NEW ADAPTIVE TRANSMISSION SCHEMES FOR MC-CDMA SYSTEMS

By

YIN-MAN LEE

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摘　要

本論文旨在探討有關多載波分碼多工（MC-CDMA）系統之傳輸優化問題，本研究的主要目的在於設計出中央式（centralized）或分散式（decentralized），並具有低功率消耗特性、排除訊道中之不完善處和減低多重使用者干擾（MAI）影響之方法。爲了達到此要求，我們提議在傳送器上的增益能隨著訊道的情形而作出適當的調整，而其相對應的接收器也能適當地作出改變，以增大訊號雜訊比（SNR）。首先被提出的是使用拉氏乘子法（Lagrange Multiplier Methods）之中央傳送器優化法，於其中我們假設有一中央控制器是負責此項計算，接著，我們提出改善之方案藉以加快其收斂至目標的速度。另外，爲了簡化使用，我們也提出分散式的傳送器優化法，在這類方法中，每一對傳送器和接收器是獨立地作出調整而不需牽涉其他傳送器和接收器的額外資料。所有以上提出的方法都可以改善多載波分碼多工系統的表現。當使用者的數目少於或等於載波的數目時，此類系統的表現就類似一個具有最優化頻率分配的分頻多工（ＦＤＭＡ）系統。而當使用者數目多於載波數目時，此類系統通常都可以支援多出來的使用者。最後，一個經過改良而在任何使用者數目多於載波數目的情形下能使訊號雜訊比逐漸地遞減的方法也被提出，令系統之表現自動維持最優化。

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Abstract

In this work, we investigate issues on the transmission optimization in multi-carrier code division multiple access (MC-CDMA) systems. The main objective is to design new algorithms, either centralized or decentralized, with low power consumption and capability of combating channel imperfection and reducing the multiple access interference (MAI). To achieve these goals, we propose that the gains in the transmitter can be adjusted according to the channel conditions. The corresponding receiver is also adaptively adjusted to maximize the signal to noise ratio (SNR). Centralized transmitter optimization based on the Lagrange multiplier methods is studied. A centralized controller is assumed to handle the computation. Modification is made on the algorithms to improve the speed of convergence. Also, decentralized transmitter optimization is provided for simplicity of implementations. Each pair of transmitter and receiver is updated independently without any information from other transmitters and receivers. All these schemes enhance the performance of MC-CDMA systems. When the number of users is smaller than or equal to the number of carriers, the systems appear to tend to frequency division multiple access (FDMA) systems with optimal frequency assignment. When the number of users is larger than the number of carriers, the systems have the potential of supporting more users under some

circumstances. A modified scheme with graceful degradation in the SNR is then derived for use whenever the number of users exceeds the number of carriers.

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Chapter 1

Introduction

1.1 Overview of MC-CDMA

Multicarrier (MC) systems are proven to be more immune to much channel imperfection than single carrier systems [1], [2]. Given the physical nature of the wireless fading channel, frequency selective fading is commonly encountered. Multicarrier modulation (MCM) is demonstrated to be an effective way to combat the negative effects of fading by dividing the frequency selective fading channel into a number of flat fading sub-channels corresponding to the carrier frequencies. Let us consider the channel with a bandwidth of B Hz. The idea of MCM is based on dividing the bandwidth B Hz into M small sub-carriers, spaced by $\frac{B}{M}$ Hz. The spectrum of the different sub-carriers mutually overlaps and the signals on different sub-carriers are orthogonal, giving therefore an optimal efficiency with small adjacent channel interference. More and more applications, such as broadcasting of digital audio, digital television and wireless local area networks (LAN), are proposed with MCM [1]. Another advantage of MCM is the possibility of efficient fast Fourier Transform (FFT) implementations [3].

In this thesis, we examine the multicarrier code division multiple access (MC-CDMA) scheme, which is a digital modulation/multiple access technique based on a combination of MCM and code division multiple access (CDMA), in wireless communication channel [4]. MC-CDMA is considered as a promising alternative to conventional DS (direct sequence)-CDMA. Since 1993 proposed for indoor wireless communication systems by Yee, Linnartz and Fettweis [4], and for mobile radio systems by Fazal and Papke [5], MC-CDMA rapidly became a hot research topic in spread spectrum communications. MC-CDMA is a suitable transmission scheme in the indoor environment where the specific character of indoor propagation [6] allows for the exploitation of this technique. In [7], Hara and Prasad categorized the MC-CDMA schemes into two groups: MCM with frequency domain spreading and MCM with time domain spreading. In the first group, the spreading operation is in the frequency domain so that a fraction of the symbol corresponding to a chip of the spreading code is transmitted through a different sub-carrier. In the second group, the spreading operation is in the time domain so that the resulting spectrum of each sub-carrier can satisfy the orthogonality condition with minimum frequency separation.

Many papers worked on the problem of enhancing MC-CDMA systems, such as detection, equalization, and combining techniques, as well as the performance evaluation in different environments [8]–[16]. The demand of good quality of service (QoS) requirements becomes an important issue in the development of MC-CDMA systems. Thus adaptive methods for the optimization of both transmitter and receiver in MC-CDMA are of interest. In this work, through applying different optimization techniques, the QoS (signal to noise ratio, SNR) requirements can be met with low power consumption.

In MC-CDMA, a data symbol is transmitted over M sub-carriers simultaneously, which allows one to perform simple and effective detection, to use the available spectrum in an efficient way, to retain many advantages of a DS-CDMA system, and to exploit frequency diversity. As many implementational problems appear solvable, MC-CDMA could be widely used and could become part of the standards.

1.2 System Model

In this section, we focus on the formulation of the MC-CDMA system model for analysis. In an uplink transmission scenario, optimal schemes are obtained based on this model. In MC technique, the total system bandwidth is divided into M sub-channels. We assume that there are K simultaneous users in the system and each user uses the same M carriers. The structure of the MC transmitter is depicted in Figure 1.1.

The kth user generates a stream of data symbols $b^{(k)}$, given by

$$b^{(k)} = (\ldots, b_0^{(k)}, b_1^{(k)}, b_2^{(k)}, \ldots). \tag{1.1}$$

The data symbols $b_i^{(k)}$ are random variables with zero mean and unit variance. For binary communications, each $b_i^{(k)}$ is either $+1$ or -1. The data stream is duplicated to M branches. To change the transmitted power in the mth branch of the transmitter, for $1 \leq m \leq M$, each sub-stream gets through a complex transmission gain. The gains for the kth user, $1 \leq k \leq K$, can be written as an M-dimensional vector, $\mathbf{c}_k = [c_1^{(k)}, c_2^{(k)}, \ldots, c_M^{(k)}]^T$. The mth sub-stream

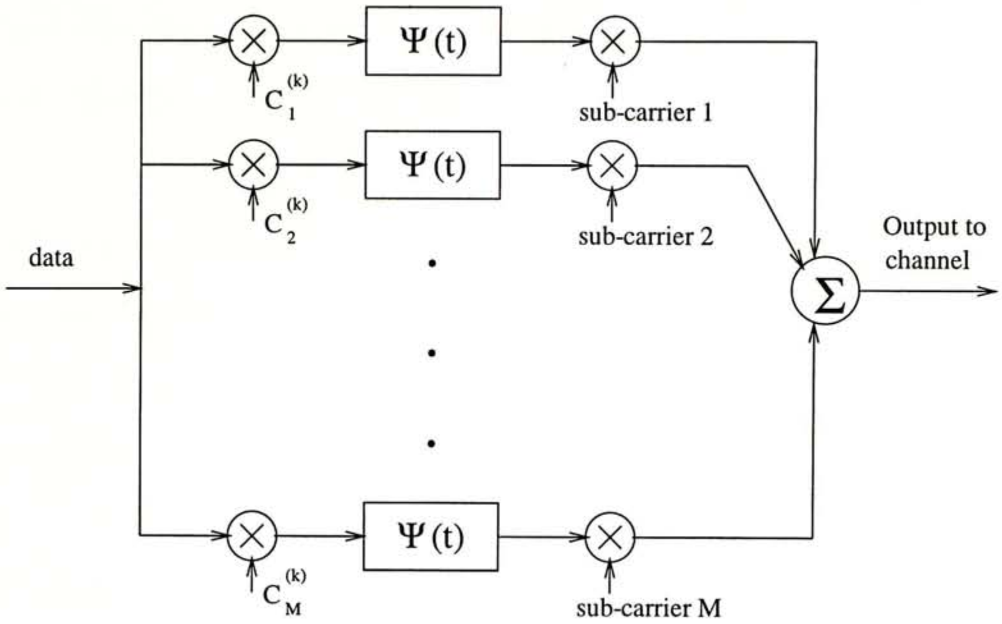

Figure 1.1: Block diagram of the MC transmitter

is filtered and carrier modulated in its branch independently. The modulation process can be accomplished through discrete time signal processing and by making use of the filtering properties of the inverse discrete Fourier Transform (IDFT). The transmitted signal of the kth user can be expressed as the real part of the following complex signal:

$$\sum_{m=1}^{M} c_m^{(k)} \left\{ \sum_{i=-\infty}^{\infty} b_i^{(k)} \psi(t - iT_s) \right\} e^{j\omega_m t}, \tag{1.2}$$

where T_s is the delay between consecutive data symbols, ω_m is the angular frequency of the mth carrier, and $c_m^{(k)}$ is chosen by the kth transmitter to vary the amplitude and the phase of the mth sub-carrier. We assume that the symbol waveform $\psi(t)$ is bandlimited, satisfies the Nyquist criterion for no intersymbol interference, and is normalized so that $\int_{-\infty}^{\infty} |\psi(t)|^2 dt = T_s$. We also assume that the sub-carrier frequencies are suitably chosen so that the signals on different sub-carriers are orthogonal and do not interfere with one other.

We now describe the channel model. We assume that the channel is a frequency selective slow Rayleigh fading channel. By suitably choosing M and the bandwidth of $\psi(t)$ [17], we can assume that each sub-carrier undergoes independent frequency non-selective slow Rayleigh fading [18]. We can use the complex-valued impulse to show the effect of the channel response as

$$h_{k,m}(t) = \alpha_{k,m}\delta(t - T_k), \tag{1.3}$$

where T_k is the received signal delay of the kth user. We introduce the complex random variables $\alpha_{k,m}$, for $k = 1, ..., K$ and $m = 1, ..., M$, which are independent and identically distributed (iid) complex Gaussian random variables with zero mean and unit variance. The amplitudes of the complex variables are, therefore, Rayleigh distributed. These channel coefficients are assumed to be invariant within the time interval for the optimization. For a particular realization, if $\alpha_{k,m} > 1$, the signal quality of the mth branch for the kth user will be enhanced. If $\alpha_{k,m} < 1$, otherwise, the signal quality will be degraded. Intuitively, much power will be placed on the sub-carriers with $\alpha_{k,m} > 1$ to achieve power efficiency. Moreover, the power concentration in a fraction of sub-carriers reduces the multiple access interference (MAI) seen by other users. We also assume the presence of additive white Gaussian noise (AWGN) with zero mean and two-sided power spectral density of $N_0/2$.

We consider the MC receiver with coherent detection as shown in Figure 1.2. The complex envelope of the received signal $r(t)$ is the convolution of the transmitted signal and the channel response $h_{k,m}(t)$, which is given by

$$r(t) = \sum_{k=1}^{K}\sum_{m=1}^{M} c_m^{(k)}\left\{\sum_{i=-\infty}^{\infty} b_i^{(k)}\psi(t - T_k - iT_s)\right\}e^{j\omega_m(t-T_k)}\alpha_{k,m} + n(t), \tag{1.4}$$

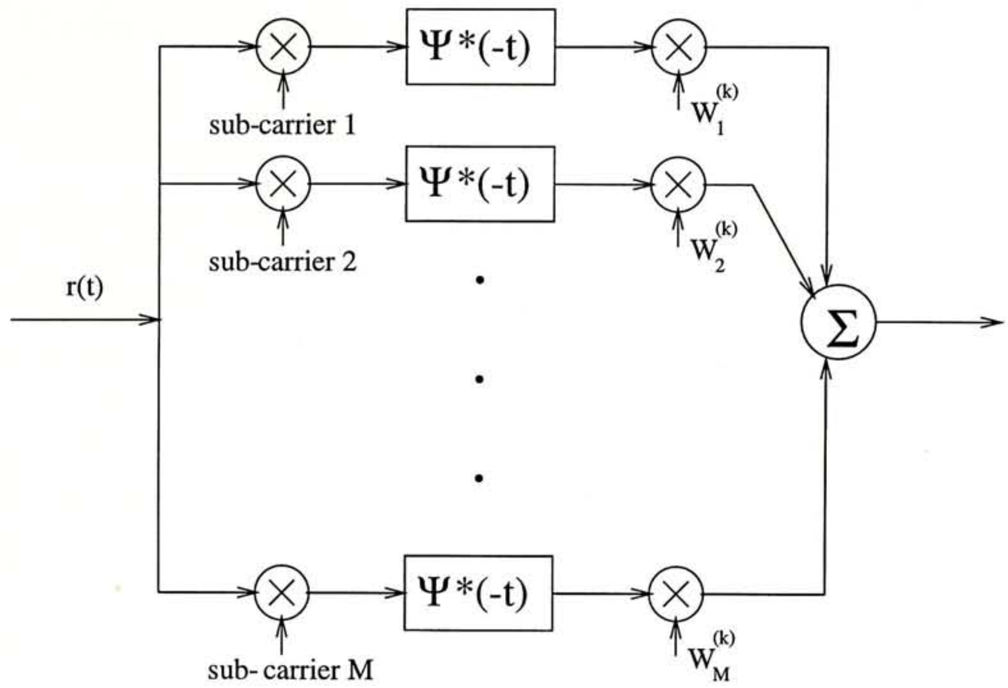

Figure 1.2: Block diagram of the MC receiver

where $n(t)$ represents AWGN. The received signal is processed by a matched filter that coherently detects the kth user signal. Similar to the transmitter, the demodulation can be performed simply with the DFT technique. Signals are weighted by the M-dimensional vector, $\mathbf{w}_k = [w_1^{(k)}, w_2^{(k)}, \ldots, w_M^{(k)}]^T$, for the kth user. After combining the contributions from the M branches, the receiver estimates the transmitted data stream.

The system model derived in this section is much similar to the model in the original work in [4]. The difference lies in the selection of $\mathbf{c}_k$ in the transmitter. N. Yee, *et al.* proposed that the value of $c_m^{(k)}$ is chosen from $\{-1, 1\}$, but we extend the possibility of $c_m^{(k)}$ to any complex number. In addition, instead of the simple combining methods, the weight vector $\mathbf{w}_k$ in the receiver is adjusted to achieve receiver optimization.

1.3 Receiver Optimization

The characteristics of the transmission channel, and the statistical properties of the noise corrupt the signals. One of the damaging impairments is linear (amplitude and phase) distortion introduced by the channel. This type of impairment is handled by an equalizer [19], which compensates, in an adaptive fashion, for the linear distortion introduced by the communication link. Some adaptive equalizers for MC-CDMA are investigated in [8]–[13]. Moreover, a receiver optimization method for MC-CDMA systems has been provided in [18]. Both training with reference signal and blind adaptive methods seem to be well-established. In [18], a blind adaptive receiver with interference suppression is proposed for MC-CDMA systems. We use it as the basis for the following analysis.

Without loss of generality, we consider the optimization of the receiver for the first user. Receiver optimization for the first user only affects the performance of the first user. We consider the detection of the symbol $b_0^{(1)}$. The output of the demodulator on the mth branch, due to the first user signal, is given by $b_0^{(1)} d_m^{(1)}$ where

$$d_m^{(1)} = T_s c_m^{(1)} \alpha_{1,m}. \tag{1.5}$$

We define an M-dimensional vector $\mathbf{d}_1 = [d_1^{(1)}\ d_2^{(1)}\ \ldots\ d_M^{(1)}]^T$. The output of the demodulator on the mth branch of the first receiver, due to the kth user signal, for $k > 1$, is given by

$$i_{k,m}^{(1)} = c_m^{(k)} e^{-\mathrm{j}\omega_m T_k} \alpha_{k,m} \sum_{i=-\infty}^{\infty} b_i^{(k)} \hat{\psi}(-iT_s - T_k), \tag{1.6}$$

where the function $\hat{\psi}(\cdot)$ is the output of the symbol waveform through the receiver filter, i.e., $\hat{\psi}(t) = \int_{-\infty}^{\infty} \psi(s)\psi^*(s-t)\, ds$. We also define M-dimensional vectors $\mathbf{i}_k^{(1)} = [i_{k,1}^{(1)} \; i_{k,2}^{(1)} \; \ldots \; i_{k,M}^{(1)}]^T$. We denote the output of the demodulator on the mth branch due to AWGN by $n_m^{(1)}$, and similarly define an M-dimensional vector $\mathbf{n}_1 = [n_1^{(1)} \; n_2^{(1)} \; \ldots \; n_M^{(1)}]^T$. The overall output of the demodulators, in vector form, is given by

$$\mathbf{z}_1 = b_0^{(1)}\mathbf{d}_1 + \mathbf{n}_1 + \sum_{k=2}^{K} \mathbf{i}_k^{(1)}. \tag{1.7}$$

Notice that the vectors $\mathbf{i}_k^{(1)}$ are uncorrelated for different k. The noise and interference correlation matrix is given by

$$\mathbf{R}_1 = \mathrm{E}_\alpha \left[\mathbf{n}_1 \mathbf{n}_1^H + \sum_{k=2}^{K} \mathbf{i}_k^{(1)} \mathbf{i}_k^{(1)H} \right], \tag{1.8}$$

where $\mathrm{E}_\alpha[\cdot]$ denotes the conditional expectation given $\alpha_{k,m}$, for $k = 1, \ldots, K$ and $m = 1, \ldots, M$, and the superscript H denotes the Hermitian operation. The decision statistic for the symbol $b_0^{(1)}$ is given by $Z = \mathbf{w}_1^H \mathbf{z}_1$. We assume that the channel coefficients $\alpha_{k,m}$ and T_k vary slowly so that they effectively remain constant within the time interval used to determine an appropriate weight vector. We determine the optimal weight vector that maximizes the signal to noise ratio (SNR) defined by

$$\mathrm{SNR}_1 = \frac{|\mathbf{w}_1^H \mathbf{d}_1|^2}{\mathrm{E}_\alpha[|\mathbf{w}_1^H(\mathbf{n}_1 + \sum_{k=2}^{K} \mathbf{i}_k^{(1)})|^2]} = \frac{|\mathbf{w}_1^H \mathbf{d}_1|^2}{\mathbf{w}_1^H \mathbf{R}_1 \mathbf{w}_1}. \tag{1.9}$$

In [20], it is shown that the optimal weight vector is given by

$$\mathbf{w}_1 = \mathbf{R}_1^{-1} \mathbf{d}_1. \tag{1.10}$$

We assume that the receiver can estimate the desired vector $\mathbf{d}_1$ and the noise and interference correlation matrix $\mathbf{R}_1$, possibly with the help of a training sequence. Through a similar approach, the weight vector $\mathbf{w}_k$ of the receiver for the kth user can also be derived as

$$\mathbf{w}_k = \mathbf{R}_k^{-1} \mathbf{d}_k, \tag{1.11}$$

where $\mathbf{R}_k$ and $\mathbf{d}_k$ are the noise and interference correlation matrix and the signal vector for the kth user, respectively. Based on this receiver optimization technique, we further the improvement for the MC-CDMA system by applying transmitter optimization.

1.4 Transmitter Optimization

A general approach to the design of multiuser communication systems is based on improving the performance. To get a good estimation of the received signals in MC-CDMA, researchers devoted much effect to the design of receivers [21], [22]. However, research work on transmitter optimization has increased recently. The motivation of it is that people would like to find some way to make the received signals more favorable for detection and estimation. The key assumption of transmitter optimization is that the optimization information from receivers can be fed back to transmitters. Sticking to the knowledge, transmitters can choose a more effective way for transmission. It is shown that transmitter optimization in addition to receiver optimization contributes significantly to efficient suppression of the MAI and other channel impairments. Full optimization of an MC-CDMA system entails optimizing both the receiver end and the transmitter end, where the second task requires optimizing the transmitted power subject to a certain set of QoS requirements [23], [24]. Yang and Roy proposed the joint transmitter-receiver optimization for multiuser communication systems with decision feedback in [25]. Other joint optimization schemes can be found in [26]–[28]. Transmitter precoding is also considered as an important branch of transmitter optimization [29]. In this work, we investigate the centralized and decentralized approaches to optimize the transmission gains. When the MC-CDMA system performs receiver optimization and transmitter optimization, the performance of the whole system will be improved.

1.5 Nonlinearly Constrained Optimization

Since we will see the optimization of MC-CDMA systems as a constrained optimization problem, we discuss the basic characteristics of this kind of problems in this section.

There are two broad approaches to the solution of nonlinearly constrained optimization (minimization) problems. In the first approach, the objective function is modified so that it has an unconstrained minimum at the minimum of the original constrained problem. We call these techniques *transformation methods.* When the modifications are performed in sequences, we call the methods *sequential*, otherwise the term *exact* will be used. The second approach involves linear approximation to the constraints followed by the application of a projection-type method and perhaps a correction procedure to maintain a kind of active set strategy. We consequently call methods of this type *projection methods.* In this thesis, we only discuss the first group of methods and leave the other in [30] for reference.

In general, the problem is posed as

$$\begin{aligned} &\min \quad f(x), \\ &\text{subject to} \quad g_j(x) = 0 \qquad j = 1, ..., J. \end{aligned} \tag{1.12}$$

One of the implementation approaches of transformation methods is by Lagrangian. The unconstrained function constructed by the Lagrange multiplier methods with constraint functions of penalty type is called the Lagrangian function. It takes the form

$$L(x) = f(x) + \sum_{j=1}^{J} \lambda_j g_j(x), \tag{1.13}$$

where λ_j for $j = 1, ..., J$ are the Lagrange multipliers. The zero gradient equation of the Lagrangian function represents the necessary condition for optimality [31], and is iteratively solved by steepest ascent/descent. The main advantage of the Lagrange multiplier methods is that constraints are virtually ignored. Furthermore, the process of handling penalties is entirely automatic and the result of the optimization with respect to the changeable variables will automatically satisfy the constraints.

1.6 Outline of Thesis

In Chapter 2, we will look at some centralized adaptive transmission schemes for MC-CDMA systems. The Lagrange multiplier methods for optimizing the centralized constrained problem are investigated. The results show that power is not wasted in the deep fading carriers after applying the transmitter optimization schemes. Working towards practicality, we will improve the centralized optimization schemes based on the use of power control for these systems. A frequency division multiple access (FDMA) system with optimal frequency assignment is derived for the purpose of comparison.

In Chpater 3, we consider the problem from a different angle. We will seek a decentralized solution to the centralized constrained optimization problem. The decentralized optimization algorithms provide the merit that no centralized information is involved in the adaptive procedures. The optimization of both transmitter and receiver for different users is performed independently. The resultant performance shows that remarkable improvements over receiver optimization only will be achieved. By seeing the transmission requirement of multimedia communications and the service requirement of wireless communications, a multirate MC-CDMA transmission system based on the decentralized transmission scheme will be proposed.

In Chapter 4, we will compare the performance of the centralized and decentralized adaptive transmission schemes. Also showed is the performance of supporting more users by the two transmission schemes after the MC-CDMA systems are heavily loaded. In particular, we will proposed a supplementary scheme for the users, in which the performance of them will descend gracefully as the number of users increases. It gives the results of averaging out the degradation in performance and letting no user break down.

In Chapter 5, conclusions for this work will be drawn and possible extensions will be discussed.

Chapter 2

Centralized Transmitter Optimization for MC-CDMA Systems

2.1 Introduction

In this chapter, we consider the use of multicarrier code division multiple access (MC-CDMA) systems over wireless communication channel. We develop several centralized adaptive transmission schemes for MC-CDMA systems, which should be well suited for wireless local area network (LAN) or wireless local loop (WLL) applications. We assume that there is a control unit with the centralized information from all users and the knowledge at a receiver can be sent back to the corresponding transmitter for the optimization of transmission. Instead of transmitting data sub-streams with uniform power through sub-channels, data sub-streams are sent over sub-channels with the special power assignment adaptively adjusted to the fading channel characteristics. The problem of determining the optimal transmitted power among sub-carriers is formulated as a constrained optimization problem. The Lagrange multiplier methods are used to solve the problem. Results show that significant improvements in performance can be achieved. When the number of users is smaller than or equal to the number of carriers, each transmitter tends to concentrate its power on a distinct carrier which does not suffer deep fading at the receiver. The MC-CDMA system with centralized transmitter optimization then tends to a frequency division multiple access (FDMA) system with optimal frequency assignment. Then we formally define this optimal FDMA system for comparison. After stating some Lagrangian approaches to solve the optimization problem, we modify them to become new algorithms with improved performance. Simulation shows that these algorithms have the merits of fast convergence and stable performance.

In Section Two, the problem of enhancing the MC-CDMA system with optimal power assignment is formulated. In Section Three, we present the Lagrangian approaches to the optimization problem. In Section Four, we derive the optimal FDMA system. In Section Five, we modify the algorithms to solve the optimization problem more efficiently. Section Six contains the simulation of the methods stated in the previous three sections. Finally, in Section Seven, we give the summary of this chapter.

2.2 Problem Development

Referring to Section Two and Section Three in Chapter 1, we have already established the system model and derived the optimal receiver for the MC-CDMA system. In [18], a blind adaptive receiver with interference suppression is proposed for MC-CDMA systems. The receiver applies

$$\mathbf{w}_k = \mathbf{R}_k^{-1}\mathbf{d}_k \tag{2.1}$$

for the kth user to weight the contributions from each branch. In this chapter, we also assume there exists a centralized controller handling the computation. The centralized controller will try to co-ordinate the effects of different users so that the required performance of each user can be achieved. In the transmitter, a complex transmission gain $\mathbf{c}_m^{(k)}$ for the mth branch of the kth user is used to adjust the transmission centrally and adaptively.

From the optimal weight vector $\mathbf{w}_k$, we see that the contributions from the sub-channels will be weighted differently according to (2.1). Assuming all users optimize the demodulated signals with their own $\mathbf{w}_k$, from (1.9), the signal to noise ratio (SNR) for the kth user is given by

$$\mathrm{SNR}_k = \mathbf{d}_k^H\mathbf{R}_k^{-1}\mathbf{d}_k, \tag{2.2}$$

where $\mathbf{R}_k$ is the noise and interference correlation matrix for the kth user and the vector $\mathbf{d}_k$ can be expressed as

$$\mathbf{d}_k = \mathbf{A}_k\mathbf{c}_k, \tag{2.3}$$

where $\mathbf{A}_k$ is an $M \times M$ diagonal matrix whose mth diagonal element is $T_s\alpha_{k,m}$.

In multiuser communication systems, the performance of one user may affect the performance of others. Given a set of target SNRs, the transmission scheme with the least average power to achieve the targets can be considered as optimum. Using less power has the advantages of saving battery life and reducing the multiple access interference (MAI) in MC-CDMA. We see that the problem is better defined as

$$\begin{aligned} &\min \quad \frac{1}{K}\sum_{k=1}^{K} \mathbf{c}_k^H \mathbf{c}_k, \\ &\text{subject to} \quad \mathbf{d}_k^H \mathbf{R}_k^{-1} \mathbf{d}_k = \gamma_k \quad \text{for all } k, \end{aligned} \tag{2.4}$$

where $\frac{1}{K}\sum_{k=1}^{K} \mathbf{c}_k^H \mathbf{c}_k$ is the average power and γ_k is the target SNR for the kth user. From (2.4), we see that it is a nonlinearly constrained optimization problem. In the next section, we apply different Lagrange multiplier methods to solve this problem.

2.3 Lagrangian Optimization Approaches

In the presence of different path losses and fading, it may be very difficult to obtain closed form solutions for (2.4) when K is reasonably large. Instead of trying to find exact closed form solutions, we consider numerical methods to treat the constrained optimization problem sequentially. The initial developments of transformation methods were motivated by the concept of minimizing the objective function with an unconstrained minimization method while maintaining *implicit* control over the violations of constraints. The principle of the methods is to add negative effect to the constructed unconstrained function at points which violate or perhaps tend to violate the constraints. In general, a constrained optimization problem can be solved with the well-developed Lagrange multiplier methods [32]. The idea is to penalize constraint violation by modifying the constraints as penalties to the objective function. Then, any technique of unconstrained optimization may be used to solve the unconstrained problem. In this section we propose different Lagrange multiplier methods to solve the problem. Simulation results of each case will be shown in Section Six.

2.3.1 Penalty Function Method

First, for the constrained optimization problem of (2.4), we consider the Lagrangian and incorporate the SNR requirements as penalty functions. The idea underlying penalty function method is to transform the problem of minimizing

$$\sum_{k=1}^{K} \mathbf{c}_k^H \mathbf{c}_k, \tag{2.5}$$

where we omit $\frac{1}{K}$ without loss of optimality, subject to certain constraints on $\mathbf{c}_k$ into the problem of finding the unconstrained minimum of

$$L_p = \sum_{k=1}^{K} [\mathbf{c}_k^H \mathbf{c}_k + \lambda_k(\gamma_k - \mathbf{d}_k^H \mathbf{R}_k^{-1} \mathbf{d}_k)^2], \tag{2.6}$$

where λ_k for $k = 1, ..., K$ are the Lagrange multipliers. The Lagrangian function L_p is considered as a function of λ_k and the components of $\mathbf{c}_k$ for $k = 1, ..., K$. If the constraints are violated, then a high value will be given to L_p so that the minimum of L_p will not arise outside the constrained region.

L_p takes on values which are greater than or equal to the corresponding values of (2.5) (the true objective function for our problem). As $\mathbf{c}_k$ moves toward feasible values, the difference between L_p and (2.5) may be reduced through letting $\mathbf{c}_k$ approach to fulfil the constraints. By choosing λ_k to be very large, we impose a very high cost for violating the constraints. On the other hand, if $\mathbf{c}_k$ takes on values, which though feasible, are close to the boundary of the constrained region, so that the constraints are satisfied or nearly satisfied, L_p and hence (2.5) will become very close. The minimum of the objective function subjected to the constraints are nearly found. Thus the operation of $\lambda_k(\gamma_k - \mathbf{d}_k^H \mathbf{R}_k^{-1} \mathbf{d}_k)^2$ with large λ_k, for all k, is to create a steep sided ridge along each of the constraint boundaries. By sticking to the constraints, so that the effect of $\lambda_k(\gamma_k - \mathbf{d}_k^H \mathbf{R}_k^{-1} \mathbf{d}_k)^2$ is still small at the minimum point, we may be able to make this unconstrained minimum point for L_p coincide with the constrained minimum of (2.4). In other words, the overall effect of minimizing the Lagrangian function is equivalent to minimize the objective function subject to constraints.

The derivative of L_p with respect to (wrt) λ_k is the requirement for the kth user SNR. The derivative of L_p wrt $\mathbf{c}_k$ can be obtained as follows. Notice that $\mathbf{R}_k^{-1}$ is not a function of $\mathbf{c}_k$ while $\mathbf{R}_j^{-1}$ for $j \neq k$ can be expressed explicitly as a function of $\mathbf{d}_k$ (and, hence, $\mathbf{c}_k$) via the matrix inversion lemma.

$$\begin{aligned} \mathbf{R}_j^{-1} &= (\mathbf{R}_{k,j} + \mathbf{d}_k \mathbf{d}_k^H)^{-1} \\ &= \mathbf{R}_{k,j}^{-1} - \frac{1}{1 + \mathbf{d}_k^H \mathbf{R}_{k,j}^{-1} \mathbf{d}_k} \mathbf{R}_{k,j}^{-1} \mathbf{d}_k \mathbf{d}_k^H \mathbf{R}_{k,j}^{-1}, \end{aligned} \tag{2.7}$$

for $j \neq k$ where

$$\mathbf{R}_{k,j} = \mathbf{n}_j \mathbf{n}_j^H + \sum_{i \neq j,k} \mathbf{d}_i \mathbf{d}_i^H. \tag{2.8}$$

where n_j is the noise vector for the jth user. Therefore, the derivative of L_p wrt $\mathbf{c}_k$ is given by

$$\begin{aligned} \frac{dL_p}{d\mathbf{c}_k} &= 2\mathbf{c}_k - 4\lambda_k(\gamma_k - \mathbf{d}_k^H \mathbf{R}_k^{-1} \mathbf{d}_k) \mathbf{A}_k^H \mathbf{R}_k^{-1} \mathbf{d}_k + \\ & \quad 4 \sum_{j \neq k} \lambda_j (\gamma_j - \mathbf{d}_j^H \mathbf{R}_j^{-1} \mathbf{d}_j) \cdot \\ & \quad \left\{ \frac{\mathbf{d}_j^H \mathbf{R}_{k,j}^{-1} \mathbf{d}_k}{1 + \mathbf{d}_k^H \mathbf{R}_{k,j}^{-1} \mathbf{d}_k} \mathbf{A}_k^H \mathbf{R}_{k,j}^{-1} \mathbf{d}_j - \frac{|\mathbf{d}_j^H \mathbf{R}_{k,j}^{-1} \mathbf{d}_k|^2}{(1 + \mathbf{d}_k^H \mathbf{R}_{k,j}^{-1} \mathbf{d}_k)^2} \mathbf{A}_k^H \mathbf{R}_{k,j}^{-1} \mathbf{d}_k \right\}. \end{aligned} \tag{2.9}$$

The Lagrangian optimization problem is usually solved by finding the saddle point of the Lagrangian function. For L_p in (2.6), it is a minimum wrt $\mathbf{c}_k$

and a maximum wrt λ_k. λ_k is taken to be a reasonably large number so that the effect on the function when a constraint is violated is to impose a penalty proportional to the amount of the violation. We consider an iterative scheme to seek a stationary point of the Lagrangian function. At each step, $\mathbf{c}_k$ and λ_k are updated according to the following rules

$$\begin{aligned} \mathbf{c}_k &\leftarrow \mathbf{c}_k - \mu \frac{dL_p}{d\mathbf{c}_k}, \\ \lambda_k &\leftarrow \lambda_k + \mu(\gamma_k - \mathbf{d}_k^H \mathbf{R}_k^{-1} \mathbf{d}_k)^2, \end{aligned} \tag{2.10}$$

i.e., a gradient descent algorithm is used to update $\mathbf{c}_k$ while a gradient ascent algorithm is used to update λ_k. The gradient algorithm is the simplest one among many sequential methods for unconstrained optimization. μ is the step size to vary the speed of convergence.

2.3.2 Barrier Function Method

This approach is suitable for inequality constraints only. The minimization of the Lagrangian function is approached from the interior of the feasible region, and this quantity is infinite on the boundary itself. Hence if we start with a feasible point and try to find the unconstrained minimum of this Lagrangian function, it will lie within the feasible region of the constrained problem. To confine the solution from the interior, any orthogonal assignment in different sub-channels for all users may be used. In addition, regardless of how much power is used initially, we can simply assign one carrier to one user to transmit data when the number of users is smaller than or equal to the number of carriers. After pouring power in the particular carrier for the user, a feasible solution from the interior can always be reached. We change the constraints of the problem to be

$$\mathbf{d}_k^H \mathbf{R}_k^{-1} \mathbf{d}_k \geq \gamma_k \quad \text{for all } k. \tag{2.11}$$

To minimize (2.5), intuitively, we see that the solution lies in the feasible region near the equalities for the target SNRs. In this part we consider the inverse barrier function. The Lagrangian function appears as

$$L_b = \sum_{k=1}^{K} [\mathbf{c}_k^H \mathbf{c}_k + \lambda_k (\mathbf{d}_k^H \mathbf{R}_k^{-1} \mathbf{d}_k - \gamma_k)^{-1}], \tag{2.12}$$

where λ_k for $k = 1, ..., K$ are the Lagrange multipliers. We can interpret the behavior of barrier function method with (2.12) in the following way. It can be seen that as any constraint tends to zero, the contribution to the penalty term in (2.12) tends to infinity. By letting λ_k to be suitably small, we avoid the term $\lambda_k(\mathbf{d}_k^H \mathbf{R}_k^{-1} \mathbf{d}_k - \gamma_k)^{-1}$, for all k, to blow up. As $\mathbf{d}_k^H \mathbf{R}_k^{-1} \mathbf{d}_k$ approaches the target SNR$_k$, an unconstrained minimum has been created within the feasible region. For the same reason said previously, the minimization of the Lagrangian function is equivalent to the minimization of the objective function subject to constraints. The minimum of the Lagrangian function may be obtained with any sequential unconstrained minimization technique (SUMT).

By using the similar approach, $\mathbf{R}_j^{-1}$ can be expressed as (2.7) by the matrix inversion lemma. To have the gradient information, we find the first derivative of L_b wrt $\mathbf{c}_k$. It follows that

$$\begin{aligned}\frac{dL_b}{d\mathbf{c}_k} &= 2\mathbf{c}_k - 2\lambda_k(\mathbf{d}_k^H \mathbf{R}_k^{-1} \mathbf{d}_k - \gamma_k)^{-2} \mathbf{A}_k^H \mathbf{R}_k^{-1} \mathbf{d}_k + \\ & \quad 2\sum_{j \neq k} \lambda_j (\mathbf{d}_j^H \mathbf{R}_j^{-1} \mathbf{d}_j - \gamma_j)^{-2} \cdot \\ & \quad \left\{ \frac{\mathbf{d}_j^H \mathbf{R}_{k,j}^{-1} \mathbf{d}_k}{1 + \mathbf{d}_k^H \mathbf{R}_{k,j}^{-1} \mathbf{d}_k} \mathbf{A}_k^H \mathbf{R}_{k,j}^{-1} \mathbf{d}_j - \frac{|\mathbf{d}_j^H \mathbf{R}_{k,j}^{-1} \mathbf{d}_k|^2}{(1 + \mathbf{d}_k^H \mathbf{R}_{k,j}^{-1} \mathbf{d}_k)^2} \mathbf{A}_k^H \mathbf{R}_{k,j}^{-1} \mathbf{d}_k \right\}. \end{aligned} \tag{2.13}$$

Notice that for this case both $\mathbf{c}_k$ and λ_k are updated using gradient descent algorithms. λ_k decreases in each step to make the difference between L_b and (2.5) decrease and to refine the optimization.

2.3.3 Powell's Method and Augmented Lagrangian Method

In the previous two parts considerable attention has been paid to the solutions of the constrained optimization problem via penalty function method and barrier function method. It is appropriate to consider the relative advantages and disadvantages of those methods [31]. The point is that both the methods have a tendency to involve very large numbers, namely λ_k or the inverse of the constraints, which causes the functions that will be minimized to be very sensitive to changes in the variables in a way that makes them difficult to manage. Therefore a number of methods have been proposed, whose general technique is that of the penalty function method, but where the functions have nice smoothness and boundedness properties.

In 1969, Powell announced Powell's method [33] for equality constraints. For this problem, we construct the Lagrangian function in the following way

$$L_{pw} = \sum_{k=1}^{K} [\mathbf{c}_k^H \mathbf{c}_k + \lambda_k(\gamma_k - \mathbf{d}_k^H \mathbf{R}_k^{-1} \mathbf{d}_k + \theta_k)^2]. \tag{2.14}$$

The required solution can usually be obtained with moderate values of the parameters. The main improvement is the introduction of the parameters θ_k, for all k, and these parameters satisfy the use of moderate values of λ_k. Powell's method usually treats $\lambda_k = \lambda$ as a constant and varies θ_k to solve the problem. $\mathbf{R}_j^{-1}$ is the same as (2.7), and the first order derivative of L_{pw} wrt $\mathbf{c}_k$ is

$$\frac{dL_{pw}}{d\mathbf{c}_k} = 2\mathbf{c}_k - 4\lambda_k(\gamma_k - \mathbf{d}_k^H \mathbf{R}_k^{-1} \mathbf{d}_k + \theta_k) \mathbf{A}_k^H \mathbf{R}_k^{-1} \mathbf{d}_k +$$

$$4\sum_{j\neq k}\lambda_j(\gamma_j - \mathbf{d}_j^H\mathbf{R}_j^{-1}\mathbf{d}_j + \theta_j)\cdot$$
$$\left\{\frac{\mathbf{d}_j^H\mathbf{R}_{k,j}^{-1}\mathbf{d}_k}{1+\mathbf{d}_k^H\mathbf{R}_{k,j}^{-1}\mathbf{d}_k}\mathbf{A}_k^H\mathbf{R}_{k,j}^{-1}\mathbf{d}_j - \frac{|\mathbf{d}_j^H\mathbf{R}_{k,j}^{-1}\mathbf{d}_k|^2}{(1+\mathbf{d}_k^H\mathbf{R}_{k,j}^{-1}\mathbf{d}_k)^2}\mathbf{A}_k^H\mathbf{R}_{k,j}^{-1}\mathbf{d}_k\right\} \quad (2.15)$$

By updating $\mathbf{c}_k$ with a gradient descent algorithm and θ_k with a gradient ascent algorithm iteratively, we will reach a similar solution as using penalty function method. If the rate of reaching targets is satisfactory, θ_k is updated and λ_k is unaltered. Otherwise, λ_k must be increased and θ_k decreased by the same factor [33]. The advantages of Powell's method are its stability and fast convergence near the optimal point.

Another modified approach of the Lagrangian function is Augmented Lagrangian method. For the same problem, we have the Lagrangian function written as

$$L_{al} = \sum_{k=1}^{K}[\mathbf{c}_k^H\mathbf{c}_k + \lambda_k(\gamma_k - \mathbf{d}_k^H\mathbf{R}_k^{-1}\mathbf{d}_k) + \frac{1}{2}s(\gamma_k - \mathbf{d}_k^H\mathbf{R}_k^{-1}\mathbf{d}_k)^2], \quad (2.16)$$

where s is a reasonably large constant which makes better behavior of reaching targets. Similarly, we derive the derivative of L_{al} wrt $\mathbf{c}_k$ as

$$\begin{aligned}\frac{dL_{al}}{d\mathbf{c}_k} = {} & 2\mathbf{c}_k - 2\lambda_k\mathbf{A}_k^H\mathbf{R}_k^{-1}\mathbf{d}_k + \\ & 2\sum_{j\neq k}\lambda_j\left\{\frac{\mathbf{d}_j^H\mathbf{R}_{k,j}^{-1}\mathbf{d}_k}{1+\mathbf{d}_k^H\mathbf{R}_{k,j}^{-1}\mathbf{d}_k}\mathbf{A}_k^H\mathbf{R}_{k,j}^{-1}\mathbf{d}_j - \frac{|\mathbf{d}_j^H\mathbf{R}_{k,j}^{-1}\mathbf{d}_k|^2}{(1+\mathbf{d}_k^H\mathbf{R}_{k,j}^{-1}\mathbf{d}_k)^2}\mathbf{A}_k^H\mathbf{R}_{k,j}^{-1}\mathbf{d}_k\right\} - \\ & 2s(\gamma_k - \mathbf{d}_k^H\mathbf{R}_k^{-1}\mathbf{d}_k)\mathbf{A}_k^H\mathbf{R}_k^{-1}\mathbf{d}_k + 2s\sum_{j\neq k}(\gamma_k - \mathbf{d}_k^H\mathbf{R}_k^{-1}\mathbf{d}_k)\cdot \\ & \left\{\frac{\mathbf{d}_j^H\mathbf{R}_{k,j}^{-1}\mathbf{d}_k}{1+\mathbf{d}_k^H\mathbf{R}_{k,j}^{-1}\mathbf{d}_k}\mathbf{A}_k^H\mathbf{R}_{k,j}^{-1}\mathbf{d}_j - \frac{|\mathbf{d}_j^H\mathbf{R}_{k,j}^{-1}\mathbf{d}_k|^2}{(1+\mathbf{d}_k^H\mathbf{R}_{k,j}^{-1}\mathbf{d}_k)^2}\mathbf{A}_k^H\mathbf{R}_{k,j}^{-1}\mathbf{d}_k\right\}. \quad (2.17)\end{aligned}$$

By using sequential technique, we update $\mathbf{c}_k$ with gradient descent and λ_k with gradient ascent to let (2.16) have a stationary point which corresponds to the constrained minimum of (2.4). Augmented Lagrangian method is also attractive because it is easy to change the parameters to generate a suitable sequence of unconstrained problem and only moderate values of λ_k are needed.

All these Lagrange multiplier methods are common and useful techniques for constrained optimization. By these methods, at least a local minimum can be found [32]. It is interesting to observe that all the above methods try to solve the problem in an FDMA way with optimal frequency assignment. Each transmitter concentrates almost all power in the least fading carrier, which does not suffer deep fading and much interference. For the case that the sub-carrier with the least fading characteristic for different users may be the same, the noise and interference correlation matrix $\mathbf{R}_k$ is also used for the decision of power allocation. For comparison, we discuss the truly optimal FDMA system in the next section.

2.4 Optimal FDMA System

In a pure FDMA system, each user gains access to a distinct carrier for transmission. In wireless communications, it is common that different carriers undergo different level of fading process. If carriers are assigned randomly to users, some deep fading sub-channels will probably be used, resulting that much more power is needed to meet the least acceptable performance. To avoid this situation, we consider a new FDMA system with optimal frequency assignment. We assume an FDMA system with K users and M carriers where $K \leq M$. We define the optimal system as an FDMA system where the minimal average power is used to achieve the target SNRs for all users. Given the sub-channel coefficients $\alpha_{k,m}$, the problem of finding the optimal FDMA system can be identified as an assignment problem, and can be solved by the well-known Hungarian method [34], which is also given in Appendix A.

For simplicity of illustration, we assume equal number of users and carriers, for $k, m = 1, ..., n$. The problem is usually described in terms of matching n objects with n other objects in a one-to-one fashion. For the fading behavior of different carriers for different users, we can build a matrix with fading coefficients as its entries

$$\alpha = \begin{bmatrix} \alpha_{1,1} & \alpha_{1,2} & \cdots & \alpha_{1,M} \\ \alpha_{2,1} & \alpha_{2,2} & & \vdots \\ \vdots & & \ddots & \alpha_{K-1,M} \\ \alpha_{K,1} & \cdots & \alpha_{K,M-1} & \alpha_{K,M} \end{bmatrix}, \tag{2.18}$$

where $K = M = n$. Under perfect power control, we also know the equation between $\alpha_{k,m}$ and the target SNR_k as follows

$$|\alpha_{k,\tilde{m}}|^2 \frac{\left|c_{\tilde{m}}^{(k)}\right|^2}{2N_0 W} = \text{target SNR}_k, \tag{2.19}$$

where $\tilde{m}$ is the carrier chosen for transmission, and W is the bandwidth of the sub-channel. Therefore the relationship between $\alpha_{k,\tilde{m}}$ and the transmission gain $c_{\tilde{m}}^{(k)}$ is

$$\left|c_{\tilde{m}}^{(k)}\right| \propto \frac{1}{|\alpha_{k,\tilde{m}}|}, \tag{2.20}$$

where the target SNR_k is assumed fixed.

We now introduce the general model. Let

n be the number of carriers (which is also assumed to be the number of users),

$\beta_{k,m}$ be proportional to the corresponding transmission gain to achieve the target SNR_k, and thus the cost matrix is written as

$$\beta = \begin{bmatrix} \beta_{1,1} & \beta_{1,2} & \cdots & \beta_{1,M} \\ \beta_{2,1} & \beta_{2,2} & & \vdots \\ \vdots & & \ddots & \beta_{K-1,M} \\ \beta_{K,1} & \cdots & \beta_{K,M-1} & \beta_{K,M} \end{bmatrix}, \tag{2.21}$$

where $\beta_{k,m} = \frac{1}{|\alpha_{k,m}|}$.

The problem is to

$$\begin{aligned} \min \quad & \textstyle\sum_{k=1}^{n} \sum_{m=1}^{n} \beta_{k,m} x_{k,m}, \\ \text{subject to} \quad & \textstyle\sum_{m=1}^{n} x_{k,m} = 1, \quad k = 1, ..., n, \\ & \textstyle\sum_{k=1}^{n} x_{k,m} = 1, \quad m = 1, ..., n, \\ & \text{where } \ x_{k,m} = 0 \text{ or } 1, \quad k, m = 1, ..., n. \end{aligned} \tag{2.22}$$

It is a standard assignment problem, and we solve it with the Hungarian method. We can view it as a kind of centralized optimization approach which also let the system obtain good performance to some extent. When $K < M$, this optimal system still works. Instead of using a K by M cost matrix, we still construct an M by M matrix with $\frac{1}{|\alpha_{k,m}|}$ in the normal entries and infinity in other indefinite entries.

2.5 Modified Centralized Optimization Schemes

In Section Three of this chapter, we studied the Lagrange multiplier methods. Actually, they are slow in reaching targets. On the other hand, the computational time of the ideal FDMA system with optimal frequency assignment is still quite large through using the Hungarian method. Actually, optimal FDMA is not optimal in all cases (i.e., $K > M$, to be considered in Chapter 4). In this section, we modify the methods in Section Three to solve the problem more efficiently. This alternative approach is taken toward the aim of increasing the speed of reaching targets. The system can still be implemented iteratively by the sequential techniques of the Lagrange multiplier methods. The idea of the modified algorithms is that after a new $\mathbf{c}_k$ is updated in the gradient process, we add a brute force step to make the $\mathbf{c}_k$ approach the target SNR much faster. It is implemented as power control in wireless communication systems. The new $\mathbf{c}_k$ is scaled by the following equation

$$\mathbf{c}_k = \frac{\tilde{\mathbf{c}}_k \sqrt{\gamma_k}}{\sqrt{\tilde{\mathbf{d}}_k^H \mathbf{R}_k^{-1} \tilde{\mathbf{d}}_k}}. \tag{2.23}$$

To avoid abrupt changing in $\mathbf{c}_k$, we may apply power control with graceful steps. For example, we use a sequence of targets $\gamma_k(n)$ in (2.23) to compute the corresponding kth gain vector in the nth iteration. The value of $\gamma_k(n)$ changes, and the final value will be the ultimate target for the kth user to arrive at eventually. For penalty function method, Powell's method and Augmented Lagrangian method, the sequence of target SNRs is set in ascent order. For barrier function method, the sequence of target SNRs is set in descent order.

So, a simple and fast recursive algorithm is designed in this way:

1. In the nth iteration, from the estimation of $\mathbf{d}_k(n)$ and $\mathbf{R}_k(n)$, the optimal weight vector $\mathbf{w}_k(n)$ is computed based on (2.1).

2. For the formulated Lagrangian function, i.e., with penalty function method, the $\mathbf{c}_k(n)$ is updated by a gradient method. Thus the new $\mathbf{c}_k(n+1)$ is found.

3. Power control is applied to scale the $\mathbf{c}_k(n+1)$ vector according to (2.23).

2.6 Performance

In this section, the system performance is evaluated using Monte Carlo simulation. By simulation, we demonstrate the behavior of the proposed centralized MC-CDMA transmission schemes and investigate the performance of those algorithms. In the first part, we analyze the typical behavior of each adaptive scheme and in the second part, we run a mass of simulation to show the average performance and draw conclusions from them.

2.6.1 Typical Behavior

First, we see the situation of only one user in the centralized MC-CDMA system with 8 carriers. The signal to thermal noise ratio (STNR) is fixed at 10dB and each carrier undergoes independent Rayleigh fading process. We set the target SNR for the user to be 10dB. Using the Lagrangian function of penalty type for the optimization, the typical behavior of the MC-CDMA system is shown in Figure 2.1. We see that much more power will be assigned to the least fading carrier after the user reached the target SNR.

Next, the behavior of the Lagrange multiplier methods is studied by the following figures. In this set of simulation, we assume that 8 users transmit data using 8 carriers and the fading pattern is assumed to be the same throughout the analysis for different methods. Again, the target SNRs are all 10dB. We apply each Lagrangian and show the behavior with the power allocation under the same fading process in the figures. From Figure 2.2 to Figure 2.5, penalty

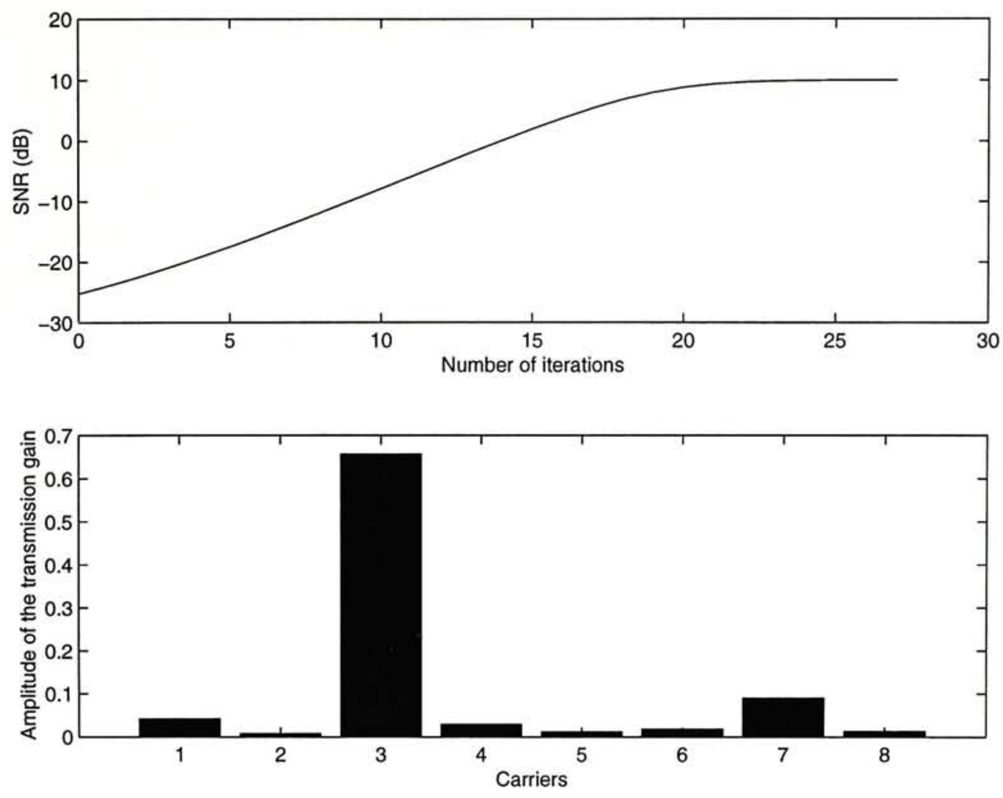

Figure 2.1: Centralized transmission scheme with a single user

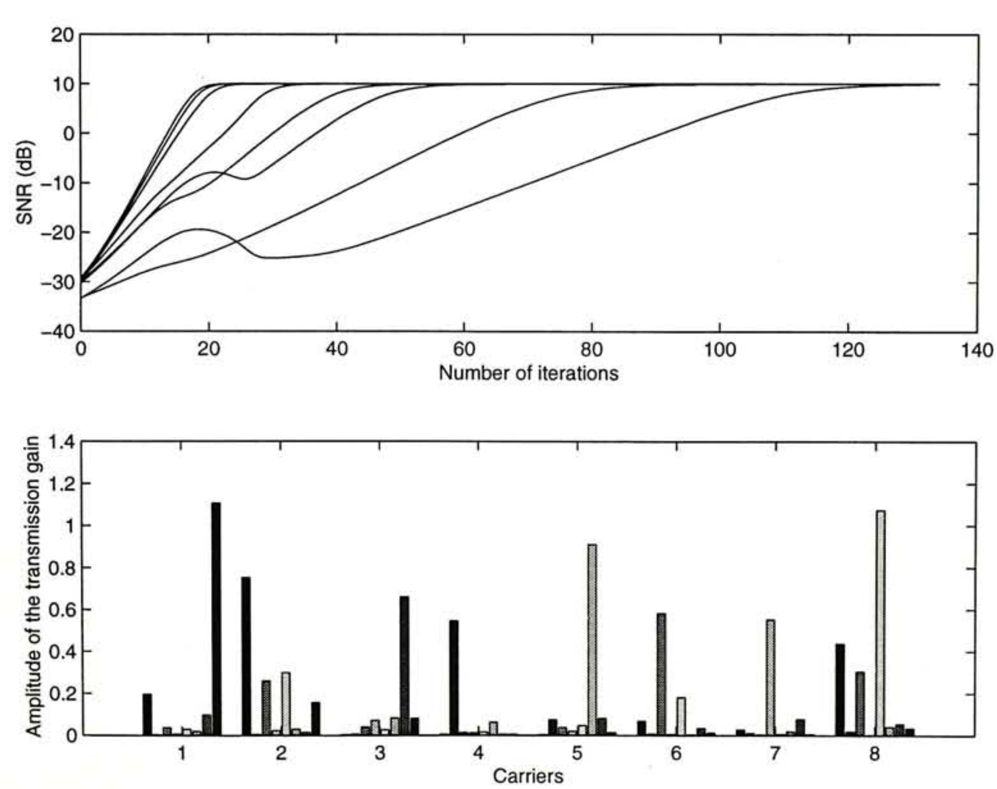

Figure 2.2: Centralized scheme incorporating penalty function method

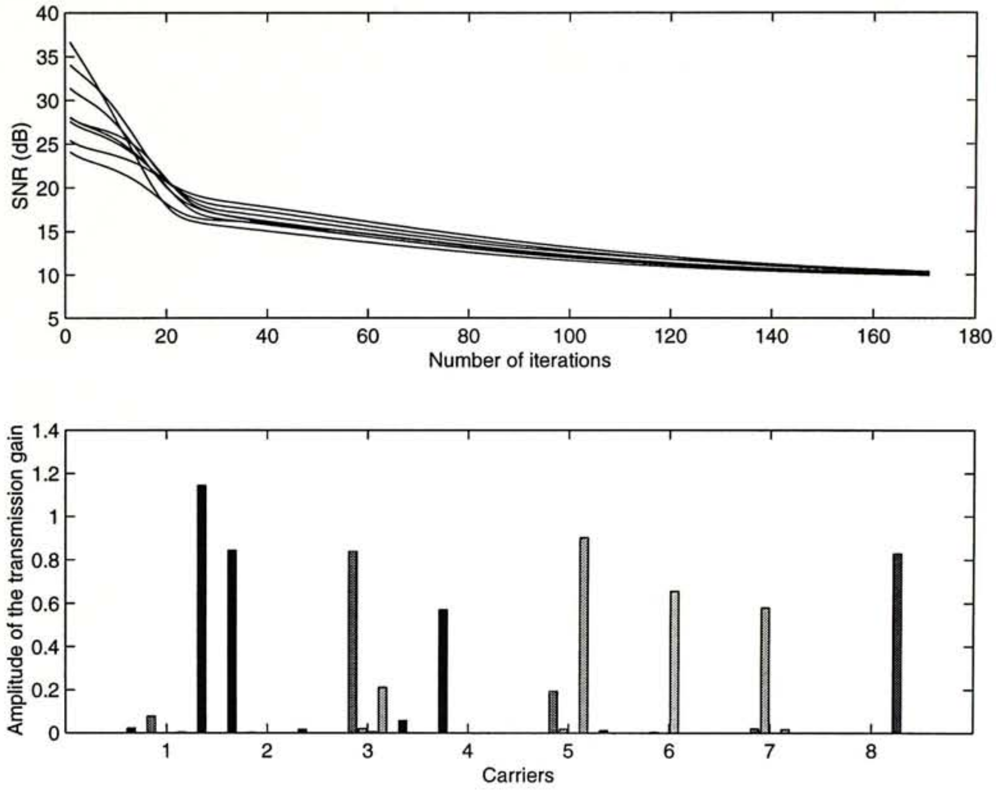

Figure 2.3: Centralized scheme incorporating barrier function method

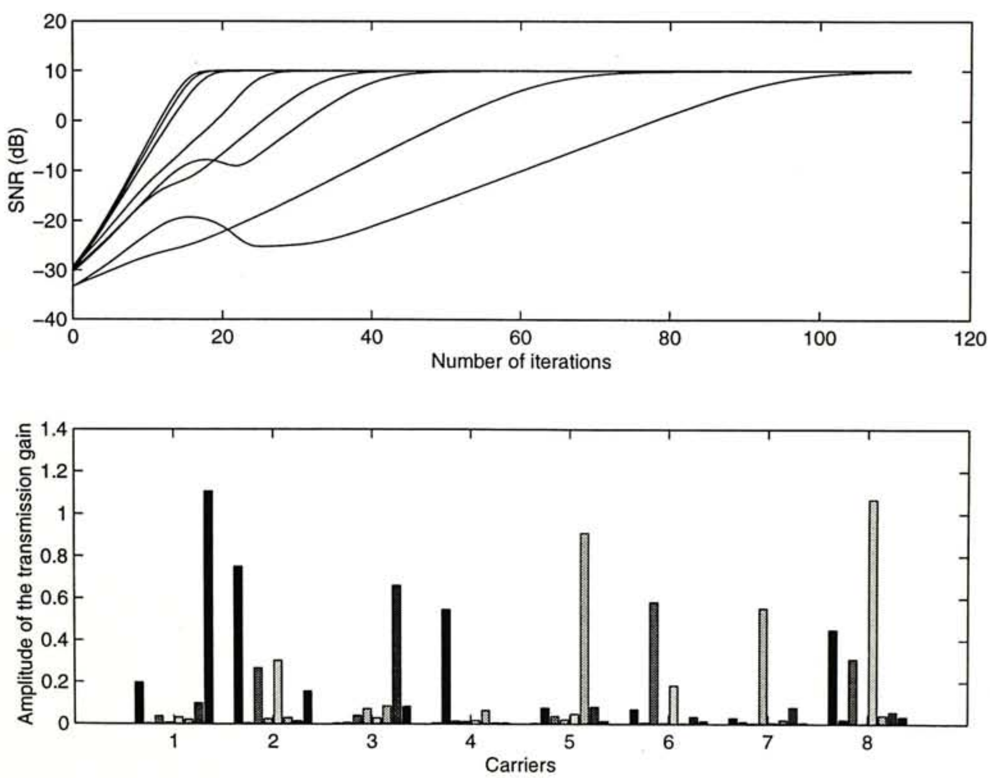

Figure 2.4: Centralized scheme incorporating Powell's method

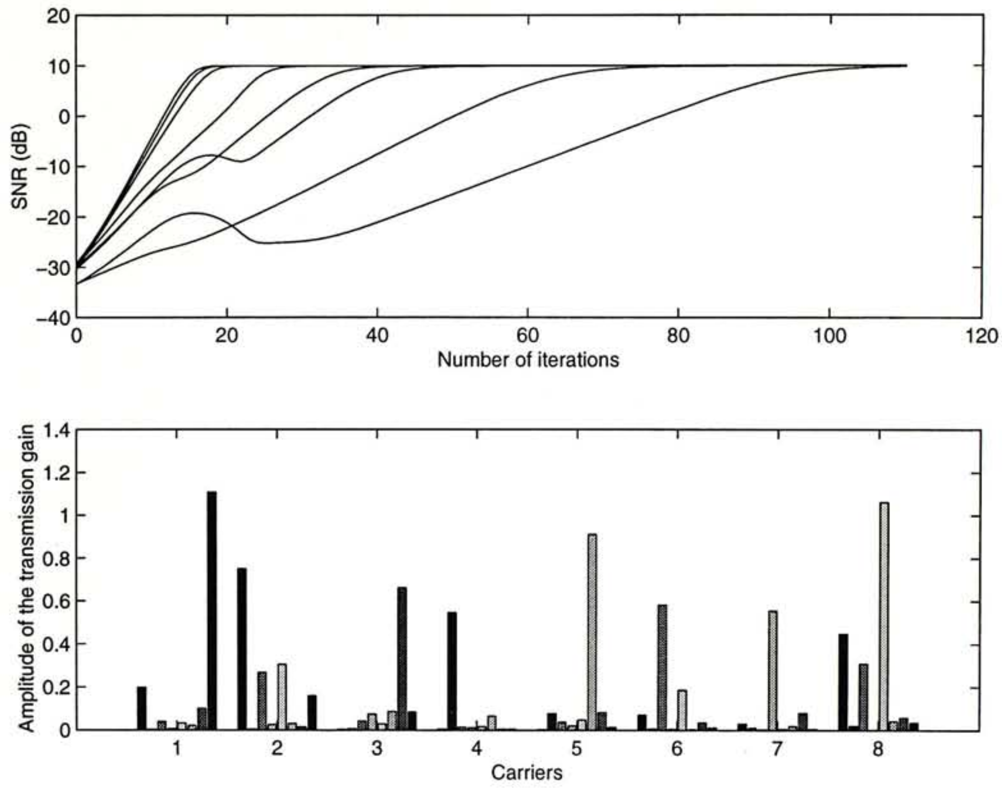

Figure 2.5: Centralized scheme incorporating Augmented Lagrangian method

function method, barrier function method, Powell's method and Augmented Lagrangian method are shown one by one. We see that all the Lagrangian functions have the performance in which every user in the system reaches the target SNR and the power settles down. On the average, the time for reaching targets is about 120 iterations for penalty function method, Powell's method and Augmented Lagrangian method. It takes more iterations for the users to reach targets with Barrier function method. We also see that each user tries to place all power in one specific carrier when $M \leq K$. While migrating to multiuser communications, this characteristic of power concentration by using the Lagrangian functions for the optimization provides the advantage of reducing the MAI. We will give a more thorough description about this interesting resultant behavior in the next part of this section.

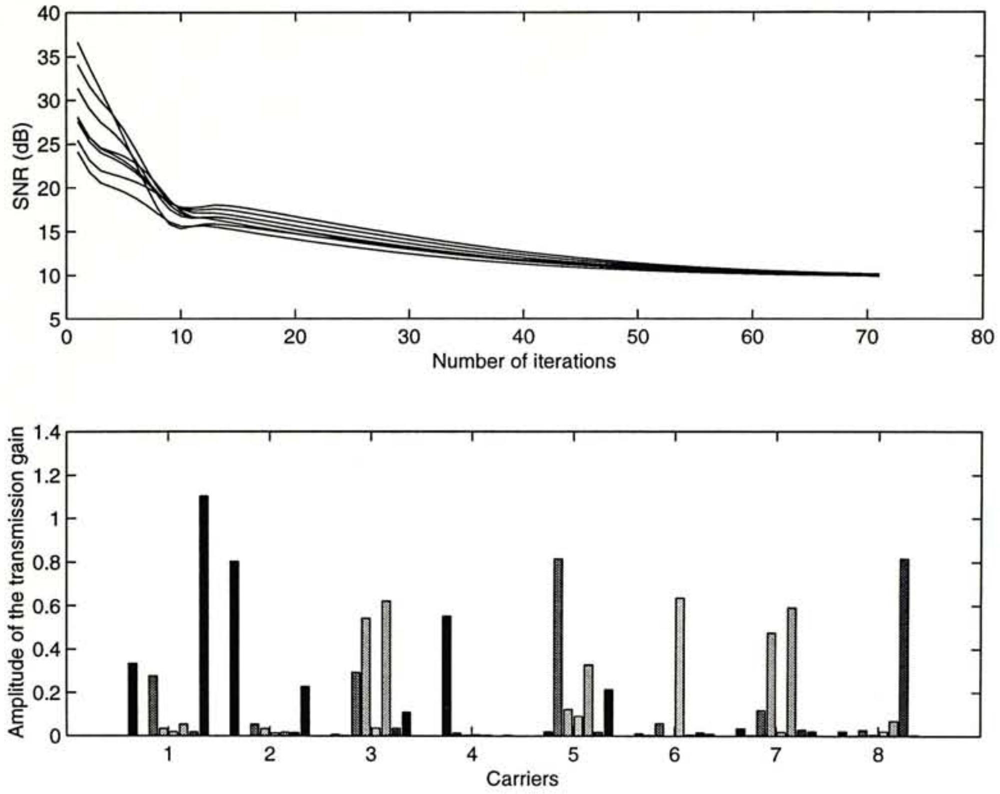

Figure 2.6: Modified centralized algorithm with barrier function

In Figure 2.3, with barrier function method, each user seems to place less power in the carriers other than the least fading carrier than the other three methods. At this stage, from the point of power consumption, barrier function method appears to be the most attractive one in the Lagrange multiplier methods since the least average power is used comparing the others. Therefore we choose it to check the typical behavior of the modified centralized optimization schemes. The result is shown in Figure 2.6. Based on the Lagrangian algorithm incorporating barrier function, the modified one shortens the time needed for reaching targets successfully, but results in a bit more power assigned in other carriers, which suffer more channel impairments than the least fading carrier. For the large number of iterations needed for reaching targets of algorithms proposed previously, it can be viewed as a trade off to give quite remarkable

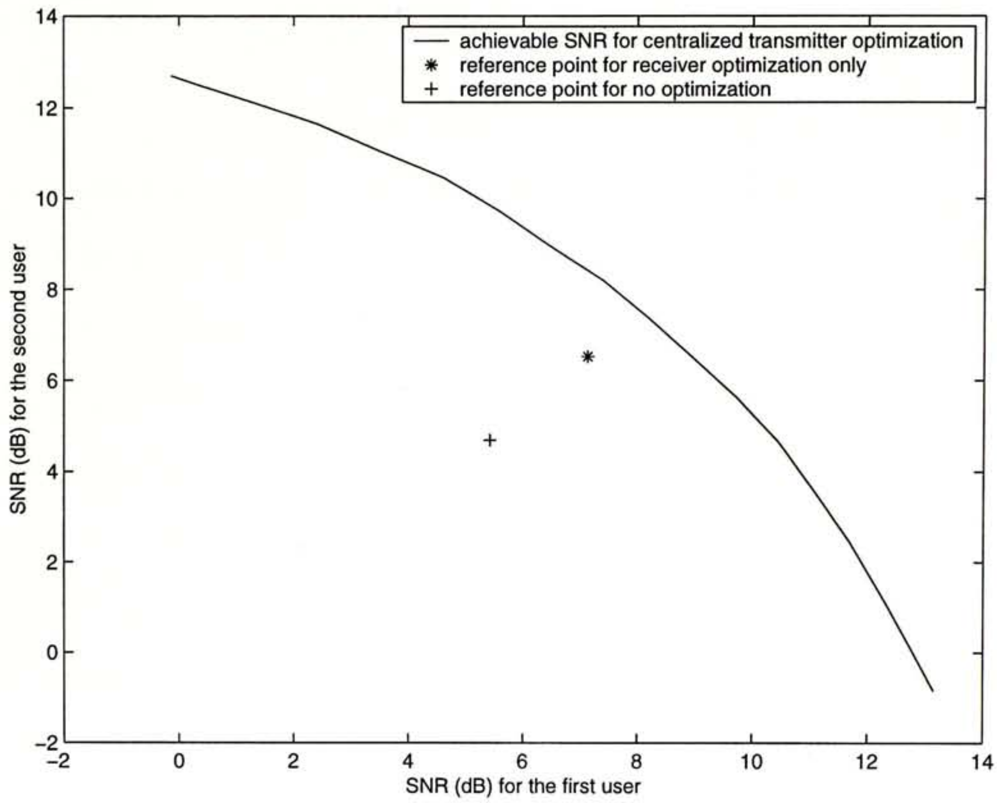

Figure 2.7: Average performance bound of two users with different optimization schemes

improvement in speed.

2.6.2 Average Performance

After illustrating the typical behavior of the methods for the optimization proposed in this chapter, we evaluate the average performance of them in this part. All figures shown are the results of 500 realizations.

For simplicity, we first consider the two-user case to give a rough picture of the improvement. We propose another criterion which suits the comparison. From the point of view of the first user, we would like to maximize SNR_1 without using more total power. We can define the problem as

$$\max \quad \mathbf{d}_1^H \mathbf{R}_1^{-1} \mathbf{d}_1,$$

$$\text{subject to} \quad \mathbf{d}_2^H \mathbf{R}_2^{-1} \mathbf{d}_2 = \gamma_2, \qquad \tfrac{1}{2}(\mathbf{c}_1^H \mathbf{c}_1 + \mathbf{c}_2^H \mathbf{c}_2) = 1, \tag{2.24}$$

where γ_2 denotes the second user's target SNR. $\mathbf{c}_k^H \mathbf{c}_k$, for $k = 1$ or 2, is the power used by the kth user and the average power of the two users is unity. As a centralized optimization scheme, this criterion is another viewpoint of the optimization problem of (2.4). We can use it to show the improvement over the cases of receiver optimization only and no optimization. Figure 2.7 presents, after performing receiver optimization, the achievable average SNR of the two users with the centralized transmitter optimization. The figure also shows the reference points of the cases of receiver optimization only and no optimization in both transmitter and receiver for completeness. The x and y axes show the SNRs of the first and second user, respectively. The number of carriers is fixed at $M = 4$ and each respective carrier of the three schemes suffers the same independent Rayleigh fading in the same realization. The sum of the two users' power is 2. The transmission gains for transmitter optimization are obtained via penalty function method. In this figure, we observe that the average SNR performance of the centralized transmitter optimization is better than the other two cases. We also see the relations of the performance of the two users. The centralized transmitter optimization provides an easy way to achieve multitargets. The users in the system can compromise with each other to give the desirable SNRs effectively.

We evaluate the power consumption of the Lagrange multiplier methods for reaching targets. In Figure 2.8 and Figure 2.9, we fix $M = 8$ and let K increase from 2 to 8. Again, we see that penalty function method, Powell's method and Augmented Lagrangian method have similar performance. The performance of

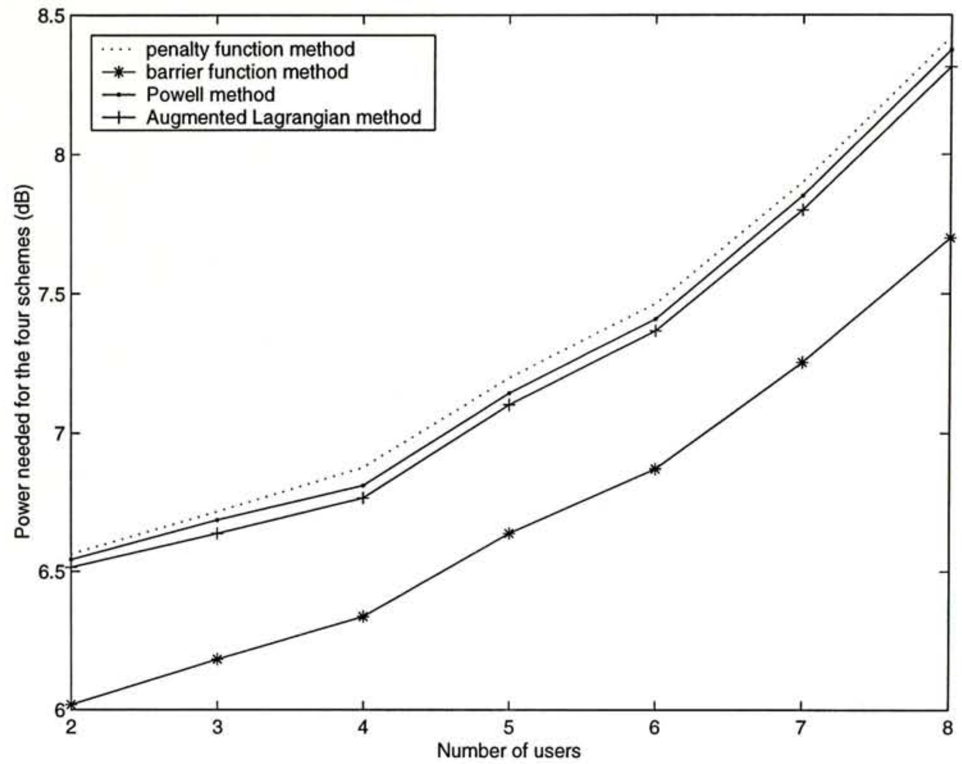

Figure 2.8: Power consumption for different centralized schemes without restrictions in iterations

barrier function method looks different. We know that the number of iterations and power needed for reaching target SNRs is affected by the initial conditions and parameters in the adaptive procedure. In Figure 2.8, we disregard the iterations needed for different centralized transmission schemes to reach targets and fine-tune the parameters to minimize the power consumption. The power shown in the figure is normalized to additive white Gaussian noise (AWGN). In terms of power needed, barrier function method is the best among the four methods.

In Figure 2.9, we try to confine the number of iterations needed for reaching targets to be around 80 in all cases. After running 500 realizations, the number of iterations in each realization for the four methods to reach targets is between 65 and 95. Some interesting results arise. When the number of carriers is much

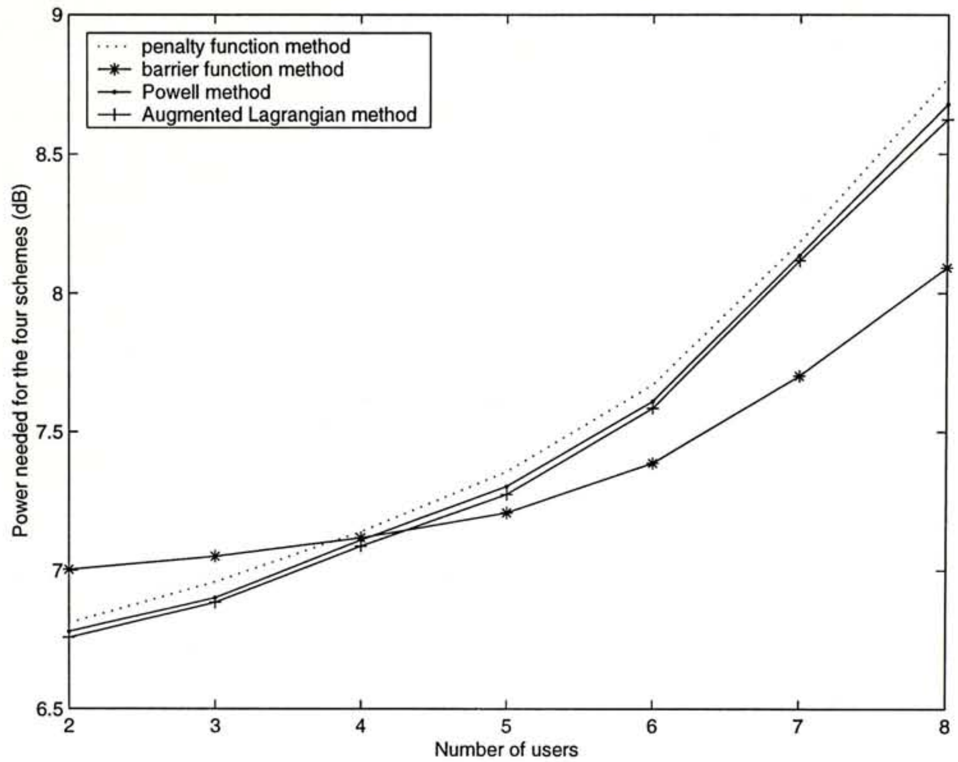

Figure 2.9: Power consumption for different centralized schemes with restrictions in iterations

more than the number of users, barrier function method does not appear as a good choice for the optimization. In this case, it requires more power than the other three methods. With small number of users, Augmented Lagrangian method is the best choice for centralized adaptive transmission with the least power consumption and most stable performance. When the number of users approaches the number of carriers, barrier function method is very attractive for power saving. As the number of users increases, penalty function method, Powell's method and Augmented Lagrangian method will need more iterations for reaching targets. On the other hand, the behavior of barrier function method looks different. For barrier function method, the number of iterations does not increase as the number of users increases. Therefore, in this situation, barrier function method is the best.

In Table 2.1 and Table 2.2, with 8 carriers and 8 users in each Lagrange multiplier method and its corresponding modified version, the performance is presented. Again, regardless of the speed, we show the power consumption for each method and its modified version in the first table. The respective number of iterations required is showed is the second table. We see that by sacrificing power, relatively speaking, about half of the iterations are saved for reaching targets in each Lagrange multiplier method. Modified algorithm based on barrier function seems to be the most attractive one because only very little additional power is needed for the improvement in iterations.

Table 2.1: Power needed for centralized schemes and their modified versions (500 realizations)

Different method	Power (original)	Power (modified)	Power increased
Penalty function	8.42 dB	9.34 dB	24 %
Barrier function	7.70 dB	7.73 dB	7 %
Powell's method	8.38 dB	9.09 dB	18 %
A L method	8.31 dB	9.08 dB	19 %

Table 2.2: Iterations needed for centralized schemes and their modified versions (500 realizations)

Different method	Iterations (original)	Iterations (modified)	Iterations saved
Penalty function	151	69	54 %
Barrier function	199	84	58 %
Powell's method	138	68	51 %
A L method	126	67	47 %

For all centralized transmission schemes shown above, we observe that every transmitter tends to select one carrier for transmission. In Section Four, we define an ideal FDMA system with optimal frequency assignment. We use it as the basis to evaluate the average power consumption of the centralized methods

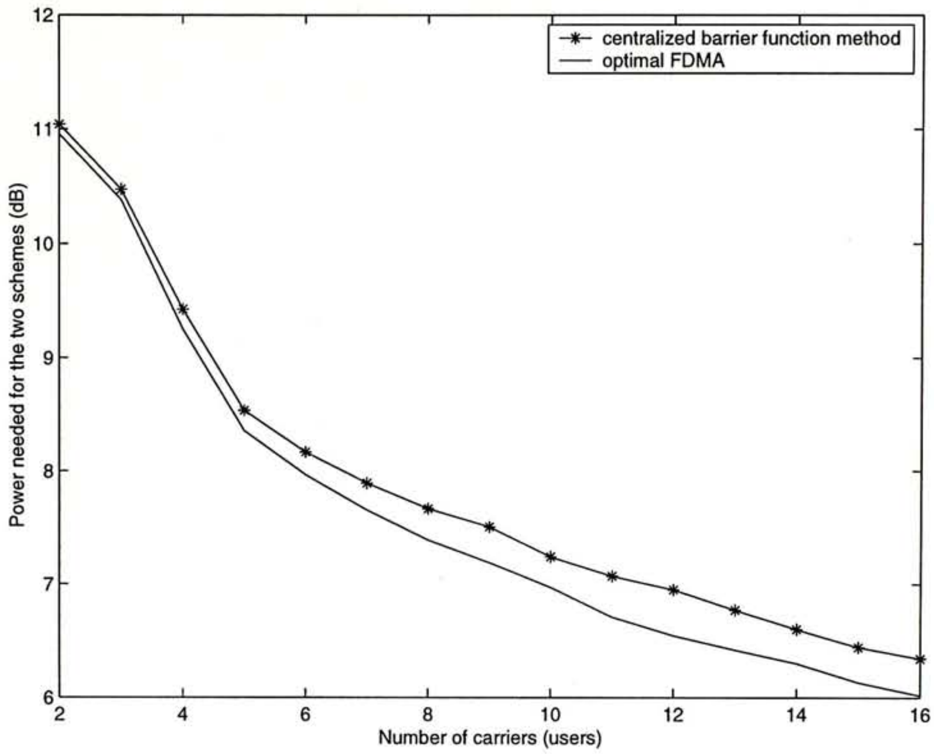

Figure 2.10: Power needed for the centralized MC-CDMA system and the optimal FDMA system (K=M)

in Figure 2.10 and Figure 2.11. We assume the same number of carriers and users in Figure 2.10 and 8 carriers in Figure 2.11. From these figures, we see that the performance of the MC-CDMA system with the centralized transmission scheme via modified version of barrier function method is nearly as good as the optimal FDMA system. In terms of power consumption, the new centralized transmission schemes are very attractive for use in implementation of real systems.

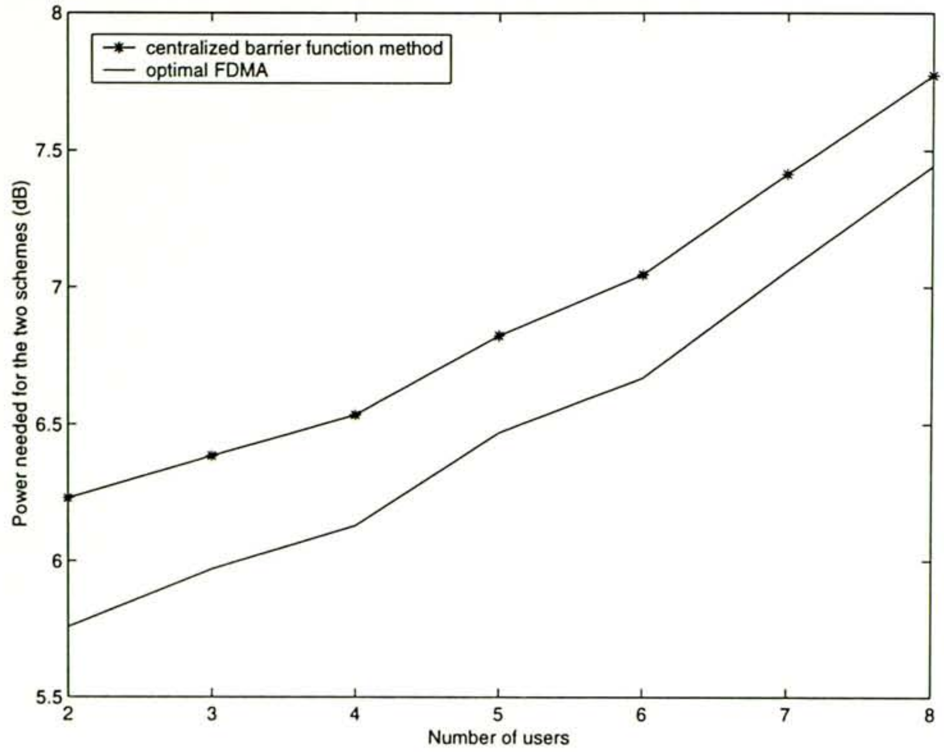

Figure 2.11: Power needed for the centralized MC-CDMA system and the optimal FDMA system (K=2 to 8, M=8)

2.7 Summary

In this chapter, joint optimization of the MC-CDMA transmitter and receiver is proposed. We use the Lagrange multiplier methods to solve the constrained optimization problem. When $K \leq M$, the goal of reaching targets can be achieved by the four Lagrangian functions. Then, modified schemes are derived to reduce the number of iterations needed to reach targets. Performance of the proposed centralized transmission schemes is compared with an optimal FDMA system. Besides approaching the behavior of the power concentration of FDMA, we further demonstrates that the power consumption of these centralized algorithms is fairly as good as the optimal FDMA system.

Chapter 3

Decentralized Transmitter Optimization for MC-CDMA Systems

3.1 Introduction

In this chapter, we consider the decentralized transmitter optimization for multicarrier code division multiple access (MC-CDMA) systems. We focus on the scenario that multiple users communicate through the same set of parallel sub-channels with different fading in different sub-carriers and users. Power is assigned to any of the sub-carriers depending on the state of the fading process among the sub-carriers. Through suitably choosing the weight for each branch in the receiver and the gain for each branch in the transmitter, the system tries to achieve the target signal to noise ratio (SNR) for each user with the minimal amount of average power.

In Chapter 2, we assume there exists a centralized controller handling the computation. Centralized knowledge is needed in the adaptation process. In this chapter, a different approach in which only decentralized information is needed for transmitter optimization is employed. The optimization process for each pair of transmitter and receiver is performed adaptively and independently. It offers the same advantages of the proposed centralized adaptive transmission schemes in terms of power consumption and multiple access interference (MAI) reduction, and it has the major improvement in performance over the conventional MC-CDMA system. Similar to the previous chapter, we observe the interesting result: the MC-CDMA system with the decentralized adaptive transmission scheme tends to an FDMA system with optimal frequency assignment in many cases.

Also consider that wireless communication systems of future generations are expected to support multimedia applications; thus, MC-CDMA systems should be able to serve integrated traffic generated by different types of sources, such as voice, video, and data. It is essential that this integrated traffic should be accommodated in a transmission efficient manner with the Quality of Service (QoS) requirements of various types of applications [35], [36]. A multirate MC-CDMA system based on the decentralized transmission scheme is proposed to provide multimedia services with graceful variation in the QoS for different usage.

A brief outline of this chapter is as follows. In Section Two, we establish the system model suitable for the following analysis. In Section Three, we study the optimization process. We consider both receiver optimization and transmitter optimization from a single user's point of view. Then we develop the decentralized adaptive transmission scheme. The modification for the decentralized

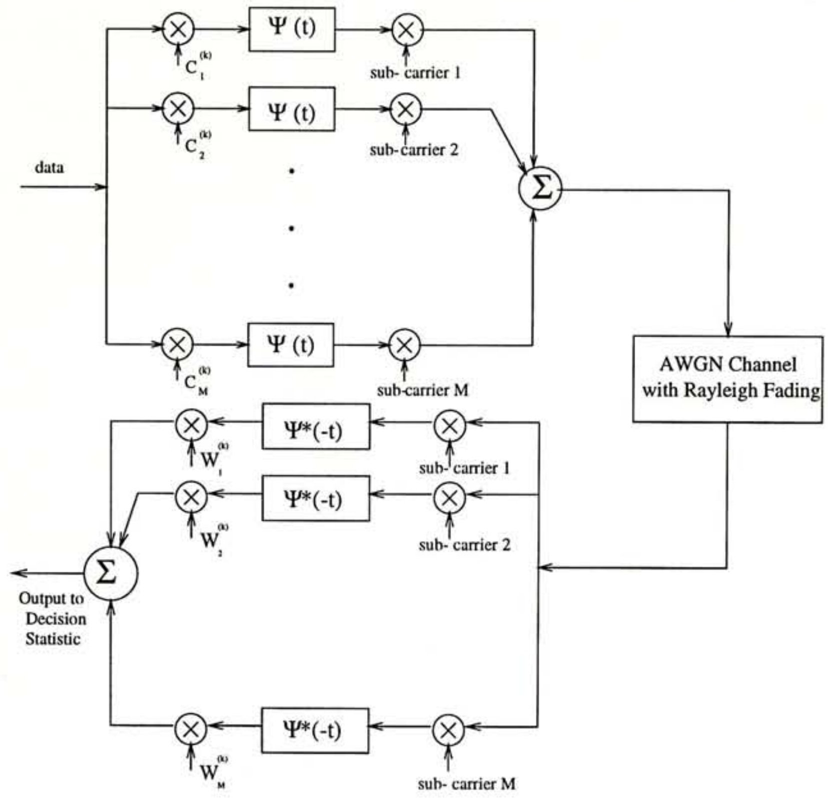

Figure 3.1: Block diagram of the MC transmission scheme

transmission scheme to support multirate services is discussed. Thus the MC-CDMA system can be used in wireless multimedia communications. Simulation results are shown in Section Four. Summary is drawn in Section Five.

3.2 System Model

In this section, we describe the model of the MC-CDMA system. We assume that there are K simultaneous users in the system, and each user uses the same M carriers.

We use the same system model derived previously for analysis. An adaptive transmission scheme for the kth user, for $1 \leq k \leq K$, of the system is shown in Figure 3.1. The input data stream is copied to all M branches. The data sub-stream in the mth branch, for $1 \leq m \leq M$, is multiplied by the gain factor $\mathbf{c}_m^{(k)}$ on each branch before it modulates the corresponding sub-carrier. For convenience, we define an M-dimensional vector

$$\mathbf{c}_k = [c_1^{(k)}, c_2^{(k)}, \ldots, c_M^{(k)}]^T \tag{3.1}$$

for the kth user.

Each receiver consists of M branches. Each branch consists of a demodulator, which is responsible for the demodulation of the sub-carrier, and an appropriate weight. We define the weight vector

$$\mathbf{w}_k = [w_1^{(k)}, w_2^{(k)}, \ldots, w_M^{(k)}]^T \tag{3.2}$$

for the kth user. It is an M-dimensional vector that combines the contributions from the M branches of the kth user to give the decision statistic.

3.3 Optimization

We first consider optimizing the receiver for the kth user by choosing an appropriate weight vector $\mathbf{w}_k$. We then consider optimizing the transmitter of the kth user from his own point of view by choosing an appropriate (complex) gain vector $\mathbf{c}_k$. The decentralized transmission scheme is obtained when all users perform the same transmitter and receiver optimization independently. Multirate transmission with this decentralized transmission scheme is derived, and will be presented in the last part of this section.

3.3.1 Receiver Optimization

Notice that optimization of the receiver for the first user only affects the performance of the first user, and does not affect the performance of other users. Since we are considering the first user, we can assume the delay T_0 of the first user to be zero. We use the method from [18] to optimize the output signals from the demodulators. Similar to the previous chapters, we consider the detection of $b_0^{(1)}$. The overall output of the demodulators, in vector form, is expressed by

$$\mathbf{z}_1 = b_0^{(1)}\mathbf{d}_1 + \mathbf{n}_1 + \sum_{k=2}^{K} \mathbf{i}_k^{(1)}, \tag{3.3}$$

where $\mathbf{d}_1 = [d_1^{(1)}\ d_2^{(1)}\ \ldots\ d_M^{(1)}]^T$ is the vector which summarize the total transmission effects in the transmitter and channel, $\mathbf{n}_1 = [n_1^{(1)}\ n_2^{(1)}\ \ldots\ n_M^{(1)}]^T$ is the AWGN noise vector and $\mathbf{i}_k^{(1)} = [i_{k,1}^{(1)}\ i_{k,2}^{(1)}\ \ldots\ i_{k,M}^{(1)}]^T$ is the interference contributed by the kth user seen by the first user. At this stage, we construct the noise and interference correlation matrix

$$\mathbf{R}_1 = \mathrm{E}_\alpha\left[\mathbf{n}_1\mathbf{n}_1^H + \sum_{k=2}^{K} \mathbf{i}_k^{(1)}\mathbf{i}_k^{(1)H}\right], \tag{3.4}$$

where $\mathrm{E}_\alpha[\cdot]$ denotes the conditional expectation given $\alpha_{k,m}$, for $k = 1, \ldots, K$ and $m = 1, \ldots, M$, and the superscript H denotes the Hermitian operation. We can express the SNR_1 in the following manner

$$\mathrm{SNR}_1 = \frac{|\mathbf{w}_1^H\mathbf{d}_1|^2}{\mathrm{E}_\alpha[|\mathbf{w}_1^H(\mathbf{n}_1 + \sum_{k=2}^{K}\mathbf{i}_k^{(1)})|^2]} = \frac{|\mathbf{w}_1^H\mathbf{d}_1|^2}{\mathbf{w}_1^H\mathbf{R}_1\mathbf{w}_1}. \tag{3.5}$$

Again,

$$\mathbf{w}_1 = \mathbf{R}_1^{-1}\mathbf{d}_1 \tag{3.6}$$

is the result that we arrive at.

3.3.2 Single-user Transmitter Optimization

We consider optimizing the transmitter of the first user from the point of view of the first user only. With the optimal weight vector, the SNR for the first user is given by

$$\mathrm{SNR}_1 = \mathbf{d}_1^H \mathbf{R}_1^{-1} \mathbf{d}_1. \tag{3.7}$$

The desired signal vector $\mathbf{d}_1$ can be rewritten as

$$\mathbf{d}_1 = \mathbf{A}_1 \mathbf{c}_1, \tag{3.8}$$

where $\mathbf{A}_1$ is an $M \times M$ diagonal matrix whose mth diagonal element is $T_s \alpha_{1,m}$. Therefore, the SNR at the receiver of the first user depends on the gain vector $\mathbf{c}_1$ according to the relationship

$$\mathrm{SNR}_1 = \mathbf{c}_1^H \mathbf{C}_1 \mathbf{c}_1, \tag{3.9}$$

where $\mathbf{C}_1 = \mathbf{A}_1^H \mathbf{R}_1^{-1} \mathbf{A}_1$. From the point of view of the first user, we would like to maximize SNR_1 without using more power. We can define the problem as follows:

$$\begin{aligned} &\max \quad \mathbf{c}_1^H \mathbf{C}_1 \ \mathbf{c}_1, \\ &\text{subject to} \quad \mathbf{c}_1^H \mathbf{c}_1 = 1. \end{aligned} \tag{3.10}$$

By the method of Lagrange multiplier, we obtain the following equation whose solution solves the optimization problem

$$\mathbf{C}_1 \mathbf{c}_1 = \lambda \mathbf{c}_1, \tag{3.11}$$

where λ is the Lagrange multiplier. It shows that $\mathbf{c}_1$ should be chosen as the eigenvector associated with the largest eigenvalue of $\mathbf{C}_1$.

In many cases, the first user only requires a certain target SNR. Alternatively, we may try to minimize the power needed to achieve the target SNR. Therefore, we can, instead of maximizing the SNR with fixed power, consider the following optimization problem:

$$\begin{aligned} \min \quad & \mathbf{c}_1^H \mathbf{c}_1, \\ \text{subject to} \quad & \mathbf{c}_1^H \mathbf{C}_1 \mathbf{c}_1 = \gamma_1, \end{aligned} \tag{3.12}$$

where γ_1 is the target SNR of the first user. Essentially, the same solution is obtained for this optimization problem. The gain vector $\mathbf{c}_1$ should be chosen as the eigenvector associated with the largest eigenvalue of $\mathbf{C}_1$. The only difference is that $\mathbf{c}_1$ should be scaled to satisfy the target SNR instead of being normalized.

Physically, the solution of the optimization problem in (3.12) can be considered as a two-step process. In the first step, the eigenvector $\tilde{\mathbf{c}}_1$ that maximizes the SNR is determined up to a constant. An iterative approach to compute the eigenvector is the well-known power method [37]. At the nth iteration, the gain vector is updated as follows:

$$\tilde{\mathbf{c}}_1(n+1) = (1-\mu)\tilde{\mathbf{c}}_1(n) + \mu \mathbf{C}_1 \tilde{\mathbf{c}}_1(n), \tag{3.13}$$

where μ is a constant. In the second step, power control is applied to scale the gain vector so that the target SNR is achieved with the minimum power:

$$\mathbf{c}_1 = \frac{\tilde{\mathbf{c}}_1 \sqrt{\gamma_1}}{\sqrt{\tilde{\mathbf{c}}_1^H \mathbf{C}_1 \tilde{\mathbf{c}}_1}}. \tag{3.14}$$

3.3.3 Decentralized Transmission Scheme

In a multiuser environment, we would like to have all of the users admitted to the system to achieve their target SNRs. (We would need to limit the number of users that can be admitted to the system.) One possible criterion for transmitter optimization is then to use the least average power to achieve the target SNRs for all users. The optimal solution to this problem would necessarily involve a centralized algorithm with co-operation from different users. While the optimal solution can be of interest, a decentralized scheme based on single-user transmitter optimization with good performance is often desirable for simplicity of implementation.

We propose the following decentralized scheme where each user adopts a greedy approach. At each iteration, each user tries to achieve his own target SNR with the minimum amount of power. More precisely, at the nth iteration, the following steps are performed by each pair of transmitter and receiver independently.

1. The receiver collects the required statistics (the desired vector $\mathbf{d}_k(n)$ and the noise and interference correlation matrix $\mathbf{R}_k(n)$) from the received signal, and computes the optimal weight vector $\mathbf{w}_k(n) = \mathbf{R}_k^{-1}(n)\mathbf{d}_k(n)$.

2. The information is also used to update $\mathbf{C}_k(n)$, which is, in turn, used to update the gain vector.

$$\mathbf{c}_k(n+1) = (1-\mu)\mathbf{c}_k(n) + \mu\mathbf{C}_k(n)\mathbf{c}_k(n), \tag{3.15}$$

   where $\mathbf{C}_k(n)$ is the current estimate of $\mathbf{C}_k$, which is the corresponding matrix that determines the SNR for the kth user.

3. Power control is applied to try to achieve the target SNR γ_k based on the current estimate of $\mathbf{C}_k$.

3.3.4 Multirate Transmission with Decentralized Transmission Scheme

In multimedia applications, video, audio and data communications are integrated. Different applications may require different data rates for transmission. In practical considerations, it is very common that some users in the system need to have higher data rates to maintain the communication quality. The decentralized transmission scheme has the behavior of choosing a least fading carrier for transmission when $K \leq M$. A least fading carrier for the kth user is described as the carrier which suffers the smallest amount of fading and interference. In this part, we modify the decentralized transmission scheme to make it suit the use of supporting multirate transmission for MC-CDMA systems.

Since we see that the decentralized transmission scheme tends to concentrate the power on a single carrier, the case that a user needs higher data rate can be engineered as more than one user in the adaptive process. For mathematical simplicity, we define r as the basic rate for transmission. The term transmission rate represents the multiple number of r can be supported by a transmitter, i.e., $1r, 2r, \ldots$. In other words, different types of traffic in the MC-CDMA system will be accomplished with different number of the decentralized adaptive resources. Multiple streams of data are generated according to the traffic and QoS. Thus multirate can be viewed as more adaptive resources are applied to the users who need higher data rates, i.e., if a user wants to double the data rate, not exceeding the system capacity [38], this user can induce the decentralized transmission schemes of two users to meet this goal. The multirate MC-CDMA system can dynamically configure the transmission to meet the QoS needs. In addition, admission control is used in order not to overload the system. To support multirate services, we assume the transmission rate of the possible kth users to be $r_k = n_k r$, where n_k is any positive integer. We define N to be the feasible number of users supported provided that

$$\sum_{k=1}^{N} n_k \leq M. \tag{3.16}$$

By satisfying (3.16), the decentralized MC-CDMA system with the additional modification can work with multiple transmission rates without difficulty. Simulation of this characteristic can be found in Section Four.

3.4 Performance

In this section, we consider the performance of this MC-CDMA system via Monte Carlo simulation. We assume that each carrier of the user undergoes independent Rayleigh fading. The signal to thermal noise ratio (STNR) is 10dB throughout the simulation.

First, we consider the advantage of both transmitter and receiver optimization over receiver optimization only. We consider a system with 8 users and 8 carriers. The first user performs both transmitter and receiver optimization. The other users perform receiver optimization only. These users without performing transmitter optimization are assumed to distribute their power uniformly across all carriers. All users transmit with unity power. The average result of 500 realizations is shown in Figure 3.2. The average SNR of the first user is shown dashed-dotted line while the average SNR of other users is shown in dotted line. It can be seen that with transmitter optimization the SNR is improved by almost 6 dB.

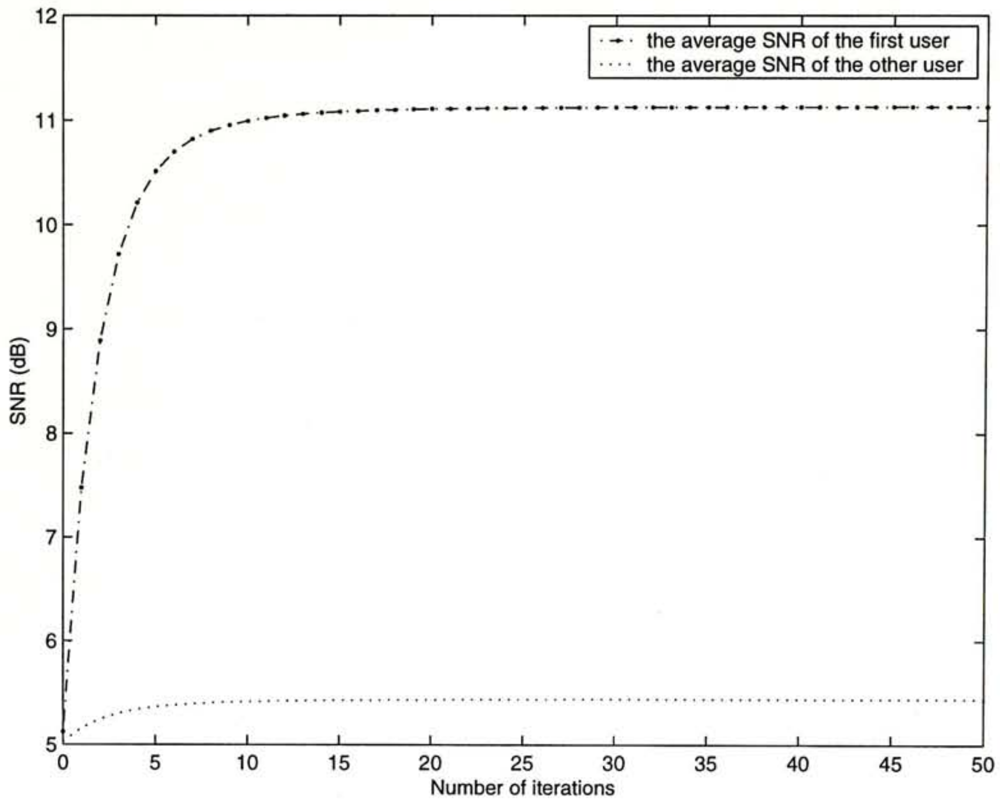

Figure 3.2: Average SNR of a single user with transmitter optimization

Again, to evaluate the performance of the MC-CDMA system with the decentralized transmitter optimization, we consider the optimization problem in (3.10) for a single user for the use of multiuser communications. The criterion for the kth user is written as

$$\max \quad \mathbf{c}_k^H \mathbf{C}_k \; \mathbf{c}_k,$$

$$\text{subject to} \quad \mathbf{c}_k^H \mathbf{c}_k = 1 \qquad \text{for all } k. \tag{3.17}$$

The optimization is that each user applies the same algorithm and tries to achieve the SNR as high as possible while holding the same amount of individual power. $\mathbf{c}_k$ should be chosen as the eigenvector associated with the largest eigenvalue of $\mathbf{C}_k$ and then scaled to give unity power. To compare the SNR performance, we assume there are three MC-CDMA systems, in which the first

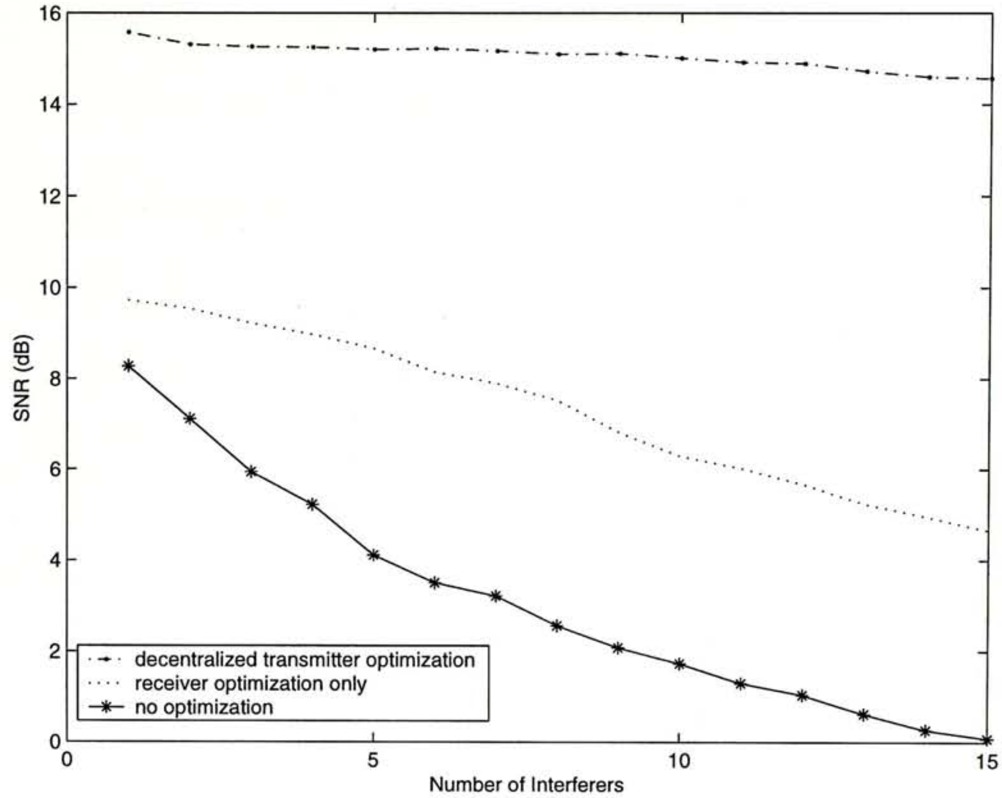

Figure 3.3: Average SNR for the users with different optimization schemes

one works with the decentralized transmitter optimization, the second one operates with receiver optimization only and the third one does no optimization in both transmitter and receiver. Figure 3.3 shows the average SNRs of the users for the three different schemes. The number of carriers is fixed at $M = 16$ and all gain vectors are normalized for a fair comparison. By letting the number of users in the systems increase from 2 to 16, we have the achievable mean SNRs over 500 realizations showed in the figure [1]. As expected, the MC-CDMA system with the decentralized transmitter optimization outperforms the other two systems, even the one with the well-established receiver optimization case for at least 5dB. In Table 3.1, we present the mean and standard deviation of the SNR performance for the three different schemes. We observe that the decentralized

[1]From the decentralized scenario, other users in the system can be treated as interferers.

transmitter optimization scheme has the largest mean SNR value, and so it is attractive for use in MC-CDMA systems with the remarkable improvement.

Table 3.1: Mean and standard deviation of the SNR for different schemes (500 realizations)

	Decen. tx optimization (μ, σ)	Rx optimization only (μ, σ)	No optimization (μ, σ)
K=2	34.51, 8.24 (15.38dB)	9.38, 2.33 (9.72dB)	6.71, 2.55 (8.27dB)
K=4	33.57, 8.11 (15.26dB)	8.34, 2.21 (9.21dB)	3.92, 1.64 (5.93dB)
K=8	32.89, 6.55 (15.17dB)	6.44, 1.88 (8.09dB)	2.09, 0.77 (3.21dB)
K=16	28.64, 5.36 (14.57dB)	2.92, 1.06 (4.65dB)	1.02, 0.37 (0.08dB)

The transient behavior of the mean SNRs of all users for the three schemes is presented in the following two figures. Again, we consider the three systems with 8 users and 8 carriers. After running 500 realizations, for the same notations, Figure 3.4 and Figure 3.5 show the mean SNR performance with and without narrowband interference. The interference is assumed to corrupt the same carrier. We see that the decentralized transmitter optimization is still the best among all the three schemes.

Next, we consider the case when all users perform both transmitter and receiver optimization according to the decentralized adaptive transmission scheme. The typical behavior of each user is shown in Figure 3.6. Regarding to the optimization criterion, the target SNRs for all users are 10 dB and the number of users and carriers is 8. All users achieve their target SNRs in just a small number of iterations. Another merit of the decentralized transmission scheme is the support of multitarget performance. In Figure 3.7, there are 8 carriers and

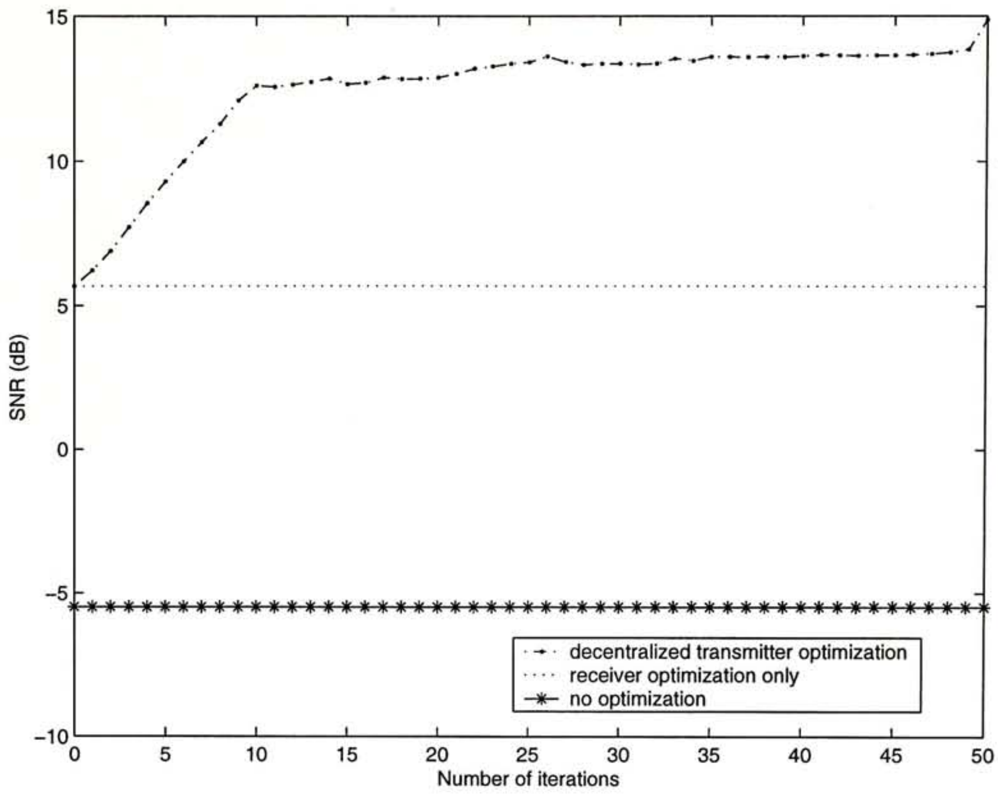

Figure 3.4: Average SNR performance without NB interference

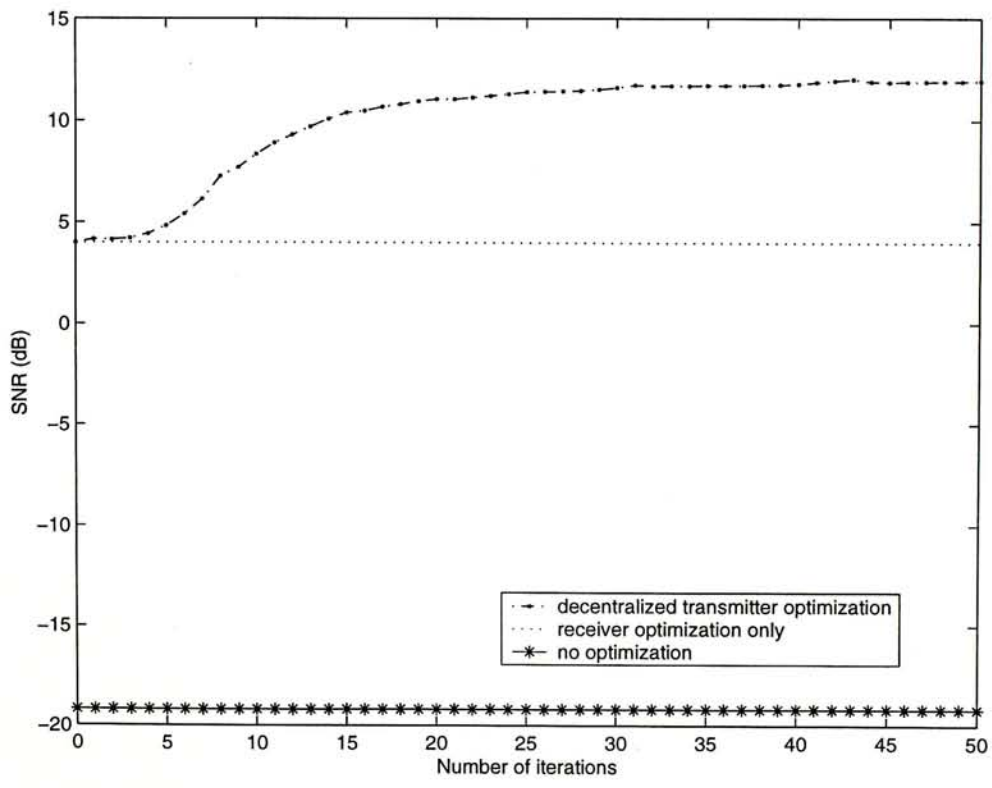

Figure 3.5: Average SNR performance with NB interference

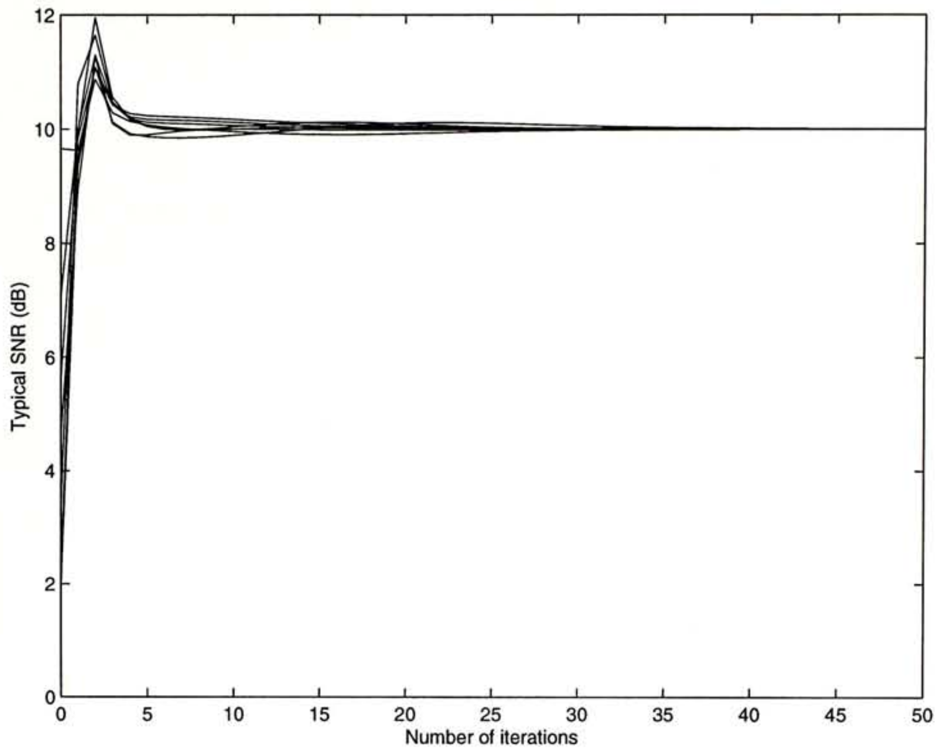

Figure 3.6: Typical behavior of the MC-CDMA system with 8 carriers and 8 users

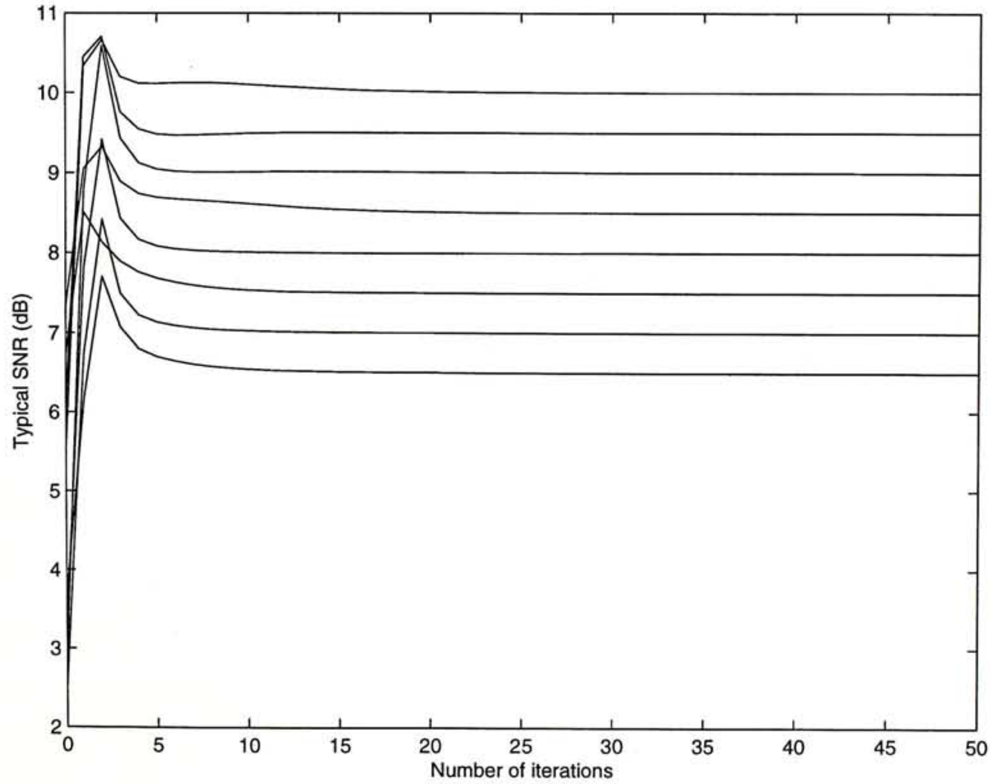

Figure 3.7: Typical behavior of multitarget transmission

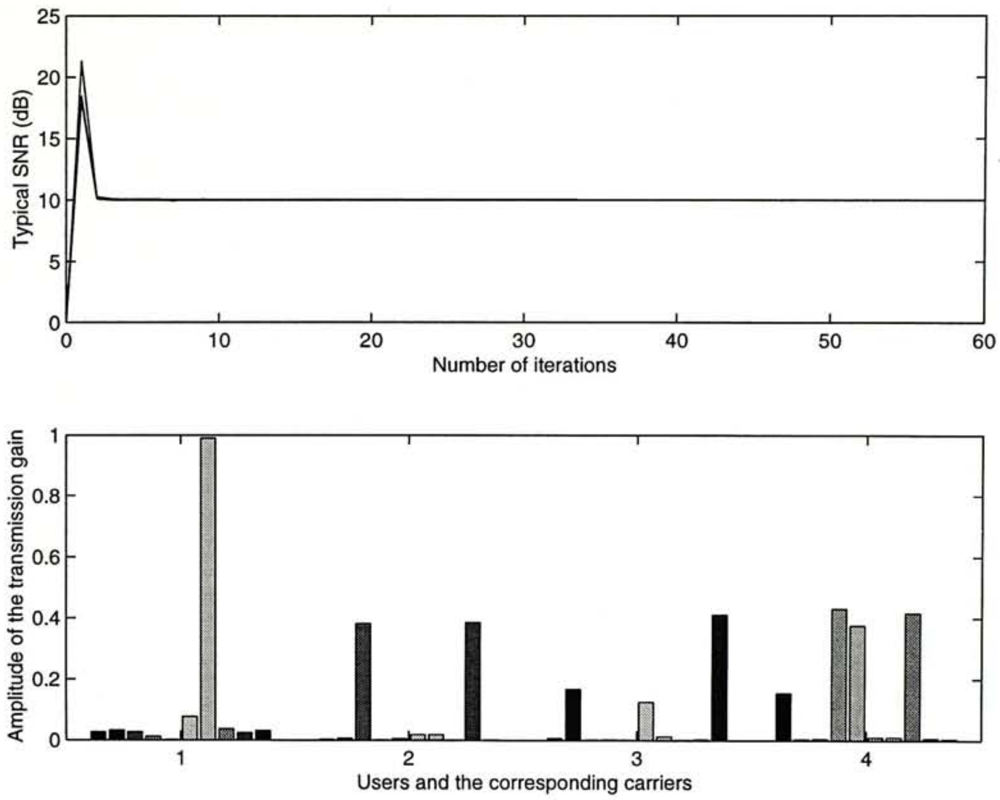

Figure 3.8: Typical behavior of multirate transmission

8 users in the MC-CDMA system. We assume different users require different target SNRs. In this case, a 0.5dB difference is set for different users. Using the simple decentralized adaptive transmission scheme, this special task can be fulfilled successfully. In the two situations shown above, an interesting behavior is observed. Each user tends to concentrate his power in a distinct carrier which does not suffer deep fading. The system then behaves like an FDMA system.

In this part, the performance of the multirate MC-CDMA with the modified decentralized adaptive transmission scheme is analyzed. We assume $M = 10$, $N = 4$ and the target SNRs are all 10dB. User 1, user 2, user 3 and user 4 require $1r$, $2r$, $3r$ and $4r$ data rates for transmission, respectively. The admission control of (3.16) is satisfied with this multirate service. In Figure 3.8, the system is modeled and simulated. Also showed is the amplitude of the transmission gains for each user. We observe that power is mainly allocated to some carriers. This behavior indicates the multirate QoS is achieved. This modification gives much flexibility in frequency planning for different multimedia applications. In Table 3.2, the percentages of power in the carriers for multirate transmission is shown. It is the result of 500 realizations. By observation, the users with data rates that are multiples of the basic rate will utilize multiple carriers to meet their transmission requirements. The number of carriers allocated much power is proportional to the data rate needed. We define them as the *main* carriers. We see that almost all power is assigned to these main carriers after applying the modified decentralized adaptive transmission scheme. With satisfaction of (3.16), no main carrier will be used by more than one user for transmission. Models derived in this chapter is useful for further improvement of the multirate MC-CDMA transmission system.

Table 3.2: Percentages of power assignment in the carriers of the multirate MC-CDMA system (500 realizations)

Data rates	Power in the main carriers (in descent order) (%)	Power in other carriers (%)
1r	87.24	12.76
2r	63.77 25.73	10.50
3r	52.29 24.72 14.88	8.11
4r	48.49 24.88 14.44 8.89	3.3

Due to observation, it is of interest to compare the MC-CDMA system employing the decentralized transmission scheme with an FDMA system. We consider a system with K users and M carriers where $K \leq M$. In an FDMA system, each user is assigned a distinct carrier. We define the optimal FDMA system as an FDMA system where the minimum average power is used to achieve the

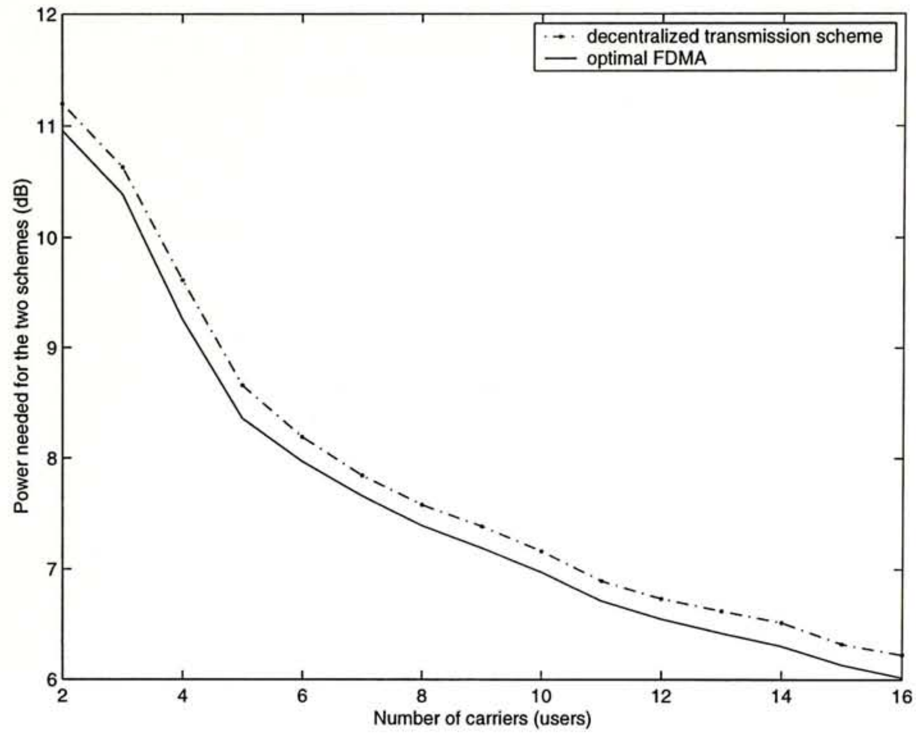

Figure 3.9: Power needed for the decentralized MC-CDMA system and the optimal FDMA system (K=M)

target SNRs for all users. We compare the power consumption of these two systems. Firstly, we consider the performance of the systems as both the number of users and carriers increases simultaneously. The average result of 500 realization is shown in Figure 3.9. The power shown is normalized to additive while Gaussian noise (AWGN). When the number of users (carriers) increases, the average power needed to achieve the target SNRs decreases. With the increased number of choices of carrier assignment, the transmitters can avoid deep fading more effectively. Secondly, we consider another situation that M is fixed at 8 and K increases from 2 to 8. Figure 3.10 is the average performance of this case. The average power required increases because the number of choices of carriers decreases. While it is not easy to conclude that the MC-CDMA system with the decentralized transmission scheme will always yield the optimal FDMA system

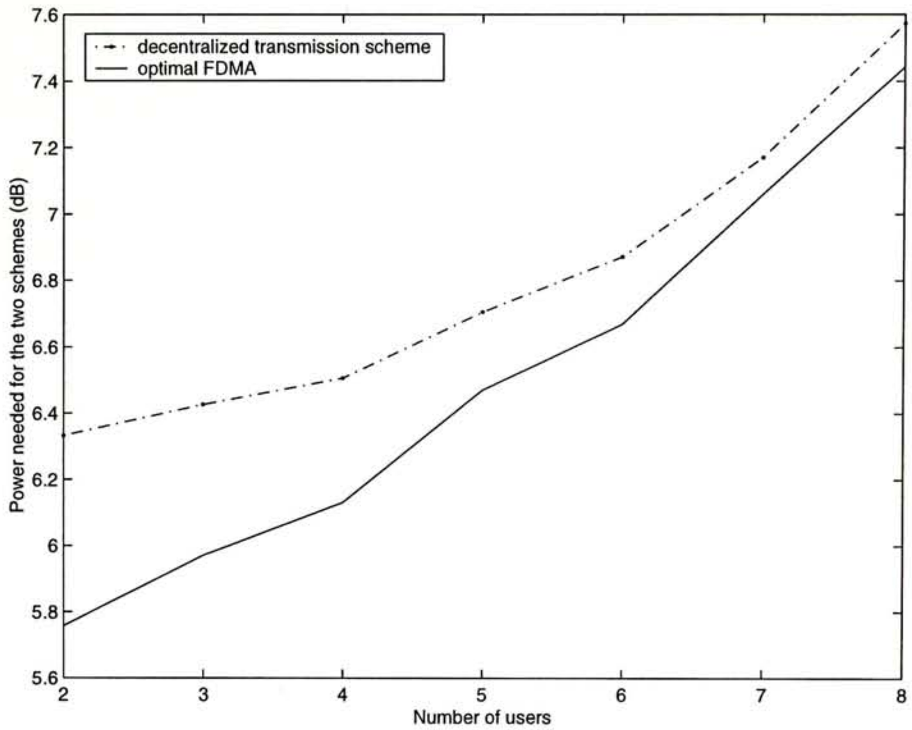

Figure 3.10: Power needed for the decentralized MC-CDMA system and the optimal FDMA system (K=2 to 8, M=8)

(although it does appear so), the results show that its performance is essentially the same as that of the optimal FDMA system.

The decentralized transmission scheme provides advantages of fast convergence, no centralized information needed and performs nearly as well as the optimal FDMA system.

3.5 Summary

In this chapter, we have developed some decentralized transmitter optimization schemes for MC-CDMA systems in frequency selective fading channels. The transmitter of each user is optimized from the point of view of the user by suitably choosing a gain vector which determines the power allocated to different carriers. Simulation shows significant improvements over receiver optimization only. When the number of users is smaller than or equal to the number of carriers, the MC-CDMA system with the decentralized transmission scheme tends to an FDMA system with optimal frequency assignment. We also analyzes the performance of the MC-CDMA system with the modified decentralized transmission scheme. Multirate transmission is achieved.

Chapter 4

Performance Evaluation of Various Adaptive Transmission Schemes

4.1 Introduction

In chapter 2 and chapter 3, we approached joint optimization of transmitter and receiver for multicarrier code division multiple access (MC-CDMA) systems from two different viewpoints: centralized and decentralized. By observation, both schemes tend to assign power in a frequency division multiple access (FDMA) way. In this chapter, we compare the performance and give comments to the advantages and disadvantages of them.

A practical problem to ask is whether the two new MC-CDMA transmission schemes can have an improvement over conventional FDMA in terms of system capacity. It is known that one possible disadvantage of an FDMA system is that it is not flexible when the system is already fully loaded. No more user can be admitted for communication after all carriers are already used. While the proposed MC-CDMA systems behave like the optimal FDMA system when the number of users is smaller than or equal to the number of carriers, they have the additional potential of supporting more users. We evaluate the two adaptive transmission schemes when the number of users exceeds the number of carriers. Of course, each user no longer concentrates his power on just one carrier.

If the number of users is over the system capacity, what we want to see is that the performance of the MC-CDMA system will degrade gracefully just like most code division multiple access (CDMA) systems. CDMA can average out the degradation. For this important feature, based on the decentralized transmission scheme, we develop another new algorithm which needs some degree of centralized information to fulfil this task. For this novel transmission scheme, it has good performance of maximizing the signal to noise ratio (SNR) for each user with some limitations, and is more flexible to the changes of environment, such as more users entering the system or sub-channels breaking down.

In Section Two, we compare the power consumption and speed of adaptation of the centralized and decentralized adaptive transmission schemes. In Section Three, we discuss the situation of adding users to the MC-CDMA systems after the available carriers are fully occupied. Simulation shows that the two proposed transmission schemes still work under this un-desirable circumstance. In Section Four, a novel adaptive transmission scheme is derived to support more users at the expense of the average performance. Summary of this chapter is presented in Section Five.

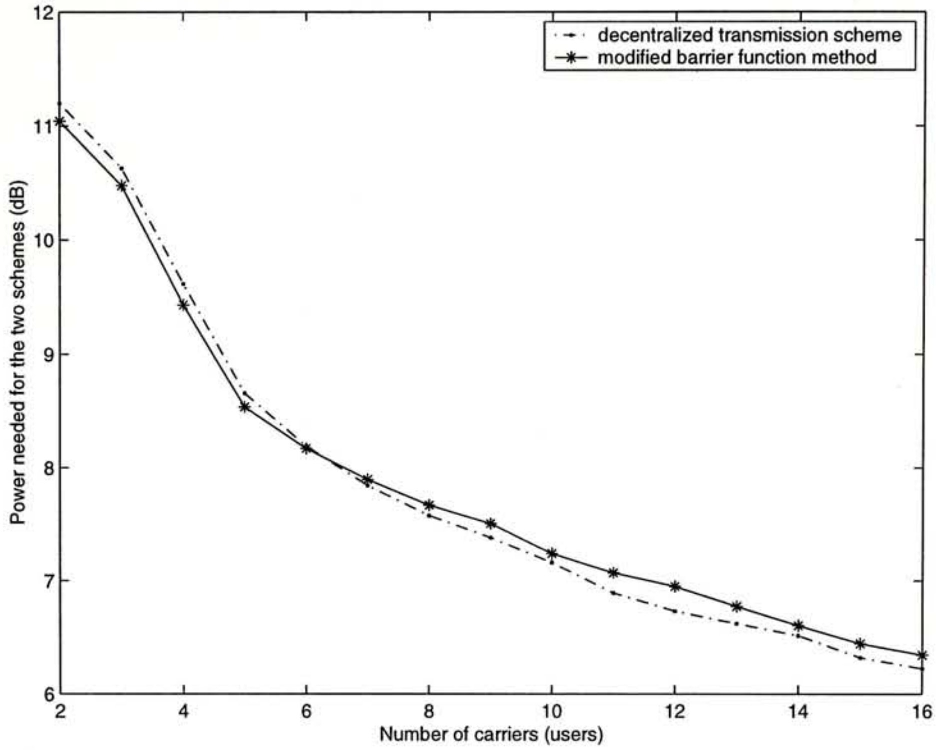

Figure 4.1: Power consumption for the centralized and decentralized schemes (K=M)

4.2 Comparison of Different Adaptive Transmission Schemes

From the previous two chapters, simulation results show that both the centralized (modified algorithm with barrier function) and decentralized transmission schemes approach the power allocation and consumption of ideal FDMA systems with optimal frequency assignment. We further our comparison which is focused on the two adaptive schemes in this section.

In this section, the simulation is the average of 500 realizations and other crucial assumptions are the same as used in previous chapters. Figure 4.1 and Figure 4.2 show the power needed of the two transmission schemes in different situations. For the two systems, we assume equal number of carriers and users

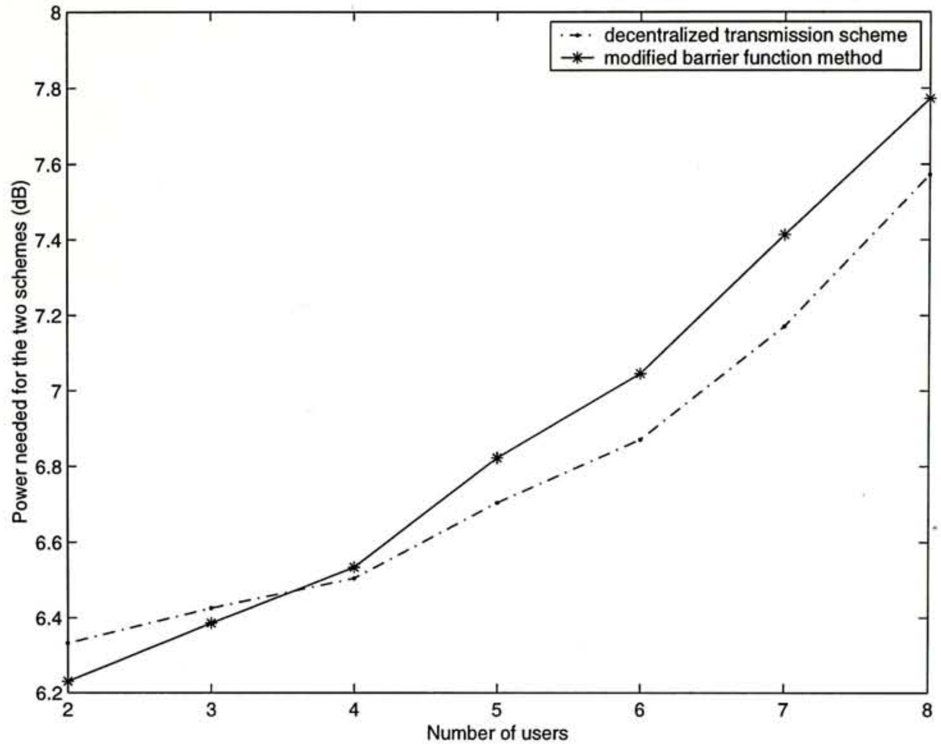

Figure 4.2: Power consumption for the centralized and decentralized schemes (K=2 to 8, M=8)

in Figure 4.1 and 8 carriers in Figure 4.2. The signal to thermal noise ratio (STNR) is fixed at 10dB. The power shown in the figures is normalized to additive white Gaussian noise (AWGN). We tune the parameters in the two adaptive processes to let the MC-CDMA systems consume as little power as possible. The two figures present that both the modified barrier function method and decentralized adaptive transmission scheme for MC-CDMA have similar low power consumption. Thus, in the view of power consumption, the two transmission schemes are equally good and give near optimal performance like the optimal FDMA scheme.

When comparing the iterations needed for reaching targets, we have different results. In this case, we let the two systems adapt to their targets as fast as possible while having power settlement at the same level. In Figure 4.3 and

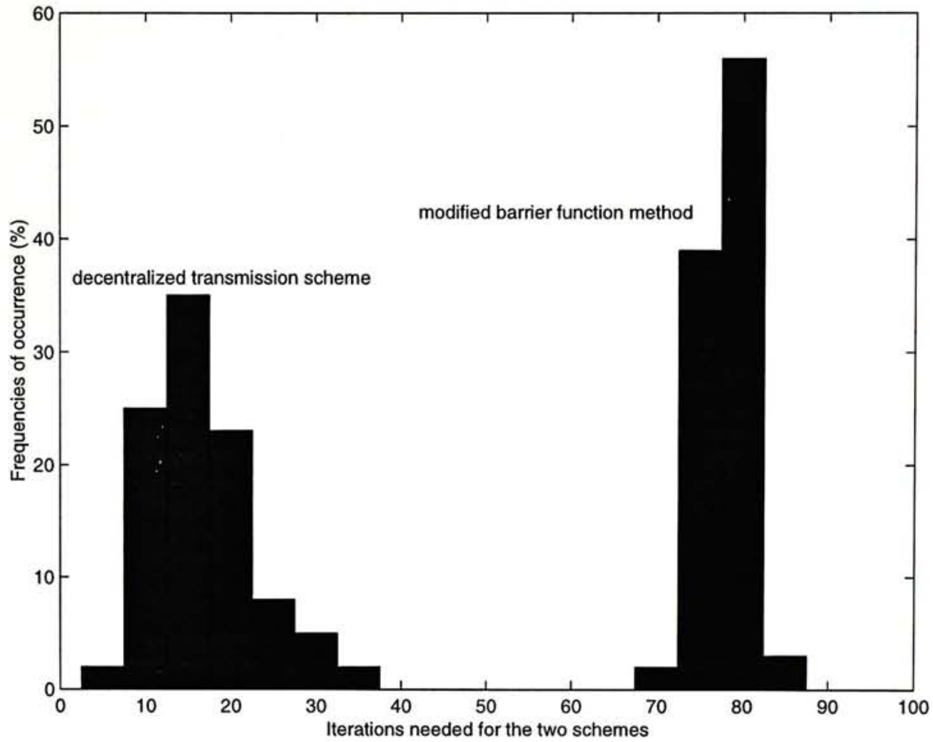

Figure 4.3: Iterations needed for the centralized and decentralized schemes (K=4)

Figure 4.4, we assume $M = 8$ and evaluate the iterations required when $K = 4$ and $K = 8$. In these histograms, we see that the number of iterations needed for the decentralized transmission scheme to reach targets is much less than the modified barrier function method. When $K \leq M$ and the number of users increases, the number of iterations needed for the decentralized transmission scheme to adapt to the surroundings increases but the modified barrier function method does not. When K approaches M, the number of iterations required for this centralized transmission scheme to reach targets is kept at the similar level with increasing variance. After checking the limit $K = M$, we see that the decentralized transmission scheme still needs smaller number of iterations for settlement than the centralized one. Also considering no centralized information is needed for the decentralized transmission scheme, we conclude that

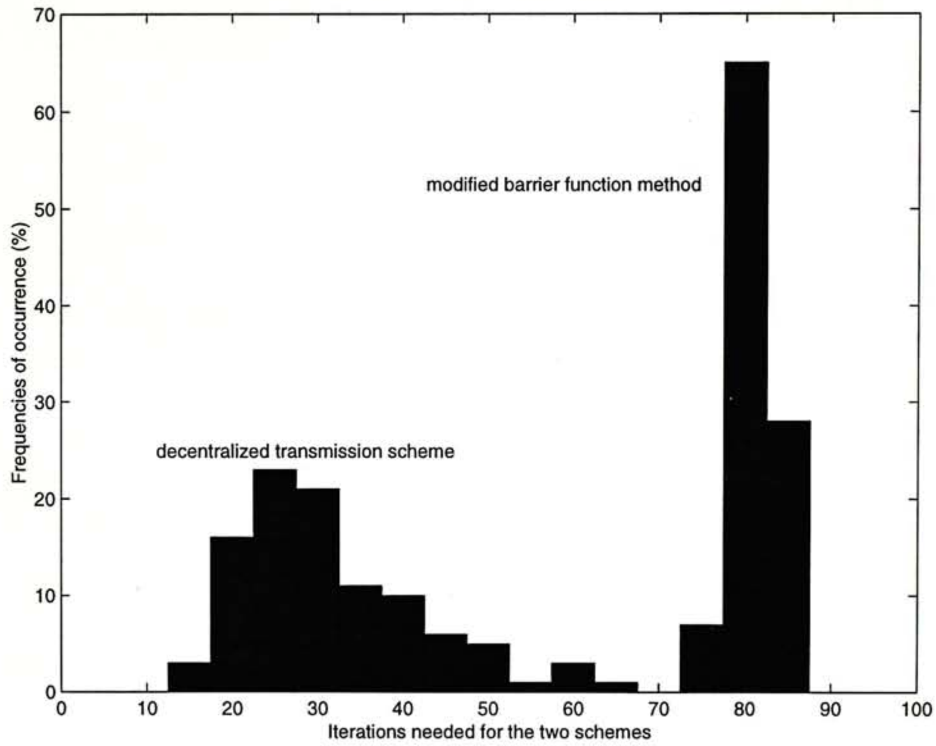

Figure 4.4: Iterations needed for the centralized and decentralized schemes (K=8)

the decentralized transmission scheme is better than the centralized modified barrier function method in practical implementations.

4.3 Adaptive Transmission Schemes with $K > M$

In multiuser environment, either centralized Lagrangian or decentralized adaptive transmission scheme for MC-CDMA systems can be used to combat imperfection in parallel fading sub-channels and eventually the processes arrive at the similar near optimal solution. They avoided the deep fading carriers and concentrate the power in the least fading carrier when $K \leq M$. Using the same system and channel model described before, when the kth receiver employs its optimal weight vector $\mathbf{w}_k$, which is calculated according to (1.11) in chapter 1, its output SNR is given by

$$\mathrm{SNR}_k = \mathbf{d}_k^H \mathbf{R}_k^{-1} \mathbf{d}_k, \tag{4.1}$$

where $\mathbf{R}_k$ is the noise and interference correlation matrix for the kth user. The desired vector $\mathbf{d}_k$ in the above equation is

$$\mathbf{d}_k = \mathbf{A}_k \mathbf{c}_k, \tag{4.2}$$

where $\mathbf{A}_k$ summarizes the fading characteristics seen by the kth user. We consider transmitter optimization by choosing the transmission gain vector $\mathbf{c}_k$ for $k = 1, ..., K$, so that the average transmitted power for all users achieving the target performance is minimized. The problem is formulated as

$$\begin{aligned} &\min \quad \frac{1}{K}\sum_{k=1}^{K} \mathbf{c}_k^H \mathbf{c}_k, \\ &\text{subject to} \quad \mathbf{d}_k^H \mathbf{R}_k^{-1} \mathbf{d}_k = \gamma_k \quad \text{for all } k, \end{aligned} \tag{4.3}$$

where γ_k is the target SNR for the kth user. From [39], the necessary and sufficient condition for the existence of a feasible solution for this optimization problem is found and the simplified version of it is stated here again for completeness

$$\sum_{k=1}^{K} \frac{\gamma_k}{1+\gamma_k} < M. \tag{4.4}$$

Notice that if $K \leq M$, the condition is always satisfied. If (4.4) is satisfied, there exists a feasible solution even when $K > M$. We apply the centralized and decentralized transmission schemes studied in chapter 2 and chapter 3 respectively to the situation $K > M$. When the condition (4.4) is satisfied, users can reach targets in many cases. After analyzing the adaptive behavior of both schemes in this situation, we see that more power is needed than the case $K \leq M$ and

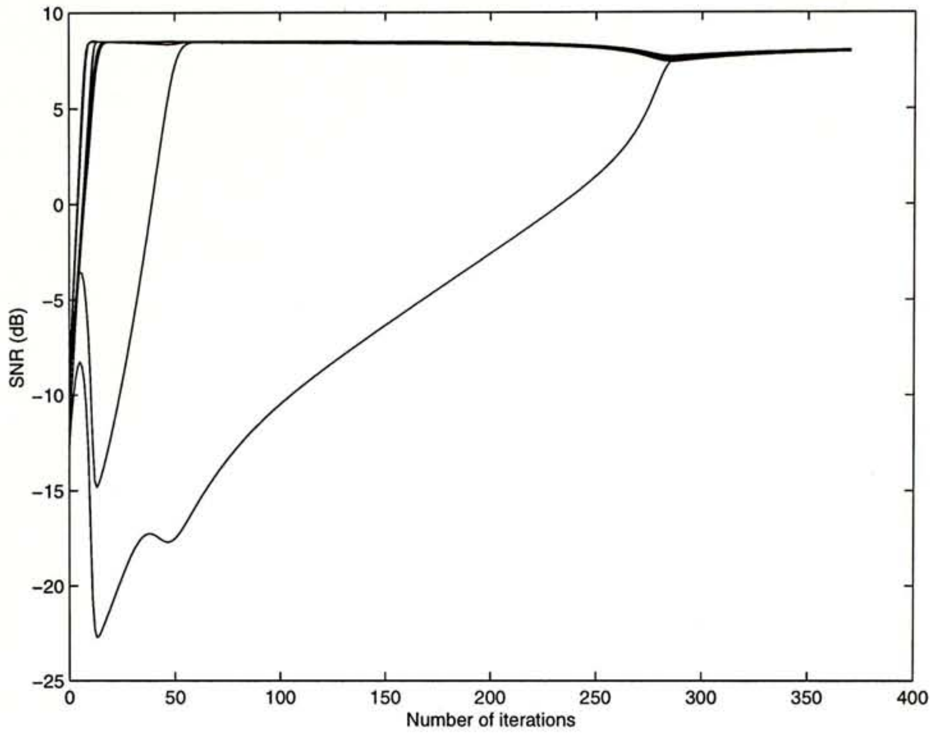

Figure 4.5: Typical behavior of the centralized MC-CDMA system with 8 carriers and 9 users

each user no longer focuses his power on just one single carrier. Moreover, the number of iterations for reaching targets increases significantly.

We consider a distinct case that there are 8 carriers and 9 users in the two MC-CDMA systems where the target SNRs for all users are 8dB. Obviously, the number of users exceeds the number of carriers, but (4.4) is satisfied. Other assumptions are the same as said in previous chapters. Under the same fading process, simulation of the typical behavior for the two adaptive transmission schemes is shown in Figure 4.5 and Figure 4.6. From the performance of the MC-CDMA systems with either centralized (co-operated with penalty function) or decentralized transmission scheme presented in the figures, we see that the target SNRs can be reached as expected. However, from these simulation figures, it is easily seen that the decentralized transmission scheme performs better

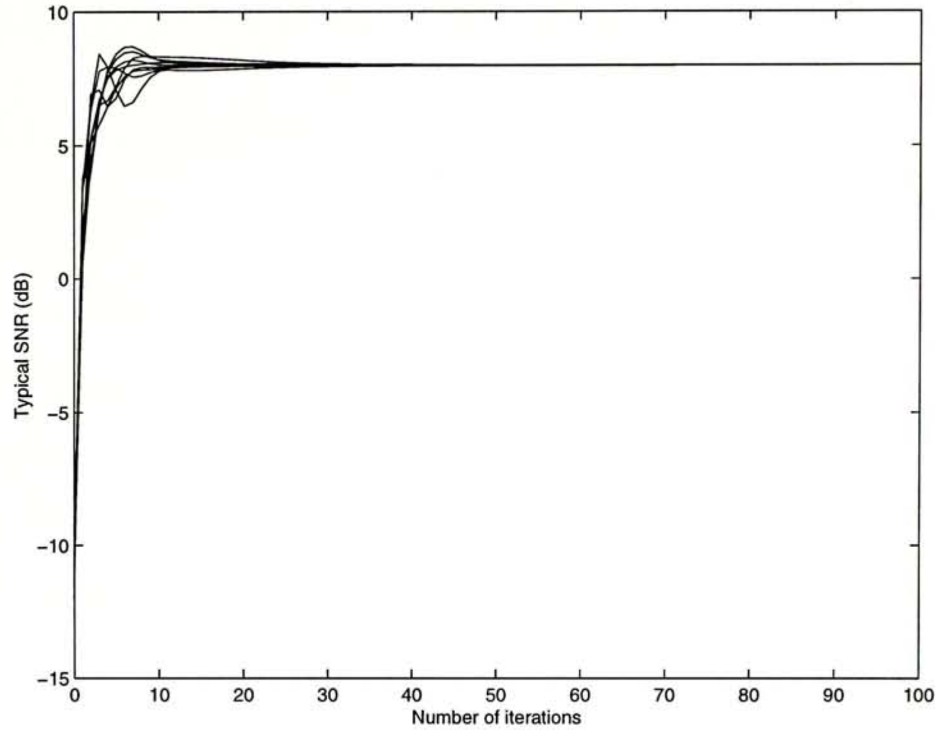

Figure 4.6: Typical behavior of the decentralized MC-CDMA system with 8 carriers and 9 users

than the centralized penalty function method. Again, given that both schemes consumes similar amount of power, the decentralized one is attractive because all users achieve their target SNRs within a reasonable number of iterations. In practical implementations, it is common to encounter the situations that more users get into the system temporarily or sub-channels fail due to narrowband interference. For the unpredictable environment, it is wasteful to introduce more bandwidth. Therefore, the two adaptive transmission schemes have additional merit in capacity which makes the MC-CDMA systems with adaptive transmission schemes better than the optimal FDMA system remarkably.

4.4 Modified Adaptive Transmission Scheme with Graceful Degradation in the SNR

The previous section shows that our centralized and decentralized transmission schemes can possibly let users reach targets as the necessary and sufficient condition (4.4) is satisfied. Sometimes the system has to use the target SNRs to compromise the way of reaching targets without over using the power. Before we present the solution, it is instructive to review the characteristics of a conventional CDMA system. For CDMA, the performance of all users will descend gracefully as the number of users increases. It is what we want to see in designing multiuser communication systems with *soft* behavior [40]. A novel adaptive transmission scheme, which can meet this demand, is discussed in this section.

We already demonstrate that the decentralized adaptive transmission scheme for MC-CDMA systems is the best among all the methods studied previously. We can use this decentralized transmission scheme in either centralized or decentralized system implementations. In centralized applications, if the condition (4.4) is not satisfied, to allow users' performance to degrade gracefully, we migrate the transmission scheme to a supplementary algorithm, which makes the MC-CDMA system work. To derive the algorithm, we have to assume, to some extent, that centralized information can be fed back to transmitters for the optimization. When the condition (4.4) is violated, the performance (target SNRs or total power consumption) of the users in the novel transmission scheme will impoverish a little. One specific solution is by allowing the total power used to be known as the centralized information. Description of the algorithm is shown as follows.

In the designing process, we pay much attention to the total power consumption for approaching the target SNRs as close as possible. In some sense it is unacceptable that the power used will be unreasonably large. Thus we modify the decentralized transmission scheme and let it suit this criterion. By giving a limit to the total power needed, we can develop a more practical iterative algorithm, which is stated as

1. In the nth iteration, the receiver formulates the desired vector $\mathbf{d}_k(n)$ and the noise and interference correlation matrix $\mathbf{R}_k(n)$ to calculate the optimal weight vector $\mathbf{w}_k(n)$ based on

$$\mathbf{w}_k(n) = \mathbf{R}_k^{-1}(n)\mathbf{d}_k(n). \tag{4.5}$$

2. From the knowledge in the previous iteration, $\tilde{\mathbf{c}}_k(n)$ is updated as the eigenvector associated with the largest eigenvalue of $\mathbf{C}_k(n)$.

$$\tilde{\mathbf{c}}_k(n+1) = (1-\mu)\tilde{\mathbf{c}}_k(n) + \mu\mathbf{C}_k(n)\tilde{\mathbf{c}}_k(n), \tag{4.6}$$

   where $\mathbf{C}_k(n)$ is the nth estimate of $\mathbf{A}_k^H\mathbf{R}_k^{-1}\mathbf{A}_k$.

3. Power control is applied to $\tilde{\mathbf{c}}_k$ according to

$$\mathbf{c}_k(n+1) = \frac{\tilde{\mathbf{c}}_k(n+1)\sqrt{\gamma_k(n+1)}}{\sqrt{\tilde{\mathbf{c}}_k^H(n+1)\mathbf{C}_k(n)\tilde{\mathbf{c}}_k(n+1)}} \tag{4.7}$$

   to achieve the target SNR γ_k.

4. If the total power used is above the upper limit, the gain vectors will be automatically multiplied by δ, which is

$$\delta = \frac{\text{upper bound of the total power}}{\text{total power in this iteration}}. \tag{4.8}$$

   Repeat the procedure and adapt to the new targets again.

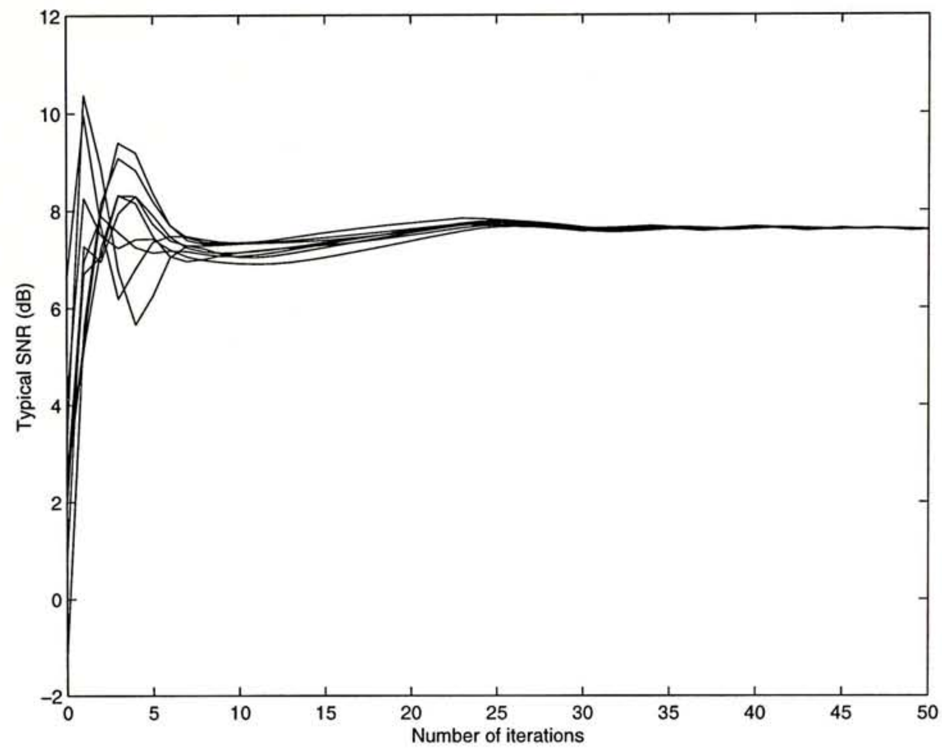

Figure 4.7: Typical behavior of the novel transmission scheme with 8 carriers and 10 users (uniform target)

In the computer simulation, we assume there are 8 carriers for serving 10 users and the target SNRs for the users are all 10dB. The maximum allowable average power consumption is limited to 2. It is the case that the condition (4.4) is violated. Figure 4.7 shows the typical behavior of this novel transmission scheme which maximizes the achievable SNRs without over using the power. It is clear from the figure that the system sacrifices about 2dB in the SNRs after the power settlement. We also test this algorithm in the multitarget situation. We assume that 10 users have different SNR requirements and only 8 carriers are available. The original targets for the 10 users are from 3dB to 12dB with a 1dB difference. Also, the condition (4.4) is violated. In Figure 4.8, it shows that the novel transmission scheme settles the performance in less than 30 iterations,

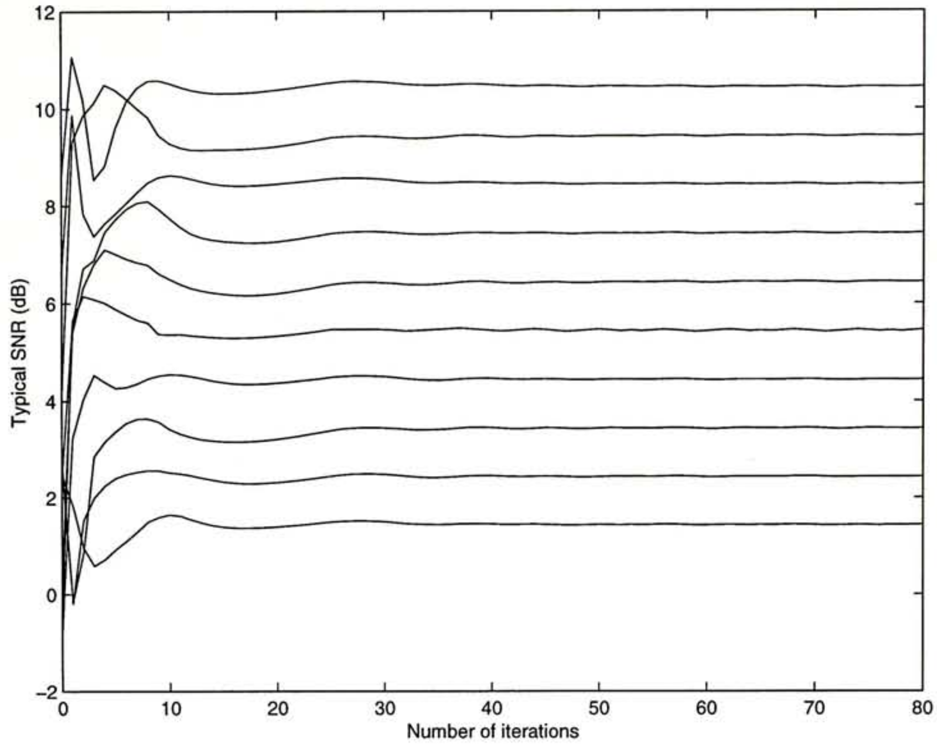

Figure 4.8: Typical behavior of the novel transmission scheme with 8 carriers and 10 users (multitarget)

and there is about 1.5dB degradation in each user's SNR. The MC-CDMA system with the novel adaptive transmission scheme is flexible to the surrounding changes and performs well in some un-desirable environment.

4.5 Summary

In this chapter, we have compared the performance of the centralized and decentralized adaptive transmission schemes. Both schemes have similar power consumption, but the decentralized transmission scheme is better than the centralized one because of the small number of iterations needed to reach targets. The capability of the centralized and decentralized transmission schemes for

MC-CDMA to support users when $K > M$ is investigated. It makes these adaptive transmission schemes more attractive than the optimal FDMA scheme. In addition, we modify the decentralized transmission scheme to meet the performance with gradual degradation as the number of users increases. Centralized information, such as the total power, is required to fulfil this task. This novel algorithm is proposed to work whenever $K > M$.

Chapter 5

Conclusions and Future Work

5.1 Conclusions

In this thesis, we have investigated the centralized and decentralized adaptive transmission schemes for the optimization of MC-CDMA systems. The Lagrange multiplier methods, possibly with power control, are used in the centralized environment to optimize the performance. On the other hand, a decentralized scheme with good performance is provided for simplicity of implementation. When the number of users is smaller than or equal to the number of carriers, the results of both kinds of schemes are that each user tends to allocate power in the least fading carrier for communication. In other words, they solve the transmission optimization problem approaching the behavior of FDMA systems with optimal frequency assignment. By doing so, we can provide an MC-CDMA system with low power consumption.

An immediate question to ask is which of the two schemes to an MC-CDMA system should be employed for a specific communication channel with better performance. Because of its practical significance, some form of answer to this question is necessary. There are at least three aspects, namely, power consumption, number of iterations needed for reaching targets and requirement of knowledge. The analysis in Chapter 4 indicates that both schemes approach the power consumption of optimal FDMA systems and perform almost equally well in that aspect. For the other two aspects, we have different results. The decentralized transmission scheme needs fewer iterations than the centralized transmission scheme to reach targets. Another benefit of the decentralized transmission scheme is that no centralized knowledge is required for the whole process of adaptation. Overall, with the decentralized transmission scheme, the proposed MC-CDMA system becomes a simple and practical choice for use in wireless communications. Furthermore, we present the capability of supporting more users after the system is already heavily loaded. In this case, the decentralized transmission scheme is still superior in the evaluation. We conclude that the decentralized transmission scheme derived in this work is more attractive than the centralized one and is useful for further improvement of the MC-CDMA system.

Since the decentralized transmission scheme outperforms the centralized Lagrangian methods, even in the centralized environment, the decentralized transmission scheme can also be applied effectively without violating the implementation principles of centralized systems. When the necessary and sufficient condition for the users to reach targets is violated, if the MC-CDMA system allows some centralized knowledge to be sent back to the transmitters, we can add a supplementary adaptive scheme to let the users approach their targets as close as possible. With this novel transmission scheme, to some extent, the MC-CDMA system performs like a DS-CDMA system with graceful degradation.

5.2 Future Work

We have repeatedly mentioned that both transmitter optimization and receiver optimization provides improvements to the communication systems. Our future work will be placed on the question: what is the optimal transmission scheme? It is interesting to find the optimal solution given the number of users, the number of carriers and the fading coefficients in the MC-CDMA system. For $K \leq M$, the results of the adaptive transmission schemes proposed in this work all approach the behavior of optimal FDMA. We try to find out whether this FDMA system with optimal frequency assignment is truly optimal.

Our research work is focused on the indoor environment, and the proposed algorithms are more suitable for indoor applications. One possible extension is the exploitation of our work to outdoor applications, i.e., cellular mobile communications.

On the other hand, transmitter optimization is an important research topic and much attention has already been drawn recently. Further research is definitely needed. An interesting topic will be extended to apply our results to find the signature sequences for spread spectrum communications. Also, we can study the effect of different chip waveforms on the performance of the MC-CDMA system. Investigation can be done on using antenna array, different modulation and channel coding schemes to the system too. Further improvements are expected with these considerations.

Appendix A

The Hungarian Method for Optimal Frequency Assignment

For the ideal FDMA system with optimal frequency assignment, it means from the cost matrix β, we should only choose one particular sub-carrier (sub-channel) for one user such that the combinatorial effect (total power needed) is minimized. One obvious, but inefficient way, to solve such a problem is to consider the $n!$ possible permutations and find the smallest. However, because of the special structure, it can be solved more efficiently by a specialized algorithm, called the Hungarian method [34], in order to avoid examining such a large number. In [41] and [42], it is showed that the Hungarian method correctly solves this kind of assignment problem for a complete bipartite graph[1] with $2n$ nodes in $O(n^3)$ arithmetic operations. The computational complexity grows polynomially rather than exponentially with respect to the size of the input.

One way of looking at the Hungarian method for the assignment problem is

[1]A graph is bipartite iff it has no circuit of odd length. Because there are disjoint subset carriers and users in this frequency assignment problem, it is complete bipartite.

in terms of a matrix. We let the cost matrix $\beta = (\beta_{i,j})$ be such that $\beta_{i,j} \geq 0$ for all $i, j = 1, ..., n$ – the assignment of the carriers of its row to the users of its column. The chosen entries are marked by asterisks. These entries must be that (i) there is exactly one asterisk entry in each row and (ii) exactly one asterisk entry is in each column. Among all valid sets of asterisk entries, we seek the set with the minimum sum. Here we state the general outline of the Hungarian method. The method is iterative in the sense that it progressively defines a series of complementary matrices with 1 and 1$*$ as their entries until a solution can be identified. The algorithm is required to find a minimum cost assignment with each carrier serving a different user.

It is well known that when the assignment problem is primal, the linear programming (LP) dual of it can be stated [42], [43]

$$\begin{aligned} &\max \quad \textstyle\sum_{i=1}^{n} u_i + \sum_{j=1}^{n} v_j, \\ &\text{subject to} \quad u_i + v_j \leq \beta_{i,j}, \end{aligned} \tag{A.1}$$

where u_1,..., u_n and v_1,..., v_n are non-negative numbers. From the Primal-Dual Algorithm [43], we try to find a feasible solution to the dual problem instead and fit it to the primal problem.

For the assignment at hand, we solve it through the following steps:

Initiation Let $a_i = \min_j \beta_{i,j}$ and $b_j = \min_i \beta_{i,j}$ for $i, j = 1, ..., n$. Also let $a = \sum_{i=1}^{n} a_i$ and $b = \sum_{j=1}^{n} b_j$. Then we define u_i and v_j using the following rule

- If $a \geq b$, $u_i = a_i$ and $v_j = 0$ for $i, j = 1, ..., n$.
- If $a < b$, $u_i = 0$ and $v_j = b_j$ for $i, j = 1, ..., n$.

For the u_i's and v_j's at hand, we construct a matrix $\mathbf{Q} = (q_{i,j})$ where

$$q_{i,j} = \begin{cases} 1 & \text{if } u_i + v_j = \beta_{i,j}, \\ 0 & \text{otherwise.} \end{cases} \tag{A.2}$$

To provide a first guess of the assignment, we mark the entries by asterisks. If $a \geq b$, the rows are examined in order and the first 1 in each row without a $1*$ in its column is changed to a $1*$. If $a < b$, the same instructions are used with rows and columns exchanging their roles.

Routine 1 In this stage, we examine the matrix $\mathbf{Q}$ according to the flow diagram shown in Figure A.1, where k and l are temporary valuables for storage. The values of the quantities for the input of Routine 1 are

$$i = j = k = l = 1, \qquad i_\nu = \epsilon_\nu = 0, \qquad \text{for} \quad \nu = 1, ..., n, \tag{A.3}$$

and

$$\epsilon_i = \begin{cases} 1 & \text{if row } i \text{ is essential}, \\ 0 & \text{if row } i \text{ is inessential.} \end{cases} \tag{A.4}$$

We can determine whether a row is essential by the flow diagram in Figure A.1. Moreover, a column is essential if it contains a $1*$ in an inessential row.

Routine 2 For all inessential rows i and columns j, we compute d, which is the minimum of $\beta_{i,j} - (u_i + v_j)$. If there are no such (i, j), the set of $1*$ in Q is the positions referring to the optimal assignment. Otherwise, $d > 0$ and there are two mutually exclusive cases to be considered.

Case 1 For all inessential rows i and $u_i > 0$, calculate the minimum among d and u_i, taken as m. Then

- $u_i \to u_i + m$ for all inessential rows i, and
- $v_j \to v_j - m$ for all essential columns j.

Case 2 For some inessential row i and $u_i = 0$, calculate the minimum among d and v_j, taken as m. Then

- $u_i \to u_i - m$ for all essential rows i, and
- $v_j \to v_j + m$ for all inessential columns j.

Repetition After changing u_i and v_j, the process should return to Routine 1. Routine 1 and Routine 2 are the two basic routines of the algorithm and the iterative procedure can be predicted according to Figure A.2

From the above iterative steps, we roughly see that the iteration takes the $O(n)$ order operations and each search and modification takes $O(n^2)$ time. So, the bound is roughly seen. It is a quite significant improvement when n is large.

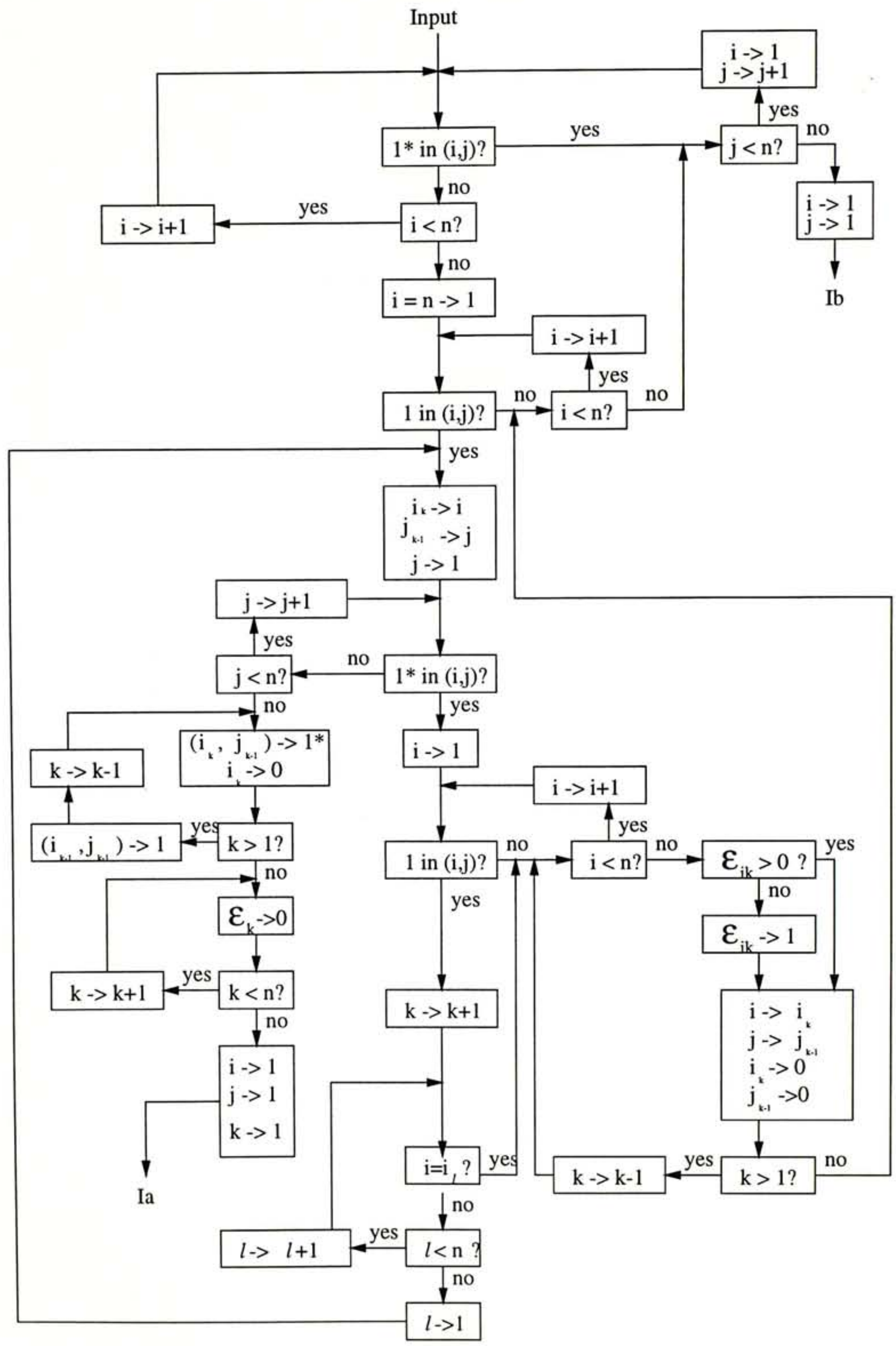

Figure A.1: Flow diagram of routine 1 [From [34]]

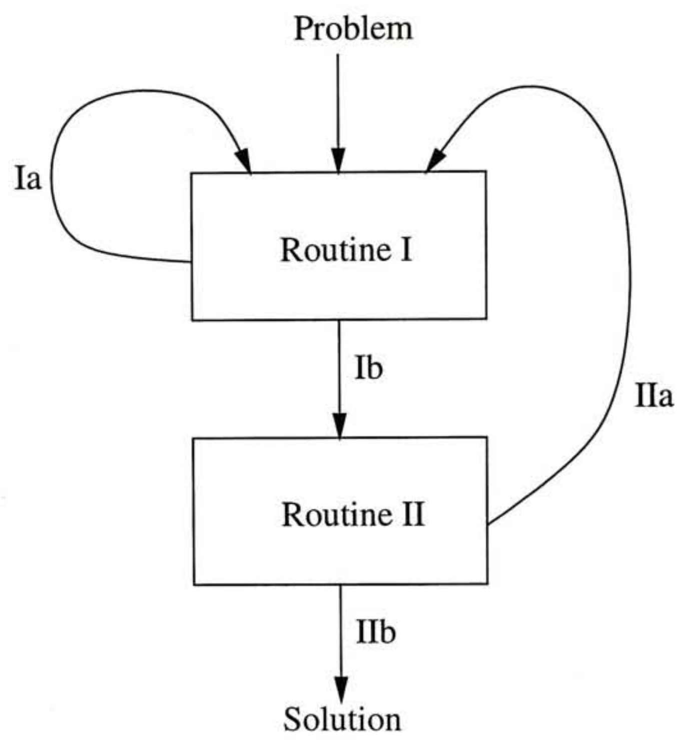

Figure A.2: Schematic description of the repetition [From [34]]

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