THE TONE SENSE MULTIACCESS PROTOCOL WITH PARTIAL COLLISION DETECTION (TSMA/PCD) FOR PACKET SATELLITE COMMUNICATIONS

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ABSTRACT

new contention protocol called the Tone Sense Multiaccess with A Partial Collision Detection (TSMA/PCD) is designed for packet satellite communications. This protocol is particularly suitable for satellite system serving an area with a dense population of earth stations. By incorporating a narrow band ground radio channel for broadcasting busy tones, the earth stations are able to avoid packet collisions by sensing for the absence of busy tones before transmitting packets. Besides, partial collision detection capability can also be achieved. The performance of TSMA/PCD is found to be somewhere between CSMA and CSMA/CD, depending on the number of busy tones N used. When N=50, singletone TSMA/PCD gives almost identical performance as CSMA/CD. While for multi-tone and slot-by-slot announcement TSMA/PCD protocols, only N=8 and N=2 respectively are sufficient to drive the system to the CSMA/CD performance. For the normalized ground channel propagation delay, a, equals to 0.01 and N=10, the nonpersistent TSMA/PCD can reach a maximum throughput of 0.933 and 1-persistent TSMA/PCD can attain a maximum of 0.922. Both the TSMA/PCD give lower delays than versions of their corresponding CSMA protocols. The performance of the protocols in a noisy satellite channel is also studied.

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UNTRODUCTION

I would like to express my deepest gratefulness to my research supervisor, Dr. T.S. YUM, for his continuous advice, guidance, encouragement and support.

TABLE OF CONTENT

1.	. INTRODUCTION	1
2.	. The TSMA/PCD Protocol	10
3.	ANALYSIS OF TSMA/PCD IN NOISE-FREE SATELLITE CHANNEL	15
	3.1 Analysis of non-persistent TSMA/PCD	15
	3.2 Analysis of 1-persistent TSMA/PCD	22
4.	ANALYSIS OF TSMA/PCD IN A NOISY SATELLITE CHANNEL	35
	4.1 Analysis of non-persistent TSMA/PCD	
	4.2 Analysis of 1-persistent TSMA/PCD	39
5.	MULTI-TONE TSMA/PCD AND SLOT-BY-SLOT AMMOUNCEMENT TEMA/PCD	45
	5.1 Analysis of Multi-tone TSMA/PCD	45
	5.2 Analysis of Slot-by-slot Announcement TSMA/PCD	
6.	NUMERICAL RESULTS AND DISCUSSION	
7.	CONCLUSION	78
RE)	FERENCES	80
API	PENDIX	0.0

"衛星通訊中TSMA/PCD小包通訊協议"

摘要

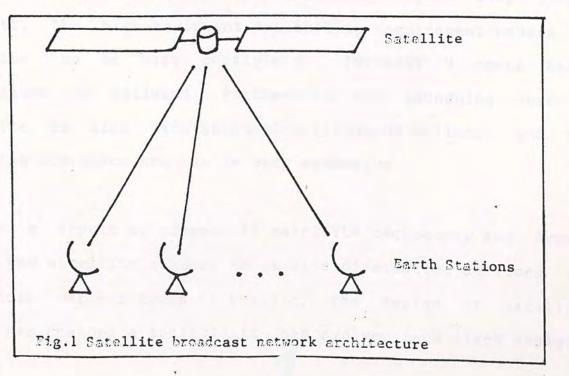
本文提出並分析一种用於衛星通訊上规 通訊協設。我们稱此為檢察信号多接觸部份碰撞 探测協议(TSMA/PCD)。这協议大適合於高密度地面站 的区域。它的最大优点是兴氛利用一條窄的地面無线 电波频道互通繁忙单音, 當地面站在發送小包前 首先檢驗確定沒有繁忙单音,便能减少小包的碰撞。 此外,地面站还可探测出部份的碰撞。视乎所採用的 单音總載N的多募,TSMA/PCD的性能介手於CSMA及 CSMA/CD之间。當N=50時,单音(single-tone)TSMA/PCD EI可 達到 CSMA/PCO 相同的性能。如果是多音 (multi-tone) TSMA/PC 或多段变音(slot by slot announcement) TSMA/PCD 的註,则分別 只须用N=8及N=2便足以達到CSMA/co的輸出率。 當地面的傳送延遲系数 a=0.01 12 N=10 時, 不持續

(non-persistent) TSMA/PCD 的輸出率可達 0.933, 持續 (1-persistent) TSMA/PCD 則可達 0.922。这两种 TSMA/PCD 協议的平均 傳送延遲 皆比其相应的 CSMA 協议為低。本文还 分析 3 在 位 道 有噪音 的 情 況 T. TSMA/PCD 協议的 搜 能。

CHAPTER 1

INTRODUCTION

Nowadays, there are many worldwide networks(e.g. S.I.T.A. Network) and local-area networks(e.g. Ethernet) based on packet switching. Moreover, the transmission of digital voice in packet networks is presently under extensive investigation. However, these wire-based conventional networks do not satisfy certain objectives, namely, the provision of large bandwidth and simple network architecture for long-haul communications, the support of mobile users, and the ease of deployment. Packet satellite network in which a digital processing satellite acts as a central switching and storage node(15) in a packet data network has been introduced to satisfy these objectives. Existing packet satellite networks have the structure shown in Fig.1 where a single satellite in geosynchronous orbit is used to connect to an arbitrary number of earth stations. The communications among the stations are done via the satellite.



several main advantages of satellite There are communications[1]. The advantage is the inherent first broadcasting capability which supports multi-destinations protocols. Secondly, the earth stations are fully connected into simple one-hop network and thus eliminating routing problem as a in multi-hop networks. Besides, the organization of stations are very flexibility. Thirdly, nowadays satellite is not merely providing a transmission means but having on-board processing capability for switching, controlling and buffering functions. Furthermore, it is capable to support mobile users and has high transmission quality.

However, there are also disadvantages[12]. The propagation delay of satellite communications is long(average of 270ms for geostationary satellite). This may be annoying to users in some applications(e.g. speech transmission, interactive computing). Besides, the transmission loss due to rain and oxygen in the atmosphere is high when the operating frequency is over 10GHz. Moreover, the high equipment reliability requirement causes the satellite to be very costly(e.g. INTELSAT V costs about 25 millions US dollars). Furthermore, the launching cost of satellite is also high(about 20 millions US dollars) and the repairing via space shuttle is very expensive.

As a result of advance in satellite technology and demand for a new satellite network to provide diversified services and to handle various types of traffic, the design of satellite system has changed a lot[13]. It has evolved from fixed assigned

frequency division multiple access (FDMA) channel switching oriented system (which was mainly used to handle telephony traffic) to demand assigned time division multiple access (TDMA) packet system to handle both data and digital voice. Recently, switched time division multiple access (SS/TDMA) satellite system[14] which incorporate both space division and time division technique is introduced to attain efficient frequency spectrum sharing. Future trends lie in the development of new network architecture to faciliate the evolution to integrated service digital network (ISDN)[1]. Potential applications include providing services to a mixed group users (such as digital voice, viedo-conferencing, transaction-based and image data traffic), interconnecting domestic and foreign networks to form a mixedmedia network.

Over the past two decades, satellite communication technology has progressed considerably and the following consequences are observed:

- (1) The cost of ground stations has dropped tremendously. Together with the ease of installation, small-size privateowned earth stations become very attractive to domestic and commerical users.
- (2) Spot beams allow the spatial reuse of the precious frequency spectrum. Therefore even a small number of stations confined to a relatively small area can communicate through the satellite economically.

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Moreover, the enormous growth in data communications has also brought in a high demand for a large bandwidth and low error rate channel. Therefore, the need for reliable and high data rate satellite channel is forseen to increase greatly. It leads to a scenario that industrial plants, campuses, government and commerical buildings, etc. could each have a rooftop antenna to communicate with others via the satellite. This is especially true for a metropolitan area having a dense population of data communications centers.

The multiaccess protocol for satellite channel is the method for conflict resolution among users desiring channel access[2]. For computer communications, data traffic is bursty in nature. This is due to the high degree of randomness in the message generation time and size and the relatively low-delay constraint required by the user. The packet-oriented protocols are more efficient for this type of traffic and nowadays emphasis is mostly on packet satellite communications. Roughly speaking, there are three major types of multiaccess protocols[2] for packet satellite communications: the reservation, the contention and the hybrid protocols.

For reservation protocols, a reservation sub-channel is used for transmission scheduling. The reservation protocols can achieve a maximum channel throughput close to unity but they inherit a minimum of two round trip propagation delay. The Reservation ALOHA is an implicit servation scheme introduced

by Crowther[6]. In this scheme, slots are organized into frames of equal size with frame duration greater than the satellite propagation delay. A station who has successfully accessed a slot in a frame is guaranteed access to the same slot in the succeeding frame until it stops using it. An unused slot is free to be accessed by all users in sloted ALOHA mode. The control is distributed and each user only need to maintain a history of the usage of each slot for just one frame duration. This protocol is effective for long multi-packet messages and stream type traffic.

First-in First-out Reservation Scheme The studied by Roberts[6] is an explicit reservation scheme where reservations made explicitly on a reservation subchannel. A slot in are the reservation subchannel is further divided into minislot for transmission of reservation packets in slotted ALOHA mode. The number of reservation slots are adaptive to the load. The control distributed, each station must maintain information on the is number of outstanding reservations and the slots at which its own reservations begin.

Under contention protocols, there is no attempt to coordinate the ready users to avoid collisions entirely. Instead, each ready user makes his own decision regarding when to access the channel but exercises caution to minimize interference with other users as much as opossible. Contention protocols do not have the queue synchronization problem and do not need to manage a global queue of requests[2] as in the reservation protocols.

They can also attain a lower packet delay when the channel is lightly utilized. However, the maximum throughput for this class of protocols is usually guite low. One common type is the slotted ALOHA protocol[2]. In this protocol, the channel is slotted in time and the data packet is of fixed length equivalent to a channel time slot. Each station transmits a data packet whenever one is ready. If transmission fails, the packet will be transmitted after a randomized delay. The maximum throughput offered by this protocol is only 0.368.

Another type of contention protocol is the Tree Algorithm Based Collision Resolution Algorithm[2]. In this scheme, contention among several active stations is resolved by searching for active users along a binary tree until all collided packets are transmitted. This is an unconditional stable scheme and Drovode a maximum throughput of 0.43.

The hybrid protocols behave like a combination of random access and reservation protocols. The recently introduced Announced Retransmission Random Access (ARRA) protocol [4] belongs to this type and can obtain a maximum throughput of 0.6 when the control overheads are ignored. In this scheme, all transmissions are accompanied by a retransmission slot announcement in the mini-slot sub-channel to announce the intended retransmission slot if the transmission fails. The aim is to prevent collision between new and retransmitted packets.

The Scheduled Retransmission Multiaccess Protocol[5] is an

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elaboration of ARRA in which the collision between retransmitted packets are also avoided by using an announcement subchannel. The retransmission reservation is scheduled and announced by the satellite via this subchannel. The Dynamic Frame (DF) version can give a throughput as high as 0.93 with the control overheads ignored. However, delay at this operating point is phenomenal.

For single-hop multiaccess broadcast networks (such 35 Ethernet, packet radio network [6]) with a small propagation delay, there is already a robust protocol called the Carrier Multiple Access (CSMA) protocol[7] which can attain high Sense throughput (greater than 0.9) and lower delay than that of slotted ALOHA. A variation of this type called the Carrier Sense Multiple Access with Collision Detection (CSMA/CD) protocol[8] can even give a better performance. It has long been thought that the CSMA type protocols cannot be used in satellite network because of the long round trip propagation delay. However, we show in this thesis that if all stations accessing a particular satellite channel are confined to a metropolitan area, the ability of sensing before transmission to avoid packet collision provided by incorporating a narrow-band ground can be radio broadcasting busy tones. In other words, when channel for 2 is transmitting a packet on the satellite channel, station it broadcasts a busy tone on the ground channel also to signify that the satellite channel is currently in use. Stations transmit only when no busy tone is sensed on the radio channel. To illustrate, consider an 8,000 bit packet transmitted on a 1Mbit/s

satellite channel. If the distance between the two farthest apart stations is 50km, the normalized propagation delay (ratio of ground propagation delay to packet transmission time (7)), a, is calculated to be 0.02. Such a value of a would guarantee a fairly accurate sensing of the channel status before packet transmission.

In this thesis, we propose a contention protocol called the Sense Multiaccess with Partial Collision Detection Tone Protocol(TSMA/PCD) for a packet satellite system serving a zone with a densed population of earth stations. The basic protocol is called the single-tone TSMA/PCD (or simply TSMA/PCD), there are two variations called the multi-tone TSMA/PCD and slot-by-slot announcement TSMA/PCD. Two versions of the protocols, the nonpersistent and the 1-persistent, are considered. We will show by analysis that this protocol, with partial collision detection capability, has a performance somewhere between CSMA and CSMA/CD. For the normalized ground channel propagation delay, a, equals to 0.01, the non-persistent TSMA/PCD can reach a maximum throughput of 0.933 while 1-persistent TSMA/PCD can attain a maximum of 0.922. Moreover, both versions of TSMA/PCD give lower delays than their corresponding CSMA protocols. In next chapter, we shall first describe the system model and the TSMA/PCD protocols. Then in chapters 3 and 4, we present the throughput and delay analysis TSMA/PCD in noise-free and noisy satellite channels of respectively. Two variations of the protocol (the multi-tone TSMA/PCD and slot-by-slot announcment TSMA/PCD) which can give a better performance are introduced and analysed in chapter 5.

Numerical results are given in chapter 6 and conclusion is given in chapter 7 together with a list of research topics for further investigation.

the non-persistent and 1-persistent versions of the TSHAPPCO protocol (it is simpled the simple-tone TSHAPPCO protocol to differenciate irms its variation, The suiti-tone TEHA/PCO protocol in Camptar 51.

System Model

peckets, the system also has a ground radio channel for accessing control purpose. Each station, when transmitting a packet, also broadcasts a busy tone to notify other stations that fine safellife channel is currently being occupied. The busy tone is candomly chosen from a pre-assigned pool of a fixed frequencies. Packet collision can be detected if the transmitting station senses another may tone like, not the busy tone is is broadcasting) on the ground channel.

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CHAPTER 2

THE TSMA/PCD PROTOCOL

In this chapter, we describe the system model and present the non-persistent and 1-persistent versions of the TSMA/PCD protocol (it is also called the single-tone TSMA/PCD protocol to differentiate from its variation, the multi-tone TSMA/PCD protocol in Chapter 5).

(A) System Model

Besides the satellite channel which is used for transmitting packets, the system also has a ground radio channel for accessing control purpose. Each station, when transmitting a packet, also broadcasts a busy tone to notify other stations that the satellite channel is currently being occupied. The busy tone is randomly chosen from a pre-assigned pool of N fixed frequencies. Packet collision can be detected if the transmitting station senses another busy tone (i.e. not the busy tone it is broadcasting) on the ground channel.

We assume the combined arrivals of new and retransmitted packets from the stations constitute a Poisson Process with rate g packets per time slot. The size of time slot is chosen to be the maximum ground propagation delay between any two stations in the system. All packets have the same length of T slots and must be transmitted at the beginning of a slot. The round trip propagation delay on the satellite channel is R slots.

satellite channel is noise-free and in a We assume the packet collision all packets involved are destroyed. Delay and throughput on a noisy channel are derived in Chapter 4. Upon receiving the first slot of a packet, the satellite can determine whether a collision has occurred. It then immediately gives an acknowledgement (positive or negative) by piggybacking it on the downlink packet. Thus, the transmitting station can receive the acknowledgement R+1 slots after transmission. When a busy tone is issued, it takes at most 1 slot for the other stations to sense. If all the transmitting stations are not sending the same busy tone, the collision can be detected by all stations in one slot. Thus, if a collision is detected, the maximum duration from the beginning of a transmission until the channel is again idle is 2 slots.

(B) Protocol Description

The TSMA/PCD protocol operates like the CSMA/CD protocol [8] except (1) the stations sense the ground radio channel for the <u>absence</u> of busy tones before transmission and (2) collision cannot be detected if all the transmitting stations choose the same busy tone, and so only a partial collision detection is possible. We descirbe below the non-persistent and 1-persistent versions of TSMA/PCD protocols.

1. non-persistent TSMA/PCD

The idea here is to limit the interference among packets by always reacheduling the packets which find the channel busy upon arrival. More precisely (refer Fig.2a), a station with a packet ready for transmission will first sense the ground channel. If the ground channel is sensed busy, the station resenses the channel after a random delay which is uniform on [1,K]. If the ground channel is sensed idle, the packet is transmitted immediately and a randomly chosen busy tone is broadcasted on the ground channel at the same time.

If a collision is detected during transmission, the transmission is aborted immediately and the station resenses the channel after a random delay (uniform on [1,K]). If collision is not detected, the sending station waits for the acknowledgement from the satellite. If a negative acknowledgement is received, the station repeats the entire procedure.

2. 1-persistent TSMA/PCD

The 1-persistent TSMA/PCD is devised to minimize delay at low traffic (when the channel traffic G in packets per packet transmission time is much less than 1) by never letting the channel idle if some stations have packets ready to be sent. This Protocol is similar to the non-persistent TSMA/PCD except that when the ground channel is sensed busy, a ready station waits until the channel goes idle and transmits its packet immediately (refer Fig.2b).

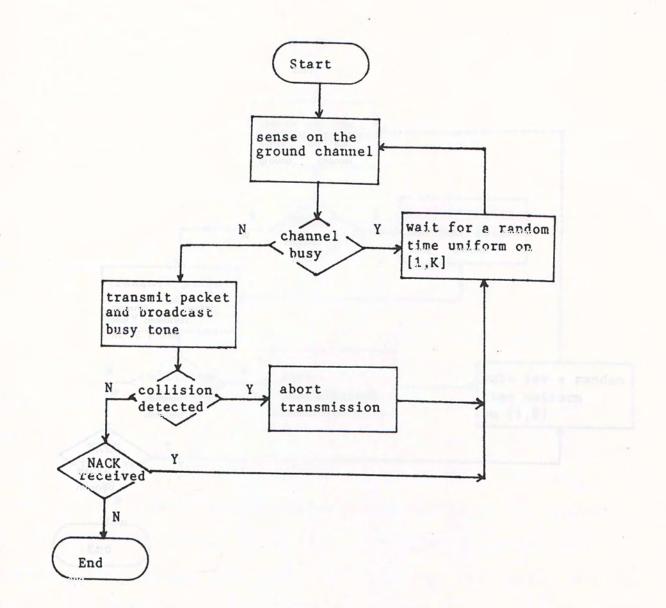


Fig.2a Procedures followed by a ready station under non-persistent TSMA/PCD

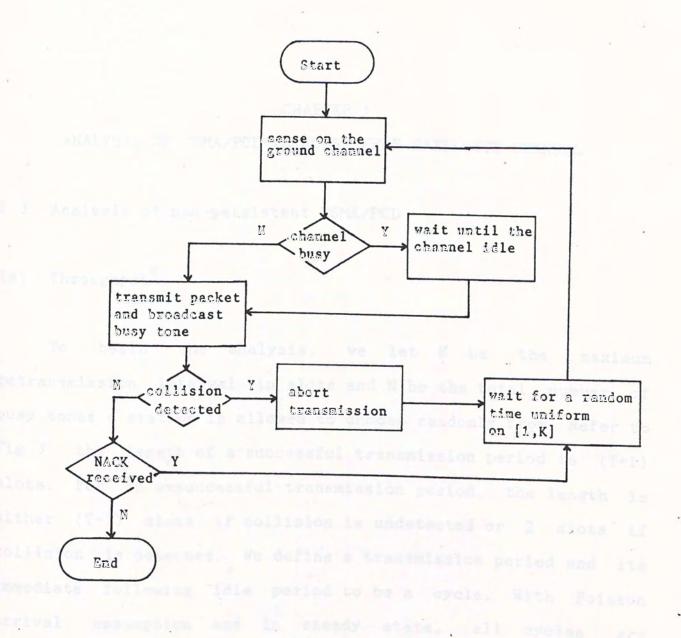


Fig.2b Procedures followed by a ready station under 1-persistent TSMA/PCD

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CHAPTER 3

ANALYSIS OF TSMA/PCD IN NOISE-FREE SATELLITE CHANNEL

3.1 Analysis of non-persistent TSMA/PCD

(A) Throughput*

To begin the analysis, we let K be the maximum retransmission interval in slots and N be the total number of busy tones a station is allowed to choose randomly from. Refer to Fig.3, the length of a successful transmission period is (T+1)slots. For an unsuccessful transmission period, the length is either (T+1) slots if collision is undetected or 2 slots if collision is detected. We define a transmission period and its immediate following idle period to be a cycle. With Poisson arrival assumption and in steady state, all cycles are statistically identical [8]. Let P_g be the probability that a transmission is successful. Then

P_s = P[exactly one arrival in the slot immediately before the transmission period given that at least one arrival occurs]

$$= \frac{ge^{-5}}{1 - e^{-g}}$$

where g is the channel traffic rate per slot.

Next, let P_u be the probability that a collision is * This throughput analysis is similar to that given in [8] except (1)

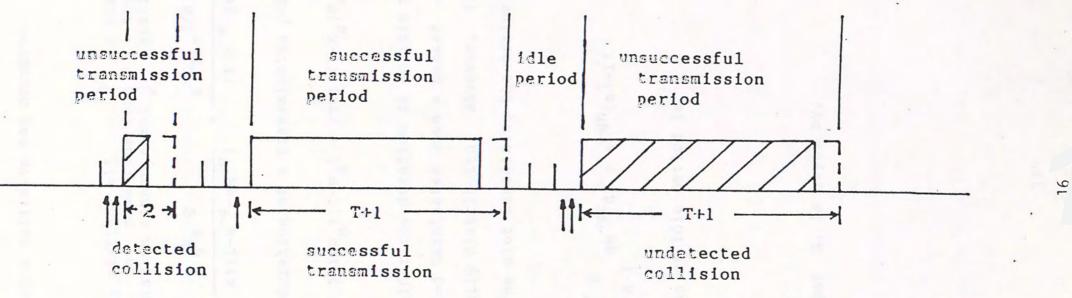


Fig.3 Transmission and idle periods in slotted non-persistent TSMA/PCD (vertical arrows represent packet arrivals) undetected given that a collision has occurred. Then P_u can be evaluated as

$$P_{u} = \sum_{k=2}^{\infty} P\left[\begin{array}{c} k \text{ ready stations all} \\ choose \text{ the same tone for} \\ transmission announcement} \end{array} \right] P\left[\begin{array}{c} k \text{ stations become ready} \\ i \text{ the number of ready} \\ stations is at least two \\ stations is at least two \\ \end{array} \right]$$

$$= \sum_{k=2}^{\infty} \left(\frac{1}{N}\right)^{k-1} \frac{g^{k}e^{-g}}{k!(1-e^{-g}-ge^{-g})} = \frac{e^{-g}N(e^{g/N}-1-g/N)}{(1-e^{-g}-ge^{-g})} \quad (1)$$

Hence, the expected duration of a transmission period, E[TP] is,

$$E[TP] = (T+1)P_{s} + 2(1-P_{s})(1-P_{u}) + (T+1)(1-P_{s})P_{u}$$

Next, we consider the duration of an idle period I. Note that an idle period terminates when a packet arrives. It is therefore exponentially distributed. However, in discrete time, we also include the slot containing the arrival in the idle period. Thus,

$$P[I=n \ slots] = \int_{n-1}^{n} ge^{-gt} dt = e^{-gn}(e^{g}-1)$$
 $n=1,2,...$

The expected duration of idle period is

$$E[I] = \frac{1}{1 - e^{-g}}$$
(2)

The channel throughput S, is given by,

$$S = \frac{P_{s}T}{E(TP) + E[I]}$$
(3)

(B) Delay

Under the Poisson arrival assumption and in steady state, the distributions of busy and idle periods seen by an arrival is their respective equilibrium distributions. Therefore, in steady state, the probability that a packet arrives at an idle period P_{I} is

$$P_{I} = \frac{E[I]}{E[TP] + E[I]}$$

The probability that a packet arrives at a busy period $P_{\rm H}$ therefore is $P_{\rm B} = 1 - P_{\rm I}$.

We define the packet delay D, to be the time elapsed in slots from the moment of its first transmission until it is successfully transmitted. Fig. 4 shows the flow graph [9] of the packet delay process of non-persistent TSMA/PCD. The circles represent the states encountered by the packet being transmitted and are described as follows :

- (i) S_o is the starting state. With probability P_I , the packet will start from state S_I , and with probability $(1-P_I)$, it will start from S_B . The transition from S_o to either S_I or S_B is instantaneous.
- (ii) S_I indicates that the packet arrives at an idle period.
- (iii) S_B indicates that either(a) the packet arrives at a busy period or

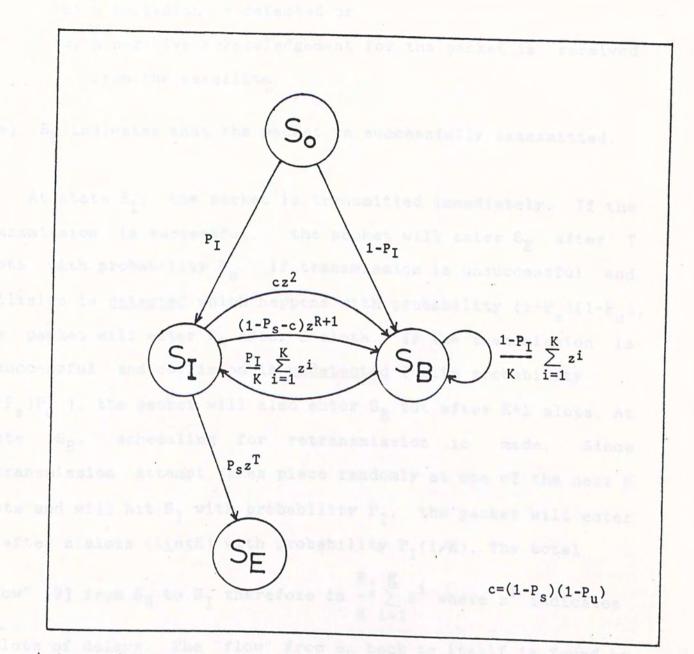


Fig.4 Flow graph of packet delay process, non-persistent TSMA/PCD

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- (b) a collision is detected or
- (c) a negative acknowledgement for the packet is received from the satellite.

(iv) S_E indicates that the packet is successfully transmitted.

At state S_I , the packet is transmitted immediately. If the transmission is successful, the packet will enter S_E after T slots with probability P_s . If transmission is unsuccessful and collision is <u>detected</u> which happens with probability $(1-P_s)(1-P_u)$, the packet will enter S_B after 2 slots. If the transmission is unsuccessful and collision is <u>undetected</u> (with probability $(1-P_s)P_u$), the packet will also enter S_B but after R+1 slots. At state S_B , scheduling for retransmission is made. Since retransmission attempt takes place randomly at one of the next K slots and will hit S_I with probability P_I , the packet will enter S_I after n slots $(1 \le n \le K)$ with probability $P_I(1/K)$. The total "flow" [9] from S_B to S_I therefore is $\frac{P_I}{K} \le z^i$ where z^i indicates is a slote in the mathematical takes.

i slots of delays. The "flow" from S_B back to itself is found by a similar argument.

The flow diagram indicates that the transition probabilities from a particular state depend only on that state; hence the Process is a Markov chain and S_E is the absorbing state. The duration of the packet delay D, is just the first passage time from S_0 to S_E . For simplicity, let $c=(1-P_s)(1-P_u)$. Then the following flow equations are readily obtained from Fig.4:

$$S_{I} = P_{I}S_{o} + \frac{P_{I}\sum_{K=i=1}^{K} z^{i}S_{B}}{S_{B}}$$

$$S_{B} = cz^{2}S_{I} + (1-P_{s}-c)z^{R+1}S_{I} + \frac{1-P_{I}\sum_{K=i=1}^{K} z^{i}S_{B} + (1-P_{I})S_{o}$$

$$S_{E} = P_{s}z^{T}S_{I}$$

The generating function of packet delay $G_D(z)$, is just the transfer function from state S_O to S_E or

$$G_{D}(z) = \frac{S_{E}}{S_{o}} = \frac{P_{I}P_{s}z^{T}}{1 - \frac{cP_{I}}{K}\sum_{i=1}^{K}z^{2+i} - \frac{1-P_{I}}{K}\sum_{i=1}^{K}z^{i} - \frac{P_{I}(1-P_{s}-c)}{K}\sum_{i=1}^{K}z^{R+1+i}}$$
$$= \frac{P_{I}P_{s}(1-z)z^{T}}{(1-z) - z(1-z^{K})(\frac{1-P_{I}}{K} + \frac{cP_{I}}{K}z^{2} + \frac{P_{I}(1-P_{s}-c)}{K}z^{R+1})}$$

The delay distribution can be evaluated by numerical inversion. The mean packet delay and delay variance are obtained by differentiating $G_{D}(z)$:

$$E[D] = \frac{dG_{D}(z)}{dz} \bigg|_{z=1}$$

= T + $\frac{1 - P_{I}P_{s}}{P_{I}P_{s}}(\frac{K+1}{2}) + \frac{2(1 - P_{s})(1 - P_{u})}{P_{s}} + \frac{(1 - P_{s})P_{u}}{P_{s}}(R+1)$

$$\sigma_{D}^{\prime}{}^{2} = \frac{d^{2}G_{D}(z)}{dz^{2}}\Big|_{z=1} + \frac{dG_{D}(z)}{dz}\Big|_{z=1} - \left[\frac{dG_{D}(1)}{dz}\Big|_{z=1}\right]^{2}$$

$$= \frac{1 - P_{I}P_{s}}{P_{I}P_{s}} \frac{K+1}{2} + \frac{2(1 - P_{s})(1 - P_{u})}{P_{s}} + \frac{(1 - P_{s})P_{u}}{P_{s}}(R+1)$$

$$+ \left[\frac{1 - P_{I}P_{s}}{P_{I}P_{s}} \frac{K+1}{2} + \frac{2(1 - P_{s})(1 - P_{u})}{P_{s}} + \frac{(1 - P_{s})P_{u}}{P_{s}}(R+1)\right]^{2}$$

$$+ \frac{1 - P_{I}}{P_{I}P_{s}} \frac{(K-1)(K+1)}{3} + \frac{(1 - P_{s})(1 - P_{u})}{P_{s}} \frac{(K+1)(K+2)(K+3) - 6}{3K}$$

$$+ \frac{(1 - P_{s})P_{u}}{P_{s}} \frac{(R+K)(R+K+1)(R+K+2) - R(R+1)(R+2)}{3K}$$

3.2 Analysis of 1-persistent TSMA/PCD

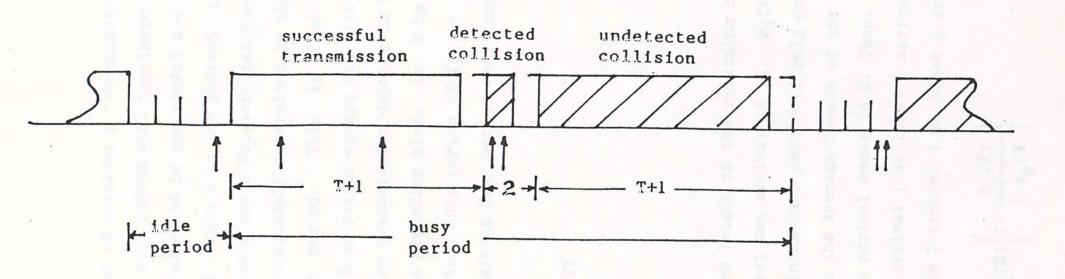
(A) Throughput **

Fig.5 shows the channel in alternating cycles of busy periods and idle periods. The busy period consists of a number of transmission periods of length T+1 slots (successful) and 2 slots (unsuccessful). Let E[U] be the expected length of the time in a busy period that the channel is engaged in successful transmission and E[BP] be the expected duration of a busy period. Then the throughput, S, is given by

$$S = \frac{E[U]}{E[BP] + E[I]}$$

(4)

** Except for eqn(6) the rest of the throughput analysis follows closely the method given in [8].



23

Fig.⁵ Busy and idle periods in 1-persistent TSMA/PCD (vertical arrows represent packet arrivals)

To calculate E[BP], let us consider a transmission period in a busy period. Whether the busy period will terminate and whether following transmission period is successful are completely the determined by the number of arrivals in the present transmission The number of arrivals has a Poisson distribution and so period. arrival statistics is completely determined by the length of the the present transmission period. This is true for all transmission periods in a busy period except the first one where the success or failure of the transmission depends on the number of arrivals in the immediate preceding slot. Let X be the length a transmission period in a busy period (X=T+1 or 2 slots); of then the length of the remaining busy period is a function of X. Define

 $q_k(X) \stackrel{\triangle}{=} P[k \text{ arrivals in } X]$ = $\frac{(gX)^k}{k} e^{-gX}$

Let B(X) be the average length of the remaining busy period given that there is at least one arrival in X, $B_1(X)$ be the average length of the remaining busy period if only one arrival occurs in X, and $B_2(X)$ be the average length of the remaining busy period if more than one arrival occur in X. Then,

B(X) = B₁(X)P[exactly 1 arrival | at least 1 arrival]

+ B2(X)P[more than 1 arrival | at least 1 arrival]

$$= B_{1}(X) \frac{q_{1}(X)}{1-q_{0}(X)} + B_{2}(X) \left[1 - \frac{q_{1}(X)}{1-q_{0}(X)} \right]$$
(5)

We first calculate $B_1(X)$. As there is only one arrival in X, the following transmission period must be successful and of length (T+1) slots. Hence,

= (T+1) + B(T+1)P[arrival occurs in the preceding T+1 slots]

$$= (T+1) + B(T+1)(1-q_0(T+1))$$

When there are more than one arrival in X, the following transmission period must be unsuccessful. Its length is (T+1) slots if the collision is undetected and 2 slots if the collision is detected. Let $P_u(X)$ be the probability that the collision is undetected given that the present transmission is unsuccessful and the length of the preceding transmission period is X. Then,

$$P_{u}(X) = \sum_{k=2}^{\infty} P \begin{cases} k \text{ ready stations all} \\ cnoose \text{ the same tone for} \\ transmission \text{ announcement} \end{cases} P \begin{cases} k \text{ stations become ready in} \\ the \text{ preceding transmission} \\ period X \mid ready \text{ stations} \ge 2 \end{cases}$$

$$= \sum_{k=2}^{\infty} \left(\frac{1}{N}\right)^{k-1} \frac{(gX)^{k} e^{-gX}}{k!(1-e^{-gX}-gXe^{-gX})} = \frac{e^{-gX}N(e^{gX/N}-1-gX/N)}{(1-e^{-gX}-gXe^{-gX})}$$
(6)

The probability of undetected collision for the first transmission period of a busy period is just $P_{ij}(1)$.

Similarly, we calculate $B_{2}(X)$ as

$$\begin{split} B_{2}(X) &= \begin{bmatrix} (T+1) + \text{period if the preceding transmission} \\ \text{period is of length T+1} \end{bmatrix} P_{u}(X) \\ &+ \begin{bmatrix} 2 + \text{period if the preceding transmission} \\ 2 + \text{period is of length 2} \end{bmatrix} \begin{bmatrix} 1-P_{u}(X) \\ 1-P_{u}(X) \end{bmatrix} \\ &= [T+1 + B(T+1)(1-q_{0}(T+1))]P_{u}(X) + [2 + B(2)(1-q_{0}(2))](1-P_{u}(X))] \\ &= [T+1 + B(T+1)(1-q_{0}(T+1))]P_{u}(X) + [2 + B(2)(1-q_{0}(2))](1-P_{u}(X))] \\ &\text{Substitute X=T+1 and X=2 into (5), we obtain two equations} \\ &\text{with two unknowns B(T+1) and B(2). After solving, we get,} \\ &B(T+1) = (dV_{1} - bV_{2})/w \\ &B(2) = (aV_{2} - cV_{1})/w \\ &\text{Where } e_{X} = \frac{q_{1}(X)}{1-q_{0}(X)} \\ &v_{1} = (T+1)e_{T+1} + (T+1)(1-e_{T})P_{u}(T+1) + 2(1-e_{T+1})(1-P_{u}(T+1)) \\ &V_{2} = (T+1)e_{2} + (T+1)(1-e_{2})P_{u}(2) + 2(1-e_{2})(1-P_{u}(2)) \\ &a = 1 - (1-q_{0}(T+1))e_{T+1} - (1-e_{T+1})(1-q_{0}(T+1))P_{u}(T+1) \\ &b = - (1-e_{T+1})(1-q_{0}(2))(1-P_{u}(T+1)) \\ &c = - (1-q_{0}(T+1))e_{2} - (1-e_{2})(1-q_{0}(T+1))P_{u}(2) \\ &d = 1 - (1-e_{2})(1-q_{0}(2))(1-P_{u}(2)) \\ &w = ad - bc \\ \end{split}$$

The expected length of busy period E[BP], is just the average remaining length of busy period given that arrival has

occurred in the slot before the busy period, and is equal to

E[BP] = B(1).

$$= (T+1 + B(T+1)[1-q_0(T+1)])e_1 + \{[T+1 + B(T+1)(1-q_0(T+1))]P_u(1) + [2 + B(2)(1-q_0(2))][1-P_u(1)]\} (1-e_1)$$
(7)

Similarly, for the calculation of E[U], we define U(X) as the average remaining time in a cycle that the channel is engaged in successful transmission given that arrival occurs in the present transmission period of length X. It is given as

$$U(X) = \begin{bmatrix} T + U(T+1)(1-q_0(T+1)) \end{bmatrix} \left(\frac{q_1(X)}{1-q_0(X)} \right) + \begin{bmatrix} U(T+1)(1-q_0(T+1))P_u(X) \\ + U(2)(1-q_0(2))[1-P_u(X)] \end{bmatrix} \left(1 - \frac{q_1(X)}{1-q_0(X)} \right)$$
(8)

Substitute X=T+1 and X=2 into (8), we have

$$U(T+1) = T(de_m - be_0)/w$$

$$U(2) = T(ae_o - ce_m)/w$$

Hence

 $\mathbf{E}[\mathbf{U}] = \mathbf{U}(1)$

 $= (T + U(T+1)[1-q_0(T+1)])e_1 + \{U(T+1)[1-q_0(T+1)]P_{11}(1)$

+ $U(2)[1-q_0(2)][1-P_u(1)]\} (1-e_1)$ (9)

Substitute (7), (9) and (2) into (4), the throughput S can be explicitly evaluated.

(B) Delay

If a collision is detected, all stations involved will retry after a random dalay which is uniform on [1,K]. If the collision is not detected, whether retransmission is needed or not is determined from the acknowledgement of the satellite after R+1 slots. Fig.6 shows the flow graph of the packet delay process under 1-persistent TSMA/PCD. The states are defined as follows:

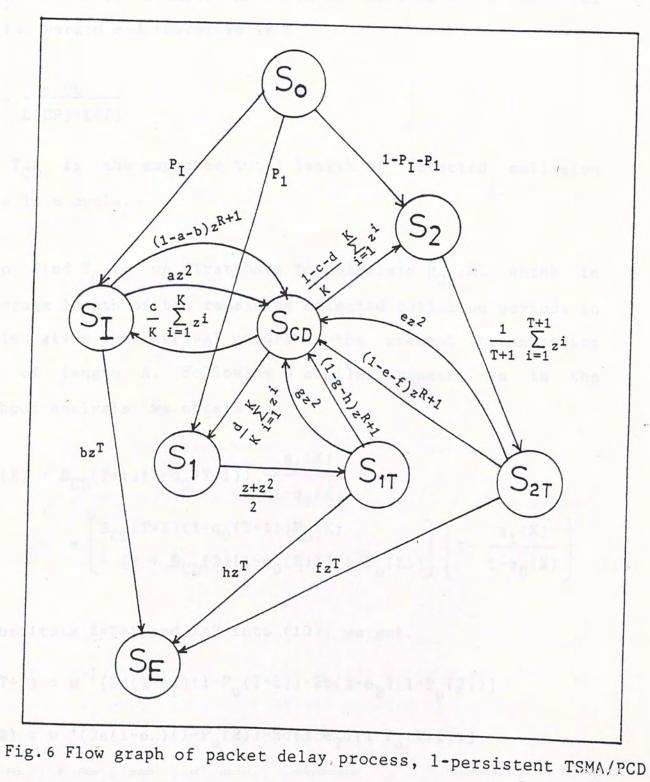
S : starting state.

S_I : the packet arrives at an idle period.

- S_{CD} : either a collision is detected or a negative acknowledgement is received.
- S₁: the packet arrives at a detected collision period.
- S₂: the packet arrives at a transmission period of length T+1.
- S_{1T} : the packet from S_1 starts to transmit.
- S_{2T} : the packet from S_2 starts to transmit.
- $S_{\underline{F}}$: the packet is successfully transmitted.

Starting from S_0 , the packet will enter S_I , S_1 and S_2 with probabilities P_I , P_1 and $1-P_I-P_1$ respectively, where

$$P_{I} = \frac{E[I]}{E[BP] + E[I]}$$





and P₁ is the probability that a packet arrives at a detected collision period and therefore is

$$P_1 = \frac{T_{CD}}{E[BP] + E[I]}$$

where T_{CD} is the expected total length of detected collision periods in a cycle.

To find T_{CD} , we first have to calculate $B_{CD}(X)$ which is the average length of the remaining detected collision periods in a cycle given that arrival occurs in the present transmission period of length X. Following a similar argument as in the throughput analysis, we obtain

$$B_{CD}(X) = B_{CD}(T+1)(1-q_0(T+1)) \frac{q_1(X)}{1-q_0(X)} + \begin{bmatrix} B_{CD}(T+1)(1-q_0(T+1))P_u(X) \\ + [2 + B_{CD}(2)(1-q_0(2))][1-P_u(X)] \end{bmatrix} \begin{bmatrix} 1 - \frac{q_1(X)}{1-q_0(X)} \end{bmatrix}$$
(10)

Substitute X=T+1 and X=2 into (10), we get,

$$B_{CD}(T+1) = w^{-1} [2d(1-e_T)(1-P_u(T+1))-2b(1-e_2)(1-P_u(2))]$$

$$B_{CD}(2) = w^{-1} [2a(1-e_2)(1-P_u(2))-2c(1-e_T)(1-P_u(T+1))].$$

correct transitation parties to and before stating in the

Personal arrival is collocally distributed on [1.Tellic one

Comparing the definitions of T_{CD} and $B_{CD}(X)$, we see that $T_{CD} = B_{CD}(1)$

$$= B_{CD}^{(T+1)(1-q_0^{(T+1)})e_1} + \{B_{CD}^{(T+1)(1-q_0^{(T+1)})P_u^{(1)}} + [2 + B_{CD}^{(2)(1-q_0^{(2)})}](1-P_u^{(1)})\} (1-e_1^{(1)})$$

Next, let $P_g(X)$ be the probability that the current transmission is successful given that the preceding transmission period is of length X and there is at least one arrival in X, or

$$P_{s}(X) = \frac{gXe^{-gX}}{1-e^{-gX}}$$

where

		if the preceding transmission is either success or an undetected collision.
X =)	2,	if the preceding transmission results in a detected collison.
- 10	1,	if the current transmission period is the first one in a busy period

As in the non-persistent version, the "flows" in the graph (Fig.4) represent the delays. These delays are random quantities and are described by their probability generating functions. To illustrate, at state S_2 , the packet will have to wait for the current transmission period to end before entering S_{2T} (or transmit). Since the remaining transmission time seen by a Poisson arrival is uniformly distributed on [1,T+1], the generating function of the delay distribution or "flow"

is $\frac{1}{T+1} \sum_{i=1}^{T+1} z^i$. The "flow" between other states are found similarly.

From the signal flow graph, the following equations are derived :

$$\begin{split} s_{I} &= cs_{o} + \frac{c}{K} \sum_{i=1}^{K} z^{i}s_{CD} \\ s_{CD} &= (az^{2} + (1-a-b)z^{R+1})s_{I} + (ez^{2} + (1-e-f)z^{R+1})s_{2T} \\ &+ (gz^{2} + (1-g-b)z^{R+1})s_{1T} \\ s_{1} &= ds_{o} + \frac{d}{K} \sum_{i=1}^{K} z^{i}s_{CD} \\ s_{1T} &= \frac{z + z^{2}}{2}s_{1} \\ s_{2} &= (1-c-d)s_{o} + \frac{1-c-d}{K} \sum_{i=1}^{K} z^{i}s_{CD} \\ s_{2T} &= \frac{1}{T+1} \sum_{i=1}^{T+1} z^{i}s_{2} \\ s_{E} &= (bs_{I} + hs_{1T} + fs_{2T})z^{T} \\ where \\ a &= [1-P_{g}(1)][1-P_{u}(1)] \\ b &= P_{g}(1) \\ c &= P_{I} \\ c &= P_{I} \\ c &= P_{I} \\ c &= P_{g}(2) \end{split}$$

The generating funcion of packet delay $G_{D}(z)$ is just the transfer function $S_{E}^{/S}/S_{O}$ and is

$$G_{D}(z) = \frac{S_{E}}{S_{O}} = V \frac{K}{K - \frac{W_{Z}(1-z^{K})}{1-z}} z^{T}$$

where $V = bc + \frac{dh(z^2+z)}{2} + \frac{f(1-c-d)}{T+1} \frac{z(1-z^{T+1})}{1-z}$

$$W = c[az^{2} + (1-a-b)z^{R+1}] + (1-c-d)\frac{ez^{2} + (1-e-f)z^{R+1}}{T+1} \frac{z(1-z^{T+1})}{1-z} + d\frac{gz^{2} + (1-g-h)z^{R+1}}{2}(z^{2}+z)$$

The mean packet delay E[D] is given as :

$$E[D] = T + \frac{1-H'}{H'} \frac{K+1}{2} + \frac{1}{H'} (\frac{T+2}{2}P_2 + \frac{3}{-P_1}) + \frac{2P'}{H'} + \frac{Q'}{H'} (R+1)$$

 $P_2 = 1 - P_1 - P_1$

$$P' = P_{I}[1-P_{s}(1)][1-P_{u}(1)] + P_{2}[1-P_{s}(T+1)][1-P_{u}(T+1)] + P_{1}[1-P_{s}(2)][1-P_{u}(2)]$$

$$Q' = P_{I}[1-P_{g}(1)]P_{u}(1) + P_{2}[1-P_{g}(T+1)]P_{u}(T+1) + P_{1}[1-P_{g}(2)]P_{u}(2)$$

$$H' = P_T P_T(1) + P_0 P_T(T+1) + P_1 P_T(2)$$

A'' = c[2a+(1-a-b)(R+1)R] + (1-c-d)[2e+(1-e-f)(R+1)R]+ (1-c-d)[2e+(1-e-f)(R+1)](T+2) + (1-c-d)(1-f)T(T+2)/3+ d[2g+(1-g-h)(R+1)R] + 3d[2g+(1-g-h)(R+1)] + d(1-h)

B'' = dh + f(1-c-d)T(T+2)/3

A' = c[2a+(1-a-b)(R+1)] + (1-c-d)[2e+(1-e-f)(R+1)]+ (1-c-d)(1-f)(T+2)/2 + d[2g+(1-g-h)(R+1)] + d(1-h)(3/2)

B' = dh(3/2) + f(1-c-d)(T+2)/2

A = 1 - B

B = bc+dh+f(1-c-d)

where

$$+ \frac{1}{B^{2}} \left[\frac{K+1}{2} + A' \right]^{2} + \frac{1}{-[B']} + \frac{K+1}{2} + A' - \left(\frac{B'}{B} \right)^{2} + \frac{K+1}{2} + \frac{K+1}{$$

2

$$\sigma_{D}^{2} = \frac{B''}{B} + \frac{1}{B} \left[\frac{(K-1)(K+1)}{3} A + (K+1)A' + A'' \right]$$

The packet delay variance is given as :

CHAPTER 4

ANALYSIS OF TSMA/PCD IN A NOISY SATELLITE CHANNEL

In this chapter, we study the performance of TSMA/PCD in a noisy satellite channel. In this case the satellite can acknowledge a packet only after the entire packet is received. The transmitting station can now expect to receive the acknowledgement R+T+1 slots after the transmission in contrast to the R+1 slots in noise-free channel case. If no positive acknowledgement is received (which may be due to packet error on the uplink or acknowledgement error on the downlink), the transmitting station will resend the packet according to the nonpersistent or 1-persistent procedures. The analyses of these two versions are as follows.

4.1 Analysis of non-persistent TSMA/PCD

(A) Throughput

To begin the analysis, let P_{cf} be the probability that a transmission is collision-free. Then,

P_{cf} = P[exactly one arrival given that at least one arrival occurs]

$$= \frac{ge^{ge}}{1-e^{-g}}$$

Also, let P_{eu} and P_{ed} be the probabilities of packet transmission error (due to channel noise) and acknowledgement transmission error respectively. Let P_e be the probability of packet retransmission due to transmission error or $P_e=1-(1-P_{eu})(1-P_{ed})$. Then, the probability that a packet transmission is successful P_e is

$$P_{s} = P_{cf}(1-P_{eu})(1-P_{ed}) = P_{cf}(1-P_{e})$$

Following the derivations of Chapter 2.1, we have

$$E[TP] = (T+1)P_{s} + 2(1-P_{cf})(1-P_{u}) + (T+1)[P_{cf}P_{e} + (1-P_{cf})P_{u}].$$

Using the same equation for E[I], S can be found explicitly from (3). Compared with the noise-free channel case, we found

which is independent of g.

(B) Delay

Fig. 7 shows the flow graph of the packet delay process for non-persistent TSMA/PCD under noisy channel condition. It is the same as Fig.4 except that (1) a delay of R+1 is replaced by R+T+1 for the flow from S_I to S_B , (2) c is now equal to $(1-P_{cf})(1-P_u)$ instead of $(1-P_s)(1-P_u)$. The following flow equations are obtained :

$$S_{I} = P_{I}S_{o} + \frac{P_{I}}{K}\sum_{i=1}^{K} z^{i}S_{B}$$

 $S_{B} = cz^{2}S_{I} + (1-P_{s}-c)z^{R+T+1}S_{I} + \frac{1-P_{I}}{K}\sum_{i=1}^{K}z^{i}S_{B} + (1-P_{I})S_{o}$

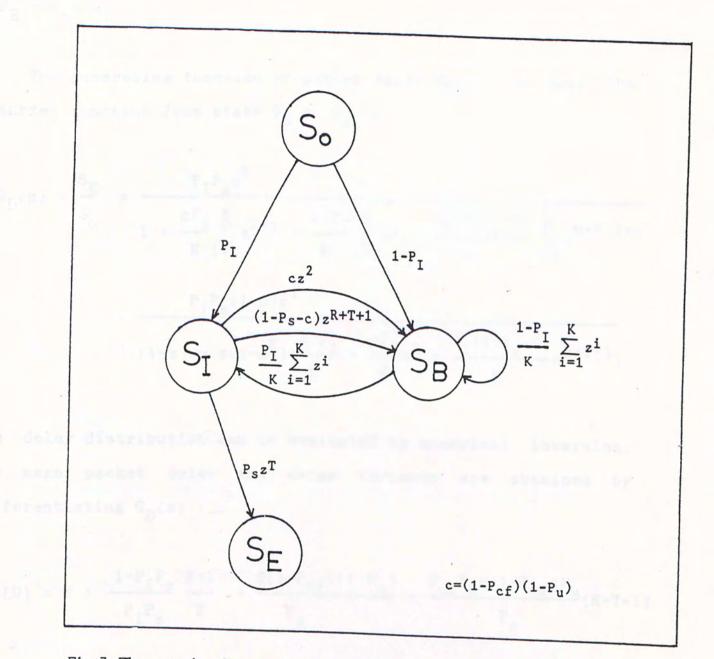


Fig.7 Flow graph of packet delay process, non-persistent TSMA/PCD in a noisy satellite channel

$$S_E = P_s z^T S_I$$

a). Delas

The generating function of packet delay ${\rm G}_{\rm D}(z),$ is just the transfer function from state ${\rm S}_{\rm O}$ to ${\rm S}_{\rm E}$ or

$$G_{D}(z) = \frac{S_{E}}{S_{o}} = \frac{P_{I}P_{s}z^{T}}{1 - \frac{cP_{I}}{K}\sum_{i=1}^{K}z^{2+i} - \frac{1-P_{I}}{K}\sum_{i=1}^{K}z^{i} - \frac{P_{I}(1-P_{s}-c)}{K}\sum_{i=1}^{K}z^{R+T+1+i}}$$
$$= \frac{P_{I}P_{s}(1-z)z^{T}}{(1-z) - z(1-z^{K})(\frac{1-P_{I}}{K} + \frac{cP_{I}}{K}z^{2} + \frac{P_{I}(1-P_{s}-c)}{K}z^{R+T+1})}$$

The delay distribution can be evaluated by numerical inversion. The mean packet delay and delay variance are obtained by differentiating $G_{\rm D}(z)$:

$$E[D] = T + \frac{1 - P_I P_s}{P_I P_s} \frac{K+1}{2} + \frac{2(1 - P_{cf})(1 - P_u)}{P_s} + \frac{P_{cf} P_e^{+(1 - P_{cf})P_u}}{P_s} (R+T+1)$$

$$\sigma'_{D}^{2} = \frac{1 - P_{I}P_{s}}{P_{I}P_{s}} \frac{K+1}{2} + \frac{2(1 - P_{cf})(1 - P_{u})}{P_{s}} + \frac{P_{cf}P_{e} + (1 - P_{cf})P_{u}}{P_{s}}(R+T+1)$$

$$+ \left[\frac{1 - P_{I}P_{s}}{P_{I}P_{s}} \frac{K+1}{2} + \frac{2(1 - P_{cf})(1 - P_{u})}{P_{s}} + \frac{P_{cf}P_{e} + (1 - P_{cf})P_{u}}{P_{s}}(R+T+1)\right]^{2}$$

$$+ \frac{1 - P_{I}}{P_{I}P_{s}} \frac{(K-1)(K+1)}{3} + \frac{(1 - P_{cf})(1 - P_{u})}{P_{s}} \frac{(K+1)(K+2)(K+3) - 6}{3K}$$

$$+ \frac{P_{cf}P_{e} + (1 - P_{cf})P_{u}}{P_{s}} \frac{(R+T+K)(R+T+K+1)(R+T+K+2) - (R+T)(R+T+1)(R+T+2)}{3K}$$

and has be are all sulliplied

4.2 Analysis of one-persistent TSMA/PCD

(A) Throughput

The throughput analysis is exactly the same as the noise-free case except

$$E[U]_{noisy} = (1-P_e)E[U]_{noise-free}$$
(11)

Substitute (2), (7) and (11) into (4), the throughput in a noisy channel is found to be

which is again independent of g.

(B) Delay

In a similar manner we let $P_{cf}(X)$ be the probability that

the current transmission is collision-free given that the preceding transmission period is of length X and there is at least one arrival in X, or

$$P_{cf}(X) = \frac{gXe^{-gX}}{1-e^{-gX}}$$

The delay flow graph shown in Fig.8 is similar to the noisefree case except that (1) $P_{g}(X)$ is replaced by $P_{cf}(X)$ and (2) the flows from S_{I} to S_{E} , S_{1T} to S_{E} and S_{2T} to S_{E} are all multiplied by a factor (1- P_{e}). The following equations are derived from the flow graph :

$$S_{I} = cS_{o} + \frac{c}{K} \sum_{i=1}^{K} z^{i}S_{CD}$$

$$S_{CD} = (az^{2} + (1-a-b)z^{R+T+1})S_{I} + (ez^{2} + (1-e-f)z^{R+T+1})S_{2T}$$

$$+ (gz^{2} + (1-g-h)z^{R+T+1})S_{1T}$$

$$S_1 = dS_0 + \frac{d}{K} \sum_{i=1}^{K} z^i S_{CD}$$

$$S_{1T} = \frac{z + z^2}{2} S_1$$

$$S_2 = (1-c-d)S_0 + \frac{1-c-d}{K} \sum_{i=1}^{K} z^i S_{CD}$$

$$S_{2T} = \frac{1}{T+1} \sum_{i=1}^{T+1} z^{i}S_{2}$$
$$S_{E} = (bS_{I} + hS_{1T} + fS_{2T})z^{T}$$

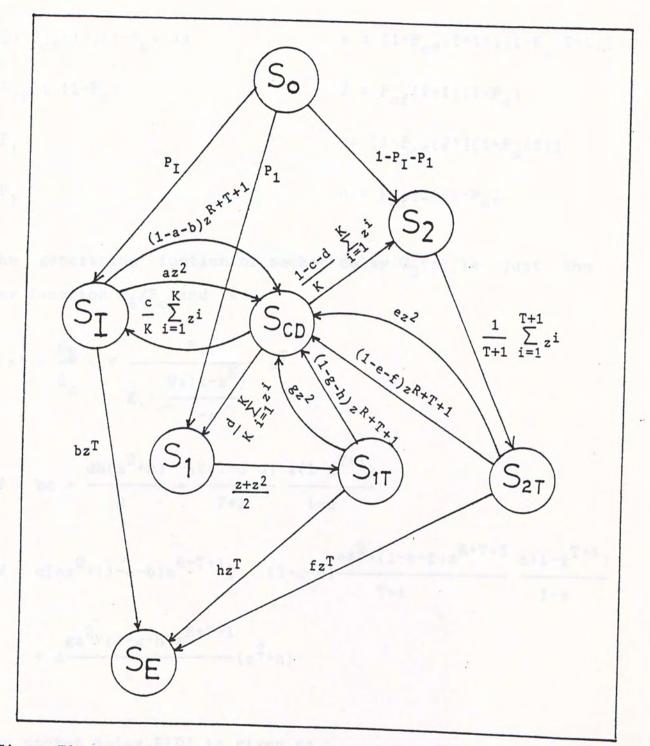


Fig.8 Flow graph of packet delay process, 1-persistent TSMA/PCD in a noisy satellite channel

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where

$$a = [1-P_{cf}(1)][1-P_{u}(1)] \qquad e = [1-P_{cf}(T+1)][1-P_{u}(T+1)]$$

$$b = P_{cf}(1)(1-P_{e}) \qquad f = P_{cf}(T+1)(1-P_{e})$$

$$c = P_{I} \qquad g = [1-P_{cf}(2)][1-P_{u}(2)]$$

$$d = P_{1} \qquad h = P_{cf}(2)(1-P_{e})$$

The generating fuction of packet delay ${\rm G}_{\rm D}(z)$ is just the transfer function ${\rm S}_{\rm E}/{\rm S}_{\rm O}$ and is

$$G_{D}(z) = \frac{S_{E}}{S_{o}} = V \frac{K}{K - \frac{Wz(1-z^{K})}{1-z}} z^{T}$$

where V = bc +
$$\frac{dh(z^2+z)}{2}$$
 + $\frac{f(1-c-d)}{T+1}$ $\frac{z(1-z^{T+1})}{1-z}$

$$W = c[az^{2} + (1-a-b)z^{R+T+1}] + (1-c-d)\frac{ez^{2} + (1-e-f)z^{R+T+1}}{T+1} \frac{z(1-z^{T+1})}{1-z}$$

+
$$d \frac{gz^2 + (1-g-h)z^{R+T+1}}{2} (z^2+z)$$

The mean packet delay E[D] is given as :

$$E[D] = T + \frac{1-H'}{H'} \frac{K+1}{2} + \frac{1}{H'} (\frac{T+2}{2}P_2 + \frac{3}{2}P_1) + \frac{2P'}{H'} + \frac{Q'}{H'} (R+T+1)$$

where

$$P_{2} = 1 - P_{I} - P_{1}$$

$$P' = P_{I} [1 - P_{cf}(1)] [1 - P_{u}(1)] + P_{2} [1 - P_{cf}(T+1)] [1 - P_{u}(T+1)] + P_{1} [1 - P_{cf}(2)] [1 - P_{u}(2)]$$

$$Q' = P_{I} [P_{e} P_{cf}(1) + (1 - P_{cf}(1)) P_{u}(1)] + P_{2} [P_{e} P_{cf}(T+1) + (1 - P_{cf}(T+1)) P_{u}(T+1)] + P_{1} [P_{e} P_{cf}(2) + (1 - P_{cf}(2)) P_{u}(2)]$$

$$H' = [P_{I} P_{cf}(1) + P_{2} P_{cf}(T+1) + P_{1} P_{cf}(2)] (1 - P_{e})$$

The packet delay variance is given as :

$$\sigma_{D}^{\prime 2} = \frac{B''}{B} + \frac{1}{B} \left[\frac{(K-1)(K+1)}{3} A + (K+1)A' + A'' \right]$$

$$+ \frac{1}{B^{2}} \left[\frac{K+1}{2} + A^{\prime} \right]^{2} + \frac{1}{B} \left[B^{\prime} + \frac{K+1}{2} + A^{\prime} \right] - \left(\frac{B^{\prime}}{B} \right)^{2}$$

where

$$B = bc+dh+f(1-c-d)$$

A = 1 - B

B' = dh(3/2) + f(1-c-d)(T+2)/2

A' = c[2a+(1-a-b)(R+T+1)] + (1-c-d)[2e+(1-e-f)(R+T+1)] + (1-c-d)(1-f)(T+2)/2 + d[2g+(1-g-h)(R+T+1)] + d(1-h)(3/2)

B'' = dh + f(1-c-d)T(T+2)/3

that of the single-tone case.

$$r_{0}(x) = \sum_{k=2} (1/c_{n}^{k}) = \frac{1}{k!(1-e^{-\frac{\pi}{2}}-\frac{\pi}{2}xe^{-\frac{\pi}{2}})} = \frac{e^{-\frac{\pi}{2}}c_{n}^{2}(e^{\frac{\pi}{2}}xe^{-\frac{\pi}{2}}xc_{n}^{2})}{(1-e^{-\frac{\pi}{2}}-\frac{\pi}{2}xe^{-\frac{\pi}{2}}x)}$$

and for the 1-persistent case,

$$P_{ij} = \sum_{k=2}^{\infty} (1/c_{ij}^{k})^{k-1} \frac{(q^{k}e^{-q})}{k!(1-e^{-q}-qe^{-q})} = \frac{e^{-q}c_{ij}^{k}(e^{q/c_{ij}^{k}}-1-q/c_{ij}^{k})}{(1-e^{-q}-qe^{-q})}$$

in the multi-tone TENA/PCC protocol, stations proposant a group of a tones to announce their transmissions rather than provide a single tone as in single-tone TENA/PCD discussed in previous chapters. These groups of a tones are chosen randomly from a pre-assigned pool of K tones. A packet collision is detected if the transmitting station nenses busy tones other than its own being broadcasted on the ground channel. For the same number of tones N, the multi-tone scheme yields a lower probability of undetected collision. For the non-persistent case.

of multi-tone TSHAPPCE

+ d[2g+(1-g-h)(R+T+1)(R+T)] + 3d[2g+(1-g-h)(R+T+1)] + d(1-h)

+ (1-c-d)[2e+(1-e-f)(R+T+1)](T+2) + (1-c-d)(1-f)T(T+2)/3

A'' = c[2a+(1-a-b)(R+T+1)(R+T)] + (1-c-d)[2e+(1-e-f)(R+T+1)(R+T)]

CHAPTER 5

MULTI-TONE TSMA/PCD AND SLOT-BY-SLOT ANNOUNCEMENT TSMA/PCD

5.1 Analysis of Multi-tone TSMA/PCD

In the multi-tone TSMA/PCD protocol, stations broadcast a group of n tones to announce their transmissions rather than broadcast a single tone as in single-tone TSMA/PCD discussed in previous chapters. These groups of n tones are chosen randomly from a pre-assigned pool of N tones. A packet collision is detected if the transmitting station senses busy tones other than its own being broadcasted on the ground channel. For the same number of tones N, the multi-tone scheme yields a lower probability of undetected collision. For the non-persistent case,

$$P_{u} = \sum_{k=2}^{\infty} (1/C_{n}^{N})^{k-1} \frac{g^{k}e^{-g}}{k!(1-e^{-g}-ge^{-g})} = \frac{e^{-g}C_{n}^{N}(e^{g/C_{n}^{N}}-1-g/C_{n}^{N})}{(1-e^{-g}-ge^{-g})}$$
(12)

and for the 1-persistent case,

$$P_{u}(x) = \sum_{k=2}^{M} (1/C_{n}^{N})^{k-1} \frac{(gx)^{k} e^{-gx}}{k!(1-e^{-gx} - gxe^{-gx})} = \frac{e^{-gx}C_{n}^{N}(e^{gx/C_{n}^{N}} - 1 - gx/C_{n}^{N})}{(1-e^{-gx} - gxe^{-gx})}$$
(13)

The value of n that minimizes P_u and $P_u(X)$ is shown in the appendix to be

$$n = \begin{cases} N/2 & , N = even \\ (N-1)/2 & , N = odd \end{cases}$$

The rest of the throughput and delay analyses follow exactly that of the single-tone case.

5.2 Slot by slot Announcement Scheme for non-persistent TSMA/PCD

The collision detection capability can also be increased by using slot by slot announcements. In this scheme, instead of continuously broadcasting the same busy tone for transmission announcement, a transmitting station now re-choose a tone (randomly from the set of N tones) to broadcast in each slot. Thus, a collision is not detected after the n-th slots if and only if all transmitting stations choose the same tone in slot 1, slot 2, ... and slot n. This probability is very small even for very small N. As before, if collision is detected, the packet and tone transmissions are aborted immediately and will retry after a random delay uniform on [1,K]. The throughput and delay analysis are as follows.

(A) Throughput

Let $P_D(n|k)$ be the probability that the collision is detected in the n-th slot from the beginning of a transmission given that there are k (k \geq 2) arrivals in the slot before the transmission period. Then,

$$P_{D}(n|k) = \left[\left(\frac{1}{N}\right)^{k-1} \right]^{n-1} \left[1 - \left(\frac{1}{N}\right)^{k-1} \right] = N^{-n(k-1)}(N^{k-1}-1)$$
 n=1,2,...T

The probability that the collision is detected in the n-th slot given that a collision has occurred therefore is

$$P_{D}(n) = \sum_{k=2}^{\infty} P_{D}(n|k) \frac{g^{k}e^{-g}}{k!(1-e^{-g}-ge^{-g})} = \frac{e^{-g}N^{n-1}(e^{gN^{1-n}}-Ne^{gN^{-n}}+N-1)}{(1-e^{-g}-ge^{-g})}$$

Next, the probability that the collision is undetected in the entire transmission period given that there are k transmissions $P_{i,j}(k)$ is

$$P_{u}(k) = \left[\begin{pmatrix} 1 \\ - \\ N \end{pmatrix}^{k-1} \right]^{T} = N^{-T(k-1)}$$

Hence the probability that the collision is undetected given that a collision has occurred P_{ij} is

$$P_{u} = \sum_{k=2}^{\infty} P_{u}(k) \frac{g^{k}e^{-g}}{k!(1-e^{-g}-ge^{-g})} = \frac{e^{-g}N^{T}(e^{gN^{-T}}-1-gN^{-T})}{(1-e^{-g}-ge^{-g})}$$

Using the same notations in Chapter 2.1, we now have E[TP] equal to

$$E[TP] = (T+1)P_{s} + (1-P_{s})\sum_{n=1}^{T} (n+1)P_{D}(n) + (T+1)(1-P_{s})P_{u}$$

As before, the throughput S is given by

$$S = \frac{P_s T}{E[TP] + E[I]}$$

(B) Delay

The packet delay flow graph is shown in Fig. 9 and the following flow equations are readily obtained :

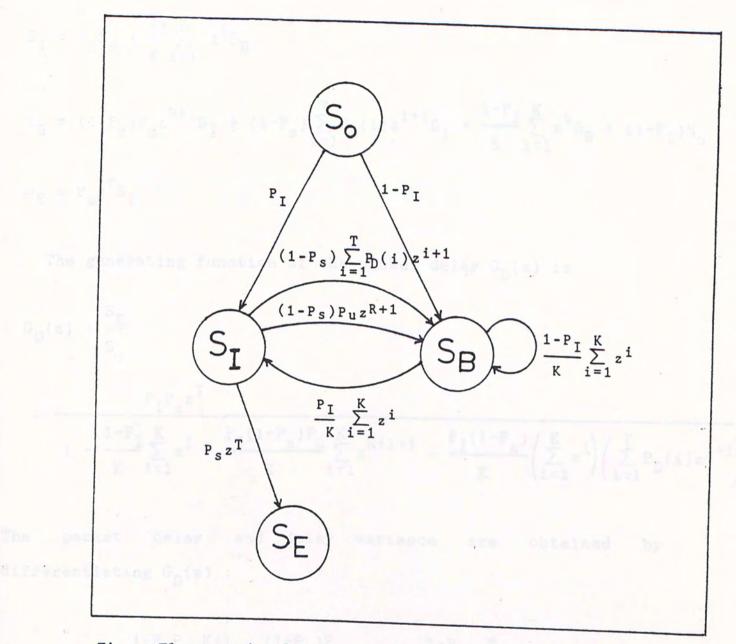


Fig. 9 Flow graph of packet delay process, slot-by-slot announcement scheme for non-persistent TSMA/PCD

$$S_{I} = P_{I}S_{o} + \frac{P_{I}}{K}\sum_{i=1}^{K} z^{i}S_{B}$$

$$S_{B} = (1-P_{s})P_{u}z^{R+1}S_{I} + (1-P_{s})\sum_{i=1}^{T}P_{D}(i)z^{i+1}S_{I} + \frac{1-P_{I}}{K}\sum_{i=1}^{K}z^{i}S_{B} + (1-P_{I})S_{o}$$

$$S_{E} = P_{s}z^{T}S_{I}$$

The generating function of the packet delay $G_{D}(z)$ is

$$G_{D}(z) = \frac{S_{E}}{S_{O}}$$

$$= \frac{P_{I}P_{s}z^{T}}{1 - \frac{1 - P_{I}}{K}\sum_{i=1}^{K}z^{i} - \frac{P_{I}(1 - P_{s})P_{u}}{K}\sum_{i=1}^{K}z^{R+1+i} - \frac{P_{I}(1 - P_{s})}{K}\left(\sum_{i=1}^{K}z^{i}\right)\left(\sum_{i=1}^{T}P_{D}(i)z^{i+1}\right)$$

The packet delay and delay variance are obtained by differentiating $G_{D}(z)$:

$$E[D] = T + \frac{1 - P_I P_s}{P_I P_s} \frac{K+1}{2} + \frac{(1 - P_s) P_u}{P_s} (R+1) + \frac{1 - P_s}{P_s} \sum_{n=1}^{T} (n+1) P_D(n)$$

$$\sigma_{D}^{\prime}^{2} = E[D] - T + (E[D] - T)^{2} + \frac{1 - P_{I}}{P_{I}P_{s}} \frac{(K-1)(K+1)}{3} + \frac{(1 - P_{s})(1 - P_{u})}{P_{s}} \frac{(K-1)(K+1)}{3} + \frac{(1 - P_{s})(1 - P_{u})}{P_{s}} \frac{(K-1)(K+1)}{3} + \frac{(1 - P_{s})(K+1)}{P_{s}} \frac{T_{s}}{P_{s}} \sum_{n=1}^{T} n(n+1)P_{D}(n) + \frac{(1 - P_{s})P_{u}}{P_{s}} \frac{(1 - P_{s})P_{u}}{2} \frac{(R+k)(R+K+1)(R+K+2) - R(R+1)(R+2)}{3K}$$

to a and for a=0.2, the throughput 5 Sebili is already higher than the tree-type collimits resolution electrics whereas for a=0.01. the maximum throughput is 0.013 for throughput performance of the 1-persistent version is similar and is shown in Fig.11

Fig. 12 shows 5 mg + function of the number of tones 8 for the non-persistent TRMA/PCD. Here, for M=1, packet collision cannot be detected and so TRMA/PCD degenerates to CRMA. For H-xcc all collisions are detected and so the throughput is the same as CSMA/CD. Note that for N=10, the throughput attains \$7.5% of the maximum and for N as small as 2, the throughput can attill be /2. of the maximum fig13 shows the same relationship for the 14 persistent version. We note that the N=2 curve is a listic stronge. That is, when 6 increases from 1 to 2.1, the increases in throughput is relatively small. This can be explained by the fact that then there is a collision involving is stations, the probability that the collision involving is stations, the probability is as the collision is undetected by is (1/2)² for N=2. For G=1, is is uncelly not large enough to drive the probability of undetected collision to a small only the is to 1

CHAPTER 6

NUMERICAL RESULT AND DISCUSSION

Fig. 10 shows the throughput S versus channel traffic G (in packets per packet transmission time) for non-persistent TSMA/PCD with a (the normalized ground channel propagation delay) as a parameter. The number of tones N is chosen to be 10 and the retransmission constant K=15T. As expected, S is very sensitive to a and for a=0.2, the throughput S (S $^{\circ}$ 0.5) is already higher than the tree-type collision resolution algorithm whereas for a=0.01, the maximum throughput is 0.933. The throughput performance of the 1-persistent version is similar and is shown in Fig.11

Fig. 12 shows S as a function of the number of tones N for the non-persistent TSMA/PCD. Here, for N=1, packet collision cannot be detected and so TSMA/PCD degenerates to CSMA. For N-7 ∞ all collisions are detected and so the throughput is the same as CSMA/CD. Note that for N=10, the throughput attains 97.6% of the maximum and for N as small as 2, the throughput can still be 92.% of the maximum. Fig13 shows the same relationship for the 1persistent version. We note that the N=2 curve is a little strange. That is, when G increases from 1 to 2.7, the increase in throughput is relatively small. This can be explained by the fact that when there is a collision involving k stations, the probability that the collision is undetected P_u is $(1/2)^{k-l}$ for N=2. For G²1, k is usually not large enough to drive the probability of undetected collision to a small number. This is

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verified in Fig.14 which shows that $P_u^{=0.5}$ when $G_{\leq 2.7}$. For $G_{<<1}$, there are relatively few collisions, so the poor collision detection condition is not severely penalized. For G>>1, the collision detection condition is greatly improved by the large value of k and so the throughput is not affected as much. Similar phenomenon does not appear in the non-persistent case because stations do not persist and so collisions are less frequent under the same load. We also see that without collision detection, the throughput of the non-persistent version drops by 12.5% (from 0.913 to 0.799) whereas for the 1-persistent case, the drop is 41.8% (from 0.901 to 0.524). Thus collision detection property is much more important to the performance of 1-persistent than non-persistent.

Figs. 15 and 16 show the throughput-delay characteristics of the non-persistent and 1-persistent TSMA/PCD respectively. Here, we choose the slot size to be 0.167ms which corresponds to a zone size of 50km across. The round trip propagation delay on the satellite channel is 270ms, and corresponds to 1617 slots. In the 1-persistent case, the sharp increase of delay for N=2 near S=0.6 is, of course, caused by the poor collision detection capability at this condition.

Fig.17 shows the coefficient of variation c_D of packet delay for the non-persistent TSMA/PCD with N as a parameter. When G>>1, the number of stations involved in a collision is usually fairly large, and so even for small N, collision detection probability

- 7

is high. Therefore, the major cause of delay is due to retransmission scheduling. In other words, for G>>1,

$$D = Y_1 + Y_2 + ... + Y_M$$

where Y_i is the i-th scheduling delay and M is the total number of scheduling before successful transmission and is geometrically distributed. In other words, if r is the probability of successful scheduling, then

$$P[M=k] = r(1-r)^{k-1}$$
 k=1,2,...

and from which we have

$$E[M] = 1/r$$
; $\sigma_M^2 = 1/r^2$ (14)

The mean and variance of D is derived in [10] to be:

E[D] = E[Y]E[M]

$$\sigma_D^2 = E[M]\sigma_Y^2 + E^2[Y]\sigma_M^2$$

By use of (14), the squared coefficient of variation is

$$c_D^2 = \frac{\sigma_D^2}{E^2[D]} = 1 + \frac{\sigma_Y^2}{E[M]E^2[Y]}$$

Given K, the retransmission scheduling constant, E[Y] and σ_Y^2 are constants independent of G. But as $G \rightarrow \infty$, $r \rightarrow 0$ and . $E[M] \rightarrow \infty$. Therefore, as $G \rightarrow \infty$, $c_D^2 \rightarrow 1$ and hence $c_D^- \rightarrow 1$. At G < 0.1, the delay variance is due primarily to Y. At $0.1 \le G \le 1$, the variance of both M and Y are significant and hence a peak occurs for c_D . The coefficient of variation for 1-persistent TSMA/PCD is

similar and is shown in Fig.18.

Fig. 19 shows the throughput performance of non-persistent TSMA/PCD in a noisy satellite channel with N=10 and a=0.02. It is shown that the channel throughput in noisy channel is dropped by a factor of $(1-P_e)$ of that of noise-free channel. The throughput performance of 1-persistent case is similar and is shown in Fig.20.

Figs. ²¹ and ²² show the throughput-delay characteristics of the non-persistent and 1-persistent TSMA/PCD in a noisy channel with P_e as a parameter. The mean delay shown is in unit of T. Here we see that there is a minimum value of mean delay for $P_e \neq 0$. This minimum delay value increases with P_e . Thus for $P_e = 0.1$, $D_{min} = 4T$. D_{min} increases to 18T and 40T when P_e is increased to 0.3 and 0.5 respectively. This minimum delay is predominately due to transmission error. It causes the total mean delay to be relatively stable until S is closed to S_{max} where the scheduling delay due to packet collision dominates.

It is obvious that the delay variance increases with the channel traffic G as well as P_e . What we now shown in Fig.23 is the coefficient of variation of packet delay c_D for the non-persistent TSMA/PCD in a noisy channel. Here, we see that an unusually large c_D is observed when P_e is between 0.01 and 0.1 for G<1. A possible explanation to this is that the delay variances due to packet re-scheduling and retransmission are both

significant in this stage whereas when $P_e \leq 0.01$ the delay variance due to scheduling dominates and when $P_e \geq 0.1$ the delay variance due to retransmission dominates. The former case consists of two types of variations. Hence c_D is expected to be larger. For $0.1 \leq G \leq 1$, a peak occurs for $P_e \leq 0.001$, and when $G \rightarrow \infty$, $c_D \rightarrow 1$. The explanations are the same as the noise-free case. The coefficient of variation of packet delay for 1-persistent version is similar and is shown in Fig. 24.

In Fig.²⁵ we compare the multi-tone and the single-tone throughput of the non-persistent TSMA/PCD. We see that the throughput of the multi-tone protocol is almost identical to CSMA/CD with N=8 whereas the single-tone protocol requires about N=50. Thus the multi-tone protocol can significantly decreases the number of tones used. For the same N=8 is used, the multitone version gives 3% higher throughput than the single-tone version. Similar phenomena are observed for the 1-persistent in Fig.²⁶.

Figs.27 compares the delays of multi-tone and single-tone protocols for non-persistent TSMA/PCD. Here, we see that the multi-tone and single tone versions offer almost identical delay performance. But this is not the case for 1-persistent TSMA/PCD as shown in Fig.28 where the multi-tone version offers a significant reduction of average delay.

The throughput of slot by slot announcement scheme for the non-persistent TSMA/PCD is shown in Fig.29. We see that for N as

small as 2, the maximum throughput(0.907) is already 99.3% of the CSMA/CD maximum throughput. The delay characteristics is identical to that of the CSMA/CD (Fig. 30).

Fig.10 The throughput performance of non-persistent TSMA/PCD with a as a parameter

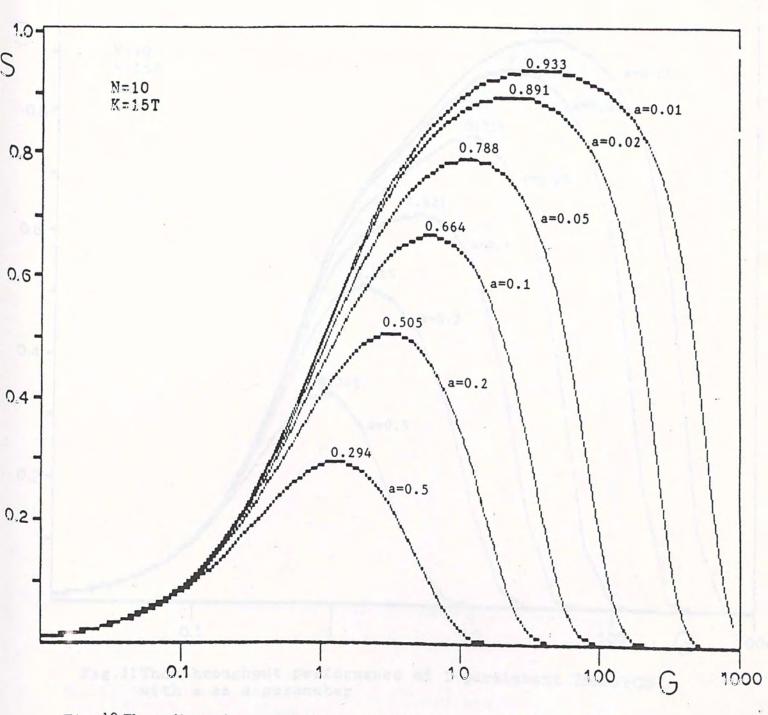


Fig.¹⁰ The throughput performance of non-persistent TSMA/PCD with a as a parameter

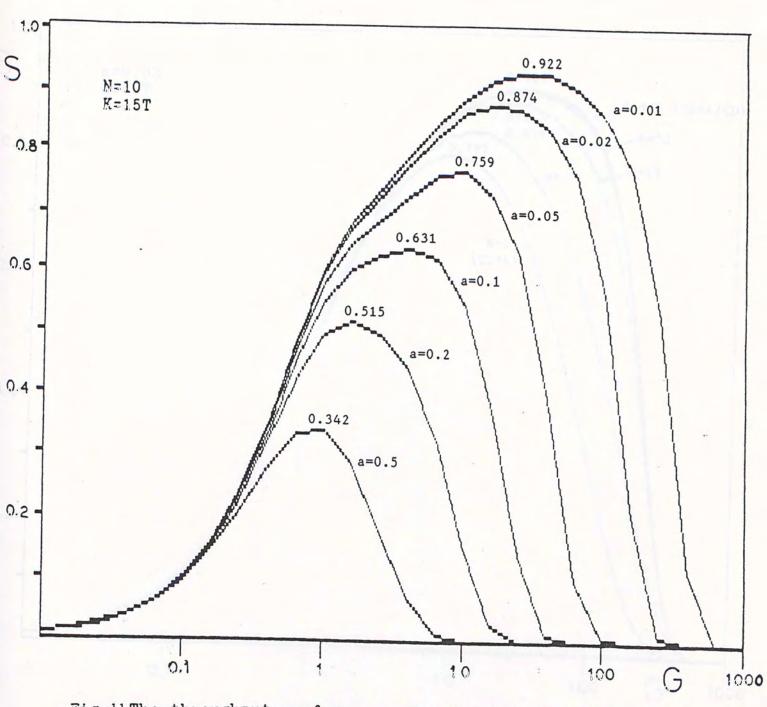


Fig.11 The throughput performance of 1-persistent TSMA/PCD with a as a parameter

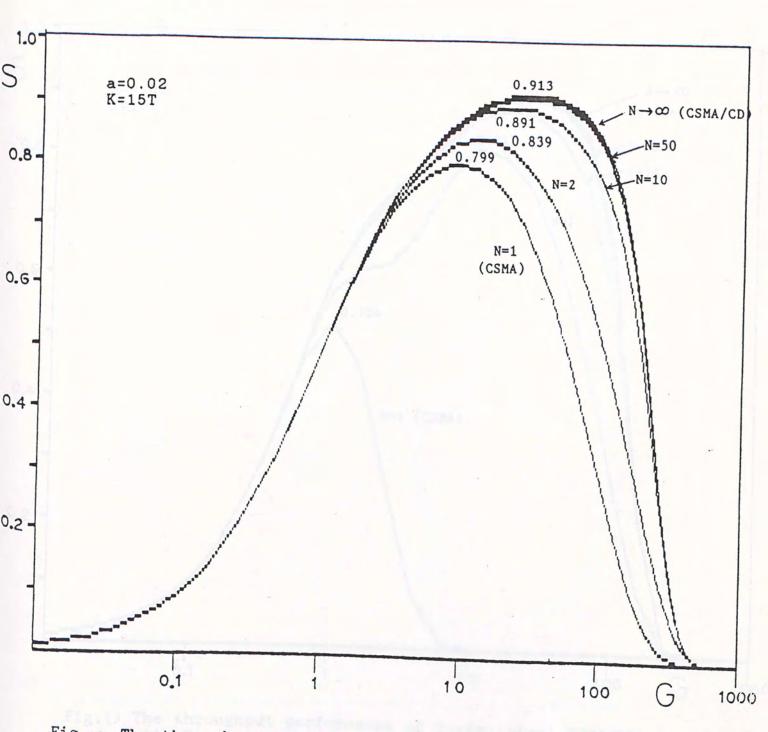


Fig.12 The throughput performance of non-persistent T SMA/PCD with N as a parameter

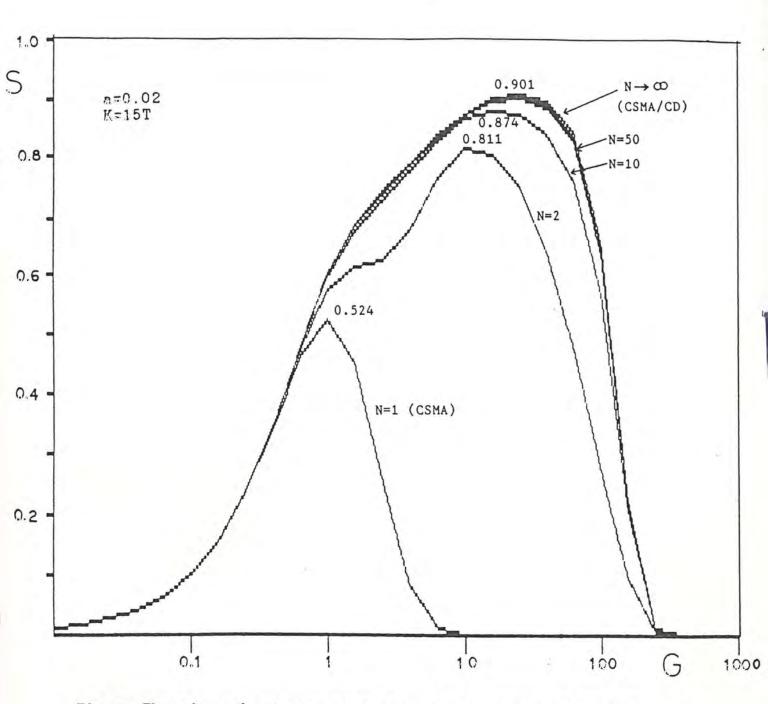
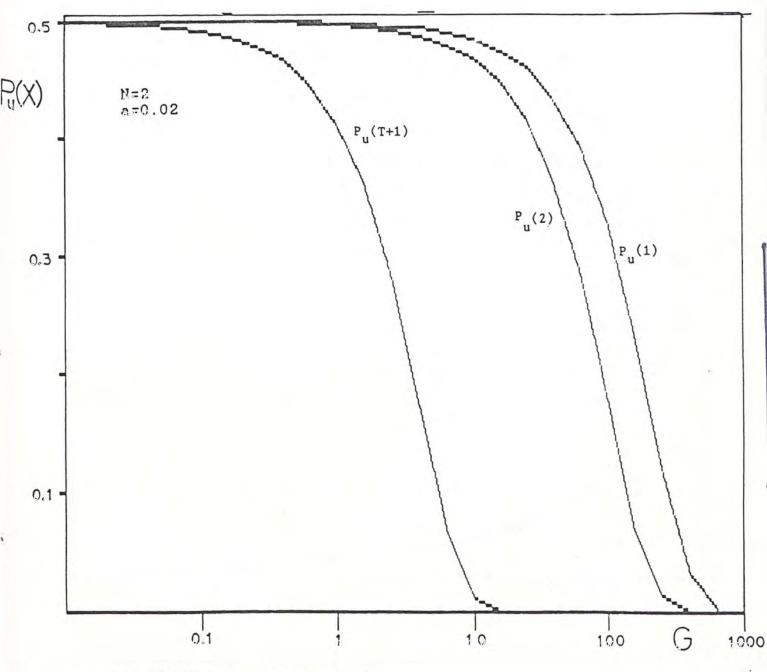
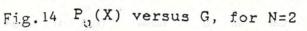
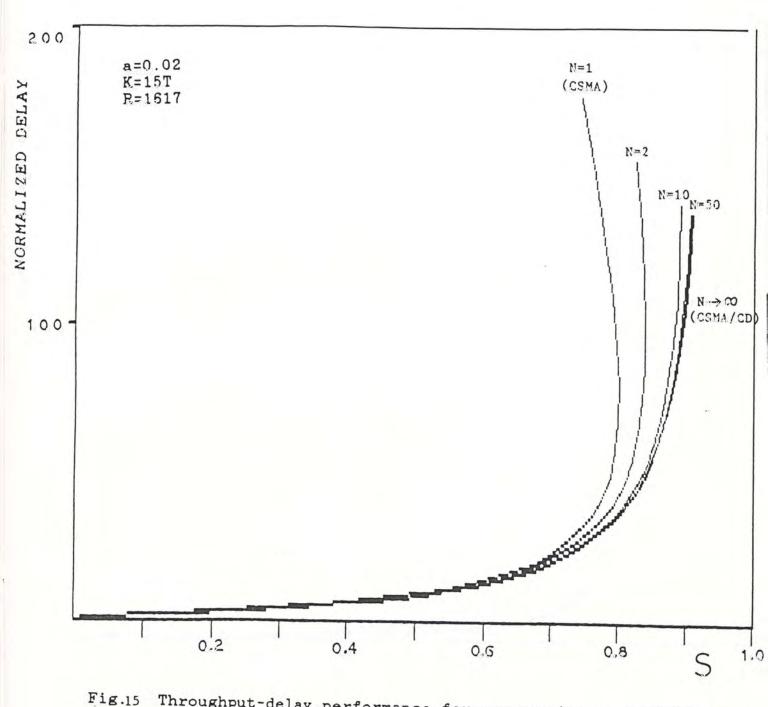


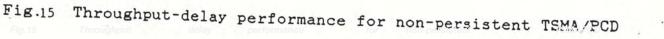
Fig.13 The throughput performance of 1-persistent TSMA/PCD with N as a parameter

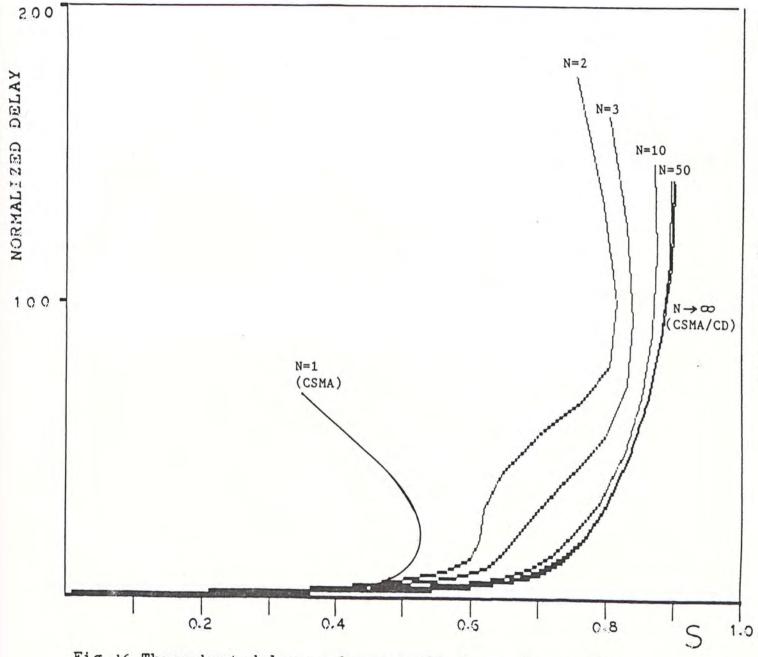
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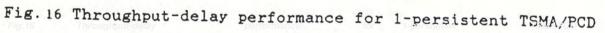




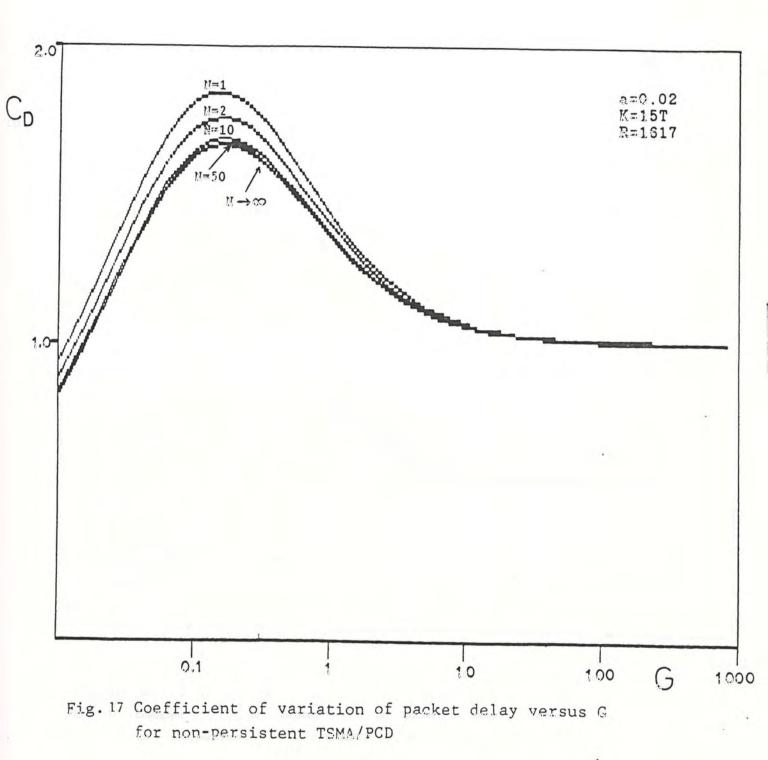


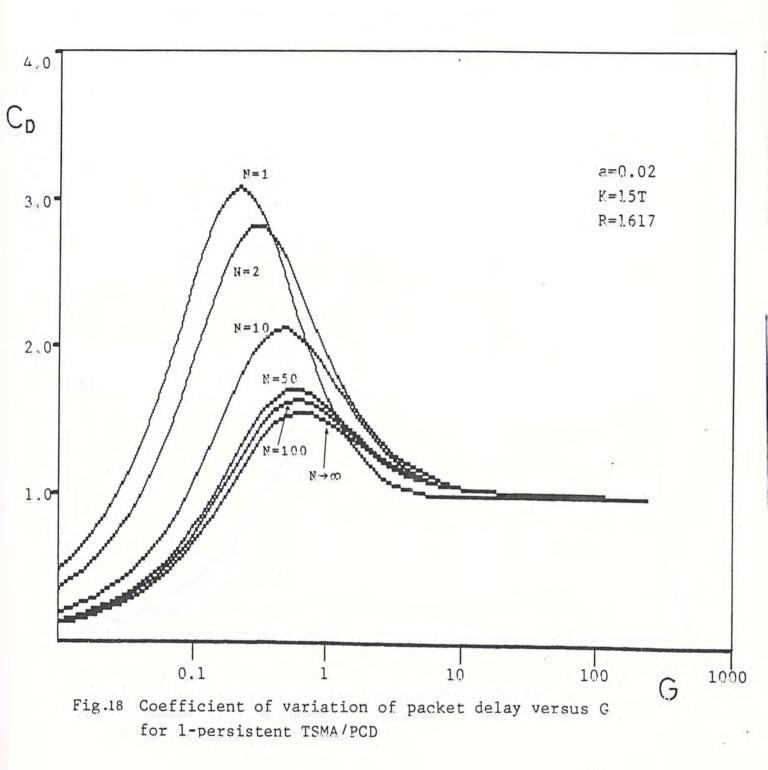


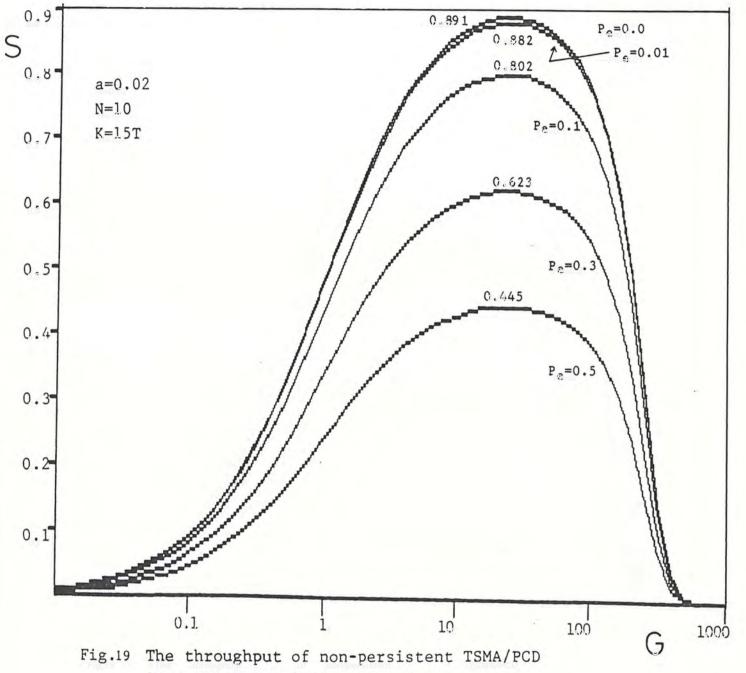




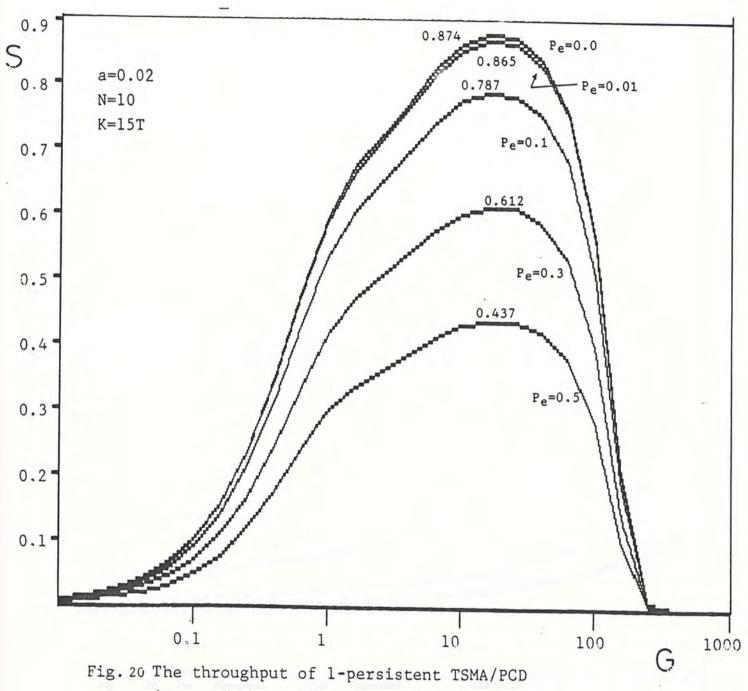
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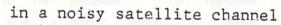


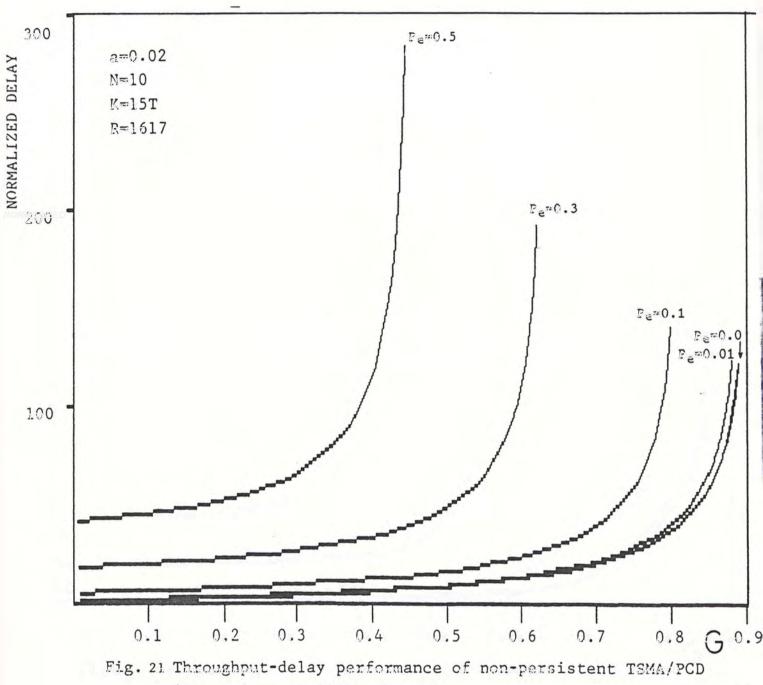


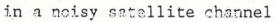


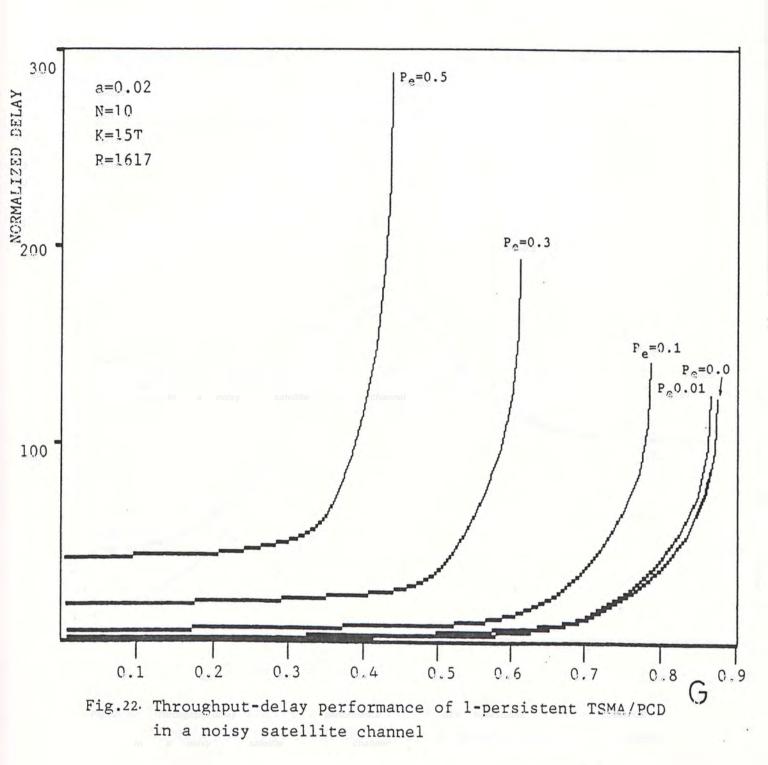
in a noisy satellite channel

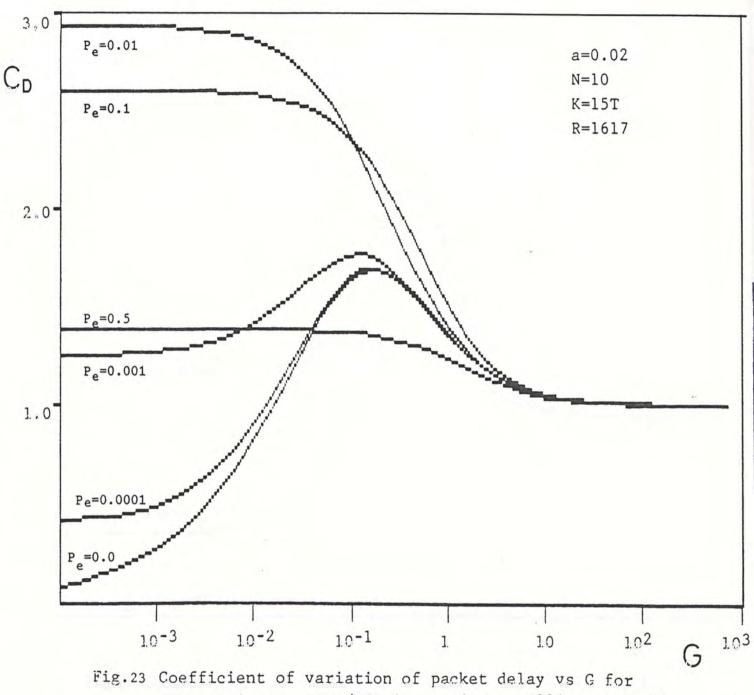




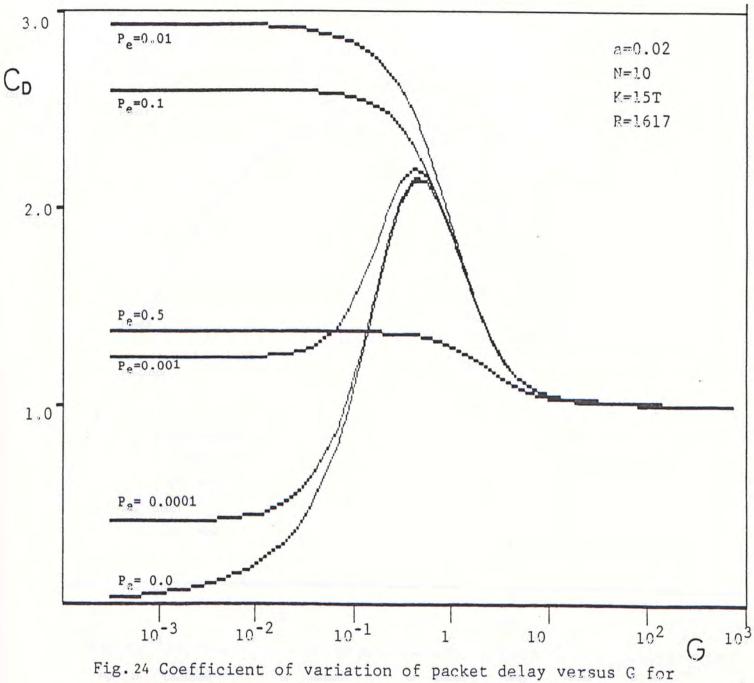




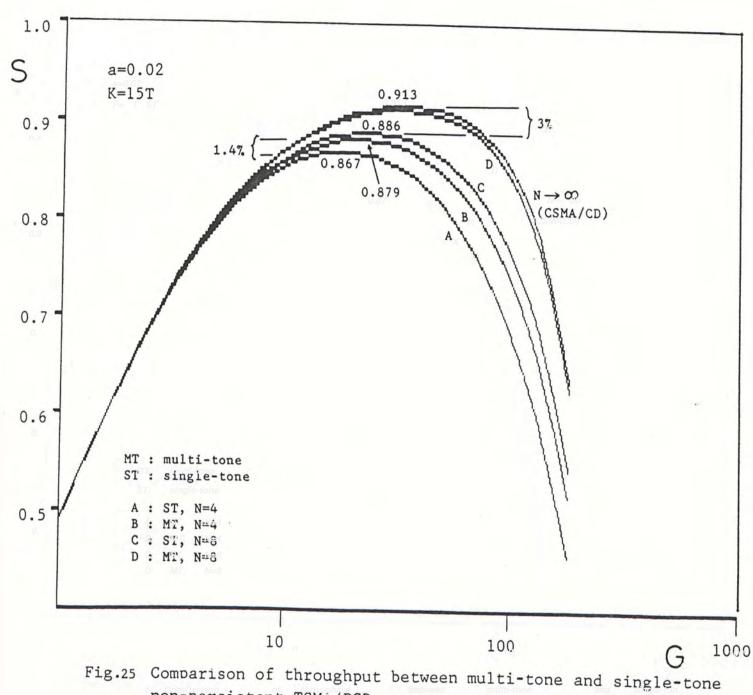




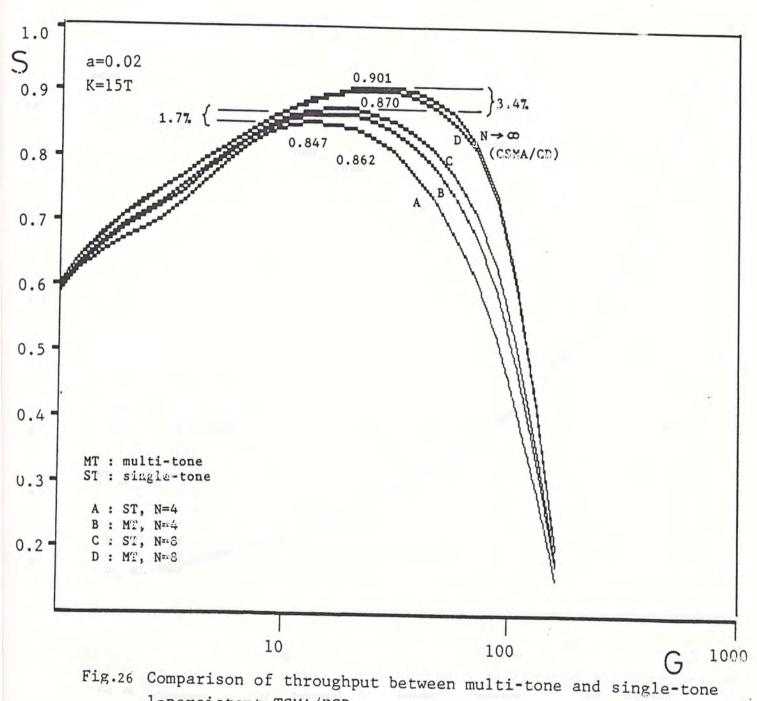
non-persistent TSMA/PCD in a noisy satellite channel



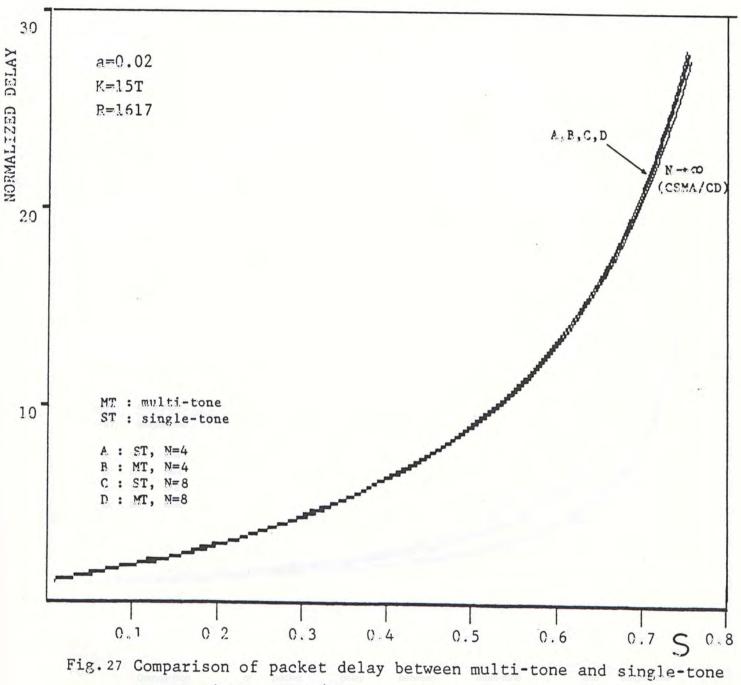
1-persistent TSMA/PCD in a noisy satellite channel



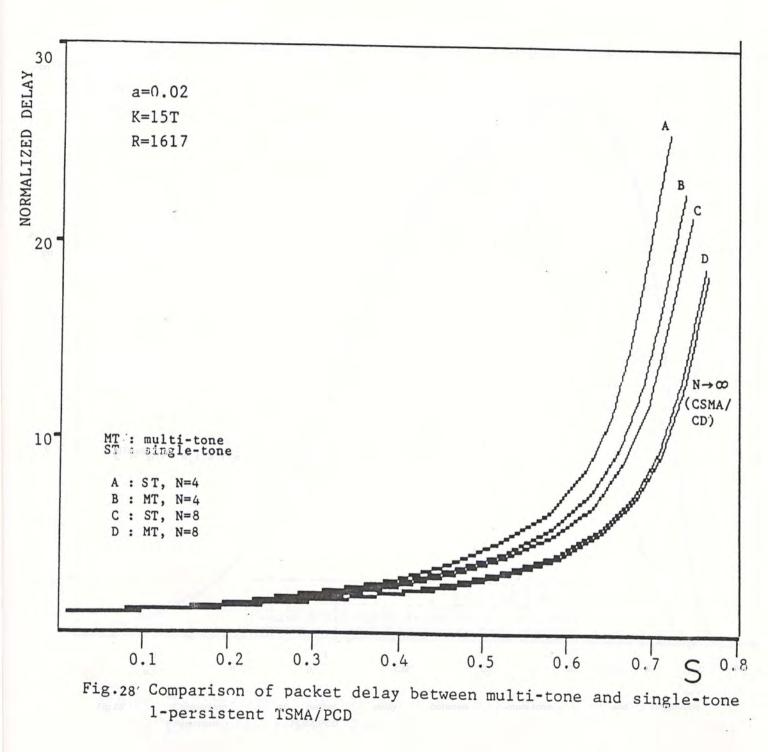
non-persistent TSMA/PCD



1-persistent TSMA/PCD



non-persistent TSMA/PCD



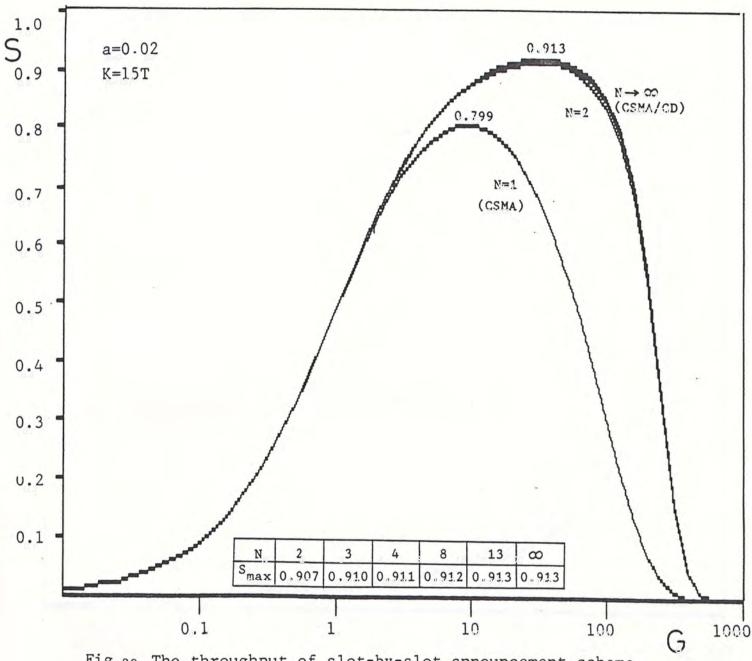
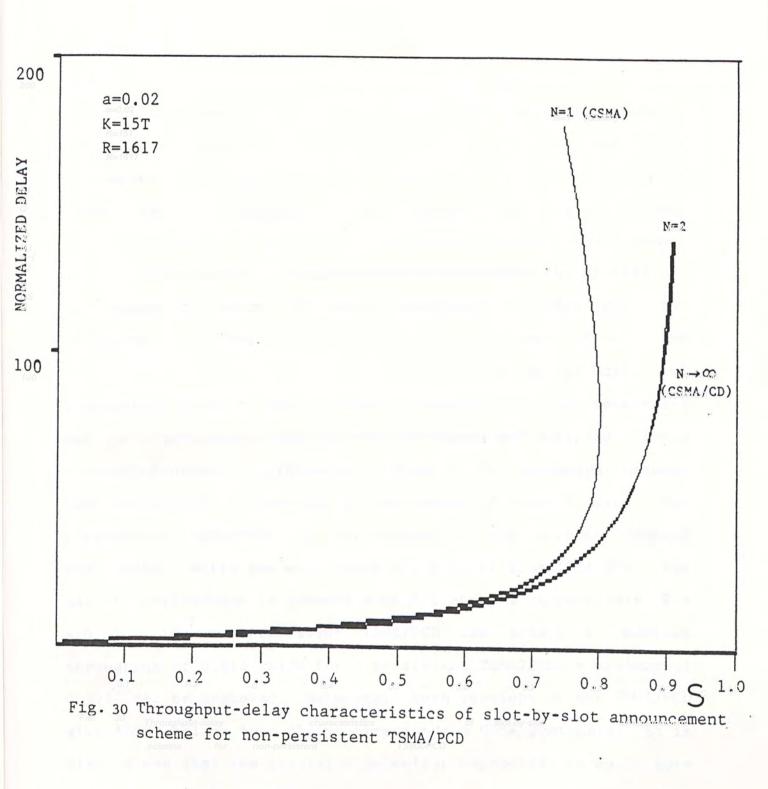


Fig.29 The throughput of slot-by-slot announcement scheme for non-persistent TSMA/PCD



CHAPTER 7

CONCLUSION

In this thesis, we have proposed a new contention protocol for packet satellite communications, called the Tone Sense Multiaccess with Fartial Collision Detection (TSMA/PCD). We have shown that by incorporating a narrow band ground radio channel for broadcasting busy tones, stations can sense the channel status before packet transmission and thus reduce the probability of packet collision. An added advantage is that when the colliding stations are not choosing the same tone, the unsuccessful transmission (or collision) can be detected and immediately aborted. Two versions of protocol, the non-persistent and the 1-persistent TSMA/PCD are introduced and analysed. Their throughput-delay performance is shown to be somewhere between CSMA and CSMA/CD, depending on the number of tones N used. For single-tone TSMA/PCD, the performance is very close to CSMA/CD when N=50. While for multi-tone and slot-by-slot TSMA/PCD, the maximum performance is reached when N=8 and N=2 respectively. For a=0.02, the non-persistent TSMA/PCD can attain a maximum throughput of 0.913 while for 1-persistent TSMA/PCD, a maximum of 0.901 can be achieved. Moreover, both versions of the TSMA/PCD give lower delays than their corresponding CSMA protocols. It is also shown that the collision detection capability is much more important to the performance of 1-persistent version. We find that without collision detection, the throughput of 1-persistent drops by 41.8% while that of non-persistent drops only by 12.5%. The performance of single-tone TSMA/PCD in a noisy channel is

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also studied. It is shown that due to transmission error, the throughput is decreased by a factor of P_e and a minimum mean delay which increases with P_e is incurred.

To conclude, we propose the following topics for further investigations. Firstly, instead of a Poisson arrival model, the finite population model can also be studied. Secondly, priority classes can be introduced into the system. Thirdly, higher level protocols can be studied. Note that in this thesis we have only considered a link level protocol. But a satellite serving a number of regions with inter-and intra-regional traffic would also involve the network and higher level protocols. The design and analysis of this is still at an infant stage.

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Appendix

To minimize P_u with respect to n, we first expand P_u as sum of non-negative terms :

$$P_u = (1/C_n^N)a_1 + (1/C_n^N)^2a_2 + (1/C_n^N)^3a_3 + \dots$$

where $a_k = (g^k e^{-g})/(k!(1-e^{-g}-ge^{-g}))$.

Therefore minimizing P_u is equivalent to mimimizing each individual term. Each term is minimum when the binomial coefficient C_n^N is maximized and this occurs at

$$n = \begin{cases} N/2 & , N = even \\ (N-1)/2 & , N = odd \end{cases}$$

