Portfolio Selection Based on Minmax Rule and Fuzzy Set Theory

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Abstract of thesis entitled:

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Most of the existing portfolio selection models are based on the probability theory. They often apply probabilistic approaches to deal with uncertainty. As known to us, uncertainty includes two aspects: randomness and fuzziness. The probabilistic approaches only partly capture the real situation. Some other techniques have also been used to handle the uncertainty of financial markets, including the fuzzy set theory. In reality, many events with fuzziness are characterized by probabilistic approaches, even though they are not random events. The fuzzy set theory has been widely used to solve many practical problems. By using fuzzy mathematical approaches, the experts' knowledge, experiences and investors' subjective opinions can be better integrated into a portfolio selection model.

The content of this thesis mainly comprises of literature review on fuzzy portfolio selection problems. Three portfolio selection models based on fuzzy theory, and minmax rule is proposed in this thesis. They also take transaction fee into consideration in fractional market. These portfolio selection models might be more efficient for practical

applications. Examples of application are provided to illustrate better performance under these three models by using real data from the Hong Kong securities markets compared with Markowitz Mean Variance model.

The main innovative results of this thesis include: Firstly, minimax rule risk is used as a risk measure. It is a more conservative and prudent risk measure compared with the whole portfolio variance, firstly proposed by Cai in 2000. Secondly, these models are all considered under frictional securities market, as transaction fee deducted. Thirdly, liquidity is considered into these models estimated by the turnover rate, which is a very important investment factor that cannot be ignored. Last but not the least, the thought of fuzzy set theory is introduced in later two models to eliminate the boundary between objectives and constraints, which is fundamental in the characteristics of membership function in fuzzy set theory. In this thesis, two typical membership functions are considered: semi-trapezoidal function and S-shape function.

中文摘要

大部分现有的投资组合模型基本上是基于概论，他们通常运用概论的方法来处理事物的非确定性。而众所周知，概论方法只能部分来描述现实情况的不确定性，事物的不确定性包括两个方面：随机性和模糊性。所以现在很多其他技术方法也被引入来解决金融市场模型的不确定性问题，例如：模糊集合论（最先由 Zadeh 于 1965 年提出概念）。而在现实中，很多事件的模糊性也笼统地被概论方法来描述，即使这些事件本身只是随机的而不是模糊的。模糊集合论被广泛地用于很多实际问题中，运用模糊数学的方法，专家的经验和投资者的主观目标能被很好地表示到投资组合优化的模型中去。

这篇论文的内容主要包括了作者在基于模糊理论的投资组合问题的文献综述总结，并在此基础上加入了改进，依次提出了三个模型。三个模型都基于极大极小规则风险衡量方法（Cai 于 2000 年提出），并且模型的建立考虑了摩擦市场下的交易费用。因此这些模型对于实际应用更为有效。同时也取得香港证券市场的实际数据运用于模型投资与恒生指数相比较，验证了模型的有效性。

这篇论文的主要创新点有：一、极大极小规则风险衡量方法在模型中的运用，这种方法相对于传统的投资组合整体的方差来表示风险更为保守和谨慎。二、模型都是基于摩擦市场下的，因为考虑了证券市场的交易费用。三、流动性作为证券投资的一个重要指标也被考虑进了模型中，在模型选取股票的换手率都衡量。四、引入了模糊集合论的思想中的隶属函数的概念去除了传统优化问题中目标和约束之间明显的界限。此文中运用了最典型的两类隶属函数半梯形函数和 S 形函数。

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Fan Yang

The Chinese University of Hong Kong
December, 2010

This work is dedicated to my friends who encourage me, support me, and guide me in my postgraduate time.

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Chapter 1

Introduction

Summary

In this chapter, we will review existing related portfolio selection literature firstly. New models based on min-max rule and fuzzy decision theory are proposed here. Three investment models give a general idea of how the models work in the real life. It is followed by the contribution of the thesis.

1.1 Literature review

The portfolio selection problem receives more and more attention in recent years. It mainly concerns people using limited funds to achieve maximum profit under risk level they can bear. Uncertainty about future events is a key factor in prediction in financial problems. As we all know, under such an uncertain and ever-changing financial environment, classic probability theories have been widely applied as the most prominent mathematical analytical tools to model both the behavior of the financial institutes and future events in financial markets, which in turn, play an influential role in practical work of financial institutions. Markowitz's mean-variance (M-V) model has been developed that allows one to determine portfolios with the highest expected returns for a given level of risk. This pioneering work has served as the basis for the development of numerous following modern financial theories. Work of Markowitz (1952) in portfolio selection has been the most influential development in mathematic finance asset management. The greatest contribution of M-V model is that it combines probability and optimization theories to model the behavior of financial agents under changing economic environment. It tries to strike a balance between maximizing the whole portfolio rate of return and minimizing the risk of investment decisions. Return is quantified by mean, and risk is characterized by variance of securities. A specific solution is related to the risk level people could bear. Investors set the risk level according to their investment preference. After M-V model, many models came out, although they have different methods on mathematical definitions of risk and return, the trade-off between return and risk has always been the major problem in portfolio selection topic. Contrary to M-V model's theoretical reputation, it is not used extensively to construct large-scale portfolios. Then Single index model is proposed by Sharpe in 1963, which is a breakthrough in portfolio selection. His contribution in this field is to greatly reduce the computational complexity from $O(N^2)$variance-covariance to a total of O(N) parameters. Furthermore, Speranza (1993) used semi-absolute deviation to measure the risk and formulated a portfolio selection model.

Semi-variance is defined as the expected value of squared "positive (or negative)" deviations from the mean. It used mean-absolute deviation as the risk of portfolio investment, and got similar performance with M-V model. Konno and Yamazaki (1991) formulated a mean absolute deviation portfolio optimization model, proposed a new risk measure l_1 and suggested that a piecewise linear function can be used to approximate this l_1. The portfolio optimization problem under l_1 risk measure can be converted into a scalar parametric linear programming problem. It turns out that the mean absolute deviation model retains the useful properties of Markowitz's model and removes most of the principal computational difficulties in solving Markowitz's model. So the implementation of the portfolio optimization problem with the l_1 risk function can be easily obtained. Yong (1998) introduced another linear programming problem model which amounts to maximizing the minimum return over time period. Both corresponding optimization problems to above linear program models are simple, and perform in a similar way to that of the mean variance model. A minimax risk function is introduced by Cai et al. (2000). By using a special format of the l_∞ risk function, explicit solution scheme is obtained for the efficient frontier of the portfolio optimization problem without having to solve any optimization problem. Such a risk function is defined as to minimize the individual risks over the maximum individual risk. The practical meaning of this risk function is that an investor wants to minimize the maximum individual risks among assets to be invested. The corresponding portfolio optimization problem is formulated as a bi-object linear programming problem. In this thesis, this minimax l_∞ risk function has been chose as risk measure in the whole thesis. The reason for choosing it as follow: firstly, it can greatly reduce computational complexity of the portfolio problem. Secondly, it is considered as a more conservative risk measurement, compared with classic variance. Thirdly, it can be easily combined with fuzzy set theory in my models, to flexibly adjust investor's objective risk preference.

Above previous works are based on probabilistic theory. Future events are uncertain, which makes the behavior of economic indicators unpredictable and brings about turbulence to financial markets. Assumptions, while allocating fund under an uncertain and ever-changing environment, are the building blocks for theories of portfolio optimization problem in finance market. Without doubt, the ground-breaking work of Markowitz's M-V model in portfolio selection has been the most impact-making development in modern mathematical finance management. However, the probabilistic approaches only partly capture the reality. Some other techniques have also been applied to handle the uncertainty in financial markets, for instance, the fuzzy set theory by Zadeh (1965). In reality, although many events are characterized by probabilistic approaches, they are not random events. The fuzzy set theory has been widely used to solve many practical problems. It has also been introduced in financial management. By fuzzy approaches, quantitative analysis, qualitative analysis, experts' knowledge and investors' subjective opinions can be integrated into portfolio selection model. A few authors such as Ramaswamy (1998), Tanaka, Guo (1999) , Inuiguchi and Ramik (2000), studied fuzzy portfolio selection problem in recent years.

Transaction cost is one of the main concerns to portfolio managers. Ignoring transaction costs would result in inefficient portfolio (Arnott and Wagner (1990)'s research). Yoshimoto's empirical analysis (1996) also drew the same conclusion. Mao (1970), Jacob (1974), Brennan (1993), Levy (1978), Patel and Subhmanyam (1982), Morton and Pliska (1995) and Mansini and Speranza (1999) studied portfolio optimization with fixed transaction costs; Pogue (1986), Chen, Jen and Zionts (1971) and Yoshimoto (1996) et al. studied portfolio optimization with variable transaction costs; Dumas and Luciano (1991), Mulvey and Vladimirou (1992) and Dantzig and Infanger (1993) incorporated the transaction costs into the multi-period portfolio selection model. Recently, Li, Wang and Deng (2000) gave a linear programming algorithm to solve a general mean variance model for portfolio selection with transaction costs. Konno and Wijayanayake (2001) studied portfolio optimization with transaction costs which can be expressed approximately as a D.C. function.

Expected return and risk are considered as two fundamental factors. In some cases, investors may consider other factors such as skew, liquidity etc. Liquidity has been measured as the degree of the probability of having the option of converting of an investment into cash without any significant loss in value. In this thesis, I will formulate optimization models for a portfolio selection problem with fuzzy liquidity constraints and transaction cost consideration, based on the minimax risk function and fuzzy set theory.

1.2 The main contribution of this thesis

In this thesis, three models are explored.

For Model 1, in the return part, transaction fee has been taken into consideration. In the risk part, I introduce the minimax risk function to describe the risk of the portfolio. Besides the return and risk, liquidity has been considered here. Liquidity is described by the turnover rate of every asset. This factor is seen as a fuzzy number (Trapezoidal fuzzy number). According to the diversify principle, we also set an upper invest limit for every asset. However, it keeps the traditional portfolio selection format: bi-criteria problem, a tradeoff between expected return and risk under such constraints.

For model 2 and model 3, based on the model 1, we do fuzzification on the expected return, risk level, liquidity with different fuzzy membership functions, which are both typical membership function in classic fuzzy set theory. The difference from model 1 is: in model 1, we just did fuzzification on liquidity constraint. The basic problem is still bi-criteria optimization as most previous work. However, in model 2 and model 3, the problems are modeled by the essential thought of fuzzy set theory. The Key parameters including expected return, risk and liquidity are all fuzzy variables, using trapezoidal fuzzy membership functions to estimate the satisfactory degree on profit, risk and liquidity of portfolio. There is no clear line between objectives and constraints. We try to maximize the total membership function value. Hence, it can achieve multi-objectives optimization.

1.3 Relations between the above three models

The first model is the simplest one. The main contribution is to propose the basic scenario of our problem and introduce the liquidity concept in portfolio selection, which is a very important and practical topic in real portfolio problem. The fuzzy set theory is then introduced to address the liquidity constraints since liquidity usually cannot be demonstrated in an explicit number. Profit stays before, estimated by expected rate of return from historical data. Risk is estimated in measure l_∞ proposed by Prof.Cai as the maximum individual risk.

The second model is a more general model of fuzzy portfolio selection. In model 1, we just introduce fuzzy number to denote liquidity constraint. In model 2, we use fuzzy membership function to denote the profit, risk and liquidity satisfactory degrees. With introduction of fuzzy number, goals and constraints become soft, there is no clear line. It becomes objectives optimal problem. However, we just use the simplest trapezoidal fuzzy membership function in model, which is linear and easy to deal with.

In order to approximate real situation closely, we revised model 2 to model 3. Since there exists limit in describing the characteristic of satisfactory degree of risk, profit and liquidity by simple trapezoidal fuzzy membership function, although it is linear and easy to deal with. Then we introduce non-linear fuzzy membership function by an S-shape function, which can make our model more practical. The numerical example also gives us a better result to demonstrate its advantages.

□ **End of chapter.**

Chapter 2

Model 1

Summary

In this chapter, we introduce the investment background and proposed our first model: portfolio selection with fuzzy constraints.

2.1 Introduction

In 1952, Markowitz's work laid the foundation of modern portfolio analysis. M-V model has served as a basis for development of the modern financial theory over the past six decades. However, contrary to its theoretical reputation, it is not used widely to deal with large-scale portfolios. The most important reason is the computational difficulty in solving large-scale quadratic programming problem with covariance matrix. Afterwards, Konno and Yamazaki proposed the absolute deviation risk function, and replace the variance function, and thus formulated a mean absolute deviation portfolio optimization model. The mean absolute deviation model keeps the advantages of Markowitz's model and greatly reduced the principal computational complexity.

Cai et al. provided a portfolio selection method under minimax rule, which introduces a new risk measure as the maximum individual risk and build up a model with a bi-criteria objective function. Following Cai et al' s work, numerous researches have been done on this minimax type risk measure as an stochastic decision problem as well as in stochastic game environment, for example, see, Deng et al. These researches only consider the case under Possibilistic assumption. However, the uncertainty can be divided into two categories: fuzzy and random. In order to describe the real case more exactly, we turn to fuzzy set theory.

Transaction cost is also taken into consideration in our model. Ignoring transaction costs would result in an inefficient portfolio as mentioned in related literatures: Yoshimoto's empirical analysis. Mao, Mansini etc. Transaction cost can be treated as fixed costs and variable costs, depending on investors' assumption.

Usually, expected return and risk are the two fundamental factors in portfolio selection problem. In this thesis, investors may also consider liquidity. Liquidity has been measured as the degree of probability of having the option of conversion of an investment into cash without any significant loss in value. Parra and Terol firstly account liquidity into criteria and used a fuzzy goal programming approach to solve the portfolio selection problem.

In this chapter, we will propose portfolio selection models with fuzzy liquidity constraints. We firstly constructed a risk function - Minimax risk function. Then we proposed optimization model with fuzzy liquidity constraints, based on the minimax risk function and fuzzy set theory.

2.2 Minimax rule risk function

Cai etc. used the minimax rule risk function to measure the risk and formulated a mean minimax portfolio selection model. In the following we will explicitly show the risk function based on Minimax rule.

Assume that an investor has initial wealth M_0, which is to be invested in n possible assets $S_j, j = 1, ..., n$. Let R_j be the return rate of the asset S_j, which is a random variable. Let $x_i \geq 0$ be the allocation from M_0 for investment to S_j. (Note that by assuming $x_j \geq 0$ we are concerned with the situation where short selling is not allowed). Thus, the feasible region for the portfolio optimization problem is

$$\mathcal{F} = \{x = (x_1, ..., x_n) : \sum_{j=1}^{n} x_j = M_0, x_j \geq 0, j = 1, ..., n\} \qquad (2.1)$$

Let $E(R)$ denote the mathematical expectation of a random variable R. Define

$$r_j = E(R_j) \ \ and \ \ q_j = E(|R_j - r_j|) \qquad (2.2)$$

Namely, r_j and q_j denote the expected return rate of the asset S_j and the expected absolute deviation of R_j from its mean, respectively.

The expected return of a portfolio $x = (x_1, ..., x_n)$ is given by

$$r(x_1, ..., x_n) = E[\sum_{i=1}^{n} R_i x_i] = \sum_{i=1}^{n} E(R_i)x_i = \sum_{i=1}^{n} r_i x_i \qquad (2.3)$$

The l_∞ measure is defined as follows,

$$l_\infty(x) = \max_{1\leq i\leq n} E(|R_i x_i - r_i x_i|) \qquad (2.4)$$

2.3 Fuzzy liquidity of asset

Liquidity is measured as the degree of probability of having an option to convert an investment into cash without any significant loss in value. The turnover rate of a asset is a proportion of turnover volume to tradable volume of the security, and is a factor which may reflect the asset's liquidity. In general case, investors prefer greater liquidity, especially in a bull market.

However, as known to us, future turnover rates cannot be accurately predicted in financial market. The possibility theory has been proposed by Zadeh and advanced by Dubois and Prade where fuzzy variables are associated with the possibility distribution. In the whole thesis, the turnover rates are modeled by possibility distributions rather than by probability distributions. That is, the turnover rates will be represented by fuzzy numbers. In this chapter, we regard trapezoidal possibility distribution as the possibility distribution of the turnover rates of the assets.

A Fuzzy number A is called trapezoidal with tolerance interval $[a, b]$

, left width and right width , if its membership function takes the following form:

$$A(t) = \begin{cases} 1-(a-t)/\alpha & , \; if \;\; a-\alpha \le t \le a\,, \\ 1 & , \; if \;\; a \le t \le b\,, \\ 1-(t-b)/\beta & , \; if \;\; a \le t \le b+\beta\,, \\ 0 & , \; otherwise\,. \end{cases} \quad (2.5)$$

We use the notation $A = [a, b, \alpha, \beta]$.It can easily be shown that $[A]^{\gamma} = [a-(1-\gamma)\alpha, b+(1-\gamma)\beta] \equiv [a_1(\gamma), a_2(\gamma)], \forall\gamma \in [0, 1]$.

Where $[A]^{\gamma}$ denotes the γ-level set of A, as shown in Figure 2.1.

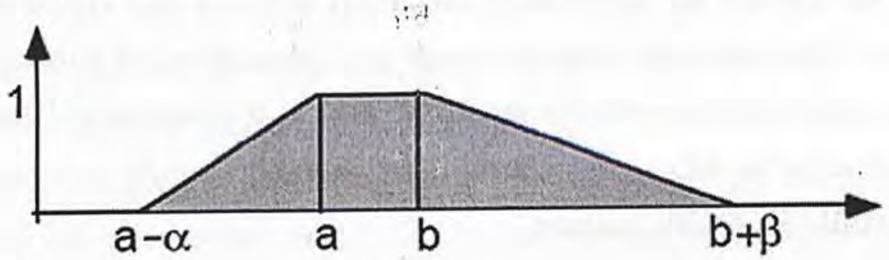

Figure 2.1: Trapezoidal fuzzy number.

Carlsson and Fuller (2001) introduced the (crisp) possibilistic mean (or expected) value of fuzzy number A as:

$$E(A) \;=\; \int_0^1 \gamma(a_1(\gamma) + a_2(\gamma))d\gamma \quad (2.6)$$

If $A = (a, b, \alpha, \beta)$ is a trapezoidal fuzzy number, then

$$\begin{aligned} E(A) &= \int_0^1 \gamma(a_1(\gamma)+a_2(\gamma))d\gamma \\ &= \int_0^1 \gamma[a-(1-\gamma)\alpha+b+(1-\gamma)\beta]d\gamma \\ &= \frac{a+b}{2}+\frac{\beta-\alpha}{6} \end{aligned} \tag{2.7}$$

The turnover rate of security j is denoted by the trapezoidal fuzzy number $\widehat{l_j} = (la_j, lb_j, \alpha_j, \beta_j)$. Then the turnover rate of portfolio $x = (x_1, x_2, ..., x_n)$ is $\sum_{j=1}^{n} \widehat{l_j} x_j$.

By the definition, the crisp possibilistic mean value of the turnover rate of security j is represented as follows:

$$E(\widehat{l_j}) = \int_0^1 \gamma[la_j-(1-\gamma)\alpha_j+lb_j+(1-\gamma)\beta_j]d\gamma = \frac{la_j+lb_j}{2}+\frac{\beta_j-\alpha_j}{6} \tag{2.8}$$

Therefore, the crisp possibilistic mean value of the turnover rate for a portfolio $x = (x_1, x_2, ..., x_n)$ can be written as follow:

$$E(\widehat{l}(x)) = E(\sum_{i=1}^{n} \widehat{l_i} x_i) = \sum_{i=1}^{n} (\frac{la_i+lb_i}{2}+\frac{\beta_i-\alpha_i}{6})x_i \tag{2.9}$$

In this thesis, we use the crisp possibilistic mean value of the turnover rate to measure the portfolio liquidity.

2.4 Notations

Suppose an investor has n assets / securities choices to allocate his/her wealth among n securities offering random rates of return. The investor starts with a existing portfolio and decides how to reallocate assets. We introduce notations here:

r_i: the expected rate of return of asset $i(i = 1, 2, ..., n)$;

x_i: the proportion of the total investment devoted to asset $i(i = 1, 2, ..., n)$;

x_i^0: the proportion of the asset $i(i = 1, 2, ..., n)$ owned by the investor;

k_i: the transaction fee ratio for asset $i(i = 1, 2, ..., n)$ in stock market;

u_i: the upper bound proportion of the total investment devoted to asset $i(i = 1, 2, ..., n)$;

$(la_j, lb_j, \alpha_j, \beta_j)$: Fuzzy turnover rate of asset $i(i = 1, 2, ..., n)$.

So, we can express the transaction costs of the asset $i(i = 1, 2, ..., n, n+1)$ can be denoted by

$$C_i(x_i) = k_i|x_i - x_i^0| \tag{2.10}$$

So the total transaction costs of portfolio $x = (x_1, x_2, ..., x_n)$ can be denoted by

$$C(x) = \sum_{i=1}^{n} C_i(x_i) = \sum_{i=1}^{n} k_i|x_i - x_i^0| \tag{2.11}$$

The total return of portfolio $x = (x_1, x_2, ..., x_n)$ in the future can be represented as

$$r(x) = \sum_{i=1}^{n} r_i x_i \tag{2.12}$$

After removing the transaction costs part, the net expected return of portfolio $x = (x_1, x_2, ..., x_n)$ can be represented as

$$f(x) = \sum_{i=1}^{n} (r_i x_i - k_i|x_i - x_i^0|) \tag{2.13}$$

The minmax rule risk of the portfolio $x = (x_1, x_2, ..., x_n)$ can be represented as

$$max \;\; q_i = E(|R_i - r_i|) \tag{2.14}$$

The turnover rate of security j is denoted by trapezoidal fuzzy number $(la_j, lb_j, \alpha_j, \beta_j)$. Then the turnover rate of portfolio $x = (x_1, x_2, ..., x_n)$ is $\sum_{i=1}^{n} \widehat{l}_i x_i$.

2.5 Model formulation

Assume the investor wants to maximize return and minimize risk after paying transaction costs. Based on the above notations, the portfolio selection problem can be formulated as the following bi-objective programming problems.

If we use the minmax rule risk function to measure risk, then we can get the following bi-objective programming problem $(P1-1)$.

$$
\begin{aligned}
(P1-1) \qquad & max & & f(x)=\sum_{i=1}^{n}(r_i x_i - k_i|x_i - x_i^0|) \\
& min\ max & & q_i x_i \\
& s.t. & & \sum_{i=1}^{n} x_i = M_0 \\
& & & 0 \le x_i \le u_i, \quad i=1,2,...,n \\
& & & \sum_{i=1}^{n} \widehat{l}_i x_i \ge \widehat{l}_0
\end{aligned}
\tag{2.15}
$$

Where $\widehat{l_0}$ is the tolerance level of the fuzzy turnover rate given by the investor. The above bi-objective programming problem can be solved by transforming it into a single objective programming problem with a parameter λ to adjust the optimization preference according to investor's demand. We saw that $(P1-1)$ can be transferred into $(P1-2)$.

$$(P1-2) \quad min \quad F_\lambda(x,y) = \lambda y + (1-\lambda)[-\sum_{i=1}^{n}(r_i x_i - k_i|x_i - x_i^0|)]$$

$$\begin{aligned} s.t. \quad & q_i x_i \leq y, \quad i = 1,2,...,n \\ & \sum_{i=1}^{n} x_i = M_0 \\ & 0 \leq x_i \leq u_i, \quad i = 1,2,...,n \\ & \sum_{i=1}^{n} \widehat{l_i} x_i \geq \widehat{l_0} \end{aligned} \tag{2.16}$$

Where $\widehat{l_0}$ is the tolerance level of the fuzzy turnover rate given by the investor. There are fuzzy inequalities in constraints. We face the problem of comparing two fuzzy numbers.

Based on possibilistic mean (or expected) value, the comparing method is shown as follow: if C and D are trapezoidal fuzzy numbers, $C \leq D$ if and only if $E(C) \leq E(D)$. According to the definition of Possibilistic mean (or expected) value, for every asset i (i=1,2,...,n).

The Possibilistic mean (or expected) value of turnover rate can be denoted:

$$E(\widehat{l_j}) = \int_0^1 \gamma[la_j - (1-\gamma)\alpha_j + lb_j + (1-\gamma)\beta_j]d\gamma = \frac{la_j + lb_j}{2} + \frac{\beta_j - \alpha_j}{6} \tag{2.17}$$

By extension principle, the Possibilistic mean (or expected) value of turnover rate for a portfolio $x = (x_1, x_2, ..., x_n)$ can be written as:

$$E(\widehat{l}(x)) = E(\sum_{i=1}^{n} \widehat{l}_i x_i) = \sum_{i=1}^{n} (\frac{la_i + lb_i}{2} + \frac{\beta_i - \alpha_i}{6}) x_i \tag{2.18}$$

So we use the crisp possibilistic mean value to the measure the portfolio liquidity, the fuzzy inequality $\sum_{i=1}^{n} \widehat{l}_i x_i \geq \widehat{l_0}$ can be transferred to crisp inequality as:

$$\sum_{i=1}^{n} (\frac{la_i + lb_i}{2} + \frac{\beta_i - \alpha_i}{6}) x_i \geq E(\widehat{l_0}) \tag{2.19}$$

hence, $(P1-2)$ can be transferred to $(P1-3)$

$$
\begin{aligned}
(P1-3) \quad & min \quad F_\lambda(x,y) = \lambda y + (1-\lambda)[-\sum_{i=1}^{n}(r_i x_i - k_i|x_i - x_i^0|)] \\
& s.t. \quad q_i x_i \leq y, \quad i = 1, 2, ..., n \\
& \qquad \sum_{i=1}^{n} x_i = M_0 \\
& \qquad 0 \leq x_i \leq u_i, \quad i = 1, 2, ..., n \\
& \qquad \sum_{i=1}^{n} (\frac{la_i + lb_i}{2} + \frac{\beta_i - \alpha_i}{6}) x_i \geq E(\widehat{l_0})
\end{aligned} \tag{2.20}
$$

where $E(\widehat{l_0})$ is the investor's lowest level of the turnover rate for portfolio.

By introducing an new variant x_{n+1}, let $\sum_{i=1}^{n} k_i|x_i - x_i^0| \leq x_{n+1}$, then the model $(P1-3)$ can be transferred to $(P1-4)$ as follow:

$$
\begin{aligned}
(P1-4) \quad min \quad & F_\lambda(x,y) = \lambda y + (1-\lambda)[-\sum_{i=1}^{n} r_i x_i + x_{n+1}] \\
s.t. \quad & \sum_{i=1}^{n} k_i |x_i - x_i^0| \le x_{n+1} \\
& q_i x_i \le y, \quad i = 1,2,...,n \\
& \sum_{i=1}^{n} x_i = M_0 \\
& 0 \le x_i \le u_i, \quad i = 1,2,...,n \\
& \sum_{i=1}^{n} (\frac{la_i + lb_i}{2} + \frac{\beta_i - \alpha_i}{6}) x_i \ge E(\widehat{l_0})
\end{aligned}
\tag{2.21}
$$

Proposition1: Portfolio $(x_1^*, \ldots, x_n^*, y^*)$ is an optimal solution of $(P1-3)$, if and only if there exist x_{n+1}^* such that $(x_1^*, \ldots, x_n^*, x_{n+1}^*, y^*)$ is an optimal solution of $(P1-4)$.

Proof:

(1) if $(x_1^*, \ldots, x_n^*, y^*)$ is an optimal solution of $(P1-3)$, let $x_{n+1}^* = \sum_{i=1}^{n} k_i |x_i^* - x_i^0|$, then $(x_1^*, \ldots, x_n^*, x_{n+1}^*, y^*)$ is a feasible solution of $(P1-4)$. Assume that $(x_1^*, \ldots, x_n^*, x_{n+1}^*, y^*)$ is not an optimal solution of $(P1-4)$, then there exists another feasible solution $(x_1^{'}, \ldots, x_n^{'}, x_{n+1}^{'}, y^{'})$ such that

$$
\lambda y^{'} + (1-\lambda)[-\sum_{i=1}^{n} r_i x_i^{'} + x_{n+1}^{'}] < \lambda y^* + (1-\lambda)[-\sum_{i=1}^{n} r_i x_i^* + x_{n+1}^*] \tag{2.22}
$$

$$
\begin{aligned}
& \lambda y' + (1-\lambda)[-\sum_{i=1}^{n}(r_i x_i' - k_i|x_i' - x_i^0|)] \\
\leq\ & \lambda y' + (1-\lambda)[-\sum_{i=1}^{n} r_i x_i' + x_{n+1}'] \\
<\ & \lambda y^* + (1-\lambda)[-\sum_{i=1}^{n} r_i x_i^* + x_{n+1}^*] \\
=\ & \lambda y^* + (1-\lambda)[-\sum_{i=1}^{n}(r_i x_i^* - k_i|x_i^* - x_i^0|)]
\end{aligned}
\tag{2.23}
$$

That is we can find another feasible solution $(x_1^*, \ldots, x_n^*, y^*)$ such that

$$
\lambda y' + (1-\lambda)[-\sum_{i=1}^{n}(r_i x_i' - k_i|x_i' - x_i^0|)] \leq \lambda y^* + (1-\lambda)[-\sum_{i=1}^{n}(r_i x_i^* - k_i|x_i^* - x_i^0|)]
\tag{2.24}
$$

This contradicts the assumption.

(2) On the other hand, if $(x_1^*, \ldots, x_n^*, x_{n+1}^*, y^*)$ is an optimal solution of $(P1-4)$, it is obvious that $(x_1^*, \ldots, x_n^*, y^*)$ is an optimal solution of $(P1-3)$. If $(x_1^*, \ldots, x_n^*, y^*)$ is not an optimal solution of $(P1-3)$, then there exist another feasible solution $(x_1'', \ldots, x_n'', y'')$ such that

$$
\sum_{i=1}^{n}(r_i x_i'' - k_i|x_i'' - x_i^0|) < \sum_{i=1}^{n}(r_i x_i^* - k_i|x_i^* - x_i^0|)
\tag{2.25}
$$

Let $x_{n+1}'' = \sum_{i=1}^{n} k_i|x_i'' - x_i^0|$.

$$
\begin{aligned}
& \lambda y^{''} + (1-\lambda)[-\sum_{i=1}^{n}(r_i x_i^{''} - k_i|x_i^{''} - x_i^0|)] \\
= \ & \lambda y^{''} + (1-\lambda)[-\sum_{i=1}^{n} r_i x_i^{''} + x_{n+1}^{''}] \\
< \ & \lambda y^* + (1-\lambda)[-\sum_{i=1}^{n}(r_i x_i^* - k_i|x_i^* - x_i^0|)] \\
\leq \ & \lambda y^* + (1-\lambda)[-\sum_{i=1}^{n} r_i x_i^* + x_{n+1}^*]
\end{aligned} \tag{2.26}
$$

This contradicts the assumption.

In $(P1-4)$, there exists absolute value term, so we do the further transformation by introducing:

$$
\begin{aligned}
d_i^+ &= \frac{|x_i - x_i^0| + (x_i - x_i^0)}{2} \\
d_i^- &= \frac{|x_i - x_i^0| - (x_i - x_i^0)}{2}
\end{aligned} \tag{2.27}
$$

Then we have:

$$d_i^+ + d_i^- = |x_i - x_i^0|,\ d_i^+ - d_i^- = x_i - x_i^0,\ d_i^+ d_i^- = 0,\ d_i^+ \geq 0,\ d_i^- \geq 0.$$

Hence, $(P1-4)$ can be equally transferred to $(P1-5)$:

$$
(P1-5) \quad min \quad F_\lambda(x,y) = \lambda y + (1-\lambda)(-\sum_{i=1}^{n} r_i x_i + x_{n+1})
$$

$$
\begin{aligned}
s.t. \quad & \sum_{i=1}^{n} k_i(d_i^+ + d_i^-) \leq x_{n+1} \\
& d_i^+ - d_i^- = x_i - x_i^0, \quad i = 1,2,...,n \\
& d_i^+ d_i^- = 0, \quad i = 1,2,...,n \\
& q_i x_i \leq y, \quad i = 1,2,...,n \\
& \sum_{i=1}^{n} x_i = M_0 \\
& \sum_{i=1}^{n} (\frac{la_i + lb_i}{2} + \frac{\beta_i - \alpha_i}{6}) x_i \geq E(\widehat{l_0}) \\
& 0 \leq x_i \leq u_i, \quad i = 1,2,...,n \\
& d_i^+ \geq 0, d_i^- \geq 0, \quad i = 1,2,...,n
\end{aligned}
\tag{2.28}
$$

Proposition2: Portfolio $(x_1^*, \ldots, x_{n+1}^*, y^*)$ is an optimal solution of $(P1-4)$, if and only if exist $d_1^{+*}, \ldots, d_n^{+*}$, $d_1^{-*}, \ldots, d_n^{-*}$ such that $(x_1^*, \ldots, x_{n+1}^*, y^*, d_1^{+*}, \ldots, d_n^{+*}, d_1^{-*}, \ldots, d_n^{-*})$ is optimal solution of $(P1-5)$. Delete the relaxing constraint $d_i^+ d_i^- = 0, i = 1,2,...,n$. $(P1-5)$ can be transferred to $(P1-6)$ as follow:

$$(P1-6) \quad min \quad F_\lambda(x,y) = \lambda y + (1-\lambda)(-\sum_{i=1}^{n} r_i x_i + x_{n+1})$$

$$
\begin{aligned}
s.t. \quad & \sum_{i=1}^{n} k_i(d_i^+ + d_i^-) \le x_{n+1} & (1)\\
& d_i^+ - d_i^- = x_i - x_i^0, \quad i = 1,2,...,n & (2)\\
& q_i x_i \le y, \quad i = 1,2,...,n & (3)\\
& \sum_{i=1}^{n} x_i = M_0 & (4)\\
& \sum_{i=1}^{n} (\frac{la_i + lb_i}{2} + \frac{\beta_i - \alpha_i}{6}) x_i \ge E(\widehat{l_0}) & (5)\\
& 0 \le x_i \le u_i, \quad i = 1,2,...,n & (6)\\
& d_i^+ \ge 0, d_i^- \ge 0, \quad i = 1,2,...,n & (7)
\end{aligned}
\qquad (2.29)
$$

Proposition3: If $(x_1^*, \ldots, x_{n+1}^*, y^*, d_1^{+*}, \ldots, d_n^{+*}, d_1^{-*}, \ldots, d_n^{-*})$ is an optimal solution of $(P1{-}5)$, then $(x_1^*, \ldots, x_{n+1}^*, y^*, d_1^{+'}, \ldots, d_n^{+'}, d_1^{-'}, \ldots, d_n^{-'})$ is an optimal solution of $(P1-6)$. In which:

$$d_i^{+'} = \begin{cases} d_i^{+*} - d_i^{-*} & , \ if \ \ d_i^{+*} > d_i^{-*} > 0 \, , \\ 0 & , \ if \ \ d_i^{-*} > d_i^{+*} > 0 \, . \end{cases} \qquad (2.30)$$

$$d_i^{-'} = \begin{cases} 0 & , \ if \ \ d_i^{+*} > d_i^{-*} > 0 \, , \\ d_i^{-*} - d_i^{+*} & , \ if \ \ d_i^{-*} > d_i^{+*} > 0 \, . \end{cases} \qquad (2.31)$$

$$\begin{pmatrix} d_i^{+'} \\ d_i^{-'} \end{pmatrix} = \begin{pmatrix} d_i^{+*} \\ d_i^{-*} \end{pmatrix}, if \ \ d_i^{+*} d_i^{-*} = 0 \qquad (2.32)$$

Proof: $(P1-5)$ and $(P1-6)$ have the same objective function and on these two points they have the same function value. And any feasible solution of $(P1-5)$ is feasible to $(P1-6)$. In order to proof the proposition, we only need to prove that $(P1-5)$, then $(x_1^*,\ldots,x_{n+1}^*,y^*,d_1^{+'},\ldots,d_n^{+'},d_1^{-'},\ldots,d_n^{-'})$ is feasible to $(P1-5)$. Obviously, this point satisfied constraint (2)-(7).

And we have

$$d_i^{+'}+d_i^{-'}=\begin{cases} d_i^{+*}-d_i^{-*}\le d_i^{+*}+d_i^{-*} & , \ if \ \ d_i^{+*}>d_i^{-*}>0 \ , \\ d_i^{-*}-d_i^{+*}\le d_i^{+*}+d_i^{-*} & , \ if \ \ d_i^{-*}>d_i^{+*}>0 \ , \\ d_i^{+*}+d_i^{-*} & , \ if \ \ d_i^{+*}d_i^{-*}=0 \ . \end{cases} \tag{2.33}$$

And

$$\sum_{i=1}^{n}k_i(d_i^{+'}+d_{i'}^{-})\le\sum_{i=1}^{n}k_i(d_i^{+*}+d_i^{-*})\le x_{n+1}^* \tag{2.34}$$

So the constraint (1) is also satisfied.

Thus, $(P1-1)$ is transformed into a standard linear programming problem $(P1-6)$. $(P1-6)$ is linear programming problem. There are many linear programming algorithms, that can be used to solve it efficiently.

2.6 Numerical example and result

We consider 40 stocks in total that constitute the Hang Seng Index during periods, which are used as our asset pool. Hence $n=40$. And we collected the real performance data from December 2006 to November 2010 including 47 months. A time unit is considered as one month, hence T is 47 ($T = 47$).

In this section, we give an example to illustrate the model for portfolio selection proposed in this chapter. We suppose that an investor wants to choose securities from the 40 different Hangseng index constituent stocks in Hong Kong securities market for his/her investment. The names of the 40 stocks are given in Table 2.1 .

Table 2.1: The names of hangseng Index constituent stocks.

Security name	Security name	Security name
1 Cheung Kong	2 CLP Hldgs	3 HK & China Gas
4 Wharf (Hldgs)	5 HSBC Hldgs	6 HK Electric
11 Hang Seng Bank	12 Henderson Land	13 Hutchison
16 SHK Prop	17 New World Dev	19 Swire Pacific 'A'
23 Bank of E Asia	66 MTR Corporation	83 Sino Land
101 Hang Lung Prop	144 China Mer Hldgs	267 CITIC Pacific
291 China Resources	293 Cathay Pac Air	330 Esprit Hldgs
386 Sinopec Corp	388 HKEx	494 Li & Fung
551 Yue Yuen Ind	688 China Overseas	700 Tencent
762 China Unicom	857 PetroChina	883 CNOOC
939 CCB	941 China Mobile	1088 China Shenhua
1199 COSCO Pacific	1398 ICBC	2038 FIH
2318 Ping An	2388 BOC Hong Kong	2600 CHALCO
2628 China Life		

The rate of transaction costs for securities is 0.003. Since we assume that the future turnover rates of the stocks are trapezoidal fuzzy numbers, we need to estimate the tolerance interval, left width and right width of the fuzzy numbers with the historical data. In the real portfolio management, these values can be obtained by using the Delphi

Method based on experts' knowledge. In this thesis numerical example, we use the frequency statistic method to estimate them based on historical data of the securities turnover rates.

In the following, we give the estimation method for the fuzzy turnover rates for Stock Cheung Kong in detail. First, we use historical data (daily turnover rates over a period, here we use 47 months real market data) to calculate the frequency of historical turnover rates. Figure 2.2 shows the frequency distribution of historical turnover rates for the stocks. By observing all the historical data, we use 0.0006 and 0.002 as the minimum and the maximum possible values of uncertain turnover rates in the future. Thus, the left width is 0.0017 and the right width is 0.0024. The fuzzy turnover rate of Stock 1 is (0.0017, 0.0024, 0.0006, 0.002). Using a similar method, we obtain the fuzzy turnover rates of all the 40 stocks. These are listed in Table 2.2.

We collect historical data of these forty constituent stocks from December, 2006 to November, 2010. The data are downloaded from the Bloomberg. Then we use one month as a period to obtain the historical rates of returns for forty-seven periods. With these historical data, the expected rates of returns of the stocks are listed in Appendix A. Based on the source data, we can get investment strategies by applying the proposed models. Assume the minimum level of liquidity is 0.003. We can obtain the optimal investment strategy by solving (P1-6). The return of the portfolio and the detailed investment strategies obtained by (P1-6) under different λ, are listed in Appendix A.

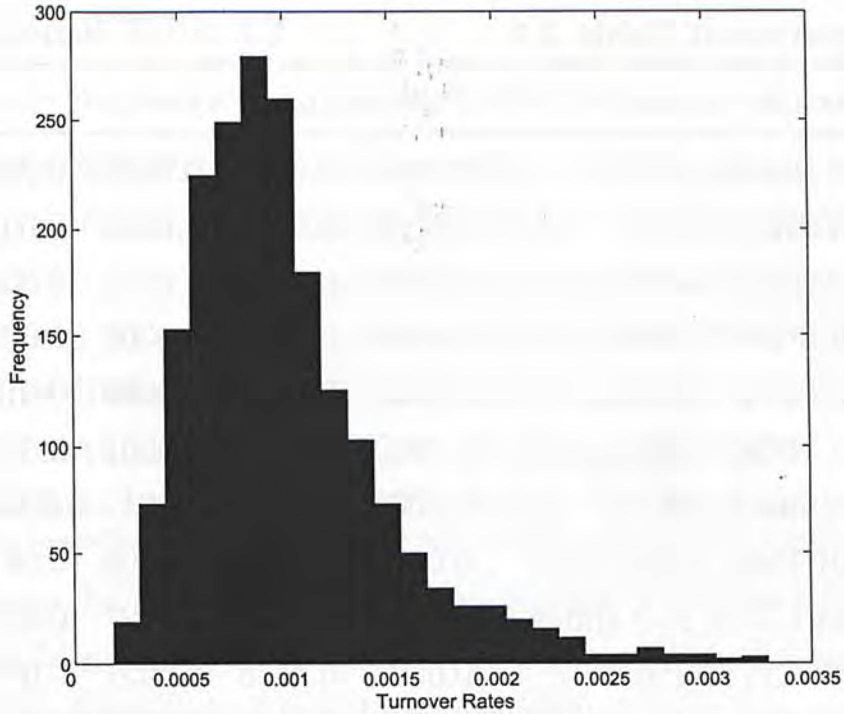

Figure 2.2: Frequency distribution of historical turnover rates for Stock Cheung Kong.

Table 2.2: Fuzzy turnover rate of stocks.

Security name	Fuzzy turnover rates			
1 Cheung Kong	0.0017	0.0024	0.0006	0.002
2 CLP Hldgs	0.0007	0.0028	0.0003	0.0071
3 HK & China Gas	0.0016	0.0029	0.0007	0.0042
4 Wharf (Hldgs)	0.0006	0.0009	0.0002	0.001
5 HSBC Hldgs	0.0001	0.0004	0.0001	0.0011
6 HK Electric	0.0009	0.002	0.0003	0.0035
11 Hang Seng Bank	0.0028	0.0045	0.0008	0.0053
12 Henderson Land	0.0012	0.0022	0.0005	0.0034

continued Table 2.2

Security name	Fuzzy turnover rates			
13 Hutchison	0.0011	0.0017	0.0005	0.0017
16 SHK Prop	0.0015	0.0038	0.0008	0.0075
17 New World Dev	0.0004	0.0006	0.0002	0.0007
19 Swire Pacific 'A'	0.0019	0.0032	0.0007	0.0042
23 Bank of E Asia	0.0021	0.0068	0.0008	0.0162
66 MTR Corporation	0.0003	0.0007	0.0001	0.0012
83 Sino Land	0.0009	0.002	0.0004	0.0036
101 Hang Lung Prop	0.0015	0.0049	0.0006	0.0119
144 China Mer Hldgs	0.0058	0.0139	0.0027	0.0272
267 CITIC Pacific	0.0084	0.0178	0.0031	0.0312
291 China Resources	0.0005	0.0019	0.0002	0.0048
293 Cathay Pac Air	0.0005	0.001	0.0002	0.0018
330 Esprit Hldgs	0.0019	0.0039	0.0007	0.0068
386 Sinopec Corp	0.0003	0.0004	0.0001	0.0005
388 HKEx	0.0011	0.0031	0.0003	0.0071
494 Li & Fung	0.0088	0.0172	0.0042	0.0264
551 Yue Yuen Ind	0.0055	0.0129	0.0025	0.0247
688 China Overseas	0.0012	0.0027	0.0005	0.0049
700 Tencent	0.0001	0.0002	0	0.0002
762 China Unicom	0.0007	0.0016	0.0003	0.0027
857 PetroChina	0.0004	0.0008	0.0002	0.0011
883 CNOOC	0	0	0	0.0001
939 CCB	0	0.0001	0	0.0002
941 China Mobile	0.0005	0.0012	0.0002	0.0022
1088 China Shenhua	0.0004	0.0118	0.0001	0

continued Table 2.2

Security name	Fuzzy turnover rates			
1199 COSCO Pacific	0.003	0.0064	0.0011	0.0112
1398 ICBC	0.0066	0.0226	0.0027	0.0558
2038 FIH	0.0007	0.0024	0.0003	0.0059
2318 Ping An	0.0048	0.0109	0.002	0.0202
2388 BOC Hong Kong	0.0099	0.0207	0.0036	0.0362
2600 CHALCO	0.0287	0.0466	0.009	0.0568
2628 China Life	0.0007	0.0011	0.0002	0.0011

Based on the real historical data, we can get the efficient frontier of the mean- Minimax rule risk function model (see Figure 2.3 and Figure 2.4) for 6 months and 12 months respectively.

After 12 months' investment, the efficient frontier can be plotted (as shown in Figure 2.3 and Figure 2.4), which is the collection of all efficient points. Every single point corresponds to one λ. Since $\lambda \in [0, 1]$, in this numerical example, we divide λ by 50. Thus the value of λ varies from $0.02, 0.04, ..., 1.00$.

From this efficient frontier graph, we can see that as λ gets smaller and smaller, the terminal wealth is getting larger and larger. That is because our objective function is

$$min\ F_\lambda(x, y) = \lambda y + (1 - \lambda)[-\sum_{i=1}^{n}(r_i x_i - k_i|x_i - x_i^0|)] \qquad (2.35)$$

If λ is smaller, it means that this investor pays less attention to risk, or the investor emphasizes more on terminal wealth. That is the

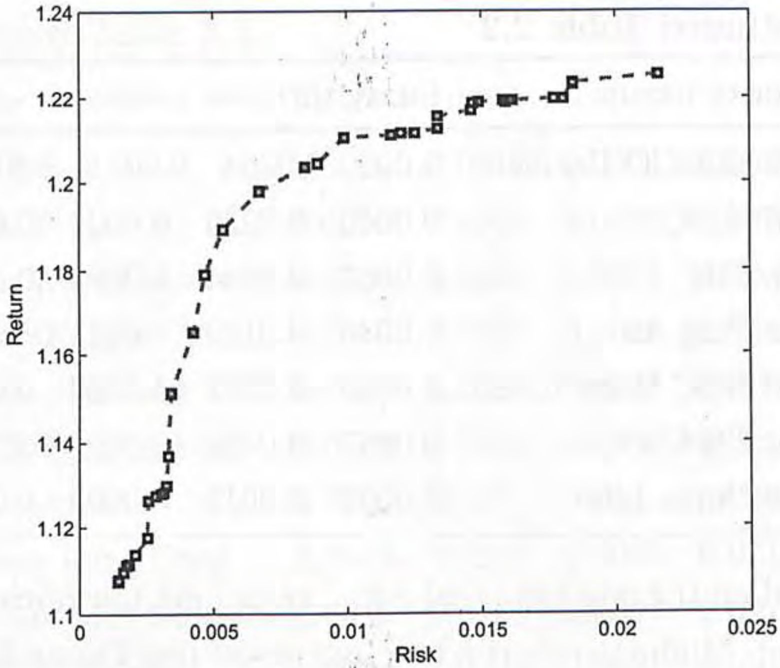

Figure 2.3: Efficient Frontier for 6 months.

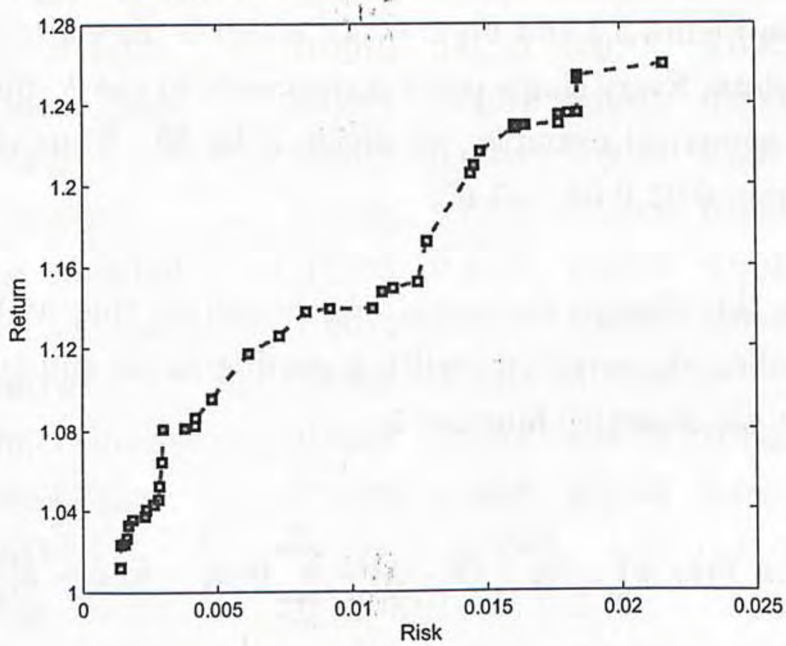

Figure 2.4: Efficient Frontier for 12 months.

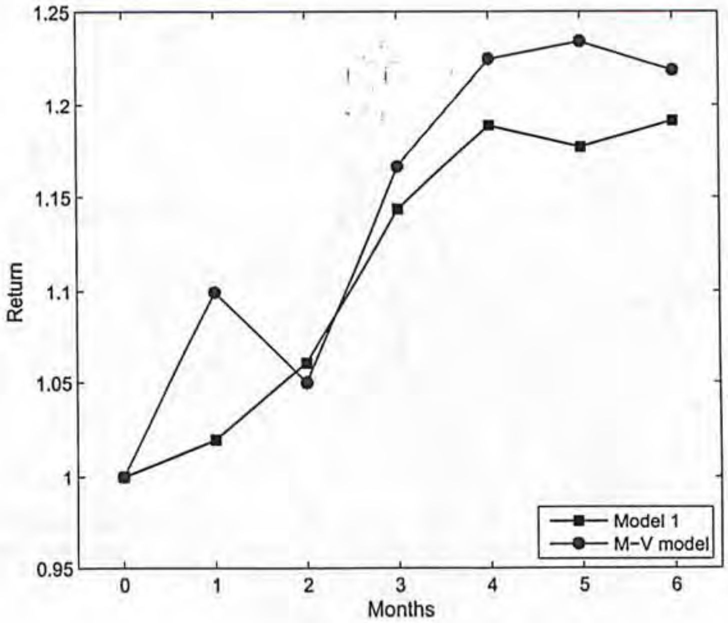

Figure 2.5: Return rate comparison between model 1 and M-V model for 6 months.

reason that $(1-\lambda)[-\sum_{i=1}^{n}(r_i x_i - k_i|x_i - x_i^0|)]$ is getting smaller if λ is getting larger. All source data of this numerical example is provided in Appendix A, including r_i and q_i.

Classic Mean-Variance model is chosen as a benchmark, we applied model 1 and MV model to get their strategy performance, two figures are listed for 6 months and 12 months when $\lambda = 0.5$, see the portfolio return rate and liquidity (turnover rate) performance comparison in Figure 2.5, Figure 2.6, Figure 2.7, and Figure 2.8. In Appendix A, the allocation strategy and the value of every period optimal total investment return are attached to the last line.

From the comparison figure above, it is clear that there is no noticeable advantage in model 1 to M-V model, because in model 1, additional two constraints are considered: deducting transaction fee

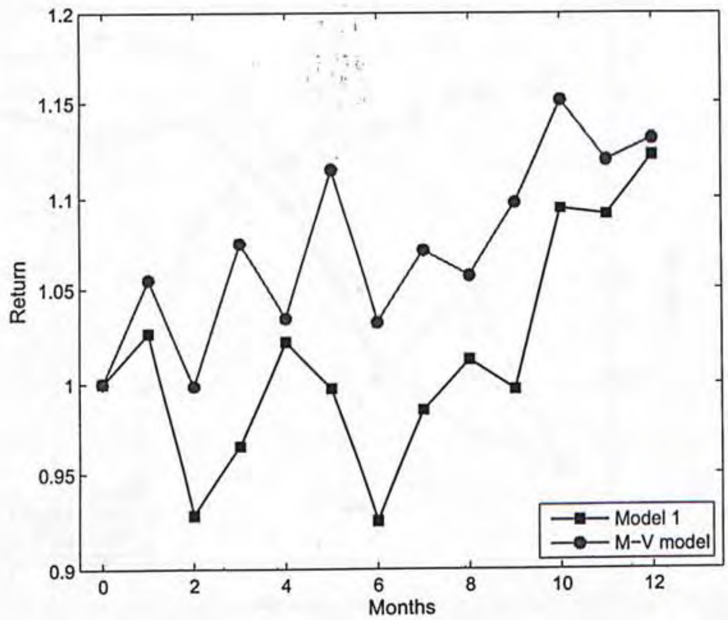

Figure 2.6: Return rate comparison between model 1 and M-V model for 12 months.

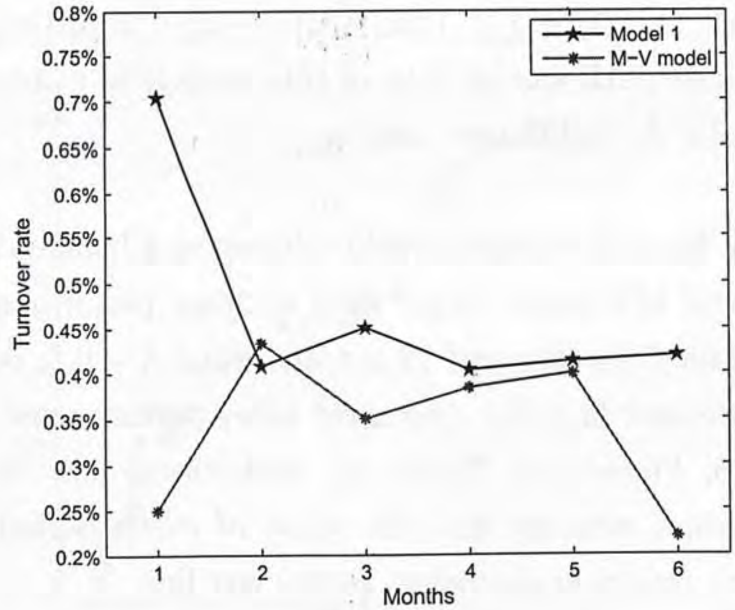

Figure 2.7: Portfolio turnover rate comparison between model 1 and M-V model for 6 months.

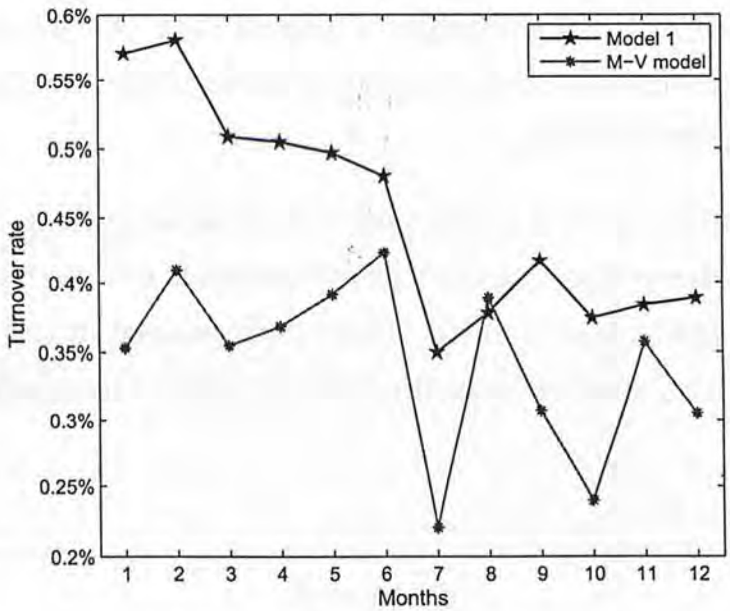

Figure 2.8: Portfolio turnover rate comparison between model 1 and M-V model for 12 months.

and considering the liquidity tolerance limit. However, if we compared the portfolio average liquidity level (estimated by turnover rate) under the two models, we can draw a conclusion that portfolio under model 1 always has better liquidity performance: higher and more stable average turnover rate. That proves the contribution of liquidity constraint.

Conclusion: according to the numerical example, we can easily summarize some points from the data result and comparison figures.

(1) As the λ getting larger, the total risk is getting smaller, but with a lower return rate.

(2) If we scrutinize the strategy allocation under different λ, it is not difficult to find that the larger λ, we can obtain a more diversified

allocation. As the larger λ means that the investor puts more attention to the risk control. Diversification is a direct way to low the portfolio risk.

(3) Comparison with performance of Markowitz Mean-Variance Model, we can see that our model's advantage is not obvious in the return rate but enjoys a much better performance in the liquidity level, which attributes to adding the liquidity constraints.

□ **End of chapter.**

Chapter 3

Model 2

Summary

In this chapter, we proposed an optimization approach based on satisfactory degree. As model 1 in last chapter, return and risk are both as satisfactory degree with fuzzy number, not random variables. By introducing this mechanism, we can formulate the portfolio selection problem as an optimization problem in satisfactory degree, which reflects experts' experience and investors' preference in a much better way.

3.1 Introduction

The expected return, risk and liquidity are vague and uncertain. In this chapter, we use a classic membership function: semi- trapezoidal function to describe the satisfactory level of expected return, risk and liquidity of the portfolio. Using the minmax rule risk function to measure the portfolio risk, we propose such a fuzzy portfolio model based on Bellman-Zadeh's maximization principle.

In this section, a brief introduction of Bellman-Zadeh's maximization principle will be given as follow. This is the key thought of fuzzy portfolio optimization model.

Definition 3.1 X is a set of all possible strategies. The fuzzy goal $\widetilde{G}$ is an unclear requirement to the objective, which is defined as a fuzzy set on X. Its fuzzy membership function reflects the satisfaction degree of strategy x to objective $\widetilde{G}$.

Definition 3.2 Fuzzy constraint $\widetilde{C}$ is a fuzzy relax limit on strategies, which is denoted as a fuzzy set on X, its membership function reflects the belonging degree to the constraint.

Definition 3.3 If $\widetilde{G}$ and $\widetilde{C}$ are fuzzy goal and fuzzy constraint respectively, then $\widetilde{D}$ is also a fuzzy set on X, where $\widetilde{D}$ is defined as intersection set of $\widetilde{G}$ and $\widetilde{C}$: $\widetilde{D} = \widetilde{G} \cap \widetilde{C}$, with membership function: $\mu_{\widetilde{D}}(x) = min\mu_{\widetilde{G}}(x), \mu_{\widetilde{C}}(x), \forall x \in X$.

Above definition can be generalized to more common scenario. Suppose that we have N fuzzy goals and M fuzzy constraints. Let $\widetilde{G}_j$ denote the j_{th} objective, $\widetilde{C}_i$ denote i_{th} constraint. If all the objectives and constraints are defined in strategies space X, they are equal in the optimization. Then fuzzy decision set $\widetilde{D}$ is the intersection of all object fuzzy sets and constraint sets, which is denoted as $\widetilde{D} = \widetilde{G} \cap \widetilde{C}$, $\widetilde{D} = (\widetilde{G}_1 \cap \widetilde{G}_2 \dots \widetilde{G}_n) \cap (\widetilde{C}_1 \cap \widetilde{C}_2 \dots \widetilde{C}_m)$, its membership function is:

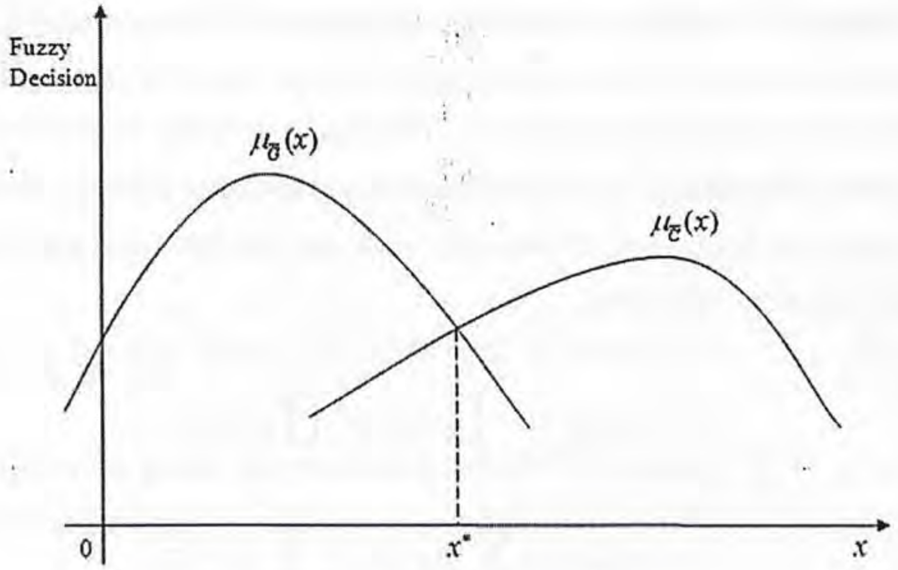

Figure 3.1: Fuzzy decision.

$$\mu_{\widetilde{D}}(x) = min\{\min_{j=1,2,\dots,n} \mu_{\widetilde{G_j}}(x), \min_{j=1,2,\dots,n} \mu_{\widetilde{C_j}}(x)\}, \forall x \in X. \tag{3.1}$$

Definition 3.4 If we need a clear strategy, those fit us requirements are decisions which achieve maximum value of membership function of $\mu_{\widetilde{G}}(x)$, we denote them as follows: $M^* = \{x_m | x \in X, \mu_{\widetilde{D}}(x_m) \geq \mu_{\widetilde{D}}(x)\}$, which is called maximum value decision set. If $\mu_{\widetilde{G}}(x)$ has only one maximum in X^*, then x^* is the only clear decision, that is the maximum decision with membership function:

$$\mu_{\widetilde{D}}(x) = \max_x \, min\{\mu_{\widetilde{G_1}}(x), \dots, \mu_{\widetilde{G_n}}(x), \mu_{\widetilde{C_1}}(x), \dots, \mu_{\widetilde{C_m}}(x)\}, \forall x \in X. \tag{3.2}$$

$$x_{max} = arg(\max_x \, min\{\mu_{\widetilde{G_1}}(x), \dots, \mu_{\widetilde{G_n}}(x), \mu_{\widetilde{C_1}}(x), \dots, \mu_{\widetilde{C_m}}(x)\}) \tag{3.3}$$

Maximum decision is not the only form of fuzzy decision, because minimum operator just considers the worst situation of the object, this is obviously pessimistic method. We need a method which can consider the best situation. So we use maximum operator, which is optimistic decision method. Sometimes, we also use product and sum to denote fuzzy decision as follow:

$$\mu_{\tilde{D}}(x) = \prod_{j=1}^{n} \mu_{\widetilde{G}_j}(x) \prod_{i=1}^{n} \mu_{\widetilde{C}_i}(x) \tag{3.4}$$

$$\mu_{\tilde{D}}(x) = \sum_{j=1}^{n} \mu_{\widetilde{G}_j}(x) + \sum_{i=1}^{n} \mu_{\widetilde{C}_i}(x) \tag{3.5}$$

3.2 Notations

Suppose that an investor has n assets/securities choice to allocate his/her wealth among n securities offering random return rate. The investor starts with an existing portfolio and decides how to reallocate assets. We introduce some notations as follows:

r_i: the expected rate of return of asset $i (i = 1, 2, ..., n)$;

x_i: the proportion of the total investment devoted to asset $i (i = 1, 2, ..., n)$;

x_i^0: the proportion of the asset $i (i = 1, 2, ..., n)$ owned by the investor;

k_i: the transaction fee ratio for asset $i(i = 1, 2, ..., n)$ in stock market;

u_i: the upper bound proportion of the total investment devoted to asset $i(i = 1, 2, ..., n)$;

$(la_j, lb_j, \alpha_j, \beta_j)$: Fuzzy turnover rate of asset $i(i = 1, 2, ..., n)$.

So, we can express the transaction costs of the asset $i(i = 1, 2, ..., n, n+1)$ can be denoted by

$$C_i(x_i) = k_i|x_i - x_i^0| \tag{3.6}$$

So the total transaction costs of portfolio $x = (x_1, x_2, ..., x_n)$ can be denoted by

$$C(x) = \sum_{i=1}^{n} C_i(x_i) = \sum_{i=1}^{n} k_i|x_i - x_i^0| \tag{3.7}$$

The total return of portfolio $x = (x_1, x_2, ..., x_n)$ in the future can be represented as

$$r(x) = \sum_{i=1}^{n} r_i x_i \tag{3.8}$$

After removing the transaction costs part, the net expected return of portfolio $x = (x_1, x_2, ..., x_n)$ can be represented as

$$f(x) = \sum_{i=1}^{n} (r_i x_i - k_i|x_i - x_i^0|) \tag{3.9}$$

The minmax rule risk of the portfolio $x = (x_1, x_2, ..., x_n)$ can be represented as

$$max\ \ q_i = E(|R_i - r_i|) \tag{3.10}$$

The turnover rate of security j is denoted by trapezoidal fuzzy number $(la_j, lb_j, \alpha_j, \beta_j)$. Then the turnover rate of portfolio $x = (x_1, x_2, ..., x_n)$ is $\sum_{i=1}^{n} \widehat{l}_i x_i$.

3.3 Model formulation

$$\begin{aligned}
(P2-1) \qquad & max && f(x) = \sum_{i=1}^{n} (r_i x_i - k_i|x_i - x_i^0|) \\
& min\ max && q_i x_i \\
& s.t. && \sum_{i=1}^{n} x_i = M_0 \\
& && 0 \le x_i \le u_i, \quad i = 1, 2, ..., n \\
& && \sum_{i=1}^{n} \widehat{l}_i x_i \ge \widehat{l}_0
\end{aligned} \tag{3.11}$$

Here, we introduce Semi-trapezoidal membership function for return rate, risk and liquidity to transfer the original model:

(1) Semi-trapezoidal membership function of return rate:

$$\mu_r(x) = \begin{cases} 0 & , \ if\ \ r(x) < r_0 \ , \\ \frac{r(x)-r_0}{r_1 - r_0} & , \ if\ \ r_0 \le r(x) \le r_1 \ , \\ 1 & , \ if\ \ r(x) > r_1 \ . \end{cases} \tag{3.12}$$

r_0 is the investor's necessary satisfactory return rate, r_1 is the investor's full satisfactory return rate.

r_0 and r_1 are set by investor from the expert empirical value.

(2) Semi-trapezoidal membership function of risk:

$$\mu_w(x) = \begin{cases} 0 & , \ if \ \ w(x) < w_0 \ , \\ \frac{w_1 - w(x)}{w_1 - w_0} & , \ if \ \ w_0 \le w(x) \le w_1 \ , \\ 1 & , \ if \ \ w(x) > w_1 \ . \end{cases} \tag{3.13}$$

w_0 is the investor's necessary satisfactory risk, w_1 is the investor's full satisfactory risk.

w_0 and w_1 are set by investor from the expert empirical value.

(3) Semi-trapezoidal membership function of liquidity:

$$\mu_{\widehat{l}}(x) = \begin{cases} 0 & , \ if \ \ E(\widehat{l}(x)) < l_0 \ , \\ \frac{E(\widehat{l}(x)) - l_0}{l_1 - l_0} & , \ if \ \ l_0 \le E(\widehat{l}(x)) \le l_1 \ , \\ 1 & , \ if \ \ E(\widehat{l}(x)) > l_1 \ . \end{cases} \tag{3.14}$$

l_0 is the investor's necessary satisfactory liquidity, l_1 is the investor's full satisfactory liquidity.

l_0 and l_1 are set by investor from the experts' empirical value.

According to the Bellman and Zadel's maximum principle:

$$\lambda = min\{\mu_r(x), \mu_w(x), \mu_{\tilde{l}}(x)\} \tag{3.15}$$

Then $(P2-1)$ can be formulated into the following problem:

$$\begin{array}{lll}(P2-2) & max & \lambda \\ & s.t. & \mu_r(x) \geq \lambda \\ & & \mu_w(x) \geq \lambda \\ & & \mu_{\tilde{l}}(x) \geq \lambda \\ & & \sum_{i=1}^{n} x_i = M_0 \\ & & 0 \leq x_i \leq u_i, \quad i = 1, 2, ..., n \end{array} \tag{3.16}$$

$(P2-2)$ can be further formulated into $(P2-3)$:

$$\begin{array}{lll}(P2-3) & max & \lambda \\ & s.t. & \sum_{i=1}^{n}(r_i x_i - k_i|x_i - x_i^0|) \geq \lambda(r_1 - r_0) + r_0 \\ & & max\ q_i x_i \leq w_1 - \lambda(w_1 - w_0), \quad i = 1, 2, ..., n \\ & & \sum_{i=1}^{n}(\frac{la_i + lb_i}{2} + \frac{\beta_i - \alpha_i}{6})x_i \geq \lambda(l_1 - l_0) + l_0 \\ & & \sum_{i=1}^{n} x_i = M_0 \\ & & 0 \leq x_i \leq u_i, \quad i = 1, 2, ..., n \end{array} \tag{3.17}$$

$$
\begin{aligned}
(P2-4) \quad & max \quad \lambda \\
& s.t. \quad \sum_{i=1}^{n}(r_i x_i - k_i|x_i - x_i^0|) \geq \lambda(r_1 - r_0) + r_0 \\
& \qquad q_i x_i \leq y, \quad i = 1, 2, ..., n \\
& \qquad y \leq w_1 - \lambda(w_1 - w_0) \\
& \qquad \sum_{i=1}^{n}(\frac{la_i + lb_i}{2} + \frac{\beta_i - \alpha_i}{6})x_i \geq \lambda(l_1 - l_0) + l_0 \\
& \qquad \sum_{i=1}^{n} x_i = M_0 \\
& \qquad 0 \leq x_i \leq u_i, \quad i = 1, 2, ..., n
\end{aligned}
\tag{3.18}
$$

In $(P2-4)$, there exists an absolute value term, so we do the further transformation by introducing

$$
\begin{aligned}
d_i^+ &= \frac{|x_i - x_i^0| + (x_i - x_i^0)}{2} \\
d_i^- &= \frac{|x_i - x_i^0| - (x_i - x_i^0)}{2}
\end{aligned}
\tag{3.19}
$$

Then we have:

$$d_i^+ + d_i^- = |x_i - x_i^0|,\ d_i^+ - d_i^- = x_i - x_i^0,\ d_i^+ d_i^- = 0,\ d_i^+ \geq 0,\ d_i^- \geq 0.$$

Hence, $(P2-4)$ can be equally transferred to $(P2-5)$:

$$
\begin{aligned}
(P2-5) \quad & max \quad \lambda \\
& s.t. \quad \sum_{i=1}^{n}(r_i x_i - k_i(d_i^+ + d_i^-)) \geq \lambda(r_1 - r_0) + r_0 \\
& \qquad d_i^+ - d_i^- = x_i - x_i^0, \quad i = 1,2,...,n \\
& \qquad q_i x_i \leq y, \quad i = 1,2,...,n \\
& \qquad y \leq w_1 - \lambda(w_1 - w_0) \\
& \qquad \sum_{i=1}^{n}\left(\frac{la_i + lb_i}{2} + \frac{\beta_i - \alpha_i}{6}\right)x_i \geq \lambda(l_1 - l_0) + l_0 \\
& \qquad \sum_{i=1}^{n} x_i = M_0 \\
& \qquad 0 \leq x_i \leq u_i, \quad i = 1,2,...,n \\
& \qquad d_i^+ \geq 0, d_i^- \geq 0, \quad i = 1,2,...,n
\end{aligned}
\tag{3.20}
$$

Conclusion: now we derived the problem to $(P2-5)$ which is a standard LP problem. There are a lot of methods and softwares to solve it.

3.4 Numerical example and result

We consider 40 stocks in total that constitute the Hang Seng Index during periods, which are used as our asset pool. Hence $n = 40$. And we collected the real performance data from December 2006 to November 2010 including 47 months. A time unit is considered as one month, hence T is 47 (T=47).

In this section, we give an example to illustrate the model for fuzzy portfolio selection proposed in this chapter. We suppose that an investor wants to choose securities from the 40 different Hangseng index constituent stocks in Hong Kong securities market for his/her investment.

Efficient frontier analysis cannot be conducted here. Because of the introduction of fuzzy set theory, we do not have a clear line between objectives and constraints. Investors do not have to choose a λ.

However, we conducted our experiments under two cases through setting different parameters in this model. One is under optimistic market anticipation, the other one is under pessimistic market anticipation.

Under pessimistic market anticipation, investors generally do not have strong confidence to the market, so they tend to choose conservative investment strategy. In this situation, the necessary satisfactory degree and fully satisfactory degree are comparative low than in the bull market. Hence, in the semi-trapezoidal membership function, we set corresponding value of the parameters: $r_0 = 0.004, r_1 = 0.013; l_0 = 0.0045, l_1 = 0.0055; w_0 = 0.0002, w_1 = 0.0035$.

Under optimistic market anticipation, investors generally have confidence to the market. They tend to choose positive investment strategy. In this situation, the necessary satisfactory degree and fully satisfactory degree are comparative high. Hence, in the semi-trapezoidal membership function, we set value of the parameters: $r_0 = 0.008, r_1 =$

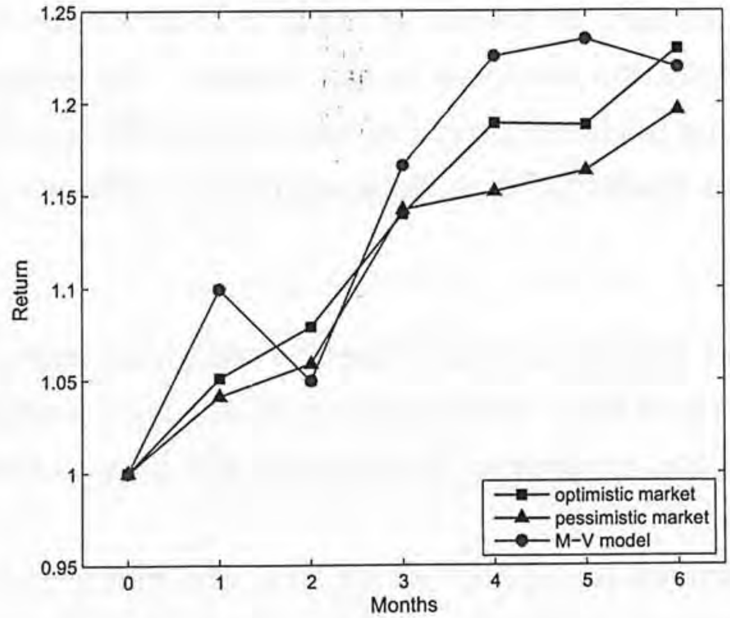

Figure 3.2: Return rate comparison between model 2 and M-V model for 6 months.

$0.025; l_0 = 0.0050, l_1 = 0.0060; w_0 = 0.0004, w_1 = 0.0040$.

We applied the model for 6 months and 12 months under two different market anticipations, and did comparison with benchmark classic Mean-Variance model, see comparison in Figure 3.2, Figure 3.3, Figure 3.4 and Figure 3.5, in Appendix B. The allocation strategy and value of every period optimal total investment return are attached to the last line.

From the numerical example data, we cannot see an obviously superior to M-V model under both market anticipation parameter sets. Because investors' preference and experts' experience are introduced in model 2, there are also additional two constraints are considered: deducting transaction fee and considering the liquidity tolerance limit. However, if we compare the portfolio average liquidity level (estimated

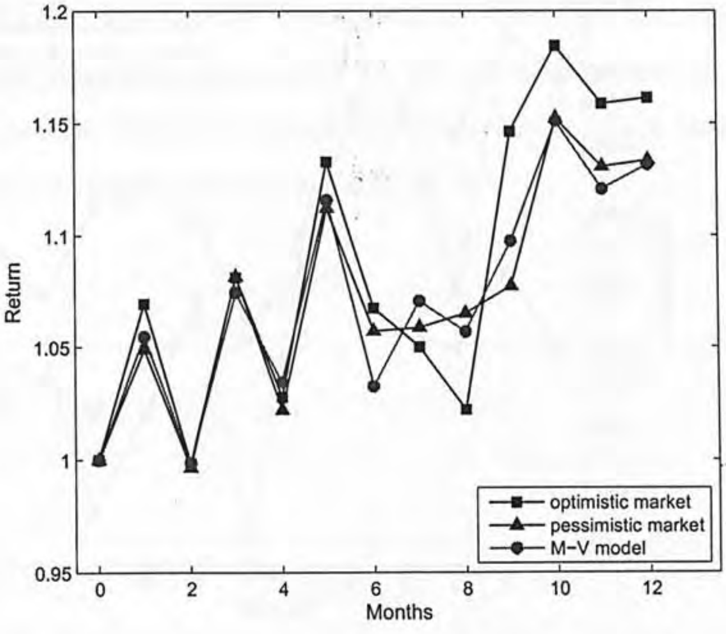

Figure 3.3: Return rate comparison between model 2 and M-V model for 12 months.

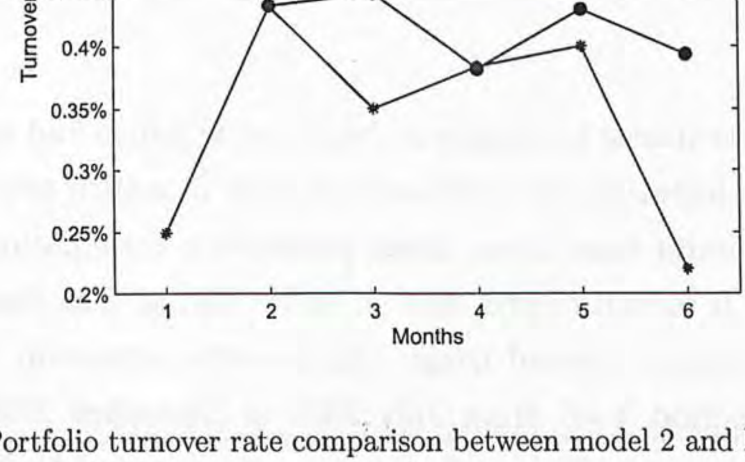

Figure 3.4: Portfolio turnover rate comparison between model 2 and M-V model for 6 months.

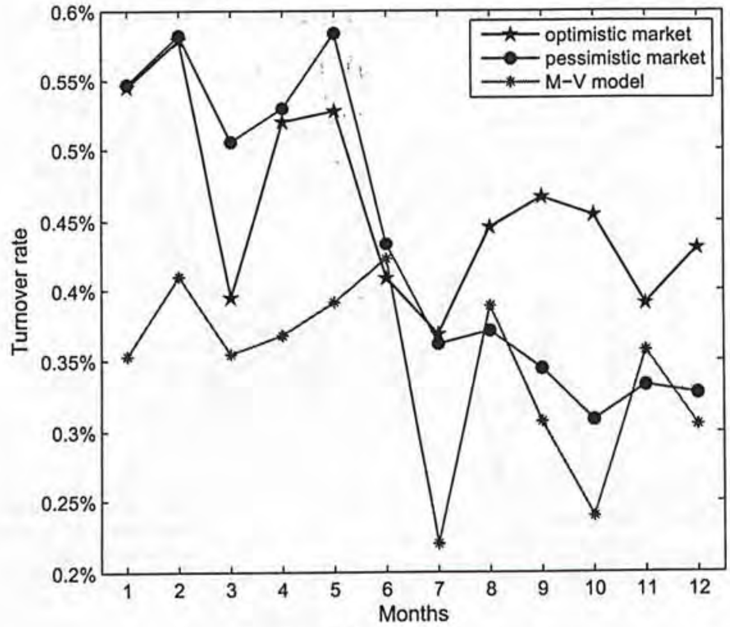

Figure 3.5: Portfolio turnover rate comparison between model 2 and M-V model for 12 months.

by turnover rate) under the two models, we can draw a conclusion that portfolio under model 2, no matter in either anticipation, always has better liquidity performance: higher and more stable average turnover rate. That proves the contribution of liquidity constraint and the fuzzy set theory.

If we scrutinize the figure in detail, we will also find under the optimistic anticipation, the performance both in return rate and turnover rate are better than those under pessimistic anticipation when $T = 6$ (the last 6 month of the 2010), since during that time period, the market shows a upward trend. On the other situation (we also chose another period T=6, from July 2008 to December 2008), it shows a downward trend. The performance under pessimistic anticipation is better than under optimistic anticipation. See the comparison graph (7/2008-12/2008) in the appendix B. So we also can summarize that: if investors set anticipation according with the market trend, it would show a better performance under model 2.

□ **End of chapter.**

Chapter 4

Model 3

Summary

In this chapter, we introduce another membership function: S shape function. Compared with linear membership function, it is a more flexible membership function to characterize the investors' preference and experts' experience.

4.1 Introduction

In model 2, we assume that membership function of three factors are linear: semi-trapezoidal membership function. That means these variables ratios are constant. However, this assumption is suspicious. Different investors have different strategy preference. Many investors' satisfactory degrees vary with return rate / risk / liquidity.

Then we introduce an S shape function to characterize the satisfaction degree

$$f(x) = \frac{1}{1 + exp(-\alpha x)} \tag{4.1}$$

4.2 Notations

Suppose that an investor has n assets/securities choice to allocate his/her wealth among n securities offering random return rate. The investor starts with an existing portfolio and decides how to reallocate assets. We introduce some notations as follows:

r_i: the expected rate of return of asset $i(i = 1, 2, ..., n)$;

x_i: the proportion of the total investment devoted to asset $i(i = 1, 2, ..., n)$;

x_i^0: the proportion of the asset $i(i = 1, 2, ..., n)$ owned by the investor;

k_i: the transaction fee ratio for asset $i(i = 1, 2, ..., n)$ in stock market;

u_i: the upper bound proportion of the total investment devoted to asset $i(i = 1, 2, ..., n)$;

$(la_j, lb_j, \alpha_j, \beta_j)$: Fuzzy turnover rate of asset $i(i = 1, 2, ..., n)$.

So, we can express the transaction costs of the asset $i(i = 1, 2, ..., n, n+1)$ can be denoted by

$$C_i(x_i) = k_i|x_i - x_i^0| \tag{4.2}$$

So the total transaction costs of portfolio $x = (x_1, x_2, ..., x_n)$ can be denoted by

$$C(x) = \sum_{i=1}^{n} C_i(x_i) = \sum_{i=1}^{n} k_i|x_i - x_i^0| \tag{4.3}$$

The total return of portfolio $x = (x_1, x_2, ..., x_n)$ in the future can be represented as

$$r(x) = \sum_{i=1}^{n} r_i x_i \tag{4.4}$$

After removing the transaction costs part, the net expected return of portfolio $x = (x_1, x_2, ..., x_n)$ can be represented as

$$f(x) = \sum_{i=1}^{n} (r_i x_i - k_i|x_i - x_i^0|) \tag{4.5}$$

The minmax rule risk of the portfolio $x = (x_1, x_2, ..., x_n)$ can be represented as

$$max\ \ q_i = E(|R_i - r_i|) \tag{4.6}$$

The turnover rate of security j is denoted by trapezoidal fuzzy number $(la_j, lb_j, \alpha_j, \beta_j)$. Then the turnover rate of portfolio $x = (x_1, x_2, ..., x_n)$ is $\sum_{i=1}^{n} \widehat{l_i} x_i$.

4.3 Model formulation

$$
\begin{aligned}
(P2-1) \qquad & max \quad & f(x) = \sum_{i=1}^{n} (r_i x_i - k_i |x_i - x_i^0|) \\
& min\, max \quad & q_i x_i \\
& s.t. \quad & \sum_{i=1}^{n} x_i = M_0 \\
& & 0 \leq x_i \leq u_i, \quad i = 1, 2, ..., n \\
& & \sum_{i=1}^{n} \widehat{l}_i x_i \geq \widehat{l}_0
\end{aligned}
\tag{4.7}
$$

Here, we introduce S-shape membership function for return rate, risk and liquidity to transfer the original model::

(1) S-shape membership function of return rate:

$$\mu_r(x) = \frac{1}{1 + exp(-\alpha_r(r(x) - r_M))} \tag{4.8}$$

r_M is the midpoint, its degree of membership is 0.5, which denotes that investor has a medium satisfaction degree with the return rate. r_M can be approximately estimated by r_0, r_1, as $r_M \approx \frac{r_0+r_1}{2}$. r_0 is the investor's necessary satisfactory return rate, r_1 is the investor's full satisfactory return rate.

r_0 and r_1 are set by investor from the experts' empirical value.

α_r is set by investor. It can reflect investor's satisfaction degree to the return rate.

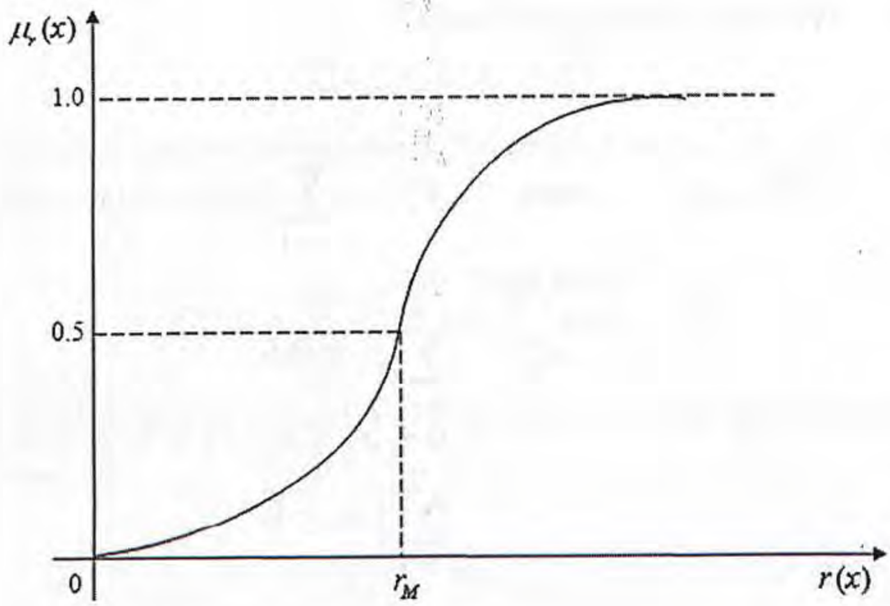

Figure 4.1: S-shape membership function of return rate.

(2) S-shape membership function of risk:

$$\mu_w(x) = \frac{1}{1 + exp(\alpha_w(w(x) - w_M))} \tag{4.9}$$

w_0 is the investor's necessary satisfactory risk, w_1 is the investor's full satisfactory risk.

w_M is the midpoint, its degree of membership is 0.5, which denotes that investor has a medium satisfaction degree with the risk. w_M can be approximately estimated by w_0, w_1, as $w_M \approx \frac{w_0+w_1}{2}$. w_0 and w_1 are set by investor from the experts' empirical value.

α_w is set by investor. It can reflect investor's satisfaction degree to the risk.

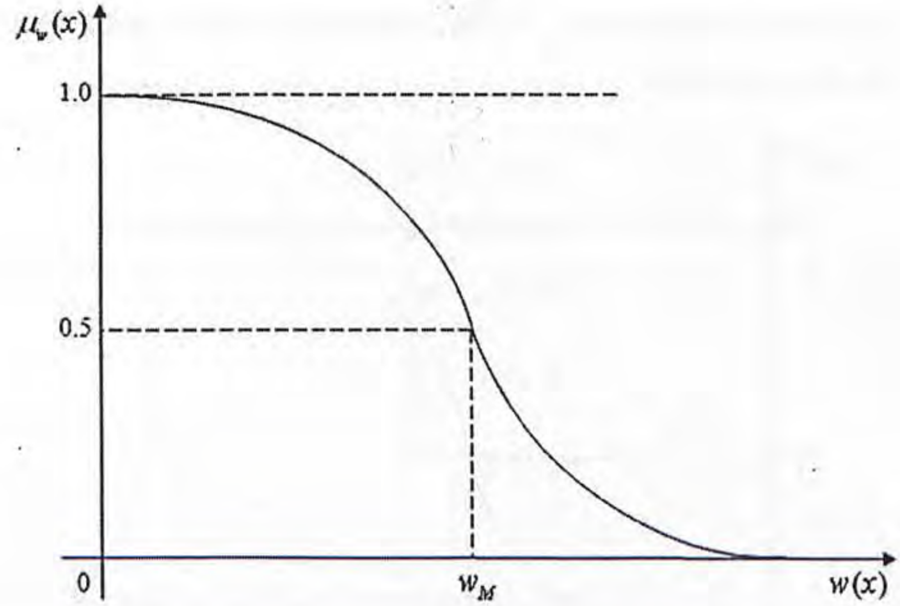

Figure 4.2: S-shape membership function of risk.

(3) S-shape membership function of liquidity:

$$\mu_l(x) = \frac{1}{1 + exp(-\alpha_l(E(\widehat{l}(x)) - l_M))} \tag{4.10}$$

l_M is the midpoint, its degree of membership is 0.5, which denotes that investor has a medium satisfaction degree with the risk. l_M can be approximately estimated by l_0,l_1, as $l_M \approx \frac{l_0+l_1}{2}$. l_0 and l_1 are set by investor from the experts' empirical value.

l_0 is the investor's necessary satisfactory liquidity, l_1 is the investor's full satisfactory liquidity.

l_0 and l_1 are set by investor from the experts' empirical value.

α_l is set by investor. It can reflect investor's satisfaction degree to the liquidity.

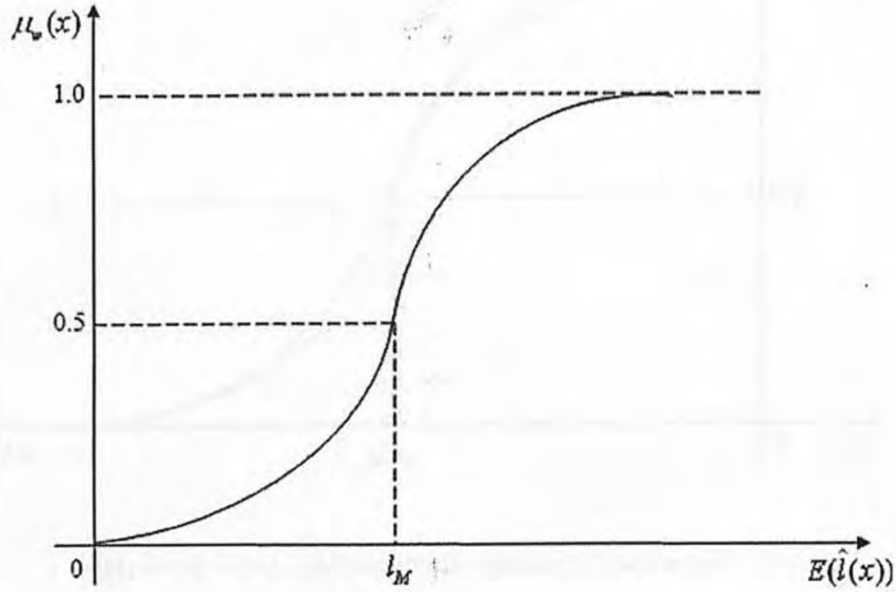

Figure 4.3: S-shape membership function of liquidity.

According to the Bellman and Zadel's maximum principle:

$$\eta = min\{\mu_r(x), \mu_w(x), \mu_{\hat{l}}(x)\} \tag{4.11}$$

$(P3-1)$ can be transferred into follow problem:

$$
\begin{aligned}
(P3-2) \quad & max \quad \eta \\
& s.t. \quad \mu_r(x) \geq \eta \\
& \qquad \mu_w(x) \geq \eta \\
& \qquad \mu_{\widehat{l}}(x) \geq \eta \\
& \qquad \sum_{i=1}^{n} x_i = M_0 \\
& \qquad 0 \leq x_i \leq u_i, \quad i = 1, 2, ..., n
\end{aligned}
\tag{4.12}
$$

$(P3-2)$ can be further transferred into $(P3-3)$:

$$
\begin{aligned}
(P3-3) \quad & max \quad \eta \\
& s.t. \quad \eta + \eta \; exp(-\alpha_r(r(x) - r_M)) \leq 1 \\
& \qquad \eta + \eta \; exp(\alpha_w(w(x) - w_M)) \leq 1 \\
& \qquad \eta + \eta \; exp(-\alpha_l(E(\widehat{l}(x)) - l_M)) \leq 1 \\
& \qquad \sum_{i=1}^{n} x_i = M_0 \\
& \qquad 0 \leq x_i \leq u_i, \quad i = 1, 2, ..., n \\
& \qquad 0 \leq \eta \leq 1
\end{aligned}
\tag{4.13}
$$

α_r, α_w and α_l are set by investor. It can reflect investor's satisfaction degree to the return rate / risk / liquidity.

Let

$$
\theta = ln \frac{\eta}{1-\eta} \tag{4.14}
$$

then

$$\eta = \frac{1}{1 + exp(-\theta)} \tag{4.15}$$

Since non-linear membership function is monotone increasing, θ achieves the maximum when η gets the maximum value. Hence, we can further formulate $(P3-3)$ into $(P3-4)$ as follows:

$$
\begin{aligned}
(P3-4) \qquad & max \quad \theta \\
& s.t. \quad \alpha_r r(x) - \theta \geq \alpha_r r_M \\
& \qquad \alpha_l E(\widehat{l}(x)) - \theta \geq \alpha_l l_M \\
& \qquad \alpha_w w(x) + \theta \leq \alpha_w w_M \\
& \qquad \sum_{i=1}^{n} x_i = M_0 \\
& \qquad 0 \leq x_i \leq u_i, \quad i = 1, 2, ..., n \\
& \qquad \theta \geq 0
\end{aligned} \tag{4.16}
$$

α_r, α_w and α_l are set by investor. It can reflect investor's satisfaction degree to the return rate / risk / liquidity.

$$
\begin{aligned}
(P3-5) \quad & max \quad \theta \\
& s.t. \quad \alpha_r \sum_{i=1}^{n} (r_i x_i - k_i |x_i - x_i^0|) - \theta \geq \alpha_r r_M \\
& \qquad \alpha_l \sum_{i=1}^{n} (\frac{la_i + lb_i}{2} + \frac{\beta_i - \alpha_i}{6}) x_i - \theta \geq \alpha_l l_M \\
& \qquad \alpha_w \{max\ q_i x_i\} + \theta \leq \alpha_w w_M, \quad i = 1, 2, ..., n \\
& \qquad \sum_{i=1}^{n} x_i = M_0 \\
& \qquad 0 \leq x_i \leq u_i, \quad i = 1, 2, ..., n \\
& \qquad \theta \geq 0
\end{aligned}
\tag{4.17}
$$

$$
\begin{aligned}
(P3-6) \quad & max \quad \theta \\
& s.t. \quad \alpha_r \sum_{i=1}^{n} (r_i x_i - k_i |x_i - x_i^0|) - \theta \geq \alpha_r r_M \\
& \qquad \alpha_l \sum_{i=1}^{n} (\frac{la_i + lb_i}{2} + \frac{\beta_i - \alpha_i}{6}) x_i - \theta \geq \alpha_l l_M \\
& \qquad q_i x_i \leq y, \quad i = 1, 2, ..., n \\
& \qquad \alpha_w y + \theta \leq \alpha_w w_M \\
& \qquad \sum_{i=1}^{n} x_i = M_0 \\
& \qquad 0 \leq x_i \leq u_i, \quad i = 1, 2, ..., n \\
& \qquad \theta \geq 0
\end{aligned}
\tag{4.18}
$$

In $(P3-6)$, there exists absolute value term, so we do the further transformation by introducing

$$\begin{aligned} d_i^+ &= \frac{|x_i - x_i^0| + (x_i - x_i^0)}{2} \\ d_i^- &= \frac{|x_i - x_i^0| - (x_i - x_i^0)}{2} \end{aligned} \tag{4.19}$$

Then we have:

$d_i^+ + d_i^- = |x_i - x_i^0|$, $d_i^+ - d_i^- = x_i - x_i^0$, $d_i^+ d_i^- = 0$, $d_i^+ \geq 0$, $d_i^- \geq 0$.

Hence, $(P3-6)$ can be equally transferred to $(P3-7)$:

$$\begin{aligned} (P3-7) \quad & max \quad \theta \\ & s.t. \quad \alpha_r \sum_{i=1}^{n} (r_i x_i - k_i(d_i^+ + d_i^-)) - \theta \geq \alpha_r r_M \\ & \qquad d_i^+ - d_i^- = x_i - x_i^0, \quad i = 1, 2, ..., n \\ & \qquad \alpha_l \sum_{i=1}^{n} (\frac{la_i + lb_i}{2} + \frac{\beta_i - \alpha_i}{6}) x_i - \theta \geq \alpha_l l_M \\ & \qquad q_i x_i \leq y, \quad i = 1, 2, ..., n \\ & \qquad \alpha_w y + \theta \leq \alpha_w w_M \\ & \qquad \sum_{i=1}^{n} x_i = M_0 \\ & \qquad 0 \leq x_i \leq u_i, \quad i = 1, 2, ..., n \\ & \qquad \theta \geq 0 \\ & \qquad d_i^+ \geq 0, d_i^- \geq 0, \quad i = 1, 2, ..., n \end{aligned} \tag{4.20}$$

Conclusion: now we derived the problem to $(P3 - 7)$ which is a standard LP problem. There are a lot of method and software to solve it.

4.4 Numerical example and result

We consider 40 stocks in total that constitute the Hang Seng Index during periods, which are used as our asset pool. Hence $n = 40$. And we collected the real performance data from December 2006 to November 2010 including 47 months. A time unit is considered as one month, hence T is 47 (T=47).

In this section, we give an example to illustrate the S shaper fuzzy optimization model proposed in this chapter. We suppose that an investor wants to choose securities from the 40 different Hangseng index constituent stocks in Hong Kong securities market for his/her investment.

Efficient frontier analysis cannot be conducted here. Because of introduce of fuzzy set theory's, we do not have a clear line between objectives and constraints. Investors do not have to choose a lambda.

However, we conducted our experiment under two cases through setting different parameters in this model. One is under optimistic market anticipation, the other one is under pessimistic market anticipation.

Under pessimistic market anticipation, investors generally do not have strong confidence to the market, so they tend to choose conservative investment strategy. In this situation, the necessary satisfactory degree and fully satisfactory degree are comparative low than in the bull market. Hence, in the S-shape membership function, we set corresponding value of the parameters: $r_M = 0.009, \alpha_r = 500, w_M = 0.0027, \alpha_w = 1000, l_M = 0.005, \alpha_l = 500$.

Under optimistic market anticipation, investors generally have confidence to the market. They tend to choose positive investment strategy. In this situation, the necessary satisfactory degree and fully satisfactory degree are comparative high. Hence, in the S-shape membership function, we set value of the parameters: $r_M = 0.013, \alpha_r = 600, w_M = 0.0032, \alpha_w = 800, l_M = 0.0055, \alpha_l = 600$.

We applied the model for 6 months and 12 months under two different market anticipations, and did comparison with benchmark classic Mean-Variance model, see the comparison in Figure 4.4, Figure 4.5, Figure 4.6 and Figure 4.7.

In Appendix C, the allocation strategy and value of every period optimal total investment return is attached to the last line.

From the numerical example data, we cannot see an obviously superior to M-V model under both market anticipation parameter sets, because investor's preference and experts' experience can be incorporated in model 3 as model 2. However, if we compare the portfolio average liquidity level (estimated by turnover rate) between model 3 and M-V model, we can also draw a conclusion that portfolio under

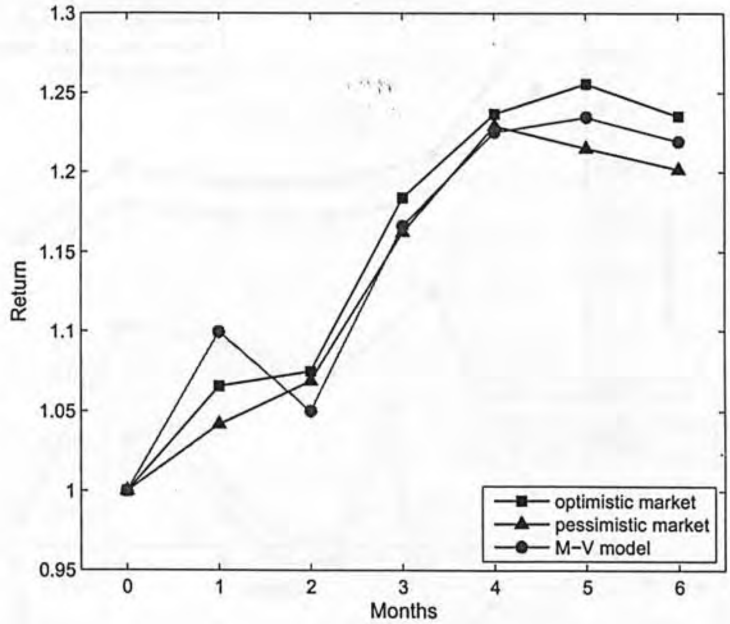

Figure 4.4: Return rate comparison between model 3 and M-V model for 6 months.

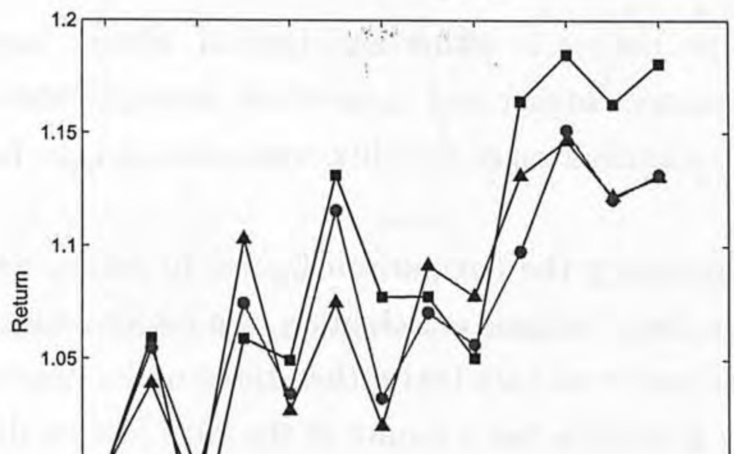

Figure 4.5: Return rate comparison between model 3 and M-V model for 12 months.

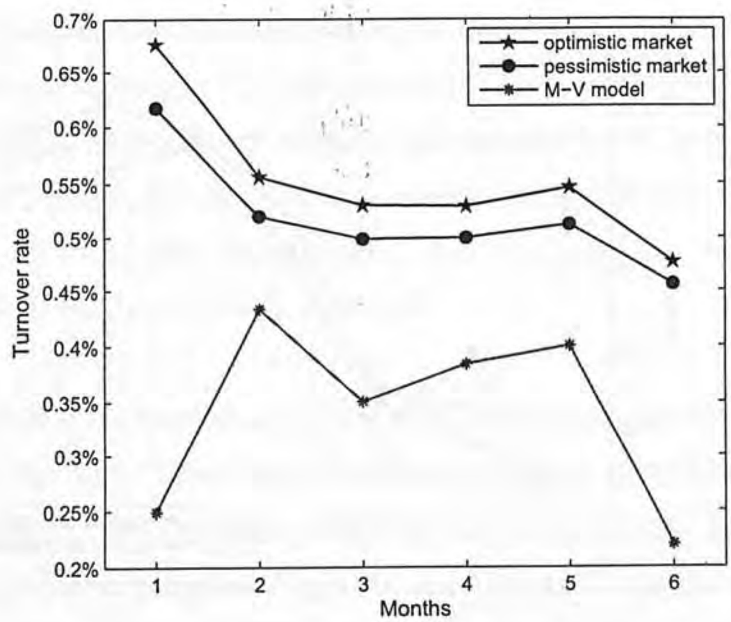

Figure 4.6: Portfolio turnover rate comparison between model 3 and M-V model for 6 months.

model 3, no matter in either anticipation, always has better liquidity performance: higher and more stable average turnover rate. That proves the contribution of liquidity constraint and the fuzzy set theory.

By scrutinizing the comparison figures in detail, we will also find that under the optimistic anticipation, the performance both in return rate and turnover rate are better than those under pessimistic anticipation when T = 6(the last 6 month of the 2010), since during that time period, the market showed a upward trend. On the other situation (we also chose another period T=6, from July 2008 to December 2008), it shows a downward trend. The performance under pessimistic anticipation is better than under optimistic anticipation. See the comparison graph (7/2008-12/2008) in the appendix C. So we also can summarize

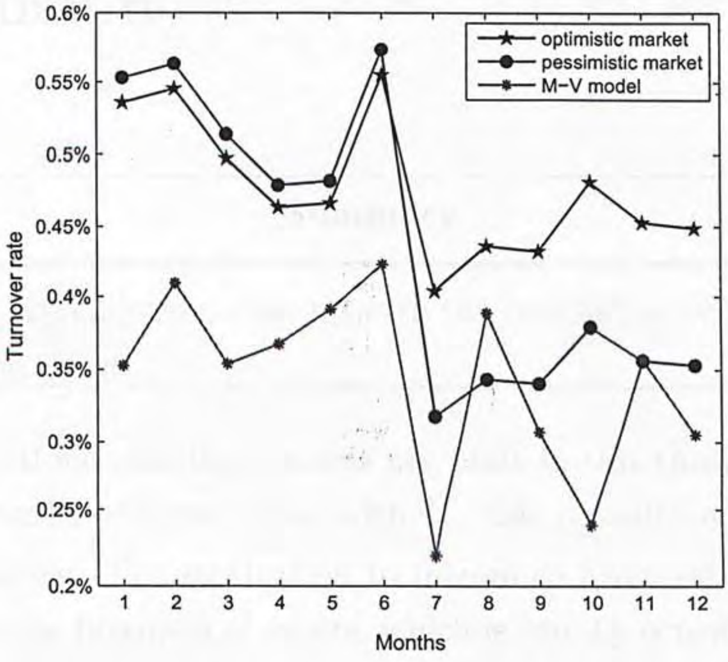

Figure 4.7: Portfolio turnover rate comparison between model 3 and M-V model for 12 months.

that: if investors set anticipation according with the market trend, it would show a better performance under model 3.

In model 3, we use the S-shape function as a membership function instead of semi-trapezoidal function in model 2, which describes the investors' preference in a more exact way. Then superiority of model 3 can be easily observed through the comparison between model 2 and 3.

□ **End of chapter.**

Chapter 5

Conclusion

Summary

The last chapter comes up with the concluding remarks of the thesis.

Three portfolio selection models are built in this thesis. They are all models under minimax rule with l_∞ risk measure and based on fuzzy set theory. The motivation to introduce fuzzy set theory is to characterize the fuzziness of events, which is usually considered as randomness, but actually is a different form of uncertainty.

The first model is the first attempt application in portfolio selection. For Model 1, transaction fee and liquidity constraints have been concerned. These Liquidity constraints are treated as trapezoidal fuzzy numbers. However, it is still a bi-criteria problem, a tradeoff between expected return rate and risk with λ. Numerical example is provided. Efficient frontier is drawn and analyzed. We choose Hang seng Index constituent stocks as our investment pool. According to the experiment results from Hong Kong securities market, model 1 is proved effective, which obtained excellent performance compared with Classic mean-variance model under different investment preference (different λ).

On the base of model 1, we proposed Model 2 and model 3. We do fuzzification not only on liquidity, but also the expected return, risk level with two kinds of classic fuzzy membership functions: semi-trapezoidal membership function and S shape function. The fuzzification is thoroughly applied in the whole model, not only in constraints. We formulated the portfolio selection problem as an optimization problem in satisfactory degree. The most obvious advantage of modeling in such an innovative way is that it can incorporat investors' preference and experts' experience. The Key parameters including expected return, risk and liquidity are fuzzy variables, using fuzzy membership functions to estimate the satisfactory degree on profit of portfolio, risk and liquidity. Different from traditional portfolio selection problem, there is no trade off parameter λ, no clear line between objectives and constraints. We just maximize the total membership function value to realize multi-goal optimization. In the numerical example part, we choose 40 hangseng index constituent stocks as our investment pool and collected 47 month real data (from December 2006 to November 2010) to simulate the model performance. In Model 2 and Model 3, parameters can be set by investors to adjust the satisfactory level according to their market anticipation. Two groups of numerical examples have been given under optimistic anticipation and pessimistic anticipation for 6 months and 12 months in 2010. Markowitz Mean-Variance Model is considered as a benchmark, which is known as the classic portfolio selection model. We also did comparison under different scenarios and got outweigh performance especially when investors' market anticipation is proved to be accord with the market trend.

□ **End of chapter.**

Appendix A

Source Data for Model 1

Table A.1: Return rate comparison between model 1 and M-V model for 6 months ($\lambda = 0.5$).

Month	Model 1	M-V model
May/2010	1	1
Jun./2010	1.0199	1.0994
Jul./2010	1.0601	1.0498
Aug./2010	1.1438	1.1661
Sep./2010	1.1887	1.2252
Oct./2010	1.1768	1.2345
Nov./2010	1.1914	1.2193

Table A.2: Return rate comparison between model 1 and M-V model for 12 months ($\lambda = 0.5$).

Month	Model 1	M-V model
Nov./2009	1	1
Dec./2009	1.0266	1.0546
Jan./2010	0.9284	0.9985
Feb./2010	0.9649	1.0745
Mar./2010	1.0221	1.0343
Apr./2010	0.9973	1.1156
May/2010	0.9255	1.0324
Jun./2010	0.9854	1.0706
Jul./2010	1.0128	1.0568
Aug./2010	0.9969	1.0976
Sep./2010	1.0942	1.1511
Oct./2010	1.091	1.1205
Nov./2010	1.123	1.1315

Table A.3: Portfolio turnover rate comparison between model 1 and M-V model for 6 months.

Month	Model 1	M-V model
Jun./2010	0.00703	0.00249
Jul./2010	0.00408	0.00433
Aug./2010	0.00450	0.00350
Sep./2010	0.00403	0.00384
Oct./2010	0.00413	0.00400
Nov./2010	0.00419	0.00220

Table A.4: Portfolio turnover rate comparison between model 1 and M-V model for 12 months.

Month	Model 1	M-V model
Dec./2009	0.00570	0.00353
Jan./2010	0.00581	0.00410
Feb./2010	0.00508	0.00354
Mar./2010	0.00505	0.00368
Apr./2010	0.00497	0.00391
May/2010	0.00481	0.00423
Jun./2010	0.00350	0.00220
Jul./2010	0.00378	0.00389
Aug./2010	0.00417	0.00307
Sep./2010	0.00374	0.00239
Oct./2010	0.00383	0.00357
Nov./2010	0.00389	0.00305

Table A.5: M-V model allocation strategy: value of x_i in 6 months.

Security name	Jun./2010	Jul./2010	Aug./2010	Sep./2010	Oct./2010	Nov./2010
1 Cheung Kong	0	0	0	0	0	0.067
2 CLP Hldgs	0	0.0604	0.0611	0.0774	0.0764	0.0483
3 HK & China Gas	0.1113	0	0	0	0	0
4 Wharf (Hldgs)	0	0	0	0	0	0
5 HSBC Hldgs	0	0	0	0	0	0
6 HK Electric	0	0	0	0	0	0
11 Hang Seng Bank	0	0	0	0	0.0823	0.051
12 Henderson Land	0	0	0.1139	0	0	0
13 Hutchison	0	0	0	0	0	0
16 SHK Prop	0	0	0	0	0	0
17 New World Dev	0	0.033	0.0334	0.0631	0.0637	0.063
19 Swire Pacific 'A'	0	0	0	0	0	0.2472
23 Bank of E Asia	0	0	0.0406	0.044	0.0398	0
66 MTR Corporation	0	0.0831	0.0819	0.0859	0.0805	0.0513
83 Sino Land	0.0605	0.0638	0	0	0.0742	0.0517
101 Hang Lung Prop	0.0822	0.091	0.1025	0.1208	0.0911	0.0575
144 China Mer Hldgs	0	0	0	0	0	0
267 CITIC Pacific	0.097	0.098	0	0	0	0
291 China Resources	0	0	0	0	0	0
293 Cathay Pac Air	0.0508	0.0567	0.0629	0.0709	0.0753	0.0475
330 Esprit Hldgs	0.0796	0.0844	0.0825	0.0919	0.0819	0.0482
386 Sinopec Corp	0	0.0837	0.0828	0.0794	0	0
388 HKEx	0	0	0	0	0	0
494 Li & Fung	0.078	0	0	0	0	0
551 Yue Yuen Ind	0	0	0	0	0	0
688 China Overseas	0	0	0	0	0	0.0586
700 Tencent	0.0794	0.0802	0.0775	0.0849	0.0779	0.0498
762 China Unicom	0.0838	0.0898	0.0911	0.0995	0.1082	0.0588
857 PetroChina	0	0	0	0	0	0
883 CNOOC	0	0	0	0	0	0
939 CCB	0	0	0	0	0	0
941 China Mobile	0	0	0	0	0	0
1088 China Shenhua	0.0945	0.0747	0.0724	0.0721	0.0609	0.0393
1199 COSCO Pacific	0	0	0	0	0	0
1398 ICBC	0	0	0	0	0	0
2038 FIH	0	0	0	0	0	0
2318 Ping An	0	0	0	0	0	0
2388 BOC Hong Kong	0.0928	0.1014	0.0975	0.1101	0.0877	0.0609
2600 CHALCO	0.0903	0	0	0	0	0
2628 China Life	0	0	0	0	0	0

Table A.6: Model 1 allocation strategy: value of x_i in 6 months.

Security name	Jun./2010	Jul./2010	Aug./2010	Sep./2010	Oct./2010	Nov./2010
1 Cheung Kong	0	0.0277	0.0323	0.0386	0.0316	0.0304
2 CLP Hldgs	0.0171	0.0155	0.0167	0.0211	0.0218	0.0217
3 HK & China Gas	0.0311	0.029	0.0329	0	0	0
4 Wharf (Hldgs)	0.0374	0.0355	0	0.0005	0.0597	0.0574
5 HSBC Hldgs	0.0185	0.0191	0.023	0	0.0198	0.0197
6 HK Electric	0.0182	0.0178	0.0206	0.0237	0.0205	0.0202
11 Hang Seng Bank	0	0	0	0.0372	0.0235	0.0229
12 Henderson Land	0.0299	0.0297	0.0312	0.0275	0.0249	0.0235
13 Hutchison	0.0212	0.0193	0.0229	0.0264	0.0217	0.0212
16 SHK Prop	0	0	0	0	0	0
17 New World Dev	0.0251	0.0234	0.0258	0.0299	0.0281	0.0283
19 Swire Pacific 'A'	0.1345	0.1067	0.1145	0.118	0.0967	0.111
23 Bank of E Asia	0.0117	0.0109	0.0111	0.012	0.0113	0
66 MTR Corporation	0.0212	0.0213	0.0225	0.0235	0.0229	0.023
83 Sino Land	0.0169	0.0164	0.0225	0.0288	0.0211	0.0233
101 Hang Lung Prop	0.0229	0.0233	0.0281	0.0329	0.026	0.0258
144 China Mer Hldgs	0	0	0	0	0	0
267 CITIC Pacific	0.0271	0.0251	0.0363	0	0	0
291 China Resources	0	0.0163	0	0	0	0.0186
293 Cathay Pac Air	0.0142	0.0145	0.0173	0.0193	0.0214	0.0213
330 Esprit Hldgs	0.0222	0.0216	0.0226	0.0251	0.0233	0.0217
386 Sinopec Corp	0.0256	0.0215	0.0227	0.0234	0.0222	0.0209
388 HKEx	0.017	0.0167	0.0184	0.0165	0	0
494 Li & Fung	0.0217	0.0214	0.0215	0	0	0
551 Yue Yuen Ind	0.0298	0.0277	0.0262	0.0259	0.0332	0.03
688 China Overseas	0.0373	0.0336	0	0	0.0328	0.0263
700 Tencent	0.0222	0.0205	0.0212	0.0232	0.0222	0.0223
762 China Unicom	0.0234	0.0229	0.025	0.0272	0.0308	0.0264
857 PetroChina	0.0245	0.0243	0.0274	0	0	0
883 CNOOC	0.089	0.0913	0.1038	0.1063	0.0996	0.1006
939 CCB	0.0243	0.0245	0.0319	0.0382	0.035	0.0353
941 China Mobile	0.0193	0	0	0.0228	0.0206	0.0102
1088 China Shenhua	0.0264	0.0191	0.0198	0.0197	0.0174	0.0177
1199 COSCO Pacific	0.0262	0.0241	0.0284	0.0295	0.0261	0.0274
1398 ICBC	0	0.0153	0.0182	0.021	0.0178	0.0189
2038 FIH	0	0.0237	0	0	0	0
2318 Ping An	0.0378	0.0353	0.0423	0.0407	0.0382	0.0393
2388 BOC Hong Kong	0.0259	0.026	0.0267	0.03	0.0249	0.0274
2600 CHALCO	0.0252	0.0314	0.0324	0.0365	0.035	0.035
2628 China Life	0.0545	0.0475	0.0539	0.0748	0.0698	0.0721

Table A.7: M-V model 1 allocation strategy: value of x_i in 12 months (1).

Security name	12/2009	1/2010	2/2010	3/2010	4/2010	5/2010	6/2010	7/2010	8/2010	9/2010	10/2010	11/2010
1 Cheung Kong	0	0	0	0	0	0	0	0	0	0	0	0
2 CLP Hldgs	0	0	0	0.0644	0	0	0	0.0552	0.0579	0.074	0.0738	0.0517
3 HK & China Gas	0.1259	0.1146	0.0951	0.0927	0.0912	0.0865	0.1014	0	0	0	0	0
4 Wharf (Hldgs)	0	0	0	0	0	0	0	0	0	0	0	0
5 HSBC Hldgs	0.0471	0	0.0505	0.0496	0	0.0431	0	0	0	0	0	0
6 HK Electric	0.0831	0.0785	0	0.0666	0.0667	0	0	0	0	0	0	0
11 Hang Seng Bank	0	0	0	0	0	0	0	0	0	0	0.0795	0.0547
12 Henderson Land	0	0	0.0707	0.0653	0	0.0751	0	0	0.1079	0	0	0
13 Hutchison	0	0	0	0	0	0	0	0	0	0	0	0
16 SHK Prop	0.0533	0.0551	0.0515	0.0463	0.0455	0.0414	0	0	0	0	0	0
17 New World Dev	0	0	0	0	0	0	0	0.0838	0.0844	0.1050	0.0955	0.0674
19 Swire Pacific 'A'	0	0	0	0	0	0	0	0	0	0	0	0
23 Bank of E Asia	0.0487	0.0439	0.0401	0.0373	0.0426	0.0401	0.0349	0.0314	0.0384	0.0421	0.0384	0
66 MTR Corporation	0	0.0668	0.062	0	0.0566	0.0490	0.0540	0.0761	0.0776	0.0821	0.0777	0.0549
83 Sino Land	0.0814	0	0	0	0	0	0.0551	0.0584	0	0	0.0717	0.0555
101 Hang Lung Prop	0	0.0808	0.08	0.0754	0.0756	0.0792	0.0749	0.0832	0.0971	0.1155	0.0881	0.0616
144 China Mer Hldgs	0.0696	0.0666	0.0666	0.0564	0.0580	0	0	0	0	0	0	0
267 CITIC Pacific	0	0	0	0	0	0	0.0883	0.0897	0	0	0	0
291 China Resources	0.0401	0.0493	0.0449	0.0411	0.0398	0.0359	0	0	0	0	0	0
293 Cathay Pac Air	0.0652	0.0596	0.0615	0.0578	0.0547	0.0419	0.0463	0.0518	0.0595	0.0677	0.0728	0.0510

Table A.8: M-V Model 1 allocation strategy: value of x_i in 12 months (2).

Security name	12/2009	1/2010	2/2010	3/2010	4/2010	5/2010	6/2010	7/2010	8/2010	9/2010	10/2010	11/2010
330 Esprit Hldgs	0	0	0	0	0.0858	0.0691	0.0724	0.0773	0.0781	0.0879	0.0791	0.0517
386 Sinopec Corp	0	0	0	0	0	0	0	0.0766	0.0784	0.0752	0	0
388 HKEx	0	0	0	0	0	0	0	0	0	0	0	0
494 Li & Fung	0	0	0	0	0	0	0.071	0	0	0	0	0
551 Yue Yuen Ind	0	0	0	0	0	0	0	0	0	0	0	0
688 China Overseas	0.0681	0.069	0.0686	0.062	0.0594	0.0613	0	0	0	0	0	0
700 Tencent	0.0697	0.0685	0.0657	0.0644	0.0676	0.0650	0.0724	0.0734	0.0734	0.0811	0.0754	0.0533
762 China Unicom	0.1005	0.1035	0.1107	0.0936	0.0904	0.0815	0.0763	0.0820	0.0863	0.0951	0.1044	0.0630
857 PetroChina	0	0	0	0	0.0264	0.0275	0	0	0	0	0	0
883 CNOOC	0	0	0	0	0	0	0	0	0	0	0	0
939 CCB	0	0	0	0	0	0	0	0	0	0	0	0
941 China Mobile	0	0	0	0	0	0	0	0	0	0	0	0
1088 China Shenhua	0.1021	0.0953	0.086	0.0827	0.0925	0.0809	0.0862	0.0684	0.0686	0.069	0.0588	0.0422
1199 COSCO Pacific	0	0	0	0	0	0	0	0	0	0	0	0
1398 ICBC	0.0453	0.0486	0.0460	0.0444	0.0473	0.0420	0	0	0	0	0	0
2038 FIH	0	0	0	0	0	0	0	0	0	0	0	0
2318 Ping An	0	0	0	0	0	0	0	0	0	0	0	0
2388 BOC Hong Kong	0	0	0	0	0	0.0807	0.0845	0.0928	0.0923	0.1053	0.0848	0.0653
2600 CHALCO	0	0	0	0	0	0	0.0823	0	0	0	0	0
2628 China Life	0	0	0	0	0	0	0	0	0	0	0	0

Table A.9: Model 1 allocation strategy: value of x_i in 12 months (1).

Security name	12/2009	1/2010	2/2010	3/2010	4/2010	5/2010	6/2010	7/2010	8/2010	9/2010	10/2010	11/2010
1 Cheung Kong	0	0	0	0.0368	0.0327	0.0349	0.0261	0.0277	0.0324	0.0387	0.0317	0.0304
2 CLP Hldgs	0.0311	0.0292	0.0239	0.0266	0.0266	0.0239	0.0164	0.0154	0.0168	0.0211	0.0218	0.0216
3 HK & China Gas	0.0505	0.0451	0.036	0.0384	0.035	0.0386	0.0299	0.0291	0.0329	0	0	0
4 Wharf (Hldgs)	0	0	0	0	0.034	0.0337	0.0354	0.0351	0	0	0.0598	0.0575
5 HSBC Hldgs	0.0188	0.0202	0.0191	0.0205	0.0219	0.0193	0.0178	0.0191	0.023	0	0.0197	0.0197
6 HK Electric	0.0332	0.0309	0.0245	0.0276	0.0256	0.0237	0.0175	0.0179	0.0206	0.0238	0.0206	0.0202
11 Hang Seng Bank	0	0	0.0386	0.0281	0.0352	0.039	0	0	0	0.0372	0.0235	0.0229
12 Henderson Land	0.0292	0.0272	0.0267	0.0271	0.0386	0.0335	0.0288	0.0297	0.0313	0.0275	0.0249	0.0235
13 Hutchison	0.0390	0.0347	0.0293	0.0319	0.0304	0.0281	0.0204	0.0194	0.0229	0.0264	0.0217	0.0213
16 SHK Prop	0.0213	0.0217	0.0194	0.0192	0.0175	0.0185	0	0	0	0	0	0
17 New World Dev	0.0382	0.0357	0.0321	0.0332	0.0296	0.0276	0.0241	0.0235	0.0259	0.0299	0.0281	0.0282
19 Swire Pacific 'A'	0	0	0	0	0	0	0.1294	0.1068	0.1146	0.1181	0.0966	0.1110
23 Bank of E Asia	0.0194	0.0172	0.0152	0.0154	0.0163	0.0178	0.0112	0.0109	0.0112	0.0120	0.0114	0
66 MTR Corporation	0.0293	0.0263	0.0235	0.0226	0.0217	0.0219	0.0204	0.0213	0.0225	0.0234	0.0229	0.0230
83 Sino Land	0.0325	0.0323	0.0301	0.0280	0.0278	0.0248	0.0162	0.0164	0.0225	0.0288	0.0211	0.0233
101 Hang Lung Prop	0.0343	0.0318	0.0303	0.0312	0.0290	0.0354	0.0221	0.0234	0.0281	0.0330	0.0260	0.0259
144 China Mer Hldgs	0.0278	0.0262	0.0252	0.0233	0.0237	0.0299	0	0	0	0	0	0
267 CITIC Pacific	0.0503	0.0507	0.0495	0.0446	0.0455	0.0432	0.0261	0.0252	0.0363	0	0	0
291 China Resources	0.0160	0.0194	0.0169	0.0170	0.0153	0.0160	0	0.0163	0	0	0	0.0186
293 Cathay Pac Air	0.0261	0.0235	0.0233	0.0240	0.0210	0.0188	0.0136	0.0145	0.0172	0.0194	0.0214	0.0214

Table A.10: Model 1 allocation strategy: value of x_i in 12 months (2).

Security name	12/2009	1/2010	2/2010	3/2010	4/2010	5/2010	6/2010	7/2010	8/2010	9/2010	10/2010	11/2010
330 Esprit Hldgs	0.0430	0.0425	0.0335	0.0348	0.0330	0.0309	0.0213	0.0217	0.0226	0.0251	0.0233	0.0216
386 Sinopec Corp	0	0.0531	0.0423	0.0472	0.0389	0.0394	0.0246	0.0215	0.0227	0.0233	0.0222	0.0209
388 HKEx	0.0322	0	0.0335	0	0.0283	0.0292	0.0163	0.0166	0.0184	0.0165	0	0
494 Li & Fung	0.0388	0.0351	0.0319	0.0335	0.0319	0.0311	0.0209	0.0205	0.0207	0	0	0
551 Yue_Yuen Ind	0	0.0354	0.0331	0.0325	0	0	0.0287	0.0277	0.0263	0.0260	0.0332	0.0301
688 China Overseas	0.0272	0.0271	0.026	0.0257	0.025	0.0305	0.0358	0.0336	0	0	0.0328	0.0263
700 Tencent	0.0279	0.027	0.0248	0.0266	0.0259	0.029	0.0213	0.0205	0.0213	0.0231	0.0222	0.0224
762 China Unicom	0.0402	0.0407	0.0418	0.0388	0.0347	0.0363	0.0225	0.023	0.0249	0.0271	0.0308	0.0264
857 PetroChina	0	0	0.0261	0.0266	0.0269	0.0318	0.0236	0.0244	0.0274	0	0	0
883 CNOOC	0	0	0	0	0	0	0.0856	0.0914	0.1038	0.1064	0.0996	0.1007
939 CCB	0.0482	0.0434	0.0384	0.0391	0.0338	0.0295	0.0234	0.0245	0.0319	0.0382	0.0351	0.0353
941 China Mobile	0.0437	0.0385	0.0328	0.0305	0.0287	0.0244	0.0186	0	0	0.0228	0.0207	0.0102
1088 China Shenhua	0.0408	0.0375	0.0325	0.0343	0.0355	0.0361	0.0254	0.0191	0.0199	0.0197	0.0173	0.0177
1199 COSCO Pacific	0	0	0	0	0	0	0.0252	0.0242	0.0284	0.0296	0.0261	0.0273
1398 ICBC	0.0181	0.0191	0.0173	0.0185	0.0181	0.0188	0.0152	0.0153	0.0182	0.0210	0.0178	0.0189
2038 FIH	0	0	0	0	0	0	0	0.0238	0	0	0	0
2318 Ping An	0.0401	0.0356	0.0362	0.0357	0.0346	0.0338	0.0347	0.0353	0.0423	0.0407	0.0382	0.0393
2388 BOC Hong Kong	0.0432	0.0431	0.0398	0.0384	0.0376	0.0360	0.0249	0.0260	0.0268	0.0300	0.0250	0.0273
2600 CHALCO	0.0589	0.0499	0.0468	0.0424	0.0395	0.0350	0.0242	0.0314	0.0324	0.0364	0.0350	0.0350
2628 China Life	0	0	0	0	0	0	0.0525	0.0475	0.0540	0.0748	0.0698	0.0721

Appendix B

Source Data for Model 2

Table B.1: Return rate comparison between M-V model and model 2 (06/2010-11/2010).

Month	M-V model	Optimistic market	Pessimistic market
May/2010	1	1	1
Jun./2010	1.0994	1.0511	1.0413
Jul./2010	1.0498	1.0789	1.0589
Aug./2010	1.1661	1.1399	1.1422
Sep./2010	1.2252	1.189	1.1517
Oct./2010	1.2345	1.1879	1.1629
Nov./2010	1.2193	1.2294	1.196

Table B.2: Return rate comparison between M-V model and model 2 (07/2008-12/2008).

Month	M-V model	Optimistic market	Pessimistic market
Jun./2008	1	1	1
Jul./2008	1.0874	1.0436	1.0523
Aug./2008	1.0589	1.0601	1.0749
Sep./2008	1.0899	1.0767	1.0796
Oct./2008	1.0876	1.0764	1.0782
Nov./2008	1.1087	1.1045	1.1065
Dec./2008	1.1091	1.1053	1.1064

Table B.3: Return rate comparison between M-V model and model 2 (12/2009-11/2010).

Month	M-V model	Optimistic market	Pessimistic market
Nov./2009	1	1	1
Dec./2009	1.0546	1.0694	1.049
Jan./2010	0.9985	0.9982	0.9967
Feb./2010	1.0745	1.0814	1.0817
Mar./2010	1.0343	1.0276	1.0219
Apr./2010	1.1156	1.1329	1.1122
May/2010	1.0324	1.0675	1.0571
Jun./2010	1.0706	1.0501	1.0587
Jul./2010	1.0568	1.0221	1.0651
Aug./2010	1.0976	1.1464	1.0775
Sep./2010	1.1511	1.1844	1.1522
Oct./2010	1.1205	1.1587	1.1307
Nov./2010	1.1315	1.1613	1.1336

Table B.4: Portfolio turnover rate comparison between M-V model and model 2 (06/2010-12/2010).

Month	M-V model	Optimistic market	Pessimistic market
Jun./2010	0.00249	0.00611	0.00592
Jul./2010	0.00433	0.00556	0.00434
Aug./2010	0.00350	0.00535	0.00444
Sep./2010	0.00384	0.00529	0.00382
Oct./2010	0.00400	0.00542	0.00430
Nov./2010	0.00220	0.00454	0.00394

Table B.5: Portfolio turnover rate comparison between M-V model and model 2 (07/2008-12/2008).

Month	M-V model	Optimistic market	Pessimistic market
Jul./2008	0.00189	0.00197	0.00199
Aug./2008	0.00207	0.00210	0.00224
Sep./2008	0.00198	0.00202	0.00211
Oct./2008	0.00190	0.00196	0.00203
Nov./2008	0.00223	0.00237	0.00239
Dec./2008	0.00186	0.00190	0.00194

Table B.6: Portfolio turnover rate comparison between M-V model and model 2 (12/2009-11/2010).

Month	M-V model	Optimistic market	Pessimistic market
Dec./2009	0.00353	0.00545	0.00547
Jan./2010	0.00410	0.00579	0.00582
Feb./2010	0.00354	0.00395	0.00505
Mar./2010	0.00368	0.00520	0.00530
Apr./2010	0.00391	0.00528	0.00584
May/2010	0.00423	0.00409	0.00433
Jun./2010	0.00220	0.00368	0.00362
Jul./2010	0.00389	0.00445	0.00371
Aug./2010	0.00307	0.00467	0.00344
Sep./2010	0.00239	0.00454	0.00308
Oct./2010	0.00357	0.00390	0.00332
Nov./2010	0.00305	0.00430	0.00327

Table B.7: Model 2 allocation strategy: value of x_i under optimistic anticipation (06/2010-11/2010) .

Security name	Jun./2010	Jul./2010	Aug./2010	Sep./2010	Oct./2010	Nov./2010
1 Cheung Kong	0	0	0	0	0	0
2 CLP Hldgs	0	0	0	0	0	0
3 HK & China Gas	0	0	0	0	0	0
4 Wharf (Hldgs)	0	0	0	0	0	0
5 HSBC Hldgs	0	0	0	0	0	0
6 HK Electric	0	0	0	0	0	0
11 Hang Seng Bank	0	0	0	0	0	0
12 Henderson Land	0	0	0	0	0	0
13 Hutchison	0	0	0	0	0	0
16 SHK Prop	0	0	0	0	0	0
17 New World Dev	0	0	0	0	0	0
19 Swire Pacific 'A'	0	0	0	0	0	0
23 Bank of E Asia	0	0	0	0	0	0
66 MTR Corporation	0	0	0	0	0	0
83 Sino Land	0	0	0	0	0	0
101 Hang Lung Prop	0	0	0	0	0	0
144 China Mer Hldgs	0	0	0	0	0	0
267 CITIC Pacific	0.0991	01533	0.1674	0.1277	0.2179	0.2439
291 China Resources	0	0	0	0	0	0
293 Cathay Pac Air	0	0	0	0	0	0
330 Esprit Hldgs	0	0	0	0	0	0
386 Sinopec Corp	0	0	0	0	0	0
388 HKEx	0	0	0	0	0	0
494 Li & Fung	0	0	0	0	0	0.0242
551 Yue Yuen Ind	0	0	0	0	0	0
688 China Overseas	0	0	0	0	0	0
700 Tencent	0	0	0	0	0	0
762 China Unicom	0	0	0	0	0	0
857 PetroChina	0	0	0	0	0	0
883 CNOOC	0	0	0	0	0	0
939 CCB	0	0	0	0	0	0
941 China Mobile	0	0	0	0	0	0
1088 China Shenhua	0	0	0	0	0	0
1199 COSCO Pacific	0	0	0	0	0	0
1398 ICBC	0.3003	0.2822	0.2775	0.2908	0.2607	0.2439
2038 FIH	0	0	0	0	0	0
2318 Ping An	0	0	0	0	0	0
2388 BOC Hong Kong	0.3003	0.2822	0.2775	0.2908	0.2607	0.2439
2600 CHALCO	0.3003	0.2822	0.2775	0.2908	0.2607	0.2439
2628 China Life	0	0	0	0	0	0

Table B.8: Model 2 allocation strategy: value of x_i under pessimistic anticipation (06/2010-11/2010).

Security name	Jun./2010	Jul./2010	Aug./2010	Sep./2010	Oct./2010	Nov./2010
1 Cheung Kong	0.0273	0.0272	0.0264	0.0294	0.0255	0.0244
2 CLP Hldgs	0.0172	0.0152	0.0137	0.016	0.0175	0.0174
3 HK & China Gas	0.0313	0.0285	0.0269	0.0236	0.0224	0.0214
4 Wharf (Hldgs)	0.0377	0.0349	0.0392	0.0573	0.0481	0.0461
5 HSBC Hldgs	0.0074	0.0188	0.0188	0.0205	0.0159	0.0158
6 HK Electric	0.0184	0.0175	0.0168	0.0181	0.0165	0.0185
11 Hang Seng Bank	0.0332	0.0331	0.0344	0.0282	0.0189	0.0184
12 Henderson Land	0.0302	0.0292	0.0255	0.0209	0.02	0.0188
13 Hutchison	0.0214	0.019	0.0187	0.02	0.0174	0.0171
16 SHK Prop	0.0159	0.0147	0.0151	0.0154	0.0133	0.0131
17 New World Dev	0.0253	0.023	0.0211	0.0227	0.0227	0.0227
19 Swire Pacific 'A'	0.1356	0.1048	0.0935	0.0896	0.0778	0.089
23 Bank of E Asia	0.0118	0.0107	0.0091	0.0091	0.0091.	0.12
66 MTR Corporation	0.0214	0.0209	0.0183	0.0178	0.0185	0.0185
83 Sino Land	0.017	0.0161	0.0183	0.0218	0.017	0.0186
101 Hang Lung Prop	0.0231	0.0229	0.0229	0.0249	0.0209	0.0207
144 China Mer Hldgs	0.0252	0.0211	0.0205	0.0194	0.0157	0.0158
267 CITIC Pacific	0.0273	0.0247	0.0297	0.025	0.0255	0.0236
291 China Resources	0.0165	0.0161	0.0178	0.0184	0.0171	0.0149
293 Cathay Pac Air	0.0143	0.0143	0.014	0.0147	0.0172	0.0171
330 Esprit Hldgs	0.0224	0.0213	0.0184	0.019	0.0188	0.0173
386 Sinopec Corp	0	0.0211	0.0185	0.0178	0.0217	0.0209
388 HKEx	0.0172	0.0164	0.0156	0.016	0.0175	0.0175
494 Li & Fung	0.0219	0.0273	0.0255	0.0246	0.0255	0.0248
551 Yue Yuen Ind	0.03	0.0272	0.0315	0.0341	0.0268	0.0241
688 China Overseas	0.0376	0.033	0.0431	0.0472	0.0264	0.0211
700 Tencent	0	0.0202	0.0173	0.0176	0.0178	0.0179
762 China Unicom	0.0236	0.0225	0.0204	0.0206	0.0248	0.0212
857 PetroChina	0.0247	0.0239	0.0224	0.0217	0.0235	0.0234
883 CNOOC	0	0.0131	0.0373	0	0.0802	0.0807
939 CCB	0	0.0241	0.026	0.028	0.0283	0.0283
941 China Mobile	0.0195	0.0189	0.0152	0.0173	0.0194	0.0194
1088 China Shenhua	0.0266	0.0188	0.0162	0.015	0.014	0.0142
1199 COSCO Pacific	0.0264	0.0237	0.0231	0.0224	0.021	0.0219
1398 ICBC	0.0159	0.0151	0.0149	0.0159	0.0143	0.0151
2038 FIH	0.0311	0.0233	0.0271	0.027	0.0279	0.0265
2318 Ping An	0.0381	0.0347	0.0345	0.0354	0.0308	0.0343
2388 BOC Hong Kong	0.0261	0.0255	0.0218	0.0228	0.0201	0.022
2600 CHALCO	0.0254	0.0309	0.0264	0.0277	0.0282	0.028
2628 China Life	0.0551	0.0466	0.044	0.0569	0.0562	0.0577

Table B.9: Model 2 allocation strategy: value of x_i under optimistic anticipation (12/2009-11/2010) (1).

Security name	12/2009	1/2010	2/2010	3/2010	4/2010	5/2010	6/2010	7/2010	8/2010	9/2010	10/2010	11/2010
1 Cheung Kong	0	0	0	0	0	0	0	0	0	0	0	0
2 CLP Hldgs	0	0	0	0	0	0	0	0	0	0	0	0
3 HK & China Gas	0	0	0	0	0	0	0	0	0	0	0	0
4 Wharf (Hldgs)	0	0	0	0	0	0	0	0	0	0	0	0
5 HSBC Hldgs	0	0	0	0	0	0	0	0	0	0	0	0
6 HK Electric	0	0	0	0	0	0	0	0	0	0	0	0
11 Hang Seng Bank	0	0	0	0	0	0	0	0	0	0	0	0
12 Henderson Land	0	0	0	0	0	0	0	0	0	0	0	0
13 Hutchison	0	0	0	0	0	0	0	0	0	0	0	0
16 SHK Prop	0	0	0	0	0	0	0	0	0	0	0	0
17 New World Dev	0	0	0	0	0	0	0	0	0	0	0	0
19 Swire Pacific 'A'	0	0	0	0	0	0	0	0	0	0	0	0
23 Bank of E Asia	0	0	0	0	0	0	0	0	0	0	0	0
66 MTR Corporation	0	0	0	0	0	0	0	0	0	0	0	0
83 Sino Land	0	0	0	0	0	0	0	0	0	0	0	0
101 Hang Lung Prop	0	0	0	0	0	0	0	0	0	0	0	0
144 China Mer Hldgs	0	0	0	0	0	0	0	0	0	0	0	0
267 CITIC Pacific	0.0991	0.1039	0	0.0589	0.1111	0.1157	0.0550	0.1141	0.1296	0.0922	0.1885	0.2396
291 China Resources	0	0	0	0	0	0	0	0	0	0	0	0
293 Cathay Pac Air	0	0	0	0	0	0	0	0	0	0	0	0

Table B.10: Model 2 allocation strategy: value of x_i under optimistic anticipation (12/2009-11/2010) (2).

Security name	12/2009	1/2010	2/2010	3/2010	4/2010	5/2010	6/2010	7/2010	8/2010	9/2010	10/2010	11/2010
330 Esprit Hldgs	0	0	0	0	0	0	0	0	0	0	0	0
386 Sinopec Corp	0	0	0	0	0	0	0	0	0	0	0	0
388 HKEx	0	0	0	0	0	0	0	0	0	0	0	0
494 Li & Fung	0	0	0	0	0	0	0	0	0	0	0	0
551 Yue Yuen Ind	0	0	0	0	0	0	0	0	0	0	0	0
688 China Overseas	0	0	0	0	0	0	0	0	0	0	0	0
700 Tencent	0	0	0	0	0	0	0	0	0	0	0	0
762 China Unicom	0	0	0	0	0	0	0	0	0	0	0	0
857 PetroChina	0	0	0	0	0	0	0	0	0	0	0	0
883 CNOOC	0	0	0	0	0	0	0	0	0	0	0	0
939 CCB	0	0	0	0	0	0	0	0	0	0	0	0
941 China Mobile	0	0	0	0	0	0	0	0	0	0	0	0
1088 China Shenhua	0	0	0	0	0	0	0	0	0	0	0	0
1199 COSCO Pacific	0	0	0	0	0	0	0	0	0	0	0	0
1398 ICBC	0.3003	0.2987	0.3359	0.3137	0.2963	0.2948	0.315	0.2953	0.2901	0.3026	0.2705	0.2535
2038 FIH	0	0	0	0	0	0	0	0	0	0	0	0
2318 Ping An	0	0	0	0	0	0	0	0	0	0	0	0
2388 BOC Hong Kong	0.3003	0.2987	0.3281	0.3137	0.2963	0.2948	0.315	0.2953	0.2901	0.3026	0.2705	0.2535
2600 CHALCO	0.3003	0.2987	0.3359	0.3137	0.2963	0.2948	0.3150	0.2953	0.2901	0.3026	0.2705	0.2535
2628 China Life	0	0	0	0	0	0	0	0	0	0	0	0

Table B.11: Model 2 allocation strategy: value of x_i under pessimistic anticipation (12/2009-11/2010) (1).

Security name	12/2009	1/2010	2/2010	3/2010	4/2010	5/2010	6/2010	7/2010	8/2010	9/2010	10/2010	11/2010
1 Cheung Kong	0.0241	0.0239	0.0247	0.0269	0.0232	0.0259	0.0286	0.0284	0.0275	0.0307	0.0257	0.0244
2 CLP Hldgs	0.0185	0.0183	0.0176	0.0196	0.0189	0.0178	0.0181	0.0158	0.0142	0.0167	0.0177	0.0174
3 HK & China Gas	0.0299	0.0283	0.0264	0.0281	0.0249	0.0287	0.0329	0.0297	0.0280	0.0247	0.0226	0.0214
4 Wharf (Hldgs)	0.0245	0.0237	0.0266	0.0270	0.0241	0.0250	0.0396	0.0364	0.0409	0.0599	0.0486	0.0461
5 HSBC Hldgs	0.0112	0.0127	0.0141	0.0151	0.0156	0.0142	0	0.0196	0.0196	0.0214	0.0161	0.0158
6 HK Electric	0.0198	0.0193	0.0181	0.0202	0.0182	0.0176	0.0193	0.0183	0.0176	0.0189	0.0167	0.0185
11 Hang Seng Bank	0.0266	0.0262	0.0283	0.0267	0.0250	0.0289	0.0348	0.0345	0.0359	0.0295	0.0191	0.0184
12 Henderson Land	0.0174	0.0171	0.0197	0.0199	0.0273	0.0249	0.0317	0.0305	0.0265	0.0218	0.0202	0.0188
13 Hutchison	0.0232	0.0217	0.0216	0.0234	0.0217	0.0208	0.0225	0.0198	0.0195	0.0209	0.0176	0.0171
16 SHK Prop	0.0127	0.0136	0.0143	0.0141	0.0124	0.0137	0.0167	0.0153	0.0157	0.0161	0.0134	0.0131
17 New World Dev	0.0227	0.0225	0.0237	0.0242	0.0211	0.0205	0	0.0240	0.0220	0.0237	0.0229	0.0227
19 Swire Pacific 'A'	0.0742	0.0793	0.0878	0.1045	0.1393	0.1329	0.1422	0.1095	0.0976	0.0936	0.0786	0.0890
23 Bank of E Asia	0.0116	0.0108	0.0111	0.0113	0.0116	0.0132	0.0124	0.0111	0.0095	0.0095	0.0092	0.012
66 MTR Corporation	0.0174	0.0165	0.0172	0.0166	0.0154	0.0162	0.0083	0.0219	0.0191	0.0186	0.0186	0.0185
83 Sino Land	0.0193	0.0202	0.0222	0.0205	0.0197	0.0184	0.0179	0.0168	0.0191	0.0228	0.0172	0.0186
101 Hang Lung Prop	0.0204	0.0199	0.0223	0.0228	0.0206	0.0263	0.0242	0.0239	0.0239	0.0262	0.0211	0.0207
144 China Mer Hldgs	0.0165	0.0164	0.0185	0.0171	0.0168	0.0221	0.0264	0.0221	0.0214	0.0203	0.0158	0.0158
267 CITIC Pacific	0.0298	0.0318	0.0363	0.0326	0.0334	0.032	0.0286	0.0258	0.0309	0.0262	0.0257	0.0236
291 China Resources	0.0095	0.0122	0.0125	0.0124	0.0108	0.0119	0.0173	0.0168	0.0186	0.0193	0.0172	0.0149
293 Cathay Pac Air	0.0155	0.0147	0.0171	0.0175	0.0149	0.0138	0.015	0.0149	0.0146	0.0153	0.0174	0.0171

Table B.12: Model 2 allocation strategy: value of x_i under pessimistic anticipation (12/2009-11/2010) (2).

Security name	12/2009	1/2010	2/2010	3/2010	4/2010	5/2010	6/2010	7/2010	8/2010	9/2010	10/2010	11/2010
330 Esprit Hldgs	0.0256	0.0266	0.0246	0.0256	0.0234	0.0229	0.0235	0.0222	0.0192	0.0199	0.0190	0.0173
386 Sinopec Corp	0.0272	0.0333	0.0312	0.0345	0.0276	0.0292	0	0.022	0.0193	0.0186	0.0219	0.0209
388 HKEx	0.0191	0.0222	0.0246	0.0234	0.0201	0.0216	0.018	0.0171	0.0163	0.0167	0.0177	0.0175
494 Li & Fung	0.0231	0.022	0.0235	0.0245	0.0226	0.0231	0.023	0.0285	0.0266	0.0257	0.0258	0.0248
551 Yue Yuen Ind	0.0224	0.0222	0.0243	0.0237	0.0267	0.0324	0.0316	0.0284	0.0329	0.0356	0.027	0.0241
688 China Overseas	0.0162	0.017	0.0191	0.0188	0.0178	0.0226	0.0394	0.0345	0.045	0.0493	0.0267	0.0211
700 Tencent	0.0166	0.0169	0.0183	0.0166	0.0184	0.0215	0	0.0158	0.0181	0.003	0.018	0.0179
762 China Unicom	0.0239	0.0255	0.0307	0.0284	0.0247	0.027	0.0248	0.0235	0.0213	0.0215	0.025	0.0212
857 PetroChina	0.0154	0.016	0.0192	0.0195	0.0191	0.0236	0.0259	0.0249	0.0233	0.0227	0.0238	0.0234
883 CNOOC	0.0511	0.0391	0	0	0.0067	0	0	0.0000	0	0	0.0706	0.0807
939 CCB	0.0287	0.0273	0.0045	0	0.0240	0.0072	0	0	0.0229	0	0.0285	0.0283
941 China Mobile	0.026	0.0241	0.0241	0.0223	0.0204	0.0181	0.0205	0.0197	0.0158	0.0181	0.0196	0.0194
1088 China Shenhua	0.0243	0.0236	0.0239	0.0251	0.0252	0.0268	0.0279	0.0196	0.0169	0.0156	0.0141	0.0142
1199 COSCO Pacific	0.067	0.0638	0.0529	0.046	0.0359	0.0291	0.0277	0.0248	0.0241	0.0234	0.0212	0.0219
1398 ICBC	0.0108	0.012	0.0128	0.0134	0.0129	0.0139	0.0167	0.0157	0.0155	0.0166	0.0144	0.0151
2038 FIH	0.0329	0.0324	0.0361	0.0338	0.0308	0.0322	0.0326	0.0243	0.0283	0.0282	0.0282	0.0265
2318 Ping An	0.0239	0.0224	0.0266	0.0261	0.0246	0.0251	0.04	0.0362	0.0360	0.0371	0.0311	0.0343
2388 BOC Hong Kong	0.0257	0.027	0.0293	0.028	0.0267	0.0267	0.0274	0.0267	0.0227	0.0238	0.0203	0.022
2600 CHALCO	0.0349	0.0313	0.0343	0.0311	0.0281	0.026	0.0266	0.0322	0.0275	0.0289	0.0285	0.028
2628 China Life	0.0398	0.0465	0.0598	0.0586	0.0492	0.0460	0.0577	0.0486	0.0459	0.0594	0.0568	0.0577

Appendix C

Source Data for Model 3

Table C.1: Return rate comparison between M-V model and model 3 (06/2010-11/2010).

Month	M-V model	Optimistic market	Pessimistic market
May/2010	1	1	1
Jun./2010	1.0994	1.0654	1.0416
Jul./2010	1.0498	1.0745	1.0685
Aug./2010	1.1661	1.1839	1.1624
Sep./2010	1.2252	1.2367	1.2291
Oct./2010	1.2345	1.2557	1.2149
Nov./2010	1.2193	1.2354	1.2016

Table C.2: Return rate comparison between M-V model and model 3 (07/2008-12/2008).

Month	M-V model	Optimistic market	Pessimistic market
Jun./2008	1	1	1
Jul./2008	1.0874	1.0517	1.0531
Aug./2008	1.0589	1.0623	1.0765
Sep./2008	1.0899	1.0759	1.0787
Oct./2008	1.0876	1.0797	1.0812
Nov./2008	1.1087	1.1065	1.1077
Dec./2008	1.1091	1.1059	1.1074

Table C.3: Performance comparison with Model 1 among Hangseng index, optimistic market and pessimistic market (12/2009-11/2010).

Month	M-V model	Optimistic market	Pessimistic market
Nov./2009	1	1	1
Dec./2009	1.0546	1.0592	1.0388
Jan./2010	0.9985	0.9642	0.9876
Feb./2010	1.0745	1.0591	1.103
Mar./2010	1.0343	1.0495	1.0267
Apr./2010	1.1156	1.1312	1.0749
May/2010	1.0324	1.0776	1.0202
Jun./2010	1.0706	1.0778	1.0915
Jul./2010	1.0568	1.0508	1.0779
Aug./2010	1.0976	1.1639	1.131
Sep./2010	1.1511	1.1847	1.1467
Oct./2010	1.1205	1.1627	1.1219
Nov./2010	1.1315	1.1806	1.1306

Table C.4: Portfolio turnover rate comparison between M-V model and model 3 (06/2010-11/2010).

Month	M-V model	Optimistic market	Pessimistic market
Jun./2010	0.00249	0.00676	0.00617
Jul./2010	0.00433	0.00556	0.00520
Aug./2010	0.00350	0.00530	0.00498
Sep./2010	0.00384	0.00529	0.00499
Oct./2010	0.00400	0.00546	0.00512
Nov./2010	0.00220	0.00476	0.00456

Table C.5: Portfolio turnover rate comparison between M-V model and model 3 (07/2008-12/2008).

Month	M-V model	Optimistic market	Pessimistic market
Jul./2008	0.00189	0.00201	0.00209
Aug./2008	0.00207	0.00212	0.00227
Sep./2008	0.00198	0.00209	0.00210
Oct./2008	0.00190	0.00190	0.00200
Nov./2008	0.00223	0.00217	0.00242
Dec./2008	0.00186	0.00194	0.00197

Table C.6: Portfolio turnover rate comparison between M-V model and model 3 (12/2009-11/2010).

Month	M-V model	optimistic market	pessimistic market
Dec./2009	0.00353	0.00535	0.00553
Jan./2010	0.00410	0.00545	0.00563
Feb./2010	0.00354	0.00497	0.00514
Mar./2010	0.00368	0.00464	0.00479
Apr./2010	0.00391	0.00467	0.00482
May/2010	0.00423	0.00555	0.00574
Jun./2010	0.00220	0.00404	0.00318
Jul./2010	0.00389	0.00436	0.00343
Aug./2010	0.00307	0.00432	0.00340
Sep./2010	0.00239	0.00481	0.00379
Oct./2010	0.00357	0.00453	0.00357
Nov./2010	0.00305	0.00449	0.00353

Table C.7: Model 3 allocation strategy: value of x_i under optimistic anticipation (06/2010-11/2010) .

Security name	Jun./2010	Jul./2010	Aug./2010	Sep./2010	Oct./2010	Nov./2010
1 Cheung Kong	0.0325	0.0328	0.0326	0.0367	0.0299	0.0281
2 CLP Hldgs	0.0205	0.0182	0.0169	0.02	0.0206	0.0201
3 HK & China Gas	0.0373	0.0343	0.0332	0.0296	0.0262	0.0246
4 Wharf (Hldgs)	0	0.0369	0.0344	0.0039	0.0564	0.0531
5 HSBC Hldgs	0	0	0	0	0.0186	0.0182
6 HK Electric	0.0219	0.0211	0.0208	0.0226	0.0195	0.0213
11 Hang Seng Bank	0.0395	0.0399	0.0426	0.0353	0.0221	0.0211
12 Henderson Land	0.036	0.0351	0.0315	0.0262	0.0234	0.0217
13 Hutchison	0.0255	0.0228	0.0231	0.025	0.0204	0.0197
16 SHK Prop	0.019	0.0176	0.0186	0.0194	0.0156	0.0151
17 New World Dev	0	0	0	0	0.0266	0.0261
19 Swire Pacific 'A'	0.1617	0.1261	0.1156	0.112	0.0913	0.1026
23 Bank of E Asia	0.0141	0.0128	0.0113	0.0114	0.0107	0.0138
66 MTR Corporation	0	0	0	0	0.0217	0.0213
83 Sino Land	0.0203	0.0193	0.0226	0.0273	0.02	0.0215
101 Hang Lung Prop	0.0276	0.0275	0.0284	0.0313	0.0245	0.0239
144 China Mer Hldgs	0.0301	0.0254	0.0253	0.0243	0.0184	0.0183
267 CITIC Pacific	0.0325	0.0297	0.0366	0.0314	0.0299	0.0271
291 China Resources	0.0197	0.0193	0.022	0.023	0.02	0.0172
293 Cathay Pac Air	0	0.0171	0.0174	0.0183	0.0202	0.0197
330 Esprit Hldgs	0.0267	0.0256	0.0228	0.0238	0.0220	0.02
386 Sinopec Corp	0	0	0	0	0.001	0.0172
388 HKEx	0.0205	0.0197	0.0193	0.02	0.0205	0.0201
494 Li & Fung	0.0261	0.0329	0.0316	0.0308	0.0299	0.0286
551 Yue Yuen Ind	0.0358	0.0328	0.039	0.0426	0.0314	0.0277
688 China Overseas	0.0448	0.0398	0.0534	0.0589	0.031	0.0243
700 Tencent	0	0	0	0	0	0
762 China Unicom	0.0281	0.0271	0.0252	0.0258	0.0290	0.0244
857 PetroChina	0	0	0	0	0.0276	0.027
883 CNOOC	0	0	0	0	0	0
939 CCB	0	0	0	0	0	0
941 China Mobile	0.0232	0.0228	0.0187	0.0216	0.0228	0.0223
1088 China Shenhua	0.0317	0.0226	0.02	0.0187	0.0163	0.0163
1199 COSCO Pacific	0.0315	0.0286	0.0287	0.028	0.0246	0.0253
1398 ICBC	0.019	0.0181	0.0184	0.0199	0.0168	0.0174
2038 FIH	0.0371	0.028	0.0335	0.0338	0.0327	0.0306
2318 Ping An	0.0455	0.0417	0.0427	0.0443	0.0361	0.0396
2388 BOC Hong Kong	0.0311	0.0307	0.027	0.0285	0.0236	0.0253
2600 CHALCO	0.0303	0.0371	0.0326	0.0347	0.033	0.0324
2628 China Life	0.0296	0.0561	0.0544	0.071	0.0659	0.0666

Table C.8: Model 3 allocation strategy: value of x_i under pessimistic anticipation (06/2010-11/2010).

Security name	Jun./2010	Jul./2010	Aug./2010	Sep./2010	Oct./2010	Nov./2010
1 Cheung Kong	0.0268	0.0267	0.0258	0.0287	0.0255	0.0244
2 CLP Hldgs	0.0169	0.0149	0.0134	0.0156	0.0175	0.0173
3 HK & China Gas	0.0307	0.0279	0.0263	0.0231	0.0223	0.0214
4 Wharf (Hldgs)	0.0370	0.0342	0.0384	0.056	0.0481	0.0461
5 HSBC Hldgs	0.0183	0.0184	0.0183	0.0201	0.0159	0.0158
6 HK Electric	0.0181	0.0171	0.0165	0.0176	0.0166	0.0185
11 Hang Seng Bank	0.0325	0.0325	0.0337	0.0276	0.0188	0.0184
12 Henderson Land	0.0296	0.0286	0.0249	0.0204	0.02	0.0189
13 Hutchison	0.021	0.0186	0.0183	0.0196	0.0174	0.0171
16 SHK Prop	0.0156	0.0144	0.0147	0.0152	0.0132	0.013
17 New World Dev	0.0248	0.0226	0.0206	0.0222	0.0227	0.0226
19 Swire Pacific 'A'	0.1331	0.1027	0.0915	0.0875	0.0778	0.0890
23 Bank of E Asia	0.0116	0.0105	0.0089	0.0089	0.0091	0.0119
66 MTR Corporation	0.021	0.0205	0.018	0.0174	0.0185	0.0185
83 Sino Land	0.0167	0.0158	0.018	0.0214	0.017	0.0187
101 Hang Lung Prop	0.0227	0.0224	0.0225	0.0245	0.0209	0.0207
144 China Mer Hldgs	0.0248	0.0207	0.02	0.0189	0.0156	0.0159
267 CITIC Pacific	0.0268	0.0242	0.029	0.0245	0.0255	0.0236
291 China Resources	0.0162	0.0158	0.0174	0.018	0.0171	0.0149
293 Cathay Pac Air	0.014	0.014	0.0138	0.0143	0.0172	0.0172
330 Esprit Hldgs	0.022	0.0208	0.0181	0.0186	0.0188	0.0173
386 Sinopec Corp	0.0077	0.0207	0.0182	0.0173	0.0217	0.0209
388 HKEx	0.0168	0.0161	0.0153	0.0156	0.0175	0.0175
494 Li & Fung	0.0215	0.0268	0.0250	0.0240	0.0255	0.0248
551 Yue Yuen Ind	0.0295	0.0267	0.0309	0.0333	0.0268	0.0241
688 China Overseas	0.0369	0.0324	0.0422	0.0461	0.0264	0.0211
700 Tencent	0	0.0198	0.0169	0.0171	0.0179	0.0179
762 China Unicom	0.0231	0.0221	0.0199	0.0201	0.0248	0.0212
857 PetroChina	0.0243	0.0235	0.0219	0.0213	0.0236	0.0233
883 CNOOC	0	0.0324	0.0578	0.0217	0.0802	0.0806
939 CCB	0	0.0236	0.0255	0.0283	0.0282	0.0283
941 China Mobile	0.0191	0.0186	0.0148	0.0169	0.0195	0.0194
1088 China Shenhua	0.0261	0.0184	0.0158	0.0146	0.0139	0.0142
1199 COSCO Pacific	0.0259	0.0233	0.0226	0.0219	0.021	0.0219
1398 ICBC	0.0156	0.0148	0.0146	0.0155	0.0143	0.0151
2038 FIH	0.0305	0.0229	0.0266	0.0264	0.0279	0.0266
2318 Ping An	0.0374	0.034	0.0338	0.0346	0.0308	0.0343
2388 BOC Hong Kong	0.0256	0.0250	0.0213	0.0223	0.0201	0.0219
2600 CHALCO	0.0249	0.0302	0.0258	0.0271	0.0281	0.0281
2628 China Life	0.0541	0.0457	0.0431	0.0555	0.0563	0.0577

Table C.9: Model 3 allocation strategy: value of x_i under optimistic anticipation (12/2009-11/2010) (1).

Security name	12/2009	1/2010	2/2010	3/2010	4/2010	5/2010	6/2010	7/2010	8/2010	9/2010	10/2010	11/2010
1 Cheung Kong	0.0266	0.0265	0.0273	0.0299	0.0266	0.0301	0.0342	0.0344	0.0341	0.0386	0.0312	0.0294
2 CLP Hldgs	0.0205	0.0203	0.0195	0.0218	0.0216	0.0206	0.0215	0.0191	0.0177	0.021	0.0214	0.0209
3 HK & China Gas	0.0332	0.0314	0.0293	0.0313	0.0284	0.0332	0.0392	0.0359	0.0348	0.031	0.0273	0.0258
4 Wharf (Hldgs)	0.0271	0.0263	0.0296	0.03	0.0276	0.0227	0	0	0	0	0.0589	0.0554
5 HSBC Hldgs	0.0124	0.0141	0	0	0	0	0	0	0	0	0	0
6 HK Electric	0.0219	0.0215	0.02	0.0225	0.0208	0.0204	0.0230	0.0221	0.0217	0.0237	0.0203	0.0222
11 Hang Seng Bank	0.0295	0.0291	0.0315	0.0297	0.0285	0.0336	0.0415	0.0418	0.0446	0.0371	0.0231	0.0221
12 Henderson Land	0.0193	0.0189	0.0218	0.0221	0.0313	0.0289	0.0377	0.0368	0.033	0.0275	0.0245	0.0226
13 Hutchison	0.0257	0.0241	0.0239	0.026	0.0247	0.0241	0.0267	0.0239	0.0242	0.0262	0.0213	0.0206
16 SHK Prop	0.014	0.0151	0.0159	0.0156	0.0142	0.0159	0.02	0.0185	0.0195	0.0203	0.0162	0.0157
17 New World Dev	0.0251	0.0249	0	0	0	0	0	0	0	0	0.004	0.0201
19 Swire Pacific 'A'	0.0821	0.0881	0.0973	0.1164	0.1592	0.1541	0.1697	0.1321	0.1211	0.1176	0.0953	0.1072
23 Bank of E Asia	0.0128	0.0121	0.0124	0.0126	0.0132	0.0154	0.0147	0.0135	0.0118	0.0119	0.0112	0.0144
66 MTR Corporation	0.0193	0.0184	0.0118	0.0061	0.0041	0	0	0	0	0	0.0226	0.0222
83 Sino Land	0.0214	0.0226	0.0246	0.0228	0.0226	0.0214	0.0213	0.0203	0.0238	0.0287	0.0208	0.0225
101 Hang Lung Prop	0.0226	0.0222	0.0247	0.0254	0.0235	0.0304	0.0289	0.0288	0.0297	0.0328	0.0256	0.0249
144 China Mer Hldgs	0.0183	0.0183	0.0205	0.019	0.0192	0.0257	0.0316	0.0266	0.0264	0.0254	0.0191	0.0191
267 CITIC Pacific	0.033	0.0354	0.0404	0.0364	0.0382	0.0372	0.0342	0.0311	0.0384	0.0328	0.0312	0.0284
291 China Resources	0.0106	0.0135	0.0139	0.0139	0.0125	0.0138	0.0207	0.0203	0.0231	0.0242	0.0209	0.018
293 Cathay Pac Air	0.0172	0.0164	0.0190	0.0195	0.0171	0.0161	0.0000	0.0087	0.007	0.0000	0.0212	0.0207

Table C.10: Model 3 allocation strategy: value of x_i under optimistic anticipation (12/2009-11/2010) (2).

Security name	12/2009	1/2010	2/2010	3/2010	4/2010	5/2010	6/2010	7/2010	8/2010	9/2010	10/2010	11/2010
330 Esprit Hldgs	0.0283	0.0296	0.0273	0.0284	0.0268	0.0266	0.028	0.0268	0.0239	0.025	0.023	0.0209
386 Sinopec Corp	0.0292	0.0176	0	0	0	0	0	0	0	0	0	0
388 HKEx	0.0212	0.0246	0.0273	0.0262	0.0230	0.0251	0.0215	0.0206	0.0202	0.021	0.0214	0.0211
494 Li & Fung	0.0256	0.0244	0.0261	0.0273	0.0259	0.0268	0.0275	0.0345	0.033	0.0323	0.0312	0.0298
551 Yue Yuen Ind	0.0248	0.0246	0.0269	0.0265	0.0306	0.0376	0.0376	0.0344	0.0408	0.0447	0.0328	0.029
688 China Overseas	0.0179	0.0189	0.0212	0.0209	0.0203	0.0263	0.047	0.0417	0.0558	0.0619	0.0324	0.0254
700 Tencent	0	0	0	0	0	0	0	0	0	0	0	0
762 China Unicom	0.0265	0.0284	0.0341	0.0316	0.0282	0.0312	0.0295	0.0284	0.0263	0.0270	0.0303	0.0255
857 PetroChina	0.0171	0.0178	0.0213	0.0217	0.0219	0	0	0	0	0	0.0288	0.0281
883 CNOOC	0	0	0	0	0	0	0	0	0	0	0	0
939 CCB	0	0	0	0	0	0	0	0	0	0	0	0
941 China Mobile	0.0288	0.0268	0.0267	0.0248	0.0233	0.0210	0.0067	0.0239	0.0196	0.0228	0.0238	0.0233
1088 China Shenhua	0.0269	0.0261	0.0265	0.0279	0.0288	0.031	0.0333	0.0237	0.021	0.0196	0.0171	0.017
1199 COSCO Pacific	0.0743	0.0710	0.0587	0.0513	0.0411	0.0338	0.0331	0.0300	0.0300	0.0294	0.0257	0.0264
1398 ICBC	0.012	0.0133	0.0142	0.015	0.0147	0.0161	0.02	0.0190	0.0193	0.0209	0.0175	0.0182
2038 FIH	0.0364	0.036	0.04	0.0376	0.0352	0.0374	0.0389	0.0295	0.0352	0.0354	0.0342	0.0319
2318 Ping An	0.0264	0.0248	0.0296	0.0290	0.0281	0.0291	0.0477	0.0438	0.0447	0.0466	0.0376	0.0413
2388 BOC Hong Kong	0.0284	0.03	0.0324	0.0313	0.0305	0.0309	0.0327	0.0321	0.0282	0.0299	0.0246	0.0264
2600 CHALCO	0.0387	0.0348	0.0381	0.0345	0.0321	0.0302	0.0318	0.0389	0.0342	0.0364	0.0345	0.0338
2628 China Life	0.044	0.0517	0.0663	0.0652	0.0562	0.0534	0	0.0588	0.057	0.0483	0.0689	0.0696

Table C.11: Model 3 allocation strategy: value of x_i under pessimistic anticipation (12/2009-11/2010) (1).

Security name	12/2009	1/2010	2/2010	3/2010	4/2010	5/2010	6/2010	7/2010	8/2010	9/2010	10/2010	11/2010
1 Cheung Kong	0.0238	0.0236	0.0244	0.0266	0.0229	0.0254	0.0280	0.0278	0.0269	0.0299	0.0255	0.0244
2 CLP Hldgs	0.0183	0.0181	0.0174	0.0193	0.0186	0.0174	0.0176	0.0155	0.0139	0.0163	0.0175	0.0173
3 HK & China Gas	0.0296	0.028	0.0262	0.0278	0.0245	0.0281	0.0321	0.029	0.0273	0.024	0.0223	0.0214
4 Wharf (Hldgs)	0.0242	0.0235	0.0264	0.0267	0.0237	0.0245	0.0386	0.0356	0.0399	0.0583	0.0481	0.0461
5 HSBC Hldgs	0.0111	0.0126	0.0139	0.0149	0.0154	0.014	0	0.0192	0.0191	0.0209	0.0159	0.0158
6 HK Electric	0.0195	0.0191	0.0178	0.0199	0.0179	0.0172	0.0189	0.0178	0.0171	0.0184	0.0166	0.0185
11 Hang Seng Bank	0.0263	0.0259	0.0281	0.0263	0.0246	0.0285	0.0341	0.0338	0.035	0.0287	0.0188	0.0184
12 Henderson Land	0.0172	0.0169	0.0194	0.0196	0.027	0.0244	0.0309	0.0298	0.026	0.0213	0.02	0.0189
13 Hutchison	0.023	0.0215	0.0213	0.0231	0.0213	0.0204	0.0219	0.0194	0.0191	0.0204	0.0174	0.0171
16 SHK Prop	0.0125	0.0134	0.0141	0.0138	0.0122	0.0134	0.0164	0.015	0.0153	0.0158	0.0132	0.013
17 New World Dev	0.0224	0.0222	0.0233	0.0239	0.0208	0.0201	0.0089	0.0235	0.0214	0.0231	0.0227	0.0226
19 Swire Pacific 'A'	0.0734	0.0784	0.0868	0.1033	0.1372	0.1305	0.139	0.1070	0.0952	0.0912	0.0778	0.089
23 Bank of E Asia	0.0114	0.0107	0.011	0.0112	0.0114	0.013	0.0121	0.0109	0.0092	0.0093	0.0091	0.0119
66 MTR Corporation	0.0172	0.0163	0.0170	0.0164	0.0152	0.016	0.022	0.0213	0.0187	0.0181	0.0185	0.0185
83 Sino Land	0.0191	0.02	0.0218	0.0202	0.0194	0.0180	0.0174	0.0164	0.0187	0.0222	0.0170	0.0187
101 Hang Lung Prop	0.0202	0.0197	0.022	0.0226	0.0203	0.0257	0.0238	0.0233	0.0234	0.0255	0.0209	0.0207
144 China Mer Hldgs	0.0164	0.0162	0.0183	0.0169	0.0166	0.0218	0.0259	0.0215	0.0208	0.0197	0.0156	0.0159
267 CITIC Pacific	0.0295	0.0315	0.036	0.0322	0.0328	0.0314	0.028	0.0252	0.0302	0.0255	0.0255	0.0236
291 China Resources	0.0094	0.0121	0.0123	0.0123	0.0107	0.0117	0.0170	0.0164	0.0181	0.0187	0.0171	0.0149
293 Cathay Pac Air	0.0153	0.0145	0.0169	0.0173	0.0147	0.0136	0.0147	0.0146	0.0143	0.0149	0.0172	0.0172

Table C.12: Model 3 allocation strategy: value of x_i under pessimistic anticipation (12/2009-11/2010) (2).

Security name	12/2009	1/2010	2/2010	3/2010	4/2010	5/2010	6/2010	7/2010	8/2010	9/2010	10/2010	11/2010
330 Esprit Hldgs	0.0253	0.0263	0.0244	0.0252	0.0231	0.0225	0.0229	0.0217	0.0188	0.0194	0.0188	0.0173
386 Sinopec Corp	0.0269	0.0329	0.0307	0.0341	0.0272	0.0287	0	0.0215	0.0189	0.018	0.0217	0.0209
388 HKEx	0.0189	0.022	0.0244	0.0232	0.0198	0.0213	0.0176	0.0167	0.0159	0.0163	0.0175	0.0175
494 Li & Fung	0.0228	0.0218	0.0232	0.0242	0.0223	0.0227	0.0225	0.0279	0.026	0.025	0.0255	0.0248
551 Yue Yuen Ind	0.0222	0.0219	0.0241	0.0235	0.0263	0.0317	0.0308	0.0278	0.0321	0.0347	0.0268	0.0241
688 China Overseas	0.016	0.0168	0.0189	0.0186	0.0175	0.0223	0.0385	0.0337	0.0439	0.048	0.0264	0.0211
700 Tencent	0.0164	0.0167	0.018	0.0193	0.0181	0.0211	0	0.0206	0.0176	0.0178	0.0179	0.0179
762 China Unicom	0.0236	0.0252	0.0304	0.0281	0.0243	0.0265	0.0242	0.023	0.0207	0.021	0.0248	0.0212
857 PetroChina	0.0153	0.0158	0.0190	0.0192	0.0188	0.0232	0.0253	0.0245	0.0228	0.0221	0.0236	0.0233
883 CNOOC	0.0616	0.0498	0	0	0.0222	0.0035	0	0	0.0202	0	0.0802	0.0806
939 CCB	0.0284	0.0269	0.0159	0.0094	0.0236	0.0215	0	0.0171	0.0265	0.0111	0.0282	0.0283
941 China Mobile	0.0257	0.0239	0.0239	0.022	0.0201	0.0177	0.02	0.0194	0.0154	0.0176	0.0195	0.0194
1088 China Shenhua	0.024	0.0233	0.0236	0.0248	0.0248	0.0263	0.0273	0.0192	0.0165	0.0152	0.0139	0.0142
1199 COSCO Pacific	0.0664	0.0631	0.0524	0.0455	0.0354	0.0286	0.0271	0.0243	0.0235	0.0228	0.021	0.0219
1398 ICBC	0.0107	0.0119	0.0127	0.0133	0.0127	0.0137	0.0164	0.0154	0.0152	0.0162	0.0143	0.0151
2038 FIH	0.0325	0.0321	0.0357	0.0334	0.0303	0.0316	0.0319	0.0238	0.0276	0.0275	0.0279	0.0266
2318 Ping An	0.0236	0.0221	0.0263	0.0257	0.0242	0.0246	0.0392	0.0354	0.0351	0.0361	0.0308	0.0343
2388 BOC Hong Kong	0.0254	0.0267	0.0289	0.0277	0.0263	0.0262	0.0268	0.026	0.0222	0.0232	0.0201	0.0219
2600 CHALCO	0.0346	0.0309	0.0340	0.0306	0.0277	0.0255	0.026	0.0315	0.0269	0.0282	0.0281	0.0281
2628 China Life	0.0393	0.0459	0.0591	0.0579	0.0484	0.0452	0.0564	0.0476	0.0448	0.0578	0.0563	0.0577

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