Coding Schemes for Multicode CDMA Systems

ZHAO Fei

A thesis submitted in Partial Fulfillment

of the Requirements for the Degree of

Master of Philosophy

in

Information Engineering

©Chinese University of Hong Kong

July 2003

The Chinese University of Hong Kong holds the copyright of this thesis. Any person(s) intending to use a part or whole of the materials in the thesis in a proposed publication must seek copyright release from the Dean of the Graduate School.

香港中文大學
大學圖書館系統
2 8 APR 2004
UNIVERSITY LIBRARY SYSTEM
THE CHINESE UNIVERSITY OF HONG KONG

Acknowledgement

Many people support my effort during my M. Phil study at CUHK. I would like to thank them for making this two-year study period a fruitful and rewarding experience.

First of all, I would like to express my deep gratitude to my supervisor, Professor Tat-Ming Lok for his guidance and advice throughout this research work. His insight and expertise guides me the way to wireless communications. It is a very valuable experience for me to have discussions with him on various research problems. I would like to thank him for his encouragement and support, as well as his trust and patience, during these two years.

Many thanks go to my friends who have a lot of stimulating discussions with me on various topics in communication systems and operational research. These include Mr. Zheng Ming, Mr. Kwan Ho-Yuet, Mr. Chen Chung Shue, Mr. Leung Kin Kwong, Dr. Yang Yang, Ms. Li Feng-Jun, Mr. Lu Guo-Wei, Mr. Hu Ke, and Mr. Li Qiang. Also, I have to thank Mr. Xie Yong-Ming for his precious advice on computer programming.

Last but not the least, I would like to thank my family for their endless love and support, especially to my parents.

Abstract

Third generation (3G) personal communication systems (PCS) will accommodate many types of traffic including voice, video, and other high-speed data. Variable data rate transmission is required to support these services. Multicode CDMA is one of the options being considered within the 3G standardization efforts for achieving variable data rate transmission. However, the major drawback of multicode CDMA is its large amplitude fluctuation and consequently the whole performance is degraded.

A general scheme of multicode CDMA system with constant amplitude transmission is presented and analyzed in this thesis. By the proposed delicate selection of Hadamard code sequences, the interference caused by the non-linear operation in this multicode scheme can be minimized. On the other hand, there are parity check bits generated by the non-linear operation in the scheme. To utilize these parity check bits efficiently, a combination of linear block code and this multicode scheme is considered. The multicode schemes combined with Hamming codes, Gallager's codes, and zigzag codes are analyzed respectively. Iterative decoding for the combined systems is efficient to recover the information bits. The iterative decoding methods are discussed in details. Through analysis and simulation, results show that the performance of combined scheme is greatly improved.

摘　　要

3G 無線個人通訊系統（PCS）將容納多種類型傳輸，包括音頻，視頻和其他高速率數據，可變數據速率傳輸對於提供這類服務是必需的。多碼 CDMA（multicode CDMA）是在 3G 標準中用於達到可變數據速率傳輸的考慮方案之一。但是，多碼 CDMA 系統的主要缺點是多碼信號有很大的幅度波動，因此，系統整體性能將降低。

本文提出並分析了一種普遍型的恆幅度傳輸多碼 CDMA 系統。由於方案中的非綫性操作所造成的干擾可以通過選擇 Hadamard 碼序列達到最小化。另一方面，方案中的非綫性操作可以產生奇偶校驗位。爲了更加有效的利用該奇偶校驗位，本文考慮綫性編碼和該多碼方案的結合方式，並且分別研究了與 Hamming 碼，Gallager's 碼和 zigzag 碼結合的方案。因爲疊代解碼對於該系統的信息恢復是非常有效的，本文將著重討論疊代解碼方法。通過分析和仿真，結果顯示通過這種結合方案，系統性能大大提高。

Contents

List of Abbreviations

APP	A Posteriori Probability
BS	Base Station
CDMA	Code Division Multiple Access
DL	Downlink (Forward Link)
DS-CDMA	Direct Sequence Code Division Multiple Access
FDD	Frequency Division Duplex
FDMA	Time Division Multiple Access
GSM	Global System for Mobile Communications (Groupe Spécial Mobile)
IMT-2000	International Mobile Telecommunications 2000
LDPC	Low-Density Parity Check (Code)
MAP	Maximum a Posterior
ML	Maximum Likelihood
MP	Message Passing
MS	Mobile Station
OCSF	Orthogonal Constant Spreading Factor
OVSF	Orthogonal Variable Spreading Factor
QoS	Quality of Service

PAP	Peak-to-Average Power (Ratio)
PCS	Personal Communication Systems
SF	Spreading Factor
SCC	Serial Concatenation Code
SPC	Single Parity Check (code)
TD-CDMA	Time Code Division Multiple Access
TDD	Time Division Duplex
TDMA	Time Division Multiple Access
UL	Uplink (Reverse Link)
UMTS	Universal Mobile Telecommunication System
UTRA	UMTS Terrestrial Radio Access
VSF	Variable Spreading Factor
WCDMA	Wideband Code Division Multiple Access

List of Tables

List of Figures

Chapter 1

Introduction

Nowadays, with the prosperity of wireless systems such as GSM, wireless multimedia applications become a part of our daily life. However current wireless systems cannot meet the consumer's growing expectation on various service and high quality of service (QoS). So in the next-generation wireless systems, it is expected that different classes of traffics are supported with their respectively required quality of service (QoS), which is drastically different from the existing second-generation wireless systems. Therefore, a necessity of variable data rate transmission for such integrated services is rapidly growing in the circle of wireless mobile communications. The proposed third generation wireless standards UMTS/IMT-2000 [1-3] uses wide-band CDMA (WCDMA) to address the higher and variable rate requirements of multimedia applications.

To achieve the specified rate, two types of approaches are being considered, of which one is the variable spreading factor (VSF) scheme using a single code and the other is multicode direct sequence CDMA (DS-CDMA) based on parallel multiple orthogonal codes (MOC). In this thesis, I will focus on multicode CDMA and its combination with error-correcting codes. One of the disadvantages of conventional multicode CDMA is its large amplitude fluctuation, since the multicode signals suffer from significant distortion through a nonlinear device. The performance of whole system degrades dramatically. In this thesis, a multicode CDMA system with constant amplitude transmission is mainly considered. In order to fully utilize the error-control capability of such a system, the combination with error-correcting coding scheme has to be investigated.

In this chapter, to provide the background knowledge for this research work, an introduction on multirate scheme is given firstly, in which basics of VSF and multicode scheme are introduced respectively. Secondly, a detailed description on multicode CDMA system is given thereafter.

1.1 Multirate Scheme

Several options are being considered within the 3G standardization efforts for achieving variable data rates. Assuming fixed bandwidth, these may be classified as varying spreading factor (VSF) CDMA [4] [5] and multicode CDMA where multiple orthogonal spreading codes are assigned to given user [6] [7]. There are some more discussions and analysis on multirate schemes in [8] [9].

Important considerations in the design of multirate schemes are good code utilization efficiency, small amplitude variance, and the ability to achieve a wide range of transmission rates [10]. Additionally, good error performance in multipath and multiuser environments is desired. In this thesis, the property of amplitude variance of multirate signals is our major concern.

1.1.1 VSF Scheme

In a DS-CDMA system, each multiple-access user is assigned a unique signature code. In the forward (base station to mobile terminal) link, the assigned codes are mutually orthogonal [11]. In the second-generation wireless CDMA system IS-95, each mobile user is assigned a single orthogonal constant spreading factor (OCSF) code. To obtain variable transmission rate, each user is assigned a single variable spreading factor (VSF) code, which is known as VSF-CDMA; usually the orthogonal spreading codes are used, so such VSF scheme is also known as OVSF-CDMA. In order to keep orthogonality among different codes, generations of orthogonal spreading codes with variable spreading factors are important. Tree-structured generation [12] is an efficient and practical method of codes generation, which is shown as follows.

OVSF codes are generated from a tree-structured set $\{C_N(n)\}_{n=1}^{N}$, which has a length of N chips [12]. Spreading codes of different lengths are deployed in channel spreading. Orthogonality is kept between different codes to provide variable spreading factors. In [12], the code generation matrix is shown below:

$$C_N = \begin{bmatrix} C_N(1) \\ C_N(2) \\ C_N(3) \\ C_N(4) \\ \vdots \\ C_N(N-1) \\ C_N(N) \end{bmatrix} = \begin{bmatrix} C_{N/2}(1)\ C_{N/2}(1) \\ C_{N/2}(1)\ \overline{C}_{N/2}(1) \\ C_{N/2}(2)\ C_{N/2}(2) \\ C_{N/2}(2)\ \overline{C}_{N/2}(2) \\ \vdots \\ C_{N/2}(N/2)\ C_{N/2}(N/2) \\ C_{N/2}(N/2)\ \overline{C}_{N/2}(N/2) \end{bmatrix}, \tag{1.1}$$

where $\overline{C}_{N/2}(n)$ is the binary complement of $C_{N/2}(n)$ with a row of $N/2$ elements.

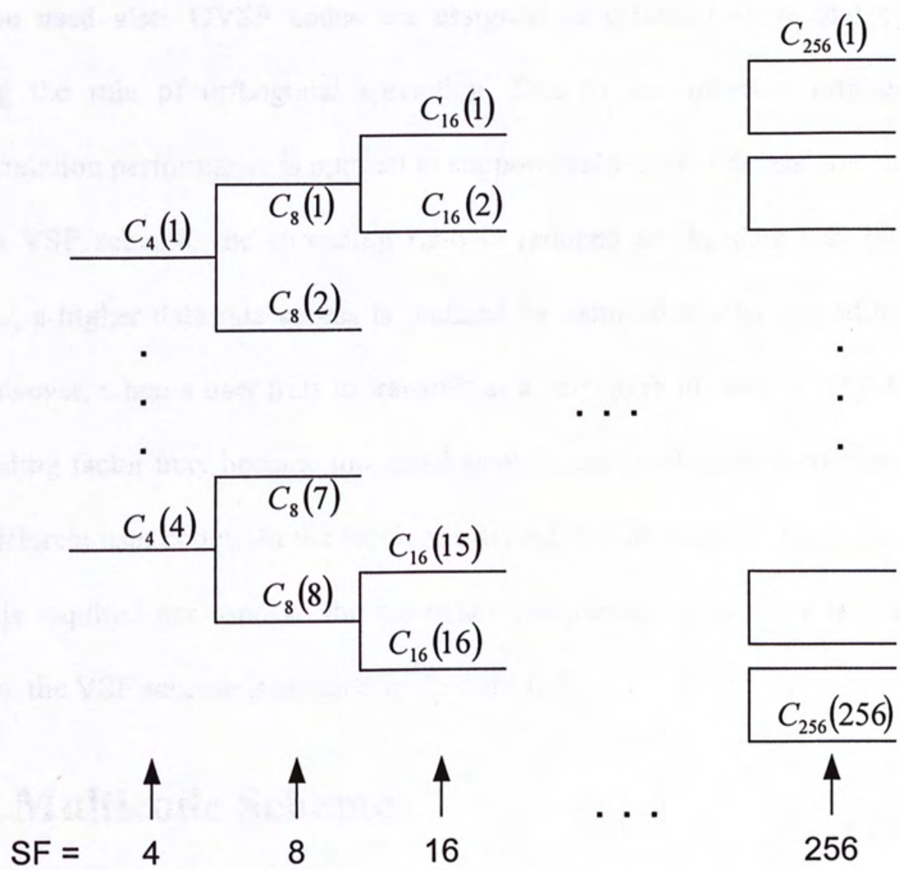


Figure 1.1: OVSF code tree

The code tree generated by code matrix C_N is illustrated in Figure 1.1. According to WCDMA, the spreading factors starting from 4 can be used. Codes from C_4 to C_{256} are shown with the corresponding spreading factors from 4 to 256. To keep orthogonality in code assignment, there is an assignment constraint. In the tree, a code is not orthogonal with its parent-code or child-codes. For example, when $C_{16}(1)$ is used, its parent-code $C_8(1)$ and child-codes $C_{32}(1)$ and $C_{32}(2)$ cannot be used. Iteratively, the parent-code of $C_8(1)$ and the child-codes of $C_{32}(1)$ and $C_{32}(2)$ cannot be used also. OVSF codes are assigned to different users and operated following the rule of orthogonal spreading. Due to the inherent orthogonality, cross-correlation performance is optimal to support multi-user code assignment.

In a VSF scheme, the spreading ratio is reduced as the data rate increases. Therefore, a higher data rate access is realized by using a smaller spreading factor code. However, when a user tries to transmit at a very high bit rate in VSF-CDMA, the spreading factor may become too small to maintain good (low) cross correlation among different user codes. At the receiver side, using VSF scheme, only one RAKE receiver is required per user, so the hardware complexity of receiver is quite low. Therefore, the VSF scheme is suitable in forward link.

1.1.2 Multicode Scheme

In the multicode scheme, additional parallel codes are allocated as the data rate increases. Figure 1.2 illustrates the multicode transmission scheme. The data stream with a high bit rate is split into N parallel channels, each with data rate R_b, which can be viewed as basic rate [3]. Walsh codes are usually used in N parallel channels, which are thus called orthogonal codes (MOC) channels since Walsh codes are mutually orthogonal. Therefore, multirate transmission is realized by assigning a specific number of code channels according to the specific data rate. More code channels are assigned when a higher data rate transmission is required [3]. Compared to VSF scheme, the spreading factor of multicode scheme keeps constant, so there is no such a problem of small spreading gain when a high rate transmission is in need.

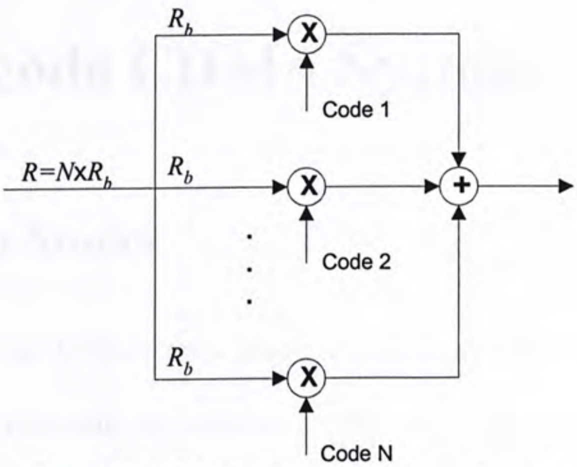


Figure 1.2: Multicode transmitter scheme

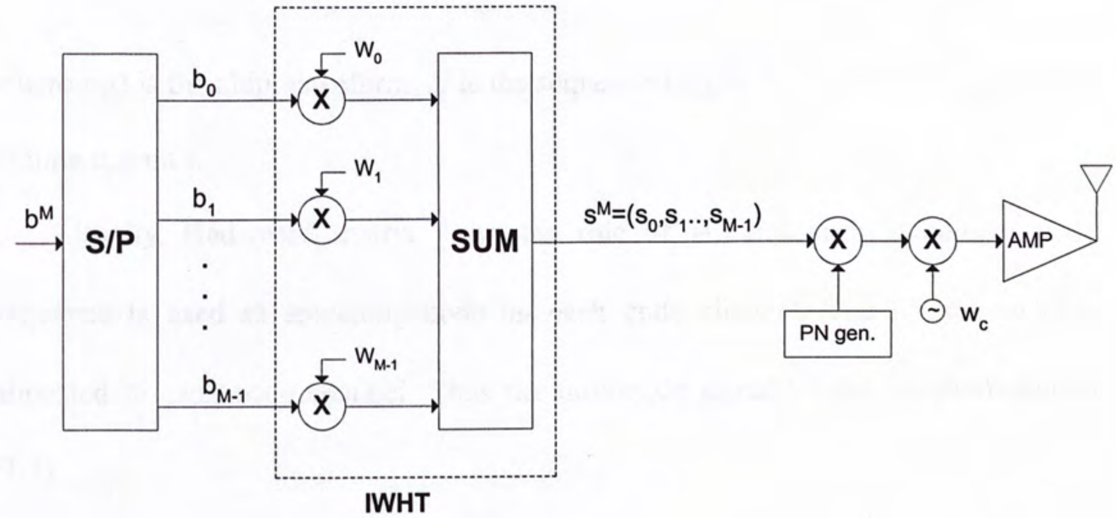
With multicode transmission, the mapping of simultaneously transmitted services into frames can be performed in two different ways: firstly, services are transmitted simultaneously in different frames and in different codes; secondly, services are mapped into the same frame, in which bits are then mapped into different codes [3].

Multicode scheme requires multiple transceiver units to support higher data rates, thus resulting in increased hardware complexity. Therefore, in terms of hardware complexity, OVSF scheme is preferable over multicode scheme. However, multicode scheme can be deployed in the reverse (mobile terminal to base station) link or uplink. The major problem in the multicode scheme is the large envelope variance in the multicode signals, namely, the large peak-to-average power ratio (PAP), because the transmitted signal is a linear sum of each parallel code channel signal. This problem will be discussed further in details in the following section.

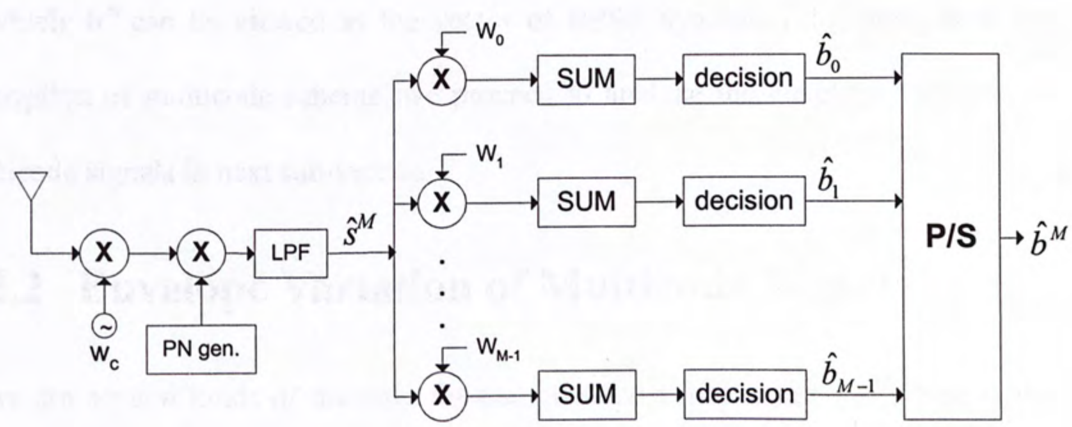
1.2 Multicode CDMA System

1.2.1 System Model

In [6], multicode CDMA was proposed as a system that realizes all the VSF-CDMA features without the problem of very small spreading gain of high rate users. A conventional multicode CDMA transmitter architecture is shown in Figure 1.3. In this multicode scheme, M code channels are assigned to the user. At receiver side, there are M RAKE receivers used to recover the signals on each code channel. Walsh codes, used in the multicode scheme, have zero cross-correlation when they are time-synchronized. However, because multipath delays can introduce significant non-zero cross-correlation between the orthogonal codes, combination of Walsh code and PN sequences is more appropriate [7][13][14], which is beyond the scope of this thesis.



(a) Transmitter



(b) Receiver

Figure 1.3: A conventional multicode CDMA system model

Now we consider this uplink multicode DS-CDMA system, where the user is assigned M code channels represented by the vector, $\mathbf{W} = [w_0, w_1, \cdots, w_{M-1}]^T$. The spreading code waveform, $w_i(t)$, is

$$w_i(t) = \sum_{i=0}^{N-1} w^{(i)} p(t - iT_c), \tag{1.2}$$

where $p(t)$ is the chip waveform, N is the sequence length, and $w^{(i)}$ is the chip used at time instant i.

Usually, Hadamard matrix plays the role of $\boldsymbol{W}$, and each Hadamard code sequence is used as spreading code on each code channel. The bit stream b^M is allocated to each code channel. Thus the multicode signal S^M can be expressed as (1.3)

$$S^M = \mathbf{b}^M \cdot \mathbf{W} = \sum_{i=0}^{M-1} b_i w_i , \tag{1.3}$$

in which $\mathbf{b}^M$ can be viewed as the vector of BPSK symbols [18]. Now, with the description of multicode scheme, we proceed to analyze the envelope variation of multicode signals in next sub-section.

1.2.2 Envelope Variation of Multicode Signal

There are several kinds of measure for the envelope variation, one of which is the peak-to-average power ratio (PAP) [31] defined as

$$\text{PAP} = \frac{\max |s(t)|^2}{E[|s(t)|^2]}, \tag{1.4}$$

in which $E[\cdot]$ is the expectation operator and $s(t)$ is the waveform of multicode signal. In the definition of PAP, the numerator stands for peak power while the denominator means the average power of multicode signal. The PAP of signal with constant amplitude is 1 (0dB), which means that there is no envelope variance in this signal. A larger PAP means a large envelope variation in the signal. Another one is the complementary cumulative distribution function (CCDF) of PAP, which is an extended approach showing the details of distribution of PAP $(\mathrm{CCDF} = \Pr(\mathrm{PAP} > \mathrm{PAP}_0))$ [31]. CCDF is a function of PAP_0, which is a base level. CCDF shows the probability of potential large PAP. However, in this thesis, a multicode signal with constant amplitude is mainly investigated, so the expression for envelope variation does not matter too much.

In this thesis, we consider the PAP of the discrete-time multicode signal instead of the continuous-time signal. Different from (1.4), PAP of the discrete-time signal can be written as

$$\mathrm{PAP} = \frac{\max |s|^2}{E[|s|^2]},$$

in which, s stands for the discrete-time signal sequence.

From (1.3), it is easy to see that

$$\max |s|^2 = \max_{k,b} |b^M \cdot w^k|^2 = M^2. \tag{1.5}$$

If we assume that both bits and code chips are random and independent and the symbols are equally probable, we have (see [18])

$$\begin{aligned} E[|s|^2] &= E_{b,w}\left[\sum_{i=0}^{M-1} b_i w_i\right]^2 \\ &= 2^{-M}\sum_{i=0}^{M}\binom{M}{i}\cdot(M-2i)^2 \\ &= 2^{-M}\sum_{i=0}^{M}\binom{M}{i}\cdot\left(M^2-4Mi+4i^2\right) \\ &= M^2-2M^2+M^2+M=M. \end{aligned} \tag{1.6}$$

Therefore, for multicode signal, PAP_{max} is M. PAP_{max} increases as the number of code channels increases. If many code channels are used for multicode scheme, the PAP_{max} will be very large. Consequently, the envelope variance of multicode signal will be large.

1.2.3 Drawback of Multicode Scheme

From previous sub-section, we note that conventional multicode signals have large envelope variation and this is the major drawback of multicode scheme. Now, we present the effect of large envelope variation to further understand the problem caused.

From the view of mobile terminals, i.e. handset, most of power is consumed by the RF power amplifier, and thus the power efficiency of this amplifier is very important. Figure 1.4 illustrates the amplitude input-output characteristics (AM-AM) of a typical RF power amplifier. Practically, a typical RF power amplifier is a nonlinear device, and such power amplifier that has high power efficiency operates near the saturation point. However, in this region the AM-AM characteristics of the amplifier are non-linear. This non-linearity has negative effects such as increased out-of-band radiation, namely, spectral spreading, and decreased performance (increased bit error rate) [18]. Therefore, a larger envelope variation in signals leads to more out-of-band radiation and thus a lower spectral efficiency. Moreover, because a large envelope variance forces the amplifier to work in the active (linear) region, such signal suffers less power efficiency [18].

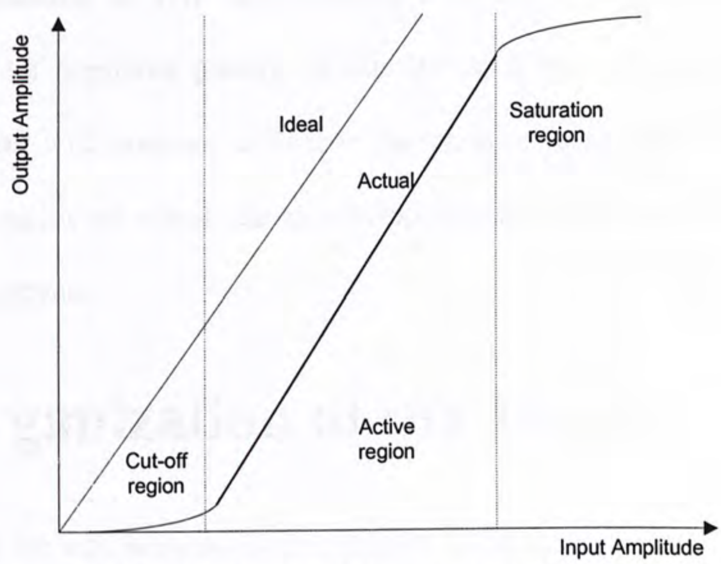


Figure 1.4: AM-AM characteristics of a typical RF power amplifier

If the transmitted signal has a large envelope variation, there are two possible options (see [18]):

1) Set the operation point of the amplifier in the saturation region, and thus get a high power efficiency, but large out-of-band radiation;

2) Use a back-off for the input signals so that operation point is set in the active (linear) region and thus get a low power efficiency but a high spectral

efficiency.

Any way between these two methods cannot radically resolve the problem caused by large envelope variance. Thus, constant (or near constant) amplitude transmission is more suitable and prevailing in wireless applications.

Conventional multicode signal having a large envelope variation suffers large out-of-band radiation and lower spectral efficiency, so the bit error rate (BER) performance is degraded. Therefore, minimizing envelope variation of multicode signals or reducing its PAP is a necessity so that the performance of multicode scheme can be improved greatly. In this thesis, a general scheme of multicode CDMA system with constant amplitude transmission is proposed and analyzed. In this system, multicode signal has no envelope variance and thus the performance of system is improved.

1.3 Organization of the Thesis

In this thesis, we will focus on the multicode CDMA system with constant amplitude transmission and its combination with efficient coding schemes. Then, we proceed to describe a multicode scheme with bounded PAP transmission, in which such multicode signal has small envelope variance and more code channels with less interference are available.

In Chapter 2, several novel studies on the minimization of envelope variation of multicode signals are introduced in details and summarized. From the related work, we can have a better understanding of methods of PAP minimization. In Chapter 3, a multicode CDMA system with constant amplitude transmission is proposed and the selection of Hadamard code sequences is also presented in details. The performance of this multicode scheme is evaluated. In Chapter 4, the multicode scheme combined with coding scheme is considered. Multicode schemes combined with Hamming codes, Gallager's codes, and zigzag codes are analyzed respectively. An iterative decoding based on message passing algorithm is discussed and the performance of the systems with this iterative decoding is evaluated. In Chapter 5, we will discuss the multicode scheme with small envelope variance. Chapter 6 contains the conclusions and future work.

Chapter 2

Related Work on Minimization of PAP of Multicode CDMA

The major problem with multicode scheme is the large envelope variation, namely, the large peak-to-average power ratio (PAP), because the transmitted signal is a sum of all the parallel code channel signals. A large variation results in a large out-of-band radiation and lower spectral efficiency of the overall system, and bit error rate performance is degraded. To solve this problem, reducing the PAP, which is a key factor in the mobile station with high power amplifier (HPA), should be well considered. Several studies have been carried out to decrease PAP. In this chapter, three efficient methods of reducing PAP are presented and summarized respectively in each subsection.

2.1 Constant Amplitude Coding

In [15], Wada proposes a constant amplitude coding scheme, by which multicode CDMA system with constant amplitude transmission can be realized. The possible rates of the multicode transmission are 1/3, 1/9, and so on. The multicode transmission of 1/3 rate is realized by using the rate 3/4 constant amplitude coding. Similarly, the rate 1/9 multicode transmission can be done by rate 9/16 constant amplitude coding [15]. Firstly, the simplest example of rate 3/4 constant amplitude coding is described as follows.

For example, the behavior of inverse Walsh-Hadamard Transform (IWHT) is shown by making use of 4×4 Hadamard matrix as below

$$H_4 = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix} \tag{2.1}$$

Thus, orthogonal code sequences are written as $w_0 = \{1,1,1,1\}$, $w_1 = \{1,-1,1,-1\}$, $w_2 = \{1,1,-1,-1\}$, and $w_3 = \{1,-1,-1,1\}$. For example, when the information bit stream $b_F^4 (=1111)$ is transmitted, the amplitude of transmitted signal s_F^4 becomes $w_0 + w_1 + w_2 + w_3 (= \{4,0,0,0\})$. Table 2.1 shows the behavior of amplitude fluctuations when 4×4 Hadamard code sequences are used.

Table 2.1: Amplitude patterns of transmitting signal using 4x4 Hadamard matrix

i	b_i^4	s_i^4	i	b_i^4	s_i^4
0	0000	-4, 0, 0, 0	**8**	**1000**	**-2, 2, 2, 2**
1	**0001**	**-2,-2,-2, 2**	9	1001	0, 0, 0, 4
2	**0010**	**-2, 2,-2,-2**	A	1010	0, 4, 0, 0
3	0011	0, 0,-4, 0	**B**	**1011**	**2, 2,-2, 2**
4	**0100**	**-2,-2, 2,-2**	C	1100	0, 0, 4, 0
5	0101	0,-4, 0, 0	**D**	**1101**	**2,-2, 2, 2**
6	0110	0, 0, 0,-4	**E**	**1110**	**2, 2, 2,-2**
7	**0111**	**2,-2,-2,-2**	F	1111	4, 0, 0, 0

In Table 2.1, the cases of constant amplitude transmissions are written in bold fonts. It is obvious that transmitted signals with constant amplitude levels are achieved when the number of "1" within the information bit stream b_i^4 is odd [15]. When the information bit stream $b_F^4 (=1111)$ is transmitted, the amplitude of transmitted signal s_F^4 becomes $w_0 + w_1 + w_2 + w_3 (= \{4,0,0,0\})$. However, when the information bit stream $b_E^4 (=1110)$ is transmitted, the amplitude of transmitted signal s_E^4 becomes $w_0 + w_1 + w_2 - w_3 (= \{2,2,2,-2\})$. From this example, we notice that the sign(s) of one code sequence or three sequences changes, the amplitude becomes –2 or 2 [15].Therefore, the signal with constant amplitude levels can be achieved when the information bit stream b_i^4 is satisfied with the following condition as shown

$$b_{i,0} \oplus b_{i,1} \oplus b_{i,2} \oplus b_{i,3} = 1 \quad . \tag{2.2}$$

where $b_i^4 = (b_{i,0} b_{i,1} b_{i,2} b_{i,3})$ and notation $\oplus$ is denoted as the operation of "exclusive OR" [15]. When any arbitrary three bits $b_{i,0}$, $b_{i,1}$, and $b_{i,2}$ are transmitted, the constant amplitude transmission can be realized by using $b_{i,3}$ generated as

$$b_{i,3} = \overline{b_{i,0} \oplus b_{i,1} \oplus b_{i,2}} \tag{2.3}$$

in which $b_{i,3}$ is made from $b_{i,0}$, $b_{i,1}$, and $b_{i,2}$ [15]. Thus, the rate 3/4 encoder can be realized based on (2.3) as shown in Figure 2.1

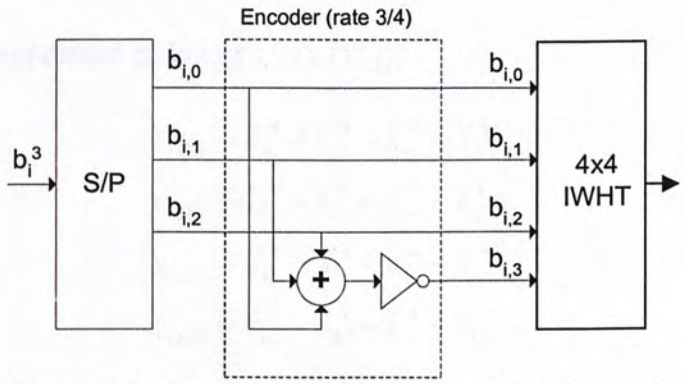


Figure 2.1: Constant amplitude encoder with rate 3/4

Based on this rate 3/4 encoder, we proceed to introduce the realization of the rate 1/9 multicode transmission with rate 9/16 encoder. In this scheme, 16×16 Hadamard matrix is involved and can be expressed as

$$H_{16} = \begin{pmatrix} H_4 & H_4 & H_4 & H_4 \\ H_4 & -H_4 & H_4 & -H_4 \\ H_4 & H_4 & -H_4 & -H_4 \\ H_4 & -H_4 & -H_4 & H_4 \end{pmatrix} \tag{2.4}$$

The amplitude of transmitted signal, s_i^{16}, can be expressed as

$$s_i^{16} = (s_{i,(0)}^4, s_{i,(1)}^4, s_{i,(2)}^4, s_{i,(3)}^4), \tag{2.5}$$

in which each element is expressed as

$$\begin{aligned} s_{i,(0)}^4 &= (s_{i,0}, s_{i,1}, s_{i,2}, s_{i,3}) \\ s_{i,(1)}^4 &= (s_{i,4}, s_{i,5}, s_{i,6}, s_{i,7}) \\ s_{i,(2)}^4 &= (s_{i,8}, s_{i,9}, s_{i,10}, s_{i,11}) \\ s_{i,(3)}^4 &= (s_{i,12}, s_{i,13}, s_{i,14}, s_{i,15}). \end{aligned} \tag{2.6}$$

Since b_i^{16} can be considered as the bit stream constructed by four of 4-bits streams, as $b_i^{16} = (b_{i_0}^4 b_{i_1}^4 b_{i_2}^4 b_{i_3}^4)$, each $s_{i,(0)}^4$, $s_{i,(1)}^4, s_{i,(2)}^4$, and $s_{i,(3)}^4$ can be calculated from (2.4) and expressed as follows (see [15]),

$$\begin{aligned} s_{i,(0)}^4 &= \widetilde{s}_{i_0}^4 + \widetilde{s}_{i_1}^4 + \widetilde{s}_{i_2}^4 + \widetilde{s}_{i_3}^4 \\ s_{i,(1)}^4 &= \widetilde{s}_{i_0}^4 - \widetilde{s}_{i_1}^4 + \widetilde{s}_{i_2}^4 - \widetilde{s}_{i_3}^4 \\ s_{i,(2)}^4 &= \widetilde{s}_{i_0}^4 + \widetilde{s}_{i_1}^4 - \widetilde{s}_{i_2}^4 - \widetilde{s}_{i_3}^4 \\ s_{i,(3)}^4 &= \widetilde{s}_{i_0}^4 - \widetilde{s}_{i_1}^4 - \widetilde{s}_{i_2}^4 + \widetilde{s}_{i_3}^4. \end{aligned} \tag{2.7}$$

From (2.7), we notice that constant amplitude transmission can be realized by the selection of $\widetilde{s}_{i_3}^4$ for any arbitrary $\widetilde{s}_{i_0}^4$, $\widetilde{s}_{i_1}^4$, and $\widetilde{s}_{i_2}^4$. Hence, we first consider the method of the constant amplitude transmission for $s_{i,(0)}^4$ by the choice of $b_{i_3}^4$, and every b_i^4 is generated by the rate 3/4 encoder. Next, the replacement of bit stream is used for construction of rate 9/16 encoder, whose rule is shown in Table 2.2

Table 2.2: Rule of the replacement of bit streams

i	b_i^4	$\tilde{s}_i^4$	br_i^4
1	0001	-2, -2, -2, 2	0001
2	0010	-2, 2, -2, -2	0100
4	0100	-2, -2, 2, -2	0010
7	0111	2, -2, -2, -2	1000
8	1000	-2, 2, 2, 2	0111
B	1011	2, 2, -2, 2	1101
D	1101	2, -2, 2, 2	1011
E	1110	2, 2, 2, -2	1110

In Table 2.2, br_i^4 is denoted as the replacement bit stream of b_i^4. The replacement is done by simply changing amplitude levels 2 and –2 into binary number 1 and 0 [15]. Thus each replacement bit stream is expressed as

$$\begin{aligned} br_{i_0}^4 &= (br_{i_0,0}br_{i_0,1}br_{i_0,2}br_{i_0,3}) \\ br_{i_1}^4 &= (br_{i_1,0}br_{i_1,1}br_{i_1,2}br_{i_1,3}) \\ br_{i_2}^4 &= (br_{i_2,0}br_{i_2,1}br_{i_2,2}br_{i_2,3}) \\ br_{i_3}^4 &= (br_{i_3,0}br_{i_3,1}br_{i_3,2}br_{i_3,3}) \end{aligned} \tag{2.8}$$

In order to achieve constant amplitude in the signal $s_{i,(0)}^4$, the replacement bit streams must satisfy with the following condition

$$br_{i_0}^4 \oplus br_{i_1}^4 \oplus br_{i_2}^4 \oplus br_{i_3}^4 = (1111). \tag{2.9}$$

This condition means that the number of "1" is always kept as an odd number. Thus, we can select the bit stream $br_{i_3}^4$ by using the equation (see [15])

$$br_{i_3}^4 = \overline{br_{i_0}^4 \oplus br_{i_1}^4 \oplus br_{i_2}^4}\ . \tag{2.10}$$

Finally, Figure 2.2 shows the constant amplitude encoder with rate 9/16.

In Wada's scheme, a multicode scheme with constant amplitude transmissions is achieved. However, the redundant bits used in the construction of multicode signals with constant amplitude are simply lost at the receiver side. Therefore, in each case of rate 9/16 and 3/4, in terms of BER performance, the quantities of the degradation are 2.50dB (= 10log(9/16)) and 1.23dB (=10log(3/4)) respectively. In case of nonlinear amplifier, Wada's scheme outperforms the conventional multicode scheme, so Wada's scheme is effective when using HPA.

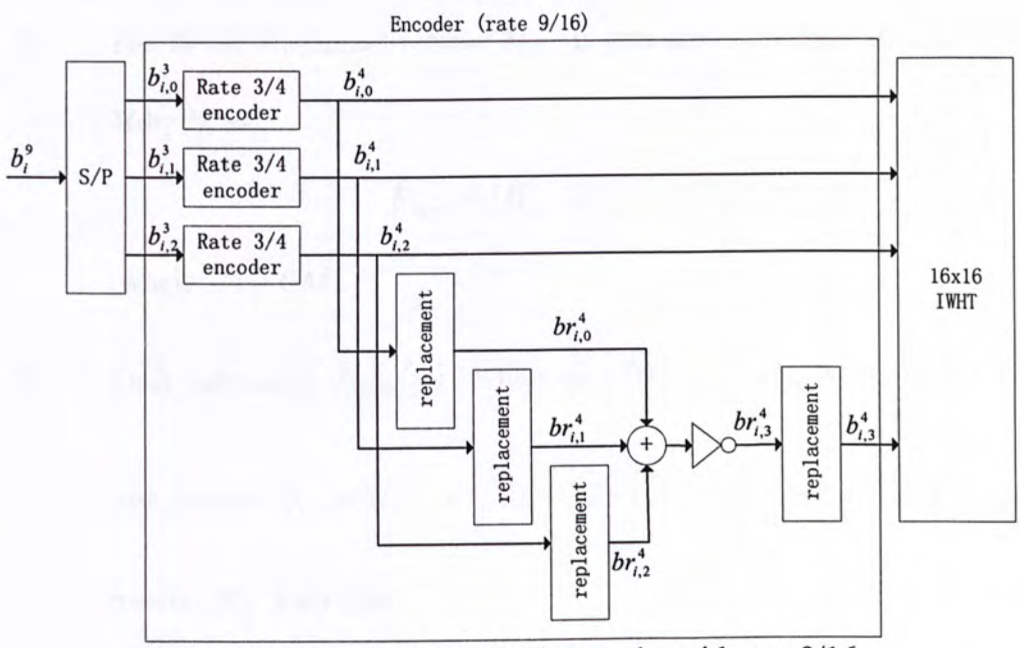


Figure 2.2: Constant amplitude encoder with rate 9/16

2.2 Multidimensional Multicode Scheme

In [16] and [17], multidimensional multicode scheme was proposed based on Wada's coding scheme. However, this scheme increases the data rate by compensating a loss in the information rate incurred by the constant envelope signal.

The multidimensional signaling based on M-parallel orthogonal multicode channels is designed as follows [16]:

1) The $M \times M$ Hadamard matrix H_M is extended into the matrix E of size M-by-N, as

$$E_{M \times N} = [H_M \mid H_M \mid \cdots \mid H_M], \tag{2.11}$$

where $N = GM$.

2) Each submatrix $E_{4 \times N}(m)$, where $m = 0,1,\cdots,\frac{M}{4}-1$, is multiplied by a row vector β_e, where $e = 0,1,\cdots,G-1$, $G = \frac{N}{M}$, of $G \times G$ orthogonal matrix W_G such that

$$\begin{aligned} E_{4 \times N}(m) \circ \beta_e^T = [&\beta_{e,0} H_{4 \times M}(m) \mid \beta_{e,1} H_{4 \times M}(m) \mid \\ &\cdots \mid \beta_{e,G-1} H_{4 \times M}(m)] \end{aligned} \tag{2.12}$$

in which, $\beta_e = (\beta_{e,0}, \beta_{e,1}, \cdots, \beta_{e,G-1})$, and the submatrics are expressed as follows

$$H_M = [H_{4\times M}^T(0) \mid H_{4\times M}^T(1) \mid \cdots \mid H_{4\times M}^T(\frac{M}{4}-1)]^T,$$
$$E_{M\times N} = [E_{4\times N}^T(0) \mid E_{4\times N}^T(1) \mid \cdots \mid E_{4\times N}^T(\frac{M}{4}-1)]^T,$$
$$E_{M\times N}(m) = [H_{4\times M}(m) \mid \cdots \mid H_{4\times M}(m)],$$

where T denotes the transpose of a matrix.

Therefore, a set of row vectors $\{\beta_e \mid e = 0,1,\cdots,G-1\}$ is able to carry additional $\log_2 G$-bit information on 4-parallel channel [16]. When 1-bit is used for constant envelope by rate 3/4 constant amplitude coding, there are 3-bit data transmitted by 4-parellel multicode signals. In Wada's scheme, the 1-bit precoding for constant envelope is simply lost. Therefore, the performance is degraded. However, in this multidimensional multicode scheme, any subset of W_G with 2^L row vectors $\{\beta_e \mid e = 0,1,\cdots,2^L-1\}$ may be selected to convey $1 \le L' \le \log_2 G$ bits, so this multidimensional signaling preserving orthogonality among all multicode signals can transmit totally $(3+L)M/4$ bits at most on the M-parallel multicode channels[17].

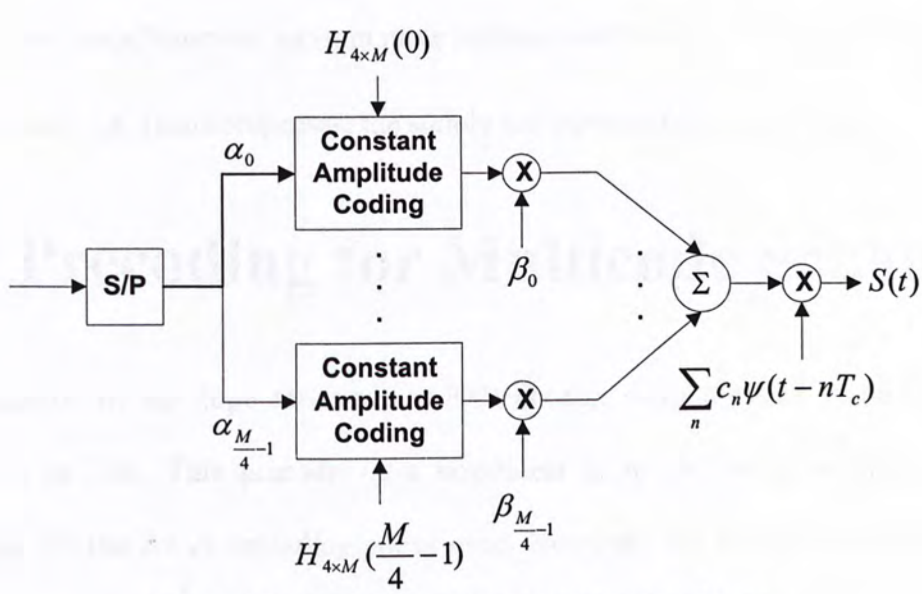


Figure 2.3: Multidimensional multicode scheme

Figure 2.3 shows the block diagram of Multidimensional multicode scheme. The output signal $S(t)$ can be thus expressed as

$$S(t)=\sum_{m=0}^{M-1} b_m \sum_{l=0}^{M-1} h_{m,l} \sum_{g=0}^{G-1} w_{<m>,g} c_{l+gM} \cdot \psi(t-(l+gM)T_c) \tag{2.13}$$

where $\{b_m\}$ denote the input M-parallel data, the sequences $[w_{<m>,0},\cdots,w_{<m>,G-1}]$ are the row vectors $\{\beta_e\}$ sending the additional data, in which $<m>=\left\lfloor \frac{m}{4} \right\rfloor$ ($\lfloor x \rfloor$ is the greatest integer not exceeding x) [16]. And $\{c_n\}$ is the user-specified scrambling code. In Figure 2.3 and (2.13), $\{\alpha_m \mid m=0,\cdots,\frac{M}{4}-1\}$ are the arbitrary data groups inputted to the module of "Constant Amplitude Coding", and $\{b_m\}$ are encoded M-parallel data. By the multidimensional signaling, the additional information data can be conveyed by

$\{\beta_e\}$. Thus this scheme can transmit more information bits than Wada's scheme in one time slot and $\{\beta_e\}$ can compensate the simply lost energy of redundant bits.

2.3 Precoding for Multicode Scheme

To minimize the envelope variations or PAP, another method called Precoding was proposed in [18]. This precoder is a non-linear high-rate block code especially designed for the set of spreading codes used. However, the precoder can be made independent of the spreading codes if a user-specific spreading code is concatenated with a set of Hadamard or conference sequences. The resulting spreading codes are orthogonal [18]. In this coding scheme, an algorithm named *envelope-decreasing algorithm* is used to design a precoder with a lower maximum envelope than that for the uncoded case. Besides Walsh codes, another set of spreading codes that has the same properties as the Hadamard matrix based on concatenation, is a concatenation based on a conference matrix. A conference matrix H_n is an $n \times n$ matrix with all diagonal elements equal to zero and the other elements of +1 or –1, which satisfies $H_n H_n^T = (n-1)I_n$. In this matrix, we can observe that a zero element (in the diagonal) means that the corresponding chip will not be transmitted. Moreover, in [18] it is proven that there exists an $(n,n\text{-}1)$-precoder such that the crest factor is reduced by at least 5.6dB compared to the uncoded system with n-1 codes and asymptotically as $n \to \infty$, the reduction in the crest factor approaches 6dB. However, finding such precoder is a really tough and complex job.

2.4 Summary

Three novel studies, *constant amplitude coding scheme*, *multidimensional multicode scheme*, and *precoding scheme*, on minimization of PAP of multicode signals are presented in details in this chapter.

Constant amplitude coding scheme (Wada's scheme) is the simplest and most efficient scheme. However, in this scheme, the redundant bits used for construction of signals with constant amplitude are not efficiently used at the receiver side, so the performance is surely degraded some decibels, although multicode scheme with constant amplitude transmissions is achieved and this scheme mitigates the effects of nonlinear device such as HPA.

In multidimensional multicode scheme based on Wada's scheme, the additional bits can be conveyed by multidimensional signaling to compensate the lost energy of redundant bits in Wada's scheme, so the degradation of performance of Wada's scheme is avoided. However, the complexity of its receiver is surely increased. More details of analysis on its receiver and performance can found in [16].

Constant amplitude transmission is realized in the first two schemes, while in precoding scheme, PAP of multicode signals can be reduced to a certain level. Although in this scheme, constant amplitude transmission cannot be achieved, it has a higher efficiency in practical application and has been adopted in current WCDMA system.

Chapter 3

Multicode CDMA System with Constant Amplitude Transmission

In Chapter 2, we have presented and summarized the three novel methods of minimizing PAP of multicode signals. In those methods, "constant amplitude coding" proposed by Wada [15] is a simplest and very efficient scheme. However, this scheme may be fulfilled only in some cases but difficult to be generalized. The 3/4 coding scheme of Wada's scheme is equivalent to a rate 3/4 single parity check code. However, other cases of Wada's scheme (i.e. 9/16 coding scheme) cannot be simply expressed by parity check code. Therefore, the data bits cannot be efficiently used, so the performance is degraded. In this chapter, we proceed to discuss a general scheme of multicode scheme with constant amplitude transmission. In this scheme, a nonlinear operator is used for construction of multicode signal with constant amplitude. This scheme is more flexible and it can be used for any odd value of M (number of code channels). Moreover, through analysis of this scheme, it is obvious that 3/4 Wada's scheme is just the special case of the proposed scheme. Unlike Wada's scheme, all the data bits can be efficiently used for the detection on information bits, so the performance of this scheme is superior to that of Wada's scheme.

3.1 System Model

The input information of k-th user is a stream of M-dimensional vector $c^{(k)}$. The k-th user is provided M binary vectors h_m, $i \le m \le M$, chosen from the columns of the Hadamard matrix of dimension N, where $N>M$. The notation H is used to denote the matrix $[H_0, H_1, \ldots, H_{N-1}]$.

Each M-dimensional vectors $c^{(k)}$ is passed through the following non-linear operator to give an N-dimensional vector [32]

$$d^{(k)} = sign(Hc^{(k)}). \tag{3.1}$$

This sign function is defined as

$$sign(x) = \begin{cases} +1, x \ge 0 \\ -1, x < 0 \end{cases}. \tag{3.2}$$

In equation (3.1), the sign function operates on each element of the vector in its argument. With this non-linear operation, each element of $d^{(k)}$ is of unit magnitude.

The Figure 3.1 illustrates the multicode transmitter scheme with constant amplitude transmissions. In this figure, c_i $(1 \le i \le M)$ is the bit being transmitted by i-th code channel assigned with h_i $(1 \le i \le M)$.

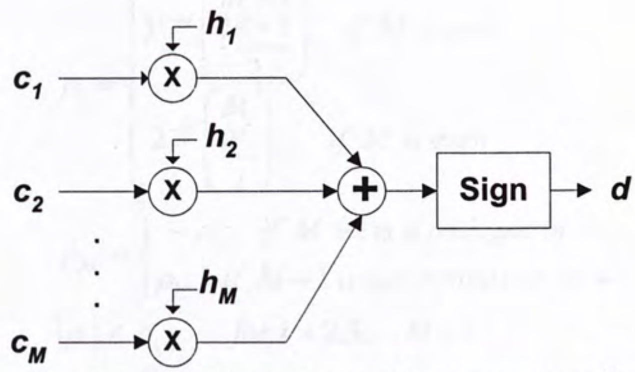


Figure 3.1: The multicode transmitter scheme with constant amplitude transmissions

Subscripts of all the elements in equation (3.1) are omitted for convenience since the case of multiuser is not considered in this thesis. From equation (3.1), we can find that each of the M elements of c is spread by a column of H, which is an N-dimensional vector. Therefore, Hc can be viewed as a multicode CDMA signal [32]. In general, a multicode CDMA signal has a non-constant envelope. The sign function in (3.1) forces a constant envelope in d.

In general, we may not be able to recover c from d. However, [32] mentions that through a careful choices of h_m, the m-th element of c, namely c_m, can be recovered from d by

$$c_m = sign(h_m^T d) \tag{3.3}$$

In fact, d can be expressed as [19]

$$\begin{aligned} d = \rho_0 + \rho_1 \sum_{m=1}^{M} c_m h_m + \rho_2 \sum_{m=}^{M} \sum_{n=m+1}^{M} c_m c_n h_m \circ h_n + \\ \dots + \rho_M c_1 c_2 \dots c_M h_1 \circ h_2 \circ \dots \circ h_M \end{aligned} \tag{3.4}$$

In equation (3.4), ° denotes the Hadamard product. The parameters ρ_m are given by

$$
\begin{aligned}
&\rho_0 = 0, \\
&\rho_1 = \begin{cases} 2^{1-M}\begin{pmatrix} M-1 \\ \frac{M-1}{2} \end{pmatrix}; & \text{if } M \text{ is odd} \\ 2^{-M}\begin{pmatrix} M \\ \frac{M}{2} \end{pmatrix}; & \text{if } M \text{ is even} \end{cases} \\
&\rho_M = \begin{cases} -\rho_1; & \text{if } M+1 \text{ is a multiple of } 4 \\ \rho_1; & \text{if } M+1 \text{ is not a multiple of } 4 \end{cases} \\
&|\rho_i| < \rho_1 \qquad \text{for } i = 2,3,\ldots,M-1
\end{aligned}
\tag{3.5}
$$

For all odd value of M, ρ_m with even subscripts vanishes. So only odd value of M is considered.

To show the properties of d, we take M=3 and 5 as an example. For M=3,

$$
d = \frac{1}{2}\sum_{m=1}^{3} c_m h_m - \frac{1}{2} c_1 c_2 c_3 h_1 \circ h_2 \circ h_3 , \tag{3.6}
$$

so d can be considered as a multicode signal with four component codes: h_1, h_2, h_3, and $h_1 \circ h_2 \circ h_3$. The data on the component codes are not independent. The data bit on the fourth component code is a parity check bit on the other three data bits. Note that this structure of coding is equivalent to 3/4 constant coding scheme.

For M=5,

$$
\begin{aligned}
d = {} & \frac{3}{8}\sum_{m=1}^{5} c_m h_m - \frac{1}{8}\sum_{m=1}^{5}\sum_{n=m+1}^{5}\sum_{p=n+1}^{5} c_m c_n c_p h_m \circ h_n \circ h_p + \\
& \frac{3}{8} c_1 c_2 \ldots c_5 h_1 \circ h_2 \circ \ldots \circ h_5 .
\end{aligned}
\tag{3.7}
$$

In this case, d can be considered as a multicode signal with six component codes: h_1, h_2,…, h_5, and $h_1 \circ h_2 \ldots \circ h_5$, together with some interference. The data bit on the last component code is a parity check bit on the other data bits.

These results show that the nonlinear operation in (3.1) can be considered as a linear code providing a parity check bit, which is followed by spreading with multicodes [32]. From the view of coding scheme, it is equivalent to single parity check (SPC) code.

In general, the resultant multicode signal (3.4 excluding ρ_0) generated by the proposed scheme consists of three components: conventional multicode signal (first term), a parity check bit (last term) and interference (middle term). When the ML sequence detector is used, there is no interference. The interference term takes negative effect, when the matched-filters are used for the scheme. In the next section, we discuss how to minimize the effect caused by interference component in order to recover c from d better using (3.3) for the case of matched-filters.

3.2 Selection of Hadamard Code Sequences

Firstly, an important property of Hadamard code sequences has to be described. In [20], Ahmed and Rao provide an exponential definition for Hadamard matrix H. We denote $H(w, v)$ for the element of w-th row and v-th column in H. Let w_i and v_i denote the i-th bit in the binary representations of integers w and v respectively, that is

$$(w)_{decimal} = (w_{n-1}w_{n-2}...w_1w_0)_{binary},$$

and

$$(v)_{decimal} = (v_{n-1}v_{n-2}...v_1v_0)_{binary}.$$

Then, the elements $H(w, v)$ of H can be generated using the relation

$$H(w,v) = (-1)^{\sum_{i=0}^{n-1} w_i v_i}, \quad w, v = 0,1,..., N-1 \tag{3.8}$$

where N is the order of Hadamard matrix H, and $n = \log_2 N$.

From equation (3.8), the product of the two functions will be

$$\begin{aligned} H(w,v)H(u,v) &= (-1)^{\sum_{i=0}^{n-1} w_i v_i} \cdot (-1)^{\sum_{i=0}^{n-1} u_i v_i} = (-1)^{\sum_{i=0}^{n-1} w_i v_i + \sum_{i=0}^{n-1} u_i v_i} \\ &= \prod_{i=0}^{n-1} (-1)^{(w_i + u_i) v_i} = \prod_{i=0}^{n-1} (-1)^{(w \oplus u)_i v_i} = (-1)^{\sum_{i=0}^{n-1} (w \oplus u)_i v_i} = H(w \oplus u, v), \end{aligned} \tag{3.9}$$

since the addition of binary terms of the same index should be carried out by Modulo-2.

From equation (3.9), this property can be extended to higher-order products using Module-2 addition (exclusive-or) of the corresponding rows of Hadamard matrix: $H(w,v)H(x,v)...H(z,v) = H(w \oplus x \oplus ... \oplus z, v)$. This property is the same for the columns of Hadamard matrix due to its symmetric structure.

In equation (3.4), there are $\sum_{i=0}^{i=(M-1)/2} \binom{M}{2i+1} = 2^{M-1}$ component codes, among which most of the energy is distributed on $h_1, h_2, ..., h_M,$ *and* $h_1 \circ h_2 ... \circ h_M$. Since the energy on the interference components is relatively small, a simple ignorance of them from resultant signal by receiver has little effect to the performance of whole system. Then, the resulting signal is made of conventional multicode signal and its parity check bit, from which the information bits can be covered. However, this result is based on the assumption that any of component codes on interference terms is different from any of $h_1, h_2, \ldots, h_M$, and $h_1 \circ h_2 \ldots \circ h_M$. Otherwise, the multicode component may be contaminated by interference component and thus the performance of system may be greatly degraded.

Followed by the property of Hadamard code sequence presented at the beginning of this subsection, we know that the product of two Hadamard code sequences, H_i [1] and H_j, produces another Hadamard code sequence, written as $H_i \circ H_j = H_{i \oplus j}$ [2] for convenience [21]. In general, this property can be extended to the case of multiple Hadamard products written as: $H_i \circ H_j \circ \ldots \circ H_z = H_{i \oplus j \oplus \ldots \oplus z}$. Since a Hadamard code sequence can be uniquely determined by its subscript, we consider the binary representation of its subscript for each Hadamard code sequence. For the Hadamard matrix of dimension N, which is a set of all N Hadamard code sequences, the set of all their N binary indexes is a vector space if each binary index is viewed as a vector of dimension $\log_2 N$. For the Hadamard matrix of dimension N=4, the vector space is {00,01,10,11}.

Since the Hadamard product is equivalent to the summation of vectors in the vector space over GF(2), the selection of Hadamard code sequence can be made

---

[1] H_i denotes $(i+1)$-th column of Hadamard matrix ($0 \le i \le N-1$), while h_j denotes selected Hadamard code sequences for j-th code channel ($0 \le j \le M$).

[2] $i \oplus j$ denotes bit-by-bit XOR operation of binary representation of i and j.

through the selection of index (subscript) of Hadamard code sequence. The basis of the vector space may be selected because summation vector of any two or more of non-zero basis vectors differs from any of basis vectors. For the Hadamard matrix of N=16, the basis of its vector space is {0000, 0001, 0010, 0100, 1000}, so H_0, H_1, H_2, H_4, and H_8 can be selected out.

Therefore, h_1, h_2,..., and h_M should be selected and each product of any odd number (3, 5, ...) out of them should not be equal to any one of them. If H_0 is selected for h_1, the criterion will be changed to that each product of any number (>1) out of h_2, h_3,..., and h_M should not be equal to any one of h_2, h_3,..., and h_M. If the index of H_i (i=0,2,..,N-1) is rewritten into binary form v_j[3] for h_j, the criterion can be put in this way that a linearly dependent set { v_2, v_3,..., v_M} of vectors should be selected for h_2, h_3,..., and h_M. We can consider all the index vectors as a vector space

$$S = \{v_1, v_2, ..., v_{N-1}\} = \{\underbrace{00..01}_{\log_2 N}, ..., \underbrace{11..11}_{\log_2 N}\}. \tag{3.10}$$

Therefore, we can choose the orthogonal basis of S as the candidate for h_2, h_3,..., and h_M.

Take M=5 and N=16 for example. v_0={0000} is selected for h_1 at first. Then, the orthogonal basis of S={0001, 0010,..., 1111}, which is {0001, 0010, 0100, 1000}, is selected for h_2, h_3, h_4, and h_5. Thus, H_0, H_1, H_2, H_4 and H_8 are used for the proposed multicode CDMA system. These codes are shown below:

---

[3] We can consider this binary form as a vector of $\log_2 N$-dimension

$H_0 =$ (1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1)
$H_1 =$ (1 -1 1 -1 1 -1 1 -1 1 -1 1 -1 1 -1 1 -1)
$H_2 =$ (1 1 -1 -1 1 1 -1 -1 1 1 -1 -1 1 1 -1 -1)
$H_4 =$ (1 1 1 1 -1 -1 -1 -1 1 1 1 1 -1 -1 -1 -1)
$H_8 =$ (1 1 1 1 1 1 1 1 -1 -1 -1 -1 -1 -1 -1 -1)

By the theory of linear algebra, we know that such M codes can be selected from at least $N=2^{M-1}$ codes ($N\times N$ Hadamard matrix) for optimal code selection (the perfect case of no interference from other component codes).

Also, from the relationship between Walsh function and Rademacher function, we can find another way to interpret this code selection. In [22], we know that a complete set of Walsh functions in natural order can be obtained from selected Rademacher function products. For n independent binary variables, there are 2^n Walsh functions, of which n+1 are Rademacher functions [23]. Hadamard matrix is a modified Walsh matrix by interchanging certain rows, so Hadamard matrix can be expressed by product series of Rademacher functions (see [23]):

$$\begin{aligned} H(w,v) &= \prod_{i=0}^{n-1} w_i R_{n-i}, \quad w = 1,2,\ldots,N-1 \\ H(0,v) &= R_0 \end{aligned} \tag{3.11}$$

For n=3, from (3.12), we can find that in this code selection only Rademacher functions are selected out for $h_1, h_2, \ldots$, and h_M.

$$H_8 = \begin{array}{c} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & - & 1 & - & 1 & - & 1 & - \\ 1 & 1 & - & - & 1 & 1 & - & - \\ 1 & - & - & 1 & 1 & - & - & 1 \\ 1 & 1 & 1 & 1 & - & - & - & - \\ 1 & - & 1 & - & - & 1 & - & 1 \\ 1 & 1 & - & - & - & - & 1 & 1 \\ 1 & - & - & 1 & - & 1 & 1 & - \end{bmatrix} \end{array} \begin{array}{c} R_0 \\ R_3 \\ R_2 \\ R_2R_3 \\ R_1 \\ R_1R_3 \\ R_1R_2 \\ R_1R_2R_3 \end{array} \tag{3.12}$$

If $X_{(w)}$ is used to denote the $(w)_{\text{decimal}}$,

$$H_{(w)} = X_{(w)} \times G_R, \tag{3.13}$$

where G_R is defined as follow

$$G_R = \begin{bmatrix} R_{M-1} \\ \dots \\ \dots \\ R_2 \\ R_1 \end{bmatrix}. \tag{3.14}$$

Therefore, if the middle terms are ignored due to their little energy, and the last term is considered only, then we can use the expression (3.12) for code selection. We can select P Hadamard sequences out of Hadamard matrix ($P>M$). Firstly, Rademacher functions R_i (i=0,1,…,M) are selected. Secondly, P-M-1 Hadamard sequences have to be selected out of the remaining columns in Hadamard matrix based on the criterion

$$X_1 \oplus X_2 \oplus \dots \oplus X_P \neq X_i \quad i = 1,2,\dots,P \tag{3.15}$$

where X_i denotes the $(i)_{\text{decimal}}$ of the i-th selected Hadamard code sequence, which is corresponding to h_i if it is used for the i-th code channel. Anyway, this code searching can be viewed as a suboptimal code selection, since there may exist more interference between multicode term and interference term. By this code selection, more suitable Hadamard code sequences could be selected out than that by optimal code selection.

As to the optimal code selection which is of most interest, in general, H_0 and H_{2^i} $(0 \le i \le \log_2 N - 1)$ may be selected, so the component code with one parity check bit is H_N. Therefore, for the Hadamard matrix of dimension N, the maximum number M of code channels is $\log_2 N+1$. By this optimal selection of Hadamard code sequences, all the 2^{M-1} component codes are different from each other. Therefore, the component of multicode signal is not affected by the interference component and the information bits can be well recovered.

3.3 The Optimal Receiver for the Multicode System

There are some redundant bits in the transmitted signals generated by nonlinear operations in the multicode scheme. Without loss of their energy, the optimal receiver is analyzed in this section, in which the maximum-likelihood (ML) sequence detector and maximum a posteriori (MAP) detector (a symbol-by-symbol detector) are considered respectively.

3.3.1 The Maximum-Likelihood Sequence Detector

Given the observation sequence $\mathbf{r} = \{r_1, r_2, ..., r_N\}$ at the receiver side, the detector determines the sequence $\mathbf{c} = \{c_1, c_2, ..., c_M\}$ that maximizes the conditional pdf $p(r_1, r_2, ..., r_N \mid \mathbf{c})$. At the transmitter side of the proposed multicode scheme, each M-dimensional vector $\mathbf{a}$ is passed through nonlinear operator to give an N-dimensional vector $\mathbf{d} = sign(\mathbf{Ha})$, in which set of code sequences is considered as a vector. In the case of an additive noise channel, the observation sequence at its receiver side can be expressed as $\mathbf{r} = \mathbf{d} + \mathbf{n}$, where $\mathbf{n}$ is the noise vector. Therefore, the conditional pdf $p(r_1, r_2, ..., r_N \mid \mathbf{c})$ is equivalent to the conditional pdf $p(r_1, r_2, ..., r_N \mid \mathbf{s})$, where $\mathbf{s} = sign(\mathbf{Hc})$.

Assuming that only an AWGN channel is considered, for any given transmitted sequence $\mathbf{s}$, the joint pdf of $r_1, r_2, ..., r_N$ may be expressed as a product of N marginal pdfs [24],

$$\begin{aligned} p(r_1, r_2, ..., r_N \mid \mathbf{s}) &= \prod_{k=1}^{N} p(r_k \mid s_k) = \prod_{k=1}^{N} \frac{1}{\sqrt{2\pi}\sigma_n} \exp\left[-\frac{(r_k - s_k)^2}{2\sigma_n^2} \right] \\ &= \left(\frac{1}{\sqrt{2\pi}\sigma_n} \right)^N \exp\left[-\sum_{k=1}^{N} \frac{(r_k - s_k)^2}{2\sigma_n^2} \right], \end{aligned} \tag{3.16}$$

where the N noise components are zero-mean uncorrelated Gaussian random variables with a common variance σ_n^2.

By taking logarithm on (3.16) and neglecting the terms that are independent of

$\mathbf{r} = \{r_1, r_2, ..., r_N\}$, it is found that an equivalent ML sequence detector which selects the sequence $\mathbf{c} = \{c_1, c_2, ..., c_M\}$ producing the sequence $\mathbf{s} = \{s_1, s_2, ..., s_N\}$ that minimizes the *euclidean distance metric* [24]

$$D(\mathbf{r}, \mathbf{s}) = \sum_{k=1}^{N} (r_k - s_k)^2 . \tag{3.17}$$

In searching the sequence to minimize the euclidean distance $D(\mathbf{r}, \mathbf{s})$, it is obvious that we must compute the distance $D(\mathbf{r}, \mathbf{s})$ for every possible sequence $\mathbf{c} = \{c_1, c_2, ..., c_M\}$ and keep the record for the minimum distance. However, since the total number of the sequences is 2^M, the complexity of computation increases exponentially with the number of code channels.

In the simulation, the cases of 5 and 7 code channels are considered. The ML detector and the hard detector are used for signal detection respectively. The optimal and suboptimal code selection are both used for the simulation. When the ML detector is used, there is no interference. However, in the second case, matched-filters with hard decision are used, so the interference term takes negative effect on the performance. The simulation result is shown in Figure 3.2. From the result, it is obvious that the case with ML receiver is superior to the case of hard decision in terms of BER performance, since the ML receiver provides the optimal performance of the proposed scheme. For the case of ML detector, the slight difference of performance between the cases of 5 and 7 channels is caused by different numbers of possible sequences. For 5 code channels, the set of possible sequences has 2^5 elements, while the set of possible sequences has 2^7 elements for 7 code channels. Therefore, the case of 5 code channels has a slightly better performance that of 7 code channels, if the lengths of output multicode signals are equal. In addition, for the second case, the degradation of performance of 7 code channels is caused not only by the bigger set of sequences, but also by the interference of code channels, because of matched-filters used in the receiver. Due to Hadamard matrix of order 16 used, 5 Hadamard code sequences are selected by the optimal code selection while 7 Hadamard code sequences can be only selected by the suboptimal code selection, which causes additional interference of code channels.

Although the optimal performance is provided, the ML receiver is hard to be implemented in the practice, because the computation will be very complicated when the number of code channels is not small. Therefore, a feasible receiver providing near-optimal performance attracts more interests and will be considered and analyzed in the next chapter.

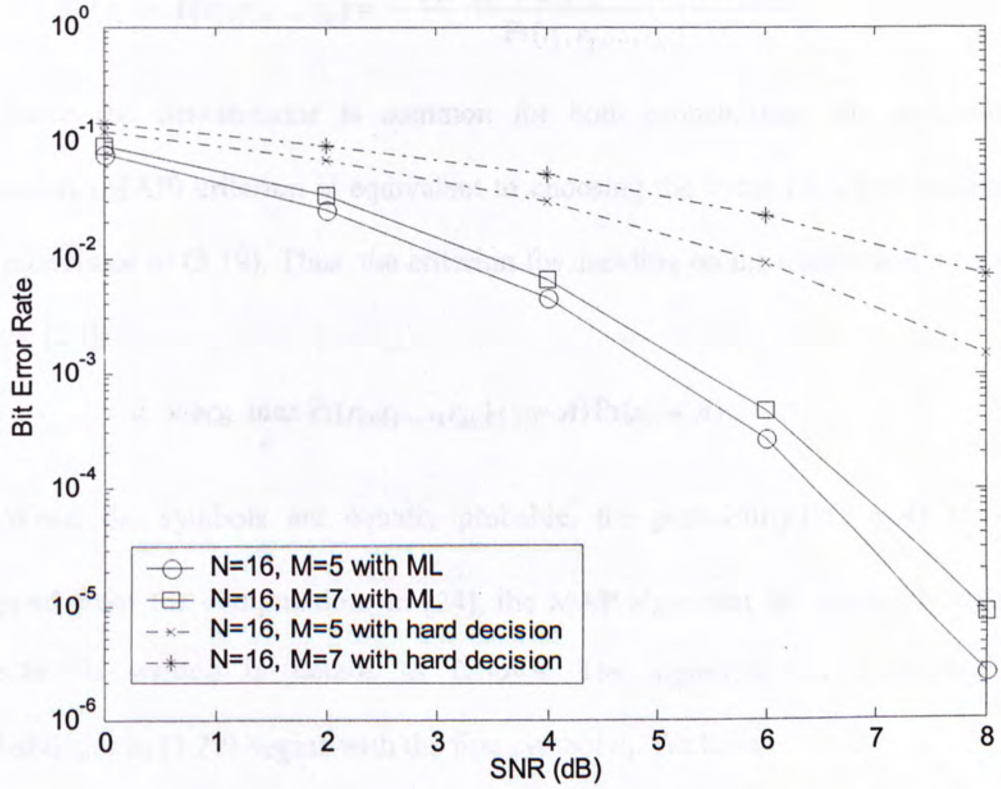


Figure 3.2: The BER performance of ML detector versus simple hard decision

3.3.2 Maximum A Posteriori Probability Detector

This detector makes symbol-by-symbol decision based on the computation of the maximum a posteriori probability (MAP) for each detected symbol. On the basis of the received sequence $\mathbf{r} = \{r_1, r_2, ..., r_N\}$, we compute the posterior probabilities

$$\Pr(c_i = A \mid r_1, r_2, ..., r_N) \quad 1 \le i \le M; A \in \{-1, +1\}, \tag{3.18}$$

for the 2 possible symbol values and choose the symbol with the largest probability [24]. The equation (3.18) can be rewritten as

$$\Pr(c_i = A \mid r_1, r_2, ..., r_N) = \frac{\Pr(r_1, r_2, ..., r_N \mid c_i = A)\Pr(c_i = A)}{\Pr(r_1, r_2, ..., r_N)}, \tag{3.19}$$

Since the denominator is common for both probabilities, the maximum a posteriori (MAP) criterion is equivalent to choosing the value of c_i that maximizes the numerator of (3.19). Thus, the criterion for deciding on the transmitted symbol c_i is (see [24])

$$\tilde{c}_i = \arg \max_{c_i} \Pr(r_1, r_2, ..., r_N \mid c_i = A)\Pr(c_i = A). \tag{3.20}$$

When the symbols are equally probable, the probability $\Pr(c_i = A)$ may be dropped from the computation. In [24], the MAP algorithm for symbol-by-symbol detector is written in details as follows. The algorithm for computing the probabilities in (3.20) begins with the first symbol c_1. We have

$$\begin{aligned} \tilde{c}_1 &= \arg \max_{c_1} \Pr(r_1, r_2, ..., r_N \mid c_1 = A)\Pr(c_1 = A) \\ &= \arg \max_{c_1} \sum_{c_2} ... \sum_{c_M} \Pr(r_1, r_2, ..., r_N \mid c_1, c_2, ..., c_M)\Pr(c_2, ..., c_M \mid c_1)\Pr(c_1) \\ &= \arg \max_{c_1} \sum_{c_2} ... \sum_{c_M} \Pr(r_1, r_2, ..., r_N \mid c_1, c_2, ..., c_M)\Pr(c_1, c_2, ..., c_M), \end{aligned} \tag{3.21}$$

where $\tilde{c}_1$ denotes the decision on c_1. The joint probability $\Pr(c_1, c_2, ..., c_M)$ may be omitted if the symbols are equally probable and statistically independent.

For detection of the symbol c_2, we have

$$\begin{aligned} \tilde{c}_2 &= \arg \max_{c_2} \Pr(r_1, r_2, ..., r_N \mid c_2 = A)\Pr(c_2 = A) \\ &= \arg \max_{c_2} \sum_{c_3} ... \sum_{c_M} \Pr(r_1, r_2, ..., r_N \mid c_2, ..., c_M)\Pr(c_3, ..., c_M \mid c_2)\Pr(c_2) \\ &= \arg \max_{c_2} \sum_{c_3} ... \sum_{c_M} \Pr(r_1, r_2, ..., r_N \mid c_2, ..., c_M)\Pr(c_2, ..., c_M). \end{aligned} \tag{3.22}$$

In general, the algorithm for detecting the symbol c_i is as follows [24]: upon reception of $\mathbf{r} = \{r_1, r_2, ..., r_N\}$, we compute

$$\begin{aligned} \tilde{c}_i &= \arg \max_{c_i} \Pr(r_1, r_2, ..., r_N \mid c_i) \Pr(c_i) \\ &= \arg \max_{c_i} \sum_{c_{i+1}} ... \sum_{c_M} \Pr(r_1, r_2, ..., r_N \mid c_i, c_{i+1}, ..., c_M) \Pr(c_i, c_{i+1}, ..., c_M). \end{aligned} \tag{3.23}$$

The joint probability $\Pr(c_i, c_{i+1}, ..., c_M)$ may be omitted if the symbols are equally probable and statistically independent.

In this simulation, the cases of 5 and 7 code channels are considered. The MAP detector and the hard detector are used for signal detection respectively. The optimal and suboptimal code selection are both used for the simulation. The simulation result is shown in Figure.3.3. MAP detector is also an optimal detector, which minimizes the symbol-by-symbol errors. However, in this simulation, information bits are transmitted randomly, and symbols "0" and "1" are equally probable, so the MAP detector and ML detector provide the identical optimal performance of the system.

Similar to ML detector, the complexity of computation of MAP detector increases exponentially with the number of code channels, so this optimal MAP detector is not practical. An iterative MAP detector involved with iterative decoding will be discussed later. This kind of detector can provide near-optimal performance and the complexity level of computation is much lower than that of optimal detector.

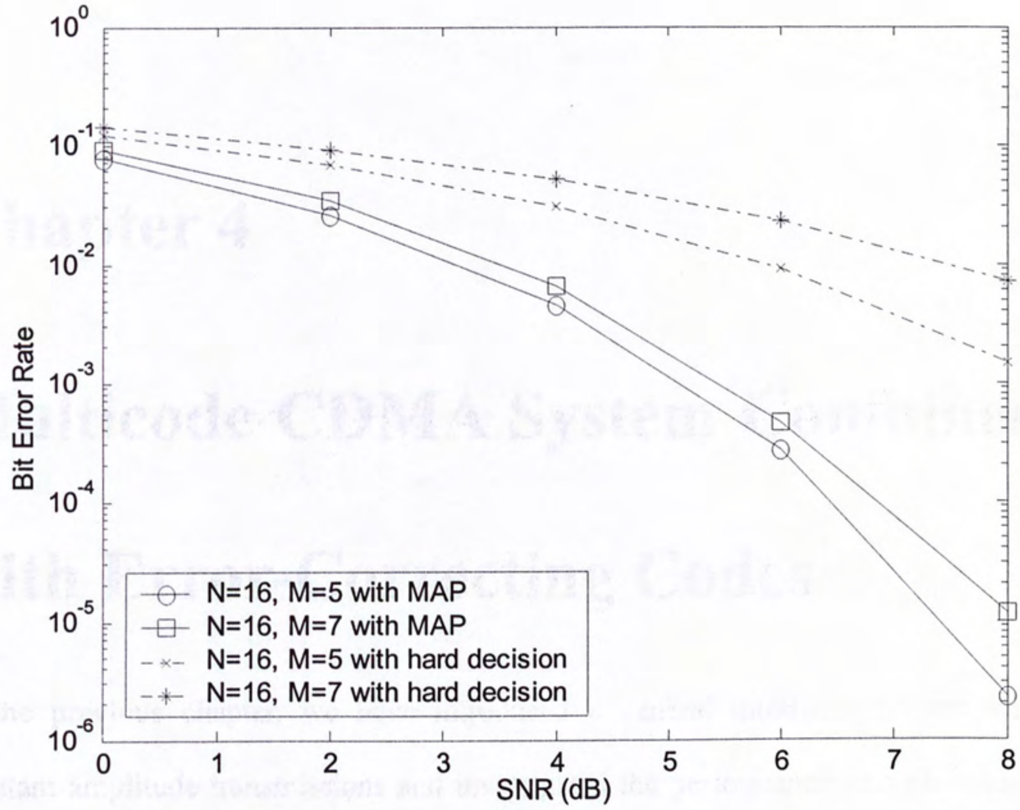


Figure 3.3: The BER performance of MAP detector versus simple hard decision

Chapter 4

Multicode CDMA System Combined with Error-Correcting Codes

In the previous chapter, we have introduced a general multicode scheme with constant amplitude transmissions and investigated the performance of such scheme with the optimal receivers. This multicode scheme can be considered as a SPC coding scheme due to one parity check bit generated. However, if we consider this multicode scheme as a standalone coding scheme, this single parity check bit can hardly improve the performance of whole system significantly. Thus, the combination of this multicode scheme and error-correcting coding scheme is inevitable. The proposed multicode system with a precoder is shown in Figure 4.1, which is a general version of multicode system combined with error-correcting codes. On the other hand, we have noticed that the complexity level of computation of the optimal detector is extremely high when the number of code channels is not small and size of the error-correcting code is big, so a suboptimal receiver providing near optimal performance with less complexity of computation is necessary. In this chapter, an iterative decoder with message passing algorithm is considered, and scenarios of Hamming code, Gallager's code, and zigzag code applied in the precoder is investigated respectively. The analysis begins with the simplest case of Hamming code.

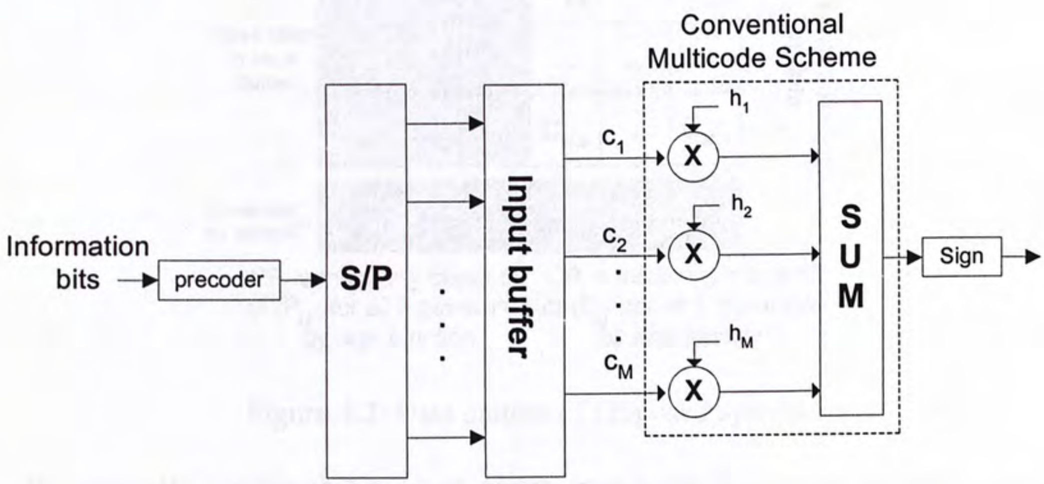


Figure 4.1: Transmitter scheme of proposed multicode system with precoder

4.1 Hamming Codes

In this scenario, Hamming encoder plays the role of the precoder in Figure 4.1. Information bits are encoded into Hamming codes by precoder and then fed into input buffer. After M encoded bit streams are stored in the buffer, the input bits rearranged by the input buffer are fed into the multicode scheme. By this multicode scheme, M more parity check bits are generated by this multicode scheme. Data pattern of system is shown in Figure 4.2, where we can find that the whole data pattern is a linear coding scheme known as a product code or a turbo block code. Therefore, the combination of Hamming code and the proposed system may greatly improve the performance of this multicode system.

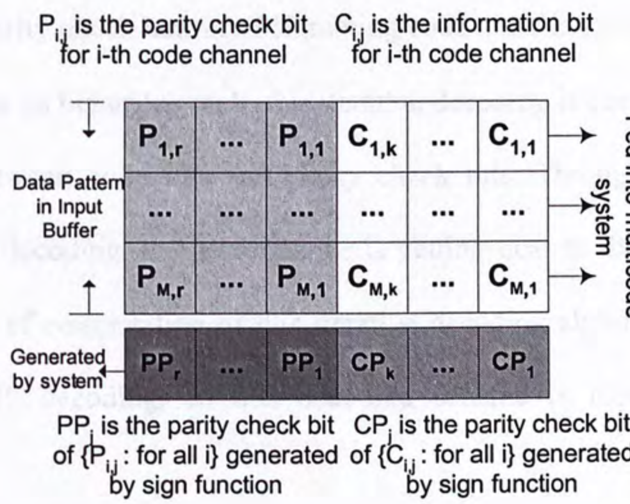


Figure 4.2: Data pattern of proposed system

To optimally decode this product codes, maximum likelihood decoding can be done by searching all possible message bits $\{C_{i,j}: 1 \leq i \leq M, 1 \leq j \leq K\}$. However, the complexity of computation increases exponentially by the size of the product codes, so it is infeasible when the size is not small. Therefore, an iterative decoding algorithm known as message passing algorithm [25] is used, which will be presented in details in the next section. Following the idea of graphic decoding given by [26], a parity check matrix of the product codes can be written out, by which the product codes can be represented in bipartite graph [26]. For M=3 and (L,K) Hamming codes are used as precoder, the parity check matrix of the product code is as

$$\begin{bmatrix} J & 0 & 0 & 0 \\ 0 & J & 0 & 0 \\ 0 & 0 & J & 0 \\ I & I & I & I \end{bmatrix}, \tag{4.1}$$

where J is the parity check matrix of Hamming code and I is an $L \times L$ identity matrix. Based on (4.1) or its bipartite graph, this iterative decoding is performed by shuttling the message between code bits and parity check bits. Through finite number of iteration of this decoding, the performance is getting near to that of ML decoding. The complexity of computation of this iterative decoding algorithm is much lower than that of ML decoding, so this decoding scheme is feasible in the actual applications.

4.2 Gallager's Codes

4.2.1 Encoding of Gallager's Codes

Gallager's code, also known as low-density parity check (LDPC) code, is firstly proposed and analyzed by Gallager [27]. A LDPC code is specified by a sparse matrix containing mostly 0's and relatively few 1's. An (n,j,k) LDPC code is a code of block length n with a parity check matrix, where each column contains a small fixed number j of 1's and each row contains a small fixed number k of 1's. Such LDPC code is often known as a regular Gallager's code. In [27], Gallager also proposes a random construction for parity check matrix, which is introduced as follows.

The (n,j,k) parity check matrix is divided into j submatrics with each containing a single 1 in each column. The first submatrix contains all its 1's in descending order; that is, the i-th row contains 1's in column $(i-1)k+1$ to ik [27]. Table 4.1 shows an example of the first submatrix for n=20, j=3, and k=4

Table 4.1: The first submatrix for n=20, j=3, and k=4

1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1

The other submatrices are merely column permutations of the first submatrix. The ensemble of (n,j,k) LDPC code is achieved by random permutations of the columns of each of the bottom j-1 submatrices with equal probability assigned to each permutation [27]. The parity check matrix of a (20,3,4) code may be (due to the random permutation) illustrated in Table 4.2.

It is obvious that a (n,j,k) LDPC parity check matrix has M=nj/k rows and at least (j-1) rows are linearly dependent. Thus the codes have a slightly higher information rate than the matrix indicates [27]. The coding rate $R \geq \dfrac{n-nj/k}{n} = 1 - j/k$.

Table 4.2: Example of parity check matrix for n=20, j=3, and k=4

1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1
1	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	1	0	0	0	0	0	0	1	0	0	0
0	0	1	0	0	0	1	0	0	0	0	0	0	1	0	0	0	1	0	0
0	0	0	1	0	0	0	0	0	0	1	0	0	0	1	0	0	0	1	0
0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1
1	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	1	0	0
0	1	0	0	0	0	1	0	0	0	1	0	0	0	0	1	0	0	0	0
0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	1	0
0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	1	0	0	0
0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1

We can also notice that this type of matrix does not have the check digits appearing in diagonal form, namely the systematic form. Therefore, encoding message block with Gallager's encoder is a problem. However, there are two ways to deal with this problem.

The first simple approach is to transform parity check matrix H into systematic form H' using Gauss-Jordan reduction. Systematic matrix H' can be expressed as $H' = [I \mid R]$, where I is an M-by-M identity matrix indicating check digits and R is a matrix of size $M \times (n\text{-}M)$. However, by this approach, the sparseness of the matrix may be lost due to the row operations involved in the Gauss-Jordan reduction. If the systematic form of H for encoding, the systematic form H' has to be used for decoding. Moreover decoding is fast if the matrix is sparse, so the decoding with H' may take longer time due to lack of sparseness.

Another approach [33] to deal with this problem is shown as follows. Given a codeword u and an M by N parity check matrix H, we have $uH^T=0$. Let us assume that the message bits, s, are located at the end of the codeword and the check bits, c, occupy the beginning of the codeword, i.e. $u=[c|s]$. Also let, $H=[A|B]$, where A is an M-by-M matrix and B is a matrix of size $M\times(n\text{-}M)$. Then

$$Ac+Bs=0\Rightarrow c=A^{-1}Bs\ ,$$

which can be used to compute the check bits as long as A is non-singular and not only when A is an identity matrix (H in systematic form). If the parity check matrix H do not have non-singular submatrix A, we can rearrange the columns of the parity check matrix in order to make A a non-singular matrix. Since no row operation is done throughout the procedure, the sparseness of the matrix is not affected. Therefore encoding by this approach is more efficient.

Another important point for Gallager's code is its graph representation by which the coding structure is more comprehensive. Every block code can be represented by bipartite graph [26], which consists of check nodes and bit nodes. Often bipartite graph is used for the representation of Gallager's code. To illustrate this coding graph, a small example is given. We consider the following parity check matrix

$$\begin{array}{ccccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1. \end{array}$$

In this parity check matrix, $x_1, x_2, \cdots, x_7$ represent the codeword bits indicating the bit nodes in bipartite graph. The 1's in each row stands for the connections between the check nodes and corresponding bit nodes. Figure 4.3 shows the bipartite graph for this coding example, in which circles stand for the check nodes indicated by parity check bits, and squares stand for the bit nodes.

This graph representation for Gallager's codes plays an important role in decoding the Gallager's codes. The message passing algorithm used in the decoding is derived based on this bipartite graph.

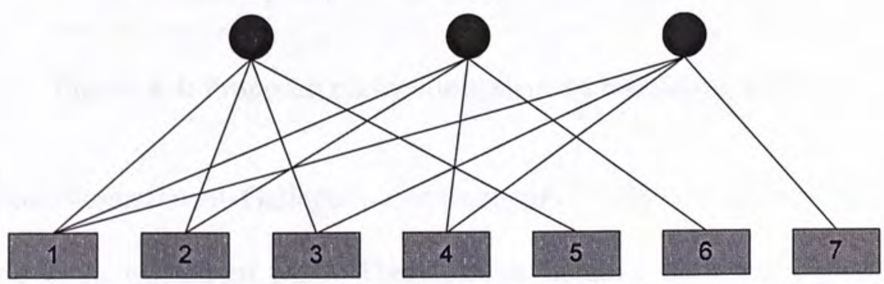


Figure 4.3: The bipartite graph for the example

4.2.2 Multicode Scheme combined with Gallager's Code

In the previous subsection, the case of Hamming code is a small example of combined scheme. In order to improve the performance of whole system dramatically, Gallager's encoder is considered as the precoder. Base on Figure 4.1, the scheme can be depicted into Figure 4.4, in which S/P module is ignored for simplicity, and input buffer is converted into a random interleaver, which sends out M bits per time slot. The output of compound (global) coding scheme of the whole system can be viewed as serial concatenation code (SCC) with Gallager's code as the outer code and SPC code as the inner code, so this coding structure contributes to improvement of the performance.

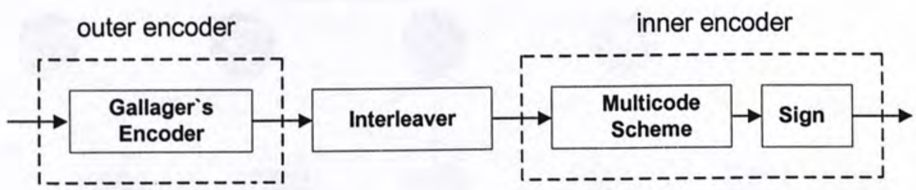
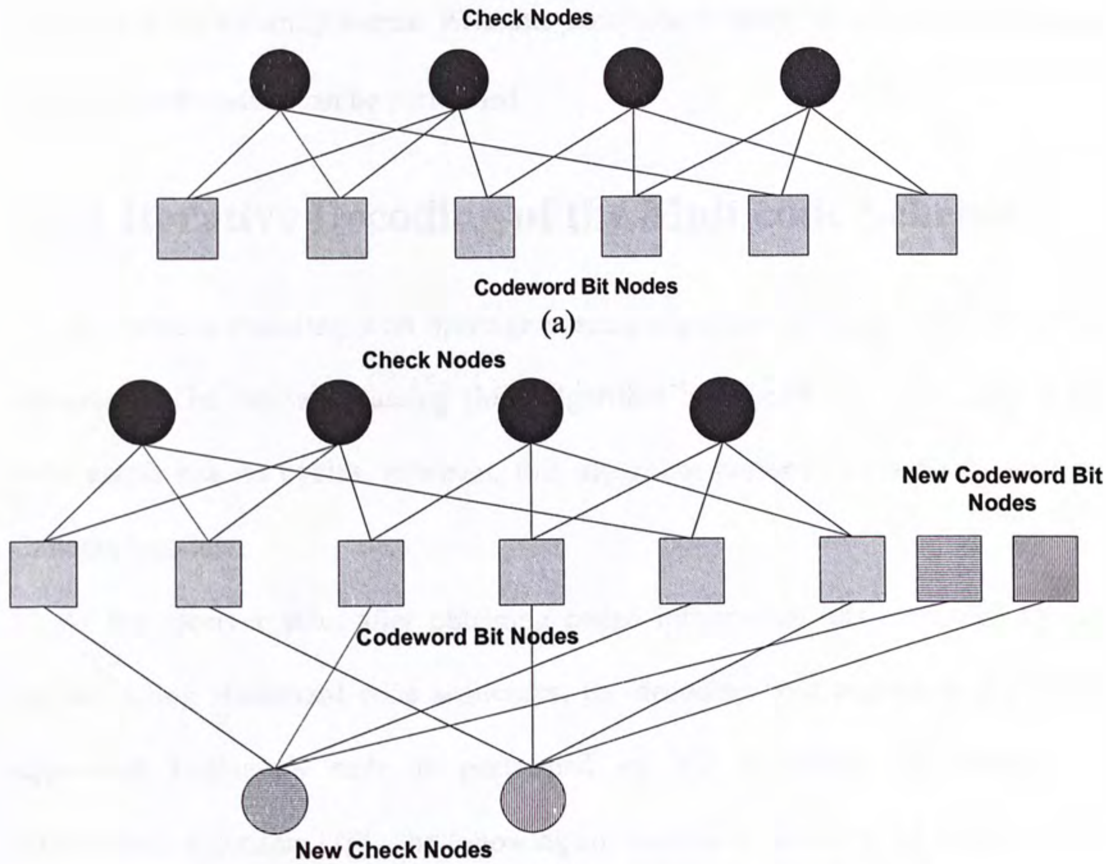


Figure 4.4: Proposed multicode system with Gallager's codes

The concatenation of Gallager's code and SPC code can be viewed as a new Gallager's code instead of SCC. Therefore an iterative decoding algorithm with respect to the new Gallager's code is used so that the parity check bits generated by the multicode scheme are fully utilized. From the bipartite graph of Gallager's code, the structure of global coding scheme is more pellucid. Take M=3 for example. If the assumed bipartite graph of original Gallager's code is shown in Figure 4.5(a), the bipartite graph of new Gallager's code may be depicted as Figure 4.5(b). Since there is a random interleaver between Gallager's encoder and the multicode scheme, the M links between original codeword bit nodes and a new parity check node are randomly set. Thus it is obvious that this construction of new Gallager's code follows the idea of random construction proposed by Gallager [27], but the new code is an irregular

Gallager's code [28].



(b)

Figure 4.5: (a) Bipartite graph of original Gallager's code;
(b) Bipartite graph of the new Gallager's code.

If the parity check matrix of original (L,R) Gallager's code (L is the length of the code and R is the number of parity check bits of the code) is written as Q, an $R\times L$ matrix, then the parity check matrix of new Gallager's code is written as

$$\begin{bmatrix} Q & 0 \\ P & I \end{bmatrix}, \tag{4.2}$$

in which P, determined by the interleaver, is an $(L/M)\times L$ matrix, which is randomly set and contains M 1's in each row and one 1 in each column, and I is an $(L/M)\times(L/M)$ identity matrix. With this parity check matrix and its bipartite graph, the iterative decoding can be performed.

4.2.3 Iterative Decoding of the Multicode Scheme

An iterative decoding with message passing algorithm is to be analyzed in this subsection. The message passing (MP) algorithm is an APP algorithm only if the code graph has no cycles, however, this algorithm performs remarkably well on Gallager's codes.

At the receiver side, after obtaining coded information bits contained by the corresponding Hadamard code sequences, the decoding with respect to the whole equivalent Gallager's code is performed by MP algorithm, an instance of sum-product algorithm [29]. The following convention is necessary for description of MP algorithm. $\{x_j\}$ stands for the codeword bits of whole equivalent Gallager's code and N channels output $\boldsymbol{r}$. Let $M(j)$ be the set of parity nodes connected to the code bit x_j, and $N(m)$ be the set of bit nodes connected to the m-th parity check node. $q_{mj}(a)$ is the probability that the bit j of $\boldsymbol{r}$ whose value is given by the information obtained via the M-1 check bits apart from check m. $r_{mj}(a)$ is the probability of check bit m being satisfied by bit j of $\boldsymbol{r}$, if r_j is suppose to be fixed at the value $a\in\{0,1\}$ and the other N-1 bits have a separable distribution given by $q_{mj'}(a): j'\in N(m)\setminus j$. Then the decoding procedure is executed by passing the message (information) between parity check bits and codeword bits, whose probability domain decoding algorithm is shown as follows (see [25] and [29]).

0) Initialization:

The variables q_{mn}^0 are initialized to

for $j = 0,\dots,N$-1

$$q_{mj}(0) = f_i^0 = \frac{1}{\sqrt{2\pi\sigma^2}} \exp^{\frac{(r_j+1)^2}{2\sigma^2}}$$

$$q_{mj}(1) = f_i^1 = \frac{1}{\sqrt{2\pi\sigma^2}} \exp^{\frac{(r_j-1)^2}{2\sigma^2}}$$

end

1) *Check node updates*: (see Figure 4.6)

for $j = 0,\dots,N$-1

for $m \in M(j)$

$$r_{mj}(0) = \frac{1}{2}\{1 + \prod_{i' \in N(m)\setminus\{j\}} [q_{mi'}(0) - q_{mi'}(1)]\}$$

$$r_{mj}(1) = \frac{1}{2}\{1 - \prod_{i' \in N(m)\setminus\{j\}} [q_{mi'}(0) - q_{mi'}(1)]\}$$

end

end

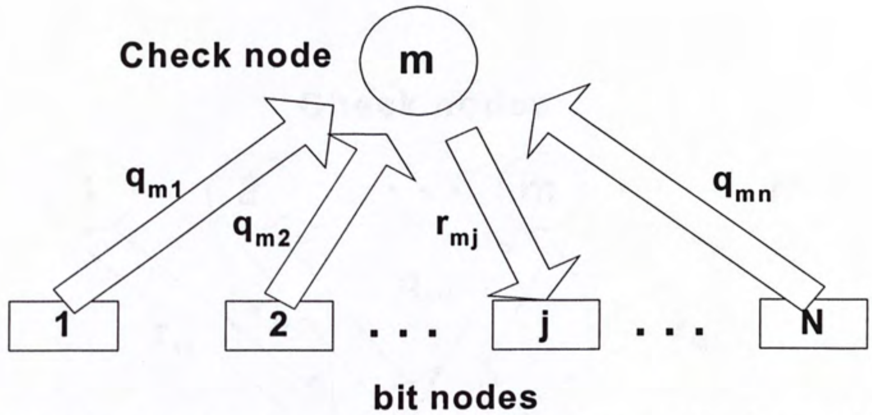


Figure 4.6: (Check node updates) Message passing from check node *m* to bit node *j*

2) *Bit node updates*: (see Figure 4.7)

The constant α and β are chosen to make $q(0)+q(1)=1$.

for *j* = 0,…,*N*-1

$$q_{mj}(0)=\alpha_{mj} f_j^0 \prod_{m' \in M(j)\backslash m} r_{m'j}(0)$$

$$q_{mj}(1)=\beta_{mj} f_j^1 \prod_{m' \in M(j)\backslash m} r_{m'j}(1)$$

end

The q_j's are updated as:

for *j* = 0,…,*N*-1

$$q_j(0)=\alpha_i f_j^0 \prod_{m' \in M(j)} r_{m'j}(0)$$

$$q_j(1)=\beta_i f_j^1 \prod_{m' \in M(j)} r_{m'j}(1)$$

end

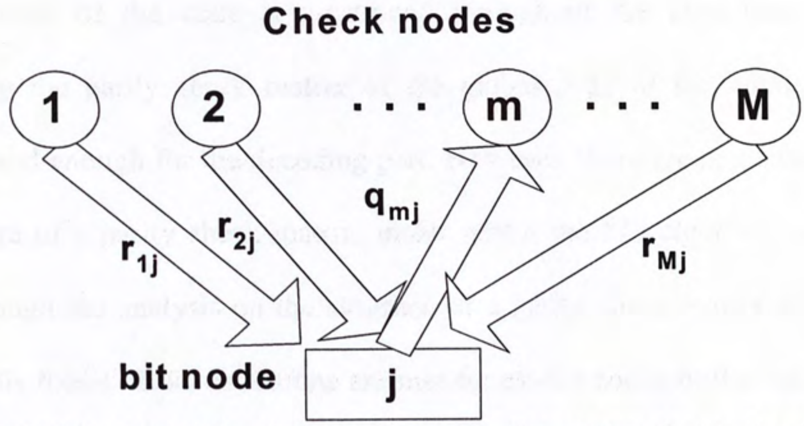


Figure 4.7: (Bit node updates) Message passing from bit node j to check node m

3) *Verify parity checks*

for $j = 0,\ldots,N-1$

    if $q_j(0) > 0.5$ then $\hat{x}_j = 0$

    else $\hat{x}_j = 1$

end

if (finite number of iteration) or ($H^T \hat{x} == 0$)

    done

else

    go to (1)

end

From above, the details of messages shuttling between check nodes and bit nodes are well unveiled. Moreover, a fact is evident that the decoding can be performed only via the parity check matrix involve in the scheme, although the bipartite graph of the code is mentioned throughout the algorithm. Therefore, determining the parity check matrix of the global code of the whole scheme is necessary and enough for the decoding part. However, there are some conditions on the structure of a parity check matrix, under which the MP algorithm can perform well. Although the analysis on the structure of a parity check matrix is beyond the scope of this thesis, these conditions are met for all the codes in this chapter due to their particular structure, which has been proved in their original work.

4.3 Zigzag Codes

Zigzag code as an instance of LDPC code has been proposed in [30]. It can be simply encoded, due to its regular structure. A zigzag code can be represented graphically as shown in Figure 4.8, in which the square nodes stand for the information bits: $\{d(i,j)\}$, $i = 1,2,\cdots,I$, $j = 1,2,\cdots,J$, and the circle nodes represent parity check bits: $\{p(i)\}, i = 1,2,\cdots,I$. The encoding procedure is straightforward. The parity check bits are generated progressively as follows

$$p(1) = \sum_{j=1}^{J} d(1,j) \bmod 2$$

$$p(j) = \sum_{j=1}^{J} d(i,j) + p(i-1) \bmod 2, \quad i = 2,3,\cdots,I. \tag{4.3}$$

The code is systematic with coding rate $J/(J+1)$. It is obvious that the zigzag code is completely parameterized by the pair (I, J). The error-correcting capability of the zigzag code itself is weak since it has a minimum distance $d_{\min} = 2$ for any pair (I, J) [30]. However, the concatenated zigzag code is more powerful and useful, which is considered in our combined multicode scheme.

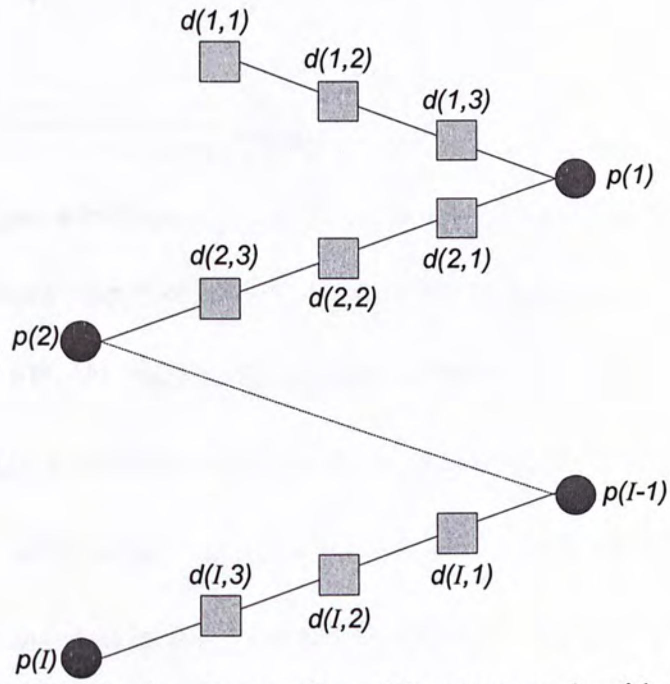


Figure 4.8: Graph representation of the zigzag code with J=3

With zigzag encoder as the precoder, the structure of this combined scheme is illustrated in Figure 4.9, in which parallel concatenated zigzag code with M constituent encoders is used before multicode scheme. The output signal may be rearranged as [output 1, output 2]. Thus, the equivalent coding scheme of whole system is a kind of hybrid-concatenated code. From aspect of the compound coding scheme, the decoding procedure is performed based on the parity check matrix of the compound code, similarly to case of Gallager's code.

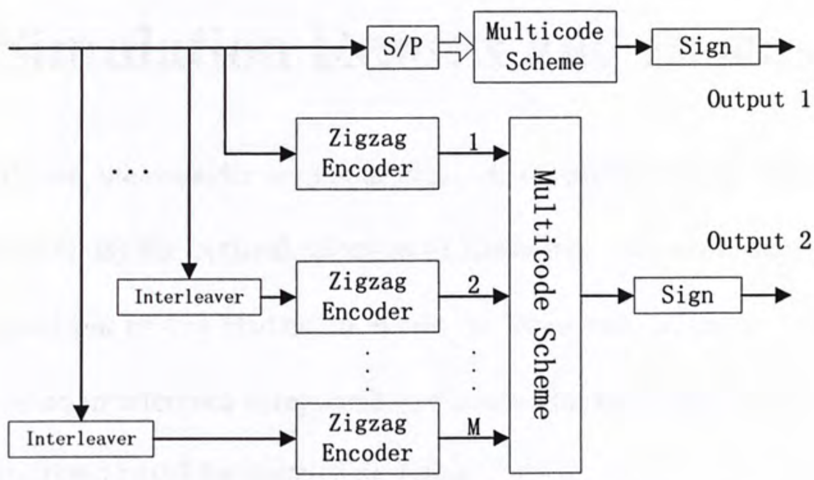


Figure 4.9: Proposed multicode system with zigzag code

If the parity check matrix of (*U*, *V*, *M*) concatenated zigzag code is written as Q, a $(V \cdot M) \times (U \cdot V + V \cdot M)$ matrix, Q can be divided into sub-matrices Q_c and Q_{p_i}, i.e. $Q = [Q_c\ Q_{p_1} ... Q_{p_M}]$, in which Q_c is the matrix with respect to the message bits and Q_{p_i} is the matrix with respect to parity check bits in *i*-th constituent code. If the output bits are arranged as [output 1, output 2], the parity check matrix of compound code is written as

$$\begin{bmatrix} Q_c & 0 & Q_{p_1} & \cdots & Q_{p_M} & 0 \\ T & I_c & 0 & \cdots & 0 & 0 \\ 0 & 0 & I_p & \cdots & I_p & I_p \end{bmatrix}, \tag{4.4}$$

in which *T*, determined by S/P module, contains *M* 1's in each row and one 1 in each column, and I_c is a $\frac{U \cdot V}{M} \times \frac{U \cdot V}{M}$ identity matrix while I_p is a $V \times V$ identity matrix.

Therefore, with the parity check matrix of (4.4), the iterative decoding algorithm can be executed according to the previous subsection.

4.4 Simulation Results and Discussion

In the simulation, we consider M=3 code channels for multicode and AWGN channel for transmission. By the optimal selection of Hadamard code sequence, H_0, H_1, and H_2 are selected out of 4×4 Hadamard matrix for three code channels. In this simple case, there is no interference component in the resultant multicode signal d. Message passing algorithm is used for iterative decoding.

Simulation A: (15,11) Hamming code is applied in the precoder. Its result is shown in Figure 4.10. In proposed scheme, the performance is improved with an increase in the number of iterations However, at BER of 10^{-3}, improvement between 10 and 15 iterations is less than 0.2 dB while more than 1 dB is achieved by 2 iterations over 1 iteration. With 15 iterations, SNR is about 5.3 dB at BER of 10^{-4}. From Figure 4.10, it is found that at BER of 10^{-3} with 15 iterations, the system with precoder achieves more than 1.6 dB over the system without precoder, which applies ML (optimal) detection. Compared with the conventional multicode scheme with identical precoder, the proposed scheme achieves about 1.3 dB at BER of 10^{-3} with 15 iterations.

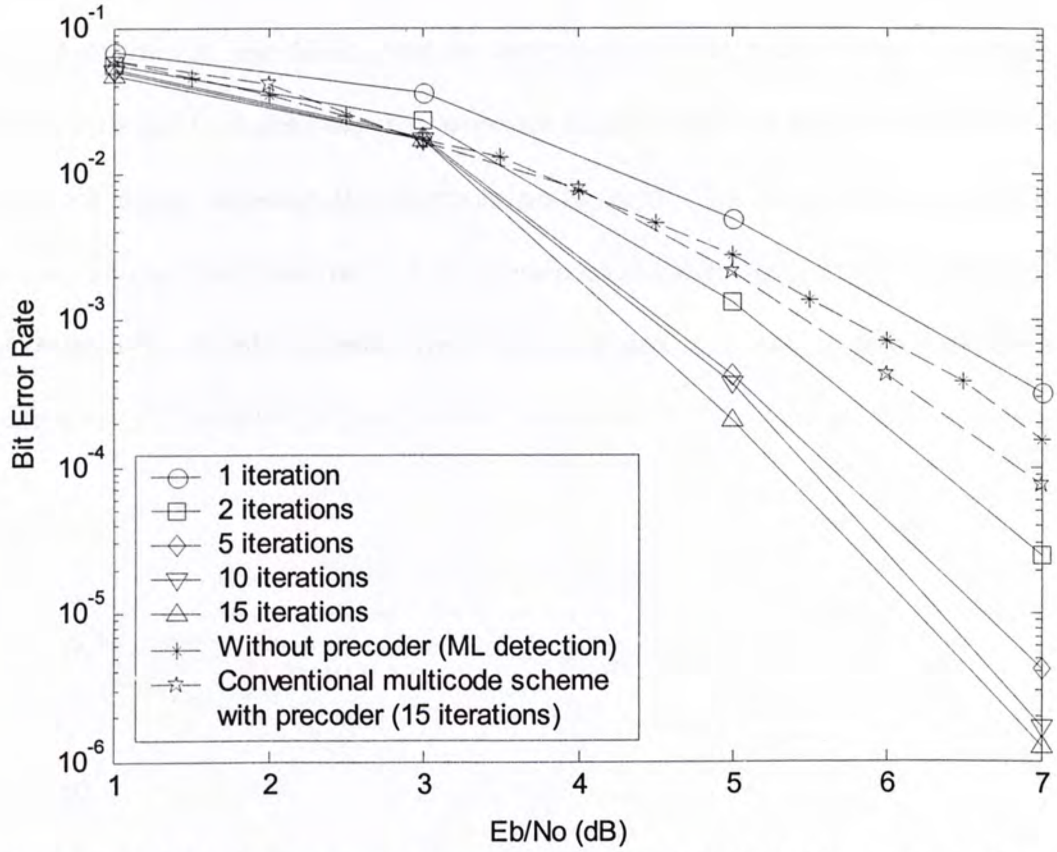


Figure 4.10: BER Performance of proposed system with (15,11) Hamming code using iterative decoding algorithm

Simulation B: (504,252) Gallager's code of coding rate-1/2 is applied in the precoder. Its result is shown in Figure 4.11. The compound code of proposed scheme is (672,420) Gallager's code of coding rate-3/8. In this scheme, the performance is improved with an increase in the number of iterations, however at BER of 10^{-2} improvement between 15 and 20 iterations is less than 0.1 dB while more than 0.9 dB is achieved by 10 iterations over 5 iterations. With 20 iterations, SNR is about 2.6 dB at BER of 10^{-4}. Compared with the system without precoder using ML (optimal) detection, multicode scheme combined with Gallager's code dramatically improves the performance at low SNR. And in comparison of the conventional multicode scheme with identical precoder, the proposed scheme achieves about 0.3 dB at 10^{-4} with 20 iterations. Although the improvement is slightly, the most important point is the signal of combined scheme with ECC has constant amplitude, while the signal of conventional multicode scheme with identical precoder has a large envelope variation.

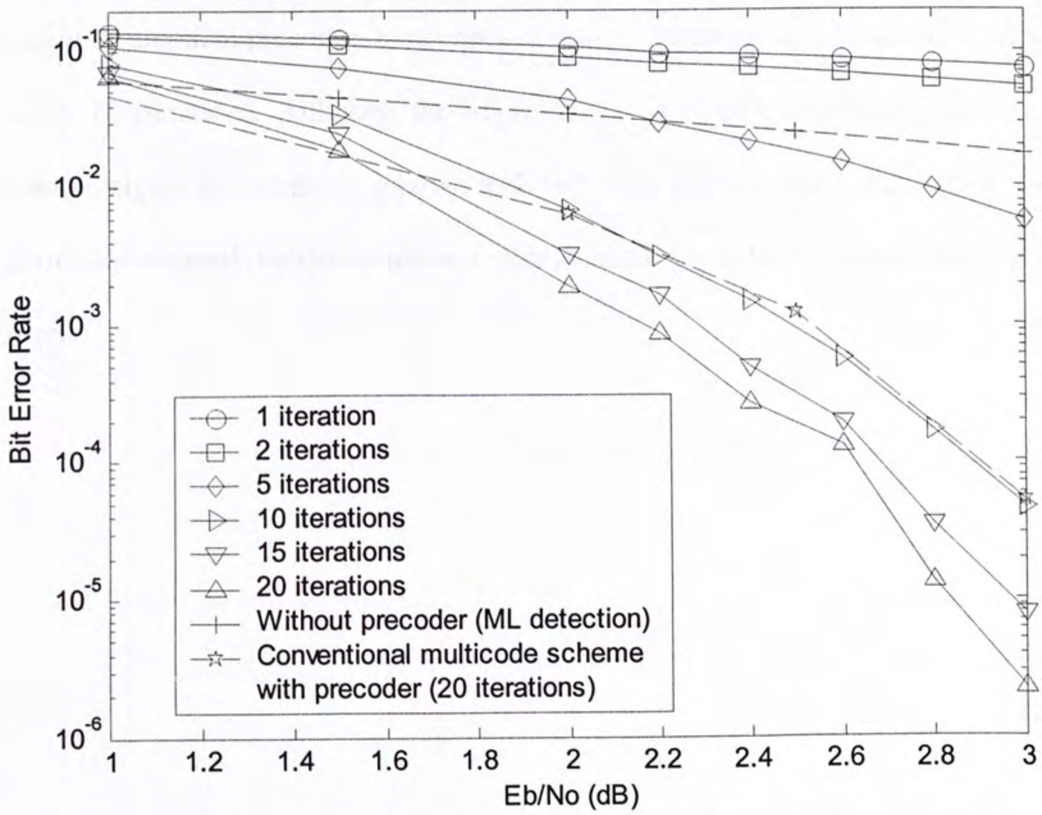


Figure 4.11: BER Performance of proposed system with (504,252) Gallager's code using iterative decoding algorithm

Simulation C: (100,3,3) zigzag code of coding rate-1/2 is applied in the precoder. Its result is shown in Figure 4.12. The coding rate of global equivalent code is 3/8. In this scheme, performance is improved with an increase in the number of iterations, however at BER of 10^{-2} improvement between 15 and 20 iterations is less than 0.1 dB while more than 0.6 dB is achieved by 10 iterations over 5 iterations. With 20 iterations, SNR is about 2.9 dB at BER of 10^{-4}. Compared with the system without precoder using ML (optimal) detection, the system with zigzag coding has a much better performance at low SNR. And compared with the conventional multicode scheme with identical precoder, the proposed scheme achieves more than 0.15 dB at 10^{-3} with 20 iterations. Although the improvement is slightly, the most important point is the signal of combined scheme with ECC has constant amplitude, while the signal of conventional multicode scheme with identical precoder has a large envelope variation.

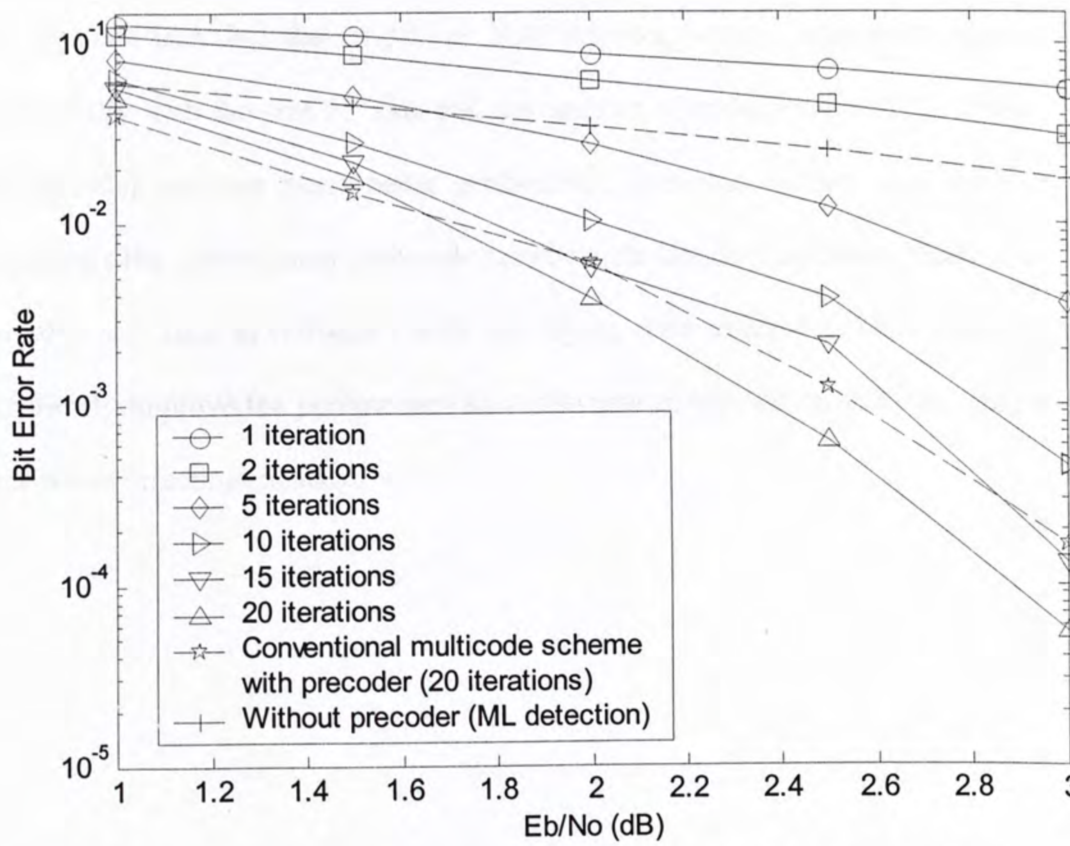


Figure 4.12: BER Performance of proposed system with (100,3,3) zigzag code using iterative decoding algorithm

In this chapter, we have investigated multicode CDMA system with constant envelope transmission combined with error-correcting codes. Hamming code, Gallager's code, and zigzag code are analyzed respectively. Through analysis, it is found that the structure of coding scheme of the combination is equivalent to that of concatenated coding scheme, which guarantees the good performance of whole system. An iterative decoding based on massage passing algorithm is described in details. The complexity of computation of this decoding scheme increases as the number of iteration increases (the performance is getting better), but the complexity level is much less than that of ML or MAP detector, whose complexity increase exponentially with the size of code and the number of code channels. The system with precoder achieves much better performance over that without precoder and outperforms the conventional multicode scheme with identical precoder. Block code with large size such as Gallager's code and zigzag code selected as outer code can dramatically improve the performance since the new compound (global) code has a better error-correcting capacity.

Chapter 5

Multicode CDMA System with Bounded PAP Transmission

In chapter 3, a general scheme of multicode CDMA system with constant amplitude transmission has been proposed and analyzed in details. In that scheme, PAP of multicode signal is 0dB, since the envelope of the signal maintains constant, but the number of code channels is small when the spreading gain (order of Hadamard matrix) is fixed. In this Chapter, two basic ideas on multicode scheme with bounded PAP transmission are introduced. In these schemes, envelope variance of multicode signal does exist, but PAP is small and upper bounded. Therefore, this kind of multicode signal is also suitable for wireless applications.

5.1 Quantized Multicode Scheme

5.1.1 System Model

In the previous multicode scheme with constant amplitude transmission [32], a sign function implements a nonlinear signal processing on the conventional multicode signal. From another point of view, we can consider this kind of signal processing as an extreme quantization on the conventional multicode signal, because it has only two quantization levels {+1, -1}. Naturally, an extreme quantization will cause more interference due to its distortion, so the interference will be less if a loose quantization is used. Now, we consider a quantization algorithm as shown below.

$$d = quanti(Hc) = quanti(s),$$
$$quanti(x) = \begin{cases} 3, & if\ \frac{V}{2} \le x \le V \\ 1, & if\ 0 \le x < \frac{V}{2} \\ -1, & if\ -\frac{V}{2} \le x < 0 \\ -3, & if\ -V \le x < -\frac{V}{2}, \end{cases} \tag{5.1}$$

in which V is the maximum value of signal amplitude. If M code sequences are selected from Hadamard matrix H, the possible maximum value of amplitude is M, and then V can be replaced by M. To better understand this quantization, Figure 5.1 gives an example.

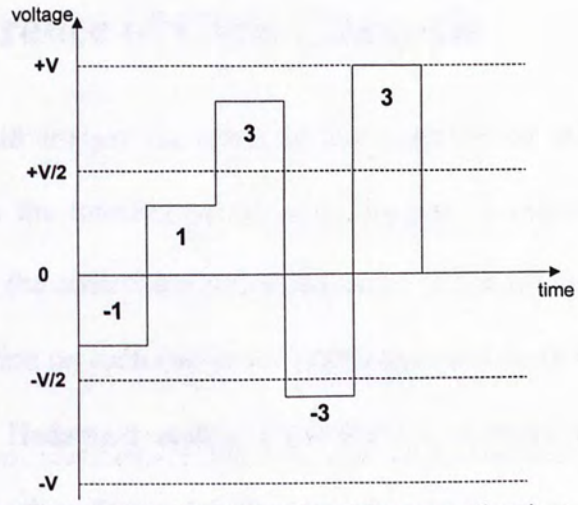


Figure 5.1: An example of the quantization

Sign function has been replaced by this kind of quantization module. The proposed multicode CDMA system should be modified and the new transmitter scheme could be depicted as Figure 5.2, in which $h_1, h_2, ..., h_M$ are the Hadamard code sequences selected from Hadamard matrix.

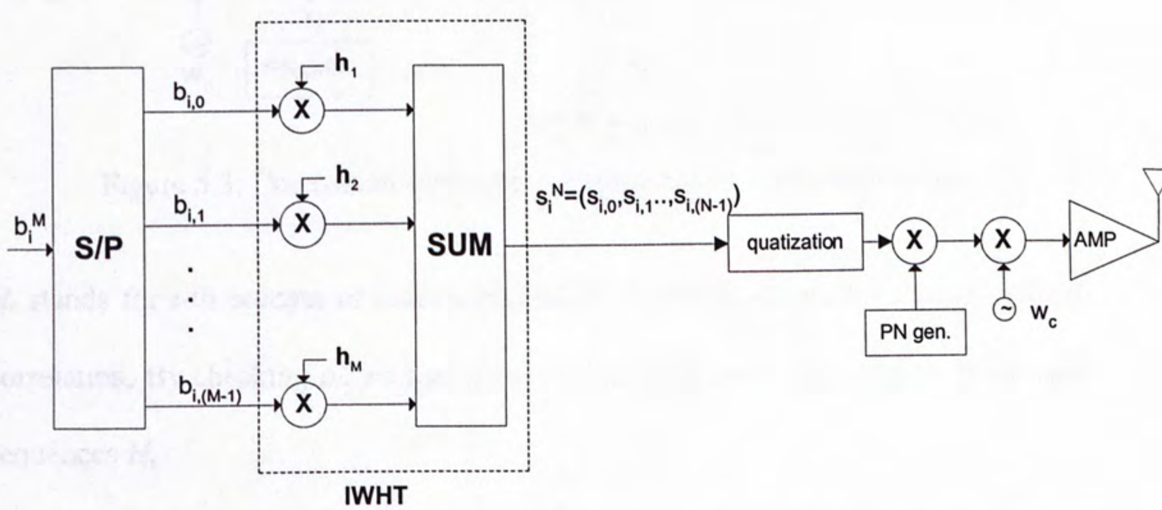


Figure 5.2: Transmitter scheme of quantized multicode CDMA system

5.1.2 Interference of Code Channels

Now we should analyze the effect of this quantization and then present code selection by which the interference of code channels is minimized. Knowing the interference among the code channels introduced by the quantization, we have to find the energy distribution on each Hadamard code sequences H_i (i=0,1,2,…,N-1), where N is the order of Hadamard matrix $(N \geq M)$ by checking every output of the correlation in the receiver. Figure 5.3 illustrates its correlation-type receiver.

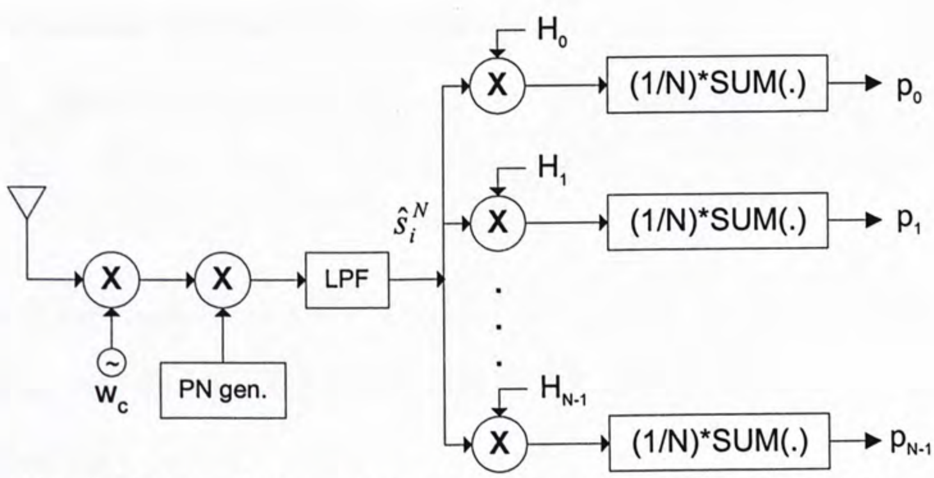


Figure 5.3: Correlation-type receiver of quantized multicode system

H_i stands for i-th column of Hadamard Matrix H, and ρ_i stands for each output of correlation. By checking ρ_i, we can know the energy distribution on each Hadamard sequences H_i.

Assuming that the code selection of multicode scheme with sign function is also valid for this quantized multicode system, we can get some results shown as follows.

Firstly, we choose Rademacher functions R_i as candidate sequences, where

$$R_i = \begin{cases} H_1 & i = 0 \\ H_{2^{i-1}+1} & i = 1,2,\ldots,\log_2 N. \end{cases} \tag{5.2}$$

And there are $\log_2 N + 1$ Rademacher functions in Hadamard matrix of order N.

Now we consider several cases of M=3, 5, and 7, since the code channels of these numbers are possibly used in practice. The interference is measured by the energy distribution on all the possible code channels, which is calculated by collecting all the possible input information bits.

In the case of M=3, the minimum order of Hadamard matrix should be 4. H_1, H_2, and H_3 are selected. The energy distribution is show in Table 5.1.

Table 5.1: The energy distribution of 3 code channels with N=4

$\|\rho_1\|$	$\|\rho_2\|$	$\|\rho_3\|$	$\|\rho_4\|$
1	1	1	0

Because in this case the multicode signal keeps unchanged from the quantization, there is no interference from other unselected Hadamard sequences and no any parity-check bit generated.

In the case of 5 code channels, Hadamard matrix of order N=16 is used. H_1, H_2, H_3, H_5, and H_9 are selected followed the basic idea of code selection. Table 5.2 shows the energy distribution.

Table 5.2: The energy distribution of 5 code channels with N=16

$\|\rho_1\|$	$\|\rho_2\|$	$\|\rho_3\|$	$\|\rho_4\|$	$\|\rho_5\|$	$\|\rho_6\|$	$\|\rho_7\|$	$\|\rho_8\|$
7/8	7/8	7/8	1/8	7/8	1/8	1/8	1/8
$\|\rho_9\|$	$\|\rho_{10}\|$	$\|\rho_{11}\|$	$\|\rho_{12}\|$	$\|\rho_{13}\|$	$\|\rho_{14}\|$	$\|\rho_{15}\|$	$\|\rho_{16}\|$
7/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8

There is little interference from other unselected Hadamard sequences and no any parity-check bit generated. The energy of multicode signal is $5\times\left(\frac{7}{8}\right)^2=3.8281$, while the energy of interference is $11\times\left(\frac{1}{8}\right)^2=0.1719$. Thus, the signal-to-interference ratio is 22.2, or 13.5dB, while signal-to-interference ratio of multicode scheme with sign function is 5.4, or 7.3dB (with parity check bit energy counted into signal energy).

In the case of 7 code channels which is of most interest, Hadamard matrix of order N=64 is used. H_1, H_2, H_3, H_5, H_9, H_{17}, and H_{33} are selected. The energy only distributes on ρ_1, ρ_2, ρ_3, ρ_5, ρ_9, ρ_{17}, and ρ_{33} plus ρ_{64} with absolute value ½. There is no interference from other unselected Hadamard sequences and one parity-check bit generated on code H_{64}. Moreover, we can also check another case of 7 code channels with N=16 instead of N=64. H_1, H_2, H_3, H_5, H_9, H_8, and H_{15} are selected as candidate code sequences. The result shows that no interference from other Hadamard sequences, and there is a parity check bit on H_7. By checking all the sets of Hadamard code sequences (when the order of Hadamard matrix is fixed and N>M), the identical results are presented. Therefore, a general expression for this case can be derived as below:

$$d = \frac{1}{2}\sum_{m=1}^{7} c_m h_m - \frac{1}{2} c_1 c_2 \ldots c_7 h_1 \circ h_2 \circ \ldots \circ h_7 .$$

From this expression, it is obvious that there is no interference and one parity check bit is generated in the case of 7 code channels. For this case, the code selection is much easier, due to the clear structure of the expression.

From checking the specific cases, which are possibly used in the practice, it found that interference among code channels is little or none, so this quantized multicode scheme is feasible and valuable. Moreover, the PAP of the output signal is upper bounded by a certain value due to the same quantization performed on every case. However, general mathematical analysis and optimal selection of Hadamard code sequences are given here, which is left for the future work.

5.2 Parallel Multicode Scheme

5.2.1 System Model

In the multicode scheme with constant amplitude transmissions, the number of code channels is small when the spreading gain (order of Hadamard matrix) is fixed. On other hand, if number of the required code channels is not small, the minimum spreading gain will be very large. To solve this problem, a parallel multicode CDMA system is proposed based on the multicode scheme with sign function, as shown in

Figure 5.4.

For the least envelope variance of resulting multicode signal, three multicode modules with sign function are connected parallel. Therefore, in this scheme, the number of possible code channels is three times of that of pure multicode scheme with sign function when the spreading gain is fixed.

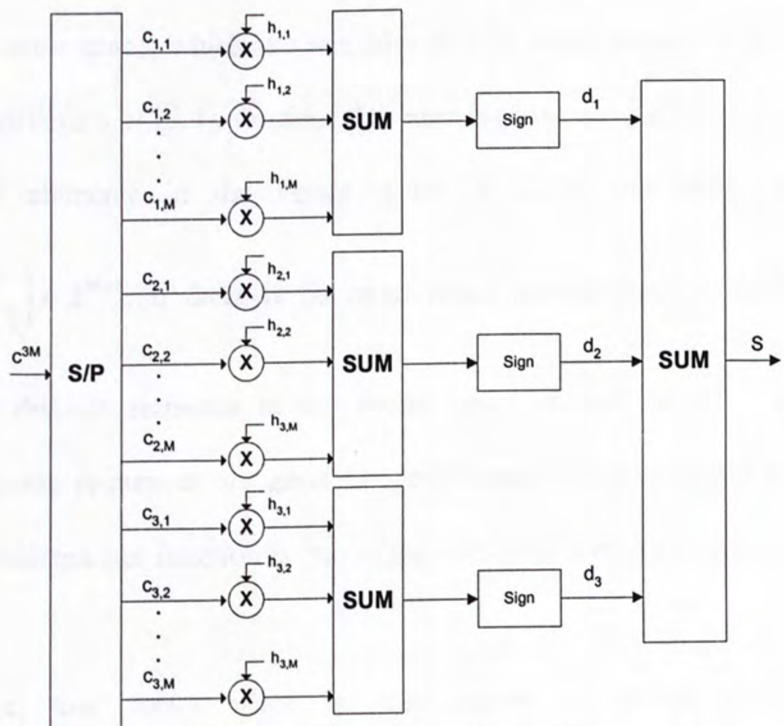


Figure 5.4: Transmitter of the parallel multicode scheme

5.2.2 Selection of Hadamard Code Sequences

A. Optimal Code Selection

By a good code selection for $\{h_m\}$, we hope that there is no interference among these terms in (3.4). By the knowledge of the property of Hadamard matrix, which can be expressed by

$$H_i \circ H_j \circ \ldots \circ H_z = H_{i \oplus j \oplus \ldots \oplus z}, \tag{5.3}$$

in which H_j is the $(j+1)$-*th* column of Hadamard matrix, we can further describe the code selection more explicitly. We can view this set of $b_1, b_2, \ldots, b_M$, a binary form of index of Hadamard sequences selected for $h_1, h_2, \ldots, h_M$, as the basis of a particular vector space, which is spanned with odd combinations of $b_1, b_2, \ldots, b_M$, due to the structure of (3.4). Because the combinations are performed in GF(2), the number of elements in the vector space is finite and upper bounded by $\sum_{i=0}^{(M-1)/2} \binom{M}{2i+1} = 2^{M-1}$. If there is no interference among these terms in (3.4), the number of distinct elements in the vector space should be 2^{M-1} and such M Hadamard code sequences are good for generating multicode signal with constant envelope. Rademacher function is one of the solutions, which is analyzed in Chapter 3.

However, this vector space is only subset of whole vector space of $\log_2 N$-dimension. Thus, there probably exist two or more such vector spaces, whose elements are totally distinct. The scenario is shown below,

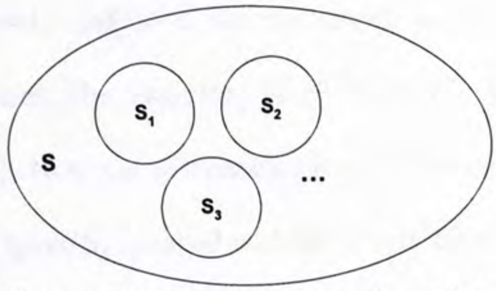


Figure 5.5: A scenario of many non-overlapped vector spaces

In Figure 5.5, S is the whole vector space containing N elements, while S_i is a vector space spanned by one of the subsets chosen from $b_1, b_2, \dots, b_N$,. Each S_i contains 2^{M-1} distinct elements. Since they do not overlap between each other, there is no interference among these sets of M Hadamard code sequences. From this idea, we can use more Hadamard sequences to produce the multicode signal. One possible scheme is that we can select three sets of M Hadamard code sequences such that three vector spaces thus spanned are non-overlapped. Each set is used to produce one multicode signal with constant amplitude, and finally we add these three multicode signals together to form the final resultant multicode signal.

According to Figure 5.4, the input information bits are $c^{3M} = \{c_1, c_2, \dots, c_M, \dots, c_{2M}, \dots, c_{3M}\}$, which are split by S/P module into three vectors $c^{(1)} = \{\mathrm{c}_{1,1}, \mathrm{c}_{1,2}, \dots, \mathrm{c}_{1,\mathrm{M}}\}^T$, $c^{(2)} = \{\mathrm{c}_{2,1}, \mathrm{c}_{2,2}, \dots, \mathrm{c}_{2,\mathrm{M}}\}^T$, and $c^{(3)} = \{\mathrm{c}_{3,1}, \mathrm{c}_{3,2}, \dots, \mathrm{c}_{3,\mathrm{M}}\}^T$. The relationship between c_i in c^{3M} and $c_{j,k}$ in $c^{(j)}$ can be clarified by $i = (j-1) * M + k$. Thus, the final multicode signal s can be expressed as $s = \sum_{j=1}^{3} sign(H^{(j)} \cdot c^{(j)})$, in which $H^{(j)} = \{h_{j,1}, h_{j,2}, \dots, h_{j,M}\}$. For example, we consider the case of N=16 and M=3. We can search for the three sets of M good Hadamard code sequences. The result may be $H^{(1)}=\{H_2, H_3, H_9\}$, $H^{(2)}=\{H_4, H_7, H_{13}\}$, and $H^{(3)}=\{H_1, H_8, H_{14}\}$. Now we will check whether they are good Hadamard code sequences. The vector space S_1 spanned with the binary form of {2, 3, 9} is {2, 3, 9, 12}. The vector space S_2 spanned with the binary form of {4, 7, 13} is {4, 7, 13,10}. The vector space S_3 spanned with the binary form of {1, 8, 14} is {1, 8, 14, 11}. We find the number of elements in each vector space is 4 and three vector spaces are non-overlapped, so they are one of the optimal code selections.

B. Suboptimal Code Selection

For M=3, it is quite easy to find such good Hadamard code sequences, however, if M is larger than 3, i.e. 5, maybe you cannot find solution using the searching method in a tolerant period of time. To solve this problem, we can use the similar algorithm to find suboptimal solutions. In (3.4), terms with ρ_1 and ρ_M have the major energy. So if there is no interference between these two terms and other terms, the case is also a good solution. Now, we can change the scenario in Figure 5.5 into another scenario, which is shown in Figure 5.6.

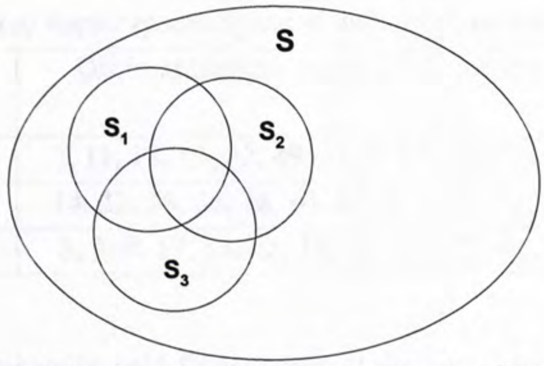


Figure 5.6: A scenario of three overlapped vector spaces

In this scenario, we find that vector spaces S_1, S_2, S_3 have some overlapped area. If the basis, which spans S_i, and the element corresponding to the parity check bit are not located in the overlapped area, we consider this code selection as a suboptimal solution. For example, we consider the case of N=64, M=5. We can also use the searching method to find three sets of M good Hadamard code sequences. In this searching procedure, only the elements corresponding to the information bits and the parity check bit are guaranteed to be out of the overlapped area, so the complexity of searching procedure is lower. The result may be

$$H^{(1)}=\{H_7, H_{11}, H_{19}, H_{13}, H_{35}\},$$

$$H^{(2)}= \{H_{14}, H_{22}, H_{38}, H_{26}, H_{28}\},$$

$$H^{(3)}= \{H_3, H_5, H_9, H_{17}, H_{33}\}.$$

Now we will check whether they are good Hadamard code sequences by Table 5.3.

Table 5.3: Three vector spaces spanned with selected sets of sequences

Index of sequences	Decimal representation of vector space thus spanned
7, 11, 19, 13, 35	**7, 11, 19, 13, 35, 49**, 31, 1, 47, 25, 55, 41, 21, 59, 37, 61
14, 22, 38, 26, 28	**14, 22, 38, 26, 28, 64**, 62, 2, 4, 50, 52, 16, 42, 44, 24, 40
3, 5, 9, 17, 33	**3, 5, 9, 17, 33, 63**, 15, 23, 39, 27, 43, 51, 29, 45, 53, 57

In Table 5.3, the numbers in bold fonts represent the corresponding Hadamard code sequences on which the major energy is distributed. We find that the elements in bold are unique in the whole vector space, so they are surely out of the overlapped area. Therefore, these three sets of Hadamard code sequences are one of suboptimal code selection.

C. Brief Discussion on PAP

Now we consider the PAP of final multicode signal s. For simplicity, the symbols of "-1" and "+1" in c^{3M} are equally probable and M is 3. Letting $r_1 = H^{(1)}c^{(1)}$, we analyze the distribution of r_1. Since "-1" and "+1" in $c^{(1)}$ are equally probable and the values of elements of $H^{(1)}$ are only"-1" and "+1", the distribution of r_1 has nothing to do with the structure of $H^{(1)}$. Thus the distribution of r_1 is shown in Table 5.4.

Table 5.4: The probability distribution of r_1

r_1	-3	-1	1	3
$\Pr\{r_1\}$	1/8	3/8	3/8	1/8

Therefore, d_1 is uniformly distributed on {-1,+1}. Similarly, we know that the probability distribution of s is the same as that of r_1 since probability distributions of r_1, r_2, and r_3 are same. The average power P_{ave} of signal s is $E[|s|^2]$, so $P_{ave} = 1\times 2\times \frac{3}{8} + 9\times 2\times \frac{1}{8} = 3$. And, the potential maximum power P_{max} of signal s is 9. Therefore, we know that PAP is 3 for this multicode signal s.

D. Summary

Although the envelope of multicode signal s is not constant, the envelope variance is small enough for transmission, and PAP is upper bounded. More importantly, Hadamard code sequences are utilized more efficiently than for the pure multicode scheme with sign function. In this scheme, a searching method, which is done by randomly searching possible solution, is used for the code selection, and the results of the cases mentioned above show that there exists a solution. However, a more efficient algorithm is necessary when the case is more complicated, and the performance of this scheme under HPA should be tested in order to evaluate the improvement over pure multicode scheme with sign function. All of these are left in the future work.

Chapter 6

Conclusions and Future Work

6.1 Conclusions

In this thesis, we mainly investigate the multicode CDMA system with constant amplitude transmission. Since large amplitude fluctuation of multicode signal is the major drawback of multicode scheme in wireless applications, the multicode scheme with constant amplitude attracts our interest. Analysis on the previous studies on minimization of the envelope variance of multicode signal give a torch guiding a way to the research on multicode CDMA system with constant amplitude transmission. Of these previous schemes, Wada's scheme is the simplest and efficient scheme. In terms of minimization of envelope variance. However, Wada's scheme is limited in some cases and the data bits are not efficiently used in its receiver. So a general scheme with high performance is necessary. In Chapter 3, a general scheme of multicode CDMA system with constant amplitude transmission is introduced. In this scheme, multicode signal of constant amplitude is realized by a nonlinear operator, sign function. Through analysis, it is found that 3/4 Wada's scheme is a special case of this general scheme. The nonlinear operation on the conventional multicode signal introduces a parity check bit and some interference. To minimize such interference, selection of Hadamard code sequences is proposed based on the property of Hadamard matrix. The simulation results show that the performance of multicode scheme is improved since the parity check bit is used in the signal detection in its ML or MAP receiver.

However, one parity check bit can hardly improved the whole performance greatly, so more efficient utilization of redundant bits attracts our focus. The combination of linear code and this scheme is the direct solution. In Chapter 4, we consider three combination cases of Hamming codes, Gallager's codes, and zigzag codes. From the view of whole combined scheme, global coding is equivalent to another linear coding scheme, serial or hybrid concatenation coding scheme. To efficiently decode the compound code, an iterative decoding method based on coding graph is thus analyzed. The performance of these three combined schemes using the iterative decoding algorithm is evaluated and simulation results show a dramatic improvement.

In multicode scheme with constant amplitude transmissions, the number of code channels is small when the spreading gain (order of Hadamard matrix) is fixed; on other hand, if the number of required code channels is not small, the minimum spreading gain will be very large. To combat this problem, two schemes with bounded PAP transmission are proposed in Chapter 5. Their common highlights are that small envelope variance is allowed, and the PAP of resulting signal is upper bounded.

As a conclusion, the research on the general scheme of multicode CDMA system with constant amplitude transmission proposes a perspective of more powerful multirate CDMA system with multicode scheme.

6.2 Future Work

Firstly, with respect to the schemes proposed in Chapter 5, general mathematical analysis is the major work, which is necessary to understand the mechanism in-depth and improve the schemes. A more efficient code selection is also needed, and the performance of these schemes under nonlinear device, i.e. HPA, should be tested in order to evaluate the improvement over pure multicode scheme with sign function.

Secondly, an idea of concatenated multicode CDMA comes into this research. In the scheme with constant amplitude transmission, the output signal can be viewed as a stream of binary bits, and this property hints a chance of a concatenation of this scheme. An example of this idea is shown in Figure 6.1, in which it is evident that more code channels can be assigned to the user while the final signal has no envelope variance. In the standalone multicode scheme with constant amplitude transmission, by the optimal code selection, at most $\log_2 N + 1$ channels can be used when the spreading gain is N, namely the order of the Hadamard matrix. However, in this concatenated multicode scheme, at most $3\cdot\left(\frac{1}{2}\cdot\log_2 N+1\right)$ code channels can be used when the spreading gain is N and the order of the Hadamard matrix in each module is $\sqrt{N}$. For example, If N=16, the number of code channels can be assigned is 9 in the concatenated multicode scheme while the number is 5 in the previous multicode scheme. Moreover, the Hadamard code sequences can be reused without causing any interference. Therefore, the underlying structure of the concatenated multicode scheme is an interesting point. If one multicode scheme is considered as a coding scheme, the global code of this whole scheme is not a simple concatenated codes but a nonlinear code. How to efficiently decode this global code is a question to be answered, and an efficient receiver structure is important to be proposed in a further work.

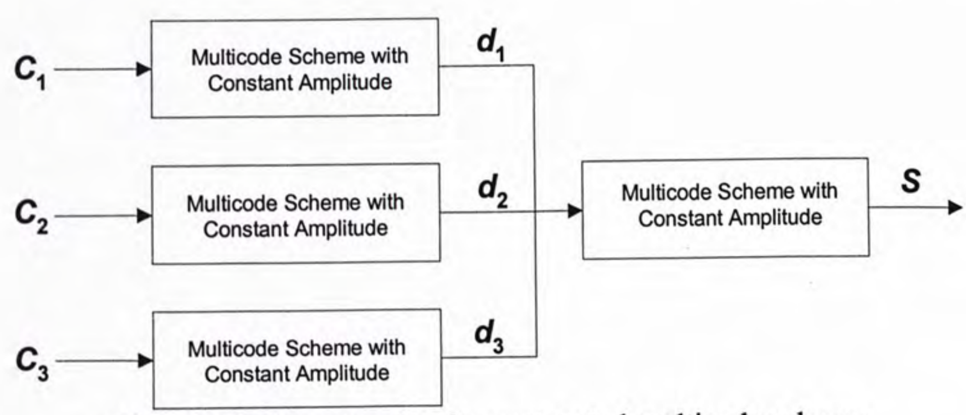


Figure 6.1: An example of concatenated multicode scheme

Looking at the whole multicode scheme, there is still a lot of work in the future. The multicode schemes proposed in this thesis should be investigated in other more complicated scenarios such as multi-user environment and multi-path channel. More aspects of multicode scheme will be considered in the future work and further improvements are expected under these considerations.

Bibliography

[1] F. Adachi, M. Sawahashi and H. Suda, "Wideband CDMA for Next-Generation Mobile Communications Systems", *IEEE Communications Magazine*, Vol. 36, pp. 56-69, Sept. 1998.

[2] E. Dahlman, B. Gudmundson, M. Nilsson, and J. Skold, "UMTS/IMT-2000 Based on Wideband CDMA", *IEEE Communications Magazine*, Vol. 36, pp.70-80, Sept. 1998.

[3] T. Ojanperä and R. Prasad, *Wideband CDMA for Third-Generation Mobile Communications*, Boston: Artech House, 1998.

[4] C -L. I and K. K. Sabnani, "Variable Spreading Gain CDMA with Adaptive Control for True Packet Switching Wireless Network," *Proc. ICC 95, Seattle*, Vol. 2, pp. 725-730, 1995.

[5] C -L. I and K. K. Sabnani, "Variable Spreading Gain CDMA with Adaptive Control for Integrated Traffic in Wireless Network," *Proc. VTC 95, Chicago*, Vol. 2, pp. 794-798, 1995.

[6] C -L. I and R. D. Gitlin, "Multi-Code CDMA Wireless Personal Communications Networks", *Proc. ICC 95, Seattle*, Vol. 2, pp. 1060-1064, 1995.

[7] K. B. Letaief, J. C. -I. Chuang, and R. D. Murch, "A High-Speed Transmission Method for Wireless Personal Communications", *Wireless Personal Communications*, Vol. 3, pp. 299-317, 1996.

[8] D. T. Harvatin and R. E. Ziemer, "Multi-rate Modulation Scheme with Controlled Peak-to-Average Power Ratio Using Balanced Incomplete Block Designs", *IEEE International Conference on Communications (ICC 2001)*, Vol. 4, pp. 1028 –1032, 2001.

[9] T. Ottosson and A. Svensson, "On Schemes for Multirate support in DS-CDMA Systems", *Wireless Personal Communications*, Vol. 6 pp. 265-287, 1998.

[10] R. E. Ziemer and D. T. Harvatin, "An Overview and Characterization of Multirate Schemes for Future-Generation Wireless Systems", *IEEE Sixth International Symp. on Spread Spectrum Tech. and Applic.*, Vol. 2 , pp.

550 –554, 2000.

[11] T. S. Rappaport, *Wireless Communications: Principles & Practice*, Englewood Cliffs, NJ: Prentice-Hall, 1996.

[12] F. Adachi, M. Sawahashi and K. Okawa, "Tree-structured Generation of Orthogonal Spreading Codes with Different Lengths for Forward Link of DS-CDMA Mobile Radio", *Electronics Letters*, vol. 33, No. 1, Jan. 1997.

[13] P. Kuganesan and K. B. Letaief, "Multicode Modulation for High Speed Wireless Data Transmission", *Proc. PIMRC'97*, Vol. 2, pp. 457-461, Sept. 1997.

[14] P. Kuganesan, K. B. Letaief and Y. Chen, "Performance Comparison of Various Multicode Modulation Schemes for High-Rate Wireless Data Transmission", *Wireless Personal Communications*, Vol. 13, pp. 257-277, 2000.

[15] T. Wada, T. Yamazato, M. Katayama, and A. Ogawa, "A Constant Amplitude Coding for Orthogonal Multi-Code CDMA Systems", IEICE Trans. Fundamental, Vol. E80-A, No. 12, Dec. 1997

[16] D. I. Kim and V. K. Bhargava, "Performance of Multidimensional Multicode DS-CDMA Using Code Diversity and Error Detection", IEEE Trans. Commun., Vol .49, No. 5, May 2001

[17] D. I. Kim, "Multidimensional Signaling & Per-Symbol Detection for High Data Rate DS-CDMA Systems", *11th IEEE International Symp. on Personal, Indoor and Mobile Radio Communications (PIMRC 2000)*, Vol. 2, pp. 1390–1394, 2000

[18] T. Ottosson, "Precoding for Minimization of Envelope Variations in Multicode DS- CDMA systems", *Wireless Personal Communications*, Vol. 13 pp. 57-78, 2000

[19] V. P. Ipatov, Y. A. Kolomensky, and R. N. Shabalin, "Reception of Majority-Multiplexing Signals", *Radio Engineering and Electronic Physics*, Vol. 20, No. 4, pp. 121-124, 1975.

[20] N. Ahmed and K. R. Rao, *Orthogonal Transforms for Digital Signal Processing*, Springer-Verlag, 1975.

[21] R. Neil Braithwaite, "Using Walsh code Selection to Reduce the Power Variance of Band-Limited Forward-Link CDMA Waveforms", *IEEE Journal on Selected Areas of Commun.*, Vol. 18, No. 11, pp. 2260-2269, Nov. 2000.

[22] K. G. Beauchamp, *Walsh Functions and Their Applications*, Academic Press, 1975.

[23] K. W. Henderson, "Some Notes on the Walsh Functions," *IEEE Trans. Electronic Computers (correspondence)*, Vol. 13, pp. 50-52, February 1964.

[24] J. G. Proakis, *Digital Communications*, New York: McGraw-Hill, 1989.

[25] S. M. Aji and R. J. McEliece, "A General Algorithm for Distributing Information in a Graph", *IEEE International Symposium on Information Theory (29 Jun-4 Jul 1997)*, pp.6, 1997.

[26] R. M. Tanner, "A Recursive Approach to Low Complexity Codes", *IEEE Trans. Inform. Theory*, Vol. IT-27, No. 5, pp. 533-547, Sept. 1981.

[27] R. G. Gallager, *Low-Density Parity-Check Codes*. Cambridge, MA: MIT Press, 1963.

[28] D. J. C. MacKay, "Good Error-Correcting Codes Based on Very Sparse Matrices", *IEEE Trans. Inform. Theory*, Vol. 45, No. 2, pp. 399-431, Mar. 1999.

[29] F. R. Kschischang, B. J. Frey, and H. -A. Loeliger, "Factor Graphs and the Sum-Product Algorithm", *IEEE Trans. Inform. Theory*, Vol. 47, No. 2, pp. 498-519, Feb. 2001.

[30] L. Ping, X. Huang and N. Phamdo, "Zigzag Codes and Concatenated Zigzag Codes", *IEEE Trans. Inform. Theory*, Vol. 47, No. 2, pp. 800-807, Feb. 2001.

[31] L. J. Cimini, Jr. and N. R. Sollenberger, "Peak-to-Average Power Ratio Reduction of an OFDM Signal Using Partial Transmit Sequences", *IEEE Communications Letters*, Vol. 4, Issue. 3, pp. 86-88, Mar. 2000.

[32] T. M. Lok, K. T. Tan, and K. K. Leung, "A Practical Non-Linear Direct Sequence CDMA System with Constant Envelope Signaling,"preprint.

[33] T. J. Richardson and R. L. Urbanke, "Efficient Encoding of Low-Density Parity-Check Codes", *IEEE Trans. Inform. Theory*, Vol. 47, No. 2, pp. 638-656, Feb. 2001

CUHK Libraries
004076636