Strategies for Minority Game and Resource Allocation

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Abstract

In multiagent environments, the environments may be dynamic and uncertain. Each agent needs to choose its action and adapt to the dynamic environments. To achieve this, this thesis introduces strategies for agents to choose their actions to adapt to the dynamic environments and then maximise their utilities in some multiagent environments, such as minority games and resource allocation.

In the traditional minority game, each agent chooses the highest-score predictor at every time step from its initial predictors which are allocated randomly. In this thesis, we study a version of the minority game in which one individual privileged agent is allowed to join the game with different memory size from the other agents and free to choose any predictor, while each of the other agents owns small number of predictors. We investigate the privileged agent's wealth in different dynamic environments. Simulations show that the privileged agent using the proposed intelligent strategy can outperform the other agents in the same model and other models proposed in previous work in terms of individual wealth. We also discuss the impacts of the parameters on the privileged agent's wealth, such as the number of predictors the privileged agent owns and its memory size. Moreover, we discuss the impact of the number of predictors the other agents possess and their memory sizes on the privileged agent's wealth.

As an application of minority games, we can model a class of multiagent resource allocation systems into minority games. The system consists of competitive agents that have to choose among several resources to complete their tasks. The capacities of resources may change gradually or abruptly. The objective of the resource allocation is that agents can adapt to the dynamic environment autonomously and make good utilisation of resources. We propose an adaptive strategy for agents to use so that agents are able to adapt to the environment with gradually or abruptly changing capacities and make good utilisation of resources. This strategy is based on individual agent's experience and prediction. Simulations show that agents using the adaptive strategy can adapt effectively to the changing capacity levels and the system as a whole results in better utilisation of resources than previous work. Finally, we also investigate how the parameters affect the performance of the strategy.

摘要

在多代理人的環境下，環境可能是變化的和不確定的。每個代理人需要選擇它們的行為並且適應動態的環境。本論文介紹了在某些多代理人環境下，例如在少數者博弈和資源分配問題的多代理人環境下，代理人使用策略選擇自己的行為，使其能夠適應動態環境從而最大化自己的利益。

在傳統的少數者博弈中，每個代理人最初時隨機分配到一些預測表，每個代理人每次選擇最高分的預測表作決策。在本論文中，我們研究一類少數者博弈，在這類博弈中，有一個擁有特權的代理人，這個代理人允許有跟其他代理人不同的記憶大小，並且可以任意選用預測表。我們研究這個特殊代理人在不同動態環境下的利益。實驗表明，採取我們所提出的智慧策略的特殊代理人所獲得的利益會比其他只擁有一定數量預測表的代理人高，並且也比使用其他策略模型的代理人高。另外，我們也討論這個特殊代理人所擁有的預測表的數量和記憶的大小對其利益的影響。此外，我們討論其他代理人所擁有的預測表的數量和記憶的大小對這個特殊代理人利益的影響。

作為少數者博弈的一個應用，我們把一類多代理人資源分配系統建模為少數者博弈問題。這個系統包含一些競爭的代理人，這些代理人必須在多個資源中選

擇其中一種以完成它們的任務。這些資源的容量可漸變也可驟變。研究這些資源分配問題的目的是讓代理人能自主地適應動態的環境並且充分的利用資源。針對這個問題，我們提出一個適應性的策略供代理人使用，使它們不僅能夠適應漸變或驟變的資源容量，而且能夠充分的利用資源。這個策略是基於各個代理人的經驗與預測。實驗表明，採用這個策略的代理人能有效地適應變化的環境並且比以前的策略能更好的利用資源。最後，我們研究各參數的變化對這個策略性能的影響。

Acknowledgments

I would like to take this opportunity to express my sincere gratitude to my supervisor Prof. Ho-fung Leung, for his generous guidance during my post-graduate study. Prof. Leung has taught me much not only on research, but also on English writing skill, presentation, teaching and so on. His numerous supervision and encouragement are extremely valuable in many aspects of my development.

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Chapter 1

Introduction

The field of Multiagent Systems (MAS) is a subfield of Artificial Intelligence. It is also interdisciplinary: it takes inspiration from such diverse area as economics, philosophy, logic, ecology, and the social science [1, 52]. It aims to provide both principles for construction of complex systems involving multiple agents and mechanisms for coordination of agents' behaviors. Each agent's behavior is governed by a set of simple rules. The collective behavior of agents in general alters the environment. Agents need to reevaluate or possibly change their strategies to compete in the altered environment more effectively.

Game theory is a mathematical theory that studies interactions among self-interested agents. The tools and techniques of game theory have found many applications in multiagent systems. Von Neumann and Morgenstem [47] found game theory to study situations in which multiple agents interact in order to each maximise an objective (payoff) function. Each agent's objective function is determined not only by its own action but also the actions of other agents. The payoffs also depend on information that is private to the individual agents if the agent has incomplete information. In recent year, decision theory and game theory have had a profound impact on computer science, such as artificial intelligence. The decision-theoretic approach provides a definition of what it means to build an intelligent agent. Meanwhile, game theory has been widely adopted as the basis for building multiagent systems.

The research addressed in this thesis focuses on individual agent's wealth (payoff) in minority games, which has attracted a lot of interest in Physics [10, 8, 13, 29, 30, 36, 51, 22, 42, 53]. The game models some specific situations in multiagent systems. It consists of multiple competing agents. Every agent wants to be on the minority side. The environment is dynamic and uncertain. The collective behavior of agents alters the environment. Researcher use the tools and techniques of game theory to solve the problems arising in minority games. In addition, one important application of minority games is to model some Multiagent Resource Allocation (MARA) problems into minority games.

1.1 Scope

Inspired by inductive reasoning, Arthur [5] introduces the El Farol Bar problem. The problem has been one of the most widely studied examples of complex adaptive systems of interacting agents. The model consists of N persons who have to decide independently whether to attend the El Farol bar every week. The bar has a limited capacity, and people try to avoid attending it when it is overcrowded. There is no explicit communication between people, and the only information available to them is the number of people that went to the bar (the attendance) in the previous weeks. Every person goes to the bar if she expects fewer than the bar's capacity to show up, or stays at home, if this is not the case. Arthur suggests that every person should own a number of predictors and use the predictor that is currently most accurate to make the decision. For example, a predictor may be: to predict next week's attendance to be the same as that of the previous week, or an average of those in the last four weeks, etc. After all people have made their decisions, those predictors that have made correct predictions are awarded one more point.

Challet and Zhang [10] formalize the El Farol Bar problem as minority games. The game consists of an odd number of N agents. At each time step, each of the N agents independently decides to join one of the two sides, labeled 0 or 1. After all agents have made their decisions, those who are on the minority side win, while the other agents belonging to the majority side lose. Instead of making decisions based on the attendance in previous time steps, which results in a lot of possible predictors, they suggest that each agent records the winning sides in the past M time steps and choose their actions based on these records. For a record of winning sides in the past M time steps, there will be 2^M possible histories of winning sides. Each prediction of a predictor maps the recent M winning sides to a prediction (side 0 or side 1). So, the total number of predictors is 2^{2^M}. The number of predictors may be too large for agents to handle even for a moderate M. Each agent randomly draws S predictors to use and keep track of. All agents always use the highest-score predictors to make decisions. After the winning side is announced, each of those predictors that have made correct predictions are added one point and those agents that on the winning side also get one point.

The minority game represents a fascinating model of a dynamical, complex adaptive system where agents of a population repeatedly compete to be on the minority side. The minority game offers a simple paradigm for the decision dynamics underlying financial markets. Johnson1 et al. [23, 21, 20, 37] and Samanidou et al. [40] study markets of heterogenous agents in the form of the minority game. Johnson1 et al. [21] suggest that the fluctuations observed in financial time-series reflect the interactions, feedback, frustration and adaptation of the markets' many participants (agents) at some levels. Marsili [35] models a market toy model as follows. Agents can buy or sell an asset at each time step. After each time step, if there are more buyers than sellers, the price is high, and if there are more sellers than buyers the price is low. If the price is high, sellers do well; if the price is low, buyers wins. The market as an adaptive competitive system is driven by the minority rule. The minority group always wins, irrespectively of whether they were buyers or sellers. However, the minority game lacks some features in a real market, such as agents may have different payoff for their actions. This makes some of the behavioral assumptions on which the minority game is based questionable when applied to financial markets.

The minority game incorporates the minority rule and agents' heterogeneity in the model as well as an agent-based approach. The minority dynamics plays an important role in adaptive competitive systems. Studying the dynamics associated with the minority rule is of great interest for understanding the fundamental general behavior of adaptive competitive systems. Also, there has been a lot of attention in the solution of general problems using multiagent models, such as Weiss [48] and Wooldridge [49].

The allocation of resources within a system of autonomous agents is an important problem in the area of computer science. The field is sometimes called Multiagent Resource Allocation (MARA). In the resource allocation system, each agent is responsible for a task. Different agents compete for limited resources to complete their tasks. An allocation is a particular distribution of resources among the agents. There are two objectives of multiagent resource allocation. One is to allocate the resources to the agents so that they can complete their tasks. The other is that the resources are fully utilised, i.e. the resources are not overloaded or underloaded.

Generally speaking, an approach for solving resource allocation problems may be centralised or distributed. For centralised approaches [38, 41], a central controlling authority or resource management is responsible for deciding on the final allocation of resources among agents. In the simplest case, agents just need to ask a central broker or dispatcher for available resources. The centrilised allocation mechanism is complex and the problem is generally NP-complete [14]. For distributed approaches [11, 32, 45], agents may cooperate or act independently. They can coordinate implicitly or explicitly with one another to achieve a consensus on the allocation of resources. In this approach, a centralised allocation mechanism is not needed.

1.2 Motivation

A lot of research have been done on minority games. Some work investigates the wealth of special agents in minority games. However, the previous work do not take into account the number of predictors agents possess. A special agent with the same number of predictors as the other agents but a larger memory size can indeed obtain more wealth than all of the other agents for smaller memory size, but the special agent receives less wealth than the other agents for larger memory size. In this thesis, we investigate a privileged agent's wealth in different environments where each of the other agents has the same number of predictors and memory size. The privileged agent can have different number of predictors and a different memory size from those of the other agents. The only available information for the privileged agent is the history information and its own predictors. We find that the privileged agent with larger memory size than the other agents and all its possible predictors can obtain more wealth than the other agents for almost all values of memory sizes.

Besides, we investigate an application for minority games. We model multiagent resource allocations into minority games. In the systems, agents choose among several resources to complete their tasks. The objective of the system aims at globally optimising the resource usage in the system. The goal of each agent is to adapt its resource selection behavior to the behavior of the other agents as well as to the changing capacities of the resources and to the changing load, without having to know what they are. Under the circumstances with different changing capacities, the resource utilisation of previous work is still very large. In this thesis, we design an adaptive strategy for agents to use in multiagent resource allocation systems with either gradually changing or abruptly changing capacities. The adaptive strategy is based on individual experience and prediction. The experience is based on the past allocation records. The prediction is based on resource capacities in previous rounds. Each agent has an attitude towards each resource. If agents detect that the current capacity differs from the previous capacity largely, agents will reset their experience and attitudes. The strategy is adaptive in that each agent's attitude is updated according to the agent's actions and the previous allocation result. Simulations show that agents using the adaptive strategy as a whole can adapt very effectively to the changing capacity levels and result in better untilisation of resources than previous work. This means that the collective behavior of agents is able to adapt to the dynamic environment.

1.3 Structure of the Thesis

This thesis is organized as five chapters. The next chapter introduces the issues related to minority games and some characteristics, and a survey is provided on current strategies for agents to use in minority games and resource allocation. An intelligent strategy for one agent to use in the minority game is described in Chapter 3 and together with the experimental analysis and comparisons with other strategies in terms of wealth. In Chapter 4, an adaptive strategy for agents to use in the resource allocation and the experimental results with comparisons in terms of resource utilisation are followed by. Finally, conclusions and future work are given in Chapter 5.

Chapter 2

Literature Review

In multiagent environments, each agent needs to choose its action and adapt to dynamic environments. Game dynamics offers a rich foundation for studying learning in multiagent systems. In the game theory formalism, each agent is characterised by a set of strategies. A strategy is a mapping from state history to action. Each agent seeks to maxmise its utility. Game dynamics studies the behavior of agents in response to games that are played many times successively. Agents in the game may cooperate or act competitively. Game dynamics has appealing properties as a control mechanism for multiagent systems in that it is distributed, flexible and scalable.

In this chapter, we first take a look at the definition of an agent and the properties of intelligent agents. Then we overview the literature of multiagent systems. Besides, we also take an overview over the literature of minority games, including the characteristics of the game and the strategies for agents to use in the game. Moreover, the literature of resource allocation is following by. A class of resource allocation is an application of minority games.

2.1 Intelligent Agents and Multiagent Systems

2.1.1 Intelligent Agents

In dictionaries, agents have a lot of meanings. The definition of the term agent presented here is suggested by Wooldrige [49]. That is, an agent is a computer system that is situated in some environment, and that is capable of autonomous action in this environment in order to meet its design objectives. Figure 2.1 gives an abstract view of an agent. An agent takes sensory input from the environment, and produces action output that affects the environment. The interaction is usually non terminating. In the most general case, agents will be acting on behalf of users with different goals and motivations. To successfully interact, they will require the ability to cooperate, coordinate, and negotiate with each other, much as people do. It is believed that agents are an appropriate software paradigm through which to exploit the possibilities presented by massive open distritbuted systems, such as the Internet.

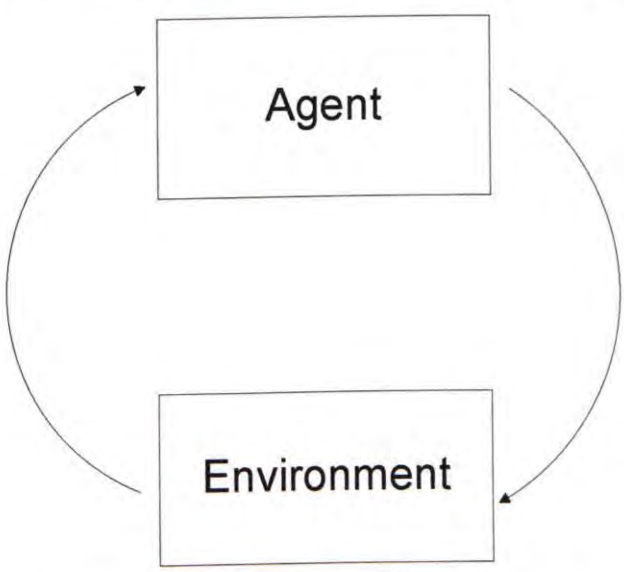


Figure 2.1: An agent in its environment

A complex class of environments can be inaccessible, non-deterministic, dynamic and continuous. Most real-world environments are not accessible for agents. There is uncertainty about the state that will result from performing an action. Agents have limited information about the environments. A dynamic environment is one that has other processes operating on it and changes in ways beyond agents' control. A continuous environment is in uncountable number of states. The environmental properties play an important role in the interaction between agent and environment.

According to the suggestion by Wooldridge and Jennings [50], an intelligent agent possesses four properties: autonomy, social ability, reactivity and pro-activeness.

- Autonomy: agents operate without the direct intervention of humans or others, and have some kind of control over their actions and internal state [6].

- Social ability: agents interact with other agents (and possibly humans) via some kind of agent communication language [3].

- Reactivity: agents perceive their environment (which may be the physical world, a user via a graphical user interface, a collection of other agents, the Internet, or perhaps all of these combined), and respond in a timely fashion to changes that occur in it.

- Pro-activeness: agents do not simply act in response to their environment, they are able to exhibit goal-directed behavior by taking the initiative.

In many circumstances, agents are exposed to the environment that changes while the procedure is executing. They are populated with more than one agent that can change the environment and there is uncertainty in the environment. The domains are too complex for an agent to observe completely. In such dynamic environments, an agent must be reactive, which is able to perceive the environments and adapt to the environments. That is, it must be responsive to events that occur in its environments, where these events affect either the agent's goals or assumptions which underpin the procedures that the agent is executing in order to achieve its goals. In such environments, a rational agent also needs be proactive to maximise its utility. The properties of pro-activeness and reactivity requires the agent to be adaptive. Adaptation is essential to cope with complex environment with agents of limited computational power.

2.1.2 Multiagent Systems

Multiagent systems has become important in both artificial intelligence and mainstream computer science. A multiagent system is a system consisting of a number of agents, which interact with one another through communication. Figure 2.2 gives a typical structure of a multiagent system [19]. In the system, different agents have different 'sphere of influence' over the environment. That means agents have control over or at least be able to influence different parts of the environment. Agents in multiagent environments are autonomous and distributed, and may be self-interested or cooperative. Multiagent environments provide an infrastructure specifying communication and interaction protocols. In multiagent environments, there is no centralized designer. Multiagent systems can differ in the agents themselves, the interactions among the agents, and the environments in which the agents act. Multiagent systems offer a way to characterize or design distributed computing systems. It is a natural metaphor for understanding and building a wide range of what we might call artificial social systems [18].

As an interacting entity, an agent may be affected by other agents or humans in pursing their goals and executing their tasks. Interaction can take place indirectly through the environment in which they are embedded by observing one another or by carrying out an action that modifies the environmental state. Interaction can also take place directly through a shared language by providing information in which other agents are interested or which confuses other agents. Distributed Artificial Intelligence focuses on coordination as a form of interaction. The interaction is particularly important with respect to goal attainment and task completion. The purpose of coordination has two aspects, which are to achieve or avoid states of affairs that are considered as desirable or undesirable by one or several agents. In order to coordinate their goals and tasks, agents have to explicitly take dependencies among their activities into consideration. Cooperation and competition are two basic patterns of coordination. For the case of cooperation, several agents work together and draw on the broad collection of their knowledge and capabilities to achieve a common goal. For the case of competition, several agents work against each other because their goals are conflicting. Cooperating agents try to accomplish what the individuals cannot do as a team. Competitive agents try to maximise their own benefit at the expense of others.

Self-interested agents in multiagent systems cannot be coordinated by externally imposing the agents's strategies. So the interaction protocol is designed so that each agent is motivated to follow the strategies that the protocol designed wants it to follow. A protocol is a negotiation protocol which determines the possible actions that agents can take at different points of interaction [48]. A strategy is a way to use the protocol. For example, the sealed-bid first price auction is a typical protocol. In the sealed-bid first price auction, each bidder is free to submit one bid for the item. This protocol rewards the highest bidder at the price of its bid. Since self-interested agents will choose the best strategy for themselves, which cannot be explicitly imposed from outside, the protocol need to be designed using a noncooperative, strategic perspective to guarantee that each agent's desired strategy is best for it.

A minority game is a game that models a multiagent system. Each agent in the game wants to be on the minority side. The environment is affected by the agents' collective behaviors, which is dynamic and uncertain. Besides,

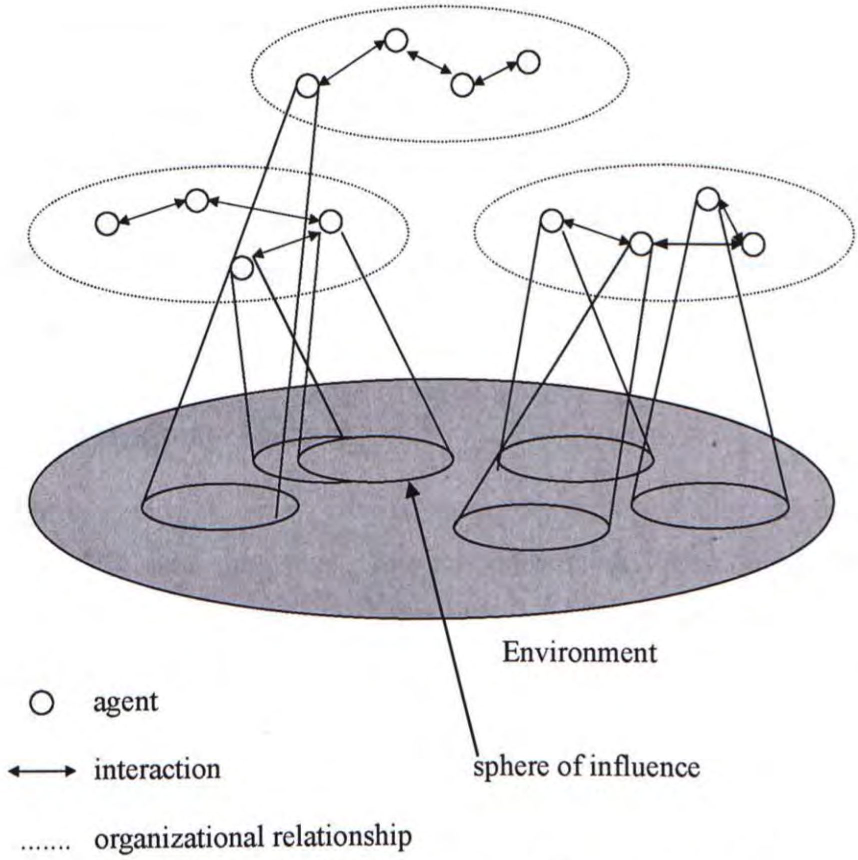


Figure 2.2: Typical structure of a multiagent system [19]

each agent is bounded rational as it is limited in its computational power and knowledge of the environment. Each agent needs to adapt to the dynamic environment. Adaptation allows for learning that enables agents to operate in uncertain environments. The dynamic for observed adaptive agent behavior can be analysed, i.e. how the behavior is changing during a history of (learning) experience. For example, the performance of actions depending on a stimulus in the present and a certain history can be modeled. We introduce minority games in the following section.

2.2 Minority Game

This section overviews previous work on agents' wealth in minority games. We first have a look at what is a minority game and the traditional method agents use in the game. Then the characteristic of the game is introduced. Besides, a lot of strategies proposed in the literature in terms of agents' wealth are presented.

2.2.1 Minority Game

In minority games, N agents have to choose between two alternatives (side 0 and side 1) at each time step. N is an odd number. After all agents have made their decisions, those who are on the minority side win. This game may seem rather simple at first glance, but it is subtle since if all agents analyse the situation in the same way, all agents will choose the same alternative and lose. Sometimes, it is not possible for an agent to have access to all the information in compliance situations and to consider how every other agent may behave. Agents may not always think rationally and may use inductive reasoning. It is reasonable to build models with agents that select a best response in complicated with forecasts and have the forecasts determined by a model of adaptive learning.

Challet and Zhang [10] suggest that each agent should keep track of a number of predictors and chooses the highest-score predictor to make the decision. A record of winning sides in the past time steps is called a memory size. A predictor of a memory size M is a lookup table consisting of 2^M entries and two columns, 'history' and 'prediction' respectively. Each entry prescribes which side to join in according to the information gathered from the recent winning sides of last M time steps, thus there are 2^M entries in each predictor. The prediction at each entry is either 0 or 1, so the total number of predictors is 2^{2^M}. An example of a predictor with $M = 3$ is shown in Table 2.1. At the beginning of the game, each agent is randomly assigned S predictors from the 2^{2^M} possible predictors. Traditionally, after all agents have made their decisions, those who are on the minority group are rewarded one point, while the other agents belonging to the majority group get nothing. All predictors which have made the correct prediction are also rewarded one point. All agents keep updating the history dynamically according to the outcome of winning side at every time step.

History	Prediction
000	1
001	0
010	0
011	1
100	0
101	1
110	1
111	0

Table 2.1: An example of a predictor with $M = 3$

2.2.2 Characteristics of Minority Game

In this section, we discuss two characteristics of minority games. First, there is a phase transition occurring in minority games. One phase is called symmetric phase and the other is called asymmetric phase. Second, due to the phase transition, there is a quasi-periodic structure with a periodicity appearing in the symmetric phase of minority games.

Phase Transition

Savit et al. [42] show that the fluctuations of the attendance size σ depends on the ratio $\rho = 2^M/N$ between the number of possible histories 2^M and the number of agents N. Challet and Marsili [8] show that there exists a phase transition of changing direction of σ^2/N locating at the point ρ_c where σ^2/N

attains its minimum. The best utilisation of resources occurs at a critical point, when the dimension of the predictor space is on the order of the number of agents, as shown in Figure 2.3. For small values of ρ, the predictor space is small, then there is much overlap of predictors among agents, hence a lot of agents will behave similarly and decide on the same action. So the population variance of agents choosing one side σ^2/N is larger when ρ is smaller. As ρ increases, the predictors space becomes larger and agents will behave relatively differently. So σ^2/N becomes smaller as ρ increases. When ρ is greater than a certain value, the predictor space becomes very large and agents will behave randomly. So σ^2/N is approaching to the value when agents just behave randomly.

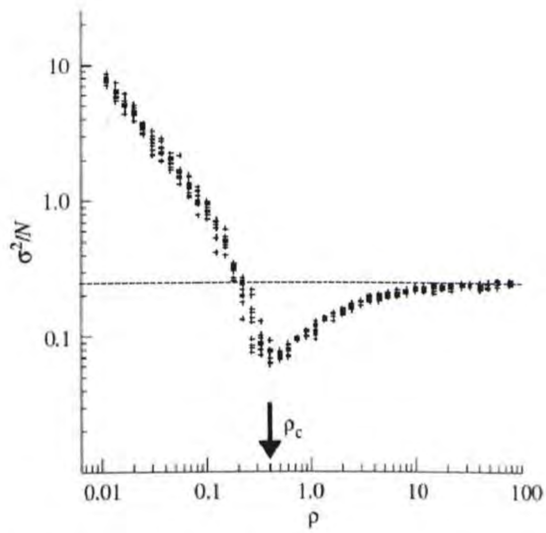


Figure 2.3: Population variance per agent [8]

To study the information content of the minority game, Savit et al. [42] consider $P(1|u_k)$, the conditional probability to have a 1 immediately following some specific history string u_k. The histogram of $P(1|u_k)$ generated by a game with $N = 101$, $k = M = 4$ and $S = 2$ is shown in Figure 2.4. The histogram is quite flat at 0.5. This means that for any predictor with memory size equal to 4, the history of minority side contains no predictive information about which will be the minority side at the next time step. In this sense, the market is efficient and no predictor using a memory size equal to or less than 4 can have a success rate better than 50%. The histogram of $P(1|u_k)$ generated by a game with $N = 101$, $k = M = 6$ and $S = 2$ is shown in Figure 2.5. As we can see the histogram is not flat. In this case, there is significant information available to the predictors of the agents playing the game with a memory size M and the market is not efficient in this sense.

When $\rho < \rho_c$, the phase is called the symmetric phase, in which no predictive information about the next minority side is available to the agents' predictors. The system is in a phase in which all information available to the agents' predictors is traded away, and agents' choices are maladaptive, resulting in a poor collective utilisation resources. When $\rho > \rho_c$, the phase is called the asymmetric phase. The system is in a phase in which the agents are able to coordinate their actions to achieve a better utilisation of resources. As the predictor space increases, the system becomes increasingly inefficient, so that there is more information in the game, while at the same time the agents' choices becomes increasingly uncoordinated and the system's behavior looks increasingly random.

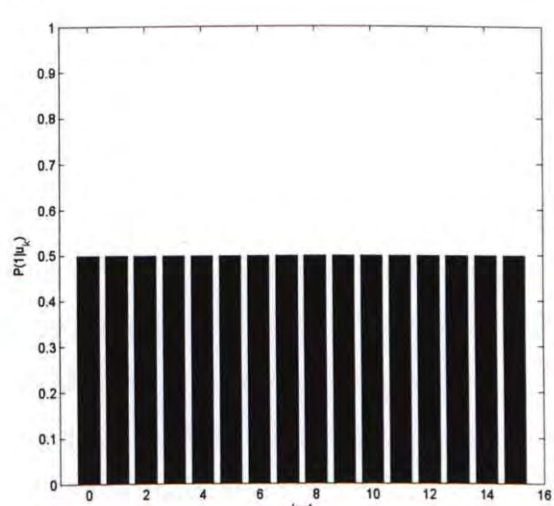


Figure 2.4: A histogram of the conditional probability $P(1|u_k)$ with $k = M = 4$

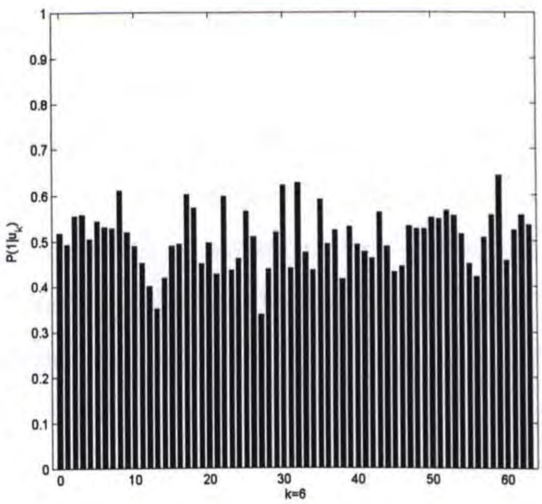


Figure 2.5: A histogram of the conditional probability $P(1|u_k)$ with $k = M = 6$

Quasi-periodic Structure

For small values of memory sizes, the predictor space is small and there is much overlap of predictors among agents, hence a crowd of agents will behave similarly and decide on the same action. Then the crowd of agents will be on the majority side and they will lose. This situation is called the crowd effect by [31]. Due to the crowd effect, a quasi-periodic structure appears in the minority game. The situation is as follows. When a particular history occurs for the first time, all agents decide randomly since both of the predictors each agent possesses have the same score at the beginning of the game. After the first occurrence of the history, agents learn that the winning outcome is a better choice. The predictors with the prediction the same as the winning outcome will gain one point. In the next occurrence of the same history, since a crowd of agents behave similarly, they will make the same decision as the winning outcome in the last occurrence. Zheng and Wang [53] point out that this leads to the winning outcome in this occurrence is opposite to that in the last occurrence of the same history. The predictors with the prediction opposite to the winning outcome in the last occurrence will gain one point.

So, both predictors of each agent will gain the same point. At the end of 2×2^M time steps, [13] assume that every history with the length of M is equally likely to occur, then both predictors of each agent will gain the same point on average. For the next round of the occurrence of the same history, the situation is equivalent to a new start of the game, similar to that of the first occurrence. Therefore, Liaw and Liu [29] point out that the quasi-periodic structure with a periodicity of length 2×2^M appears in the symmetric phase of the minority game.

2.2.3 Strategies for Agents in Minority Game

A lot of research have been done on minority games, such as [7, 12, 9, 28, 2, 25, 39]. In this section, we focus on the work on agents' wealth in minority games. Due to the crowd effect, a lot work have been done to enhance some special agents in a population of agents with smaller memory size M and small number of predictors S. [51] consider some special agents that participate in the minority game with a probability q per turn to enhance its success rate. The special agents do not participate in the game every turn, so they can successfully escape in over-adapting to the history created by the other agents. [30] propose an opposite strategy for a privileged agent which is free to choose any possible predictors to maximise its personal gain. It is to use the highest-score predictor when ρ is larger than ρ_c, and use the opposite strategy when ρ is smaller than ρ_c. The opposite strategy is to use the prediction in each entry opposite from that of the highest-score predictor. The opposite strategy makes good use of the quasi-periodic structure of the minority game. The work of [22] and [9] demonstrates the importance of the memory size in the minority game. It pays to increase the special agent's memory size M to increase its payoff. Details of these strategies are given as follows.

Manuca et al. [33] investigate the distribution of wealth (the number of points) to the agents under the condition that all agents have the same memory size and the same number of predictors. They point out that in the symmetric phase, agent wealth is strongly correlated with intra-agent distance. Intra-agent distance is defined as the Hamming distance between an agent's two predictors [9]. In this phase, the more similar an agent's two predictors, the wealthier the agent tends to be. Since there is no predictive information available to the agents' predictors in this phase, agents that ignore the information of the collective behavior of the other agents do better. So agents whose predictors are more similar are less adaptive, they will not respond to the misleading signals of the collective behavior by choosing to join different minority sides at different times. In the asymmetric phase, there is a strong correlation between agent wealth and inter-agent distance. Inter-agent distance describes the average behavioral distance of an agent from all other agents playing the game. The further an agent is, on average, from all other agents in behavior space, the wealthier it tends to be. In this phase, the agents use the real information in the minority game to coordinate their choices. Those agents having predictors that allow them to behave maximally differently from the other agents will more often find themselves in the minority side, and will accumulate more points.

Sometimes it pays to increase the agent's memory size M. Johnson et al. [22] study a mixed population of adaptive agents with smaller and larger memory sizes, but all agents own the same number of predictors and choose the highest-score one to make decisions. They find that the average winning per agent with larger memory size within a mixed-ability population can be greater than 0.5 by uncovering and exploiting hidden information in the system's recent history left by the agents with smaller memory size. The average winning per agent is defined as the total number of points awarded divided by the total number of agents. This demonstrates the importance of the memory size in the system. If the system contains a pure population, the agents cannot access any additional correlations. Challet et al. [9] consider the case of a pure population with a memory size M and one agent with larger memory size M'. They point out that the special agent with larger memory size can obtain larger gain than all of the other agents in the symmetric phase since the agent can exploit the hidden information, while the gain cannot be increased further more if the agent increases its memory size. Furthermore, in the asymmetric phase, the special agent receives a lower payoff than the average payoff of the other agents. Both of these two pieces of work, Johnson et al. [22] and Challet et al. [9], demonstrate the importance of the memory size in the minority game, but they ignore the influence of the number of predictors.

Yip et al. [51] consider special agents who participate in the game with a probability q per turn. That means these agents have a probability q of joining the game in each turn and a probability of $1-q$ of staying out of the game in a turn. The other agents participate in the game every turn. For all agents, they choose the highest-score predictors to make the decisions. Besides joining the game only with probability q, the special agents differ from the other agents in that they only assess the performance of their strategies in the turns that they participate. For the turns that the special agents decide not to play, they do not reward or subtract points to their predictors, regardless of the outcome. They find that for small values of memory sizes, these special agents achieve higher success rate than the average of all other agents when ρ is small. The success rate is the ratio of the number of winning turns to the number of turns the agent has actually participated. The special agents do not participate in the game every turn, so they successfully escape in over-adapting to the history created by action of the other agents. Since for small M, due to the small predictor space and substantial overlap of predictors among agents, this crowd tends too large to be the minority side. However, this method is a passive one because the special agents do not participate in the game for all turns. They only enhance their winning probability, but not enhance their overall payoffs.

Liu and Liaw [30] consider the gain of a privileged agent that is free to choose any predictor at every time step. They propose the opposite strategy for the privileged agent to maximise its personal gain. It is to use the highest-score predictor among all 2^{2^M} possible predictors when ρ is larger than ρ_c, and use the opposite strategy when ρ is smaller than ρ_c. The opposite strategy is to use the prediction in each entry opposite from that of the highest-score predictor. It is shown that the winning probability of this special agent using the opposite strategy can be larger than 0.5 for almost all values of ρ. The reason that the opposite strategy can enhance the winning probability lies in that it makes use of the quasi-periodic structure of the game: the winning outcome of an even occurrence of any history is most likely opposite to that of the odd occurrence of this history in the case of small ρ. They also point out that there is no need for the privileged agent to know ρ_c or N in advance. The privileged agent can simply use the highest-score predictor initially and switch to the opposite strategy when it finds its gain is smaller than 0.5. For small M, there is another option for the privileged agent. The privileged agent can simply chooses different side from the winning outcome in the last occurrence of the current history.

Lam and Leung [26] propose an adaptive behavioral strategy for the minority game according to the winning histories h and the net payoff u for choosing side 0 or 1. Each agent has two initial attitudes a_x towards choosing side 0 or 1 and two respective adaptive parameters. At each time step, each agent calculates the attractiveness ($= (1 - a_x) \times h + a_x \times u$) of side 0 and 1 to make the prediction. If side 0's attractiveness is larger than side 1's, it will choose side 0, and vice versa. At the end of each round, each agent updates its attitudes: if it has chosen side 0 and wins, then its attitude towards side 0 will be increased by the increasing adaptive parameter; if it has chosen side 0 and loses, then its attitude towards side 0 will be decreased by the decreasing adaptive parameter. Effectively, these agents do not use explicit predictors. Simulations show that agents using the adaptive behavioral strategy perform well. However, the performance of the agents with the adaptive behavioral strategy relies on each other because of the limitation of the strategy itself. The strategy can work well only if there are enough agents using it, because the agents update their attitudes according to the winning outcome. The winning outcome need enough agents using the adaptive behaviorial strategy to affect itself so that the agents can update their attitudes in the right way.

2.3 Resource Allocation

A multiagent resource allocation with one resource can be modeled as a minority game. Suppose there are N agents, all of them need to use the resource to complete their task. At each round, each of N agents decides whether to use the resource or not. The resource has a fixed capacity. If the number of agents choosing to use the resource exceeds its capacity, the resource is overloaded. After all agents have made their decisions, if the resource is not overloaded, those agents who has chosen to use the resource get a point. Otherwise, those agents who have chosen not to use the resource get a point. At the end of each round, the agents are informed whether the resource is overloaded or not. As we can see, this is a single-choice model.

The single-choice model can be extended to a multi-choice model. Suppose there are N agents, each of them need to use one of the Q resources to complete their tasks. Each resource has a fixed capacity. In each round of a multi-choice game, each of N agents decides to use one of the Q resources. If an agent chooses to use a resource and the resource is not overloaded, the agent gets a payoff. Details of strategies for multiagent resource allocation are given below.

There have been a lot of work on resource allocation techniques in recent years. Centralised [14, 38, 41] and distributed [11, 32, 45] solutions are the two main approaches. In the following section, we focus on introducing the strategies for multiagent resource allocations in the literature.

2.3.1 Strategies for Agents in Multiagent Resource Allocation

Galstyan et al. [17] study the resource allocation games with changing capacities. They propose that agents use a set of predictors to decide which resource to choose. Different from the traditional minority game model [10], a predictor is a lookup table based on the actions of agents' neighbors in the previous round and recommends agents to choose which resource in this round. For example, an agent use the actions of its two neighbors to make a choice in the next round among three resources. A predictor is shown in Table 2.2. In round t, if the actions of the two neighbors are 2 and 0, the the agent will choose 2, which means that it will choose the third resource. In each round, each agent chooses the predictor with the highest score to make the decision. At the end of each round, each agent assesses the performance of its predictors, adding (subtracting) a point if the predictor has predicted the right (wrong) choice. The right (wrong) choice is that the resource the agent chooses is not overloaded (overloaded).

$S_{k_1}(t)$	$S_{k_2}(t)$	$S_i(t+1)$
0	0	2
0	1	0
0	2	1
1	0	0
1	1	1
1	2	0
2	0	2
2	1	1
2	2	0

Table 2.2: An example of a predictor with two neighbors $K = 2$

Schlegel et al. [45] propose a self-organising load balancing approach for a single server with mobile agents. The server has a resource with constant or abruptly changing capacity. Mobile agents can decide to meet at the same server or communicate only locally without generating any network traffic except of the migration. Each agent uses a set of predictors to predict the next resource load based on the history values of resource load. For example, a predictor can predict a value to be the same as n^{th}-last history value, or the average of all values in a window of the last n history values, or the interpolated value that considers the last n history values. If all agents predict the same resource load, than this would invalidate their beliefs. So it is important to ensure agents make different decisions in the environment. The probability that a predictor is chosen increases with the predictor's accuracy. The accuracy of a predictor is based on the number of correct predictions it has made in the past. A correct prediction means that a predictor has predicted that the resource is not overloaded and the agent decided to migrate to the server. Predictors that make a correct prediction receive a positive rating, otherwise the responsible active predictors receive a negative rating.

According to Prospect Theory [24] suggesting that people have their own attitudes towards risk, Lam and Leung [27] propose an adaptive strategy for resource allocation with constant capacities. The strategy is based on the history information h and the net payoff u_x, which is the payoff for choosing a not overloaded resource x. The history information h is the ratio of the number of times that a resource x is not overloaded to the history size H. Each agent has an initial attitude a_x towards each resource. In each round, each agent calculates the attractiveness $((1-a_x) \times h + a_x \times u_x)$ of each resource and chooses the resource with the largest attractiveness. At the end of each round, according to the theory of Conditions of Learning in psychology [15], each agent updates its attitudes by adjusting parameters based on the resource allocation result. If the agent has chosen a not overloaded resource, then the attitude towards the resource is increased; if the agent has chosen an overloaded resource, then the attitude towards the resource is decreased.

Chapter 3

Individual Agent's Wealth in Minority Game

In this chapter, we first introduce the model of the minority game we study. Then we described our motivation to study such minority games. Inefficiency information in the minority game is introduced followed by. Besides, we present an intelligent strategy for an agent to use to enhance its wealth. Moreover, we implement some simulations to investigate the performance of the intelligent strategy. Finally, some comparison with related work are conducted.

3.1 The Model

The model we discuss here consists of N agents playing the minority game. In this paper, we focus on investigating the wealth of one individual agent which is different from the other agents in the game. At the beginning of the game, each of the other agents is randomly assigned S (generally equal to two) of the 2^{2^M} possible predictors of memory size M. Each of the other agents has the same memory size. One special agent can have larger or smaller memory size M' than the other agents ($M' > M$ or $M' < M$). The number of predictors S' the special agent owns can be different. For example, $S' = 1$, $S' = 2$, $S' = 2^{2^{M'}}$ or $S' > 2$. Such a special agent is called a privileged agent. At each time step, each agent chooses from among its predictors the one that has had the best performance over the history of the game up to that time, i.e. each agent chooses the highest-score predictor to make its decision and makes random choice at ties. After each time step, the cumulative performance of each of an agent's predictors is updated by comparing each predictor's prediction with the current minority side. Also, if an agent is on the minority side, the agent's wealth is increased by one. An agent's wealth is the number of points it obtain during playing in the game. An agent's wealth is also referred to its payoff.

The game is adaptive in that agents can choose to play different predictors at different time in response to the changes in their environment. That is in response to new history entries in the time series of minority sides as the game proceeds. The time series of minority sides is created by the collective action of the agents themselves. Although the system is adaptive, it is not evolutionary. The predictors do not evolve during the game, and the agents play with the same set of predictors that are assigned at the beginning of the game.

3.2 Motivation

The motivation of the work is that there is a common phenomenon: a crowd effect in the minority game when ρ is small ($N \gg 2^M$). The problem for agents is how to escape from the crowd effect and maximise personal wealth based on the history information and their own predictors. What will happen if the agent is more intelligent, i.e. having larger memory size or more predictors? In previous studies as described in Section 2, [9], [51], [22] and [30] propose different methods to escape from the crowd and enhance the winning probability. Based on the previous work of [9] and [22] that an agent has longer memory, we anticipate that if a privileged agent has larger memory size M' than the other agents and is free to choose any strategy at every time step while the other agents are using their highest-score predictors drawn randomly from the 2^{2^M}

possible predictors, then the privileged agent can also escape from the crowd and hence enhance the success rate. Intuitively, this mechanism can achieve the performance because the privileged agent with longer memory and more predictors is more intelligent than the other agents. On the other hand, the resource allocation problem can be modeled as minority games, such as the work of [16] and [27], which we introduce in the next chapter.

3.3 Inefficiency Information

In order to study the information content of the minority game, we consider $P(1|h_k)$, the conditional probability to have a winning outcome of side 1 immediately following some specific history string h_k of k bits introduced by [42] and [33]. That means when the history string h_k with length of k occurs, the probability of the winning outcome to be side 1 is $P(1|h_k)$. [51] define the inefficiency ε as follows:

$$\varepsilon = \frac{1}{2^M} \sum_{i=0}^{2^M-1} |P(1|h_k) - \frac{1}{2}| \tag{3.1}$$

where the sum is over all 2^M possible winning history strings of M bits. The inefficiency ε measures the information left in the winning history strings that a privileged agent uses to assess its predictors. If $P(1|h_k)$ is larger than 0.5, then the predictors with the prediction of side 1 at that specific history h_k are rewarded more points. If $P(1|h_k)$ is smaller than 0.5, then the predictors with the prediction of side 0 at that specific history h_k are rewarded more points. The agent decides whether to reward points to its predictors based on the winning outcome at the past winning history and chooses the highest-score predictor to make the decision.

The predictive information is about which will be the minority group at the next time step. [42] and [33] have shown that in the symmetric phase of the minority game, the winning history strings with length less than or equal to the

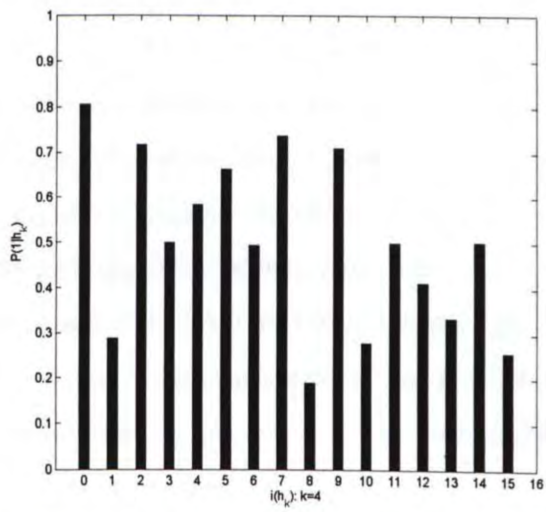


Figure 3.1: A histogram of the conditional probability $P(1|h_k)$ with $k = 4$ for the game played with $M = 3$

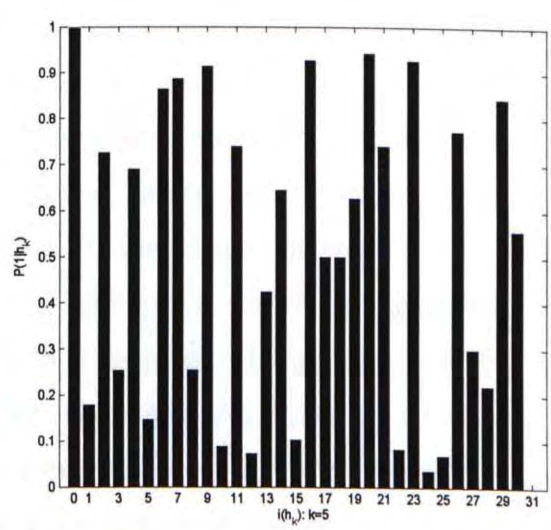


Figure 3.2: A histogram of the conditional probability $P(1|h_k)$ with $k = 5$ for the game played with $M = 3$

memory size contain no predictive information. That means $P(1|h_k) = 0.5$ for any history and hence $\varepsilon = 0$. In Figures 3.1 and 3.2, we plot $P(1|h_k)$ generated by a game with $N = 101$. One is the privileged agent with larger memory size and all possible predictors and the others have $M = 3$ and $S = 2$. $i(h_k)$ is the corresponding integer value of the binary history string h_k of length k. Figure 3.1 shows the histogram of $P(1|h_k)$ for the privileged agent having one longer memory than the memory other agents have, i.e. $k = M + 1 = 4$. Figure 3.2 shows the histogram of $P(1|h_k)$ for the privileged agent having two longer memory than the memory other agents have, i.e. $k = M + 2 = 5$. From the histograms, we can see that $P(1|h_k)$ for $k = 5$ is more closer to 1 than $P(1|h_k)$ for $k = 4$ when $P(1|h_k)$ is greater than 0.5. On the other hand, $P(1|h_k)$ for $k = 5$ is more closer to 0 than $P(1|h_k)$ for $k = 4$ when $P(1|h_k)$ is smaller than 0.5. These imply that the distinguished hidden information becomes larger when the privileged agent has longer memory, Using the figures in Figures 3 and 4 and Eq. (1), we can get the numerical results of the inefficiency ε: $\varepsilon_1 = 0.154$ for Figure 3 and $\varepsilon_2 = 0.299$ for Figure 4. Obviously, $\varepsilon_2 > \varepsilon_1$.

Thus we are led to the intriguing idea that a privileged agent can make good use of this information to maximise its own utility by having longer memory and owning all possible predictors. At each time step, the side within the highest-score strategy at that specific history is selected to make the decision. After each time step, if the winning outcome is side 1, then the predictor's score with side 1 at that specific history is increased by one. If the winning outcome is side 0, then the predictor's score with side 0 at that specific history is increased by one. The probability $P(1|h_k) > 0.5$ means that the probability of the winning outcome to be side 1 at the history h_k is greater than 0.5, then the probability of the winning outcome to be side 0 at the history h_k is smaller than 0.5. That means the winning outcome of side 1 at the history h_k occurs more often than side 0. After some learning steps, the predictor's score with side 1 at that specific history will be greater than the predictor's' score with side 0 at that specific history and this situation lasts throughout the game. So the agent with longer memory and all possible predictors will always choose side 1 if $P(1|h_k) > 0.5$. This implies that the probability P_{win} that the agent will win through the game is approximately equal to $P(1|h_k)$. Conversely, the agent will always choose side 0 if $P(1|h_k) < 0.5$, because the winning outcome to be side 0 occurs more often than side 1. The probability P_{win} that the agent will win is approximately equal to $1 - P(1|h_k)$. Concluding the above analysis, we can get the following equation:

$$P_{win} \simeq \begin{cases} P(1|h_k) & P(1|h_k) \geq 0.5 \\ 1 - P(1|h_k) & P(1|h_k) < 0.5 \end{cases} \tag{3.2}$$

Combining Eqs. (1) and (2), we have

$$P_{win} \simeq \frac{1}{2} + \varepsilon \tag{3.3}$$

Therefore the probability that the privileged agent wins for all occurrences of histories will be greater than 0.5 if $\varepsilon \neq 0$. The larger inefficiency ε is, the larger winning probability is. From these two figures, we can conclude that the privileged agent can increase the memory size to get more inefficiency information.

3.4 An Intelligent Strategy

In the traditional minority game, all agents keep the same memory size M and the same number of predictors S. As described in Section 1, there is a crowd effect in the symmetric phase, all agents behave similarly and obtain similar payoff. So it is hard to distinguish one from others. How can one individual agent manage to outperform the other agents in terms of individual payoff? Intuitively, the agent should be intelligent enough to avoid the crowd effect. The only available information it can use is the history information and its predictors. So how can the agent make good use of the information to maximise its payoff? Does it need to increase its memory size or the number of predictors it owns?

[9] suggest that the payoff of the agent with $M' = M + 1$ and $S' = 2$ cannot be increased furthermore if the agent increases its memory size. This result is applicable when the agent has the same number of predictors as the other agents but longer memory than the others. However, in addition to having longer memory than the others, if the agent also has larger number of predictors, the situation maybe change.

Inspired by the inefficiency information described in Section 3.1, we propose an intelligent strategy for the privileged agent to maximise its own payoff. That is the privileged agent with larger memory size M' than the other agents and free to choose any predictor at each time step. In the present model, we consider a population of N agents in which there is a privileged agent using the intelligent strategy. The other agents have the same memory size M ($M' > M$) and are only assigned S predictors drawn randomly from all the 2^{2^M} possible predictors. For all agents, they choose the highest-score predictors to make the decisions. For predictors with the same score, the agents make random choice. After each time step, the winning outcome is announced to the public. Each agent's payoff is increased by one if it makes the accurate decision. All the predictors' score are also updated. If the prediction at the specific history in one predictor is the same as the winning outcome, then the predictor is rewarded one point.

3.5 Experiment Analysis

In Figure 3.3 and Figure 3.4, we plot the payoff of the privileged agent using the intelligent strategy versus the average payoff of the other agents as a function of different memory sizes M. The experiment setting is as follows: the number of total agents is $N = 101$ for Figure 3.3 and $N = 1001$ for Figure 3.4, the number of predictors each traditional agent owns is $S = 2$, the range of the memory size M is the integer value between 1 and 15. The memory M' of the privileged agent ranges among $M+1$, $M+2$, $M+3$, $M+4$, $M+5$, $M+7$ and $M+10$ independently. Note that the memory the privileged agent has is longer than the other agents' memory. All agents are using the highest-score predictor in hands. For each value of M, each data point is the average of 10 independent runs with different initial random distributions of predictors and each runs 10^6 rounds. The purpose for doing so is to cover as many situations as possible because the initial predictors are randomly generated.

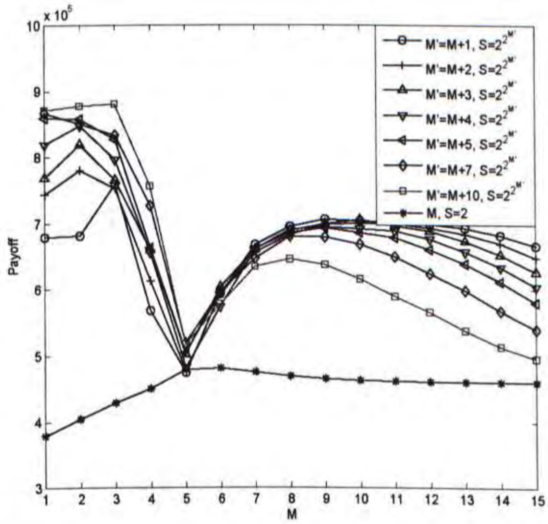


Figure 3.3: The privileged agent's payoff with M' and $S' = 2^{2^{M'}}$ versus the average payoff of the other agents with M and $S = 2$ as a function of M. ($N = 101$)

From Figure 3.3 and Figure 3.4, we can see that the privileged agent with longer memory performs significantly better than the average of the other agents for almost all values of M, no matter whether it is in the symmetric phase ($\rho < \rho_c$) or asymmetric phase ($\rho > \rho_c$). That means the privileged agent can outperform others for almost all values of ρ ($\rho = 2^M/N$). The phase transition occurs at $M_c = 5$ and $M_c = 8$ respectively. Qualitatively, the maximal utility of the privileged agent comes from a successful escape in fully

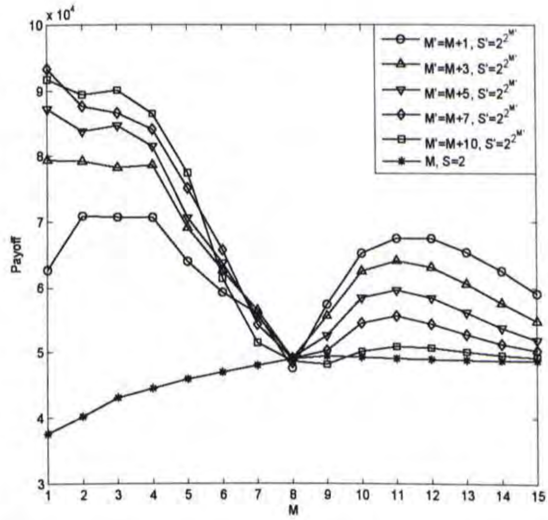


Figure 3.4: The privileged agent's payoff with M' and $S' = 2^{2^{M'}}$ versus the average payoff of the other agents with M and $S = 2$ as a function of M. ($N = 1001$)

adapting to the history information created by the other agents, and hence it does not become part of the crowd. Furthermore, the interesting result is that in the symmetric phase, the agent with longer memory obtains more payoff. As described in Section 3.1, the inefficiency information ε is larger for the privileged agent with longer memory. According to Equation (3.3), the privileged agent's winning probability P_{win} is larger, so it is able to obtain larger payoff. However, in the asymmetric phase, the privileged agent with smaller memory size performs better than the one with larger memory size. In this phase, the memory size M is larger, so the predictor space 2^{2^M} is larger, thus the other agents do not behave similarly. So there is no crowd effect in this phase. The privileged agent cannot make use of any further information by increasing its memory size.

Therefore, we can conclude that the privileged agent using the intelligent strategy can maximise its personal utility with larger memory size M' in the symmetric phase and smaller memory size M'' in the asymmetric phase. Both M' and M'' are larger than the others agents'.

Also, we investigate the effects that the parameter N have on the dynamic phase transition point M_c of such a system with a population with a memory M and one privileged agent with longer memory M' and all its possible predictors. From Figure 3.3 and Figure 3.4, we can see that the privileged agent's payoff drops as M increases and reaches a minimum around $M_c = 5$ for $N = 101$ and $M_c = 8$ for $N = 1001$. Then the privileged agent's payoff increases as M increases for $M > M_c$. This implies that when N increases, M_c increases. [42] show that a phase transition of changing direction of σ^2/N locates at the point ρ_c ($\rho = 2^M/N$) where σ^2/N attains its minimum. So when N increases, M_c drifts to a larger value.

3.6 Discussions and Analysis

The privileged agent works better than the other agent when all agents are endogenous. Keeping larger memory size makes the privileged agent have the inefficiency information, which the other agents do not have. The inefficiency information indicts that the winning side to be side 0 or 1 occurs more often at each history. When the inefficiency information is larger than 0.5 at some history, it indicts that side 1 occurs more often than side 0. Besides, the privileged agent possesses all possible predictors to make decisions based on the inefficiency information. The privileged agent will always choose side 1 when the inefficiency information is larger than 0.5 at each history. This makes the privileged agent obtain larger payoff than the other agents. But the inefficiency information is dependent on the behaviors of the other agents. If the other agents are random agents, namely, randomly choosing side 0 or 1, it is useless for the privileged agent to record larger memory size, because the history is created by random agents.

In this section, an Experience method for the privileged agent is introduced when it has all possible predictors. Then we discuss the impact of the memory size and the number of predictors the privileged agent and the other agents have on the payoffs of the privileged agent and the other agents. Besides, we investigate the impact of the increasing number of such privileged agent on its payoff. Moreover, we compare the performance of the privileged agent with other agents proposed previously.

3.6.1 Equivalence to the Experience method

Obviously, if an agent owns all 2^{2^M} predictors, the number of predictors will be too large for the agent to handle even when M is moderate. In this section, we present a simple Experience method, and show that agents employing Experience method have the same behavior as agents employing the traditional method with all 2^{2^M} predictors.

The Experience method is as follows. Instead of using any of the 2^{2^M} predictors, an agent simply records, for each immediate past history of length M, the number of times side 0 has won and the number of times side 1 has won. The number of times side 0 or 1 has won is said to be the score of the respective side. To make a decision given an immediate past history of length M, an agent chooses the side with the highest score, and makes random choice at ties.

Let $E^i_x(h)$ denote the score of side x (0 or 1) at time step i for an immediate past history h of length M. Formally, the experience method can be expressed as follows:

$$E^i_x(h) = \begin{cases} 0 & i = 0 \\ E^{i-1}_x(h) & i > 0 \text{ and} \\ & \text{side } x \text{ loses at time step } i \\ E^{i-1}_x(h) + 1 & i > 0 \text{ and} \\ & \text{side } x \text{ wins at time step } i \end{cases} \tag{3.4}$$

At time step i, if the immediate past history is h, an agent chooses side 0 if

$E_0^i(h) > E_1^i(h)$, or side 1 if $E_0^i(h) < E_1^i(h)$, or a random choice between 0 and 1 if $E_0^i(h) = E_1^i(h)$. This Experience strategy is intuitively simple and easy to implement. However, the following theorem proves that agents employing such an Experience method are behaviorally equivalent to agents employing the traditional method with all predictors.

Theorem *The behavior of an agent using the Experience method is equivalent to the behavior of an agent using the traditional method with all possible predictors.*

Proof: Consider an agent using the traditional method, which has all 2^{2^M} predictors. For any predictor P, let $P(h)$ denote the prediction made by predictor P with history h. Choose any two predictors P_1 and P_2. Suppose at time step i with history h, P_1 has the highest score S_1^i and P_2 has the score S_2^i ($S_1^i \geq S_2^i$). Then we have $E_{P_1(h)}^i(h) \geq E_{P_2(h)}^i(h)$ for the following reason. Suppose $E_{P_1(h)}^i(h) < E_{P_2(h)}^i(h)$. As the agent has all possible predictors, there must exist a strategy P_3 with the same prediction in P_2 at the history h ($P_3(h) = P_2(h)$) and with the same predictions in P_1 at all the other histories ($P_3(h') = P_1(h')$ iff $h' \neq h$). So P_3's score $S_3^i = S_1^i - E_{P_1(h)}^i(h) + E_{P_2(h)}^i(h)$. Then $S_3^i > S_1^i$, which contradicts to the fact that S_1^i is the highest score. Therefore, we have $E_{P_1(h)}^i(h) \geq E_{P_2(h)}^i(h)$. In other words, P_1 scores weakly better than any other predictor P_2 for each h.

If both P_1 and P_2 are the highest-score predictors at time step i ($S_1^i = S_2^i$), then we have $E_{P_1(h)}^i(h) \geq E_{P_2(h)}^i(h)$ and $E_{P_1(h)}^i(h) \leq E_{P_2(h)}^i(h)$, hence $E_{P_1(h)}^i(h) = E_{P_2(h)}^i(h)$.

In summary, $S_1^i \geq S_2^i$ if and only if $E_{P_1(h)}^i(h) \geq E_{P_2(h)}^i(h)$, and vice versa.

Therefore, the agent that uses the Experience method and chooses the side with the highest score at each history is actually using the traditional method with all possible predictors. So their behaviors are equivalent.

3.6.2 Impact of M' and S'

In this section, we discuss the impact of the privileged agent's memory size M' and number of predictors S' on its payoff. In Figure 3.5, we plot the privileged agent's payoff with all possible predictors $2^{2^{M'}}$ and M' ranging among $M+1$, M, $M-1$, $M-2$, and $M-3$ independently versus the average payoff of the other agents with M and $S=2$ as a function of M for $N=101$. For each value of M, the data point is the average of 10 independent runs with different initial random distributions of predictors and each runs 10^6 rounds. From this figure, we can get three results in the symmetric phase. The first one is that the privileged agent with $M'=M$ performs the worst and even achieves less payoff than the average payoff of the other agents. The reason is that the privileged agent has a memory of the same length as the other agents but owns all the possible predictors. That means the privileged agent will always follow the crowd and become a loser most of the time. The second result is that the payoff of the privileged agent with shorter length of memory than the other agents is smaller than the average payoff. The third result is that the privileged agent with shorter length of memory than the other agents, such as $M'=M-1$, behaves better than the privileged one with $M'=M$. The reason is that the privileged agent has a memory size M' which is smaller than that of the other agents. This cause that the privileged agent does not fully adapt to the collective history information created by the other agents. So it can avoid being in the crowd sometimes. Thus its payoff is a little larger than the one with $M'=M$. In the asymmetric phase, there is no crowd effect. The agent with $M'<M$ gets less history information about the game than the agent with $M'\geq M$. This is not good for predictions, so it gets less payoff.

Next we investigate how the number of predictors the privileged agent owns affects its performance. In Figure 3.6, we plot the payoff of two kinds of privileged agents with $M'=M+1$ versus the average payoff of the other

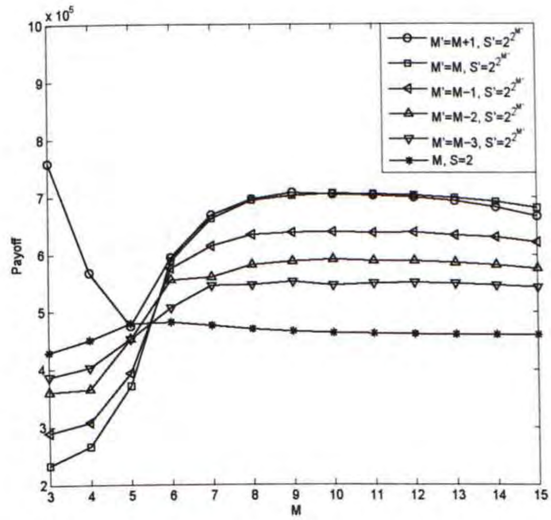


Figure 3.5: The privileged agent's payoff with M' and $S' = 2^{2^{M'}}$ versus the average payoff of the other agents with M and $S = 2$ as a function of M. $(N = 101)$

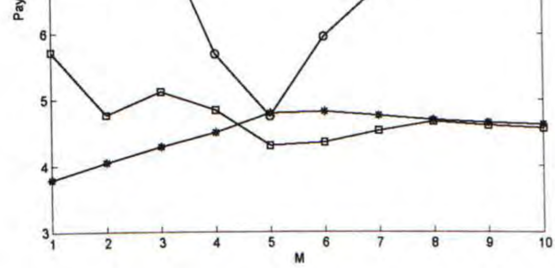


Figure 3.6: The privileged agent's payoff with $M' = M + 1$ and $S' = 2^{2^{M'}}$ and another one with $M' = M + 1$ and $S' = 2$ versus the average payoff of the other agents with M and $S = 2$ as a function of M. $(N = 101)$

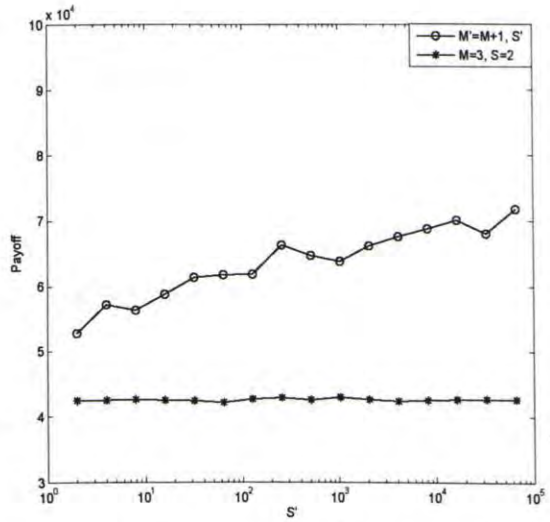


Figure 3.7: The privileged agent's payoff $M' = M + 1 = 4$ and S' versus the average payoff of the other agents with $M = 3$, and $S = 2$ as a function of S'. ($N = 101$)

agents with M and $S = 2$ for $N = 101$: the first privileged agent with all the $2^{2^{M'}}$ possible predictors, and the second privileged agent with $S' = 2$. S' ranges from 2 to 2^{2^4} and samples 16 values by multiplying 2 every time. We can see that the first privileged agent always outperforms the second privileged one. The only difference between the two privileged agents is the difference between the number of predictors they have, which causes the first privileged agent achieves higher payoff. We further investigate the impact of S on the privileged agent's payoff. In Figure 3.7, we plot the privileged agent's payoff with $M' = M + 1$ and S' versus the average payoff of the other agents with $M = 3$ and $S = 2$ as a function of S' for $N = 101$. We can see that the larger the number of predictors the privileged agent has, the more payoff it obtains. The reason is that if an agent has more predictors, it has more opportunities to explore in the predictor-space and thus predict more accurately. We can also observe that the privileged agent's payoff may decrease as the number of predictors increases. The reason is that the agent behaves based on its predictors, so its payoff is strongly related to the initial distribution of the predictors. If the initially assigned predictors do not predict well, the agent will not perform well. However, this does not affect the principal changing trend: the larger the number of predictors the privileged agent has, the more payoff it obtains.

3.6.3 Impact of M and S

First, we investigate the impact of the other agents' memory size on the privileged agent's payoff. As we can see in Figure 3.3 and Figure 3.5, if the other agents' memory size is smaller than that of the privileged agent, the privileged agent with $S' = 2^{2^{M'}}$ can increase its payoff by increasing its memory size in the symmetric phase. If the other agents' memory size is greater than or equal to the privileged agent', the privileged agent with $S' = 2^{2^{M'}}$ obtains lower payoff than the average payoff of the other agents in the symmetric phase. In the asymmetric phase, no matter the other agents' memory size is smaller or greater than the privileged agent', the privileged agent's payoff is higher than the average payoff of the other agents.

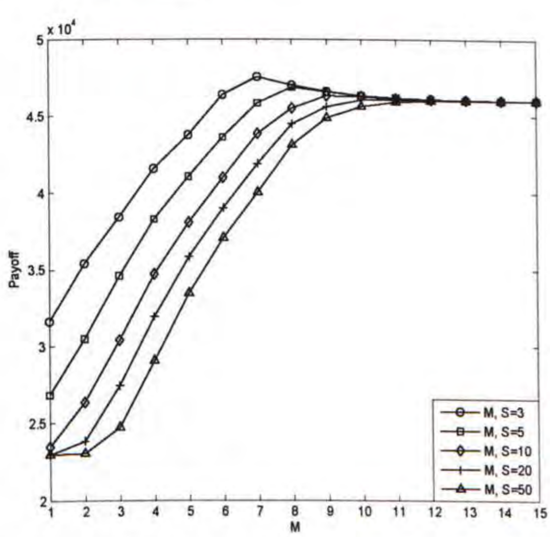


Figure 3.8: The average payoff of the other agents with M and different number of predictors as a function of M. $(N = 101)$

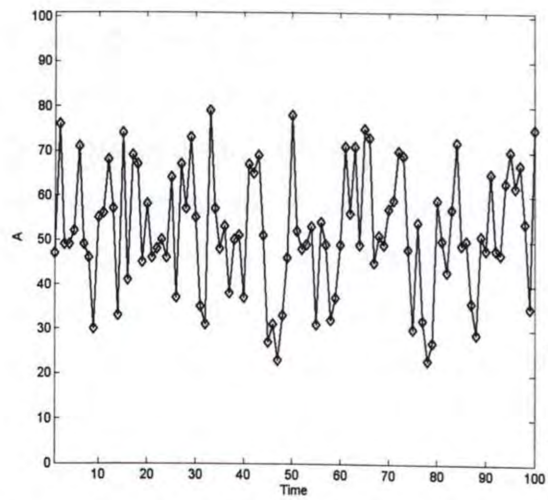


Figure 3.9: Time series of the number of agents choosing side 1 when each of the other agents has $M = 3$ and $S = 3$. ($N = 101$)

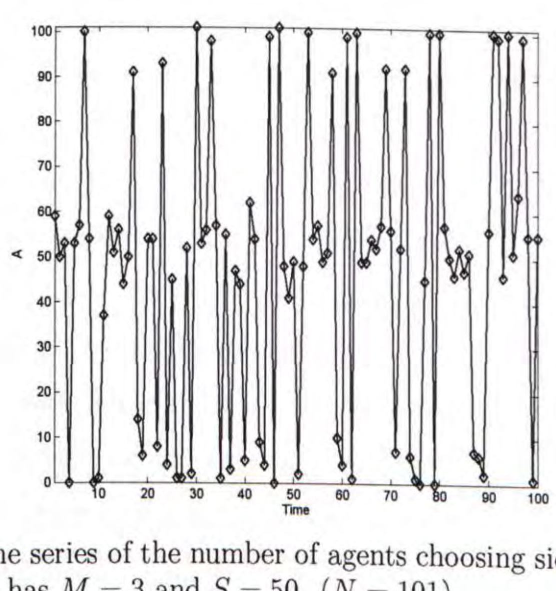


Figure 3.10: Time series of the number of agents choosing side 1 when each of the other agents has $M = 3$ and $S = 50$. ($N = 101$)

Next, we investigate the impact of the number of predictors S the other agents possess on the privileged agent's payoff and the average payoff of the other agents. We plot the payoff of the privileged agent with $M' = M + 1$ and $S' = 2^{2^{M'}}$ and the average payoff of the other agents with M and S in a population of $N = 101$ agents for different S. M ranges from 1 to 15. In Figure 3.8, the five lines represent the average payoff of the other agents, each of them having $S = 3$, $S = 5$, $S = 10$, $S = 20$ and $S = 50$ respectively. From this figure, we can see that for smaller M, the average payoff of the other agents decreases as the number of predictors they possess increases. To find out the reason, we plot the time series of the number of agents choosing side 1 A when each of the other agents has $M = 3$, $S = 3$ and $S = 50$ in Figure 3.9 and 3.10 respectively. Comparing these two figures, we can see that the more predictors each of the other agents has, the more agents will choose side 0 and A is more close to 0, or the more agents will choose side 1 and A is more close to the total number of agents. That means the more predictors each of the other agents has, the more agents will choose the same side (side 0 or side 1). So there will be more agents on the majority side and these agents will lose. Therefore, if each of the other agents have more predictors, their average payoff will lower.

Why do agents tend to choose the same side when they have more predictors? As described in Section 3.1, if an agent has all possible predictors, it simply records, for each immediate past history of length M, the number of times side 0 has won and the number of times side 1 has won. At each history entry, each agent will choose the side with the higher score and make random choice at ties. After all agents have made their decisions, the score of the side with the same as the winning outcome will be increased by one. Each agent will have the same score of side 0 at each history entry and the same score of side 1. Then they will all choose side 0 or side 1. whichever has a higher score, with ties broken randomly. So all agents' behaviors will be almost the same.

Then this will cause most of them to lose.

If an agent with all possible predictors uses the traditional method, at each history entry, it chooses the highest-score predictors to make the decision. The distribution of predictions in the highes-score predictors could be purely side 0 (100% of side 0) or purely side 1 (100% of side 1) or side 0 and side 1 but with the same number (50% of side 0 and 50% of side 1). We analyse it as follow: In Section 4.1, we have proven that if both P_1 and P_2 are the highest-score predictors at time step i, then $E^i_{P_1(h)}(h) = E^i_{P_2(h)}(h)$, where $P(h)$ denote the prediction made by predictor P at history entry h and $E^i_x(h)$ denote the score of side x (0 or 1) at time step i for an immediate past history h. That means in the highest-score predictors, at each history entry, the predictions' scores are all the same. The predictions could be purely side 0 or purely side 1, and it satisfies that all predictions' scores are the same. If the predictions have both side 0 and side 1, since the predictions' scores are all the same, i.e. the scores of side 0 and side 1 are the same. The number of side 0 and side 1 appearing in the highest-score predictors at some history entry are also the same. The reason is that the agent has all possible predictors and it can always find predictors that have the same predictions at the other history entries and only different predictions at some history entry. Therefore, if an agent having all possible predictors use the traditional method, the distribution of predictions in the highest-score predictors are purely side 0 or purely side 1 or side 0 and side 1 with the same number.

For an agent that does not have all possible predictors, the more predictors it has, the greater possibility it has to have similar distribution of predictions in the highest-score predictors to the agent with all possible predictors. Then its behavior will be more similar to the behavior of the agent with all possible predictors. When each of the other agents has more predictors, their behaviors will be more similar to the agent with all possible predictors. Since the behaviors of agents having all possible predictors are almost the same, each of the other agents will have similar behavior and then it will cause most of them to lose, so their average payoff will be lower.

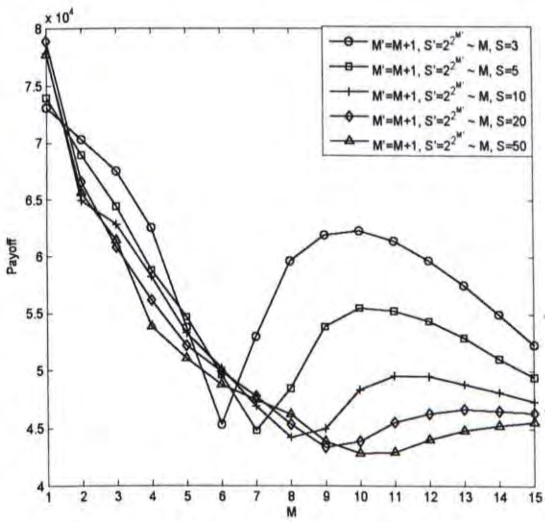


Figure 3.11: The privileged agent's payoff with $M' = M + 1$ and $S' = 2^{2^{M'}}$ as a function of M under different cases of the other agents having different number of predictors. ($N = 101$)

In Figure 3.11, the five lines represent the privileged agent's payoff with $M' = M + 1$ and $S' = 2^{2^{M'}}$ in the cases that each of the other agents posses $S = 3$, $S = 5$, $S = 10$, $S = 20$ and $S = 50$ respectively. For smaller M, the privileged agent's payoff slightly decreases as the number of the other agents' predictors increases. For larger M, the privileged agent's payoff greatly decreases as the number of the other agents' predictors increases. Since the other agents' behaviors become more similar to the privileged agent's when each of them has more predictors, this causes the side the privileged agent chooses to be the majority side, so the privileged agent's payoff decreases.

Next, we investigate the impact of S on the position of phase transition. We plot the privileged agent's payoff versus the average payoff of the other agents in the cases that each of the other agents possess $S = 3$ in Figure 3.12, $S = 5$ in Figure 3.13, $S = 10$ in Figure 3.14 and $S = 50$ in Figure 3.15. We can see that in Figure 12, the phase transition occurs at $M_c = 6$. The phase

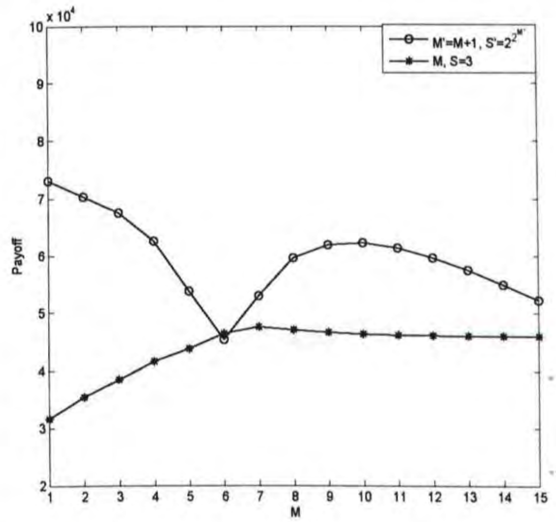


Figure 3.12: The privileged agent's payoff with $M' = M + 1$ and $S' = 2^{2^{M'}}$ versus the average payoff of the other agents with M and $S = 3$ as a function of M. ($N = 101$)

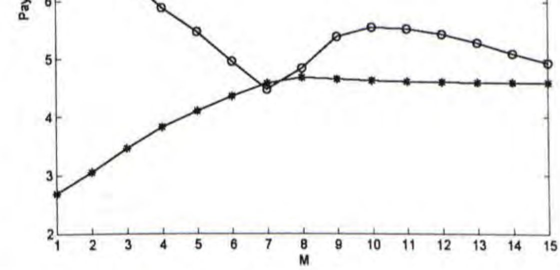


Figure 3.13: The privileged agent's payoff with $M' = M + 1$ and $S' = 2^{2^{M'}}$ versus the average payoff of the other agents with M and $S = 5$ as a function of M. ($N = 101$)

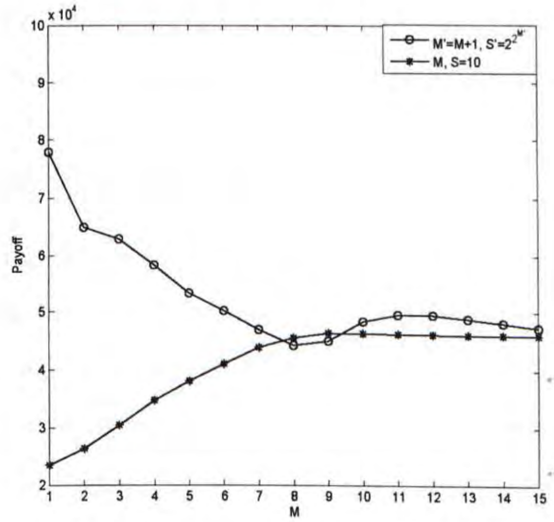


Figure 3.14: The privileged agent's payoff with $M' = M + 1$ and $S' = 2^{2^{M'}}$ versus the average payoff of the other agents with M and $S = 10$ as a function of M. ($N = 101$)

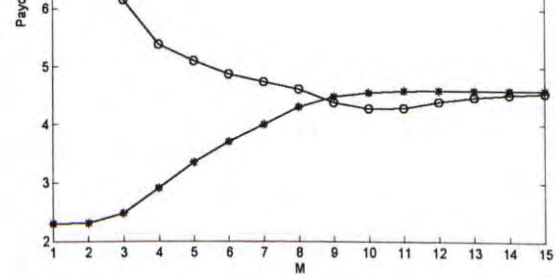


Figure 3.15: The privileged agent's payoff with $M' = M + 1$ and $S' = 2^{2^{M'}}$ versus the average payoff of the other agents with M and $S = 50$ as a function of M. ($N = 101$)

transition occurs at $M_c = 7$ in Figure 15. In Figure 16 and 17, the transition point occurs nearly $M_c = 8$ and $M_c = 9$. We notice that the transition position M_c increases as the number of predictors S the other agents possess increases.

3.6.4 Impact of Larger Number of Privileged Agents

In the situation that there are larger number of the privileged agents playing the minority game, what is the impact on the privileged agents' payoff? Each of the privileged agents has $M' = M + 1$ and $S' = 2^{2^{M'}}$. Their behaviors are the same after playing for sometime although there are some differences at the beginning of the game. Each of the other agents has M and $S = 2$. M ranges from 1 to 15. The total number of agents is $N = 101$. In Figure 3.16, we plot three pairs of payoffs between the average payoffs of the privileged agents and the other agents. The first pair is when the number of the privileged agents is $n = 1$. The second one is when the number of the privileged agents is $n = 5$, namely, 5% of the total number of agents. The third one is when the number of the privileged agents is $n = 10$, namely, 10% of the total number of agents. We can see that the average payoff of the privileged agents decreases as the percentage of such agents increases. These privileged agents behave in the same way and they choose the same side at the same time. If the percentage of the privileged agents is large enough to affect the minority side, the side the privileged agents choose is probable to be the majority side. These privileged agents will always be the loser. So the average payoff of these agents will decrease and become smaller than the average payoff of the other agents.

3.6.5 Comparisons with Related Work

In this section, we discuss some simulation results using different approaches which all enhance one individual agent's wealth. These agents all escape from the crowd effect. The model proposed by [51] is a passive way to avoid the

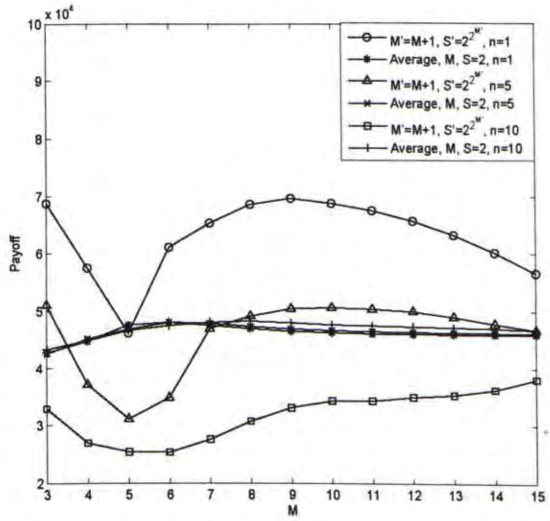


Figure 3.16: The average payoff of different number of the privileged agents with $M' = M + 1$ and $S' = 2^{2^{M'}}$ versus the average payoff of the other agents with M and $S = 2$ as a function of M. ($N = 101$)

over-adaptation to the history produced by the collective behavior of the other agents. It assumes that the particular agent decides to whether to participate in the game with a probability $q < 1$ and assesses the performance of its predictors only in the turns that it participates. So, the particular agent's payoff is at most half of the total turns when $q = 0.5$, so the success rate for $q = 0.5$ is at most 0.5. In addition, Yip et al. [51] also show that the enhanced success rate for $q < 1$ takes on similar values, so the success rate is at most 0.5 even if q is close to 1. Thus, the payoff is at most half of the number of the total turns for any q. The achievable payoff is not large enough.

In Figure 3.17, we compare the payoff of the privileged agent using the intelligent strategy with the payoff of another agent using the adaptive behavioral strategy proposed by [26]. The experiment setting is as follows: the number of total agents is $N = 101$, the number of predictors each of the other agents own is $S = 2$, the range of the memory size M is the integer values between 1 and 15. The memory size M' of the agent using the intelligent strategy is $M + 1$.

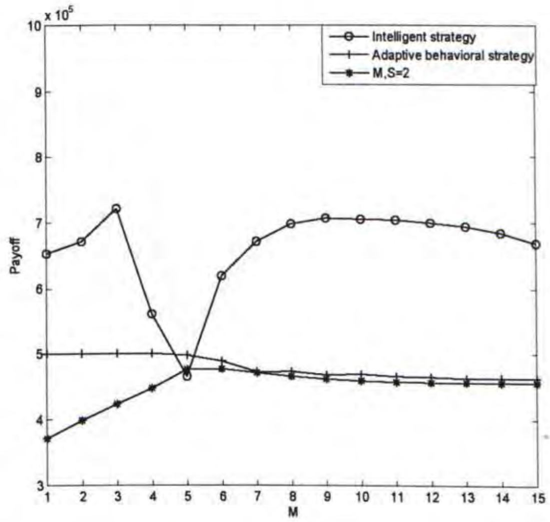


Figure 3.17: The privileged agent's payoff with $M' = M + 1$ and $S' = 2^{2^{M'}}$ and the adaptive behavioral agent's payoff versus the average payoff of the other agents with M and $S = 2$ as a function of M. ($N = 101$)

The agent's initial attitude towards side 0 and 1 and adaptive parameters using the adaptive behavioral strategy are randomly generated at the beginning of the game. The other agents are using the highest-score predictors in hands. For each value of M, the data point is the average of 10 independent runs with different initial random distributions of predictors and each runs 10^6 rounds. This figure illustrates that the privileged agent using the intelligent strategy achieves larger payoff than the agent using the adaptive behavioral strategy for all most values of M. As we have discussed in Section 2.3.3, since there is only one agent using the adaptive behavioral strategy in the experiment, its decision affect little on the winning outcome. Thus the agent may not update its attitudes in the right way. So the agent cannot obtain large payoff.

In Figure 3.18, we compare the payoff of the privileged agent using the intelligent strategy with the payoff of another agent using the opposite strategy of Liu and Liaw [30]. The experiment setting is the same as the previous one. The memory M' of the agent using the intelligent strategy is $M + 10$ when $M \leq 5$ and $M + 1$ when $M > 5$. We can see from Figure 3, the agent with

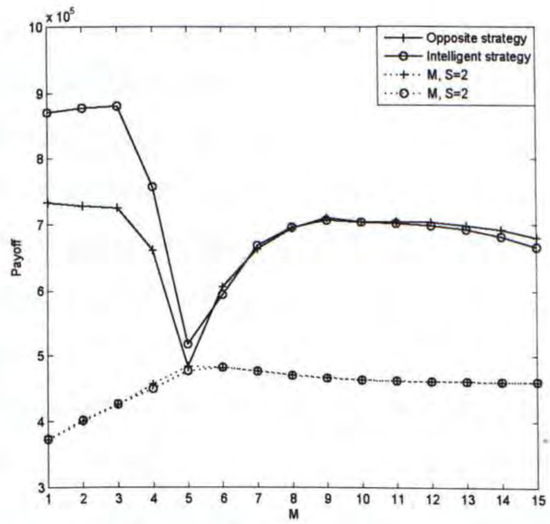


Figure 3.18: The privileged agent's payoff with the intelligent strategy and the one with the opposite strategy versus the average payoff of the other agents with M and $S = 2$ as a function of M. ($N = 101$)

$M + 10$ performs better in the symmetric phase and the agent with $M + 1$ performs better in the asymmetric phase. For the agent using the opposite strategy, it uses the highest-score strategy when $M > 5$ and uses the opposite strategy when $M \leq 5$. The opposite strategy is the one with the prediction opposite from that of the highest-score strategy at any entry. From this figure, we can see that the agent using the intelligent strategy obtains more payoff than the one using the opposite strategy in the symmetric phase. In the asymmetric phase, the payoff of the agent using the intelligent strategy and the payoff of the agent using the opposite strategy are more or less the same. In fact, as described in Section 2.3.2, Zheng and Wang [53] point out that the winning outcome of an even occurrence of any history is most likely opposite to that of the odd occurrence of this history in the symmetric phase. Since the agent using the opposite strategy use the prediction opposite from that of the highest-score predictor, so it can almost wins for the even occurrence of any history. For the odd occurrence of any history, it has a probability of 0.5 to win. So its winning probability on average will be approximately equal to 0.75. On the other hand, in Section 3.1, statistical results reveal that the inefficiency information contained in $M+2$ is $\varepsilon > 0.25$. Since longer memory can lead to larger inefficiency information ε, so the inefficiency information ε contained in $M+10$ is greater than 0.25. Then according to the relationship $P_{win} \simeq \frac{1}{2} + \varepsilon$, the winning probability will be greater than 0.75. Therefore, the agent using the intelligent strategy can obtain more payoff than the agent using the opposite strategy.

From these comparisons, we can conclude that the privileged agent using the intelligent strategy is able to make more accurate predictions with larger memory size in the symmetric phase. However, if the agent does not know when the phase transition will occur, it can just lengthen its memory size by one, i.e. keep longer memory than the other agents' by one, no matter in the symmetric phase or the asymmetric phase. It is because the agent with $M+1$ performs well for all most values of M. If the agent knows where the phase transition occurs, it can lengthen its memory size more than one in the symmetric phase.

Chapter 4

An Adaptive Strategy for Resource Allocation

In this chapter, we first introduce the specification of multiagent resource allocation we study. Then we present an adaptive strategy for agents to use to adapt to the changing environments. Besides, we describe the advantage and disadvantage of the adaptive strategy. Moreover, we implement some simulations to investigate the performance of the adaptive strategy. Finally, we compare the adaptive strategy with some related work.

4.1 Problem Specification

The following resource allocation problem we consider is similar to the problem studied in [16, 26]. There are Q available resources, each having different capacities $C = \{C_1, ..., C_Q\}$. There is a set $A = \{A_1, ..., A_N\}$ of N agents, each having one task to execute in each round. Each agent only needs one unit of resources and gets its task completed in one round. The resources can be shared by multiple agents. All agents compete for the resources to execute their tasks. The capacities of the provided resources can be constant, but they generally vary over time. The total amount of capacities is equal to or greater than the number of agents at any time, which means that there are always sufficient resources. This is because if the total amount of capacities is less than the number of agents, then at least one resource will be overloaded all the time. We are only interested in the case of sufficient resources in this paper since this case can make it possible that all agents are able to complete their tasks at the end of each round. If the number of agents choosing a resource is less than or equal to the capacity of the resource, then the resource is not overloaded and agents choosing the resource can complete their task. Otherwise, the resource is overloaded and not all agents choosing the resource can complete their tasks.

Similar to [16, 26], there is no central information and no communication among agents in the system. The history information from the past resource allocation records and the resource capacities during the previous rounds are the only information available to all agents for making a resource choice decision. A past resource allocation record is a record that whether a resource is under-utilised or over-utilised in the previous round. Since the system completes a resource allocation for each round, we can consider the system as a sequence of multi-choice games, i.e. each of N agents decides to choose one of the Q resources in each round, and a run of the system consists of R rounds.

To make good utilisation of resources in the dynamic environment is one of the objectives in the resource allocation problem. Good utilisation of resources means that the number of agents choosing the resource is close to the resource capacity, which results in little or no under-utilisation or over-utilisation. Dynamics means that the capacities of resources vary over time. They can change gradually or abruptly. If agents in the system are able to coordinate themselves well, then agents as a whole can adapt effectively to the dynamic environment.

4.2 An Adaptive Strategy

Since there is no communication between agents, agents need to make good use of the available information to make correct decisions. A correct decision means that agents choose a not overloaded resource. Based on the work by Lam and Leung [26], we design an adaptive strategy to tackle the resource allocation problem with not only gradually changing capacities but also with abruptly changing capacities. The strategy is based on individual agent's experience and prediction. Each agent keeps an experience for each resource. Using this strategy, each agent records the number of correct decisions in the past for each resource statistically. The *experience* e_x^r for resource x in round r is defined as follows:

$$e_x^r = \frac{n_x^r}{r} \tag{4.1}$$

where n_x^r is the number of times that the agent has chosen resource x and resource x is not overloaded.

Each agent also keeps a prediction for each resource. For simplicity, the predicted capacity is approximated by a linear function of the resource capacities in previous rounds. In round $r = 1$ and $r = 2$, we assume that the predicted capacity of each resource is equal to its current capacity. The *prediction* p_x^r for resource x in round r is defined as follows:

$$p_x^r = \begin{cases} \frac{C_x^r}{\sum_{i=1}^{Q} C_i^r} & r = 1 \text{ or } r = 2 \\ \\ \frac{[C_x^{r-1} + (C_x^{r-1} - C_x^{r-2})] \times f}{\sum_{i=1}^{Q} C_i^{r-1}} & r > 2 \end{cases} \tag{4.2}$$

where C_x^r, C_x^{r-1} and C_x^{r-2} is the capacities of resource x in round r, $r-1$ and $r-2$, C_i^r and C_i^{r-1} denotes the capacity of resource i in round r and $r-1$, and f is the scaling factor.

Some agents may rely more on the experience and some may rely more on the prediction. According to Prospect Theory [24], each agent has an attitude towards a choice. So each agent is associated with an attitude $a_x \in [0, 1]$

to calculate the weighted value of the experience and the prediction. The attitude is used as a weight. If an agent's attitude is close to 0, the agent tends to choose the resource which is not overloaded most of the time. If an agent's attitude is close to 1, the agent biases the selection towards the resource with the largest capacity. The weighted value is called the attractiveness of the resource. The resource with the highest value of attractiveness is the most attractive to agents. So each agent chooses the resource with the highest value of attractiveness in each round. Formally, the *attractiveness* $attr_x^r$ of resource x in round r is calculated as follows:

$$attr_x^r = (1 - a_x^r) \times e_x^r + a_x^r \times p_x^r \tag{4.3}$$

where a_x^r is the attitude towards resource x in round r.

According to the theory of Conditions of Learning in psychology [15], attitudes will be changed by favorable or unfavorable experiences. So each agent using this strategy adjusts its attitudes at the end of each round. If the agent has chosen a not overloaded resource, the attitude towards the resource a_x^r is increased by a_+. This means that the agent has made a correct decision, and it can put more weight on considering the prediction. If the agent has chosen an overloaded resource, then a_x^r is decreased by a_-. This means that the agent has made a wrong decision, then it should put more weight on considering the experience. Both a_+ and a_- are adjusting rates.

In addition, the current capacities of resources are announced to all agents at the end of each round. To deal with the abruptly changing capacities, we introduce the threshold θ. If an agent detects that the difference between the current capacity and the previous capacity is larger than the threshold value θ, i.e. $|C_x^r - C_x^{r-1}| > \theta$, then the agent will consider that it encounters abruptly changed capacities. To eliminate the influence of the past resource allocation records, the agent will reset its experience value to zero and reset its attitudes to initial attitudes. The reason for doing resetting is that the agent accumulates its experience and adjusts its attitudes according to the resource allocation result after each round. When the experience is reset, the agent relies only on the prediction p_x^r to make decisions and record the experience e_x^r from scratch. Then it will acquire the experience again. Since the capacity is considered changing abruptly when $|C_x^r - C_x^{r-1}| > \theta$, as long as the slope of the gradually changing capacity is within the threshold θ, the agent will not wrongly consider gradually changing as abruptly changing.

4.3 Remarks of the Adaptive Strategy

The merit of the strategy lies in that the threshold can tackle the abruptly changing capacities. However, the limitation of the strategy also lies in the threshold, because the value of the threshold is critical. If the threshold is too small, agents may wrongly consider gradually changing as abruptly changing. Since if the slope of the gradually changing capacity is large, the difference of capacities between the two consecutive rounds will be large too. However, if the threshold is too large, agents cannot detect the abruptly changing capacities, while the capacities change abruptly within a small amount actually.

Therefore, the value of the threshold depends on different situations. If the resource capacities change by a large amount, agents may set the threshold larger. In this situation, even though agents wrongly take gradually changing as abruptly changing, resetting the current experience and attitudes may be beneficial since the capacities change largely. On the other hand, if the resource capacities change within a small amount, then agents may set the threshold smaller. In this situation, even though agents do not detect the abruptly changing, the current experience and attitudes may be useful since the capacities do not change largely.

In addition, the appropriate adjusting rate and scaling factor are also important to agents' adaptation. If the adjusting rate is too small, agents may not adapt fast enough to the dynamic environment; if the adjusting rate is too large, agents may over-adapt. For the scaling factor, if it is too large, it may cause the value of prediction to dominate the value of experience in the calculation of attractiveness; if the scaling factor is too small, it may have little effect on the value of the prediction.

4.4 Experiment Analysis

4.4.1 Simulations

In dynamic environments where the resource capacities change either gradually or abruptly, we want to investigate the overall performance of the system where agents use the proposed adaptive strategy described in Section 3. We define the number of agents choosing a resource as the resource load. Similar to [16, 26], we consider the situation that the total amount of capacities is equal to the number of agents. The good overall performance means that the number of agents choosing a particular resource is always close to the capacity of the resource even though the capacity varies over time, which indicates that the resource load follows the resource capacity very well.

The experiment setting is as follows. The number of agents is $N = 1000$. The number of resources is $Q = 2$. The two resources have capacities of C_1^r and C_2^r respectively, which change gradually or abruptly. The total amount of capacities remains constant over the experiment with $C = C_1^r + C_2^r = 1000$. To account for agents' heterogeneity, the initial attitudes a_x of each agent are generated randomly. The adjusting parameters for all agents are $a_+ = a_- = 0.01$. The scaling factor is $f = 2$. The threshold is $\theta = 10$. The parameters may have some impact on the performance of the resource allocation system.

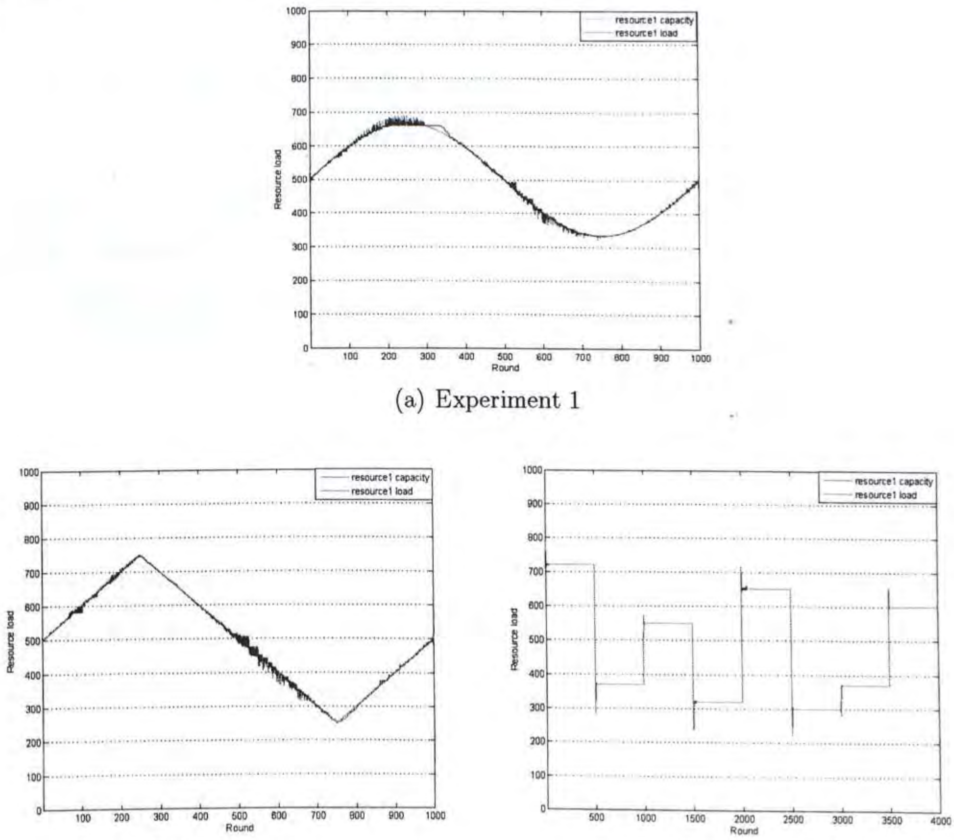
The first experiment is similar to that in [16] that is conducted in the dynamic environment with a constant number of agents and gradually changing capacities. The capacity of the first resource varies over time: $C_1^r = (\frac{1}{2} + \frac{1}{6}\sin\frac{2\pi r}{R}) \times C$, where $R = 1000$. The capacity of the second resource is $C_2^r = C - C_1^r$. The slope of the changing function is gradually changing. Figure 4.1(a) shows the resource load of the first resource in this environment. We plot the resource load for the first resource only since the second one is fully determined by the first one. It can be seen that the resource load follows the resource capacity very well. This means that the number of agents choosing a particular resource is almost always close to the capacity of the resource. This results in very little under-utilisation or over-utilisation of resources.

The second experiment is also conducted in the dynamic environment with gradually changing capacities, but the slope of the changing function remains constant for some rounds and then changes to another slope. The capacity of the first resource is as follows:

$$C_1^r = \begin{cases} 500 + r & r < 250 \\ 750 - (r - 250) & 250 \leq r < 750 \\ 250 + (r - 750) & 750 \leq r < 1000 \end{cases}$$

The capacity of the second resource is $C_2^r = C - C_1^r$. The resource load of the first resource in this environment is shown in Figure 4.1(b). It can be seen that the agents are able to adapt to the dynamic environment too. The resource load also follows the resource capacity very well.

The third experiment is conducted in the dynamic environment with abruptly changing capacities. The capacity of first resource abruptly changes every 500 rounds. The capacity C_1^r is a random number between 0 and 1000 and it remains constant within 500 rounds. The capacity of the second resource is $C_2^r = C - C_1^r$. Figure 4.1(c) shows the resource load of the first resource in this environment. We can see that at the beginning of every 500 rounds, the agents detect that the available resource capacities change abruptly and reset their experience and attitudes, and then after only few rounds, the agents self-organise themselves again and find a stable solution for the allocation of the resources. This result also indicates that the adaptive strategy can work in static environments.



(a) Experiment 1

(b) Experiment 2

(c) Experiment 3

Figure 4.1: Resource load using the adaptive strategy

To measure the performance of the system quantitatively, we use Equation (4.4) introduced by Galstyan et al. [17] to calculate the average deviation of resource utilisation for these three experiments.

$$\sigma_{avg}^2 = \frac{1}{Q}\sum_{i=1}^{Q}\sigma_i^2 \tag{4.4}$$

where Q is the number of provided resources and σ_i^2 is resource i's cumulative deviation of the number of agents choosing the resource A_i^r from the resource capacity C_i^r over certain rounds R, which is defined as follows:

$$\sigma_i^2 = \frac{1}{R} \sum_{r=r_0}^{r_0+R} (A_i^r - C_i^r)^2 \tag{4.5}$$

The average deviation for each experiment is averaged over 100 independent runs. The results are shown in the Figure 4.1 column of Table 4.1. These numbers are very small, which indicates that the system is very close to the optimal allocation.

Experiment	Figure 4.1	Figure 4.4	Figure 4.5	Figure 4.6
1	53.76	17895.89	23870.19	3233.47
2	74.49	20344.37	29129.27	3138.79
3	40.16	21744.78	29912.64	2654.75

Table 4.1: Averaged deviation

The simulation results suggest that the adaptive strategy enables agents to adapt well to the dynamic environments with changing capacities autonomously and adaptively. The system results in very litter under-uitlisation or over-utilisation. Agents use the threshold θ to detect whether they encounter abruptly changing capacities and reset their experience and attitudes to eliminate the influence of the past resource allocation records. The varying capacities can be either gradually changing with the slope less than the threshold or abruptly changing with the difference of capacities between two consecutive rounds greater than the threshold.

Next, we investigate how the adjusting rate and scaling factor affect the system's performance. In Figure 4.2, we plot the average deviation of resource utilisation versus the scaling factor with different adjusting rates for Experiment 1. The scaling factor ranges from 2 to 10. The adjusting rate ranges from 0.01 to 0.04. The results are averaged over 100 independent runs of Experiment 1. From this figure, we can see that the minimal average deviation occurs when the adjusting rate is equal to 0.01 and the scaling factor is equal to 2. Other parameters result in larger average deviation.

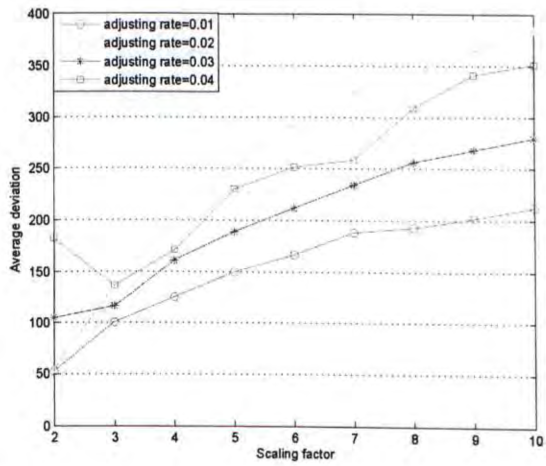


Figure 4.2: Average deviation vs scaling factor

The average deviations of resource utilisation versus the adjusting rate with different scaling factors are plotted in Figure 4.3. The adjusting rate ranges from 0.01 to 0.1. The scaling factor ranges from 2 to 5. It can be seen that when the scaling factor is equal to 2 and the adjusting rate is equal to 0.01, the system achieves the minimal average deviation. The average deviation increases as the adjusting rate increases. Large scaling factor and adjusting rate both result in large average deviation. The system is studied numerically for different cases with a wide range of parameters. Some parameters can lead to the minimal average deviation. However, the relationship between the appropriate parameters and the capacities is under investigation.

4.4.2 Comparisons with Related Work

In this section, we conduct the simulations using other strategies in the environments of the above three experiments. We first implement the simulations using Lam and Leung's strategy [26]. The strategy of Galstyan et al. [17] is another approach. We then use their strategy to do the simulations. Finally, we use the strategy of Schlegel et al. [45] to implement the simulations. We compare the simulation results with the results using the proposed adaptive

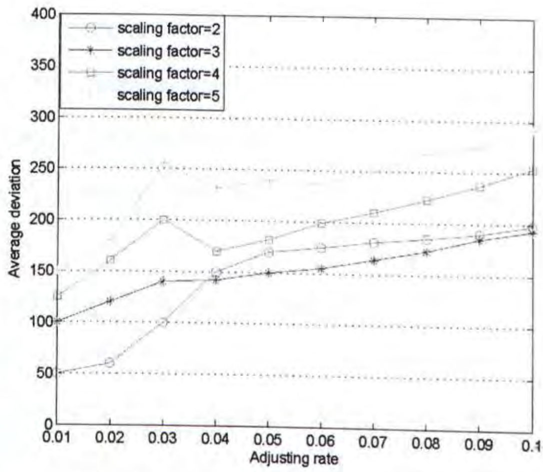


Figure 4.3: Average deviation vs adjusting rate

strategy. We also calculate the average deviation for these strategies. We want to investigate whether the system using the adaptive strategy can make better utilisation of resources than other strategies.

Comparison with Lam and Leung's strategy

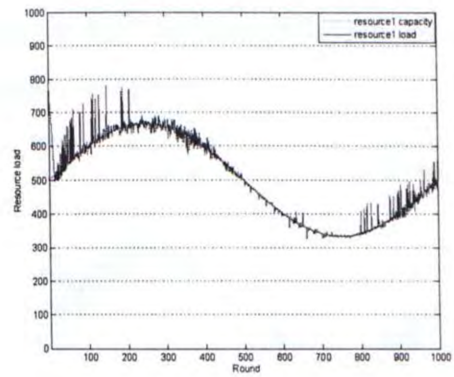
In the situation that agents may have preference over the resources, Lam and Leung [26] consider the payoff of choosing a not overloaded resource as the preference over the resource. They generate the preferences for each agent randomly at the beginning. The preference is fixed once it is generated. To account for agents' heterogeneity, agents' initial attitudes towards each resource are generated randomly. The adjusting parameters for all agents are $a_+ = a_- = 0.02$. The history size is $H = 5$.

From Figure 4.4(a) to Figure 4.4(c), we plot the resource load using Lam and Leung's strategy in the environments of the three experiments in Section 4.4.1. We can see that agents using Lam and Leung's strategy do not adapt very effectively to the dynamic environment. Although the fluctuation of the resource load fluctuating around the resource capacity becomes small after some rounds, the fluctuation becomes large again when the capacity changes on the opposite direction or changes abruptly. The average deviations of resource utilisation over 100 independent runs are calculated using Equation (4.5), which are shown in the Figure 4.4 column of Table 4.1. It can be seen that the average deviations are much larger than those using the adaptive strategy.

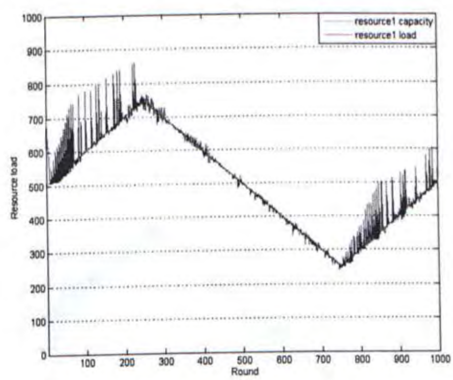
The adaptive strategy is based on individual experience and prediction. It takes the predicted capacities of resources into account. This is important in the resource allocation with changing capacities. The prediction reflects the changing of capacities to some degree and also changes with time. Agents using the adaptive strategy learn whether they have made correct decisions in the previous round and adjust their attitudes adaptively. In addition, when agents detect that the difference of capacities between two consecutive rounds is larger than the threshold, agents will reset their experience and attitudes to eliminate the influence of the past resource allocation records. This is important in the environment with abruptly changing capacities. Although agents using Lam and Leung's strategy also adjust their attitudes, the preference is fixed once it is generated at the beginning of the game rather than changing with time. This may be the reason that agents cannot adapt to the changing capacity level very effectively as the capacity is varying over time. So Lam and Leung's strategy may work very well in the static environment, but not very well in the dynamic environment.

Comparison with the strategy of Galstyan et al.

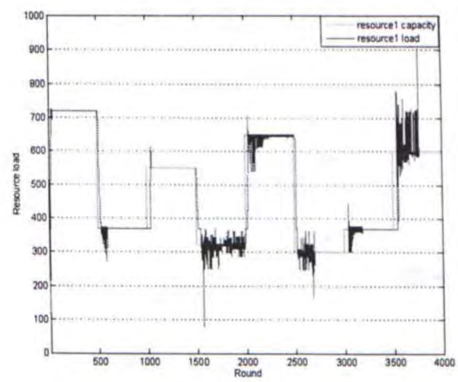
We conduct the simulations using the strategy of Galstyan et al. [17] in this section. At the beginning of the game, each agent randomly chooses two neighbors ($K = 2$) and also randomly generates two strategies ($S = 2$), which means the actions that recommend agents which resource to choose in the next round are also generated randomly. Each agent's neighbors are fixed throughout the game, but each agent updates its neighbors' actions at the end



(a) Experiment 1



(b) Experiment 2

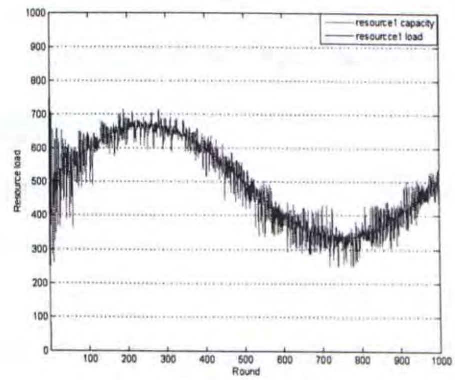


(c) Experiment 3

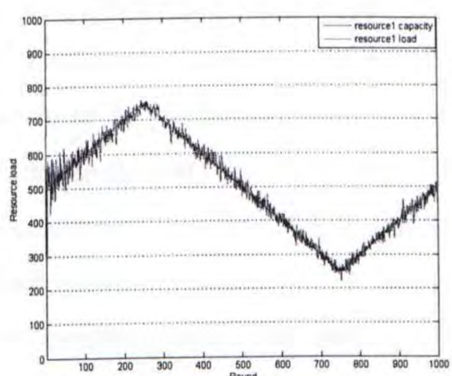
Figure 4.4: Resource load using Lam and Leung's strategy

of each round.

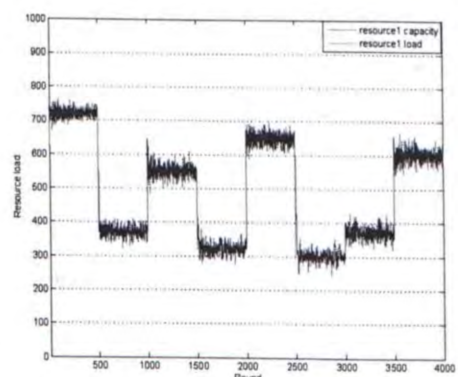
Using the strategy of Galstyan et al., we plot the resource load for the three experiments from Figure 4.5(a) to Figure 4.5(c). It can be sees that although the resource load fluctuates around the resource capacity, the fluctuation is much larger compared with the results in Figure 4.5. The average deviations for the three experiments are shown in the Figure 4.5 column of Table 4.1. From this table, we can see that the average deviations are also much larger than those using the adaptive strategy.



(a) Experiment 1



(b) Experiment 2



(c) Experiment 3

Figure 4.5: Resource load using the strategy of Galstyan et al.

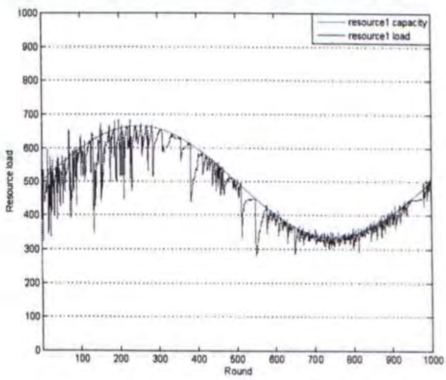
The reason agents using the strategy of Galstyan et al. can sense the changing trend of capacities may be because they assess the performance of their strategies after each round and choose the highest-score strategy to make decisions. Agents update their neighbors' actions after each round. This may make agents can coordinate their behaviors among themselves. However, the strategies are randomly generated, which is not sensible. Also, the strategies do not change once they are generated at the beginning. The agents are limited to choose the resources appearing in the strategies. This may be the reason that the agents cannot adapt effectively enough to the changing capacities. The performance could be worse than the results in Figure 5 if the randomly generated strategies predict badly.

Comparison with the strategy of Schlegel et al.

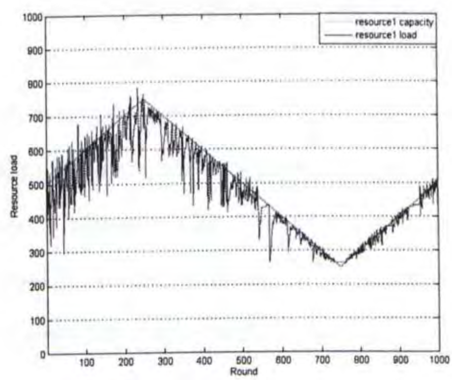
Schlegel et al. [45] suggest agents use a set of predictors to make decisions. The number of predictors for the simulations is 6. They include: same value as the last history value, same value as n^{th} last history value, average value of last n history values, the trend of last n history values, random value in the interval between the minimum and maximum of last n history values and average value of the minimum and maximum of last n history values, where $n = 5$. The history value is the past resource load. It is useful when the set of predictors is not the same for all agents. So each agent randomly chooses two of these predictors to use. The resource load information is updated only if the agent migrates to the server. If an agent predicts no free resources and decides not to go, no new historical information will be updated. This will lead to that the agent may always predict resource over-utilisation. To solve this problem, each agent uses a random retry after the agent does not go for a while [44]. This is different for agents, so that they do not go all together at the same time.

The resource load using the strategy of Schlegel et al. for each of the three experiments is shown from Figure 4.6(a) to Figure 4.6(c). It can be seen that the resource load fluctuates around the resource capacity largely. Agents using the strategy of Schlegel et al. make decisions based on predictors' predictions.

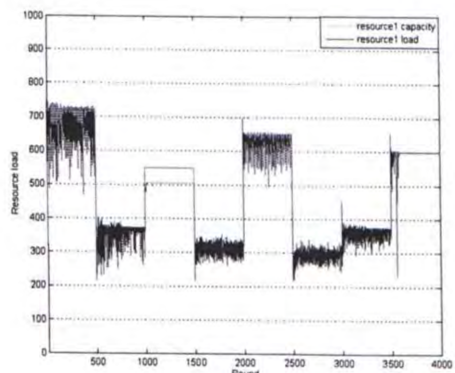
Although the predictors are not randomly generated, the average deviations shown in the Figure 4.6 column of Table 4.1 are still larger than those using the adaptive strategy. Agents in the system coordinate their behaviors among themselves so that they can follow the trend of the changing capacities. However, each agent uses a random retry after the agent does not go for a while, this may be the reason that the resource load fluctuates around the resource capacity largely.



(a) Experiment 1



(b) Experiment 2



(c) Experiment 3

Figure 4.6: Resource load using the strategy of Schlegel et al.

Chapter 5

Conclusions and Future Work

5.1 Conclusions

This thesis introduces strategies for agents to choose their actions in multiagent environments. Agents in minority games need to be reactive to the dynamic environment and adapt to it. Some multiagent resource allocation can be modeled into minority games. We investigate these two research topics in this thesis. We have the following conclusions.

For minority games, first, we study the performance of one privileged agent with larger memory size M' and free to choose any possible predictor in a population with a memory M and $S = 2$. We find some results. The privileged agent outperforms the other agents for almost all values of M in terms of payoff. In the symmetric phase, the privileged agent with larger memory size can obtain more payoff than the one with smaller memory size but still larger than the others'.

In addition, we investigate the impact of the number of predictors S' on the payoff of the privileged agent with larger memory size. The larger number of predictors the privileged agent possesses, the more payoff it obtains. We also investigate the impact of the memory size M' on the payoff of the privileged agent with all possible predictors. There are three results in the symmetric phase. First, the privileged agent with the same memory size as the other agents achieves less payoff than the average payoff of the other agents. Second, the payoff of the privileged agent with smaller memory size than the other agents is smaller than the average payoff. Third, the privileged agent with smaller memory size than the other agents behaves better than the privileged agent with the same memory size as the other agents. We then discuss how the number of predictors S the other agents possess affect the privileged agent's payoff and the average payoff of the other agents. For smaller M, the average payoff of the other agents decreases as S increases. For larger M, as S increases, the privileged agent's payoff decreases. We also find that if the number of the privileged agents increases, the average payoff of the privileged agents decreases.

Moreover, we compare the payoff the privileged agent using the intelligent strategy with the payoff of another agent using the adaptive behavioral strategy proposed by [27]. The result shows that the privileged agent can outperform the agent using the adaptive behavioral strategy for all most values of M. We also compare the payoff of the agent using the intelligent strategy with the payoff of another agent using the opposite strategy proposed by [30]. The result also shows that the intelligent agent can outperform the agent using the opposite strategy in the symmetric phase. Therefore, the privileged agent using the intelligent strategy we propose outperforms the other agents in the same model and other models proposed in previous work in terms of individual payoff. Finally, we present a simple Experience method for agents with all possible predictors, and prove that agents employing Experience method have the same behavior as agents employing the traditional method with all predictors.

For multiagent resource allocation, we propose an adaptive strategy to tackle the resource allocation problem with not only gradually changing capacities but also with abruptly changing capacities. We investigate the performance of the system using the strategy under different situations. The varying capacities can be either gradually changing with the slope less than the threshold or abruptly changing with the difference of capacities between two consecutive rounds greater than the threshold. The simulation results demonstrate that agents using the adaptive strategy as a whole can adapt very effectively to the changing capacity levels and result in very little under-utilisation or over-utilisation.

Moreover, we also compare our results with some related work. The simulation results show that agents using the adaptive strategy are able to make better utilisation of resources, i.e. the average deviation of resource utilisation from the optimal allocation is smaller than those of related work. The adaptive strategy is parameterised by the adjusting rate and scaling factor. We are going to investigate further how to determine the values of these parameters. In addition, the value of the threshold depends on the changing amount of the capacities.

5.2 Future Work

For minority games, there are some aspects for future work. First, we are going to investigate further the relationship between the number of predictors the agents own and the position of transition. We hope that we can find a function that can express the relationship, such that [33] find that the transition occurs when the dimension of the predictor space is of the order of the number of agents playing the game. Second, there is no communication among agents in minority games we study, what will the situation be if we allow additional communication among agents? Third, it is also interesting to explore the relationship between the individual agent's predictors and its wealth if the predictors the agents own are evolutionary, i.e. agents' predictors are not fixed at the beginning of the game. There are some research on this aspect [4, 25]. Fourth, [16] and [27] have modeled the resource allocation problem as minority games. In the resource allocation problems, there may be not only one resource. The resource capacity may vary over time. Agents may need bundles of resources. So each agent does not make a binary decision, but uses some predictors to predict the resource load to decide which resource to choose. In real life, agents may have different kinds of predictors, or the agents may have preference over the predictors. We are going to extend the model to more complicated multi-agent systems in real-world environments. [32] and [34] have done the research on applications in sensor network and grid computing.

For multiagent resource allocation, we have assumptions that each task only needs one unit of resources to complete in one round and the resources can be shared among agents, which are similar with those in [16, 27]. In real resource allocation systems, the situation may be more complicated. Each agent may have multi-task and each task may need more than one unit of resources to complete in more than one round. The resources may not be available if some tasks already occupy them. So another aspect of future work is to extend the adaptive strategy in more complicated environments, such as resource allocation problems in load balance [43][46].

Appendix A

List of Publications

1. Y. N. She and H. F. Leung. Maximising Personal Utility Using Intelligent Strategy in Minority Game. In *ATC '08: Proceedings of the 5th International Conference on Autonomic and Trusted Computing. Lecture Notes in Computer Science, volume 5060, pages 191-205*, 2008.

2. Y. N. She and H. F. Leung. An Adaptive Strategy for Allocation of Resources with Gradually or Abruptly Changing Capacities. In *ICTAI '08: Proceedings of the 20th IEEE International Conference on Tools with Artificial Intelligence. Los Alamitos: IEEE Computer Society*, 2008.

3. Y. N. She and H. F. Leung. An adaptive Strategy for Resource Allocation with Changing Capacities. *Accepted to the First International Conference on Complex Sciences: Theory and Applications (Complex '09)*.

4. Y. N. She and H. F. Leung. Individual Agent's Wealth in Minority Games. *Submitted to International Journal of Autonomous and Adaptive Communications Systems (IJAACS)*.

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