Cointegration Pairs Trading Strategy on Derivatives

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Declaration

No portion of the work referred to in this thesis has been submitted in support of an application for another degree or qualification of this or any other university or other institution of learning.

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Abstract of thesis entitled:

Cointegration Pairs Trading Strategy on Derivatives

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Abstract

The notion of cointegration has been widely used in finance and econometrics, in particular in constructing statistical arbitrage strategies in the stock market. In this thesis, an arbitrage trading strategy for derivatives based on cointegration is studied to account for the volatility factor. Pairs of short dated at-the-money straddles of European options with positive net carry (i.e. theta) are used to capture the mean-reverting property of the linear combinations of implied volatilities. Furthermore, modeling and forecasting realized volatility are also considered as a supplement to the trading strategy. Implied-Realized Criertion and Gamma-Vega Criterion are introduced to improve the trading strategy. A performance analysis is conducted with a 3-year historical data of Foreign Exchange Options. From the empirical results, the portfolio based on the cointegration strategy makes a profit, where Vega plays a dominant role, and either the Implied-Realized Criertion or the Gamma-Vega Criterion is effective.

摘要

在現今的社會，協整技術已被廣泛應用於金融和計量經濟領域，特別用於構建股票市場的統計套利策略。在這一篇論文中，我們主要考察在衍生品市場中，基於協整技術的套利交易策略，這一策略的主要研究對象是隱含波動率。利用隱性波動率的線性組合的均值回歸的特性，通過配對兩隻帶有正利差（如theta）的短期平價歐式跨式期權來獲利。同時，構建實際波動率的模型和預測未來實際波動率的模型將會用於補充這一交易策略的不足，隱性－實際條件和Gamma-Vega條件被引入來提高交易策略的效率。這一策略的績效分析是基於三年的歷史外匯期權數據。從實證數據中，基於協整技術的策略能賺取利潤，而且Vega在利潤中起著重要的作用，並且無論是隱性－實際條件還是Gamma-Vega條件都是有效的。

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Chapter 1

Introduction

The notion of cointegration was introduced by Robert Engle and Clive Granger in 1987, see Engle and Granger (1987). Since then, cointegration is widely used in statistical arbitrage for pairs-trading strategy. There are two popular methodologies to construct the cointegration relationship: Engle-Granger methodology and Johansen's methodology, see Johansen (1988).

Investors take advantage of the idea of cointegration in the stock market to obtain additional profit, see Chan (2010). However, this technique is seldom used in the derivative markets due to market complexity. The purpose of this thesis is to generalize the pairs-trading strategy based on cointegration to the derivative market.

Pairs-trading is pioneered by Nunzio Tartaglia's group at Morgan Stanley in the 1980's. This strategy is said to be market neutral and is based on the mean-reverting property. Simple pairs-trading aims to find out two historically correlated stocks and captures the divergence of the original trend of these two stocks. For example, assume that stock A and stock B are highly correlated. When the trend of the price of stock A deviates from that of stock B, a position is initiated until the spread between two stock prices eventually converges to the historical level. Similar to this idea, cointegration pairs can replace highly correlated pairs to construct a statistical arbitrage strategy. More details can be found in Gatev, Goetzmann and Rouwenhorst (2006).

The idea of cointegration can be applied to most simple financial instruments, including stock prices, commodity prices, and spot exchange rates. Prices of derivatives, however, are not suitable quantities to apply cointegration because it is a function of several factors, including strike price, expiration time, implied volatility, dividend rate and risk-free rate. By the Johansen test, we found that strong cointegration relationships exist among the implied volatilities in both Hang Seng Index constituents options and Foreign Exchange options from 2009 to 2010. An alternative way to introduce cointegration into the derivative market is trading volatilities by trading at-the-money (ATM) straddles. It is not as simple as trading stocks, however. Besides vega, which measures the sensitivity to the volatility, different aspects related to the straddle price, such as delta, gamma and theta, need to be considered. We aim to make the vega as large as possible while others have negligible impacts. Through a series of settings, pairs of short dated ATM straddles of European options with positive net carry (i.e. theta) are developed.

In our analysis, we found that the portfolio may suffer a huge loss from gamma if the actual change of the underlying asset deviates too much from the expected change. The cointegration criterion, however, does not consider this discrepancy. To improve the efficiency of the strategy, an additional criterion on the volatility that captures the difference between the actual and expected variation is necessary. Realized volatility is used to measure the actual variation of the underlying asset, but it cannot be observed. We need to model and forecast the realized volatility so that the additional criterion can be implemented. Modelling and forecasting realized volatility are explored by many authors, see Garman and Klass (1980) and Andersen, Bollerslev, Diebold and Labys (2003).

Among various models, High-Frequency Estimation and ARFIMA model are chosen to implement two additional criteria: Implied-Realized Criterion and Gamma-Vega Criterion.

In this thesis, the cointegration strategy is applied to the implied volatility using Johansen's methodology by trading at-the-money FX straddles. Two criteria: Implied-Realized Criterion and Gamma-Vega Criterion are introduced to improve the trading strategy. The empirical study is conducted with the Foreign Exchange Options from 2009 to 2011. In Chapter 2, basic ideas of Johansen's methodology and trading strategy are presented. The main idea of cointegration pairs trading strategy in derivatives is studied in Chapter 3. In Chapter 4, improvements of the strategy are discussed. Finally, conclusion and further discussion are presented in Chapter 5.

Chapter 2

Basic Ideas

This chapter discusses the basic concepts of the trading strategy. Because the trading strategy is based on the notion of cointegration, the concepts of cointegration are presented. Among different ideas of cointegration, Johansen's methodology is used to identify the cointegration relationship, from which a pairs trading strategy for underlying assets using cointegration will be explored. In the analysis below, we found that losses may be incurred if the actual variation of the underlying asset is far from the expectation, To avoid initiating these "bad" trades, an additional criterion based on the difference between realized volatility and implied volatility [1] is introduced. However, realized volatility cannot be observed. Methods for modelling and forecasting realized volatility will be studied.

2.1 Cointegration and Johansen's Methodology

2.1.1 Cointegration

The concept of cointegration was introduced by Robert Engle and Clive Granger in 1987, see Engle and Granger (1987). They aim to remove the common

[1] Realized volatility measures the actual change in the underlying asset while implied volatility is the assessment of future variation.

stochastic trend of a multivariate time series based on identifying a linear relationship.

Definition $\{X_t\}$ is said to be $I(d)$ process, where d is the order of integration, if $(1-L)^d X_t$ is stationary, where $LX_t = X_{t-1}$.

Definition Let $\{X_t\}$ be $n \times 1$ vector and $I(d)$ process. If there exists an $n \times r$ $(r < n)$ matrix H, $H \neq 0$, such that $H'X_t$ is stationary, then X_t is said to be cointegrated of order d. When d is an integer, it is called "cointegration". When d is a fraction, it is called "fractional cointegration".

There are two popular methodologies to test the cointegration relationship: Engle-Granger's methodology and Johansen's methodology. In our analysis, we use the latter method because it is more efficient. For simplicity, a dimension $n = 2$ is used in our empirical study.

2.1.2 Johansen's Methodology

Johansen's methodology is based on the Vector Autoregressive (VAR) process. Consider a p-dimensional non-stationary $I(1)$ time series $\{X_t\}$, which follows a VAR(k) process:

$$X_t = \Phi_1 X_{t-1} + \Phi_2 X_{t-2} + \cdots + \Phi_k X_{t-k} + \varepsilon_t, t = \ldots, -1, 0, 1, \ldots,$$

where $\Phi_1, \Phi_2, \ldots, \Phi_k$ are $p \times p$ matrices, and ε_t is Gaussian random vector with mean 0 and covariance matrix Ω.

Note that the above equation can be rewritten as a Vector Error Correction Model (VECM):

$$\Delta X_t = \Gamma X_{t-1} + \Gamma_1 \Delta X_{t-1} + \cdots + \Gamma_{k-1} \Delta X_{t-k+1} + \varepsilon_t,$$

where $\Gamma = \sum_{i=1}^{k} \Phi_i - I, \Gamma_l = -\sum_{j=l+1}^{k} \Phi_j, l = 1, \ldots, k-1$. Hence, Γ_l, $l = 1, \ldots, k-1$ are unrestricted.

Johansen's methodology aims to test whether the matrix Γ can be expressed as the multiplication of two suitable $p \times r(r < p)$ matrices α and β. If $\Gamma = \alpha\beta'$, then $\beta' X_t$ is stationary, where α is the adjustment coefficient and β is the cointegrating vector. Johansen proposed two methods to test if $r < p$ is true: the trace test and the maximum eigenvalue test. The corresponding likehood ratio tests statistics (L) and hypothesises are as follows:

Trace test:

$$L_{trace} = -N \sum_{i=r+1}^{p} \log(1 - \hat{\lambda}_i),$$

$$H_0 : K_c = r \quad vs \quad H_1 : K_c = p.$$

Maximum eigenvalue test:

$$L_{eig} = -N \log(1 - \hat{\lambda}_{r+1}),$$

$$H_0 : K_c = r \quad vs \quad H_1 : K_c = r + 1.$$

Here N is the sample size, $\hat{\lambda}_i$ is the i-th largest canonical correlation and K_c is the number of coinegrating vector.

The parameters are estimated by maximum likelihood estimation. The details of the procedure are given in Johansen (1988).

2.2 Cointegration Pairs Trading Strategy

In the past, it was common to use principal component analysis (PCA) to identify the common trend of two stocks based on its correlation coefficient. However, the PCA assumes the stock returns data are i.i.d.. Cointegration, which is used to identify the common stochastic trend, would be a better technique to identify the linear relationship between two stochastic processes. Profitability of the cointegration pairs trading strategy in the stock market has been reported, see Chan (2010). The details of the strategy are as follows.

Let X_t and Y_t be the prices of two stocks at time t. Assume that $\log X_t$ and $\log Y_t$ are cointegrated, i.e. there exists two constant a, b such that the linear combination of the log prices of the two stocks $a \log X_t + b \log Y_t$ is stationary.

Based on Taylor Series expansion evaluated at time t_0,

$$\begin{aligned} a \log X_t + b \log Y_t &\approx a(\log X_{t_0} + \frac{X_t - X_{t_0}}{X_{t_0}}) + b(\log Y_{t_0} + \frac{Y_t - Y_{t_0}}{Y_{t_0}}) \\ &= \frac{a}{X_{t_0}} X_t + \frac{b}{Y_{t_0}} Y_t + a(\log X_{t_0} - 1) + b(\log Y_{t_0} - 1). \end{aligned}$$

Because $a(\log X_{t_0} - 1) + b(\log Y_{t_0} - 1)$ is a constant, the stationarity of $a \log X_t + b \log Y_t$ implies that $\frac{a}{X_{t_0}} X_t + \frac{b}{Y_{t_0}} Y_t$ is approximately stationary, i.e. $\frac{a}{X_{t_0}} X_t + \frac{b}{Y_{t_0}} Y_t$ should exhibit the mean-reverting property. By virtue of this property, let a trading portfolio Π_t be

$$\Pi_t = \tfrac{a}{X_{t_0}} X_t + \tfrac{b}{Y_{t_0}} Y_t.$$

If the portfolio continues to be governed by this mean-reverting process and $\Pi_{t_0} < \mathbb{E}(\Pi_t)$, one should long the portfolio at time t_0 (buy $\frac{a}{X_{t_0}}$ shares of X_t and sell $\frac{b}{Y_{t_0}}$ shares of Y_t) and realizes the gain when the portfolio value returns to its mean level. If $\Pi_{t_0} > \mathbb{E}(\Pi_t)$, then one should short the portfolio at time t_0 (sell $\frac{a}{X_{t_0}}$ shares of X_t and buy $\frac{b}{Y_{t_0}}$ shares of Y_t) and realizes the gain when the portfolio value returns to its mean level. Chan (2010) conducted an empirical study on the constituents of the Hang Seng Index in 2007. The result suggests that such a strategy can be profitable.

2.3 Modelling and Forecasting Realized Volatility

Trading volatilities is more complex than trading stocks. The portfolio used to trade volatility is affected by serval factors including: strike price, expiration time, implied volatility and risk-free rate. In the analysis below, we found that the difference between the implied volatility and realized volatiltiy affects the portfolio P/L. The cointegration criterion does not consider this aspect, however. To remedy such a problem, modelling and forecasting realized volatility are necessary. Andersen, Bollerslev, Diebold and Labys (2003) introduced a *High-Frequency Realized Volatility Estimation*. The detail is as follow.

Let $\{r_{n,t}, n = 1, ..., N, t = 1, \ldots, T\}$ be the log-return of an underlying asset at the n-th data point (equally-spaced interval) of the t-th day. Assume that $r_{n,t}$ are i.i.d. with zero mean and constant variance σ^2/N. Define *High-Frequency Realized Volatility Estimation* as

$$s_t^2 = \sum_{n=1}^{N} r_{n,t}^2, \text{ for } t = 1, 2, \ldots, T.$$

Then,

$$\mathbb{E}(s_t^2) = \mathbb{E}([\sum_{n=1}^{N} r_{n,t}]^2) = \mathbb{E}(\sum_{n=1}^{N} r_{n,t}^2) = \sum_{n=1}^{N} \frac{\sigma^2}{N} = \sigma^2.$$

The quantity s_t^2 is an unbiased estimate of σ^2, based on the information on the t-th day only (avoids the problem of lagging). Estimates of realized volatilities during the whole day are required, and thus, this estimation takes advantage of the market which are traded 24 hours/day (such as FX market). In order to obtain a smoother estimate of s_t^2 over time, Andersen, Bollerslev, Diebold and Labys (2003) recommended a data frequency of 30-minute intervals, i.e. $N = 48$.

Meanwhile, Andersen, Bollerslev, Diebold and Labys (2003) compared the one-day-ahead and ten-day-ahead of estimates based on different long-memory time series models on realized volatilities and found that the best model in terms of R^2 is ARFIMA model on the log of realized volatilities $y_t = \log s_t$:

$$\Phi(L)(1-L)^d(y_t - \mu) = \epsilon_t,$$

where d is the order of integration and ϵ_t is a vector white noise process.

Because of the advantages of High Frequency Estimation (avoids the problem of lagging and benefits from FX market), High Frequency Estimation and ARFIMA model are used to model and forecast the realized volatility in the empircal study.

Chapter 3

Cointegration Pairs Trading Strategy On Derivatives

Implied volatility is the assessment of the future variation of an underlying asset. Even though prices between two underlying assets may not be cointegrated, the volatility of these two underlying assets may still exhibit a cointegration phenomenon because they may be affected by the same exogenous event so that people have the same perspective to the variations of the underlying assets. It is therefore possible that there exist cointegration pairs in the implied volatilities. By the Johansen test, we found that strong cointegration relationships exist among the implied volatilities of Hang Seng Index constituents options and Foreign Exchange options from 2009 to 2010. By virtue of this observation, a deviation from the mean level of the cointergration pairs can be captured to generate possible profits.

3.1 Trading On Implied Volatility

The cointegration trading strategy is applied to the volatility instead of the stock price. In the current market, no products trade the volatility of an individual underlying asset directly. Alternatively, there are several methods to implicitly trade volatility by derivatives. One of the most commonly used methods is trading a straddle. For simplicity, at-the-money straddles are considered, as it is mainly driven by the volatility of the underlying asset, while it is relatively immued to the price movement of the underlying asset.

Definition A long (short) straddle is long (short) a call option and a put option with the same strike price and the expiration date.

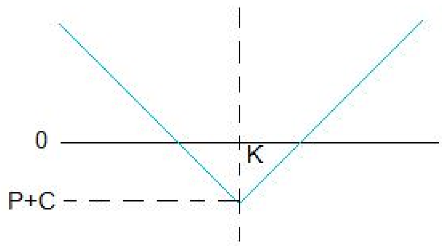

Figure 3.1: Payoff function of longing a straddle.

Recall that the payoff function of a straddle (Figure 3.1) is equal to the absolute value of the moneyness of the option (stock price - strike) at expiration. If we buy an at-the-money straddle at time t, then we can make a profit as long as the change of stock price in the future is large enough, i.e. the volatility is large. Longing (shorting) a straddle is equivalent to longing (shorting) the volatility until expiration. Furthermore, in Appendix A, it is shown that the approximated price of an at-the-money (ATM) straddle (ST_t) is given by

$$ST_t = C_t + P_t = \sqrt{\frac{2}{\pi}} S_t \sigma \sqrt{T-t} + o(S_t \sigma^2 (T-t)), \tag{3.1}$$

which is approximately linear in both S_t and σ, when the expiration duration $(T-t)$ is small (say less than one year). From equation (3.1), we observe that trading an at-the-money straddle is like trading the corresponding volatilities.

Hence, similar to the trading strategy in the stock market, we introduce the notion of cointegration in the derivative market by trading at-the-money straddle (trading the cointegration pairs of implied volatilities).

3.2 Cointegration Trading Strategy

To find out when we should initiate a trade, suppose that the trading signal is

$$TS_t = a\sigma_t^x - b\sigma_t^y \sim I(0),$$

where σ_t^x and σ_t^y are implied volatilites of underlying assets x and y at time t, a and b are the coefficients of the cointegration pairs ($0 < a, b \leq 1$) and the z-score of the trading signal is

$$Z_t = \tfrac{TS_t - \mu}{\sigma},$$

where μ and σ are the mean and standard deviation of the trading signal TS_t.

In the preceding section, we found that option price and implied volatilities are concordant. Hence, if TS_t (or Z_t) is far below the mean level, then the option price of a straddle for x is too cheap comparing to that of y in the historical term. We should long a portfolio that is able to replicate TS_t, i.e. a long position on ST_t^x and a short position on ST_t^y. We expect it to return to the historical mean level. On the other hand, if TS_t (or Z_t) is much higher than the mean level, then a portfolio which replicates TS_t should be sold, i.e. a long position on ST_t^y and a short position on ST_t^x.

Define the portfolio Π_t consisting of these two straddles,

$$\Pi_t = A \times (C_t^x + P_t^x) - B \times (C_t^y + P_t^y), \quad t \in (t_0, T), \tag{3.2}$$

where C_t^x, P_t^x, C_t^y, P_t^y are the price of ATM options at time t, T is the option expiry and the values of A and B will be determined later.

3.3 Greek Letters

We are concerned if the strategy makes money. But it is difficult to determine the future value of the portfolio. On the other hand, the expected value of the portfolio can be estimated. If cointegration is found in the implied volatility of two specific FX options, then one can show that the expected return of the portfolio is greater than 0 under certain conditions. The details are as follows.

3.3.1 Requirements of the Trade

To approximate the P/L of the portfolio $\triangle\Pi_t = \Pi_t - \Pi_{t_0}$ at time t, where $t \in (t_0, t_0+\varepsilon)$, define $\triangle t = t - t_0$, $\triangle S_t^x = S_t^x - S_{t_0}^x$, $\triangle S_t^y = S_t^y - S_{t_0}^y$, $\triangle\sigma_t^x = \sigma_t^x - \sigma_{t_0}^x$ and $\triangle\sigma_t^y = \sigma_t^y - \sigma_{t_0}^y$. Based on the Taylor expansion, the change in portfolio $\triangle\Pi_t$ can be approximated by

$$\begin{aligned}\triangle\Pi_t \approx \Theta \times \triangle t + \triangle_x \times \triangle S_t^x + \triangle_y \times \triangle S_t^y + \frac{1}{2}\Gamma_{xx} \times (\triangle S_t^x)^2 \\ + \frac{1}{2}\Gamma_{yy} \times (\triangle S_t^y)^2 + \nu_x \times \triangle\sigma_t^x + \nu_y \times \triangle\sigma_t^y,\end{aligned} \tag{3.3}$$

where $\Theta = \frac{\partial\Pi_t}{\partial t}$, $\triangle_x = \frac{\partial\Pi_t}{\partial S_t^x}$, $\triangle_y = \frac{\partial\Pi_t}{\partial S_t^y}$, $\Gamma_{xx} = \frac{\partial^2\Pi_t}{\partial(S_t^x)^2}$, $\Gamma_{yy} = \frac{\partial^2\Pi_t}{\partial(S_t^y)^2}$, $\nu_x = \frac{\partial\Pi_t}{\partial\sigma_t^x}$, and $\nu_y = \frac{\partial\Pi_t}{\partial\sigma_t^y}$.

Note that the last two terms of (3.3), namely the P/L due to the ***vegas*** of x and y, are highly related to the trading signal TS_t mentioned in the previous section. Vega measures the sensitivity to the volatility, the quantity that we want to trade, and therefore, we aim to mainly trade vega. We select options with dominating terms for vegas in (3.3), while the P/L impact of the other five terms in (3.3) should either be small or have a positive impact to the portfolio P/L.

The impact of vega in the trade can be maximized while others (delta, gamma, theta) either are negligible or have a positive impact if the trade fulfills the following requirements:

1. ***S*hort-term period of trading;**

   Delta of an at-the-money straddle is close to zero, but increases to 1 as the moneyness increases, and decreases to -1 when the moneyness decreases. Therefore, the delta of P/L could have a significant impact on $\triangle\Pi_t$ as time goes on. In practice, one can minimize its P/L impact based on dynamic hedging in delta. By the end of each trading session (or each week), we rebalance the deltas ($\triangle_x$ and $\triangle_y$) of the portfolio to zero, based on longing or shorting the underlying S_t^x and S_t^y. However, dynamic hedging could be costly due to the transaction cost incurred. Alternatively, one can trade the straddle portfolio for a shorter period of time to reduce the impact of the volatility of moneyness to the delta of P/L.

2. ***U*sing long-dated option;**

   Gamma of longing an at-the-money straddle is maximized, but is diminished to zero when the moneyness increases. In addition, a long-dated straddle would have lower gamma (and higher vega) than a short-dated straddle because of the $\sqrt{T-t}$ term in the denominator of gamma (but at the numerator of vega). Thus, we focus on long-dated straddles in the trading strategy to minimize the impact of gamma.

3. ***T*he mean of the trading signal is negative (positive), the position is initiated when the trading signal is too low (high). Otherwise, the position should not be initiated.**

   In the derivation below, it is shown that at time t,

$$\Theta = \frac{\partial \Pi_t}{\partial t} \approx \begin{cases} -\frac{TS_t}{2\sqrt{T-t}}, & \text{long the portfolio,} \\ \frac{TS_t}{2\sqrt{T-t}}, & \text{short the portfolio,} \end{cases}$$

where $TS_t = Z_t\sigma + \mu$, μ and σ are the mean and standard deviation of the trading signal TS_t.

In the case of longing the portfolio, the trade is initiated when TS_t is too low (Z_t is too negative). If μ is negative, TS_t remains negative during the trading period so that theta is positive all the time. Otherwise, if μ is positive, TS_t is positive when TS_t is close to the mean level so that there exists a loss from theta. To ensure a positive profit from theta, the trade requires this condition.

If these requirements can be met, the trade will be dominated by vega.

3.3.2 Approximation of the Expected P/L

Withou loss of generality, assume that the mean of TS_t is smaller than 0 and the trading signal TS_{t_0} is too low compared to historical terms (with z-score of say -2.0) in the derivation below. In that case, we would long A units of straddle ST_t^x and short B units of straddle ST_t^y at time $t = t_0$ to capture the mean-reverting property. On top of the assumptions mentioned in section 3.3.1, if we further assume that the trade will be implemented within a short-term period starting at t_0, the Greek Letters can be approximated. Let the units to be long for stock x and y in (3.2) be $A = \sqrt{\frac{\pi}{2}}\frac{a}{S_{t_0}^x}$ and $B = \sqrt{\frac{\pi}{2}}\frac{b}{S_{t_0}^y}$, where a and b are the coefficients of the cointegration and let

$$\begin{aligned}
&(Vega)\ I = \nu_x \times \triangle\sigma_t^x + \nu_y \times \triangle\sigma_t^y, \\
&(Delta)\ II = \triangle_x \times \triangle S_t^x + \triangle_y \times \triangle S_t^y, \\
&(Gamma)\ III = \frac{1}{2}\Gamma_{xx} \times (\triangle S_t^x)^2 + \frac{1}{2}\Gamma_{yy} \times (\triangle S_t^y)^2, \\
&(Theta)\ IV = \Theta \times \triangle t.
\end{aligned}$$

Then $\triangle\Pi_t = I + II + III + IV$ and we are concerned if $\mathbb{E}(\triangle\Pi_t) > 0$.

We apply Taylor expansion of $\Phi(d_1)$ and $\Phi(d_2)$ at $d_1 = d_2 = 0$:

$$\Phi(d_1) = \tfrac{1}{2} + \tfrac{\sigma\sqrt{T-t}}{\sqrt{2\pi}} + o(d_1^2),\ \Phi(d_2) = \tfrac{1}{2} - \tfrac{\sigma\sqrt{T-t}}{\sqrt{2\pi}} + o(d_2^2),$$

and the Geometric Brownian Motion S_t satisfies $\frac{dS_t}{S_t} = rdt + \sigma^r dW_t$, where the realized volatility σ^r is the actual variation of the underlying asset, in the approximation. The expected values of I, II, III and IV can be approximated as follows:

$$\begin{aligned}
&\mathbb{E}(I) \approx \sqrt{T-t_0}(\mathbb{E}(TS_t) - TS_{t_0}),\\
&\mathbb{E}(II) \approx \frac{\Delta t\sqrt{T-t_0}r_{t_0}TS_{t_0}}{2},\\
&\mathbb{E}(III) \approx \frac{\Delta t}{2\sqrt{T-t_0}}(\frac{a\mathbb{E}(\sigma_x^r)^2}{\sigma_{t_0}^x} - \frac{b\mathbb{E}(\sigma_y^r)^2}{\sigma_{t_0}^y}),\\
&\mathbb{E}(IV) \approx -\frac{\Delta t \times TS_{t_0}}{2\sqrt{T-t_0}},
\end{aligned}$$

where $\Delta t = t - t_0$, r_{t_0} is the risk-free rate, $(\sigma_x^r)^2$ and $(\sigma_y^r)^2$ are the average squared annual realized volatilities, defined as

$$(\sigma_x^r)^2 = \tfrac{1}{t-t_0}\int_{t_0}^t (\sigma_{x,t}^r)^2\ dt,\ (\sigma_y^r)^2 = \tfrac{1}{t-t_0}\int_{t_0}^t (\sigma_{y,t}^r)^2\ dt.$$

Note that part I (Vega Part) is the main profit to be captured in the trading strategy. Should the trading signal reverts back to its mean value, then $\mathbb{E}(I) > 0$. In general, if the speed of mean-reversion of the trading signal is fast enough, then the trade will be closed for profit taking before the option expires. Furthermore, we can prove that $\mathbb{E}(\Delta\Pi_t) > 0$ under certain conditions.

Proposition 3.3.1. *If the trade is in a short-time period,* $\mathbb{E}(II) + \mathbb{E}(IV) > 0$.

Proof. In section 3.3.1, we know that theta (IV) is a positive net carry in the trade. Compared the expected value of theta (IV) with that of delta (II),

$$\left|\tfrac{\mathbb{E}(IV)}{\mathbb{E}(II)}\right| = \tfrac{1}{(T-t_0)r_{t_0}}.$$

Because $T - t_0$ is smaller than one (year) and r_{t_0} is very small (0%-5% in most countries), $|\mathbb{E}(II)| << |\mathbb{E}(IV)|$. Hence,

$$\mathbb{E}(II) + \mathbb{E}(IV) \geq |\mathbb{E}(IV)| - |\mathbb{E}(II)| > 0,$$

i.e. the loss due to delta is likely to be compensated by the positive net carry from IV. $\square$

Proposition 3.3.2. *Assume that*

1. *Annualized volatilities (include implied volatility and realized volatility) of the underlying assets are smaller than 80% ;*

2. $\frac{1}{c}\sigma_{t_0}^i < \sigma_i^r < c\sigma_{t_0}^i$*, for* $i = x, y$*,* $c > 1$*;*

3. $\sigma(TS_t) > 1$.

Then $\mathbb{E}(I) + \mathbb{E}(III) > 0$.

From the historical data, most of the volatilities are smaller than 80% unless the company bankrupts. Furthermore, in the short-term period, the realized volatility should not dramatically differ from its corresponding implied volatility, otherwise there exists an obvious arbitrage which will be absorbed by the market immediately. Thus, we believe that assumptions (1) and (2) are reasonable. Based on these two assumptions, the boundary of $\mathbb{E}(III)$ can be deduced. On the other hand, the trade is initiated when TS_{t_0} is too negative (postive), say over two standard deviations from the mean level, i.e. $|\mathbb{E}(TS_t) - TS_{t_0}| > 2\sigma(TS_t)$. Assumption (3) is used to ensure that $\mathbb{E}(TS_t) - TS_{t_0}$ is large enough. In the empirical study, most pairs fulfill this assumption.

Proof. Under assumptions (1) and (2), one can show that

$$|\mathbb{E}(III)| = |\tfrac{\Delta t}{2\sqrt{T-t_0}}(\tfrac{a\mathbb{E}(\sigma_x^r)^2}{\sigma_{t_0}^x} - \tfrac{b\mathbb{E}(\sigma_y^r)^2}{\sigma_{t_0}^y})| \leq \tfrac{0.4c^2\triangle t}{\sqrt{T-t_0}}.$$

Recall that the trade requires long dated options and trades in a short-term period. To strike a balance between the trading requirements and options liquidity, the maturity of the options should be at least three months and the trade should not last over one month. i.e.

$$\frac{T-t_0}{\Delta t} \geq 3.$$

Vega is the main profit in the trade. Compared the expected value of gamma with that of vega,

$$\left|\frac{\mathbb{E}(I)}{\mathbb{E}(III)}\right| = \left|\frac{\sqrt{T-t_0}(\mathbb{E}(TS_t)-TS_{t_0})}{\frac{\Delta t}{2\sqrt{T-t_0}}(\frac{a\mathbb{E}(\sigma_x^r)^2}{\sigma_{t_0}^x}-\frac{b\mathbb{E}(\sigma_y^r)^2}{\sigma_{t_0}^y})}\right| \geq \frac{7.5}{c^2}(\mathbb{E}(TS_t) - TS_{t_0}).$$

The trade is initiated when TS_{t_0} is too negative comparing to the mean level ($\mathbb{E}(TS_t)$), and c is close to 1. Hence,

$$\frac{7.5}{c^2}(\mathbb{E}(TS_t) - TS_{t_0}) > \frac{15}{c^2}\sigma(TS_t) >> 1,$$

and therefore

$$\mathbb{E}(I) + \mathbb{E}(III) \geq |\mathbb{E}(I)| - |\mathbb{E}(III)| > 0.$$

$\square$

In conclusion, if the implied volatilities of the underlying assets are not too high and deviate too far from the corresponding realized volatilities, and if $\mathbb{E}(TS_t) - TS_{t_0}$ is large enough, then

$$\mathbb{E}(\Delta\Pi_t) = \mathbb{E}(I) + \mathbb{E}(II) + \mathbb{E}(III) + \mathbb{E}(IV) > 0.$$

3.4 Foreign Exchange Options

Based on the aforementioned analysis, we select a pair of cointegrated implied volatilities to examine the day-by-day performance of the portfolio. Derivatives of Foreign Exchange (FX) options are the best candidates for our analysis because of the following reasons:

1. FX options are traded in both exchanges and over-the-counter (OTC) market with high liquidity.

2. Tick-by-tick data of the spot exchange rates are available in some public database (e.g. http://ratedata.gaincapital.com/).

3. FX options are European options which are concordant with our analysis.

For our trading strategy to be effective, the following criteria are set when selecting and trading the FX options:

(i) Short-term Period of Trading

Because a short-term period of trading is assumed in the analysis before, the speed of mean-reversion of the trading signal should be fast. In our analysis, two year of historical data were used to identify the cointegration relationship, with the data for the subsequent three months using for trade initialization. This measure makes the trading signal more stable and the speed of the mean-reverting faster.

(ii). Long-dated options

Balancing the liquidity of the options and the requirement, we use the options with 3-month expiry.

3.4.1 Cointegration Pairs

Recall that our trading requirement is trading in a short-term period and using long-dated option. To strike a balance between the trading requirements and options liquidity, FX options of 30 currency pairs (e.g. EUR/USD) with 3-month expiry for all the strikes from 2009Q4 to 2011Q4 are examined to illustrate the trading strategy. The data from 2009Q4–2011Q3 is used to identify possible cointegration in any two currency pairs by Johansen test, and the cointegration pairs identified will be used as the trading signals for 2011Q4.

According to the results of Johansen test, the implied volatilities of GBPNZD and GBPUSD are significantly cointegrated with the parameters $a = 0.6762$

and $b = 1.0000$, i.e. the trading signal is

$$TS = 0.6762 \times \sigma_{GBPNZD} - \sigma_{GBPUSD}.$$

The details of two implied volatilities of two currencies during 2009Q4–2011Q3 are given in Table 3.1 and Figure 3.2.

From Table 3.1, after the linear combination, the half-life [1] of the trading signal, which is used to measure the speed of the mean-reversion, is smaller than the half-life of two individual currencies. Meanwhile, from Figure 3.2, prior to the orange line (vertical line on Sep 30, 2011), the historical z-scores moves stably around the mean level. We believe the z-score will keep moving in this pattern in the next three months. If there exists a TS_t deviated from its mean, it can be captured to make a profit.

The average of the trading signal is positive, therefore, the position should be initiated when the trading signal is too positive. The z-score of the TS_t on 03/10/2011 is above 2, which is the threshold we set, and the z-score is expected to return to the mean level. Hence, we initiate a position with the three-month expiry options, i.e. the options expire on 03/01/2012.

	GBPNZD Implied (in%)	GBPUSD Implied (in%)	TS	z-score
Average	13.14	11.43	5.41	0.00
Stdev	1.49	1.61	0.95	1.00
Minimum	10.69	8.78	3.37	-2.13
Maximum	17.75	16.35	8.46	3.19
Half-Life(Days)	19.43	39.46	**14.63**	**14.63**

Table 3.1: Details of the implied volatilities.

[1] Half-life is used to measure the duration that a quantity falls in half of its value at the beginning. It can be defined as T such that $E(X_{t_0+T}) = \frac{1}{2}X_{t_0}$. Assume that X_t follow AR(1) model $X_t = \phi X_{t-1} + \varepsilon_t$, then $E(X_{t+T}) = \phi^T X_t = \frac{1}{2}X_t \Rightarrow T = -\frac{\log 2}{\log \phi}$.

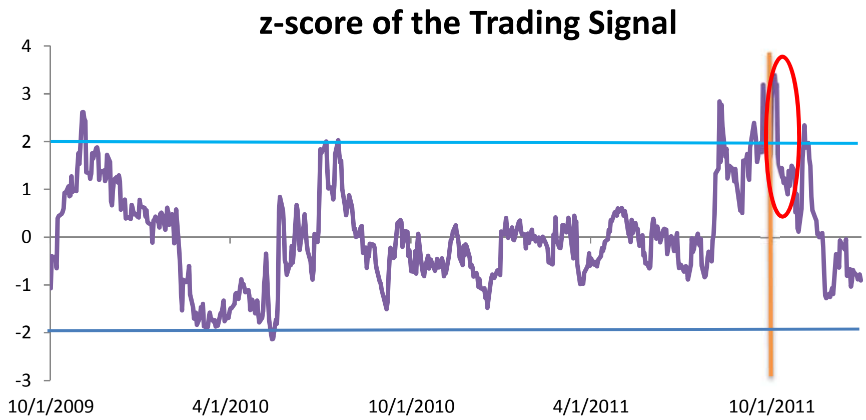

Figure 3.2: z-score of the trading signal.

3.4.2 Trading Process

The z-score of the Trading Signal on 03/10/2011 is 3.39. It means that, relative to the historical data, the current implied volatility of GBPNZD is too high compared with that of GBPUSD. Therefore, the price of the portfolio is too high and we should sell them. According to the definition of the units to be long or short for the underlying assets (A and B) as above, we calculate how many shares of GBPNZD straddle we should sell and how many shares of GBPUSD straddle we should buy. To clearly identify the effect of the Greek Letters, introduce the scale of 100,000. More details of the setting of the trade can be found in Appendix B.

The trading signal gradually falls back to the mean level since 03/10/2011. On 07/10/2011, the z-score of the Trading Signal is 1.52, which is smaller than half of that on 03/10/2011 and within the threshold we set. Therefore, we close the portfolio on that day. Figure 3.3 illustrates the performance of this strategy.

From Figure 3.3, the cumulative P/L of the portfolio is led by the vega at first and the other Greek Letters are very small. After a few days, the Trading Signal

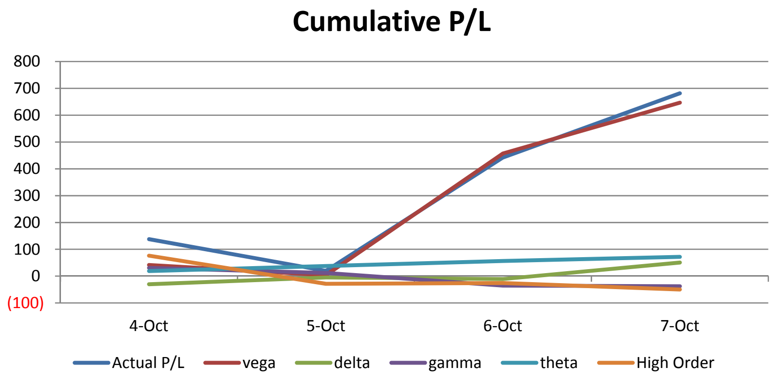

Figure 3.3: Change of P/L due to Greek Letters.

returns to its mean level and the position is closed on 07/10/2011. The value of the portfolio is -USD3,465 on 03/10/2011, and then the value increases to -USD2,783 on 07/10/2011 (the details of the portfolio can be found in Appendix B). Finally, we gain +USD682 by this strategy.

3.4.3 More Examples

In this section, more trades based on the proposed strategy are explored. We continued to use the FX options. We first consolidated the data to estimate the implied volatilities of at-the-money straddle for each currency pair. Then we identify possible cointegration in any two currency pairs, based on the data from 2009Q1-2010Q4. The cointegration pairs identified will be used as trading signals for 2011Q1. We repeated the same procedure to identify possible cointegration pairs during 2009Q2-2011Q1, 2009Q3-2011Q2 and 2009Q4-2011Q3, which would serve as the trading signals for 2011Q2, 2011Q3 and 2011Q4.

In order for the trade to be equally weighted in terms of P/L, the notional amount of each trade will be adjusted so that the expected P/L obtained from vega is USD1,000. For example, if the expected profit from vega is USD400 in a certain trade, then the notional amount should be increased 2.5 times so that the expected profit from vega is USD1,000.

On the other hand, if the trading signal does not return to the mean level, we cannot close the position. If the trading period is too long or a huge loss is incurred, it is possible that there is a loss. Hence, criteria for stopping a trade should be set up.

- **Profit Taking**

  We initiate the trade at time t_0 when there is an outlier in the trading signal TS_t and close it when TS_t reverts back to the value as $\frac{1}{2}TS_{t_0}$, where t_0 is the time we initiated the trade.

- **Stop Loss Limit**

  Stop loss limit directly reflects the profit or loss of the trade. Our expected profit from vega is USD1,000, and therefore, stop loss limit is set as $-$USD1,000, i.e. if the loss in the trade is over $-$USD1,000, the trade will be closed at the end of that day (because we only have the historical implied volatility at the end of the day).

- **Trade Unwind**

  The speed of the mean-reverting should be fast and the trading period does not last too long. In general, the speed of the mean-reverting in the historical data is around 12-13 days. Hence, we set the maximum of the trading period as 25, i.e. if the trade lasts over 25 days, we assume that the trading signal will not return to the mean level and the position is closed immediately.

After all settings have been made, the empirical study can be conducted. Based on the data, there are 118 potential trades within the trading period. However, most of the trades are highly correlated with each other. For instance, during the period Jan 3, 2011 to Jan 27, 2011, 6 out of 7 cointegration pairs involved the currency pair of EURNZD. To avoid the P/L being magnified by the strong correlations, we would not initiate a position with one of the currency pairs

Figure 3.4: Criterion for closing the trade.

overlapping with any of the existing trades. After filtering the correlated trades, the performance of the cointergration strategy on FX options is shown in Table 3.2.

From this table, we see that the strategy generated positive P/L in 2011Q1, 2011Q2 and 2011Q4. P/L in 2011Q3, however, incurred huge losses as the trades failed to revert back to their historical mean levels. The possible reason is that the foreign exchange rate is sensitive to the policy and the economy of the country. These factors are difficult to capture from the historical data. Obviously, the global economy was unstable during 2011Q3 so that the realized volatility of the FX options has changed to different patterns, which may drive the implied volatility pattern away from the expected ones. It violates the assumptions made in estimating the gamma of P/L. Hence, the pevious strategy does not work when the volatility encounters exogenous factors. Besides the cointegration criterion, additional criteria in terms of the difference between the implied volatility and realized volatility for trading FX option are necessary. In the next chapter, we will introduce the Implied-Realized criterion and the Vega-Gamma criterion to remedy such problems.

Product 1	a	Product 2	b	Signal	Start	End	Profit
CADJPY	0.6428	EURNZD	1.0000	-2.6940	1/3/11	1/21/11	-1,545
GBPJPY	0.5870	NZDCAD	1.0000	-2.0715	1/27/11	2/21/11	772
EURJPY	0.7251	EURNZD	1.0000	-2.1267	2/22/11	3/4/11	-1,610
NZDCAD	1.0000	NZDJPY	0.4415	2.1924	2/24/11	3/10/11	-1,098
AUDJPY	0.5300	NZDCHF	1.0000	-2.0530	2/24/11	3/14/11	1,964
GBPNZD	1.0000	USDCAD	0.9726	2.0069	3/8/11	3/16/11	1,733
GBPUSD	0.9411	NZDCAD	1.0000	-2.3959	3/15/11	3/18/11	1,759
USDCHF	1.0000	USDSEK	0.2250	2.5874	5/6/11	5/9/11	1,568
AUDCAD	1.0000	NZDCHF	0.8077	-2.0895	6/15/11	7/8/11	-278
CADCHF	1.0000	EURCAD	0.7780	2.1004	7/7/11	7/15/11	-1,232
CADJPY	0.3954	NZDCHF	1.0000	-2.6521	7/12/11	7/21/11	-1,419
AUDCHF	1.0000	EURAUD	0.9516	2.0714	7/15/11	7/21/11	-1,047
CADCHF	1.0000	EURUSD	0.6740	2.1752	7/25/11	8/2/11	-2,717
EURAUD	0.6977	NZDCHF	1.0000	-2.0884	7/27/11	8/4/11	-2,498
AUDCHF	1.0000	AUDUSD	0.3987	2.0745	8/2/11	8/4/11	-1,029
USDCHF	1.0000	USDSEK	0.1757	-2.8074	8/2/11	8/5/11	-1,687
CADCHF	1.0000	GBPCHF	0.6654	2.4183	8/5/11	8/9/11	-1,253
GBPUSD	0.4198	NZDCAD	1.0000	-2.4273	8/5/11	8/11/11	-1,031
AUDCHF	1.0000	USDCHF	0.4939	2.1452	8/8/11	8/9/11	-1,339
GBPCAD	0.8110	GBPNZD	1.0000	-3.2243	8/9/11	8/10/11	446
CADJPY	0.3096	EURNZD	1.0000	-2.9409	8/9/11	8/23/11	1,458
AUDCAD	0.7553	EURNZD	1.0000	-2.3284	9/2/11	9/27/11	1,727
AUDJPY	1.0000	NZDJPY	0.9958	-2.8884	9/8/11	9/9/11	2,067
EURAUD	0.2479	NZDCAD	1.0000	-2.0820	9/23/11	9/28/11	863
EURNZD	1.0000	GBPJPY	0.2413	2.6896	10/3/11	10/14/11	886
GBPNZD	1.0000	USDJPY	0.9827	3.4003	10/3/11	10/25/11	488
GBPUSD	0.3897	NZDCAD	1.0000	-2.4143	10/3/11	10/6/11	1,666
CADJPY	0.5754	GBPAUD	1.0000	-2.1848	10/4/11	10/17/11	958
						Total Profit	-1,427

Table 3.2: Cointegration Strategy Performance.

Chapter 4

Further Trading Strategies

In Chapter 3, we introduced the cointegration strategy on derivatives of a pair of foreign exchange rates, with the P/L of the portfolio comprising of four parts: **vega**, **delta**, **gamma** and **theta**. A loss is incurred when the realized volatility and implied volatility are affected by some non-anticipated factors. Under this situation, assumptions made in estimating the gamma of P/L is violated and the P/L of gamma, which is sensitive to the realized volatility, becomes an issue. In this chapter, we introduced two criteria to the trading signal TS_t from cointegration, which enable us to take into account the magnitudes of the realized volatilities of the two exchange rates. These criteria choose the trades without a huge expected loss of gamma and enhances the P/L of trading FX options.

4.1 Estimation of Realized Volatility

Recall that $\mathbb{E}(III) = \frac{\Delta t}{2\sqrt{T-t_0}}(\frac{a\mathbb{E}(\sigma_x^r)^2}{\sigma_{t_0}^x} - \frac{b\mathbb{E}(\sigma_y^r)^2}{\sigma_{t_0}^y})$, where $(\sigma_x^r)^2$ and $(\sigma_y^r)^2$ are the average squared future annual realized volatilities for longing a portfolio. The future realized volatilities cannot be observed at the starting time of the trade. To determine the value of $\mathbb{E}(III)$ and to decide whether we should initiate a trade, modelling and forecasting realized volatility are needed. The following models are adopted to model and forecast realized volatility.

Modelling: High-Frequency Estimation s_t^2:

$$s_t^2 = \sum_{n=1}^{N} r_{n,t}^2, \text{ for } t = 1, 2, \ldots, 48, \text{ i.e. 30-minute intervals.}$$

Forecasting: ARFIMA model:

$$\Phi(L)(1-L)^d(y_t - \mu) = \epsilon_t,$$

where $y_t = \log s_t$, d is the order of integration and ϵ_t is a vector white noise process.

Becasue $(\sigma^r)^2 = \frac{1}{t-t_0}\int_{t_0}^{t}(\sigma_s^r)^2\, ds$, t should be determined before forecasting. As the trade will not last too long, we believe that 25 days would be a suitable period to forecast the realized volatility $(\sigma^r)^2$, which is equal to the average of annual realized volatility for the next 25 days, i.e.

$$(\tilde{\sigma^r})^2 \approx \frac{1}{25}\sum_{t=1}^{25}(\tilde{\sigma_t^r})^2.$$

4.2 Implied-Realized Criterion

In the previous analysis, we show that the quantity III is one of the major factors in contributing the variability of the P/L. Besides the cointegration signal, we would like to add an additional criterion to control the quantity III. When the mean of trading signal TS is smaller than 0,

$$\mathbb{E}(III) = \frac{\Delta t}{2\sqrt{T-t_0}}\left(\frac{a\mathbb{E}(\sigma_x^r)^2}{\sigma_{t_0}^x} - \frac{b\mathbb{E}(\sigma_y^r)^2}{\sigma_{t_0}^y}\right).$$

Otherwise,

$$\mathbb{E}(III) = \frac{\triangle t}{2\sqrt{T-t_0}}\left(\frac{b\mathbb{E}(\sigma_y^r)^2}{\sigma_{t_0}^y} - \frac{a\mathbb{E}(\sigma_x^r)^2}{\sigma_{t_0}^x}\right).$$

Obviously, if $\mathbb{E}(III)$ is too negative, i.e., the difference between $\frac{a\mathbb{E}(\sigma_x^r)^2}{\sigma_{t_0}^x}$ and $\frac{b\mathbb{E}(\sigma_y^r)^2}{\sigma_{t_0}^y}$ is far from the corresponding expected value, then the trade should not be initiated. Hence, we define the **Implied-Realized Criterion** as the ratio of this two terms:

$$K = \frac{a\mathbb{E}(\sigma_x^r)^2}{\sigma_{t_0}^x} / \frac{b\mathbb{E}(\sigma_y^r)^2}{\sigma_{t_0}^y} \begin{cases} \geq d, & \text{if } TS < 0, \\ \leq u, & \text{if } TS > 0, \end{cases} \tag{4.1}$$

where d and u are to be determined.

When $TS < 0$ and $K \geq 1$,

$$\tfrac{a\mathbb{E}(\sigma_x^r)^2}{\sigma_{t_0}^x} \geq \tfrac{b\mathbb{E}(\sigma_y^r)^2}{\sigma_{t_0}^y} \Rightarrow \mathbb{E}(III) > 0.$$

This does not often happen. Furthermore, we do not expect there is a profit from gamma. A small loss in gamma can be tolerated. Thus, it is more desirable to use a softer limit, such as $d = 0.7$. It would not make the trade suffer a big loss becasue the positive vega covers part of the losses of gamma. Similarly, u can be set as 1.3. The $K - value$ in the **Implied-Realized Criterion** can be calculated using $(\tilde{\sigma^r})^2$.

Now, we use two examples to show how the criterion works. According to the cointegration signal, two potential trades are identified as follows:

Product 1	a	Product 2	b	TS	Start	$K - value$
CADJPY	0.6428	EURNZD	1.0000	-2.6940	1/3/11	0.5311
NZDCAD	1.0000	NZDJPY	0.4415	2.1924	2/24/11	1.9376

TS in the first trade is smaller than 0 and the $K - value$ is 0.5311, which is smaller than the limit d $(= 0.7)$. This trade should not be initiated. Similarly, in the second trade, TS is greater than 0. Whether the trade is initiated depends on whether $K - value$ is smaller than u $(= 1.3)$. $K - value$ in the second trade is 1.9376, which is greater than 1.3. Hence, this trade should not be initiated. This criterion is used to eliminate the trade with a huge expected loss from gamma. We use the data in Section 3.4.3 to examine the performance of this

Implied-Realized Criterion. The performance is shown in Table 4.1.

The strategy with the **Implied-Realized Criterion** is much better than the performance of the original strategy. By imposing the **Implied-Realized Criterion**, some trades with enormous losses were eliminated. Because the **Implied-Realized Criterion** only considers the P/L of gamma while ignores the impact from vega, this criterion is too strict so that part of the potential "good" trades cannot be initiated. In the next section, we will introduce another criterion to improve the P/L of the portfolio based on the trade-off between the vega and gamma P/L.

4.3 Gamma-Vega Criterion

The **Implied-Realized Criterion** introduced in the previous section only controls the gamma P/L and ignores the vega P/L. It is possible that gamma P/L can be compensated by vega P/L. To consider the portfolio P/L effectively, we use the ratio of vega and gamma as the **Gamma-Vega Criterion**, which is expressed as

$$K = |\frac{Vega}{Gamma}| = \frac{|\sqrt{T-t_0}(\mathbb{E}(TS_t) - TS_{t_0})|}{|\frac{\triangle t}{2\sqrt{T-t_0}}(\frac{a\mathbb{E}(\sigma_x^r)^2}{\sigma_{t_0}^x} - \frac{b\mathbb{E}(\sigma_y^r)^2}{\sigma_{t_0}^y})|} \geq d, \tag{4.2}$$

where d is to be determined.

If K is significantly greater than 1, the trade is more likely to be initiated. From the analysis in Chapter 3, if the trading signal TS shows the mean-reverting property, then there is a high probability that the ratios of vega P/L and gamma P/L are greater than 1. Hence, if $d = 1$, the **Gamma-Vega Criterion** will not have a significant effect in choosing a suitable trade. From the historical data, we found that $d = 11$ is a better lower bound. In this empirical study, we use $d = 11$ to be the lower bound for the **Gamma-Vega Criterion**.

Product 1	a	Product 2	b	TS	Start	End	K $(\sigma^2_{(1)})$	P/L (in USD)
CHFJPY	0.9063	EURNZD	1.0000	-2.10	1/14/11	2/8/11	1.03	248
NZDCAD	0.8365	USDJPY	1.0000	-2.97	3/17/11	3/21/11	1.59	457
NZDCAD	0.7957	USDCHF	1.0000	-2.52	5/6/11	5/31/11	1.06	3,415
AUDCAD	1.0000	NZDCHF	0.8077	-2.09	6/15/11	7/8/11	0.79	-278
AUDNZD	0.9516	USDCHF	1.0000	-2.39	8/3/11	8/5/11	0.91	-1,885
EURGBP	1.0000	GBPNZD	0.8435	-2.46	8/9/11	8/10/11	0.84	792
AUDCAD	0.7553	EURNZD	1.0000	-2.33	9/2/11	9/27/11	0.81	1,727
AUDJPY	1.0000	NZDJPY	0.9958	-2.89	9/8/11	9/9/11	0.88	2,067
AUDNZD	0.8166	EURNZD	1.0000	-2.11	10/3/11	10/12/11	0.75	1,143
					Total P/L (in USD)			7,686

Table 4.1: Strategy Performance (Implied-Realized criterion).

There are three unknown parameter in equation (4.2): $\mathbb{E}(TS_t)$, $\mathbb{E}(\sigma_x^r)^2$ and $\mathbb{E}(\sigma_y^r)^2$. The last two quantities can be estimated by $(\tilde{\sigma^r})^2$ in Section 4.1. To estimate $\mathbb{E}(TS_t)$ starting at TS_{t_0}, we apply the following time series models on the z-score of the trading signal Z_t, where $Z_t = \frac{TS_t - \mu}{\sigma}$ (μ and σ are the mean and standard deviation of the trading signal). We can then estimate $\mathbb{E}(TS_t) = \mathbb{E}(Z_t)\sigma + \mu \approx \frac{\sigma}{n}\sum_{i=1}^{n} \tilde{Z}_i + \mu$.

1. AR(1)

   Consider an AR(1) Model: $Z_t = aZ_{t-1} + \varepsilon_t$, where ε_t is white noise.

   Note that the trading signal is a stationary series, as it is constructed by a linear combination of cointergration pairs. In order to model Z_t, we use the simplest model AR(1) for forecasting. After 1000 simulations, the mean of the forecasting would be the estimation of $\mathbb{E}(Z_t)$.

2. Vasicek Model

   Consider the Vasicek Model: $dZ_t = a(b - Z_t)dt + \sigma dW_t$, where a is the speed of the mean-reversion and b is the mean of Z_t.

   Because the trading signal is supposed to be mean-reverting, Vasicek model, which includes the mean-reverting effect, is appropriate. The mean of the z-score of the trading signal is supposed to be 0. Hence, $b = 0$. The standard deviation of the historical trading signal is regarded as σ. a is equal to the coefficient of the regression of Z_n on Z_{n-1} minus 1. Under 1000 simulations, the mean of the forecasting by Vasicek model is the estimation of $\mathbb{E}(Z_t)$.

3. Deterministic Method

   In this method, we assume Z_t would mean-revert to some target values (e.g. 0 or $\frac{1}{2}Z_{t_0}$) at time t, and therefore the trades will be closed. In our analysis, we set $\mathbb{E}(Z_t) = 0$ for equation (4.2).

Then the P/L in each trade based on these three methods above are shown in Tables 4.2–4.4.

From these Tables, observe that the performance of the strategy with Vasicek model and AR model are similar, while the Deterministic method seems to be inferior in terms of generating the smallest P/L. On the other hand, because the **Gamma-Vega Criterion** accounts for the effect of vega, which can balance different aspects of the P/L in each trade, there are more trades initiated under the **Gamma-Vega Criterion** than the **Implied-Realized Criterion**. In the next section, we will compare the performance of these two criteria.

4.4 Summary

Trading strategy on FX options based on cointegration itself does not perform well. One of the main reasons is that, when a sharp increase in implied volatility is observed, the increase in implied volatility is likely to be driven by an even sharper increase in realized volatilities. Even though the cointegration relationship suggests that the implied volatility should mean-revert with its implied volatilities counterpart, in the short run, however, the cointegration relationship may break down. This situation is not uncommon, especially when major shifts in monetary policy or economy is observed/announced. Because of this, the **Implied-Realized Criterion** is necessary to ensure that the dislocation in implied volatilities observed in the cointegration signal is not driven by the realized volatilities. In addition, to increase the number of trades, the **Gamma-Vega Criterion** is also imposed to select more trades with smaller gamma P/L against vega P/L. In this chapter, two different criteria with different methods have been introduced. The comparison of their performance is summarized in Table 4.5.

From Table 4.5, observe that the strategy without any supplementary criterion

Product 1	a	Product 2	b	TS	Start	End	AR(1)	P/L (in USD)
CADJPY	0.6428	EURNZD	1.0000	-2.70	1/3/11	1/21/11	23.62	-1,545
GBPJPY	0.5870	NZDCAD	1.0000	-2.07	1/27/11	2/21/11	13.06	772
EURJPY	0.7251	EURNZD	1.0000	-2.13	2/22/11	3/4/11	23.97	-1,610
NZDCHF	1.0000	NZDJPY	0.5499	2.16	2/24/11	3/16/11	11.99	1,372
USDCAD	0.8321	USDJPY	1.0000	-2.04	3/14/11	3/15/11	25.97	-1,121
GBPUSD	0.9411	NZDCAD	1.0000	-2.40	3/15/11	3/18/11	22.99	1,759
NZDCAD	0.7957	USDCHF	1.0000	-2.52	5/6/11	5/31/11	278.86	3,415
AUDCAD	1.0000	NZDCHF	0.8077	-2.09	6/15/11	7/8/11	27.45	-278
AUDNZD	0.9516	USDCHF	1.0000	-2.39	8/3/11	8/5/11	23.59	-1,885
GBPCAD	0.8110	GBPNZD	1.0000	-3.22	8/9/11	8/10/11	13.61	446
AUDCAD	0.7553	EURNZD	1.0000	-2.33	9/2/11	9/27/11	30.72	1,727
AUDJPY	1.0000	NZDJPY	0.9958	-2.89	9/8/11	9/9/11	17.58	2,067
GBPNZD	1.0000	USDJPY	0.9827	3.40	10/3/11	10/25/11	12.48	488
EURNZD	1.0000	GBPCHF	0.4607	2.61	10/3/11	10/12/11	11.40	1,098
CADJPY	0.5754	GBPAUD	1.0000	-2.18	10/4/11	10/17/11	12.21	958
							Total P/L (in USD)	7,663

Table 4.2: Strategy Performance (AR(1)).

Product 1	a	Product 2	b	TS	Start	End	Vasicek	P/L (in USD)
CADJPY	0.6428	EURNZD	1.0000	-2.70	1/3/11	1/21/11	23.83	-1,545
GBPJPY	0.5870	NZDCAD	1.0000	-2.07	1/27/11	2/21/11	12.65	772
EURJPY	0.7251	EURNZD	1.0000	-2.13	2/22/11	3/4/11	23.45	-1,610
NZDCHF	1.0000	NZDJPY	0.5499	2.16	2/24/11	3/16/11	11.68	1,372
USDCAD	0.8321	USDJPY	1.0000	-2.04	3/14/11	3/15/11	19.11	-1,121
GBPUSD	0.9411	NZDCAD	1.0000	-2.40	3/15/11	3/18/11	21.90	1,759
NZDCAD	0.7957	USDCHF	1.0000	-2.52	5/6/11	5/31/11	274.01	3,415
AUDCAD	1.0000	NZDCHF	0.8077	-2.09	6/15/11	7/8/11	27.08	-278
AUDNZD	0.9516	USDCHF	1.0000	-2.39	8/3/11	8/5/11	20.96	-1,885
GBPCAD	0.8110	GBPNZD	1.0000	-3.22	8/9/11	8/10/11	13.82	446
AUDCAD	0.7553	EURNZD	1.0000	-2.33	9/2/11	9/27/11	30.61	1,727
AUDJPY	1.0000	NZDJPY	0.9958	-2.89	9/8/11	9/9/11	17.39	2,067
GBPNZD	1.0000	USDJPY	0.9827	3.40	10/3/11	10/25/11	12.26	488
EURNZD	1.0000	GBPCHF	0.4607	2.61	10/3/11	10/12/11	11.03	1,098
CADJPY	0.5754	GBPAUD	1.0000	-2.18	10/4/11	10/17/11	11.71	958
							Total P/L (in USD)	7,663

Table 4.3: Strategy Performance (Vasicek Model).

Product 1	a	Product 2	b	TS	Start	End	DE	P/L (in USD)
CADJPY	0.6428	EURNZD	1.0000	-2.69	1/3/11	1/21/11	23.83	-1,545
GBPJPY	0.5870	NZDCAD	1.0000	-2.07	1/27/11	2/21/11	12.74	772
EURJPY	0.7251	EURNZD	1.0000	-2.13	2/22/11	3/4/11	23.56	-1,610
NZDCAD	1.0000	NZDJPY	0.4415	2.19	2/24/11	3/10/11	11.12	-1,098
USDCAD	0.8321	USDJPY	1.0000	-2.04	3/14/11	3/15/11	24.38	-1,121
GBPUSD	0.9411	NZDCAD	1.0000	-2.40	3/15/11	3/18/11	23.19	1,759
NZDCAD	0.7957	USDCHF	1.0000	-2.52	5/6/11	5/31/11	280.83	3,415
AUDCAD	1.0000	NZDCHF	0.8077	-2.09	6/15/11	7/8/11	27.56	-278
AUDNZD	0.9516	USDCHF	1.0000	-2.39	8/3/11	8/5/11	22.57	-1,885
GBPCAD	0.8110	GBPNZD	1.0000	-3.22	8/9/11	8/10/11	13.83	446
AUDCAD	0.7553	EURNZD	1.0000	-2.33	9/2/11	9/27/11	30.73	1,727
AUDJPY	1.0000	NZDJPY	0.9958	-2.89	9/8/11	9/9/11	17.50	2,067
GBPNZD	1.0000	USDJPY	0.9827	3.40	10/3/11	10/25/11	12.40	488
EURNZD	1.0000	GBPCHF	0.4607	2.61	10/3/11	10/12/11	11.19	1,098
CADJPY	0.5754	GBPAUD	1.0000	-2.18	10/4/11	10/17/11	11.80	958
							Total P/L (in USD)	5,193

* DE means Deterministic

Table 4.4: Strategy Performance (Deterministic).

	No Second Criterion	Implied-Realized Criterion	Gamma-Vega criterion AR(1)	Vasicek	DE*
# of trades	28	9	15	15	15
# of positive trades	14	7	10	10	9
Rate of positive trades	50%	78%	67%	67%	60%
Total P/L	-1,427	7,686	7,663	7,663	5,193
Average P/L per trade	-51	854	511	511	346
S.D.	1,500	1,508	1,535	1,535	1,568
Max	2,067	3,415	3,415	3,415	3,415
Min	-2,717	-1,885	-1,885	-1,885	-1,885

* DE means Deterministic.

Table 4.5: Summary of different strategies.

would initiate most trades, but the average P/L is the lowest. The open trades under this strategy are not necessarily "good" trades. If we initiate a "bad" trade, then the potential "good" trade may not be initiated. It is possible that without a supplementary criterion, the open trades under the strategy would reduce the number of "good" trades.

On the other hand, the strategy with the **Implied-Realized Criterion** and the **Gamma-Vega Criterion** are better than that without additional criterion. Comparing the performance of **Implied-Realized Criterion** and **Gamma-Vega Criterion**, the total profit under these criteria is similar, but the number of trades in the strategy with **Gamma-Vega Criterion** is larger than that with **Implied-Realized Criterion**. Hence, a supplementary criterion for the trading strategy is useful.

Chapter 5

Conclusion and Further Discussion

The notion of cointegration technique has been widely used in the stock market using pairs trading strategy. However, the notion is seldom introduced in the derivative market due to the market complexity. In this thesis, pairs trading strategies on straddles are proposed, based on the cointegration relationship of implied volatiles between ATM foreign exchange straddles.

Based on our analysis, if the implied volatilities do not deviate dramatically from their corresponding realized volatilities, then the pairs trading strategy on the ATM straddles of two foreign exchange rates is profitable. In order to ensure the trade does not suffer a huge loss from gamma, which is sensitive to the ratio of implied volatility and realized volatility, we extended our strategy to take the modelling and forecasting realized volatilities into account. Balancing the advantages and disadvantages of existing methods, the *High-Frequency Estimation* of the realized volatility of Andersen, Bollerslev, Diebold and Labys (2003) is adopted. We introduced two supplementary criteria to improve this strategy: Implied-Realized Criterion and Gamma-Vega Criterion. Comparing the profit under different criteria, the result shows that the cointegration pairs trading strategy with supplementary criteria is effective and profitable.

In conclusion, we have extended the cointegration pairs trading strategy to the derivative markets and the empirical study shows that this strategy is profitable. To further extend this strategy, the number of traded underlying assets can be bigger than two. Furthermore, the dynamic hedging can be imposed in the strategy to minimize the impact of Delta. This method may weaken the "short-term period trading" restriction. Last but not least, the method for modeling and forecasting realized volatility can be revised, such as the methodology proposed by Luciani and David (2011), and the notion of fractional cointegration may be pursued for further study.

Appendix A

In this appendix, it is shown that the price of an at-the-money (ATM) straddle and the corresponding implied volatility σ_t is approximately a linear relationship.

Consider the prices of European Call and Put option.

$$C_t = S_t e^{-q(T-t)}\Phi(d_1) - Ke^{-r(T-t)}\Phi(d_2),$$
$$P_t = Ke^{-r(T-t)}\Phi(-d_2) - S_t e^{-q(T-t)}\Phi(-d_1),$$

where $d_1 = \frac{\log(\frac{S_t}{K})+(r-q+\frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}}$, $d_2 = d_1 - \sigma\sqrt{T-t}$ and σ is the implied volatility. Using Taylor expansion,

$$\Phi(d_1) = \Phi(0) + \phi(0)d_1 + \phi'(0)d_1^2 + o(d_1^2) = \tfrac{1}{2} + \tfrac{d_1}{\sqrt{2\pi}} + o(d_1^2),$$

Similarly, $\Phi(d_2) = \frac{1}{2} + \frac{d_2}{\sqrt{2\pi}} + o(d_2^2)$.

Given that $K = S_t e^{(r-q)(T-t)}$ for ATM option, we have

$$d_1 = \frac{\log(\frac{S_t}{K})+(r-q+\frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}} = \frac{\sigma\sqrt{T-t}}{2}.$$

Similarly, $d_2 = -\frac{\sigma\sqrt{T-t}}{2}$. Then

$$C_t = S_t e^{-q(T-t)}\Phi(d_1) - Ke^{-r(T-t)}\Phi(d_2) = S_t e^{-q(T-t)}[\Phi(d_1) - \Phi(d_2)]$$
$$= S_t e^{-q(T-t)}[\frac{d_1 - d_2}{\sqrt{2\pi}} + o(\sigma^2(T-t)],$$

Finally,

$$\begin{aligned} C_t &= \tfrac{S_t e^{-q(T-t)}}{\sqrt{2\pi}} \sigma\sqrt{T-t} + o(S_t\sigma^2(T-t)), \\ P_t &= \tfrac{S_t e^{-q(T-t)}}{\sqrt{2\pi}} \sigma\sqrt{T-t} + o(S_t\sigma^2(T-t)). \end{aligned}$$

For the price of an ATM straddle (ST_t)

$$ST_t = C_t + P_t = \sqrt{\frac{2}{\pi}} S_t\sigma\sqrt{T-t} + o(S_t\sigma^2(T-t)).$$

Appendix B

The following Table contains the details of the setting of the portfolio.

Currency Ticker	**GBPNZD**	**GBPUSD**		
Trading	a	b	**TS**	**Decision**
Signal	1.0000	0.6762	3.93	short
Straddle	sell	buy		**Scale(US$)**
Weights	28792.0	43561.2		100,000
Strike	2.05	1.55		

Table B.1: Details of the trade.

The following Table is the value of the trading signal. From Table B.2, the position should be initiated on 03/10/2011 and closed on 07/10/2011.

Date	**GBPNZD**	**GBPUSD**	**TS**	**z-score**
03/10/2011	16.52	11.64	8.65	3.39
04/10/2011	16.88	12.47	8.44	3.18
05/10/2011	16.92	12.51	8.46	3.20
06/10/2011	15.34	12.24	7.07	1.74
07/10/2011	16.92	12.51	8.46	1.52

Table B.2: Trading signal of the trade.

The following Tables are the performance of the strategy and the change of cumulative P/L due to Greek Letters.

Date	GBPNZD Call(1)	GBPNZD Put(1)	GBPUSD Call(1)	GBPUSD Put(1)	Portfolio
03/10/2011	1,148	1,148	361	361	-3,465
04/10/2011	1,229	1,075	307	452	-3,327
05/10/2011	1,100	1,247	354	406	-3,445
06/10/2011	838	1,331	343	397	-3,023
07/10/2011	876	1,160	374	333	-2,783

(1) This is the price of 10,000 shares option.

Table B.3: Performance of the strategy.

Date	Vega	Theta	Delta	Gamma	High Order	Actual P/L
03/10/2011	–	–	–	–	–	–
04/10/2011	42	19	-30	-31	76	138
05/10/2011	3	38	-5	12	-28	20
06/10/2011	457	57	-11	-35	-26	442
07/10/2011	647	72	51	-38	-50	682

Table B.4: Change of cumulative P/L due to Greek Letters.

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