TESTS FOR LINEARITY IN TIME SERIES : A COMPARATIVE STUDY

By

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DECLARATION

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ABSTRACT

Many methods have been developed to detect the nonlinearity of time series, including the technique of nonparametric regression, Subba Rao-Gabr's test, Hinich's test and Keenan's test. We begin with a general description of these methods and a simulation study is presented to examine their power and their discriminant ability. Test results on real data will be discussed.

A new method which mainly based on the likelihood ratio test is proposed. After a simulation study, the merits and limitations of our test are discussed.

Finally, a comparsion among all tests on the simulation results, applications to real data, sensitivity to some special time series and computational efficiency is presented.

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CHAPTER I INTRODUCTION

It is a remarkable fact that linear Gaussian models have dominated the development of time series model building for the past five decades. Time series data from a variety of sources are often analyzed under the explicit or implicit assumption that they are generated by linear Gaussian processes. These processes are finite parameter linear stationary stochastic processes. The general linear process representation of $\{ X_r \}$ is of the form

$$X_r = \sum_{s=0}^{\infty} h_s e_{r-s}, \qquad (1.1)$$

where e_t are Gaussian, independent identically distributed random innovations with $E[e_t] = 0$. Although many successful examples can be found in linear Gaussian model building, it has been pointed out that there are still some limitations on this modelling technique (See, e.g., Tong 1983).

Recently, several special non-linear models in Time Series have been developed, including Bilinear Models (Granger and Andersen 1978, Subba Rao and Gabr 1984), Threshold Models (Tong 1983) and Exponential Autoregressive Models (Haggan and Ozaki 1978). A general non-linear time series model can be regarded as the "output" of a non-linear system whose "input" is a stationary random process $\{ e_t \}$. The output is of the form

$$X_t = f [e_t, e_{t-1}, \dots], \qquad (1.2)$$

where f is a nonlinear function that does not depend on t. Nonlinear time series data can exhibit limit cycles, clipping, hysteresis, and so on. We may often have to face the problem of deciding whether a given set of data are generated from a linear or non-linear process.

Diagnostic checking in the traditional Box and Jenkins approach is not designed to reveal non-linearity of the time series data. A simple graphical method that can be used to detect nonlinearity is nonparametric kernel regression (See, e.g., Watson 1964). However the accuracy of this method may be affected by the subjective choice of bandwidth and type of window. Nevertheless, nonparametric regression is valuable as a preliminary examination of the data.

Today, businessmen, engineers and sociologists may face a lot of time series data; they need a systematic method of discriminating between linear and non-linear time series.

CHAPTER II EXISTING METHODS

Recently, a few statistical methods designed to detect certain types of nonlinearity in a time series have appeared in the literature. Some of them are nonparametric methods, while others are parametric tests.

Subba Rao and Gabr (1980) presented a test for nonlinearity using a sample estimates of the bispectrum of time series. Hinich (1982) presented a nonparametric test that also uses the sample bispectrum, but which takes advantage of the asymptotic properties of the bispectrum estimator. Keenan (1985) described a Tukey nonadditivity - type test for nonlinearity in time series. The test is regarded as a diagnostic for linearity versus a second-order Volterra expansion. In this chapter, we are going to investigate existing methods individually.

2.1 SUBBA RAO-GABR'S TEST

Let $\{ X_t \}$ be a time series with third order moments.

Define

$$T_{ij} = \frac{| f(w_i,w_j) |^2}{f(w_i)\, f(w_j)\, f(w_i+w_j)} \qquad (2.1.1)$$

where

$f(w)$ is the spectral density function of $\{ X_t \}$

$$f(w) = \frac{1}{2\pi} \sum_{s=-\infty}^{\infty} R(s)\, e^{isw} \qquad (2.1.2)$$

with R(s) is the autocovariance function of $\{ X_t \}$

$$R(s) = E [X_t X_{t+s}] \tag{2.1.3}$$

$f(w_1, w_2)$ is the bispectral density function of $\{ X_t \}$

$$f(w_1, w_2) = \frac{1}{(2\pi)^2} \sum_{r,s=-\infty}^{\infty} C(r,s)\, e^{-irw_1} e^{-isw_2} \tag{2.1.4}$$

with C(r,s) is the third-order central moment of $\{ X_t \}$

$$C(r,s) = E [X_t X_{t+r} X_{t+s}] \tag{2.1.5}$$

Subba Rao-Gabr (1980) showed that the T_{ij} should be constant over all w_i and w_j if $\{ X_t \}$ is given by a linear representation , i.e.

$$X_t = \sum_{r=-\infty}^{\infty} a_r e_{t-r} \tag{2.1.6}$$

where $\{ e_t \}$ is a sequence of independent identically distributed random variables with zero mean, constant variance σ_e^2 . The actual test statistic is based upon the complex-valued analogue of Hotelling's T^2 test of the mean vector lying on the equiangular line. Subba Rao-Gabr use the asympototic complex-normality of the bispectrum in a certain triangular region of $[0, 2\pi]^2$ (Van Hess 1966). They, however, use as their estimated covariance matrix the usual sample second moment matrix of multivariate analysis, treating the bispectral estimates as the data, rather than using the known asymptotic covariance matrix of the bispectral estimates.

The hypothesis of Subba Rao-Gabr's test are :

H_0 : $f(w_i, w_j)$ is constant for all w_i and w_j

H_1 : $f(w_i, w_j)$ is not constant for some w_i and w_j

Acceptance of H_0 is consistent with linearity while rejection of H_0 implies that the process is not linear.

In order to obtain $\hat{T}_{ij}$, we need to estimate f(w) and $f(w_i,w_j)$ of $\{ X_t \}$. Subba Rao-Gabr (1984) considered the estimation of the spectral and bispectral density function using the spectral window approach (See, e.g., Priestley 1981). The natural estimates of $E[X_t]$ and R(s), repectively, are

$$\bar{X} = \frac{1}{N} \sum_{i=1}^{N} X_t \qquad (2.1.7)$$

$$\hat{R}(s) = \frac{1}{N} \sum_{t=1}^{N-|s|} (X_t-\bar{X})(X_{t+|s|}-\bar{X}) \qquad (2.1.8)$$

where $s = 0, \pm 1, \pm 2, \ldots, \pm (N-1)$

The estimated spectral density function is

$$\hat{f}(w) = \frac{1}{2\pi} \sum_{s=-n}^{n} V[s/M]\, \hat{R}(s) \cos ws \qquad (2.1.9)$$

where M is the truncation point and V is the lag window function. Some of the windows which will be used in the simulation study in Section 4 of this chapter are given in table 2.1.

Table 2.1 here

The estimated bispectral density function is

$$\hat{f}(w_1,w_2) = \frac{1}{(2\pi)^2} \sum_{r=-n}^{n} \sum_{s=-n}^{n} V_2[\tfrac{r}{M},\tfrac{s}{M}]\, \hat{C}(r,s)\, e^{-irw_1} e^{-isw_2} \qquad (2.1.10)$$

where n = N-1

$$\hat{C}(r,s) = \frac{1}{N} \sum_{i=1}^{N-P} (X_t-\bar{X})(X_{t+r}-\bar{X})(X_{t+s}-\bar{X}), \quad r,s \geq 0 \qquad (2.1.11)$$

$P = Max(0,r,s)$

V_2 is the two dimensional spectral window

$$V_2(p,q) = V(p)\ V(q)\ V(p-q) \qquad (2.1.12)$$

Subba Rao-Gabr (1984) have compared different type of one-dimensional and two-dimensional lag windows. In Section 2.4, we have performed their test under different types and bandwidths of lag windows. Some interesting results were discovered.

2.2 HINICH'S TEST

Hinich (1982) improved on Subba Rao-Gabr's test by using the known asymptotic covariance matrix of the bispectral estimates and also proposed a test based on the interquartile range of square modulus of the sample bispectrum over a certain triangular region $[\ 0,\ 2\pi\ ]^2$. The test statistic is the interquartile range of a subset of $\{2|T_{ij}|^2\}$, which is approximately normal distributed under the linearity hypothesis and certain conditions.

There are many ways to average the bispectral density function f(i,j) to obtain a consistent estimate of the bispectrum on a lattice of points in the triangular grid region. Subba Rao-Gabr use the standard window approach, but Hinich prefers to smooth the f(i,j) in a square of M^2 points, where $M = N^c$ for N is number of data, $0.5 < c < 1$. The parameter c controls the trade off between bias and variance.

This nonparametric test proposed by Hinich is based on a robust measure of dispersion. The Subba Rao-Gabr's F test can be sensitive to outliers in the T_{ij} due to small estimates of the spectrum at certain frequences. Also, the power of Hinich's test is high when rN is large (r is the average skewness).

2.3 KEENAN'S TEST

An important insight to the nature of the general nonlinear model is provided by the discrete time Volterra series expansion :

$$X_t = \sum_{i=0}^{\infty} a_i e_{t-i} + \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} a_{ij} e_{t-i} e_{t-j}$$

$$+ \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} a_{ijk} e_{t-i} e_{t-j} e_{t-k} + \ldots \qquad (2.3.1)$$

The a_i, a_{ij}, a_{ijk}, ... are coefficients of the series and the e_t input process is usually considered to be unobservable. Wiener (1958), in a classic study of nonlinear systems, used the continuous time version of (2.3.1) to express the nonlinear relationship between input and output of a physical system. When $\{ e_t \}$ is a purely random process with zero mean, the first term in (2.3.1) is a general linear model and the successive terms are usually referred to as the "quadratic", "cubic", ... components.

When restricting to the quadratic component of the Volterra series expansion, Keenan (1985) argues that a test of linearity is equivalent to test whether all a_{ij} in (2.3.1) are equal to zero. An analogue of Tukey's (1949) one degree of freedom for nonadditivity test provides a framework to achieve this target.

The hypothesis of Keenan's test are :

$H_0 : a_{ij} = 0$ where a_{ij} are coefficients in (2.3.1)

$H_1 : a_{ij} \neq 0$

The steps to calculate the test statistic of Keenan's test are as follows :

(i) Regress X_s on { 1, X_{s-1}, ... , X_{s-M} } and calculate the fitted values { $\hat{X}_s$ } and the residuals, { $\hat{e}_s$ } , for s = M+1, ..., n , and the residual sum of squares, $< \hat{e},\hat{e} > = \Sigma \hat{e}_s^2$.

(ii) Regress $\hat{X}_s^2$ on { 1, X_{s-1}, ... , X_{s-M} } and calculate the residuals { $\hat{E}_s$ }, for s = M+1, ... , n.

(iii) Regress $\hat{e} = (\hat{e}_{M+1}, \ldots , \hat{e}_n)$ on

$\hat{E} = (\hat{E}_{M+1}, \ldots , \hat{E}_n)$

and obtain N^2 and F via

$$N^2 = N_0^2 \left[\sum_{t=M+1}^{n} \hat{E}_t^2 \right]$$

where N_0 is the regression cofficient and

$$F = \frac{N^2 [n - 2M - 2]}{< \hat{e},\hat{e} > - N^2}$$

Keenan proved that for a large n (sample size) and a large, fixed M, F is asymptotically Chi-square distributed with one degree of freedom.

2.4 A SIMULATION STUDY

To investigate each existing tests, nine sample models have been selected as the basis for examining the discriminant ability of each method.

APPENDIX I here

Models 1 and 2 are linear models, second-order moving average and second-order autoregressive, respectively. The others are all nonlinear models; models 3,4,5 and 6 are some special nonlinear time series models (Priestley 1981, Section 11.6). Models 7,8 and 9 are some nonlinear MA models which have been discussed by Keenan (1985). The parameter values of the sample models are fairly representative, in the sense of not being close to the boundary. The { e_t } are all Gaussian with zero mean and unit variance.

A. Nonparametric Regression

In Chapter I, we have mentioned that people usually use nonparametric regression as a preliminary technique to detect nonlinearity in time series. A series with 10,000 data was generated for each sample model. By using G. S. Watson's (1964) approach, we performed the nonparametric regression analysis for each of them. $E[X_t|X_{t-1}]$ are plotted.

Figs 2.1a to 2.1i here

The nonparametric regression seems quite reasonable for detecting nonlinearity in the sample series. Fig 2.1 a and b show the linear characteristics of models 1 and 2. The nonlinear elements of models 3, 4, 5, 6, 8 and 9 can be identified by means of this technique. However, model 7 has hidden its nonlinearity under nonparametric regression.

Although the performance of nonparametric regression in our sample models is quite satisfactory, there are still some deficiencies of this technique. How to choose a suitable type of window and its width ? Different choices among them may give different conclusions. Moreover, the scale of the graph $E[X_t|X_{t-1}]$ is very important, different scales may reveal different shapes. Finally, in the above simulation, we have chosen a very large sample size (N=10,000). The results are not satisfactory when the sample size is less than 300.

B. Subba Rao-Gabr's Test

Subba Rao-Gabr (1984) provided a FORTRAN program to perform their test. However, we discovered that there are some minor bugs within the program :

```
line 58   original  : R(I)=V(I)*SUM/FLOAT(N-I)
          corrected : R(I)=V(I)*SUM/FLOAT(N)

line 93   original  : S=V(J1)*V(J2)*V(J1-J2)*C(J1,J2)
          corrected : S=V(J1)*V(J2)*V(J1-J2+1)*C(J1,J2)

line 231            : Subroutine FO4ASF is missing
```

FO4ASF is replaced by IMSL subroutines which are designed to perform matrix operations. After correcting these minor mistakes, their program will run. Moreover, we added to the program two subroutines DANIEL and BRTPRI which evaluate Daniell and Bartlett-Priestley windows respectively.

In order to examine the Subba Rao-Gabr's test, we have generated one linear time series from model 1 (Series A) and one nonlinear time series from model 9 (Series B), each one with size N=500. These series are plotted in Fig 2.2a and Fig 2.2b.

Figs 2.2a and 2.2b here

In estimating the spectral and bispectral density functions, we have attempted to use different types of window and different M. The parameters K, L, d, r, p and n for constructing the F-statistic are as follows

$$K=6 \quad L=4, \quad P=7, \quad r=2 \quad n=9, \quad d=15.$$

The critical value is upper 5% point of F with (6,3) degrees of freedom (8.94). The calculated F values for Series A and Series B are given in Tables 2.2a and 2.2b

Tables 2.2a and 2.2b here

The F-statistics vary a lot under different M and types of window. It reveals that the F-statistic under Subba Rao-Gabr's test is extremely unstable and heavily depends on the choices of M and type of window. As a typical example, if we choose one and two dimensional Daniell window, M=30 for Series A, the conclusion of Subba Rao-Gabr's test is linear (1.10 < 5.89). But if we choose Parzen window with M=30, the conclusion will be reversed (15.95 > 5.89) !

The main problem of Subba Rao-Gabr's test is the erratic behaviour of $\hat{T}_{ij}$ under different types of window and M values. From (2.1.2)

$$T_{ij} = \frac{| f(w_i,w_j) |^2}{f(w_i)\, f(w_j)\, f(w_i+w_j)}$$

the values of $f(w_i)$, $f(w_j)$ and $f(w_i+w_j)$ may be very close to zero at certain frequencies and T_{ij} will blow up. At different M values and types of window, the location of these frequencies are different. The F-statistic is also very sensitive to outliers in the T_{ij} and will lead to instability in Subba Rao-Gabr's test.

Moreover, Subba Rao-Gabr (1984) chose a distance "d" so that the bispectral estimates at neighbouring points on the fine grid are assumed to be uncorrlated. This assumption may not be always true. Also, their test requires the existence of third-order moment of the data, which may not always hold without appropriate instantaneous transformation. These are some of the limitations of their test.

C. Hinich's Test

The test is examined under the nine sample models in Appendix I.

APPENDIX I here

To analyse the discriminant ability of Hinich's test, 100 replications were performed for each sample model with N=204. When we set α =0.05, the critical values are $\pm$ 1.96. Table 2.3 gives the results.

Table 2.3 here

Hinich's test seems unable to detect the nonlinearity in Threshold and Exponential Autoregressive models (models 3,5, and 6). Its discriminant ability for other non-linear models is not very high (models 4,7, and 8), even less than 40%. Fortunately, Hinich's test can detect the linearity in models 1 and 2 quite satisfactory.

D. Keenan's Test

For the convenience of comparsion, we also used the nine sample models in Appendix I to examine Keenan's test. We maintained N=204, 100 replications were performed for each sample model. The summary are given in Table 2.4.

Table 2.4 here

Keenan's test performed quite well in models 1, 2, 4, 8 and 9, but it failed to detect the nonlinearity in models 3,5 and 7. The original design of Keenan's test is to detect the existence of "quadratic" component in Volterra series expansion. However, some nonlinear time series models may without "quadratic" compenent but have other higher order terms. For an example, consider a general Exponential AR(1) model:

$$X_t = [\ a + b\ \exp(-g\ X_{t-1}^2)\ ]\ X_{t-1} + e_t$$

$$= \{\ a + b\ [\ 1 + (-g\ X_{t-1}^2) + 1/2!\ (-g\ X_{t-1}^2)^2 + \ldots]\ \}\ X_{t-1} + e_t$$

$$= \{\ aX_{t-1} + bX_{t-1} - gbX_{t-1}^2 + 0.5g^2bX_{t-1}^4 + \ldots\}\ X_{t-1} + e_t$$

$$= p_1\ X_{t-1} + p_2\ X_{t-1}^3 + p_3\ X_{t-1}^5 + \ldots + e_t$$

where p_1, p_2, p_3, ... are constants in terms of a, b and g

i.e. $X_t = \sum_{i=0}^{\infty} q_i e_t + \sum_{i,j,k=0}^{\infty}\sum^{\infty}\sum^{\infty} q_{ijk} e_{t-i} e_{t-j} e_{t-k} + \ldots$ for some q_i, q_{ijk}

Therefore, the Volterra series expansion of model 3 is only in odd power terms. Keenan's test cannot detect its nonlinearity.

Moreover, some symmetric with respect to the origin SETAR models, as model 5, are approximately close to those Exponential AR(1) models.

It may explain the failure of detecting the nonlinearity in model 5 by Keenan's test. Figs 2.3a and 2.3b show the shapes in E[X_t] vs X_{t-1} of models 3 and 5 respectively.

Figs 2.3a and 2.3b here

2.5 APPLICATION TO REAL DATA

The test were also applied on some well known time series data. The series considered are

(i) the Canadian lynx data ;

(ii) Wölfer sunspot numbers ; and

(iii) Nicholson's blowfly data.

The first series we considered is the annual number of Canadian lynx trapped in the Mackenzie River district of North-west Canada for the years 1821-1934, giving a total of 114 observations. These numbers are given in Appendix II and plotted in Fig 2.4a. There is an obvious cycle of approximately ten years with varying amplitude. We tested both raw and logarithm of the data. The second series we considered is the Wölfer sunspot for the years 1700-1955 (Waldmeirer,1961), giving 256 observations. These numbers are given in Appendix III and plotted in Fig 2.4b. This series has a certain historic interest for statisticians, see, e.g. Yule(1927), Bartlett(1950), Whittle(1954) and Brillinger and Rosenblatt(1967). It is believed by many scientists that this series has an eleven year cycle. We applied the raw, logarithm and a special square root (proposed by Tong, 1983) transformed data to the tests. The last series we considered is a laboratory data -- Nicholson's blowfly data. Nield in an unpublished M.Sc. dissertation (University of Manchester, 1982; see also Tong, 1983, S5.4) has seperated the data set into two halves :

BLOWFLY A	$20 \leq t \leq 145$
BLOWFLY B	$218 \leq t \leq 299$

The full set of blowfly data are given in Appendix IV. Fig 2.4c gives the data plot, from whcih the time intervals of BLOWFLY A and BLOWFLY B are specified. Chan and Tong (1985) has made the following comments to the blowfly data :

(i) that there is a sharply defined change point, i.e. threshold, in the generating mechanism for the first half of the data set (BLOWFLY A) and that the generating mechanism is best taken to be non-linear ;

(ii) that, for the second half of the data set (BLOWFLY B), there is no evidence of non-linearity.

Figs 2.4a, 2.4b and 2.4c here

Nonparametric regression were applied to each raw series and the results are plotted.

Figs 2.5a to 2.5d here

The graphs suggest that all the input series are generated from non-linear process, even BLOWFLY B data set.

We tried Subba Rao-Gabr's test for each series. The values of K was chosen, as in Section 2.4, to be K=6, which implies that L=4 and P=7.

Also r=2, which implies that n=9 for all the above real series. The critical value is upper 5% point of F distribution with (6,3) degrees of freedom (8.94).

Tables 2.5a to 2.5d here

The computation of Subba Rao-Gabr's F-statistic again behaves quite erratically when applying to the real series.

The results of Hinich's test for the real series are given in a condensed table.

Table 2.6 here

The test concluded that lynx and sunspot data are generated from linear processes, regardless of the transformation. In BLOWFLY A data, Hinich's test detects nonlinearity after the logarithm transformation. His test accepts the linearity in BLOWFLY B data.

We also applied Kaneen's test to each real series, the results are summarized at the table 2.7.

Table 2.7 here

The test classifies the lynx data into linear time series. For BLOWFLY A, Keenan's test concluded the raw data as linear but the transformed data are non-linear. Our experience suggests that instantaneous transformation has a useful role to play in detecting non-linearity. Now, tests for nonlinearity are usually based on the assumption of homogeneous variance. A suitable instantaneous transformation may be needed to stablize the variance. The non-linear behaviour of the series could then become more obvious.

2.6 COMMENTS

Various tests for linearity have been proposed although none is totally satisfactory. After the study of these existing tests, we have found out some weakness of each method.

A. Subba Rao-Gabr's test

Because of the numerical instability of Subba Rao-Gabr's F-statistic, it is difficult to interpret results of their test.

B. Hinich's test

Although an improvement over Subba Rao-Gabr's test, Hinich's test seems unable to detect some Threshold or Exponential AR models. The discriminant ability of his test is rather low. It quite often draws misleading conclusion.

C. Keenan's test

The test fails to detect those nonlinear time series models which do not have the quadratic component in Volterra series expansion.

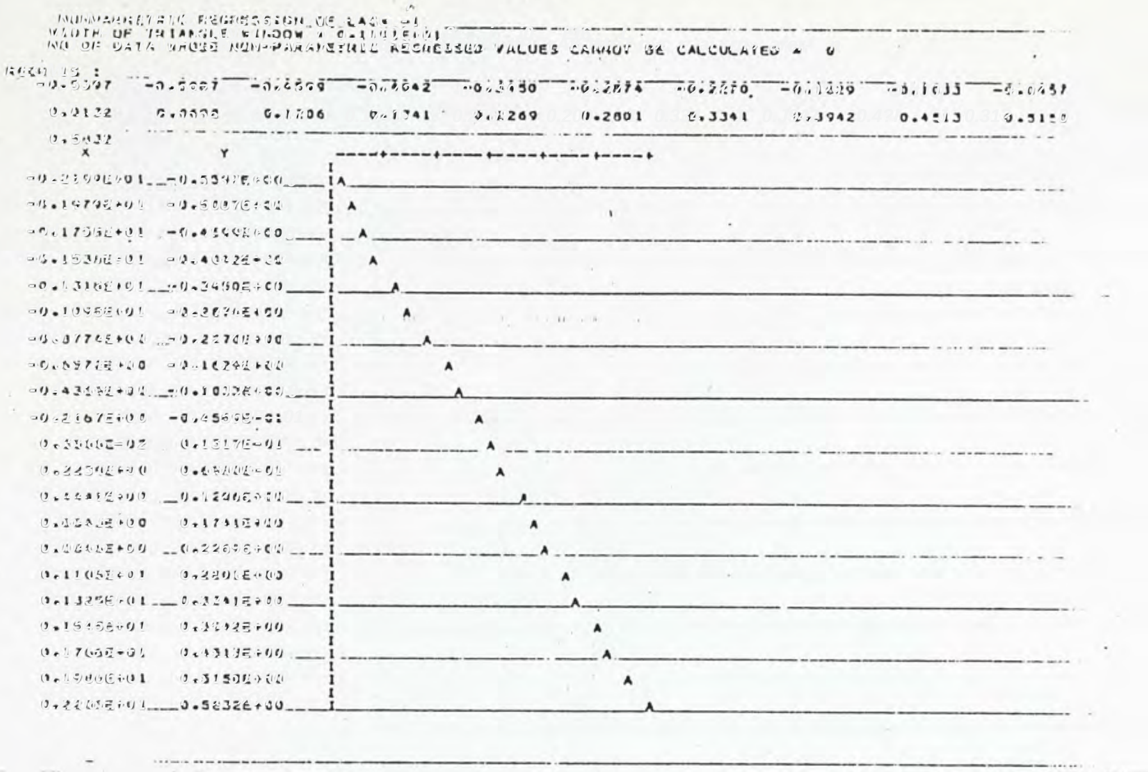


Fig 2.1a $E[X_t|X_{t-1}]$ for Model 1

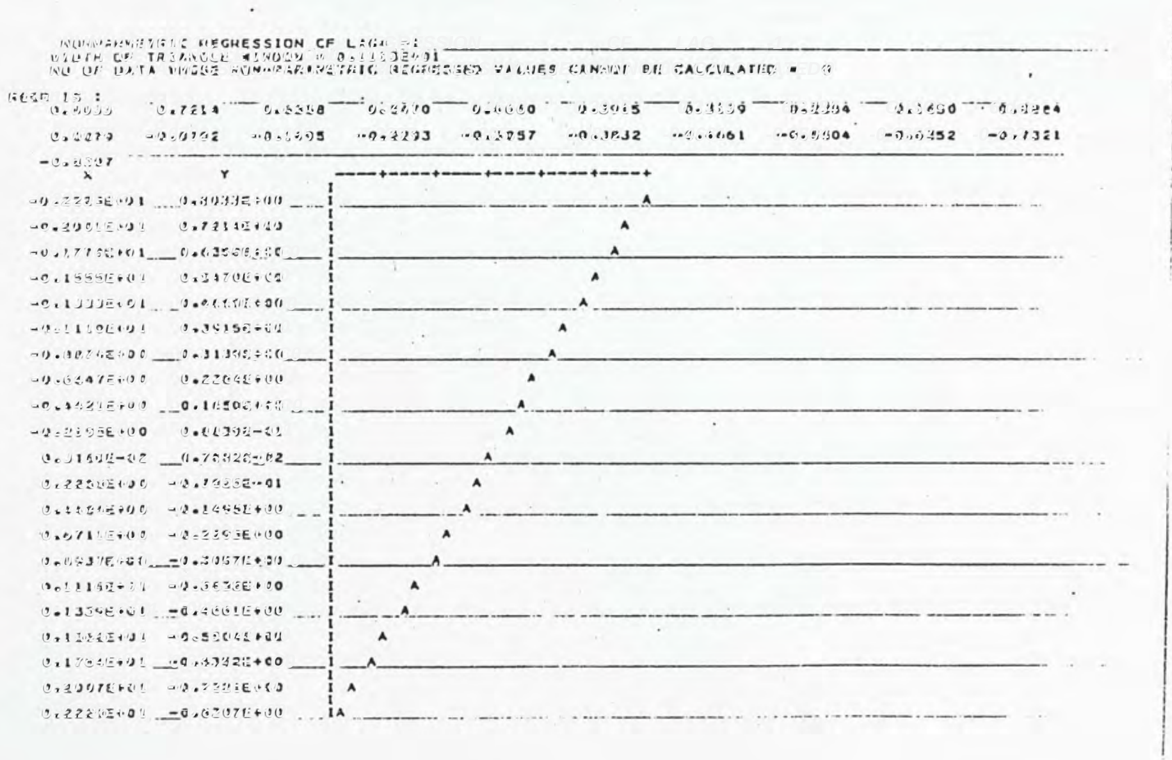


Fig 2.1b $E[X_t|X_{t-1}]$ for Model 2

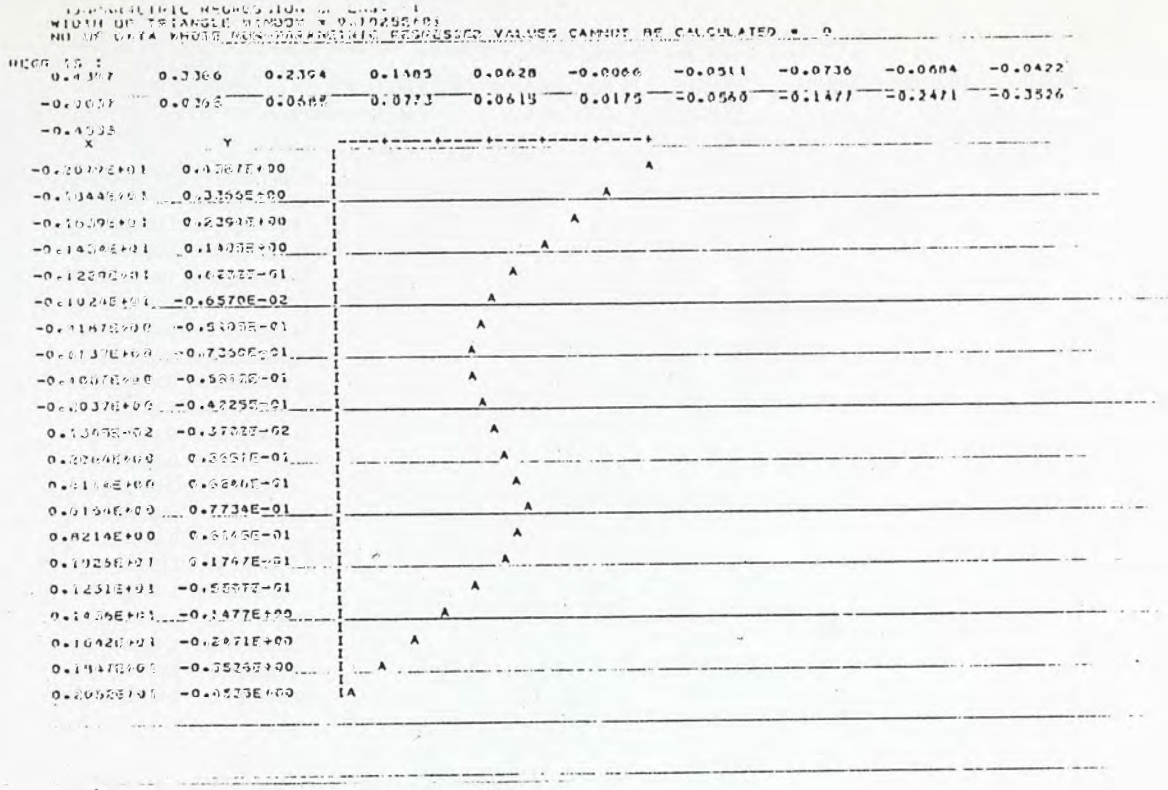


Fig 2.1c $E[X_t | X_{t-1}]$ for Model 3

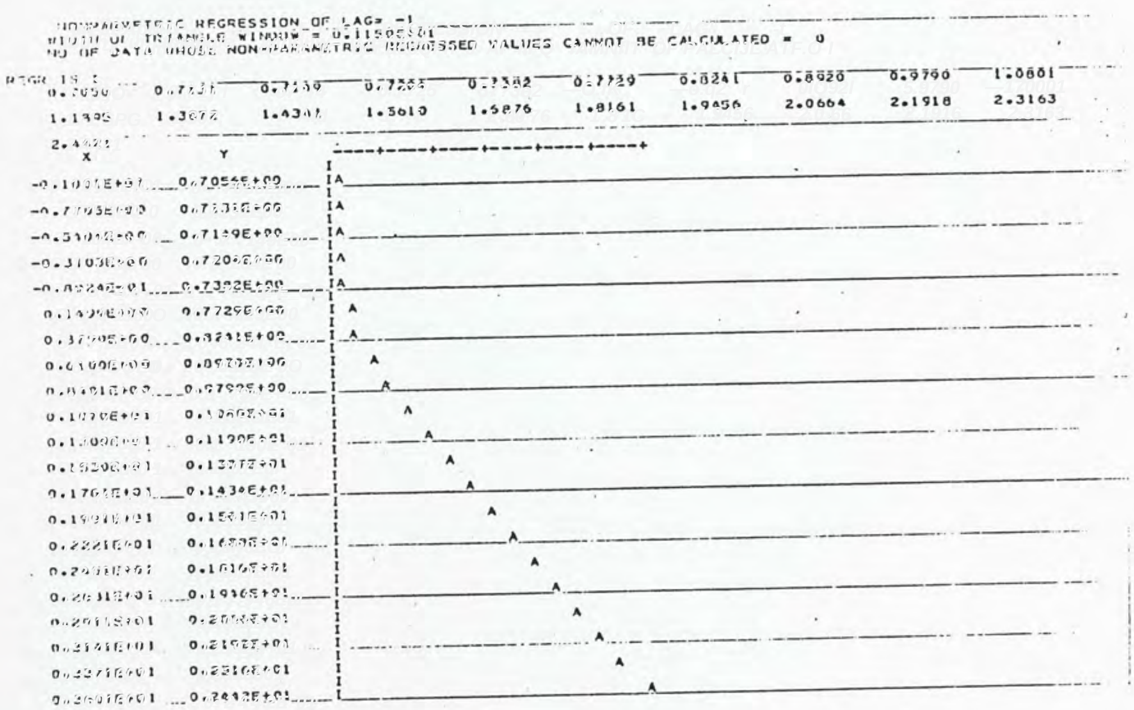


Fig 2.1d $E[X_t | X_{t-1}]$ for Model 4

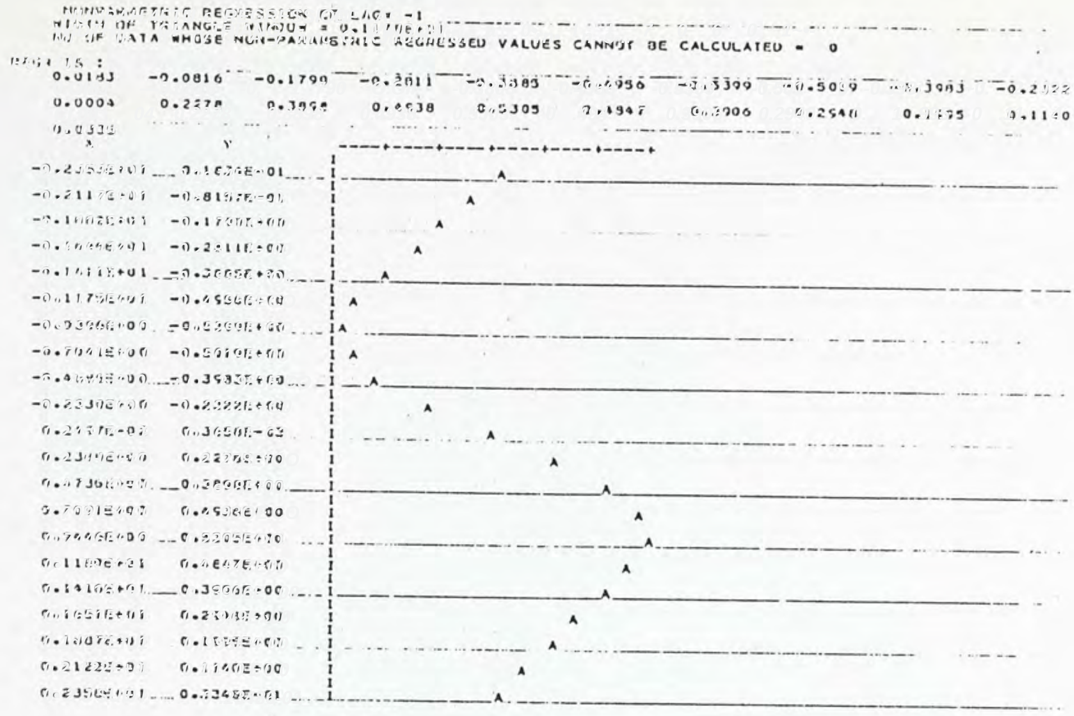


Fig 2.1e $E[X_t|X_{t-1}]$ for Model 5

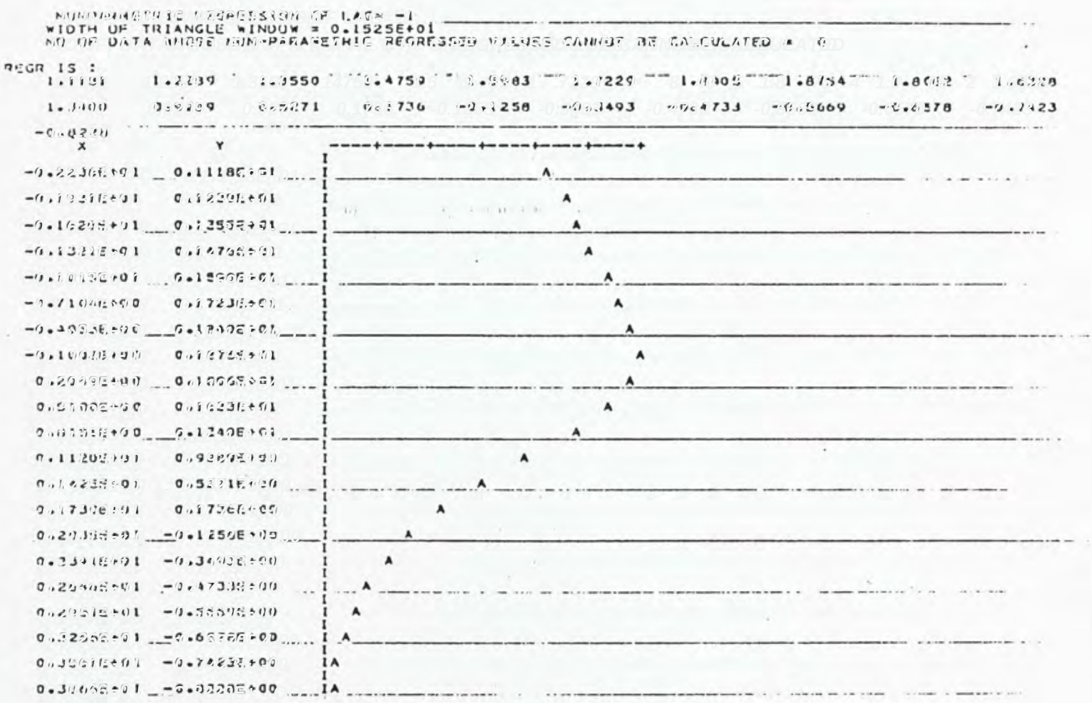


Fig 2.1f $E[X_t|X_{t-1}]$ for Model 6

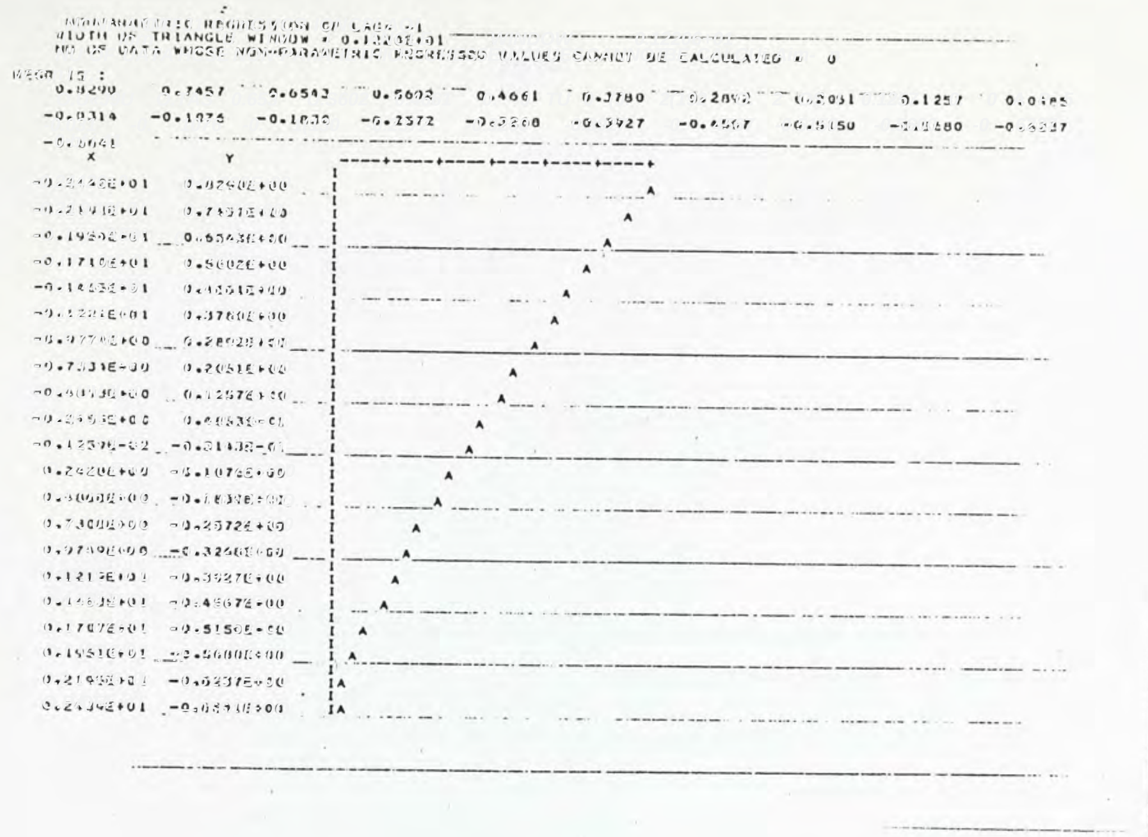


Fig 2.1g $E[X_t|X_{t-1}]$ for Model 7

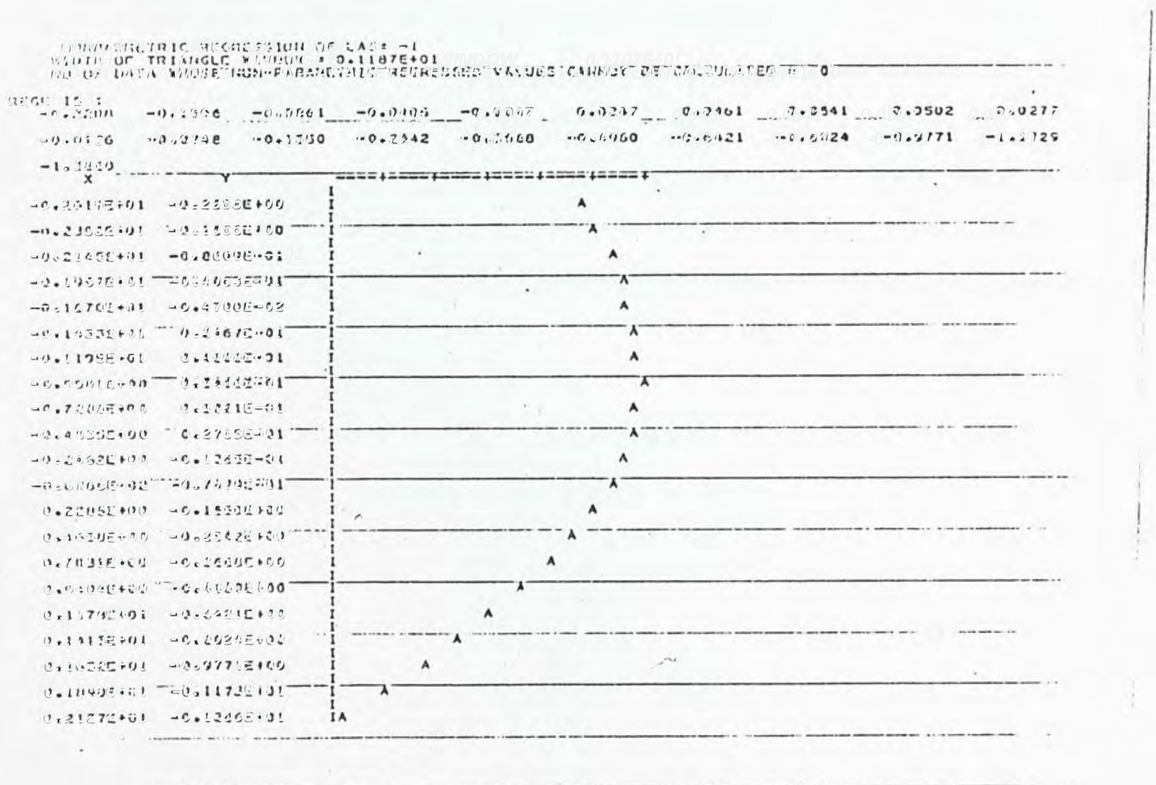


Fig 2.1h $E[X_t|X_{t-1}]$ for Model 8

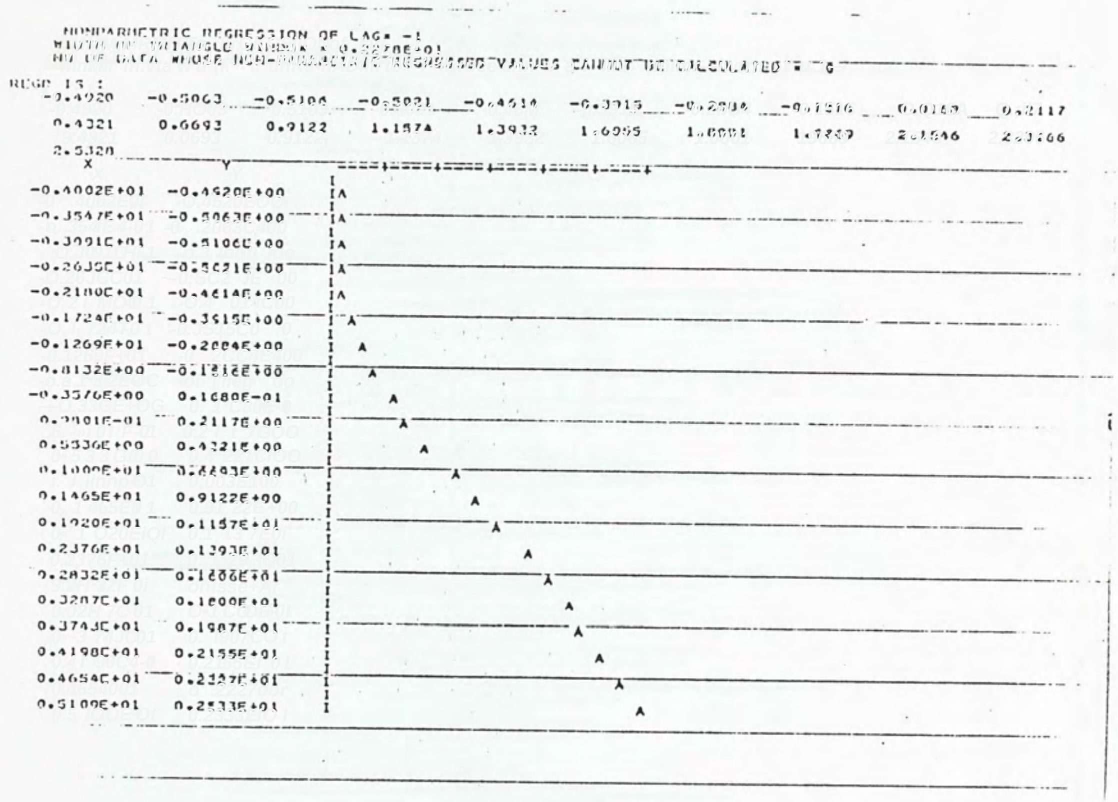


Fig 2.11 $E[X_t | X_{t-1}]$ for Model 9

Fig 2.2a Series A, simulated from the model 1, N=500

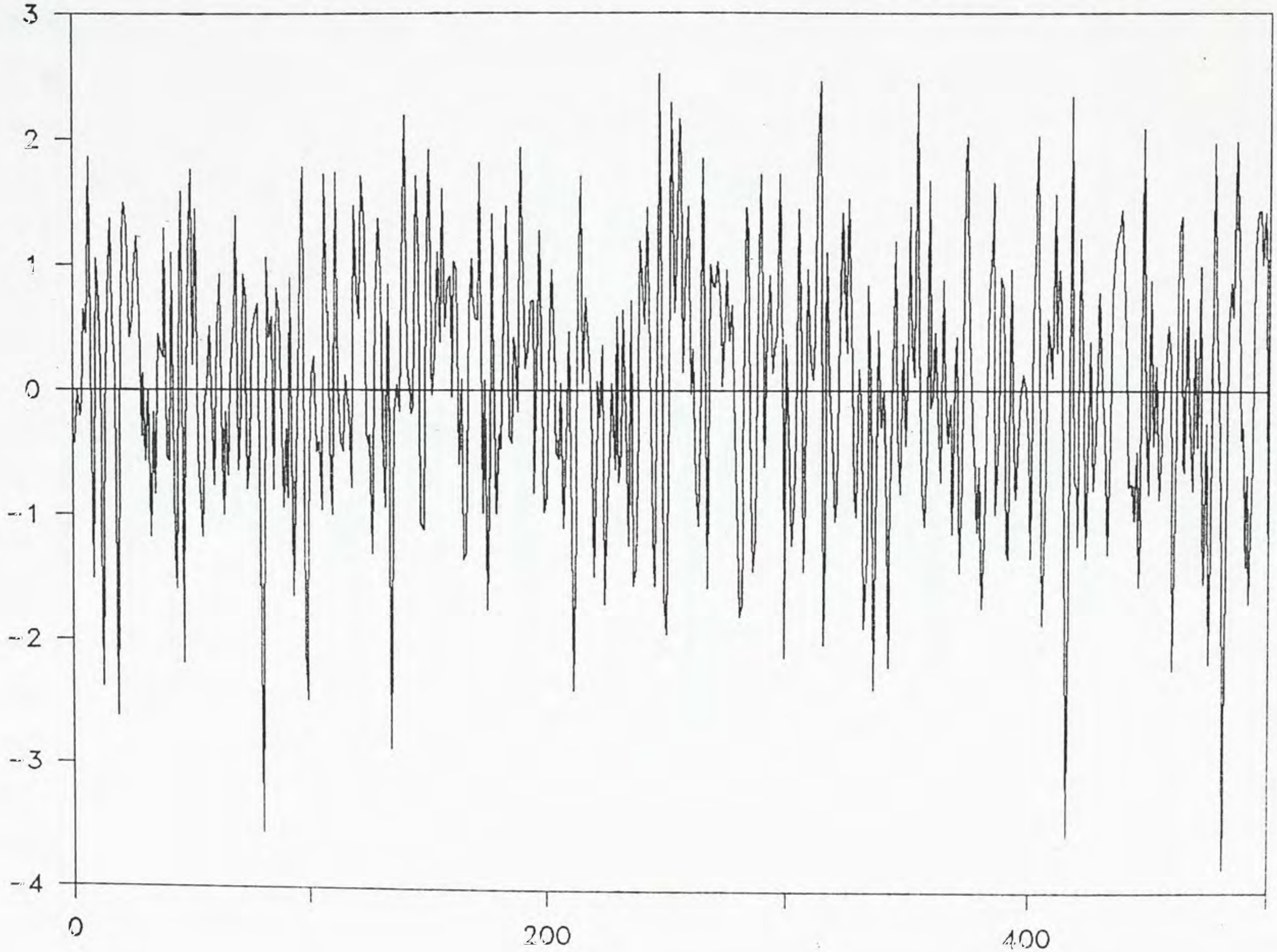
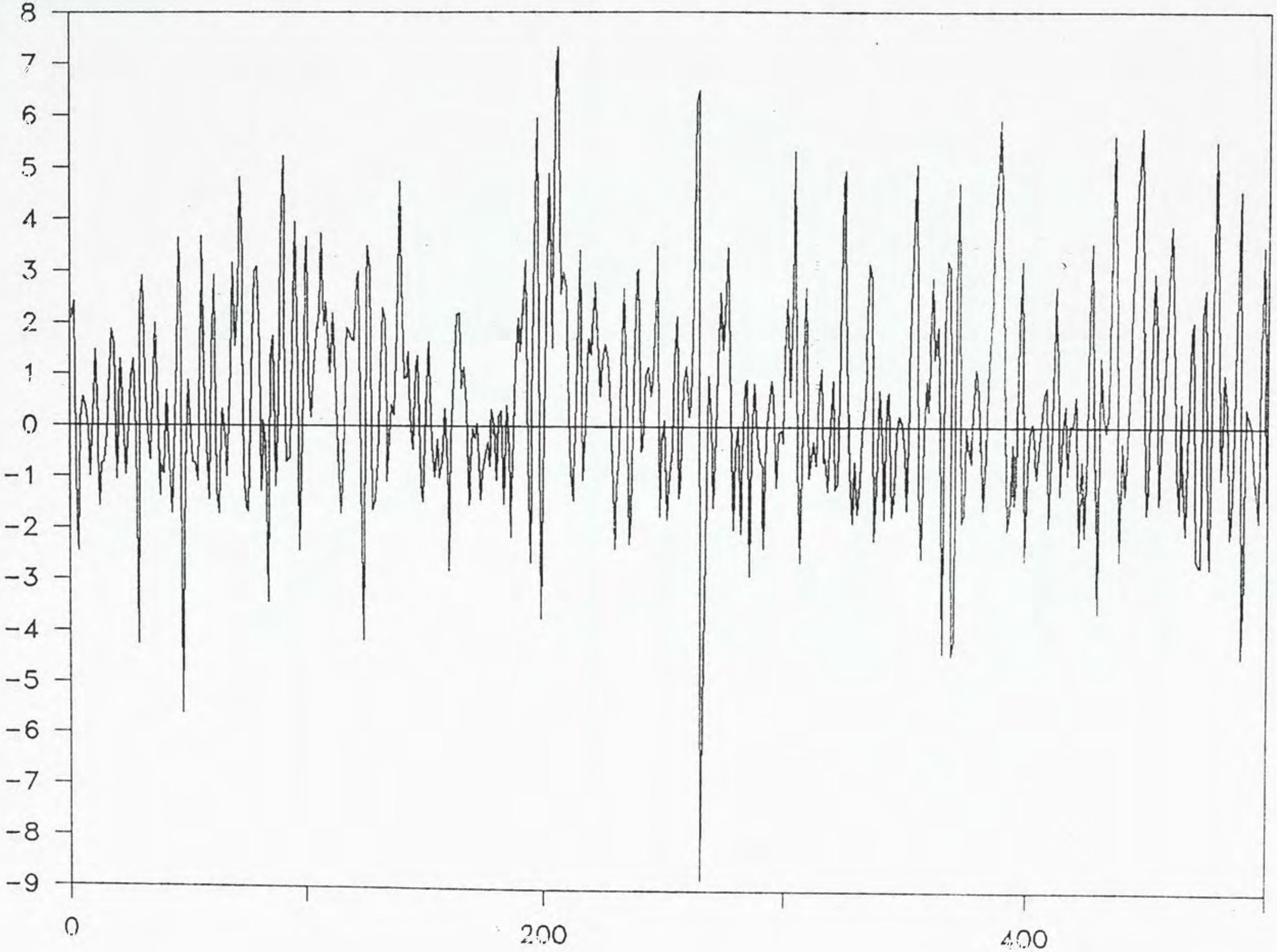


Fig 2.2b Series B, simulated from model 9, N=500



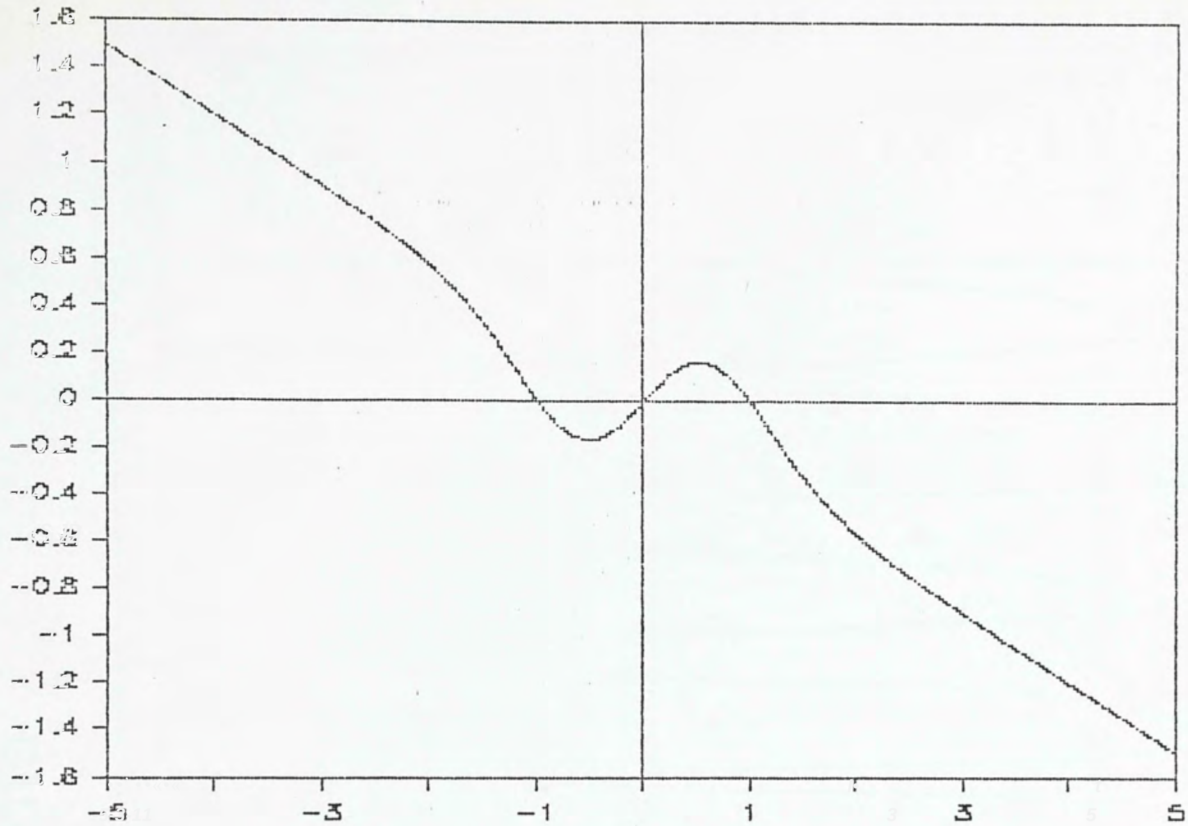


Fig 2.3a $E[X_t]$ Vs X_{t-1} for model 3

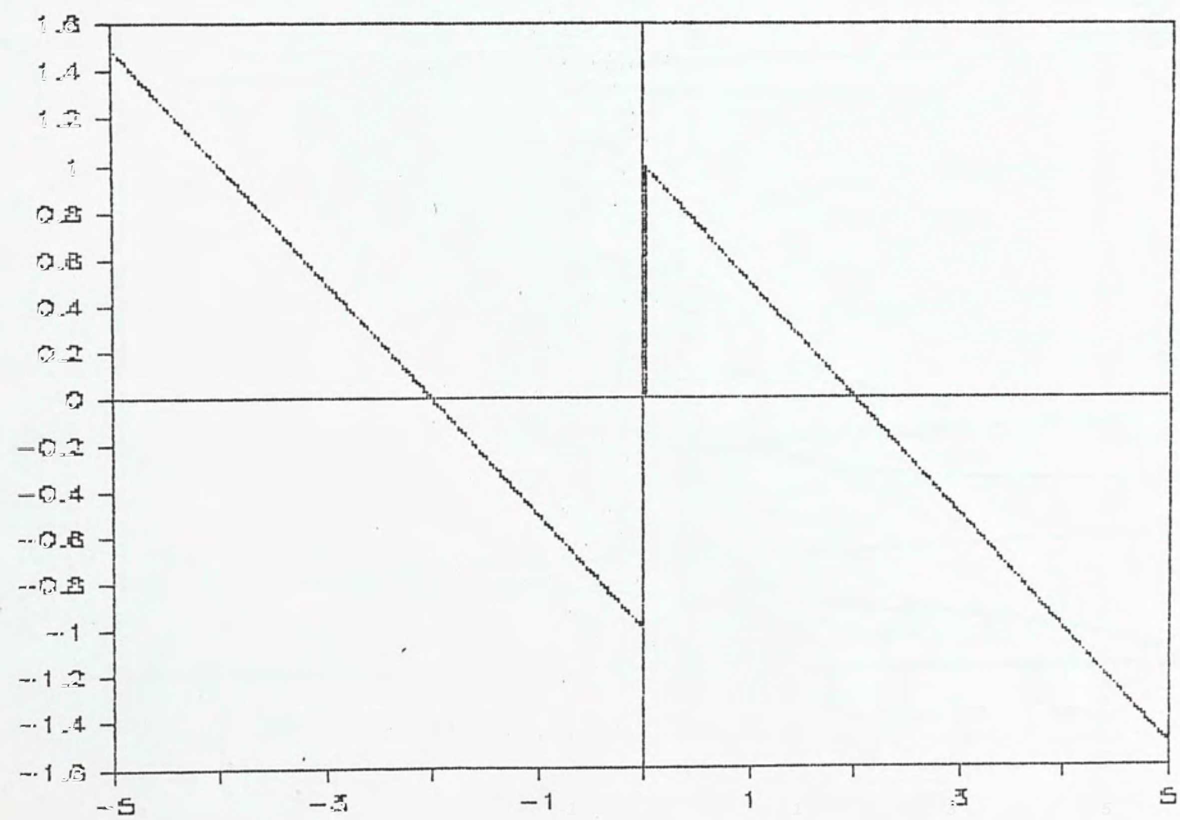


Fig 2.3b $E[X_t]$ Vs X_{t-1} for model 5

Fig2.4a Lynx Data

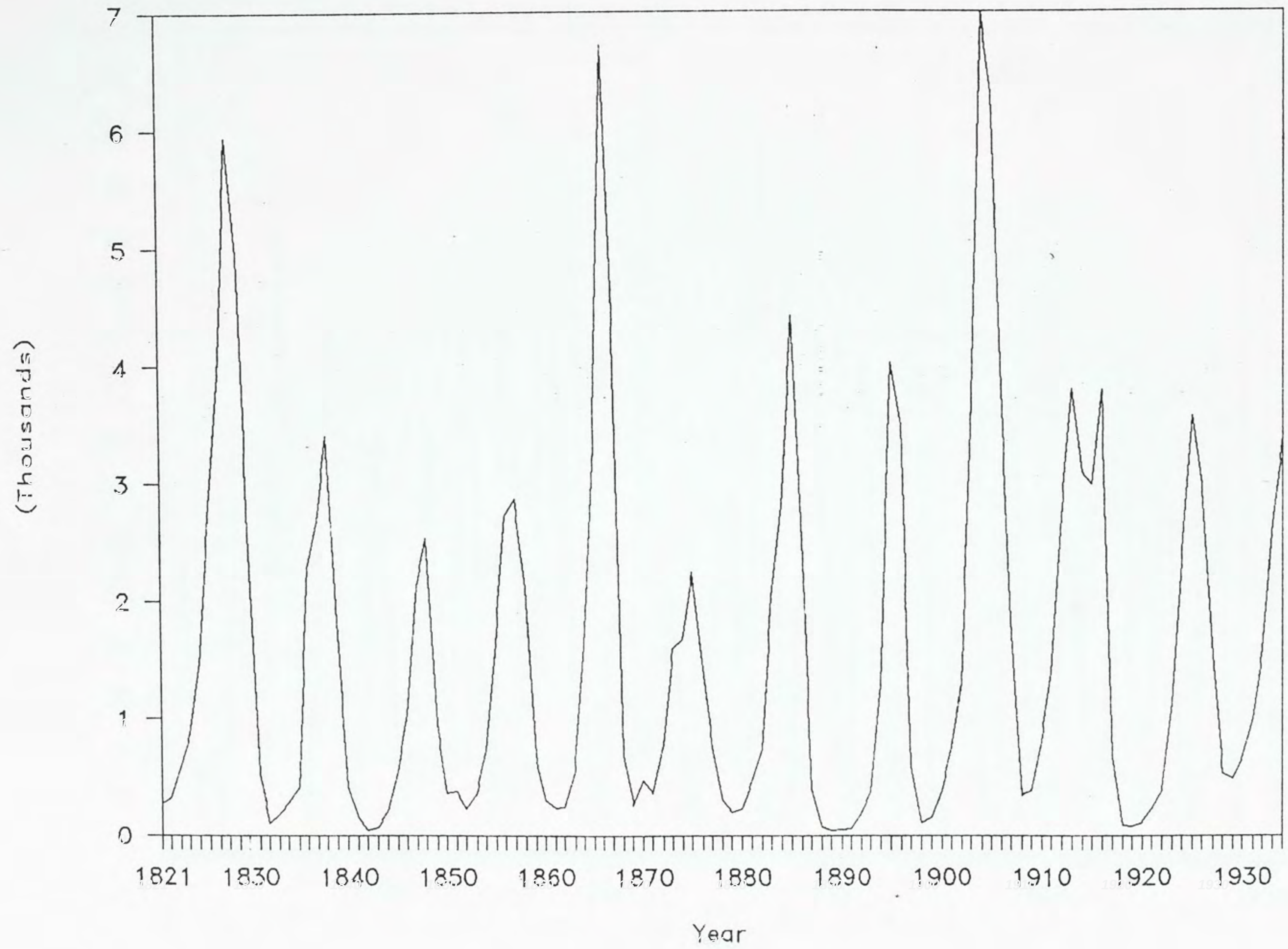


Fig2.4b Sunspot Data

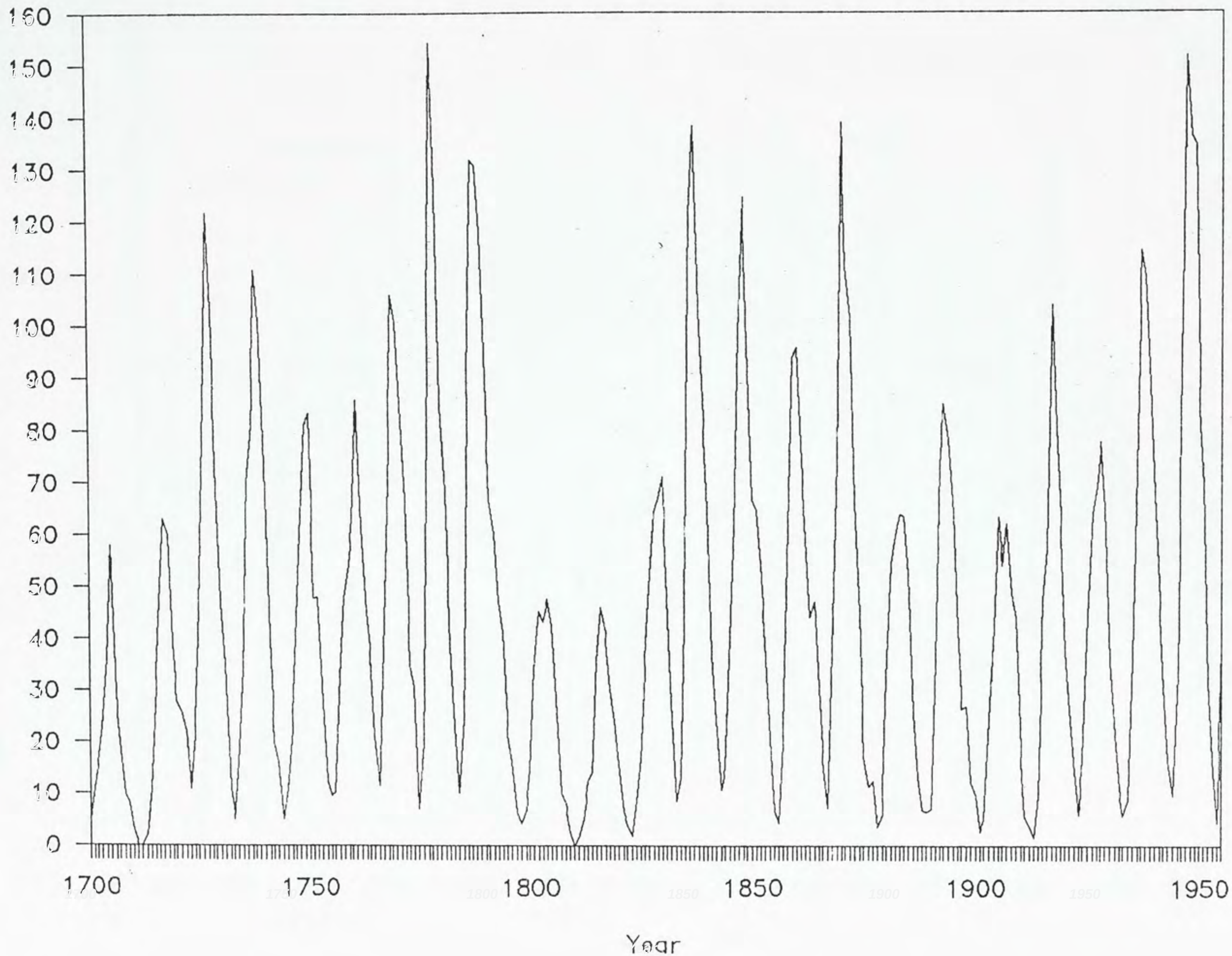
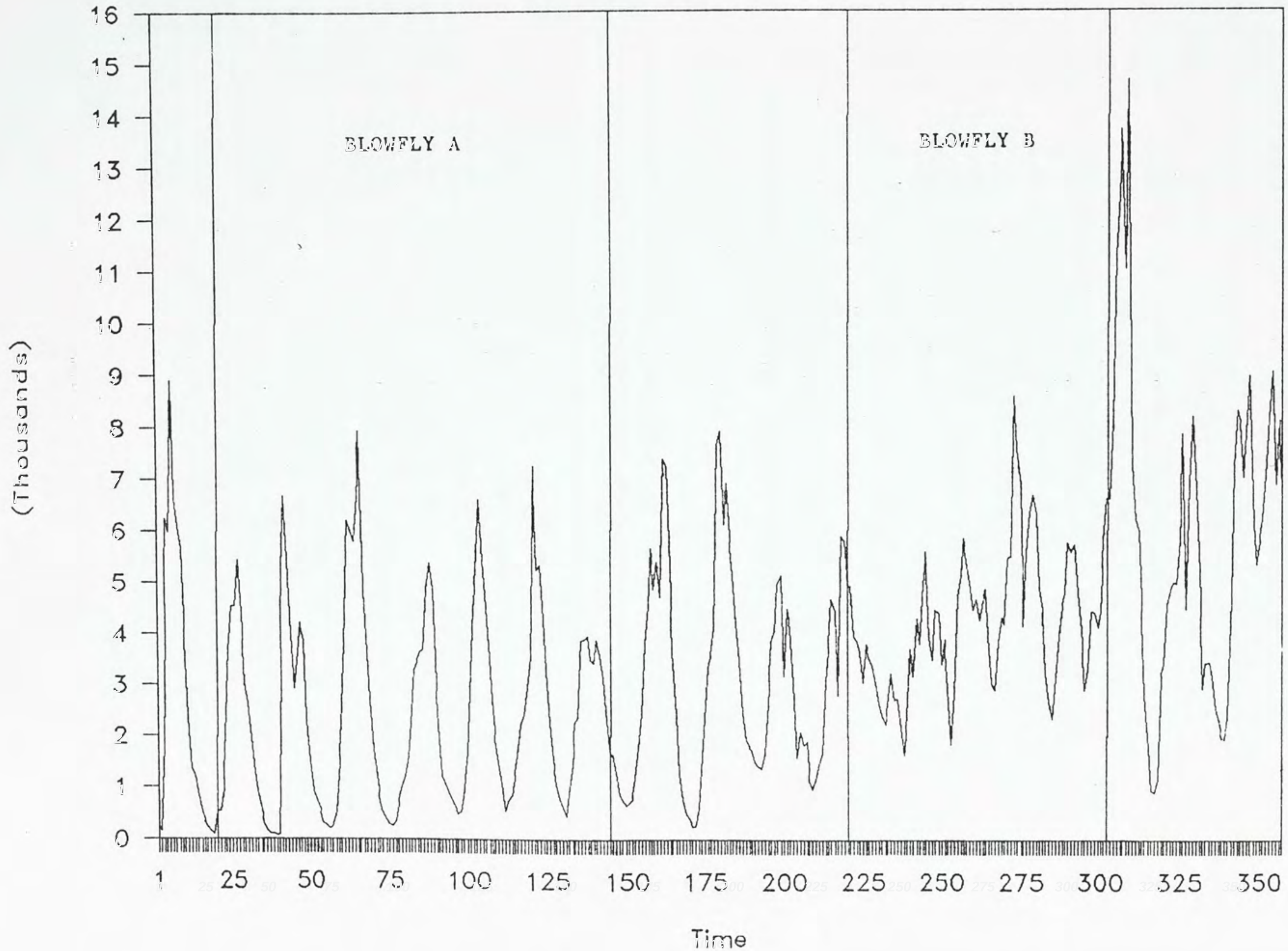


Fig2.4c Blowfly Data



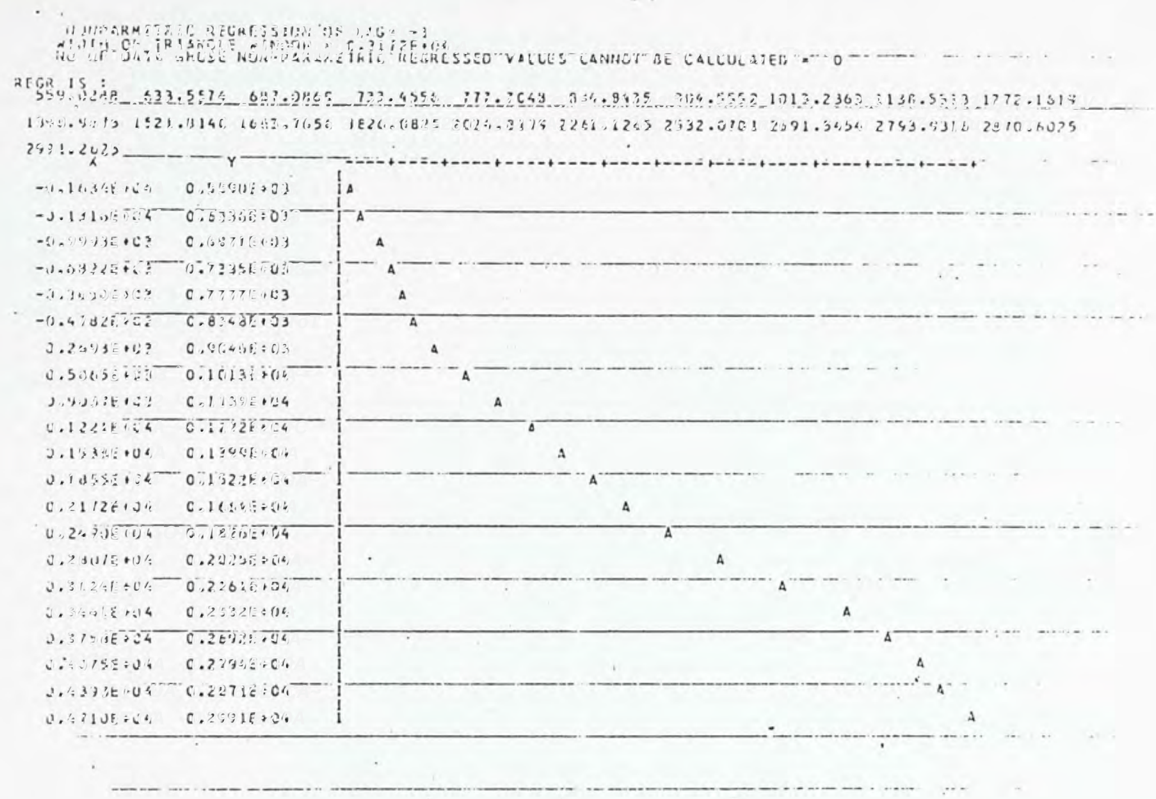


Fig 2.5a $E[X_t | X_{t-1}]$ for lynx data

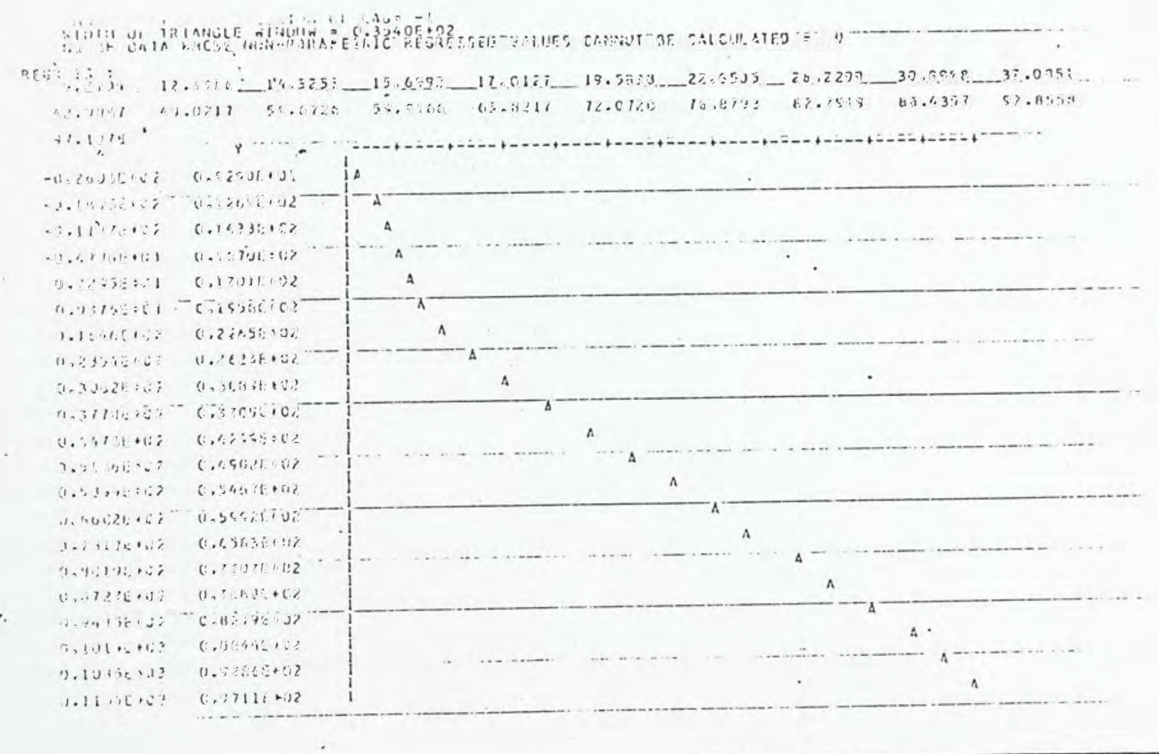


Fig 2.5b $E[X_t | X_{t-1}]$ for sunspot data

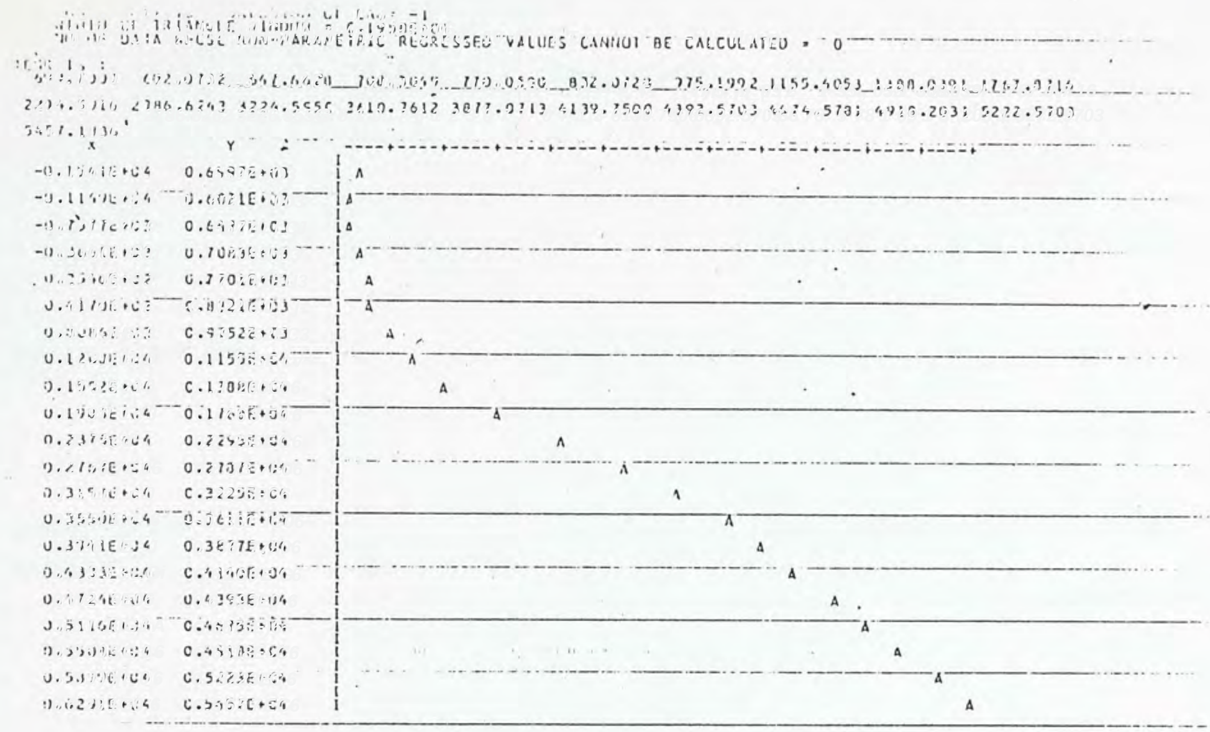


Fig 2.5c $E[X_t|X_{t-1}]$ for Blowfly A data

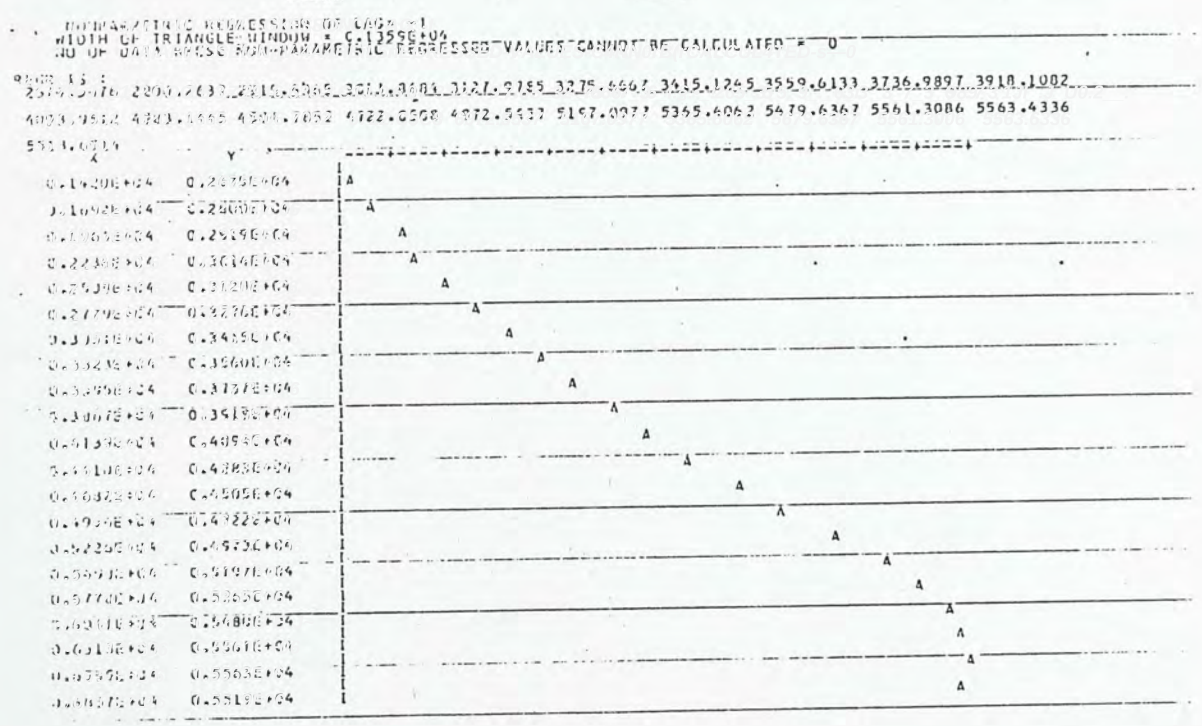


Fig 2.5d $E[X_t|X_{t-1}]$ for Blowfly B data

TABLE 2.1 Lag windows V(s)

Daniell window	$V(s) = \dfrac{\sin(s\pi)}{s\pi}$
Tukey-Hamming window	$V(s) = \begin{cases} 0.54 + 0.46 \cos \pi s & \lvert s \rvert \leq 1 \\ 0 & \text{otherwise} \end{cases}$
Parzen window	$V(s) = \begin{cases} 1 - 6s^2 + 6\lvert s \rvert^3 & \lvert s \rvert \leq \frac{1}{2} \\ 2(1 - \lvert s \rvert)^3 & \frac{1}{2} \leq \lvert s \rvert \leq 1 \\ 0 & \text{otherwise} \end{cases}$
Bartlett-Priestley window	$V(s) = \dfrac{3}{(\pi s)^2} \left\{ \dfrac{\sin \pi s}{\pi s} - \cos \pi s \right\}$
Bartlett window	$V(s) = \begin{cases} 1 - \lvert s \rvert & \lvert s \rvert \leq 1 \\ 0 & \text{otherwise} \end{cases}$

TABLE 2.2a F-STATISTICS FOR SUBBA RAO-GABR'S TEST FOR MODEL 1

Lag	WINDOW					
M	Daniell Optimum	Bartlett	Tukey	Daniell	Parzen	Bartlett Priestley
17	90.26	146.45	55.67	20.17	804.11	87.57
20	1.25	24.93	14.52	19.39	83.47	5.74
25	0.61	8.38	5.27	2.31	12.70	1.55
30	0.68	3.11	1.53	1.10	15.95	1.54

TABLE 2.2b F-STATISTICS FOR SUBBA RAO-GABR'S TEST FOR MODEL 9

Lag	WINDOW					
M	Daniell Optimum	Bartlett	Tukey	Daniell	Parzen	Bartlett Priestley
17	3.79	435.05	93.88	141.59	821.47	7.19
20	19.59	12418.30	174.76	188.84	2648.87	19.37
25	6.61	10315.09	270.28	153.88	436.49	21.17
30	4.39	1810.75	56.61	53.04	246.57	2.97

TABLE 2.3 SUMMARY : HINICH'S TEST FOR MODEL 1-9 (N=204,M=15) 100 REPLICATIONS

MODEL NUMBER	ORIGINAL TYPE	FREQUENCY OF CORRECT DECISIONS (α =0.05)
1	L	91
2	L	97
3	NL	10
4	NL	38
5	NL	5
6	NL	2
7	NL	34
8	NL	27
9	NL	87

REMARKS : L - LINEAR ; NL - NONLINEAR

TABLE 2.4 SUMMARY : KEENAN'S TEST FOR MODEL 1-9 (N=204,M=8) 100 REPLICATIONS

MODEL NUMBER	ORIGINAL TYPE	FREQUENCY OF CORRECT DECISIONS (α =0.05)
1	L	95
2	L	98
3	NL	6
4	NL	88
5	NL	8
6	NL	37
7	NL	10
8	NL	91
9	NL	93

REMARKS : L - LINEAR ; NL - NONLINEAR

TABLE 2.5a SUBBA RAO-GABR'S TEST FOR RAW LYNX DATA (F-STATISTICS)

Lag	WINDOW					
M	Daniell Optimum	Bartlett	Tukey	Daniell	Parzen	Bartlett Priestley
13	1.09	96.84	36.87	9.35	3002.15	1.95
16	2.03	4959.02	7.64	26.66	429.48	0.46
20	8.67	185.23	1.51	26.68	829.37	1.64
25	75.39	10.23	3.23	5.32	53.06	1.42

REMARKS : $\alpha=0.05$, Critical Value = 3.94

TABLE 2.5b SUBBA RAO-GABR'S TEST FOR RAW SUNSPOT DATA (F-STATISTICS)

Lag	WINDOW					
M	Daniell Optimum	Bartlett	Tukey	Daniell	Parzen	Bartlett Priestley
17	2.82	545.62	4.04	116.27	54396.73	0.83
20	2.37	343.46	1245.46	109.86	17180.14	1.99
25	7.26	247.43	60.60	95.91	4955.32	0.49
30	1.48	143.11	1.89	92.33	1876.83	1.64

REMARKS : $\alpha=0.05$, Critical Value = 8.94

TABLE 2.5c SUBBA RAO-GABR'S TEST FOR RAW BLOWFLY A (F-STATISTICS)

Lag	WINDOW					
M	Daniell Optimum	Bartlett	Tukey	Daniell	Parzen	Bartlett Priestley
13	1.38	7.54	2.96	23.87	3283.80	0.90
16	1.77	4.92	29.39	49.27	1537.53	0.83
20	38.30	14.18	3.53	18.61	143.87	7.90
25	7.99	7.84	2.00	90.09	245.44	2.16

REMARKS : α=0.05, Critical Value = 3.94

TABLE 2.5d SUBBA RAO-GABR'S TEST FOR RAW BLOWFLY B (F-STATISTICS)

Lag	W I N D O W					
M	Daniell Optimum	Bartlett	Tukey	Daniell	Parzen	Bartlett Priestley
8	0.67	55.57	11.59	8.02	23720.70	2.99
10	4.44	9.15	14.09	5.51	69.19	0.83
13	6.85	36.31	2.39	4.31	382.44	1.73
16	0.86	8.89	4.64	18.45	43.27	0.75

REMARKS : $\alpha = 0.05$, Critical Value = 8.94

TABLE 2.6 HINICH'S TEST FOR REAL DATA

Data	Transformation	Test Statistics[1]	Conclusion[2]
Lynx N=114	RAW	1.4588	L
	LOGTEN	-1.1824	L
Sunspot N=256	RAW	0.3785	L
	2[Sq(X+1) - 1]	-0.2394	L
BlowflyA N=126	RAW	0.4834	L
	SQRT	-0.6705	L
	LOGTEN	2.8977	NL
BlowflyB N=82	RAW	1.1816	L
	SQRT	-0.5868	L
	LOGTEN	-0.3768	L

REMARKS : (1) α =0.05, Critical Values = $\pm$ 1.96

(2) L -- Linear ; NL -- Nonlinear

TABLE 2.7 KEENAN'S TEST FOR REAL DATA

Data	Transformation	Test Statistics[1]	Conclusion[2]
Lynx N=114	RAW	0.1221	L
	LOGTEN	1.3661	L
Sunspot N=256	RAW	14.5364	NL
	2[Sq(X+1) - 1]	5.8320	NL
BlowflyA N=126	RAW	1.8994	L
	SQRT	3.8706	NL
	LOGTEN	5.5964	NL
BlowflyB N=82	RAW	0.2282	L
	SQRT	0.1914	L
	LOGTEN	0.1242	L

REMARKS : (1) α=0.05, Critical Value = 3.84

(2) L -- Linear ; NL -- Nonlinear

CHAPTER III A NEW PROPOSED METHODS

3.1 INTRODUCTION

In a recent paper, Tong and Lim (1980) state that " the new era of practical non-linear time series modelling is, without doubt, long-overdue ". To accompany this claim, they introduce a family of non-linear models, called threshold autoregessive (TAR) models, and demonstrate their applicability to practical problems by examples.

Although Tong and Lim invited us to enter the world of non-linear models, there is no reason for us to abandon all models in the territory of linearity. The theory of linear models is as yet much better developed than that of non-linear models, and many problems are easier to deal with in the linear framework. We believed that there should be ways of choosing the right (linear or non-linear) family of models on the basis of the evidence contained in the data.

3.2 THE IDEA

The idea of using piecewise linear models in a systematic way for the modelling of discrete time series data was first mentioned in Tong (1977) and reported in Tong (1978a, 1978b, 1980). A Comprehensive account, together with numerous applications and discussion, is available in Tong and Lim (1980). Our new proposed test is based on choosing between Autoregressive (AR) and Self-Exciting Threshold Autoregressive (SETAR) models. Tong (1983) provided a general reference for SETAR modelling.

Consider a general SETAR(2;k,k) model :

$$X_t = \begin{cases} a_0^{(1)} + \sum_{i=0}^{k} a_i^{(1)} X_{t-i} + e_t & \text{if } X_{t-d} \le r \\ a_0^{(2)} + \sum_{i=0}^{k} a_i^{(2)} X_{t-i} + e_t & \text{if } X_{t-d} > r \end{cases} \qquad (3.2.1)$$

Let

$$\underset{\sim}{A}^{(1)} = (a_0^{(1)}, a_1^{(1)}, \ldots, a_k^{(1)}) \text{ and}$$
$$\underset{\sim}{A}^{(2)} = (a_0^{(2)}, a_1^{(2)}, \ldots, a_k^{(2)}).$$

Given X_0, X_1, ..., X_N, we consider testing the interesting null hypothesis

$$H_0 : \underset{\sim}{A}^{(1)} = \underset{\sim}{A}^{(2)}$$

against

$$H_1 : \underset{\sim}{A}^{(1)} \neq \underset{\sim}{A}^{(2)}$$

H_1 states that the generating machanism is non-linear in being piecewise linear as specified in (3.2.1). Note that, under H_0, the nuisance parameter r is not present and the SETAR(2;k,k) model will collapse to a AR(k) model. However, for fixed r, ignoring the transient effect of initial observations (as is usually done), the classical result of likelihood ratio test is shown to hold (Chan and Tong, 1985).

Let

$$\lambda = [(\hat{\sigma}_L^2)^{-N/2} \quad / \quad (\hat{\sigma}_{NL}^2)^{-N/2}] \qquad (3.2.2)$$

Where $\hat{\sigma}_L^2$ is the estimated innovation variance of the AR(k) model and $\hat{\sigma}_{NL}^2$ is the estimated innovation variance of the non-linear SETAR(2;k,k) model. Under H_0, $-2 \ln\lambda$ is asymptotically χ^2_{k+1}.

It is intuitively clear that the test thus provided is useful only when the alternative is in the form (3.2.1) with a fixed r. However, in the original setup, r is seldom known beforehand. Chan and Tong suggest that a natural approach is to consider instead $Y = \max_{r \in R} -2 \ln \lambda$ or, if we have reason to believe that $r \in S \subseteq R$ $Y_1 = \max_{r \in S} -2 \ln \lambda$.

Tong and Lim (1980) employ Akaike's Information Criterion (AIC, Akaike ,1974) in choosing the $\hat{r}$ and $\hat{d}$ to replace the fixed threshold (r) and the delay parameter (d) in (3.2.1) respectively. However, Teräsvirta and Luukkonen (1983) have suggested that the Schwarz's Bayesian Information Criterion (SBIC), cf. Schwarz (1978), is a viable alternative.

Unfortunately, even if we obtain $\hat{r}$ and $\hat{d}$ to replace r and d in calculating the likelihood ratio statistic Y or Y_1, the classical result no longer holds. Feder (1975) pointed out that in these cases the parameter estimates are not asymptotically normal and -2 ln λ is not asymptotically χ^2 with the " appropriate " number of degree of freedom. Although the distribution of Y or Y_1 is not asymptotically χ^2 and rather different to obtain, we can always appeal to the Monte Carlo technique. Silverman (1985) has applied similar approach in specification of regression models.

The algorithm of our new proposed test is as follows :

(1) Input the test data, $X_1, X_2, \ldots, X_N$.

(2) Use SBIC to choose a " best fitted " linear $AR(k)^*$ model and obtain the appropriate $\hat{\sigma}_L^2$.

(3) Use SBIC to choose a " best fitted " non-linear SETAR(2;k,k) model and obtain $\hat{\sigma}^2_{NL}$.

(4) Calculate the likelihood ratio test statistic

$$T^* = -2 \ln \lambda = (N \ln \hat{\sigma}^2_L - N \ln \hat{\sigma}^2_{NL})$$

(5) Simulate the null distribution :

(i) $X_1^*, X_2^*, \ldots, X_N^*$ are simulated from the AR(k)* linear model specified by the step (2).

(ii) Repeat steps (2), (3) and (4) to obtain an observation, say $\hat{T}_i$, from the null distribution.

(iii) Repeat (i) and (ii) 100 times and obtain a sample from the null distribution in form of $T_1, T_2, \ldots, T_{100}$, from which the null distribution may be estimated.

(6) T^* is compared with the simulated null distribution and a conclusion is drawn. Let { $T_{(1)}, T_{(2)}, \ldots, T_{(100)}$ } be the order statistic of { $T_1, T_2, \ldots, T_{100}$ }. If we take α =0.05, the decision rule is :

$T^* > T_{(95)}$ conclusion : non-linear

$T^* \leq T_{(95)}$ conclusion : linear

The Monte Carlo results reveal that our new proposed test is quite roboust among the choices of k (maximum lag in fitting the linear and non-linear models). Therefore, we prefer to use k=2. In this case, the estimation of the AR(k) and SETAR(2;k,k) models are quite simple.

3.3 A SIMULATION STUDY

Our simulation study, again, is based on the nine sample models in Appendix I.

Appendix I here

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204 simulated data are generated from each sample model. We perform our test with 100 replications. Number of correct decision for each sample model are recorded in table 3.1.

Table 3.1 here

We have a very high detection rate among models 1 to 6, models 8 and 9. However, our test fails to detect the non-linearity in model 7. Consider model 7 :

$$X_t = e_t - 0.4e_{t-1} + 0.3e_{t-2} + 0.5e_t e_{t-2}$$

$$E[X_t | X_{t-1}, \ldots] = -0.4\, e_{t-1} + 0.3\, e_{t-2}$$

The conditional mean of model 7 is in a linear form. It may be considered a limitation of our test which seems unable to detect non-linear time series models with a conditional mean linear in the unobservable e_t's. The following simulation experiment tends to support our observation. Consider

MODEL A : $e_t - 0.4e_{t-1} + 0.3e_{t-2} + 0.5e_{t-1}e_{t-2}$

MODEL B : $e_t - 0.4e_{t-1} + 0.3e_{t-2} + 0.5e_{t-1}^2$

MODEL C : $e_t - 0.4e_{t-1} + 0.3e_{t-2} + 0.5e_t e_{t-1}$

MODEL D : $e_t - 0.4e_{t-1} + 0.3e_t e_{t-1} + 0.5e_t e_{t-2}$

We test each model with 100 replications, the results are given in table 3.2.

Table 3.2 here

Models A and B, with a quadratic conditional mean, can be identified by the test. While models C and D, with a linear conditional mean, cannot be detected by our test.

3.4 APPLICATION TO REAL DATA

We apply the same real series used in section 2.5 to our new proposed test. The results are given in table 3.3.

Table 3.3 here

The test concluded that the lynx, sunspot and BLOWFLY A data are non-linear time series, regardless of transformation. These results suggest that fitting a SETAR model to lynx data, blowfly A data and sunspot data is not unreasonable. This conclusion leads some support to non-linear modelling of these data. For BLOWFLY B data, the test agrees with Chan and Tong's (1985) conclusion that the data are generated from a linear machanism.

3.5 COMMENTS

This new proposed test seem to have a high degree of reliability in detecting non-linearity in time series data. From the simulation study in section 3.3, when the generating machanism is piecewise linear as specified in (3.2.1), we have a 100% detection rate ! Even when the generating machanism of the test data is non-linear in the form of bilinear, exponential AR or non-linear MA, which is different from the working setup (SETAR), our test still tends to classify the data as being generated by a non-linear model. Also, when applying to some well known real series, the test draws conclusions which tends to support non-linear modelling of them. But there is a limitation of using the test. It is unable to identify some non-linear time series models with conditional mean linear in the e_t's. Since we use Monte Carlo technique to obtain the null distribution of our test, it is difficult to assign the exact level of significance. But it does not seem a great barrier in practice.

TABLE 3.1 SUMMARY : OUR TEST FOR MODEL 1-9 (N=204) 100 REPLICATIONS

MODEL NUMBER	ORIGINAL TYPE	FREQUENCY OF CORRECT DECISIONS (α =0.05)
1	L	99
2	L	96
3	NL	94
4	NL	100
5	NL	96
6	NL	100
7	NL	14
8	NL	96
9	NL	98

REMARKS : L - LINEAR ; NL - NONLINEAR

TABLE 3.2 SUMMARY : OUR TEST FOR MODEL A-D (N=204) 100 REPLICATIONS

MODEL NUMBER	ORIGINAL TYPE	FREQUENCY OF CORRECT DECISIONS (α =0.05)
A	NL	95
B	NL	97
C	NL	14
D	NL	12

REMARKS : L - LINEAR ; NL - NONLINEAR

TABLE 3.3 OUR TEST FOR REAL DATA

Data	Transformation	Test Statistics	Critical Value α = 5%	Conclusion
Lynx N=114	RAW	27.4200	7.0310	NL
	LOGTEN	26.4254	6.3744	NL
Sunspot N=256	RAW	68.1915	8.8388	NL
	2[Sq(X+1) - 1]	20.0074	9.0814	NL
BlowflyA N=126	RAW	23.4344	6.9814	NL
	SQRT	7.2725	7.0105	NL
	LOGTEN	17.9827	8.5457	NL
BlowflyB N=82	RAW	2.5966	10.3386	L
	SQRT	0.1748	9.7058	L
	LOGTEN	1.4368	7.9038	L

REMARKS: L -- Linear ; NL -- Nonlinear

CHAPTER IV A COMPARSION

4.1 SIMULATION RESULTS

In Section 2.4 of Chapter II, the Monte Carlo results reveal that Subba Rao-Gabr's F-statistic is extremely unstable and cannot draw consisent conclusions. Some simulated data are used to test their method and the results are given in Table 2.2a and 2.2b. We input the same test data into Hinich's test, the results are recorded in Table 4.1.

Tables 2.2a, 2.2b and 4.1 here

From Table 4.1, Hinich's test can draw correct and consistent conclusions under different choices of M. Therefore, based on the simulation study, we can conclude that Hinich (1982) has improved Subba Rao-Gabr's test significantly.

We are going to compare Hinich's test, Keenan's test and our new proposed method. The comparsion will be mainly based on the performance of each test to the nine sample models.

Appendix I here

Number of correct decision among each test in 100 replications are given in Table 4.2.

Table 4.2 here

All of the test can identify the linearity elements in models 1 and 2 successfully. But none of them can detect the non-linearity in model 7 satisfactory. Hinich's test seems perform better in testing model 7, but its detection rate for other non-linear sample models are quite poor. Keenan's test is sensitive to non-linear MA models, but its ability in detecting non-linear AR models (e.g. SETAR, EXPAR,..) are weak. From the simulation results, our test is satisfactory but not perfect. The disability of detecting some non-linear models with linear conditional mean are disclosed in testing model 7.

4.2 REAL DATA

An ideal test is not only perform satisfactory under simulation experiments, but also can draw reasonable conclusions when applying to the real data. Some well known time series have been selected to examine the tests. The lynx, sunspot and BLOWFLY A data are widely accepted as non-linear time series. While BLOWFLY B data are generated from linear machanism. In previous chapters, individual tests had been applied to each real series and the results were discussed. In Table 4.3, we summarize the conclusions of the tests to each real series.

Table 4.3 here

The conclusions of our test agree with the widely accepted results. Keenan's test classifies lynx data to linear time series, but his test can draw quite reasonable conclusions among other series. Hinich's test concludes that all tested series are generated from linear machanism, except logarithm transformed BLOWFLY A data. Our test seems better than other methods in applying to the real series.

4.3 SOME SPECIAL TIME SERIES

A. Non-Gaussian Time Series

After five decades of domination by linear Gaussian models, the time is certainly ripe for a serious study of ways of removing the many limitations of these models. Once we decide to incorporate features in addition to the autocovariances, the class of models would have to be greatly enlarged to include those besides the Gaussian ARMA models. We may retain the general ARMA framework and allow the white noise to be non-Gaussian.

We are interested in comparing the ability of each test in detecting the linearity of linear non-Gaussian models. Consider a non-Gaussian MA(1) model

$$X_t = e_t - a\, e_{t-1}$$

where

$$e_t \sim U\,(-1.7321, 1.7321).$$

We perform a simulation experiment with 100 replications, the number of correct decision among each test are reported in table 4.4.

Table 4.4 here

Keenan's test and Hinich's test can detect the linearity in this linear non-Gaussian model successfully. However, our test misclassify it into non-linear territory. Consider the conditional mean of this non-Gaussian MA(1) model, after some manipulation, $E[X_t|X_{t-1}]$ is shown to be non-linear (Tong, 1983). As previous conclusion in Chapter III, our test is mainly based on testing the conditional mean of the model. In this case, our test will misclassify these models.

B. Linear Time Series Models With Random Coefficients

Consider the following random coefficient AR(1) model

$$X_t = (a + r b) X_{t-1} + e_t$$

where

$$r \sim N (0,1)$$

$$e_t \sim N (0,1)$$

Random coefficient AR models have been included as a sub-class of non-linear models (e.g. Nicholls and Quinn 1982). However, it is arguable if these models are truly non-linear because the conditional means, E[X_t | past X's], are linear in the past X's (See the discussion paper by Lawrance and Lewis 1985). We apply the tests to the model with 100 replications, the results are given in table 4.5. All tests draw the same conclusion, random coeficient AR(1) models are linear. Therefore, as far as Hinich's (a fortified Subba Rao-Gabr's) test, Keenan's test and our test are concerned, these models are linear.

Table 4.5 here

C. Time Series With Small Sample Size

We are going to examine the tests with small sample. A linear AR(2) (model 1) and a non-linear model (model 9) are chosen as basis. Hinich's test finds it difficult to deal with small samples. When N < 60, the number of squares within prinicpal domain which are used to smooth the spectrum is less than 2. Therefore, Hinich's test is not applicable to time series with observations less than 60. When we use Keenan's test in model 9, we discover the design matrix of regression in Tukey's non-additivity-type test-framework is near singlar. Therefore, we cannot draw valid conclusion. However, when the data are generated from model 1 (linear model), Keenan's test appears to operate well, even when N < 50. Our test seems to work normally when applying to time series with small sample size. Table 4.6 gives the results of the simulation experiment.

Table 4.6 here

D. White Noise With Different Innovation Variance

Consider a series of white noise $e_t \sim N(0, \sigma_e^2)$. An ideal test can detect its linearity independent of any σ_e^2. We apply the tests to these series of white noise with different magnitude of σ_e^2, the results are given in table 4.7. All tests appear to identify the linearity of each series, regardless of the magnitude of σ_e^2 within the range of (1,500).

Table 4.7 here

4.4 CPU TIME REQUIRED

We are going to compare the computational efficiency of each tests. A comparsion of CPU time required by each test to process a time series with 204 observations are given in table 4.8.

Table 4.8 here

Hinich's test and Keenan's test only need 15* and 17* CPU seconds respectively. Because of the complexity in computing the spectral and bispectral estimates, Subba Rao-Gabr's test requires 60* seconds. Our test needs 152* CPU seconds to calculate the test statistic and simulate the null distribution. About 95% of the required CPU time for our test is taken up in simulating the null distribution. Although our new proposed method requires nearly 3 minutes CPU time, it is not a barrier for us to use the test. The advanced computer technology today can handle our test without any difficulty.

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* All computer programs are written in FORTRAN, executed by IBM-3031, VM370-OSVS1 system at the computer centre, the Chinese University of Hong kong.

TABLE 4.1 HINICH'S TEST FOR SIMULATED DATA

M	MODEL 1 (L)		MODEL 9 (NL)	
	Test Statistic	Conclusion	Test Statistic	Conclusion
17	-0.2345	L	8.3921	NL
20	0.0417	L	11.6327	NL
25	0.5677	L	6.4612	NL
30	-0.0595	L	2.2404	NL

REMARKS : L - Linear ; NL - Non-Linear,
$\alpha = 0.05$; Critical values = ± 1.96

TABLE 4.2 NUMBER OF CORRECT DECISION AMONG EACH TEST (100 REPLICATIONS)

Model Number	Original Type	Hinich's Test	Keenan's Test	Our Test
1	L	91	95	99
2	L	97	98	96
3	NL	10	6	94
4	NL	38	88	96
5	NL	5	8	100
6	NL	2	37	100
7	NL	34	10	14
8	NL	27	91	96
9	NL	87	93	98

REMARKS : L - Linear ; NL - Non-Linear,
We set $\alpha = 0.05$.

TABLE 4.3 A COMPARSION FOR ALL TESTS TO REAL DATA

Data	Transformation	Hinich's Test	Keenan's Test	Our Test
Lynx N=114	RAW	L	L	NL
	LOGTEN	L	L	NL
Sunspot N=256	RAW	L	NL	NL
	2[Sq(X+1) - 1]	L	NL	NL
BlowflyA N=126	RAW	L	L	NL
	SQRT	L	NL	NL
	LOGTEN	NL	NL	NL
BlowflyB N=82	RAW	L	L	L
	SQRT	L	L	L
	LOGTEN	L	L	L

REMARKS: L -- Linear ; NL -- Nonlinear
We set α =0.05.

TABLE 4.4 TEST RESULTS FOR NON-GAUSSIAN MA(1) MODELS

a	Test (α = 0.05)	Number of correct decision per 100
0.7	Hinich's	99
	Keenan's	97
	Our test	12
-0.7	Hinich's	89
	Keenan's	96
	Our test	14

TABLE 4.5 TEST RESULTS FOR RANDOM COEFFICIENT AR(1) MODELS

a,b	Test (α = 0.05)	Number of correct decision per 100
a=0.5 b=0.1	Hinich's	95
	Keenan's	94
	Our test	97
a=0.0 b=0.1	Hinich's	91
	Keenan's	93
	Our test	98
a=-0.5 b=0.1	Hinich's	93
	Keenan's	96
	Our test	94

TABLE 4.6 TEST RESULTS FOR SMALL SAMPLE TIME SERIES

DATA	Number of Correct Decision per 100 replications		
	Hinich's test	Keenan's test	Our test
Model 1 (N=30)	X	95	97
Model 1 (N=50)	X	94	94
Model 9 (N=30)	X	X	88
Model 9 (N=50)	X	X	91

REMARKS : X - The test is not operational.

TABLE 4.7 TEST RESULTS FOR WHITE NOISE WITH DIFFERENT VARIANCE

DATA	Number of Correct Decision per 100 replications		
	Hinich's test	Keenan's test	Our test
N(0,1)	96	98	95
N(0,5)	92	98	92
N(0,10)	94	96	93
N(0,100)	95	90	99
N(0,500)	97	96	95

TABLE 4.8 CPU TIME REQUIREMENT FOR EACH TEST (N=204)

	CPU Time* (seconds)
Lag Regression	6
Subba Rao-Gabr's test	60
Hinich's test	15
Keenan's test	17
Our test	152

* The FORTRAN program is executed by IBM-3031, VM370-OSVS1 system at the Computer Centre, the Chinese University of Hong Kong.

CHAPTER V CONCLUSION

Various tests for linearity have been proposed although none is totally satisfactory. Keenan's test seems quite reasonable as a diagnostic for linearity versus a second-order Volterra expansion. Such a test would be time domain based and computationally less complex than the frequency domain based alternatives. However, his test is not sensitive to detect non-linear AR models. It is also a limitation of Keenan's test that it cannot deal with some non-linear time series model without " quadratic " component in Volterra series expansion. The computational instability in Subba Rao-Gabr's F-statistic is a fatal weakness of their test. Although Hinich has improved Subba Rao-Gabr's test significanlty and proposed a non-parametric method, the new test is not very powerful. It is quite often that his test misclassifies a non-linear time series into linear territory. In this thesis, we have proposed a new parametric method in testing the linearity in time series. The simulation studies show that our test is quite powerful. Although the test fails to detect the non-linearity of some non-linear time series models with linear conditional mean, we are satisfied with the overall performance of this test in our study.

In practice, we may use nonparametric regression as a preliminary technique. If one prefers nonparametric method, we recommend Hinich's test. Otherwise, we suggest to use our new proposed parametric method.

Further development of test for linearity of time series data may concentrate on detecting the non-linearity of time series models with linear conditional mean (e.g. model 7). None of the existing methods can handle this kind of models satisfactorily.

APPENDIX I : SAMPLE MODELS

MODEL 1 : [Linear AR(2)]

$$X_t = 0.4\, X_{t-1} - 0.3\, X_{t-2} + e_t$$

MODEL 2 : [Linear MA(2)]

$$X_t = e_t - 0.4\, e_{t-1} + 0.3\, e_{t-2}$$

MODEL 3 : [Exponential AR(1)]

$$X_t = (\, 0.3 - 0.8 \exp(-1\, X_{t-1}^2))\, X_{t-1} + e_t$$

MODEL 4 : [Bilinear Model BL(1,0,1,1)]

$$X_t = 0.5 - 0.4\, X_{t-1} + 0.4\, X_{t-1} e_{t-1} + e_t$$

MODEL 5 : [Threshold Model SETAR(2;1,1)]

$$X_t = \begin{cases} 1 - 0.5\, X_{t-1} + e_t & X_{t-1} < 0 \\ -1 - 0.5\, X_{t-1} + e_t & \text{otherwise} \end{cases}$$

MODEL 6 : [Threshold Model SETAR(2;1,1)]

$$X_t = \begin{cases} 2 + 0.5\, X_{t-1} + e_t & X_{t-1} < 1 \\ 0.5 - 0.4\, X_{t-1} + e_t & \text{otherwise} \end{cases}$$

MODEL 7 : [Nonlinear MA Model]

$$X_t = e_t - 0.4\, e_{t-1} + 0.3\, e_{t-2} + 0.5\, e_t e_{t-2}$$

MODEL 8 : [Nonlinear MA Model]

$$X_t = e_t - 0.3\, e_{t-1} + 0.2\, e_{t-2} + 0.4\, e_{t-1} e_{t-2} - 0.25\, e_{t-1}^2$$

MODEL 9 : [Bilinear Model]

$$X_t = 0.4\, X_{t-1} - 0.3\, X_{t-2} + 0.5\, X_{t-1} e_{t-1} + 0.8\, e_{t-1} + e_t$$

APPENDIX II THE CANADIAN LYNX DATA (1821 - 1934)

YEAR	1	2	3	4	5	6	7	8	9	10
1821 - 1830	269	321	585	871	1475	2821	3928	5943	4950	2577
1831 - 1840	523	98	184	279	409	2285	2685	3409	1824	409
1841 - 1850	151	45	68	213	546	1033	2129	2536	957	361
1851 - 1860	377	225	360	731	1638	2725	2871	2119	684	299
1861 - 1870	236	245	552	1623	3311	6721	4254	687	255	473
1871 - 1880	358	784	1594	1676	2251	1426	756	299	201	229
1881 - 1890	469	736	2042	2811	4431	2511	389	73	39	49
1891 - 1900	59	188	377	1292	4031	3495	587	105	153	387
1901 - 1910	758	1307	3465	6991	6313	3794	1836	345	382	808
1911 - 1920	1388	2713	3800	3091	2985	3790	674	81	80	108
1921 - 1930	229	399	1132	2432	3574	2935	1537	529	485	662
1930 - 1934	1000	1590	2657	3396						

APPENDIX III ANNUAL SUNSPOT NUMBERS (1700 - 1955)

YEAR	1	2	3	4	5	6	7	8	9	10
1700 - 1709	5.0	11.0	16.0	23.0	36.0	58.0	29.0	20.0	10.0	8.0
1710 - 1719	3.0	0.0	0.0	2.0	11.0	27.0	47.0	63.0	60.0	39.0
1720 - 1729	28.0	26.0	22.0	11.0	21.0	40.0	78.0	122.0	103.0	73.0
1730 - 1739	47.0	35.0	11.0	5.0	16.0	34.0	70.0	81.0	111.0	101.0
1740 - 1749	73.0	40.0	20.0	16.0	5.0	11.0	22.0	40.0	60.0	80.9
1750 - 1759	83.4	47.7	47.8	30.7	12.2	9.6	10.2	32.4	47.6	54.0
1760 - 1769	62.9	85.9	61.2	45.1	36.4	20.9	11.4	37.8	69.8	106.1
1770 - 1779	100.8	81.6	66.5	34.8	30.6	7.0	19.8	92.5	154.4	125.9
1780 - 1789	84.8	68.1	38.5	22.8	10.2	24.1	82.9	132.0	130.9	118.1
1790 - 1799	89.9	66.6	60.0	46.9	41.0	21.3	16.0	6.4	4.1	6.8
1800 - 1809	14.5	34.0	45.0	43.1	47.5	42.2	28.1	10.1	8.1	2.5
1810 - 1819	0.0	1.4	5.0	12.2	13.9	35.4	45.8	41.1	30.1	23.9
1820 - 1829	15.6	6.6	4.0	1.8	8.5	16.6	36.3	49.6	64.2	67.0
1830 - 1839	70.9	47.8	27.5	8.5	13.2	56.9	121.5	138.3	103.2	85.7
1840 - 1849	64.6	36.7	24.2	10.7	15.0	40.1	61.5	98.5	124.7	96.3
1850 - 1859	66.6	64.5	54.1	39.0	20.6	6.7	4.3	22.7	54.8	93.8
1860 - 1869	95.8	77.2	59.1	44.0	47.0	30.5	16.3	7.3	37.6	74.0
1870 - 1879	139.0	111.2	101.6	66.2	44.7	17.0	11.3	12.4	3.4	6.0
1880 - 1889	32.3	54.3	59.7	63.7	63.5	52.2	25.4	13.1	6.8	6.3
1890 - 1899	7.1	35.6	73.0	85.1	78.0	64.0	41.8	26.2	26.7	12.1
1900 - 1909	9.5	2.7	5.0	24.4	42.0	63.5	53.8	62.0	48.5	43.9
1910 - 1919	18.6	5.7	3.6	1.4	9.6	47.4	57.1	103.9	80.6	63.6
1920 - 1929	37.6	26.1	14.2	5.8	16.7	44.3	63.9	69.0	77.8	64.9
1930 - 1939	35.7	21.2	11.1	5.7	8.7	36.1	79.7	114.4	109.6	88.8
1940 - 1949	67.8	47.5	30.6	16.3	9.6	33.2	92.6	151.6	136.3	134.7
1950 - 1955	83.9	69.4	31.5	13.9	4.4	38.0				

APPENDIX IV BLOWFLY DATA

248	146	1801	6235	5974	8921	6610	5973	5673	3875
2361	1352	1226	912	521	363	229	142	82	542
939	2431	3687	4543	4543	5441	4412	3022	2656	1967
1295	915	551	313	167	96	93	60	68	5259
6673	5441	3987	2952	3648	4222	3889	2295	1509	928
739	566	303	274	192	226	519	1224	2236	3818
6208	5996	5789	6652	7939	4868	3952	2712	1734	1224
703	508	366	279	243	343	761	1025	1221	1600
2267	3290	3471	3637	3703	4876	5364	4890	3029	1950
1225	1076	905	772	628	473	539	825	1702	2868
4473	5221	6592	5400	4752	3521	2719	1931	1500	1082
549	774	864	1308	1624	2224	2423	2959	3547	7237
5218	5311	4273	3270	2281	1549	1081	795	610	445
894	1454	2262	2363	3847	3876	3936	3479	3415	3861
3571	3113	2319	1630	1297	861	761	659	701	762
1188	1778	2428	3806	4519	5646	4851	5374	4713	7367
7236	5245	3636	2417	1258	766	479	402	248	254
604	1340	2342	3328	3599	4081	7643	7919	6098	6896
5634	5134	4188	3469	2442	1931	1790	1722	1488	1416
1369	1666	2627	3840	4044	4929	5111	3152	4462	4082
3026	1589	2076	1829	1888	1149	968	1170	1465	1676
3075	3815	4639	4424	2784	5860	5781	4897	3920	3835
3618	3050	3772	3517	3350	3018	2625	2412	2221	2619
3203	2706	2717	2175	1628	2388	3677	3156	4272	3771
4955	5584	3891	3501	4436	4369	3394	3869	2922	1843
2837	4690	5119	5838	5389	4993	4446	4651	4243	4620
4849	3664	3016	2881	3821	4300	4168	5448	5477	8579
7533	6884	4127	5546	6316	6650	6304	4842	4352	3215
2652	2330	3123	3955	4494	4780	5753	5555	5712	4786
4066	2891	3270	4404	4398	4112	4401	5779	6597	8091
11282	12446	13712	11017	14683	7258	6195	5962	4213	2775
1781	936	898	1160	3158	3386	4547	4823	4970	4940
5793	7836	4457	6901	8191	6766	5165	2919	3415	3431
3162	2525	2290	1955	1936	2384	4666	7219	8306	8027
7010	8149	8949	6105	5324	5766	6214	7007	8154	9049
6883	8103	6803							

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