Fast RFID Counting under Unreliable Radio Channels

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A fast RFID counting algorithm with performance guarantee can be used as a fundamental building block for other more sophisticated RFID query protocols and operations. Recently, Kodialam et al. propose various low-latency RFID counting schemes with accuracy guarantees [1, 2] based on a probabilistic counting approach which does not require explicit identification of individual tags. However, the proposed schemes all assume a perfect communication channel between the reader and the tags which is unlikely to be true in practice. On the contrary, as demonstrated by recent empirical measurement studies, the radio communications between an RFID reader and a set of seemingly "in-range" tags are rather non-deterministic and can even be unreliable at times due to varying radio conditions. In this thesis, we investi-

gate fast RFID counting algorithms which can take the effects of radio channel unreliability into account.

In the first approach, we assume the channel has been characterized so that the first two moments of the successful response probability distribution of the tags in the tag-set are known. In particular, such indirect two-parameter characterization of the channel is derived by modeling the spatial distribution of tags and the corresponding channel fading effects. Based on such characterization, we analyze the new requirements on the algorithm parameters used in [2] (e.g., number of reader polling cycles, frame-size and persistent probability) in order to achieve a desired level of estimation accuracy based on the channel model. A key observation is that, unlike the perfect channel case where one can indefinitely reduce the estimation error by increasing the number of reader polling cycles, with an unreliable radio channel, there is a lower-bound on the estimation error due to the inherent variation in the spatial distribution of the tags and the radio channel conditions. Towards this end, we have derived an expression for this lower-bound. We also demonstrate the efficacy of our analytical results and their corresponding guarantees in estimation accuracy via a simulation study.

In the second approach, we develop efficient and fast estimation schemes that can provide good estimates of the cardinality of the tag-set while assuming no prior knowledge of the parameters of the unreliable radio channel between the reader and the tags. This approach is based on a novel interpretation of the capture-recapture models [3] from the field of ecology/ biostatistics. By leveraging the rich estimation techniques in this field, we extend the probabilistic counting framework introduced by [1, 2] to tackle the challenge of a lossy channel with unknown characteristics. The variance of the resultant tag-set cardinality estimators are then characterized analytically. We also demonstrate the performance of the proposed schemes under various system parameters and channel conditions.

摘要

一個具履約保證的快速RFID數量估計算法可以用作發展其他更尖端的RFID技術查詢協定。Kodialam等人 [1, 2] 最近提出了各種以概率計算為基礎的RFID數量估計方法。它們不但提供準確性保證，而且不需要明確辨認任何個別RFID標籤，大大縮短估計時間。

然而，它們都作出了不切實際的假設：RFID閱讀器和RFID標籤之間有著完善的溝通渠道。相反，近日不斷有實證測量研究指出，閱讀器和標籤之間的無線電通信是相當不穩定，甚至是不可靠的。在此論文中，我們會將無線電通信納入考慮，並建議如何顧及其不可靠性對快速RFID數量估計算法的影響。

在第一種方法中，我們假設知道無線電通信頻道的特性。透過頻道衰落和標籤空間分佈的建模，我們可以得到標籤成功回應閱讀器的概率的第一和第二階矩。基於這樣的特性，我們重新分析了算法 [2] 的要求，包括閱讀器輪詢週期、幀大小和持續概率，以實現理具履約保證的快速RFID數量估計算法。一個重要的觀察是，與假設完善溝通渠道的結論不同：在完善溝通渠道的情況下，人們可以透過增加閱讀器輪詢週期以無限地減少估計誤差。相反，在不可靠的無線電頻道下，由於標籤空間分佈的固有變化及無線電頻道的不可靠性，估計誤差是有下限的。為此，我們得出這個下限的表達式。我們也透過分析和仿真研究證明該算法的結果及其相應的履約保證。

在第二個辦法，我們發展出一種不需事先了解無線電頻道的快速、有效的數量估計算法。這種方法是運用生態學 生物統計學的捕獲一再捕獲模型 [3] 。利用在這一領域豐富的數量估計技術，我們伸延了 [1, 2] 的概率計算框架，於不需事先了解無線電頻道下，仍能作出數量估計。我們分析了其估值的差額，並展示這算法在不同系統參數和頻道衰落的性能。

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This work is dedicated to my family.

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Chapter 1

Introduction

Radio Frequency Identification (RFID) tags have become increasingly popular in supply-chain management applications to improve processing latency while reducing costs for various logistics operations. For instance, the frequent check-in and check-out of a large number of goods can be streamlined via RFID tagging and reader polling at various points of interest along the supply chain. Under standard RFID protocols, upon the receipt of queries from a reader, the tags attached to the goods typically respond with their unique identifiers based on which the specifics of the goods can be collected and tracked. However, such explicit identification of individual tags may be an overkill for a large set of common operations, e.g., determining the quantities of a given type of goods. Not only would explicit tag identification increase processing latency (as it requires serialization of the responses from all the relevant tags via some multiple access protocol), it can also lead to unnecessary privacy violations. As shown in [1, 2], accurate tag-set cardinality estimation can be done in a much shorter time without explicit tag-identification using a frame-based slotted ALOHA-like protocol. This approach can also alleviate some of the privacy concerns which has threatened the widespread acceptance of RFID technologies.

To meet their low-cost requirement, most RFID tags, especially the passive ones, do not contain any power source or support any sophisticated error-correction/ retransmission protocol to ensure reliable communications with the reader. The objective of this thesis is to develop an efficient and fast estimation scheme that will provide a good estimate of the cardinality of the tag-set while taking non-deterministic/ unreliable radio channel conditions into account. The proposed algorithm is based on the schemes developed in [2] which take an probabilistic counting approach. Like its predecessors, our scheme only requires simple modifications to standard RFID tags/readers and is easily implementable using current technologies with minimal increase in tag/reader cost.

Counting the number of RFID tags with probabilistic guarantee instead of having exact count can reduce the latency involved. Requiring exact count involves serializing the tags and the latency involved depends on the number of tags. For example, counting 10000 tags would require the reader to receive all 10000 distinct tag responses. The minimum size of the tag responses hence has to be at least $log_2(10000)$=14bits long. Hence, the minimum latency involved would be 140000 slots. This latency has not taken the imperfect MAC scheduler and unknown tag set size into account. On the other hand, if we are interested in providing probabilistic guarantee: within the actual number of tag 2.5% for 99% of the time. Then the latency can be reduced to 7018 slots [2]. Suppose there are 4000 slots in 1 second, exact counting would require the tags to be within the read range for at least 35 seconds while probabilistic counting requires only 2 seconds. Given the limited read range of the (passive) RFID readers, exact counting requires the goods to wait for a much longer time than using probabilistic counting. Moreover, probabilistic counting allows adjusting the trade-off between latency and accuracy. Higher accuracy requires higher latency. This controllable accuracy/latency allows operators to optimize the (logistics) operations. One example is controlling the queue length in supermarket as to maximize the revenue.

In addition, probabilistic counting requires no explicit tag identification. The information we are interested in (count) is collected during the air interface. The amount of data stored/processed by the server (middleware) would be greatly reduced. An example is to monitor the stock of goods in the supermarket. When the number of goods on the shelf is less than some threshold, it will trigger refilling. In this case, all we are interested in is whether the number of goods is higher than the threshold or not. We dont́ really care the exact number of goods is 20 or 21, as long as it is greater than 5.

As demonstrated in [1, 2], accurate tag-set cardinality estimation can be done in a much shorter time without explicit tag-identification using a frame-based slotted ALOHA-like protocol. This approach can also alleviate some of the privacy concerns which has threatened the widespread acceptance of RFID technologies. One of the major limitation of the proposed schemes is that they all assume a perfectly reliable communications between the reader and the RFID tags. Unfortunately, such assumption is practically unachievable given the current technology and cost constraints: To meet their low-cost requirement, most RFID tags do not support any sophisticated error-correction or recovery scheme to maintain reliable communications with the reader. This becomes a severe problem when the reader needs to operate over a lossy wireless channel caused by noise, fading, or physical blockage: under such circumstances, the reader faces uncertainity over the number of probes that are needed to reliably count and/or identify the number of tags in its range.

The objective of this thesis is to develop efficient and fast estimation schemes that will provide a good estimate of the cardinality of the tag-set while taking unreliable radio channel conditions into account.

In the first approach, we assume the channel has been characterized so that the first two moments of the successful response probability distribution of the tags in the tag-set are known. In particular, such indirect two-parameter characterization of the channel is derived by modeling the spatial distribution of tags and the corresponding channel fading effects. Based on such characterization, we analyze the new requirements on the algorithm parameters used in [2] (e.g., number of reader polling cycles, frame-size and persistent probability) in order to achieve a desired level of estimation accuracy based on the channel model. A key observation is that, unlike the perfect channel case where one can indefinitely reduce the estimation error by increasing the number of reader polling cycles, with an unreliable radio channel, there is a lower-bound on the estimation error due to the inherent variation in the spatial distribution of the tags and the radio channel conditions.

Towards this end, we have derived an expression for this lower-bound. We also demonstrate the efficacy of our analytical results and their corresponding guarantees in estimation accuracy via a simulation study.

In the second approach, we develop efficient and fast estimation schemes that can provide good estimates of the cardinality of the tag-set while assuming no prior knowledge of the parameters of the unreliable radio channel between the reader and the tags. This approach is based on a novel interpretation of the capture-recapture models [3] from the field of ecology/ biostatistics. By leveraging the rich estimation techniques available for the capture-recapture models, we extend the probabilistic counting framework introduced by [1, 2] to tackle the challenge of a lossy channel with unknown characteristics. The variance of the resultant tag-set cardinality estimators are then characterized analytically. We also demonstrate the performance of the proposed schemes under various system parameters and channel conditions. To the best of our knowledge, this is the first set of schemes that can support fast RFID tag-cardinality estimation in lossy wireless channels without requiring a priori knowledge of the channel parameters.

All of our proposed algorithms require only simple modifications to standard RFID tags/readers similar to those proposed in [1] and can be easily implemented using current technologies with minimal increase in tag/reader cost.

The organization of the thesis is as follows: In Chapter 2, we discuss the related work. In Chapter 3, we propose the fast RFID counting scheme which requires the first two moments of the successful response probability distribution of the tags in the tag-set to be known. Chapter 4 covers RFID counting schemes which assume no prior knowledge of the characteristics of the unreliable radio channel. We conclude the thesis in Chapter 5 and discuss possible future work.

□ **End of chapter.**

Chapter 2

Background and Related Work

A low-latency RFID counting scheme with accuracy guarantees can be used as a fundamental building block to support more elaborated RFID query operations. RFID counting algorithms of such nature have been proposed recently by Kodialam et al. [1, 2]. One distinct feature of these schemes is that they do not require the reader to explicitly identify individual tags and thus can help to preserve privacy of the RFID users. However, these schemes all assume a perfect communication channel between the reader and the tags which is not achievable in practice. In fact, empirical measurement studies have found that the radio communications between an RFID reader and a set of seemingly "in-range" tags are still unreliable and non-deterministic due to ever-changing radio channel conditions. Worse still, given the stringent cost constraint, it is unlikely that standard channel estimation procedures can be applied for individual tags. In this thesis, we will extend the fast probabilistic RFID counting framework pioneered by Kodialam et al to handle lossy wireless channels of known or unknown characteristics.

Recent measurement studies [4, 5, 6] have demonstrated empirically the limited reliability of communications between passive RFIDs and a reader. [4] studied the impact of read distance for tags with different physical sizes under various types of surrounding objects, operating radio frequencies and reader types. It showed that even with same output (transmission) power, some readers can read some types of tags better than the others. Their experiments also compared outdoor and indoor environment and showed that there was a clear bias of improved performance in read distance for outdoor (open) environment. [5] investigated the performance of the tags against different reader operating modes defined in the EPC Class 1 Gen 2 UHF standards. They found that different modes of reader operation resulted in different error rate against reader-tag distance. [7] provided comprehensive discussions on the performance limitations of the passive UHF RFID systems. The above studies, however, do not quantify the impact of unreliable reader-tag communications on the application level performance of an RFID system. Nor do they provide any remedial schemes to combat or offset inaccuracies caused by unreliable tag readings.

Several schemes have been proposed to handle the unreliability/ uncertainties involved in RFID data acquisition. [8] viewed the raw RFID acquired data as a data stream with potential missing items and made corrections by imposing time-averaged windowing to compensate for missing data items. The paper applied an adaptively-sized time-window filter to capture the tag dynamics while reducing false positive reading rate. Per-tag cleaning operations were performed to filter readings obtained from each individual tag. One drawback of the proposed approach is its heuristic nature. The correctness of the filtering operation relied on the assumption of the time-scale of movements associated with the tags. Another limitation of the proposed cleaning scheme was that it required explicit identification of individual tags which increased the processing latency substantially. [9] defined a high-level application event in an RFID system (e.g., a group of people are having a meeting in a room) as a series of lower level events related to the detection or data-acquisition from the RFID tags involved. To address the uncertainties caused by RFID data-acquisition, instead of giving a definitive declaration on the occurrence of an event, every event observed is reported with an associated probability.

The probability of occurrence of a high-level event can be derived based on the uncertainties associated with its "component" low-level events.

Estimating the unknown population size of species is a well studied problem in the field of ecology/ biostatistics, and has implications to the problem under consideration. One of the techniques is capture-recapture [10], where some of animals in a closed population are first caught, marked and released. In the recapture process, some of the animals are caught and the number of marked/ unmarked animals will provide some clues on the population size. A sample application can be the estimation of the number of fishes in a pool using the Lincoln-Petersen method[11] based on (1) the number of fishes caught in each of two separate catches, and (2) the portion of fishes caught in both catches. In the most basic model (M_0), all animals have the same likelihood of being captured. The basic model can be extended along multiple dimensions: capture probabilities can vary by time, by animal, by number of times an animal has been previously captured, and any combination thereof. The relevant model to our RFID problem is the so-called heterogeneous catchability [12] model, commonly denoted by M_h, where each animal has a distinct and unknown capture probability which does not vary with time or capture history. Besides the rich set of practical estimators developed for M_h [3, 12, 13, 14] by the biostatistics community, there are also recent results [15, 16] on the theoretical front regarding the identifiability of population-size (N) under the M_h model. In [16], Holzmann et al. provided a general criterion for identifiability of the population-size (N) in the M_h model under different classes of capturing-probability distributions. In particular, they established the identifiability within commonly used families of capturing-probability distributions including the Uniform, Beta and discrete distribution with finite support points.

□ **End of chapter.**

Chapter 3

RFID Tag-set Cardinality estimation based on a Two-parameter implicit Channel Model

In this chapter, we extend the algorithms in [2] by taking into account the effects of radio channel unreliability. We model the spatial distribution of tags and the corresponding channel fading effects to result in an indirect characterization of the channel effects via the first two moments of the successful respond probability distribution of the tags in the tag-set of interest. Based on this two-parameter characterization, we analyze the new requirements on the algorithm parameters used in [2] (e.g., number of reader polling cycles, frame-size and persistent probability) in order to achieve a desired level of estimation accuracy. Another key observation is that, unlike the perfect channel case where one can indefinitely reduce the estimation error by increasing the number of reader polling cycles, with an unreliable radio channel, there is a lower-bound on the estimation error due to the inherent variation in the spatial distribution of the tags and the radio channel conditions. Towards this end, we have derived an expression for this lower-bound. We also demonstrate the efficacy of our analytical results and their corresponding guarantees in estimation accuracy via an simulation study.

3.1 System Model

Consider a reader which broadcasts a probing request to t RFID tags located within a warehouse. Assume that the location of each tag (represented by its (x, y) coordinates) follows an independent, identical spatial distribution $w(x, y)$. We are interested in estimating the value of t to a desired level of accuracy while considering the non-deterministic nature of radio channels between the reader and the tags. Let $\mathcal{P}(x, y)$ be the probability that a tag located at coordinates (x, y) can communicate with the reader. In general, $\mathcal{P}(x, y)$ depends on the reader transmission power, antenna gain, minimum power requirement (sensitivity) of the tag, its backscattering efficiency as well as the radio channel and fading model parameters. In Section 3.4 we will demonstrate how $\mathcal{P}(x, y)$ can be derived based on this list of parameters. Define $F_R(\cdot)$ to be the c.d.f. of the tag responding probability for reader R over all realizations of spatial distribution of a tag within the warehouse. We have:

$$F_R(p) = \int_0^p f_R(p')dp' = \iint_{\mathcal{P}(x,y)\leq p} w(x, y)dxdy, \qquad (3.1)$$

where $f_R(\cdot)$ represents the corresponding p.d.f.. Denote the mean and variance of $f_R(\cdot)$ by μ and σ^2 respectively.

We assume that the RFID system follows the *q-persistent framed-slotted ALOHA* model described in [1, 2]. Denote by n the total number of reader polling cycles required per estimation. In each polling cycle, the reader will broadcast the frame size f, persistent probability q and a random seed. Depending on the forward (reader-to-tag) radio channel condition, some tags may not receive sufficient power to process the probing request. A tag receiving sufficient power will pick an integer within the range of $[1, f]$ and respond in that slot with probability q. Depending on the reverse (tag-to-reader) radio channel condition, the reader may not be able to detect the reply from a responding tag. The reader will then count the number of empty slots, i.e., the slots in which no response is observed. Denote by Z_i the number of empty slots observed by the reader during the i-th polling cycle. The reader will repeat the polling cycle for n times, each with a different seed. Denote by $Y = \sum_i Z_i/n$ the average number of empty slots per frame over the n polling cycles. Suppose the actual cardinality of the tag-set is t. The estimator will take Y as an input and return an estimate $\hat{t}$ satisfying the following accuracy requirement:

$$Prob\left[\frac{\hat{t}}{t} \in \left(1 - \frac{\beta}{2}, 1 + \frac{\beta}{2}\right)\right] \geq 1 - \alpha,$$

where $\beta > 0$ specifies the error bound and $0 < \alpha < 1$ gives the failure probability. We use Z_α to denote the α-percentile of the Unit Normal distribution $\mathcal{N}(0,1)$ where $\mathcal{N}(a,b)$ represents a Normal distribution with mean a and variance b respectively. Table 3.1 summarizes the definition of the symbols and parameters used in our derivation.

3.2 Number of Empty Slots Observed by the Reader

In this section, we characterize the number of empty slots observed by a reader under an unreliable radio channel. Our tag-set cardinality estimator relies on the average empty-slot count. The accuracy of the estimator depends on the variance of the empty-

Variables	Meaning
t	True number of tags in the set
$\hat{t}$	Estimated number of tags given by the estimator
α	Failure probability of the estimator satisfying the accuracy requirement. The scheme will fail with probability $1-\alpha$
β	Error bound for specifying accuracy requirement of the estimator. $\hat{t}/t \in [1-\beta/2, 1+\beta/2]$
p_i	Probability for tag i to have a perfect channel with the reader R
$\boldsymbol{p}$	the set of $\{p_i\}$
$f_R(p)$	With probability $f_R(p)\,dp$, a tag in the warehouse will have a responding probability within the range $[p+dp]$
μ	The average responding probability (due to imperfect channel) for a tag in the warehouse to respond to reader R
σ^2	The variance of the responding probability of a tag with respect to reader R
f	frame size/ number of slots in each frame
n	number of polling cycles per estimation
q	A tag receiving the query will choose to content with persistent probability q
ρ	loading factor $= t/f$
$N_0(t)$	random variable for empty-slot count
Z_i	empty-slot count for the i-th reader polling cycle
Y	average empty-slot count $= \sum_i Z_i/n$

Table 3.1: Notations

slot count, which is determined by the following two factors: (1) the inherent variation in the spatial distribution of tags and the radio conditions, (2) the randomness in the slotted-ALOHA-based polling scheme.

Denote by $\boldsymbol{p} = \{p_1, ...p_t\}$ a realization of the set of responding probabilities of t tags within the warehouse. In general, $\boldsymbol{p}$ depends on the realization of the spatial locations of the t tags within the warehouse and the radio channel propagation parameters. Notice that a different realization of $\boldsymbol{p}$ will result in a different distribution of the number of empty slots observed by the reader.

Denote by $N_0(t)$ the empty-slot count of one single polling cycle. The expected value of $N_0(t)$ is derived in Lemma 3.2.1 below.

Lemma 3.2.1 *Given that there are t tags within the warehouse, the expected number of empty slots observed by the reader, $E[N_0(t)]$, is given by $fe^{-\rho q \mu}$.*

Proof 1 *Define X_j to be an indicator random variable with $X_j =$ 1 corresponding to the event that the j-th slot is not chosen by any of the responding tag(s). $X_j = 0$ represents the event that the j-th slot is chosen by at least one of the responding tags. The* *probability that tag i will not respond in slot j is given by:*

$$1 - p_i + p_i \left(1 - q + \frac{f-1}{f} q\right) = 1 - \frac{p_i q}{f}.$$

Thus, we have:

$$Prob\left[X_j = 1 | \boldsymbol{p}\right] = \prod_{i=1}^{t} \left(1 - \frac{p_i q}{f}\right)$$

$$E\left[N_0\left(t\right)|\boldsymbol{p}\right] = E\left[\sum_{j=1}^{f} X_j | \boldsymbol{p}\right] = f \cdot \prod_{i=1}^{t} \left(1 - \frac{p_i q}{f}\right). \quad (3.2)$$

Taking $e^x \approx 1+x$ for $|x| \ll 1$, the R.H.S. of (3.2) *can be rewritten as*

$$f e^{-\rho q \mu} \left(1 + \sum_{k=1}^{\infty} \frac{q^k}{k! f^k} \left(\sum p_i - t\mu\right)^k\right). \quad (3.3)$$

Note that, for large t, $\sum p_i - t\mu$ follows a Gaussian distribution $\mathcal{N}\left(0, t\sigma^2\right)$ and $E\left[\left(\sum p_i - t\mu\right)^k\right] = E\left[\left(\left(\sum p_i - t\mu\right) - 0\right)^k\right]$, is the k-th central moment of the Gaussian distribution which is equal to 0 *when k is an odd number. The* $2k$*-th central moment is given by:*

$$E\left[\left(\sum p_i - t\mu\right)^{2k}\right] = \frac{(2k)!}{2^k k!} \left(t\sigma^2\right)^k$$

By the linearity of expectation, we have:

$$E\left[N_0\left(t\right)\right] = f e^{-\rho q \mu} \left(1 + \sum_{k=1}^{\infty} \frac{q^k \sigma^{2k}}{2^k k!} \left(\frac{\rho q}{f}\right)^k\right) \quad (3.4)$$

With f selected according (3.13) *which we will discuss later, we have*

$$1+\sum_{k=1}^{\infty}\frac{q^k\sigma^{2k}}{2^k k!}\left(\frac{\ln\frac{f}{5}}{\mu f}\right)^k \geq 1+\textstyle\sum_{k=1}^{\infty}\frac{q^k\sigma^{2k}}{2^k k!}\left(\frac{\rho q}{f}\right)^k \geq 1$$

$$1+\sum_{k=1}^{\infty}\frac{1}{k!}\left(\frac{q\sigma^2\ln\frac{f}{5}}{2\mu f}\right)^k \geq 1+\textstyle\sum_{k=1}^{\infty}\frac{q^k\sigma^{2k}}{2^k k!}\left(\frac{\rho q}{f}\right)^k \geq 1$$

$$1+\sum_{k=1}^{\infty}\left(\frac{q\sigma^2\ln\frac{f}{5}}{2\mu f}\right)^k \geq 1+\textstyle\sum_{k=1}^{\infty}\frac{q^k\sigma^{2k}}{2^k k!}\left(\frac{\rho q}{f}\right)^k \geq 1$$

$$1+\frac{1}{\frac{1}{\frac{q\sigma^2\ln\frac{f}{5}}{2\mu f}}-1} \geq 1+\textstyle\sum_{k=1}^{\infty}\frac{q^k\sigma^{2k}}{2^k k!}\left(\frac{\rho q}{f}\right)^k \geq 1$$

$$1+\frac{1}{\frac{2\mu f}{q\sigma^2\ln\frac{f}{5}}-1} \geq 1+\textstyle\sum_{k=1}^{\infty}\frac{q^k\sigma^{2k}}{2^k k!}\left(\frac{\rho q}{f}\right)^k \geq 1$$

$$1+\zeta \geq 1+\textstyle\sum_{k=1}^{\infty}\frac{q^k\sigma^{2k}}{2^k k!}\left(\frac{\rho q}{f}\right)^k \geq 1$$

Denote ζ as $\frac{1}{\frac{2\mu f}{q\sigma^2\ln\frac{f}{5}}-1}$. Noted that $\zeta \ll 1$ since $\frac{f}{\ln\frac{f}{5}} \gg 1$ and $\frac{1}{q} \geq 1$ and $\frac{\sigma^2}{\mu} \leq 1$ for a p.d.f. with support $\in [0,1]$ (See proof below). Hence $1+\sum_{k=1}^{\infty}\frac{q^k\sigma^{2k}}{2^k k!}\left(\frac{\rho q}{f}\right)^k \approx 1$. For example, when $f = 200$, $\sigma^2/\mu = 1$, $q = 1$, $\zeta = 0.0075 \ll 1$.

Hence, we have $E\left[N_0(t)\right] \approx fe^{-\rho q\mu}$

Lemma 3.2.2 *$\frac{\sigma^2}{\mu} \leq 1$ for any p.d.f. with support $\in [0,1]$.*

Proof 2

$$\frac{\sigma^2}{\mu} = \frac{E[X^2]-E^2[X]}{E[X]} \leq \frac{E[X^2]}{E[X]} \leq 1 \quad \textit{for } X \textit{ with support } \in [0,1]$$

Lemma 3.2.3 *The variance of the number of empty slots for an actual tag-set of size t is given by:*

$$Var[N_0(t)] = \frac{tq^2\sigma^2}{e^{2\rho q\mu}} + fe^{-\rho\mu q}\left(1 - \left(1 + \rho q^2\left(\mu^2 + \sigma^2\right)\right)e^{-\rho\mu q}\right).$$

Proof 3 *In order to capture the variance of the number of empty slots, we need to compute (1) $Var[E[N_0(t)|p]]$, the variance due to different realizations of $\boldsymbol{p}$, and (2) $E\left[Var\left[N_0\left(t\right)|\boldsymbol{p}\right]\right]$, the variance due to the slotted-ALOHA-based polling scheme.*

Consider only the first and second order terms in (3.3) *and* (3.4), *we have:*

$$\begin{aligned} &Var\left[E\left[N_0\left(t\right)|\boldsymbol{p}\right]\right] = E\left[\left(E\left[N_0\left(t\right)|\boldsymbol{p}\right]\right)^2\right] - \left(E\left[N_0\left(t\right)\right]\right)^2 \\ \approx &f^2e^{-2\rho q\mu}\left(1 + \frac{2q^2}{f^2}t\sigma^2 - 1 - \frac{q^2}{f^2}t\sigma^2\right) = \frac{tq^2\sigma^2}{e^{2\rho q\mu}} \end{aligned} \tag{3.5}$$

Higher order terms are ignored because we pick f according to (3.13). *As shown in the proof following* (3.4), *the sum of the terms are small enough to be ignored.*

The variance caused by the polling scheme is given by:

$$Var\left[N_0\left(t\right)|\boldsymbol{p}\right] = E\left[\left(\sum_{j=1}^{f} X_j\right)^2 |\boldsymbol{p}\right] - E\left[\sum_{j=1}^{f} X_j|\boldsymbol{p}\right]^2,$$

where X_i is defined in the proof of Lemma 3.2.1. For $i \neq j$,

$$E[X_iX_j|\boldsymbol{p}] = Prob[X_i = 1, X_j = 1|\boldsymbol{p}] = \prod_{i=1}^{t}\left(1 - \frac{2p_iq}{f}\right).$$

For $i = j$, *we have* $E\left[X_j^2|\boldsymbol{p}\right] = E\left[X_j|\boldsymbol{p}\right]$. *Hence, we have:*

$$
\begin{aligned}
&Var\left[N_0(t)|\boldsymbol{p}\right] = f(f-1)\prod_{i=1}^{t}\left(1-\frac{2p_iq}{f}\right)\\
&+f\prod_{i=1}^{t}\left(1-\frac{p_iq}{f}\right)-f^2\prod_{i=1}^{t}\left(1-\frac{2p_iq}{f}\right)^2\\
\approx&\left(-f+\frac{(2f-f^2)}{f^2}\sum_{i=1}^{t}p_i^2q^2\right)e^{-\frac{2t}{f}\mu q}+fe^{-\frac{t}{f}\mu q} && (3.6)\\
=&fe^{-\frac{t}{f}\mu q}\left(1-\left(1+\frac{f-2}{f^2}\left(\sum_{i=1}^{t}p_i^2q^2\right)\right)e^{-\frac{t}{f}\mu q}\right) && (3.7)\\
\approx&fe^{-\frac{t}{f}\mu q}\left(1-\left(1+\frac{1}{f}\left(\sum_{i=1}^{t}p_i^2q^2\right)\right)e^{-\frac{t}{f}\mu q}\right) && (3.8)
\end{aligned}
$$

Notice that the proof of (3.6) is given in Appendix A.

Step (3.8) *is true because* $f \gg 2$.

Taking expectation of (3.8) *to yield:*

$$E\left[Var\left[N_0(t)|\boldsymbol{p}\right]\right] = fe^{-\rho\mu q}(1-(1+\rho q^2(\mu^2+\sigma^2))e^{-\rho\mu q}). \quad (3.9)$$

Substitute Equations (3.5) *and* (3.9) *into the "Conditional Variance Formula" [17], the variance of the number of empty slots can be shown to be:*

$$
\begin{aligned}
&Var\left[N_0(t)\right] = Var\left[E\left[N_0(t)|\boldsymbol{p}\right]\right] + E\left[Var\left[N_0(t)|\boldsymbol{p}\right]\right]\\
=&\frac{tq^2\sigma^2}{e^{2\rho q\mu}} + fe^{-\rho\mu q}\left(1-\left(1+\rho q^2\left(\mu^2+\sigma^2\right)\right)e^{-\rho\mu q}\right).
\end{aligned}
$$

Theorem 1 *Given that there are* t *tags. Consider* $Y = \sum_{i=1}^{n} Z_i/n$.

The expectation and variance of Y are given by:

$$E[Y] = fe^{-q\rho\mu}$$

and

$$Var[Y] = \frac{tq^2\sigma^2}{e^{2\rho q\mu}} + \frac{f}{n}e^{-\rho\mu q}(1 - (1 + \rho q^2(\mu^2 + \sigma^2))e^{-\rho\mu q})$$

Proof 4 *Denote by $Z_1, ...Z_n$ the empty-slot counts observed over the n polling cycles. Recall that the mean and variance of $Z_i \sim N_0(t)$ have been derived in Lemma 3.2.1 and 3.2.3 respectively. Since the $\{Z_i\}'s$ are obtained from the same (i.e.,* a single*) realization of $\mathbf{p}$, the $\{Z_i\}$'s are dependent on each other. Such dependency is removed by conditioning on $\mathbf{p}$. We have:*

$$E[Y] = E[N_0(t)] = fe^{-q\rho\mu};$$

$$\begin{aligned} Var[Y] &= Var[E[Y|\mathbf{p}]] + E[Var[Y|\mathbf{p}]] \\ =&\frac{tq^2\sigma^2}{e^{2\rho q\mu}} + \frac{1}{n}fe^{-\rho\mu q}\left(1 - \left(1 + \rho q^2\left(\mu^2 + \sigma^2\right)\right)e^{-\rho\mu q}\right) \end{aligned}$$

Note that we can reproduce Theorem 4 of [1] as the special case of a perfect communication channel by setting $\mu = 1$, $\sigma^2 = 0$ in Theorem 1.

The estimator proposed depends highly on value of μ. Suppose the value of μ is deviated from the actual value by 10%, the estimated number of tags would also deviate from the actual value by 10%.

Exact distribution and Operating range of the estimator

The operating range of our estimator is defined as the valid range of tag-set cardinality t under which the resultant estimate $\hat{t}$ will satisfy the desired accuracy requirement. Notice that if all the slots in the response frame as observed by the reader are occupied, our estimator would not be able to provide an upper bound on the tag size. Hence, we determine the maximum number of tags that the estimator can support by requiring that, with high probability (w.h.p.), at least 1 empty slot will be observed in the response frame. Towards this end, we need to first characterize the exact distribution of the number of empty slots observed in a response frame. We follow [1, 18, 19] to derive this exact distribution. In particular, the probability of no empty slot observed in a frame is given by:

$$Prob(N_0 = 0) \approx (1 - e^{-\mu q \rho})^f, \tag{3.10}$$

where $\rho = t/f$. Here $e^{-\mu q \rho}$ is the probability that a slot is empty. Hence, the probability of having no empty slot for all f slots is given by (3.10). Notice that (3.10) gives the upper end of the operating range for the estimator such that at least one empty slot can be observed in the response frame. If we want this to happen with probability at least γ, the target tag-set size t must satisfy:

$$Prob\left[N_0 = 0\right] = \left(1 - e^{-\rho\mu q}\right)^f \leq 1 - \gamma. \tag{3.11}$$

For $\gamma = 0.99$, we have

$$\left(1 - e^{-t\mu q/f}\right) \leq e^{-\frac{5}{f}} \approx 1 - \frac{5}{f} \tag{3.12}$$

$$\Rightarrow e^{t\mu q/f} \leq \frac{f}{5}. \tag{3.13}$$

By solving (3.13) numerically, we can obtain an upper bound on the effective range of t.

3.3 Estimator Accuracy and Performance Analysis

In Section 3.2, we have shown that the mean and variance of the average empty-slot count is given by Theorem 1. Here we describe our estimation scheme.

Theorem 2 *Based on the observed average empty-slot count* $Y = \sum_i Z_i/n$, *we propose an estimator*

$$\hat{t} = g(Y) = -\frac{f}{\mu q} \log\left(\frac{Y}{f}\right),$$

which has the following properties:

$$\begin{aligned} E\left[g\left(Y\right)\right] &= t + \frac{\left(e^{\rho\mu q} - \left(1 + \rho q^2\left(\mu^2 + \sigma^2\right)\right)\right)}{2n\mu q} \\ Var\left[g\left(Y\right)\right] &= \frac{t\sigma^2}{\mu^2} + \frac{f\left(e^{\rho\mu q} - \left(1 + \rho q^2\left(\mu^2 + \sigma^2\right)\right)\right)}{n\mu^2 q^2} \end{aligned}$$

Proof 5 *The expected number of empty slots* $E[Y] = fe^{-\rho q\mu} = \theta$. *Consider the Taylor series expansion of* $g(Y)$ *around* θ, *we have:*

$$g(Y) \approx g(\theta) + (Y - \theta)\, g'(\theta) + \frac{(Y - \theta)^2}{2!} g''(\theta)$$

Compute the individual terms: $g'(Y) = -\frac{f}{\mu q Y}$, $g''(Y) = \frac{f}{\mu q Y^2}$, $g(\theta) = t$, $g'(\theta) = -\frac{e^{\rho\mu q}}{\mu q}$, $g''(\theta) = \frac{e^{2\rho\mu q}}{f\mu q}$, *to yield:*

$$E[g(Y)] \approx t + \frac{Var[Y]}{2} \frac{e^{2\rho\mu q}}{f\mu q}.$$

Similarly, by considering the first two terms of the Taylor series expansion, we get:

$$Var[g(Y)] \approx Var[Y] \left(g'(\theta)\right)^2.$$

The proof can be completed by substituting back the terms listed above.

The R.H.S. of the expression of $Var[g(Y)]$ in Theorem 2 implies a lower-bound on the estimation error which cannot be reduced by increasing the number of reader polling cycles n. An increase in n can only reduce the randomness caused by the probabilistic response probability (q) and the varying radio channel conditions but not the inherent variation in the spatial distribution of the tags within the warehouse. This is because all the polling cycles within an estimation share the same realization of tag locations. For a given a load factor ρ, the optimal persistent probability q for minimizing the variance is given by:

$$q^* = \min\left(1, \frac{1.59}{\mu\rho}\right).$$

Determining the parameters for a range of tag population

Typically, the exact loading factor ρ, (i.e., the size of the tag-set) is unknown. Instead, we are given an upper bound t_u and lower bound t_l on the number of tags in the set. Based on these bounds, we want to determine the polling parameters such that the estimation accuracy requirement can always be satisfied. If we want the estimation error $\left|t - \hat{t}\right|$ to be within $\pm\beta t/2$, we need

$$Z_\alpha \sqrt{\delta_0} \leq \frac{\beta t}{2}$$

where $\delta_0 = Var\left[g\left(Y\right)\right]$ as given in Theorem 2.

Theorem 3 *For a given spatial distribution of the tags, a radio channel model (summarized by μ and σ^2 of the corresponding $f_R(\cdot)$ function), and a set of accuracy requirements (defined by α and β), there exists a lower-bound on the size of the tag-set (t_{crit}) below which the accuracy requirement of the estimator cannot be satisfied regardless of the settings of the other protocol parameters, e.g., n, f and q.*

Proof 6 *Rewriting δ_0 shown above, we have:*

$$\delta_0 = \frac{t^2}{nf}\left(\frac{e^{\mu q \rho} - 1}{\mu^2 q^2 \rho^2} - \frac{1}{\rho}\left(1 + \frac{\sigma^2}{\mu^2}\right)\right) + \frac{t\sigma^2}{\mu^2} \leq \frac{\beta^2 t^2}{4Z_\alpha^2} \tag{3.14}$$

Note that the necessary condition for satisfying the accuracy requirement is $\frac{\sigma^2}{\mu^2} < \frac{\beta^2 t}{4Z_\alpha^2}$, which does not depend on the number of reader polling cycles. This is because the estimator requires a minimum number of tags, t_{crit}, present in the tag-set to compensate, via the Law of Large Number, the variation caused by different realizations of tag locations within the warehouse in order to achieve the required estimation accuracy:

$$t > \frac{4Z_\alpha^2 \sigma^2}{\beta^2 \mu^2} = t_{crit}.$$

For any $t > t_{crit}$, the number of polling cycles required is given by:

$$n \geq \frac{\left(\frac{e^{\mu q \rho}-1}{\mu^2 q^2 \rho^2} - \frac{1}{\rho}\left(1 + \frac{\sigma^2}{\mu^2}\right)\right)}{\frac{f\beta^2}{4Z_\alpha^2} - \frac{1}{\rho}\frac{\sigma^2}{\mu^2}}. \tag{3.15}$$

In Fig. 3.1, we plot the number of reader polling cycles required for $\mu = 0.8820$ and $\sigma^2 = 0.0089$, with $f = 165$, $q = 0.65408$, $n = 41$ and estimation accuracy requirement of $\alpha = 0.99$ and $\beta = 0.1$. The range t is varied from 50 to 1200 with $t_{crit} = 31$. Under the 4000 slots per second assumption, the estimation takes around 1.7 seconds. Observe that as the number of tags approaches t_{crit} from above, the number of reader polling cycles approaches infinity.

As long as the number of tags is larger than t_{crit}, the accuracy requirement (α and β) can always be satisfied. However, when the number of tags approaches t_{crit}, the variance due to the unknown realization of the tag position will also become larger (since there are less tags), and there is only a small allowance for the variance of protocol randomness. Toward this end, as the number of tags approaches to t_{crit}, the variance allowance for protocol randomness will approaches to 0. This implies the number of polling cycles/latency will become larger and larger in order to reduce the randomness in the protocol (since we are using probabilistic counting).

To support a tag-set of size $t \in [t_l, t_u] \Leftrightarrow \rho \in [\rho_l = \frac{t_l}{f}, \rho_u = \frac{t_u}{f}]$ with minimum response latency while meeting estimation error requirement, we can solve the following optimization problem:

$$\begin{aligned} &\text{minimize} \quad n \cdot f \\ &\text{s.t.} \quad n \geq \max_{\rho_l \leq \rho \leq \rho_u} \frac{\left(\frac{e^{\mu q \rho}-1}{\mu^2 q^2 \rho^2} - \frac{1}{\rho}\left(1 + \frac{\sigma^2}{\mu^2}\right)\right)}{\frac{f\beta^2}{4Z_\alpha^2} - \frac{1}{\rho}\frac{\sigma^2}{\mu^2}} \\ &\qquad 0 < q < 1, \quad f > 0 \end{aligned} \tag{3.16}$$

Since the R.H.S. of (3.16) reaches its maximum when either $\rho = \rho_u$ or $\rho = \rho_l$, we set

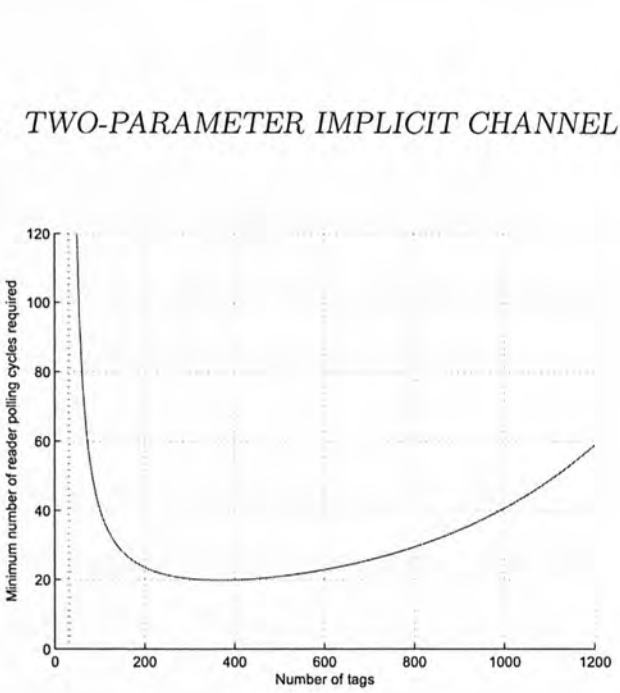


Fig 3.1: Number of polling cycles vs tag-set size with optimal parameters for range [100, 1000] ($f = 165$, $q = 0.65408$, $n = 41$). The number of slots involved is 6765 slots. Suppose there are 4000 slots in one second, the latency involved would be 1.7 seconds.

$$\frac{\left(\frac{e^{\mu q \rho_l}-1}{\mu^2 q^2 \rho_l^2} - \frac{1}{\rho_l}\left(1+\frac{\sigma^2}{\mu^2}\right)\right)}{\frac{f\beta^2}{4Z_\alpha^2} - \frac{1}{\rho_l}\frac{\sigma^2}{\mu^2}} = \frac{\left(\frac{e^{\mu q \rho_u}-1}{\mu^2 q^2 \rho_u^2} - \frac{1}{\rho_u}\left(1+\frac{\sigma^2}{\mu^2}\right)\right)}{\frac{f\beta^2}{4Z_\alpha^2} - \frac{1}{\rho_u}\frac{\sigma^2}{\mu^2}}. \tag{3.17}$$

Define $s = t_l/t_u$ and treat (3.13) as an equality and substitute it into (3.17), we have:

$$\frac{\frac{\left(\frac{f}{5}\right)^s-1}{\left(s\ln\frac{f}{5}\right)^2} - \frac{1}{s\rho_u}\left(1+\frac{\sigma^2}{\mu^2}\right)}{\frac{f\beta^2}{4Z_\alpha^2} - \frac{1}{s\rho_u}\frac{\sigma^2}{\mu^2}} = \frac{\frac{\frac{f}{5}-1}{\left(\ln\frac{f}{5}\right)^2} - \frac{1}{\rho_u}\left(1+\frac{\sigma^2}{\mu^2}\right)}{\frac{f\beta^2}{4Z_\alpha^2} - \frac{1}{\rho_u}\frac{\sigma^2}{\mu^2}} \tag{3.18}$$

Upon solving (3.18) numerically for f, the optimal persistent probability q can be obtained from (3.13). The number of reader polling cycles required can be found by using (3.15). The minimum response latency can then be calculated. However, solving

(3.13) may give $q > 1$. In such cases, we put $q = 1$ and solve (3.17) and (3.15) for f and n respectively. This will give the polling parameters for our estimator with an operating range of $t \in [t_l, t_u]$ and a latency of $n \cdot f$.

The latency for estimating the tag-set cardinality can be further reduced by applying the sub-ranging techniques proposed in Section VB of [2]. Since Theorem 3 suggests that when t_l is close to t_{crit}, the number of polling cycles required approaches infinity, this motivates us to minimize the latency when $t = t_l$. As a larger value of q will reduce the required number of polling cycles for a given accuracy requirement, we set $q = 1$ and find the optimal value of f to minimize the estimation latency when $t = t_l$:

$$f = \arg(\min_f \frac{e^{\mu\rho_l} - 1}{\mu^2 \rho_l^2} - \frac{1}{\rho_l}\left(1 + \frac{\sigma^2}{\mu^2}\right))$$

The maximum tag-set cardinality supported by the operating range, with $q = 1$ together with the f shown above, is given by (3.12):

$$t_{u'} \leq \frac{\ln \frac{f}{5}}{\mu} f$$

This gives the polling parameters for the sub-range $[t_l, t_{u'}]$. When $t_{u'} < t_u$, an additional sub-range will be required to guarantee the accuracy requirement of the estimator over the entire range of t. In such cases, replace t_l by $t_{u'}$ and repeat the procedure described above to determine the parameters for the next sub-range. Further sub-range optimization can be performed using techniques described in [2]. However, the overall estimation latency is typically dominated by the polling operations for the lowest sub-range, i.e., when t is close to t_{crit}.

3.4 Results and Discussions

Before presenting the results, we introduce the channel model used in the simulation. Here, we assume the channel to be dependent only on the distance, d, between the reader and the tag. We derive the probability for receiving a tag response as a function of d under different channel fading models. Denote by $C_{P_{in},d}(\cdot)$ the p.d.f. of the received signal power at distance d from a transmitter transmitting at a power of P_{in}. In general, the distribution $C_{P_{in},d}(\cdot)$ depends on the actual channel model. In our simulation, we consider the Lognormal and Rayleigh fading [40] channels. Both the forward and reverse link effects are accounted for. A tag response is observed by the reader only if (1) the tag can receive sufficient power from the reader probe, and subsequently, (2) the reader can receive sufficient power to detect the tag response. The probability that the reader will receive a response from a tag at a distance

d is given by:

$$\begin{aligned}&Prob\,(\text{tag at distance } d \text{ will response to the reader})\\&= \int_{P_t}^{\infty} C_{P_r,d}\,(w_s) \cdot \int_{S_r}^{\infty} C_{E_t \cdot w_s,d}\,(w_r)\, dw_r dw_s,\end{aligned} \tag{3.19}$$

where P_r is the reader transmitting power, P_t is the minimum power required to power up a tag, S_r is the receiving sensitivity of the reader, $G_r(G_t)$ is the reader(tag) antenna gain and E_t is the backscatter efficiency. The outer integral specifies the condition that enough power is received by the tag during the forward link transmission while the inner integral corresponds to the power requirement for successful detection for the reverse link transmission.

In Fig. 3.2, we plot the tag responding probability against distance between the reader and a tag for the Lognormal and Rayleigh fading channels with path loss exponent $r = 3$ and 4. We assume $P_r = 36dBm$, $P_t = -10dBm$, $S_r = -80dBm$, $G_r = 6dBi$, $G_t = 1dBi$, $E_t = -20dB$.

Here, we validate the estimation accuracy guarantees derived in Section 3.3 by simulation: A reader is placed at the center of a $5m \times 5m$ room and each tag is randomly placed in the room according to the i.i.d. uniform distribution. We compare the Lognormal and Rayleigh channel fading models under different path loss exponent $r = 3$, 3.5 and 4. Assume the accuracy requirement

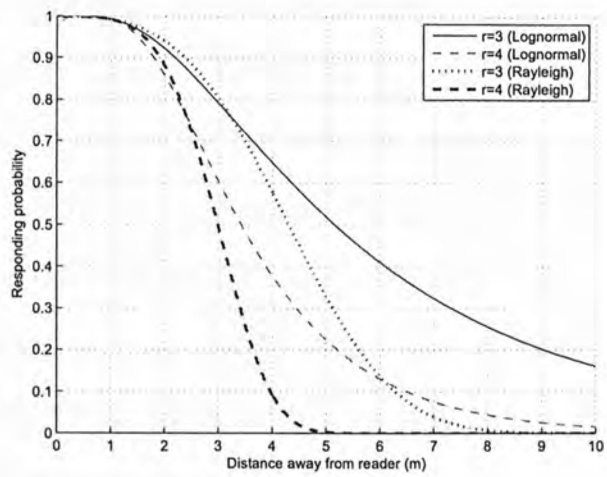


Fig 3.2: Tag responding probability against distance

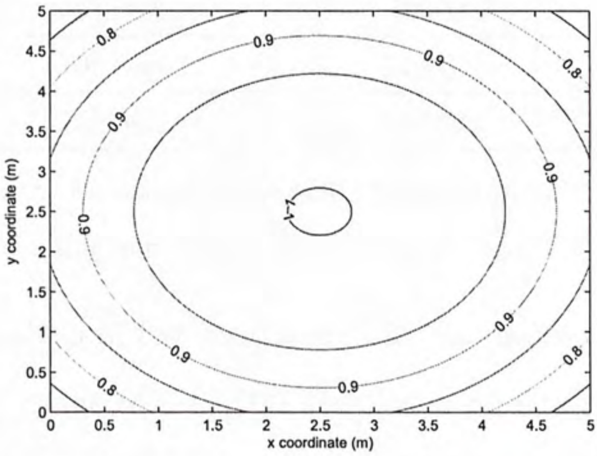
is $\alpha = 0.99, \beta = 0.1$, i.e., to estimate the number of tags within $\pm 5\%$ of its actual value for 99% of the time. As discussed in Theorem 3, there exists a minimum tag requirement t_{crit} in order to satisfy the given level of estimation accuracy. As shown in Table 3.2, t_{crit} generally increases with the path loss exponent r.

We set the operating range of the estimator to be $10^2 \leq t \leq 10^5$ and compute the corresponding polling latency in order to achieve the stated accuracy requirement. Table 3.2 also compares the results when sub-ranging technique is applied. The number in the bracket corresponds to the number of sub-ranges used (m). As shown in Fig. 3.2, the Rayleigh fading channel drops off faster than the Lognormal one for $r = 4$. As such, more time slots will be needed for the former. Notice that the overall latency increases dramatically especially when the lower-end of the operating range is close to t_{crit} (e.g., for Rayleigh fading with $r = 4$). By applying the sub-ranging technique to split the operating range of the estimator into multiple smaller sub-ranges, the overall latency can be reduced by at least 75% for all cases. However, even with sub-ranging, the latency for the Rayleigh fading model with $r = 4$ is still extremely large when compared with other scenarios where t_{crit} is far smaller than the lower-end of the operating range. In particular, 92% of the latency is due to the polling for the lowest sub-range. In Table 3.3, we plot the minimum latency for the conventional MAC protocol for different tag-set size between 10^2 to 10^5. We assume prefect channel, prefect scheduling, prefect information of tag size in computing the latency. To estimate a tag-set with 10^5 tags, conventional MAC protocol takes at least 425 seconds, compared with 6 seconds in our proposed scheme (under the unreliable channel model we considered).

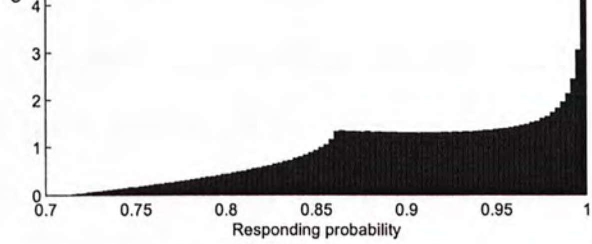
Next, we examine the simulation results for the case of a Log-normal shadowing channel with $r = 3.5$, $\sigma = 8.7dB$. The level set of the responding probability distribution of the room $(\mathcal{P}(x, y))$ and the corresponding histogram are depicted in Fig. 3.3(a) and (b) respectively. In Fig. 3.4, we show the accuracy of the proposed estimator against a given tag-set of size = 500 under dif-

path loss exponent r	3	3.5	4
Channel	Lognormal ($\sigma = 8.7dB$)		
μ	0.9149	0.8820	0.8426
σ^2	0.0043	0.0089	0.0164
t_{crit}	13.77	30.47	61.24
without sub-ranging (slots)	101290	105110	126660
latency (s)	25.3	26.3	31.7
with m sub-ranges (slots)	22785 ($m = 3$)	23444 ($m = 3$)	24426 ($m = 3$)
latency (s)	5.7	5.9	6.1
Channel	Rayleigh		
μ	0.9275	0.8893	0.8350
σ^2	0.0038	0.0106	0.0260
t_{crit}	11.73	35.64	99.07
without sub-ranging (slots)	100830	106340	3289392
latency (s)	25.2	26.6	822.3
with m sub-ranges (slots)	22844 ($m = 3$)	23726 ($m = 3$)	256571 ($m = 4$)
latency (s)	5.7	5.9	64.1

Table 3.2: t_{crit} and latency for operating range from 10^2 tags to 10^5 tags, with accuracy requirement $\alpha = 0.99$, $\beta = 0.1$ under different channel model, $P_r = 36dBm$, $P_t = -10dBm$, $S_r = -80dBm$, $G_r = 6dBi$, $G_t = 1dBi$, $E_t = -20dB$



(a) Visualization of the response probability distribution



(b) aggregated response probability over the readers

Fig 3.3: Testing Scenario: Reader at position $(2.5, 2.5)$, channel: Lognormal with $r = 3.5$, $\sigma = 8.7dB$

Conventional MAC protocol	latency (slots)	latency (s)
10^2 tags	700	0.2
10^3 tags	10000	2.5
10^4 tags	140000	35
10^5 tags	1700000	425

Table 3.3: Latency for conventional MAC protocol (Assuming prefect channel, prefect scheduling and prefect information of tag-set size)

ferent realizations of tag locations. The estimator is designed for an operating range of $t \in [100, 1000]$ and no sub-ranging is performed in this case. The optimal set of parameters are computed to be: $n = 41$, $q = 0.6541$ and $f = 165$, which takes around 1.7 seconds. The solid curve shows the analytical distribution of $\hat{t}$ predicted by Theorem 2 and the circles represent the simulation results. The outer dotted curve corresponds to the desired level of estimation accuracy for $\alpha = 0.99$ and $\beta = 0.1$. The best achievable accuracy guarantee (by polling infinite number of cycles) is shown as the inner dotted curve. Notice that, with $t = 500$, we only require $n = 22$ polling cycles (or equivalently 0.9 second) to achieve the required level of accuracy according to Fig. 3.1. On the other hand, 41 polling cycles (or equivalently 1.7 second) are required for $t = 100$ or 1000.

In Fig. 3.5, we show the estimation results as the size of the tag-set varies from 50 to 1300 while the operating range of the

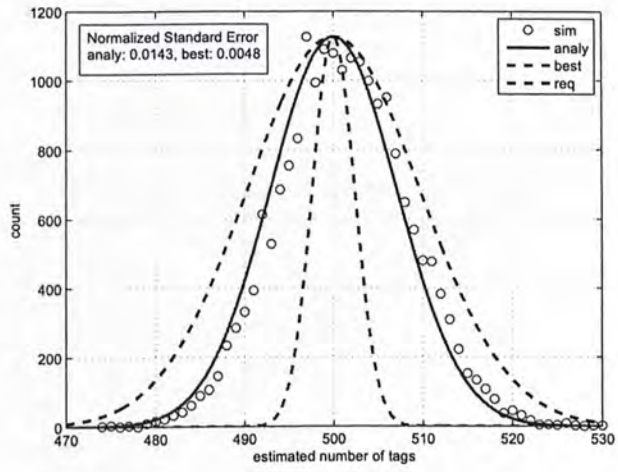
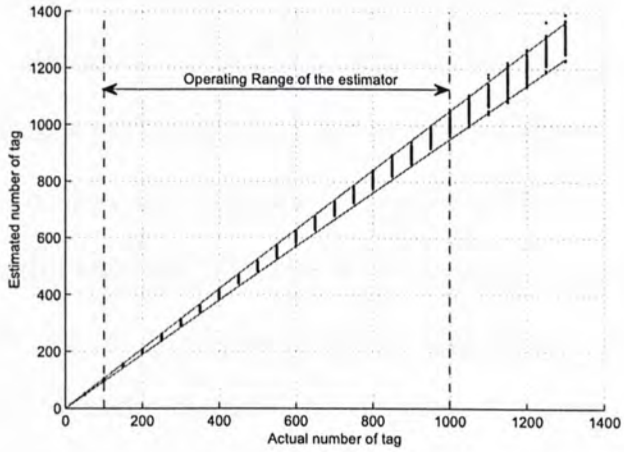
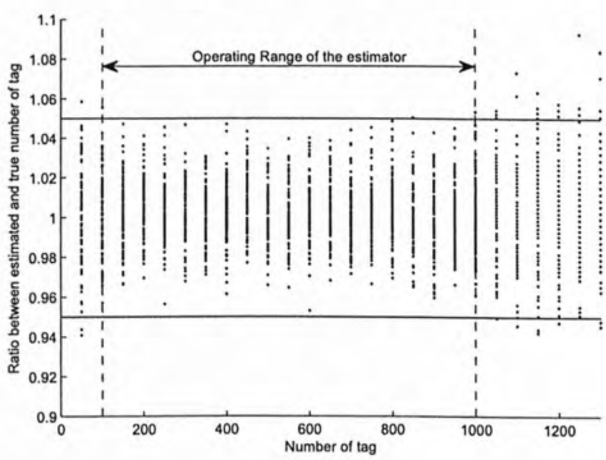


Fig 3.4: Simulation result of 10000 trials of 500 tags distributed uniformly under the map of Fig. 3.3

estimator is designed to be $t \in [100, 1000]$. Fig. 3.5a plots the estimate value $\hat{t}$ against the actual value t while Fig. 3.5b depicts the ratio $\hat{t}/t$ against t. In both cases, the pair of solid lines in the graph correspond to the $\pm 5\%$ target error bounds. According to our design, on average, only 1 out of the 100 simulation experiments will have an estimate lying outside the error bounds. This is in line with the observation from Fig. 3.5a and b. Observe from Fig. 3.5b that when t falls out of the designed operating range of the estimator (i.e., when $t < 100$ or $t > 1000$), the number of experiments which cannot produce an estimate meeting the accuracy requirements (manifested in form of dots lying outside the error-bound lines) increases only gradually.



(a) actual number of tags vs estimated number of tags



(b) ratio between estimated and actual number of tags

Fig 3.5: Simulation result showing 99% confident within $\pm 5\%$ interval for the estimated number of tag with size [100, 1000], the range predicted by the estimator. Latency: 1.7 seconds.

3.5 Chapter Summary

In this chapter, we extend the algorithms in [2] by taking into account the effects of radio channel unreliability. Unlike the perfect channel case where one can indefinitely reduce the estimation error by increasing the number of reader polling cycles, with an unreliable radio channel, there is a lower-bound on the estimation error due to the inherent variation in the spatial distribution of the tags and the radio channel conditions. We have demonstrated the efficacy of our analytical results and their corresponding guarantees in estimation accuracy via an simulation study.

□ **End of chapter.**

Chapter 4

RFID Tag-set Cardinality estimation over Unknown Channel

In Chapter 3, the RFID counting algorithms in [2] are extended to account for the effects of radio channel unreliability. However, the work there implicitly assumes the knowledge of the channel by requiring the first two moments of the tag responding probability distribution to be known. As such, the focus has been on the effects of radio channel variations on the latency and accuracies of RFID counting algorithms. In this chapter, we propose a set of new algorithms to count RFID tags over unknown, lossy wireless channels. In particular, the proposed schemes can provide good estimates of RFID Tag-set cardinality while assuming no prior knowledge of channel parameters. This makes this work applicable in a wide variety of situations where the reader is unaware of channel conditions and also cannot afford to perform explicit channel estimation. We will demonstrate the efficacy and accuracy of the proposed schemes via extensive simulation studies.

4.1 System Model

Consider a reader which broadcasts a probing request to t RFID tags located within a region of interest. The tags send a response of fixed length back to the reader. The probability that a tag i can successfully[1] communicate with the reader is p_i. In general, p_i depends on the reader transmission power, antenna gain, minimum power requirement (sensitivity) of the tag and reader, backscattering efficiency as well as the radio channel and fading model parameters. Unlike the previous chapter (and [20]), we hereby assume p_i's of each tag to be sampled from a common *unknown* probability distribution. We are interested in obtaining good estimates for the value of t to while accounting for the non-deterministic, unknown nature of radio channels between the reader and the tags.

We assume that the RFID system follows the same *q-persistent*

[1]The success probability decreases with the length of the message.

framed-slotted ALOHA model introduced in Chapter 3: In each probe, the reader will broadcast the frame size f, persistent probability q and a random seed. Since tags select the slot in a frame based on a hashing scheme initialized by the random seed, we need multiple random seeds to reduce the variability due to the hashing scheme. Denote by m the total number of random seeds required for the probes.

Depending on the forward (reader-to-tag) radio channel condition, some tags may not receive sufficient power to process the probing request. A tag receiving sufficient power will randomly select an integer within the range of $[1, f]$ and respond in that corresponding slot with probability q. Depending on the reverse (tag-to-reader) radio channel condition, the reader may not be able to detect the reply from a responding tag. The reader detects whether a slot is empty or not (indicated by SNR levels). The reader will count the number of empty slots, i.e., the slots in which no response is observed.

The reader cannot silence an individual tag based on the status of its response, as is common with passive tags that do not possess any active memory or power source. In order to account for the unreliable channel, the reader sends out multiple (r) probes with the same seed. Thus, the total number of probes performed (as well as the number of frames received) by the reader is $m \times r$. Our goal is to estimate the tag-set cardinality $\hat{t}$ with the fewest possible number of probes such that the following constraint on the accuracy of $\hat{t}$ is met:

$$Prob\left[\frac{\hat{t}}{t} \in \left(1-\frac{\beta}{2}, 1+\frac{\beta}{2}\right)\right] \geq 1-\alpha,$$

where $\beta > 0$ specifies the error bound and $0 < \alpha < 1$ gives the failure probability.

Similar to the arguments in [1], since tags cannot be silenced individually, if we let tags transmit their identifiers, then any scheme that attempts to identify individual tags will take a large amount of time to complete. Thus, we are interested in a simpler counting algorithm that uses only a few bits ($\in \{1, 10\}$) bits as response from each tag without individually identifying them.

4.2 Baseline: The Union-based approach

Before we study the capture-recapture-based approach, we first consider a simple baseline solution for the problem under consideration. We analyze its bounds for comparison with the capture-recapture-based estimators.

4.2.1 Motivation

A tag i with $p_i < 1$ cannot guarantee that its response to a reader probe will reach the reader. In fact, the reader should probe at least $\frac{1}{p_i}$ times in order to achieve in the average case, one response per tag, can be registered at the reader. Assume that $p_{min} \leq p_i, \forall i$. If we assume that each tag picks a distinct slot in a frame without collisions, it is safe to say that we need at least $1/p_{min}$ probes to count all the tags on average. In order to get much stronger performance guarantees while considering the presence of collisions between tag responses, we extend the slot-based estimator proposed in [2], the details of which are described next.

4.2.2 Union Algorithm

In the original slotted-ALOHA model [2], for each probe, the reader will announce a seed and the persistent probability q to the tags such that the tags can pick one slot out of the f slots and respond to the reader with probability q. The reader will then observe the number of empty slots (slots that no tags picked) and hence estimate the number of tags. The presence of channel loss/ fading will make some of tags fail to register their response to the reader and hence resulting in observing more empty slots (which are marked as zeros) than it should be. To provide a reliable estimate, we will need to distinguish slots that no tags picked versus slots that seems to be empty simply because tags responses in those slots cannot be observed by the reader due to channel loss. A simple approach is to repeat the probe with the same seed multiple times and take the slot-wise logical OR operation across those frame responses based on the empty slots. As long as the tag response is recorded by the reader in one of the probes of the same seed, we will be able to take the tag into account via this so-called *Union-based approach*. By repeating the probes multiple times, the probability of failing to detect a presented tag will be reduced exponentially.

We define the set of r probes as a single *experiment*. The outcome of each experiment is an estimate of the actual number of empty slots in a given frame (seed), which can be used to estimate the number of tags. In order to reduce the variance of the estimate, we need to perform multiple experiments using different seeds to realize different tag-to-slot mappings. We denote the number of experiments (and thus seeds) by m.

4.2.3 Analysis of the Union algorithm

As described in the previous subsection, the number of probes with the same seed is given by r. There are t tags in the system.

We provide the worst-case bound for the cardinality estimation time by assuming all the tags have minimum success probability p_{min}. The probability that the response of a tag i is received by the reader after r probes is given by

$$q\phi_i \triangleq q(1-(1-p_i)^r) \geq q(1-(1-p_{min})^r)$$

Summing this over all tags i, and using the fact that $\phi_i \leq 1$, we get

$$t \geq \sum_{i=1}^{t} \phi_i \geq t\,(1-(1-p_{min})^r)$$

For accurate capture of all tags responses, we require

$$t\,(1-p_{min})^r < 1$$

Therefore, the required number of probes in each experiment is given by

$$r \geq \frac{-\log t}{\log(1-p_{min})} \approx \frac{\log t}{p_{min}} \tag{4.1}$$

Let X_j be the random variable representing the number of empty slots after aggregating the r frames of an experiment. It is easy to see that

$$E[X_j] = f\ \Pi_{i=1}^{t}(1-\frac{q\phi_i}{f}) \approx fe^{-q\rho}$$

where $\rho = \frac{\sum_i \phi_i}{f}$. We now derive a couple of results similar to those in [1] that characterize X_j and $Y = \frac{1}{m}\sum_1^m X_j$ as normal distributions.

Theorem 4.2.1 *If each of the t tags picks randomly among f slots and transmit in that same slot with probability q in each of the r probes of an experiment, then $X_j \sim \mathcal{N}\left[\mu_0, \sigma_0^2\right]$,*
where $\mu_0 = fe^{-q\rho}$, $\sigma_0^2 = fe^{-q\rho}\left(1 - \left(1 + q^2\rho\right)e^{-q\rho}\right)$ and $\rho = \frac{\sum_i \phi_i}{f}$
$\diamond$

Theorem 4.2.2 *Consider each of the t tags picks a slot randomly among f slots in the j-th experiment $j = 1, 2, \ldots, m$ and transmit in the same chosen slot with probability q in all r probes of this experiment. If*

$$Y = \sum_{j=1}^{m} X_j/m$$

then,

$$Y \sim \mathcal{N}\left[\mu_0, \sigma_0^2/m\right]$$

where μ_0 and σ_0 are given in Theorem 4.2.1. $\diamond$

The proofs of these two theorems are similar to those in [2].

The reader computes the estimate $\hat{t}$ of the tag set size, t, based on $y = \sum_j x_j/m$ as described below where y and x_j's are the realizations of Y and X_j respectively for this set of experiments. In particular, we know that the expected number of empty slots is $f\ e^{-q\rho}$, or the fraction of empty slots is $e^{-q\rho}$. Thus the reader can determine $\hat{\rho}$, an estimate of ρ, by:

$$\hat{\rho} = -\frac{1}{q}\log\left(\frac{y}{f}\right) = -\frac{1}{q}\log\left(\frac{\sum_{j=1}^{m} x_j}{m\ f}\right). \tag{4.2}$$

Since r is chosen based on Equation (4.1), $\sum_i \phi_i \approx t$, and hence, we can set $\hat{t} = f\hat{\rho}$. This is the *Union-bound estimator.* The estimation algorithm is stated in Algorithm 1.

Algorithm 1 Adaptive scheme based on union algorithm

1. Determine $r = -\frac{\log t}{\log(1-p_{min})}$.
2. Probe the tag set with same seed for r times.
3. Determine the number of empty slots for which no response was received in any of the r probes.
4. Compute the average number of empty slots over all previous experiments and use Union-bound estimator to compute tag estimate.
5. Repeat step 2-4 until the required accuracy is achieved, based on the observed variance.

Note that μ_0 is a continuous, monotonically decreasing, function of t, and hence it has an inverse function denoted by $g(\cdot)$, i.e., $g(\mu_0(t)) = t$. The mean and variance of the estimator $g(\cdot)$ can be derived along the lines of Theorem 3 in [2], and are stated as follows.

Theorem 4.2.3 *Given* $g(Y) = -f\log(Y/f)$,

$$E[g(Y)] = t + \frac{\left(e^{q\rho} - \left(1 + q^2\rho\right)\right)}{2mq^2}$$

$$Var[g(Y)] = \frac{f\left(e^{q\rho} - \left(1 + q^2\rho\right)\right)}{mq^2} = \delta_0$$

$As\ m \to \infty, E[g(Y)] \to t.$ $\diamond$

We know from the results in [2] that given a ρ, we want to select the probing probability q to minimize the variance rather than increasing m. By differentiating δ_0 with respect to q, and numerically solving the resultant expression, we can show that given any ρ, the variance of the Union-bound estimator is minimized when $q\rho = 1.59$. In other words, for minimum variance, we set

$$q^* = \min\left(1, \frac{1.59}{\rho}\right) = min\left(1, -\frac{1.59\,q}{\log(\frac{y}{f})}\right) \tag{4.3}$$

We estimate the optimal persistent probability q based on the current estimate of the number of empty slots in the frame, which in itself is a function of the number of tags. We can start with an initial value of q and as we progressively increase the number of experiments, we can refine the value of q. With each successive estimate of $\hat{t}$, we can use minimum variance combining ([2]: Theorem 4) across different $\hat{\rho}$'s (and thus $\hat{t}$) obtained using different values of q in Eq. 4.2 to produce a unified estimate until we reach our desired variance bound.

In summary, the key of the union-based approach is that by repeating the same probe (with the same seed) over and over again within an experiment, we eliminate the effects of the lossy wireless channel on the tags' responses. This helps us to obtain the final count accurately. The drawback of this baseline scheme is that the number of probes required is $O(\frac{\log t}{p_{min}})$. We now propose a set of schemes that aims to use far fewer number of probes to determine the tag set cardinality.

4.3 Probabilistic Tag-counting over Lossy, Unknown channels via the M_h model

Recall the Capture-Recapture model with heterogeneous catchability M_h introduced in Chapter 2. In this section, we will describe how to formulate a key sub-problem of the RFID counting over a lossy, unknown channel based on the M_h model and then leverage the rich estimation techniques developed by the biostatistics community to extend the probabilistic counting framework in [2] to address the uncertainties caused by the unknown, lossy wireless channel.

4.3.1 Novel Interpretation of M_h for RFID Counting over Lossy, Unknown Channels

In most facilities where RFIDs are deployed, the responding probabilities across different tags can vary widely, e.g., due to the hardware characteristics of the tag, its location and communication channel properties. As such, one may attempt to directly apply the M_h model for estimating the RFID tag population size by mapping tags as animals whose population is to be estimated using capture-recapture techniques. However, such application would require one to explicitly identify and record individual tags responded (captured) in each probing and thus substantially increase operational latency. Instead of mapping tags as animals and estimating the tag population size directly, we map slots as animals and count the number of slots occupied by the tags during each probe. By using the same seed to probe the tags, tags will always select the same slot in a frame. This tags-slot mapping is fixed for r probes when the same seed is used. Although the mapping is fixed, the frame responses for the r probes may be different due to the unreliable radio channels. Slots with tags occupied may have no tag responded for some probes, resulting in idle slots. The probability of being idle, however, depends on the actual number of tags selecting the slot and the responding probabilities of those tags, and is not the same for all slots with tags. This is an analogy to the animal capturing in the M_h model where animals have heterogeneous capture probabilities and each of the r reader probes using the same seed corresponds to a round of animals-trapping, i.e., a catch, under the capture-recapture model. The occupancy history of each slot in a frame across these multiple probes correspond to the statistics of capture history of marked animals caught across multiple trappings. We can then apply various population estimators for the M_h model to determine the actual number of occupied slots in the frame for the current tag-to-slot mapping as if the channel were perfectly reliable. In particular, most M_h model-based estimators have the "projection power" which allows us to estimate the ultimate number of slots with at least one tag under the current tag-to-slot mapping, even though some of the tags might have failed to respond in any one of the r probes due to the lossy channel. With the number of occupied slots (N) estimated, we can then determine the corresponding number of idle slots ($N_0 = f - N$) under the current tag-to-slot mapping. Once this sub-problem is solved, we can use different random seeds to realize different tag-to-slot mappings as in the case of [1, 2] and apply the one of the estimators in [1, 2] to estimate the total the number of tags.

To determine the actual number of empty slots in a frame as if the channel were perfect, we will first consider two capture-recapture estimators, namely, the Moment Estimator proposed in [21, 22] and the Sample Coverage Estimator introduced in [3]. While there are other M_h-based estimators which can serve the purpose, e.g., the u_i-based estimators in [3, 13], we select these two estimators due to their relatively low implementation complexity, near best-of-class performance, and more importantly, complementary strength under different types of distributions of the heterogeneous capture probabilities.

Both the Moment and Sample Coverage estimators rely on the "capture" frequencies of the slots, n_i as sufficient statistics for the "hidden" non-empty-slot population. In particular, n_i is the number of slots which has received one or more responses in i out of the r reader probes using the same random seed, for $i = 1, 2, \ldots, r$. Denote by S the number of slots with at least one response out of the r reading probes, i.e., $S = \sum_{i=1}^{r} n_i$. One key power of these capture-recapture-based estimators is to provide an estimate of n_0.

4.3.2 The Moment Estimator

The Moment estimator assumes that the capture probability p_i of the i-th animal is sampled from the same unknown distribution which is valid in our model where we assume individual tag response probabilities are i.i.d. according to some common, unknown distribution F_t. Because each tag selects a slot uniformly random from the f slots, if there is no hash collision, the "capture probability" distribution F, of the actually-non-empty slots in a frame should follow the same distribution of F_t. In case of hash collision(s), part of the probability mass of F_t will shift towards 1 to result in F.

Below, we summarize the derivation of the Moment estimator by following the development in [22]. Consider the expected value of n_i's given by:

$$E(n_i) = N \int_0^1 \binom{r}{i} u^i (1-u)^{r-i} \, dF(u), \quad \text{for } i = 0, 1, \ldots, r. \tag{4.4}$$

where N is the actual number of non-idle slots in the current frame given the current seed under perfect channel and F is the c.d.f. of the capture probability of the slot.

By combining the Cauchy-Schwarz inequality with Eq. 4.4 for $i = 0, 1$ and 2, we have:

$$\left[\int (1-u)^r \, dF(u)\right] \left[\int u^2 (1-u)^{r-2} \, dF(u)\right] \geq \left[\int u (1-u)^{r-1} \, dF(u)\right]^2,$$

which gives

$$E[n_0] \geq \left(\frac{r-1}{r}\right) \left\{\frac{[E^2[n_1]]}{2E[n_2]}\right\}.$$

When r is large, an estimator $\hat{N}_M$ for N is given by the following theorem:

Theorem 4.3.1 *The estimate of the moment estimator is given by:*

$$\hat{N}_M = S + n_1^2/(2n_2).$$

The variance of $\hat{N}_M$ is given by:

$$Var\left[\hat{N}_M\right] = n_2\left[0.25\left(\frac{n_1}{n_2}\right)^4 + \left(\frac{n_1}{n_2}\right)^3 + 0.5\left(\frac{n_1}{n_2}\right)^2\right].$$

Note that the estimator relies on the first two frequency counts to predict the number of uncaptured animals (n_0). It is reasonable, as pointed out in [21], that those animals with small capture probabilities will be missed or show up a few times only, and the first few frequency counts contain most information about n_0. The estimator therefore is particularly suitable for scenarios where most animals are having small capture probabilities. However, when $n_2 = 0$, the estimator will fail to produce an estimate. A modified estimator $\tilde{N}_M$ for $n_2 = 0$ is therefore proposed in [24]:

$$\tilde{N}_M = S + \frac{n_1(n_1-1)}{2}$$

where

$$Var\left[\tilde{N}_M\right] = 0.25n_1(2n_1-1)^2 + 0.5n_1(n_1-1) - \frac{0.25n_1^4}{\tilde{N}_M}.$$

4.3.3 Sample Coverage Estimator

Another capture-recapture-based population estimator for M_h relies on the idea of Sample Coverage[3]. The Sample Coverage C is defined as the proportion of the total individual capture probabilities of the captured animals. Under the equal-individual-catchability assumption, we have an estimate of the population size

$$\hat{N} = S/C. \tag{4.5}$$

The basic motivation in [14, 3] is that sample coverage usually can be well estimated in general populations even under heterogeneous capturing probabilities. By finding the discrepancy between S/C and $\hat{N}$ in (4.5) under heterogeneous scenario, we can have an estimate of the population size. Based on the derivation from [3], a sample-coverage-based estimator for the M_h model is given by the following theorem:

Theorem 4.3.2 *The estimate of the sample coverage estimator is given by:*

$$\hat{N}_{SC} = \frac{S}{C} + \frac{n_1}{C}\gamma^2,$$

where

$$\begin{aligned} C &= 1 - \frac{n_1}{\sum_{k=1}^{r} k n_k}, \\ \gamma^2 &= \max\left\{ \frac{\frac{S}{C} r \sum_{k=1}^{r} k\,(k-1)\,n_k}{(r-1)\left(\sum_{k=1}^{r} k n_k\right)^2} - 1, 0 \right\}. \end{aligned}$$

The exact computation of the variance of $\hat{N}_{SC}$ requires the knowledge of individual animal capturing probabilities, i.e., the *values of p_i's, which we do not have. With the weaker assumption that the p_i's are random variables from the same distribution F, the variance can be computed approximately via the delta method as follows:*

$$Var\left[\hat{N}_{SC}\right] = \sum_{k=1}^{r}\sum_{l=1}^{r} H_k H_l COV_{k,l}(n),$$

where H_k denote $(\partial/\partial n_k)\hat{N}_{SC}$ and $COV(n)$ is the covariance matrix of n whose (k,l)-th entry, $COV_{k,l}(n)$ is given by:

$$covariance(n_k, n_l) = \begin{cases} n_k\left[1 - n_k/\hat{N}\right] & if\, k = l, \\ -n_k n_l/\hat{N} & if\, k \neq l. \end{cases}$$

The error associated with this approximation will be quantified in the later section.

4.3.4 Estimating the overall Tag population t

After applying the M_h model to estimate the number of occupied slots N under a given random seed (which dictates the corresponding tag-to-slot mapping in the frame), we will perform m experiments, each using a different seed to estimate the overall tag population t according to the approach given in [2]. In particular, the use of m different seeds is for reducing the estimation variability due to different tag-to-slot mappings per the hashing scheme in [2].

Using one of the capture-recapture-based population estimators described in the previous subsections, we compute $\hat{N}_j$, the estimated number of occupied slots in the frame when the j-th seed is used. Represent by $\hat{X}_j = f - \hat{N}_j$ the corresponding estimate of the number of idle slots in the frame under the j-th seed. Denote by N_{0j} the true number of idle slots under the j-th seed. From [2], we know N_{0j} is a realization of a Gaussian random variable N_0 with

$$E[N_0] = fe^{-q\rho_1}$$

and

$$Var\left[N_0\right] = \delta_0 = fe^{-q\rho_1}\left(1 - \left(1 + q^2\rho_1\right)e^{-q\rho_1}\right)$$

where $\rho_1 = t/f$ is defined as the load factor.

Denote by $\theta_j = \hat{N}_j - (f - N_{0j})$ the noise introduced by either one of the capture-recapture estimators for N_{0j} as described in the previous subsection. We have

$$\hat{X}_j = N_{0j} - \theta_j.$$

Let $\delta_j = Var[\theta_j]$ be the variance of the capture-recapture estimators given in the previous subsection. Define

$$g\left(y\right) = -\frac{1}{qf}\log\left(\frac{y}{f}\right)$$

to be our estimator for the overall tag population t, where

$$y = \frac{1}{m}\sum_{j=1}^{m} \hat{X}_j$$

is the average of the number of estimated empty slots obtained from the capture-recapture estimator across m different seeds. Based on results from [2], together with the additional Gaussian assumption of θ_j, y is the realization of a Gaussian random variable Y where

$$\begin{aligned} E[Y] &= \sum \frac{E[N_{0j}] + E[\theta_j]}{m} \\ &= E[N_0] + \sum \frac{E[\theta_j]}{m} = fe^{-q\rho_1} + \sum \frac{E[\theta_j]}{m}; \\ Var[Y] &= \frac{Var[N_0]}{m} + \sum \frac{Var[\theta_j]}{m^2} = \frac{\delta_0}{m} + \frac{\sum \delta_j}{m^2}. \end{aligned}$$

Although the variance of the Moment and Sample Coverage estimators, and thus, the variance of θ_j can be approximately characterized as described in the previous subsections, these estimators are both biased in nature and no analytical characterization of their bias, i.e., $E(\theta_j)$, can be found in the literature except that extensive numerical studies, e.g., those in [3, 12, 13, 14], have demonstrated the bias to be acceptable under a wide range of capture-probability distributions and population sizes. As such, we have to ignore the bias of the estimator for now and resort to numerical studies in the later section to evaluate the impact of the bias in practice.

By ignoring the bias of the capture-recapture estimators, i.e., assuming $\sum \frac{E[\theta_j]}{m} = 0$, we apply the Taylor series expansion of $g(Y)$ around $E[Y]$ to obtain the first two moments of our estimator for t from the following theorem:

Theorem 4.3.3 *Let $g(Y) = -\frac{1}{qf}\log\left(\frac{Y}{f}\right)$ be our estimator, where $Y = \sum \frac{X_j}{n}$ to be the average number of empty slots estimated by the capture-recapture estimator. Assume that the estimators are unbiased in the previous subsections are unbiased. We have*

$$E\left[g\left(Y\right)\right] = t + \frac{Var\left[Y\right]}{2}\frac{e^{2q\rho_1}}{qf} = t + \frac{\sum_{j=1}^{m}\left(\delta_j+\delta_0\right)e^{2q\rho_1}}{2m^2qf};$$

$$Var\left[g\left(Y\right)\right] = Var\left[Y\right]\frac{e^{2q\rho_1}}{q^2} = \frac{\sum_{j=1}^{m}\left(\delta_j+\delta_0\right)e^{2q\rho_1}}{m^2q^2}.$$

The unbiased assumption does not affect the variance devised, but only the bias of the estimator.

4.4 Performance Validation and Comparison

To compare the effectiveness of the estimators proposed in Section 4.2 and 4.3, we will apply them to three capture probability distributions F of different characteristics. In particular, the capture probabilities, i.e., the p_i's of the tags will be distributed as follows: Case I: uniform between 0.1 and 1, Case II: uniform between 0.5 and 1, and Case III: mostly concentrated at 0.1. (probably the most challenging case, especially for Union-based) Case IV: F based on realistic Rayleigh fading model inside a room with path loss exponent equals to 4.

For each instance of the problem, 1000 tags are introduced, each with a fixed p_i drawn from the capture-probability distribution F from one of the above cases. The frame-size is set to 629 slots, which is optimal for the perfect channel case. 100 instances are simulated for each set of the parameters and the difference between the estimate and the actual number of tags across these 100 instances are recorded. We plot the 99%-tile worst case on the graphs.

We present the results in form of contour graphs to show the performance of each of the estimators in terms of (1) the accuracy level they claims to support (assuming zero-bias from the underlying capture-recapture estimator), and (2) the actual accuracy level they can deliver during the simulation study by comparing the difference between their estimates and the actual value of t.

For each graph, contour-lines of values 0.02, 0.05 and 0.1 are plotted. Those contours are the accuracy levels that the estimators claimed to support. For example, areas enclosed by the 0.02 contour-line corresponds to the accuracy level of $\pm 2\%$, namely the estimator will provide an estimate will differ from the actual value by at most 2%. We plot on the same graph the actual level of accuracy achieved by the estimator using different number of seeds (m) and different number of probes (r) for the same seed. The minimum number of seeds to achieve an accuracy level of $\pm 2\%$, 99% of the time under the perfect channel case is also plotted as a horizontal dotted line in each graph.

Four colors (from light to dark in black-and-white printing) are used to represent the differences between the estimate and the actual value of the tag-set size over 100 number of problem instances Yellow, Orange and Red are used to represent accuracy-level of at least 2, 5 and 10% respectively. Black is used for estimation errors of larger than 10%.

By looking at these graphs, we are able to determine how good the estimator is. For example, area enclosed by the contour 0.02 should be filled up with yellow color if the estimators are accurate. If there are a lot of orange, red, or even black color areas, that means the estimator failed to achieve what it promised.

The moment estimator shows superior performance over sample coverage for the mostly-0.1 cases (Case III) as expected; otherwise, sample coverage typically achieves better accuracy given the same latency in other distribution cases.

From the color panel, we see that the sweet zone can be reached sooner when capture-recapture estimators are used, i.e., smaller value of $m * r$ will be required to reach the accuracy requirement when compared with the union algorithm. Typically, r has to be at least 5 for those capture-recapture based estimators to function properly. The number of seeds to use (m) has to be at least the number of seeds required under prefect radio channels. With m and r satisfying the requirements, the capture-recapture based estimators can return an estimate within 5% for most of the time.

The assumption on having an unbaised estimate for the sample coverage estimator makes the accuracy requirement cannot be satisfied. This only happens for small r.

In Table 4.1, we listed the latency involved in Case IV for achieving $\pm 5\%$ using different estimator proposed. We also calculate the latency involved when conventional MAC protocol is used. We assume prefect channel, prefect scheduling and prefect information of tag-set size in the calculation of the latency of conventional MAC protocol.

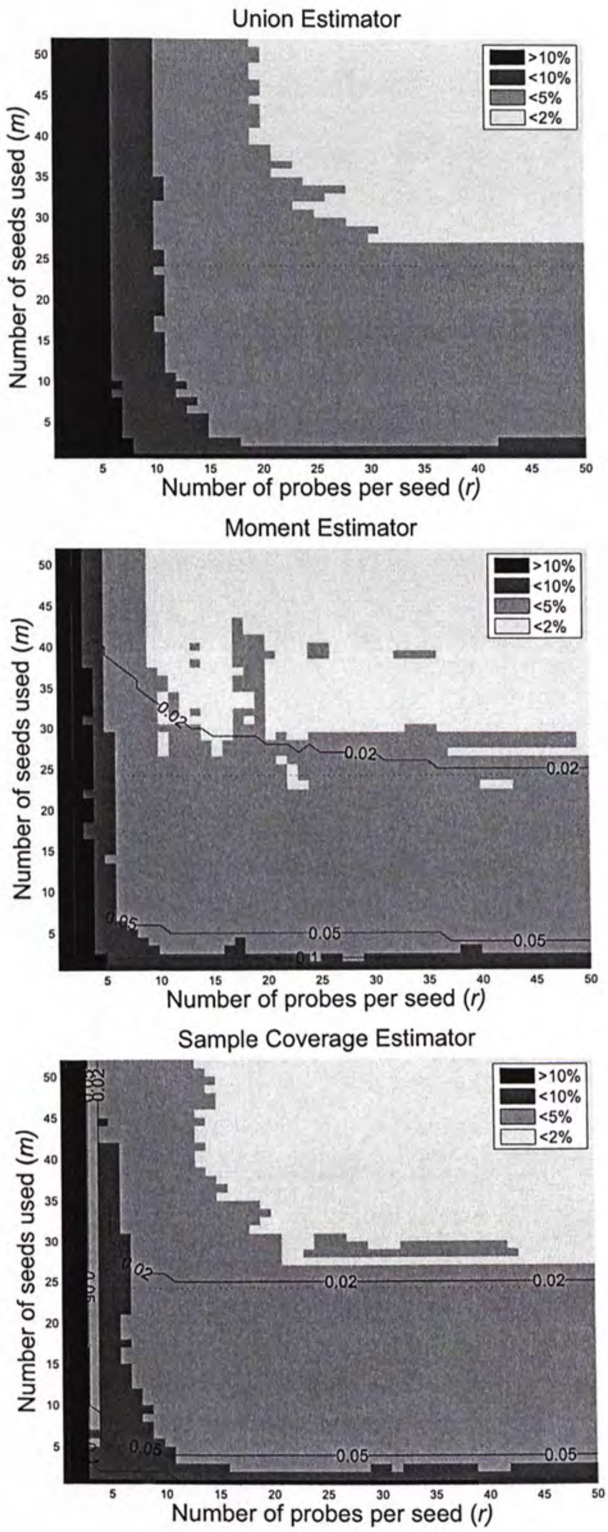


Fig 4.1: Case I: p_i's are uniformly distributed between 0.1 and 1

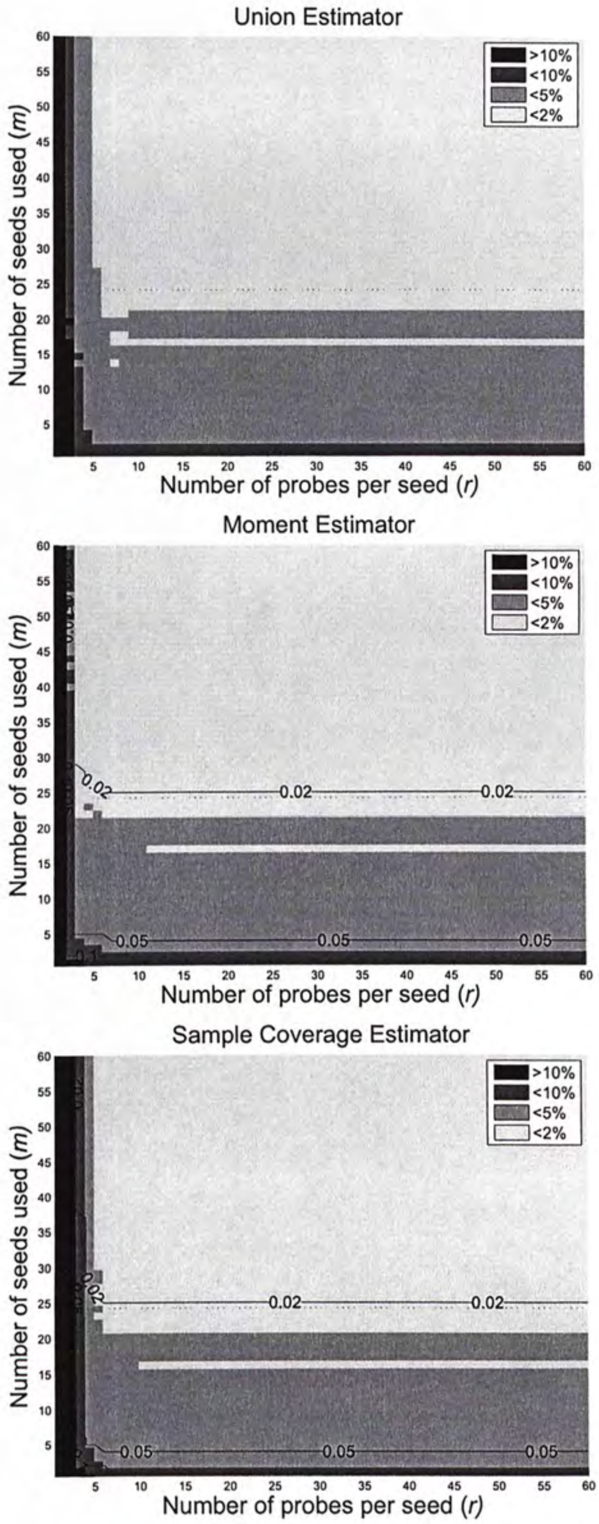


Fig 4.2: Case II: p_i's are uniformly distributed between 0.5 and 1

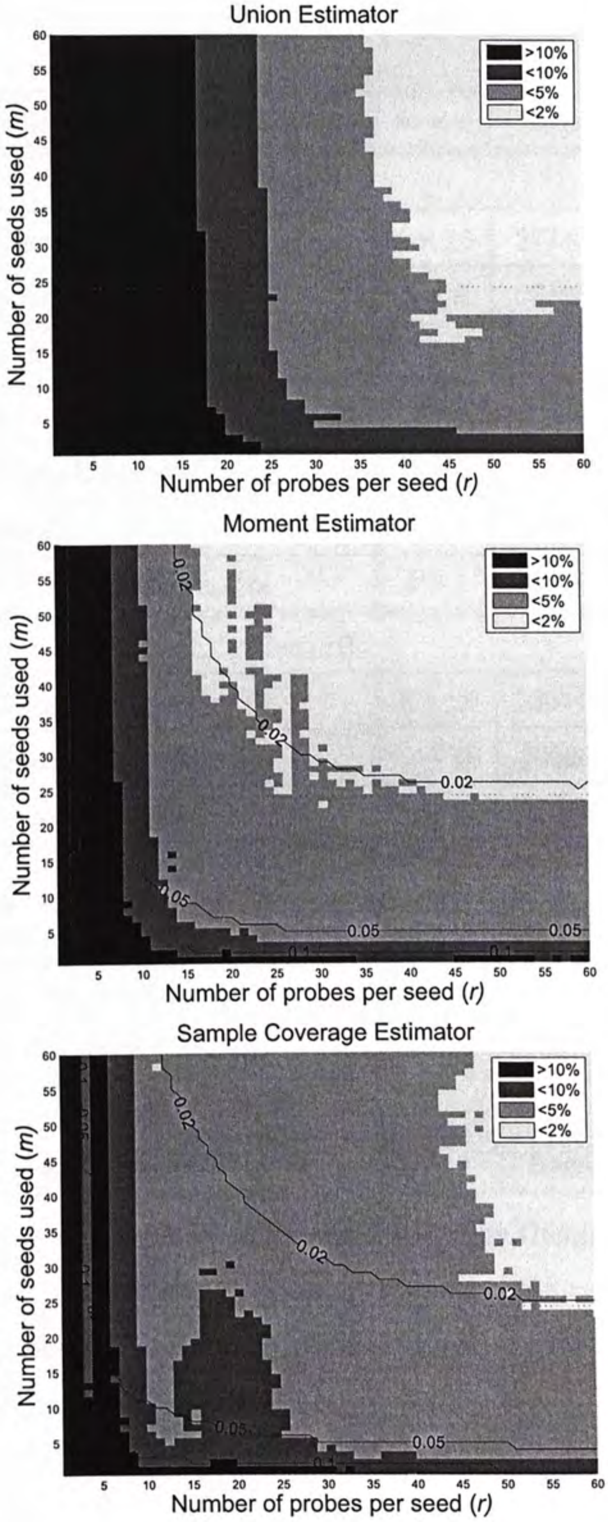


Fig 4.3: Case III: p_i's are concentrated at 0.1

Latency for achieving ±5% for 1000 tags	$m \times r$	Slots	Latency (s)
Case I			
Union Estimator	4×15	37740	9.4
Moment Estimator	5×9	28305	7.1
Sample Coverage Estimator	4×11	27676	6.9
Case II			
Union Estimator	3×5	9435	2.4
Moment Estimator	4×4	10064	2.5
Sample Coverage Estimator	3×5	9435	2.4
Case III			
Union Estimator	6×29	109446	27.4
Moment Estimator	5×16	50320	12.6
Sample Coverage Estimator	6×25	94350	23.6
Case IV			
Union Estimator	5×5	15725	3.9
Moment Estimator	3×3	5661	1.4
Sample Coverage Estimator	4×4	10064	2.5
Conventional MAC (with assumptions)			
Conventional MAC	/	10000	2.5

Table 4.1: Latency requried for achieving ±5% using Union Estimator, Moment Estimator, Sample Coverage Estimator. Latency for conventional MAC protocol is calculated by assuming prefect channel, prefect scheduling and prefect information of tag-set size

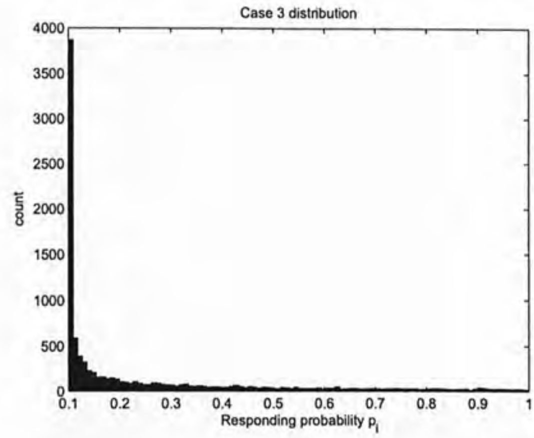


Fig 4.4: Case III distribution of the p_i

4.5 Chapter Summary

In this chapter, we have developed efficient and fast estimation schemes that can provide good estimates of the cardinality of the tag-set while assuming no prior knowledge of channel characteristics based on a novel interpretation of the capture-recapture models [3] from the field of ecology/ biostatistics. In particular, by leveraging the estimation techniques available for the capture-recapture model with heterogeneous catchability, we extend the probabilistic counting framework introduced by [1, 2] to tackle the challenge of a lossy channel with unknown characteristics. The variance of the resultant tag-set cardinality estimators are then characterized analytically. We also demonstrate the performance of the proposed schemes under various system parameters and

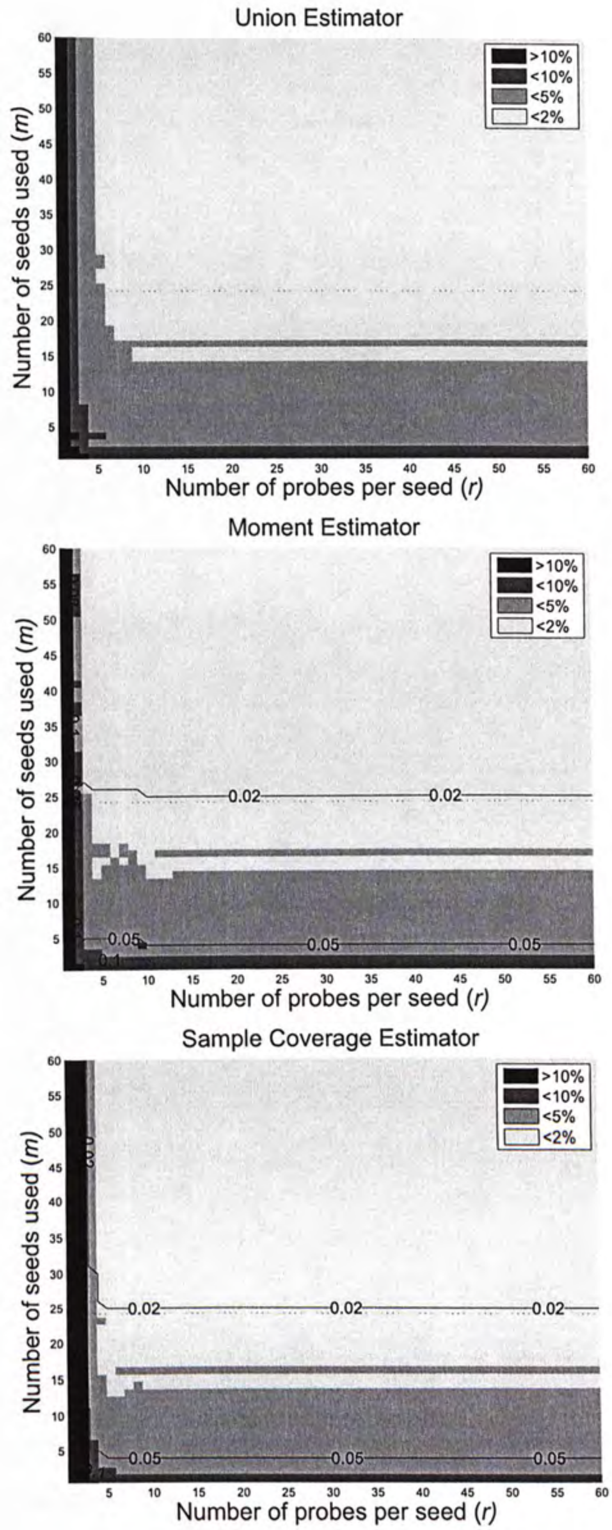


Fig 4.5: Case IV: Realistic Rayleigh Fading with path loss exponent 4, other parameters remains the same as in the previous section

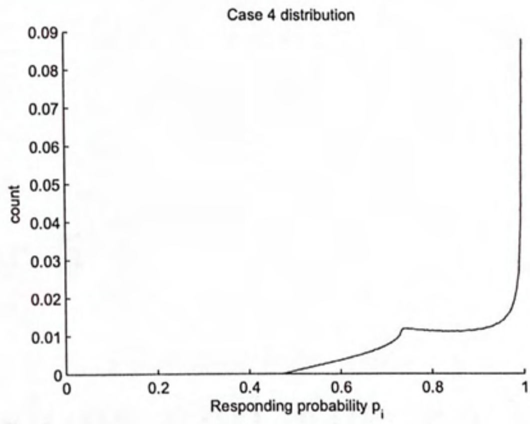


Fig 4.6: Case IV distribution of the p_i

channel conditions.

□ **End of chapter.**

Chapter 5

Conclusions and Future Work

In this thesis, we have proposed fast probabilistic RFID counting schemes which can provide accurate estimates of the tag-set cardinality even under non-deterministic, unreliable radio channels. The first approach requires the channel to have been characterized so that the first two moments of the successful response probability distribution of the tags in the tag-set are known. In the second approach, we provide fast, good estimates of the cardinality of the tag-set while assuming no prior knowledge of the parameters of the unreliable radio channel based on a novel interpretation of the capture-recapture models [3] from the field of ecology/ biostatistics. By leveraging the rich estimation techniques available for the capture-recapture models, we extend the probabilistic counting framework pioneered by [1, 2] to tackle the challenge of a lossy, unknown channel. We also demonstrate efficacy and quantify the performance of the proposed schemes via extensive simulation studies under various system parameters and channel conditions.

Our future work includes the extension of the single reader scenario into multiple readers scenario and to provide tighter analytical characterization of the performance guarantees under the unknown channel scenario. Another dimension of investigation is to extend our techniques to tackle dynamic membership changes for the tag-set being probed. The study of optimal stochastic stopping rules [23] for the reader-probing process over lossy channels given latency cost is also a problem of practical interest. Another promising direction is to leverage the species-identifying techniques [24, 25, 26, 27] from biostatistics for solving the RFID tag category identification problem [28] under lossy channels.

□ **End of chapter.**

Appendix A

Proof of Equation (3.6) in Chapter 3

Lemma A.0.1 $A\prod_{i=1}^{t}(1-\frac{2p_i}{f}) - B\prod_{i=1}^{t}(1-\frac{p_i}{f})^2 \approx (A - B - \frac{B-2A}{f^2}(\sum_{i=1}^{t} p_i^2))e^{-\frac{2}{f}t\mu}$ *for any* $A \in \mathbb{R}$, $B \in \mathbb{R}$.

Proof 7 *By expanding the exponential function, we have:*

$$
\begin{aligned}
& A\prod_{i=1}^{t}\left(1-\frac{2p_i}{f}\right)-B\prod_{i=1}^{t}\left(1-\frac{p_i}{f}\right)^2 \\
= \; & A\prod_{i=1}^{t}\left(e^{-\frac{2p_i}{f}}-\frac{4p_i^2}{2f^2}+\frac{8p_i^3}{3!f^3}-\dots\right) \\
& -B\prod_{i=1}^{t}\left(e^{-\frac{p_i}{f}}-\frac{p_i^2}{2f^2}+\frac{p_i^3}{3!f^3}-\dots\right)^2 \\
\approx \; & A\prod_{i=1}^{t}\left(e^{-\frac{2p_i}{f}}-\frac{4p_i^2}{2f^2}\right)-B\prod_{i=1}^{t}\left(e^{-\frac{p_i}{f}}-\frac{p_i^2}{2f^2}\right)^2 & \text{(A.1)} \\
\approx \; & A\prod_{i=1}^{t}\left(e^{-\frac{2p_i}{f}}-2\frac{p_i^2}{f^2}\right)-B\prod_{i=1}^{t}\left(e^{-\frac{2p_i}{f}}-e^{-\frac{p_i}{f}}\frac{p_i^2}{f^2}\right) & \text{(A.2)} \\
\approx \; & \left(A-\frac{2A}{f^2}\left(\sum_{i=1}^{t}p_i^2\right)-B+\frac{B}{f^2}\left(\sum_{i=1}^{t}p_i^2\right)\right)e^{-\frac{2}{f}\sum_{i=1}^{t}p_i} \\
& (\textit{Consider only the first two terms}) & \text{(A.3)} \\
\approx \; & \left(A-B-\frac{B-2A}{f^2}\left(\sum_{i=1}^{t}p_i^2\right)\right)e^{-\frac{2}{f}t\mu} & \text{(A.4)}
\end{aligned}
$$

A.1 is true because $\frac{p_i}{f} \ll 1$ *and hence we ignore higher order terms.*

In A.2, we ignored the term $\left(\frac{p_i^2}{2f^2}\right)^2$ *as it is much smaller then* $e^{-\frac{2p_i}{f}}$ *and* $p_i \ll f$.

In A.3, we ignored the terms with p_i *with order 4 or above as* $\left(\frac{p_i^2}{f^2}\right)^2$ *is very small because* $p_i \ll f$.

In A.4, we relies on the fact that p_i *are with mean* μ. *Hence, sum of* p_i *is approximately* $t\mu$.

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