Capital Constrained Supply Chain Problem

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Abstract

In this thesis, we consider a supply chain problem with one supplier and one retailer where the retailer is facing financial constraints. We examine the optimal decisions of the retailer and the supplier under different financing schemes, such as bank loan or trade credit.

We analytically characterize the retailer's best response under the multi-period setting and illustrate the supplier's optimal decisions in the simplified single period problem. We also demonstrate the impacts of different financing sources on the supply chain performance. Our numerically studies show that as long as the default risk of the capital constrained retailer is sufficiently low, the supply chain can achieve a higher total profit under the trade credit policy than under the bank loan scheme.

Key Words: capital constrained, supply chain management, trade credit, bank loan

摘　要

在本論文中，我們主要研究一個包含單一供應商與單一零售商的多期供應鏈模型。其中零售商面臨資金短缺的問題，供應商負責起草批發價契約。本論文意在探討在不同的貸款方案下供應商與零售商的最優決策。貸款方案之一為零售商，即被資金約束者，向第三方金融機構，例如銀行等，提出貸款；其二為供應商向零售商提出的較早結算折扣合約，令其可以延遲交付貨款之時間從而緩解現金流的壓力。

在給定批發價合約的情況下，我們刻畫了零售商的最優回應，并闡釋了供應鏈各成員在一期模型中的最優決策。本文之研究結果發現，資金的限制會導致供應商向零售者收取較高的批發價，從而引發供應鏈雙方的利潤損失。比外，我們亦利用數據分析的方法以演示不同貸款方案對供應鏈整體效率的影響。研究結果表明，只要零售商的現金狀況不是極為緊張導致其有極大概率破產，供應鏈雙方能在貿易信貸的作用下更好的協調配合，因而提高整個供應鏈的效率。

關鍵詞: 資金約束，供應鏈管理，貿易信貸，銀行貸款

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Chapter 1

Introduction

A supply chain is a network of partners that produces raw materials, subassemblies, and then distributes the finished products to end customers. Along this chain, there are three major flows: material, information, and capital. The traditional research of supply chain management has long focused on the flows of materials and information to improve supply chain efficiencies — sharply reduced lead times, lower inventories, more collaboration on planning and forecasting and improved customer service, while ignoring the capital flows. Most of the prior studies almost inevitably assume that there is sufficient working capital available. In reality, however, there are many situations in which suppliers or retailers are facing capital constraints and therefore limited in their operational decisions. An important case arises when a firm is the small-sized or nascent. For example, PenAgain, which is a small pen making firm started by two young 34-year old entrepreneurs, struggled with a copious orders (over 470,000 units) from Wal-Mart because of the lack of budget. For complementary reasons, financial crisis and economic downturns are also a lead to firms' illiquidity. In these circumstances, the supply chain management neglecting the financial constraints is not sound enough to carry out its function, which exercises negative influence over the overall supply chain performance. One natural question arises, "what are the impacts of capital constraints on the supply chain?"

In the presence of capital constraints, firms usually finance to maintain their daily operations. According to the report by Aberdeen Research, supply chain finance is gaining in more and more importance. Around 74% of respondent corporations have already introduced or planning to do supply chain finance to optimize both the material and financial flows within the supply chain. An obvious financing source is through short-term loans or corporate bonds from third-party financial institutions, such as banks. For example, Motorola, a fortune 100 company, had $9.775 billion in equity while $15.825 billion in debt in 2009. However, SMEs (Small and Medium Enterprises) usually have limited access to short-term bank loan or been offered a relatively high interest rate, as a result of their lack of collateral and the tenuous nature of their business establishment (Vandenberg 2003). They resort to factors or trade credit financing to fund their operations. Not only the nascent or SMEs, according to the first report on the supply chain finance published by Demica, 73% of large European corporations are looking to extend payment terms with their suppliers in order to conserve cash. In the UK, 63% of companies overall are trying to extend credit, compared with 48% in Germany. On the other hand, suppliers are eager to sell their products to the intermediaries and end-customers. Due to the harmful effects on supply chain performance resulted from capital constraints, many suppliers are willing to collaborate with the retailers financially. In reality, allowing retailers to delay payment for goods already delivered is a common business practice (Peterson and Rajan 1997). For example, Wal-Mart, a fortune 500 Corporation, uses trade credit as a preferable financing source. In its balance sheet on January, 2009, Wal-Mart had \$28.8 billion accounts payable amounting to 75% of its total inventory. In addition, Hewlett-Packard (referred to as HP), IBM, Sony etc., all provide trade credit financing to their distributors or resellers. Especially for HP, the financing of its products could equal up to 100% of the value of the borrower's inventory (Zhou and Groenevelt 2007). One advantage of using trade credit to mitigate the impacts of financial limitations is that as an inter-supply chain contract, the amount of debt and the timing of payback is closely related to the timing of procurement and selling such that the retailer will face lower pressure from paying back the debt before his profit is realized.

Banking financing and trade credit financing differ in two main aspects. One is the party who takes the credit risk. Due to the uncertainty of demand, it is possible that the borrower cannot fully pay off the debt and in this circumstance, the debt holder will bear some loss. The other one is the flexibility for the supplier. In trade credit financing, the supplier decides both the early payment discount and the wholesale price such that she has the opportunity to balance her profit and the corresponding credit risk. On the other hand, in bank financing scheme, the supplier offers a traditional wholesale price contract in which the retailer must pay in advance.

These differences bring several interesting problems to the forefront of managing capital constrained supply chain: What is the optimal wholesale price under a trade credit or a bank financing scheme? What are the impacts of different financing channels on the borrower's operational decisions, supply chain efficiency and the profits of each player?

To answer the above questions, we set up a capital constrained supply chain model with one supplier and one capital constrained retailer under three different financing schemes: basic setting - capital constrained without financing; bank loan setting - bank financing scheme; and trade credit setting - supplier financing scheme. After reviewing the literature related to this thesis in Chapter 2, we describe the models in Chapter 3. In Chapter 4, we study the retailer's problem for a given wholesale price and bank loan/trade credit interest rate. We demonstrate that, in the presence of bank loan or trade credit, the retailer's optimal ordering quantity may not be monotone in his initial budget. We also extend this problem to a two-period setting. Under the assumption that the demand has a log-concave density, we show the the retailer's profit function is still unimodal in his ordering quantity and financial decision. In Chapter 5, we introduce the mathematical formulation and show the results from the supplier's perspective. We show that in the basic setting where the retailer is unable to raise fund, the supplier's profit is unimodal, while in the bank loan and trade credit case, the supplier's objective function has at most two modes. In Chapter 6, numerical experiments are conducted to demonstrate our analytical results and generate more managerial insights. We find that if the interest rate of bank loan and trade credit are the same and exogenously given, then there is no dominating financial scheme for the retailer. For the supply chain, its total profit is much larger when the retailer can seek external financing than that when the retailer can not. Finally, we summarize our main findings and propose several future research directions in Chapter 7.

Chapter 2

Literature Review

Our model relates to three bodies of literature. In §2.1, we review the literature on models related to operations and finance interface; in §2.2, we go over the literature on trade credit, and the papers related to supply chain contracts are listed in §2.3.

2.1 Operations and Finance Interface

2.1.1 Single Period Setting

Recently, a growing trend towards jointly considering financial and operational decisions has arisen in the operations management community. An early semina working paper by Xu and Birge (2004) brings capital constraints into a traditional newsvendor model. Their model focuses on the operational and financial decisions of the capital constrained retailer, and demonstrates how a firm's operational decisions are influenced by the financial constraints under its optimal capital structure. They first show that capital structure has no influence on the production decisions in a perfect market, which follows from the Modigliani and Miller (1958)'s theory (This is the seminal work in capital structure which shows that a firm's investment and financial decisions can be made separately, referred to as the M&M theory). Then their work reveals the importance of making production and financial decisions simultaneously by demonstrating the significant impacts of misidentifying the company's optimal leverage ratio on the firm's value. Another work by Ding, et al. (2004) also studies an integrated operational and financial model. They focus on the hedging decisions faced by a global company who sells to both home and foreign markets. Our work is distinguished from theirs by focusing on the whole supply chain rather than the financial constrained retailer.

Buzacott and Zhang (2004) investigate a model incorporating asset-based financing into production decisions. They first introduce a multi-period inventory model with deterministic demands to explore the relationship between finance and operations via the integrations between systems to control the material flows and cash flows. Then they study a Stackelberg game between a capital-constrained newsvendor and a bank, who makes a lending decision based on the retailers' assets which are monitored by their balance sheet and associated accounts. Their work suggests that banks are better off using asset-based financing with appropriately chosen parameters, while retailers are able to enhance their cash return over what it would be if the retailers only use their own capital. Dada and Hu (2008) study a similar problem which analyzes the game between a bank and a capital constrained retailer. They show that if the cost of borrowing is not too high, in the Stackelberg game equilibrium, the capital-constrained newsvendor borrows funds to order to an inventory level which is lower than the traditional newsvendor's optimal ordering amount, and the bank charges an interest rate which decreases in the initial capital level of the newsvendor.

In some recent working papers, researchers start to look at how financial constraints and sources influence the supply chain performance. Lai et al. (2009) consider a financial constrained supply chain operating under different modes: preorder mode, consignment mode and the combination of these two. In their model, the supplier makes a take-it-or-leave-it offer to the retailer. They show that if the supplier is capital constrained, the combination mode is preferred because the inventory investment sharing between the supplier and the retailer can lower the inventory risk. Caldentey and Haugh (2009) study the performance of a stylized "selling to the newsvendor" model with a retailer whose profit partially depends on a financial market. In their work, the supplier offers a list of supply contracts identifying the execution time and the wholesale prices. The retailer makes decision on the time for executing the contract. Specifically, in the flexible contract with hedging, the retailer does have access to financial markets, so he can dynamically trade in the financial market to mitigate the effects of capital constraints. They extend the existing literature by including financial markets as a source of public information upon which procurement contracts can be written, and a way of financial hedging to mitigate the effects of capital constraints.

Babich et al. (2006) also study a single period financial constrained supply chain problem. The manufacturer can finance externally or borrow from the suppliers by trade credit. In their model, the financial decision of the capital constrained manufacturer is to decide how many suppliers to contract with.

They show that different financing sources, such as bank loans and trade credit, are substitutable so that the firm contracts with more suppliers if bank financing is not available. Different from our model, the wholesale prices in the supply contracts are exogenously given in their work.

The papers by Caldentey and Chen (2008), Yang and Birge (2009), Zhou and Groenevelt (2007) and Kouvelis and Zhao (2009) are most related to our simplified single period problem. They all consider a two-echelon supply chain in which the retailer is capital constrained, and exam the supply chain performance under different short-term supply contracts.

In Caldentey and Chen (2008), they consider two financing schemes for the capital constrained retailer. One is "internal financing" in which the supplier collects only a fraction of the wholesale price when the contract is signed, and receives the remaining payment after the demand is realized. This contract is slightly different from ours. In their financing scheme, the supplier sets an asset-based credit limit and the retailer receives zero interest loans from the supplier, while in our model the supplier charges interest for the delayed payment. The other financing scheme is "external financing" in which a bank offers loans to the retailer. The bank sets an interest rate based on a competitive financial market. They show that in most instances both the supplier and the retailer are better off by using internal financing than by relying on external financing. In addition, the value of offering internal financing decreases in the retailer's initial capital level.

Yang and Birge (2009) study a stylized single period supply chain problem in which the cash constrained retailer has multiple financing sources. Facing a trade credit contract, the retailer finances his inventory using a portfolio of initial cash, trade credit and short-term loans. They demonstrate that trade credit plays an essential role to act as a risk-sharing mechanism and provides some empirical evidence to support their conclusion.

In the bank financing schemes mentioned above, the supplier is not connected to the bank. While in Zhou and Groenevelt (2008), the supplier teams up with a bank, offers interest-free loans to the capital constrained retailer and pays interests for the retailer. The bank only sets a credit limit of the loan based on the retailer's assets. In their model, the interest rate is exogenous given. They also investigate open account financing in which the supplier allows partially delayed payment, which is equivalent to offering a loan to the retailer. Their numerical results lead to the conclusion that the joint supplier and bank financing is preferable to open account financing for the entire supply chain.

In Kouvelis and Zhao (2009), they exam the impacts of different financing schemes in the presence of bankruptcy risk on a capital constrained retailer. Their bank financing setting follows Xu and Birge's in which the loan interests are priced at the risks involved, therefore, the retailer's operational and financial decisions can be made separately in their model. They draw a conclusion that it is in the supplier's best interest to offer credits to the retailer at an interest rate which is less than or equal to the risk free interest rate, and the retailer will always prefer to finance from the supplier rather than the bank if the optimal structured financing scheme is offered. Different from their model, we assume the interest rate is exogenous given, and as a result, the retailer's best response is much more complicated (since operational and financial decisions cannot be made separately). Under our assumptions, if the retailer's financial constraint is sufficiently tight, the supply chain is better off in bank loan case since the high bankruptcy risk is shifted to an external institution.

2.1.2 Multi-Period Setting

Besides the single period models mentioned above, there are also several works studying topics on operations and finance interface under the multi-period setting. Li, et al. (2005) (first draft in 1997) present a multi-period model in which the objective is to maximize the expected present value of shareholders' dividends by simultaneously deciding operational and financial decisions. The firm's decisions include dividends, capital subscriptions, short-term borrowing and physical inventories. In their model, the discount rate and interest rate for the bank loan are exogenous given. They show the optimality of myopic policy and characterize the base-stock level explicitly in the special case of linear inventory and default cost. They also conclude that in this case, the base-stock level in a dividend-maximizing firm is lower than that in a profit-maximizing firm. Hu and Sobel (2007) study a Clark and Scarf multi-echelon extension of Li et al (2005). They provide a counter-example to show that there is no optimal echelon base-stock policy in the presence of financial constraints in a supply chain system with two or more echelons.

Chao et al. (2008) consider a multi-period periodic-review inventory control problem in which the replenishment decisions of the retailer are constrained by cash flow (they focus on the lost sale case). In their model, the retailer is self-financing and therefore he is unable to raise external fund. They show the optimality of a capital-dependent base-stock policy and present an algorithm for computing the optimal base stock level for each period. Another work of Chao et al. (2008) addresses the borrowing and lending actions in their model. They make assumptions as follows. If the retailer has insufficient cash, then he can borrow from the bank. On the other hand, if he has some cash leftover after the orders are placed, he should deposit it. Similar to their previous work, they show that the capital-dependent base-stock policy is optimal in the presence of lending and borrowing. Different from their papers, our model places emphasis on the entire supply chain rather than only on the financial constrained retailer.

Xu and Birge (2006) propose an integrated corporate planning model which considers an optimization-based valuation framework to make production and financial decisions simultaneously. They develop an efficient algorithm to solve the integer stochastic programming problem with nonlinear constraints and demonstrate the importance of jointly considering financial and production decisions.

It is also worth mentioning several related research investigating the operations and finance interface. Hu and Sobel (2005) examine the interdependence of a firm's capital structure and its short-term operational decisions concerning inventories, dividends and liquidity. In their model, the firm's objective is to maximize the expected present value of dividends. They conclude that the optimal inventory policy is independent on the firm's financial leverage, and the optimal capital structure consists of only debt or equity depending on the financial parameters. In most of these models, it is the retailer who is facing financial constraint while the supplier is not. In contrast, Babich (2007) studies the situation where the manufacturer is stronger than the supplier financially. He answers the question that whether it is profitable to give financial subsidies to the weaker supplier, when the supplier is facing financial problems.

2.2 Trade Credit

Besides operations and finance interface, our paper also addresses the effects of trade credit in supply chain management. Years ago, people have already noticed the importance of trade credit. In Peterson and Rajan (1997), they state that trade credit is a very important financing source in short-term for the companies in United State. They find that firms with less opportunities to raise bank loans are more likely to take early payment discount. Giannetti and Burkart (2007) also demonstrate this importance empirically from the operational and marketing point of view. They reveal that trade credit can not only help retailer as a financing source, but also provide the supplier informational advantage. Several studies have considered this issue. For example, Haley and Higgins (1973), study the relationship between inventory policy and trade credit policy in an EOQ model. In their model, the retailer decides not only the ordering quantity but also the time for payment. Aggarwal and Jaggi (1995) study an EOQ model of deteriorating products under a permissible-delay-payments scheme. Carlson et al. (1996) investigate an EOQ model under both all-units and incremental-quantity discounts on date-terms supplier credit. Date-terms is a trade credit scheme which allows the amount of supplier credit received to vary with the timing of the purchase. Recently, Gupta and Wang (2009) present a discrete time, joint inventory-financing model with stochastic demand. They show that in the presence of trade credit, the order-up-to inventory policy is still optimal and they provide an algorithm for finding the optimal base-stock level in a continuous review model. Other related works include Chapman et al. (1984) and Huang (2003).

2.3 Supply Chain Contracts

In our model, the only contract term is the wholesale price. In current literature, several types of supply chain contracts have been studied for supply chain management problems with two echelons. For example, the buy back contracts (Pasternack 1985), the revenue sharing contracts (Cachon and Lariviere 2003), .etc. A detailed review is provided by Cachon (2002). Van Mieghem (1999) values the effects of subcontracting in improving financial performance and coordinating of the supply chain by analyzing a stochastic investment game between two competing players. He shows that a state-dependent contract can coordinate not only production decisions but also capacity investment decisions. Cachon (2004) also studies the allocation of inventory risk in supply chain under three different contracts: push, pull and advance-purchase discount contracts. In push and pull contracts, the retailer and the supplier, who holds the inventory, bears the inventory risk respectively. While in advance-purchase discount contract, two wholesale prices are provided and the risk is shared between the retailer and the supplier. He concludes that under the consideration of advance-purchase discount contract, the supply chain can be coordinated and its profit can be allocated arbitrarily. Similar to our work, Lariviere and Porteus (2001) consider a price-only supply chain contract and examine how supply chain performance, including the retailer's price sensitivity and the wholesale price, depends on the relative variability (i.e., the coefficient of variation) of demand. This setting has been extended by Bernstein et al. (2006) to a two-echelon case with a network of retailers who compete with each other by selecting sales quantities. They identify a sufficient condition, refer to as echelon operational autonomy (EOA), under which perfect coordination is feasible through simple wholesale pricing contract. The papers discuss supply chain coordination with risk of random demand are referred to Choi et al. (2008) and Qi et al. (2004). Papers studying supply chain contracts are numerous, our work is complementary to this direction by taking financial constraints into consideration.

Chapter 3

The Model

This section is organized as follows. In §3.1, we introduce the notation, present the model and the assumptions we make. After that, in §3.2, we discuss the specific properties of demand distribution we need in our model.

3.1 Model Description

We consider a two-period supply chain problem where the supply chain consists only one supplier (with pronoun "she") and one retailer (with pronoun "he"), between whom a single type of goods is produced and transferred. We assume that both the supplier and the retailer are in a quantity competitive industry and risk-neutral, which means they only concern about the expected return and are completely indifferent to the risk involved. Different from the classical supply chain problems, we focus on the situation where the retailer is facing a financial constraint. We will analyze this problem under three different settings: the basic setting, in which the capital constrained retailer is unable to raise any external funds; the bank loan setting, where the retailer is allowed to raise short-term loan from a bank (or other financial institutions); the trade credit setting, as the words imply, in which the retailer can trade with the supplier based on credit.

3.1.1 The Basic Setting

In the basic setting, the retailer faces a financial constraint and is unable to raise fund through a financial institution (e.g. a bank) or extend credit from the supplier. This may happen when the retailer is a new entrant to this industry. Figure 3.1 illustrates the sequence of events in this setting. Specifically, at the beginning of the first period, the supplier proposes a

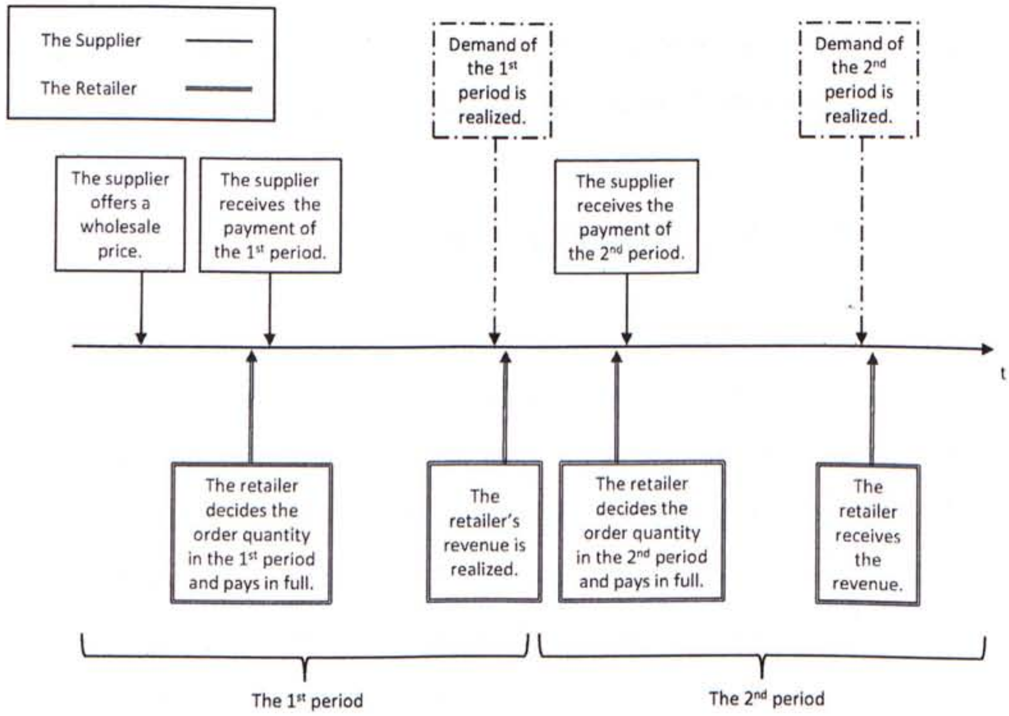

Figure 3.1: Sequence of Events - the Basic Setting

wholesale price contract, which identifies the wholesale price in each period (we denote as w in our model). For simplicity, we assume the wholesale prices are identical between two periods. Note that similar results can be obtained if the wholesale prices are different between two time periods, and they are decided in the beginning of the first period. Based on the contract the retailer determines an ordering quantity in the first period, we denote as Q_1. Note that the retailer is facing a financial constraint and unable to raise any external fund. Suppose his initial capital level is k, it is obvious the retailer cannot order more than k/w units of product. The supplier then starts to produce the products at a unit cost c and receives the payment for the transaction in the first period. At the same time, the products produced are delivered to the retailer, who sells them at a market price $p \geq (1 + r_f)c$ in this period, where r_f is the risk-free interest rate for one period of trading. Without loss of generality, we assume $r_f = 0$ in our model, that is, we ignore the time value of money.

We assume the demands $D_i, i = 1, 2$ in these two periods are independent and identically distributed with probability density function $f(\cdot)$. Unless otherwise noted, we assume the support of D_i is $[0, +\infty)$. After the realization of the demand in the first period, the retailer sells $\min(Q_1, D_1)$ units of product and receives the revenue. Unsatisfied demand is lost. In the second period, the decision process of the retailer stays the same except the initial capital is updated from k to including the realized profit of the first period. To simplify the problem, we assume the products are perishable or fashion goods without salvage value, which means all leftover inventory is salvaged at price 0. In addition, we assume that all unmet demand is lost, and no additional penalty cost is incurred. In the later analysis, we illustrate that depending on the tightness of the capital constrains, the effects of the financial constraint can be significant.

3.1.2 The Bank Loan Setting

In reality, firms facing financial constraints can often raise funds from banks to finance their operations. We assume the capital constrained retailer is able to finance additional unsecured debt B from a bank at an interest $r \geq 0$ and this financing scheme is only allowed at the beginning of the first period. Moreover, in the situation where the retailer raises a loan, the marginal revenue of selling a product should be more than the marginal cost, i.e., $p > w(1+r), i = 1, 2$ as otherwise, the retailer should not borrow any money to finance his ordering. Figure 3.2 illustrates the sequence of events in this setting. When the first selling season starts, the retailer receives the loan B and fully pays off the procurement cost to the supplier by his on-hand cash and the loan. The term of the loan is two periods, thus the retailer has to repay both the principal and the interest, that is, $(1+r)^2B$ to the bank at the end of the second period, and receives the residual revenue, if any. If unfortunately, the demand is too little, and the retailer cannot fully pay off the loan, then he gets zero revenue while the bank, who is the debt-holder in this setting, pays the bankruptcy costs and receives the residual revenue.

3.1.3 The Trade Credit Setting

It is often assumed the retailer must fully pay off the procurement cost once the order is received. However, allowing retailers to delay payment for goods already delivered is a common business practice. And the retailers, especially

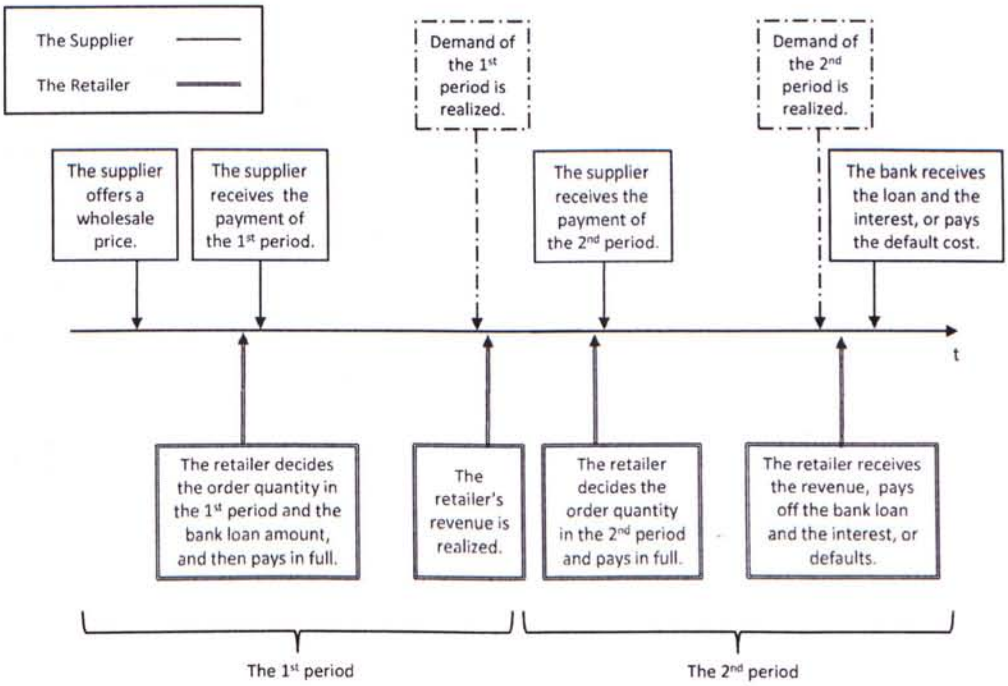

Figure 3.2: Sequence of Events - the Bank Loan Setting

small businesses, rely on this practice as a source of short-term funds since most of them have a limited number of financing opportunities. Figure 3.3 illustrates the sequence of events in this setting. At the beginning of each selling season, the retailer places an order and pays the procurement cost partially. Similar to the basic setting, this early payment must satisfy the capital constraint. After the revenue is realized, the retailer has to pay the rest plus some interest. Because we ignore the time value of money, the retailer will always pay the supplier all the cash he has upon delivery to lower the marginal cost. Mathematically, if the operational decision of the retailer is constrained by his financial status $wQ > k$, he pays all his on-hand cash k to the supplier and receives Q units of product, and the remainder $(wQ-k)^+$ is considered to be a kind of loan, so both the principal and interest must

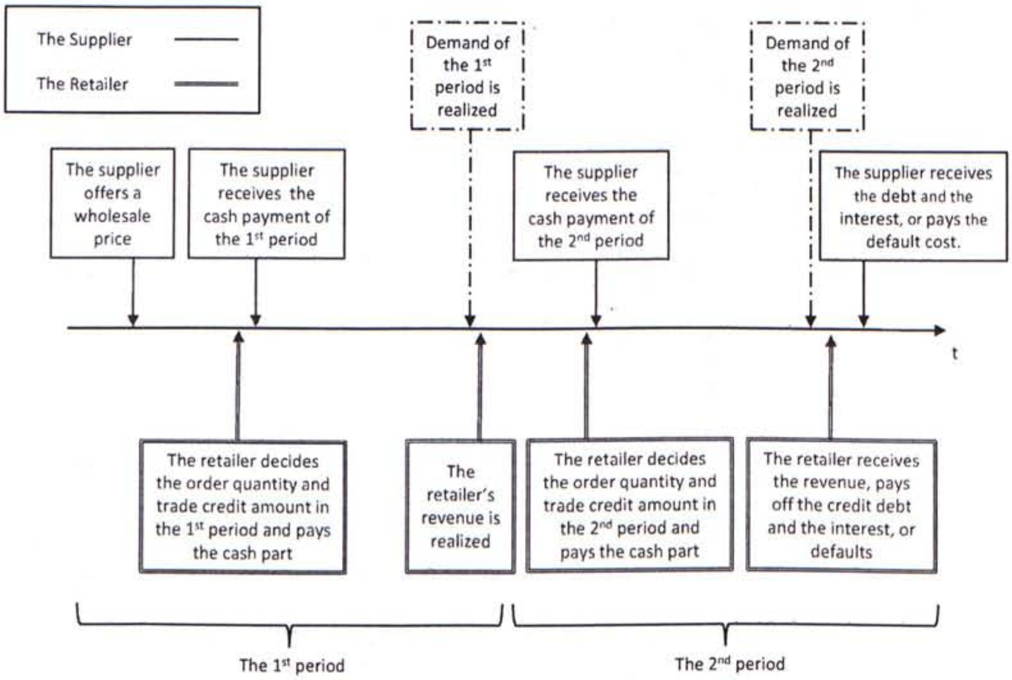

Figure 3.3: Sequence of Events - the Trade Credit Setting

be paid in full at the end of the second period. Such a financing scheme is almost identical as the traditional "early payment discount", i.e. the retailer can pay early (before the products are delivered) to get a discount price, or he can pay the full wholesale price after sales. Note that in the bank loan setting, the retailer only has a single opportunity to do financing, and therefore, he may borrow some capital for future use. While in the trade credit case, the retailer can finance in both periods and as a result, he only borrows just enough to cover the procurement cost, i.e., $B_i = (wQ_i - k_i)^+$, $i = 1, 2$.

3.2 Demand Distribution Properties

Let random variable D_i be the demand in period $i, i = 1, 2$. The probability density function (PDF) of D_i is $f(\cdot)$, cumulative distribution function (CDF) is $F(\cdot)$ and complimentary CDF is $\bar{F}(\cdot)$. We will make the following assumptions about $F(\cdot)$ throughout the thesis.

Assumption 3.2.1. *The demand distribution function $F(\cdot)$ satisfies the following properties:*

(i) $F(\cdot)$ is differentiable, strictly increasing and $F(0) = 0$, $F(+\infty) = 1$.

(ii) D_i has a finite mean, and

(iii) its failure rate function $h(\cdot) \triangleq f(\cdot)/\bar{F}(\cdot)$ is increasing.

Definition 3.2.1. Let X be a nonnegative random variable with distribution F. X has a convex increasing failure rate (CIFR) and F is an CIFR distribution if $h(\xi)$ is increasing in the weak sense, and convex for all ξ such that $F(\xi) < 1$.

Among the commonly used distributions, it can be shown that Uniform, Exponential, Normal, and Beta ($\alpha \geq 1$)distributions are CIFR while in general, Gamma distributions is not.

Definition 3.2.2. A function g is said to be log-concave on the interval (a, b) if the function $\ln g$ is a concave function on (a, b), which is to say that

$$g(\mathbf{x_1}) \cdot g(\mathbf{x_2}) \leq g\left(\frac{\mathbf{x_1} + \mathbf{x_2}}{2}\right)^2.$$

In some of our theoretical results, we need the density function of demand to be log-concave. A list of log-concave distributions is given in the appendix. Note that if a random variable has a log-concave density function, then it has an increasing failure rate.

Remark 3.2.1. Throughout the thesis we interpret function properties such as increasing, decreasing, convex and concave in the non-strict sense.

Chapter 4

Retailer's Perspective

In this chapter, we analyze the problem from the retailer's perspective. In §4.1, we present the single period model and characterize the retailer's optimal strategy under different situations. After that, in §4.2, §4.3 and §4.4, we present the two-period model formulations and the results we obtain in the basic setting, in the bank loan setting and in the trade credit setting, respectively.

4.1 The Single Period Problem

To set the stage for the later analysis of the two-period problem, we first present the model and results in the single-period case in this section.

We consider a supply chain where a retailer has a single opportunity to place an order to satisfy the stochastic demand D which is realized at the end of the selling season. This model is essentially a classical newsvendor problem with financial constraints k. Suppose the retailer is a new entrant to the market and has limited financing opportunity. Therefore, for any given wholesale price w, the retailer's ordering quantity Q is constrained by $Q \leq k/w$ and he may not always be able achieve his targeted ordering quantity. Figure 4.1 illustrates the cash flow in this setting.

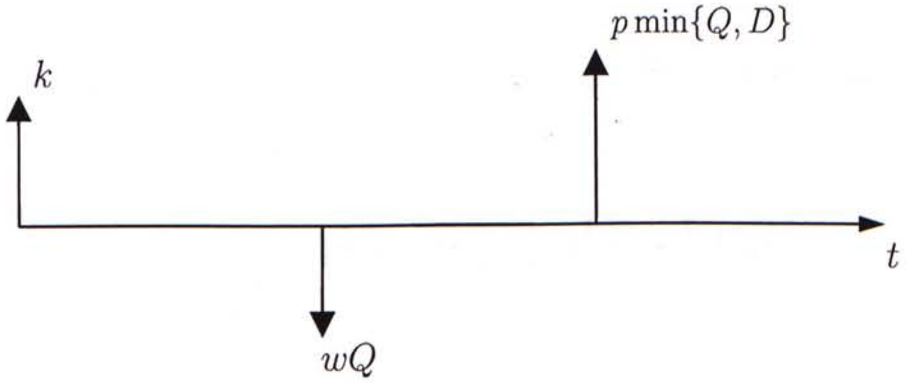

Figure 4.1: Cash Flow - Single Period Basic Setting

To find the optimal ordering quantity Q, the retailer's optimization problem is

$$\begin{aligned} \max_{Q} \quad & \mathsf{E}\big[p\min\{Q, D\} - wQ + k\big], \\ s.t. \quad & 0 \leq wQ \leq k. \end{aligned} \tag{4.1.1}$$

As it is not hard to show the retailer's profit function is concave and therefore the retailer's optimal ordering quantity $Q^*(w)$ is determined as follows:

$$Q^*(w) = \min\left\{Q_0, \frac{k}{w}\right\}, \tag{4.1.2}$$

where $Q_0 = F^{-1}\left(\frac{p-w}{p}\right)$.

If $Q_0 \leq k/w$, the financial constraint does not affect the optimal operational strategy; otherwise, the retailer can only use up his capital and order k/w units.

In other situations, firms facing financial constraints can often raise funds from banks to support their operations. We assume the capital constrained retailer is able to finance additional unsecured debt B from a bank at an interest $r_2 \geq 0$. When the selling season starts, the retailer receives the loan B and fully pays off the procurement cost to the supplier by his initial cash and the loan. At the end of the selling season, the retailer repays $(1 + r_2)B$ of his revenue to the bank and receives the residual revenue, if any. If the retailer cannot fully pay off the loan, he gets zero revenue while the bank receives all the residual revenue. Figure 4.2 illustrates the cash flow in this setting.

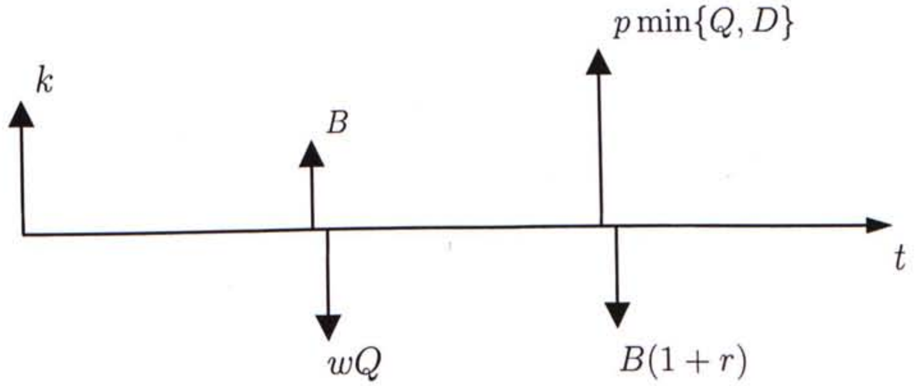

Figure 4.2: Cash Flow - Single Period Bank Loan Setting

Thus, the retailer's optimization problem is

$$\begin{aligned} \max_{Q,B} \quad & \mathsf{E}\Big[p\min\{Q, D\} - wQ - rB + k\Big]^+, \qquad (4.1.3) \\ s.t. \quad & 0 \leq wQ \leq k + B, \\ & B \geq 0. \end{aligned}$$

In (4.1.3), we also ignore the time value of on-hand cash. It is easy to justify that, if the retailer decides to raise fund, he will only borrow just enough money to cover the procurement cost, i.e., $B = (wQ - k)^+$ at optimum.

The following lemma identifies the optimal ordering quantity and debt level with initial capital k.

Lemma 4.1.1. *(Buzacott and Zhang, 2004) The optimal ordering quantity of the retailer when he can borrows from the bank is*

$$Q_b^*(w) = \begin{cases} Q_0(w), & \text{if } Q_0(w) \le \dfrac{k}{w}; \\ \dfrac{k}{w}, & \text{if } F^{-1}\Big(\dfrac{p - w(1+r)}{p}\Big) \le \dfrac{k}{w} < Q_0(w); \\ \hat{Q}(w), & \text{otherwise,} \end{cases} \tag{4.1.4}$$

where $\hat{Q}(w)$ *satisfies* $p\bar{F}(\hat{Q}) = w(1+r)\bar{F}\left(\dfrac{(w\hat{Q} - k)(1+r)}{p}\right)$, *and the resulting level of debt is*

$$B^*(w) = \begin{cases} 0, & \text{if } F^{-1}\Big(\dfrac{p - w(1+r)}{p}\Big) \le \dfrac{k}{w}; \\ w\hat{Q}(w) - k, & \text{otherwise.} \end{cases} \tag{4.1.5}$$

We would like to point out that, as we do not consider the salvage value of the remaining inventory at the retailer, the optimal solution is slightly simpler than that in Buzacott and Zhang (2004). In the first case of (4.1.4), the capital level of the retailer is relatively high so that the financial constraint becomes redundant and the retailer purchases the desired ordering quantity by his internal fund. While in the second case, the retailer just uses up his equity k. This may occur when the interest rate is too high so that it is not profitable to raise a loan. Alternatively, if the interest rate is sufficiently low, then after using up his internal capital, the retailer would seek additional fund to order $\hat{Q}$.

It can be observed from (4.1.3) that the retailer's optimal profit is decreasing in w. Intuitively, if the suppler offers a higher wholesale price, the retailer should always be worse off regardless whether he is capital constrained or not. One may expect that $\hat{Q}(w)$ is always less than or equal to unconstrained solution Q_0 as the retailer should order less if he needs to raise fund for his ordering expenses. However, the following result shows that this may not be true sometimes.

Proposition 4.1.2. *For given w, if* $\frac{1}{1+r} \geq \bar{F}\left(\frac{(wQ_0(w)-k)(1+r)}{p}\right)$, *then* $\hat{Q}(w) \geq Q_0(w)$.

Proof. For given w, if the retailer' internal cash level is high, i.e., $k/w \geq Q_0(w)$, then the retailer orders exactly $Q_0(w)$. So it is only possible to order more than the unconstrained case when the retailer raises an external fund. In that case, the retailer's optimization problem can be rewritten as

$$\max_{Q\geq 0} \Pi_r^b(Q) = p\int_{\frac{(wQ-k)(1+r)}{p}}^{Q} xdF(x) + pQ\bar{F}(Q) - B(1+r)\bar{F}\left(\frac{(wQ-k)(1+r)}{p}\right) - k,$$

which is unimodal in Q (see Buzacott and Zhang 2004). So the optimal ordering decision, $\hat{Q}(w)$ is determined by the first order condition

$$\frac{d\Pi_r^b(Q)}{dQ} = w(1+r)\bar{F}(Q)\left(\frac{p}{w(1+r)} - \frac{\bar{F}((wQ-k)(1+r)/p)}{\bar{F}(Q)}\right) = 0.$$

The optimal ordering quantity is more than the unconstrained case, that is, $\hat{Q}(w) \geq Q_0(w)$ if and only if $d\Pi_r^b(Q_0)/dQ_0 \geq 0$, which can be simplified as

$$\frac{d\Pi_r^b(Q_0)}{dQ_0} \geq 0 \Leftrightarrow 1 \geq (1+r)\bar{F}\left(\frac{(wQ_0-k)(1+r)}{p}\right),$$

where $\bar{F}(Q_0) = w/p$. □

when the loan rate is sufficiently low, the condition in (4.1.2) is satisfied because the complementary cumulative distribution function is always less than one. This means if the retailer borrows cash with little extra cost, he would order more because the more inventory on hand, the more market demands the retailer can capture. Similarly, if the unit price of the products is sufficiently low, it is also very likely that the condition is satisfied and the retailer tends to order more. Intuitively, when the selling price is sufficiently low, the probability of the retailer's bankruptcy is relatively high and the retailer will be so desperate that orders even more than the unconstrained procurement quantity to catch as much revenue as possible.

Remark 4.1.1. The above result is different from that in Kouvelis and Zhao (2008). In their analysis for the same wholesale price w, the ordering quantity in the bankruptcy region is always larger than the newsvendor solution. This is because in their definition of the newsvendor problem, the retailer purchases the products at an undiscounted price $w(1 + r)$ (following our notation), while he pays unit price w in our setting.

Next we analyze how $\hat{Q}(w)$ changes with the wholesale price w. In traditional newsvendor problem, the optimal ordering quantity is decreasing in the wholesale price, which is very intuitive because with a higher wholesale price, the marginal cost of overstock is higher while the under-stock cost is constant. In the following, we will show that in the presence of loan raising and default risk, a higher wholesale price still lowers the retailer's procurement quantity. In Kouvelis and Zhao (2008)'s work, they derive a similar result.

Proposition 4.1.3. *$\hat{Q}(w)$ is decreasing in w.*

To prove this Proposition 4.1.3, we need some preliminary results. Define for any fixed $\beta \geq 0$,

$$G(Q,\alpha) \triangleq \bar{F}(Q) - \alpha\bar{F}(\alpha Q - \beta),$$

where $1 \geq \alpha \geq 0$, then we can obtain the following properties.

Lemma 4.1.4. *(a) $G(Q,0) \geq 0$ and $G(Q,1) \leq 0$.*

(b) $(G(Q,0))'_\alpha \leq 0$, and $G(Q,\alpha)$ is negative unimodal function of α.

(c) If $G(Q_1,\alpha) = 0$, then $G(Q_2,\alpha) \leq 0$ for all $Q_2 \geq Q_1$,

Proof.

(a) Substituting zero and one into $G(Q,\alpha)$, we can easily prove that

$$\begin{aligned} G(Q,0) &= \bar{F}(Q) \geq 0, \quad \text{and} \\ G(Q,1) &= \bar{F}(Q) - \bar{F}(Q-\beta) \leq 0, \end{aligned}$$

because β is positive.

(b) Taking derivative of $G(Q,\alpha)$ w.r.p. to α yields

$$(G(Q,\alpha))'_\alpha = -\bar{F}(\alpha Q - \beta) + \alpha f(\alpha Q - \beta)Q.$$

It is easy to see that $(G(Q,0))'_\alpha = -\bar{F}(-\beta) \leq 0$. To prove $G(Q,\alpha)$ is a negative unimodal function in α, we only have to show that $(G(Q,\alpha))'_\alpha$ crosses zero at most once, which is equivalent to show that if there exists some α_1 satisfies $(G(Q,\alpha_1))'_\alpha \geq 0$, then for any $\alpha_2 \geq \alpha_1$, $(G(Q,\alpha_2))'_\alpha \geq 0$. If $(G(Q,\alpha_1))'_\alpha \geq 0$, which is

$$-\bar{F}(\alpha_1 Q - \beta) + \alpha_1 Q f(\alpha_1 Q - \beta) \geq 0 \Leftrightarrow \frac{\alpha_1 Q f(\alpha_1 Q - \beta)}{\bar{F}(\alpha_1 Q - \beta)} \geq 1.$$

Then for any $\alpha_2 \geq \alpha_1$, with the IFR property of $F(\cdot)$, it is easy to verify that, for $Q \geq 0$

$$\frac{\alpha_2 Q f(\alpha_2 Q - \beta)}{\bar{F}(\alpha_2 Q - \beta)} \geq \frac{\alpha_2 Q f(\alpha_1 Q - \beta)}{\bar{F}(\alpha_1 Q - \beta)} \geq 1,$$

which implies the desired result.

(c) To prove $G(Q_2,\alpha) \leq 0$, it is equivalent to show that $\bar{F}(Q_2) - \alpha \bar{F}(\alpha Q_2 - \beta) \leq 0$. Since $\alpha = \bar{F}(Q_1)/\bar{F}(\alpha Q_1 - \beta)$, we only need to prove that

$$G(Q_2,\alpha) = \bar{F}(\alpha Q_2 - \beta)\left(\frac{\bar{F}(Q_2)}{\bar{F}(\alpha Q_2 - \beta)} - \frac{\bar{F}(Q_1)}{\bar{F}(\alpha Q_1 - \beta)}\right) \leq 0.$$

As $\bar{F}(\alpha Q_2 - \beta)$ is positive, it is sufficient to show that $\bar{F}(Q)/\bar{F}(\alpha Q - \beta)$ is decreasing in Q. Taking the first order derivative yields

$$\left(\frac{\bar{F}(Q)}{\bar{F}(\alpha Q - \beta)}\right)' = \bar{F}(Q)\left(\frac{\alpha f(\alpha Q - \beta)}{\bar{F}(\alpha Q - \beta)} - \frac{f(Q)}{\bar{F}(Q)}\right) \Big/ \bar{F}(\alpha Q - \beta) \leq 0,$$

because the demand distribution function is IFR.

□

Now we are ready to prove Proposition 4.1.3

Proof. We let $\alpha = w(1+r)/p$ and $\beta = k(1+r)/p$, it is easy to see that $1 \geq \alpha \geq 0$ and $\beta \geq 0$, which satisfy the condition in Lemma 4.1.4. With a given α, there exists a $\hat{Q}(\alpha)$ satisfies

$$G_{\hat{Q}}(\alpha) \triangleq \bar{F}(\hat{Q}) - \alpha \bar{F}(\alpha \hat{Q} - \beta) = 0,$$

while for a given $\hat{Q}$, we also can find $\hat{\alpha}(\hat{Q})$ satisfying

$$G_{\hat{Q}}(\hat{\alpha}) \triangleq \bar{F}(\hat{Q}) - \hat{\alpha}\bar{F}(\hat{\alpha}\hat{Q} - \beta) = 0.$$

To show $\hat{Q}(w)$ is a decreasing function of w, we only need to prove that: first, there is a one-to-one correspondence between $\hat{Q}(\alpha)$ and $\hat{\alpha}(\hat{Q})$; second, $\hat{\alpha}(\hat{Q})$ is a decreasing function.

First we prove the one-to-one correspondence. To show this, we only need to prove that $G_{\hat{Q}}(\alpha)$, as a function of α, crosses zero only once, which implies that there is a unique $\hat{\alpha}$ satisfying $G_{\hat{Q}}(\hat{\alpha}) = 0$. Because $G_Q(0) \geq 0$, $G_Q(1) \leq 0$, $dG_Q(\alpha)/d\alpha\big|_{\alpha=0} \leq 0$ and $G_Q(\alpha)$ is negative unimodal in α, it is easy to verify that $G_Q(\alpha)$ will cross zero at most once by plotting the figure of $G_Q(\alpha)$, where $\alpha \in [0, 1]$.

Secondly, we prove $\hat{\alpha}(\hat{Q})$ is a decreasing function of $\hat{Q}$. It is equivalent to show that if given $\hat{Q}_1$ and $\hat{Q}_2$, where $\hat{Q}_1 < \hat{Q}_2$, then $\hat{\alpha}_1(\hat{Q}_1) > \hat{\alpha}_2(\hat{Q}_2)$, where $\hat{\alpha}$ satisfies $G_Q(\hat{\alpha}) = 0$. Following part (*b*.) in Lemma 4.1.4, $G_{\hat{Q}_2}(\hat{\alpha}_1) \leq G_{\hat{Q}_1}(\hat{\alpha}_1) = 0$, which means, the curve $G_{\hat{Q}_2}(\alpha)$ has already crossed zero, implying $\hat{\alpha}_2(\hat{Q}_2) < \hat{\alpha}_1(\hat{Q}_1)$. □

In the single-period problem, for the retailer, trading on credit is just like raising some loans. Once the wholesale price is given, his borrowing decision only depends on the loan rate. In other words, the retailer feels indifferent between raising a loan from a bank or using trade credit with the supplier. Mathematically, given the same wholesale price w and interest rate r, the retailer's optimal decisions are summarized in (4.1.4) and (4.1.5). However, from the supplier's point of view, if the retailer deals with the supplier on credit, the supplier bears the retailer's default risk because of the demand uncertainty. Therefore, the supplier may need to adjust her pricing strategy to take this risk into account, which in turn will affect the retailer's ordering quantity. We will focus on the problems of the supplier in the next chapter.

In the following part of this chapter, we will analyze the two-period problem from the perspective of the retailer under different settings. It shall be seen that although in the thesis we focus on the two-period model, similar results can be obtained for the general multi-period case.

4.2 The Basic Setting

This section concerns the model introduced in Section §3.1.1. We consider the periodic-review problem with two planning periods. To better illustrate, the first period is numbered 1 and the second period is 2. The only connection between these two periods is the cash carry-over. In our model, we focus on the lost-sale setting and assume the lead time is zero. In the basic setting, our model is similar to Chao et al. (2008)'s work except that we do not consider inventory carry over between the two consecutive periods. In the later sections, our setting is different from theirs in that we take financing issues into account, such as external financing and bankruptcy risk.

Let k_i be the capital level and Q_i be the procurement decisions at the beginning of period i, respectively, where $i = 1, 2$, and k_3 be the terminal wealth at the end of the planning horizon. Because the retailer is capital constrained and self-financed, the procurement decision satisfies the cash flow constraint

$$0 \le Q_i \le k_i/w, \quad i = 1, 2. \tag{4.2.1}$$

and the revenue from sales in period i is $p\min\{Q_i, D_i\}$. Hence the total capital level at the end of period i, which is also the capital level at the beginning of period $i+1$, is

$$k_{i+1} = p\min\{Q_i, D_i\} - wQ_i + k_i. \tag{4.2.2}$$

Therefore, the retailer's problem is to decide an ordering quantity to maximize the expected terminal wealth at the end of the second period, given initial capital level k_1, subject to the cash flow constraint in each period. That is, the retailer's decision problem is

$$\max_{Q_1,Q_2} \mathsf{E}[k_3], \tag{4.2.3}$$

subject to (4.2.1) and (4.2.2).

Next we analyze this problem using backward induction. In the second period which is also the last period, the ending wealth is $k_3 = p\min\{Q_2, D_2\} - wQ_2 + k_2$. Denote by $V_2(k_2)$ the maximum expected ending wealth given that the initial capital level at the beginning of period two is k_2, then the retailer's optimization problem is

$$V_2(k_2) = \max_{0\le wQ_2\le k_2} \Pi_2(k_2, Q_2), \tag{4.2.4}$$

where

$$\Pi_2(k_2, Q_2) = \mathsf{E}_{D_2}\big[p\min\{Q_2, D_2\} - wQ_2 + k_2\big].$$

This problem is essentially the single period problem we studied in the previous section. Therefore, the optimal ordering quantity in the second period, Q_2^* is determined as

$$Q_2^* = \min\left\{Q_0, \frac{k_2}{w}\right\}, \tag{4.2.5}$$

where $Q_0 = F^{-1}\left(\frac{p-w}{p}\right)$.

Now let's go backwards to the first period. The initial state of the second period is the ending wealth of this period. Similarly, we denote $V_1(k_1)$ as the maximum expected revenue given the initial cash level is k_1, then the optimization problem is

$$V_1(k_1) = \max_{wQ_1 \le k_1} \Pi_1(Q_1), \tag{4.2.6}$$

with

$$\Pi_1(Q_1) = \mathsf{E}_{D_1} V_2 \left(k_1 - wQ_1 + p\min\{Q_1, D_1\}\right).$$

The trade-off in the optimization problem above is between ordering products and saving cash for the future. When products are ordered, the retailer is exposed to the risk of not selling them and as a result, loses the opportunity of saving the cash for the next period. Intuitively, the more cash on hand, the more expected revenue the retailer can get, since the higher initial cash level, the larger chance for him to order up to the optimal base-stock level. According to Chao et al. (2008), $\Pi_1(Q_1)$ is a concave function in Q_1, therefore, the optimal ordering quantity is determined by the first order condition. The following propositions characterize some properties of the optimal ordering quantity.

Proposition 4.2.1. $Q_1^* \le Q_0$.

Proof. To show the proposition is true, we only need to show the unconstrained optimal ordering quantity in the first period, we denote as Q_1^{u*}, is less than or equal to the newsvendor fractile Q_0, since the optimal solution should be the minimum between the quantity that makes the constraint binding and Q_1^{u*}. Notice that for a given w, the unconstrained single period objective function for the retailer is

$$\Pi^u(Q_1) = \mathsf{E}_{D_1} g(Q_1, D_1),$$

where $g(Q_1, D_1) \triangleq [p \min\{Q_1, D_1\}] - wQ_1 + k_1$.
Then we can rewrite $\Pi_1(Q_1)$ as $\mathsf{E}_{D_1} V_2\big(g(Q_1, D_1)\big)$. To show $Q_1^{u*} \le Q_0$, we only need to show that

$$\begin{aligned} &\text{if} \quad \mathsf{E}_{D_1} \partial g(Q_1, D_1)/\partial Q_1 = 0, \\ &\text{then } \mathsf{E}_{D_1} \left(V_2'\big(g(Q_1, D_1)\big) \frac{\partial g(Q_1, D_1)}{\partial Q_1} \right) \le 0. \end{aligned}$$

It is not hard to prove that $V_2'\big(g(Q_1, D_1)\big)$ is decreasing in D_1, since $V_2(\cdot)$ is a concave function and $g(Q_1, D_1)$ is increasing in the second dimension. We also can show that

$$\frac{\partial g(Q_1, D_1)}{\partial Q_1} = \begin{cases} -w, & \text{if } D_1 \le Q_1; \\ p - w, & \text{otherwise,} \end{cases}$$

which indicates that $\partial g(Q_1, D_1)/\partial Q_1$ is increasing in D_1. According to the Rearrangement Inequality (Theorem 11.1 in Ross (2000)), when $\mathsf{E}_{D_1} \partial g(Q_1, D_1)/\partial Q_1 = 0$, we have

$$\begin{aligned} \mathsf{E}_{D_1} \left(V_2'\big(g(Q_1, D_1)\big) \frac{\partial g(Q_1, D_1)}{\partial Q_1} \right) &\le \mathsf{E}_{D_1} V_2'(g(Q_1, D_1)) \cdot \mathsf{E}_{D_1} \frac{\partial g(Q_1, D_1)}{\partial Q_1} \\ &= 0, \end{aligned}$$

which finishes our proof. □

Intuitively, if the retailer has more than enough cash, he will order as many as the newsvender solution since in this case our problem becomes two separable newsvendor problems. In contrast, if the retailer' cash level is very low and the financial constraint is binding, then it is impossible for him to order Q_0. Another case is that the retailer has enough cash on hand, but not that much to cover the procurement in the second period, then he had better save some capital for future use and order a smaller quantity comparing to the newsvendor solution.

Similar to the traditional single period newsvendor problem, the order quantity decreases in the wholesale price in a two-period problem:

Proposition 4.2.2. *$Q_1^*(w)$ decreases in w.*

Proof. To show the monotonicity of Q_1^*, we only need to show that $\Pi_1(w, Q_1)$ is submodular. We can write the full expression of $\Pi_1(w, Q_1)$ as

$$\begin{aligned}\Pi_1(w,Q_1) &= \mathsf{E}_{D_1}\Bigg\{p\,\mathsf{E}_{D_2}\left[\min\left\{\min\left\{\frac{k_1 - wQ_1 + p\min\{Q_1, D_1\}}{w}, Q_0\right\}, D_2\right\}\right] \\ &\quad - \min\left\{k_1 - wQ_1 + p\min\{Q_1, D_1\}, wQ_0\right\} \\ &\quad + k_1 - wQ_1 + p\min\{Q_1, D_1\}\Bigg\}.\end{aligned}$$

Then it is easy to verify that for any given D_1 and D_2, $\partial^2\Pi_1(w, Q_1)/\partial w \partial Q_1$ is non-positive. □

4.3 The Bank Loan Setting

This section concerns the model introduced in Section §3.1.2. Different from the basic setting, we now have an additional variable B, which is the debt the retailer borrows in the first period. We need to track this information because although the financing decision is only made in the first period, the repayment is due at the end of the decision horizon. Figure 4.3 illustrates the cash flow in this setting.

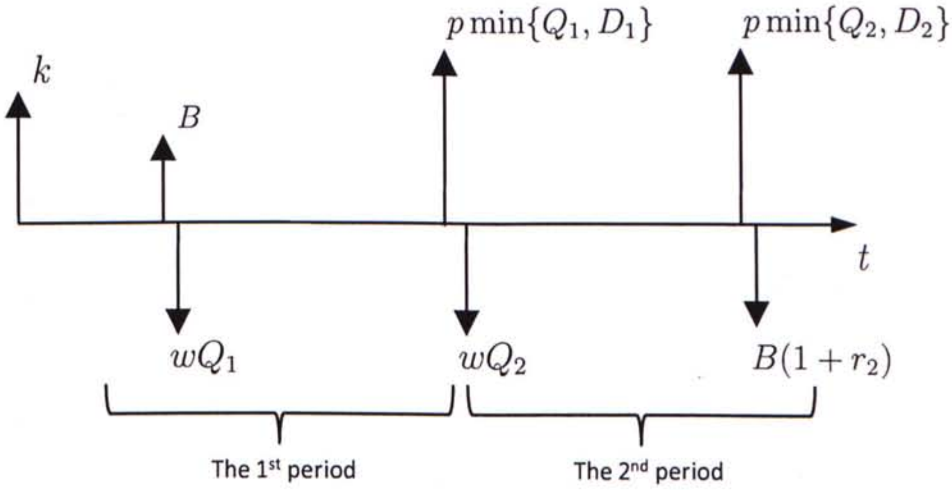

Figure 4.3: Cash Flow - Two-Period Bank Loan Setting

Similar to the basic setting, if we denote k_i as the capital level, Q_i be the procurement decision at the beginning of period i respectively, where $i = 1, 2$, then in this case

$$k_2 = p\min\{Q_1, D_1\} - wQ_1 + k_1 + B, \tag{4.3.1}$$

then the retailer's optimality equation in the second period is

$$V_2^b(k_2, B) = \max_{0 \le wQ_2 \le k_2} \Pi_2^b(k_2, B, Q_2), \tag{4.3.2}$$

where

$$\Pi_2^b(k_2, B, Q_2) = \mathsf{E}_{D_2}\big(p\min\{Q_2, D_2\} - wQ_2 + k_2 - (1+r_2)B\big)^+.$$

Proposition 4.3.1. *If D_2 has an increasing failure rate, then $\Pi_2^b(k_2, B, Q_2)$ is unimodal in Q_2.*

Proof. By taking derivative, we have

$$\frac{\partial \Pi_2^b(k_2, B, Q_2)}{\partial Q_2} = p\bar{F}(Q_2)\left(1 - \frac{w\bar{F}(D_s(Q_2))}{p\bar{F}(Q_2)}\right),$$

where $D_s(Q_2) = (wQ_2 - k_2 + (1+r)B)/p$. To prove the unimodality, we only need to prove that the second term in the brackets is increasing in Q_2. Taking derivative to this term, yields

$$\left(d\frac{w\bar{F}(D_s(Q_2))}{p\bar{F}(Q_2)}\right) \Big/ (dQ_2) = \frac{w\bar{F}(D_s(Q_2))}{p\bar{F}(Q_2)}\left(\frac{f(Q_2)}{\bar{F}(Q_2)} - \frac{wf(D_s(Q_2))}{p\bar{F}(D_s(Q_2))}\right).$$

It is easy to show that under the optimal condition, Q_2 must be greater than or equal to $D_s(Q_2)$, then with the assumption of IFR, the equation above is always greater than zero, which proves our result. □

According to the proposition above, the optimal solution can be fully characterized by:

$$Q_2^{b*} = \min\left\{\hat{Q}_2^b(k_2, B), \frac{k_2}{w}\right\}, \tag{4.3.3}$$

where $\hat{Q}_2^b(k_2, B)$ satisfies

$$p\bar{F}(\hat{Q}_2^b) = w\bar{F}\left(\frac{w\hat{Q}_2^b - k_2 + (1+r_2)B}{p}\right). \tag{4.3.4}$$

This result is similar to that in the capital constrained newsvender problem, where the optimal ordering quantity is the minimum of a newsvendor fractile and the maximum amount the retailer can order using the on hand cash. The difference is mainly resulted from the financing issues. Because of the capital constraints, the retailer would seek additional financing in the first period, and he has to take bankruptcy issues into account. The following propositions identify some properties of the optimal order quantity.

Proposition 4.3.2. *$\hat{Q}_2^b(k, B)$ is increasing in B.*

Proof. By implicit function theorem, we have

$$\frac{\partial \hat{Q}_2^b(k,B)}{\partial B} = \left(\frac{1+r}{p}\frac{f(\phi(B))}{\bar{F}(\phi(B))}\right) \Big/ \left(\frac{f(\hat{Q}_2^b(k,B))}{\bar{F}(\hat{Q}_2^b(k,B))} - \frac{w}{p}\frac{f(\phi(B))}{\bar{F}(\phi(B))}\right),$$

where $\phi(B) = \dfrac{w\hat{Q}_2^b - k + (1+r_2)B}{p}$.

Since we assume the demand distribution function is IFR, and the numerator is always non-negative, this partial derivative is greater than zero if and only if $\hat{Q}_2^b(k, B) \geq \phi(B)$. We will show this condition must be satisfied by contradiction. If it is not, i.e., $w\hat{Q}_2^b - k + (1 + r_2)B > p\hat{Q}_2^b$, which means even if all the retailer's products are sold out, the profit is not sufficient to pay off the debt. In another word, the retailer will bankrupt unavoidably. In this case, it is profitable for the retailer to order more because for some Q_2^b which is greater than $\hat{Q}_2^b$ and satisfies $wQ_2^b - k + (1 + r_2)B < pQ_2^b$, there exists some chance that the retailer will make some profit, which makes

$$\Pi_2^b(k, B, Q_2^b) > \Pi_2^b(k, B, \hat{Q}_2^b) = 0.$$

This contradicts with the optimality of $\hat{Q}_2^b$, so it must be true that $\hat{Q}_2^b(k, B) \geq \phi(B)$. □

If the capital constrained retailer has a higher debt level, he should order more. This is intuitive because the retailer with a higher debt level will be more aggressive to capture as much revenue as he can to pay off the debt and avoid bankruptcy.

Next we go backwards to the first period's problem. Similarly, we denote $V_1^b(k_1)$ as the maximum expected revenue given the initial cash level is k_1, then the maximization problem is

$$\max_{0 \le wQ_1 \le k_1 + B} \quad \Pi_1^b(B, Q_1),$$

with

$$\Pi_1^b(B, Q_1) = \mathsf{E}_{D_1} V_2\big(k_1 + B - wQ_1 + p\min\{Q_1, D_1\}, B\big).$$

The trade-off in the optimal problem above is as follows: On one hand, if the retailer borrows more, he has more capital on hand and a bigger chance to order to the optimal ordering level, however, he runs a higher risk of bankruptcy; on the other hand, similar to the basic setting, the more products are ordered, the higher the overstock risk the retailer is exposed to and he loses the opportunity of saving cash for the future. Note that for the single period bank loan setting, the retailer always uses up the capital on hand, or borrows just enough to cover the procurement. However, in the two-period case, the retailer may save some cash for the next period as he does not have the opportunity to borrow money in the second period.

With additional assumption on the demand distribution, we can show the objective function of the retailer at the first period is log-concave. It is intuitive that log-concavity implies unimodality.

Proposition 4.3.3. *If the demand density function $f(\cdot)$ is log-concave, then $\Pi_1^b(B, Q_1)$ is jointly log-concave in Q_1 and B.*

To show this, we need a lemma:

Lemma 4.3.4. *If $f(\mathbf{x}, y)$ is log-concave, $\mathbf{x} \in \mathbb{R}^n$, then $\mathsf{E}_D f(\mathbf{x}, D)$ is log-concave in $\mathbf{x}$, if the pdf of D is log-concave.*

Proof. By the definition of log-concavity, we only need to show for any $\mathbf{x}_1, \mathbf{x}_2$,

$$\mathsf{E}_D f(\mathbf{x}_1, D)\mathsf{E}_D f(\mathbf{x}_2, D) \leq \left(\mathsf{E}_D f(\frac{\mathbf{x}_1 + \mathbf{x}_2}{2}, D)\right)^2,$$

which is equivalent to

$$\int_t f(\mathbf{x}_1, t)\phi(t)dt \int_t f(\mathbf{x}_2, t)\phi(t)dt \leq \left(\int_t f(\frac{\mathbf{x}_1 + \mathbf{x}_2}{2}, t)\phi(t)dt\right)^2. \quad (4.3.5)$$

Consider three functions:

$$\begin{aligned} g_1(t) &= f(\mathbf{x}_1, t)\phi(t); \\ g_2(t) &= f(\mathbf{x}_2, t)\phi(t); \\ g_3(t) &= f(\frac{\mathbf{x}_1 + \mathbf{x}_2}{2}, t)\phi(t), \end{aligned}$$

then for any t_1, t_2,

$$\begin{aligned} g_3\left(\frac{t_1 + t_2}{2}\right)^2 &= \left(f(\frac{\mathbf{x}_1 + \mathbf{x}_2}{2}, \frac{t_1 + t_2}{2})\phi(\frac{t_1 + t_2}{2})\right)^2 \\ &\geq f(\mathbf{x}_1, t_1)\phi(t_1)f(\mathbf{x}_2, t_2)\phi(t_2) \\ &= g_1(t_1)g_2(t_2). \end{aligned}$$

Now applying Prekopa-Leindler's inequality (see the remark after this proof),

$$\int_t g_1(t)dt \int_t g_2(t)dt \leq \left(\int_t g_3(t)dt\right)^2,$$

which is exactly (4.3.5). □

Now we apply the lemma to show the proposition. We have denoted $k + B - wQ_1 + p\min\{Q_1, D_1\}$ as $k_2(D_1, Q_1)$, we only need to show that $V_2(k_2(D_1, Q_1), B)$ is jointly log-concave in B, Q_1 and D_1. This is not hard to prove because $V_2^b(k, B)$ is jointly log-concave and increasing in the first dimension, while $k_2(D_1, Q_1, B)$ is jointly concave.

Remark 4.3.1. (Prékopa-Leindler's Inequality) Let $0 < \lambda < 1$ and let $f, g, h : \mathbb{R}^n \rightarrow [0, +\infty)$ be non-negative real-valued measurable functions defined on n-dimensional Euclidean space $\mathbb{R}^n$. Suppose that these functions satisfy

$$h\left((1-\lambda)x + \lambda y\right) \geq f(x)^{1-\lambda} g(y)^{\lambda},$$

for all x and y in $\mathbb{R}^n$. Then

$$\|h\|_1 = \int_{\mathbb{R}^n} h(x)\,\mathrm{d}x \geq \left(\int_{\mathbb{R}^n} f(x)\,\mathrm{d}x\right)^{1-\lambda} \left(\int_{\mathbb{R}^n} g(x)\,\mathrm{d}x\right)^{\lambda} = \|f\|_1^{1-\lambda} \|g\|_1^{\lambda}.$$

As we stated before, this result can also be extended to the multi-period situation. With this unimodality property, the optimal ordering quantity in each period can be fully characterized by the first order condition and it guarantees that we can find the optimal solutions quickly.

4.4 The Trade Credit Setting

This section concerns the model introduced in Section §3.1.3. For the trade credit scheme, we consider the early payment discount model. This is equivalent to model it as a bank financing setting except the debt holder is the supplier. Figure 4.4 illustrates the cash flow in this setting. In the beginning of each period, the retailer decides how much to borrow from the supplier (if necessary) and his ordering quantity. Note that we assume the retailer will repay all the debt only at the end of the planning horizon. If unfortunately, the retailer is unable to pay off the unpaid cost plus the interests, he will be forced into bankruptcy and his debt holder, which is the supplier here, will take all the residual value and suffer from the bankruptcy cost, if any.

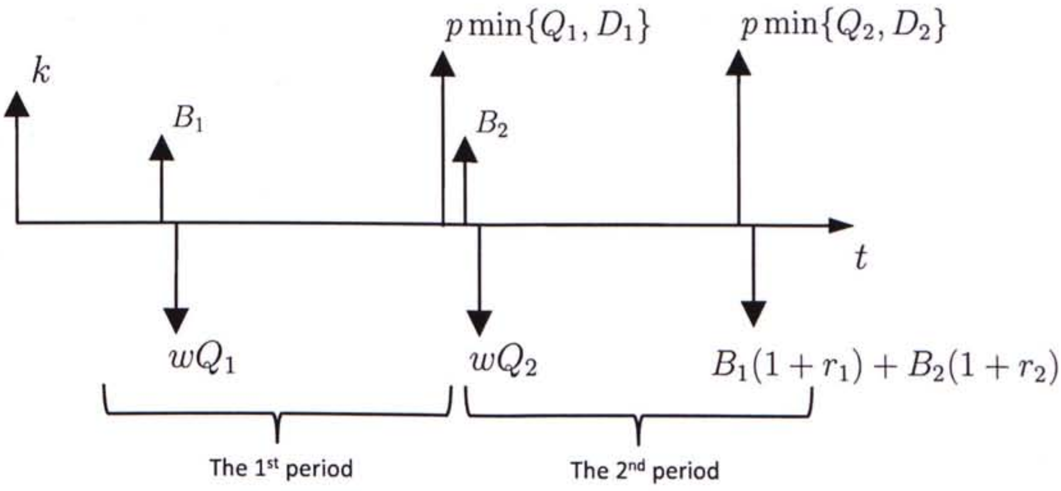

Figure 4.4: Cash Flow - Two-Period Trade Credit Setting

Therefore, in this case, the supplier will be the one who shares default risk with the retailer.

Next we analyze this problem from the second period. We denote $V_2(k_2, B_1)$ as the retailer's expect profit in the second period given the borrowing amount in the first period is B_1 and the on hand capital level at the beginning of the second period is k_2. Then the optimization problem can be formulated as the following:

$$\begin{aligned} V_2(k_2, B_1) &= \max_{Q_2 \geq 0, B_2 \geq 0} \Pi_2^t(k_2, B_1, Q_2, B_2), \qquad (4.4.1) \\ s.t. \quad & 0 \leq wQ_2 \leq k_2 + B_2, \\ & B_2 \geq 0, \end{aligned}$$

with

$$\Pi_2^t(k_2, B_1, Q_2, B_2) = \mathsf{E}_{D_2}\big(p\min\{Q_2, D_2\} - wQ_2 + k_2 - (1+r_1)B_1 - (1+r_2)B_2\big)^+.$$

In (4.4.1), obviously there is no incentive for him to borrow more than he need, that is, $B_2 = (wQ_2 - k_2)^+$, because the retailer has financing opportunity in each period and we ignore the time value of money. Similar to the bank loan case, it is easy to show that $\Pi_2^t(k_2, B_1, Q_2, B_2)$ is a jointly unimodal function, thus, the optimal ordering quantity in the trade credit setting, Q_2^{t*} can be fully characterized by the first order condition. The following expression indicates the optimized decision of the retailer:

$$Q_2^{t*} = \tag{4.4.2}$$
$$\begin{cases} \hat{Q}_2^b, & \text{if } F_2^{-1}\Big(\frac{p - w\bar{F}_2(\frac{r_1 B_1}{p})}{p}\Big) \leq \frac{k}{w}; \\ \frac{k}{w}, & \text{if } F_2^{-1}\Big(\frac{p - w(1+r)\bar{F}_2(\frac{r_1 B_1}{p})}{p}\Big) \leq \frac{k}{w} < F_2^{-1}\Big(\frac{p - w\bar{F}_2(\frac{r_1 B_1}{p})}{p}\Big); \\ \hat{Q}_2^t, & \text{otherwise,} \end{cases}$$

where $\hat{Q}_2^t$ satisfies $p\bar{F}_2(\hat{Q}_2^t) = w(1+r)\bar{F}_2\left(\frac{(w\hat{Q}_2^t - k)(1+r_2) + r_1 B_1}{p}\right)$.

In the first case of (4.4.2), the retailer has more than enough cash for their operations and will not borrow in the second period. While in the second case of (4.4.2), the retailer's capital level is not sufficient but is also not too low and the interest rate in this period is relatively high, then he will use all the cash he has to finance his procurement and will not borrow. Finally, in the third case, if the initial cash of the retailer is very low or the interest rate is sufficiently low, after exhausting his equity, the retailer would seek additional financing from the supplier to order $\hat{Q}_2^t$. Note that in the formulation (4.4.1), the state variable k_2 is a function of the realized demand, the ordering quantity and the debt level in the first period, that is

$$k_2(k_1, B_1, Q_1, D_1) = p\min\{Q_1, D_1\} - wQ_1 + B_1 + k_1. \tag{4.4.3}$$

Therefore, the problem of the first period is

$$\begin{aligned} \max_{Q_1,B_1} \quad & \Pi_1^t(B_1,Q_1), && (4.4.4)\\ s.t. \quad & 0 \le wQ_1 \le k_1 + B_1, \\ & B_1 \ge 0, && (4.4.5) \end{aligned}$$

where

$$\Pi_1^t(B_1,Q_1) = \mathsf{E}_{D_1} V_2\big(k_1 + B_1 - wQ_1 + p\min\{Q_1, D_1\}, B_1\big).$$

Similar to the bank loan case, we can derive the following result.

Proposition 4.4.1. *If the demand density function $f(\cdot)$ is log-concave, then $\Pi_1^t(B_1,Q_1)$ is jointly log-concave in Q_1 and B_1.*

To prove this proposition, we need the following lemma.

Lemma 4.4.2. *If $V(\mathbf{x},y)$ is jointly log-concave and increasing in the first dimension, $f(\mathbf{x},y)$ is jointly concave, then $V(f(\mathbf{x},y),y)$ is jointly log-concave.*

Proof. Following the property of the concave function, we have

$$\begin{aligned} V(f(\mathbf{x}_1,y_1),y_1)\cdot V(f(\mathbf{x}_2,y_2),y_2) &\le V\left(\frac{f(\mathbf{x}_1,y_1)+f(\mathbf{x}_2,y_2)}{2}, \frac{y_1+y_2}{2}\right)^2 \\ &\le V\left(f\left(\frac{\mathbf{x}_1+\mathbf{x}_2}{2}, \frac{y_1+y_2}{2}\right), \frac{y_1+y_2}{2}\right)^2, \end{aligned}$$

where the second inequality follows from the increasing property of $V(\cdot)$. □

According to the above result, $V_2(k_2, B_1)$ is jointly log-concave, and increasing in the first dimension. We also know that $k_2(k_1, B_1, Q_1, D_1)$ is jointly concave, then it is easy to see that $V_2\big(k_2(k_1,B_1,Q_1,D_1),B_1\big)$ is jointly log-concave. Apply Lemma 4.3.4, we can prove the proposition.

4.5 Summary

In this chapter, we first study the model under a single-period setting and characterize the optimal operational and financial decisions which provide some managerial insights on how to manage products and financial flows simultaneously. We find that the optimal ordering quantity may not be monotone in the retailer's initial budget.

We then extend this problem into a two-period dynamic setting. Under some mild assumptions on the demand distribution, we show the unimodality of the retailer's objective function, which is very important for finding the optimal ordering quantity.

Chapter 5

Supplier's Perspective

In this chapter, we analyze the problem from the supplier's perspective. In §5.1, we present the theoretical results of the supplier's problem in a simplified single-period model. After that, in §5.2, we extend our problem to the two-period setting.

5.1 Single Period Results

Following the retailer's best strategy in the single-period basic setting, we next investigate how financial constraint of the retailer affects the supplier. Intuitively, the supplier's revenue is influenced by the insufficient initial cash of the retailer due to the ordering decrement. In our model, the supplier needs to decide a wholesale price contract to optimize his own profit. Note that in our model, we assume the supplier does not face financial constraints.

5.1.1 The Basic Setting

In the basic setting, the supplier's problem is essentially the same as "selling to the newsvendor" problem, the only difference is that for any given wholesale price, the retailer's optimal response decision is constrained by his initial cash. Then it is easy to derive the supplier's optimization problem, which is

$$\max_{w} \ \Pi_s(w) = (w-c)Q^*(w), \tag{5.1.1}$$

where $Q^*(w)$ denotes the optimal ordering quantity identified in equation (4.1.2). The supplier should never set the wholesale price w lower than the production cost c or higher than the selling price p, because if so, either the supplier or the retailer has no incentive to sell the product.

Lemma 5.1.1. *There are at most two values of w satisfying $F^{-1}\left(\frac{p-w}{p}\right) = \frac{k}{w}$.*

Proof. To prove this lemma, we only need to show $wF^{-1}((p-w)/p)$ is first strictly increasing in w then decreasing in w after some point, i.e., a strictly unimodal function. Because there is a one-to-one mapping from w to $Q_0(w)$, where $w = p\bar{F}(Q_0)$, we only need to show that $p\bar{F}(Q_0)Q_0$ is a strictly unimodal function of Q_0. By taking the derivative w.r.p. to Q_0, we have $p\bar{F}(Q_0)(1 - Q_0 f(Q_0)/\bar{F}(Q_0))$. Because $p\bar{F}(Q_0)$ is nonnegative, it is easy to see that when demand distribution is IFR, $1 - Q_0 f(Q_0)/\bar{F}(Q_0)$ is decreasing as $Q_0 \geq 0$. Therefore, $p\bar{F}(Q_0)Q_0$ first increases with Q_0 then decreases and the result follows. □

According the retailer's ordering quantity, we analyze the supplier's problem as follows. First we define $w_1 \leq w_2$ such that

$$F^{-1}\left(\frac{p-w_i}{p}\right) = \frac{k}{w_i}, \quad i=1,2. \tag{5.1.2}$$

Note that w_1, w_2 may be equal. If there is no solution of the above equation, then let $w_1 = w_2 = -\infty$.

Proposition 5.1.2. *(Caldentey and Chen, 2008) (a)* $\Pi_s(w)$ *is unimodal;*

(b) the optimal wholesale price $w^* = \max\{w_0, w_2\}$*, where* w_0 *satisfies*

$$Q_0(w_0) = \frac{w_0 - c}{p}\frac{1}{f\big(Q_0(w_0)\big)}. \tag{5.1.3}$$

Proof. To prove (a), we analyze two cases. The first case is that the retailer is never constrained by his capital, i.e., $\frac{k}{w} \geq F^{-1}\left(\frac{p-w}{p}\right)$ for all w. It then follows that $w_2 = -\infty$. Note that $F(Q_0) = (p-w)/p$. Since $F(\cdot)$ is strictly increasing and continuous, there exists a one-to-one mapping between w and Q_0. Let $w(Q_0)$ be the unique wholesale price which induces the retailer to purchase Q_0 units, then we have $w(Q_0) = \bar{F}(Q_0)p$. We can rewrite the supplier's profit function as follows:

$$\Pi_s\big(Q_0, w(Q_0)\big) = \big(w(Q_0) - c\big)Q_0.$$

Taking the first derivative with respect to Q_0,

$$\begin{aligned}(\Pi_s(w(Q_0), Q_0))' &= w(Q_0) - c + Q_0 w'(Q_0)\\ &= p\bar{F}(Q_0)\left(1 - Q_0\frac{f(Q_0)}{\bar{F}(Q_0)} - \frac{c}{p\bar{F}(Q_0)}\right).\end{aligned}$$

We know $c/\big(p\bar{F}(Q_0)\big)$ is increasing, so as long as the demand distribution is IFR, $Q_0 f(Q_0)/\bar{F}(Q_0)$ is increasing, then the supplier's profit is unimodal.

Secondly, if there exist a finite w_2, then the supplier's objective function becomes $\Pi_s(w) = (w-c)\min\{Q_0(w), k/w\}$. Note that $(w-c)k/w$ is an increasing concave function in w. By the definition of unimodality, it is not hard to show the minimum of two unimodal functions is still unimodal. Thus, part (a) is proved.

For (b), we also need to consider two separate cases. Note that if $w \geq w_2$ or $w \leq w_1$, then $k/w \geq Q_0(w)$. And it is not hard to see that $w_1 \leq w_0$ as the left-sided derivative of $\Pi_s(w)$ at $w = w_1$ is greater than or equal to 0. Specifically, for a small $\epsilon > 0$,

$$\frac{(w_1 - c)Q_0(w_1) - (w_1 - \epsilon - c)Q_0(w_1 - \epsilon)}{\epsilon} \geq$$
$$\frac{(w_1 - c)k/w_1 - (w_1 - \epsilon - c)k/(w_1 - \epsilon)}{\epsilon} \geq 0,$$

because of the definition of w_1 such that for $w < w_1$, $Q_0(w) < k/w$. Thus,

(i) if $w_0 \geq w_2$, clearly $\Pi_s(w)$ is increasing in $w \leq w_0$ and decreasing in $w > w_0$ so w_0 is optimal.

(ii) If $w_2 > w_0$, then as $\Pi_s(w)$ is increasing in $w \leq w_2$ and decreasing in $w > w_2$, the optimal whole sale price $w^* = w_2$.

Therefore, $w^* = \max\{w_0, w_2\}$. □

Figure 5.1 shows the supplier's profit function in the basic setting.

The above result implies that when the retailer is financial constrained, the supplier tends to set a higher wholesale price than in the case the retailer has sufficient cash. From the supply chain coordination point of view, a higher wholesale price intensifies the double marginalization effect and reduces the supply chain efficiency.

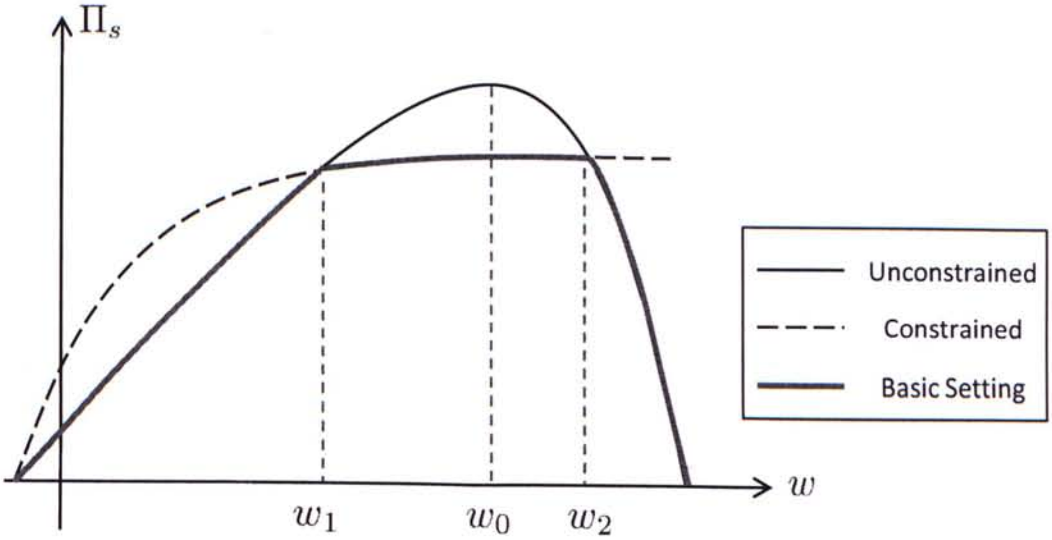

Figure 5.1: Single Period Supplier's Profit Function in Basic Setting

Special Case - Uniformly Distributed Demand

If the market demand is uniformly distributed between $[0, 1]$, then the retailer's optimal ordering quantity Q, and the supplier's optimal wholesale price w can be determined as follows.

(i) If $p \leq 4k$,

$$Q^*(w) = \frac{p-w}{p} \text{ , } w^* = \frac{p^2-c^2}{4p}.$$

(ii) If $p > 4k$,

$$Q^*(w) = \begin{cases} \dfrac{p-w}{p}, & \text{if } w \leq w_1 \text{ or } w \geq w_2; \\ \dfrac{k}{w}, & \text{otherwise,} \end{cases}$$

$$w^* = \begin{cases} \dfrac{p+c}{2}, & \text{if } k \geq \dfrac{p^2-c^2}{4p}; \\ w_2, & \text{otherwise,} \end{cases}$$

where $w_1 = \dfrac{p-\sqrt{p^2-4pk}}{2}$, $w_2 = \dfrac{p+\sqrt{p^2-4pk}}{2}$.

5.1.2 The Bank Loan Setting

Next we consider the case where the retailer finances his operations by raising external loan from a bank with an interest rate r.

Given the retailer's best response to the wholesale price in Lemma 4.1.1, we now consider the supplier's problem. Note that the debt holder in this setting is the bank, therefore, when the retailer issues a loan, the supplier does not share any risk and the profit function for the supplier is the same as that in the basic setting, except that $Q^*(w)$ is substituted by $Q_b^*(w)$.

$$\max_w \quad \Pi_s^b(w) = (w-c)Q_b^*(w). \tag{5.1.4}$$

When the supplier decides the wholesale price, she needs to consider the corresponding response of the retailer. In the first case of (4.1.4), i.e., $F^{-1}(1-w/p) \leq k/w$, then the optimal ordering quantity is the same as that in traditional newsvendor problem, it has been proved that profit function of the supplier is unimodal. On the other hand, if the internal fund of the retailer is insufficient and it is not profitable for him to seek additional funds, the profit function of the supplier is an increasing function of the wholesale price. Next we will examine the last situation, where the retailer's capital level is insufficient and the interest rate is low enough so that he will raise some loan.

Proposition 5.1.3. *If the demand distribution function $F(\cdot)$ is CIFR (convex increasing failure rate), then the profit function of the supplier $\Pi_s^b(w)$ is unimodal in the range*

$$w \in \left\{ w : \frac{k}{w} < F^{-1}\left(\frac{1-w(1+r)}{p}\right)\right\}.$$

Proof. When the retailer raises a fund, the optimal ordering quantity is $\hat{Q}(w)$, determined in (4.1.4). Taking the first derivative of $\Pi_s^b(w)$ with respect to w yields

$$\frac{d\Pi_s^b(w)}{dw} = (w-c)\frac{d\hat{Q}(w)}{dw} + \hat{Q}(w).$$

Since $\hat{Q}(w)$ satisfies $p\bar{F}(\hat{Q}(w)) = w(1+r)\bar{F}\left(\frac{(w\hat{Q}(w)-k)(1+r)}{p}\right)$, by the implicit function theorem, the derivative of $\hat{Q}(w)$ can be expressed as follows

$$\frac{d\hat{Q}(w)}{dw} = (1+r)\frac{p\bar{F}(\delta(w)) - w(1+r)f(\delta(w))\hat{Q}(w)}{w^2(1+r)^2 f(\delta(w)) - p^2 f(\hat{Q}(w))},$$

where we denote $(w\hat{Q}(w)-k)(1+r)/p$ as $\delta(w)$ for notational convenience. By substituting $d\hat{Q}(w)/dw$ into $d\Pi_s^b(w)/dw$, we can easily get

$$\begin{aligned}\frac{d\Pi_s^b(w)}{dw} = &\left(\frac{w-c}{w} - \hat{Q}(w)\left(\frac{f(\hat{Q}(w))}{\bar{F}(\hat{Q}(w))} - \frac{c}{p}\frac{f(\delta(w))}{\bar{F}(\delta(w))}\right)\right)\\ &\Bigg/\left(\frac{w(1+r)}{p}\frac{f(\delta(w))}{\bar{F}(\delta(w))} - \frac{f(\hat{Q}(w))}{\bar{F}(\hat{Q}(w))}\right).\end{aligned}$$

To proof the unimodality of $\Pi_s^b(w)$, we only need to show the first derivative of supplier's profit function changes sign at most once. It is easy to verify that if the demand distribution function is IFR, then the denominator will be always less than zero because $\delta(w) \le \hat{Q}(w)$ and $w(1+r)/p \le 1$. To prove the proposition, we only need to prove that the numerator is an increasing function of w.

Because $(w-c)/w$ is increasing and $\hat{Q}(w)$ is decreasing in w, if we can show the latter term in the first bracket is decreasing (since it is positive as $F(\cdot)$ is IFR), that is,

$$\zeta(w) \triangleq \frac{f(\hat{Q}(w))}{\bar{F}(\hat{Q}(w))} - \frac{cf(\delta(w))}{\left(p\bar{F}(\delta(w))\right)} \tag{5.1.5}$$

is decreasing, then this proposition is proved. Taking the first order derivative of $\zeta(w)$ w.r.p to w, yields

$$\frac{d\zeta(w)}{dw} = h'(\hat{Q}(w))\hat{Q}'(w) - \frac{c(1+r)}{p^2}h'(\delta(w))\left(w\hat{Q}'(w) + \hat{Q}(w)\right),$$

where $h(\cdot)$ denotes failure rate, i.e., $h(\cdot) = f(\cdot)/\bar{F}(\cdot)$. With the assumption that demand distribution function is IFR, the first term in the expression is always less than or equal to zero. To prove $\zeta(w)$ is decreasing, we will analyze in the following two cases:

(i) If $w\hat{Q}'(w) + \hat{Q}(w) \geq 0$, the second term is positive, then we can easily see that the first derivative of $\zeta(w)$ is negative.

(ii) If $w\hat{Q}'(w) + \hat{Q}(w) < 0$, it is easy to verify that

$$\begin{aligned} |\hat{Q}'(w)| &\geq |c(1+r)w\hat{Q}'(w)/p^2| \\ &\geq \left|c(1+r)\left(w\hat{Q}'(w) + \hat{Q}(w)\right)/p^2\right| \end{aligned}$$

as $\hat{Q}(w) \geq 0$. Therefore as long as $h'(\delta(w)) \leq h'(\hat{Q}(w))$, the first order derivative of $\zeta(w)$ is still negative. Because CIFR implies that $h'(\delta(w)) \leq h'(\hat{Q}(w))$, if the demand distribution function satisfies CIFR, the proposition is proved. □

On the other hand, if the retailer does not raise any fund, then we can easily find that the corresponding profit function of the supplier is also unimodal.

Next we will examine the overall supplier's profit function and identify the optimal wholesale price. Similar to the previous subsection, we can show that there are at most two w satisfies the equation

$$F^{-1}\left(\frac{p - w(1+r)}{p}\right) = \frac{k}{w}.$$

We denote the two solutions as w_3 and w_4 with $w_3 \leq w_4$. If there is no solution, then we define $w_3 = w_4 = -\infty$. For any given w, $wF^{-1}(1 - w/p)$ is strictly greater than $wF^{-1}(1 - w(1+r)/p)$, therefore, we have $w_4 < w_2$.

Figure 5.2 shows the supplier's profit function in this setting and the following theory determines the optimal decision for the supplier.

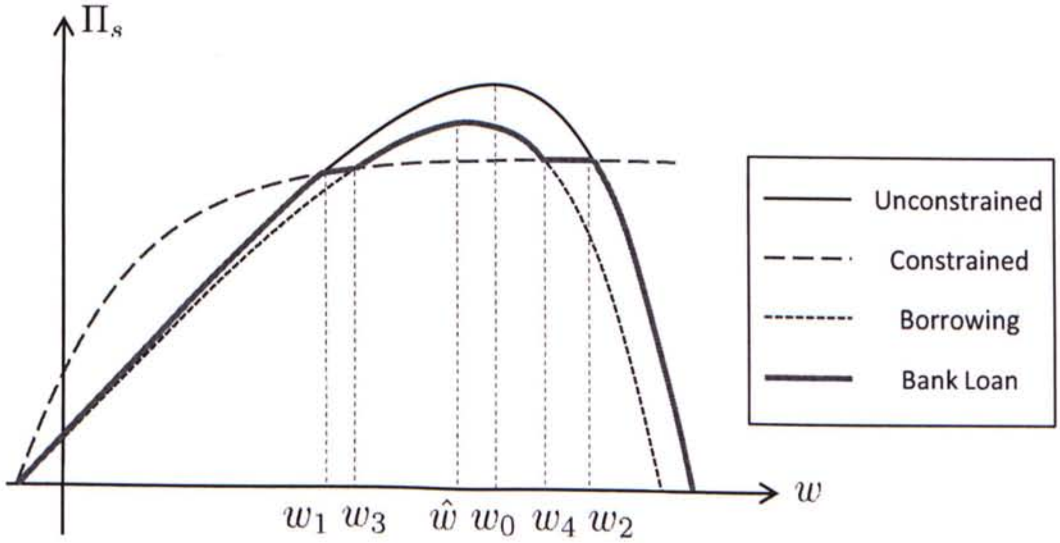

Figure 5.2: Single Period Supplier's Profit Function in Bank Loan Setting

Theorem 5.1.4. *(a) If $w_4 = -\infty$, then the supplier's profit function $\Pi_b^s(w)$ is unimodal; otherwise, if $w_4 > -\infty$, $\Pi_b^s(w)$ has at most two modes.*

(b) The supplier's optimal wholesale price, w_b^*, is determined as follows.

(i) If $w_2 = -\infty$, the optimal wholesale price $w_b^* = w_0$.

(ii) If $w_4 = -\infty$, but $w_2 > -\infty$, then the optimal wholesale price $w_b^* = w_2$.

(iii) If $w_4 > -\infty$, then

- *if $w_0 \in [w_1, w_2]$, the optimal wholesale price, w_b^*, will be either w_2 or $\hat{w}$;*
- *if $w_0 \notin [w_1, w_2]$, the optimal wholesale price, w_b^*, will be either w_0 or $\hat{w}$, where $\hat{w}$ is defined as*

$$\hat{w} = \max\{w \in [w_3, w_4] : (w - c)(\hat{Q}(w))' + \hat{Q}(w) \geq 0\} \quad (5.1.6)$$

Proof. According to the retailer's optimal ordering quantity with different w, the profit function of the supplier can be characterized as follows.

(1.) When $w \in [c, w_3]$, $\Pi_s^b(w)$ is increasing. If $w_3 > -\infty$, we know $w_2 > w_3 \geq w_1$. We know $w_0 > w_1$, hence $\Pi_s(w)$ is increasing in $[c, w_1)$. Moreover, as $w_3 < w_2$, $\Pi_s(w)$ is increasing in $[w_1, w_3]$ as well.

(2.) When $w \in (w_3, w_4)$, $\Pi_s^b(w)$ is unimodal, which has been shown in Proposition 5.

(3.) When $w \in (w_4, w_2)$, $\Pi_s^b(w)$ is increasing as $\Pi_s^b(w) = (w - c)k/w$ in this range.

(4.) When $w \in (w_2, p)$, $\Pi_s^b(w)$ is unimodal. This can be argued as follows. If $w_0 \geq w_2$, then $\Pi_s^b(w)$ first increase until w_0 and then decreases. If $w_0 < w_2$, $\Pi_s^b(w)$ is decreasing in w for $w > w_2$.

Because $\Pi_s^b(w)$ is continues, $\Pi_s^b(w)$ is at most a Bi-modal function with modes at $\hat{w}$ and w_0 or $\hat{w}$ and w_2. If there exist no w_3 or w_4, then $\Pi_s^b(w)$ is a unimodal function.

In the first two cases, the supplier's optimization problem is the same as the previous section. In the last one, the supplier's profit function is Bi-modal and the optimal solution is either $\arg\max_w\{(w-c)\hat{Q}(w)\}$ or $\arg\max_w\{(w-c)\min\{Q_0(w), k/w\}\}$. □

Since the supplier's profit function has at most two peaks, depending on the tightness of financial constraints, the optimal wholesale price will be either the solution in the basic setting or the optimal one when the retailer raises some loans. This theorem is very important since it makes the search procedure of the optimal solution much more efficient than an exhaustive search. Specifically, to find the optimal wholesale price, we first make sure the shape of the supplier's profit curve. If it has two modes, which is the more complex case, we only have to solve Equation 5.1.2 and 5.1.6 by Bisection method for w_2 and $\hat{w}$, and then substitute these two values to the supplier's function. The one achieves a higher value will be the optimal solution.

From above analysis, we can see if the supplier offers a sufficiently high or low wholesale price, the retailer does not borrow because in either of these two cases, the internal capital level of the retailer is relatively high and therefore, the financial constraint becomes redundant. On the other hand, if the supplier sets a wholesale price in the middle range, and the interest rate is relatively low, then the retailer will seek for external financing and there will exist a unique optimal wholesale price for the supplier. The supplier compares the profits under these two optimal wholesale prices and chooses the one leads to a higher profit.

This result implicates that the tightness of the retailer's financial constraint will also affect the wholesale price provided by the supplier and the external financing option may somehow improve the supply chain performance, which we will present in our numerically analysis later.

Special Case - Uniformly Distributed Demand

If the demand distribution is uniform on $[0, 1]$, then we can characterize the optimal wholesale price in a sharper form.

(i) If $4k(1+r) \geq p > 4k$,

$$Q_b^*(w) = Q^*(w),\ w_b^* = w^*.$$

(ii) If $p > 4k(1+r)$,

$$Q_b^*(w) = \begin{cases} \dfrac{p-w}{p}, & \text{if } w \leq w_1 \text{ or } w \geq w_2; \\ \dfrac{k}{w}, & \text{if } w_1 < w \leq w_3 \text{ or } w_4 \leq w < w_2; \\ \hat{Q}, & \text{otherwise,} \end{cases}$$

where $\hat{Q} = \dfrac{p^2 - w(1+r)p - wk(1+r)^2}{p^2 - w^2(1+r)^2}$,

$$w_3 = \frac{p - \sqrt{p^2 - 4pk(1+r)}}{2(1+r)} \text{ and } w_4 = \frac{p + \sqrt{p^2 - 4pk(1+r)}}{2(1+r)}.$$

$$w_b^* = \begin{cases} \arg\max\{\Pi_s^b(\hat{w}), \Pi_s^b(\frac{p+c}{2})\}, & \text{if } k \geq \frac{p^2-c^2}{4p}; \\ \arg\max\{\Pi_s^b(\hat{w}), \Pi_s^b(w_2)\}, & \text{otherwise,} \end{cases}$$

$$\text{where } \hat{w} = \frac{p\left(p^2 + (c+k)p(1+r) - \sqrt{k(1+r)(p^2 - c^2(1+r)^2)(2p + k(1+r))}\right)}{(1+r)\left(p^2 + cp(1+r) + ck(1+r)^2\right)}.$$

Proposition 5.1.5. *If the demand distribution is Uniform on [0,1], in the case where the retailer raises an external fund from a bank, if*

(i) $p^2 - pk(1+r) - c(1+r)\sqrt{p(p-4k)} \leq 0$, *or*

(ii) c *satisfies*

$$\begin{aligned} &-c^2(1+r)^2\left(p^2 + 2kp((r-1) + k^2(1+r)^2\right) \\ &+2cp(1+r)(p - k(1+r))\sqrt{p(p-4k)} - p^4 + 4kp^3(1+r) \geq 0, \end{aligned}$$

then $w_b^* = \tilde{w}_2$*; otherwise,* $w_b^* = \hat{w}$.

5.1.3 The Trade Credit Setting

In this subsection, we discuss the supplier's problem in the trade credit setting. First, we assume that if the retailer has insufficient cash to pay for the procurement cost, he has to give the supplier all the internal cash on hand as a deposit and pay off the rest after demand realization. At the end of the period, if the realized revenue is more than enough to cover the unpaid part, the retailer pays it off and receives the residual profit. In this case, the supplier receives full payment plus interests. However, if unfortunately, the revenue is insufficient to pay off the debt, the retailer goes to bankruptcy and the supplier only acquires the residual value of the firm. Therefore, the supplier's payoff function in trade credit case can be written as the following.

$$\max_{w} \quad \Pi_s^t(w) = \begin{cases} (w-c)Q_b^*(w), & \text{if } F^{-1}\left(\dfrac{p-w(1+r)}{p}\right) \leq \dfrac{k}{w}; \\ \Big(wQ_b^*(w) + rB^*(w)\Big)\bar{F}\big(\frac{B^*(w)(1+r)}{p}\big) & \\ + \int_0^{B^*(w)(1+r)/p}(px+k)f(x)dx - cQ_b^*(w), & \text{otherwise.} \end{cases} \tag{5.1.7}$$

The first case of (5.1.7) indicates the expected profit of the supplier when the retailer' initial cash is sufficient to pay the procurement cost in full so that the supplier does not need to extend trade credit to the retailer. While in the second case, the retailer trades with the supplier on credit and therefore, he is exposed to the bankruptcy risk. The supplier, who is the debt holder, may also bear some loss. We can easily verify that if the retailer does not borrow from the supplier, the optimal action for the supplier is the same as that in the basic setting. As a result, to fully characterize the optimal decision of the supplier, we need to know some properties of the supplier's objective function in the second case. Toward this end, we first need the following result.

Lemma 5.1.6. *If there exists some $\bar{w}$ satisfies $\left\{\bar{w} : 1-\hat{Q}(\bar{w})\dfrac{f\big(\hat{Q}(\bar{w})\big)}{\bar{F}\big(\hat{Q}(\bar{w})\big)} = 0\right\}$, then $w(\hat{Q}(w))' + \hat{Q}(w) \geq 0$ if and only if $w \leq \bar{w}$.*

Proof. We substitute the expression of $\hat{Q}'(w)$ into $w\hat{Q}'(w) + \hat{Q}(w)$, then we have

$$\begin{aligned}
w\hat{Q}'(w)+\hat{Q}(w) &= \frac{w(1+r)\bar{F}\big(\delta(w)\big)-w^2(1+r)^2 f\big(\delta(w)\big)\hat{Q}(w)/p}{w^2(1+r)^2 f\big(\delta(w)\big)/p-pf(\hat{Q}(w))}+\hat{Q}(w)\\
&= \frac{\left(1-\hat{Q}(w)\frac{f(\hat{Q}(w))}{\bar{F}(\delta(w))}\frac{p}{w(1+r)}\right)}{\left(\frac{w(1+r)}{p}\frac{f(\delta(w))}{\bar{F}(\delta(w))}-\frac{p}{w(1+r)}\frac{f(\hat{Q}(w))}{\bar{F}(\delta(w))}\right)}\\
&= \frac{\left(1-\hat{Q}(w)\frac{f(\hat{Q}(w))}{\bar{F}(\hat{Q}(w))}\right)}{\left(\frac{w(1+r)}{p}\frac{f(\delta(w))}{\bar{F}(\delta(w))}-\frac{f(\hat{Q}(w))}{\bar{F}(\hat{Q}(w))}\right)}.
\end{aligned}$$

Recall that $\delta(w)=(w\hat{Q}(w)-k)(1+r)/p$.

We assume that the demand distribution function is IFR, then the denominator is always less than zero. Therefore, $w\hat{Q}'(w)+\hat{Q}(w)$ is greater than zero if and only if the numerator is less than zero, that is,

$$1-\hat{Q}(w)\frac{f(\hat{Q}(w))}{\bar{F}(\hat{Q}(w))}\leq 0.$$

We have proved that $\hat{Q}(w)$ is a decreasing function, and IFR implies IGFR (increasing generalized failure rate), that is, $xf(x)/\bar{F}(x)$ is an increasing function if x is nonnegative. As a result, $\hat{Q}(w)f(\hat{Q}(w))/\bar{F}(\hat{Q}(w))$ is a decreasing function of w. Then it is easy to verify that as long as w is smaller than some specific w which satisfies

$$1-\hat{Q}(w)\frac{f(\hat{Q}(w))}{\bar{F}(\hat{Q}(w))}=0,$$

$w\hat{Q}'(w)+\hat{Q}(w)$ is greater than zero. □

By using the above lemma, we can obtain the following result.

Proposition 5.1.7. *If the retailer finances his ordering using trade credit,* *that is, $w \in \{w : k/w < F^{-1}(1 - w(1+r)/p)\}$, and the demand distribution function $F(\cdot)$ is CIFR, then the profit function of the supplier $\Pi_s^t(w)$ is unimodal over this range of w.*

Proof. First, taking the first derivative of supplier's profit function w.r.p to w yields

$$(\Pi_s^t(w))' = \hat{Q}'(w)\left((1+r)\bar{F}(\delta(w))\Big(w + \frac{\hat{Q}(w)}{\hat{Q}'(w)}\Big) - c\right).$$

To prove it is unimodal, we only need to prove that

(a.) if $w\hat{Q}'(w) + \hat{Q}(w) \geq 0$, then $d\Pi_s^t/dw \geq 0$;

(b.) if $w\hat{Q}'(w) + \hat{Q}(w) < 0$, then $\Pi_s^t(w)$ is unimodal.

Because we have proved that when w is less than or equal to some specific number, $w\hat{Q}'(w) + \hat{Q}(w)$ will be greater than zero, which means Π_s^t increases first and then becomes a unimodal function, which implies that Π_s^t a unimodal function.

It is easy to see that part $(a.)$ is true, so we omit the proof here. Now we focus on part $(b.)$. Because $\hat{Q}'(w)$ is less than zero, to prove the unimodality, we only need to show the term in the bracket is increasing, which is equivalent to show that $\bar{F}(\delta(w))\left(w + \hat{Q}(w)/\hat{Q}'(w)\right)$ is increasing. Furthermore, we know in this case, $w\hat{Q}'(w) + \hat{Q}(w) < 0$, which implies $w + \hat{Q}(w)/\hat{Q}'(w) \geq 0$ and $\bar{F}(\delta(w))$ is an increasing function of w, therefore we only need to show that when $w\hat{Q}'(w) + \hat{Q}(w) < 0$, $w + \hat{Q}(w)/\hat{Q}'(w)$ is also increasing.

Now we analyze this term in detail, we have

$$\begin{aligned}
\frac{\hat{Q}(w)}{\hat{Q}'(w)} &= \frac{w^2(1+r)^2\hat{Q}(w)f(\delta(w)) - p^2\hat{Q}(w)f(\hat{Q}(w))}{(1+r)p\bar{F}(\delta(w)) - w(1+r)^2 f(\delta(w))\hat{Q}(w)} \\
&= \frac{\left(\frac{w^2(1+r)}{p}\hat{Q}(w)\frac{f(\delta(w))}{\bar{F}(\delta(w))} - \frac{p}{1+r}\hat{Q}(w)\frac{f(\hat{Q}(w))}{\bar{F}(\delta(w))}\right)}{\left(1 - \frac{w(1+r)}{p}\frac{f(\delta(w))}{\bar{F}(\delta(w))}\hat{Q}(w)\right)} \\
&= \hat{Q}(w)w\frac{\left(\frac{w(1+r)}{p}\frac{f(\delta(w))}{\bar{F}(\delta(w))} - \frac{f(\hat{Q}(w))}{\bar{F}(\hat{Q}(w))}\right)}{\left(1 - \frac{w(1+r)}{p}\frac{f(\delta(w))}{\bar{F}(\delta(w))}\hat{Q}(w)\right)} \\
&\leq 0.
\end{aligned}$$

Because of IFR, we know the numerator is always negative, and we also know that $\hat{Q}(w)/\hat{Q}'(w)$ is negative, then it is easy to see that the denominator must be non-negative, that is,

$$1 - \frac{w(1+r)}{p}\frac{f(\delta(w))}{\bar{F}(\delta(w))}\hat{Q}(w) \geq 0.$$

Following the previous analysis, to prove the unimodality, we need to show that

$$w + \frac{\hat{Q}(w)}{\hat{Q}'(w)} = w\left(1 - \hat{Q}(w)\frac{f(\hat{Q}(w))}{\bar{F}(\hat{Q}(w))}\right) \Bigg/ \left(1 - \frac{w(1+r)}{p}\hat{Q}(w)\frac{f(\delta(w))}{\bar{F}(\delta(w))}\right) \geq 0$$

is increasing. We have proved that denominator is non-negative, therefore the numerator should be also non-negative, i.e.,

$$w\left(1 - \hat{Q}(w)\frac{f(\hat{Q}(w))}{\bar{F}(\hat{Q}(w))}\right) \geq 0.$$

By taking derivative, we have

$$\left(w + \frac{\hat{Q}(w)}{\hat{Q}'(w)}\right)' = \frac{1 - \hat{Q}(w)\frac{f(\hat{Q}(w))}{\bar{F}(\hat{Q}(w))}}{1 - \frac{w(1+r)}{p}\hat{Q}(w)\frac{f(\delta(w))}{\bar{F}(\delta(w))}} + w\left(\frac{\Delta(w)}{\left(1 - \frac{w(1+r)}{p}\hat{Q}(w)\frac{f(\delta(w))}{\bar{F}(\delta(w))}\right)^2}\right),$$

where

$$\Delta(w) = -\hat{Q}'(w)\left(\frac{f(\hat{Q}(w))}{\bar{F}(\hat{Q}(w))} + \hat{Q}(w)\Big(\frac{f(\hat{Q}(w))}{\bar{F}(\hat{Q}(w))}\Big)'\right)\left(1 - \frac{w(1+r)}{p}\hat{Q}(w)\frac{f(\delta(w))}{\bar{F}(\delta(w))}\right)$$
$$+\frac{1+r}{p}\left(w\hat{Q}'(w) + \hat{Q}(w)\right)\left(\frac{f(\delta(w))}{\bar{F}(\delta(w))} + \frac{w(1+r)}{p}\hat{Q}(w)\Big(\frac{f(\delta(w))}{\bar{F}(\delta(w))}\Big)'\right)$$
$$\cdot\left(1 - \hat{Q}(w)\frac{f(\hat{Q}(w))}{\bar{F}(\hat{Q}(w))}\right).$$

Because we have shown that the first term is non-negative, we only need to prove that $\Delta(w)$ is also non-negative. And it is easy to verify that

(1). $-\hat{Q}'(w) \geq 0$, $(1+r)\big(w\hat{Q}'(w) + \hat{Q}(w)\big)/p \leq 0$ and

$$\left|\hat{Q}'(w)\right| \geq \left|\frac{1+r}{p}(w\hat{Q}'(w) + \hat{Q}(w))\right|.$$

(2). Because of IFR and CIFR,

$$\frac{f(\hat{Q}(w))}{\bar{F}(\hat{Q}(w))}+\hat{Q}(w)\Big(\frac{f(\hat{Q}(w))}{\bar{F}(\hat{Q}(w))}\Big)' > \frac{f(\delta(w))}{\bar{F}(\delta(w))}+\frac{w(1+r)}{p}\hat{Q}(w)\Big(\frac{f(\delta(w))}{\bar{F}(\delta(w))}\Big)' \geq 0.$$

(3). Because we assume the demand distribution function is CIFR,

$$1 - \frac{w(1+r)}{p}\hat{Q}(w)\frac{f(\delta(w))}{\bar{F}(\delta(w))} > 1 - \hat{Q}(w)\frac{f(\hat{Q}(w))}{\bar{F}(\hat{Q}(w))} \geq 0,$$

which implies that $\Delta(w)$ is non-negative. □

Compared to the bank loan case, the trade credit case is more complicated because the financing decision of the retailer not only affects his own payoff, but also makes the supplier, the Stackelberg game leader involved with the credit risk. With this risk sharing effect, as long as the retailer raises a loan, the supplier's wholesale price decision ends differently and the performance of the supply chain will be different.

Similar to the bank loan case, the supplier's payoff function is piecewise with at most five pieces. Its shape is illustrated in Figure 5.3.

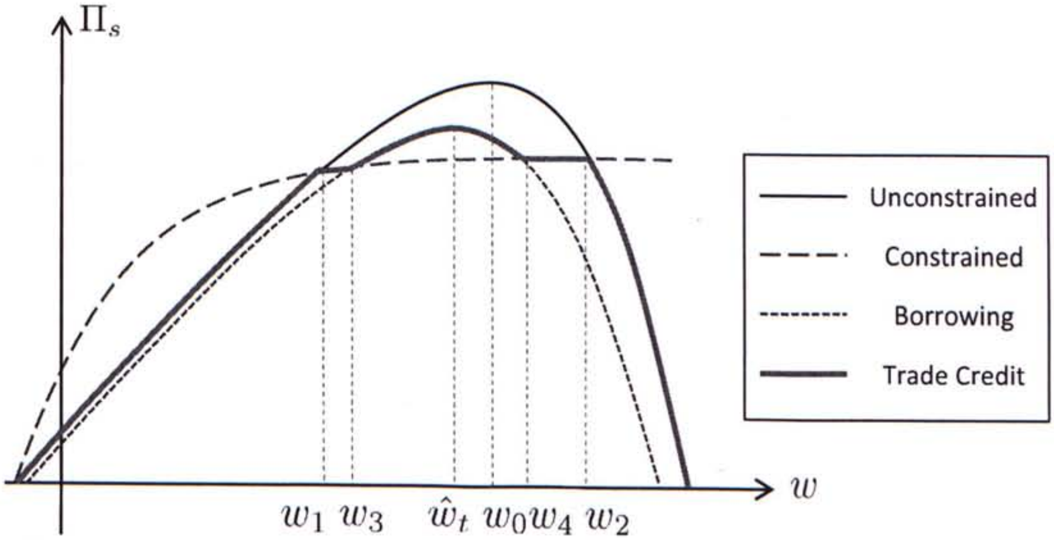

Figure 5.3: Single Period Supplier's Profit Function in Trade Credit Setting

Theorem 5.1.8. *(A similar result is derived by Kouvelis and Zhao (2008))* *(a) If* $w_4 = -\infty$*, the supplier's profit function* $\Pi_t^s(w)$ *is unimodal; otherwise, if* $w_4 > -\infty$*,* $\Pi_t^s(w)$ *is a Bi-modal function of* w*.*

(b) The supplier's optimal wholesale price w_t^* *is characterized in the following.*

(i) If $w_2 = -\infty$*, then the optimal wholesale price* $w_t^* = w_0$*.*

(ii) If $w_4 = -\infty$ *but there exists some* $w_2 > -\infty$*, then the optimal wholesale price* $w_t^* = w_2$*.*

(ii) If there $w_4 > -\infty$, then

- *if $w_0 \in [w_1, w_2]$, the optimal wholesale price, w_t^*, will be either w_2 or $\hat{w}_t$;*
- *if $w_0 \notin [w_1, w_2]$, the optimal wholesale price, w_t^*, will be either w_0 or $\hat{w}_t$;*

where $\hat{w}_t$ is defines as

$$\begin{aligned}\hat{w}_t &= \max\Big\{w \in [w_3, w_4] : c\hat{Q}'(w) \\ &= (1+r)\bar{F}\Big((w\hat{Q}(w)-k)(1+r)/p\Big)\big(w\hat{Q}'(w)+\hat{Q}(w)\big)\Big\}.\end{aligned} \tag{5.1.8}$$

Proof. The proof is very similar to that in the bank loan case, so we omit it here.

From the above theorem, we see that the structure of the optimal wholesale price in the trade credit setting is similar to that in the bank loan setting. Comparing the supplier's optimal decisions between the basic setting and the bank loan or the trade credit setting, we can obtain the following result.

Proposition 5.1.9. *$w_b^* \leq w^*$ and $w_t^* \leq w^*$.*

Proof. We prove the result for three different cases.

(i) If $w_2 = -\infty$, which means, regardless of the wholesale price, the retailer has sufficient initial capital so that his procurement decision will never be constrained. In this situation, $w_b^* = w^* = w_0$.

(ii) If $w_4 = -\infty$, but $w_2 > -\infty$, which means the retailer's decision may be constrained but he will not raise a debt. In this case, still, $w_b^* =$

$w^* = w_2$, because whether borrowing externally is allowed or not does not make any difference.

(iii) Except the two cases above, it may be profitable for the retailer to raise a loan. In this case, define

$$\Omega = \left\{ w : k/w \leq F^{-1}(1 - w(1+r)/p) \right\}.$$

When retailer decides to borrow, i.e., $w_b^* \in \Omega$. Because w^* is greater than any point in Ω. i.e. $w_b^* \geq w^*$. □

Indeed, when the retailer is capable of doing external financing, the supplier will always set a smaller wholesale price to induce the retailer to order more even if the supplier bears the retailer's default risk. This also suggests the double marginalization effect is reduced in the bank loan or the trade credit setting as compared to in the basic setting.

When the interest rates provided by the bank and by the supplier are the same, the key factor that determines the retailer's financing channel is the wholesale price. So it is interesting to compare the optimal wholesale prices between these two different financing schemes. For a given wholesale price, if the retailer does not rely on any external fund, it is clear that the supplier feels indifferent between the bank loan and the trade credit case. Therefore, from the previous analysis, for a w such that $F^{-1}(1-w(1+r)/p) \geq k/w$, the supplier's profits are the same under these two cases. Next we will focus on the wholesale price w at which the retailer raises fund from external sources.

Proposition 5.1.10. *When $k/w \leq F^{-1}(1 - w(1+r)/p)$, $w_t^* \leq w_b^*$ if and only if $\hat{Q}(w_b^*) \leq F^{-1}(1 - w_b^*/p)$.*

Proof. To compare the optimal solution of these two cases, we compare the first-order derivative of the profit function. Taking derivative yields

$$\begin{aligned} d\Pi_r^b(w)/dw &= \hat{Q}(w) + w\hat{Q}(w)' - c\hat{Q}(w)' = 0, \\ d\Pi_r^t(w)/dw &= (1+r)(\hat{Q}(w) + w\hat{Q}(w)')\bar{F}\left(\frac{(w\hat{Q}(w)-k)(1+r)}{p}\right) - c\hat{Q}'(w). \end{aligned}$$

We have proved that $\hat{Q}(w)$ is decreasing in w, therefore when $d\Pi_r^b(w)/dw = 0$, $\hat{Q} + w\hat{Q}' \leq 0$. To have $w_t^* \leq w_b^*$, it is sufficient to verify that $d\Pi_r^t(w)/dw \leq d\Pi_r^b(w)/dw$. From the equations above, $d\Pi_r^t(w)/dw \leq d\Pi_r^b(w)/dw$ if and only if

$$(1+r)\bar{F}\big((w\hat{Q}(w)-k)(1+r)/p\big) \geq 1.$$

Because we know that $\hat{Q}(w)$ satisfies

$$p\big(1-F(\hat{Q}(w))\big) = w(1+r)\bar{F}\big((w\hat{Q}(w)-k)(1+r)/p\big),$$

obviously, the condition can be rewritten as if $p\bar{F}(\hat{Q}(w))/w \geq 1$. As $\Pi_r^b(w)$ and $\Pi_r^t(w)$ are both unimodal functions in this case, $p\bar{F}(\hat{Q}(w_b^*)/w_b^* \geq 1$ implies $w_t^* \leq w_b^*$. □

When the retailer finances externally, the supplier sets a lower wholesale price in the trade credit case than in the bank loan case if the retailer orders less than the newsvendor solution given the wholesale price w_b^*.

5.2 Two-Period Problem

In this section, we consider the supplier's problem in the two-period setting. We assume the supplier offers a single wholesale price contract in which the term is two periods, and announces it at the beginning of the first period. However, under the two-period setting, the interaction between the retailer and the supplier becomes a dynamic Stackelberg game in which there may exist multiple solutions, and extremely difficult to solve. Therefore, we present the mathematical formulations in this section. Related numerical study will be presented in the next chapter.

Depending on the retailer's best response we have derived in Sections 4.2 and 4.3, the realization of demand in the first period will certainly affect the ordering decision of the retailer in the second period. As a result, different from the traditional "selling to the newsvendor" supplier who faces deterministic orders, the profit of the supplier in our dynamic setting is also affected by the demand uncertainty to some extent. In addition, in both the basic and the bank loan setting, the supplier does not play a role as a debt-holder, therefore, the procurement cost is always fully paid regardless whether the retailer is forced into bankruptcy or not. Since we ignore the time value of cash, the supplier's objective function in this two-period problem can be formulated as the following:

$$\max_{w} \quad (w-c)\big(Q_1^*(w) + \mathsf{E}_{D_1} Q_2^*(w, D_1)\big), \tag{5.2.1}$$

where $Q_i^*(w)$ is substituted by $Q_i^{b*}(w)$ in bank loan setting.

In contrast, in trade credit financing, the supplier not only issues the contract but also offers credit to the budget constrained retailer. As a debt holder, she collects interests and bears credit risks. In one case, if demands are sufficiently high, the supplier collects both the payment for the products and interests,

$$w\big(Q_1^{t*}(w) + Q_2^{t*}(w)\big) + B_1^{t*}(w) r_1 + B_2^{t*}(w) r_2.$$

If unfortunately, the retailer is unable to pay off the loan(s), the supplier only receives the residual value of the retailer, that is

$$k + B_1(w) + B_2(w) - w\big(Q_1^{t*}(w) + Q_2^{t*}(w)\big) + p\min\{Q_1^{t*}(w), D_1\} + p\min\{Q_2^{t*}(w), D_2\}.$$

As the result for two above cases, the final formulation of the supplier's optimization problem can be written as:

$$\begin{aligned}\max_{w} \quad & \mathsf{E}_{D_1}\Bigg(\mathsf{E}_{D_2}\min\Big\{k + B_1(w) + B_2(w) - w\big(Q_1^{t*}(w) + Q_2^{t*}(w)\big) \\ & +p\min\{Q_1^{t*}(w), D_1\} + p\min\{Q_2^{t*}(w), D_2\}, w\big(Q_1^{t*}(w) + Q_2^{t*}(w)\big) \\ & +B_1^{t*}(w)r_1 + B_2^{t*}(w)r_2\Big\} - cQ_2^{t*}(w)\Bigg) - cQ_1^{t*}(w).\end{aligned}$$

5.3 Summary

In this section, we study the supplier's problem in the capital constrained supply chain under three financing schemes. We characterize the structure of her optimal wholesale prices in the single-period setting. Furthermore, we also formulate the supplier's optimization problems under a two-period setting.

Chapter 6

Numerical Study and Insights

In this chapter, we conduct numerical experiments for the analytical models in the previous chapters to generate further managerial insights. In §6.1 we study the single period problem, where the issues we concern include the retailer's and the supplier's optimal decisions, the supply chain efficiency, etc. In §6.2, we focus on the retailer's problem in the two-period setting. The impacts of different financing schemes and dynamics on the retailer's optimal decisions will be discussed.

6.1 The Single Period Supply Chain

In this section, we discuss the impact of capital constraints on the supply chain in the single period setting. The default parameters are as follows. We choose the market demand to be a normal distribution with mean $\mu = 10$ and standard deviation $\sigma = 2$ or 4. We set the unit price of the product $p = 10$, and the production cost $c = 3.5$. The loan interest rate $r = 5\%$ or 10% and the initial capital level $k = 30$. In the experiments, we will vary some of the parameters while keeping the others fixed as default.

6.1.1 Impact of Different Financing Schemes

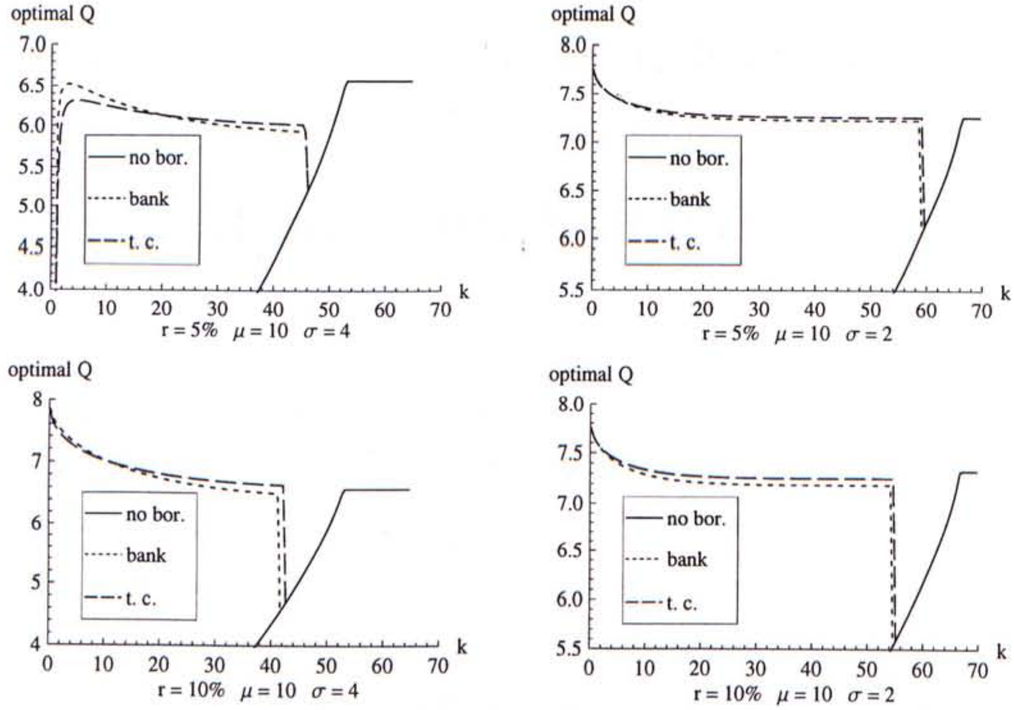

Figure 6.1: Impact of initial capital on optimal procurement quantity

Figure 6.1 depicts the retailer's optimal procurement quantity as a function of his initial capital level under the optimal wholesale price offered by the supplier. In each subplot, when the initial capital of the retailer is low, he seeks for external loans or credits extended by the supplier. As the initial capital increases, the optimal ordering quantity drops down and coincides with the one in the basic setting (which also reflects a jump of the optimal wholesale price between the two peaks, as we will see later). If the capital level is sufficiently high, the financial constraint of the retailer becomes redundant. Interestingly, when the standard deviation of the market demand is large ($\sigma = 4$), the financial constrained retailer orders less than the unconstrained ordering level when the interest rate is low ($r = 5\%$), while he tends to order more when the interest rate is high ($r = 10\%$). One of the possible explanations is that because when the market demand is more volatile and the loan rate is relatively high, the retailer is desperate to order more so that he can catch as much revenue as possible, while in other cases, the retailer is more rational. It is also worth mentioning that in the top left subplot, the optimal ordering quantity of the retailer in the trade credit case is not always larger than that in the bank loan case. When the retailer's initial capital level is very low, he orders more when his financing source is a bank. In addition, the cross point increases in the standard deviation of the demand and decreases in the interest rate level.

Figure 6.2 shows the supplier's optimal wholesale price in the (w, k) space. The jump-up in each subplot reflects the switch between the two peaks, which we have mentioned in Theorem 5.1.4 and 5.1.8. As the demand standard deviation increases or as the interest rate decreases, the range in which the supplier offers a higher wholesale price in trade credit than in the bank loan case expands.

We also study the impact of interest rate on the optimal decisions of the retailer and the supplier. It follows from Figure 6.3 that the retailer's optimal procurement quantity is decreasing in the interest rate provided by the bank or the supplier, which is very intuitive. On the other hand, the optimal wholesale price is non-monotone in the interest rate. Intuitively,

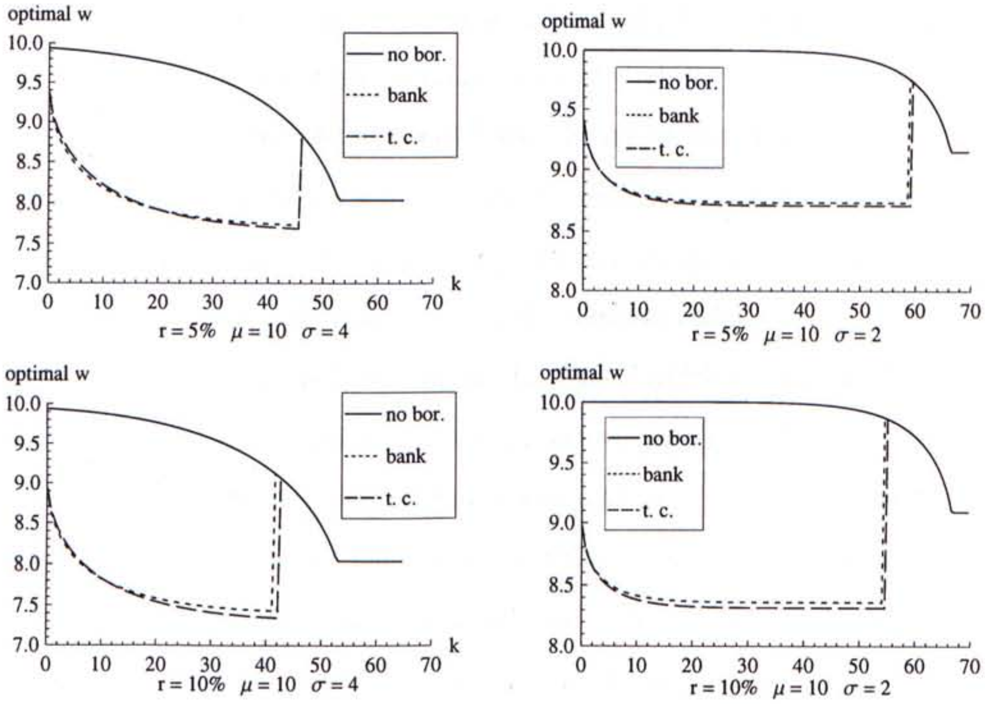

Figure 6.2: Impact of initial capital on optimal wholesale price

as the interest rate increases, the supplier will lower the wholesale price to induce the retailer to order more. However, in the bottom left subplot, where the retailer has a relatively high initial cash level, there is an upward jump which implies that it is no longer profitable for the supplier to induce the retailer to raise any loan, because she can make enough profit from his initial cash. It is worth mentioning that the optimal wholesale price in the trade credit case jumps at a higher interest rate, because the supplier, as a debt holder in this case, is more likely to set a lower wholesale price to induce the retailer to borrow money from her such that she would benefit from the interests. From the right panel, we also observe that among different interest rates, there is no dominant financing scheme for the retailer in terms of his

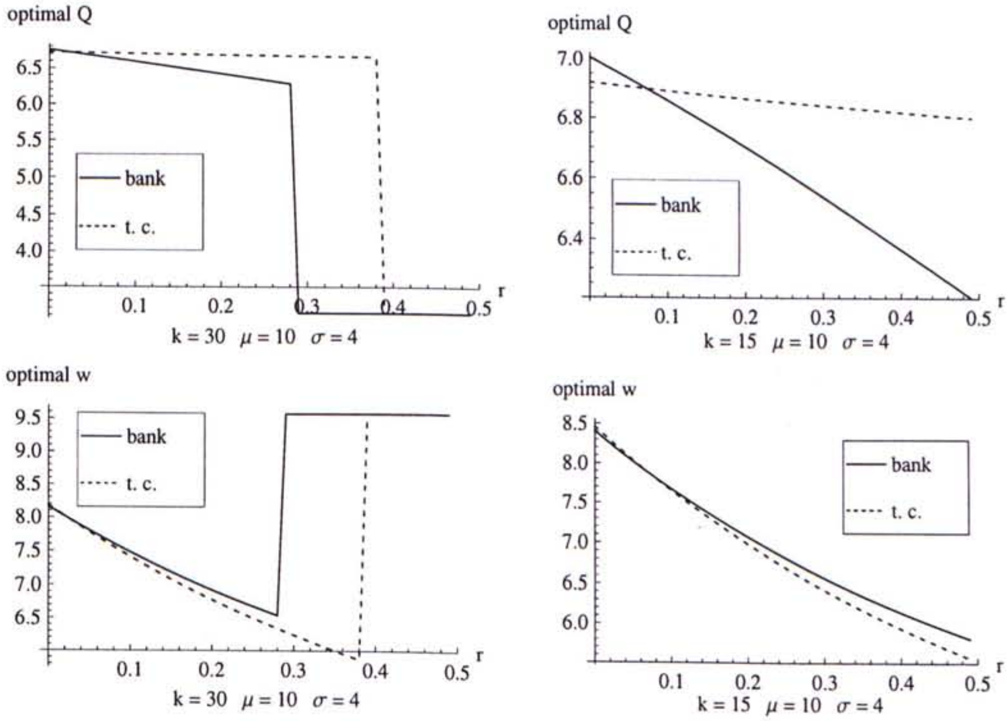

Figure 6.3: Impact of interest rate on optimal procurement quantity and optimal wholesale price

profit, since the optimal wholesale price in the bank loan case may or may not be larger than that in the trade credit case. When r is sufficiently low, the supplier offers a lower wholesale price in the bank loan case, and the retailer orders more accordingly. This is because when the demand variance is large but interest earned by trade credit is relatively low, the supplier will avoid taking risk by increasing the wholesale price a bit compared to that in the bank loan case.

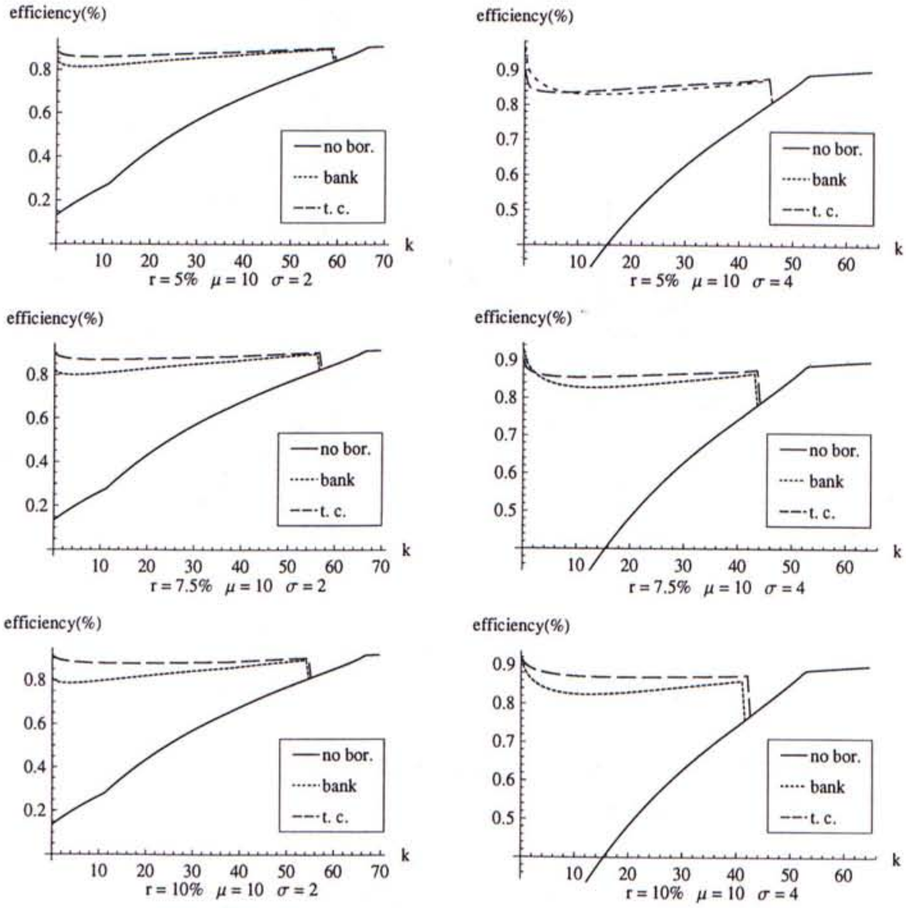

Figure 6.4: Impact of initial capital on supply chain efficiency

6.1.2 Supply Chain Efficiency

In this section, we examine how the financial constraints affect the supply chain performance and whether the loss of channel efficiency can be reduced by appropriately financing the retailer.

To see the impacts of financial constraint on the supply chain's profit, we plot the supply chain efficiency with respect to the initial capital level of the retailer in Figure 6.4, where the supply chain efficiency is defined as follows:

$$\text{efficiency} = \frac{\text{total supply chain profit}}{\text{total supply chain profit without retailer's constraint}} \times 100\%.$$

From each subplot we observe that in the basic setting, the supply chain efficiency is strictly increasing in the initial capital level. Moreover, when the retailer's initial cash level is very low, the efficiency can be even lower than 15%, which indicates the significant impacts of retailer's financial constraints. Intuitively, financing, no matter by bank loan or by trade credit, provides the constrained retailer an opportunity to achieve the unconstrained operational decisions, therefore, the performance will be always better than in the basic setting, which is confirmed in our experiments.

We also find that, in the three subplots on the left, where the standard deviation of the demand is relatively low, i.e., $\sigma = 2$, the supply chain always performs better in the trade credit case. In addition, when the initial capital level is not high, as the interest rate becomes higher, the difference between the efficiencies in the bank loan case and that in the trade credit case becomes larger. The reason behind is: if the supplier is the debt holder, she will offer a lower wholesale price to reduce the default risk due to the high interest rate, which gives the retailer an incentive to order more and results in a higher supply chain efficiency. On the other hand, if the debt holder is an external intermediary, higher interest rate incurs more internal cash flowing out of the supply chain, which certainly lowers the efficiency. Observing from the three subplots on the right in Figure 6.4, the supply chain under the trade credit scheme may not always achieve a higher performance than under the bank financing scheme when the variance of the demand is large. When the interest rate is low ($r = 5\%$), the supply chain performs better in the bank loan case if the initial cash of the retailer is sufficiently low. Intuitively, when the value of risk is higher than the interests paid, the supply chain is always better off if the debt holder is someone outside the supply chain.

6.2 Capital Constrained Retailer in Two-Period Setting

In this section, we numerically examine the retailer's optimal strategy under the two-period setting. Since it is practically difficult to approach a two-dimensional dynamic programming with a continuous state space, we choose the market demand in each period to be discrete-uniformly distributed between 0 and 9. We set selling price $p = 9$, the wholesale price $w = 4$, and interest rate $r = 10\%$, which is identical between two periods.

6.2.1 Impacts of Different Financial Schemes on the Retailer

First we investigate the retailer's optimal ordering decision under different financing schemes. Figure 6.5 illustrates his optimal ordering quantity with respect to his initial cash level. As we can see from this figure, the retailer who is able to do financing always orders more than he does in the basic setting. When the retailer has very limited initial cash, he orders even more than the optimal newsvendor ordering quantity. This is because once the retailer facing very tight financing constraint borrows, he bears essentially high credit risk, and therefore, he has to order more to capture as much

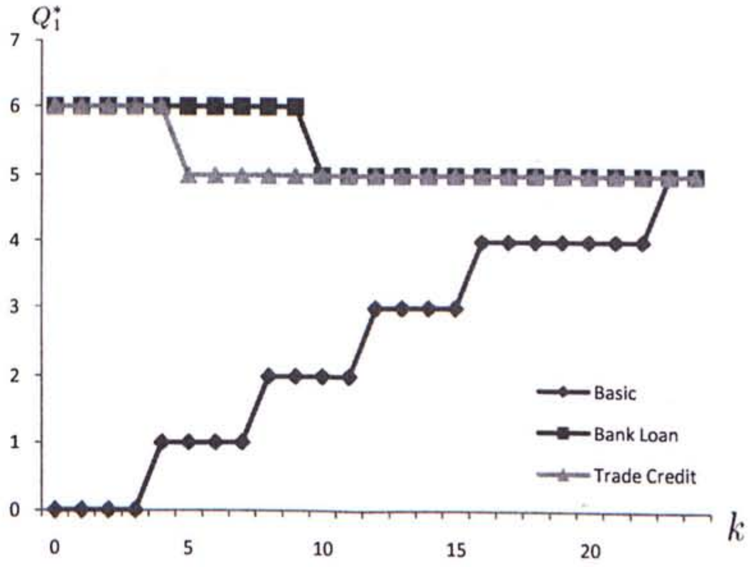

Figure 6.5: Optimal Ordering in the First Period v.s. Initial Cash Level

revenue as possible to pay off his debt. Interestingly, as his initial cash level is in the low middle range, the retailer in the bank loan setting keeps on ordering more, while in the trade credit case, his ordering quantity drops to the unconstrained optimal quantity Q_0. Intuitively, the retailer in the trade credit setting only cares about this period, since no matter how bad his financial status is in the beginning of the second period, he still has an opportunity to borrow. However, in the bank loan case, the retailer does not have the second chance to raise loans, as a result, he is more aggressive in his procurement decision.

Figure 6.6 shows the retailer's optimal profit efficiency as a function of his initial cash level. Note that in this figure, the efficiency ϕ is defined as follows.

$$\phi = \frac{\text{Retailer's optimal profit}}{\text{Retailer's unconstrained profit}} \times 100\%,$$

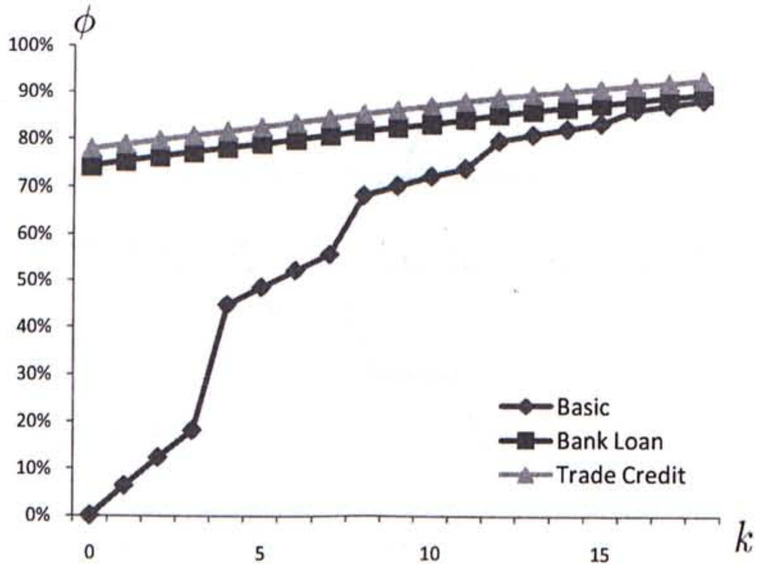

Figure 6.6: Efficiency v.s. Initial Cash Level

where the retailer's unconstrained profit is obtained by optimizing the same objective function of the retailer without considering the financial constraints. As this figure reveals, the retailer's optimal profits in all three cases are increasing in k, which is intuitive, and the retailer' payoffs are much better in the bank loan and the trade credit case than that in the basic setting, which demonstrates the significant impact of the financial status on the retailer's profitability. We also notice that for the cash constrained retailer, the trade credit financing scheme dominates the other two. Hence, we draw a conclusion that under two-period setting, the retailer always prefers trade credit when the wholesale price is exogenous given.

6.2.2 Saving for the Future

Under the multi-period bank loan setting, intuitively, the retailer will save some cash for the future use. Figure 6.7 shows the retailer's saving amount as a function of his initial cash level. We define the saving for the future δ

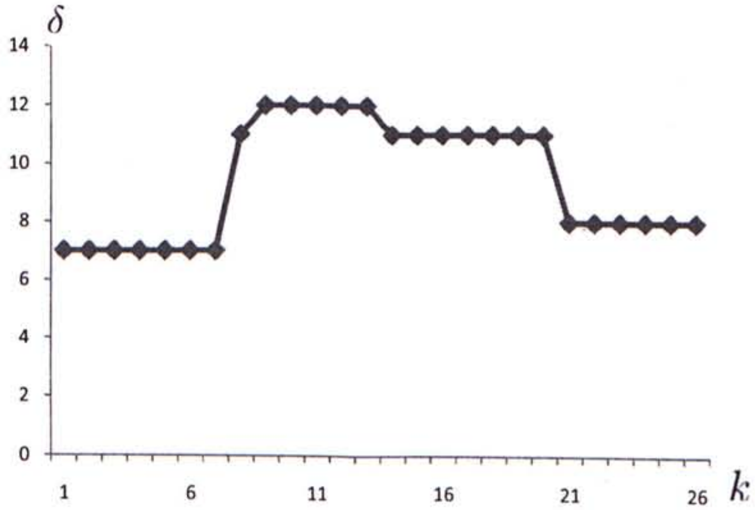

Figure 6.7: Cash Saving v.s. Initial Cash Level

by the following formula.

$$\delta = k + b_1^* - wQ_1^*$$

The interesting result is that the retailer's saving amount is not monotone in k. He cares more about next period when his initial capital level is moderate. The reason is when the retailer is facing a very tight financing constraint, he is exposed to a very high risk of bankruptcy so that he may be too desperate to plan for the future. In contrast, if the retailer has sufficient cash on hand, he does not have to save much for the future since most likely his revenue would be able to cover his next period's procurement cost.

6.2.3 Comparison of the Single- and Two-Period Settings

Next we compare the retailer's optimal ordering quantities of the single-period and the two-period case. Figure 6.8 plots the ordering quantities in

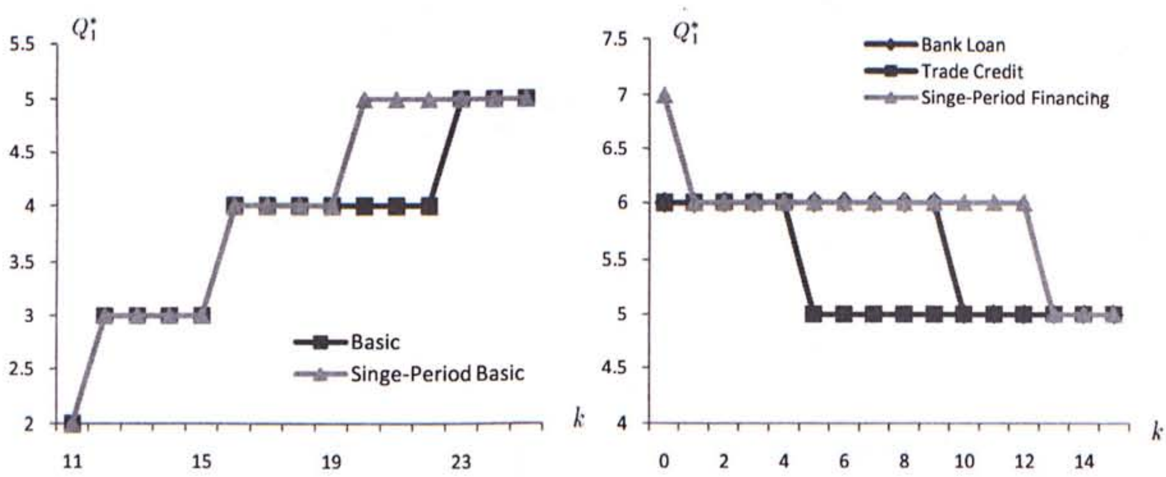

Figure 6.8: Optimal Ordering Quantity - Single Period v.s. Two Period

the basic setting (left panel) and in the settings in which the retailer has financing opportunities (right panel), as a function of his initial cash. From these figures, the retailer orders more in the single period setting in both cases. The intuition for this is that the retailer in the multi-period setting is always so conservative that orders less (weak sense) comparing to he does in the single-period case. As the retailer's initial cash increases, his optimal ordering quantity in the two-period financing setting reaches newsvendor solution earlier than in the single-period case. This is because the retailer is less sensitive to the tightness of financial constraints due to the extra earning opportunity in the second period.

Chapter 7

Conclusion and Future Research

Finance plays a vital role in running a business. Lack of capital severely decreases a company's profitability and such harm may ultimately plague the entire supply chain. In this thesis, we make several contributions to the literature of supply chain models with financial constraints. Specifically, we study a supply chain model with a capital constrained retailer. We investigated the impacts of financial constraints and different financing schemes on the profitability of the retailer, the supplier and the supply chain, and how the retailer and the supplier should optimally react to the financial constraint.

We first study the retailer's problem for a given wholesale price and bank loan/trade credit interest rate. We find that in the single period setting, the retailer's profit is unimodal in his ordering quantity and financial decision if the demand distribution has an increasing failure rate. We also demonstrate that, in the presence of bank loan or trade credit, the retailer's optimal ordering quantity may not be monotone in his initial budget. We then extend this problem to a two-period dynamic setting. Under the assumption that the demand has a log-concave density, we show the retailer's profit function is still unimodal in his ordering quantity and financial decision. Such result can be extended to the general multi-period setting and is very important for the retailer to maximize his profit.

Next we study how the supplier decides an optimal wholesale price to maximize her profit given the retailer is financial constrained. Since the supplier's problem in a two period setting is a dynamic Stackelberg game and is in general intractable, we only focus on the single period setting. We show that in the basic setting where the retailer is unable to raise fund, the supplier's profit is unimodal and the optimal wholesale price is always at least as the optimal wholesale price when the retailer is unconstrained. In the bank loan and trade credit case, we show that the supplier's objective function has at most two modes. It means that the supplier will either set a wholesale price such that the retailer will use up his capital without exerting external financing or a smaller wholesale price to encourage the retailer to seek financial help and order more.

To gain more managerial insights, we conduct several numerical studies. Our main findings are that if the interest rate of bank loan and trade credit are the same and exogenously given, then there is no dominating financial scheme for the retailer, i.e. the retailer may prefer bank loan to trade credit, or vise versa, depending on his initial capital. For the supply chain, its total profit is much larger when the retailer can seek external financing than that when the retailer can not.

In the future, we intend to investigate other common used financing schemes such as bonds with dividends and securities, and the impacts of market imperfections on trade credit and bank financing channels. It is also of interest to research the capital structure and operations management interface under the long term setting.

In our model, we assume all the debts are repaid at the end of the second period. It is also very interesting to study the case the loans are settled in each period. In this situation, the borrower can be bankrupt at the end of each period and the ending time of the planning horizon will be stochastic.

Another possible extension is to address the information asymmetry issue. Through our model, we assume that initial budget of the retailer is a public information that is available to all players in the supply chain. In reality, however, this may not be true. As a result, in trade credit practice, the supplier may not be able to offer the optimal contract terms, and retailer may misrepresent his initial capital to obtain a more favorable contract and improve his own profit.

Appendix A

Log-concavity of Some

Common Distributions

Distribution	Log-Concave (Density)	IFR
Uniform	yes	yes
Normal	yes	yes
Chi-Squared	yes	yes
Exponential	yes	yes
Laplace	yes	yes
Weibull ($c \geq 1$)	yes	yes
Power Function ($\beta \geq 1$)	yes	yes
Gamma ($m \geq 1$)	yes	yes
Beta ($a \geq 1, b \geq 1$)	yes	yes
Log Normal	log-concave on the interval (0,1), log-convex on the interval (1,∞)	no
Power Function ($\beta < 1$)	log-convex	no
Weibull ($c < 1$)	log-convex	no
Gamma ($m < 1$)	log-convex	no
Beta ($a = 0.5, b = 0.5$)	log-convex	no
Beta ($a = 2, b = 0.5$)	neither log-convex nor log-concave	yes

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