of the issue. In this context, increasing the maturity of borrowing is a major issue of the debt management strategy.

Recently stochastic simulation based risk analysis has been adopted to compare cost and risk characteristics of alternative borrowing policies and to develop a strategic benchmark policy. Risk is measured as Cost-at-Risk. Treasury does not announce borrowing benchmarks, that is risk limits, in terms of duration targets or fixed rate share for the debt stock. However, within the framework of strategic benchmark practice, determined on account of the cost and risk analysis, announces borrowing objectives. For the year 2004, objectives were defined as to raise funds mainly in TL, to use fixed rate TL instruments as a major source of domestic borrowing and to increase average maturity of domestic borrowing over a year taking into account market conditions.

III. The Stochastic Model

Stochastic model, in which random variables that affect the debt cost are modeled, comprises the first module of the simulation model. This is the part where source of randomness is introduced into the simulation framework. Random variables to be modeled are determined by the structure of the debt stock and the scope of the analysis. In the literature, these models range from simple stochastic processes (Valendia, 2002) to parsimonious macro models (Bergström et al., 2002) depending on the structure of the debt stock and the scope of the analysis. In our study, set of borrowing strategies are compared with respect to the interest rate risk, hence our focus is on modeling the interest rates of different maturities. One way of doing this is the term structure or yield curve modeling.[10]

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[10] Methods other than term structure modeling were used in the models developed by the World Bank and the Swedish National Debt Office (SNDO). In the World Bank Model, interest rate along with other financial variables is modeled as a simple stochastic process (Valendia, 2002). Whereas in the SNDO model short term and long term interest rates are modeled under a parsimonious macroeconomic framework, in which short term rate with three month maturity is determined on the basis of a monetary policy rule that central bank assumes to follow, the Taylor rule and After that, to obtain the long-term rate, in the first version of the model, spread between the three-month rate and the ten-year rate is modeled as a regime switching autoregressive process (Bergström and Holmlund, 2000). Whereas in the extended model nominal long-term yield is modeled on the basis of real return requirement, which depend on its lagged values and the capacity utilization in the economy (Bergström et al., 2002). The

Yield curve or term structure of interest rates is the set of interest rates for different maturities. Term structure model to be used in the risk management analysis need to fit to cross-sectional set of observations and at the same time capture the inter-temporal dynamics of the term structure of interest rates. In order to serve this purpose, for modeling and simulating the yield curve, we use the dynamic framework proposed by Diebold and Li (2002). In this framework three factor Nelson-Siegel (1987) yield curve model is used to fit the yield curve in each period. After that, each factor of the yield curve is estimated as an autoregressive model, and the yield curve is forecasted by forecasting the factors. Nelson-Siegel(1987) model that is utilized in our risk analysis framework does not belong to the class of dynamic yield curve models used in sovereign risk analysis (Bolder (2002), Danish National Bank (1998, 2001)), infact it is a static model. However under Diebold and Li(2002) approach the model is structured in a dynamic framework.[11]

Nelson-Siegel Yield Curve Model

Nelson-Siegel (1987) term structure model is capable of producing humped, monotonic and S-shaped yield curves using four parameters.[12] In this approach, initially forward rate function is drawn as a solution to a second order differential or a difference equation of the form represented in equation (1), where *r(m)* is the instantaneous rate at maturity *m*.[13]

$$r(m) = \alpha_1 r(m-1) + \alpha_2 r(m-2) + \alpha_0 \tag{1}$$

The forward rate, solution to the difference equation for the case of equal roots is:

$$r(m) = \beta_0 + \beta_1 \, exp(-m/\tau) + \beta_2 \left[(m/\tau) \, exp(-m/\tau) \right] \tag{2}$$

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nominal yields for the maturities in between the three-month rate and the ten-year rate are obtained through interpolation.

[11] In the early version of the sovereign risk model developed by Danish National Bank (DNB), Nelson Siegel yield curve model was utilized, however not in a dynamic framework as in Diebold and Li (2002). In the DNB framework, initially, historical yield curves were estimated for each time increment using Nelson-Siegel model. Afterwards, the curves that will be used in the simulation were randomly chosen from the estimated yield curves (Danish National Bank, 1998).

[12] Nelson-Siegel model is capable of generating curves with one hump. Svensson (1994) model is the extended version of the Nelson-Siegel model and it is capable of fitting yield curve shapes with two humps or u-shapes.

[13] Instantaneous rate is the rate of interest with an extremely short maturity.

Yield to maturity on a bill, denoted *R(m)* is the average of the forward rates *r (m)*,

$$R(m) = 1/m \int_0^m r(x)dx$$

$$R(m) = \beta_0 + (\beta_1 + \beta_2)[1 - exp(-m/\tau)]/(m/\tau) - \beta_2 \, exp(-m/\tau) \tag{3}$$

$\{\beta_0, \beta_1, \beta_2, \tau\}$ are the parameters of the model and *m* stands for maturity. In these functions, τ is a positive constant determined by α_1 and α_2; β_0, β_1, β_2 are constants determined by the initial conditions on the forward rate. The parameter τ determine the rate at which the terms *exp(-m/ τ)* decay to zero. Limiting value of *R(m)* and *r(m)* as *m* approach to infinity is β_0 and as m approach to zero is $(\beta 0 + \beta 1)$.

Figure 3.1.Components of the Forward Curve

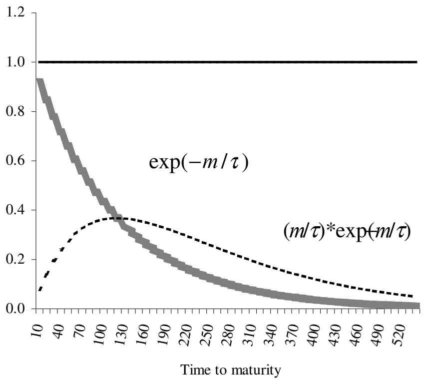
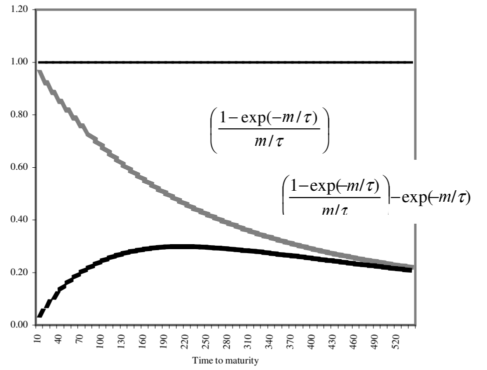


Figure 3.2. Components of the Yield curve (Diebold-Li Specification)



Note: Paths of the forward rate components are computed using τ value equal to 121, which is the average τ value for our sample period.

Forward rate curve (equation 2) is composed of long term, short term and medium term components. Long-term component, weighted with β_0, is a constant and does not decay to zero in the limit. Short-term component, weighted with β_1, starts at value one and decays to zero and the medium-term component, weighted with β_2, starts out at zero and decays to zero (Figure 3.1). At the point where medium component is maximized maturity is equal to the value of τ. Hence, τ specifies the position of the hump on the curve. Weight of the medium term component, β_2, determines the magnitude and direction of the hump. If β_2 is positive (negative), hump (u-shape) will occur at τ (Bolder and Streliski, 1999). Given these characteristics of the yield curve components, with the appropriate choice of parameters, which are weights for the components, Nelson-Siegel model is capable of generating shapes including humps, S shapes, and monotonic curves.

Diebold and Li (2002) propose different interpretation to the parameters of the slightly modified specification of the Nelson-Siegel model. Re-specified model is given below.

$$R(m) = \beta_0 + \beta_1\left[\frac{1-exp(-m/\tau)}{(m/\tau)}\right] + \beta_2\left[\frac{1-exp(-m/\tau)}{(m/\tau)} - exp(-m/\tau)\right] \tag{4}$$

Model parameters β_0, β_1, β_2, which were defined as weights for the long term, short term and medium term components of the yield curve in Nelson-Siegel (1987) are now interpreted as level, slope, curvature factors. In equation (4) values multiplied by parameters β_0, β_1, β_2 are defined as loadings. In that respect loading of β_0 is constant, 1, in the limit it doesn't approach to zero. Increase in β_0, increases all yields equally, hence β_0 control the level of the yield curve. Loading of β_1, denominated as $\left(\frac{1-exp(-m/\tau)}{(m/\tau)}\right)$ starts at value one and approaches to zero in the limit. Therefore β_1 is evaluated as the short-term factor. Short-term factor controls the slope of the yield curve. Increase in β_1 increases short yields relatively more compared to long yields, consequently changing the slope of the yield curve. Loading of β_2, $\left(\frac{1-exp(-m/\tau)}{(m/\tau)} - exp(-m/\tau)\right)$, starts at zero, increases with maturity and after a point starts to decrease again. Hence, β_2 is interpreted as the medium term factor. And medium term factor is related to the curvature, given that change in β_2 will have little effect on the very short and very long yields, but will affect medium term yields. Thus change in β_2 will alter the curvature of the yield curve. These interpretations were supported by the calculations of Diebold and Li (2002) using their database, where they have demonstrated that level, slope and the curvature of the yield curve were affected in major part by the long-term, short-term and the medium term components respectively. Therefore β_0, β_1, and β_2 were treated as level, slope and curvature of the yield curve.

In figure 3.2 yield curve loadings that prevail in our analysis are plotted.[14] Thus, figure 3.2 displays the loadings that prevail in our analysis for the average τ value of the sample. Even though loadings of β_0 and β_1 enable interpretation of these factors as level and slope, path displayed by the loading of β_2, given the short maturity range, necessitates modification on the interpretation of the medium term factor. It is observed that the loading of β_2 after reaching a maximum decays back to zero at a slow pace. Change in β_2 does not only effect the medium term but also long term yields.

Nelson-Siegel method provides a static curve fitting tool. Diebold and Li approach has given a dynamic framework to the model by interpreting yield curve factors as time varying variables. As a result yield curve equation (4) turns into a dynamic equation. Specification of the simulation framework that is used in our analysis is given below:

$$R(m) = \beta_{0_t} + \beta_{1_t}\left(\frac{1-\exp(-m/\tau)}{(m/\tau)}\right) + \beta_{2_t}\left(\frac{1-\exp(-m/\tau)}{(m/\tau)} - \exp(m/\tau)\right), \text{ t=1, ..,60} \tag{5}$$

$$\left.\begin{aligned}
R\beta_{0_t} &= \delta_0 + \delta_1 R\beta_{1_{t-1}} + \varepsilon_t^{R\beta_0} \\
\beta_{1_t} &= \eta_0 + \eta_1 \beta_{1_{t-1}} + \varepsilon_t^{\beta_1} \\
\beta_{2_t} &= \gamma_0 + \gamma_1 \beta_{2_{t-1}} + \varepsilon_t^{\beta_2} \\
\beta_{0_t} &= \left[\left(1 + {}^{R\beta_{0_t}}\!\!\diagup_{100}\right)\left(1 + {}^{\pi_t^{\,e}}\!\diagup_{100}\right) - 1\right] * 100
\end{aligned}\right. \qquad , \text{ t=1,…,60.}$$

where, $\pi_t^{\,e}$ is the expected inflation.

Residuals of the above equations have the following specification:[15]

$$\begin{bmatrix} \varepsilon_t^{R\beta_0} \\ \varepsilon_t^{\beta_1} \\ \varepsilon_t^{\beta_2} \end{bmatrix} = \begin{bmatrix} \sigma_{R\beta_0} \\ -\sigma_{\beta_1} \\ \sigma_{\beta_2} \end{bmatrix} * v_t \text{ , t=1, …, 60.}$$

where, $\nu_t \sim N(0,1)$

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[14] Yield curves are plotted for τ=121, and maturity range of 10 to 540 days.

[15] Residual of the slope equation is taken as the negative of the random shock νt on account of the negative correlation of the slope coefficient with the level and the curvature (Appendix IV, Table IV.1).

β_{0t}, β_{1t}, and β_{2t} are the time varying counterparts of β_0, β_1, and β_2 parameters respectively.[16] In order to forecast yield curves, Nelson-Siegel factors are estimated as univariate autoregressive processes. Random variable ν_t is the source of randomness in the simulation model.

IV. Strategy Simulation

Second part of the simulation model is the strategy simulation, in which under each borrowing strategy debt is rolled over on top of simulated term structure of interest rates. Strategy simulation is the core of the stochastic simulation analysis. In each step, role of the strategy simulation is to determine the amounts that will be borrowed at different maturities. Moreover, strategy simulation also keeps track of the debt cost and the debt service (principal and interest payments) of each period.

In sovereign risk analysis, there is no common framework for strategy simulation. Methods applied by the practitioners are shaped by the debt management practice and objectives. A simple simulation framework is to assume exogenous or zero government budget balance and roll over the maturing debt under a static financing strategy defined in terms of vector of fixed weights as in Hahm and Kim (2003). The level of complexity of the simulation framework could be extended in various ways. One extension is to include debt buybacks as in the simulation framework developed by Swedish National Debt Office (Bergström et.al , 2002) and Danmarks National Bank (Danish Government Borrowing and Debt 1999). Bolder (2003) contains a comprehensive strategy simulation framework, which is based on a stochastic model where the evolution of the term structure of interest rates, macroeconomic business cycles and government's financial position are jointly modeled. Distinguished feature of the model is that it takes into account the effect of financing strategy on government's financial position and interest rates.

Under strategy simulation, borrowing strategy and the initial portfolio choice are the determined by the practitioner. Since these variables are under control of the practitioner, they are referred to as control variables. There are two possible initial portfolio alternatives. One of them is to use the actual portfolio and its maturity structure. Drawback of this approach is that starting with the actual portfolio can make

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[16] In the analysis, level coefficient β_{0t} is non-stationary. Therefore we use the level coefficient deflated by inflation, defined as $R\beta_{0t}$

the results from different debt management strategies less definite, since starting from a common portfolio will influence the results. Using the actual portfolio will become less of a concern if the maturity of the actual debt stock short and the period of the analysis is long. The other alternative is to work with steady state portfolio. Initial portfolio is in steady state if the proportions of debt instruments in the overall portfolio are identical to the weights of the borrowing instruments in the financing strategy vector. Steady state portfolio is preferred to actual portfolio when the motivation of the simulation analysis is comparison of the long-term cost and risk characteristics of different debt portfolios rather than moving from one portfolio to the other (Bolder, 2003, Bergström et al, 2000). However use of steady state portfolio disregards the cost and time required to transform actual portfolio to the steady state portfolio.

Along with the initial debt portfolio, the other control variable is the financing strategy. Financing strategy indicates how much of the borrowing requirement to allocate among borrowing instruments. Financing strategies can be formulated in various ways. The strategy could be defined in terms of duration target or a target for allocation of debt stock between different denominations as in Bergström and Holmlund (2000) and Bergström et al. (2002). Alternatively, financing strategies could be defined in terms of a fixed borrowing structure, such as fixed vector of weights allocated to each debt instrument (Bolder, 2003 and Hahm and Kim, 2003). A common practice is to work with predetermined strategies. In other words, decision on which borrowing instrument to issue or the duration of the debt stock is not conditional on the realization of the random variables affecting the cost of the debt stock. Public debt management practice is assumed to be exercised within a transparent and predictable manner. In that respect, government adheres to the predetermined borrowing policy, despite fluctuations in the random variables

Within the strategy simulation framework utilized in this paper, the actual initial domestic currency denominated debt portfolio and the implied maturity structure is used. Since maturity of the debt stock is short, drawback arising from the use of actual portfolio is not expected influence the results in a significant magnitude. Every strategy is run for 1000 simulated interest rate paths and each simulation is run for 5 years in monthly steps, that is 60 periods. At each step government finances net borrowing requirement in line with the predetermined financing strategy defined as fixed vector of weights allocated to debt instruments. Net borrowing requirement of the government consists of maturing debt, interest payment and the primary (non-interest) deficit.

Strategy simulation starts with an initial debt portfolio represented in terms of a maturity matrix of dimension 60 by 2. The information embedded in the maturity matrix is the debt service of the subsequent 60 months. Each row of the matrix corresponds to the time increments, i.e. months. Principal payments and the interest payments are located at the first and the second column respectively. For each financing strategy simulation is repeated for 1000 times. In each simulation run debt portfolio is rolled over for 5 years in monthly steps. In each step following tasks are performed;

1. Borrowing requirement for the current month is calculated as the total debt service (interest + principal) minus the primary budget surplus of the government.[17] Debt service is the sum of the first row of the maturity matrix.
2. Once the debt service of the current month is calculated, first row of the maturity matrix is eliminated and a new row of zeros is added to the end of the matrix. Thereby, size of the maturity matrix is kept unchanged and it is prepared for the subsequent step. After these arrangements first row of the matrix now represents debt service of the next period
3. Total borrowing requirement is distributed among borrowing instruments in accordance with associated weights.
4. Nominal and real cost are computed.
5. Next, accrued interest and principal payments are distributed over the related cells in the maturity matrix.

Steps of strategy simulation along with the MATLAB codes are provided in the Appendix II.

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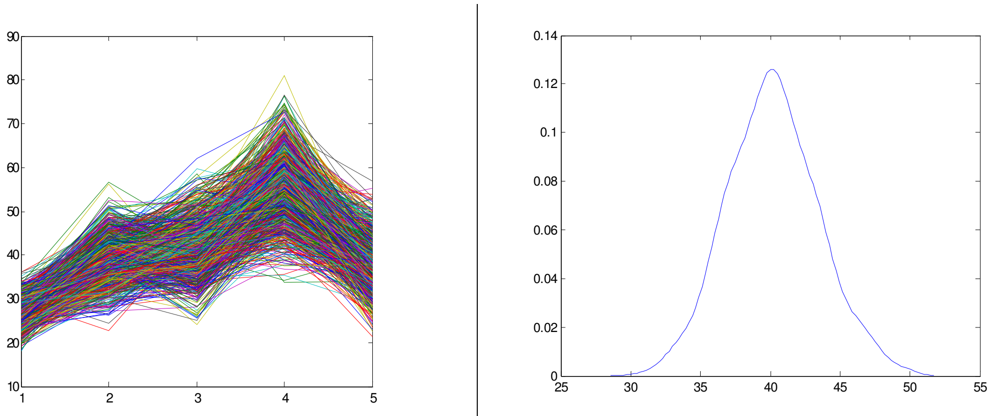
[17] Primary budget surplus is exogenous to the model. It is calculated as 5 percent of the GNP of the year. Our assumption is that 50 percent of the primary surplus will be used to finance debt service arising from the TL denominated portion of the debt stock.

Figure 4.1.Illustration of the Simulation Output



Under each financing strategy, simulation provides an output in the form of cost distributions for each month. Figure 4.1 displays an example simulation output. The average cost distribution (right panel) is obtained by taking the average of each cost path (left panel). In other words, 1000 simulated paths are summarized into a single distribution representing average cost. The expected cost of the strategy is then defined as the median (50th percentile) of the average cost distribution. In the remaining of this section cost and risk definitions are clarified.

Cost measure covers the interest expenditure of discounted securities and coupon bonds. It is measured under cash flow basis and it is recorded under accrual terms as defined by the ESA95 criteria[18]. Under cash flow basis costs only occur when money is paid out, thus under this framework mark-to-market effects stemming from varying interest rates are not included. Besides the coverage and the type of measuring, another issue is the recording period: cost can be recorded in period it is paid or in period it belongs. Under the latter approach, for each bond total cost is distributed over its existence period. Thus, cost over a given period is limited to the cost of bonds that fall within the period considered. Measuring cost in accrual terms provides a better comparison of alternative strategies in a specified period. To give an example, suppose

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[18] ESA95 Manual on General Government deficit and debt http://www.imf.org/external/bopage/pdf/99-35.pdf

that costs are recorded in the period payment is made in a risk analysis with one-year simulation horizon. In this case bonds with maturity higher than a year will have no interest cost. As a result, short-term debt will appear more costly.

Under stochastic simulation framework two dimensions of risk can be specified: the scenario risk and the time series risk (Bergström et al., 2002). In this paper alternative financing strategies are compared on the basis of scenario risk, which is the risk that overall debt cost will exceed a certain amount in a specified period. [19] Once the average cost distribution is obtained scenario risk can be calculated using various risk measures. These measures comprise absolute Cost-at-Risk, relative Cost-at-Risk (Bolder, 2003) and relative risk (Bergström and Holmlund, 2000). In the following, we describe these measures one by one.

Absolute Cost-at-Risk (ACaR) is the largest amount of government debt cost over a given time horizon that is not exceeded with probability 1-p, using statistical terminology $P(X \leq \text{CaR}) = 1 - \text{p}$. When p is set to 0.01, absolute CaR is the 99th percentile of the debt cost distribution, implying that absolute CaR is not exceeded by 99 percent of the debt cost observations. Relative CaR (RcaR) is the distance between absolute CaR (99th percentile) and the median of the distribution. Comparative analysis of alternative financing strategies based on ACaR and RCaR entangle a bias. When these measures are used, expensive financing strategies also appear more risky compared to financing strategies dominated by less costly short-term borrowing.

Relative risk measure proposed in Bergström et al. (2002) is similar to RCaR. Instead of the absolute deviation, percentage deviation is used to overcome the abovementioned bias associated with RCaR. Relative risk measure is computed as the relative distance between the 99th and the 50th percentiles of the simulated distribution:

$$\text{Re}\,lative\ risk = \frac{P99(Cost)}{P_{50}(Cost)} - 1$$

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[19] Time series risk defines another risk dimension, which is the variability of costs between the years

Bolder (2003) proposes an alternative risk measure that incorporates time dimension into the risk analysis. Proposed risk measure is calculated from a conditional debt cost distribution. In this method debt cost is estimated as an autoregressive time series model. A forecast error from the model that is captured by conditional volatility provides a notion of risk. This measure provides a measure of uncertainty relating to the debt charges of the subsequent period given debt charge of the current period.

V. Emprical Analysis: Yield Curve Estimation for Turkey

In this part of the analysis, three factor Nelson-Siegel model is estimated for each month of the period June 2001-July 2004 using Turkish secondary market data for government securities.[20] Afterwards the extracted level, slope and curvature components of the yield curve are modeled as autoregressive processes. Then, we proceed to the simulation of these factors and thereby the yield curves for the 60-month period into the future.

Data

Data used in the estimations are monthly continuously compounded yields and their corresponding maturities. Yields are calculated from end-of-month prices of the government securities from the Turkish Secondary Government Securities Market, from June 2001 through July 2004. Data is obtained from daily bulletin of the Istanbul Stock Exchange (ISE). Only discounted securities are used in the estimations which for the 1992-2004 period comprise on average 95 percent of the secondary government securities market in Turkey (Alper et al., 2004b). Securities with a maturity of less than a month are excluded from the estimations, hence minimum maturity is 30 days.[21]

Fitting the Yield Curves

We fit the yield curve using Nelson-Siegel Model (1987) as in equation (4). Nonlinear least squares and ordinary least squares (OLS) estimation methods are used in a complementary framework for estimating the yield curve parameters {β0, β1, β2, τ}.[22]

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[20] Yoldaş(2002) and Alper et al.(2004a) have estimated yield curves in Turkey using Nelson-Siegel method with secondary market government securities data.

[21] Data filtering is done in a similar manner in Yoldaş(2002) and Alper et al.(2004a). In Yoldaş(2002) floating rate bonds, coupon bonds, inflation linked bonds and T-bills with time to maturity of less than a month were excluded from the sample. In Alper et al.(2004), data sample includes discounted securities with time to maturity greater than ten days.

[22] Different methods have been used to fit the yield curves using Nelson-Siegel Model. Yield curve equation (Equation 1) becomes a linear model when the value of τ is given. Thus for a given value of τ, remaining parameters of the model can be estimated using ordinary least squares (OLS). Hence, in Nelson-Siegel (1987), best-fitting values of the yield curve parameters is found by repeating OLS estimation over a grid of values for τ. Diebold and Li (2002) uses nonlinear least squares estimation method. In Bolder and Streliski (1999) Nelson-Siegel curves are not determined through statistical estimation but in pure optimization framework.

Initially the yield curve is estimated using non-linear least squares for various initial τ values in a range of 30 to 100. Given the initial values, two different results are identified; i) non-linear estimation converges to a single solution, ii) non-linear estimation converges to multiple results, more specifically low (high) initial τ values converge to results in which estimated τ value is low (high).[23] Estimation procedure is finalized if a statistically and economically significant single result is obtained. If the parameters of the single solution are not significant OLS estimation is carried out over a grid of values for τ, and the value of τ that provides the best fit among the significant parameter estimates is chosen. In the latter case, where there are multiple results, initially we chose the estimated solution with the lowest sum of squared residuals (SSR). Then, if the non-linear estimation results are of the high (low) τ value we proceed by decreasing (increasing) τ by increments of 10 and estimating the yield curve trough OLS until significant results are obtained.[24] Estimation results are presented in Appendix III. Estimated parameters are all statistically significant for each month of the Jun 2001-July 2004 period. In the estimations we control for the correlation, heteroscedasticity and normality in the residuals. Diagnostics of the yield curve equations reveal that assumptions related to the disturbances hold except for few dates where the heteroscedasticity and/or the normality assumptions are not satisfied. For the estimated parameters t-statistics are computed using White-Heteroscedasticity consistent covariances.

Along with statistical significance we impose constrains on the values that β_0 and β_1 can take on account of the economic interpretation attached to these parameters. When maturity approaches to infinity and zero, spot yield is given by β_0 and $\beta_0+\beta_1$ respectively. Thus constraints $R(0) \geq 0$ and $R(\infty) \geq 0$ which correspond to $\beta_0+\beta_1 \geq 0$ and $\beta_0 \geq 0$ apply. In our estimations we impose an additional constraint on β_1, which is the negative of the slope of the yield curve. β_1 is constricted to be negative (positive) when curve had upward (downward) slope. When the curve is upward sloped, this constraint implies that as maturity increases yield will not fall below the shortest yield.

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[23] We initially estimated yield curves by excluding securities with remaining maturity of less than 10 days. In that case we observed a higher tendency for small τ values to converge to solutions with small values of τ.

[24] Existence of heteroscedasticity in the error terms necessitates use of heteroscedasticity consistent error terms. These statistical issues were taken into account when deciding upon the statistical soundness of the estimated curves.

In the yield curve estimation, value of τ affects the fit of the curve, thus it is an important choice. Value of τ determines the location of the hump in the forward curve (and thereby the yield curve).[25] Small values of τ correspond to rapid decay in the regressors and therefore the estimated curve will be able to fit curvature at low maturities well while being unable to fit excessive curvature over longer maturity ranges. Likewise, large values of τ result in slow decay in the regressors, in which case the estimated curve can fit curvature over longer maturity ranges but it will be unable to follow extreme curvature at short maturities (Nelson-Siegel, 1987). Average τ value for our sample is 121. Given that our sample is in the range of 30-523 days, average value corresponds to the 23 percent of the upper limit of the maturity range.[26]

Our concern is not static curve fitting that is to estimate the best fitting curve at one point in time but to obtain consistent values for the model parameters so as to capture the dynamics of the yield curve and to obtain plausible forecasts for the future periods. Value of τ that provide the best fit could vary considerably and taking different values for τ each period could lead to fluctuating parameters and little gain in precision. In order to overcome fluctuations in estimated parameters Nelson-Siegel (1987) and Diebold and Li (2002) have used same τ value for the whole sample period. Nelson-Siegel has pointed out the value of τ is best chosen by fitting across data sets rather than by selecting the value for each individual data set. Employing the same approach in our study resulted in higher volatility in the parameters. Therefore we proceeded using the estimated parameters obtained from the solutions where value of τ provided best fit.

In Appendix IV, fitted curves are plotted with raw yields for some selected dates. For the period under consideration, estimated yield curves are mostly upward sloped and humped or concave. Downward sloping curve is observed only in December 2003. In general Nelson Siegel model provided good fit for the raw yield curves in our sample period.

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[25] At the point where medium term component of the forward curve is maximized value of τ is equal to the maturity (Figure 4.1).

[26] Estimated τ values are lower in Alper et al. (2004a) compared to our estimation results. In our opinion difference stems from the maturity range used in the estimations. Alper et al (2004a) includes maturities within the range of 10 to 30 days. We had initially estimated the yield curves by including securities with remaining maturity of greater than 10 days. In this case, when short maturities are included, nonlinear estimation converges to solutions with low τ values. The implication is that, at low τ values, estimated yield curve fits the curvature located in the short end of the curve. This leads to a bias, since curvature at the short end of the yield curve does not reflect the overall curvature of the curve.

Figure 5.1 Estimated yield curves for the period June 2001 – July 2004

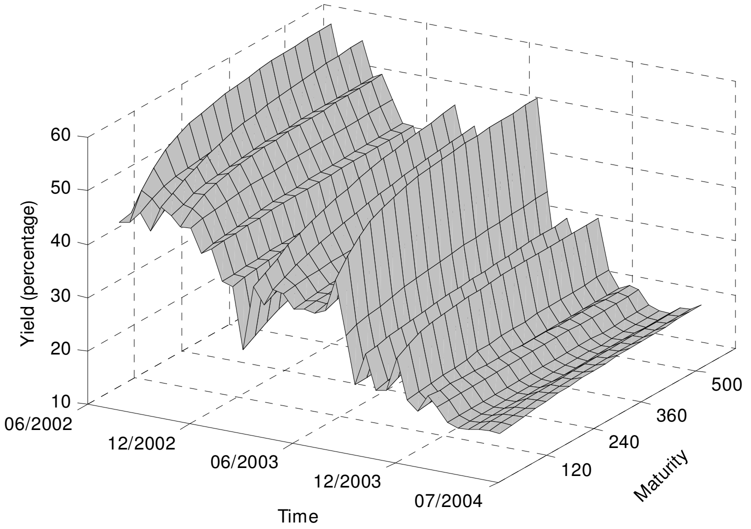

Table 5.1. Descriptive Statistics of the Estimated Yield Curve

Maturity (Days)	Mean	St. dev.	Min.	Max.	ρ (1)	ρ (12)
30	31.4	10.4	16.9	47.8	0.90	0.72
90	32.8	11.3	17.1	51.7	0.89	0.69
180	34.1	12.0	17.3	56.2	0.87	0.67
270	34.9	12.4	17.4	58.0	0.87	0.66
360	35.4	12.6	17.5	58.9	0.86	0.65
450	35.8	12.7	17.6	59.5	0.86	0.64
540	36.0	12.8	17.6	59.8	0.86	0.64
B_0	37.5	13.4	17.8	61.7	0.84	0.59
B_1	-7.3	6.2	-25.2	0.8	0.40	0.25
B_2	29.4	18.5	-10.1	85.4	0.48	0.41
τ	121.0	47.5	40.0	190.0	0.43	-0.13

Note: ρ(1) and ρ(12) are the 1st and the 12th order autocorrelations.

Graphical representation and descriptive statistics of the estimated yield curves are presented in Figure 5.1 and Table 5.1. In terms of volatility of the interest rates our estimation results depart from the typical observation that short end of the term structure is more volatile than the long end of the yield curve (Bolder, 2003; Diebold and Li, 2002). Our estimation results validate the contrary; volatility in the long end is higher compared to the short end. This results from the high and negative correlation among β_0

and β_1, such that variance of β_1 is less than the covariance of β_0 and β_1 in absolute terms (Appendix V).[27] Correlation of β_2 with the other two factors are low, whereas correlation between β_0 and β_1 is high implying that when there is a shock to the level component yield curve not only shifts but also its slope changes Table V.1 (Appendix V). Estimated yields and the estimated level factor, β_0, are persistent and highly variable relative to their mean. Compared to the level coefficient β_1 and β_2 are less persistent.

Time series plots of the yield curve factors are displayed in Appendix V. Level factor of the yield curve, β_0, which in the limit corresponds to the long-term interest rate, is plotted together with the primary market average compounded rate for discounted securities. Level coefficient displays a declining trend during the July 2001- July 2004 period, as interest rates are falling from considerably high levels that prevailed in the previous years due to the favorable economic environment established within the framework of the stabilization program that initiated following the 2000 and 2001 crises.

Declining path of the factor β_0 reveals non-stationary nature of the series for the sample period. ADF test suggest that for the given sample period β_1 and β_2 are stationary, whereas β_0 is non-stationary for the sample under consideration (Appendix V).[28]

Modeling Yield Curve Factors

Estimated yield curve factors are modeled as autoregressive processes (Table 5.2). However due to the non-stationary nature of the level coefficient, it is deflated with the expected inflation and the resulting deflated stationary series is modeled, which we refer to as real β_0 $(R\beta_0)$ (Appendix V). Since the level coefficient refers to the long-term interest rate, when deflating it, we imitated the Fisher equation, where reel interest rate is obtained by deducting inflation expectations from the nominal interest rate (Dornbush and Fisher, 1998).

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[27] Variance of the long-term yield is given by $VAR(R(\infty))=VAR(\beta_0)$ and variance of the short rate is given by $VAR(\beta_0+\beta_1) = VAR(\beta_0)+VAR(\beta_1)+2COV(\beta_0, \beta_1)$.Then $VAR(R(\infty))>VAR(R(0))$ when $VAR(\beta_1)+COV(\beta_0, \beta_1) <0$ that is $VAR(\beta_1)< -COV(\beta_0, \beta_1)$.

[28] Alper et al (2004b) finds all the Nelson-Siegel yield curve factors to be stationary for the period 1992-2004.

Table 5.2: Estimation Results of the Yield Curve Factors

	Equations*		
	$R\beta_0$	β_1	β_2
C	4.89 (4.37)	-2.73 (-3.11)	18.33 (5.01)
$R\beta_0(-1)$	0.3 (2.44)	-	-
$\beta_1(-1)$	-	0.38 (4.22)	-
$\beta_2(-1)$	-	-	0.37 (3.56)
D0107	-	-20.87 (-6.19)	-
D0109	-	-9.91 (-3.98)	-
D0206	-	-10.02 (-3.69)	52.48 (4.49)
D0207	11.52 (3.07)	-12.65 (-2.98)	-
D0303	15.41 (4.02)	-13.39 (-2.95)	-33.19 (-2.82)
D0309	-	-	-36.38 (-3.11)
R-squared	0.52	0.77	0.65
Std. dev. of residuals	3.52	3.02	10.83
	Diagnostics**		
Breusch-Godfrey Serial Correlation LM Test	2.51 [0.29]	0.30 [0.86]	0.81 [0.67]
White Heteroskedasticity Test	1.58 [0.81]	8.59 [0.28]	3.51 [0.62]
Jarque-Bera Normality Test	0.51 [0.77]	3.62 [0.16]	1.15 [0.56]

* In parenthesis are the t-statistics.

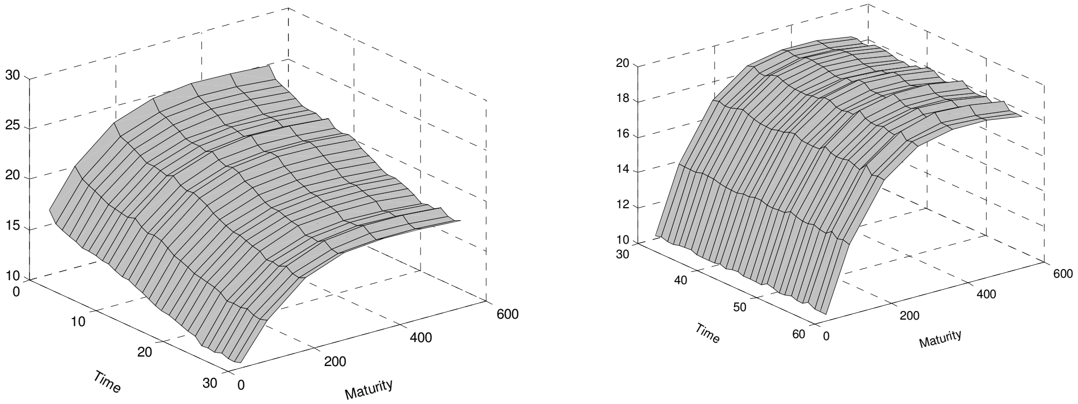
**In parenthesis are the probabilities.

Note: D0107, D0109, D0206, D0207, D0303, D0309 are the dummy variables. First two digits after the letter D stand for the year, following two digits stand for the month.

Yield Curve Simulation Framework and the Simulation Results[29]

Once the factors are modeled, yield curve simulation is exercised as in the simulation framework presented in section III, equations (5)-(9).[30] During simulation, level coefficient is derived from the simulated real level series, given the assumptions regarding the inflation expectations that will prevail for the simulation horizon. On account of the declining inflation, expected annual inflation is assumed to be 8 percent for the end of year 2005 and 5 percent for the remaining period. Initial values for the parameters are taken from the final yield curve equation estimated for July 2004. Simulation is carried out for 1000 times over 5 years in monthly steps, that is 60 periods. In each simulation run yields for 3, 6, 9, 12, 15, 18 months maturities are calculated. Consequently, for each period 1000 yield curves are simulated.

Figure 5.2 Average Simulated Yield Curves for the 2005-2009 Period



Average of the simulated yield curves and their descriptive statistics are plotted in Figure 5.2 and Table 5.3 respectively. Average simulated yield curve in major part possesses the characteristics of the average estimated curve. Simulated curve is upward sloped and concave, as the average estimated curve. However unlike the estimated curve, simulated curve has a steeper short end and the long end of the simulated curve slopes downwards. Yields display declining trend till the 24th period, afterwards follow

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[29] For yield curve simulation, MATLAB Software is used.

[30] Residual of the slope equation is taken as the negative of the random shock ν_t on account of the negative correlation of the slope coefficient with the level and the curvature (Appendix IV, Table IV.1).

a stable path. This results from fixing the expected inflation at 5 after 2006. Volatility of the simulated yields is lower compared to the estimated period, however volatility in the long end is still higher than the volatility in the short end of the curve. Moreover, persistency of the simulated yields has declined compared to the estimated yields for the June 2001-July 2004 period. Another prominent feature of the simulated yields is that the yields with a maturity of 12 months and greater have similar characteristics in terms of volatility and persistency (Table 5.3). Thus characteristics of the yield curves that will be used in the debt simulation exercise can be summarized as follows; i) Yield curve is on the average upward sloping and concave. Long-term yields are higher than short term yields. However after maturity of 12 months, yields slightly decline, ii) Long term yields are more volatile compared to short term yields, iii) Yields decline for two-year period until the year 2007 and thereafter follow a stable path.

Table 5.3. Descriptive Statistics of the Simulated Yield Curve

Maturity (Days)	Average	St. dev.	Min.	Max.	ρ(1)	ρ(12)
30	11.2	2.4	6.2	18.3	0.52	0.19
90	14.9	3.8	5.8	24.8	0.38	0.05
180	18.0	5.1	5.3	31.0	0.35	0.02
270	19.4	5.8	5.0	34.0	0.35	0.01
360	19.8	6.1	4.7	35.1	0.35	0.00
450	19.8	6.2	4.4	35.3	0.35	0.00
540	19.5	6.1	4.2	34.8	0.35	0.00

Note: ρ(1) and ρ(12) are the 1st and the 12th order autocorrelations

VI. Illustrative Results of the Risk Analysis

In this section results of the strategy simulation are presented. As mentioned previously market risk is only one of the debt management objectives. Hence results obtained under the risk analysis framework in this paper are not the final words on the design of the financing strategy. Instead it provides a tool for debt management to evaluate cost and risk characteristics of alternative debt structures. Moreover it is limited to the domestic currency denominated portion of the debt stock and considers only the risk associated with the movements in the interest rate. Thus, results obtained from the stochastic simulation analysis should be evaluated under this framework.

Under the strategy simulation framework, financing strategies are defined as static vector of weights assigned to each borrowing instruments. There are 12 borrowing strategies formulated using 14 different borrowing instruments: 6 discounted securities with maturity of 3 to 18 months and 8 coupon bonds. Coupon bonds are formulated using different combinations of the following three features: i) maturity (2 year, 3 year), ii) coupon type (flexible, fixed) and iii) coupon period (quarterly, semiannually). Interest rates to be applied to the discounted bonds are computed directly from the simulated yield curves. On the other hand, coupon payments are computed using the information provided from the yield curve with some additional assumptions, due to the restriction posed by the short maturity range of the estimated yield curves. Estimations were carried out using discounted bonds with a maturity range of 30-523 days. Therefore, on account of the short maturity range, yields greater than 18 months are not forecasted from the estimated yield curve. Specifically, coupon rate is computed over the simulated 3 or 6-month yield with an additional return that is assumed to reflect investors risk perception for increased maturity.[31] In a financing strategy that involves a coupon bond, 15 percent of the total borrowing is restricted to be of the discounted bond with maturity equal to the coupon period. Reference interest rate for the coupon rate, which is the interest rate of the discounted bond with maturity equal to the coupon period, is set three or six months before the time of the coupon payment. This is inline with the current practice in debt management in Turkey.

In the remaining of this section, financing strategies are compared with respect to their average cost and risk characteristics. In Figure 6.1 and Figure 6.2 results from the simulation of the twelve strategies are shown.[32] Each strategy is located as a point on the cost and risk surface. Cost is expressed both in nominal terms, in terms of domestic currency and in real terms as a share of GNP. Real cost measure attributes higher relative weight to the interest expenditure that accrues in a shorter period of time and thus provides more adequate assessment of the debt burden on government budget.[33] Each of the risk measures contains different information. ACaR provides a

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[31] Additional return is assumed to be 1.5 percentage points for all types of coupon bonds, irrespective of the maturity and the coupon period.

[32] Computed cost and risk measures, both in real and nominal terms are real and nominal terms are also presented in Table VI (Appendix VI) and Table VII (Appendix VII) respectively.

[33] When the cost is measured in nominal terms, equal weight is attributed the interest payments that accrue in different times from now. Given this equal weighting scheme, comparison of alternative strategies by means of nominal cost would be biased as the same amount of debt service at different dates

measure to assess the amount of maximum total cost government would have to undertake at the specified probability, whereas RCaR and Relative Risk measures focus on the risk that government debt cost will be significantly higher than expected. The higher the 99th percentile value or the deviation from expected cost, higher is the risk of a strategy. Given our motivation to consider the cost and risk that are actually affecting the budget, main focus of the analysis is on real cost and Absolute CaR(ACaR) measure (Figure 6.2).

Expected average real cost associated with financing strategies range from 6.8 % of GNP in strategy 1 to 11.2 % of GNP in strategy 8. This 4.4 % average cost range per annum indicates that considerable cost savings can be accomplished by the appropriate choice of a strategy. Major remarks of the analysis can be summarized as follows:

1. Strategies are clustered in to three groups; discounted securities, bonds with quarterly and semiannual coupon payments. Clustering is more apparent in terms of the relative risk measure in percentage deviations, which ony reflects volatilitiy of the interest rates (Figure 6.2.c).In real terms, discounted securities have the lowest cost and ACaR. In nominal terms however cost of discounted secutities with a maturity greater than 9 months overlap with that of cuopon bonds (Figure 6.1).
2. A prominent observation is that, excluding the case where risk is expressed as percentage deviation from expected cost, there are no cost and risk trade-offs among the strategies. There are two main reasons underlying this result. First of all, along the yield curve within the 3-15 month maturity range, volatility of the yield increases with maturity. Secondly, when risk is measured in absolute terms (ACaR) or as absolute deviation (RCaR) scenarios with high cost also appear to be more risky. Therefore under these biased risk measures, high cost associated with coupon bonds dominates. As a result of this phenomenon, even though discounted securities possess the highest risk in terms of relative risk and relative CaR, because they are the least costly strategy group they are characterized with lowest Absolute CaR. On the other hand, in terms of relative risk measure, which only reflects the volatility in the interest rates, there appears to be a tradeoff between borrowing strategies containing discounted securities

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in the future doesn't correspond to equal amount of burden from the governments perspective, since governments' capacity to service debt increases through time along with economic growth.

and coupon bonds. Shifting from former to the latter increases the cost but reduces the risk. Relative risk of quarterly coupon bonds falling under this group is around 10 percent. Implying that at 1 percent significance level, average cost for the period 2005-2009 can be over 10 percent of the expected cost. Second group of strategies with semiannual coupon bonds have higher risk; in the range of 12-14 percent of the expected cost. Lastly, risk of discounted securities lie within the range of 14-16 percent. This risk scale, reflects the volatility structure of the yield curve. Costs associated with coupon bonds are exposed to volatility in either three or six month yield. On the other hand, discounted bonds, which are issued in the maturity range of 9 to 18 months, are exposed to higher volatility.

3. Inline with our expectations, floating-rate bonds are less costly compared to fixed couponed bonds. Under an economic environment of declining interest rates, fixing the coupon rate at the time when bond is issued is a factor increasing the cost of a strategy. Floating-rate bods are also less risky compared to fixed rate bonds. However, one would expect the contary since floating-rate debt exposes debt stock to fluctuations in the interest rates. Results of the analysis reveal that this fact doesn't have a dominant effect in our analysis. In fact, this counterintuitive result might be explained by the fact in the model interest volatility doesn't change over time. The risk gap between fixed and flaoting rate bonds increases in size for RCaR and ACaR, since for these measures strategies with high cost appear to have higher risk.
4. Strategies with semiannual coupon payments are more risky compared to securities with quarterly payments. When cost is measured in real terms, for floating rate bonds, risk difference between bonds that differ only with respect to coupon period are 0.5, 0.5 and 5.45 percentage points in terms of ACaR, RCaR and relative risk respectively.
5. There is not significant cost and risk difference among floating rate strategies of different maturities. Within strategies including fixed rate bonds, real cost, ACar and RCaR increase with maturity and cuopon period. On the other hand, response of relative risk to the change in maturity and cuopon period is not significant.

Figure 6.1: Cost and Risk of Alternative Financing Strategies (Cost is expressed in nominal terms)

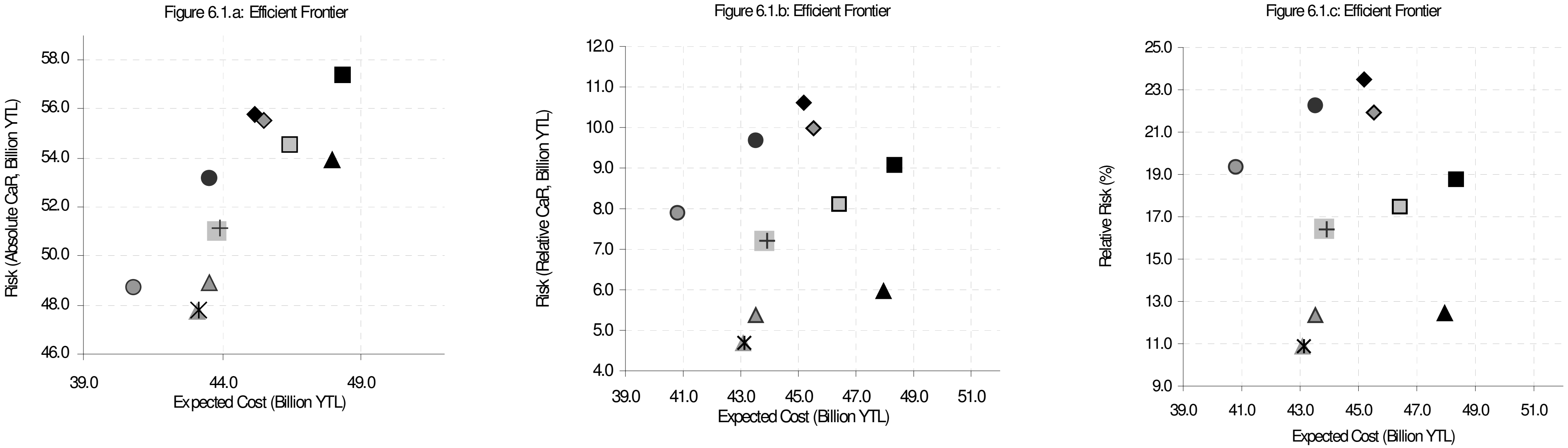

Figure 6.1.a: Efficient Frontier

Figure 6.1.b: Efficient Frontier

Figure 6.1.c: Efficient Frontier

○	S1: 100 % 9-month discounted securities	●	S2: 100 % 12-month discounted securities	◆	S3: 100 % 15-month discounted securities	◇	S4: 100 % 18-month discounted securities
▲	S5: 15% 3-month discounted &85% 2-year, floating rate securities; coupon period 3 months	△	S6: 15% 3-month discounted & 85% 2-year, fixed rate securities; coupon period 3 months	✻	S7: 15% 3-month discounted & 85% 3-year, floating rate securities; coupon period 3 months	▲	S8: 15% 3-month discounted & 85% 3-year, fixed rate securities; coupon period 3 months
■	S9: 15% 6-month discounted &85% 2-year, floating rate securities; coupon period 6 months	□	S10: 15% 6 month discounted 85% 2-year , Fixed rate; coupon period 6 months	+	S11: 15% 6-month discounted & 85% 3-year, floating rate securities; coupon period 6 months	■	15% 6-month discounted & 85% 3-year, fixed rate securities; coupon period 6 months

Figure 6.2: Cost and Risk of Alternative Financing Strategies (Cost is expressed in real terms, as a percentage of GDP)

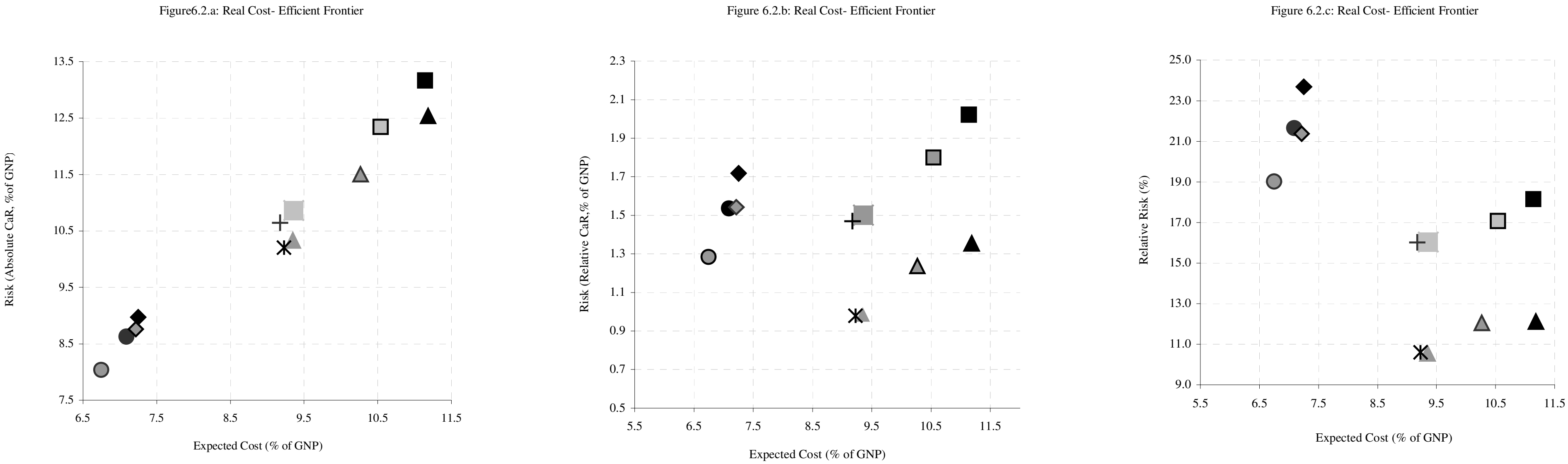
Figure6.2.a: Real Cost- Efficient Frontier

Figure 6.2.b: Real Cost- Efficient Frontier

Figure 6.2.c: Real Cost- Efficient Frontier

○	S1: 100 % 9-month discounted securities	●	S2: 100 % 12-month discounted securities	◆	S3: 100 % 15-month discounted securities	◇	S4: 100 % 18-month discounted securities
▲	S5: 15% 3-month discounted &85% 2-year, floating rate securities; coupon period 3 months	△	S6: 15% 3-month discounted & 85% 2-year, fixed rate securities; coupon period 3 months	*	S7: 15% 3-month discounted & 85% 3-year, floating rate securities; coupon period 3 months	▲	S8: 15% 3-month discounted & 85% 3-year, fixed rate securities; coupon period 3 months
■	S9: 15% 6-month discounted &85% 2-year, floating rate securities; coupon period 6 months	□	S10: 15% 6 month discounted 85% 2-year , Fixed rate; coupon period 6 months	+	S11: 15% 6-month discounted & 85% 3-year, floating rate securities; coupon period 6 months	■	15% 6-month discounted & 85% 3-year, fixed rate securities; coupon period 6 months

Given that absolute CaR is the maximum cost that will be undertaken by the government, among other risk measures it is the most adequate one to assess the market risk. Strategies including discounted securities are characterized with highest volatility however with lowest expected cost. Consequently, these strategies are characterized with lowest risk in terms of absolute CaR. Under this framework if the government is not faced with any other risk in terms of fulfilling its borrowing requirement and there is no cost burden on the government arising from debt sustainability concerns then the government would be inclined to borrow in terms of discounted securities. However given the short maturity of the government domestic debt stock in Turkey and high risk premium embedded in the interest rates, improvement of the debt structure necessitates extension of the borrowing maturity. In order to accomplish this structural change government has to issue long-term coupon bonds. Therefore we can evaluate the results of our analysis by constraining available strategies to the coupon bonds. Among strategies containing coupon bonds, strategy groups with lowest cost and risk are the floating rate bonds with quarterly coupon payments. If the government has the objective of increasing the maturity further. Then, under declining interest rates given the lower volatility at the short end of the yield curve, our analysis suggests the issuance of bonds with quarterly coupon payments. However it should also be emphasized that for floating rate bonds, choice on the coupon period doesn't make a significant difference in terms of costs and ACaR. If the confidence among the investors that interest rates will decrease in the forthcoming period is constituted, additional yields imposed on the coupon bonds would decline. Consequently the government would be able to increase the maturity by issuing coupon bonds without incurring such high cost as implied by the results of our analysis.

VI. Conclusion

In this study stochastic simulation based risk analysis is applied to the domestic currency denominated portion of the Turkish sovereign debt stock. Simulation horizon is five years and covers the 2005-2009 period. Risk analysis is based on three different risk measures, namely, Relative Risk, Absolute Cost-at-Risk (CaR) and Relative CaR. Relative risk, measured as the percentage deviation from the expected cost, enables one to compare alternative strategies on the basis of volatility of the expected cost. On the other hand, Absolute and Reative CaR provide a measure to assess the amount of maximum total cost and the maximum excess cost over the expected cost that government would have to undertake at the specified probability. The only source of uncertainty in the model is the term structure of interest rates. It is simulated by using the dynamic yield curve framework in Diebold and Li (2002), which is founded on Nelson-Siegel(1987) model.

Strategy simulation, in which debt is rolled over under each strategy, is applied on top of simulated term structure of interest rates. Formulated strategies comprise discounted securities and coupon bonds. Yield of the discounted securities are computed directly from the yield curve. Whereas yield of the coupon bonds are computed by imposing an additional return over the coupon rate obtained from the yield curve. Results of the risk analysis are highly depended on the characteristics of the simulated term structure of interest rates and the additional yield imposed on the coupon bonds. Relative risk point out to the tradeoff among the strategies. Shifting from discounted bonds to coupon bonds increases cost but reduces volatility, i.e. relative risk. However in terms of absolute CaR, determined both by the level and the volatility of expected cost, there are no tradeoffs. Risk and cost increase with maturity, as strategies with high costs also appear to be more risky. Among the coupon bonds, strategy group with the lowest cost and risk are the floating rate bonds with quarterly coupon payments. If the government has the objective of increasing the maturity further, then given the lower volatility of the three-month rate compared to six month rate our analysis suggest the issuance of bonds with quarterly coupon payments.

Risk analysis in this paper points out that results of the strategy simulation are dependent on the characteristics of the simulated term structure of interest rates. Hence it is necessary to be able to adequately model the nature of term structure of interest rates, in order to perform risk management analysis. Yield curve modeling approach that we have adopted displayed a good performance in terms of fitting the data for the 2001 July-2004 June period and producing simulated curves having similar characteristics with the estimated yield curves. One drawback of the analysis is the shortness of the estimation period. However this does not degrade the outcome of the strategy simulation, since results are evaluated on account of the random environment generated from the stochastic model acknowledging that different random environment would lead to different results.

Strategy simulation framework adopted in this paper could be extended in various aspects. One extension would be to cover greater portion of the public debt stock by including debt instruments denominated in terms of foreign currency. Another extension would be to lengthen the simulation horizon. This would not only improve the scenario risk analysis but would also provide a framework to evaluate the trend in the cost paths associated with the financing strategies.

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APPENDIX I

Table I.1 : CURRENCY AND MATURITY COMPOSITION OF OUTSTANDING DEBT OF CENTRAL GOVERNMENT (Million USD)

	2000		**2001**		**2002**		**2003**		**2004**	
	Million USD	***Share(%)***	Million USD	***Share(%)***	Million USD	***Share(%)***	Million USD	***Share(%)***	Million USD	***Share(%)***
Central Government Total Debt Stock	93,740	***100.0***	123,581	***100.0***	148,490	***100.0***	202,670	***100.0***	235,803	***100.0***
Central Government Domestic Debt Stock	54,217	***57.8***	84,857	***68.7***	91,691	***61.7***	139,262	***68.7***	167,262	***70.9***
TL	49,763	***53.1***	54,657	***44.2***	62,217	***41.9***	108,745	***53.7***	137,859	***58.5***
Fixed	30,426	***32.5***	12,326	***10.0***	22,989	***15.5***	49,156	***24.3***	70,733	***30.0***
Floating	19,336	***20.6***	42,331	***34.3***	39,228	***26.4***	59,588	***29.4***	67,126	***28.5***
FX Denominated&FX Indexed	4,454	***4.8***	30,200	***24.4***	29,474	***19.8***	30,517	***15.1***	29,404	***12.5***
Fixed	-	-	-	-	10,287	***6.9***	11,762	***5.8***	15,380	***6.5***
Floating	-	-	-	-	19,187	***12.9***	18,755	***9.3***	14,023	***5.9***
Central Government Foreign Debt Stock	39,523	***42.2***	38,724	***31.3***	56,799	***38.3***	63,408	***31.3***	68,541	***29.1***
Fixed	-	-	-	-	33,532	***22.6***	38,145	***18.8***	40,799	***17.3***
Floating	-	-	-	-	23,267	***15.7***	25,263	***12.5***	27,742	***11.8***
	Months	***Share(%)***	Months	***Share(%)***	Months	***Share(%)***	Months	***Share(%)***	Months	***Share(%)***
Maturity of Domestic Debt Stock	15.5	***100.0***	38.5	***100.0***	32.1	***100.0***	25.1	***100.0***	20.6	***100.0***
Cash	9.4	***80.8***	19.2	***47.4***	12.8	***59.6***	12.4	***67.1***	11.8	***73.8***
Non-Cash	41.4	***19.2***	55.9	***52.6***	60.4	***40.4***	51.2	***32.9***	45.5	***26.2***
Maturity of Domestic Borrowing	17.3	***100.0***	30.9	***100.0***	20.6	***100.0***	18.1	***100.0***	17.3	***100.0***
Cash	14.0	***78.4***	18.1	***37.0***	11.1	***80.4***	14.7	***90.5***	15.0	***96.1***
Non-Cash	29.2	***21.6***	38.5	***63.0***	59.5	***19.6***	50.4	***9.5***	73.9	***3.9***
Duration										
TL-Denominated Domestic Cash Debt Stock (Months)	-	-	-	-	-	-	5.2 Months		6.6 Months	
Foreign Bond Stock (Years)	-	-	-	-	-	-	3.4 Years		4.1 Years	
For information										
$ FX (USD Buying rate)	671,765		1,439,567		1,634,501		1,395,835		1,342,100	

Source: Treasury, CBRT.

APPENDIX II: Strategy Simulation

Control variable

$W = [w_1 \cdots w_{14}]$ → Borrowing strategy vector
$w_i, \quad i = 1, \cdots, 14$ → weight associated with each borrowing instrument.

Exogeneous variables

$GNPN_{1\times 96} \rightarrow$ GNP vector. Vector elements are the annual GNP at monthly frequency for the period 2005-2012.

$F\ell_{60\times 1000} \rightarrow$ Interest rate matrix. Where $\ell = 90, 180, 270, 360, 450, 540$.
Elements of the matrix are the 60 period interest rate simulations of the corresponding maturity.

Each column stands for a simulation run. Matrices are the output of yield curve simulation.

$FZ\ell_{60\times 1} \rightarrow$ Interest rate vector. Where $\ell = 90, 180, 270, 360, 450, 540$.

$M_{72\times 2} \rightarrow$ Initial maturity matrix

Debt Simulation

```
for j=1:1000
  $FZ\ell = F\ell(1:60, j);$
for k=1:n
Step 1:Total financing need for the $k^{th}$ period of the $j^{th}$ simulation run is computed:
  $N = sum(M(1,:)) - (GNP(1,1)*0.05/24);$
Step 2: Interest and the principal payment of the $k^{th}$ period of the $j^{th}$ simulation run is computed:
   F(j,k)=M(1,2);
   AN(j,k)=M(1,1);
Step 3: Maturity matrix is adjusted. First row is eliminated and row of zeros is added to the end of the matrix

   M(1,:)=[ ];
   M(72,1)=0;
```

Step 4: Financing need is distributed according to the financing strategy.

$Addi2, \quad i = 3,6,9,\ldots,36$ are the total interest payments of the ith subsequent month.

```
   M(3,1)=M(3,1)+N*w1;
   Add32= N*w1*FZ90(1,1)+N*w7*FZ90 (1,1)+ N*w8*FZ90(1,1) +
N*w9*FZ90(1,1)
   +N*w10*FZ90(1,1);
   M(3,2)=M(3,2)+Add32;

   M(6,1)=M(6,1)+N*w2;
   Add62=N*w2*FZ180(1,1)+          N*w7*FZ90(3,1)          +N*w8*FZ90(1,1)
+N*w9*FZ90(3,1)
  +N*w10*FZ90(1,1) + N*w11*FZ180(1,1)+N*w12*FZ180(1,1)
  +N*w13*FZ180(1,1) + N*w14*FZ180(1,1);
   M(6,2)=M(6,2)+Add62;
                                  •
                                  •
                                  •
   Add362=N*w9*FZ90(33,1) +N*w10*FZ90(1,1) +N*w13*FZ180(30,1)
+N*w14*FZ180(1,1);
   M(36,2)=M(36,2)+Add362;
   M(36,1)=N*w9+N*w10+N*w13+N*w14;
```

Step 5: Nominal and real costs are computed. CSTN and CSTR are the nominal and real cost for the kth period of the jth simulation

```
CSTN=Add32+Add62+Add92+Add122+Add152+Add182+Add212+Add242+Add2
72+Add302+Add332+Add362;
CSTR= (Add32/GNPN(1,3))+(Add62/GNPN(1,6))+ (Add92/GNPN(1,9))+
(Add122/GNPN(1,12))+
(Add152/GNPN(1,15))+(Add182/GNPN(1,18))+(Add212/GNPN(1,21))+
(Add242/GNPN(1,24))+
(Add272/GNPN(1,27))+(Add302/GNPN(1,30))+(Add332/GNPN(1,33))+(Add362/G
NPN(1,36));
```

Step 6: Interest rates and GNP are adjusted for the next period

```
    FZℓ(1, :) = [ ];
    FZℓ(60 ,1) = 0;
    GNPN(:,1)     = [ ],
end
end
```

APPENDIX III

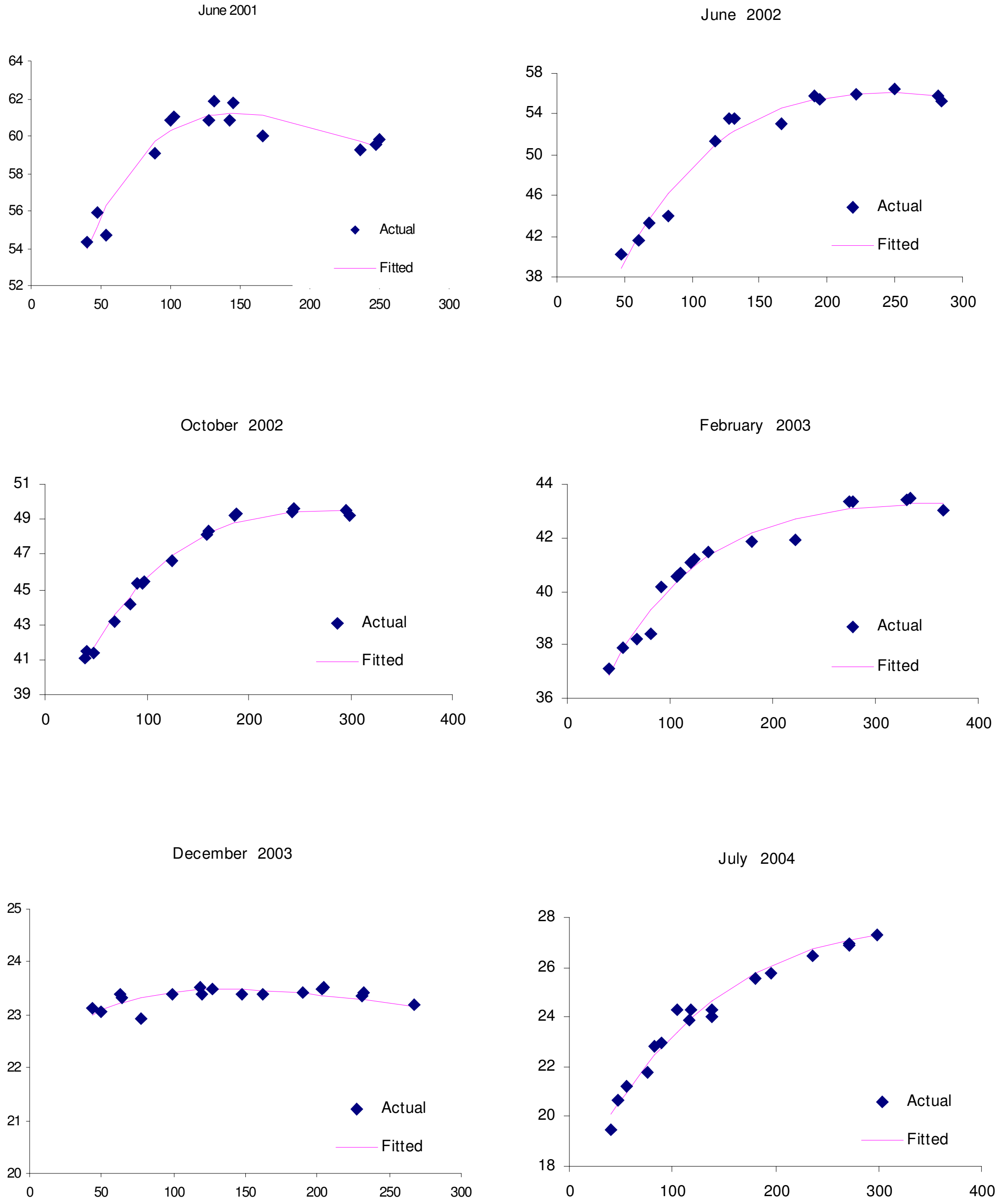
		Parameters				Diagnostics*				Maturity
						Ser. Corr.			Number of	range
		B0	B1	B2	Tau	LM Test	Heteroscedasticity	Jarque-Bera	variables	(Days)
2001	Jun	46.9	-4.2	54.2	74	1.30 (0.52)	2.01 (0.36)	1.57 (0.46)	14	40-250
	Jul	61.7	-25.2	40.9	40	0.51 (0.77)	1.34 (0.51)	2.74 (0.25)	14	36-218
	Aug	51.5	-7.5	42	42	2.75 (0.25)	2.40 (0.66)	1.33 (0.51)	15	42-189
	Sep	55.6	-15.5	50.9	70	0.38 (0.82)	2.21 (0.32)	0.58 (0.75)	12	40-159
	Oct	60.8	-14.9	27.5	110	0.10 (0.94)	2.62 (0.26)	2.86 (0.24)	13	42-233
	Nov	52.4	-6.9	25.5	80	3.98 (0.13)	1.60 (0.44)	0.78 (0.68)	12	37-208
	Dec	52.9	-11.3	12.9	40	4.53 (0.10)	3.96 (0.13)	2.06 (0.36)	11	44-191
2002	Jan	48.4	-4.1	21.7	80	1.15 (0.56)	0.26 (0.87)	0.77 (0.68)	14	34-217
	Feb	44.8	-4.6	37.1	110	3.57 (0.16)	7.27 (0.02)	0.14 (0.93)	19	33-342
	Mar	44.1	-2.7	19.3	110	1.22 (0.54)	2.34 (0.30)	1.55 (0.46)	11	40-313
	Apr	37.8	-2	20.5	90	3.63 (0.16)	0.26 (0.87)	0.69 (0.71)	15	50-344
	May	40.1	-5.1	39.6	140	2.29 (0.31)	1.99 (0.57)	1.43 (0.49)	18	37-313
	Jun	36.9	-14.7	85.4	110	1.28 (0.52)	3.22 (0.19)	0.48 (0.79)	14	47-285
	Jul	55.5	-21	44.2	180	4.14 (0.12)	5.30 (0.07)	0.6 (0.74)	17	35-252
	Aug	43.8	-11.8	47.6	160	1.28 (0.52)	0.78 (0.67)	1.47 (0.48)	13	55-251
	Sep	50.6	-15.3	45.8	180	0.73 (0.69)	4.95 (0.17)	1.45 (0.48)	17	37-275
	Oct	39	-3.4	40.3	140	1.70 (0.42)	2.86 (0.23)	0.6 (0.74)	18	40-300
	Nov	36.4	-3.2	21.4	130	2.07 (0.35)	2.20 (0.69)	0.24 (0.89)	15	40-369
	Dec	36.1	-2.6	32.5	130	1.94 (0.37)	1.29 (0.52)	3.75 (0.15)	15	36-337

APPENDIX III (Cont'd)

		Parameters				Diagnostics*			Number of variables	Maturity range (Days)
		β0	β1	β2	τ	Ser. Corr. LM Test	Heteroscedasticity	Jarque -Bera		
2003	Jan	39.8	-6.9	29.9	190	0.44 (0.80)	1.55 (0.45)	0.89 (0.64)	23	30-362
	Feb	41.9	-8.6	15.1	111	1.97 (0.37)	1.17 (0.55)	4.78 (0.09)	18	40-369
	Mar	54.9	-19.4	-9.3	50	1.85 (0.39)	7.22 (0.30)	1.82 (0.40)	23	49-364
	Apr	38.7	-9.1	27.6	170	0.37 (0.82)	2.49 (0.28)	2.07 (0.35)	23	49-364
	May	24.1	-2.9	56.9	160	0.61 (0.73)	11.5 (0.04)	11.77 (0.00)	26	33-404
	Jun	33.9	-6.8	38	190	2.26 (0.32)	8.98 (0.10)	1.19 (0.55)	22	43-373
	Jul	28.3	-7.8	48	170	1.85 (0.39)	2.49 (0.28)	2.55 (0.28)	22	48-384
	Aug	30.4	-10.5	21.6	90	1.78 (0.41)	7.76 (0.02)	2.16 (0.34)	25	40-390
	Sep	33.3	-7.6	-10.1	40	2.72 (0.25)	10.6 (0.00)	1.16 (0.56)	25	22-390
	Oct	25.1	-5.1	15.6	190	2.99 (0.22)	5.93 (0.05)	1.1 (0.59)	23	33-362
	Nov	21.2	-2.7	20	140	1.58 (0.45)	9.20 (0.10)	0.81 (0.67)	18	54-383
	Dec	21.2	0.8	6.1	90	3.49 (0.17)	3.06 (0.38)	1.03 (0.60)	24	43-393
2004	Jan	21	-1.0	10.2	90	4.17 (0.12)	1.91 (0.38)	1.2 (0.55)	22	33-418
	Feb	18.6	-1.0	13.1	130	1.86 (0.39)	3.72 (0.44)	0.61 (0.74)	23	40-425
	Mar	17.8	-1.0	12.1	100	1.72 (0.42)	4.87 (0.30)	2.72 (0.26)	22	35-511
	Apr	17.9	-1.0	16.5	160	0.10 (0.94)	0.94 (0.62)	0.75 (0.69)	25	40-523
	May	19.6	-2.2	36.1	170	0.24 (0.88)	4.13 (0.38)	0.5 (0.78)	26	37-492
	Jun	19.6	-2.1	33.3	160	1.42 (0.49)	3.68 (0.59)	2.46 (0.29)	22	42-462
	Jul	21.6	-4.6	26.8	180	0.65 (0.72)	11.6 (0.00)	0.02 (0.99)	27	41-496

APPENDIX IV

Figure 1. Examples of Fitted Yield Curves



APPENDIX V: Yield Curve Factors

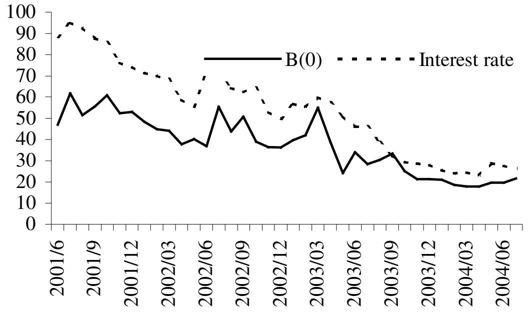
Table V.1. Covariance/Correlation Matrix of the Estimated Parameters

		Correlation		
Variance-Covariance		B_0	β_1	β_2
	β_0	178.9	-0.7	0.2
	β_1	-59.4	38.3	-0.3
	β_2	50.5	-28.8	343.3

Table V.2. ADF Test Results for Levels of the Estimated Yield Curve Factors

	β_0	$R\beta_0$	β_1	β_2
	ADF(1)	ADF(0)	ADF(0)	ADF(0)
Test statistic	-1.31	-4.1	-3.88	-3.67
5% Critical value	-2.95	-2.94	-2.94	-2.94

Figure V.1 Interest Rate* and Level Factor of the Yield Curve



* Average compounded primary market rate for discounted securities.

Figure V.2. Slope Factor of the Yield Curve

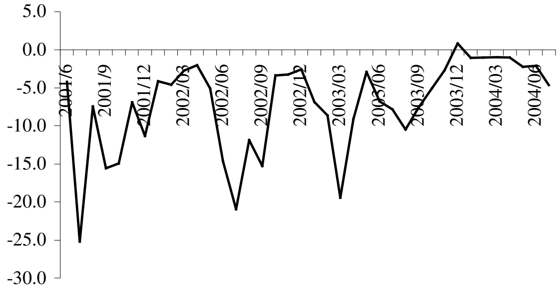


Figure V.3. Curvature Factor of the Yield Curve

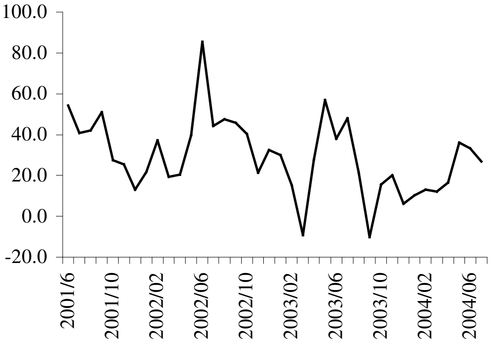
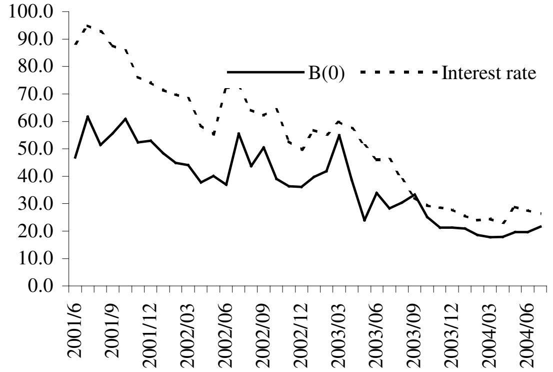


Figure V.4 Interest Rate* and Level Factor of the Yield Curve



* Average compounded primary market rate for discounted securities.

APPENDIX VI

Table VI.1. Computed Nominal Cost and Risk Measures (Billion YTL)

	Definitions	Expected Cost-50th percentile (1)	Absolute CaR-99th percentile (2)	Relative CaR (2)-(1)	Relative Risk ((2)-(1))/(1)*100 (Percentage)
Strategy 1	100 % 9 Month discounted securities	40.81	48.70	7.90	19.35
Strategy 2	100 % 12 Month discounted securities	43.52	53.21	9.68	22.25
Strategy 3	100% 15 Month discounted securities	45.19	55.80	10.61	23.48
Strategy 4	100 % 18 Month discounted securities	45.52	55.50	9.98	21.93
Strategy 5	15% 3 month discounted securities85% 2 Year, Floating rate; coupon payments 3 months	43.08	47.76	4.68	10.87
Strategy 6	15% 3 month discounted securities 85% 2 Year , Fixed rate; coupon payments 3 months	43.52	48.90	5.37	12.35
Strategy 7	15% 3 month discounted securities85% 3 Year , Floating rate; coupon period 3 months	43.13	47.82	4.69	10.88
Strategy 8	15% 3 month discounted securities85% 3 Year , Fixed rate; coupon period 3 months	47.96	53.92	5.97	12.44
Strategy 9	15% 6 month discounted securities 85% 2 Year , Floating rate; coupon period 6 months	43.83	51.03	7.20	16.43
Strategy 10	15% 6 month discounted securities85% 2 Year , Fixed rate; coupon period 6 months	46.42	54.53	8.11	17.47
Strategy 11	15% 6 month discounted securities 85% 3 Year , Floating rate; coupon period 6 months	43.91	51.12	7.20	16.41
Strategy 12	15% 6 month discounted securities 85% 3 Year , Fixed rate; coupon period 6 months	48.34	57.41	9.07	18.77

Appendix VII

Table VIII.1. Computed Nominal Cost and Risk Measures (Quadrillion TL)

	Definitions	Expected Cost-50th percentile (1)	Absolute CaR-99th percentile (2)	Relative CaR (2)-(1)	Relative Risk ((2)-(1))/(1)*100 (Percentage)
Strategy 1	100 % 9 Month discounted securities	6.75	8.03	1.28	19.02
Strategy 2	100 % 12 Month discounted securities	7.09	8.63	1.54	21.65
Strategy 3	100% 15 Month discounted securities	7.25	8.97	1.72	23.69
Strategy 4	100 % 18 Month discounted securities	7.22	8.76	1.54	21.38
Strategy 5	15% 3 month discounted securities85% 2 Year, Floating rate; coupon payments 3 months	9.35	10.33	0.99	10.55
Strategy 6	15% 3 month discounted securities 85% 2 Year , Fixed rate; coupon payments 3 months	10.27	11.50	1.24	12.05
Strategy 7	15% 3 month discounted securities85% 3 Year , Floating rate; coupon period 3 months	9.23	10.21	0.98	10.61
Strategy 8	15% 3 month discounted securities85% 3 Year , Fixed rate; coupon period 3 months	11.18	12.54	1.36	12.12
Strategy 9	15% 6 month discounted securities 85% 2 Year , Floating rate; coupon period 6 months	9.36	10.86	1.50	16.03
Strategy 10	15% 6 month discounted securities85% 2 Year , Fixed rate; coupon period 6 months	10.54	12.34	1.80	17.07
Strategy 11	15% 6 month discounted securities 85% 3 Year , Floating rate; coupon period 6 months	9.17	10.64	1.47	16.02
Strategy 12	15% 6 month discounted securities 85% 3 Year , Fixed rate; coupon period 6 months	11.14	13.16	2.02	18.15