RESOURCE ALLOCATION IN MOBILE CELLULAR SYSTEMS

By

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Abstract

The channel assignment problem (CAP) in mobile cellular systems is considered. We assume that the multiple access scheme used is either time division or frequency division, and thereof channels refer to either time slots or frequency carriers. The assignment of channels can be performed in a static or dynamic manner. Static assignment means that channels are nominally assigned to cells. Each cell has a fixed number of nominal channels. Dynamic assignment means that there is no fixed relation between channels and cells. When a call is initiated in a cell, a channel is requested from a central pool. Basically, every cell can use every channel.

In this thesis, both types of assignment method will be considered. In static assignment, graph coloring algorithms are usually employed. These algorithms, though sometimes can yield optimal solution, do not supply any information on how far it is from the optimum or on which situations an optimal solution can be found. In view of these undesirable features, two relevant results are presented in Chapter 2. First of all, a lower bound for the minimum number of total channels required for the fulfillment of the demand of each cell is derived. This lower bound is tighter than the existing ones under certain conditions and can be used as a supplement of those approximate algorithms. Secondly, we propose an efficient algorithm called Sequential Packing (SP) to solve this problem. Though the CAP is NP-complete in general, our algorithm provides optimal solution for a special class of networks. For general networks, promising results are obtained and numerical examples show that our algorithm has a better performance than the existing algorithms.

Dynamic channel assignment method is more suitable for the future microcellular systems. One such method called Distributed Packing (DP) is proposed and described in Chapter 3. It aims at packing cochannel cells close to each other using only local information. When a cell site has no available channel for a new call, it requests its

neighboring cells to reassign channels instead of blocking the call. Simulation results show that this algorithm outperforms other existing algorithms. Its performance is also compared with Maximum Packing (MP), which is the optimal strategy among this class of channel assignment scheme. In our examples, DP attains 95% and 84% of the maximum capacity under uniform and non-uniform traffic respectively.

In Chapter 4, a performance analysis of cellular systems with single channel is presented. This preliminary study serves as a stepping stone towards the challenging multi-channel case.

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Chapter 1

Introduction

1.1 Design Issues in Mobile Communication Systems

Personal mobile communication is one of the essential elements in the information era. The demand for mobile services has become larger and larger. This triggers a lot of research in designing mobile systems of high capacity, ubiquitous coverage and good quality. To achieve this goal, a number of design issues must be addressed by system designers. The two most important issues are radio transmission and network planning.

Reliable transmission via the radio channel is not an easy task. Due to the multi-path characteristic, it is common for radio signals to experience fading. It means that the signal strength may be dropped by 40-50 dB in a fraction of a second [7]. A large bit-error rate (BER) may result. Therefore, special techniques must be employed to combat the severe transmission conditions.

Another important design issue is network planning. It includes radio resource management, mobility management, cell coverage, etc. In this thesis, we focus only on the part of radio resource management. Since the spectrum is limited, we have to utilize the allocated bandwidth in the most efficient manner such that a high-capacity system can be realized.

1.2 Radio Resource Management

Given an allocated spectrum, we have to utilize it in an efficient manner. For analog systems, the spectrum is divided into frequency channels. Each user is assigned a channel. This kind of resource sharing is called *frequency division multiple access* (FDMA). Nowadays, digital transmission is more popular for cellular systems. A major advantage is the robustness of digital signals. With channel coding, impairments introduced by the transmission medium, such as distortions and errors, can be corrected. In other words, the original digital signal can be regenerated. In digital systems, other multiple access schemes are possible. The most well-known ones are *time division multiple access* (TDMA) and *code division multiple access* (CDMA). In TDMA systems, users may share the same frequency band but each one is allocated a dedicated time slot for transmission. In direct-sequence CDMA systems, each user is assigned a well designed code such that the interference among users are minimized. In this thesis, we will consider only TDMA and FDMA systems. We use the term *channels* to refer to either time slots or frequency carriers. Channel is the basic unit in our consideration. The resource management task is concerned with the use of channels to support users in the system.

1.2.1 Constraint: Radio Interference

If there are only two parties communicating with each other in the system, then the natural or man-made noise of the environment will dominate the performance. The system is said to be *noise-limited.* In cellular systems, however, each transceiver unit has to cope not only with the environmental impairments but also with the interference produced by other transmitters. Typically, the effects of interference is much larger than that of noise. It is said to be *interference-limited* [7].

There are two kinds of interference which may degrade the system performance. The dominant one is the *cochannel interference.* Due to the scarcity of spectrum, it is necessary to reuse the channels, which means that the same channel is used by more than one base station at the same time. As a result, cochannel interference occurs. The carrier-to-cochannel-interference ratio, C/I, is often used as an important parameter for system planning. For example, a C/I ratio larger than 18 dB is required in AMPS system for good voice quality [34]. For second generation system, which is digital in nature, the C/I requirement can be derived from the acceptable bit error rate (BER). For example, in GSM system with frequency hopping, a C/I ratio of 9 dB is required [37]. Once the C/I requirement is known, the reuse factor, which represents the minimum separation between cochannel cells, can be calculated.

Another form of interference is *adjacent channel interference.* This kind of interference occurs when signal energy from one channel spills over into an adjacent channel or when the filter on the receiver is too "loose" and captures energy from a broader band than it really needs to [7]. However, it can be reduced by using filter with a sharp falloff slope [32]. The cochannel interference, therefore, is the single most important constraint on the channel assignment task.

1.2.2 Objective: High Capacity and Good Quality

To evaluate the performance of a digital communication system, the most commonly used measure is the bit error rate (BER). The BER of a system depends on the transmission technique employed. For example, modulation, coding, equalization etc. are all factors which affect the BER performance. In interference-limited system, C/I ratio sometimes is used instead of BER. To provide a high C/I among a group of users using the same channel, power control is needed. An optimum power control scheme which maximizes the minimum C/I ratio under a given channel allocation is presented in [52]. Later, distributed implementation of its are proposed [47] and [53]. These algorithms aim at providing a good voice quality to users.

Another important consideration of a system is the capacity. Capacity is usually measured in Erlangs per cell or Erlangs per unit area under the constraint of a maximum blocking probability. Sometimes, the amount of traffic is given and the average blocking probability is used as a quality of measure (QOS). The ultimate goal is to design a system with high capacity and good quality using the limited resources.

1.3 Channel Assignment

Given a certain number of channels to a system, how to use them such that a system with high capacity and good quality can be acheieved is generally referred to as the *channel assignment problem.*

Channel assignments can be performed statically or dynamically. Static means that channels are nominally assigned to cells. Each cell has a fixed number of nominal channels. However, the number of channels assigned to each cell is not necessarily equal. Some cells may have more channels assigned due to a higher traffic intensity. Dynamic assignment means that there is no fixed relation between channels and cells. When a call is initiated in a cell, a channel is requested from a central pool. Channels are granted on a call by call basis. Basically, every cell can use every channel. In this thesis, both types of assignment method will be considered.

1.3.1 Static Channel Assignment

To perform the channel assignment task, a theoretical model for cellular networks is needed. The most widely accepted one is the hexagonal tiling of cells. In this model, the base station is assumed to be located at the center of the hexagon. Each hexagon represents the radio coverage of that base station. A main reason in choosing hexagon to represent the cell shape is that hexagon is a regular polygon which closely approximates the shape of a circle and can cover a plane with no gaps or overlaps [34]. This model has an advantage that it is easy to understand and its regular structure has been explored by some researchers to develop algorithms for channel assignment [18], [55]. However, this model is too ideal. In practice, the centers of the hexagons are rarely available as base station sites [20]. Besides, traffic variations may require cells of different sizes. Assuming that the radio coverage is hexagonal in shape is inadequate for practical planning purpose.

In light of the limitations of the hexagonal approach, some researchers take into consideration the actual propagation characteristics, which base on measurement results or detailed propagation models. One such model can be found in [45]. The channel assignment problem is formulated as a nonlinear optimization problem. The transmitter power level and the frequency assignment plan is used as control variables and the objective is to optimize the radio qualities. However, the solution space of the problem is too enormous and no efficient methods exist.

Another approach is formulating the problem as a graph coloring problem [40], [41] and [43]. For each pair of cells, it is determined whether they can share the same channel simultaneously. A graph can then be formed where the vertices represent the cells and the edges indicate that frequency reuse is prohibited. Coloring the vertices is equivalent to assigning channels to cells. This approach has an advantage that there is no need to deal with the geometric structure and the geographical location of the cells. However, it is well known that the graph coloring problem is NP-complete. So, optimal solutions cannot be found in polynomial time for general networks.

Later, there are some other approaches, which employs neural network [16] [30], simulated annealing [12] [33] and set theory approach [8]. However, they all have their own limitations.

In this thesis, we will follow the graph coloring approach to tackle the channel assignment problem. Our results are presented in Chapter 2.

1.3.2 Dynamic Channel Assignment

A classification of different dynamic channel assignment (DCA) algorithms was given in [3]. DCA-algorithms are mainly divided into three classes based on the adaptability. The first one is *adaptability to traffic*. Most of the efforts thus far are put in this class. For example, see [11] [14] [26] [38] [42] [48] [51] and [54]. The assignment of channels in this class of DCA is based on a *compatibility matrix*, which states that whether a given pair of cells can use the same channel at the same time. This matrix is determined by propagation models or measurement results. It was shown that DCA of this class has a capacity gain relative to fixed channel assignment. However, when the traffic load becomes larger and larger, the capacity gain will decrease and the asymptotic performance of it is the same as that of fixed channel assignment [29], [35].

In traffic-adaptive DCA, compatibility is usually based on the worst case scenario such that mobiles located near the cell boundary can still maintain a reliable radio link to the base station. To acheive this, a C/I margin is needed and this results in a capacity penalty. Another class of DCA which has *adaptability to interference* is proposed [1] [36]. The assignment of channels is based on the real-time interference measurement. If the interference power is small, channel is granted. This class of DCA has the advantage that the C/I margin can be removed because there is no need to take into account the worst case situation. Besides, network planning task is simplified. However, there is little general results known for this class. No framework is available for an estimation of the capacity gain.

The third class of DCA bases on the *adaptability to channel reusability*, which means that channel reuse is based on the location of mobiles rather than the worst case situation. It is easy to understand that a mobile closer to a base station can tolerate a higher interference power than a mobile at the cell boundary. Some researchers then propose an algorithm called *reuse partitioning* where a cell is divided into different zones, each having a different reuse pattern [25] [39] [44].

In this thesis, we will focus only on the traffic-adaptive DCA. In this class of DCA, algorithms with centralized and distributed control can both be found in the literature. In Chapter 3, we will propose a new distributed algorithm, which has a better performance than existing ones.

1.4 Review of Previous Results and Motivation

In [13], a channel assignment strategy called *Maximum Packing* (MP) is proposed. It is a method originally used for dynamic channel allocation. The policy is that every arriving call will be accepted if the given channels can be arranged in a way such that the reuse constraints are not violated. This assignment strategy is attractive because it is analytically tractable and it provides a lower bound for other dynamic channel assignment strategies. However, this strategy may require rearrangements of calls in progress in a global basis. The number of rearrangements to accept a call is not bounded. So it is not practical to implement MP.

Even worse, the authors of [13] wrongly believes that the call admission to a cell can be determined by examining the number of calls in progress in that cell and its neighbors. This errata was pointed out by [28]. In fact, the decision for call admission in MP is NP-complete for ordinary hexagonal cell structure.

Let us consider an example. Take a look at Figure 1.1. The number '1' denotes that there is one call in progress in the corresponding cell. Otherwise, there is no call in that cell. Assume a cluster size of seven and total number of channels in the system equal to two. If there is a new call arrived at cell A and a complete channel rearrangement of the ongoing calls is allowed, can the call be accepted?

It was suggested in [13] that the call admission can be determined by the *clique condition*. A clique here stands for a group of cells in which every pair of cells are mutually interfered with each other. The clique condition says that if the channel demands, including the new call arrival, in each clique does not exceed the total number of channels in the system, the call can be accepted.

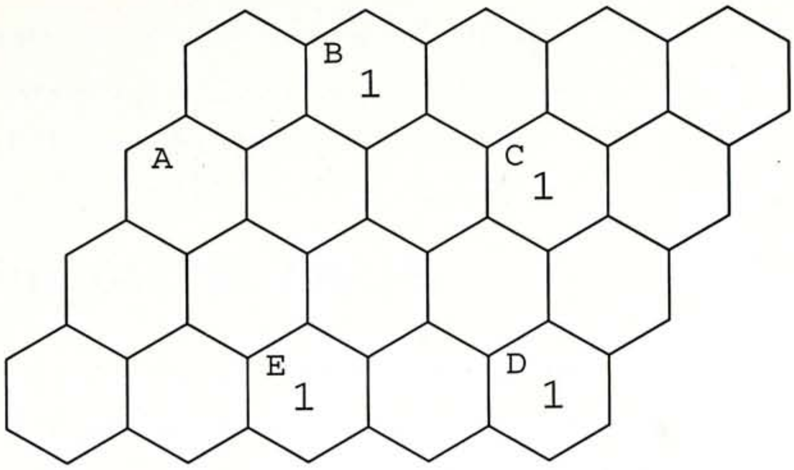

Figure 1.1: Call admission cannot be determined by examining local condition.

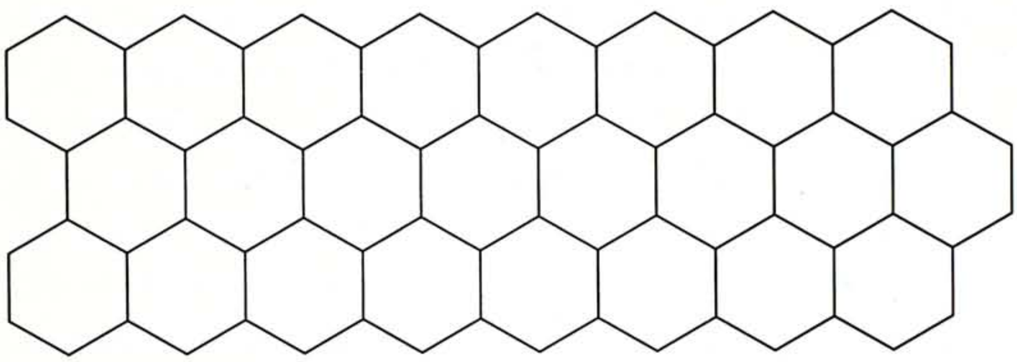

Figure 1.2: An example of a 3-stripe system.

In Figure 1.1, $\{A, B\}, \{B, C\}, \{C, D\}, \{D, E\}$ and $\{E, A\}$ are cliques in the system. The demands in these cliques are all equal to two. Since there are two channels in the system, according to the clique condition, the call can be accepted. However, it is obvious that at least three channels are needed to satisfy all the demands. It is pointed out by [28] that the clique condition, in general, is a necessary but *not* sufficient condition for call admission. A local sufficient condition is given in [28]. However, the admission requirement is too strict. Many calls may be rejected even though they can actually be accepted. This renders it of little practical value.

Until now, no general necessary and sufficient condition for call admission has been found. The clique condition, however, is necessary and sufficient if the underlying graph of the cellular system is *perfect* [28]. The question is that which kinds of network topology has a perfect underlying graph. By observation, we find that a system which consists of three rows of hexagonal cells and has a reuse factor of seven has a perfect underlying graph. We call it a *3-stripe system* (see Figure 1.2). This observation motivates us to search for a fast algorithm which can assign channels to cells of a 3-stripe system in an optimal way. Such an algorithm has been found and will be described in the next chapter.

1.5 Outline of the Thesis

The scope of this thesis is on the resource allocation in mobile cellular systems. In Chapter 2, we will tackle the static channel assignment problem, which concerns the assignment of nominal channels to cells according to their traffic load. We propose an algorithm called *Sequential Packing*, which is proved to be optimal in the 3-stripe system. Besides, promising results are obtained if we apply the algorithm to a general network topology. Further discussion on the optimality of sequential packing is postponed to Appendix A.

Although frequency planning serves as a useful tool in the traditional cellular system, it is not practicable to apply it to the future microcellular environment. Not only is it difficult to determine the reuse compatibility of cells, but also it is hard to estimate the traffic load of each microcell. Therefore, dynamic channel allocation method is more preferable. In Chapter 3, we will present a new algorithm called *Distributed Packing.* Numerical results show that our algorithm is better than a previously proposed one.

In Chapter 4, we will analyse the performance of a linear system and a 3-stripe system. As a preliminary study, we will consider the case that there is only one channel in the system. Part of the analysis is given in the Appendix B. Closed-form solution is obtained in the linear case. This analysis may act as a stepping-stone to the multi-channel analysis.

Finally, a conclusion will be given in Chapter 5. We will summarize the results obtained and suggest directions for further research.

Chapter 2

Static Channel Assignment

2.1 Introduction

The limiting availability of radio spectrum imposes an inherent bound on the capacity of a mobile cellular system. As demands for various mobile communication services grow, the question of how to utilize the valuable bandwidth in the most efficient way becomes more and more critical. To maximize system capacity, one typically tries to reuse the frequencies as much as possible. However, this may increase the mutual interferences among the cellular users. To maintain a certain quality of service, one has to keep the interference below a predefined level. For systems using Frequency Division Multiple Access (FDMA) or Time Division Multiple Access (TDMA), this requirement usually translates into compatibility constraints, stating for an arbitrary cell site what channels may be used for new calls based on what channels are currently used in other cell cites. Allocating the channels in an efficient way which does not violate the compatibility constraints is the main objective of the channel assignment problem (CAP). A lot of research can be found in the literature. Among them, most of the investigations are based on graph theoretic or heuristic approaches [6], [17], [18], [41], [55]. Recently, algorithms employing neural networks [16], [30] and simulated annealing [12], [33] have also been proposed. However, neural network based algorithms typically yield only suboptimal solutions [31]. The simulated annealing approach, although maybe more flexible, has difficulty in controlling the quality of the obtained solution. Besides, it is easy to be trapped in the local minima which requires a lot of computation time to escape from [12]. So far, the graph theoretic approach is still the most powerful and efficient way to handle this class of problems.

In the simple formulation of the CAP, only cochannel interference is considered and is known to be equivalent to the classical graph coloring problem. Since this problem is NP-complete [23], an exact search for the optimal solution is impractical for a large-scale system due to its exponentially growing computation time. Hence, most of the efforts are spent in developing approximation algorithms [6], [17], [41]. These algorithms, occasionally can find optimal solutions, but in general provides only suboptimal solutions with no information provided on how far away they are away from the optimal solution. In view of this undesirable feature, Gamst derives some lower bounds for the minimum number of channels required [19]. Our paper will provide another lower bound, which will be proved to provide a tighter bound in some cases. We also propose an algorithm which always finds the optimal solution for a special class of cellular networks. This optimality not only is significant in its own right, but it also yields clue on which circumstances our algorithm has good performance. Finally, an overall better performance compared to other existing competitive algorithms will be demonstrated by numerical examples.

2.2 Problem Formulation

Frequency sharing among different users is an important issue in mobile cellular systems. Many different multiple access schemes have been proposed. Among them, the most popular ones are frequency division multiple access (FDMA), time division multiple access (TDMA) and code division multiple access (CDMA). In FDMA systems, the spectrum is divided into non-overlapping frequency bands. Each user is allocated a dedicated frequency band for information transmission. In TDMA systems, each user is allocated a dedicated time slot for transmission and different users may share the same frequency band. In CDMA systems, each user is assigned a well designed code such that the interference among users are minimized. The entire time frame as well as the entire spectrum can be used for transmission. For simplicity, in this paper, we focus on channel assignments in FDMA systems. In this context, channel is referred to as frequency channel. However, the idea may as well be applied to TDMA systems provided that we refer to channel as time slot and define the compatibility matrix mentioned below in an appropriate way.

We assume that channels are equally spaced in the frequency domain and ordered from low frequency band to high frequency band with number 1,2,3...etc. A system of n cells is represented by an n-vector $X = [x_1, x_2, ..., x_n]$. Each cell x_i requires m_i channels. This forms a requirement vector $M = [m_1, m_2, ..., m_n]$. The assignment of the channels to the cells subjects to three different types of constraints:

- cochannel constraint (c.c.c): the same channel is not allowed to assign to certain pairs of cells simultaneously;
- adjacent channel constraint (a.c.c): channels adjacent in number are not allowed to assign to certain pairs of cells (normally, adjacent cells) simultaneously;
- co-site constraint (c.s.c): any pairs of channels assigned to the same cell must be separated by a certain number.

The constraints can be represented by an $n \times n$ non-negative symmetric matrix C, the so-called *compatibility matrix.* If any pair of cells x_i and x_j subject to the cochannel constraint or adjacent channel constraint, we have $c_{ij} = 1$ or 2 respectively. The co-site constraint is represented by the diagonal elements c_{ii}. Typically, c_{ii} is greater than or equal to 5.

The channel assignment problem is specified by the 3-tuple $P = (X, M, C)$. Let $\{1, 2, \ldots, M\}$ be a set of channels and F_i be the set of channels assigned to cell x_i. The objective of the problem is to find the minimum value of M such that there exists an assignment pattern, $F = \{F_1, F_2, \ldots, F_n\}$, which satisfies the following conditions:

$$|F_i| = m_i, \quad \text{for all } i$$

and

$$|f - f'| \geq c_{ij}, \quad \text{for all } i, j, f \in F_i, f' \in F_j.$$

This problem is equivalent to a generalized graph coloring problem [41]. Represent each cell by a vertex with weight $w_i = m_i$. If $c_{ij} > 0$, the vertices v_i and v_j are joined together by an edge with label c_{ij}. The resulting graph is called an *interference graph.* The channel assignment problem is equivalent to assigning positive integers, $\{1, 2, \ldots, M\}$, to the vertices such that each vertex has w_i integers assigned. The difference of the integers assigning to the vertices connected by an edge must not be less than the edge label. The objective is to minimize the maximum integer used. In the special case that only cochannel interference is considered, the c_{ij}'s are either 0's or 1's. This reduces the problem to the classical graph coloring problem. In the next section, we will consider this special case first.

2.3 Pure Cochannel Interference Case

The pure cochannel interference problem can be defined by a *topology graph*, G, with n vertices representing the n cells; each vertex has weight w_i ($1 \leq i \leq n$). A feasible coloring solution assigns color to the vertices with the constraint that no two adjacent vertices can have the same color. Moreover, a vertex, v_i, with weight w_i, needs to be colored w_i times, each time with a different color. The objective of the problem is to find a solution with the minimum number of colors. The optimal policies are termed the Maximum Packing (MP) assignments [13].

Unfortunately, for an arbitrary graph, the problem of determining an MP assignment is NP-complete. Hence, MP is an ideal concept rather than a practical solution. However, for graphs of special structures, efficient algorithms to compute MP assignments may exist and we call them *MP algorithms.*

In this paper, a heuristic algorithm is proposed. It has the property of yielding solutions with performance close to the MP assignments. Moreover, for a special class of network topology, it can be proved that this heuristic method is an MP algorithm.

Before we proceed, we have to define some terms. First of all, define the neighborhood of v, $N(v)$, as the set of v's adjacent vertices. A set of vertices in a graph which are all connected is called a *clique.* For every clique, we define its *clique weight* as the sum of weights of all the vertices inside it. A vertex typically belongs to more than one clique, the clique containing v with the maximum clique weight is called the *maximum clique weight* of v, denoted as $W(v)$. At times it is necessary to make the corresponding graph G explicit, we write it as $W(v|G)$.

Basically, our algorithm uses the *requirement exhaustive strategy* [17]. We pick up a color c_i and assign it to the vertices one by one until no further assignment of that color is possible. Then the next color c_{i+1} is used and the procedure is started over again. The question is how to determine which vertices should be colored by c_i. The vertex with greatest weight is chosen as the first vertex. To choose the subsequent vertices, the principle of *maximum overlap of denial areas* as defined in the third method in [17] is used. This principle states that a channel should be assigned to the cell whose denial area has maximum overlap with the already existing denial area of that channel. (A denial area for a cell, c, is the set of neighboring cells which cannot share the same frequency with c due to cochannel interference.) Our algorithm differs from the algorithm defined in [17] in the way the overlap is defined. In our algorithm, we define the overlap as the number of cells within the intersection of the two denial areas. In [17] overlap is defined as the sum of the requirements of the cells within the intersecting areas. Our definition ensures that the cells to which a channel assigned can be packed as close to each other as possible. When there is a tie, we break it by choosing the vertex with the largest maximum clique weight with respect to the topology graph induced by the intersection of the denial areas. The rationale of this rule is that the larger the maximum clique weight, the more difficult it is for that vertex to be colored. We call our algorithm *sequential packing algorithm* (SP) and state it as follows in psuedocode. We use A to denote the set of vertices being colored by the current color c and F to denote the vertices which are forbidden to be colored by c.

procedure SP (G(V,E): graph)
    $c := 1$;
    while G has 1 or more vertices do
        let A and F be empty sets ;
        let v be a vertex in $\{v_i : v_i \in V \text{ and } w_i = \max_{v_j \in V} w_j\}$;
        repeat
            put v into A ;
            $F := F \cup \{v\} \cup N(v)$;
            $m := \max_{v_i \in V \setminus F} |N(v_i) \cap F|$;
            if $m = 0$
                let v be a vertex in $\{v_i : v_i \in V \setminus F \text{ and } w_i = \max_{v_j \in V \setminus F} w_j\}$;
            else
                $K := \{ v_i : v_i \in V \setminus F \text{ and } |N(v_i) \cap F| = m\}$;
                if $|K| = 1$
                    then let v be the only vertex in K ;
                    else

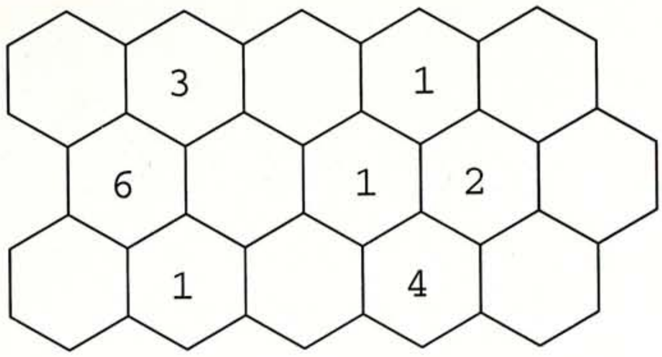

Figure 2.1: A seven-cell system for the demonstration of SP algorithm

```
                for each $v_i \in K$ do
                    construct a subgraph $S_{v_i}$ induced by the vertex set
                        $(N(v_i) \cap F) \cup \{v_i\}$ ;
                let $v$ be a vertex in
                    $\{v_i : v_i \in K$ and $W(v_i|S_{v_i}) = \max_{v_j \in K} W(v_j|S_{v_j})\}$ ;
            end{else}
        end{else}
    until $F = V$ ;
    for each $v_i \in A$ do
        color it with $c$ ;
        increase $c$ by 1 ;
        decrease $w_i$ by 1 ;
        if $w_i = 0$
            then delete from $G$ the vertex $v_i$ and all the edges connecting to $v_i$ ;
    end{for}
  end{while}
end
```

Example

In Figure 2.1, a seven-cell system is shown. The channel requirement of each cell is specified inside the corresponding hexagon. This system can be represented by a graph as shown in Figure 2.2.

1. First of all, we use c_1 to color vertex A which has the greatest weight among all the vertices. The denial area S becomes $\{B, C, D\}$.

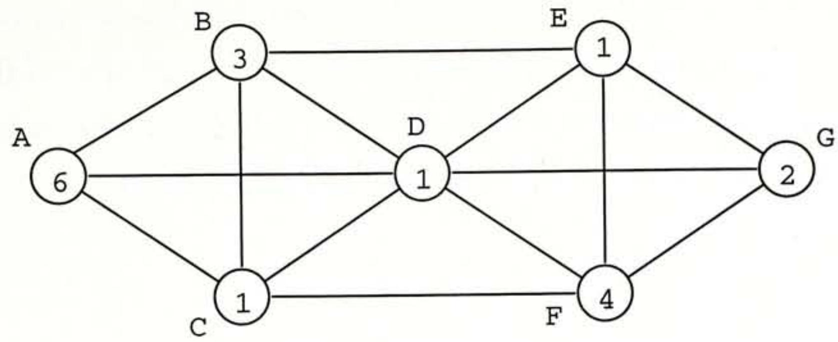

Figure 2.2: The corresponding graph of the seven-cell system

2. Next, we choose the vertex whose denial area has maximum overlap with the current denial area. Totally, there are three candidates. They are vertex E, F and G.

   - (Denial area of $E) \bigcap S = \{B, D\}$.
   - (Denial area of $F) \bigcap S = \{C, D\}$.
   - (Denial area of $G) \bigcap S = \{D\}$.

3. We choose the one with maximum cardinality. However, there is a tie between vertex E and F. To break the tie, we first form a subgraph for vertex E and F. The subgraph is induced by the candidate vertex and the overlapping of the denial areas. Then, we choose the vertex which has a larger maximum clique weight in the corresponding subgraph.

   - For vertex E, consider the subgraph induced by $\{B, D, E\}$. The maximum clique weight of E is 5.
   - For vertex F, consider the subgraph induced by $\{C, D, F\}$. The maximum clique weight of F is 6.

   Therefore, vertex F is chosen and colored by c_1. The new denial area S becomes $\{B, C, D, E, G\}$.

4. No more vertices can be colored by c_1. So c_2 is used and the procedure is repeated.

In the next section, we will show that this algorithm yields an optimal solution for this simple example. Besides, it is worth noting that the most time-consuming task in this algorithm is the calculation of clique weight. However, for interference graph arising from cellular systems, it is shown in [21] that the number of cliques grows only linearly with the network size and thus can be implemented as a routine operation in the stage of network design. Numerical experiences also show that our algorithm is quite efficient.

2.4 Systems of Special Structure

Although the channel assignment problem $P(X, M, C)$ is NP-complete in general, for the pure cochannel case, there it turns out that the SP algorithm is an efficient algorithm to allocate channels optimally for certain networks with special types of structure.

From now on, to facilitate the discussion, we will make the standard technical assumption that cells are laid out in the regular hexagonal tiling pattern. Define a system consisting of i rows of cells an *i-stripe cellular system.* Notice that a linear cellular system is a 1-stripe system. A 3-stripe system is shown in Figure 2.3. Furthermore, we assume that the cochannel constraint is equivalent to a cluster size N_c of 7.

Given any graph, we call any feasible coloring of vertices a *realization.* The realization which requires the minimum number of colors is called an *MP realization.* Given a realization, it is possible to obtain another realization by simply relabeling some or all of the colors. We call realizations which can be obtained from one another by relabeling colors *equivalent.* For an arbitrary graph, the MP realization is not necessarily unique. This is clear in view of the possibility of equivalent realizations. However, some graphs may allow multiple MP realizations that are not equivalent. For those realizations which require the same number of colors, we define that they are *similar.* Therefore, if a realization is similar to an MP realization, it is also MP.

One way to construct examples of similar realizations that are not eqivalent is to use the following color swapping operation for a 3-stripe system with reuse factor of 7. Assume that Ψ is a realization in which v_j is colored by c_b and v_k is colored by c_a and $v_j \in N(v_k)$. Define the operation $\text{Swap}_r(v_j, v_k, c_b, c_a)$ by swapping color c_a and c_b for cells v_j and v_k and all the cells on the right-hand-side of either of these two cells. Similarly, we can define $\text{Swap}_l(v_j, v_k, c_b, c_a)$ by swapping c_a and c_b for cells v_j

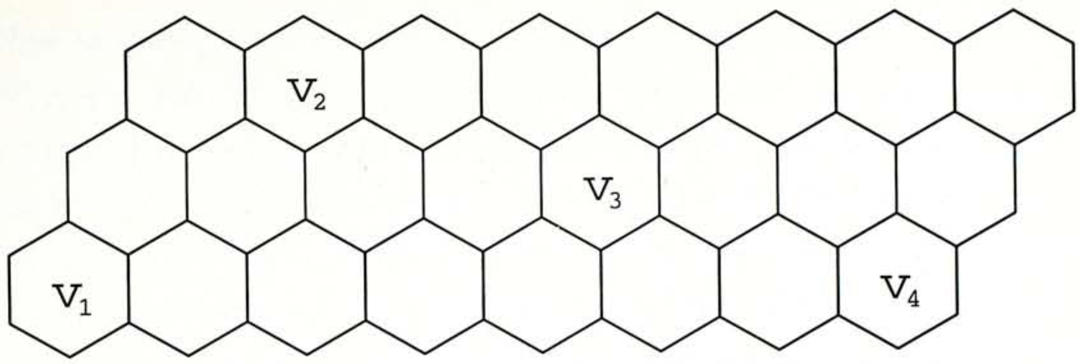

Figure 2.3: Example of a 3-stripe system. If v_1, v_2, v_3 and v_4 are colored by c_a, then $S_a = (v_1, v_2, v_3, v_4)$.

and v_k and all the cells on the left-hand-side of either of these two cells. If we apply the operation $\text{Swap}_r(v_j, v_k, c_b, c_a)$ to a realization and this swapping operation does not violate the channels assigned for cells on the left-hand-side of either v_j or v_k, then the realization obtained after the swapping is similar to the original realization. If at least one of the colors has been used on cells on the left-hand-side of either v_j or v_k, then the two realizations are not equivalent.

Theorem 1 *For a graph arising from an i-stripe cellular system with i less than or equal to three and cluster size equal to seven, the sequential packing algorithm (SP) always yields a solution which has the smallest possible number of colors used. In other words, it is an MP algorithm for this special class of graphs.*

Proof : A 1- or 2-stripe cellular system can be embedded into a 3-stripe system. An assignment problem for a 1- or 2- stripe system can be viewed as a problem on a 3-stripe system if cells outside of the original system are considered to have no channel demands. Therefore, it suffices to prove only the 3-stripe case.

Let Ψ_{SP} be a realization obtained from the SP. If a color, c_a, is used in a realization, Ψ, let $K_a(\Psi)$ be the set of vertices colored by c_a in the Ψ. We claim that there exists an MP realization, Ψ_{MP}, such that

$$K_a(\Psi_{SP}) = K_a(\Psi_{MP})$$

If this claim holds, we can use it to prove the theorem statement by using the following induction argument: If Ψ_{SP} has only one color, then it must be an MP realization. Suppose the theorem statement holds for all SP realizations using n

colors. Now consider a problem $P = (X, M, C)$. Suppose that Ψ_{MP} and Ψ_{SP} uses $k+1$ and $n+1$ colors ($k \leq n$) respectively. Let c_a be the first color used in the SP algorithm. For each vertex in $K_a(\Psi_{SP})$, subtract one from the corresponding component of the original requirement, M, to obtain M'. By the definition of the SP algorithm, the realization it yields for $P' = (X, M', C)$ is equivalent to Ψ_{SP} without c_a, and hence requires n colors. If the claim holds, MP will use k colors for the reduced problem P'. By the induction assumption, n is the minimal number of color needed for P' and hence n must be equal to k. So Ψ_{MP} uses $n+1$ colors and this shows that Ψ_{SP} is also MP.

Before proving the claim, we note that for a 3-stripe system and a reuse factor of 7, cells (vertices) colored by the same color, c_a, can be labeled in a left-to-right order, S_a, with no ambiguity (see Figure 2.3). Let $R_a(v)$ be the succeeding vertex of v in S_a, if it exists. Similarly define $R_a^{-1}(v)$ to be the preceeding vertex of v in S_a, if it exists. Notice that the order induced by S_a is not necessarily identical to the order the SP algorithm assigns the vertex to color c_a. However, for a 3-stripe system with reuse factor of 7, the following property, $\mathcal{P}$, holds:

Suppose that vertex i is assigned before vertex j. If $R_a(v_i) = v_j$ and $N(v_i) \cap N(v_j)$ is not empty, then v_j is the first node on the right-hand-side of v_i that is assigned after v_i. Similarly, if $R_a^{-1}(v_i) = v_j$ and $N(v_i) \cap N(v_j)$ is not empty, then v_j is the first node on the left-hand-side of v_i that is assigned after v_i.

Suppose that, according to SP, vertex v_0 is the first vertex colored by c_a. Notice that there exists an MP realization Ψ_{MP} in which v_0 is colored by c_a since v_0 must be colored by at least one color and one can relabel one of the colors to c_a. If the vertices referenced by $R_a^l(v_0)$ for both realizations are identical for all integer l, then the claim holds. Suppose, on the other hand, that l is the integer with the smallest absolute value such that the vertices referenced by $R_a^l(v_0)$ are not identical for the two realizations. Without loss of generality, we may assume that l is positive and $R_a^l(v_0)$ for Ψ_{SP} and Ψ_{MP} refers to v_j and v_k respectively with $j \neq k$. We claim that one can construct another MP realization, $\Psi_{MP'}$, so that all the vertex assignment to the left of and up to $R_a^{l-1}(v_0)$ are identical for Ψ_{MP} and $\Psi_{MP'}$ and $R_a^l(v_0)$ for $\Psi_{MP'}$ refers to v_j.

For notation simplicity, let us denote $R_a^{l-1}(v_0)$ by v_i. This node is colored by c_a in both realizations. Notice that if $R_a(v_i)$ is well defined for Ψ_{SP} but not for Ψ_{MP}, then

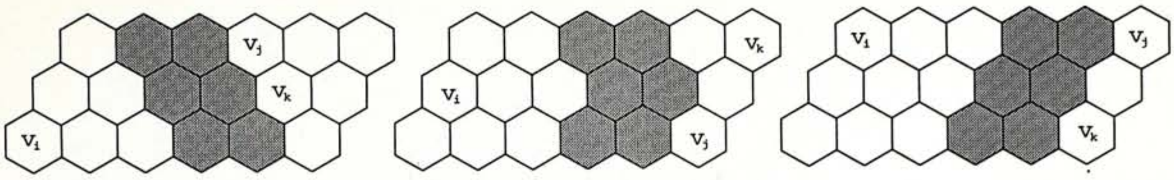

Figure 2.4: If $N(v_j) \cap F$ is empty, the shaded cells must not have any channel demand.

one can pick an arbitrary color used for v_j in Ψ_{MP} and replace it with c_a. This defines the new realization $\Psi_{MP'}$ as claimed. On the other hand, if $R_a(v_i)$ is well defined for Ψ_{MP}, it must be also well defined for Ψ_{SP} due to the nature of the SP algorithm which stops the assignment of a color only when there is no candidate cell available. Hence, we may assume v_j and v_k are well defined.

If v_k is also colored by c_a in the SP realization, then v_k must be on the right-hand-side of v_j. Moreover, v_k is not contained in $N(v_j)$. Hence, one can use c_a to color v_j in Ψ_{MP} without causing any violation on the right-hand-side of v_j. There is also no violation on the left-hand-side of v_j because the first cell on the left-hand-side on v_j colored by c_a in Φ_{MP} is v_i. Hence, we can assume that v_k is colored by c_b in Ψ_{SP} with $c_a \neq c_b$. Without loss of generality, we may assume that v_j is also colored by c_b in Ψ_{MP}.

Let F denote the set of *denial area* just before the assignment to v_j is made in the SP algorithm. Notice that

$$|N(v_k) \cap F| < |N(v_j) \cap F|$$

if not, then v_k will be picked by the SP algorithm to be colored by c_a. There are three possible cases for further consideration:

Case 1: $N(v_j) \cap F$ is empty.

This implies $N(v_k) \cap F$ is also empty. Due to the special topology of a 3-stripe, $N(v_j) \cap F = N(v_j) \cap N(v_i)$. It follows from Figure 2.4 that, there must be some layers of cells between v_i and v_j which does not have any channel demand. Hence, it is possible to do a swapping operation $\text{Swap}_r(v_j, v_k, c_b, c_a)$ on Ψ_{MP} without causing any violation on the left-hand-side of either v_j or v_k. The resulting realization is $\Psi_{MP'}$ and satisfied the claimed property.

Case 2: $|N(v_k) \cap F| < |N(v_j) \cap F|$

As observed before, this implies $|N(v_k) \cap N(v_i)| < |N(v_j) \cap N(v_i)|$. Recall that both v_j and v_k are on the right-hand-side of v_i. It follows from the special topology

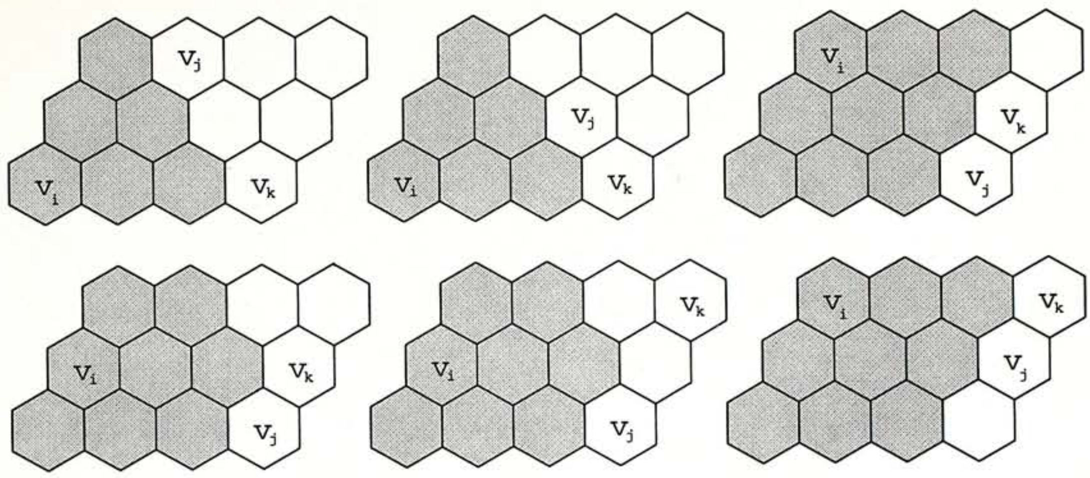

Figure 2.5: Some examples showing that $(N(v_k) \cap F) \subset (N(v_j) \cap F)$. The denial area is shaded.

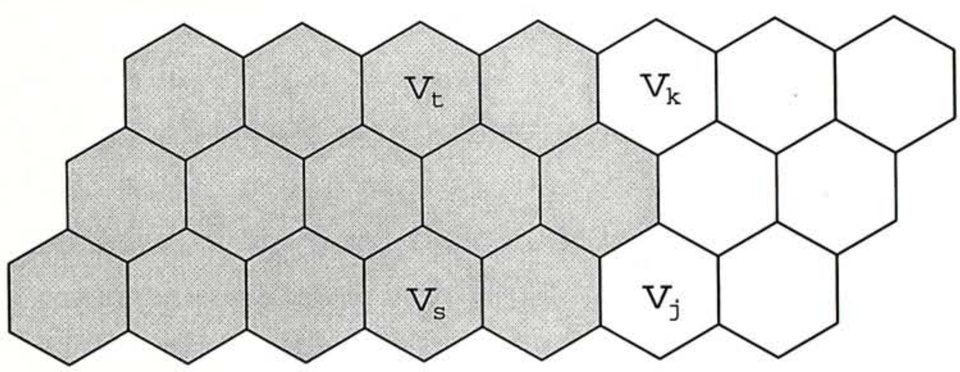

Figure 2.6: Let the shaded cells be the denial area F. The only possible location for v_j and v_k is shown.

of a 3-stripe that $(N(v_k) \cap F) \subset (N(v_j) \cap F)$ for all possible v_i (see Figure 2.5). Furthermore, if an arbitrary color can be used to color v_j in a realization, it can also be used to color v_k without causing any violiation for vertices on the left-hand-side of v_k. Perform a swapping operation $\mathrm{Swap}_r(v_j, v_k, c_b, c_a)$ on Ψ_{MP}. This operation does not casue any violation. The resulting realization is $\Psi_{MP'}$ and satisfied the claimed property.

Case 3: $|N(v_k) \cap F| = |N(v_j) \cap F|$

Let G_v be the subgraph induced by the vertex set $(N(v) \cap F) \cup \{v\}$. In this case, $W(v_k|G_{v_k}) \leq W(v_j|G_{v_j})$, otherwise v_k will be colored by c_a instead of v_j. If $N(v_k) \cap F = N(v_j) \cap F$, we can perform the operation $\mathrm{Swap}_r(v_j, v_k, c_b, c_a)$ as in case 2. So it is only necessary to consider the complementary situation.

Now suppose that $N(v_k) \cap F \neq N(v_j) \cap F$. The only possible location for v_j and

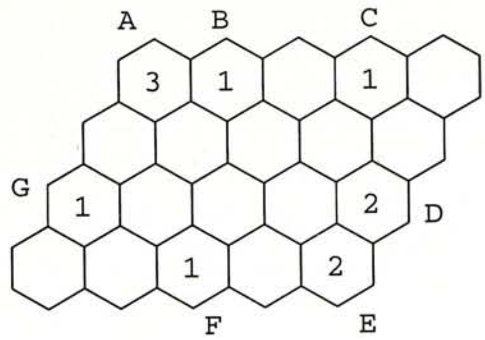

Figure 2.7: SP realization may not be optimal in a 4-stripe cellular system.

v_k is shown in Figure 2.4. From the Figure, it is clear that v_s and v_t are the only cells on the left-hand-side of v_j and v_k which may experience violation if a swapping operation Swap_r is performed.

If $w_j > w_t$, then in Ψ_{MP}, v_j is colored by more colors than v_t. Therefore, we can find a color, say c_d, that is used to color v_j but not v_t. Then we can perform $\text{Swap}_r(v_j, v_k, c_d, c_a)$ as before to obatin $\Psi_{MP'}$.

On the other hand, if $w_j \leq w_t$, then $w_s \geq w_k$. This is because $W(v_k|G_{v_k}) \leq W(v_j|G_{v_j})$ by the given assumption and $W(v_j|G_{v_j}) - W(v_k|G_{v_k}) = w_j + w_s - w_k - w_t$. In Ψ_{MP}, one of the colors assigned to v_k is c_a and c_a cannot be assigned to v_s. Therefore, we can always find a color, say c_d, in v_s which is not assigned to v_k. Hence, the operations $\text{Swap}_r(v_j, v_k, c_b, c_a)$ and $\text{Swap}_l(v_t, v_s, c_b, c_d)$ can be applied to Ψ_{MP}. Notice that the operation Swap_l does not alter the vertices which are colored by c_a and so will not affect the previous coloring of c_a. The resulting realization is $\Psi_{MP'}$.

Hence, in all three cases, we can find $\Psi_{MP'}$ with the claimed property. Repeating this argument if necessary for the case where l is negative, one can then guarantee that there is an MP realization which has an identical sequence of vertices, $R_a^i(v_0)$ as Ψ_{SP} up to $|i| \leq l$. Hence, there exists an MP realization which has the same set of vertices colored by c_a as Ψ_{SP} and this proves the claim. As a result, the theorem is proved.

□

This theorem cannot be generalized to an i-stripe system for $i > 3$. This can be seen from the example shown in Figure 2.7. As before, we assume a reuse factor of seven. Table 2.1 shows that SP uses five colors while MP uses only four. So the SP

Cell	Ψ_{SP}	Ψ_{MP}
A	c_1, c_2, c_4	c_1, c_2, c_3
B	c_3	c_4
C	c_1	c_3
D	c_2, c_4	c_1, c_2
E	c_3, c_5	c_3, c_4
F	c_1	c_1
G	c_3	c_4

Table 2.1: The realization of SP and MP for the example shown in Figure 2.7.

realization is not optimal. It has been shown that MP is NP-hard. Since SP is a polynomial time algorithm, this should not come as a surprise.

2.5 Generalization of SP

The SP algorithm stated in the previous section can only be used in the pure cochannel case. In this section, we will generalize it to include the adjacent channel and co-site constraint.

In the pure cochannel case, cells using the same channel are packed closely to each other such that the utilization of each channel is maximized. However, this may not be a good policy when there is adjacent channel constraint, for the reason that a closely packed channel will leave little room for its adjacent channels. Because of this mutual intervention, we pack the channels in a two by two basis.

Now we describe our Generalized SP (GSP) in graph theoretic terms. Suppose now we have two colors c_i and c_{i+1}. We first find the vertices with greatest weight. If it can be colored by c_i, we color it using c_i. Otherwise, we use c_{i+1}. Then, we try to color the remaining vertices in a round robin fashion. We first find the set of vertices which are allowed to be colored by c_i but not c_{i+1}. If the set is empty, it becomes c_{i+1}'s turn. If not, we choose the vertex to be colored using the same criterion of SP, i.e. first by the principle of maximum overlap of denial area and then by the maximal clique weight if there is a tie. Afterwards, we continue the coloring using c_{i+1}. If the considered set of both c_i and c_{i+1} are empty, we start the process again using the maximum weight criterion. This procedure repeats until no more vertices can be colored by either c_i or c_{i+1}.

In this generalized SP, we have two variations. In the first variation, when no further coloring by c_i or c_{i+1} is possible, we use the next two colors c_{i+2} and c_{i+3}. The second one, however, will 'uncolor' the vertices already colored by c_{i+1}. It starts the procedure again using c_{i+1} and c_{i+2} instead. We call these two variations GSP1 and GSP2 respectively.

So far we have not considered the co-site constraint. To deal with it, we use the simplest strategy. When the vertex which determined to be colored according to our criterion is restricted by the co-site constraint, we simply choose the next most favorable vertex according to the same criterion.

Before evaluating the algorithms by numerical examples, we derive a lower bound in the next section first.

2.6 A Lower Bound for the General Case

In [19], several lower bounds are found. However, we find that in some cases, the result obtained by our proposed GSP is still quite far away from the lower bound. This motivates us to improve the lower bound. Here, we will derive another bound which, in some cases, is tighter than those given by [19].

As in [19], we use $S_0(P)$ to denote the minimum number of channels used for problem P and we call a subset Q of X *v-complete* if

$$c_{ij} \geq v, \quad \text{for all } x_i, x_j \in Q.$$

The concept of v-complete subset is just a generalization of clique.

Theorem 2 *Let $P = (X, M, C)$ be a channel assignment problem and Q be a 1-complete subset of X. Let $x_i \in Q$ and assume $c_{ii} = k > u > 1$ and there exists $R \subseteq Q$ $(R \neq \phi, x_i \notin R)$ such that*

$$c_{ij} \geq u, \quad \text{for all } x_j \in R$$

Furthermore, let $m_R = \sum_{j \in R} m_j$.
If $k - 2u + 1 \leq 0$,

$$S_0(P) \geq (m_i - 1)k + 1 + m_R \tag{2.1}$$

else

$$S_0(P) \geq (m_i - 1)k + 1 + \max(m_R - (m_i - 1)(k - 2u + 1), 0). \tag{2.2}$$

Proof: Define $P' = (X, M', C')$ with M' having only two nonzero components

$$m'_i = m_i \ \ (x_i \in Q)$$

and

$$m'_j = m_R \ \ (x_j \in R).$$

The entries of the compatibility matrix C' are

$$c'_{ii} = k, c'_{ij} = c'_{ji} = u \text{ and } c'_{jj} = 1.$$

By lemma 4 and 5 in [19], $S_0(P) \geq S_0(P')$. Now we want to find $S_0(P')$. It is obvious that

$$S_0(P') \geq (m'_i - 1)k + 1$$

since the channels used in x_i has to be separated with minimum distance k.

For any two channels in x_i spaced with distance k, the number of usable channels between the gaps is

$$\begin{aligned} n &= (k-1) - 2(u-1) \\ &= k - 2u + 1 \end{aligned}$$

If $n \leq 0$, no gap exists and m'_j more channels are needed. Hence we obtain (2.1).

On the other hand, if $n > 0$, there are $(m'_i - 1)n$ spaces left inside all the gaps. If $m'_j \leq (m'_i - 1)n$, no additional channel is needed. Otherwise, we need $m'_j - (m'_i - 1)n$ more channels. Hence we obtain (2.2).

□

dimension of system layout	#. of times optimal solution found (out of 30)	worst case performance	average performance
10×10	16	7.40%	1.22%
5×20	22	3.59%	0.30%

Table 2.2: Performance of SP under different system layouts (cochannel case)

2.7 Numerical Examples

Example 1: Pure cochannel case

In the pure cochannel case, it is well-known that the *clique number*

$$\rho = \max_{Q:clique} \sum_{x_i \in Q} m_i$$

provides a lower bound for the number of channels needed since all the channels assigned to the same clique must be different. Hence, we will use this bound to judge the performance of our algorithm SP under different system topologies.

We compare two different layouts of hexagonal cells. The first one we considered is a 10×10 system and the second one is 5×20. We assume a cluster size N_c of 7. The channel requirement in each cell is generated randomly, ranging from 1 to 100. Thirty instances of each system are obtained by varying the seed of our random number generator. Generally, we do not know the optimal solution, except for the case that the number of channels used by our algorithms is exactly the same as the lower bound. So, we use the percentage of additional channels required relative to the lower bound as the performance measure.

The result is shown in Table 2.2. It can be seen that SP performs better in the 5×20 system. An optimal solution is found 22 out of 30 times. Both the worst case and average performance is much better. It is reasonable to expect that the 'narrower' the network structure, the better the performance of SP.

In general, the performance of SP is acceptable, in light of the fact that it is NP-complete to find a fast algorithm which can gaurantee the obtained solution exceeding the optimal value by less than 100% [22].

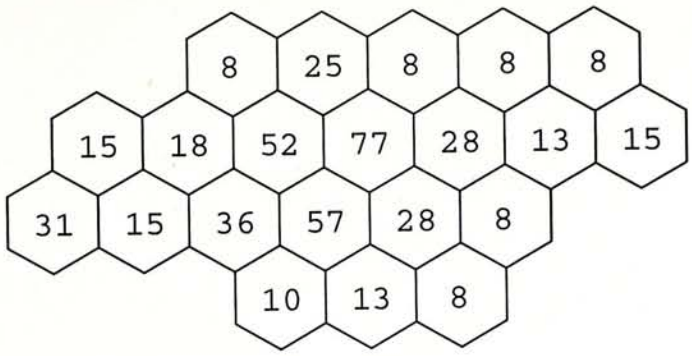

Figure 2.8: Case 1 of example 3. The numbers in the cells represent the corresponding m_i.

Example 2: General case (lower bound)

We take an example from [19] to demonstrate that the lower bound presented in the previous section can be tighter than that in [19]. The cellular layout is given in Figure 2.8. The numbers in the cells represent the corresponding channel requirements. As in [19], we assume co-channel constraints equivalent to a 12-cell cluster, adjacent channel constraints for adjacent cells, and the co-site constraint $c_{ii} = 5$.

We apply Theorem 2 with $k = 5$ and $u = 2$. Let x_i be the cell requiring 77 channels. Take R as the set containing all of the x_i's adjacent cells and hence $m_R = 198$. Equation (2.2) gives a lower bound of 427, tighter than the best lower bound given by [19], which is just 414.

Example 3: General case (algorithmic results)

Now we compare our algorithms GSP1 and GSP2 with that proposed by Box [6] and Sivarajan *et al.* [41]. The examples we use are taken from [41], which originate from [19]. They are shown in Figure 2.8 and 2.9.

The original algorithm proposed by Box attempts to satisfy the requirements using a given number of channels N. It is an iterative algorithm, which starts with an arbitrary initial order of the requirement list. Each requirement is associated with a real number which represents the assignment difficulty. Assignment is made according to the order, using the first channel which is compatible with previous assignments. If a requirement cannot be satisfied, the assignment difficulty of that requirement is increased by a random amount drawn uniformly from [0.5, 1.5]. After an iteration, the

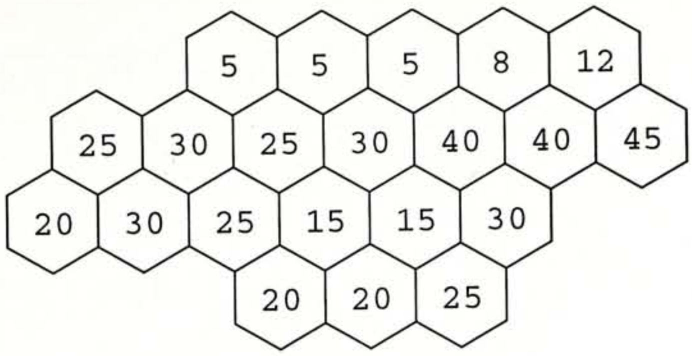

Figure 2.9: Case 2 of example 3. The numbers in the cells represent the corresponding m_i.

Problem #.	N_c	a.c.c	c_{ii}	LB	Sivarajan's	Box's (50 iterations)	Box's (100 iterations)	GSP1	GSP2
P1	12	2	5	427	460	449	446	440	450
P2	7	2	5	427	447	446	445	436	444
P3	12	2	7	533	536	533	533	565	533
P4	7	2	7	533	533	535	533	563	533
P5	12	1	5	381	381	381	381	381	381
P6	7	1	5	381	381	381	381	381	381
P7	12	1	7	533	533	533	533	533	533
P8	7	1	7	533	533	533	533	533	533

Table 2.3: Algorithmic results for Figure 2.8

requirement list is rearranged in decreasing order of the assignment difficulty. Then the procedure is repeated.

Here we modify this algorithm slightly to suit our needs. We use the tightest lower bound as the input parameter N. Additional channels will be used if a requirement cannot be satisfied by the first N channels. The algorithm terminates when all the requirements can be satisfied by N channels or the number of iterations reaches a prescribed maximum value.

The algorithms proposed by Sivarajan is a class of algorithms based on different ordering strategies. For a detailed description, see [41].

The results are shown in Table 2.3 and 2.4. The best result obtained among the whole class of Sivarajan's algorithms are reproduced from [41]. The Box's algorithm is performed twice. One is limited to a maximum of 50 iterations and the other is limited to 100.

Problem #.	N_c	a.c.c	c_{ii}	LB	Sivarajan's	Box's (50 iterations)	Box's (100 iterations)	GSP1	GSP2
Q1	12	2	5	258	283	274	274	291	273
Q2	7	2	5	258	270	273	273	273	268
Q3	12	2	7	309	310	309	309	314	309
Q4	7	2	7	309	310	309	309	323	309
Q5	12	2	12	529	529	529	529	530	529

Table 2.4: Algorithmic results for Figure 2.9.

It can be seen that most of the algorithms find the optimal solution in Problem P3-8 and Q3-5. In these cases, the lower bound are obtained by

$$\max_{x_i \in X}\{(m_i - 1)c_{ii}\} + 1$$

which implies that these problems are limited by the co-site constraint. This class of problems can be well solved by GSP2 since it always assigns the smallest possible color to the vertex with maximum weight. Hence optimal solutions are found in all the cases.

On the other hand, Problem P1-2 and Q1-2 are relatively hard to solve. The best results are obtained either by GSP1 or GSP2. Although GSP1 cannot deal with the problem limited by co-site constraints adequately, it does have the best performance in P1 and P2. Problem P2 is just the same problem considered in the previous example. As stated in [19], Box's algorithm gives a solution of 445, which is the best result at that time. However, both GSP1 and GSP2 yield a better solution.

In general, GSP2 gives satisfactory results in all the cases. Its performance is better than Sivarajan's algorithms. If compared to Box's heuristic, it requires more channels only for Problem P1. Besides, Box's heuristic is computationally more expensive due to its iterative nature.

Example 4: Comparison with neural network approach

Recently, neural network approach is used to solve CAP [16], [30]. In [16], eight problems were used for testing the proposed neural network parallel algorithm. It was found that the optimal solutions are obtained in all these cases. We have tried to solve these problems using GSP2 and we find that our algorithm also yields the optimal solutions.

Taking a closer look at these problems, we find that four of them are identical to Problem P4, P6, P8 and Q4, which we have already examined in the previous example. Another two have the same cellular network as shown in Figure 9 but with different constraints: $N_c = 7$, a.c.c=1 and $c_{ii} = 5$ or 7. It is worth noting that all these six problems are limited by the co-site constraint, which is relatively easy to solve as we have pointed out already. In fact, seven out of the eight problems are co-site constraint limited.

The remaining problem is taken from [30]. The data is obtained from a real-world network, which consists of 25 cells. Co-site constaints have not been considered. In this case, optimal solution is found by both the neural network algorithm and GSP2.

Since most of the testing problems are co-site constraint limited, we suspect whether the neural network approach can deal with other more difficult problems adequately such as Problem P1-2 and Q1-2 in Example 3.

2.8 Summary

In this chapter, we have proposed an approximate algorithm to solve the channel assignment problem. The algorithm is non-iterative, and converge much faster than the iterative one proposed by Box. This is especically favorable when the scale of the system is large. Numerical examples from [41] are used to test our algorithms and results are convincing.

Additionally, we have derived a lower bound for the minimum number of channels required. It is tighter than those proposed by Gamst in some cases. However, there is still a gap between the lower bound and the best solution known. Further improvements may be possible.

Chapter 3

Dynamic Channel Assignment

3.1 Introduction

The frequency spectrum, due to its scarcity, is the most valuable resource for wireless personal communication. To support a large number of mobile users in a cellular system, an efficient bandwidth sharing method is essential. Traditionally, the spectrum is divided into frequency channels, and a fixed set of nominal channels is assigned to each cell. However, this fixed channel assignment (FCA) method lacks the ability to cope with the temporal and spatial traffic variation. This undesirable feature of FCA brings the dynamic channel assignment (DCA) methods into consideration. A lot of DCA schemes have been proposed and most of them fall into two classes, namely interference adaptation and traffic adaptation. Algorithms that take account of the actual propagation loss, fading effects and inteference conditions belong to the first class. A few studies have shown that this class of algorithms can provide a capacity gain over the latter one [36], [46]. However, there is no general framework which can be used to analyse or bound the performance of these algorithms. Besides, it is difficult to compare the performance of different algorithms because their models and assumptions differ from one another. On the contrary, analytic results of some traffic adaptive DCA schemes have been obtained [9], [13], [35], [49]. A consensus framework is available such that comparison between different algorithms is simple. Among this class, the one which of special interest is the Maximum Packing (MP) algorithm [13]. For one thing, the steady state probability of a system which uses MP has a product form solution. For the other, it is optimal in the sense that a call will not be blocked if there exists a feasible rearrangement of channels which can carry all the ongoing calls plus the new call. But unfortunately, MP is an ideal concept rather than a practical algorithm, for the determination of whether a call can be accepted is equivalent to the graph coloring problem, which is well known to be NP-complete [28]. Therefore, quests for efficient traffic adaptive DCA algorithms continue.

In general, DCA algorithms can be implemented in either a centralized [11], [42] [48] or a distributed [9], [26], [54] manner. However, as the demand of mobile services grows, systems composed of a large number of microcells are expected. A distributed approach will become more favorable since it can greatly relieve the burden of the central switch. In this chapter, we will focus on traffic adaptive DCA scheme with distributed control.

3.2 Distributed Packing Algorithm

In our proposed Distributed Packing (DP) algorithm, the selection of channels is performed by the base stations. Each base station maintains a Channel Status (CS) table which records the channel usability of itself and its interfering neighboring cells. Table 3.1 shows a CS table at cell site c_i. We assume that there are M channels in the system. Totally, there are $n_i + 1$ rows, provided that there are n_i interfering neighbors of c_i. For example, $n_i = 18$ if the reuse factor is seven. Each row contains the information of the channel usability of the corresponding cell. A '1' denotes that the channel is currently in use in that cell while a '-1' denotes that the channel is prohibited to use subjecting to the reuse constraint. An empty entry indicates that it is a free channel in that cell.

At each cell site, whenever there is a channel assigned or released, an update of the CS table is needed. This information should be broadcasted to all its neighboring cells immediately in order to ensure database consistency.

When a new call or a handoff call arrived at cell c_i, channel selection will be based on the information provided by the CS table. The assignment principle of DP is: "Use the channel that most of your neighbors cannot use."

For each cell c_i, we define the assignment index of channel k, $I(k)$, as the number of neighboring cells of c_i which are not allowed to use channel k due to the reuse constraint. Then we choose channel k which can maximize the index $I(k)$. When

cell site	Channels					
	1	2	3	4	$\cdots$	M
c_i	1		-1		$\cdots$	
c_1	-1	-1	1		$\cdots$	-1
c_2	-1	-1	-1		$\cdots$	1
c_3	-1				$\cdots$	-1
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
c_{n_i}	-1			-1	$\cdots$	

Table 3.1: Channel Status table at cell site c_i

there is a tie, we break it arbitrarily.

If there is no free channel in cell c_i, channel reassignment will be attempted. First, the cell site c_i looks for a column of the CS table with only a single '1' among the n_i rows. If found, it means that the corresponding channel, say channel k, is used by only one of its neighbors. Then, it identifies that neighboring cell and check whether that cell has a free channel. If this is the case, it requests that cell to handoff the ongoing call from channel k to another channel. The requested cell will select a free channel according to the assignment index as described above. Consequently, the call arriving at c_i can be accepted.

However, if there is no column with only a single '1', the DP algorithm will look for columns with two '1's. If still none is found, it proceeds to find columns with three '1's. The procedure repeats until it finds a reassignment method or it finds out that the call cannot be accepted by local reassignments.

In the above procedure, the number of reassignments per call is upper bounded. For regular hexagonal layout with reuse factor seven, there are at most four reassignments per call. Since the channel reassignments are performed at call arrival instants, a fast handoff process is needed.

The DP algorithm is similar to the Local Packing (LP) algorithm proposed in [26]. For example, both algorithms require the maintainance of the CS table at each cell site. Besides, both algorithms may reassign channels at call arrival instants. However, a major difference is that we select channels according to the assignment index instead of arbitrary selection. Another difference is that we allow more than one channel reassignment per arriving call, while LP allows only one. In the next section, we will show that DP has better performance than LP in terms of blocking probability and average number of reassignments.

3.3 Performance Evaluation

In this section, we study the performance of DP by simulation. The simulated cellular system consists of 81 hexagonal cells arranged in a 9×9 array. We consider only cochannel constraint with a reuse factor of seven. We let the number of channels in the system be $M = 105$. Assume that the call arrival to each cell is a Poisson process and the call duration is exponentially distributed with a mean of 3 minutes. Mobility is not modeled and so there is no inter-cell handoff.

For the purpose of comparison, we also investigate the performance of the following channel assignment methods.

Fixed Channel Assignment (FCA) A fixed number of channels is assigned to each cell. In our examples, each cell has fifteen channels. The curves are plotted using the Erlang-B formula.

First Available DCA (FA-DCA) We assume that the channels are ordered in a list. When there is a new call arrived, the first available channel will be assigned. If no available channel, the call will be blocked.

Local Packing (LP) This algorithm is proposed by [26]. The similiarities and discrepancies between LP and DP have been mentioned in the previous section.

Maximum Packing (MP) A call is accepted if there exists a feasible arrangement of channels which can carry all the ongoing calls plus the new call. It is optimal among all traffic adaptive DCA methods. However, to determine whether a call can be accepted is shown to be NP-complete [28]. Therefore, we use the next two methods to bound its performance.

Clique Packing (CP) A set of cells which interfere with all the others is called a clique. The clique condition requires the total number of calls in the same clique be less than or equal to the total number of channels M. This is a necessary

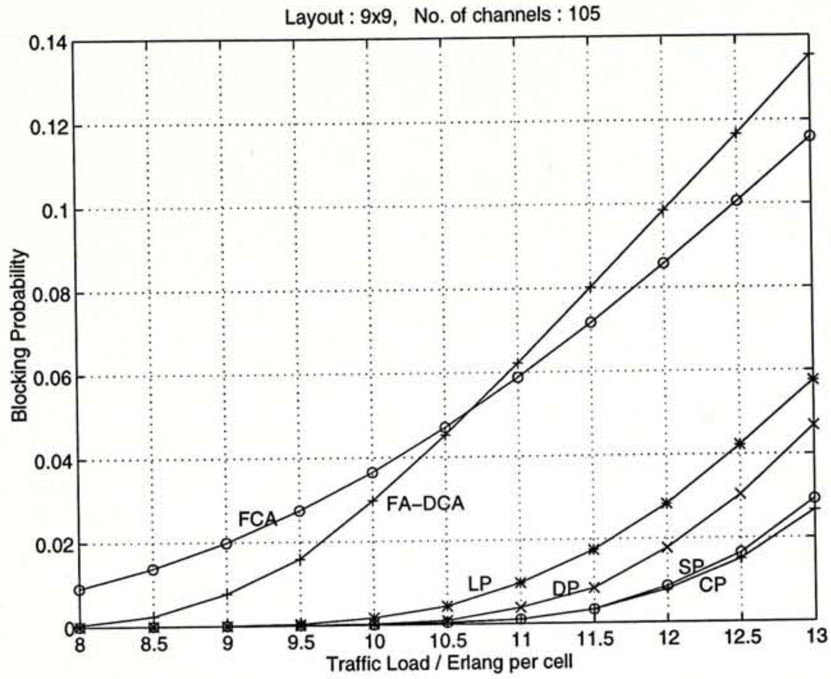

Figure 3.1: Blocking probablity for uniform traffic distribution

condition for the admission of a call. However, it is not sufficient even if we allow unlimited number of reassignments. Under CP, a call will be accepted if the clique condition is satisfied. Though it may sometimes violate the reuse constraint, its performance (in terms of blocking probability) can be used as a lower bound of the Maximum Packing (MP). For details, see [28] and [38].

Sequential Packing (SP) This algorithm is proposed in Chapter 2, originally used as a nominal channel allocation algorithm. If a new call arrives, we assign channels using FA-DCA strategy. If there is no free channel, we allow a complete reassignment of all the ongoing calls for the accommodation of the new call. Although this is not practical due to the large number of reassignments, its performance can be used as an upper bound of MP. [1]

First, we start with a uniform spatial traffic distribution in the 81 cells. The average blocking probability of the system under various channel assignment schemes is plotted as a function of traffic load in Figure 3.1. It can be seen that DP offers a lower blocking probability than all other practical algorithms under the considered

[1]It was proved in Chapter 2 that for a special class of network, SP has the same performance as MP.

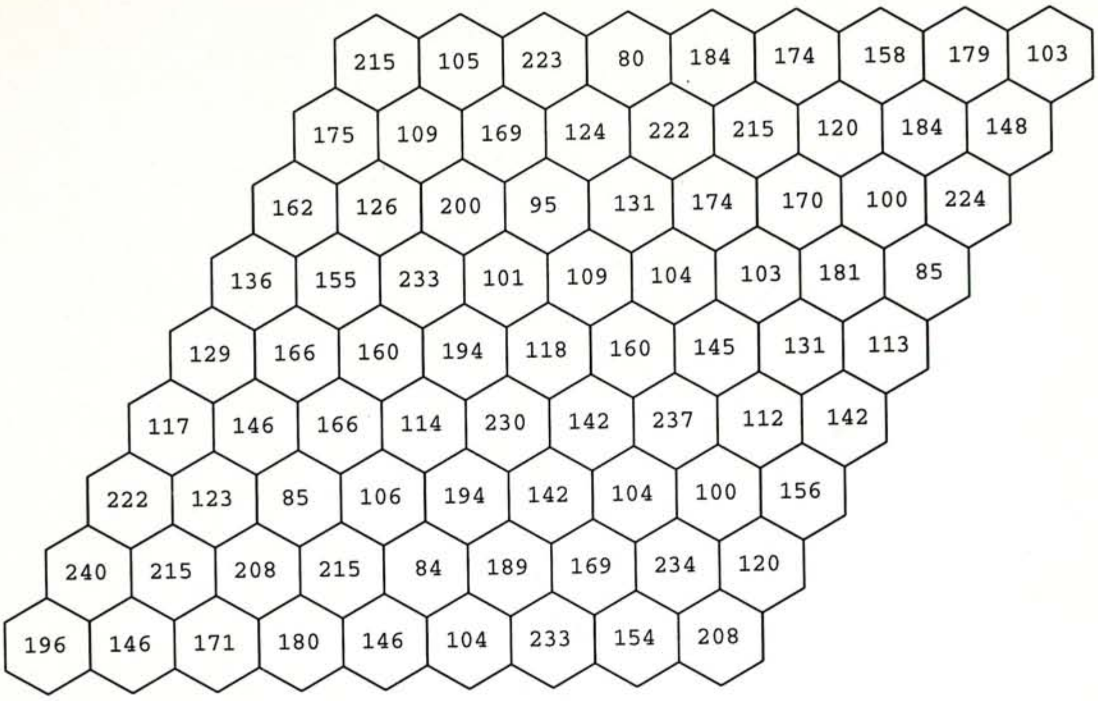

Figure 3.2: Non-uniform traffic distribution in a 81-cell system

region of traffic loading. Typically, we compare the system capacities achieved by different algorithms at a blocking probability P_B of 0.02. The capacities of FCA, FA-DCA , LP and DP are respectively 9, 9.7, 11.6 and 12.1 erlangs/cell. If we use FCA as a reference, the capacity gain of FA-DCA, LP and DP are 8%, 29% and 34% respectively. This implies that DP can provide an additional 5% capacity gain over LP.

At this point, a question arises naturally. What is the difference between DP and the optimal strategy MP? In Figure 3.1, the blocking probability of CP is plotted. It provides a lower bound for us. However, in the literature, there is no information on how tight this lower bound is. In the same figure, we also plot the curve for SP. Though this algorithm requires too much reassignments for practical applications, its call admissions, unlike CP, do not violate the reuse constraint. So, it provides an upper bound for the blocking performance of MP. From Figure 3.1, it can be seen that the performance of SP and CP is very close, especially in the light traffic region. This suggests that CP can serve as an good approximation of MP.

Now we compare the capacities of systems using DP and CP. At $P_B = 0.02$, the carried traffic of DP and CP are about 12.1 and 12.7 erlangs/cell respectively. So, DP can acheive about 95% of the maximum capacity.

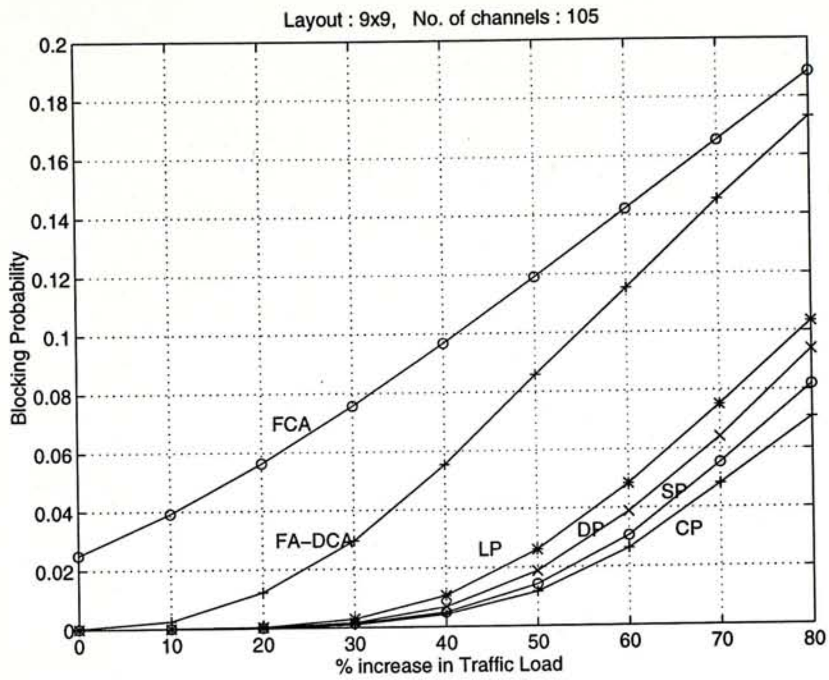

Figure 3.3: Blocking probablity for non-uniform traffic distribution

Next, we consider a non-uniform spatial traffic distribution in the 81 cells. The distribution is shown in Figure 3.2. The numbers inside the cells represent the call arrival rates (in calls/hour) to the corresponding cell. They are generated uniformly from 80 to 240. The blocking performances of the various channel assignment schemes are plotted in Figure 3.3. As expected, DP offers the lowest blocking probability among all the practical strategies. At $P_B = 0.02$, again DP can carry 5% more traffic than LP. Moreover, capacity of DP is about 84% of the maximum capacity.

The average number of reassignments per accepted call under uniform and non-uniform traffic are shown in Figure 3.4 and 3.5 respectively. Although DP may require more than one reassignment to accomodate a single call in some circumstances, surprisingly, under normal operating region (low traffic region where $P_b < 0.04$), DP requires less reassignments than LP in both cases. The reason is that the channel selection of DP is based on the assignment index while that of LP is arbitrary. As a result, DP can produce a more compact channel pattern which requires less reassignments in average.

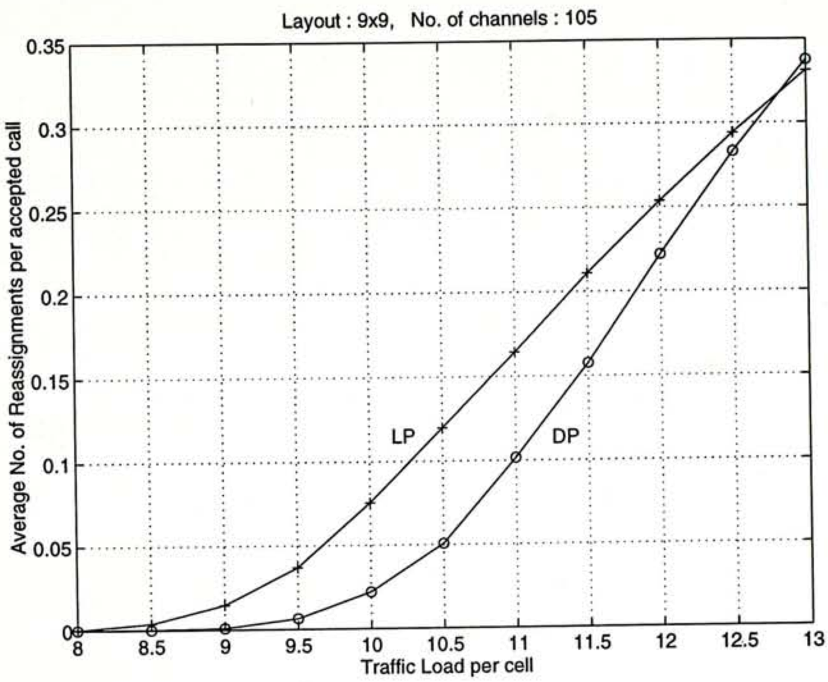

Figure 3.4: Number of reassignments per accepted call for uniform traffic distribution

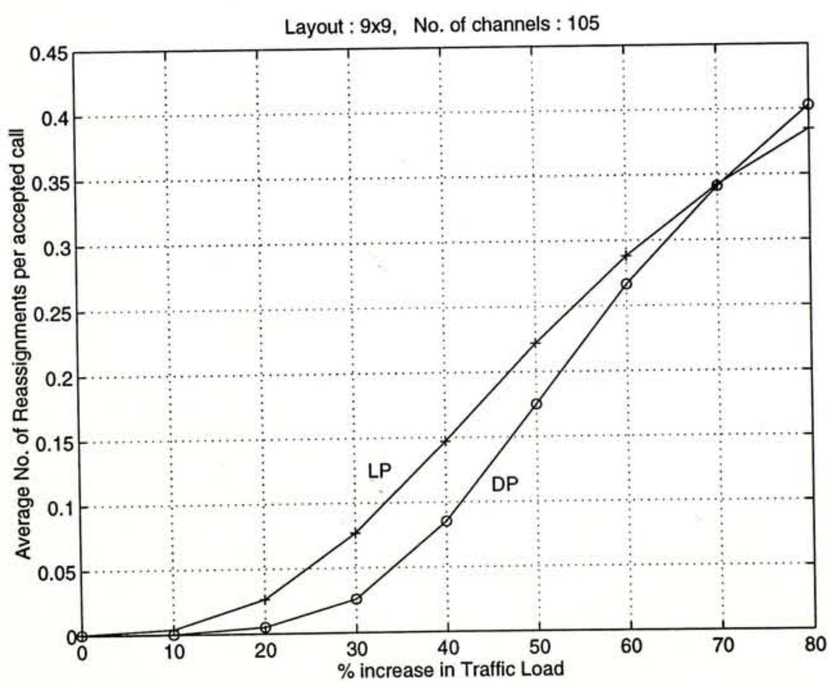

Figure 3.5: Number of reassignments per accepted call for non-uniform traffic distribution

3.4 Summary

The Distributed Packing (DP) algorithm is proposed in this paper. We have shown that in both uniform and non-uniform traffic situations, a larger system capacity can be obtained if compared to the Local Packing (LP) algorithm. Besides, its performance is close to the optimal one. It attains 95% and 84% of the maximum capacity under uniform and non-uniform traffic. Since DP needs only local information for channel assignment, it is very suitable to be employed in systems consisting of a large number of microcells. Moreover, DP can produce a channel pattern which is more compact than LP due to a more appropriate utilization of the available information. As a result, the number of channel reassignments in the system is reduced without extra cost.

Chapter 4

Single-Channel User-Capacity Calculations

4.1 Introduction

Capacity or blocking probability is often used as a measure to compare systems using different channel assignment algorithms. For static channel assignment, the blocking probability of each cell can be obtained by the famous Erlang-B formula. However, for dynamic channel assignemnt, analytical results are difficult to obtain. In the literature, most of the studies on DCA are based on simulation model. Theoretical studies are relatively rare. A channel borrowing scheme is investigated in Yeung and Yum [48] while a hybrid allocation method is examined by Yue [50]. In Kelly [29] and in McEliece and Sivarajan [35], asymptotic performance of a class of DCA are obtained. Besides, Cimini *et. al.* [10] derives analytical result for the capacity of a linear cellular system using single channel.

In this chapter, we re-derive the result of Cimini *et. al.* for a linear system using another method first suggested by C.Buyuhhor. In addition, we use this method to derive the capacity for a 3-stripe cellular system.

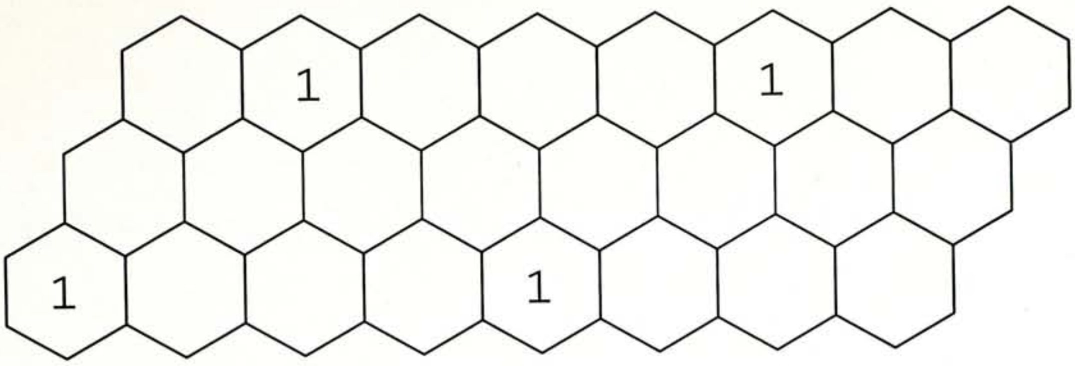

Figure 4.1: The most compact way to use the channel. The '1's in the cells denote that the cell is using the channel.

4.2 Capacity as a Performance Measure

In [10], only single-channel system is considered and *capacity* refers to the average proportion of cells which use the channel simultaneously. This concept can be generalized for the multi-channel system by defining capacity as the average number of users per channel per cell.

Consider the single channel case. It is obvious that the capacity cannot exceed one because of the reuse constraint. However, a tighter upper bound can be obtained if we consider the topology of the celluar system and the reuse constraint together. In [10], it is said that the maximum capacity C_{max} of an infinite linear cellular system under single-cell buffering is $\frac{1}{2}$. This can be achieved if the channel is reused in the most compact way. For example, all the odd-numbered cells use the channel but none of the even-numbered cells use it. Then half of the cells use the channel and the capacity equals $\frac{1}{2}$. This result can be generalized for R-cell buffering. In a planar celluar system, the situation is similar. For single-cell buffering, C_{max} is $\frac{1}{3}$ while for double-cell buffering, C_{max} is $\frac{1}{7}$.

In later sections, we will consider the capacity of a 3-stripe system under double-cell buffering. From Figure 4.1, it can be seen that the maximum capacity C_{max} is $\frac{1}{6}$, which is slightly greater than the general planar case under the same reuse constraint.

In [10], Poisson arrival, exponential call holding time and uniform traffic in all cells are assumed. A channel will be granted to a new call if there is free channel available. A relation between capacity C and blocking probability P_b is then derived. Both of them are functions of the offered traffic per cell ρ and they are related by equation (4.1).

$$P_b(\rho) = 1 - \rho^{-1}C(\rho) \tag{4.1}$$

4.3 Capacity of a Linear Celluar System

In this section, we calculate the capacity of a linear cellular system with an infinite number of cells. We assume that there is only one channel available for use. We adopt the standard traffic model which assumes that the call arrival process is Possion with rate λ and the channel holding times are exponentially distributed with mean $1/\mu$. Besides, we assume single-cell buffering which means that the same channel cannot be used simultaneously in neighboring cells.

First of all, assume that the cellular system is composed of n cells. Let $(Q_1, Q_2, \cdots, Q_n)$ be the binary occupancy vector where

$$Q_i = \begin{cases} 1 & \text{if cell } i \text{ is using the channel} \\ 0 & \text{otherwise} \end{cases}$$

If there is no restriction on how to use the channel, it is obvious that the solution is given by a product-form solution. However, since the restriction is simply a truncation of the state space and this truncation only cancels the transitions in the local balance equation, the solution is still in product form.

$$\Pr(Q_1 = q_1, Q_2 = q_2, \cdots, Q_n = q_n) = \alpha_n \rho^{q_1+q_2+\cdots+q_n} \tag{4.2}$$

where α_n is a normalization constant.

Therefore,

$$\Pr\left(\text{number of cells using the channel} = i\right) = \alpha_n c_i(n) \rho^i \tag{4.3}$$

where $c_i(n)$ is the number of possible arrangement that among the n cells, i cells are using the channel.

Since the probabilities must sum to one,

$$\alpha_n = \Big(\sum_{i=0}^{\lfloor (n+1)/2 \rfloor} c_i(n) \rho^i \Big)^{-1}$$

where $\lfloor x \rfloor$ denotes the largest integer smaller than x.

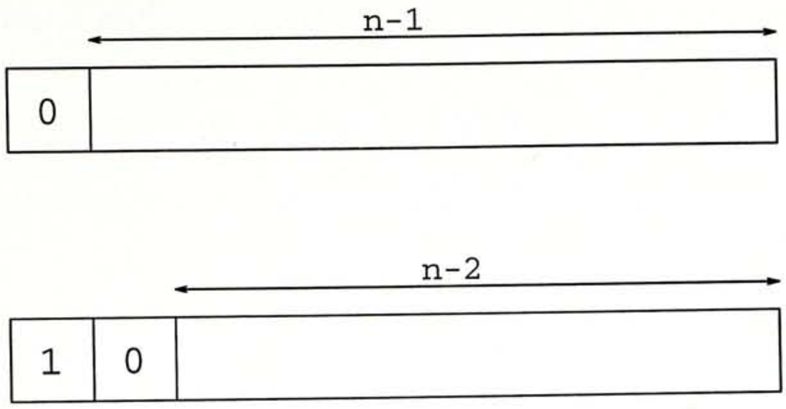

Figure 4.2: There are two possibilities, concerning the channel occupancy in the first cell.

Define $C_n(\rho)$ as the E[number of cells using the channel]$/n$.

$$nC_n(\rho) = \frac{\sum_{i=0}^{\lfloor (n+1)/2 \rfloor} i c_i(n) \rho^i}{\sum_{i=0}^{\lfloor (n+1)/2 \rfloor} c_i(n) \rho^i} \tag{4.4}$$

Let M_n and A_n be the numerator and denominator of the expression on the right hand side of equation (4.4) respectively, i.e.

$$M_n = \sum_{i=0}^{\infty} i c_i(n) \rho^i$$

$$A_n = \sum_{i=0}^{\infty} c_i(n) \rho^i$$

The upper limits of M_n and A_n are changed into infinity. When i is larger than $\lfloor \frac{n+1}{2} \rfloor$, $c_i(n) = 0$ by definition.

To determine A_n and M_n, we set up a difference equation of $c_i(n)$ first. Among the n cells, i of them are using the channel. If the first cell is not one of the i cells, then all the cells using the channel must be among the remaining $n-1$ cells. The number of arrangements in this case is $c_i(n-1)$. On the other hand, if the first cell is one of the i cells, then the second cell cannot use the channel, due to the constraint of single-cell buffering. As a result, there are $i-1$ cells which are using the channel among the last $n-2$ cells. The number of arrangements in this case is $c_{i-1}(n-2)$ (see Figure 4.2). Therefore, we have

$$c_i(n) = c_i(n-1) + c_{i-1}(n-2) \quad i \geq 1,\ n \geq 2 \tag{4.5}$$

Multiplying with ρ_i and sum up the terms over i,

$$\begin{aligned}
\sum_{i=1}^{\infty} c_i(n)\rho^i &= \sum_{i=1}^{\infty} c_i(n-1)\rho^i + \sum_{i=1}^{\infty} c_{i-1}(n-2)\rho^i \\
A_n - c_0(n) &= A_{n-1} - c_0(n-1) + \rho A_{n-2} \\
A_{n+1} &= A_n + \rho A_{n-1}
\end{aligned}$$

since $c_0(n) = c_0(n-1) = 1$.

Examining the boundary condition on $c_i(n)$, we get $A_1 = 1+\rho$ and $A_2 = 1+2\rho$.

Define $A(s) = \sum_{i=0}^{\infty} A_{i+1}s^i$. Using the standard technique, we get

$$\begin{aligned}
A(s) &= \frac{1+\rho+\rho s}{1-s-\rho s^2} \\
&= \frac{1}{\rho}[\frac{1+\rho+\rho\alpha}{(\alpha-\beta)(\alpha-s)} - \frac{1+\rho+\rho\beta}{(\alpha-\beta)(\beta-s)}]
\end{aligned}$$

where α and β are the roots of $1-s-\rho s^2 = 0$.

Taking inverse transform, we obtain

$$A_{i+1} = \frac{1}{\rho}[\frac{1+\rho+\rho\alpha}{\alpha-\beta}(\frac{1}{\alpha^{i+1}}) - \frac{1+\rho+\rho\beta}{\alpha-\beta}(\frac{1}{\beta^{i+1}})]$$

Let α be the root with a smaller magnitude (i.e. $|\alpha| < |\beta|$).

When n goes to infinity,

$$A_n \asymp \frac{1}{\rho}[\frac{1+\rho+\rho\alpha}{\alpha-\beta}(\frac{1}{\alpha^n})] \tag{4.6}$$

Next we have to compute M_n. Multiplying $i\rho^i$ to equation (4.5) and sum up the terms over i, we have

$$\sum_{i=1}^{\infty} ic_i(n)\rho^i = \sum_{i=1}^{\infty} ic_i(n-1)\rho^i + \sum_{i=1}^{\infty} ic_{n-1}(n-2)\rho^i$$

$$\sum_{i=1}^{\infty} ic_i(n)\rho^i = \sum_{i=1}^{\infty} ic_i(n-1)\rho^i + \rho(\sum_{i=1}^{\infty}(i-1)c_{i-1}(n-2)\rho^{i-1} + \sum_{i=1}^{\infty} c_{i-1}(n-2)\rho^{i-1})$$

$$M_{n+1} = M_n + \rho(M_{n-1} + A_{n-1})$$

Again, examining the boundary condition of $c_i(n)$, we get $M_1 = \rho$ and $M_2 = 2\rho$.

Similary, define $M(s) = \sum_{i=0}^{\infty} M_{i+1}s^i$.

$$\begin{aligned} M(s) &= \frac{\rho[1+s+s^2A(s)]}{1-s-\rho s^2} \\ &= \frac{\rho}{(1-s-\rho s^2)^2} \end{aligned}$$

As before, using partial fraction and taking inverse transform,

$$\begin{aligned} M_{i+1} &= \frac{1}{\rho}[\frac{2}{(\alpha-\beta)^3}(\frac{1}{\alpha^i+1}) + \frac{1}{(\alpha-\beta)^2}(\frac{i+1}{\alpha^{i+2}}) \\ &\quad -\frac{2}{(\alpha-\beta)^3}(\frac{1}{\beta^{i+1}}) + \frac{1}{(\alpha-\beta)^2}(\frac{i+1}{\beta^{i+2}})] \end{aligned}$$

When n goes to infinity,

$$M_n \asymp \frac{1}{\rho(\alpha-\beta)^2}(\frac{n}{\alpha^{n+1}}) \tag{4.7}$$

By equation (4.4), (4.6) and (4.7),

$$\begin{aligned} C(\rho) &= \lim_{n\to\infty} C_n(\rho) \\ &= \lim_{n\to\infty} \frac{M_n}{nA_n} \\ &= \frac{1}{\alpha(\alpha-\beta)(1+\rho+\rho\alpha)} \\ &= \frac{1}{2}[1-(1+4\rho)^{-1/2}] \end{aligned}$$

Hence, we arrive at a solution which is the same as [10]. Note that $\lim_{\rho\to 0} C(\rho) = \rho$ and $\lim_{\rho\to\infty} = \frac{1}{2}$. The first limit is obvious since if the offered traffic is sufficiently light, the reuse constraint has negligible effect on the use of the channel. The second limit implies that the maximum capacity C_{max} is achievable, on the condition that the offered traffic is very heavy.

4.4 Capacity of a 3-stripe Cellular System

In this section, we calculate the capacity of a 3-stripe system. We assume that there are $3n$ cells in the system, with n cells in each row. The layout is shown in Figure 4.3.

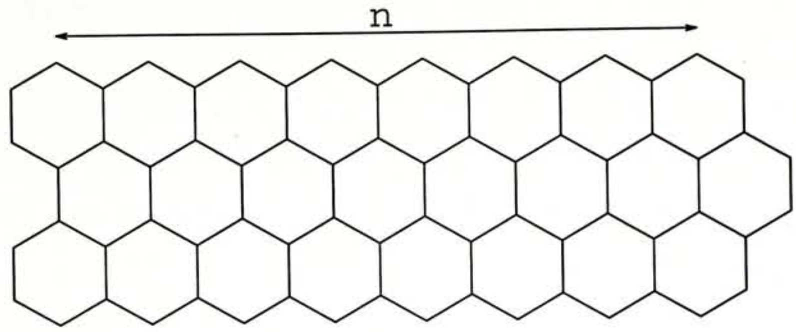

Figure 4.3: There are three possibilities, concerning the channel occupancy in the first cell of each row.

We adopt the same traffic model as the previous section. This time we assume that double-cell buffering is used, which means that the channel occupied by a cell cannot be reused in its first two-tier's neighbors.

By the same argument in the previous section, the product form solution holds.

$$\Pr(\text{number of cells using the channel} = i) = \alpha_n a_i(n)\rho^i \tag{4.8}$$

where α_n is a normalization constant and $a_i(n)$ is the number of possbile arrangements that among the $3n$ cells, i cells are using the channel.

By the same technique as before, we can obtain the capacity of a 3-stripe system composed of infinite number of cells.

$$C(\rho) = \frac{1}{\alpha}\left(\frac{3\rho + 2\rho^2\alpha^2 - \rho^3\alpha^4}{1 + 11\rho + (3\rho + 11\rho^2)\alpha + (2\rho + 17\rho^2)\alpha^2 + (\rho^2 - 3\rho^3)\alpha^3 + (\rho^2 + 8\rho^3)\alpha^4}\right) \tag{4.9}$$

where α is the root which has the smallest magnitude among those of the equation $1 - z - \rho z^2 - 2\rho z^3 - \rho^2 z^5 = 0$.

For clarity, we show the derivation of equation (4.9) in Appendix B.

Note that when $\rho \to 0$, $\alpha \to 1$ and $C \to \rho$. This result is the same as the linear case in previous section. When $\rho \to \infty$, $\rho\alpha^2 \to 1$. Substituting the result in equation (4.9), we obtain $C \to \frac{1}{6} = C_{max}$.

4.5 Summary

The single-channel capacities of an infinite linear and 3-stripe system have been calculated. Closed form results are obtained for the linear case. It was shown that the maximum capacity can be approached when the traffic load is sufficiently heavy.

Chapter 5

Conclusion

5.1 Summary of Results

The channel assignment problem has been studied by many researchers for many years. A lot of heuristic algorithms have been proposed. However, little general results have been known. In 1986, Gamst proposed some lower bounds for this problem [19]. From the lower bound calculated, one can know whether the optimal solution has been obtained or the solution yielded is far away from it. This eases the evaluation of the performance of different channel assignment method. In Chapter 2, we have proposed another lower bound which is tighter in some instances. Examples have been given for demonstration.

The lower bound is useful in performance evalution. However, it is equally important to tackle the problem in a constructive way. A lot of algorithms can be found in the literature. All of them have the undesirable feature that there is no information on how well it can perform. Our proposed algorithm, called sequential packing, has been proved optimal in a special class of cellular systems called 3-stripe system. When applying it to general systems, promising results are obtained.

In the future microcellular systems, it is difficult to carry out the traditional frequency planning. Therefore, dynamic channel assignment with distributed control is expected. In Chapter 3, we proposed a distributed algorithm called Distributed Packing (DP). It has a better performance than a previously proposed one. Besides, its performance is also compared with Maximum Packing (MP), which is the optimal strategy among this class of channel assignment scheme. In our examples, DP attains 95% and 84% of the maximum capacity under uniform and non-uniform traffic respectively.

Finally, in Chapter 4, a preliminary analysis of the performance of DCA in special cellular systems is given. We consider only the single channel case, which is a first step to solve the more challenging multi-channel systems.

5.2 Suggestions for Further Research

In this thesis, we have considered static and dynamic channel assignment for mobile cellular systems. The power control issue has not been addressed. In previous papers such as [15],[47],[52] and [53], power control is performed after channel allocation. The objective is to save battery life or improve transmission quality. However, as Ishii and Yoshida pointed out, power control can take an active role in the channel allocation process [27]. A good channel assignment algorithm should allocate a channel to a new call such that power control can be applied effectively. An integration of channel assignment and power control is an interesting area for further research.

Another issue worth considering is the voice activity. For normal conversational speech, the duty cycle is about 43% [24]. If statistical multiplexing is used, a maximum gain of 2.35 can be achieved. A scheme called packet reservation multiple access (PRMA) is described in [24]. This scheme can be regarded as a combination of slotted Aloha with reservation and TDMA. The performance is evaluated by a simulation model in [24]. A multiplexing gain can be achieved. However, since it is a contention based protocol, system will become unstable if the traffic load is too high. It is desirable to design a protocol which works well in both high traffic and low traffic region. In addition, the incoporation of voice and data should also be considered.

Appendix A

On the Optimality of Sequential Packing

Graph coloring is one of the most famous areas in combinatorics. In general, it is known to be NP-complete [23]. A lot of heuristics have been proposed. One class of heuristics is called the *greedy* (or *sequential*) method [5]. One version of its is stated as follows:

Go through the vertices in order, assigning the first color to every vertex for which it is available; repeat for the second color, and so on, until all the vertices are colored.

In chapter 2, we proposed the Sequential Packing (SP) algorithm, which belongs to this class of heuristics. In applying SP to channel assignment problem in cellular systems, we have proved that SP is optimal for a class of network topologies called i-stripe system for $i \leq 3$. Now we will use graph theoretic terms to discuss the optimality of the algorithm. First of all, we formulate the graph multi-coloring problem. Then, we will show that the algorithm is optimal for all triangulated graphs.

A.1 Graph Multi-coloring Problem

Given a graph $G = (V, E)$, we associate each vertex $v_i \in V$ with a positive integer w_i. We call the resulting graph G_w a *weighted graph* with G being its *underlying graph.* In the classical graph coloring problem, the objective is to color all the vertices using a minimum number of colors such that each vertex is colored once and vertices joining by an edge are colored by different colors. This problem can be generalized to a *multi-coloring problem* where each vertex v_i needs to be colored w_i times. If $w_i = 1$

for all i, it is reduced to the traditional coloring problem.

Now we introduce some standard graph theoretic terms. Most of them are taken from [2] and [4].

- A path between two vertices v_1 and v_r in a graph is a finite sequence of vertices and edges of the form $v_1, e_1, v_2, e_2, \cdots, e_r, v_r$, where e_k is an edge between v_k and v_{k+1}.

- A path which does not encounter the same vertex twice is called an *elementary path.*

- A graph is said to be connected if there is a path between every pair of vertices in it.

- The distance $d(v_i, v_j)$ between two vertices v_i and v_j is defined as the minimum path length between them.

- A graph $G' = (V', E')$ is called a subgraph of $G = (V, E)$ if V' and E' are subsets of V and E respectively.

- If A is a subset of V, the subgraph of G induced by A is the graph $G_A = (A, E')$ where e' is an edge of E' if $e' = \{v_1, v_2\}$, where e' is in E and both v_1 and v_2 are in A.

- A *clique* K of a graph $G = (V, E)$ is defined as a subgraph of G such that all the vertices of K is fully connected. If K is not a proper subgraph of another clique of G, it is a *maximal clique.*

- A *stable set* S is defined as a subset of vertices such that no two vertices in S are adjacent.

- The *clique number* $\omega(G)$ of a graph G is defined as the maximum number of vertices in any clique of G.

- The *chromatic number* $\gamma(G)$ of a graph G is defined as the minimum number of colors needed to color G. It is clear that $\omega(G) \leq \gamma(G)$ because all the vertices in a clique must be colored by different colors.

- The *stability number* $\alpha(G)$ is defined as the maximum cardinality of a stable set.
- The minimum number of cliques that partition the vertex set V is denoted as $\theta(G)$.
- A graph G is said to be α-perfect if

$$\alpha(G_A) = \theta(G_A) \text{ for all } A \subseteq V.$$

- A graph G is said to be γ-perfect if

$$\gamma(G_A) = \omega(G_A) \text{ for all } A \subseteq V.$$

The above concepts can as well be applied to a weighted graph with slight modifications. Consider a weighted graph G_w. We define the *clique weight* of a clique K as the sum of the weights of all vertices in K. The *clique number* $\omega(G_w)$ of G_w is defined as the maximum clique weight of any clique in G_w. The *chromatic number* $\gamma(G_w)$ of G_w is defined as the minimum number of colors to *multi-color* the graph G_w. In the next section, we will describe the Sequential Packing algorithm.

A.2 Sequential Packing Algorithm

Sequential Packing (SP) uses the greedy method to color a graph. The most crucial question is how to determine the ordering of the vertices. Before answering it, we define some terms first.

- *Denial area of a vertex* v is defined as the set of vertices which is joined to v by an edge.
- Suppose that color c is chosen to color the vertices. If $v_1, v_2, \cdots v_i$ has just been colored by c, the *current denial area* is defined as the union of the denial area of $v_1, v_2, \cdots v_i$.

In SP, the first vertex being colored by c is the one with maximum weight. Tie is broken arbitrarily. The subsequent vertices is determined as follows. Choose the vertex v such that the cardinality of

$$\text{denial area of } v \cap \text{current denial area}$$

is maximized. If there is a tie, we break it arbitrarily. (In chapter 2, the tie is broken by the clique weight condition. This condition is imposed to ensure the optimality in 3-stripe system. However, it is immaterial in our discussion here.)

The algorithm proceeds until no more vertices can be colored by c. Then we choose the next color and repeats.

A.3 Optimality of Sequential Packing

In Chapter 2, it was shown that SP yields optimal solution for a special class of network structures called i-stripe system for $i \leq 3$. In this section, we prove that SP is optimal for a special class of graphs called *triangulated graph.* A graph G is defined to be a triangulated graph if each cycle of length greater than three possesses a chord. Before proving the optimality, we need the following lemmas.

Lemma 1 *A triangulated graph is γ-perfect.*

Proof: See Berge [4], pp. 369.

□

Lemma 2 *Pick up a color c and use it to color the vertices of a connected and weighted graph G_w according to the SP algorithm. Then, at any stage of the coloring and for any pair of colored vertices, there exists an elementary path which joins the two vertices and encounters colored and uncolored vertices alternately.*

Proof: We prove the lemma by induction. Let v_i be the i-th vertex colored by c. Assume that v_1 has just been colored. To determine v_2, we choose the vertex whose denial area has maximum overlapping with the existing denial area. If $d(v_1, v_2) > 2$, there is no overlapping between the denial areas and v_2 will not be chosen because G_w is connected. Notice that $d(v_1, v_2) \neq 1$, otherwise they cannot share the same color. Therefore, $d(v_1, v_2) = 2$ which implies that the lemma is true when only two vertices have been colored.

Now assume that k vertices have been colored and the lemma holds up to this stage. Next, we have to determine v_{k+1}. If no more vertices can be colored by c, the lemma follows immediately. If not, the denial area of v_{k+1} must have some overlapping with the existing denial area, otherwise it will not be chosen according to SP. Therefore, we can find vertex v_i where $i \leq k$ such that $d(v_i, v_{k+1}) = 2$. Let v^* be a vertex joining to both v_i and v_{k+1}. For any vertex v_j where $j \leq k$, there exists a color alternating path P which joins v_j and v_{k+1} via v_i. Since the path joining v_j and v_i is elementary, v^* is visited at most twice in P. If v^* is visited once, P is an elementary path. If v^* is visited twice, there exists another color alternating path P' joining v_j and v_{k+1} via v^*. Note that P' is elementary.

□

Lemma 3 *Pick up a color c and use it to color the vertices of a weighted graph G_w according to the SP algorithm. If G, the underlying graph of G_w, is a triangulated graph, then inside any maximal clique of G, one and only one vertex is colored by c.*

Proof: An unconnected graph can be viewed as several connected subgraphs. For SP algorithm, coloring an unconnected graph is the same as coloring several connected subgraphs one after another. So there is no loss of generality to assume that G is connected.

Assume that there exists a maximal clique K in which no vertex is colored by c. Let v_j be the last vertex (or one of the last vertices) in K which is denied coloring by c. Let this denial be caused by the coloring of a_j (i.e. there is an edge joining a_j and v_j). Note that there exists $v_i \in K$ such that there is no edge joining v_i and a_j, otherwise $\{a_j\} \cup K$ is a clique which contains K as a subset and K is not maximal.

Since v_j is the last vertex (or one of the last vertices) in K being denied coloring by c, we can find a vertex a_i which is connected to v_i by an edge and is colored before a_j. Obviously, there is no edge joining a_i and v_j.

Assume that a_j is the m-th vertex being colored by c. At the time when the $(m-1)$-th vertex has just been colored, denote the intersection of the denial area of a_j and the current denial area by R. For any $v_k \in R$, there exists a path P_k of length 2 which joins a_j, v_k and another colored vertex, say a_k. By Lemma 2, we can find a color alternating path joining a_i and a_k (a_i and a_k may be the same vertex). Denote this path as P_1. Note that P_1 does not contain a_j because at this moment, a_j has not been colored and it has no colored adjacent vertex. By the same argument, P_1 does not pass through v_j. If P_1 does not pass through v_i, then for all k such that $v_k \in R$, the paths P_k, P_1 together with the path $[a_i, v_i, v_j, a_j]$ forms a cycle of length greater

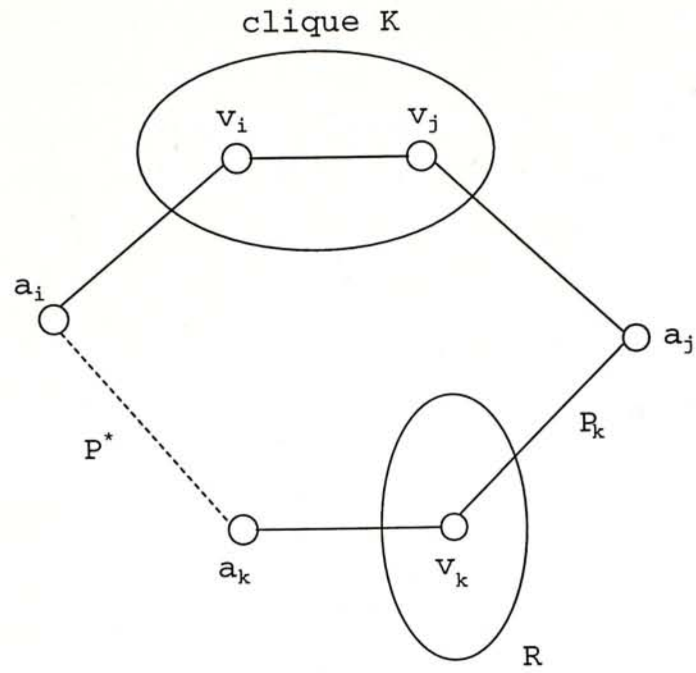

Figure A.1: If $a_i = a_k$, the length of the cycle is 5.

than or equal to five (see Figure A.1). On the contrary, if P_1 passes through v_i, we can find a path P_2 joining v_i and a_k. THen the paths P_k, P_2 and $[v_i, v_j, a_j]$ form a cycle of length greater than or equal to five.

Since G is triangulated, for all $v_k \in R$, there exists an edge joining v_k and v_j. Denote the intersection of denial area of v_j and the current denial area by R'. Then $R \subseteq R'$. Since $v_i \in R'$ and $v_i \notin R$, $R \subset R'$. According to SP, v_j, instead of a_j, should be the m-th vertex being colored. This implies a contradiction. Hence, for any maximal clique, at least one of the vertices inside it is colored. Since no two vertices in the same clique can be colored by the same color c, we conclude that one and only one vertex is colored by c.

□

Lemma 4 *If G is the underlying graph of a weighted graph G_w, $\omega(G) = \gamma(G)$ implies $\omega(G_w) = \gamma(G_w)$.*

Proof: Consider the graph G' formed from G_w as follows. Expand each vertex v_i of G_w into w_i vertices, each connected by an edge to one another and to all those vertices originally connected to v_i. The order of expansion is immaterial. Note that

$\omega(G_w) = \omega(G')$ and $\gamma(G_w) = \gamma(G')$. In addition, it can be seen that $\alpha(G) = \alpha(G')$ and $\theta(G) = \theta(G')$. Lovász's Perfect Graph Theorem (see [4]) states that a graph is α-perfect if and only if it is γ-perfect. Therefore, $\alpha(G) = \theta(G)$ implies that $\alpha(G') = \theta(G')$. This further implies that $\gamma(G') = \omega(G')$. Hence, $\omega(G_w) = \gamma(G_w)$.

□

Theorem 3 *If G, the underlying graph of G_w, is a triangulated graph, the sequential packing (SP) algorithm colors G_w in an optimal way, which means that the number of colors used is minimum.*

Proof: We prove the theorem by induction. If $\gamma(G_w) = 1$, then there is no edge in G. Obviously, SP yields an optimal solution for G_w. Assume that the statement is true for all weighted graph G_w with $\gamma(G_w) = k$. Now consider a weighted graph G_w where $\gamma(G_w) = k + 1$. We pick up a color c and then apply SP algorithm. The weight of the colored vertices will then be decreased by one. By Lemma 3, every maximal clique of G has one vertex being colored. Since cliques of maximum weight in G_w must be maximal cliques in G, $\omega(G_w)$ is reduced by one after coloring using c. By Lemma 1, G, being a triangulated graph, is γ-perfect. By Lemma 4, G_w is also γ-perfect. This implies that $\gamma(G_w)$ is also reduced by one. So the chromatic number of the reduced graph is k and by the induction hypothesis, SP will color it use another k colors. Totally, $k + 1$ colors are used. So SP is optimal for G_w if $\gamma(G_w) = k + 1$. Hence, by the principle of induction, SP yields an optimal solution for all triangulated graphs.

□

A.4 Concluding Remarks

We have shown that the sequential packing algorithm is optimal for triangulated graphs. Examples of triangulated graphs include i-stripe cellular system with reuse factor 7 for $i \leq 2$ and graphs of tree structure.

Appendix B

Derivation of the Capacity of 3-stripe system

The derivation of the capacity of 3-stripe system is similar to the linear case. Define $C_{3n}(\rho)$ as the E[# of cells using the channel]/$3n$. Since the product form solution holds and by equation (4.8),

$$3nC_{3n}(\rho) = \frac{\sum_{i=0} i a_i(n)\rho^i}{\sum_{i=0} a_i(n)\rho^i} \tag{B.1}$$

where $a_i(n)$ is the number of possible arrangements that among the $3n$ cells, i cells are using the channel.

Similarly, let M_n and A_n be the numerator and denominator of the expression on the right hand side of equation (B.1) respectively.

To determine A_n and M_n, we need to set up a difference equation of $a_i(n)$. Before that, we define two auxillary variables first. Let $b_i(n)$ be the number of possible arrangements that i cells are using the channel, on the condition that the first cell in the second row is using the channel. Let $c_i(n)$ be the number of possible arrangements that i cells are using the channel, on the condition that the first cell in the first row is using the channel (see Figure B.1).

Considering the channel occupancy of the first cell in each row, there are altogether four cases: none of the cells occupy the channel, the cell in first row, second row or third row occupies the channel. Notice that the second and the fourth case is the same by symmetry. Therefore, we have equation (B.2). Equation (B.3) and (B.4) can be obtained similarly.

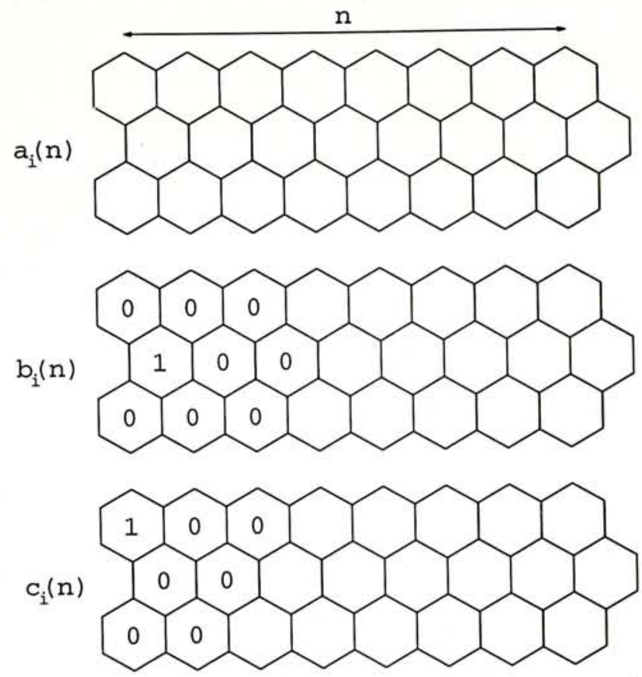

Figure B.1: The figures show the situations corresponding to $a_i(n)$, $b_i(n)$ and $c_i(n)$.

$$\begin{aligned} a_i(n) &= 2c_i(n) + b_i(n) + a_i(n-1) && \text{(B.2)} \\ b_i(n) &= a_{i-1}(n-3) && \text{(B.3)} \\ c_i(n) &= c_{i-1}(n-2) + b_{i-1}(n-2) + a_{i-1}(n-3) && \text{(B.4)} \end{aligned}$$

The boundary conditions of the above set of difference equations are given by $a_0(n) = 1, b_0(n) = c_0(n) = 0$.

Define $A_n = \sum_{i=0}^{\infty} a_i(n)\rho^i, B_n = \sum_{i=0}^{\infty} b_i(n)\rho^i$ and $C_n = \sum_{i=0}^{\infty} c_i(n)\rho^i$.

From equation (B.2), (B.3) and (B.4),

$$\begin{aligned} A_n &= 2C_n + B_n + A_{n-1} && \text{(B.5)} \\ B_n &= \rho A_{n-3} && \text{(B.6)} \\ C_n &= \rho(C_{n-2} + B_{n-2} + A_{n-3}) && \text{(B.7)} \end{aligned}$$

Define $A(z) = \sum_{n=1}^{\infty} A_n z^{n-1}, B(z) = \sum_{n=1}^{\infty} B_n z^{n-1}$ and $C(z) = \sum_{n=1}^{\infty} C_n z^{n-1}$. From equation (B.5), (B.6) and (B.7), we can solve for $A(z)$.

$$A(z) = \frac{1 + 3\rho + 3\rho z + (2\rho + \rho^2)z^2 + \rho^2 z^3 + \rho^2 z^4}{1 - z - \rho z^2 - 2\rho z^3 - \rho^2 z^5} \tag{B.8}$$

Next we need to solve for M_n. Define $M_n = \sum_{i=0}^{\infty} i a_i(n)\rho^i$, $P_n = \sum_{i=0}^{\infty} i b_i(n)\rho^i$ and $Q_n = \sum_{i=0}^{\infty} i c_i(n)\rho^i$. From equation (B.2), (B.3) and (B.4),

$$
\begin{aligned}
M_n &= 2Q_n + P_n + M_{n-1} && \text{(B.9)}\\
P_n &= \rho(M_{n-3} + A_{n-3}) && \text{(B.10)}\\
Q_n &= \rho(Q_{n-2} + C_{n-2} + P_{n-2} + B_{n-2} + M_{n-3} + A_{n-3}) && \text{(B.11)}
\end{aligned}
$$

Define $M(z) = \sum_{n=1}^{\infty} M_n z^{n-1}$, $P(z) = \sum_{n=1}^{\infty} P_n z^{n-1}$ and $Q(z) = \sum_{n=1}^{\infty} Q_n z^{n-1}$. Using equation (B.5)-(B.7) and (B.9)-(B.11), we can solve for $M(z)$.

$$M(z) = \frac{3\rho + 2\rho^2 z^2 - \rho^3 z^4}{(1 - z - \rho z^2 - 2\rho z^3 - \rho^2 z^5)^2} \tag{B.12}$$

Let $f(z) = 1 - z - \rho z^2 - 2\rho z^3 - \rho^2 z^5$ and α be the root that has smallest magnitude among all the roots of $f(z) = 0$. From equation (B.8) and (B.12), we can use method of partial fraction and inverse transform to obtain

$$
\begin{aligned}
\lim_{n\to\infty} A_n &= \frac{1}{g(\alpha)}(1 + 3\rho + 3\rho\alpha + (2\rho + \rho^2)\alpha^2 + \rho^2\alpha^3 + \rho^2\alpha^4)(\frac{1}{\alpha^n})\\
\lim_{n\to\infty} M_n &= \frac{1}{g^2(\alpha)}(3\rho + 2\rho^2\alpha^2 - \rho^3\alpha^4)(\frac{n}{\alpha^{n+1}})
\end{aligned}
$$

where

$$
\begin{aligned}
g(\alpha) &= -f'(z)|_{z=\alpha}\\
&= 1 + 2\rho\alpha + 6\rho\alpha^2 + 5\rho^2\alpha^4
\end{aligned}
$$

Hence, we can find the capacity $C(\rho)$.

$$
\begin{aligned}
C(\rho) &= \lim_{n\to\infty} C_{3n}(\rho)\\
&= \lim_{n\to\infty} \frac{M_n}{3nA_n}\\
&= \frac{1}{3\alpha g(\alpha)}(\frac{3\rho + 2\rho^2\alpha^2 - \rho^3\alpha^4}{1 + 3\rho + 3\rho\alpha + (2\rho + \rho^2)\alpha^2 + \rho^2\alpha^3 + \rho^2\alpha^4})\\
&= \frac{1}{3\alpha}(\frac{3\rho + 2\rho^2\alpha^2 - \rho^3\alpha^4}{1 + 11\rho + (3\rho + 11\rho^2)\alpha + (2\rho + 17\rho^2)\alpha^2 + (\rho^2 - 3\rho^3)\alpha^3 + (\rho^2 + 8\rho^3)\alpha^4})
\end{aligned}
$$

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