Gossip Mechanisms for Distributed Database Systems

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Gossip Mechanisms for Distributed Database Systems

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for the Degree of Master of Philosophy
in the Department of Computer Science and Engineering, CUHK

Abstract

The research project studies various gossip mechanisms and their abilities to solve common network database system problems. Novel *two-tier gossip mechanisms*, which in general refer to a gossip mechanisms that adjust the gossipee-selection criteria for once as the communication algorithm proceeds, are proposed and studied. In particular, it can solve the Clustered Destination Problem within time $\log_{2-\delta}(n/b)+\log_{2-\delta}\ln\ln(n/b)+O(\lg^{1+\varepsilon}(2r))$, where n is the number of nodes in the network, b and r are the number of nodes and radius of any moderately-sized bounding hypersphere for the cluster respectively, δ and ε are arbitrarily small positive constants. Moreover, the destinations are informed within a "short" period of time (a notion to be made precise in this thesis) involving small communication distances, both of which cannot be achieved with uniform gossip techniques. The second result is for data dissemination for the entire network, which can be accomplished within time $\lg(n/b)+\ln(n/b)+O(\lg^{1+\varepsilon}(2r))$. Several two-tier and n-tier gossip mechanisms are also evaluated in situations in which only a limited number of gossip rounds are allowed.

Besides two-tier gossip mechanisms, *semantic-dependent gossip mechanisms* are also studied. Semantic-dependent gossip mechanisms are probabilistic communication paradigms in which the selection of communication peers depends on information obtained from the application layer. Sensor networks that detect environmental temperature are mainly studied and the efficiency of locating heat sources with the gossip mechanisms is presented. Distance from the heat source(s) can be estimated with the help of the local temperature detected, and the two-tier technique, which allows a gossip mechanism to switch a gossipee-selection scheme that tends to select nearby nodes, is employed for further improvement. With uniform gossip as the benchmark, an overall improvement in both time and communication distances required is observed.

Gossip Mechanisms for Distributed Database Systems

網絡數據庫系統的流言傳播方法

任承忠

摘要

這研究計劃旨在調查不同“流言傳播方法”的特性，及它們處理常見網絡資料庫系統問題的能力。我們提出並研究能在傳播過程中調整一次選擇通訊對象的條件的“二階流言傳播方法”。它能以$\log_{2-\delta}(n/b)+\log_{2-\delta}\ln\ln(n/b)+O(\lg^{1+\varepsilon}(2r))$的時間將資料從一個地點送到一組地點，其中$n$是網絡的通訊地點數目、$b$和$r$分別是能包含接收地點組的任一適當大小的球體的大小及半徑、δ和ε是任意小的數值。而且，它能在一“短”時間內（本文將會解釋“短”時間的定義）通知所有接收地點，而這兩者皆是“均勻流言傳播方法”所不能辦到的。另一個結論是它能以$\lg(n/b)+\ln(n/b)+O(\lg^{1+\varepsilon}(2r))$的時間以高機會率通知每一個網絡地點。不同的二階及多階流言傳播方法在有限的回數中的能力亦是此研究計劃的對象。

除二階流言傳播方法外，“取決意義流言傳播方法”是另一個研究目標。它們會以網絡上層所收集到的資料來決定選擇通訊對象的方法。我們對它在偵察溫度的網絡上尋找發熱源頭的能力作了研究工作。偵察到的溫度能用來推測發熱源頭的的距離以增加成功率，然後配合二階流言傳播方法再加以改良。相比於均勻流言傳播方法，取決意義流言傳播方法能減低所需的時間及通訊距離。

Acknowledgement

I would like to take this opportunity to thank my supervisor, Professor Man-Hon Wong, for his invaluable opinions to my research directions, as well as insights for the research project. Without his help this research project would not have been completed.

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Section 1 Introduction

Communication mechanisms, which specify how information is disseminated in a network, are an area of intense research. A particular type of communication mechanism studied in this research project is the *gossip mechanism*, which involves informed nodes passing on a piece of information to randomly-selected nodes. The number of informed nodes (informed nodes) initially increases exponentially, and the nodes are "infected" as in an epidemic; hence, a gossip mechanism is also known as an "epidemic algorithm."

In this research project we consider a network in which each node can directly communicate with another node selected according to predefined probability distribution functions. The network nodes can communicate with others only according to these probability distribution functions; hence, their communication capability is limited.

The research project investigates how gossip mechanisms perform when confronting specific problems. The main problems we studied are the Clustered Destination Problem and location detection of heat sources. The Clustered Destination Problem asks for data dissemination from one node, known as the source node, to a set of nodes, known as the destination nodes, with high probability (w.h.p.)[1]. The result is also extended to inform any node in the whole network instead of only a set of destinations. The heat location detection problem asks a network to find all network nodes that are above a given

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[1] An event occurs "with high probability" if the probability of occurrence $= 1 - O((\ln n)^{-x})$, where $x > 0$, for a network with n nodes. The phrase "w.h.p." is used throughout this thesis.

temperature, where the network nodes are placed in an environment in which one or more heat sources are present.

1.1 Motivation

The research project proposes two novel classes of gossip mechanisms — *two-tier* and *semantic-dependent*.

The proposed *two-tier* gossip mechanism allows nodes to adjust the probability density distribution from which gossipees are chosen once. Gossip mechanisms that are not two-tier have been well-studied over the years. Uniform gossip [27] and spatial gossip [15] are *one-tier* in the sense that their gossipee-selection scheme never changes throughout the communication process. The classical time bound using uniform gossip for network broadcasts is $\lg n + \ln n + O(1)$ with high probability (w.h.p.) [27].

The idea of the proposed gossip mechanism is to initially inform as many nodes as possible across the whole network, and then to "intensify" by switching to a gossipee-selection scheme that tends to select nodes near to the gossipers. This intensification allows a thorough search within a cluster, while the first phase allows the cluster to be located efficiently. Using this method enables the Clustered Destination Problem to be solved within $\log_{2-\delta}(n/b) + \log_{2-\delta} \ln \ln(n/b) + O(\lg^{1+\varepsilon}(2r))$ rounds, where b is the number of nodes in a bounding hypersphere of the clustered destination, r is the hypersphere's radius, and δ and ε are arbitrarily small positive constants. The time bound depends on the hypersphere's size and, hence, the destination. Because of this, the time can be optimized by choosing a suitable hypersphere. This is contrary to classical methods, which employ network broadcasts regardless of the destination size. Extending this notion shows that, instead of only a set of clustered destinations, any node in a given network can be informed in time $\lg(n/b)+\ln(n/b)+O(\lg^{1+\varepsilon}(2r))$ with high probability.

Another problem we investigated is the location detection of heat sources. Consider an environment with one or more heat sources. Static temperature sensors are deployed in this environment, forming a sensor network in which nodes can communicate with each other. The temperature detected by the nodes around these heat sources is higher, while other nodes are cooler. A typical query might involve finding the network nodes with a temperature above a given value.

A classical solution to this problem employs uniform gossip to spread a query message from informed nodes ("gossipers") to randomly-selected network nodes ("gossipees"), which takes place in rounds and terminates when the desired nodes are found. Consider the example in Figure 1. The uniform gossip mechanism is initiated by node 1, and in the first round of gossip this node randomly passes the query message to another node (node 2 in this case) according to a uniform distribution (hence the name "uniform gossip") over all the network's nodes. In the second round, each node again randomly passes the message to a randomly-selected node, which are nodes 3 and 4. The arrows show that the messages are to be passed to the next gossipees chosen in the third round. This goes on until the hot nodes receive the message. In the worst case, nearly all nodes have to be informed before the hot nodes are found. A scenario using uniform gossip to find hot nodes is shown in Figure 1.

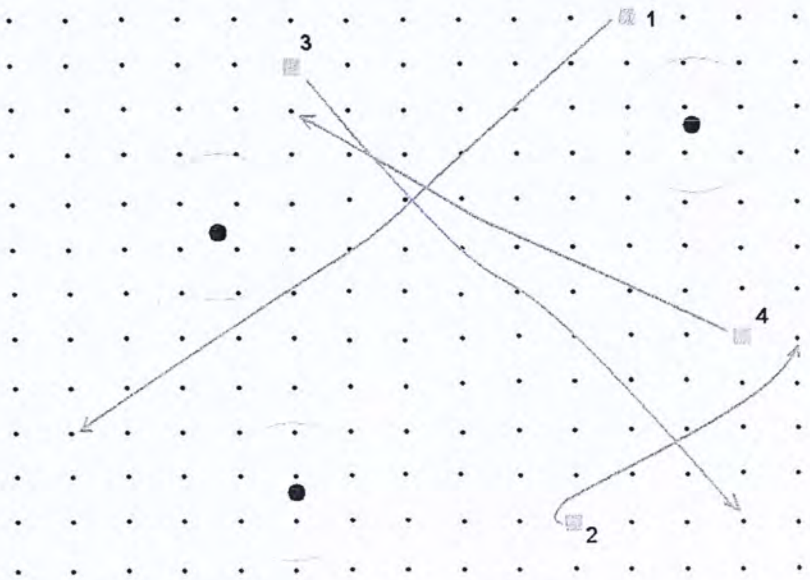


Figure 1: Locating hot nodes with Uniform Gossip

Solid circles are heat sources and their surrounding nodes have a high temperature. Black dots are uninformed network nodes, and rectangles are informed nodes (gossipers). The arrows point to the gossipers' randomly-selected gossipees.

The above method works but is inefficient. First, it does not utilize the nodes' sensor values at all. Second, the nodes continue to randomly select gossipees, which can be far away, even when it is indicated that the node(s) desired to be found might be nearby. To overcome this inefficiency, our proposed semantic-dependent gossip mechanism uses locally detected sensor values to estimate the distance from the heat sources, increasing the probability of locating hot nodes. This mechanism can also be combined with two-tier gossip mechanisms to yield better results than either would yield alone. Figure 2 shows a scenario using our proposed gossip mechanism to find hot nodes, in which nearby nodes are chosen when there is an indication that a heat source is nearby.

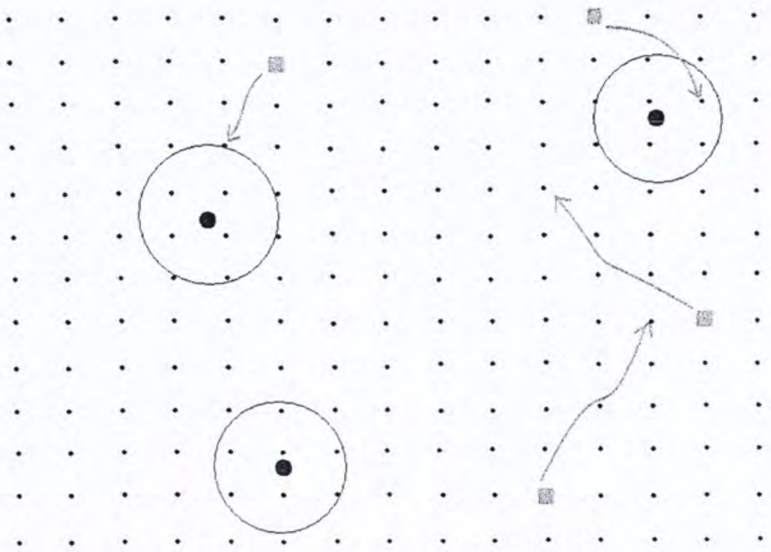


Figure 2: Locating hot nodes with the Semantic-dependent Gossip Mechanism

1.2 Thesis Organization

This thesis is organized as follows. Section 2 is a literature review of state-of-the-art technology in various communication models and applications, with a focus on gossip mechanisms and related communication paradigms. This section presents basic definitions and examples of gossip mechanisms.

Section 3 gives preliminary knowledge of all the network and computational models that are being studied and, in particular, introduces the two-tier and semantic-dependent gossip mechanisms. This section also explains gossipee-selection schemes and presents some common examples.

Section 4 presents some new results concerning two-tier gossip mechanisms, and specifies the network model defined in Section 3, on which the results of this research are based.

Section 5 presents the results of the semantic-dependent gossip mechanism.

Section 6 concludes the results and presents possible research directions.

Section 2 Literature Review

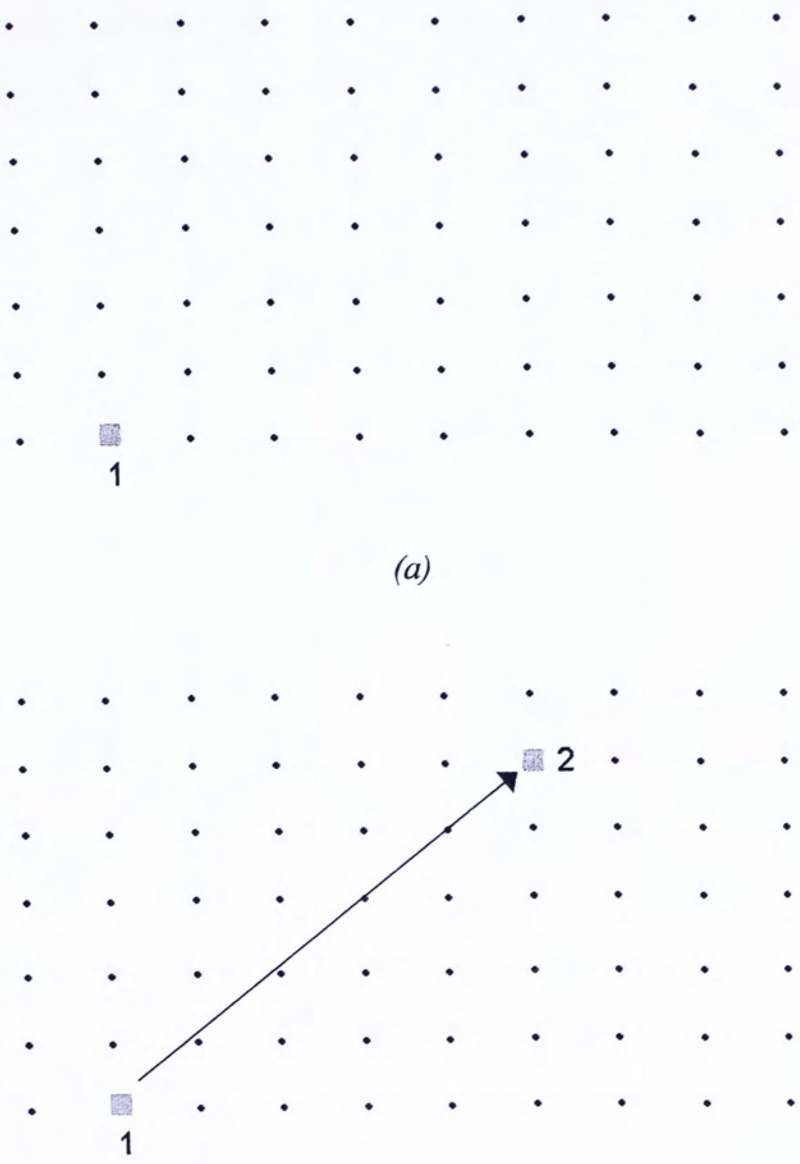
Deterministic algorithms that compute aggregate values or capture snapshots have long been investigated; for example, Chandy's distributed snapshot algorithm [3]. These algorithms were designed to achieve perfect accuracy but would apply only to networks providing a considerable quality of services (e.g., FIFO, reliable network layer, no topological changes).

For networks that are unreliable and that have rapid and frequent topological changes, probabilistic algorithms such as gossip (or a gossip mechanism) have been proposed. Gossip is a simple communication mechanism that has a wide variety of applications. This section briefly reviews various gossip mechanisms, explains where are they applied, and discusses some problems that they can solve as proposed in the literature.

2.1 Data Sharing and Dissemination

Gossip mechanisms can be used in a wide variety of contexts. In the case of information dissemination, a node initially holds a piece of information and would like to spread this information to one, some or all nodes in the network, depending on the nature of the problem being considered. The nature of this piece of information is irrelevant—it can be a message that the destination nodes need, a query message that is injected into the network or any other application data. This node would randomly choose a number of peer nodes (gossipees) for communication. In each subsequent round the nodes that hold the information choose other nodes to pass on the information to, and eventually the requests/data are spread to the whole network in an epidemic fashion. The informed nodes are called "gossipers," and the nodes they choose to communicate with in each round are "gossipees." Hence the mechanism starts with one gossiper and ends when all the destination nodes have become gossipers. The term "gossip mechanism" actually encompasses a large class of communication mechanisms, with one common concept—probabilistic selection of communication peers to minimize the adverse effects of the hostile network conditions typical in unreliable networks.

An example of data dissemination with uniform gossip is shown in Figure 1. As shown in Figure 3(a), node 1 initially holds a piece of information. In round 1, node 1 randomly chooses another node, node 2 in this case, and passes the information to it, as shown in Figure 3(b). In round 2, node 1 chooses node 3 and node 2 chooses node 4; hence, after this round there are four gossipers, as shown in Figure 3(c). In round 3, nodes 1, 2, 3 and 4 choose nodes 5, 6, 7 and 8, respectively, and pass on the information to them as shown in Figure 3(d). This process is repeated until the required destinations receive the information. Note that because the probability of choosing a node which has already received the information increases with the number of informed nodes in the network, the rate of increase in the number of gossipers eventually falls as the algorithm proceeds.



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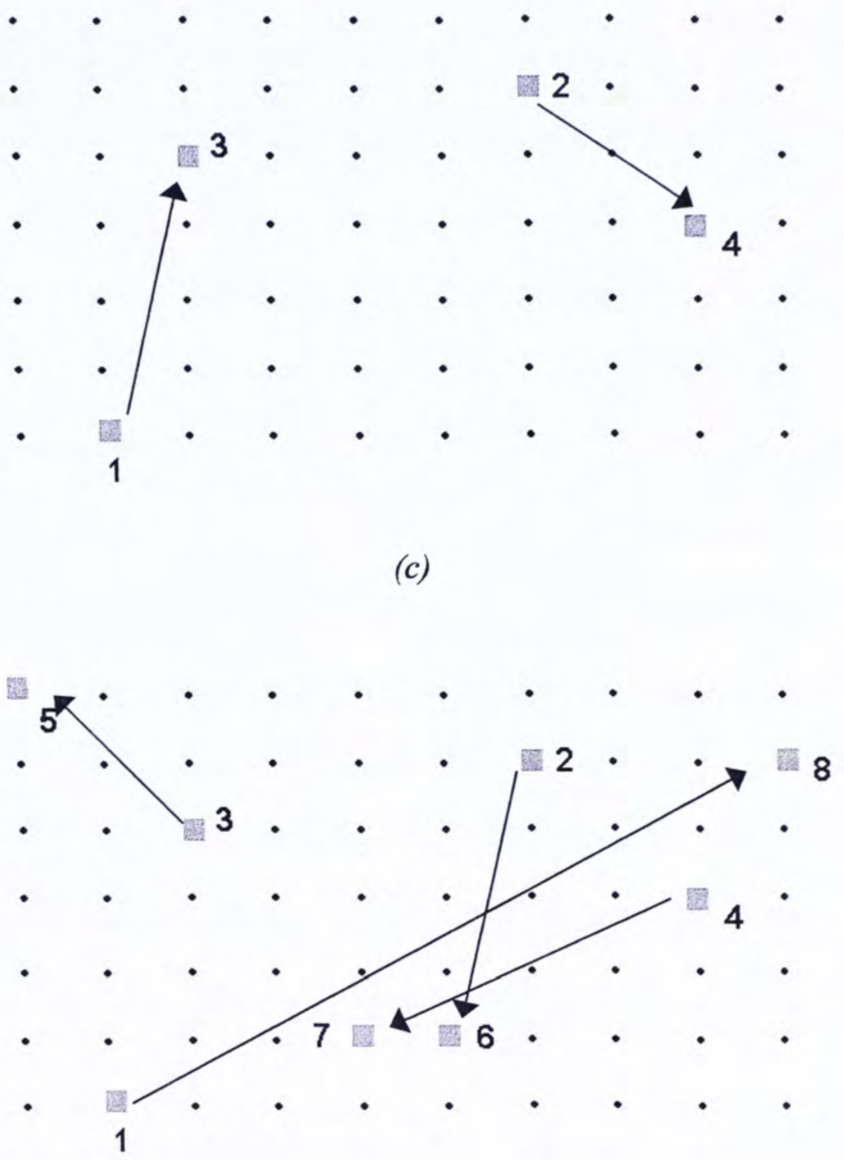

(c)

(d)

Figure 3: Data dissemination with uniform gossip[2]

(a) At round 0, there is only one gossiper

(b) Round 1

(c) Round 2

(d) Round 3

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[2] In these examples, the physical location of the network nodes is irrelevant. Hence it is not a problem to have the nodes placed as grids, randomly, or in any other fashions.

An extensive study of gossip mechanisms that rely on the distance between nodes can be found in [15, 16]. These two papers investigate *spatial gossip* with exponent ρ, in which gossipees that are distance d away from the gossipers are selected with probability proportional to $d^{-\rho D}$, where D is the network's dimensionality.

Network nodes reach no consensus as to which gossipees are to be chosen because gossip is a completely distributed communication mechanism. No centralized server(s) or even regional leaders guide the dissemination process. There is always a chance (usually slim, though) that some nodes might not receive the information being disseminated around the network after an undesirably long period of time. Hence the "with high probability" criterion is common in the analysis of gossip mechanisms. The time bounds for gossip mechanisms usually take the form of the time required to satisfy certain conditions, for example, destination nodes receiving the desired information with high probability.

Many forms of data sharing and finding also employs gossip, for example, in [5, 8, 12, 13, 23, 24, 25, 28, 29]. These studies focus on different network conditions and assumptions of those networks' capabilities. They also explore or rely on various aspects of gossip to solve network problems, such as data downloading [24] and finding, propagation of updates [5], resource availability information [8], periodically finding available resources, and so on. Gossiping is also used to overcome Byzantine failures in data dissemination processes [7]. On the other hand, a number of probabilistic multicast algorithms make use of gossiping [2, 20, 26].

2.2 Data Aggregation

Gossip can be used to collect data from a network in which nodes hold specific values. For example, consider computing data aggregations, e.g., finding MAX, MIN, AVG, SUM or other functions of values that are stored in network nodes. A network's ability to aggregate data is highly important if energy conservation is a concern. Without it the network nodes would need to forward a large amount of data to the control station.

Many data aggregation algorithms have been proposed. The Push-Sum Protocol [17] uses uniform gossip (i.e., choosing a gossipee from a uniform distribution of all the nodes) and local computation based on each node's received message to allow all nodes to obtain aggregate values such as SUM, MAX, MIN and AVG. The same paper also proves that the values converge to the true values exponentially fast.

Gupta's Grid Box Hierarchy approach enabled the design of algorithms that divide nodes into a hierarchical structure [10]. This approach arranges nodes in grid boxes of various sizes in a hierarchical manner. Gossip messages are first exchanged among nodes within the smallest grid boxes, then in larger ones and finally, the aggregate values of the whole network are obtained.

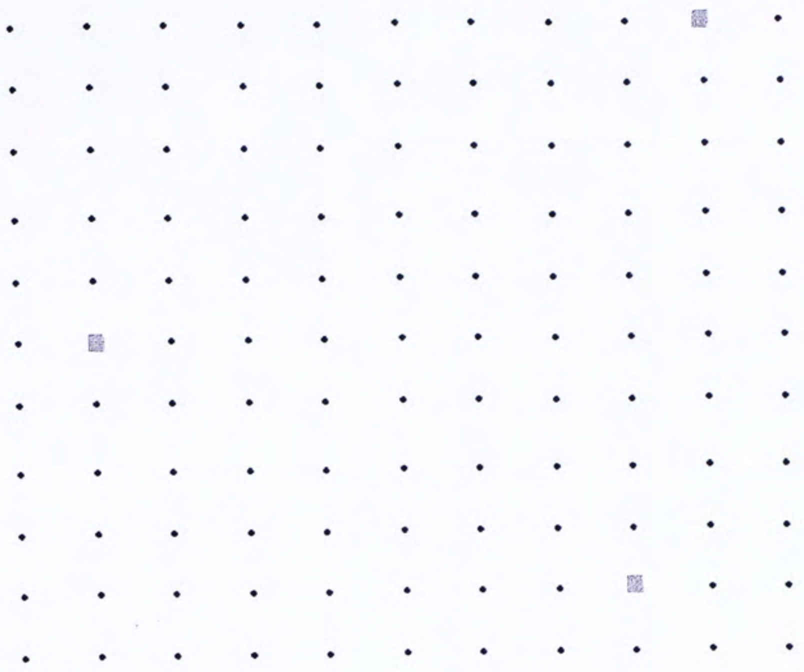
2.3 Network Database Systems

The idea of treating networks as distributed database systems is not new. The possibility of supporting database operations such as query engines, query optimizations, application-independent query operators, data replication and data consistency has been investigated by various researchers, e.g., Govindan [9] and Xu [30]. These database systems usually require that high quality service be provided by the underlying network in order to support high-level database-like functionalities.

Much research recently has been performed in the area of data retrieval in network database systems. Bonnet outlined three types of queries that can be performed in the network, namely, historical queries, snapshot queries and long-running queries [1]. Data aggregation algorithms thus can be characterized by their ability to support these types of queries.

Database systems have to maintain data consistency, and this applies to network database systems as well. A possible way to achieve data consistency is to have nodes exchange information periodically. This is useful in, for example, when updates are performed in certain sites. In this case, nodes may hold information for the same entity but of different versions—this leads to data inconsistency. The "communication" involved between nodes can be a push of information, in which the node informs the gossipees, a pull of information if the node is informed by its gossipees, or a hybrid of both. In a push scheme, each informed node contacts another node and passes the information to it. In a pull scheme, all nodes periodically query other nodes for new information. Because gossipers are always informed nodes in the push case, the two terms are used interchangeably in this case.

Demers *et al* [6] discussed the idea of using gossip mechanisms (also known as epidemic algorithms) to perform distributed database updates. Instead of ensuring that all database sites receive the nearest updates (which incurs too much overhead), the idea is to, with high probability, maintain an acceptably high level of consistency among databases. His paper [6] studied two proposals to achieve this. One is anti-entropy, in which each database site chooses another database site, and they exchange their entire database contents to resolve all discrepancies between them. An example of this is shown in Figure 4. Sites periodically perform inconsistency resolutions and, in Figure 4(a), three sites decide to perform these at that time. Figure 4(b) shows the respective gossipees selected for the operation. In Figure 4(c) a direct connection is established between the pairs of gossipers and gossipees, and their entire database contents are exchanged to resolve possible inconsistencies.



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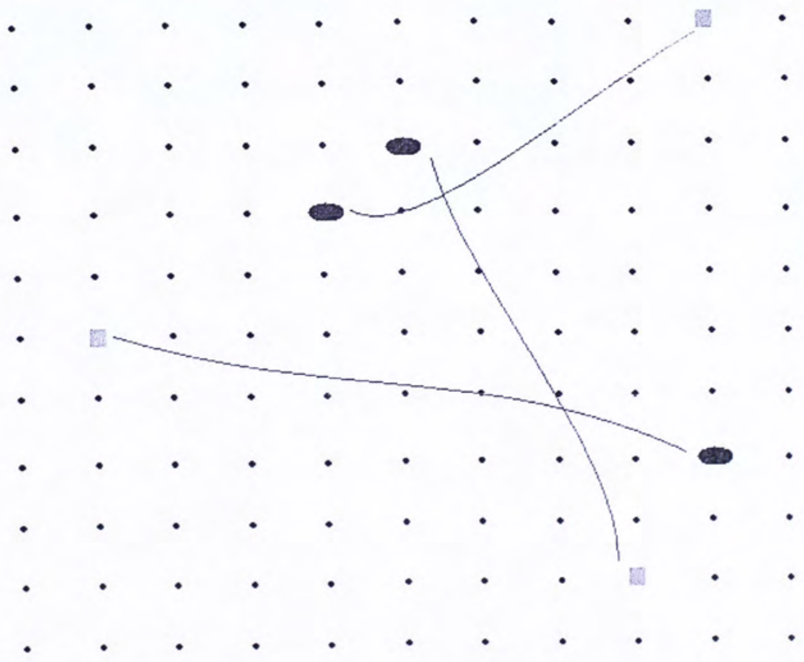

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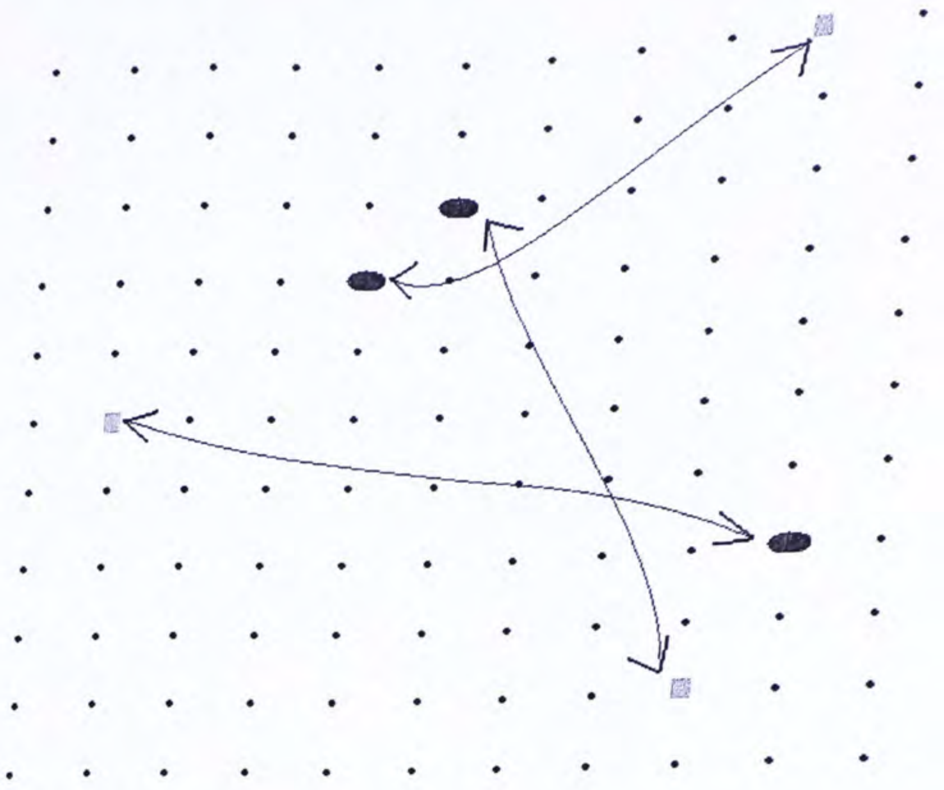


(c)

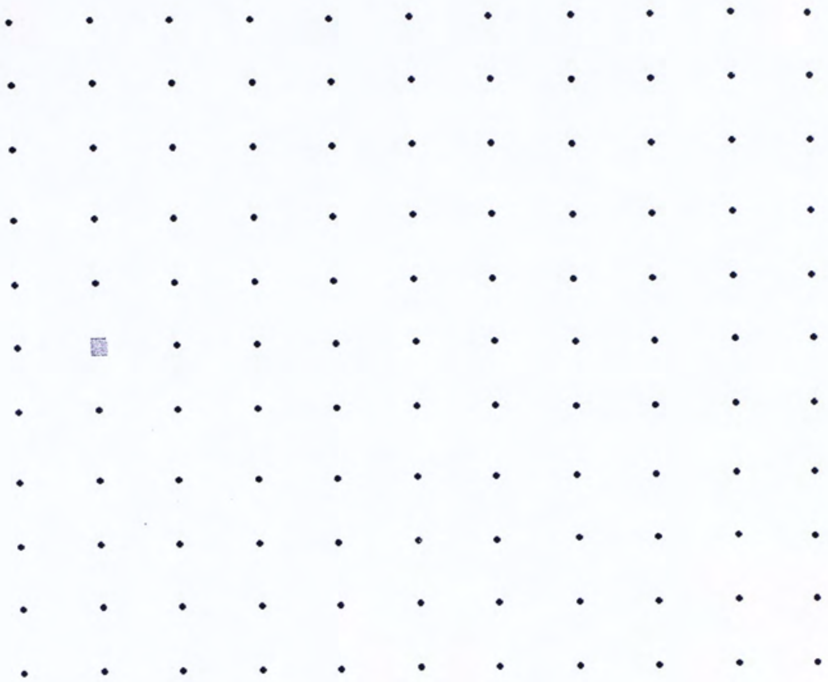
Figure 4: Using Anti-Entropy to Resolve Database Inconsistencies

(a) Three sites decide to resolve possible inconsistencies

(b) Random selection of communication peers

(c) Exchange of entire database contents

Another algorithm proposed is rumor mongering [6]. In this case a database site that receives an update proactively and randomly selects other nodes and passes the new information to them. As illustrated in Figure 5(a), a node receives an update (e.g., from users or other agents outside the network). The other nodes do not know of this update.



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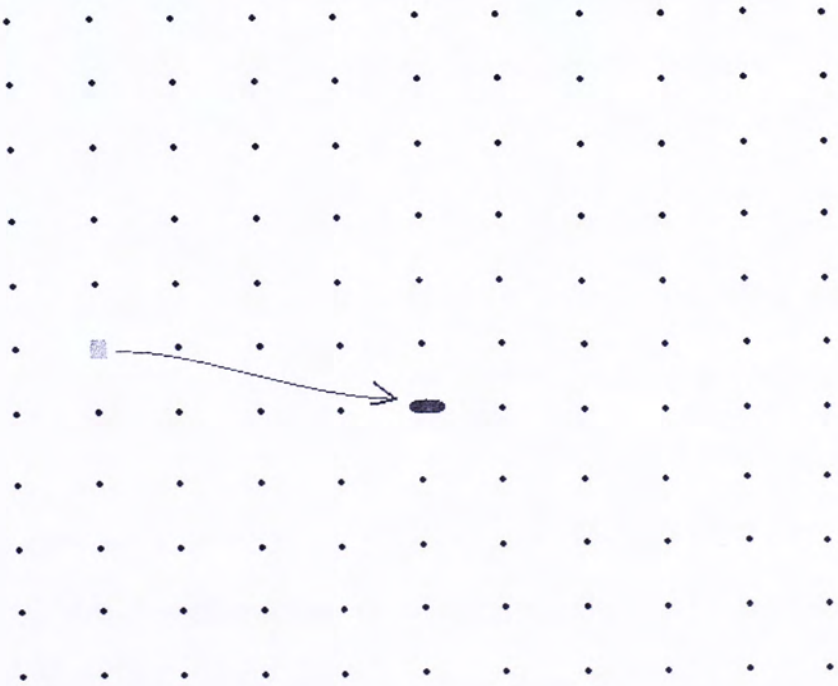

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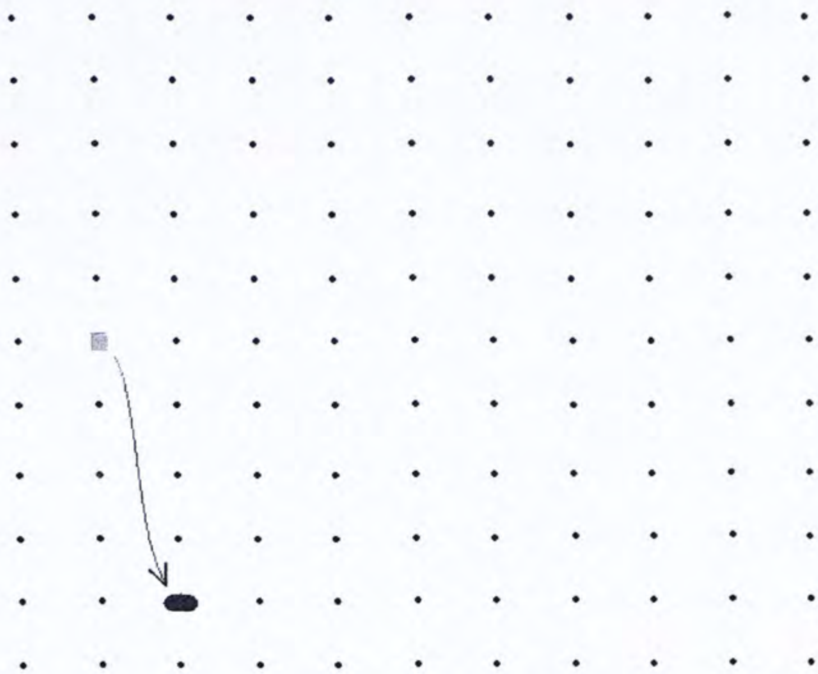


(c)

Figure 5: Using Rumor Mongering to Resolve Database Inconsistencies

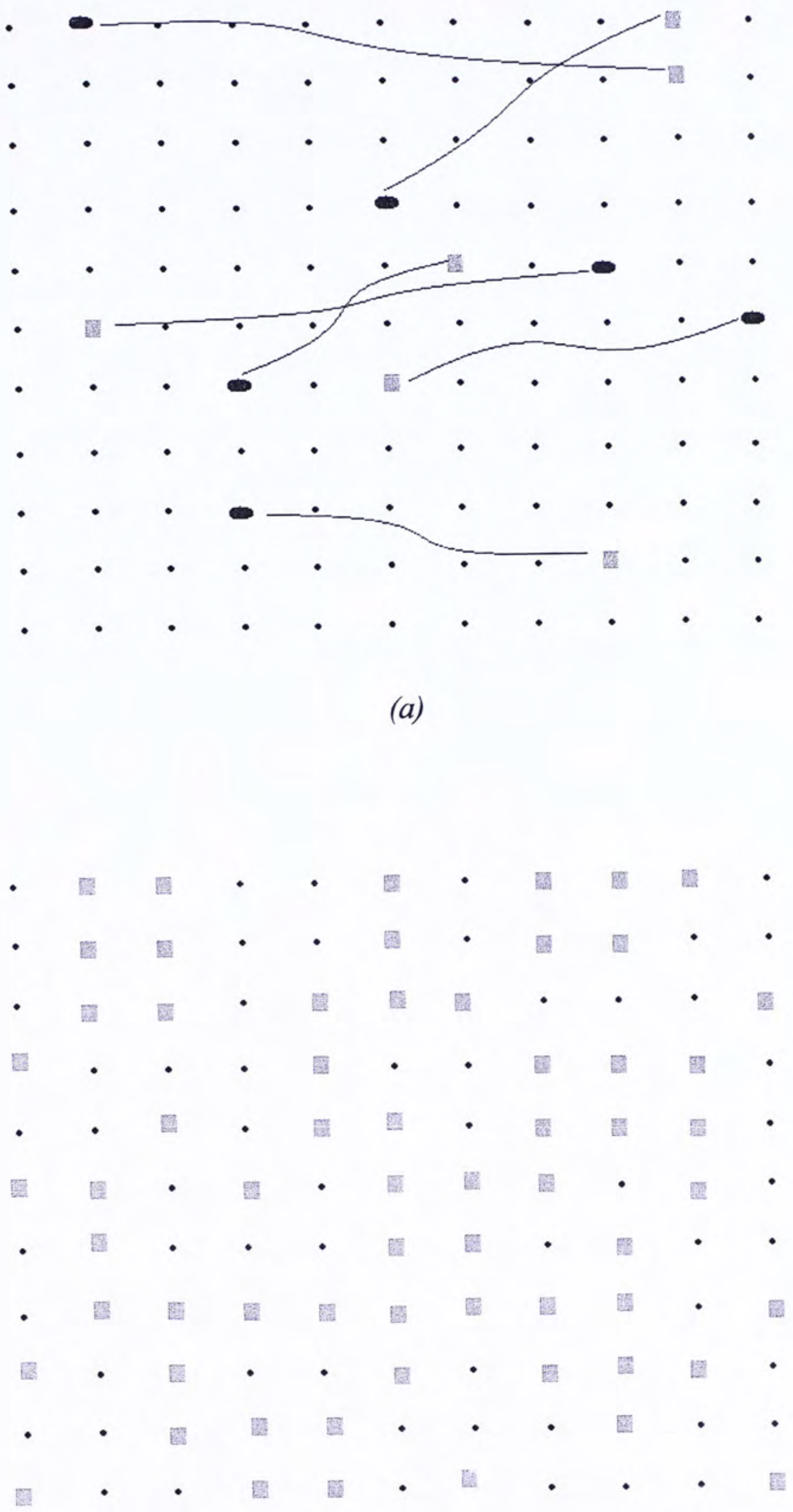
(a) A node receives an update

(b) Passing the new information to a randomly-selected node

(c) Passing it to another randomly-selected node

Apparently the former mechanism, anti-entropy, induces a large amount of network communication while achieving a high level of consistency. The latter mechanism requires much less communication, especially when the database is large, because only the relatively small amount of recently updated information is sent. Moreover, only the updated site proactively communicates with other database sites, so the undesirable situation of two already updated sites exchanging information is less likely to occur than in the former mechanism. The choice between these two mechanisms entails tradeoffs between the degree of consistency and network traffic, a common dilemma in probabilistic communication mechanisms.

Karp *et al* [14] furthered Demers's work and focused on the gossip mechanisms' efficiency. Consider the use of gossip mechanisms to perform network broadcasting, with one initial gossiper. If a push scheme is used, the number of gossipers initially increases exponentially, and gradually slows down because the number of uninformed nodes decreases with time. In fact, once the number of gossipers reaches half of the network's nodes, it is likely that an already informed node will be selected as a gossipee because uninformed nodes start to outnumber informed nodes. At this time the rate of increase in informed nodes slows down significantly, not following the exponential rate. This phenomenon is illustrated in Figure 6, which shows a situation with few gossipers in a network. Because most other nodes are uninformed, the gossiper is likely to have uninformed gossipees. Figure 6(b) shows a large number of gossipers; hence, it is likely that another gossiper is chosen as a gossipee when each of these gossipers selects a communication peer.



(a)

(b)

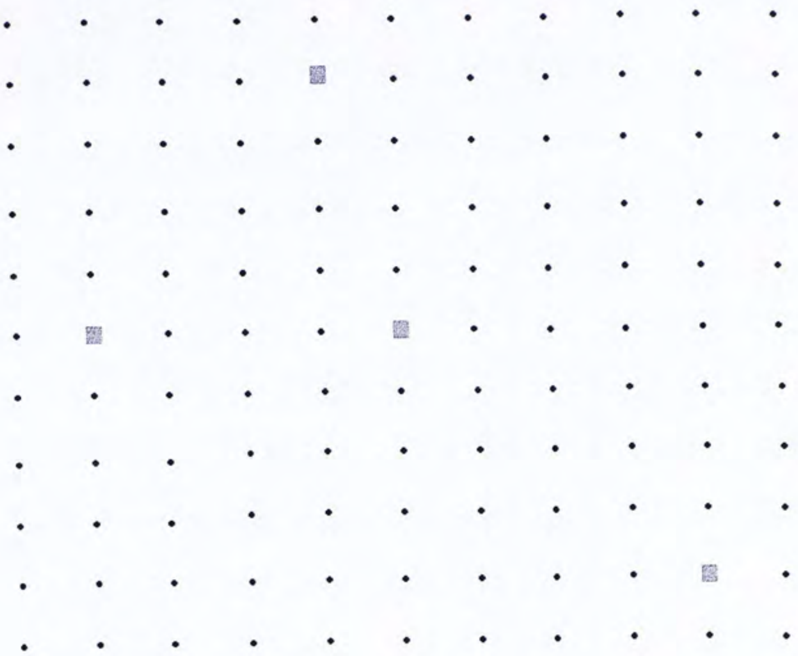
Figure 6: Probability of selecting informed or uninformed nodes with push scheme

Rectangles are gossipers (informed nodes), black dots are uninformed nodes, and ovals are their respective selected gossipees.

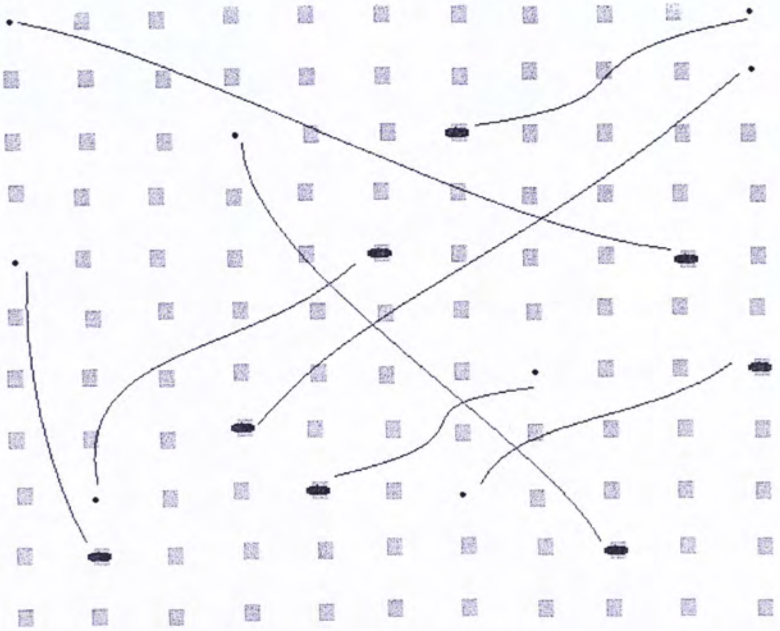
(a) Small number of gossipers

(b) Large number of gossipers

The situation for the pull scheme version of gossip is different. Figure 7, which shows a small number of informed nodes, illustrates this. Now as the uninformed nodes (periodically) query others for new information, it is likely that they will select an uninformed node. Hence, initially, when there are few informed nodes, the number of informed nodes increases slowly. But as the number of informed node increases, the chances of selecting informed nodes for querying also increase. As illustrated in Figure 7(b), when there are many informed nodes, it is highly likely that the query target chosen by each uninformed node holds the desired piece of information.



(a)



(b)

Figure 7: Probability of selecting informed or uninformed nodes with pull scheme

Rectangles are informed nodes, black dots are uninformed nodes, and ovals are gossipees chosen by uninformed nodes.

(a) Small number of gossipers

(b) Large number of gossipers

Karp *et al* showed that these two opposite paradigms can be combined to form a gossip mechanism, which they call the push&pull-scheme, that is more efficient than either of them alone. It can inform all players with high probability in time $\log_3 n + O(\ln \ln n)$ using only $O(n \ln \ln n)$ messages, compared to $\Theta(n \ln n)$ messages required by the push scheme alone [14].

2.4 Data Routing and Networking

In researching ad-hoc routing, Haas simulated the use of a gossip mechanism to broadcast messages in random networks, and showed a bimodal effect, i.e., the probability of very few nodes getting the message, and the probability of almost all nodes getting the message, are both high [11]. An example of dynamic adjustments in the nodes' forwarding probability according to the network's topology was proposed by Lin [19], in whose research each node maintains a table of weights, which is defined as the number of edges to be removed in order to disconnect that neighbour from its neighbouring nodes. The probability of forwarding a message to a neighbour with a high weight will be lowered accordingly, and vice versa.

Gossip mechanism is also employed in maintaining ad-hoc networks in hostile environments [18].

2.5 Other Applications

Distributed network security algorithms [21, 31] also use gossip. On the other hand, mathematical problems such as computing separable functions can also be solved probabilistically with the help of gossip mechanisms [22]. There are also applications in distributed automatic target recognition (ATR) [4].

Section 3 Preliminaries

3.1 Probability Distribution and Gossipee-selection Schemes

Gossip mechanisms are characterized by their probabilistic selection of communication peers, hence, by the probability distributions they employ to select gossipees. In particular, in this research project we consider the probability distribution over *distance* from other nodes.

Our use of the distance metric as the criterion to select communication peers is not new. For example, in the neighbourhood flooding scheme, nodes pass on information to their nearest node. On the other hand, uniform gossip does not consider the distance between nodes—it selects all nodes with equal probability. The gossipee-selection scheme can be fully represented by a probability distribution. The probability density function for uniform gossip is shown in Figure 8. Every node follows this function for gossipee selection.

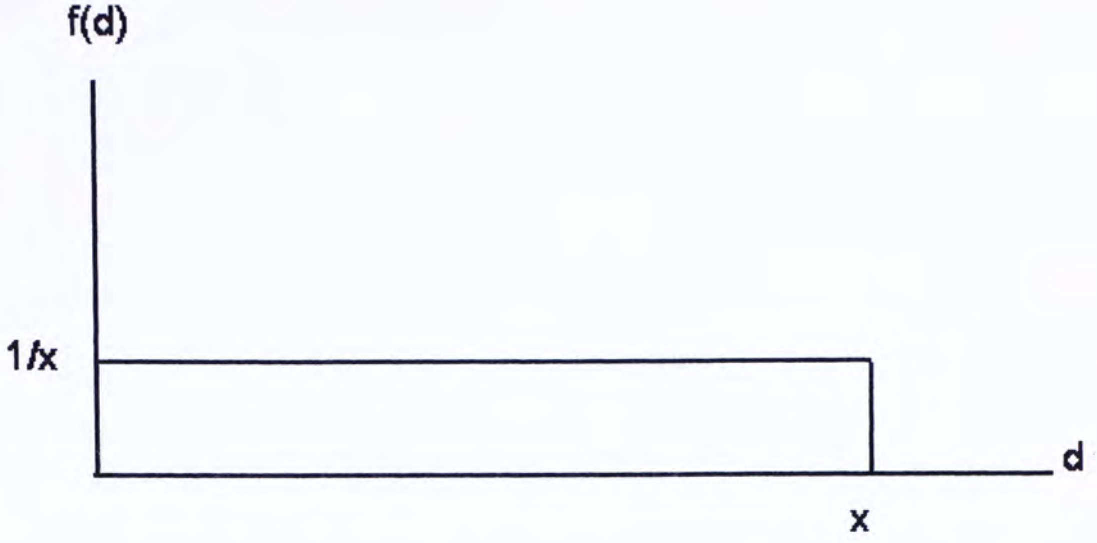


Figure 8: Probability Density Function for Uniform Gossip

x is the network's diameter.

Between these two extremes, mechanisms also fall into the middle of the spectrum. Spatial gossip [17] and truncated-uniform gossip are two examples. Spatial gossip is a class of gossip mechanisms with a positive parameter ρ. Figure 9 shows the probability density function for spatial gossip. The probability of selecting nearby nodes is higher than that of distant nodes.

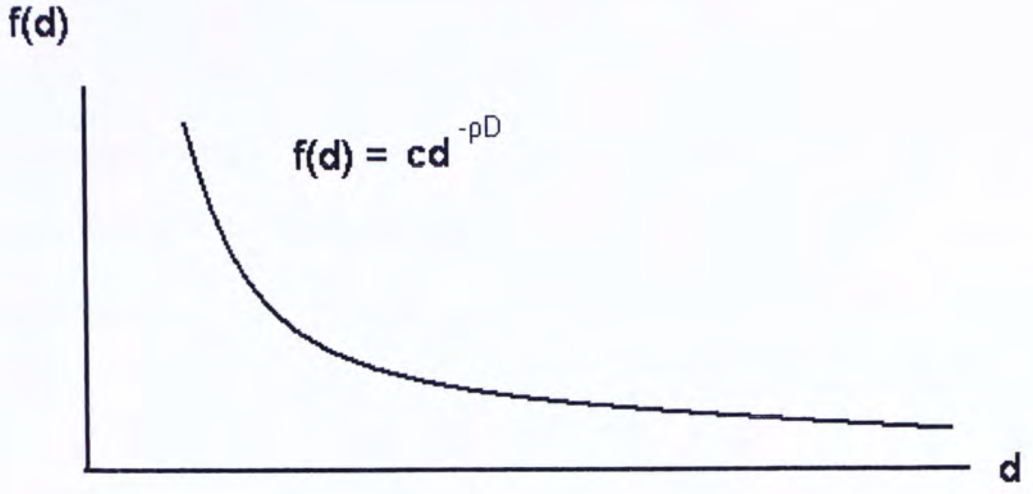


Figure 9: Probability Density Function for Spatial Gossip with Parameter ρ

c is a constant; D is the network's dimensionality.

Figure 10 shows the probability density function for truncated uniform gossip. Nodes within distance *u* are selected with equal probability, while the other nodes are not selected at all.

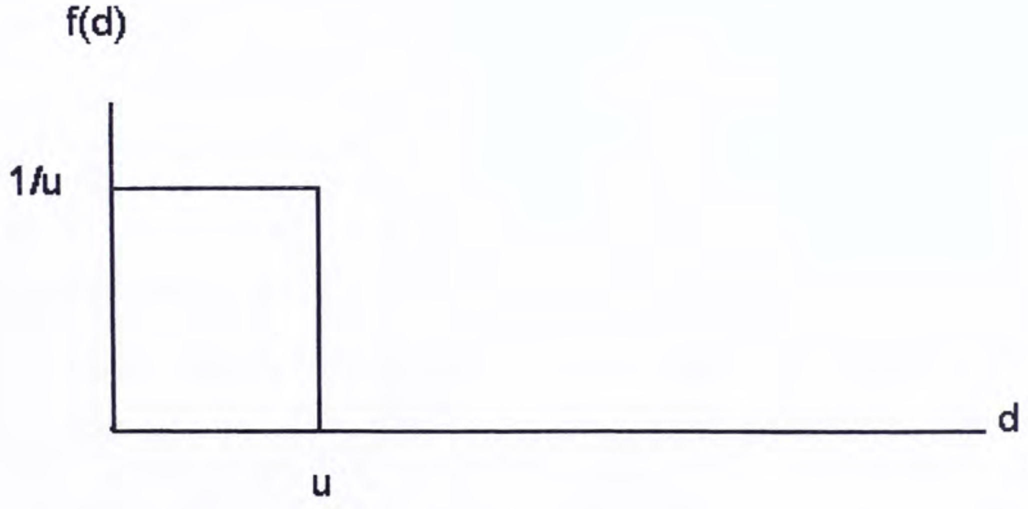


Figure 10: Probability Density Function for Truncated Uniform Gossip with Parameter u

In most applications, the parameters u or ρ are set to values such that for each gossipee-selection only nearby nodes are selected. In these cases, these short-distance gossip mechanisms are intended to transmit information to nearby nodes with high probability, and are not suitable for transmission to a (potentially far) node an unknown distance away. Take truncated-uniform gossip as an example. If the truncation threshold is u, i.e., the maximum transmission distance is u, at least d/u rounds are required to transmit the information to a node d units of distance away.

The two-tier gossip mechanisms proposed below have the advantage of uniform and short-range gossip mechanisms such as spatial and truncated-uniform gossip. By employing a uniform gossip first, gossipers spread across the whole network sparsely so that most uninformed nodes are within a short distance of at least one gossiper. Then a short-distanced gossip mechanism is employed, so that each gossiper informs its nearby nodes. This is more efficient than employing only the uniform gossip mechanism.

3.2 The Network Models

In this section we discuss the network models for the results in Sections 4 and 5. The models consist of two essential elements, namely, the distribution of network nodes, and the network nodes' communication and computational capabilities.

This research project studies both the class of uniformly distributed networks, and networks whose nodes are placed at integral coordinates, a special case of uniformly distributed networks. The uniformly distributed network is also studied in the literature; for example, in [15, 16, 27]. This model considers a network with uniformly distributed nodes in the D-dimensional real-number hyperspace (R^D). A *uniformly distributed network* has between αr^D and βr^D nodes in any hypersphere with radius $r \geq 1$ where α and β are constants. Let N be the set of all network nodes. Denote the number of nodes as n if N is a finite network. Networks whose nodes are placed at integral coordinates are, in a restricted sense, uniformly distributed.

Each node can directly communicate with any other nodes in the network. However, it is crucial to specify the network nodes' communication capabilities, as this is one major assumption that determines what classes of protocols are allowed to run. The capabilities can be specified by the *gossipee-selection scheme* that a node can follow, which is a probability distribution function over the network nodes. For example, in the gossip mechanism T to be defined in Section 4.1 (which is the main mechanism discussed in Section 4), the nodes can perform both uniform distribution and spatial gossip. As shown in Section 4.4, the network nodes can select gossipees according to an upper-truncated uniform distribution with an adjustable truncation threshold $\in [0, \infty]$. No specific restrictions limit the nodes' computational capabilities. The network nodes' communication capabilities will be explicitly specified in the section that presents the experimental/theoretical results.

In this research project, we study protocols in which each node is allowed to send one message per round and that message is delivered to the recipient before the end of the round. Let $d(u, v)$ be the (Euclidean) distance between nodes u and v. For any node $u \in N$, u can select another node from all nodes $v \in N$ following the probability density function $f(d(u, v))$. A new node is selected in each round. Asynchronous clocks, random node failures and random communication link failures do not alter the mechanism's correctness, but the algorithmic time might need to be longer in these situations. The correctness in these cases is apparent because the proposed protocol is a simple *gossip* mechanism—any number of packets lost can be tolerated as long as there is at least one packet still circulating around in the network, which would be eventually distributed to any node in the network if we wait long enough. We define the *message length complexity* of a given communication mechanism as the order of the length of each of these additional headers.

3.3 Objective and Problem Statement

The above network model provides a general framework for studying how particular communication mechanisms behave and, hence, their abilities to solve network problems. Here we define the *Clustered Destination Problem* as follows. Initially a ("source") node holds a piece of information. A set of "destination" nodes has to acquire this piece of information. The problem requires a method to disseminate this information to the destination nodes. This problem is the delivery of a message from a node to the set of destination nodes. Normally we consider the target nodes to be clustered within a small region, rather than sparsely distributed over the whole network; hence, it is called the Clustered Destination Problem. The following is a formal definition of the problem.

Definition 1 [Clustered Destination Problem (CDP)]: Given a set of network nodes N distributed in a D-dimensional Euclidean space, a source node $s \in N$ and a set of destination nodes $K \subseteq N$ of which the location is unknown to s,

the Clustered Destination Problem is how to distribute the piece of information from s to each node $x \in K$ w.h.p.

$\square$

Recall that an event occurs "with high probability" (w. h. p.) if the probability of occurrence $= 1 - O((\ln n)^{-x})$, where $x > 0$, for a network with n nodes. In Section 4 we shall present a mechanism that satisfies this high probability requirement.

3.4 Two-tier Gossip Mechanism

As described in the introduction, our proposed two-tier gossip mechanism allows network nodes to change their gossipee-selection scheme once during the communication process. Normally the first gossipee-selection scheme disseminates the information sparsely throughout the whole network, while the second tends to select nearby nodes. Some "intensification" is achieved in the sense that this algorithm initially tries to randomly inform as many nodes as possible across the whole network, and "intensifies" to do a more thorough search within small areas. This is different from traditional one-tier mechanisms, whose gossipee-selection scheme does not change.

Each node selects a node to communicate with, and this selection depends solely on the probability density function. Originally the function is set to the initial probability density function. It is then changed to the second probability density function when a specific criterion (predetermined at compile time) is satisfied. Each node is supposed to have a copy of this code at its communication mechanism layer. By abusing terminology we denote "a scheme in which each gossiper selects gossipees according to the probability density function $f(d)$" as simply a gossipee-selection scheme $f(d)$.

3.5 Semantic-dependent Gossip Mechanism

Sensor network database systems are formed when network nodes with sensing abilities are deployed into an environment and can communicate with each other. Because the queries almost always involve the values measured by the sensors, protocols to be run on these networks can take the advantage of the sensing values, the aggregate of which is usually being queried. Common queries include MAX, MIN, AVG, COUNT and the location of nodes that satisfy specified criteria.

A semantic-dependent gossip mechanism considers the sensor's value when choosing a gossipee. In other words, the gossipee-selection scheme depends on the sensor's value. An example of this is sensor networks that detect temperature. By utilizing the temperature detected, the distance from possible heat sources can be estimated without knowledge of the global network status. More will be discussed on this in the results of the semantic-dependent gossip mechanism.

Section 4 Results of Two-tier Gossip Mechanisms

4.1 Background

In the two-tier gossip mechanism proposed in this research project, the initial gossipee-selection scheme is uniform gossip, which runs for *k* rounds (*k* will be specified later). After that, spatial gossip [17] with $\rho \in (1,2)$ is used for $O(\lg^{1+\varepsilon}(2r))$ rounds, where ε is an arbitrarily small positive constant.

For the following sections, let *B* be any circle/sphere/hypersphere that contains *K* and whose number of nodes is $b \geq n^{1-\lg(2-\delta)}\gamma^{\lg(2-\delta)} \ln \ln n$, n is the number of nodes in the whole network, δ is an arbitrarily small positive constant, for any $\gamma > 1$. Define *r* = radius of B.

The proposed two-tier gossip mechanism requires knowledge of either *b* or *r* by the source node. Bounding values of *b* and *r* can be derived from each other using information of network *N*, because there are between αr^D and βr^D nodes in any hypersphere with $r \geq 1$ in any uniformly distributed networks. Hence, $\alpha r^D \leq b \leq \beta r^D$. Other information, e.g., the location of *B*, is not necessary to execute the mechanism. Our two-tier gossip mechanism is denoted as *T* and is formally defined as follows.

Definition 2 [gossip mechanism *T*]: The two-tier gossip mechanism *T* is defined as a mechanism that is initialized by a node (the first gossiper) in the network, and each node which becomes a gossiper at round *i*:

i) performs uniform gossip for (*m*-*i*) rounds, and then

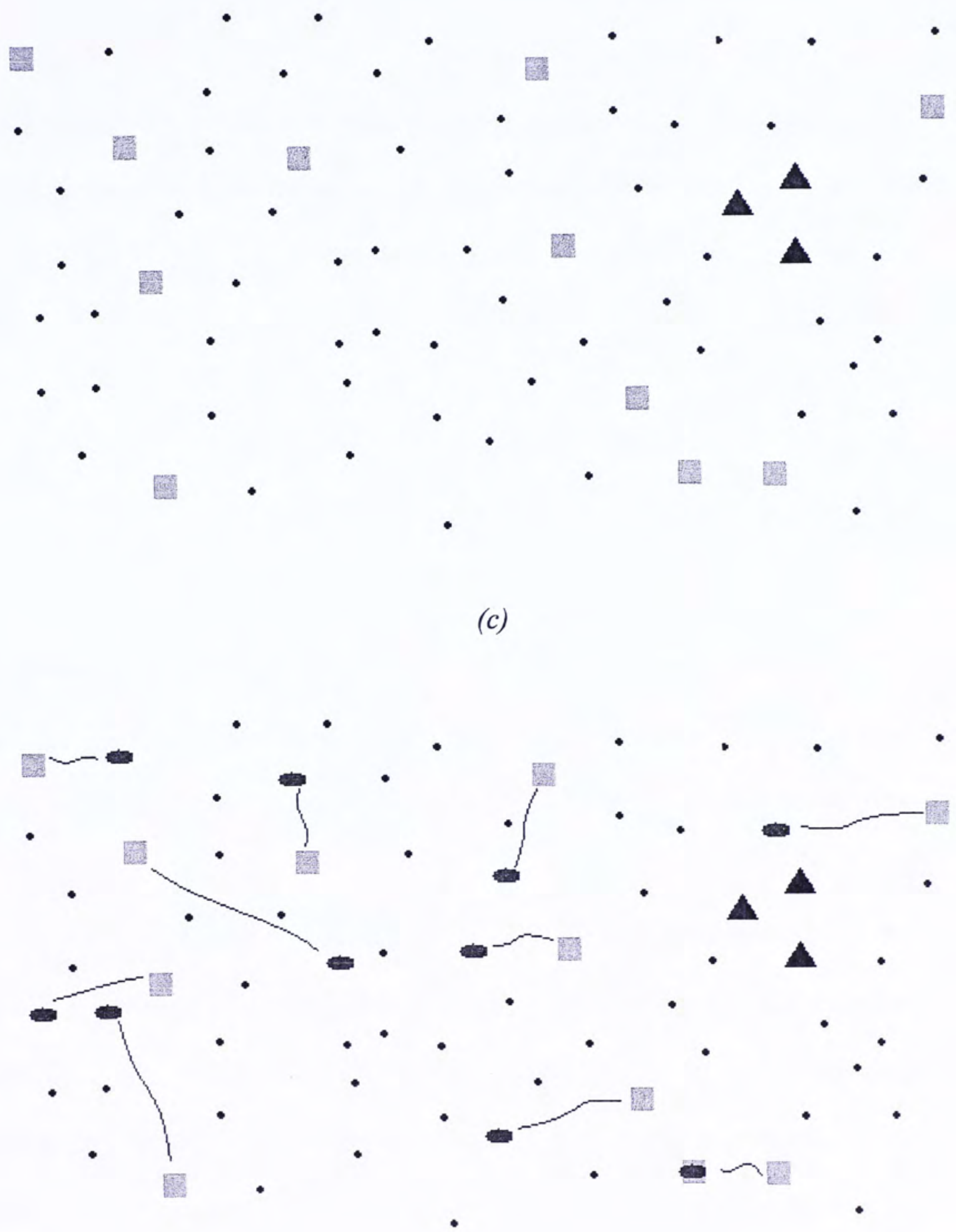
ii) performs spatial gossip for *n* rounds. ☐

The following figures illustrate the mechanism of *T*. Consider the network scenario presented in Figure 2, which has three destination nodes randomly distributed across this two-dimensional Euclidean space (however, theorems 1 and 2 discussed below can also be applied to multi-dimensional Euclidean spaces). Node 1 in Figure 2(a) is the initial gossiper and holds the information to be passed to the destination nodes. Node 1 has no knowledge of the destination nodes' location, and its only communication capability is to perform truncated-uniform gossip. It randomly selects another node as gossipee (node 2) and passes the information to it, as shown in Figure 2(b).

In the next round both gossipers choose gossipees and pass on the information. Suppose that after *k* rounds the network's situation is as depicted in Figure 2 (c). There are a considerable number of gossipers, but none of the destinations has received the information. According to the definition of *T*, the gossipee-selection scheme will now switch to spatial gossip. Hence each gossiper hitherto employs spatial gossip to select gossipees. As spatial gossip tends to select nearby nodes, the communication distance is smaller on average than in the previous rounds. Figure 2(d) shows the network's situation after performing the (m+1)-th round of gossip, i.e., the first round of spatial gossip. Spatial gossip will be employed for, at most, *n* rounds.

(a)

(b)



(c)

(d)

Figure 11: An example of using gossip mechanism T to solve the Clustered Destination Problem

Black dots are (uninformed) network nodes.

Rectangles are informed nodes (gossipers) that receive the message during the first phase of T.

Ovals are gossipers that receive the message during the second phase of T.

Lines represent the dissemination of information from gossipers to gossipees.

Triangles are destination nodes.

(a) Round 0

(b) Round 1

(c) Round m

(d) Round m+1

The following theorem 1 shows the time bound of using gossip mechanism *T* to solve the Clustered Destination Problem, specifying the number of rounds of phase 1 and phase 2 gossip, so that the destination nodes receive the message w.h.p. after the time bound.

Theorems 1 and 2 in Subsections 4.2 and 4.3 are for this version of CDP:

There is no restriction on the number of elements in *N*, which node is the source node *s* or which nodes are in *K*. The only restriction on the distribution of *N* across the Euclidean space is that the nodes are uniformly distributed (as defined in Section 3.1). The network nodes' only communication capability is to perform uniform and spatial gossip. The time required to solve CDP is to be minimized.

4.2 A Time Bound for Solving the Clustered Destination Problem with *T*—Theorem 1

In this subsection we will show that the two-tier gossip mechanism *T* can solve the Clustered Destination Problem within time $\log_{2-\delta}(n/b)+\log_{2-\delta}\ln\ln(n/b)+O(\lg^{1+\varepsilon}(2r))$.

The intuitive idea behind the proof is that due to the large number of gossipers after *k* rounds, a gossiper exists in the bounding hypersphere *B* (and, hence, in the clustered destination *K*) w.h.p. After that, with the help of spatial gossip [17], this gossiper can inform any node in the destination. The first phase of *T* is carried out for *k* rounds, and the second phase is carried out for $O(\lg^{1+\varepsilon}(2r))$ rounds. The precise value of *k* will be defined in the proof that follows.

Lemmas 1 to 5 show that after *k* rounds, there is at least one gossiper in *B* w.h.p. Here we use Pittel's deterministic approximation of the number of gossipers *I(t)* under uniform gossip.

$x_n(t)$ is defined as:

Definition 3 [$x_n(t)$]: $x_n(t+1)=1-(1-x_n(t))e^{-x_n(t)}$ for $t = 0, 1, 2\ldots$

and $x_n(0)=\frac{1}{n}$. □

Let *I(t)* be the number of gossipers ("informed nodes") in the network after *t* rounds of uniform gossip.

[27] shows that n is the number of network nodes and t is the number of rounds after one node has initiated uniform gossip, hence, $x_n(t) = I(t)/n$ (refer to [27] for this function's derivation). Define $f(x_n(t)) = x_n(t+1)$ in the following proof.

With these we can begin to prove the theorem. The following lemmas leading to Lemma 5 show that after k rounds of uniform gossip, a gossiper exists in the hypersphere B w.h.p. with an appropriate value of k. In other words, we would like to guarantee that with probability at least

$$1-(\ln n)^{-c},$$

there is a gossiper in the hypersphere B with b nodes, for appropriate values of b, k and $c>0$.

The first two Lemmas establish a lower bound for the number of gossipers after rounds of uniform gossip, namely, that $x_n(t) \geq (2-\delta)^t c/n$ for a constant c, as long as $x_n(t)$ is bounded by a constant $m < 1$. Lemmas 3 and 4 then show that, for our application of uniform gossip (i.e., run for k rounds), it is indeed true that $x_n(t)$ is smaller than a constant $m < 1$, for all $t \leq k$, and hence the results of Lemmas 1 and 2 can be applied to prove Lemma 5. With these results, Lemma 5 then establishes the probability guarantee of $1-(\ln n)^{-c}$.

Lemma 1: If $x_m(s) \geq x_n(t)$, then $x_m(s+i) \geq x_n(t+i)$ for all natural numbers m, n, s, t and i.

Proof

Here Mathematical Induction on variable *i* is used.

Base case [i = 0]:

For any *m, n, s* and *t*, if $x_m(s) \geq x_n(t)$, then $x_m(s+0) \geq x_n(t+0)$.

Induction:

Assume that Lemma 1 is true for $i = \alpha$, where $\alpha \in \{0, 1, 2, \ldots\}$. That is, if $x_m(s) \geq x_n(t)$, then $x_m(s+\alpha) \geq x_n(t+\alpha)$ for all *m, n, s* and *t*.

Consider the case when $i = \alpha + 1$.

$$x_m(s+\alpha+1) = f(x_m(s+\alpha)) \qquad \text{(definition of } f(x)\text{)}$$

$$\geq f(x_n(t+\alpha)) \qquad (f(x) \text{ is strictly increasing})$$

$$= x_n(t+\alpha+1) \text{ for all } m, n, s \text{ and } t.$$

Hence if Lemma 1 is true for $i = \alpha$, it implies it is true for $i = \alpha + 1$.

By induction, Lemma 1 is proven. □

Lemma 2: For all *n* and *t* such that $x_n(t)$ is upper-bounded by a constant $m < 1$, there exists $c > 0$ independent of *n* such that $x_n(t) \geq (2-\delta)^t c/n$, where $\delta < 1$ is an arbitrarily small positive constant.

Proof

This Lemma establishes a lower bound for the growth of the function $x_n(t)$. Before we start to prove Lemma 2, we define the following two functions, $g_n(t)$ and $h(x_n(t))$, based on the function $x_n(t)$.

$g_n(t) = \frac{x_n(t+1)}{x_n(t)}$, where $t = 0, 1, 2...$

Define $h(x_n(t)) = g_n(t)$.

Note that $g_n(t)$ and $h(x)$ are strictly decreasing functions and $1 < g_n(t) < 2$ for $0 < x_n(t) < 1$.

Also, we define the sequence $\{a_q\}$ as $a_q = g_n(t-q-1)/2-\delta$ for all $0 \le q \le t-1$.

The function $x_n(t)$ can be expressed in terms of a_is.

For any n and t,

$x_n(t) = [g_n(t-1)g_n(t-2)...g_n(0)]x_n(0)$

$= (2-\delta)^t(a_0a_1...a_{t-1})x_n(0)$.

Define $c_n(t) = a_0a_1...a_{t-1}$.

Then $x_n(t) = (2-\delta)^t c_n(t).x_n(0)$ for any n. ---[Result 4.1]

Define $c_n = \min(\{\min c_n(t)\colon x_n(t) \le m\}, 1)$.

We can now start to prove Lemma 2. In the following we consider only the case when $x_n(t) \leq m$. Mathematical induction on α is employed for $n = (2-\delta)^{\alpha} q$, with $\alpha = 0$ (i.e., $n = q$) as the base case and q as a constant.

Base case [α = 0]:

Choose any positive integer q such that $g_q(0) \geq 2-\delta$. From Result 4.1,

$$x_q(t) = (2-\delta)^t c_q(t) x_q(0)$$

$$\geq (2-\delta)^t c_q x_q(0) \qquad \text{(Definition of } c_n\text{)}$$

$$= (2-\delta)^t c_q / q \qquad \text{(Definition of } x_n(t),\ t = 0\text{)}$$

a constant exists c_q independent of n and t.

Induction:

Here we show that c_q in the expression $(2-\delta)^t c_q / n$ is indeed "constant," i.e., it does not increase as n increases.

Induction Hypothesis: Assume Lemma 2 is true for $n = (2-\delta)^{\alpha} q$, i.e., constant c exists such that $x_{(2-\delta)^{\alpha} q}(t) \geq \frac{(2-\delta)^t c}{(2-\delta)^{\alpha} q}$ for any t such that $x_{(2-\delta)^{\alpha} q}(t) \leq m$ where $\alpha \in \{0,1,2,\ldots\}$.

Moreover we claim that $c = c_q$.

Consider the case when $n = (2-\delta)^{\alpha+1} q$. First we shall show that $x_{(2-\delta)^{\alpha+1} q}(1) \geq x_{(2-\delta)^{\alpha} q}(0)$ (i.e., Result 4.2). Then with the application of Lemma 1 we shall prove that Lemma 2 indeed holds for $n = (2-\delta)^{\alpha+1} q$.

Proved in the Appendix of this thesis,

$x_{(2-\delta)^{\alpha+1}q}(1) \geq x_{(2-\delta)^{\alpha}q}(0)$. ---[Result 4.2]

By Lemma 1, because $x_{(2-\delta)^{\alpha+1}q}(1) \geq x_{(2-\delta)^{\alpha}q}(0)$ (by Result 4.2),

$x_{(2-\delta)^{\alpha+1}q}(t) \geq x_{(2-\delta)^{\alpha}q}(t-1)$ ---[Result 4.3]

for any natural number *t*.

For the case $t > 0$,

$$x_{(2-\delta)^{\alpha+1}q}(t) \geq x_{(2-\delta)^{\alpha}q}(t-1)$$

$$\geq \frac{(2-\delta)^{t-1}c_q}{(2-\delta)^{\alpha}q} \qquad \text{(Induction Hypothesis)}$$

$$= \frac{(2-\delta)^{t}c_q}{(2-\delta)^{\alpha+1}q} \qquad \text{---[Result 4.4]}$$

For the case $t = 0$,

$$x_{(2-\delta)^{\alpha+1}q}(0) = \frac{1}{(2-\delta)^{\alpha+1}q} \qquad \text{(Definition of } x_n(t)\text{)}$$

$$\geq \frac{(2-\delta)^{0}c_q}{(2-\delta)^{\alpha+1}q} \qquad (c_q \leq 1)$$

This nearly completes the inductive step, with a slight complication. Recall that $x_n(t)$ is bounded by the constant *m* as required in Lemma 2. Above we have proven that Result 2.5 holds for *t* such that $x_{q(2-\delta)^{\alpha}}(t-1) \leq m$, but what should be proven is that it is true for all t such that $x_{q(2-\delta)^{\alpha+1}}(t) \le m$. Let p be the largest integer such that $x_{(2-\delta)^{\alpha} q}(p) \le m$. Result 4.4 is shown to hold for t = 0, 1, 2, …, p+1 above. There is no need to consider the cases in which $t \ge p+2$, because

$$x_{(2-\delta)^{\alpha+1} r}(p+2) \ge x_{(2-\delta)^{\alpha} r}(p+1) \qquad \text{(Result 4.3)}$$

$$> m \qquad (x_n(t) \text{ is strictly increasing})$$

Hence Result 4.4 holds for any t such that $x_{(2-\delta)^{\alpha+1} q}(t) \le m$.

By induction, Lemma 2 is true for $n = (2-\delta)^{\alpha} q$ for all natural numbers α, and q is a constant. The extension of this result to any positive integer n is straightforward. $\square$

In order to use Lemma 2 we need to show that our application of uniform gossip requires only a limited number of rounds (i.e., that $x_n(t)$ is bounded by a constant m<1 or, equivalently, that $I(k) \le n/\gamma$ where γ is a constant). Lemmas 3 and 4 handle this issue.

Lemma 3: The number of gossipers, $I(t)$, is bounded by $I(t) \le 2^t$ after t rounds of uniform gossip, for all $t \ge 0$ and $t \in N \cup \{0\}$, where N is the set of natural numbers {1, 2, 3, …}.

Proof

This Lemma is obvious because the number of gossipers can, at most, double in each round. $\square$

With foresight, the number of rounds of uniform gossip in gossip mechanism T is set to $k = \log_{2-\delta}(\frac{n}{b}\ln\ln n)$, so that after this number of rounds, at least one gossiper exists in clustered destination B w.h.p. The following Lemma shows that after k rounds of uniform gossip, a portion of informed nodes remain bounded by a constant smaller than 1.

Lemma 4: $I(k) \leq \frac{n}{\gamma}$ where $k = \log_{2-\delta}(\frac{n}{b}\ln\ln n)$ and $\gamma > 1$.

Proof

Recall that $b \geq n^{1-\lg(2-\delta)}\gamma^{\lg(2-\delta)}\ln\ln n$ as defined in Section 4.1.

$I(k) = I(\log_{2-\delta}(\frac{n}{b}\ln\ln n))$

$\leq 2^{\log_{2-\delta}(\frac{n}{b}\ln\ln n)}$ (by Lemma 3)

$= (\frac{n}{b}\ln\ln n)^{\frac{1}{\lg(2-\delta)}}$

$\leq (\frac{n}{n^{1-\lg(2-\delta)}\gamma^{\lg(2-\delta)}\ln\ln n}\ln\ln n)^{\frac{1}{\lg(2-\delta)}}$ (by definition of b)

$= \frac{n}{\gamma}$. □

Because $I(k) \leq \frac{n}{\gamma}$, $x_n(k) \leq \frac{1}{\gamma} < 1$ by the definition of $x_n(t)$. Note that as $\delta \to 0$, the minimum value for $b \rightarrow \gamma \ln\ln n$.

The following Lemma proves that at least one gossiper exists in B after k rounds of uniform gossip, w.h.p.

Lemma 5: There is at least one gossiper in B after $k = \log_{2-\delta}(\frac{n}{b}\ln\ln n)$ rounds of uniform gossip with probability at least $1-(\ln n)^{-c}$, where c is a positive constant.

Proof

The probability of selecting *I(k)* nodes from the entire network without replacement such that none of the selected nodes lies in B is equivalent to one minus the probability of having at least one gossiper in B after k rounds of uniform gossip.

Hence after k rounds, the probability of having at least one gossiper in B =

$$= 1 - \prod_{p=0}^{I(k)-1} \left(\frac{n-b-p}{n-p}\right)$$

(selection of *I(k)* nodes without replacement)

$$\geq 1 - \prod_{p=0}^{(2-\delta)^k c-1} \left(\frac{n-b-p}{n-p}\right)$$

(Lemmas 2 and 4)

$$\geq 1-\left(1-\frac{b}{n}\right)^{(2-\delta)^k c}$$

(selection of $2^k c$ nodes with replacement)

$$= 1-\left(1-\frac{b}{n}\right)^{n\left(\frac{(2-\delta)^k c}{n}\right)}$$

$$\geq 1-e^{-\frac{b}{n}(2-\delta)^k c}$$

(formula for *e*)

$$= 1 - e^{-\frac{bc}{n}\frac{n \ln \ln n}{b}}$$

$$= 1 - e^{-c \ln \ln n}$$

$$= 1 - (\ln n)^{-c}.$$ □

The following lemma is a result derived in [17].

Lemma 6: Let x and x' be nodes in the network which are $2r$ apart. If x initializes spatial gossip for $O(\lg^{1+\varepsilon}(2r))$ rounds, where ε is an arbitrarily small positive constant, then x' is informed with a probability independent of the network's size.

Proof

See [17]. □

Theorem 1: The two-tier gossip mechanism T can solve the Clustered Destination Problem within time $\log_{2-\delta}(n/b) + \log_{2-\delta} \ln \ln(n/b) + O(\lg^{1+\varepsilon}(2r))$, where n is the number of network nodes, b and r are the number of nodes and the radius of any bounding hypersphere for the clustered destination with $b \geq 2n^{1-\lg(2-\delta)}\gamma^{\lg(2-\delta)} \ln \ln n$, respectively, and ε and δ are arbitrarily small positive constants.

Proof

By Lemma 5, a gossiper exists in *B* after *k* rounds of uniform gossip, with probability at least $1-(\ln n)^{-c}$.

By Lemma 6, this gossiper can inform any node in *B* in time $O(\lg^{1+\varepsilon}(2r))$ for an arbitrarily small ε >0, with probability related only to the distances between nodes and independent of the network's size, a result shown in [17]. This result is independent of the location of the gossiper in *B* because any node in *B* is, at most, 2*r* from this gossiper. Now, because *B* completely contains *K*, it implies that any node in *K* is informed. Moreover, the message that is forwarded by this gossiper propagates to each node in *B* within a (short) time of $O(\lg^{1+\varepsilon}(2r))$, independent of the network's size.

The overall time is $k+O(\lg^{1+\varepsilon}(2r))$

$=\log_{2-\delta}(n/b)+\log_{2-\delta}\ln\ln(n/b)+O(\lg^{1+\varepsilon}(2r))$. ☐

4.3 Further Results—Theorem 2

With slight modification for gossip mechanism *T*, we can inform each node in the whole network *N*, instead of just the nodes in the cluster. Here, the initial gossip scheme (i.e., uniform gossip) of *T* is run for $k=\lg(n/b)+\ln(n/b)+O(1)$ rounds instead of the original value from Section 4.2. All other parameters remain unchanged.

To prove Theorem 2 below, we first define a network *N'* which consists of *n*/*b* nodes.

Each of these nodes in *N'* can be viewed as "representative" of *b* nodes of *N*.

Definition 4 [network *N'*]: Network *N'*, with $\frac{n}{b}$ nodes, is formed under the mapping $f: N \to N': f(p_x) = p'_{\left\lfloor \frac{x}{b} \right\rfloor} \forall p_x \in N$. □

Definition 5 [gossip mechanism *T'*]: Gossip mechanism *T'* is the gossip mechanism that runs on network *N'*, as *T* runs on *N*. □

Theorem 2: *T* can inform any node in network *N* in time $\lg(n/b) + \ln(n/b) + O(\lg^{1+\varepsilon}(2r))$ w.h.p.

Proof

The idea is to choose a natural number *b* and divide the network into $\lceil n/b \rceil$ subsets $N_1, \ldots, N_{\lceil n/b \rceil}$, each with, at most, *b* nodes. Let *2r* denote the maximum diameter of these subsets. Label the nodes in *N* such that p_{sb+i} exists in subset N_s, $0 \le i \le b-1$, $\forall p_{sb+i} \in N$.

As gossip mechanism *T* is initialized by any node in network *N*, label a node $p' \in N'$ as "informed" if $\exists p \in N : f(p) = p'$ and *p* is informed. Compare the effect of *T'* and uniform gossip on network *N'*. *T'* can spread information faster than uniform gossip can because, given any network *N'*, the probability of selecting an uninformed node in *N'* is the same in both cases; but in *T'* the number of actual gossipers (i.e., in *N*) is larger than or equal to that in *N'*. If uniform gossip is applied to network *N'*, all network nodes are informed in time $\lg(n/b)+\ln(n/b)+O(1)$ w.h.p. because that network has, at most, $\lceil n/b \rceil$ nodes, using Pittel's result [27]. This corresponds to having at least one gossiper in each subset N_s of *N*, $1 \le s \le \lceil n/b \rceil$. Hence, *T* can also guarantee the existence of at least one gossiper in each subset N_s of *N* in time $\lg(n/b)+\ln(n/b)+O(1)$ w.h.p.

After $\lg(n/b)+\ln(n/b)+O(1)$ number of rounds, spatial gossip can inform any node *h* in all the subsets by their respective gossiper *g* in time $O(\lg^{1+\varepsilon}(2r))$ with a probability independent of the network's size [17]. The guarantee that *g* can inform *h* w.h.p. using only nodes in the smallest hypersphere containing *g* and *h* [17] ensures that "interferences" between subsets of N_s do not affect our result, because all gossipers can inform each node in the subsets they are responsible for with no help from outside or remote nodes.

Combining the time requirement for both phases gives an overall time of $\lg(n/b)+\ln(n/b)+O(\lg^{1+\varepsilon}(2r))$.

□

4.4 Experimental Results for Two-tier and N-tier Gossip Mechanisms

This set of experiments is designed to simulate data dissemination with various gossip mechanisms. In all experimental setups, a piece of data exists at one of the nodes in the network. In each subsequent round, each gossiper chooses a gossipee and passes on the information to it. Both experiments below model a one-dimensional network in which nodes are placed at unit-interval grids. The network's nodes can perform only truncated-uniform gossip with an adjustable truncation threshold $\in [0, \infty]$.

The number of intended destinations informed and the total communication distance are used to evaluate these gossip mechanisms' effectiveness. Communication distance is closely related to energy consumption, as long-distance communications consume more energy than short communications.

4.4.1 Performance Evaluation of Two-tier Gossip Mechanisms

This experiment was designed to investigate the ability to inform clustered destinations and the communication distances involved, using various gossip mechanisms. A two-tier gossip mechanism is used, and uniform gossip is employed as a comparison. A truncated uniform distribution is used instead of spatial gossip for the second gossipee-selection scheme. Under this probability density function all nodes within distance *ut* inclusively are selected with equal probabilities, where *ut* is known as the *truncation threshold.* The following is the two-tier gossip mechanism, which is the focus of this experiment.

Definition 6 [Two-tier gossip mechanism *M(f, s)*]: The two-tier gossip mechanism *M(f, s)* is defined as a mechanism that is initiated by a node (the first gossiper) in the network, and each node which becomes a gossiper at round *i*:

i) performs uniform gossip for $(f - i)$ rounds, and then

ii) performs truncated-uniform gossip for *s* rounds. ☐

We use $M(f, 0)$ to denote uniform gossip for f rounds.

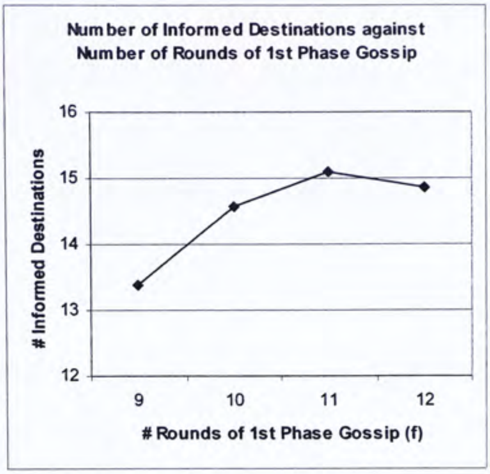
There are 20,000 network nodes in this experiment. The number of clustered destinations b is set at $20\log n$, the number of gossiping rounds permitted t_{max} is set to $(\lg n + \ln n)/2$, and the truncation threshold ut for the second gossipee-selection scheme is $b/6$, with round-offs. The actual values are shown in Table 1.

number of nodes in the network (n)	20,000
number of destination nodes in the cluster (*b*)	86
number of rounds of gossip permitted (t_{max})	12
truncation threshold in the second gossipee-selection scheme of *M(f, s)* (*ut*)	14

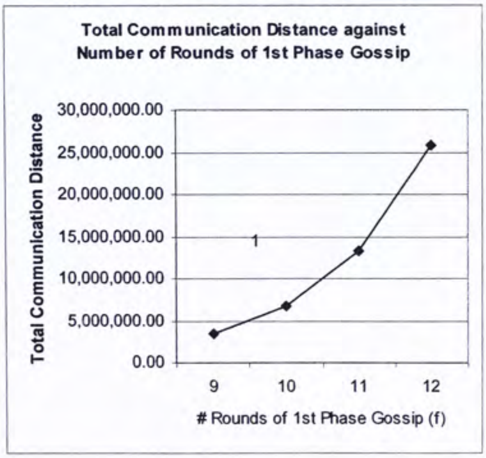
Table 1: Parameters in the experiment for Two-tier Gossip

The value f in $M(f, t_{max}-f)$ is varied to obtain different gossip mechanisms, and the total communication distance and number of informed destinations are investigated. The total communication distance is the sum of the distance of all communication events in all the gossip rounds.

Figure 12 shows the experimental results. The entries along the x-axis represent the gossip mechanisms *M*(9, 3), *M*(10, 2), *M*(11, 1) and *M*(12, 0). All values of the dependent variable are averages over 200 runs of the same program.



(a)



(b)

Figure 12: Results for n = 20,000, gossip mechanism = M(f, 12 -f)

(a) Number of destinations $\in B$ informed against f

(b) Total communication distance against f

It is clear that the two-tier gossip mechanism $M(t_{max}-1, 1)$ outperforms the other gossip mechanisms investigated, including uniform gossip, in terms of the number of informed destinations. However, a premature switch to the second gossipee-selection scheme (this corresponds to $M(f, t_{max}-f)$ with a small f value) could decrease the number of informed destinations.

The total communication distance required for uniform gossip is larger than for the two-tier gossip mechanisms $M(f, s)$. This is because uniform gossip involves many long-range communications across the whole network, while $M(f, s)$ need long-range communications only in the first f rounds.

In summary, the two-tier gossip mechanism can greatly reduce the communication distance needed compared to uniform gossip, and performs approximately the same as uniform gossip in terms of the number of informed destinations. Hence, it is most suitable for networks in which energy conservation is crucial, as long-distance communications require more energy than short-distance ones.

4.4.2 Performance Evaluation of N-tier Gossip Mechanism

The concept of two-tier gossip mechanisms can be extended to N-tier gossip mechanisms, where N is a natural number. One example of this is to have each gossiper choose gossipees that are nearer and nearer as the number of rounds increases. The following defines two N-tier gossip mechanisms, N(q) and G(q), investigated in this experiment. The former's truncation threshold decreases according to arithmetic progression (AP), while the latter follows a geometric progression (GP).

Definition 7 [N-tier gossip mechanism N(*q*)]: In the *i*-th round each gossiper chooses a gossipee uniformly from all nodes within distance $d-\frac{d-ut}{q-1}(i-1)$ inclusively $\forall i \in [1, q]$, where *d* is the network's diameter and *ut* is the final truncation threshold. □

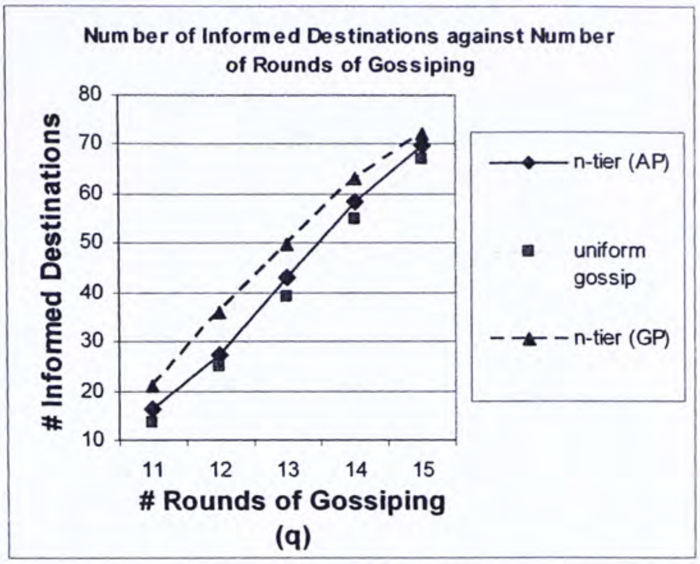
Definition 8 [N-tier gossip mechanism G(q)]: In the *i*-th round each gossiper chooses a gossipee uniformly from all nodes within distance $d(\frac{ut}{d})^{\frac{i-1}{q-1}}$ inclusively $\forall i \in [1, q]$, where *d* is the network's diameter and *ut* is the final truncation threshold. □

Uniform gossip with the corresponding same number of rounds is chosen for comparison. The truncation threshold is chosen to be equal to *b*/6 where *b* is the clusters' size. The values of the parameters in this experiment are shown in Table 2.

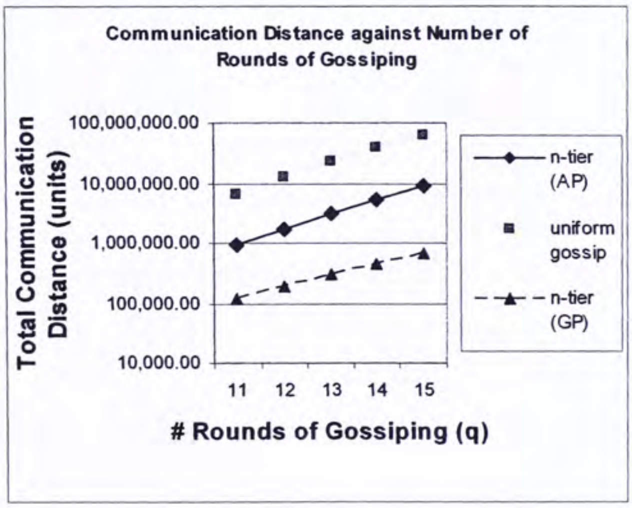
Parameter	Value
number of nodes in the network (n)	10000
number of destination nodes in the cluster (*b*)	80
final truncation threshold (*ut*)	13
Number of tiers (N)	Number of rounds

Table 2: Parameters in the experiment for N-tier Gossip

The results are shown in Figure 13. All values of the dependent variables are averages of 200 runs of the same program.



(a)



(b)

Figure 13: Results for n = 10,000, b=80,

gossip mechanism = N(q), G(q) and uniform gossip

(a) Number of destinations $\in B$ *against q*

(b) Total communication distance against q

The number of destinations informed using either of the N-tier gossip mechanisms is greater than that of uniform gossip in all five experimental cases. The percentage improvement in the number of destinations informed is significant for smaller number of rounds, and decreases as the number of rounds increases. For example, a 20.3% improvement is observed for 11 rounds of *N(q)*, compared to 3.61% for 15 rounds. The improvement in *G(q)* is even more effective than that of N(q) in terms of the number of informed destinations, in all five test cases. Moreover, the total communication distance increases quickly in uniform gossip as the number of rounds increases, but it increases slowly in the case of the N-tier gossip mechanisms.

4.5 Discussion

Traditional uniform gossip does not consider the distance between nodes in gossipee-selection, so message delivery to destinations that are clustered in a small area is as hard as to destinations that are scattered across the network. On the other hand, gossipee-selection schemes that tend to select nearby nodes take time to propagate messages to distant regions. For example, spatial gossip with any $\rho \in (1, \infty]$ takes at least an order of lg(*d*) time to reach a node distance *d* away [17] w.h.p., which is ineffective if the cluster is far away (note that the analysis of the gossip mechanism T discussed above makes no assumptions about the location of the cluster of destinations). A special case of this is neighbourhood flooding, equivalent to spatial gossip with $\rho = \infty$, which takes an order of *d* time to deliver a message.

As revealed in Sections 4.2 and 4.3, the effectiveness of the two-tier gossip mechanism *T* depends on the size of *B*. If *B* is too large (e.g., comparable to the size of the whole network), the time for the second phase dominates because the radius r of the bounding hypersphere is large. In this case the problem is similar to a network broadcast, so simple uniform gossip may be more appropriate.

On the other hand, if *B* is very small (e.g., in the extreme case, only one node) then the problem is similar to a unicast routing problem. Again, in this case *T* is ineffective compared to uniform gossip. However, in most applications of the Clustered Destination Problem, the cluster's size lies within these extremes.

The shape of cluster *B* can also affect the time required to complete this gossip mechanism. A long and narrow cluster requires a large hypersphere to bound, meaning that more time is required for the mechanism's second phase. In fact, the mechanism is most efficient if the cluster is close in shape to a hypersphere.

A direct implementation for our proposed gossip mechanism is to attach a header to a message indicating the number of rounds that this message has been propagating (this avoids the need for synchronous clocks). In this implementation the message length complexity of *T* is $O(\lg(n/b))$.

Section 5 Results for Semantic-dependent Gossip Mechanisms

5.1 Background

The following heat source location-detection problem is studied:

Problem: Given a network with randomly-located heat sources, a query with parameter T is initiated by a network node to find at least one node, if any, with temperature > T. □

The network nodes' only communication capability is to pass on information to another node selected from an upper-truncated uniform distribution of inter-node distance, with a dynamically adjustable truncation threshold $\in [0, \infty]$. The nodes have no idea of the global network's condition and can detect only the local temperature. Each node can send, at most, one message per round. All nodes can detect the intensity of the radiation emitted by heat source(s). The network's nodes are placed at integral coordinates of a two-dimensional Euclidean space.

The algorithms discussed below take place in rounds, but their accuracy is not affected by asynchronous clocks or network failures.

A classical solution to the above heat source location-detection problem is to employ uniform gossip to spread a query message from informed nodes ("gossipers") to randomly-selected network nodes ("gossipees"), which takes place in rounds and terminates when the desired nodes are found. Consider the example in Figure 1, which shows the network situation at round *m* for a certain positive integer *m*. Nodes with a temperature above 40 units are considered "hot" and have to be found. At this time nodes 1 and 2 are informed. At the (*m*+1)-th round, nodes 1 and 2 uniformly and randomly select other nodes to pass on the information to, and nodes 5 and 6 are selected, respectively. At the (*m*+2)-th round, nodes 1, 2, 5 and 6 select nodes 7, 8, 9 and 10, respectively, to pass on the information to. This goes on until the hot nodes are found. Some of the informed and uninformed nodes are shown in Table 1.

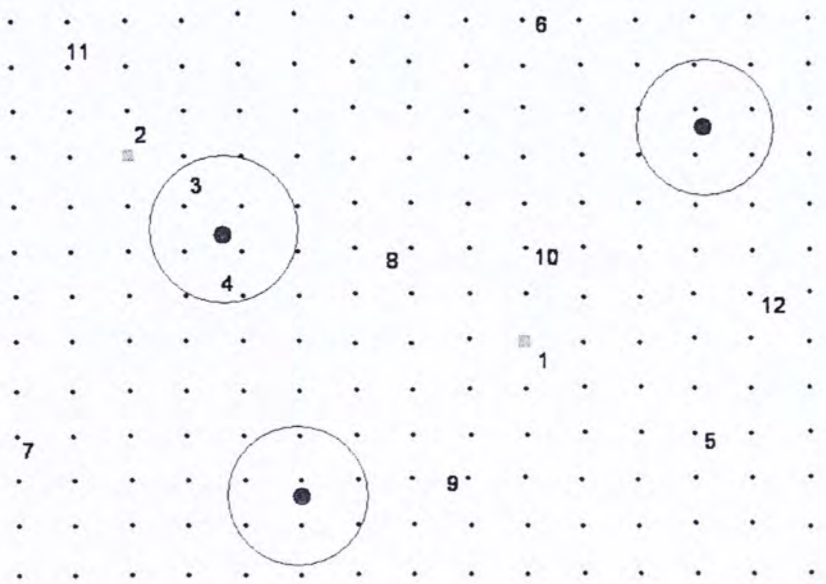


Figure 14: Network situation at gossip round m

Solid circles are heat sources and their surrounding nodes, inside the circular regions, have a high temperature.

Black dots are uninformed network nodes, and rectangles are informed nodes (gossipers).

Node	Temperature	Round m	Round m+1	Round m+2
1	10	1	1	1
2	25	1	1	1
3	90	0	0	0
4	85	0	0	0

Table 3: Nodes informed at various rounds of uniform gossip

1=informed, 0=uninformed

The above method works but is inefficient. First, it does use the nodes' sensor values at all. Second, the nodes continue to randomly select gossipees, which can be far away, even when there is an indication the desired node(s) to be found might be nearby. To overcome this inefficiency, our proposed gossip mechanism uses locally detected sensor values to estimate the distance from the heat sources, so that the probability of locating hot nodes is higher than in the above method.

Consider the same example as depicted in Figure 1, but that uses the semantic-dependent two-tier gossip mechanism discussed extensively in Section 4.2.2, instead of uniform gossip. At round (m+1), node 1, having a low temperature, employs uniform gossip and selects node 11 as gossipee. Node 2 has a moderate temperature and uses its detected temperature to estimate its distance from a nearby heat source. It performs truncated-uniform gossip according to the estimation, and selects node 3 as gossipee. At round (m+2), node 3 performs truncated-uniform gossip and selects node 4 as gossipee (while nodes 1 and 2 select their own gossipees). The situation is summarized in Table 2.

		Informed?		
Node	Temperature	Round m	Round m+1	Round m+2
1	10	1	1	1
2	25	1	1	1
3	90	0	1	1
4	85	0	0	1

Table 4: Nodes informed at various rounds of semantic-dependent gossip

1=informed, 0=uninformed

Hot nodes 3 and 4 are informed within two rounds with the semantic-dependent two-tier gossip mechanism, but uniform gossip fails to inform any hot nodes within the same time scale. The example above makes it clear that semantic-dependent two-tier gossip mechanisms can improve the efficiency of locating hot nodes because they use sensor data to select nodes that are likely to be hot.

5.2 Theory

The radiation intensity obeys the following rule:

$$I = \frac{C}{d^2} \tag{5.1}$$

where I is the radiation intensity detected at the sensor node, C is a constant and d is the distance between the sensor node and the heat source.

Our proposed gossip mechanism uses the information obtained in the application level, namely, the intensity I detected, to alter the gossipee-selection criterion.

Definition 9 [S(u)]: The semantic-dependent gossip mechanism *S(u)* is defined as a gossip mechanism that employs upper-truncated uniform gossip with a truncation threshold = u. □

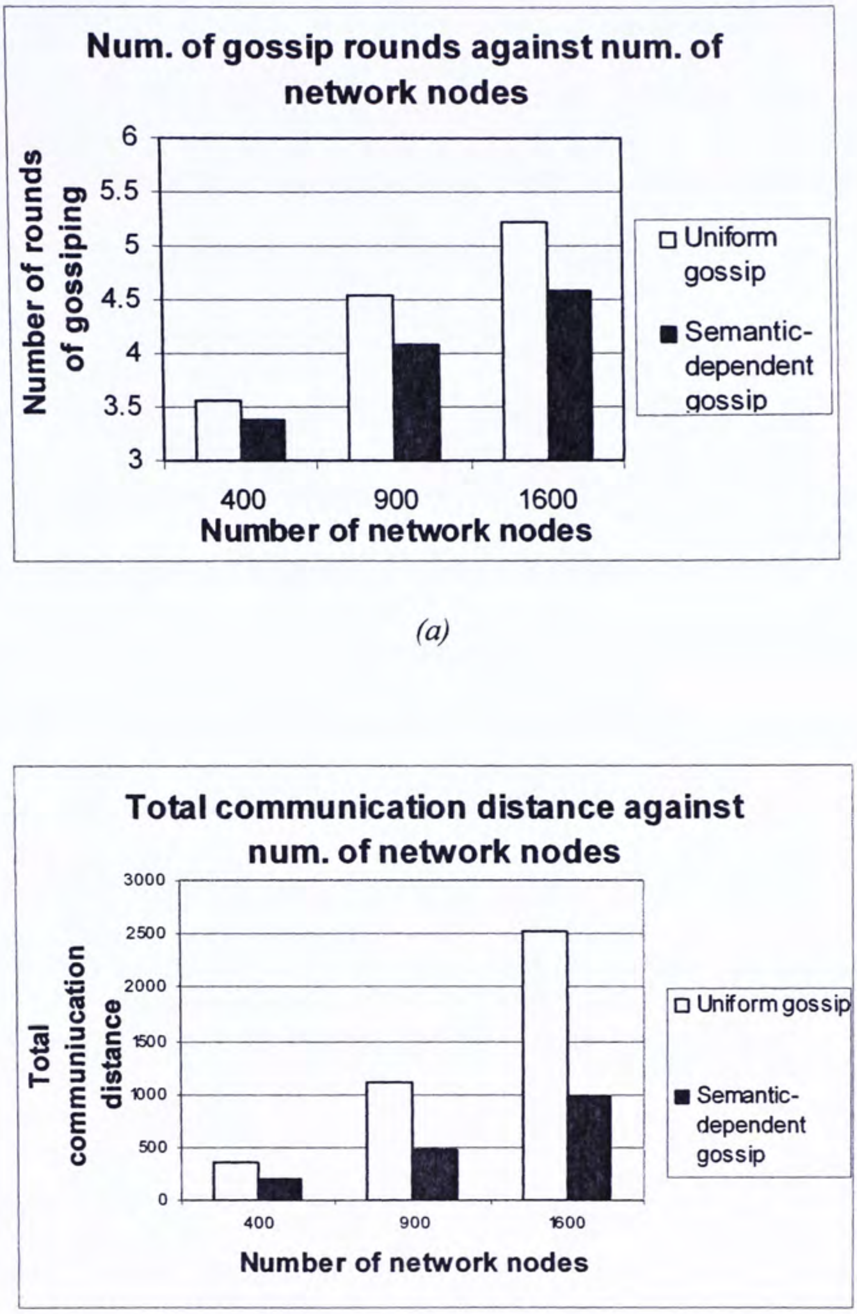
Note that this upper-truncated uniform gossip has a mean value of $u/2$. For example, consider the gossip mechanism $S(2\sqrt{C/I})$. Here, I is the intensity detected locally and C is known to the node that initializes the gossip event. This gossip mechanism directly takes advantage of equation 5.1 by employing truncated uniform gossip with a mean = $\sqrt{C/I}$, which is promised to be the distance to the heat source, assuming there is only one heat source and the sensor is not affected by noise. In the following experiment we employ a slight variant of this mechanism.

5.3 Detection of Single Moving Heat Source with $S(\max(2\sqrt{C/I}, \sqrt{C/h}))$

This experiment evaluates gossip mechanisms for their ability to detect the location of a moving heat source of known temperature. It considers two-dimensional networks with nodes placed at integral coordinates. A node is defined "hot" whenever its detected intensity exceeds a predefined threshold h. The location of the heat source is said to be detected whenever a hot network node is found. Both the communication distances involved and the number of rounds of gossip are used as evaluation metrics.

The performance of $S(\max(2\sqrt{C/I}, \sqrt{C/h}))$ is evaluated. Here we introduce the term $\sqrt{C/h}$, which is the radius of the "hot" region around the heat source, to avoid having the truncation threshold set at an undesirably small value. Three test cases are considered; they differ only in the number of network nodes. 1500 trials are performed for each test case and the average values of both evaluation metrics are obtained. The constant C, which characterizes the heat source's temperature, is set at 300, while a node is considered hot if its detected intensity exceeds 40. The heat source moves at a pace of 4 distance units per round. The algorithm terminates whenever at least one hot node is detected, and the number of rounds of gossip and communication distance involved are recorded. Uniform gossip is employed as the benchmark.

The experimental results are shown in Figure 15.



(a)

(b)

Figure 15: Results for location detection of a moving heat source with $S(max(2\sqrt{C/I}, \sqrt{C/h}))$

(a) Number of gossip rounds

(b) Average communication distance

These results suggest that adjusting the truncation threshold according to the detected intensity reduces, compared to uniform gossip, the time required to detect a static heat source's location,. The improvement is observed in all three test cases. Moreover, the communication distances needed are reduced significantly. Improvement in both metrics becomes more significant as the number of network nodes increases.

5.4 Detection of Multiple Static Heat Sources with Two-tier Gossip mechanism

The semantic-dependent gossip mechanism in Section 4.2.1 relies on the fact that we can calculate the distance from the heat source using the locally detected intensity. However, when the number of heat sources is unknown, the detected intensity is the sum of radiation from all the sources, so this technique cannot be applied in that case.

Although the actual distance from the heat source cannot be calculated, it can be estimated. One key observation is that, if the local temperature is high, indicating the existence of a nearby heat source, this high temperature would be due mostly to this source, and the effect of other sources becomes negligible. This applies to networks whose heat sources are sparsely distributed across the network's space, so that a high temperature would be due to one nearby heat source, rather than the sum of two or more. The estimation is therefore expected to work well in large networks (i.e., with a large network space) with a small number of sparsely-distributed heat sources.

The two-tier gossip mechanism to be employed is defined below.

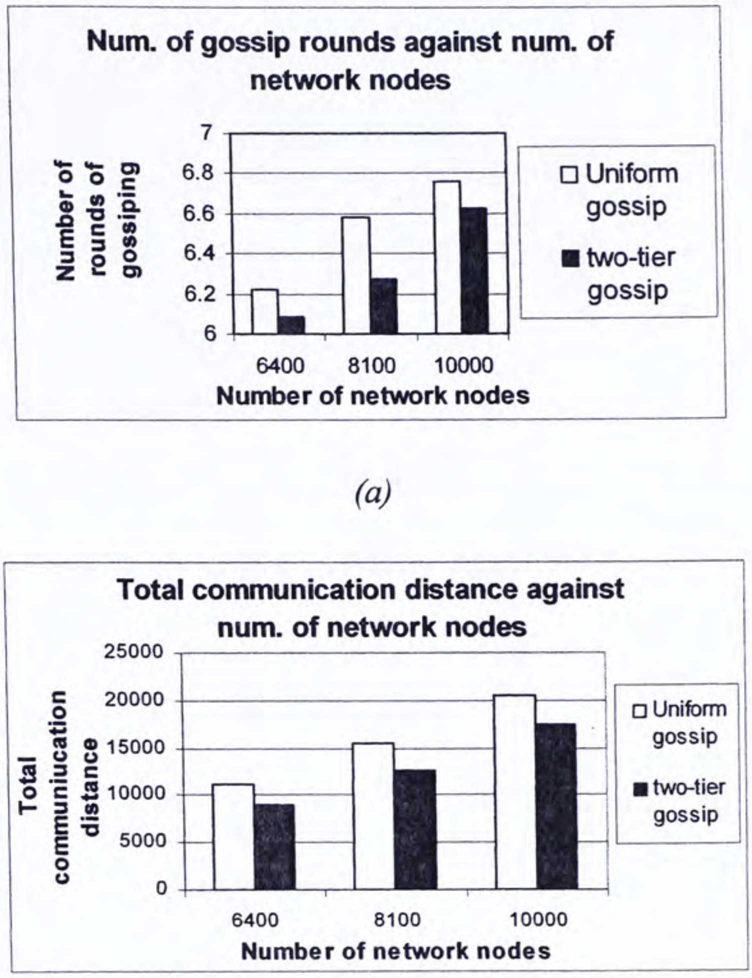
Definition 10 [T(*m*; *S*(*u*), *w*)]: The two-tier gossip mechanism T(m; $S(u)$, w) is defined as a mechanism that is initiated by a node (the first gossiper) in the network, and in which each node that becomes a gossiper at round i:

i) performs uniform gossip for $(m-i)$ rounds if $m > i$, and then

ii) performs semantic-dependent gossip $S(u)$ afterward if the detected intensity >= w, and performs uniform gossip otherwise.

In other words, if a network node's detected intensity < w, then it carries out uniform gossip from the first to the last round. Otherwise, the node would first employ uniform gossip, then switch to $S(u)$ from the $(m+1)$-th round onward.

Three sets of experiments are carried out, which differ only in their number of network nodes. All the parameters are the same as in the experiment in Section 4.2.1, except that three static heat sources (all randomly located in the network's space) are now considered. The network's nodes are unaware of the number of heat sources. The algorithm terminates when at least one hot node is detected. The two-tier gossip mechanism $T(4; S(\max(2\sqrt{C/I}, \sqrt{C/h})), 20)$ is evaluated against uniform gossip, and the results are shown in Figure 16.



(a)

(b)

Figure 16: Results for location detection of multiple static heat sources with

$T(4;\ S(max(2\sqrt{C/l}, \sqrt{C/h})),\ 20)$

(a) Number of gossip rounds

(b) Average communication distance

The results are encouraging in the sense that although the number of heat sources is not known, and hence no accurate calculation of the distance from any of the heat sources can be made, estimation can lead to improvements, compared to uniform gossip, in both the number of rounds and the communication distances involved.

5.5 Discussion

The efficiency and improvement of semantic-dependent gossip mechanisms compared to uniform gossip are presented. Distance-estimating heat sources, using local sensor values and switching to a gossipee-selection scheme that favours nearby nodes increase the probability of finding nodes that match a given query and, hence, reduce the time required to find these nodes. Communication distances required are reduced, too, because reliance on long-term communications is reduced.

Section 6 Conclusion

Novel two-tier gossip mechanisms are presented, and their ability to solve the Clustered Destination Problem and data dissemination in general are discussed. These mechanisms can disseminate information sparsely over the whole network, followed by intensive disseminations over small regions. The mechanisms display a variety of beneficial properties, including a reduction in long-range communications compared to one-tier mechanisms, for example, uniform gossip.

As one possible extension of this concept, examples of N-tier gossip mechanisms as well as semantic-dependent gossips are studied experimentally and their effectiveness over uniform gossip is presented.

Many issues surrounding communication mechanisms have not yet been explored by the research community. For example, what are the inherent advantages of two-tier or N-tier gossip mechanisms over one-tier ones? What are these communication mechanisms' general limitations, in terms of lower bounds for number of messages generated, query time, average communication length, robustness and so on? What additional performance guarantees can these gossip mechanisms provide? What are the classes of protocols that can be built on top of data dissemination making use of gossiping? What are the relationship between the truncation threshold for truncated uniform gossip in two-tier gossip mechanisms and the network size? How sensitive is the results to different network node densities, and when the nodes are not randomly placed but are consisted of dense clusters? How would the gossip mechanism perform in the case of detection of multiple moving heat sources?

Although gossip mechanisms are simple to define, they are shown to be highly robust to network and nodal failures, due to their probabilistic nature. By investigating communication mechanisms as simple as gossip mechanisms, it is hoped that efficient and robust protocols can be developed.

Section 7 References

[1] Bonnet, P. and Gehrke, J. and Seshadri, P., *Querying the Physical World* (2000), IEEE Pers. Commun., vol. 7, Oct. 2000, p. 10--15

[2] Chandra, R. and Ramasubramanian, V. and Birman, K., *Anonymous gossip: Improving multicast reliability in mobile ad-hoc networks*, Proc. 21st International Conference on Distributed Computing Systems, 2001, pp. 275–283

[3] Chandy, K. M. and Lamport, L., *Distributed snapshots: Determining global states of distributed systems*. ACM Transactions on Computer Systems, 3(1), February 1985, p. 63-75

[4] Dasgupta, P., *Distributed Automatic Target Recognition Using Multiagent UAV Swarms*, AAMAS, May 2006, Hakodate, Hokkaido, Japan

[5] Datta, A., *SoS: self-organizing substrates*, Thèse EPFL, no 3615 (2006)

[6] Demers, A. and Greene, D., et al., *Epidemic algorithms for replicated database maintenance*, Sixth Annual ACM Symposium on Principles of Distributed Computing, Vancouver, British Columbia, Canada, 1987

[7] Drabkin, V. and Friedman, R. and Segal, M., *Efficient Byzantine Broadcast in*

Wireless Ad-Hoc Networks, DSN 2005: 160-169

[8] Gerla, M. and Chen, L.-J. and Lee, Y.-Z. and Zhou, B. and Chen, J.-W. and Yang G. and Das, S., *Dealing with Node Mobility in Ad Hoc Wireless Network*, Formal Methods for Mobile Computing, pp 69-106, Springer-Verlag LNCS, 2005

[9] Govindan, R. and Hellerstein, J. and Hong, W. and Madden, S. and Franklin, M. and Shenker. S., *The Sensor Network as a Database*, Technical Report 02-771, Computer Science Department, University of Southern California, September 2002

[10] Gupta, I. and van Renesse, R. and Birman, K. P., *Scalable Fault-Tolerant Aggregation in Large Process Groups*, Proceedings of the 2001 International Conference on Dependable Systems and Networks, 2001

[11] Haas, Z. and Halpern, J. and Li, L., *Gossip-based ad hoc routing*, IEEE InfoCom Proceedings 2002, vol. 3, June 2002, p. 1707-1716

[12] Hawick, K. A., *Managing Online Data Archives with the DISCWorld Metacomputing Environment*, Distributed and High Performance Computing Group, 1998, Technical Report 046

[13] Iamnitchi, A. and Ripeanu, M. and Foster, I., *Locating Data in (Small-World?) Peer-to-Peer Scientific Collaborations*, 1st International Workshop on Peer-to-Peer Systems, Cambridge, MA, USA, 2002

[14] Karp, R. M. and Schindelhauer, C. and Shenker, S. and Vöcking, B., *Randomized Rumor Spreading*. IEEE Symp. on Foundations of Computer Science, 2000, p. 565-574

[15] Kempe, D. and Kleinberg, J. M. and Demers, A., *Spatial Gossip and Resource Location Protocols*, JACM, Vol 51/6, Nov. 2004, p. 943-967, also in Proceedings of STOC 2001, Crete, Greece

[16] Kempe, D. and Kleinberg, J. M. and Demers, A. J., *Protocols and Impossibility Results for Gossip-Based Communication Mechanisms*, Proceedings of FOCS 2002, Vancouver

[17] Kempe, D. and Dobra, Alin and Gehrkey, Johannes, *Gossip-Based Computation of Aggregate Information*, Proceedings of the 44th Annual IEEE Symposium on Foundations of Computer Science, 2003, p.482

[18] Lau, R. and Demers, S. and Ling, Y. and Siegell, B. and Vollset, E. and Birman, K. and van Renesse, R. and Shrobe, H. and Bachrach, J. and Foster, L., *Cognitive Adaptive Radio Teams*, 2nd International Workshop on Wireless Ad-hoc and Sensor Networks, June 2006

[19] Lin, Meng-Jang and Marzullo, Keith, *Directional Gossip: Gossip in a Wide Area Network*, Proceedings of the Third European Dependable Computing Conference on Dependable Computing, p.364-379, 1999, September 15-17

[20] Luo, J. and Eugster, P. and Hubaux, J.-P., PILOT: ProbabilistIc Lightweight grOup communication sysTem for Mobile Ad Hoc Networks, IEEE Transactions on Mobile Computing, Volume: 03 , Issue: 2 , April 2004

[21] Martucci, L. A. and Schweitzer, C. M. and Venturini, Y. R. and de Brito Carvalho, T. C. M. and Ruggi, W. V., *A Trust-Based Security Architecture for Small and Medium-Sized Mobile Ad Hoc Networks*, Proceedings of MED-Hoc-Net 2004 Third Annual Mediterranean Ad Hoc Networking Workshop. Yavuz - Turkey: Med-Hoc-Net 2004, 2004. vol. 1, p. 278-290

[22] MoskAoyama, *D. and Shah, D., Computing Separable Functions via Gossip*, PODC 2006

[23] Mosk-Aoyama, D. and Shah, D., *Information Dissemination via Network Coding*, 2006 IEEE International Symposium on Information Theory

[24] Nandan, A. and Das, S. and Pau, G. and Gerla, M. and Sanadidi, M. Y., *Co-operative downloading in vehicular ad-hoc wireless networks*, Wireless On-demand Network Systems and Services, 2005.

[25] Ning, N. and Wang, D. S. and Ma, Y. Q. and Hu, J. F. and Sun, J. and Gao, C. N. and Zheng, W. M., *Genius: Peer-to-Peer Location-Aware Gossip Using Network Coordinates*, International Workshop on Grid Computing Security and Resource Management Atlanta, USA. May 2005

[26] Özkasap, Ö. and Genç, Z., *Peer-to-Peer Epidemic Algorithms for Reliable Multicasting in Ad Hoc Networks*, International Conference on Information Technology, Istanbul, December 2004.

[27] Pittel, B., *On Spreading a Rumor*, SIAM Journal on Applied Mathematics, Vol. 47, No. 1, Feb., 1987, p. 213-223

[28] Vollset, E. and van Renesse, R. and birman, K., *ChickWeed: Group Communication for Embedded Devices in Opportunistic Networking Environments*, 3rd International Workshop on Dependable Embedded Systems, in conjunction with 25th Symposium on Reliable Distributed Systems (SRDS 2006), October 2006

[29] Xing, B. and Lazardis, I. and Hore, B. and Venkatasubramanian, N. and Mehrotra, S., *CREW: A Gossip-based Flash-Dissemination System*, 26th IEEE International Conference on Distributed Computing Systems (ICDCS'06) p. 45

[30] Xu, B. and Wolfson, O. and Chamberlain, S., *Spatially distributed databases on sensors,* Geographic Information Systems, Proceedings of the 8th ACM international symposium on Advances in geographic information systems

[31] Zhang, G. and Parashar M., *Cooperative Defence against DDoS Attacks*, Journal of Research and Practice in Information Technology, Vol. 38, No. 1, February 2006

Appendix Prove of Result 4.2

This is a proof of Result 4.2 which is used for proving Lemma 2. The definition of $x_n(t)$, $g_n(t)$ and $h(x)$ can be found in Section 4.2.

To prove: $x_{(2-\delta)^{\alpha+1}r}(1) \geq x_{(2-\delta)^{\alpha}r}(0)$, where α and r are natural numbers and $\delta < 1$ is a constant.

Proof

$g_{(2-\delta)^{\alpha+1}r}(0) = h(x_{(2-\delta)^{\alpha+1}r}(0))$ (Definition of $h(x)$)

$> \mathrm{h}(x_{(2-\delta)^{\alpha}r}(0))$ ($\because x_{(2-\delta)^{\alpha+1}r}(0) < x_{(2-\delta)^{\alpha}r}(0)$)

$= g_{(2-\delta)^{\alpha}r}(0)$

$\geq g_r(0)$

$\geq 2-\delta$. (Definition of r)

$g_{(2-\delta)^{\alpha+1}r}(0) \geq 2-\delta$ ---[Result 2.2]

$x_{(2-\delta)^{\alpha+1}r}(1) = g_{(2-\delta)^{\alpha+1}r}(0)x_{(2-\delta)^{\alpha+1}r}(0)$ (Definition of $g_n(t)$, $t = 0$)

$= g_{(2-\delta)^{\alpha+1}r}(0).\dfrac{1}{(2-\delta)^{\alpha+1}r}$ (Definition of $x_n(t)$, $t = 0$)

$> (2-\delta).\dfrac{1}{(2-\delta)^{\alpha+1}r}$

(by Result 2.2)

$= \dfrac{1}{(2-\delta)^{\alpha}r}$

$= x_{(2-\delta)^{\alpha}r}(0)$, as required.