On Optimization of the Resource Allocation in Multi-cell Networks

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In multi-cell networks where mobile stations perceive different channel gains to different base stations, it is important to associate a MS with the right BS so as to achieve a good communication quality with limited bandwidth resources. Oftentimes, the already-challenging BS association problem is further complicated by the need of transmission power control, which is an essential component to manage co-channel interference in many wireless communications systems. Despite its importance, the joint optimal BS association and power control (JBAPC) problem has remained largely open, mainly because its non-convex nature that makes the global optimal solution difficult to obtain.

In this thesis, we propose a novel algorithm, referred to as BARN, to solve the JBAPC problem efficiently and optimally, in the sense that the system revenue is maximized while the total transmission power is minimized. In particular, we propose a single-stage formulation that simultaneously captures the two objectives in discussion. Then the problem is transformed in a way that can be efficiently solved using BARN algorithm that is derived from the classical Benders Decomposition. Finally, we derive a closed-form analytical formula to characterize the effect of the termination criterion on the obtained solution and the optimal one. For practical implementation, we proposed an Accelerated BARN (A-BARN) algorithm that can significantly reduce the

computational time. By carefully choosing the termination criteria, both BARN and A-BARN are guaranteed to converge to the global optimal solution.

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I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.

CHEN Jieying

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Chapter 1

Introduction

1.1 Motivation

Wireless communication has experienced remarkable growth after more than 20 years' research and operation. Contrary to the wire counterpart, wireless systems have the unique aspect of providing ubiquitous and broadband access to the users. Meanwhile, the qualities of wireless services have also experienced explosive improvement. For example, Protocol 802.11b specified the maximum raw data rate of 54 megabits per second (Mbit/s) in 1999, while 802.11n, released in 2007, increases the maximum raw data rate up to 600 Mbit/s [1]. Consequentially, wireless networks are expected to offer a comprehensive solution to fulfill the ever-increasing communication demands.

However, due to the broadcast nature of wireless communication that the wireless channel behaves a random-like fashion and simultaneous transmissions in the same channel interfere each other, the network performance is severely limited. Oftentimes, the already-undesirable situation is further degraded by the limited radio resource (i.e., radio spectrum, transmission technologies,etc.) as well as the heavy user expectations for 'high-speed, high-quality' services. Accordingly, unprecedented challenges are posed to the design of future wireless systems that must cope with these deficiencies.

It is well known that fully utilizing the wireless resource, in any of the dimensions allowed by the multiple access technologies (time or frequency slots, codes, etc.), would largely improve the spectral efficiency and thus increase network capacity. At the heart of those resource utilization issues lies the need to efficiently allocate channels and power in a way that good communication quality is retained on each link at a minimum cost. In this respect, each time when a new generation of the wireless systems is proposed, some technological revolutions in resource management are made to overcome the limitations of the predecessors. In the following, a historical overview of the technological revolutions in wireless cellular systems is presented.

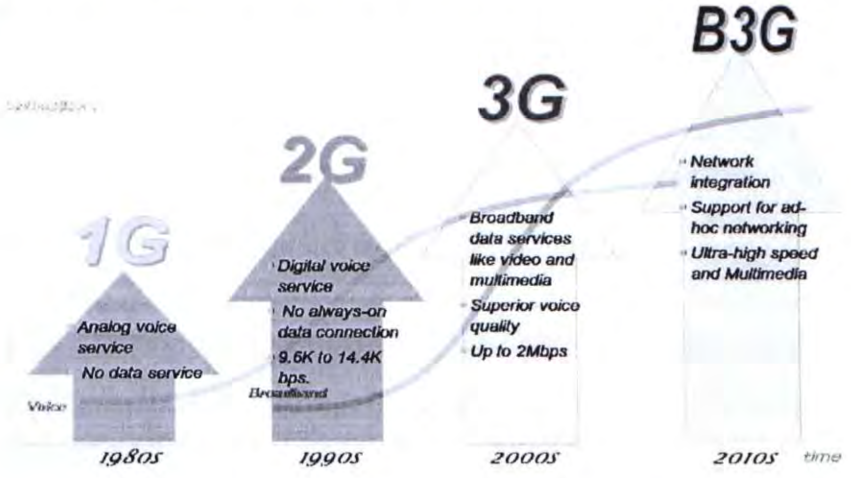

Figure 1.1: Evolution of Wireless Networks

The first-generation cellular systems ($1G$) are designed for analog voice data. The frequency-modulated (FM) analog technology is adopted, where bandwidth is divided into a series of non-overlapped frequency channels to carry analog signals. To avoid the interference caused by simultaneous transmissions in the same channel (co-channel interference), the same frequency cannot be reused among cells unless their centers are greater than certain distance, for that the signal power depletes with distance. This situation leads to the frequency assignment problem which is extensively discussed in [2].

In second-generation cellular systems (*2G*), radio signals are changed from analog to digital for encoding and decoding. There are two dominant techniques adopted in *2G*. One is termed as TDMA/FDMA (*time division multiple access/frequency division multiple access*) technology where a whole bandwidth is divided in both the frequency domain and the time domain. For example, in the standard termed as the Global System for Mobile Communication(GSM), a 25MHz bandwidth is divided into 124 channels (plus one guard channel) and each channel is divided into eight time slots. Both the channels and the time slots can be allocated to the mobile users. To efficiently allocate the time and frequency slots is one of the the key issues in GSM, as indicated in [3]. The other technique, known as *code division multiple access*(CDMA), allows the entire transmission bandwidth to be shared by all the users at all time, therefore no explicit schedule of time or frequency slots are necessary. However, power control becomes especially important for providing reliable and energy efficient communications to the mobile users. An efficient power control will limit the transmitted power on both the mobile stations' side (uplink) and the base stations' side (downlink), and hereby enable successful reception on a link while mitigating the interference to the other links. Since the power transmitted is exponential to the distance between the transmitter and receiver, the problem of selecting a proper base station location for a mobile user is also indispensable. An evaluation of problems in base station selection and power control in 2G can be found in [4].

The third-generation cellular systems (*3G*), which have been put into commercial operation recently, are able to transmit various types of data in high speeds and high qualities. The transmitted data can be voice data (the phone call) as well as the non-voice data, such as music, photographs, video, email, instant messaging, etc. Unlike 2G which is built mainly on voice data with slow transmission rate, 3G provides more communication bandwidth for each user and thus supports higher speed data services, at an extreme up to 100 times those of 2G [5]. *3G* is mainly based on the CDMA technology and companied with a variety of wireless standards including

WCDMA(*Wideband CDMA*), CDMA2000, UMTS(*Universal Mobile Telecommunications System*) and EDGE(*Enhanced Data rates for GSM Evolution*). Note that relatively limited bandwidth is available for $3G$ comparing with the heavy service demand, it is clear that efficiently utilizing the spectrum is indispensable for increasing the system capacity. As indicated in [6], network operators in $3G$ should make considerable investments in the infrastructures to reduce the reach of each base station (i.e., the distance of a base station to the mobile stations it serves) and increase the cellular density(i.e., the number of mobile stations served by a base station) in the service area.

Today the mobile community moves toward fourth-generation (also known as the beyond third-generation) systems ($4G/B3G$) . The $B3G$ systems are expected to be a complete replacement for the current networks, providing comprehensive and secure solutions where voice, data, and streamed multimedia can be given to users on an 'Anytime, Anywhere' style, with much higher data rates than previous generations [7]. Based on the orthogonal frequency division multiple access (OFDMA) technology, mobile stations in $B3G$ systems are multiplexed by frequency, and their datas are transmitted on a subset of the orthogonal sub-carriers. The design of $B3G$ systems has to take into account that the dominant load in B3G wireless channels will be high-speed burst-type and heterogenous traffics [8], and thus reformative power control strategies as well as proper base station selection schemes are needed. An generalization of the technological revolutions is shown in Figure 1.1,

In this thesis, we address the resource allocation problem in multi-cell networks. The so-called multi-cell network is a system where multiple base stations (BSs) are allowed to cooperate in terms of joint resource allocation. Herein, the resource allocation denotes for the power control and BS association, where the former is to establish spectrum-efficient connections between mobile stations (MSs) and BSs according to the location of MSs and the traffic load distribution in the geometric area, the latter is to ensure that each link transmits an appropriate amount of power to maintain its link quality without imposing excessive interference on other links. We should mention that while it is widely admitted that a joint optimization of BS association and power control(JBAPC) would offer enormous potential to improve overall system performance, it comes across extraordinary challenges that such a joint optimization problem is largely intractable due to its non-convex and combinatorial nature.

1.2 Literature Review

In the past decades, much attention has been paid on the design of spectral efficient systems through carefully allocating the system resources, such as carriers, time slots, spread codes, power, etc. Among those system resources, transmission power represents as an important tunable parameter. A recent survey [9] has generalized the main aims of power control, including: manage the interference caused by simultaneous transmissions; manage the energy consumption of the mobile stations considering their limited battery budgets, and maintain logical connectivity for the signal receivers so that they can stay connected with the signal transmitters and estimate the channel states. Initially, the work on power control concerns a single service. For example, [10] considers the voice service in a CDMA network while users have identical data rate requirement. Latter, some work focuses on multimedia services including voice, image, video, file transfer, interactive data and so on. For example, in [11], the proposed W-CDMA cellular system offers conventional power assignment mechanisms for multimedia services requiring heterogenous data rates. In [12], different types of multimedia services, such as heterogeneous requirements of the minimum transmission power, are investigated in a novel power control structure. In [13], multimedia services with various service quality requirements are concerned, while the main threads and ideas in the recent development for minimizing the power usage are elaborately discussed.

Another key degree of freedom is the interference and energy management in which BS association is the main issue. Recent studies have shown that in a multi-cell system, MSs are typically not uniformly distributed, and thus some BSs are prone to suffering from heavy load while some adjacent BSs may carry only light load or be idle [14–16]. Therefore, intelligently assigning MSs with proper BSs are crucial to overcome such an imbalanced situation. In [14] the long-time average system throughput is maximized by dynamically providing service to its MSs. In [15], the proposed BS association scheme is based on the assumption that the required data rates of all the mobile stations are identical. In [16], the BS association problem is formulated as a pricing-based non-cooperative game where no explicit communications are allowed among the BSs and each user maximizes its own utility function by selecting a strategic action according to its observation of the actions of other links.

Moreover, recent years have seen the rise of interests on the Joint problem of Bs Association and Power Control(JBAPC) [17–29]. Currently, many researches in this area are based on the concept of "divide and conquer": decouple the multi-cell BS association from the optimization of per-link transmission power (e.g., BS association assuming fixed transmit power [17,18] and power control assuming fixed BS association [19–22]) and then solve the two problems sequentially. Those researches have shown the potential of improving the overall system performance by a scheme to joint the two optimizing problems together. However, since JBAPC problem is largely intractable due to its non-convex and combinatorial nature, most schemes attempt to find an equilibrium (or stable) solution or a suboptimal solution through heuristics. In [23], the author proposes a heuristic algorithm which decomposes transmission power control, BS association, and user admission. In [24–26], joint BS association and power control is formulated as several pricing-based non-cooperative games. With well designed prices, the proposed games can converge to some Nash equilibrium points, which, however, have no guarantee to be unique or optimal. Furthermore, some researches seek for suboptimal solutions through heuristic methods. Several related algorithms on the JBAPC problem have been proposed in [30,31]. For example, in [30], the JBAPC problem is based on the condition that all the MSs in the system should get served simultaneously. Such a condition would sometimes be the key factor for a severe performance degradation. The work in [31] proposes a utility-based resource allocation algorithm to solve the JBAPC problem through elaborately designing the utility function for each individual BS.

1.3 Contributions Of This Thesis

In this thesis, we aim to maximize the overall system revenue while minimizing the total transmission power under the constraint of meeting the signal-to-interference-and-noise radio (SINR) target on each link. The challenges and contributions are listed as follows:

- BS association and power control interact with each other, and hence cannot be optimized separately. In this thesis, we transform the two problems into a single-stage problem that can simultaneously optimize the objective functions of both problems.
- The single-stage optimization problem is a mixed-integer linear program (MIP), and thus difficult to solve. In this thesis, we propose the BS Association and poweR Control (BARN) algorithm, to optimally and efficiently solve the JBAPC problem. In particular, BARN is derived from the classical Benders Decomposition method, which has nice convergence properties. By doing so, our proposed algorithm is guaranteed to converge to the global optimal solution.
- To strike a balance between computational complexity and system performance, an error tolerance is introduced to terminate the BARN algorithm before it converges to the global optimal solution. In this thesis, we obtain a closed-form expression characterizing the effect of the error tolerance on system performance. This result provides a convenient trade-off in the system design. More interestingly, our analysis shows that there exists a threshold, below which the error tolerance does not degrade the system revenue that we aim to maximize.
- To further expedite the computation of the JBAPC problem, a novel algorithm, referred to as Accelerated BARN (A-BARN), is proposed. A-BARN avoids solving

the integer optimization problem in each iteration, and thus largely reduces the computational time. By carefully choosing the termination criteria, A-BARN is guaranteed to converge to the global optimal solution. Similar to the BARN algorithm, A-BARN allows a convenient trade-off between computational complexity and the optimality of the solution.

- Throughout this thesis, we focus on the downlink communication of multi-cell networks. However, the problem formulation and analysis can be extended to the uplink case as well. The extension is discussed in the chapter 6 of the thesis.

1.4 Structure Of This Thesis

Following this introductory chapter, Chapter 2 provides the system model and transforms the JBAPC problem into a single-stage optimization problem through parametrization. Chapter 3 describes the preliminary mathematics and proposes the BARN algorithm to solve the single-stage problem, together with the analysis of the tradeoff between its performance and convergence time. In Chapter 4, the A-BARN algorithm to expedite the calculation of JBAPC problem is proposed. The performance of BARN and A-BARN is evaluated through several simulations in Chapter 5. In Chapter 6, we extend the JBAPC problem to uplink multi-cell networks and finally, we conclude this thesis in Chapter 7.

□ **End of chapter.**

Chapter 2

Problem Formulation

In this chapter, we describe the system model and the general formulations of the BS association and power control problems. Then the two problems are transformed into a single-stage optimization problem through parametrization.

2.1 The JBAPC Problem

We consider the downlink communication (i.e., transmission from BS to MS) of a multi-cell network consisting of a set $\mathcal{J} = \{1, \cdots, J\}$ of base stations (BS) and a set $\mathcal{I} = \{1, \cdots, I\}$ of mobile stations (MS). In particular, a MS is classified as being served if there exists a BS communicating with it at a satisfactory SINR level. Let x_{ij} be a binary indicator variable. x_{ij} is equal to one when MS i is served by BS j, and is equal to zero otherwise. Likewise, let p_{ij} denote the transmission power with which BS j communicates to MS i, and g_{ij} denote the channel gain between BS j and MS i. To condense the notations, we define $\boldsymbol{X} = [x_{ij}]$, $\boldsymbol{G} = [g_{ij}]$ and $\boldsymbol{P} = [p_{ij}]$ to represent the BS association matrix, the channel gain matrix, and the transmission power matrix, respectively. With the above notations, the received SINR at MS i from the BS j is given as follows

$$SINR_{ij} = \frac{\overbrace{p_{ij}x_{ij}g_{ij}}^{\text{received power}}}{\underbrace{\sum_{\forall i' \neq i}\sum_{\forall j' \neq j} p_{i'j'}x_{i'j'}g_{ij'}}_{\text{inter-cell interference}} + \zeta \underbrace{\sum_{\forall i' \neq i} p_{i'j}x_{i'j}g_{ij}}_{\text{intra-cell interference}} + \sigma_i^2}, \tag{2.1}$$

where the received power corresponds to the power received by MS i from BS j, the inter-cell interference corresponds to the received interference from all the other BSs j', $j' \neq j$, and the intra-cell interference corresponds to the received interference from the same BS j. Besides, σ_i^2 is the thermal noise at the MS i, and ζ is the orthogonality factor representing the ability of intra-cell interference cancelation at the receiver side. Without loss of generality, the value of ζ spans between [0,1]. Specifically, $\zeta = 0$ stands for the perfect intra-cell interference cancelation (i.e., no interference within the intra-cell), and $\zeta = 1$ stands for no intra-cell interference cancelation (i.e., full interference within the intra-cell).

In this paper, we aim to jointly optimize BS association and power control. In particular, our paper is to maximize the system revenue while minimizing the total transmission power under the constraint of meeting the SINR target of each MS in service. Suppose that MS i generates a 'revenue' λ_i once it gets served. Then, the system revenue is the weighted sum of the number of served MSs, i.e., $\sum_{i=1}^{I}\sum_{j=1}^{J}\lambda_i x_{ij}$. In practice, λ_i can be the data rate of MS i, and then $\sum_{i=1}^{I}\sum_{j=1}^{J}\lambda_i x_{ij}$ is the throughput of the network. Likewise, when λ_i is equal to 1 for all i, $\sum_{i=1}^{I}\sum_{j=1}^{J}\lambda_i x_{ij}$ is the total number of served MSs in the network. In general, the BS association problem can be formulated as follows.

$$[\boldsymbol{BA}]: \quad \max_{\boldsymbol{X},\boldsymbol{P}} \sum_{i=1}^{I}\sum_{j=1}^{J} \lambda_i x_{ij}$$

$$s.t.\ 0 \le \sum_{i=1}^{I} p_{ij}x_{ij} \le P_j^{\max}, \tag{2.2}$$

$$\frac{p_{ij}x_{ij}g_{ij}}{\sum\limits_{\forall i'\neq i}\sum\limits_{\forall j'\neq j} p_{i'j'}x_{i'j'}g_{ij'} + \zeta \sum\limits_{\forall i'\neq i} p_{i'j}x_{i'j}g_{ij} + \sigma_i^2} \ge \Gamma_i x_{ij}, \tag{2.3}$$

$$\sum_{j=1}^{J} x_{ij} \le 1, \tag{2.4}$$

$$x_{ij} \in \{0,1\}, \tag{2.5}$$

$$0 \le p_{ij} \le p_{ij}x_{ij}, \quad \forall i \in \mathcal{I}, \forall j \in \mathcal{J}. \tag{2.6}$$

Herein, $P_j^{\max}$ is the maximum transmission power of BS j, $\Gamma_i > 0$ is the minimum SINR requirement for MS i to be in service, and constraints (P1.4) ensures that p_{ij} is equal to 0 if MS i is not served by BS j, i.e., $x_{ij} = 0$. Through solving [BA], we can obtain the optimal BS associations, which support the maximum total number of MSs in service. We should mention that the optimal BS association may not be unique, since different association strategy may result in the same total number of served MSs. These optimal solution set of [BA] is denoted as

$$\Omega = \{(\boldsymbol{X},\boldsymbol{P}) \mid \forall(\boldsymbol{X},\boldsymbol{P}) \text{ is the solution to P1 }\}. \tag{2.7}$$

To reduce power consumption of the system, it is desirable to find one solution $(\boldsymbol{X},\boldsymbol{P})$ in Ω that consumes the minimum amount of power. The minimum transmission power needed by a given BS association can be obtained by solving the following power control problem.

$$[\boldsymbol{PC}]: \quad \min_{(\boldsymbol{X},\boldsymbol{P})\in\Omega} \sum_{i=1}^{I}\sum_{j=1}^{J} p_{ij}\bar{x}_{ij}$$

$$s.t.\ 0 \le \sum_{i=1}^{I} p_{ij}\bar{x}_{ij} \le P_j^{\max}, \tag{2.8}$$

$$\frac{p_{ij}\bar{x}_{ij}g_{ij}}{\sum_{\forall i'\neq i}\sum_{\forall j'\neq j} p_{i'j'}\bar{x}_{i'j'}g_{ij'} + \zeta \sum_{\forall i'\neq i} p_{i'j}\bar{x}_{i'j}g_{ij} + \sigma_i^2} \ge \Gamma_i \bar{x}_{ij}, \tag{2.9}$$

$$0 \le p_{ij} \le p_{ij}\bar{x}_{ij}, \quad \forall i \in \mathcal{I}, \forall j \in \mathcal{J}. \tag{2.10}$$

Once we obtain the solutions $\boldsymbol{P}$ of [PC] where $\boldsymbol{P}$ yields the minimum power consumption for each specific $\boldsymbol{X}$, it is easy to find, among all $(\boldsymbol{X}, \boldsymbol{P})$'s in Ω, the one that requires the minimum amount of transmission power. In other words, through solving [BA] and [PC] sequentially, we can simultaneously maximize the system revenue and minimize the total transmission power consumed by the MSs in service. Unfortunately, a close look at [BA] indicates that it is a mixed-integer *non-convex* problem due to the products of optimization variables $p_{ij}x_{ij}$ in the constraints. Thus, it is difficult to find the global optimal BS association even in a centralized fashion. In the next Section, we will show that problems [BA] and [PC] can be combined into a mixed integer *linear* optimization problem, which can then be solved efficiently by the BARN algorithm presented in Chapter III.

2.2 The Single-Stage Reformulation

In this section, we propose a novel methodology to combine problems [BA] and [PC] into an single-stage optimization problem. By doing so, the mixed integer *non-convex* optimization problem can be simplified as a mixed integer *linear* optimization problem, which is critical for developing the BARN algorithm in Section IV.

This single-stage optimization problem is formulated as

$$[\boldsymbol{SSP}]: \quad \min_{\boldsymbol{X},\boldsymbol{P}} \Phi(\boldsymbol{P},\boldsymbol{X}) = \epsilon \sum_{i=1}^{I}\sum_{j=1}^{J} p_{ij} + (1-\epsilon)\sum_{i=1}^{I}\sum_{j=1}^{J} w_{ij}(1-x_{ij})$$

$$s.t. \; 0 \leq \sum_{i=1}^{I} p_{ij} \leq P_j^{\max}, \tag{2.11}$$

$$\frac{p_{ij}g_{ij} + \delta^{-1}(1-x_{ij})}{\sum_{i'=1}^{I}\sum_{\forall j' \neq j} p_{i'j'}g_{ij'} + \zeta \sum_{\forall i' \neq i}\sum_{j=1}^{J} p_{i'j}g_{ij} + \sigma_i^2} \geq \Gamma_i, \tag{2.12}$$

$$\sum_{j=1}^{J} x_{ij} \leq 1, \tag{2.13}$$

$$x_{ij} \in \{0,1\}, \tag{2.14}$$

$$p_{ij} \geq 0, \forall i \in \mathcal{I}, \forall j \in \mathcal{J}. \tag{2.15}$$

Specifically, w_i is obtained by scaling up all λ_i's by the same constant ρ, so that all w_i's are positive integers, i.e.,

$$\rho \triangleq \frac{w_i}{\lambda_i} = \frac{w_{i'}}{\lambda_{i'}} \; \forall i \in \mathcal{I}. \tag{2.16}$$

This is tenable as long as λ_i's are rational numbers. Moreover, ϵ and δ are constants satisfying

$$0 \leq \epsilon < \frac{1}{\sum_{j=1}^{J} P_j^{\max}/\breve{w} + 1}; \tag{2.17}$$

and

$$0 < \delta \leq \min_i \frac{\Gamma_i^{-1}}{(I-1)\hat{p}\cdot\hat{g} + \sigma_i^2}, \tag{2.18}$$

where $\hat{p} = \max_j\{P_j^{max}\}$, $\hat{g} = \max_{ij}\{g_{ij}\}$ and $\breve{w} = \min_i\{w_i\}$. Note that there must exist a set of w_{ij}'s that satisfies (2.16) as long as λ_i's are rational numbers. Given parameters w_{ij}'s, ϵ and δ satisfying (2.16), (2.17) and (2.18), respectively, Lemma 1 shows that solving [SSP] is equivalent to solving the joint BS association problem (i.e., [BA]) and the power control problem (i.e., [PC]) simultanously.

Lemma 1. The solution to [SSP] yields the maximum system revenue and minimum total transmission power in the same time. In other words, the solution to [SSP] simultaneously optimizes problems [BA] and [PC].

The proof of Lemma 1 is deferred to the Appendix A.1.

[SSP] is a mixed-integer linear programming problem. Furthermore, it is easy to see that [SSP] reduces to [BA] when $\epsilon = 0$. This implies that even if there is no need to minimize the total power consumption, it is still desirable to reformulate the non-convex BS association problem [BA] to the linear [SSP] using the technique described above. However, it is worth nothing that [SSP] is still NP-hard due to the presence of integer variables. In the next section, we will propose an efficient algorithm, referred to as BARN, to solve [SSP] through Benders Decomposition.

Before leaving this chapter, we note that the single-stage formulation technique was also used in [32] in a different context. Unlike our work, their work solved a joint multiuser beamforming control problem.

□ **End of chapter.**

Chapter 3

The BARN Algorithm

In this chapter, we propose the BARN algorithm to efficiently solve [SSP] based on its special characteristics. The key idea of this algorithm largely comes from the Benders Decomposition method [33]. Instead of directly solving the mixed integer linear programming in [SSP], BARN tackles the problem by iteratively solving a linear optimization problem with IJ continuous variables, and an integer linear optimization problem with IJ 0/1 integer variables. By doing so, the computational complexity can be largely reduced.

The main structure of this chapter is as follows: Section 3.1 introduces the preliminary mathematics, including the theory for linear optimization (LP) and the Benders Decomposition method. The BARN algorithm is proposed in Section 3.2. The tradeoff between performance and convergence time of BARN is finally analyzed in Section 3.3.

3.1 Preliminary Mathematics

3.1.1 Duality Of The Linear Optimization Problem

Consider the following linear programming (LP) problem:

$$\begin{aligned} &\min_{\mathbf{x}} \mathbf{c}^T\mathbf{x} \\ s.t.\ &\mathbf{A}\mathbf{x} \geq \mathbf{b} \\ &\mathbf{x} \geq \mathbf{0}. \end{aligned} \tag{3.1}$$

where $\mathbf{x}$ represents the vector of variables, $\mathbf{c}$ and $\mathbf{b}$ represent the vectors of coefficients and $\mathbf{A}$ is a matrix of coefficients, $\mathbf{c}^T\mathbf{x}$ is so-called the objective function and constraints $\mathbf{A}\mathbf{x} \geq \mathbf{b}$ together with $\mathbf{x} \geq \mathbf{0}$ specify a polyhedron known as the feasible set, over which the objective function is minimized. In particular, the feasible set can be bounded, unbounded or infeasible, to represent that (3.1) is bounded (i.e., objective function value is finite), unbounded (i.e., the objective function value is infinite), or infeasible (i.e., (3.1) has no solution), respectively. Herein, each $\mathbf{x}$ that lies in the feasible set of (3.1) is called the feasible solution, and the one that generates a finite minimal objective value is termed as the optimal solution. Otherwise, any $\mathbf{x}$ outside the feasible set is called the infeasible solution.

In the theory about the LP problem lies the concepts of extreme points and extreme rays, both of which have rigorous definitions in the domain of convex optimization. Interested readers are recommended to refer to book [34] for details. Herein, we termed the extreme points as the solutions in the feasible set that generate the finite minimal (for minimum optimization problem) or maximal (for maximum optimization problem) objective value for certain objective functions, and the extreme rays are the rays on which the optimal solution goes to unboundness at the maximum gradient.

An example of extreme points and extreme rays is illustrated in Figure 3.1 under the problem structure of (3.1), where the solution $\mathbf{x}$ consists of two real variables, i.e., $\mathbf{x} = [x1, x2]$. The dashed polyhedral is the feasible region of (3.1). The blue curves represent the objective function which achieves its optimal value at the red point. This red point is an extreme point of the feasible set. The green curves represent the objective function which is unbounded, and the red ray is the corresponding extreme ray.

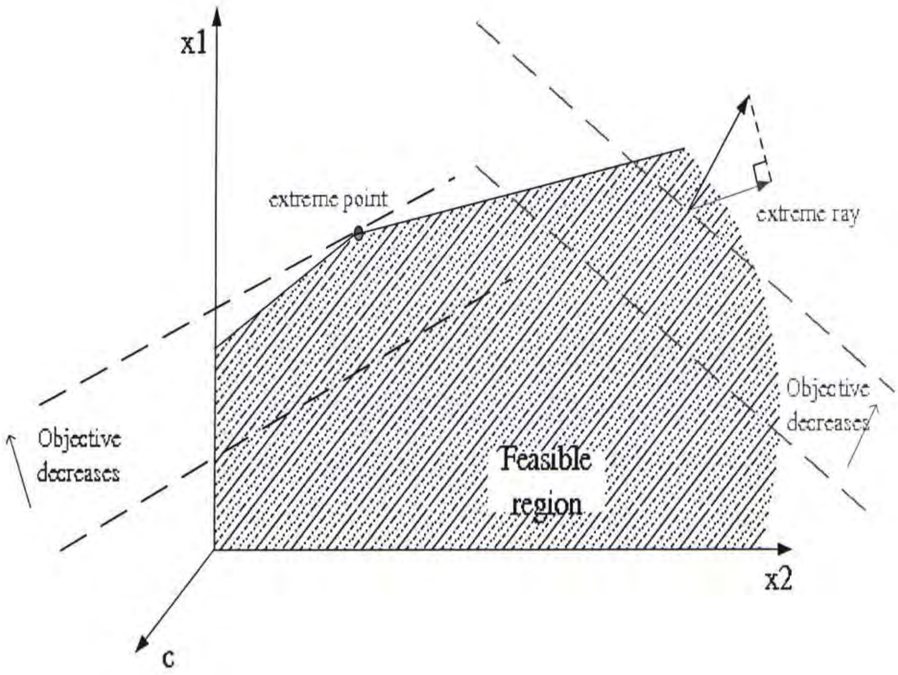

Figure 3.1: The feasible set and solution.

Having introduced the extreme points and extreme rays, now we consider the dual problem of a linear problem. Specifically, the dual problem corresponding to (3.1) is in the form

$$\begin{aligned} &\max_{\lambda} \mathbf{b}^T\lambda \\ &s.t.\ \mathbf{A^T}\lambda \leq \mathbf{c} \\ &\lambda \geq \mathbf{0}. \end{aligned} \tag{3.2}$$

where λ represents the vector of variables.

There is a fundamental duality theory [34] that the objective function value of (3.2) at any point of its feasible set is always smaller than or equal to that of (3.1). As showed in Lemma 2.

Lemma 2. Let the feasible set of problem (3.1) and (3.2) be $\mathcal{L}$ and $\mathcal{D}$ respectively. If

$\mathcal{L} \neq \emptyset$ or $\mathcal{D} \neq \emptyset$ then it follows that

$$\underset{\mathbf{x}\in\mathcal{L}}{inf}\ \mathbf{c}^T\mathbf{x} = \underset{\lambda\in\mathcal{D}}{sup}\ \mathbf{b}^T\lambda. \tag{3.3}$$

If one of the two problems is solvable, then the other is also solvable, and the strong duality holds:

$$\underset{\mathbf{x}\in\mathcal{L}}{min}\ \mathbf{c}^T\mathbf{x} = \underset{\lambda\in\mathcal{D}}{max}\ \mathbf{b}^T\lambda. \tag{3.4}$$

Corollary 1. If either Problem (3.1) or (3.2) has an unbounded objective value, then the other problem possesses no feasible solution.

3.1.2 Benders Decomposition

Benders Decomposition is one of the efficient iterative approaches for solving problems that involve a mixture of either different types of variables or different types of functions [33, 35], such as the Mixed Integer Linear programming (MIP) problems and the mixed linear/nonlinear problems. In the following section, we provide an exposition of the Benders Decomposition method.

Consider a generic MIP problem with integer variables and positive continuous variables in the following form:

$$\begin{aligned} &\min_{\mathbf{p},\mathbf{s}}\ \mathbf{c}^T\mathbf{p} + \mathbf{f}^T\mathbf{s} \\ s.t.\ &\mathbf{A}\mathbf{p} + \mathbf{B}\mathbf{s} \geq \mathbf{b} \\ &\mathbf{D}\mathbf{s} \geq \mathbf{t} \\ &\mathbf{p} \geq \mathbf{0}, \mathbf{s} \geq \mathbf{0}\ \textit{and integer}. \end{aligned} \tag{3.5}$$

where $\mathbf{p}$ represents the vector of continuous variables, $\mathbf{x}$ represents the vector of integer variables, $\mathbf{c}$, $\mathbf{f}$, $\mathbf{b}$ and $\mathbf{t}$ are vectors of coefficients and $\mathbf{A}$, $\mathbf{B}$, $\mathbf{D}$, are matrices of coefficients. $\mathbf{Ds} \geq \mathbf{t}$ represent the constraints which, if any, involve the integer vector $\mathbf{s}$ only.

Problem (3.5) can be rewritten as

$$\begin{aligned} &\min_{\mathbf{s}} \mathbf{f^T s} + \psi(\mathbf{s}) \\ s.t.\ &\mathbf{Ds} \geq \mathbf{t} \\ &\mathbf{s} \geq \mathbf{0} \textit{ and integer.} \end{aligned} \tag{3.6}$$

where $\psi(\mathbf{s})$ is

$$\begin{aligned} &\psi(\mathbf{s}) = \min_{\mathbf{p}} \mathbf{c}^T\mathbf{p} \\ s.t.\ &\mathbf{Ap} \geq \mathbf{b} - \mathbf{Bs} \\ &\mathbf{p} \geq \mathbf{0}. \end{aligned} \tag{3.7}$$

The dual of (3.7) is given as

$$\begin{aligned} &\phi(\mathbf{s}) = \max_{\lambda} (\mathbf{b} - \mathbf{Bs})^T\lambda \\ s.t.\ &\mathbf{A^T}\lambda \leq \mathbf{c} \\ &\lambda \geq \mathbf{0}. \end{aligned} \tag{3.8}$$

By Lemma 2, if problem (3.7) has the optimal solution for a particular $\mathbf{\bar{s}_1}$, then the strong duality holds, i.e.,

$$\psi(\mathbf{\bar{s}_1}) = \min_{\mathbf{p}}\{\mathbf{c^T p}|\mathbf{Ap} \geq \mathbf{b}-\mathbf{B\bar{s}_1}, \mathbf{p} \geq \mathbf{0}\} = \max_{\lambda}\{(\mathbf{b}-\mathbf{B\bar{s}_1})^T\lambda|\mathbf{A^T}\lambda \leq \mathbf{c}, \lambda \geq \mathbf{0}\} = \phi(\mathbf{\bar{s}_1}) \tag{3.9}$$

Let λ_p be the extreme point of (3.8) for this particular $\mathbf{\bar{s}_1}$, i.e., $\phi(\mathbf{\bar{s}_1}) = \max_\lambda(\mathbf{b} - \mathbf{B\bar{s}_1})^T\lambda = (\mathbf{b} - \mathbf{B\bar{s}_1})^T\lambda_p$, and let Ξ_p indicates the set of the extreme point. By substituting $\psi(\mathbf{s})$ with $\phi(\mathbf{s})$ in problem (3.6), we can get

$$\begin{aligned} &\min_{\mathbf{s}} \mathbf{f^T s} + \upsilon \\ s.t.\ &(\mathbf{b} - \mathbf{Bs})^T\lambda_p \leq \upsilon, \qquad \lambda_p \in \Xi_p \\ &\mathbf{Ds} \geq \mathbf{t} \\ &\mathbf{s} \geq \mathbf{0} \textit{ and integer.} \end{aligned} \tag{3.10}$$

On the other hand, if $(\mathbf{b}-\mathbf{B}\bar{\mathbf{s}}_2)^T\lambda_r > 0$ for a particular $\bar{\mathbf{s}}_2$, where λ_r is the extreme ray in the feasible set of (3.8), then $m\lambda_r$ is also in the feasible set for any positive scalar m. Note that (3.8) is linear in λ_r, which implies that $(\mathbf{b}-\mathbf{B}\bar{\mathbf{s}}_2)^T m\lambda_r = m(\mathbf{b}-\mathbf{B}\bar{\mathbf{s}}_2)^T\lambda_r$. Thus, $(\mathbf{b}-\mathbf{B}\bar{\mathbf{s}}_2)^T\lambda_r > 0$ implies that $\max_\lambda\ (\mathbf{b}-\mathbf{B}\bar{\mathbf{s}}_2)^T\lambda_r$ goes to infinity. Based on Corollary 1, when (3.8) is unbounded, (3.7) is infeasible. Therefore, constraint $(\mathbf{b}-\mathbf{B}\mathbf{s})^T\lambda_r \leq \mathbf{0}$ should be added to (3.6) in order to exclude this unacceptable $\bar{\mathbf{s}}_2$, i.e.,

$$\begin{aligned}
&\min_{\mathbf{s}}\ \mathbf{f^T s} \\
s.t.\ &(\mathbf{b}-\mathbf{Bs})^T\lambda_r \leq 0, \qquad \lambda_r \in \Xi_r \\
&\mathbf{Ds} \geq \mathbf{t} \\
&\mathbf{s} \geq \mathbf{0}\ \textit{and integer.}
\end{aligned} \tag{3.11}$$

where Ξ_R indicates the set of the extreme rays. The first constraint of problem (3.11) provides necessary and sufficient conditions on $\mathbf{s}$ to generate a new solution different from $\bar{\mathbf{s}}_2$.

Note that the feasible set of (3.8) is irrelevant with $\mathbf{s}$, and thus, in general, with all the extreme points and extreme rays obtained, problem (3.6) can be equivalently write as

$$\begin{aligned}
&\min_{\mathbf{s}}\ \mathbf{f^T s} + \upsilon \\
s.t.\ &(\mathbf{b}-\mathbf{Bs})^T\lambda_p \leq \upsilon, \quad \forall\lambda_p \in \Xi_P \\
&(\mathbf{b}-\mathbf{Bs})^T\lambda_r \leq 0, \quad \forall\lambda_r \in \Xi_R \\
&\mathbf{Ds} \geq \mathbf{t} \\
&\mathbf{s} \geq \mathbf{0}\ \textit{and integer.}
\end{aligned} \tag{3.12}$$

Hereafter, constraints $(\mathbf{b}-\mathbf{Bs})^T\lambda_p \leq \upsilon$ and $(\mathbf{b}-\mathbf{Bs})^T\lambda_r \leq \mathbf{0}$ are termed as the optimality cuts and the feasibility cuts, respectively.

A close look at (3.8), there are considerably large number of extreme points in Ξ^P and extreme rays in Ξ^R, which make problem (3.12) intractable. Fortunately, applying the Benders Decomposition only needs partial of the optimality cuts and feasibility cuts. In particular, Benders Decomposition adopts a strategy of *'learning from ones mistakes'*: starting with an initialization $\mathbf{s}^{(0)} = \mathbf{0}$, we check whether the subproblem (3.7) is feasible (or equivalently, whether its dual (3.8) is bounded) for $\mathbf{s}^{(0)}$. If it is, add an optimality cut $(\mathbf{b} - \mathbf{B}\mathbf{s})^T \lambda_p \leq v$ to (3.12), where λ_p is the maximizer of (3.8) for $\mathbf{s}^{(0)}$. Otherwise, if (3.7) is infeasible (or equivalently, (3.8) is unbounded), then add a feasibility cut $(\mathbf{b} - \mathbf{B}\mathbf{s})^T \lambda_r \leq \mathbf{0}$, where λ_r is a point on the extreme ray for $\mathbf{s}^{(0)}$. With the newly added constraints, (3.12) is solved to obtain an updated $\mathbf{s}^{(0)} = \mathbf{s}^{(1)}$. The iteration continuous until certain stopping criterion is satisfied.

3.2 Solving The JBAPC Problem Using BARN Algorithm

The single-stage optimization problem [SSP] falls within the generic structure of the MIP problem(3.6), and thus can be exploited by Benders Decomposition method.

For a fixed matrix $\mathbf{X}$, [SSP] reduces to

$$
\begin{aligned}
[PSP] \qquad & \vartheta(\boldsymbol{X}) = \min_{\boldsymbol{P}} \sum_{i=1}^{I} \sum_{j=1}^{J} p_{ij} \\
& s.t.\ 0 \leq \sum_{i=1}^{I} p_{ij} \leq P_j^{\max}, \\
& \frac{p_{ij} g_{ij} + \delta^{-1}(1 - x_{ij})}{\sum_{i'=1}^{I} \sum_{\forall j' \neq j} p_{i'j'} g_{ij'} + \zeta \sum_{\forall i' \neq i} \sum_{j=1}^{J} p_{i'j} g_{ij} + \sigma_i^2} \geq \Gamma_i, \\
& p_{ij} \geq 0, \forall i \in \mathcal{I}, \forall j \in \mathcal{J},
\end{aligned}
\tag{3.13}
$$

while the dual of [PSP] is

$$
\begin{aligned}
[DSP] \quad & \max_{\boldsymbol{\alpha},\boldsymbol{\beta}} g(\boldsymbol{X}, \boldsymbol{\alpha}, \boldsymbol{\beta}) \\
& = \max_{\boldsymbol{\alpha},\boldsymbol{\beta}} \sum_{j=1}^{J} (-\alpha_j P_j^{\max}) + \sum_{i=1}^{I} \sum_{j=1}^{J} (\delta^{-1}(x_{ij} - 1)) + \Gamma_i \sigma_i^2) \beta_{ij} \\
& s.t.\ 1 + \alpha_j + \zeta \sum_{\forall i' \neq i} \sum_{j'=1}^{J} \Gamma_{i'} \beta_{i'j} g_{i'j} \\
& \quad + \sum_{i'=1}^{I} \sum_{\forall j' \neq j} \Gamma_{i'} \beta_{i'j'} g_{i'j} - \beta_{ij} g_{ij} \geq 0, \forall i, \forall j, \\
& \boldsymbol{\alpha} = (a_j, \forall j) \succeq 0, \boldsymbol{\beta} = [\beta_{ij}, \forall i, \forall j] \succeq 0.
\end{aligned}
\tag{3.14}
$$

Intuitively, the single-stage problem [SSP] can be solved using search methods by evaluating $\vartheta(\boldsymbol{X})$ for any $\boldsymbol{X}$ of interest through solving [PSP]. However, one major drawback of such process is that some $\boldsymbol{X}$'s may result in an empty feasible set for [PSP]. Once such an $\boldsymbol{X}$ is fed into [PSP], no meaningful $\vartheta(\boldsymbol{X})$ can be obtained and the algorithm cannot proceed.

To avoid this issue, we resort to Benders Decomposition described in the above section, which, instead of obtaining $\vartheta(\boldsymbol{X})$ directly through [PSP], generates optimality cuts and feasibility cuts to shrink the feasible region of [SSP] by solving the dual of [PSP], i.e.,[DSP], in each iteration. In particular, an optimality cut is added to [SSP] once [DSP] is bounded (or [PSP] is feasible), and a feasibility cut is added when [DSP] is unbounded (or [PSP] is infeasible). Following this procedure, we obtain the master problem:

$$
\begin{aligned}
[MSP] \quad & \min_{\boldsymbol{X},\theta} \epsilon\theta + (1-\epsilon)\sum_{i=1}^{I}\sum_{j=1}^{J} w_{ij}(1-x_{ij}) \\
s.t.\ & g(\boldsymbol{X}, \boldsymbol{\alpha}_p^{(m)}, \boldsymbol{\beta}_p^{(m)}) \leq \theta, \forall m = 1, \cdots, k_1, \\
& g(\boldsymbol{X}, \boldsymbol{\alpha}_r^{(l)}, \boldsymbol{\beta}_r^{(l)}) \leq 0, \forall l = 1, \cdots, k_2, \\
& \sum_{j=1}^{J} x_{ij} \leq 1, \forall i \in \mathcal{I}, \\
& x_{ij} \in \{0,1\}, \forall i \in \mathcal{I}, \forall j \in \mathcal{J}, \\
& \theta \geq 0,
\end{aligned}
\tag{3.15}
$$

where the new constraints $g(\boldsymbol{X}, \boldsymbol{\alpha}_p^{(m)}, \boldsymbol{\beta}_p^{(m)})$ and $g(\boldsymbol{X}, \boldsymbol{\alpha}_r^{(l)}, \boldsymbol{\beta}_r^{(l)})$ are added when problem [DSP] is bounded and unbounded, respectively. Obviously, the total number of added constraints $k_1 + k_2 = k$.

The BARN algorithm works as follows. Starting with an initialization $\boldsymbol{X}^{(0)} = \mathbf{0}$, the algorithm checks whether [PSP] is feasible (or equivalently, whether its dual [DSP] is bounded) for $\boldsymbol{X}^{(0)}$. If it is, a constraint $g(\boldsymbol{X}, \boldsymbol{\alpha}_p, \boldsymbol{\beta}_p) \leq \theta$ is generated and added to [MSP]. Note that $(\boldsymbol{\alpha}_p^{(0)}, \boldsymbol{\beta}_p^{(0)})$ is an extreme point in the feasible set of [DSP], because [DSP] is a linear problem. Otherwise, if [PSP] is infeasible (or equivalently, [DSP] is unbounded), find a point $(\boldsymbol{\alpha}_r^{(0)}, \boldsymbol{\beta}_r^{(0)})$ on the extreme ray [1] and add a feasibility cut $g(\boldsymbol{X}, \boldsymbol{\alpha}_r^{(0)}, \boldsymbol{\beta}_r^{(0)}) \leq 0$ to [MSP]. With the newly added constraint, we solve [MSP] to obtain an updated $\boldsymbol{X}^{(0)} = \boldsymbol{X}^{(1)}$. The iteration continuous until certain stopping criterion is satisfied.

Let the objective values of [DSP] and [MSP] at the kth iteration be $U^{(k)}$ and $L^{(k)}$ respectively. Lemma 2 shows that $U^{(k)}$ and $L^{(k)}$ can be used to calculate the upper and lower bounds of the optimal objective value of [SSP].

Lemma 3. Let LB denote the lower bound of the optimal objective value of [SSP] and UB denote the upper bound. The upper and lower bounds can be improved iteratively

[1] As in an unbounded polyhedral set, the extreme ray represents the ray that cannot be represented as a positive combination of other rays in the set.

as follows: in iteration k, $LB^{(k)} = L^{(k)}$ is a lower bound of the optimal value of [SSP], while $UB^{(k)} = \min_{0 \le s \le k} \{U^{(s)}\epsilon + (1-\epsilon)\sum_{i=1}^{I}\sum_{j=1}^{J} w_{ij}(1 - x_{ij}^{(s)})\}$ is an upper bound.

The proof of Lemma 3 is deferred to the Appendix A.2.

The BARN algorithm adopts the gap between $UB^{(k)}$ and $LB^{(k)}$ as the termination criteria. The algorithm stops when $UB^{(k)} - LB^{(k)} \le \tau$, where $\tau \ge 0$ is the so-called error tolerance. In particular, if $\tau = 0$ then the exact global optimal solution to [SSP] is obtained when the algorithm terminates.

Having introduced the basic operations, we now formally present the BARN algorithm in Algorithm 1 and the flow chart of BARN is illustrated in Figure 3.2, with $\tau = 0$.

Algorithm 1 The BARN algorithm with $\tau = 0$

1: **Initialization**: Set $k = 0$.

2: **repeat**

3: If $k = 0$, set $\boldsymbol{X}^{(0)} = \boldsymbol{0}$; otherwise, solve the optimization [MSP] to obtain the optimal solution $\boldsymbol{X}^{(k)} = [x_{ij}^{(k)}]$ and the lower bound $LB^{(k)}$.

4: Solve the optimization [DSP] to obtain the upper bound $UB^{(k)}$ according to $\min_{0 \le s \le k} \{U^{(s)}\epsilon + (1-\epsilon)\sum_{i=1}^{I}\sum_{j=1}^{J}(1 - x_{ij}^{(s)})\}$. Then, add the constraint $g(\boldsymbol{X}, \boldsymbol{\alpha}_p^{(k_1)}, \boldsymbol{\beta}_p^{(k_1)}) \le \theta$ into the optimization [MSP] if the optimization [DSP] is bounded, and add $g(\boldsymbol{X}, \boldsymbol{\alpha}_r^{(k_2)}, \boldsymbol{\beta}_r^{(k_2)}) \le 0$ into the optimization [MSP] otherwise.

5: $k = k + 1$.

6: **until** $UB^{(k)} - LB^{(k)} = 0$;

7: Compute the optimal power allocation $\boldsymbol{P}^*$ through solving the optimization [PSP] with $\boldsymbol{X}^{(k)}$. And thus $(\boldsymbol{X}^*, \boldsymbol{P}^{(k)})$ is a global optimal solution to problem [SSP].

3.3 Performance And Convergence

After proposing BARN algorithm, we analyze the global optimality as well as the tradeoff between performance and converge time in BARN.

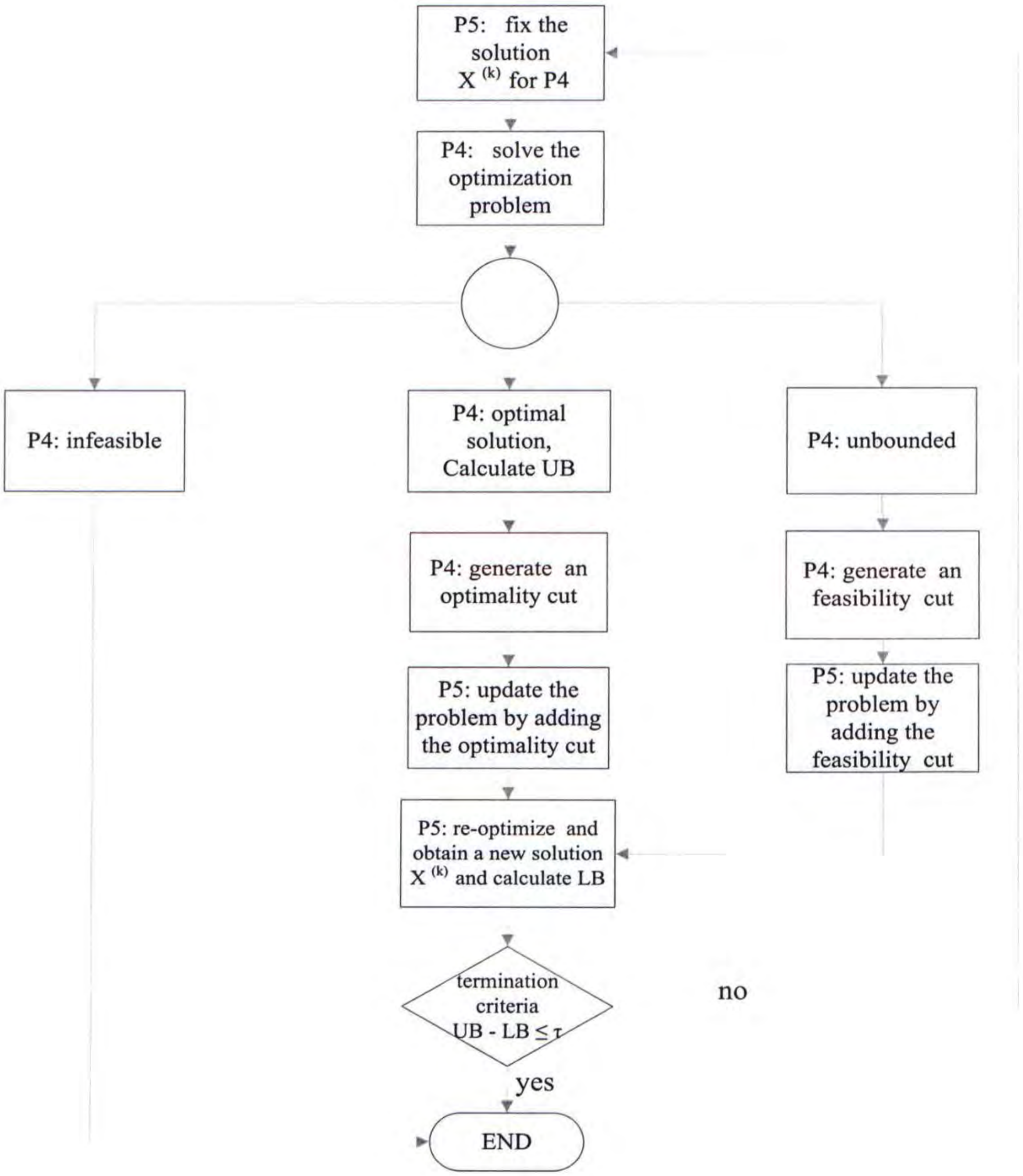

Figure 3.2: The flow chat of BARN

3.3.1 Global Convergence

In this subsection, we propose the following theorem to elucidate the convergence of BARN algorithm.

Theorem 1. The BARN algorithm globally converges to a global optimal solution of Problem [SSP] with finite number of iterations.

Proof: Immediate from the theories about the Benders Decomposition method [33].

3.3.2 BARN With Error Tolerance

In Algorithm 1, we have forced τ to be 0. Alternatively, we can set τ to be a small positive value to speed up the convergence of BARN. That is, when BARN terminates with $UB^{(k)} - LB^{(k)} \leq \tau,\ \tau > 0$, the power allocation $\boldsymbol{P}'$ is computed through solving [PSP] with $\boldsymbol{X}^{(\tilde{s})}$, where $\boldsymbol{X}^{(\tilde{s})}$ is the solution to [MSP] that generates the current upper bound, i.e., $\tilde{s} = \underset{1\leq s\leq k}{\operatorname{argmin}}\{U^{(s)}\epsilon + (1-\epsilon)\sum_{i=1}^{I}\sum_{j=1}^{J}(1-x_{ij}^{(s)})\}$. By doing so, we have obtained an approximate solution to [SSP], denoted as $(\boldsymbol{X}', \boldsymbol{P}')$, where $\boldsymbol{X}' = \boldsymbol{X}^{(\tilde{s})}$. It is obvious that $\Phi(\boldsymbol{X}', \boldsymbol{P}') = UB^{(k)}$, according to the definition of $UB^{(k)}$.

The BARN algorithm with a positive error tolerance is formally stated in Algorithm 2

3.3.3 Trade-off Between Performance And Convergence Time

In this subsection, we analyze the effect of τ on the system performance.

Definition 1 (τ-optimal solution)**.** Let $\Phi(\boldsymbol{X}^*, \boldsymbol{P}^*)$ denote the optimal solution of [SSP]. We say that a solution $(\boldsymbol{X}', \boldsymbol{P}')$ to [SSP] is an τ-optimal solution of [SSP] if $(\boldsymbol{X}', \boldsymbol{P}')$ satisfies all the constrains of [SSP] and $\Phi(\boldsymbol{X}^*, \boldsymbol{P}^*) - \Phi(\boldsymbol{X}', \boldsymbol{P}') \leq \tau$.

Theorem 2. With the termination criteria $UB^{(k)} - LB^{(k)} \leq \tau$, the solution obtained by the BARN algorithm is an τ-optimal solution.

Algorithm 2 The BARN algorithm with $\tau > 0$

1: **Initialization**:Set $UB^{(0)} = +\infty$ and $LB^{(0)} = -\infty$. Let $k = 0$.
2: **repeat**
3: If $k = 0$, set $\boldsymbol{X}^{(0)} = \boldsymbol{0}$; otherwise, solve the optimization [MSP] to obtain the optimal solution $\boldsymbol{X}^{(k)} = [x_{ij}^{(k)}]$ and the lower bound $LB^{(k)}$.
4: Solve the optimization [DSP] to obtain the upper bound $UB^{(k)}$ according to $\min_{0\leq s\leq k}\{U^{(s)}\epsilon + (1-\epsilon)\sum_{i=1}^{I}\sum_{j=1}^{J}(1-x_{ij}^{(s)})\}$. Then, add the constraint $g(\boldsymbol{X}, \boldsymbol{\alpha}_p^{(k_1)}, \boldsymbol{\beta}_p^{(k_1)}) \leq \theta$ into the optimization [MSP] if the optimization [DSP] is bounded, and add $g(\boldsymbol{X}, \boldsymbol{\alpha}_r^{(k_2)}, \boldsymbol{\beta}_r^{(k_2)}) \leq 0$ into the optimization [MSP] otherwise.
5: $k = k + 1$.
6: **until** $UB^{(k)} - LB^{(k)} = 0$;
7: Compute the optimal power allocation $\boldsymbol{P}'$ through solving the optimization [PSP] with $\boldsymbol{X}^{(\tilde{s})}$, where $\tilde{s} = \operatorname{argmin}_{1\leq s\leq k}\{U^{(s)}\epsilon + (1-\epsilon)\sum_{i=1}^{I}\sum_{j=1}^{J}(1-x_{ij}^{(s)})\}$. Denote $\boldsymbol{X}^{(\tilde{s})}$ as $\boldsymbol{X}'$, and thus $(\boldsymbol{X}', \boldsymbol{P}')$ is the solution to [SSP].

Proof: Let $(\boldsymbol{X}', \boldsymbol{P}')$ be the solution when BARN terminates at the kth iteration with $UB^{(k)} - LB^{(k)} \leq \tau$. Then

$$LB^{(k)} \leq \Phi(\boldsymbol{X}', \boldsymbol{P}') = UB^{(k)}. \tag{3.16}$$

Since $LB^{(k)} \leq \Phi(\boldsymbol{X}^*, \boldsymbol{P}^*) \leq UB^{(k)}$ and $UB^{(k)} - LB^{(k)} \leq \tau$, (3.16) implies that $\Phi(\boldsymbol{X}', \boldsymbol{P}') - \Phi(\boldsymbol{X}^*, \boldsymbol{P}^*) \leq \tau$. In other words, $\Phi(\boldsymbol{X}', \boldsymbol{P}')$ is an τ-optimal solution to [SSP]. ■

By tuning the error tolerance τ, we can achieve an appropriate tradeoff between the convergence time and the obtained system revenue. The following proposition characterizes such a tradeoff.

Proposition 1. Let $\epsilon = \frac{\kappa}{\sum_{j=1}^{J} P_j^{\max}+1}$ where $0 \leq \kappa < 1$. The system revenue $\sum_{i=1}^{I}\sum_{j=1}^{J}\lambda_i x_{ij}$ of [SSP] obtained by the BARN algorithm with a positive τ is at most $\frac{1}{\rho} \cdot \lfloor\frac{\tau+\kappa-\epsilon}{(1-\epsilon)}\rfloor$ less [2] than the maximum system revenue obtained by setting $\tau = 0$.

[2] $\lfloor x \rfloor$ rounds x to the nearest integer less than or equal to x.

Proof: Assume that BARN terminates at the kth iteration with $UB^{(k)} - LB^{(k)} \leq \tau$ and obtains a τ-optimal solution $(\boldsymbol{X}', \boldsymbol{P}')$. Let Z' be the corresponding objective value, $(\boldsymbol{X}^*, \boldsymbol{P}^*)$ be the optimal solution for [SSP] and Z^* be the optimal objective value. By the definitions of $UB^{(k)}$ and $LB^{(k)}$, we have $Z' = UB^{(k)}$ and $Z^* \geq LB^{(k)}$. Thus

$$Z' - Z^* \leq Z' - LB^{(k)} = UB^{(k)} - LB^{(k)} \leq \tau. \tag{3.17}$$

On the other hand, $\epsilon = \frac{\kappa}{\sum_{j=1}^{J} P_j^{\max}+1}$ implies that $\sum_{i=1}^{I}\sum_{j=1}^{J} p'_{ij} - \sum_{i=1}^{I}\sum_{j=1}^{J} p^*_{ij} \geq -\sum_{j=1}^{J} P_j^{\max} = -\frac{\kappa-\epsilon}{\epsilon}$. Hence, we can get

$$\begin{aligned}
Z' - Z^* &= \big(\epsilon\sum_{i=1}^{I}\sum_{j=1}^{J} p'_{ij} + (1-\epsilon)\sum_{i=1}^{I}\sum_{j=1}^{J} w_{ij}(1-x'_{ij})\big) \\
&\quad - \big(\epsilon\sum_{i=1}^{I}\sum_{j=1}^{J} p^*_{ij} + (1-\epsilon)\sum_{i=1}^{I}\sum_{j=1}^{J} w_{ij}(1-x^*_{ij})\big) \\
&= \epsilon\big(\sum_{i=1}^{I}\sum_{j=1}^{J} p'_{ij} - \sum_{i=1}^{I}\sum_{j=1}^{J} p^*_{ij}\big) \qquad (3.18)\\
&\quad + (1-\epsilon)\big(\sum_{i=1}^{I}\sum_{j=1}^{J} w_{ij}(1-x'_{ij}) - \sum_{i=1}^{I}\sum_{j=1}^{J} w_{ij}(1-x^*_{ij})\big) \\
&\geq -(\kappa-\epsilon) + (1-\epsilon)\big(\sum_{i=1}^{I}\sum_{j=1}^{J} w_{ij}x^*_{ij} - \sum_{i=1}^{I}\sum_{j=1}^{J} w_{ij}x'_{ij}\big),
\end{aligned}$$

From (3.17) and (3.18), it follows that $\sum_{i=1}^{I}\sum_{j=1}^{J} w_{ij}x^*_{ij} - \sum_{i=1}^{I}\sum_{j=1}^{J} w_{ij}x'_{ij} \leq \frac{\tau+\kappa-\epsilon}{1-\epsilon}$. Since w_{ij}, x^*_{ij} and x'_{ij} are all integers, the difference between $\sum_{i=1}^{I}\sum_{j=1}^{J} w_{ij}x^*_{ij}$ and $\sum_{i=1}^{I}\sum_{j=1}^{J} w_{ij}x'_{ij}$ satisfies $\sum_{i=1}^{I}\sum_{j=1}^{J} w_{ij}x^*_{ij} - \sum_{i=1}^{I}\sum_{j=1}^{J} w_{ij}x'_{ij} \leq \lfloor\frac{\tau+\kappa-\epsilon}{1-\epsilon}\rfloor$. Thus we have $\sum_{i=1}^{I}\sum_{j=1}^{J} \lambda_i x^*_{ij} - \sum_{i=1}^{I}\sum_{j=1}^{J} \lambda_i x'_{ij} \leq \frac{1}{\rho}\cdot\lfloor\frac{\tau+\kappa-\epsilon}{(1-\epsilon)}\rfloor$. ∎

Note that if $\frac{\tau+\kappa-\epsilon}{1-\epsilon} < 1$, $\sum_{i=1}^{I}\sum_{j=1}^{J} w_{ij}x'_{ij}$ is exactly equal to $\sum_{i=1}^{I}\sum_{j=1}^{J} w_{ij}x^*_{ij}$. In other words, the system revenue $\sum_{i=1}^{I}\sum_{j=1}^{J} \lambda_j x'_{ij}$ is exactly equal to the optimal one $\sum_{i=1}^{I}\sum_{j=1}^{J} \lambda_j x^*_{ij}$. Thus, we have the following remark.

Remark 1. With $\tau < 1 - \kappa$, the gap between $\sum_{i=1}^{I}\sum_{j=1}^{J} w_{ij}x_{ij}^*$ and $\sum_{i=1}^{I}\sum_{j=1}^{J} w_{ij}x_{ij}'$ is 0 (i.e., $\lfloor \frac{\tau+\kappa-\epsilon}{1-\epsilon} \rfloor = 0$), and thus the gap between $\sum_{i=1}^{I}\sum_{j=1}^{J} \lambda_i x_{ij}^*$ and $\sum_{i=1}^{I}\sum_{j=1}^{J} \lambda_i x_{ij}'$ is 0.

In general, the smaller τ, the longer the algorithm runs, and the more accurate the solution is. Theorem 2 and Proposition 1 have pointed to a convenient tradeoff between convergence time and system performance.

Before leaving this section, we emphasize that the key idea of BARN is to decompose a mixed-integer problem [SSP] into two smaller problems with fewer variables. In particular, subproblem [DSP] is a linear program that can be solved easily. Moreover, the master problem is composed of binary integer variables x_{ij}'s plus one continuous variable θ. Such a problem can be solved with the need of considering the integer variables only, for example, the methods proposed in [36, 37].

□ **End of chapter.**

Chapter 4

Accelerating BARN

In this chapter, we propose a novel algorithm, referred to as A-BARN, to reduce the computational complexity of the BARN algorithm. In particular, we relax the master problem into a linear optimization problem which, together with the Feasibility Pump method introduced latter, play an important role in A-BARN. Our analysis shows that A-BARN is guaranteed to achieve the global optimal solution despite its reduced computational cost.

The structure of this chapter is as follows: in Section 4.1 we relax the master problem of BARN, and propose an important property of this relaxed problem. In Section 4.2 we introduce the Feasibility Pump method to generate an integer solution from the relaxed master problem. And finally, we propose the A-BARN algorithm to solve the JBAPC problem in Section 4.3,.

4.1 The Relaxed Master Problem

A close look at the BARN algorithm shows that the computational complexity is dominated by the cost of solving the integer linear programming problem [MSP] in each iteration. A-BARN reduces the complexity by relaxing the integer constraints of [MSP] in the intermediate iterations. In particular, in the kth iteration, the master problem

[MSP] is relaxed into the following linear programming (LP) problem.

$$
\begin{aligned}
[LMSP] \qquad & \min_{\boldsymbol{X},\theta} \epsilon\theta + (1-\epsilon)\sum_{i=1}^{I}\sum_{j=1}^{J} w_{ij}(1-x_{ij}) \\
s.t.\ & g(\boldsymbol{X}, \boldsymbol{\alpha}_p^{(m)}, \boldsymbol{\beta}_p^{(m)}) \le \theta, \forall m = 1, \cdots, k_1, \\
& g(\boldsymbol{X}, \boldsymbol{\alpha}_r^{(l)}, \boldsymbol{\beta}_r^{(l)}) \le 0, \forall l = 1, \cdots, k_2, \\
& \sum_{j=1}^{J} x_{ij} \le 1, \forall i \in \mathcal{I}, \\
& x_{ij} \in [0,1], \forall i \in \mathcal{I}, \forall j \in \mathcal{J}, \\
& \theta \ge 0,
\end{aligned}
\tag{4.1}
$$

where x_{ij} is now a continuous variable ranging between [0,1]. [LMSP] are can be solved very fast, due to its linear programming nature, a lot of existing algorithms can be applied such as the simplex method [38], the ellipsoid method [39], and the interior matrix method [40]. Hereafter, we say that $\boldsymbol{X}$ is an integer matrix when all its entries x_{ij}'s are 0/1 integers, and $\boldsymbol{X}$ is not an integer matrix or a fractional matrix if some entries in it are not integers.

Lemma 4. Let $(\hat{\boldsymbol{X}}, \hat{\theta})$ denote an arbitrary feasible solution of [LMSP]. The optimality cut or feasibility cut generated by solving [DSP] with $\hat{\boldsymbol{X}}$ does not exclude the optimal solution $(\boldsymbol{X}^*, \boldsymbol{P}^*)$ to [SSP], from the remaining feasible set of [LMSP].

Proof: If [DSP] is bounded with $\hat{\boldsymbol{X}}$, an optimality cut

$$g(\boldsymbol{X}, \hat{\boldsymbol{\alpha}}_p, \hat{\boldsymbol{\beta}}_p) \le \theta. \tag{4.2}$$

is generated, where $(\hat{\boldsymbol{\alpha}}_p, \hat{\boldsymbol{\beta}}_p)$ is the optimal solution to [DSP] with $\hat{\boldsymbol{X}}$. Otherwise, a feasibility cut

$$g(\boldsymbol{X}, \hat{\boldsymbol{\alpha}}_r, \hat{\boldsymbol{\beta}}_r) \le 0. \tag{4.3}$$

is generated where $(\hat{\boldsymbol{\alpha}}_r, \hat{\boldsymbol{\beta}}_r)$ is a point on the extreme ray of [DSP] with $\hat{\boldsymbol{X}}$. To prove Lemma 4, we show in the following that in either case, the optimal solution $(\boldsymbol{X}^*, \boldsymbol{P}^*)$ to

Problem [SSP] does not violate constraints (4.2) or (4.3), and hereby not be excluded from the feasible set by the cuts.

Let $(\boldsymbol{\alpha}_p^*, \boldsymbol{\beta}_p^*)$ be the optimal solution of Problem [DSP] with $\boldsymbol{X}^*$. The corresponding optimal objective function value of [DSP], denoted by θ^*, is hereby given as $\theta^* = g(\boldsymbol{X}^*, \boldsymbol{\alpha}_p^*, \boldsymbol{\beta}_p^*)$. Suppose that $(\boldsymbol{X}^*, \theta^*)$ violates (4.2) when [DSP] is bounded. Then, we have, $g(\boldsymbol{X}^*, \hat{\boldsymbol{\alpha}}_p, \hat{\boldsymbol{\beta}}_p) > \theta^*$. This contradicts the fact that $(\boldsymbol{\alpha}_p^*, \boldsymbol{\beta}_p^*)$ is the optimal solution to [DSP] with $\boldsymbol{X}^*$, and that θ^* should be the maximum objective value of [DSP]. Hence, the optimality cut (4.2) cannot be violated by $\boldsymbol{X}^*$.

In the case when [DSP] is unbounded, we suppose that $\boldsymbol{X}^*$ violates the feasibility cut (4.3). That is, $g(\boldsymbol{X}, \hat{\boldsymbol{\alpha}}_r, \hat{\boldsymbol{\beta}}_r) > 0$. Since $(\hat{\boldsymbol{\alpha}}_r, \hat{\boldsymbol{\beta}}_r)$ is a point on the extreme ray of the feasible region of problem [DSP], $(m\hat{\boldsymbol{\alpha}}_r, m\hat{\boldsymbol{\beta}}_r)$ is also in the feasible set for any positive scalar m. Note that if function $g(\boldsymbol{X}, \hat{\boldsymbol{\alpha}}_r, \hat{\boldsymbol{\beta}}_r)$ is linear in $(\hat{\boldsymbol{\alpha}}_r, \hat{\boldsymbol{\beta}}_r)$ then $g(\boldsymbol{X}, m\hat{\boldsymbol{\alpha}}_r, m\hat{\boldsymbol{\beta}}_r) = mg(\boldsymbol{X}, \hat{\boldsymbol{\alpha}}_r, \hat{\boldsymbol{\beta}}_r)$. Therefore, $g(\boldsymbol{X}^*, \hat{\boldsymbol{\alpha}}_r, \hat{\boldsymbol{\beta}}_r) > 0$ implies that the objective value of problem [DSP] is also unbounded with $\boldsymbol{X}^*$. This contradicts the fact that θ^* should be a finite value of [DSP] with $\boldsymbol{X}^*$. Therefore, $\boldsymbol{X}^*$ cannot violate the feasibility cut (4.3). ∎

Lemma 4 implies that we can safely replace [MSP] by [LMSP] in the BARN algorithm without compromising the optimality of the solution. However, one should note that when [DSP] is solved with $\hat{\boldsymbol{X}}$, the upper bound calculated from its objective function value may not be a valid upper bound for [SSP], for that $\hat{\boldsymbol{X}}$ may be a fractional matrix and is infeasible to [MSP]. One way to solve this problem is to round $\hat{\boldsymbol{X}}$ to the nearest integer matrix that is feasible to [lMSP] (and hence feasible to [MSP]). Such an integer matrix can be efficiently found using the Feasible Pump (FP) method proposed in [41].

4.2 The Feasibility Pump Method

Let $(\hat{\mathbf{X}}, \hat{\theta})$ be the solution of [LMSP], and $\mathring{\mathbf{X}}$ be an integer point with the same dimension as $\hat{\mathbf{X}}$. Using denotation $dist\langle\hat{\mathbf{X}}, \mathring{\mathbf{X}}\rangle$ to represent the l_1-norm distance between

$\hat{\mathbf{X}}$ and $\mathring{\mathbf{X}}$, i.e.,

$$dist\langle\hat{\mathbf{X}},\mathring{\mathbf{X}}\rangle \triangleq \{\|\hat{\mathbf{X}}-\mathring{\mathbf{X}}\|_1 \mid \hat{\mathbf{X}} \in \textit{the feasible set of } [LMSP] \ \}.$$

Given an integer matrix $\mathring{\mathbf{X}}$, the matrix $\hat{\mathbf{X}}$ within the feasible set of [LMSP] and the minimal distance of $\mathring{\mathbf{X}}$ can therefore be determined by $arg\min_{\mathbf{X}}\ dist\langle\mathbf{X},\mathring{\mathbf{X}}\rangle$. Note by intuition that if $dist\langle\hat{\mathbf{X}},\mathring{\mathbf{X}}\rangle = 0$, then $\hat{\mathbf{X}}$ is an integer point equivalent to $\mathring{\mathbf{X}}$, and thus feasible to [MSP]. Additionally, an integer matrix nearest to a fractional matrix $\hat{\mathbf{X}}$ can be easily determined by scalar rounding all the entries of $\hat{\mathbf{X}}$ to the nearest integer point. These observations lead to the FP method.

The FP method can be described as a linear searching cycle. At the kth iteration, denote the solution of [LMSP] as $(\hat{\mathbf{X}}^{(k)}, \hat{\theta}^{(k)})$, denote $\hat{\mathbf{X}}^{(k)}$ as $\hat{\mathbf{X}}(0)$. Firstly, an integer matrix, denoted as $[\hat{\mathbf{X}}(0)]$, is obtained by scalar rounding all the entries of $\hat{\mathbf{X}}(0)$ to the nearest integer point. Then a solution $\hat{\mathbf{X}}(1)$ is obtained through solving $\min_{\mathbf{X}}\ dist\langle\hat{\mathbf{X}}(1), [\hat{\mathbf{X}}(0)]\rangle$, and $[\hat{\mathbf{X}}(1)]$ is obtained through rounding $\hat{\mathbf{X}}(1)$ to the nearest integer. After that a new cycle begins. This process stops until an integer matrix $\hat{\mathbf{X}}(v)$ is obtained at the vth cycle with $dist\langle\hat{\mathbf{X}}(v), [\hat{\mathbf{X}}(v-1)]\rangle = 0$ and this integer matrix has not been fathomed in the previous iterations. $\hat{\mathbf{X}}(v)$ is therefore adopted to solve the dual subproblem [DSP] to generate a new constraint for the later iterations. The procedure of the FP method is formally given in Algorithm 3

Before leaving this section, we have proposed the basic procedure needed for A-BARN. One should note that the linear searching cycle in Algorithm 3 terminates as long as one feasible integer solution $\hat{\boldsymbol{X}}(v)$ is found, or when the maximum cycles limit V is met. In the latter case, FP fails to generate a feasible integer solution, which is *'not surprising, for that finding a feasible solution of an integer problem is NP-hard in general'* [41]. However, we should mention that when FP is adopted in A-BARN algorithm, such a failure will not cause the algorithm break down, due to the special structure of A-BARN, as shown in the next section.

Algorithm 3 The Feasibility Pump method

Initialization: Input $\hat{\mathbf{X}}^{(k)}$. Let $\hat{\mathbf{X}}^{(k)} = \hat{\mathbf{X}}(0)$ and the maximum cycle limit be V. Set $v = 0$.
repeat
    find an integer matrix $[\hat{\mathbf{X}}(v)]$;
    $\hat{\mathbf{X}}(v+1) = arg\min_{\mathbf{X}} \; dist\langle \hat{\mathbf{X}}, [\hat{\mathbf{X}}(0)] \rangle$
    If $\hat{\mathbf{X}}(v+1)$ is an integer matrix, return $\hat{\mathbf{X}}(v+1)$;
    $v = v + 1$;
until v=V;
return False.

4.3 A-BARN Algorithm For Solving The JBAPC Problem

Based on the discussion in the above two sections, the A-BARN algorithm is formally presented in Algorithm 4.

Note *step*5 in A-BARN, $LB^{(k)}$ may typically be generated by a fractional matrix $\hat{\boldsymbol{X}}^{(k)}$ while $UB^{(k)}$ is generated by an integer matrix $[\hat{\boldsymbol{X}}^{(k)}]$. Thus, the speed for the gap between $UB^{(k)}$ and $LB^{(k)}$ to shrink from a small value to zero is slow. To avoid such a situation, we add Phase-II which runs the same procedure as BARN, except for that Phase-II is initialized with the cuts generated in Phase-I. Actually there are variety of choices to decide when to transfer from Phase-I to Phase-II [37], In this paper, we just set that Phase-I goes to Phase-II after K iterations, which implies that K cuts have been generated from Phase-I. Numerical results in the next section will show that, with a proper K, only a few iterations are needed in Phase-II before the algorithm converges. Besides, when FP fails to find an integer solution $\boldsymbol{X}$ through the FP procedure, the algorithm directly jumps to Phase-II, in which we start the identical procedure as BARN algorithm, therefore avoids the A-BARN breaking down that would be brought by the failure of FP failure.

□ **End of chapter.**

Algorithm 4 The A-BARN algorithm

1: **Initialization**:Set $UB^{(0)} = +\infty$, $LB^{(0)} = -\infty$ and $\boldsymbol{X}^{(0)} = \mathbf{0}$. Let $k = 0$.

Phase − I

2: **repeat**

3: Solve the LP relaxed optimization problem [LMSP], and obtain the solution $(\hat{\boldsymbol{X}}^{(k)}, \hat{\theta})$, get the relaxed lower bound $LB^{(k)}$.

4: If find an integer matrix $[\hat{\boldsymbol{X}}^{(k)}]$ through the FP procedure, solve [DSP] to obtain the upper bound $UB^{(k)}$ according to $\min\{\min_{0\le s\le k}\{U^{(s)}\epsilon + (1-\epsilon)\sum_{i=1}^{I}\sum_{j=1}^{J}(1 - x_{ij}^{(s)})\}, UB^{(0)}\}$. Then, add the constraint $g(\boldsymbol{X}, \boldsymbol{\alpha}_p^{(k_1)}, \boldsymbol{\beta}_p^{(k_1)}) \le \theta$ into the optimization [LMSP] if the optimization [DSP] is bounded, or add $g(\boldsymbol{X}, \boldsymbol{\alpha}_r^{(k_2)}, \boldsymbol{\beta}_r^{(k_2)}) \le 0$ into the optimization [LMSP] if the optimization [DSP] is unbounded.

5: if failed to find an integer solution $\boldsymbol{X}$ through the FP procedure, go to *Step 9*.

6: $k = k + 1$;

7: If $UB^{(k)} - LB^{(k)} \le \tau$, go to *Step 14*.

8: **until** $k = K$.

Phase − II

9: **repeat**

10: Solving the optimization [MSP] added with the constraints generated in Phase-I, and obtain the optimal solution $\boldsymbol{X}^{(k)} = [x_{ij}^{(k)}]$ and the lower bound $LB^{(k)}$.

11: Solve the optimization [DSP] to obtain the upper bound $UB^{(k)}$ according to $\min\{\min_{0\le s\le k}\{U^{(s)}\epsilon + (1-\epsilon)\sum_{i=1}^{I}\sum_{j=1}^{J}(1 - x_{ij}^{(s)})\}, UB^{(0)}\}$. Then, add the constraint $g(\boldsymbol{X}, \boldsymbol{\alpha}_p^{(k_1)}, \boldsymbol{\beta}_p^{(k_1)}) \le \theta$ into the optimization [MSP] if the optimization [DSP] is bounded, and add $g(\boldsymbol{X}, \boldsymbol{\alpha}_r^{(k_2)}, \boldsymbol{\beta}_r^{(k_2)}) \le 0$ into the optimization [MSP] otherwise.

12: $k = k + 1$.

13: **until** $UB^{(k)} - LB^{(k)} \le \tau$.

14: Compute the optimal power allocation $\boldsymbol{P}'$ through solving the optimization [PSP] with $\boldsymbol{X}^{(\tilde{s})}$, where $\tilde{s} = \operatorname{argmin}_{1\le s\le k}\{U^{(s)}\epsilon + (1-\epsilon)\sum_{i=1}^{I}\sum_{j=1}^{J}(1 - x_{ij}^{(s)})\}$. Denote $\boldsymbol{X}^{(\tilde{s})}$ as $\boldsymbol{X}'$, and thus $(\boldsymbol{X}', \boldsymbol{P}')$ is the solution to [SSP].

Chapter 5

Computational Results

In this chapter, we illustrate the effectiveness of BARN and A-BARN through several examples. Herein, the two algorithms are implemented in MATLAB 7.0. Simulations are conducted using an HP Compaq dx7300 desktop with two (2.40GHz,1.60GHz) processors and 1Gb of RAM. The propagation gains are calculated by the *Log-Distance Path Loss Model* [42]:

$$g_{ij} \propto \left(\frac{d_{ij}}{d_0}\right)^{-n}. \tag{5.1}$$

where n is the path loss exponent, d_0 is so-called the reference distance and d_{ij} is the distance between BS j and MS i. Eqn.(5.1) indicates that the propagation gain decreases at the rate n as d_{ij} decreases. Herein, n is set to 4 to represent the shadowed urban environment, and d_0 is set to 1 meter as suggested in [42].

5.1 Global Optimality And Convergence

We consider a multi-cell network with two BSs and six MSs in Fig. 5.1. The BSs and MSs are placed in a 10m-by-10m area. The channel matrix is

$$\boldsymbol{G}_1 = \begin{bmatrix} 0.0894 & 0.0019 & 0.0007 & 0.0442 & 0.0010 & 0.0021 \\ 0.0033 & 0.0894 & 0.0007 & 0.0131 & 0.0143 & 0.3974 \end{bmatrix}^T.$$

Let the minimum SINR requirement of each MSs be $(\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4) = (0.5, 0.5, 0.65, 0.65, 0.5, 0.65)$dB, the maximum BSs power be $(P_1^{\max}, P_2^{\max}) = (200, 100)$mW, the orthogonality factor ζ be 0.1, the revenue λ_i of each MS i be 1, and the thermal noise of each MS be 0.001mW. K is set to be 15 in Algorithm 4.

Fig. 5.2 illustrates the convergence of BARN and A-BARN. It can be seen that both algorithms converge to the global solution quickly. The gap between the upper bound and the lower bound shrinks to 0 at the 44th iteration with BARN and at the 36th iteration with A-BARN, respectively. Besides the reduced number of iterations, A-BARN runs much faster than BARN in terms of the total computational time, because the average time spent on each iteration is reduced in A-BARN. In this particular example, BARN converges in 265.5 seconds, while A-BARN algorithm converges in 123.6 seconds.

5.2 Average Convergence Time

We consider a set of network topologies, where two BSs and a number of MSs are randomly placed in a 15m-by-15m area. Let the minimum SINR requirement of each MS be 1dB, and the maximum transmission power of each BS be 200mW. Likewise, let the orthogonality factor ζ be 0.1, the revenue λ_i of each MS i be 1 and the thermal noise be 0.001mW.

The number of MSs, I, varies from 1 to 13. The computational time is plotted against the number of MSs in Fig. 5.3. For fair comparison, we introduce the so-called the *normalized error tolerance* η, such that both BARN and A-BARN algorithms stop when $UB^{(k)} - LB^{(k)} \leq \eta LB^{(k)}$. By doing so, when the algorithm terminates at iteration k, the relative difference between the obtained objective value $Z^{(k)}$ and the exact optimal objective value Z^* is bounded as

$$|\frac{Z^{(k)} - Z^*}{Z^*}| \leq \frac{UB^{(k)} - LB^{(k)}}{Z^*} \leq \frac{UB^{(k)} - LB^{(k)}}{LB^{(k)}} \leq \eta. \tag{5.2}$$

when $LB^{(k)} > 0$. Otherwise when $LB^{(k)} \leq 0$, we have $UB^{(k)} - LB^{(k)} \leq \eta LB^{(k)} \leq 0$. In the later case, algorithms will not stop. Note that the objective value of Problem P3 is always positive, the algorithms will always stops when (5.2) is satisfied. Therefore, through setting η other than the error tolerance τ in this example, we can compare the relative error of the solution for different network topologies. The normalized error tolerance η is set as 0 and 0.2 in this example.

From the figure, we observe that the convergence time of A-BARN is always lower than that of BARN. For example, with $\eta = 0$ (i.e., $\tau = 0$), A-BARN terminates in around 83000 seconds with 10 MSs. In contrast, the convergence time of BARN is several orders of magnitude longer. The figure also shows that the convergence time can be significantly reduced by increasing the error tolerance. For example, when η is set to 0.2, the computational times for BARN and A-BARN reduce to around 87000 seconds and 1000 seconds, respectively.

5.3 Trade-off Between Performance And Convergence Time

Consider a multi-cell network with two BSs and nine MSs, where all BSs and MSs are randomly placed in a 15m-by-15m area. Assume that the minimum SINR requirements of the MSs are $(0.5, 0.5, 0.65, 0.65, 0.5, 0.65, 0.5, 0.55, 0.65)dB$, and λ_i's are $(0.5, 0.5, 0.65, 0.65, 0.5, 0.65, 0.5, 0.55, 0.65)$. Let the orthogonality factor ζ be 0.1 and the thermal noise of each MS be 0.001mW. Set $\epsilon = \frac{0.7}{\sum_{j=1}^{J} P_j^{\max}/\tilde{w}+1}$. Likewise, assume that the maximum power of BSs is $(P_1^{\max}, P_2^{\max}) = (200.0, 200.0)$mW.

We run BARN and A-BARN with different normalized error tolerance η. The system revenues and the convergence times are plotted in Fig. 5.4 and Fig. 5.5, respectively. Form Fig. 5.4, it can be seen that the system revenue reaches the maximum value when η is smaller than or equal to $0. = 2$, which is consistent with Proposition 1. Besides, it is not surprising that the system revenue increases as the error tolerance decreases. On the other hand, Fig. 5.5 shows that the convergence time increases as the error tolerance decreases. Moreover, we observe that A-BARN is likely to terminate during

Phase-I when the error tolerance is large. As such, it completely avoids the calculation of integer optimization.

5.4 Average Algorithm Performance Of BARN and A-BARN

Consider a set of network topologies where J BSs and I MSs are randomly placed in a 15m-by-15m area. Assume that the minimum SINR requirement Γ_i of each MS is randomly selected between [-1,1]dB. Let the orthogonality factor ζ be 0.1, the thermal noise of each MS be 0.001mW, the revenue λ_i of each MS i be 1, and the maximum power of each BS be 200.0mW.

We compare the performance of BARN and A-BARN through testing the same 15 cases in Table 5.1 and 5.2, respectively. Through the two tables, we observe that A-BARN becomes more efficient as the complexity of the problem (in terms of the number of MSs multiplied by BSs) increases. For the small scale cases, such as case 1 and case 6, A-BARN requires a slightly more computational time. This is because that in cases of only a few variables solving the IP problem [MSP] in BARN costs less than solving [LMSP] in A-BARN where additional time is spent on the FP cycle. For the middle scale cases, such as cases $3-5$, $7-11$ and $13-15$, A-BARN outperforms BARN in terms of the computational time. This is not surprising, for that solving the relaxed master problem [LMSP] in those instants could cost much less time than solving the integer problem [MSP] and thus reduced the total computational time. This is reflected in the last column of Table 5.2, which enumerates the average computational time required for each case. From this column, we observe that in case 10 the computational time per iteration is reduced by approximately 23.6%, and on the other extreme, the computational time per iteration for case 3 is reduced by 5.15%.

□ **End of chapter.**

Table 5.1: Average performance of the BARN algorithm

Case	*BS*	*MS*	*Total Iter*	Time	Time per Iter
1	2	4	15	3.042	0.2028
2	2	5	27	8.432	0.3123
3	2	6	65	39.78	0.6120
4	2	7	166	201.2	1.212
5	2	8	347	838.1	2.415
6	3	3	31	10.73	0.3460
7	3	4	82	35.73	0.4357
8	3	5	321	632.4	1.970
9	3	6	630	1233	1.957
10	3	7	1215	3469	2.855
11	3	8	2301	9243	4.017
12	4	3	63	31.96	0.5073
13	4	4	259	579.6	2.238
14	4	5	1543	4169	2.702
15	4	6	2853	9466	3.318

Table 5.2: Average performance of the A-BARN algorithm

Case	*BS*	*MS*	LP Iter	IP Iter	Time	Total Iter	Time per Iter
1	2	4	25	1	3.698	26	0.1422
2	2	5	25	1	6.738	26	0.2592
3	2	6	50	4	31.35	54	0.5805
4	2	7	50	2	52.05	52	1.001
5	2	8	178	43	411.7	221	1.863
6	3	3	50	1	16.41	51	0.3218
7	3	4	50	5	22.39	55	0.4070
8	3	5	100	72	167.9	172	0.9763
9	3	6	200	102	482.9	302	1.599
10	3	7	318	183	1091	502	2.173
11	3	8	377	298	2136	675	3.164
12	4	3	50	14	23.80	64	0.3719
13	4	4	200	19	452.5	219	2.066
14	4	5	500	212	1638	712	2.301
15	4	6	784	611	4332	1395	3.105

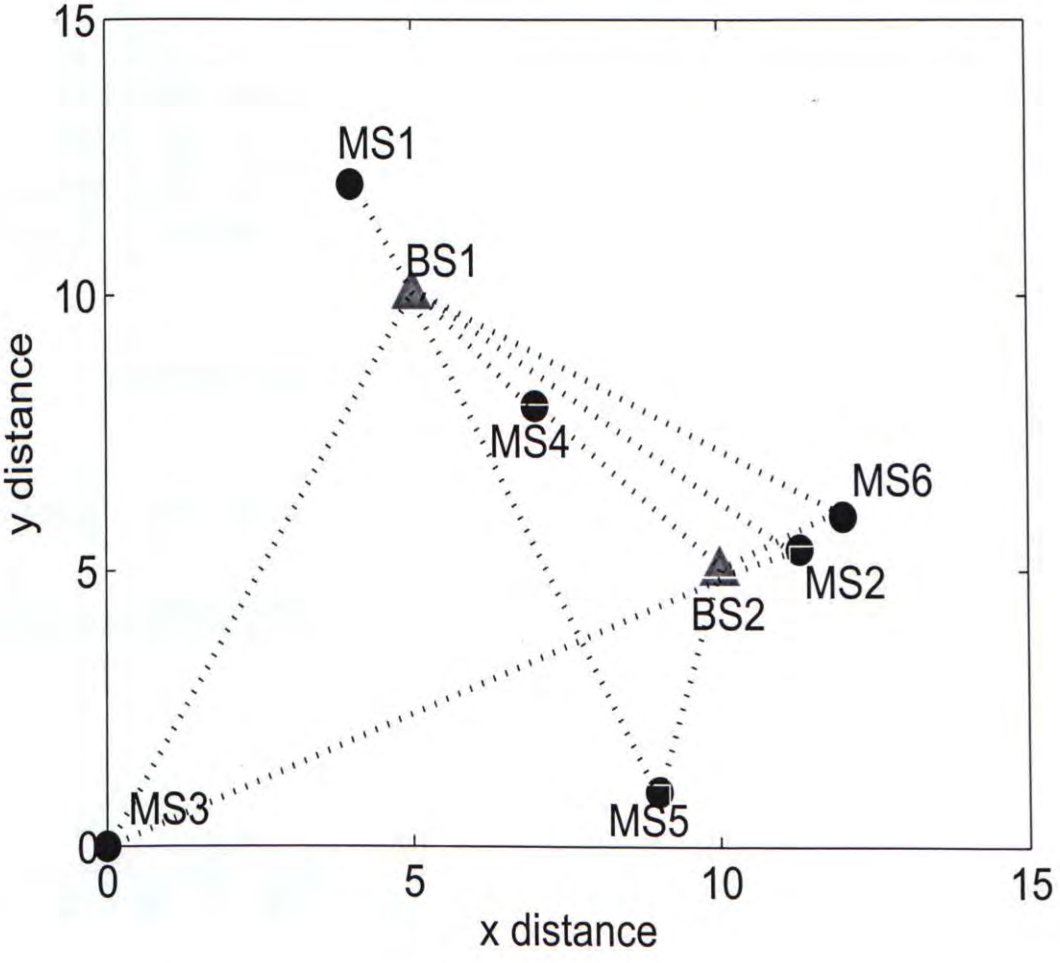

Figure 5.1: A multi-cell network with two BSs and four MSs

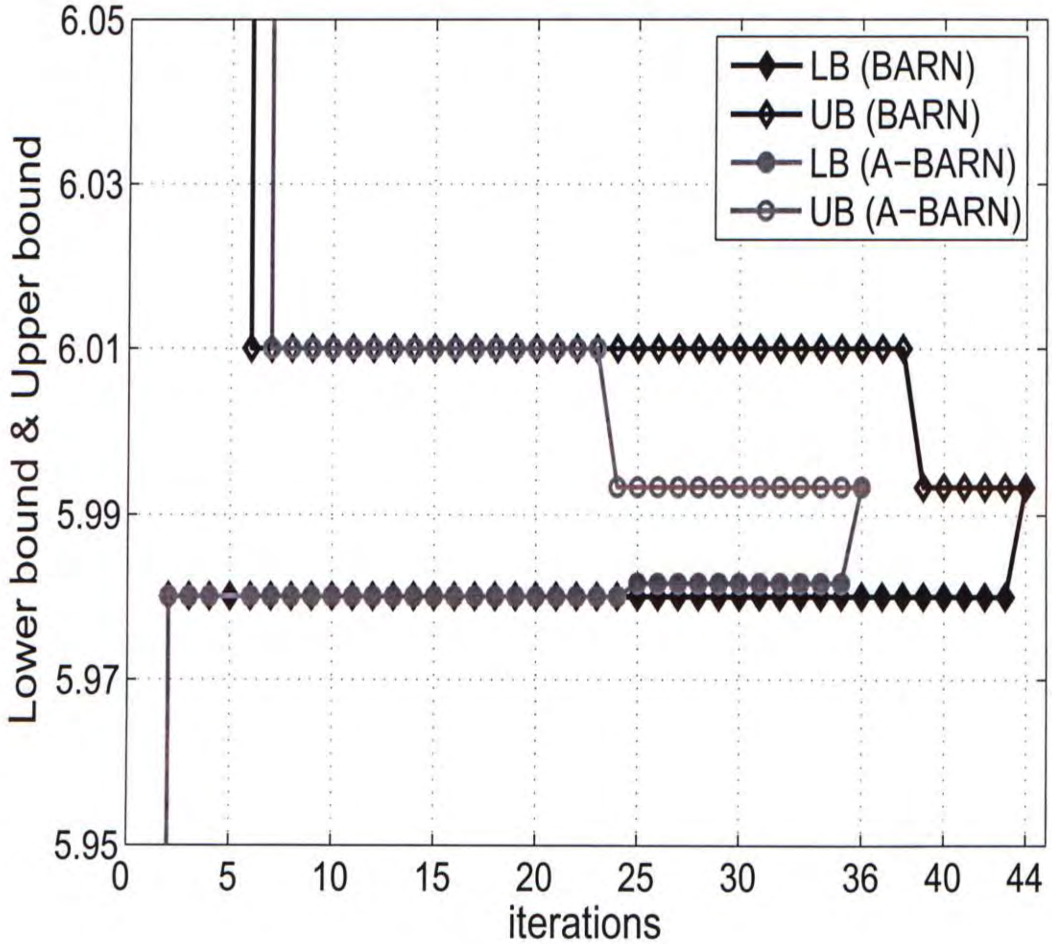

Figure 5.2: The converge condition of BARN and A-BARN

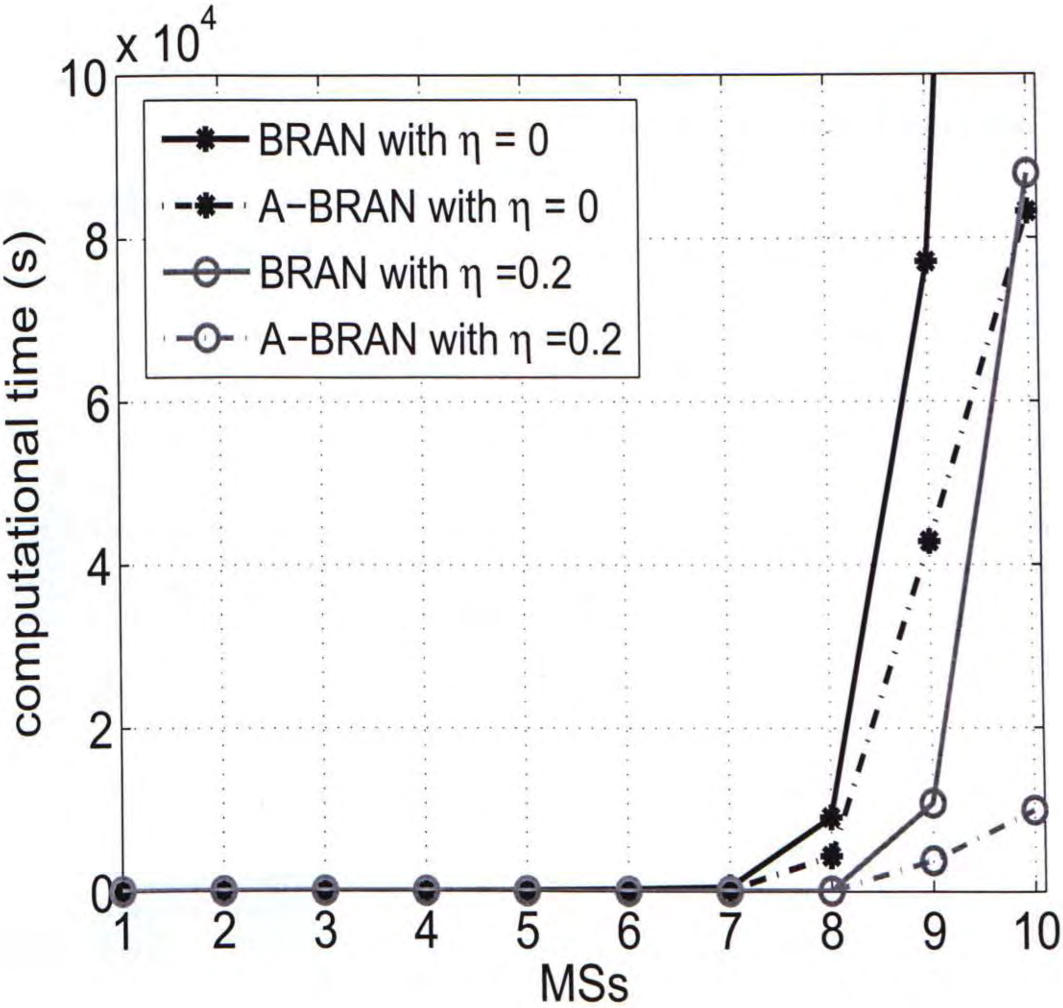

Figure 5.3: Time complexity versus the network topology

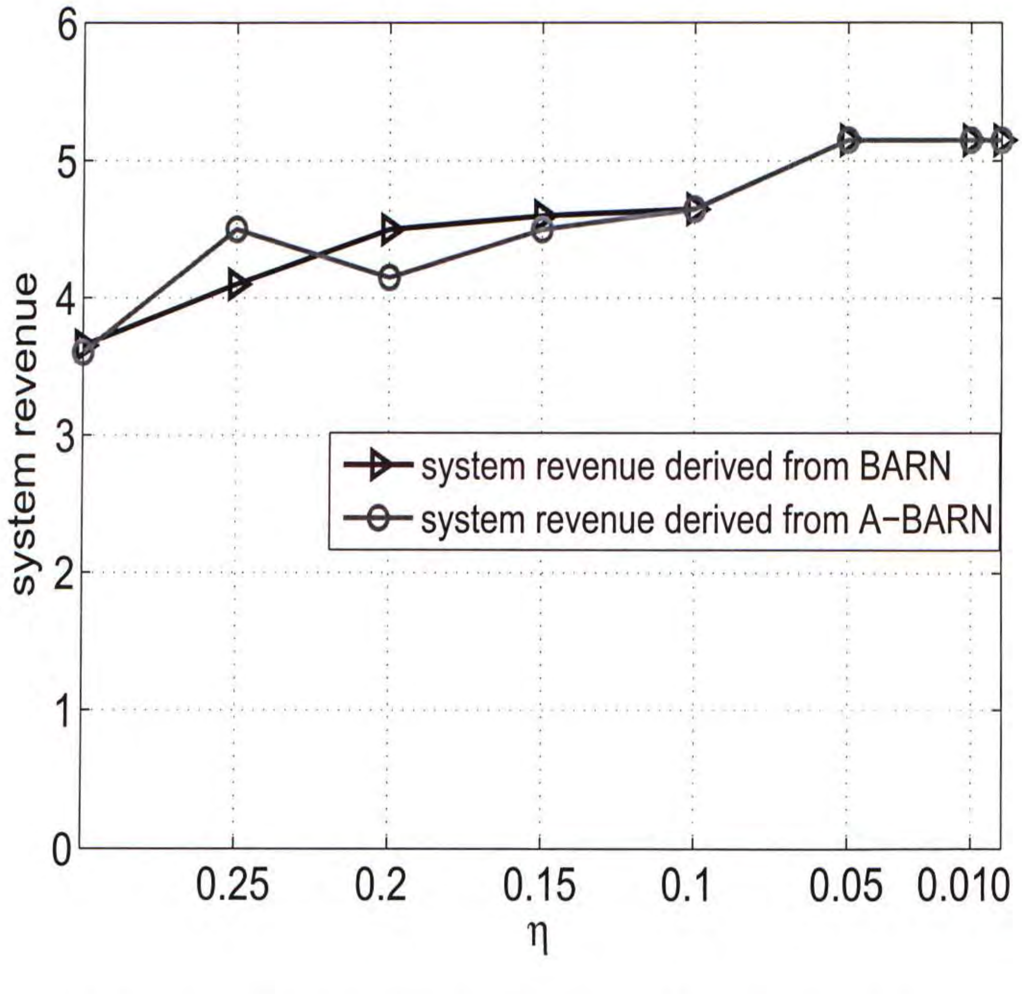

Figure 5.4: The system revenue versus normalized error tolerance η.

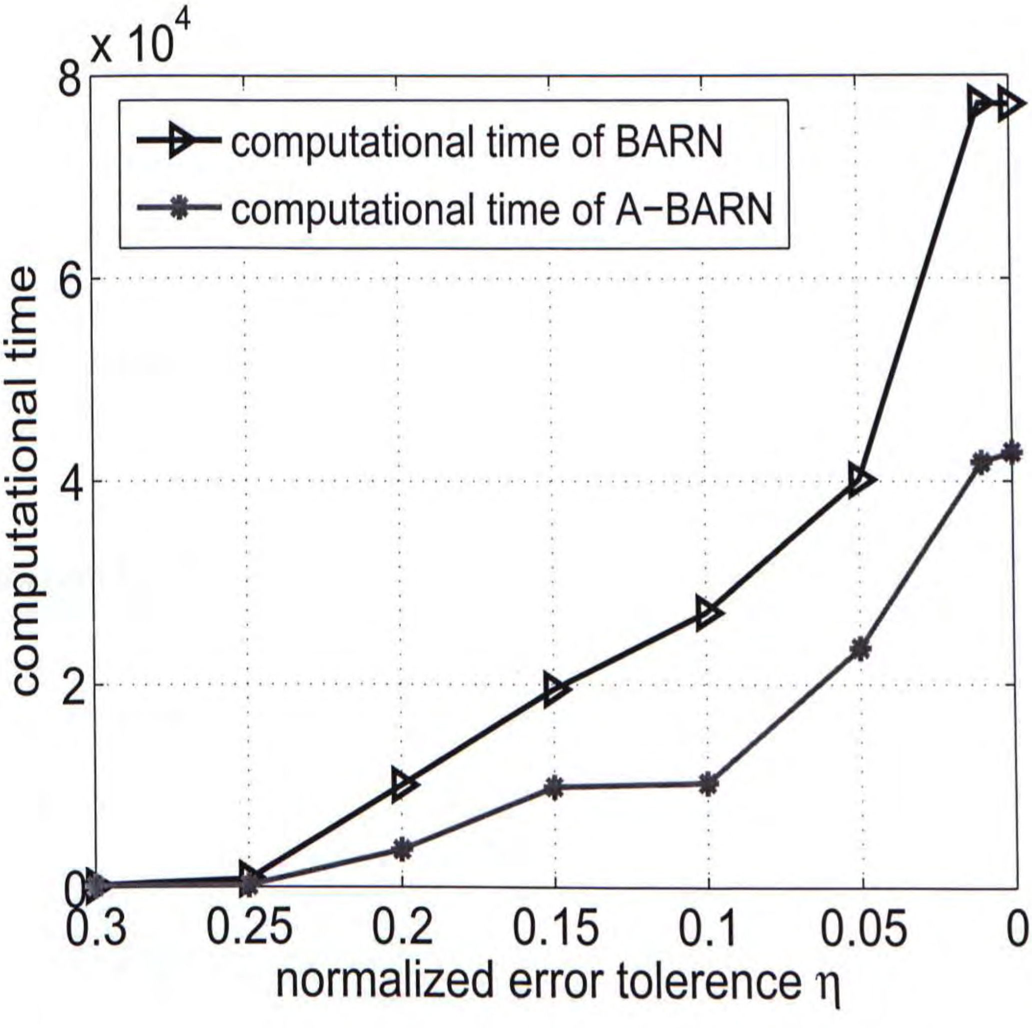

Figure 5.5: The computational time versus normalized error tolerance η.

Chapter 6

Discussions

So far, we have focused on the downlink communication of a multi-cell network.In this chapter, we extent the single-stage formulation to the uplink communication (i.e., transmission from MS to BS) in multi-cell networks.

6.1 Resource Allocation In The Uplink Multi-cell Networks

The JBAPC problem in the uplink case can be formulated as

$$
\begin{aligned}
[\boldsymbol{UBA}]: \quad & \max_{\boldsymbol{X},\boldsymbol{P}} \sum_{i=1}^{I}\sum_{j=1}^{J} \lambda_i x_{ij} \\
s.t.\ & 0 \leq \sum_{j=1}^{J} p_{ij} x_{ij} \leq P_i^{\max}, && (6.1)\\
& \frac{p_{ij} x_{ij} g_{ij}}{\sum_{\forall i' \neq i} \sum_{\forall j' \neq j} p_{i'j'} x_{i'j'} g_{i'j} + \zeta \sum_{\forall i' \neq i} p_{i'j} x_{i'j} g_{i'j} + \sigma_i^2} \geq \Gamma_i x_{ij}, && (6.2)\\
& \sum_{j=1}^{J} x_{ij} \leq 1,\\
& x_{ij} \in \{0,1\};\\
& 0 \leq p_{ij} \leq p_{ij} x_{ij}, \quad \forall i \in \mathcal{I}, \forall j \in \mathcal{J}..
\end{aligned}
$$

Herein, x_{ij} has the same meaning as that in the downlink case, p_{ij} denotes the transmission power with which MS i communicates to BS j, g_{ij} denotes the channel gain between MS i and BS j in the uplink case and $P_i^{\max}$ denotes the maximum transmission power of MS i. Constraints (6.1) corresponds to the maximum uplink power of the MSs, and (6.2) corresponds to the minimal SINR requirement of the MSs in the uplink case.

Correspondingly, the power control problem is formulated as

$$\begin{aligned}
[\boldsymbol{UPC}]: \quad & \min_{\boldsymbol{P}} \sum_{i=1}^{I} \sum_{j=1}^{J} p_{ij}\bar{x}_{ij} \\
& s.t.\ 0 \le \sum_{j=1}^{J} p_{ij}\bar{x}_{ij} \le P_i^{\max}, \\
& \frac{p_{ij}\bar{x}_{ij}g_{ij}}{\sum_{\forall i' \ne i} \sum_{\forall j' \ne j} p_{i'j'}\bar{x}_{i'j'}g_{i'j} + \zeta \sum_{\forall i' \ne i} p_{i'j}\bar{x}_{i'j}g_{i'j} + \sigma_i^2} \ge \Gamma_i \bar{x}_{ij}, \\
& 0 \le p_{ij} \le p_{ij}\bar{x}_{ij}, \quad \forall i \in \mathcal{I}, \forall j \in \mathcal{J}.
\end{aligned} \tag{6.3}$$

6.2 JBAPC Problem In The Uplink Multi-cell Networks

After presenting the JBAPC problem formulation for the uplink case, we propose the single-stage formulation.

$$\begin{aligned}
[\boldsymbol{USSP}]: \quad & \min_{\boldsymbol{X},\boldsymbol{P}} \Phi(\boldsymbol{P},\boldsymbol{X}) = \epsilon \sum_{i=1}^{I} \sum_{j=1}^{J} p_{ij} + (1-\epsilon) \sum_{i=1}^{I} \sum_{j=1}^{J} w_{ij}(1-x_{ij}) \\
& s.t.\ 0 \le \sum_{j=1}^{J} p_{ij} \le P_i^{\max}, \\
& \frac{p_{ij}g_{ij} + \delta^{-1}(1-x_{ij})}{\sum_{\forall i' \ne i} \sum_{\forall j' \ne j} p_{i'j'}g_{i'j} + \zeta \sum_{\forall i' \ne i} p_{i'j}g_{i'j} + \sigma_i^2} \ge \Gamma_i, \\
& \sum_{j=1}^{J} x_{ij} \le 1, \\
& x_{ij} \in \{0,1\}, \\
& p_{ij} \ge 0, \forall i \in \mathcal{I}, \forall j \in \mathcal{J}.
\end{aligned}$$

Similar to the downlink case, [USSP] is equivalent to [UBA] and [UPC] with properly chosen the parameters w_{ij} ϵ and δ. Specifically,

w_i is obtained by scaling up all λ_i's by the same constant ρ, so that all w_i's are positive integers, i.e.,

$$\rho \triangleq \frac{w_i}{\lambda_i} = \frac{w_{i'}}{\lambda_{i'}} \; \forall i \in \mathcal{I}. \tag{6.4}$$

This is tenable as long as λ_i's are rational numbers. Moreover, ϵ and δ are constants satisfying

$$0 \leq \epsilon < \frac{1}{\sum_{i=1}^{I} P_i^{\max} + 1}; \tag{6.5}$$

and

$$0 < \delta \leq \min_i \frac{\Gamma_i^{-1}}{(I-1)\hat{p} \cdot \hat{g} + \sigma_i^2}, \tag{6.6}$$

where $\hat{p} = \max_j\{P_j^{max}\}$, $\hat{g} = \max_{ij}\{g_{ij}\}$ and $\breve{w} = \min_i\{w_i\}$.

To prove that problems [UBA] and [UPC] are equivalent to the single-stage problem [USSP], one can refer to the proof of Lemma 1 for the downlink case.

Note that the structure of [USSP] is similar to that of [SSP]. Therefore, both BARN and A-BARN can be used to solve [USSP].

□ **End of chapter.**

Chapter 7

Conclusion

7.1 Conclusion Of This Thesis

This paper considers the JBAPC problem in wireless multi-cell systems. By transforming the BS association and power control problems into the single-stage optimization problem [SSP], we can simultaneously maximize the system revenue and minimize the total transmission power consumption. The single-stage problem [SSP] is efficiently and optimally solved by the proposed BARN algorithm. An error tolerance τ is introduced to facilitate a convenient trade-off between system performance and computational time. Our analysis shows that when τ is sufficiently small, the proposed algorithm converges to a solution that yields the same system revenue as the global optimal value. To further reduce the computational complexity, a novel algorithm, A-BARN, is proposed. In particular, A-BARN solves a relaxed master problem instead of an integer programming problem in each iteration. Our numerical results show that A-BARN can converge within a much shorter computational time than the BARN algorithm. Besides, both BARN and A-BARN are guaranteed to converge to a τ-optimal solution of the JBAPC problem. By setting $\tau = 0$, both algorithms converge to the exact global optimal solution within finite time.

7.2 Future Work

The proposed analytic model of the JBAPC problem helps network operators achieve business objectives such as maximizing the MSs in service, increasing the network revenue and improving the service quality.

Besides, the single-stage formulation in this paper is one example of solving two correlated optimization problems. It is an interested future research direction to study the optimization of multiple correlated optimization problems.

Another future research topic would be the development of variants of the BARN algorithm to expedite the convergence and reduce the computational complexity.

□ **End of chapter.**

Appendix A

The Proof

A.1 Proof of Lemma 1

Let $(\tilde{\boldsymbol{X}}, \tilde{\boldsymbol{P}})$ be the optimal solution of [BA] and $(\boldsymbol{X}^*, \boldsymbol{P}^*)$ be the optimal solution to [SSP]. To prove Lemma 1, we prove that the maximum system values obtained by $\tilde{\boldsymbol{X}}$ and $\boldsymbol{X}^*$ are equal. That is,

$$\sum_{i=1}^{I}\sum_{j=1}^{J}\lambda_i\tilde{x}_{ij} = \sum_{i=1}^{I}\sum_{j=1}^{J}\lambda_i x^*_{ij}. \tag{A.1}$$

Firstly, we need to prove that $(\tilde{\boldsymbol{X}}, \tilde{\boldsymbol{P}})$ is also feasible to [SSP] [1] while $\sum_{i=1}^{I}\sum_{j=1}^{J}\lambda_i\tilde{x}_{ij} \leq \sum_{i=1}^{I}\sum_{j=1}^{J}\lambda_i x^*_{ij}$. According to constraints (2.6), $\tilde{p}_{ij} = 0$ for each $\tilde{x}_{ij} = 0$ in [BA]. Therefore, when $\tilde{x}_{ij} = 1$, constraints (2.3) are reduced to

$$\frac{\tilde{p}_{ij}g_{ij}}{\sum\limits_{\forall i'\neq i}\sum\limits_{\forall j'\neq j}\tilde{p}_{i'j'}g_{ij'} + \zeta\sum\limits_{\forall i'\neq i}p^*_{i'j}g_{ij} + \sigma_i^2} \geq \Gamma_i. \tag{A.2}$$

This is equivalent to constraints (2.12) with $\tilde{x}_{ij} = 1$. Therefore, $(\tilde{\boldsymbol{X}}, \tilde{\boldsymbol{P}})$ satisfies con-

[1] The term 'feasible' denotes that a solution satisfies all the constraints of the considered optimization problem

straints (2.12) for $\tilde{x}_{ij} = 1$. Otherwise, since $\delta \leq \min_i \frac{\Gamma_i^{-1}}{(I-1)\hat{p}\cdot\hat{g}+\sigma_i^2}$ (see (2.18)), we have

$$\frac{\tilde{p}_{ij}g_{ij} + \delta^{-1}}{\sum_{i'=1}^{I}\sum_{\forall j' \neq j} \tilde{p}_{i'j'}g_{ij'} + \zeta \sum_{\forall i' \neq i}\sum_{j=1}^{J} \tilde{p}_{i'j}g_{ij} + \sigma_i^2} \geq \frac{\Gamma_i\big((I-1)\hat{p}\cdot\hat{g} + \sigma_i^2\big)}{\sum_{i'=1}^{I}\sum_{\forall j' \neq j} \tilde{p}_{i'j'}g_{ij'} + \zeta \sum_{\forall i' \neq i}\sum_{j=1}^{J} \tilde{p}_{i'j}g_{ij} + \sigma_i^2} \geq \Gamma_i. \tag{A.3}$$

when $\tilde{x}_{ij} = 0$. This implies that $(\tilde{\boldsymbol{X}}, \tilde{\boldsymbol{P}})$ satisfies constraints (2.12) for $\tilde{x}_{ij} = 0$. Besides, $(\tilde{\boldsymbol{X}}, \tilde{\boldsymbol{P}})$ also satisfies constraints (2.11) and (2.13)-(2.15) in [SSP], respectively, for that these constraints are consistent with constraints (2.2), (2.4)-(2.6). Therefore, $(\tilde{\boldsymbol{X}}, \tilde{\boldsymbol{P}})$ is a feasible solution to [SSP].

Let $(\boldsymbol{X}^*, \boldsymbol{P}^*)$ be a global optimal solution to [SSP]. Suppose that $\tilde{\boldsymbol{X}}$ yields a system revenue greater than $\boldsymbol{X}^*$ does. That is,

$$\sum_{i=1}^{I}\sum_{j=1}^{J} \lambda_i \tilde{x}_{ij} > \sum_{i=1}^{I}\sum_{j=1}^{J} \lambda_i x_{ij}^*. \tag{A.4}$$

Since $\rho = \frac{w_{ij}}{\lambda_i}$ (see (2.16)), by multiplying ρ at the both sides of (A.4), we get

$$\sum_{i=1}^{I}\sum_{j=1}^{J} w_{ij} \tilde{x}_{ij} > \sum_{i=1}^{I}\sum_{j=1}^{J} w_{ij} x_{ij}^*. \tag{A.5}$$

With $\tilde{x}_{ij}$'s and x_{ij}^*'s being binary variables, and w_{ij} being positive integers, (A.5) implies that $\tilde{\boldsymbol{X}}$ achieves a system revenue that is at least one more unit than that is achieved by $\boldsymbol{X}^*$. As a result, (A.5) can be tightened to

$$\sum_{i=1}^{I}\sum_{j=1}^{J} w_{ij} \tilde{x}_{ij} - 1 \geq \sum_{i=1}^{I}\sum_{j=1}^{J} w_{ij} x_{ij}^*. \tag{A.6}$$

Note that $0 \leq \epsilon < \frac{1}{\sum_{j=1}^{J} P_i^{\max}+1}$ (see (2.17)). Hence, we have $\epsilon \sum_{i=1}^{I}\sum_{j=1}^{J} p_{ij} \leq \epsilon \sum_{j=1}^{J} P_j^{\max} < (1-\epsilon)$. This, together with (A.6), imply that the objective function values corresponding to $(\boldsymbol{X}^*, \boldsymbol{P}^*)$ and $(\tilde{\boldsymbol{X}}, \tilde{\boldsymbol{P}})$ satisfy

$$\begin{aligned}
&\epsilon \sum_{i=1}^{I}\sum_{j=1}^{J} \tilde{p}_{ij} + (1-\epsilon)\sum_{i=1}^{I}\sum_{j=1}^{J} w_{ij}(1-\tilde{x}_{ij}) \\
&\leq \epsilon \sum_{i=1}^{I}\sum_{j=1}^{J} \tilde{p}_{ij} + (1-\epsilon)\sum_{i=1}^{I}\sum_{j=1}^{J} w_{ij}(1-x^*_{ij}) - (1-\epsilon) \\
&< (1-\epsilon)\sum_{i=1}^{I}\sum_{j=1}^{J} w_{ij}(1-x^*_{ij}) \\
&\leq \sum_{i=1}^{I}\sum_{j=1}^{J} p^*_{ij} + (1-\epsilon)\sum_{i=1}^{I}\sum_{j=1}^{J} w_{ij}(1-x^*_{ij}).
\end{aligned} \tag{A.7}$$

Consequently, $(\tilde{\boldsymbol{X}}, \tilde{\boldsymbol{P}})$ yields a smaller objective value than $(\boldsymbol{X}^*, \boldsymbol{P}^*)$ does. Since $(\tilde{\boldsymbol{X}}, \tilde{\boldsymbol{P}})$ is also a feasible solution to [SSP], the inequality (A.7) contradicts the assumption that $(\boldsymbol{X}^*, \boldsymbol{P}^*)$ is a global optimal solution of [SSP]. Therefore, we have

$$\sum_{i=1}^{I}\sum_{j=1}^{J} \lambda_i \tilde{x}_{ij} \leq \sum_{i=1}^{I}\sum_{j=1}^{J} \lambda_i x^*_{ij}. \tag{A.8}$$

Next, we show that $(\boldsymbol{X}^*, \boldsymbol{P}^*)$ is feasible to [BA] and $\sum_{i=1}^{I}\sum_{j=1}^{J} \lambda_i x^*_{ij} \leq \sum_{i=1}^{I}\sum_{j=1}^{J} \lambda_i \tilde{x}_{ij}$. Note that for each $x^*_{ij} = 0$, constraints (2.12) are reduced to

$$\frac{p^*_{ij} g_{ij} + \delta^{-1}}{\sum_{i'=1}^{I}\sum_{\forall j' \neq j} p^*_{i'j'} g_{ij'} + \zeta \sum_{\forall i' \neq i}\sum_{j=1}^{J} p^*_{i'j} g_{ij} + \sigma_i^2} \geq \frac{\Gamma_i\big((I-1)\hat{p}\cdot\hat{g} + \sigma_i^2\big)}{\sum_{i'=1}^{I}\sum_{\forall j' \neq j} p^*_{i'j'} g_{ij'} + \zeta \sum_{\forall i' \neq i}\sum_{j=1}^{J} p^*_{i'j} g_{ij} + \sigma_i^2} \geq \Gamma_i. \tag{A.9}$$

Considering that $\hat{p} = \max_i\{P_i^{max}\}$ and $\hat{g} = \max_{ij}\{g_{ij}\}$, inequations (A.9) always hold regardless of the corresponding power p^*_{ij}. Hence, the minimization nature of [SSP] will force p^*_{ij} to be 0. Therefore, the denominator of (2.12) is equal to the denominator of (2.3). When $x^*_{ij} = 1$, (2.12) is reduced to

$$\frac{p^*_{ij} g_{ij}}{\sum_{i'=1}^{I}\sum_{\forall j' \neq j} p^*_{i'j'} g_{ij'} + \zeta \sum_{\forall i' \neq i}\sum_{j=1}^{J} p^*_{i'j} g_{ij} + \sigma_i^2} = \frac{p^*_{ij} x^*_{ij} g_{ij}}{\sum_{\forall i' \neq i}\sum_{\forall j' \neq j} p^*_{i'j'} x^*_{i'j'} g_{ij'} + \zeta \sum_{\forall i' \neq i} p^*_{i'j} x^*_{i'j} g_{ij} + \sigma_i^2} \geq \Gamma_i x^*_{ij}, \tag{A.10}$$

which is equivalent to (2.3). Similarly, we can also show that $(\boldsymbol{X}^*, \boldsymbol{P}^*)$ satisfies constraints (2.2) and (2.4)-(2.6), respectively. This implies that $(\boldsymbol{X}^*, \boldsymbol{P}^*)$ is feasible to [BA]. Since $(\tilde{\boldsymbol{X}}, \tilde{\boldsymbol{P}})$ is the optimal solution to [BA], we have

$$\sum_{i=1}^{I}\sum_{j=1}^{J}\lambda_i x_{ij}^* \leq \sum_{i=1}^{I}\sum_{j=1}^{J}\lambda_i \tilde{x}_{ij}. \tag{A.11}$$

(A.8) and (A.11) imply that $\sum_{i=1}^{I}\sum_{j=1}^{J}\lambda_i x_{ij}^* = \sum_{i=1}^{I}\sum_{j=1}^{J}\lambda_i \tilde{x}_{ij}$. In other words, the optimal solution to [SSP] yields the maximum system revenue of [BA].

We now prove that $\boldsymbol{P}^*$ yields the minimal transmission power among the $\hat{\boldsymbol{X}}$'s in [PC], where $\hat{\boldsymbol{X}}$'s have the same system value, i.e., $\sum_{i=1}^{I}\sum_{j=1}^{J}\lambda_i \hat{x}_{ij} = \sum_{i=1}^{I}\sum_{j=1}^{J}\lambda_i x_{ij}^*$. We can prove that $(\hat{\boldsymbol{X}}, \hat{\boldsymbol{P}})$ is feasible to [SSP], and $\boldsymbol{P}^*$ is feasible to [PC] with the particular $\boldsymbol{X}^*$, following the similar process of the proof for the solutions of [BA] and [SSP]. Then, to prove that $\boldsymbol{P}^*$ is the minimum transmission power required to generate the maximum system revenue, we need to show that $\boldsymbol{P}^*$ yields the minimum transmission power, i.e.

$$\sum_{i=1}^{I}\sum_{j=1}^{J}p_{ij}^* \leq \sum_{i=1}^{I}\sum_{j=1}^{J}\hat{p}_{ij}. \tag{A.12}$$

Since $(\boldsymbol{X}^*, \boldsymbol{P}^*)$ is a global solution to [SSP], we have

$$\Phi(\hat{\boldsymbol{X}}, \hat{\boldsymbol{P}}) \geq \Phi(\boldsymbol{X}^*, \boldsymbol{P}^*). \tag{A.13}$$

Together with $\sum_{i=1}^{I}\sum_{j=1}^{J}\lambda_i \hat{x}_{ij} = \sum_{i=1}^{I}\sum_{j=1}^{J}\lambda_i x_{ij}^*$, it follows that $\sum_{i=1}^{I}\sum_{j=1}^{J}p_{ij}^* \leq \sum_{i=1}^{I}\sum_{j=1}^{J}\hat{p}_{ij}$.

On the other hand, since $\hat{\boldsymbol{P}}$ is the optimal solution to [PC] for any $\hat{\boldsymbol{X}}$ satisfying $\sum_{i=1}^{I}\sum_{j=1}^{J}\lambda_i x_{ij}^* = \sum_{i=1}^{I}\sum_{j=1}^{J}\lambda_i \tilde{x}_{ij}$, we have $\sum_{i=1}^{I}\sum_{j=1}^{J}p_{ij}^* \geq \sum_{i=1}^{I}\sum_{j=1}^{J}\hat{p}_{ij}$. Therefore, equation $\sum_{i=1}^{I}\sum_{j=1}^{J}p_{ij}^* = \sum_{i=1}^{I}\sum_{j=1}^{J}\hat{p}_{ij}$ holds, and the optimal solution to [SSP] yields the minimum transmission power of Problem 2 for a particular $\boldsymbol{X}^*$. ■

A.2 Proof of Lemma 3

Let $(\boldsymbol{X}^*, \boldsymbol{P}^*)$ be the optimal solution of [SSP], Z^* be the corresponding objective value and θ^* be the dual value of $\boldsymbol{P}^*$. To prove Lemma 2, we show that $LB^{(k)} \leq Z^* \leq UB^{(k)}$ for all k. First, we prove that $LB^{(k)}$ is the lower bound of [SSP].

Let $(\theta^{(k)}, \boldsymbol{X}^{(k)})$ be the solution of [MSP] at the kth iteration. Assume that $LB^{(k)} > Z^*$. Since [MSP] is a relaxed problem, $(\theta^*, \boldsymbol{X}^*)$ should be one of the feasible solutions of [MSP]. Besides, $(\theta^{(k)}, \boldsymbol{X}^{(k)})$ is the optimal solution at the kth iteration. Hence, we have $Z^* = \epsilon\theta^* + (1-\epsilon)\sum_{i=1}^{I}\sum_{j=1}^{J}(1-x_{ij}^*) \geq \epsilon\theta^{(k)} + (1-\epsilon)\sum_{i=1}^{I}\sum_{j=1}^{J}(1-x_{ij}^{(k)}) = L^{(k)} = LB^{(k)}$, which is a contradiction to the assumption that $LB^{(k)} > Z^*$. Therefore, $LB^{(k)}$ is the lower bound of [SSP].

Next, we prove that $UB^{(k)}$ is the upper bound of [SSP]. Note that $U^{(k)}$ is either finite or infinite depending on the boundness of [DSP]. If $U^{(s)} = +\infty$, $\forall 0 \leq s \leq k$, then $UB^{(k)} = \min_{0\leq s\leq k}\{U^{(s)}\epsilon + (1-\epsilon)\sum_{i=1}^{I}\sum_{j=1}^{J} w_{ij}(1-x_{ij}^{(s)})\} = +\infty$. In this case, it is trivial that $UB^{(k)}$ is the upper bound of [SSP]. On the other hand, when [DSP] is bounded and $UB^{(k)} < +\infty$, to prove that $UB^{(k)}$ is the upper bound of [SSP], we first assume $UB^{(k)} < Z^*$. Due to the strong duality

$$U^{(\bar{s})} = g(\boldsymbol{X}^{(\bar{s})}, \boldsymbol{\alpha}^{(\bar{s})}, \boldsymbol{\beta}^{(\bar{s})}) = \sum_{i=1}^{I}\sum_{j=1}^{J} p_{ij}^{(\bar{s})}, \tag{A.14}$$

where $\boldsymbol{P}^{(\bar{s})} = [p_{ij}^{(\bar{s})}]$ is the optimal solution to [PSP] with $\boldsymbol{X}^{(\bar{s})}$. Hence, we have

$$\begin{aligned} UB^{(k)} &= \epsilon U^{(\bar{s})} + (1-\epsilon)\sum_{i=1}^{I}\sum_{j=1}^{J} w_{ij}(1-x_{ij}^{(\bar{s})}) \\ &= \epsilon\sum_{i=1}^{I}\sum_{j=1}^{J} p_{ij}^{(\bar{s})} + (1-\epsilon)\sum_{i=1}^{I}\sum_{j=1}^{J} w_{ij}(1-x_{ij}^{(\bar{s})}) \\ &< Z^* = \epsilon\sum_{i=1}^{I}\sum_{j=1}^{J} p_{ij}^* + (1-\epsilon)\sum_{i=1}^{I}\sum_{j=1}^{J} w_{ij}(1-x_{ij}^*), \end{aligned} \tag{A.15}$$

where the inequality is due to the assumption that $UB^{(k)} < Z^*$. (A.15) implies that there exists a solution $(\boldsymbol{X}^{(\bar{s})}, \boldsymbol{P}^{(\bar{s})})$ that yields a smaller objective value than $(\boldsymbol{X}^*, \boldsymbol{P}^*)$ does. This is a contradiction to the fact that $(\boldsymbol{X}^*, \boldsymbol{P}^*)$ is the optimal solution to [SSP].

Hence, we can claim that $\min_{0 \le s \le k} \{U^{(s)}\epsilon + (1-\epsilon)\sum_{i=1}^{I}\sum_{j=1}^{J} w_{ij}(1-x_{ij}^{(s)})\}$ is the upper bound of [SSP]. ■

□ **End of chapter.**

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