Resource Allocation for Wireless Networks: Learning, Competition and Coordination

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Resource Allocation for Wireless Networks: Learning, Competition and Coordination

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Resource allocation plays a significant role in designing efficient and reliable wireless networks. However, a generic approach is still not available due to the challenging wireless environments, the many degrees of freedom of the wireless resources, the heterogeneity of wireless networks, etc. We in this thesis investigate several resource allocation problems for typical wireless transmission scenarios. In particular, the thesis illustrates the roles of learning, competition, and coordination in multiuser communication systems.

We first study the distributed power control problem for stochastic parallel Gaussian interference channels while, to the best of our knowledge, all the existing works only consider deterministic scenarios. We resort to learning theory and propose two distributed learning algorithms. Sufficient conditions are provided to guarantee the convergence of the proposed algorithms. We further show that the algorithmic convergence speed is "exponential" in some sense.

We further consider the one-to-many transmission scenarios, extended from the previous one-to-one cases. The distributed power control problem now becomes a generalized Nash equilibrium problem. Resorting to variational inequality theory, we show the existence of generalized Nash equilibrium. Identifying the variational equilibrium as the network operation point, we study the sufficient conditions for the uniqueness issue. Then we propose a penalty-based distributed algorithm along with convergence analysis.

We then take a reverse approach, compared to the previous two studies. In particular, we present a framework for distributed flow allocation in multiple access networks, where the end users can seek wireless flows from multiple access points. Interestingly, this reverse approach helps us investigate the problem in question as a convex one. Consequently, the two proposed distributed algorithms can converge not only to the unique Nash equilibrium but also the globally optimal solution.

We finally consider joint relay assignment and admission control for cooperative networks. This problem illustrates the value of coordination when it comes to complicated resource allocation problems. Indeed, the problem in question is NP-hard. We decompose the problem into two subproblems. A good final solution can be obtained by iteratively solving the two subproblems. We also propose a simple heuristic algorithm to solve the problem in a distributed fashion.

摘 要

資源分配在設計高效可靠的無線網絡中起著十分重要的作用。然而，由於無線環境的複雜性，無線資源的多樣性，無線網絡的異構性等因素，通用的資源分配方法尚未被提出。本篇論文研究了數個在典型的無線通信環境中的資源分配問題。論文尤其展示了學習，競爭和協調在多用戶無線通信系統中的作用。

論文首先研究隨機並行高斯干擾信道中的分佈式功率控制問題。對該問題，現有的研究只考慮確定性的情況。通過應用學習理論，我們提出了兩個分佈式學習算法。論文中給出了充分條件以保證算法的收斂。論文進而論證了算法的指數收斂速度。

在單對單通信問題的基礎上，論文進一步考慮單對多的通信情況。所研究的分佈式功率控制問題本質上是一個廣義納什均衡問題。通過應用變分不等式理論，論文證明了廣義納什均衡的存在。論文同樣研究了變分平衡的單一性問題並給出了充分條件。接著，論文提出了基於懲罰的分佈式算法並分析了算法的收斂性。

與前兩個功率控制問題相比，論文接著採取了流量分配的逆向方法。具體來說，論文為多接入網絡提出了一個分佈式流量分配的解決方案。該方案允許終端用戶接收來自多個無線接入點的信息流。有趣的是，這種方法將原本非凸的問題轉換為一個凸問題。因此，兩個提出的分佈式算法最終能夠收斂到全局的最優解。

論文最後聯合考慮協同在網絡中中繼選擇和接入控制的問題。該問題的研究展示了協調在復雜資源分配問題上的作用。論文將所研究的 NP 困難問題其分解為兩個子問題。一個良好的解可以通過迭代求解兩個子問題獲得。論文同時也為該問題提出了一個簡單的分佈式算法。

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This work is dedicated to my family for their unconditional love !

Contents

List of Figures

List of Tables

Chapter 1

Introduction

If I have seen a little further it is by standing on the shoulders of Giants.

—— Isaac Newton

1.1 Motivation

During the past two decades, we have witnessed an ever increasing demand of wireless services ranging from delay-sensitive applications (real-time multimedia transmission, for one) to delay-insensitive applications. To meet these demands, various wireless techniques and standards have been proposed. For instance, the cellular networks, one of the most well-known wireless networks, has gone through three generations [66]. Nevertheless, a systematic and generic approach for efficient design of wireless networks is still far from being achieved if not impossible.

The fundamental contributor that makes the wireless network design challenging is the wireless channel. Transmission signals are susceptible to path loss, shadowing, noise, and interference in wireless channels. Worse still, these impediments vary over time due to user mobility, fading, and many other dynamic factors. Another major challenge comes from the fact that the wireless communication resources such as the transmission bandwidth and power are often limited. Besides, unlike the wired networks, the transmissions of wireless users are not independent of each other. Indeed, one user's signal often acts as interference to other users that share the same channel. The quality of wireless communications would be severely deteriorated without appropriate interference management techniques. Consequently, equipped with limited resources, communications in complex and dynamic wireless environments make it error-prone and thus challenging.

With the above difficulties in wireless communications, resource allocation plays a significant role in designing efficient and reliable wireless networks [27]. Though significant progress has been made for the resource allocation in wireless networks, a generic approach is still not available. In fact, it is quite unlikely we would arrive at a generic approach for the resource allocation in wireless networks due to the inherent nature of the problem in question. Firstly, there exist a large number of degrees of freedom in wireless networks. In particular, typical wireless resources include time slots, frequency bands, orthogonal codes, space, and transmit powers in traditional wireless networks. New resources also become available when relatively new techniques were proposed these years. For example, relays can also be considered as resources that need to be allocated carefully in cooperative networks [36]. Another example is the antenna resources in wireless networks employing multiple-input-multiple-output (MIMO) techniques [24]. With so many different wireless resources present, a joint optimal allocation would lead to a prohibitively difficult problem though significant performance gains might be guaranteed. As a result, researchers and network designers often just design the allocation schemes for different resources separately.

Furthermore, the heterogeneity of wireless networks makes the resource allocation more challenging. Indeed, different wireless networks may have different design goals. For example, how the available resources should be allocated so that the transmission delay can be minimized is a critical issue for wireless networks that supports delay-sensitive wireless applications. In contrast, how the limited power resource should be carefully exploited to maximize the network life time should be given higher priority in wireless sensor networks [2]. In this scenario, minimizing transmission delay is not the fundamental objective. Also in some networks users move in a fast fashion with opportunistic spectrum access while users use a large portion of the spectrum in networks employing ultra-wide band (UWB) technology but with little mobility [76]. In a word, different application scenarios in general require different design approaches for efficient resource allocation in wireless networks.

Besides, most existing resource allocation schemes are essentially carried out in a centralized fashion. These schemes implicitly assume the existence of central controllers in the corresponding wireless networks. Centralized schemes may be appropriate in wireless networks such as cellular networks, where the base stations can allocate the available network resources to the end users. Nevertheless, the centralized resource allocation schemes possess several disadvantages. First and foremost, the central controllers become the network bottleneck. As the size of the network grows, the central controllers must manage more and more computing and storage elements. Thus the wireless network does not scale well. Moreover, information exchange between central controllers and other network nodes also causes a loss of system resources. Last but not the least, central controllers may not even exist in some networks (ad hoc networks, for one) where centralized scheme becomes infeasible. Therefore, distributed protocols and algorithms for wireless resource allocation are often desired in many scenarios. However, the requirement for distributed resource allocation schemes often imposes further design difficulties including limited communications among the distributed decision makers, the incomplete information at the distributed users, the dynamic changing environments, and so on.

This thesis does not aim to and also cannot provide a comprehensive answer to the wireless resource allocation which is such a broad issue as described above. Instead, we investigate several resource allocation problems for typical wireless transmission scenarios in the hope of shedding some lights on the basic principles and techniques for the allocation of wireless resources. In particular, we study both centralized and distributed wireless resource allocation schemes depending on the specific application scenarios. Nevertheless, we pay more attention to the distributed schemes since they possess obvious advantages over their centralized counterparts.

1.2 Background

1.2.1 Wireless Communication Schemes

A wireless network consists of many transmitters and receivers. Given a channel transition matrix that characterizes the effects of interference and noise, the problem of interest is to decide the optimal communication scheme that achieves the network capacity [16]. However, this general problem so far is still open. In this thesis, we consider several special communication schemes. It should be pointed out that the communication schemes considered in this thesis may or may not be optimal in the sense of network information theory. Nevertheless, as seen later, the considered schemes are of practical interest. In this subsection, we briefly describe the communication schemes studied in this thesis.

1. Gaussian Interference Channel

   The resource allocation problems studied in Chapter 2 and Chapter 3 are based on Gaussian interference channel model. In a Gaussian interference channel, each communication pair communicates in the presence of interference from other independent communication pairs as well as background additive Gaussian noise. For illustration purpose, we consider a two-user Gaussian interference channel scenario for the time being. The extension to the general multiple-user scenario is completely natural.

   In a two-user Gaussian interference channel, the inputs X_1 and X_2 and the outputs Y_1 and Y_2 are related by

$$\begin{aligned} Y_1 &= X_1 + bX_2 + Z_1 \\ Y_2 &= aX_1 + X_2 + Z_2 \end{aligned} \tag{1.1}$$

   where a and b are given interference coefficients, and Z_1 and Z_2 are Gaussian noise with zero-mean and variances N_1 and N_2, respectively. Each user $k, k = 1, 2$, has an average power constraint of P_k.

Even in this simple scenario, the determination of the associated capacity region is still an open problem. The only known case is the strong interference scenario whose capacity region is the same as if there were no interference [25]. A recent breakthrough made in [18] shows that a simple Han-Kobayashi type scheme can achieve to within one bit of the capacity for arbitrary interference scenarios.

Fewer results have been obtained in the parallel Gaussian interference channels which model a network where the network communication pairs share a number of independent channels. Each of these independent channels is a Gaussian interference channel.

Instead of pursuing the optimal communication scheme for (parallel) Gaussian interference channels, we consider a suboptimal scheme where each individual communication pair is only interested in its own signal and simply treats interference as noise when decoding, i.e., not allowing joint encoding/decoding and interference cancellation techniques. This communication scheme is very appealing in practice due to the simplicity and distributiveness. Indeed, receivers in current practical communication systems generally treat interference as noise though substantial research works have been carried out on interference-aware receivers and significant performance gains are promised by multi-user techniques [6].

By adopting single user detection scheme where interference is simply treated as noise, the set of rates (R_1, R_2) in the above two-user Gaussian interference channel setting is given by the capacity of single-user Gaussian channels of user 1 and user 2, respectively. Specifically,

$$\begin{aligned} R_1 &= \log(1+\frac{P_1}{bP_2+N_1}), \\ R_2 &= \log(1+\frac{P_2}{aP_1+N_2}). \end{aligned} \tag{1.2}$$

The extension to the parallel Gaussian interference channels is straightforward. In particular, the set of rates (R_1, R_2) now is given by

$$\begin{aligned} R_1 &= \sum_{i\in\mathcal{I}} \log(1+\frac{P_1^i}{b^i P_2^i+N_1^i}), \\ R_2 &= \sum_{i\in\mathcal{I}} \log(1+\frac{P_2^i}{a^i P_1^i+N_2^i}) \end{aligned} \tag{1.3}$$

where we use notation i to refer to any one of the independent parallel Gaussian interference channels denoted by the set $\mathcal{I}$. Each user $k, k = 1, 2$, has a total power constraint P_k over all the parallel Gaussian interference channels, i.e.,

$$\sum_{i \in \mathcal{I}} P_1^i \leq P_1,$$
$$\sum_{i \in \mathcal{I}} P_2^i \leq P_2. \tag{1.4}$$

Intuitively, this simple suboptimal communication scheme provides an inner bound of the capacity region of parallel Gaussian interference channels. If the channel coefficients and power constraints satisfy a certain condition, a somewhat surprising result recently given in [60] is that this simple communication scheme indeed achieves the sum-rate capacity of the parallel Gaussian interference channels. This result further justifies our interest in studying parallel Gaussian interference channels with single user detection scheme where interference is simply treated as noise. For more details, we refer to Chapter 2 and Chapter 3.

2. Gaussian Multiple Access Channel

   The flow allocation problem studied in Chapter 4 is based on Gaussian multiple access channel model. In a Gaussian multiple access channel, multiple independent transmitters send information to a common receiver in the presence of additive Gaussian noise. Unlike the Gaussian interference channel, the capacity region of Gaussian multiple access channel has been characterized. For ease of exposition, we again consider a two-user scenario for the time being.

   In a two-user Gaussian multiple access channel, the inputs X_1 and X_2 and the output Y are related by

$$Y = X_1 + X_2 + Z \tag{1.5}$$

   where Z is Gaussian noise with zero-mean and variance N_0. Each user $k, k = 1, 2$, has an average power constraint of P_k.

   The associated capacity region in two-user Gaussian multiple access channel is the set of rates (R_1, R_2) satisfying the following three constraints (see, e.g., [66]):

$$R_1 \leq \log(1 + \frac{P_1}{N_0})$$
$$R_2 \leq \log(1 + \frac{P_2}{N_0})$$
$$R_1 + R_2 \leq \log(1 + \frac{P_1 + P_2}{N_0}). \tag{1.6}$$

These constraints are quite natural. In particular, the first constraint describes that user 1 cannot transmit at a rate higher than the capacity of single-user Gaussian channel with user 2 absent from the system. Similar interpretation with an exchange of the roles of user 1 and user 2 can be carried over for the second constraint. The third constraint describes that the sum rate of user 1 and user 2 cannot be higher than the capacity of single-user Gaussian channel with the sum of the received powers of the two users.

The next question that naturally arises is how the maximum sum capacity, i.e., $\log(1+\frac{P_1+P_2}{N_0})$, can be achieved in this two-user Gaussian multiple access channel. The answer hinges on a key idea: successive interference cancellation.

For illustration of the idea of successive interference cancellation, we describe an approach that achieves the set of rates

$$(R_1, R_2)^* = (\log(1+\frac{P_1}{N_0}), \log(1+\frac{P_2}{P_1+N_0})).$$

It is clear the above set of rates achieves the maximum sum capacity $\log(1+\frac{P_1+P_2}{N_0})$. We next describe how this set of rates can be achieved by the network. At the transmitter side, each user independently encodes its data adopting a capacity-achieving Gaussian channel code. At the receiver side, the decoding consists of three steps. In the first step, the receiver decodes the data of user 2 by simply treating user 1's signal as noise. Thus the rate attained by user 2 is $\log(1+\frac{P_2}{P_1+N_0})$. In the second step, the receiver reconstructs and subtracts user 2's signal from its aggregate received signal. In the third step, the receiver decodes the data of user 1 in the presence of only background Gaussian noise. Thus, the rate attained by user 1 is $\log(1+\frac{P_1}{N_0})$.

It should be pointed out that the above approach is not the only optimal scheme in terms of sum capacity. Clearly, we also can achieve the maximum sum capacity $\log(1+\frac{P_1+P_2}{N_0})$ by reversing the order of the above interference cancellation which achieves the set of rates

$$(R_1, R_2)^\dagger = (\log(1+\frac{P_1}{P_2+N_0}), \log(1+\frac{P_2}{N_0})).$$

Furthermore, a convex combination of the above two sets of rates, i.e.,

$$(R_1, R_2)^\theta = \theta(R_1, R_2)^* + (1-\theta)(R_1, R_2)^\dagger, \theta \in (0, 1),$$

is also optimal in the sense of achieving the maximum sum capacity. Thus, we can time share the above two orders of interference cancellation for other system design concerns (fairness, for one) while preserving optimality.

The idea of successive interference cancellation will be utilized in the flow allocation problem studied in Chapter 4.

3. Cooperative Communications

Cooperative communications is a new paradigm that allows communication nodes in the network to help each other. However, cooperative communications is a rather vague and broad concept that has been extensively used in both wireline and wireless networks (see, e.g., [33]). Nevertheless, we use this concept in this thesis only to refer to the several low complexity cooperative protocols proposed in [36] to achieve spatial diversity for wireless networks. Indeed, cooperative communications can improve transmission diversity by allowing single-antenna users to exploit other users' antennas, generating a virtual MIMO system [36].

We next briefly describe how cooperative communications works. Let us consider a classic three-node relay model which works in a time-division way. In particular, the communication between the source and the destination is carried out in two time slots. In the first time slot, the source transmits a signal to the destination which is overheard by the relay as well due to the broadcast nature of wireless communications. Then in the second time slot the relay forwards its overheard signal to the destination based on some cooperative protocol. Based on how the relay functions during the cooperative transmission, two basic cooperative protocols: amplify-and-forward (AF) and decode-and-forward (DF), were proposed in [36] by Laneman *et al.*

For illustration, we in the sequel take a more careful look at the AF cooperative protocol while we refer to [36] for DF scheme. In the first time slot, the signals $Y_{s,d}$ and $Y_{s,r}$ received at the destination and the relay are

$$\begin{aligned} Y_{s,d} &= \sqrt{P_s} h_{s,d} X + Z_{s,d}, \\ Y_{s,r} &= \sqrt{P_s} h_{s,r} X + Z_{s,r}, \end{aligned} \tag{1.7}$$

where

(a) $h_{s,d}$ and $h_{s,r}$ are the channel coefficients of the source-relay and source-destination channels, respectively;

(b) P_s is the transmission power used by the source;

(c) X denotes the transmission symbol of the source, the power of which is normalized, i.e., $\mathbb{E}(XX^*) = 1$;

(d) $Z_{s,d}$ and $Z_{s,r}$ are the white Gaussian noises of the source-relay and source-destination channels. We denote by $N_{s,d}$ and $N_{s,r}$ the variances of $Z_{s,d}$ and $Z_{s,r}$, respectively.

In the second time slot, the relay normalizes its received signal and forwards it to the destination. The signal $Y_{r,d}$ received at the destination from the relay is

$$Y_{s,d} = \sqrt{P_r} h_{r,d} X_r + Z_{r,d} \tag{1.8}$$

where

(a) X_r denotes the normalized signal, i.e.,

$$X_r = \frac{Y_{s,r}}{\mathbb{E}[|Y_{s,r}|^2]} = \frac{\sqrt{P_s} h_{s,r} X + Z_{s,r}}{\sqrt{|h_{s,r}|^2 P_s + N_{s,r}}};$$

(b) $h_{r,d}$ is the channel coefficient of the relay-destination channel;

(c) P_r is the transmission power used by the relay;

(d) $Z_{r,d}$ is the white Gaussian noise of the relay-destination channel, the variance of which is denoted by $N_{r,d}$.

The capacity expression for the above AF scheme is given by [36]

$$C_{AF} = \frac{1}{2} \log_2 \left(1 + \frac{|h_{s,d}|^2 P_s}{N_{s,d}} + f\left(\frac{|h_{s,r}|^2 P_s}{N_{s,r}}, \frac{|h_{r,d}|^2 P_r}{N_{r,d}}\right) \right), \tag{1.9}$$

where $f(x, y) := \dfrac{xy}{1 + x + y}$. Recall the baseline capacity attained by the point-to-point direct transmission scheme:

$$C_{DT} = \log_2 \left(1 + \frac{|h_{s,d}|^2 P_s}{N_{s,d}} \right). \tag{1.10}$$

By comparing the two capacity expressions above, it is clear that the AF scheme has the potential to achieve more diversity gain than the direct transmission scheme. Nevertheless, it should be pointed out that there is also a spectral efficiency loss in the above simple AF scheme. Thus, it seems that cooperative protocols (at least the described AF) do not have absolute advantage over the traditional direct transmission scheme. A natural question arising from this observation is that when we should employ cooperative communications in a network setting. We refer to Chapter 5 for more details.

1.2.2 Mathematical Preliminaries

We in this subsection briefly describe the main mathematical techniques involved in this thesis in solving the resource allocation problems. In general, we resort to the relevant optimization tools. Nevertheless, different optimization tools are often required in designing different wireless networks. Here we briefly introduce the major optimization tools used throughout this thesis.

1. Convex Optimization

   Convex optimization is a special class of mathematical optimization, which in general has the form given by [63]

$$\begin{aligned} \text{minimize} \quad & f(x) \\ \text{subject to} \quad & x \in \Phi \end{aligned} \tag{1.11}$$

   where x is the *optimization variable* constrained on the set Φ, and $f(x)$ is the *objective function*. The aim of this problem is to find the optimal solution $x^* \in \Phi$ such that $f(x^*) \leq f(x), \forall x \in \Phi$. If the objective function $f(x)$ is convex with x constrained on a convex set Φ, then it is called *convex optimization problem*[1].

   Convex optimization is a relatively well-studied area. Efficient algorithms such as interior-point methods have been developed for convex optimization problems. Since convex optimization problems possess certain (convexity) structures, its prevalent application power was beyond people's previous imagination. Communication network design is one of the many applications where convex optimization has been widely used. We refer to [47] for the wide application of convex optimization for communication network design.

   Nevertheless, the well development of convex optimization theory does not imply that applying it to the resource allocation problems in wireless network design is a simple task. First, identifying a resource allocation problem as a convex one is not as easy as it looks. Indeed, sophisticated reformulation is often required to arrive at a convex formulation of the problem in question. Moreover, how to utilize convex optimization theory to shed some lights on the structure of the resource allocation problem (may or may not be convex) also requires careful analysis and great efforts. Besides, in order to develop efficient methods for specific application scenarios (large-scale problems, for one), researchers and designers often need to carefully exploit the special structures of the specific resource allocation problems even they turn out to be convex optimization problems.

   Though most of the problems studied in this thesis are in general not convex, convex optimization theory is still used throughout this thesis. For example, some subproblems of the resource allocation problems studied may turn out to be convex ones. Besides, convex optimization is also used to find bounds on optimal objective values and/or approximate solutions in some resource allocation problems studied.

[1]For the detailed definitions of convex function and convex set, we refer to [63].

2. Integer Programming

   Integer programming is also a special class of mathematical optimization. In particular, it aims at solving optimization problems with integer variables. It in general has the form given by [73]

$$\begin{aligned} \text{minimize} \quad & f(x, y) \\ \text{subject to} \quad & x \in \Phi_1 \subseteq \mathbb{R}^m, y \in \Phi_2 \subseteq \mathbb{Z}^n. \end{aligned} \tag{1.12}$$

   Here x is the *continuous optimization variable* constrained on the set Φ_1 which is a subset of m-dimensional real vectors. y is the *integer optimization variable* constrained on the set Φ_2 which is a subset of n-dimensional integer vectors. $f(x, y)$ is the *objective function*. More precisely, the above problem is called *mixed integer programming problem*. It would be called *pure integer programming problem* if there were only integer variables y present.

   Obviously, one can find wide applications of integer programming in all walks of life since many decision variables in real life are often indivisible. Indeed, many resource allocation problems such as scheduling in wireless network design can be formulated as integer programs. Unfortunately, unlike convex optimization, it is in general difficult to find the optimal solution to integer programs and/or even to check whether a given feasible solution is optimal. Indeed, many integer programs are in general NP-hard. Therefore, relaxation and decomposition are often required in solving integer programs. Sometimes one can even only obtain heuristic algorithms.

   The relay assignment problem in cooperative networks with quality-of-service (QoS) guarantee studied in this thesis turns out to be integer program. For more details on the application of integer programming in this thesis, we refer to Chapter 5.

3. Game Theory

   Game theory is a branch of applied mathematics that is used to study the interaction among distributed decision makers. It has been widely used in economics. Starting from the early 1990's, researchers in communication community became increasingly interested in applying game theory to networking problems. Indeed, the potentials of game theory in modeling the iteraction among the distributed decision makers make it a natural tool to model and design dynamic communication networks. It becomes even more powerful in designing self-organizing networks (ad hoc networks, for one). A comprehensive description on game theory can be found in [46] and is certainly beyond the scope of this thesis. We here only briefly introduce some key concepts in game theory and focus on strategic form game.

   A strategic form game can be described by a 3-tuple $\mathcal{G} = \{\mathcal{N}, \{\Phi_i\}_{i\in\mathcal{N}}, \{U_i\}_{i\in\mathcal{N}}\}$ where

(a) $\mathcal{N} = \{1, 2, ..., N\}$ is the set of N *players*,

(b) $\Phi_i \subseteq \mathcal{R}^{M_i}$, with $\mathcal{R}^{M_i}$ being the M_i-dimensional Euclidean space, is the set of *actions* of player i, and

(c) $U_i : \Phi \mapsto \mathbb{R}$ with $\Phi = \prod_{i\in\mathcal{N}} \Phi_i$ is the *utility* function of player i.

Note that we implicitly assume that the action sets of the players are continuous. In this thesis, we focus on the class of noncooperative games, where each user only cares about its[2] own utility without cooperation with other players. A widely accepted (though debatable) rational outcome of the noncooperative games is called Nash equilibrium (NE). We formally define NE as follows.

Definition 1.1 *An action profile $\boldsymbol{a}^* = (a_1^*, ..., a_N^*)$ is called Nash equilibrium of the noncooperative game $\mathcal{G}$ if and only if*

$$a_i^* \in arg\max\{U_i(a_i, a_{-i}^*) : a_i \in \Phi_i\}, \forall i \in \mathcal{N}, \tag{1.13}$$

where $arg\max\{U_i(a_i, a_{-i}^*) : a_i \in \Phi_i\}$ denotes the set of actions that maximize $U_i(a_i, a_{-i}^*)$ with given a_{-i}^*, and a_{-i} is formed by the actions of all players other than player i.

Clearly, no player can increase its utility by unilaterally changing its action at an NE. Several issues deserving careful investigation exist in game $\mathcal{G}$. Firstly, a basic question is the existence of NE. Another classical issue is the uniqueness of NE of game $\mathcal{G}$. Besides, how to arrive at an NE from initially nonequilibria states is of practical interest. Indeed, one in general cannot expect the distributed players to choose the equilibrium actions right away. Instead, these distributed players must be offered time to *learn* and update their actions according to some rules. These predetermined updating rules are termed as *strategies* of the players. In particular, we denote by $B(a_{-i})$ the strategy of player i responding to other players' actions a_{-i}. Commonly used strategies in the noncooperative games are as follows.

(a) Best Response:

$$B(a_{-i}) \in \arg\max\{U_i(a_i, a_{-i}) : a_i \in \Phi_i\}; \tag{1.14}$$

(b) Gradient Projection Response:

$$B(a_{-i}) = \mathcal{P}_{\Phi_i}(a_i + \alpha_i \nabla_{a_i} U_i(a_i, a_{-i})), \tag{1.15}$$

[2]The choice for the gender of the players concerned is debatable. For simplicity, we simply use "its" throughout this thesis.

where $\mathcal{P}_{\Phi_i}(x)$ denotes the Euclidean projection of x on the set Φ_i and α_i is a positive step size.

Note that the action set Φ_i of player i may or may not depend on the actions of other players. If Φ_i is independent of $a_{-i}, \forall i \in \mathcal{N}$, then the problem of finding the NE of game $\mathcal{G}$ is termed as Nash equilibrium problem (NEP). In contrast, the problem in question is is termed as generalized Nash equilibrium problem (GNEP) if Φ_i depends on a_{-i}, i.e., $\Phi_i(a_{-i}) : \prod_{j \neq i, j \in \mathcal{N}} \Phi_j \mapsto \mathcal{R}^{M_i}$ is a set-value mapping, for some $i \in \mathcal{N}$.

The majority of this thesis on distributed resource allocation design for wireless network is based on the above game-theoretical framework. Indeed, the power allocation in parallel Gaussian interference channels is formulated and studied as a stochastic noncooperatve game in Chapter 2. The distributed information flow allocation problem in multiple access networks and the power allocation for one-to-many transmission networks are both formulated as GNEPs. Nevertheless, different issues exist in these resource allocation problems considered in this thesis and thus need to be addressed on a case by case basis.

1.3 Outline of the Thesis

In this thesis, we in general focus on the design of the various resource allocation schemes in wireless communications and networks, with a special emphasis on distributed schemes. The outline of each chapter is as follows.

In chapter 1, we give the motivation of the resource allocation in wireless communications and networks. We then describe our objective in this thesis. The various optimization techniques used throughout this thesis are also briefly presented to facilitate understanding of the remaining chapters.

Chapter 2 deals with the distributed power control issue for parallel Gaussian interference channels. This issue recently draws great interests. However, all existing works only studied this problem under deterministic communication channels and required certain perfect information to carry out their proposed algorithms. We instead study this problem for stochastic parallel Gaussian interference channels. In particular, we take into account the randomness of the communication environment and the estimation errors of the desired information, and thus formulate a stochastic noncooperative power control game. We then propose a stochastic distributed learning algorithm SDLA-I to help communication pairs learn the Nash equilibrium. A careful convergence analysis on SDLA-I is provided based on stochastic approximation theory and projected dynamical systems approach. We further propose another learning algorithm SDLA-II by including a simple iterate averaging idea into SDLA-I to improve the algorithmic convergence performance.

Chapter 3 extends the distributed power control problem for one-to-one transmissions in Gaussian interference channels to one-to-many transmission scenarios. We assume a user-centric wireless network where the end users play the roles of decision makers. We formulate the power control problem as a noncooperative game. New challenges arise due to the coupling issues among power strategy spaces of distributed end users, which make standard Nash equilibrium based noncooperative game approach inapplicable. Indeed, our problem turns out to be a GNEP. Resorting to variational inequality theory, we show several fundamental properties of the GNEP. Then we propose a penalty-based distributed algorithm IP^2JA, which possesses favorable properties for practical implementation.

Chapter 4 considers multiple access transmissions, which can be regarded as a special communication scenario of the more general interference channel model considered in Chapter 2 and 3. We resort to a flow allocation approach rather than power control for multiple access transmissions. As it will become clear later, this "dual" approach possesses some particular advantages over the power control method. In particular, we aim to minimize the power consumption while satisfying each end user's minimum data rate requirement but not violating peak power constraint of each access point and interference constraint monitored by regulatory agents. Toward this end, we model the flow allocation problem as a game which is proved to be a best-response potential game. Then based on potential game theory, we show the existence and uniqueness of NE in the formulated game. Moreover, we demonstrate that the NE is actually the globally optimal solution to our problem. Besides, we propose two distributed algorithms along with convergence analysis for the network to obtain the NE. Meanwhile, we also remark the interesting layered structure of the flow allocation problem considered.

Chapter 5 studies the newly emerging cooperative networks, where relays play a fundamental role to fully explore the potentials of cooperative communication technique. However, adaptive cooperation with conflict-free relay assignment in a network setting is a challenging problem. The problem becomes even more difficult when users have QoS requirements but the available spectrum resource is limited, where admission control may be required. In this chapter, we jointly study relay assignment and admission control in cooperative networks. A one-stage optimization problem is formulated to integrate our multiple objectives. Since the problem in question is prohibitively difficult, we resort to an appropriate decomposition approach after a careful analysis on the structure of the formulated problem. A simple distributed algorithm is also proposed to overcome the inherent drawbacks of the centralized scheme.

Chapter 6 concludes the thesis by summarizing the main results and discussing further research directions.

□ **End of chapter.**

Chapter 2

Learning for Parallel Gaussian Interference Channels

There is no royal road to learning; no short cut to the acquirement of any art.
—— *Anthony Trollope*

The interference channel has long drawn interests from both information theory and communication communities [16]. Indeed, the interference channel provides a good model for many communication systems from digital subscriber lines to wireless communication systems. Nevertheless, its capacity region is still unknown in general even in the Gaussian scenario. Moreover, compared to the flat interference channel, fewer works have been done in frequency-selective interference channels. We refer to [69] for an overview on interference channels.

In this chapter we focus on power control in frequency-selective interference channels with Gaussian noise, i.e., parallel Gaussian interference channels. It has been shown recently in [39] that obtaining globally optimal solution to maximizing the network sum rates is NP-hard in general. Nevertheless, a distributed game-theoretic approach originally proposed in [78] becomes increasingly popular. The key assumption is that each individual communication pair is only interested in its own signal and simply treats interference as noise when decoding, i.e, not allowing joint encoding/decoding and interference cancellation techniques.

After the seminal work [78], different approaches have been applied to study the distributed power control in parallel Gaussian interference channels when the channel power gains are deterministic. Specifically, [58] [57] [56] are based on contraction mapping, [62] is based on piecewise affine mapping, [48] resorts to variational inequality theory, and [38] formulates an equivalent linear complementary problem. These works focused on characterizing the Nash equilibrium (NE) such as existence and uniqueness and devising distributed algorithms along with convergence analysis. Indeed, the proposed iterative water-filling algorithm (IWFA) has become a popular candidate for distributed power control in parallel Gaussian interference channels.

Nevertheless, a common assumption in existing works is that a communication pair is just interested in maximizing its immediate transmission rate. Besides, it is assumed that communication channels remain unchanged during the algorithmic iterations [78] [58] [57] [56] [48] [38]. However, the communication time scale is usually large in common applications such as video transmission in wireless data networks [40]. During the whole communication period, it is unlikely that channels would remain the same. In these scenarios, a communication pair may be more interested in maximizing its long term transmission rate rather than the immediate one. Besides, existing works require the knowledge of exact CSI and/or interference levels to be fed back to the corresponding transmitters during the algorithmic iterations. Unfortunately, none of these can be easily obtained in practical communication systems if not impossible. The convergence results of existing schemes such as IWFA are no longer valid or at least unknown when relevant estimation errors exist.

In this chapter we take into account the randomness of the communication environment and estimation errors of the desired information. We assume each communication pair is concerned about the long term transmission rate, i.e., the expected transmission rate. We first propose a basic stochastic distributed learning algorithm SDLA-I to help distributed communication pairs learn the NE in stochastic transmission environments. The desired information in implementing SDLA-I is also allowed to be subject to errors. A careful convergence analysis on SDLA-I is also provided based on stochastic approximation theory [51] [35] and projected dynamic systems (PDS) approach [44]. Inspired by the recent developments in stochastic approximation theory [50] [34], we propose another learning algorithm SDLA-II by including a simple iterate averaging idea into the basic learning algorithm SDLA-I to improve the algorithmic convergence performance.

The power control algorithms proposed in this chapter belong to the class of stochastic power control algorithms. Existing stochastic power control algorithms (see, e.g., [68] [70] [79] and references therein) cannot be applied to the parallel Gaussian interference channels considered in this chapter. Note that the recent work [13] studied the distributed power control for time-varying parallel Gaussian interference channels. Nevertheless, the model formulated in [13] is essentially a deterministic one. So IWFA could still be applicable in [13]. In contrast, as explained in section 2.1, it would be extremely difficult and/or inconvenient to apply IWFA in our model if not impossible. Besides, [13] also requires the knowledge of exact CSI and interference levels to be fed back to the corresponding transmitters during each iteration of power update.

The rest of this chapter is organized as follows. Section 2.1 describes the specific system model and the problem formulation. In section 2.2, the basic learning algorithm SDLA-I is described along with a careful convergence analysis. The PDS approach is adopted in section 2.3 to study the rate of convergence of SDLA-I. We further include the idea of iterate averaging and propose SDLA-II in section 2.4. Section 2.5 presents some numerical results.

2.1 System Model and Problem Formulation

We consider a scenario consisting of a set of N source-destination pairs indexed by $\mathcal{N} = \{1, 2, ..., N\}$. These communication pairs share a common set $\mathcal{K} = \{1, 2, ..., K\}$ of frequency-selective unit-bandwidth channels so that their transmissions may interfere with each other. Specifically, the received signal at destination j on the k-th channel can be described by the baseband signal model

$$y_j^k = h_{jj}^k \sqrt{p_j^k} x_j^k + \sum_{i \neq j, i \in \mathcal{N}} h_{ji}^k \sqrt{p_i^k} x_i^k + z_j^k, \tag{2.1}$$

where h_{ji}^k denotes the channel coefficient from source i to destination j on the k-th channel, p_j^k denotes the transmission power used by source j on the k-th channel, x_j^k denotes the normalized transmission symbol of source j on the k-th channel, and z_j^k denotes the white Gaussian noise with variance n_j^k at destination j on the k-th channel.

For later use, we let $g_{ji}^k = |h_{ji}^k|^2$. In time-varying communication scenarios, channel coefficients are obviously random variables. We denote by $\boldsymbol{G}$ the random vector composed of all the random channel power gain coefficients, i.e., $\boldsymbol{G_{ji}^k}, \forall k \in \mathcal{K}, \forall j, i \in \mathcal{N}$. For the sake of greater applicability we shall make no assumption on the specific underlying statistical distribution of $\boldsymbol{G}$. We simply assume that $\boldsymbol{G}$ is bounded almost surely and different realizations $\boldsymbol{g}$'s of $\boldsymbol{G}$ are independent and identically distributed (i.i.d.). This i.i.d. assumption on $\boldsymbol{G}$ is reasonable in large scale networks.

We further assume that each user is only interested in its own signal and treats interference as noise. Thus, we can write the signal-to-interference-plus-noise-ratio (SINR) at destination j on the k-th channel with realization $\boldsymbol{g}$ as

$$\gamma_j^k = \frac{g_{jj}^k p_j^k}{\sum_{i \neq j, i \in \mathrm{N}} g_{ji}^k p_i^k + n_j^k}, \forall k \in \mathcal{K}, \forall j \in \mathcal{N}. \tag{2.2}$$

The corresponding maximum achievable rate R_j for user j is given by Shannon formula [16]

$$R_j(\boldsymbol{p_j}, \boldsymbol{p_{-j}} | \boldsymbol{g}) = \sum_{k=1}^{K} \ln(1 + \gamma_j^k), \tag{2.3}$$

where $\boldsymbol{p_j} = [p_j^1, p_j^2, ..., p_j^K]^T$ denotes the power allocation strategy of user j, and $\boldsymbol{p_{-j}}$ denotes the power allocation strategies of all the other users. The power allocation strategy of each user should satisfy certain constraints. Specifically, $\boldsymbol{p_j}$ is regulated by spectral mask constraints, i.e., $0 \leq p_j^k \leq \bar{p}_j^k$, as well as a total power constraint, i.e., $\sum_{k\in\mathcal{K}} p_j^k \leq p_j^{max}$. In order to avoid trivial cases, we assume for all $j \in \mathcal{N}$ that $\bar{p}_j^k < p_j^{max}, \forall k \in \mathcal{K}$, and $p_j^{max} < \sum_{k\in\mathcal{K}} \bar{p}_j^k$.

We now formulate the following noncooperative game to characterize the interaction among the users in question:

$$\mathcal{G} = \{\mathcal{N}, \{\Phi_j\}_{j\in\mathcal{N}}, \{\bar{R}_j(\boldsymbol{p_j}, \boldsymbol{p_{-j}})\}_{j\in\mathcal{N}}\}. \tag{2.4}$$

In game $\mathcal{G}$, $\mathcal{N}$ is the set of players, i.e., communication pairs. $\bar{R}_j(\boldsymbol{p_j}, \boldsymbol{p_{-j}})$ is the utility function of user j given by

$$\bar{R}_j(\boldsymbol{p_j}, \boldsymbol{p_{-j}}) = \mathbb{E}_{\boldsymbol{G}}[R_j(\boldsymbol{p_j}, \boldsymbol{p_{-j}}|\boldsymbol{G})], \tag{2.5}$$

where $\mathbb{E}_{\boldsymbol{G}}[\cdot]$ denotes the expected value with respect to $\boldsymbol{G}$. We shall in the sequel drop the subscript to write $\mathbb{E}[\cdot]$ instead of $\mathbb{E}_{\boldsymbol{G}}[\cdot]$ when not leading to confusion. Here we implicitly assume that $\bar{R}_j(\boldsymbol{p_j}, \boldsymbol{p_{-j}})$ exists. We further assume $\bar{R}_j(\boldsymbol{p_j}, \boldsymbol{p_{-j}})$ is continuous with respect to $\boldsymbol{p}$. Φ_j is the strategy space of user j defined as

$$\Phi_j = \{\boldsymbol{p_j} \in \mathbb{R}^K : \sum_{k\in\mathcal{K}} p_j^k \leq p_j^{max}, 0 \leq p_j^k \leq \bar{p}_j^k, \forall k \in \mathcal{K}\}. \tag{2.6}$$

For later use, we denote by Φ the product space $\Phi_1 \times ... \times \Phi_N$.

Due to the uncertainty of channel power gains, player j in stochastic game $\mathcal{G}$ wishes to maximize its expected transmission rate $\bar{R}_j$ by choosing appropriate power allocation strategy $\boldsymbol{p_j}$. Mathematically, player j solves the following optimization problem

$$\begin{array}{ll} \text{maximize} & \bar{R}_j(\boldsymbol{p_j}, \boldsymbol{p_{-j}}) \\ \text{subject to} & \boldsymbol{p_j} \in \Phi_j \end{array}$$

where $\bar{R}_j(\boldsymbol{p_j}, \boldsymbol{p_{-j}})$ and Φ_j are given in (2.5) and (2.6), respectively. Note that this is a stochastic optimization problem [63].

We are interested in understanding if and how the players in stochastic game $\mathcal{G}$ can achieve NE, which is a widely adopted rational outcome of noncooperative games. We formally define NE of the stochastic power control game $\mathcal{G}$ as follows.

Definition 2.1 *A power allocation profile $\boldsymbol{p}^* = (\boldsymbol{p_1^*}, ..., \boldsymbol{p_N^*})$ is called an NE of the stochastic power control game $\mathcal{G}$ if and only if*

$$\boldsymbol{p_j^*} \in arg\max\{\bar{R}_j(\boldsymbol{p_j}, \boldsymbol{p_{-j}^*}) : \boldsymbol{p_j} \in \Phi_j\}, \forall j \in \mathcal{N}. \tag{2.7}$$

Game $\mathcal{G}$ has been extensively studied when the channel power gains are deterministic. Nevertheless, new challenges arise due to the randomness in the channel power gains caused by the stochastic communication environments. Indeed, player j in stochastic game $\mathcal{G}$ may not even be able to know its utility function $\mathbb{E}[R_j(\boldsymbol{p_j}, \boldsymbol{p_{-j}}|\boldsymbol{G})]$ due to the following reasons. Firstly, the distribution of $\boldsymbol{G}$ is unknown though $R_j(\boldsymbol{p_j}, \boldsymbol{p_{-j}}|\boldsymbol{g})$ is known. Thus, it is impossible to evaluate $\mathbb{E}[R_j(\boldsymbol{p_j}, \boldsymbol{p_{-j}}|\boldsymbol{G})]$ analytically or numerically. Indeed, even if the distribution of $\boldsymbol{G}$ was known, it would require player j to obtain global knowledge to evaluate $\mathbb{E}[R_j(\boldsymbol{p_j}, \boldsymbol{p_{-j}}|\boldsymbol{G})]$, which may result in an unacceptable level of communication overhead. Furthermore, even further assuming that player j has the global knowledge about the distribution of $\boldsymbol{G}$, evaluation of $\mathbb{E}[R_j(\boldsymbol{p_j}, \boldsymbol{p_{-j}}|\boldsymbol{G})]$ involves multi-dimensional integration and is thus computationally expensive. Since player j does not know its exact utility function $\mathbb{E}[R_j(\boldsymbol{p_j}, \boldsymbol{p_{-j}}|\boldsymbol{G})]$, it is impossible for player j to compute a best response, which is an essential component in IWFA. So IWFA cannot be applied to the stochastic game $\mathcal{G}$ investigated in this chapter.

2.2 Stochastic Algorithm for Learning

2.2.1 Algorithm Design

We aim to design a distributed scheme so that an NE of the stochastic game $\mathcal{G}$ can be obtained even with so many difficulties described in the previous section. Obviously, such a distributed scheme makes sense only when NE exists. Thus, we first address the existence of NE in the following proposition.

Proposition 2.1 *At least one NE exists in the stochastic power control game $\mathcal{G}$.*

Proof It is obvious that $\boldsymbol{\Phi}$ is a convex, nonempty, and compact set. Besides, $\bar{R}_j(\boldsymbol{p_j}, \boldsymbol{p_{-j}})$ is jointly continuous by assumption. Noting further that $R_j(\boldsymbol{p_j}, \boldsymbol{p_{-j}}|\boldsymbol{g})$ is concave with respect to $\boldsymbol{p_j}$, we conclude that $\bar{R}_j(\boldsymbol{p_j}, \boldsymbol{p_{-j}}) = \mathbb{E}[R_j(\boldsymbol{p_j}, \boldsymbol{p_{-j}}|\boldsymbol{G})]$ is also concave with respect to $\boldsymbol{p_j}$ since expectation operation preserves concavity. The existence of NE thus follows from standard results in game theory [46].

In stochastic communication environments, a desired distributed scheme must offer users time to "learn" the environments gradually. Hopefully, an NE can be achieved as users in game $\mathcal{G}$ keep taking adaptive strategies during the learning process. Toward this end, we first define f_j^k as

$$f_j^k = \frac{g_{jj}^k p_j^k}{\sum_{i\in\mathcal{N}} g_{ji}^k p_i^k + n_j^k}, \tag{2.8}$$

which represents the ratio of the received energy of user j's signal to the total received signal energy at destination j on the k-th channel. We let $\boldsymbol{f_j} = [f_j^1, ..., f_j^K]^T$. Since $R_j(\boldsymbol{p_j}, \boldsymbol{p_{-j}}|\boldsymbol{g})$

Step 1: Initialization:
Each player $j \in \mathcal{N}$ starts with an arbitrarily feasible power allocation vector, i.e., $\boldsymbol{p_j}(0) \in \Phi_j$. Set $n := 0$.

Step 2: Computation:
Each player $j \in \mathcal{N}$ computes $\boldsymbol{p_j}(n+1)$ by

$$\boldsymbol{p_j}(n+1) = \mathcal{P}_{\Phi_j}[\boldsymbol{p_j}(n) + a_j(n)\frac{\hat{\boldsymbol{f}}_j(n)}{\boldsymbol{p_j}(n)}], \tag{2.9}$$

where $\mathcal{P}_{\Phi_j}[\cdot]$ denotes the projection onto Φ_j with respect to the Euclidean norm, $(a_j(n))_{n=0}^{\infty}$ is step size sequence.

Step 3: Convergence Verification:
If stopping criteria are satisfied, then stop; otherwise, set $n := n+1$, and go to Step 2.

Table 2.1: Detail steps of SDLA-I

is concave with respect to $\boldsymbol{p_j}$ and $\boldsymbol{f_j}/\boldsymbol{p_j}$ is the associated gradient, we have for any $\boldsymbol{p_j} \in \Phi_j$

$$R_j(\boldsymbol{q_j}, \boldsymbol{p_{-j}}|\boldsymbol{g}) \leq R_j(\boldsymbol{p_j}, \boldsymbol{p_{-j}}|\boldsymbol{g}) + (\boldsymbol{f_j}/\boldsymbol{p_j})^T(\boldsymbol{q_j} - \boldsymbol{p_j}), \forall \boldsymbol{q_j} \in \Phi_j.$$

Now we are in a position to describe the distributed learning algorithm SDLA-I for game $\mathcal{G}$ to reach NE. We formally summarize SDLA-I in Tabel 2.1.

In equation (2.9), $\hat{\boldsymbol{f}}_j(n) = (\hat{f}_j^1(n), ..., \hat{f}_j^K(n))^T$ where $\hat{f}_j^k(n)$ is an approximate estimate of $f_j^k(n+1)$. Thus, receiver j can just locally measure the total received signal energy and extract its own signal energy on each subchannel. Then receiver j notifies transmitter j through control channel the corresponding ratio vector $\hat{\boldsymbol{f}}_j$. Note that SDLA-I does not require an exact estimate. Mathematically,

$$\begin{aligned} R_j(\boldsymbol{p_j}, \boldsymbol{p_{-j}}(n)|\boldsymbol{g}(n+1)) \leq & R_j(\boldsymbol{p_j}(n), \boldsymbol{p_{-j}}(n)|\boldsymbol{g}(n+1)) \\ & + (\hat{\boldsymbol{f}}_j(n)/\boldsymbol{p_j}(n))^T(\boldsymbol{p_j} - \boldsymbol{p_j}(n)) + \epsilon_j(n), \forall \boldsymbol{p_j} \in \Phi_j, \end{aligned} \tag{2.10}$$

where $\epsilon_j \geq 0$ measures the accuracy of the estimation $\hat{\boldsymbol{f}}_j$. Note that all existing algorithms for distributed power control in parallel Gaussian interference channels require the knowledge of exact CSI and/or interference level to be fed back to the corresponding transmitters[78] [58] [57] [56] [48] [38] [13]. Unfortunately, it is hard to obtain perfect knowledge of these information in practical communication systems if not impossible. The convergence results on existing schemes such as IWFA are no longer valid or at least unknown when relevant estimation errors exist. Thus, as described above, our proposed SDLA-I is more robust and requires less communication overhead. Nevertheless, the estimation errors of $\hat{\boldsymbol{f}}_j$ should not be too "bad". We later will formalize the quantitative criteria which specify how exact $\hat{\boldsymbol{f}}_j$

should be.

A careful reader may concern about the computation complexity of SDLA-I since each player needs to conduct a projection operation during every iteration and projection is in general time-consuming. We address this issue through the following proposition which in fact provides a close form solution for the projection operation in (2.9), implying that SDLA-I can be carried out efficiently.

Proposition 2.2 *The close form solution for the projection operation (2.9) is given by*

$$p_j^k(n+1) = [p_j^k(n) + a_j(n)\frac{\hat{f}_j^k(n)}{p_j^k(n)} - \lambda_j]_0^{\bar{p}_j^k}, \forall k \in \mathcal{K}, \tag{2.11}$$

where $[x]_a^b = \max(a, \min(x, b))$, *and* $\lambda_j \geq 0$ *is chosen to satisfy* $\sum_{k\in\mathcal{K}} p_j^k(n+1) = p_j^{max}$.

Proof We can prove this proposition by analyzing the well-known Karush-Kuhn-Tucker (KKT) conditions in optimization theory [57] [63]. To begin with, note that the projection operation (2.9) is equivalent to the following optimization problem:

$$\begin{aligned} &\text{minimize } \frac{1}{2} \parallel \boldsymbol{p_j}(n+1) - (\boldsymbol{p_j}(n) + a_j(n)\frac{\boldsymbol{\hat{f}_j}(n)}{\boldsymbol{p_j}(n)}) \parallel_2^2 \\ &\text{subject to } \sum_{k\in\mathcal{K}} p_j^k(n+1) \leq p_j^{max}, \\ &\qquad 0 \leq p_j^k(n+1) \leq \bar{p}_j^k, \forall k \in \mathcal{K}. \end{aligned} \tag{2.12}$$

This quadratic optimization problem is strictly convex. Therefore, the corresponding solution $\boldsymbol{p_j}(n+1)$ can be obtained from the KKT conditions which are both necessary and sufficient for the optimality [63]. Toward this end, consider the Lagrangian:

$$\begin{aligned} \mathcal{L}(\boldsymbol{p_j}(n+1), \lambda_j, \boldsymbol{u_j}, \boldsymbol{v_j}) &= \frac{1}{2}\sum_{k\in\mathcal{K}}(p_j^k(n+1) - (p_j^k(n) + a_j(n)\frac{\hat{f}_j^k(n)}{p_j^k(n)}))^2 \\ + \lambda_j(\sum_{k\in\mathcal{K}} p_j^k(n+1) - p_j^{max}) &+ \sum_{k\in\mathcal{K}} u_j^k(p_j^k(n+1) - \bar{p}_j^k) - \sum_{k\in\mathcal{K}} v_j^k p_j^k(n+1), \end{aligned} \tag{2.13}$$

where λ_j, $\boldsymbol{u_j} = [u_j^1, ..., u_j^K]^T$, $\boldsymbol{v_j} = [v_j^1, ..., v_j^K]^T$ are the associated Lagrangian multipliers. Then the KKT conditions are given by

$$\begin{aligned} p_j^k(n+1) - (p_j^k(n) + a_j(n)\frac{\hat{f}_j^k(n)}{p_j^k(n)}) + \lambda_j + u_j^k - v_j^k = 0, \forall k \in \mathcal{K} \\ u_j^k \geq 0,\ p_j^k(n+1) \leq \bar{p}_j^k,\ u_j^k(p_j^k(n+1) - \bar{p}_j^k) = 0, \forall k \in \mathcal{K} \\ v_j^k \geq 0,\ p_j^k(n+1) \geq 0,\ v_j^k p_j^k(n+1) = 0, \forall k \in \mathcal{K} \\ \lambda_j \geq 0, \sum_{k\in\mathcal{K}} p_j^k(n+1) \leq p_j^{max}, \lambda_j(\sum_{k\in\mathcal{K}} p_j^k(n+1) - p_j^{max}) = 0. \end{aligned} \tag{2.14}$$

Now for any $k \in \mathcal{K}$, we observe that if $p_j^k(n+1) = 0$, we have $u_j^k = 0$ by complementary slackness condition. Furthermore, we have $-v_j^k = p_j^k(n) + a_j(n)\frac{\hat{f}_j^k(n)}{p_j^k(n)} - \lambda_j \leq 0$. By a similar argument, we can obtain that $p_j^k(n+1) = p_j^k(n) + a_j(n)\frac{\hat{f}_j^k(n)}{p_j^k(n)} - \lambda_j$ if $0 < p_j^k(n+1) < \bar{p}_j^k$, and $p_j^k(n) + a_j(n)\frac{\hat{f}_j^k(n)}{p_j^k(n)} - \lambda_j \geq \bar{p}_j^k$ if $p_j^k(n+1) = \bar{p}_j^k$. This completes the proof.

2.2.2 Convergence Analysis

In this subsection, we study the convergence property of SDLA-I. Toward this end, we first introduce some further notations for ease of exposition. We denote by $D(n) = diag(D_1(n), ..., D_N(n))$ the $NK \times NK$-dimensional block diagonal matrix where $D_j(n) = diag(a_j(n), ..., a_j(n))$ is a $K \times K$-dimensional diagonal matrix with uniform diagonal entry $a_j(n)$. Then the iteration step (2.9) in SDLA-I can be rewritten in a compact form given by

$$\boldsymbol{p}(n+1) = \mathcal{P}_{\Phi}[\boldsymbol{q}(n)], \tag{2.15}$$

where $\boldsymbol{q}(n) = \boldsymbol{p}(n) + D(n)\frac{\hat{\boldsymbol{f}}(n)}{\boldsymbol{p}(n)}$ with $\hat{\boldsymbol{f}}(n) = (\hat{\boldsymbol{f}}_1(n), .., \hat{\boldsymbol{f}}_N(n))^T$. Denote by $\boldsymbol{s}_j(n) = \boldsymbol{f}_j(n)/\boldsymbol{p}_j(n)$, $\hat{\boldsymbol{s}}_j(n) = \hat{\boldsymbol{f}}_j(n)/\boldsymbol{p}_j(n)$ and $\bar{\boldsymbol{s}}_j = \mathbb{E}[\nabla_{\boldsymbol{p}_j} R_j(\boldsymbol{p}_j, \boldsymbol{p}_{-j}|\boldsymbol{G})]$. We group all the $\boldsymbol{s}_j(n)$'s, $\hat{\boldsymbol{s}}_j(n)$'s, and $\bar{\boldsymbol{s}}_j$'s into column vectors $\boldsymbol{s}(n)$, $\hat{\boldsymbol{s}}(n)$, and $\bar{\boldsymbol{s}}$, respectively. We further denote by $\boldsymbol{\Gamma}$ the $N \times N$-dimensional matrix with $[\boldsymbol{\Gamma}]_{ij}$ defined as

$$[\boldsymbol{\Gamma}]_{ij} = \begin{cases} 1 & \text{if } i = j, \\ -\max_{k\in\mathcal{K}}\left(\frac{g_{ij}^k}{g_{jj}^k} \cdot \frac{n_j^k + \sum_{j^\dagger\in\mathcal{N}} g_{jj^\dagger}^k \bar{p}_{j^\dagger}^k}{n_i^k}\right) & \text{if } i \neq j. \end{cases}$$

With these notations in mind, the following lemma summarizes some main (in)equalities, which will be used in the later proofs of the convergence results of SDLA-I.

Lemma 2.1 *The following (in)equalities hold:*

(i) A power allocation profile $\boldsymbol{p}^* \in \Phi$ is an NE of the stochastic game $\mathcal{G}$ if and only if for any $a_j > 0$

$$\boldsymbol{p}_j^* = \mathcal{P}_{\Phi_j}[\boldsymbol{p}_j^* + a_j\frac{\bar{\boldsymbol{f}}_j}{\boldsymbol{p}_j^*}], \forall j \in \mathcal{N}, \tag{2.16}$$

where $\bar{\boldsymbol{f}}_j = \boldsymbol{p}_j^*\bar{\boldsymbol{s}}_j(\boldsymbol{p}^*)$.

(ii) For any $\boldsymbol{p}, \bar{\boldsymbol{p}} \in \mathbb{R}^{NK}$,

$$\| \mathcal{P}_{\Phi}(\boldsymbol{p}) - \mathcal{P}_{\Phi}(\bar{\boldsymbol{p}}) \| \leq \| \boldsymbol{p} - \bar{\boldsymbol{p}} \| . \tag{2.17}$$

(iii) For any $\boldsymbol{p} \in \mathbb{R}^{NK}$ *and* $\bar{\boldsymbol{p}} \in \Phi$,

$$(\bar{\boldsymbol{p}} - \boldsymbol{p})^T(\mathcal{P}_\Phi(\boldsymbol{p}) - \boldsymbol{p}) \geq 0. \tag{2.18}$$

Proof (i). We know that if $\boldsymbol{p}^* \in \Phi$ is an NE, then for any $a_j > 0$ [7]

$$\boldsymbol{p}_j^* = \mathcal{P}_{\Phi_j}[\boldsymbol{p}_j^* + a_j \nabla_{\boldsymbol{p}_j} \mathbb{E}[R_j(\boldsymbol{p}_j^*, \boldsymbol{p}_{-j}^* | \boldsymbol{G})]], \forall j \in \mathcal{N}. \tag{2.19}$$

The result follows if the interchange of mathematical expectations and gradient signs is justified. Recall that the realization $\boldsymbol{g}$ is bounded by assumption. Then it is straightforward to verify $\| \nabla_{\boldsymbol{p}_j} R_j(\boldsymbol{p}_j, \boldsymbol{p}_{-j}|\boldsymbol{g}) \|$ is also bounded. Thus, $\nabla_{\boldsymbol{p}_j} \mathbb{E}[R_j(\boldsymbol{p}_j^*, \boldsymbol{p}_{-j}^*|\boldsymbol{G})] = \mathbb{E}[\nabla_{\boldsymbol{p}_j} R_j(\boldsymbol{p}_j^*, \boldsymbol{p}_{-j}^*|\boldsymbol{G})]$ [53].

(ii). This is a well-known result on the nonexpansive property of projection, the proof of which can be found in, e.g., [7].

(iii). Since $\mathcal{P}_\Phi(\boldsymbol{p})$ minimizes $\frac{1}{2} \| \bar{\boldsymbol{p}} - \boldsymbol{p} \|_2^2$ over all $\bar{\boldsymbol{p}} \in \Phi$, we have

$$(\bar{\boldsymbol{p}} - \mathcal{P}_\Phi(\boldsymbol{p}))^T(\mathcal{P}_\Phi(\boldsymbol{p}) - \boldsymbol{p}) \geq 0, \forall \boldsymbol{p} \in \mathbb{R}^{NK}, \tag{2.20}$$

by optimality condition [7]. Noting another obvious fact:

$$(\mathcal{P}_\Phi(\boldsymbol{p}) - \boldsymbol{p})^T(\mathcal{P}_\Phi(\boldsymbol{p}) - \boldsymbol{p}) \geq 0, \forall \boldsymbol{p} \in \mathbb{R}^{NK}, \tag{2.21}$$

we conclude that $(\bar{\boldsymbol{p}} - \boldsymbol{p})^T(\mathcal{P}_\Phi(\boldsymbol{p}) - \boldsymbol{p}) \geq 0$ for any $\boldsymbol{p} \in \mathbb{R}^{NK}$ and $\bar{\boldsymbol{p}} \in \Phi$.

The following lemma inspired by [48] provides another inequality (2.22) that will be used later. As a byproduct, we also characterize the uniqueness property of NE in game $\mathcal{G}$ with deterministic channel power gains in the following lemma. We refer to [58] [48] and references therein for a more detail discussion on the uniqueness property of NE in deterministic game $\mathcal{G}$.

Lemma 2.2 *For given channel power gain realization* $\boldsymbol{g}$, *if* $\boldsymbol{\Gamma} \succ \boldsymbol{0}$ *(positive definite), then there exists a unique NE* $\boldsymbol{p}^*(\boldsymbol{g}) \in \Phi$, *and*

$$\boldsymbol{s}(\boldsymbol{p}|\boldsymbol{g})^T(\boldsymbol{p}^*(\boldsymbol{g}) - \boldsymbol{p}) \geq \tau(\boldsymbol{s}) \| \boldsymbol{p} - \boldsymbol{p}^*(\boldsymbol{g}) \|_2^2, \forall \boldsymbol{p} \in \Phi, \tag{2.22}$$

with

$$\tau(\boldsymbol{s}) = \frac{\lambda_{\min}(\boldsymbol{\Gamma})}{\max_{i \in \mathcal{N}} \max_{k \in \mathcal{K}} (\kappa_i^k)^2} \tag{2.23}$$

where $\lambda_{\min}(\boldsymbol{\Gamma}) > 0$ *denotes the minimal eigenvalue of the symmetric part of* $\boldsymbol{\Gamma}$, *and* $\kappa_i^k = (n_i^k + \sum_{j \in \mathcal{N}} g_{ij}^k \bar{p}_j^k)/g_{ii}^k$.

Proof Following Proposition 2 in [48], if $\mathbf{\Gamma} \succ \mathbf{0}$ under given $\boldsymbol{g}$, then

$$(\boldsymbol{s}(\boldsymbol{q}|\boldsymbol{g}) - \boldsymbol{s}(\boldsymbol{p}|\boldsymbol{g}))^T(\boldsymbol{p} - \boldsymbol{q}) \geq \tau(\boldsymbol{s}) \parallel \boldsymbol{p} - \boldsymbol{q} \parallel_2^2, \forall \boldsymbol{p}, \boldsymbol{q} \in \Phi, \tag{2.24}$$

with $\tau(\boldsymbol{s})$ specified in (2.23). That is, $\boldsymbol{s}(\cdot|\boldsymbol{g})$ is strongly monotone on Φ. The uniqueness of NE $\boldsymbol{p}^*(\boldsymbol{g}) \in \Phi$ follows (see, e.g., [21]).

Furthermore, by the equivalence of standard NE problem and variational inequality (VI)[1], we have

$$-\boldsymbol{s}(\boldsymbol{p}^*|\boldsymbol{g})^T(\boldsymbol{p} - \boldsymbol{p}^*) \geq 0, \forall \boldsymbol{p} \in \Phi. \tag{2.25}$$

Substituting $\boldsymbol{p}^*$ for $\boldsymbol{q}$ in (2.24), we obtain

$$(\boldsymbol{s}(\boldsymbol{p}^*|\boldsymbol{g}) - \boldsymbol{s}(\boldsymbol{p}|\boldsymbol{g}))^T(\boldsymbol{p} - \boldsymbol{p}^*) \geq \tau(\boldsymbol{s}) \parallel \boldsymbol{p} - \boldsymbol{p}^* \parallel^2, \forall \boldsymbol{p} \in \Phi. \tag{2.26}$$

Thus, the desired inequality (2.22) immediately follows from (2.25) and (2.26). This completes the proof.

We now describe the convergence results of SDLA-I in the following theorem.

Theorem 2.1 *Let $\mathcal{F}_n$ be the σ-field generated by $(\boldsymbol{g}(m), \boldsymbol{p}(m))_{m=0}^n$. Assume that:*

(i) $\mathbf{\Gamma} \succ \mathbf{0}$ holds almost surely.

(ii) The step sizes $a_i(n) \geq 0$ satisfy:

$$\sum_{n=0}^{\infty} \min_{i \in \mathcal{N}} a_i(n) = +\infty, \tag{2.27}$$

$$\sum_{n=0}^{\infty} a_i^2(n) < +\infty, \forall i \in \mathcal{N}. \tag{2.28}$$

(iii) The estimation errors $\epsilon_i(n) \geq 0$ satisfy:

$$\sum_{n=0}^{\infty} \mathbb{E}[a_i(n)\epsilon_i(n)|\mathcal{F}_n] < +\infty, \forall i \in \mathcal{N}. \tag{2.29}$$

Then $(\boldsymbol{p}(n))_{n=0}^{\infty}$ generated by SDLA-I converges to the unique NE $\boldsymbol{p}^*$ of the stochastic game $\mathcal{G}$ in the mean square sense, i.e., $\lim_{n\to\infty} \parallel \boldsymbol{p}(n) - \boldsymbol{p}^* \parallel^2 = 0$ almost surely.

Proof See Appendix A.

We make the following remarks on the assumptions in Theorem 2.1:

[1] Given a set $K \subseteq \mathbb{R}^n$ and a mapping $F : K \to \mathbb{R}^n$, the variational inequality $VI(K, F)$ is to find a vector $x \in K$ such that $(y - x)^T F(x) \geq 0, \forall y \in K$ [21].

Remark 2.1 *Assumption (i) is the major requirement for the convergence of SDLA-I. A careful thinking reveals that this assumption is indeed intuitive. On the one hand, from the game theory point of view,* $\mathbf{\Gamma} \succ \mathbf{0}$ *implies that each player* j *has a more significant influence on its utility than other players do. From the communication point of view,* $\mathbf{\Gamma} \succ \mathbf{0}$ *imposes upper bounds on the interference received and/or caused by communication pair* j*. Under mild interference conditions, communication pair* j*'s achievable transmission rate is not heavily influenced by other communication pairs. Note that all existing algorithms even for deterministic distributed power control such as IWFA require more or less similar conditions to ensure convergence [78] [58] [57] [56] [48] [38] [13].*

Remark 2.2 *Assumption (ii) is quite standard in stochastic approximation algorithms. Indeed, condition "*$\sum_{n=0}^{\infty} \min_{i\in\mathcal{N}} a_i(n) = +\infty$*" ensures that SDLA-I can cover the entire time axis to reach the NE in stochastic parallel Gaussian interference channels. Meanwhile, the choice of step sizes such that* $\sum_{n=0}^{\infty} \mathbb{E}[a_i^2(n)|\mathcal{F}_n] < +\infty$ *can asymptotically suppress error variance during the learning process.*

Remark 2.3 *Assumption (iii) provides a quantitative answer to the question on how well the estimation* $\hat{\boldsymbol{f}}$ *should be. Specifically,* $\sum_{n=0}^{\infty} \mathbb{E}[a_j(n)\epsilon_j(n)|\mathcal{F}_n] < +\infty$ *implies that the total estimation errors can be controlled. This assumption is reasonable especially in slow to medium time-varying communication environments. Nevertheless, the estimation* $\hat{\boldsymbol{f}}$ *may not be good enough in fast time-varying scenarios. In this regard, better estimates may be required. Noting that* $\hat{\boldsymbol{f}}(n)$ *only utilizes the last feedback* $\boldsymbol{f}(n)$*, one possible solution is to take advantage of empirical distribution after having observed many realization* $\boldsymbol{f}$*'s. That is, distributed communication pairs gradually learn more about the environment. Thus, better estimates* $\hat{\boldsymbol{f}}(n)$ *may be obtained. Nevertheless, we do not aim to explore this topic which is beyond the scope of this chapter.*

To further appreciate how SDLA-I works, let us consider a particular scenario where the difference between $\hat{\boldsymbol{f}}_j(n)/\boldsymbol{p}_j(n)$ and $\mathbb{E}[\nabla_{\boldsymbol{p}_j} R_j(\boldsymbol{p}_j(n), \boldsymbol{p}_{-j}(n)|\boldsymbol{G})]$ is captured by random vector $\boldsymbol{\theta}_j(n)$, i.e.,

$$\hat{\boldsymbol{f}}_j(n)/\boldsymbol{p}_j(n) = \mathbb{E}[\nabla_{\boldsymbol{p}_j} R_j(\boldsymbol{p}_j(n), \boldsymbol{p}_{-j}(n)|\boldsymbol{G})] + \boldsymbol{\theta}_j(n), \forall j \in \mathcal{N}. \tag{2.30}$$

As usual, we group all $\boldsymbol{\theta}_j$'s into a column vector $\boldsymbol{\theta}$, i.e., $\boldsymbol{\theta} = (\boldsymbol{\theta}_1, ..., \boldsymbol{\theta}_N)^T$. In other words, we simply use an online estimate $\hat{\boldsymbol{f}}_j(n)/\boldsymbol{p}_j(n)$ to approximate $\mathbb{E}[\nabla_{\boldsymbol{p}_j} R_j(\boldsymbol{p}_j(n), \boldsymbol{p}_{-j}(n)|\boldsymbol{G})]$ though we are not able to evaluate $\mathbb{E}[\nabla_{\boldsymbol{p}_j} R_j(\boldsymbol{p}_j(n), \boldsymbol{p}_{-j}(n)|\boldsymbol{G})]$. The approximation difference is captured by $\boldsymbol{\theta}_j$. We will show that SDLA-I converges as long as this simple approximation is not too "bad". We will formalize these ideas in Theorem 2.2. Toward this end, we first prove a simple lemma as follows.

Lemma 2.3 *The mapping $\bar{\boldsymbol{s}}(\boldsymbol{p})$ where $\bar{\boldsymbol{s}}_j(\boldsymbol{p}) = \mathbb{E}[\nabla_{\boldsymbol{p}_j} R_j(\boldsymbol{p}_j, \boldsymbol{p}_{-j}|\boldsymbol{G})]$ is Lipschitz continuous almost surely. That is, there exists a positive constant L such that $\forall \boldsymbol{p}, \boldsymbol{q} \in \Phi$,*

$$\| \bar{\boldsymbol{s}}(\boldsymbol{p}) - \bar{\boldsymbol{s}}(\boldsymbol{q}) \| \leq L \| \boldsymbol{p} - \boldsymbol{q} \| \tag{2.31}$$

holds almost surely.

Proof Note that we assume that $\boldsymbol{G}$ is bounded almost surely. Given bounded realization $\boldsymbol{g}$, it is straightforward to verify that $\boldsymbol{s}(\boldsymbol{p}|\boldsymbol{g})$ where $\boldsymbol{s}_j(\boldsymbol{p}|\boldsymbol{g}) = \nabla_{\boldsymbol{p}_j} R_j(\boldsymbol{p}_j, \boldsymbol{p}_{-j}|\boldsymbol{g})$ has bounded derivative and thus Lipschitz continuous. It follows that $\bar{\boldsymbol{s}}(\boldsymbol{p})$ is Lipschitz continuous almost surely.

Theorem 2.2 *Let $\mathcal{F}_n$ be the σ-field generated by $(\boldsymbol{g}(m), \boldsymbol{p}(m))_{m=0}^n$. Assume that:*

(i) $\boldsymbol{\Gamma} \succ \boldsymbol{0}$ *holds almost surely.*

(ii) The step sizes $a_i(n) \geq 0$ satisfy:

$$2\tau(\bar{\boldsymbol{s}}) \min_{i \in \mathcal{N}} a_i(n) \geq L^2 \max_{i \in \mathcal{N}} a_i^2(n) + \delta(n), \tag{2.32}$$

where $\delta(n)$ is any bounded positive constant.

(iii) The difference random vector $\boldsymbol{\theta}(n)$ satisfy:

$$\mathbb{E}[\boldsymbol{\theta}(n)|\mathcal{F}_n] = \boldsymbol{0}, \tag{2.33}$$

$$\sum_{n=0}^{\infty} \sum_{i \in \mathcal{N}} a_i^2(n) \mathbb{E}[\| \boldsymbol{\theta}_i(n) \|^2 |\mathcal{F}_n] < +\infty. \tag{2.34}$$

Then $(\boldsymbol{p}(n))_{n=0}^{\infty}$ generated by SDLA-I converges to the unique NE $\boldsymbol{p}^*$ of the stochastic game $\mathcal{G}$ in the mean square sense, i.e., $\lim_{n\to\infty} \| \boldsymbol{p}(n) - \boldsymbol{p}^* \|^2 = 0$ almost surely.

Proof See Appendix B.

Note that distributed algorithms based on the gradient projection mapping for deterministic parallel Gaussian interference channels have been proposed in [57]. Nevertheless, the convergence behaviors of those algorithms in [57] are only shown for deterministic scenarios and thus cannot be applied to stochastic case. Indeed, Theorem 2 establishes a theoretical foundation for the convergence of those deterministic algorithms under stochastic scenarios. The key conditions are included in assumption (iii) in Theorem 2. That is, the naive estimate $\hat{\boldsymbol{f}}_j(n)/\boldsymbol{p}_j(n)$ for $\mathbb{E}[\nabla_{\boldsymbol{p}_j} R_j(\boldsymbol{p}_j(n), \boldsymbol{p}_{-j}(n)|\boldsymbol{G})]$ should not be too "bad" in the sense of assumption (iii) in Theorem 2.

Besides, the requirement (2.32) imposed on step sizes is also reasonable. Consider a common step size choice for every communication pair, i.e., $a_i(n) = \bar{a}(n), \forall i \in \mathcal{N}$. Ignoring the arbitrarily small constant $\delta(n)$ for ease of exposition, condition (2.32) is then reduced to $\bar{a}(n) \leq \frac{2\tau(\bar{\boldsymbol{s}})}{L^2}, \forall n$. That is, larger step size can be taken if $\bar{\boldsymbol{s}}$ is more strongly monotone (i.e., larger $\tau(\bar{\boldsymbol{s}})$). In contrast, smaller step size should be taken if $\bar{\boldsymbol{s}}$ changes more significantly with respect to $\boldsymbol{p}$ (i.e., larger Lipschitz constant L).

Though Theorem 2 is of interest in theory, we remark that assumptions in Theorem 2 may not be easily verified. For instance, it is hard to know if the difference random vector $\boldsymbol{\theta}$ could satisfy assumption (iii) if little is known about the distribution of $\boldsymbol{G}$ in real communication systems. Besides, requirement (2.32) imposed on step sizes involves strongly monotone modulus $\tau(\bar{\boldsymbol{s}})$ and Lipschitz constant L, both of which depend on the specific channel gain distribution $\boldsymbol{G}$. In contrast, the step sizes choice in Theorem 1 is relatively standard. The requirement there is that the total error in the stochastic gradient obtained by local communication pair could be properly controlled. This requirement may be easily satisfied when the parallel Gaussian interference channels do not change too fast.

2.3 Continuous Time Approximation

Note that previous convergence results do not provide insights on the speed of convergence of SDLA-I. Indeed, they may be considered as study of the accuracy of SDLA-I. Equally important is the convergence rate of SDLA-I. In this section, we shall shed some lights on this question. We note that an exact analysis on the convergence rate of SDLA-I is extremely difficult if not impossible due to the various stochastic factors. Therefore, we resort to a PDS approach which approximates but still captures the essential behaviors of SDLA-I to help us appreciate the convergence speed. Note that a PDS formulation for transient behavior analysis for deterministic cognitive radio networks was also briefly described in [59].

To begin with, we recall some basic concepts of PDS from [44] to facilitate further discussions. Consider a closed convex set $\mathcal{K} \in \mathcal{R}^M$ and a vector field $\mathcal{F}$ whose domain contains $\mathcal{K}$. Recall $\mathcal{P}_{\mathcal{K}}$ denotes the norm projection. Then define the projection of $\mathcal{F}$ at $\boldsymbol{x}$ as

$$\prod_{\mathcal{K}}(\boldsymbol{x}, \mathcal{F}) = \lim_{\delta \to 0} \frac{\mathcal{P}_{\mathcal{K}}(\boldsymbol{x} + \delta\mathcal{F}) - \boldsymbol{x}}{\delta}. \tag{2.35}$$

Now we formally define PDS as follows.

Definition 2.2 *The following ordinary differential equations*

$$\dot{\boldsymbol{x}}(t) = \prod_{\mathcal{K}}(\boldsymbol{x}(t), -\mathcal{F}(\boldsymbol{x}(t))) \tag{2.36}$$

with an initial value $\boldsymbol{x}(0) \in \mathcal{K}$ is called projected dynamical system $PDS(\mathcal{F}, \mathcal{K})$.

Note that the right hand side in (2.36) is discontinuous on the boundary of $\mathcal{K}$ due to the projection operator, which is different from classical dynamical systems.

Now let us consider $PDS(\bar{\boldsymbol{s}}, \Phi)$ given by $\dot{\boldsymbol{p}}(t) = \prod_{\Phi}(\boldsymbol{p}(t), \bar{\boldsymbol{s}}(\boldsymbol{p}(t)))$ with initial value $\boldsymbol{p}(0) \in \Phi$. The key results of this PDS are summarized in the following proposition.

Proposition 2.3 *The $PDS(\bar{\boldsymbol{s}}, \Phi)$ with initial value $\mathbf{p}(0)$ has the following properties:*

(i) It has a unique solution $\mathbf{p}(t)$ which continuously depends on the vector field $\bar{\mathbf{s}}$ and initial value $\mathbf{p}(0)$;

(ii) A vector $\mathbf{p}^* \in \Phi$ is the NE of the stochastic game $\mathcal{G}$ if and only if it is a stationary point of $PDS(\bar{\boldsymbol{s}}, \Phi)$, i.e., $\dot{\mathbf{p}}^*(t) = 0$;

(iii) If $\boldsymbol{\Gamma} \succ \mathbf{0}$ holds almost surely, then stationary point $\mathbf{p}^*$ of $PDS(\bar{\boldsymbol{s}}, \Phi)$ is unique and globally exponentially stable, i.e., $\| \boldsymbol{p}(t) - \boldsymbol{p}^* \| \leq \| \boldsymbol{p}(0) - \boldsymbol{p}^* \| \exp(-\min_{\boldsymbol{s}} \tau(\boldsymbol{s}) \cdot t)$ with $\tau(\boldsymbol{s})$ given in (2.23).

Proof (i). Note that $\bar{\boldsymbol{s}}(p)$ is Lipschitz continuous by Lemma 2.3. Hence $PDS(\bar{\boldsymbol{s}}, \Phi)$ is well posed and the results follow from Theorem 2.5 in [44].

(ii). The equivalence of NE in game $\mathcal{G}$ and the set of stationary points in PDS can be shown by observing that Φ is convex polyhedron by following [44]. We provide a sketch of the proof here for completeness. Define a variational inequality problem $VI(\bar{\boldsymbol{s}}, \Phi)$, the aim of which is to find a vector $\boldsymbol{p}^*$ such that

$$(\boldsymbol{p} - \boldsymbol{p}^*)^T \bar{\boldsymbol{s}}(\boldsymbol{p}^*) \leq 0, \forall \boldsymbol{p} \in \Phi. \tag{2.37}$$

It is known that $\boldsymbol{p}^*$ is a solution to $VI(\bar{\boldsymbol{s}}, \Phi)$ if and only if it is an NE of the game $\mathcal{G}$ (see, e.g., [21]). Noting that Φ is convex polyhedron, the stationary points of $PDS(\bar{\boldsymbol{s}}, \Phi)$ coincide with the solutions of $VI(\bar{\boldsymbol{s}}, \Phi)$ by Theorem 2.4 in [44].

(iii). Recall the condition that $\boldsymbol{\Gamma} \succ \mathbf{0}$ holds almost surely implies the strongly monotonicity of $\bar{\boldsymbol{s}}(\boldsymbol{p})$. Then the uniqueness and globally exponential stability follow from Theorem 3.7 in [44]. Indeed, we can associate a Liapunov function $\| \boldsymbol{p} - \boldsymbol{p}^* \|$ for $PDS(\bar{\boldsymbol{s}}, \Phi)$ to obtain the stability result.

$PDS(\bar{\boldsymbol{s}}, \Phi)$ is the underlying idealized version of SDLA-I. In other words, we can view SDLA-I as a stochastic approximation of $PDS(\bar{\boldsymbol{s}}, \Phi)$ [44]. Thus, the iteration process $(\boldsymbol{p}(n))_{n=0}^{\infty}$ in SDLA-I approximates or tracks the solution $\boldsymbol{p}(t)$ of $PDS(\bar{\boldsymbol{s}}, \Phi)$. From the above proposition, we know that $\boldsymbol{p}(t)$ converges to $\boldsymbol{p}^*$ at an exponential rate. Note that the stationary point $\boldsymbol{p}^*$ of $PDS(\bar{\boldsymbol{s}}, \Phi)$ is also the limit point of $(\boldsymbol{p}(n))_{n=0}^{\infty}$. So we can expect that $\boldsymbol{p}(n)$ moves in an approximately (subject to the inherent stochastic variations) monotone fashion to $\boldsymbol{p}^*$ at an exponential rate. This understanding of the iteration process in SDLA-I is also instrumental in exploring the idea of iterate averaging, which is detailed in the next section.

2.4 Learning with Averaging

Fast convergence performance of distributed learning algorithm for obtaining an NE of the power control game $\mathcal{G}$ is clearly desirable in real communication systems. From previous discussions, we can see that the choice of good step sizes $a_i(n)$ has a profound effect on the convergence performance of SDLA-I. In this section, we discuss how we can improve the convergence performance of SDLA-I so that distributed communication pairs can learn the NE in a faster fashion.

Among various approaches proposed in stochastic approximation theory, the concept of iterate averaging reported in [50] is an especially appealing and simple way to improve the convergence performance. It was shown in [50] that the averaged sequence $\sum_{m=0}^{n-1} \boldsymbol{p}(m)/n$ converges to its limit if step size sequence $a(n)$ decays more slowly than $\mathcal{O}(\frac{1}{n})$ used in the original Robbins-Monro formulation [51]. This iterate averaging method is optimal in terms of convergence rate. We will take advantage of this appealing technique to improve the convergence performance of SDLA-I.

Using the concept of iterate averaging, we add an averaging operation to the basic recursion (2.9) in SDLA-I, i.e.,

$$\begin{aligned} \boldsymbol{p}_j(n+1) &= \mathcal{P}_{\Phi_j}[\boldsymbol{p}_j(n) + a_j(n)\frac{\hat{\boldsymbol{f}}_j(n)}{\boldsymbol{p}_j(n)}], \\ \tilde{\boldsymbol{p}}_j(n+1) &= \frac{1}{n+1}(n\tilde{\boldsymbol{p}}_j(n) + \boldsymbol{p}_j(n+1)). \end{aligned} \tag{2.38}$$

The stochastic learning algorithm with the above modified recursion will be referred to as SDLA-II. It can be shown $(\tilde{\boldsymbol{p}}(n))_{n=0}^{\infty}$ generated by SDLA-II converges to the unique NE $\boldsymbol{p}^*$ of the stochastic game $\mathcal{G}$ in the mean square sense, i.e., $\lim_{n\to\infty} \parallel \boldsymbol{p}(n) - \boldsymbol{p}^* \parallel^2 = 0$ almost surely, as long as $a_j(n)$ is a suitable decreasing sequence or even fixed step size sequence with small enough value. A detail proof for the convergence of SDLA-II can be carried out by following similar arguments as [50] and is thus omitted here. We instead provide an intuitive exposition on why SDLA-II has faster convergence rate than SDLA-I. The idea behind SDLA-II is that we can use larger step size in the basic online recursion for $\boldsymbol{p}(n)$ and the increased noise effects due to larger step size can be smoothed out by the offline averaging recursion for $\tilde{\boldsymbol{p}}(n)$. As a result, SDLA-II converges faster with larger step size and is less likely to get stuck at the first few iterations [35]. Indeed, our numerical results demonstrate the convergence rate improvement of SDLA-II over SDLA-I.

A careful reflection on the power allocation trajectory $(\boldsymbol{p}(n), \tilde{\boldsymbol{p}}(n))_{n=0}^{\infty}$ generated by SDLA-II may reveal a potential handicap in guaranteeing better convergence performance of SDLA-II over SDLA-I. Specifically, with arbitrarily initial starting point $\boldsymbol{p}(0)$ which is quite unlikely near the desired solution of NE $\boldsymbol{p}^*$, it is expected that $\boldsymbol{p}(n)$ moves in an approximately monotonic fashion to the NE $\boldsymbol{p}^*$ at the early stage since the channel power gains of parallel Gaussian interference channels satisfy $\boldsymbol{\Gamma} \succ \mathbf{0}$ and thus the underlying driving force $\bar{\boldsymbol{s}}$ of the recursion of $\boldsymbol{p}(n)$ is strongly monotone. Nevertheless, the noise due to the randomness plays a relatively significant role in the recursion compared to the underlying driving force $\bar{\boldsymbol{s}}$ after sufficient number of iterations. In other words, $\boldsymbol{p}(n)$ starts to hover randomly around the NE $\boldsymbol{p}^*$ when $\boldsymbol{p}(n)$ is near NE $\boldsymbol{p}^*$. Only at this stage can the iterate averaging $\tilde{\boldsymbol{p}}(n)$ be successful since the averaging in this stage can produce a mean solution that is nearer to the NE $\boldsymbol{p}^*$. This implies that the communication pairs should transmit with power level $\boldsymbol{p}(n)$ generated by the basic recursion at the initial stage, and transmit with power level $\tilde{\boldsymbol{p}}(n)$ generated by averaging after SDLA-II has sufficiently converged.

The above reflection does not imply that SDLA-II is of limited use in practical communication systems. Indeed, SDLA-II has few gains in terms of convergence rate over SDLA-I when the initial stage is relatively long compared to the communication period. Nevertheless, the communication time scale can be large in common applications such as video transmission in wireless data networks [40]. Thus, SDLA-II will yield better estimate of NE $\boldsymbol{p}^*$ over SDLA-I in the long run.

2.5 Numerical Results

We provide some numerical results in this section for illustration purposes. Simulation parameters are chosen as follows unless specified otherwise. Inspired by [38] and [59], we set both the number of users and number of channels to be 4. The channel power gains g_{ij}^k are chosen randomly from the intervals $(\bar{g}_{ij}^k(1-\upsilon), \bar{g}_{ij}^k(1+\upsilon))$ with $\upsilon \in \{10\%, 20\%, 30\%, 40\%, 50\%\}$. Clearly, perturbation parameter υ can serve as an indicator for the time varying rates of parallel Gaussian interference channels. In particular, larger υ implies faster channel varying rate. We further let $\bar{g}_{ij}^k = 15$ if $i = j$ and 0.75 otherwise. With this choice of simulation parameters, one can verify that $\boldsymbol{\Gamma} \succ \mathbf{0}$ almost surely if $\upsilon \in \{10\%, 20\%, 30\%\}$. For clarity, we relax the spectral constraints, i.e., $\bar{p}_i^k = +\infty, \forall i \in \mathcal{N}, \forall k \in \mathcal{K}$. The total power constraint $p_i^{max} = 10 * K = 40, \forall i \in \mathcal{N}$. Besides, the background noise level $n_i^k = 0.1/N = 0.025, \forall i \in \mathcal{N}, \forall k \in \mathcal{K}$. We also choose common step size for all users. So we simply write $a_i(n)$ as a_n in this section.

We first compare our proposed SDLA-I with the popular IWFA. We let users using IWFA have the perfect CSI and interference levels at the corresponding transmitters in every power update, while users implementing SDLA-I only have stochastic gradients subject to errors. Due to the limited space, we only show the power evolution of user 1 on channel 1 as a function of iteration index in Fig. 2.1. As expected, even with perfect CSI and interference level, the power evolution generated by IWFA fluctuates significantly. In contrast, users in SDLA-I are more concerned about the long term transmission rates. Consequently, the power evolution only fluctuates mildly after sufficiently long period of learning about the environment. Another interesting observation here is that constant step sizes also lead to convergence of SDLA-I. Indeed, one can show that $(\boldsymbol{p}(n))_{n=0}^{\infty}$ converges to the neighborhood of the unique NE $\boldsymbol{p}^*$ with the choice of sufficiently small constant step sizes. We omit the details due to limited space.

Though both constant step size $\mathcal{O}(1)$ and decreasing step size $\mathcal{O}(\frac{1}{n})$ can lead to the convergence of SDLA-I in numerical experiments, we observe that a tradeoff exists between the convergence rate and exactness of the converged value, which is evaluated by the standard normalized squared error (NSE) defined as

$$NSE(n) = \parallel \boldsymbol{p}(n) - \boldsymbol{p}^* \parallel / \parallel \boldsymbol{p}^* \parallel . \tag{2.39}$$

The numerical results are shown in Fig. 2.2. As shown, decreasing step size $\mathcal{O}(\frac{1}{n})$ has better convergence rate than constant step size $\mathcal{O}(1)$ since $\mathcal{O}(\frac{1}{n})$ goes to 0 very fast and thus new channel power gain realization has little effect on power update. However, the solution obtained by decreasing step size is not as exact as those by constant step sizes. Nevertheless, an appropriate choice of constant step size is necessary to trade off the convergence rate and exactness of the converged value. Indeed, the convergence rate with $a_n = 0.01$ is very slow as shown in Fig. 2.2. Besides, numerical results in Fig. 2.2 also demonstrate the exponential convergence rate predicted by the continuous time approximation using PDS in section 2.3.

We show the impact of time-varying rate in parallel Gaussian interference channels on the convergence performance of SDLA-I in Fig. 2.3. As described, perturbation parameter υ can be used to model the time varying rate of parallel Gaussian interference channels in our setting. The power evolutions of user 1 on channel 1 as a function of iteration index are plotted with different υ's in Fig. 2.3. It is shown that the power allocation does not converge when $\upsilon \in \{40\%, 50\%\}$. Indeed, one can verify that $\boldsymbol{\Gamma} \succ \boldsymbol{0}$ can not hold almost surely with $\upsilon \in \{40\%, 50\%\}$. Thus, the convergence of SDLA-I is not guaranteed by Theorem 1. Note that $\boldsymbol{\Gamma} \succ \boldsymbol{0}$ is also required in one way or another in existing distributed power control algorithms including IWFA for deterministic parallel Gaussian interference channels. We in this numerical example also observe the importance of condition $\boldsymbol{\Gamma} \succ \boldsymbol{0}$ for the power control in stochastic parallel Gaussian interference channels.

We next show the performance improvement by iterate averaging in terms of convergence rate. In Fig. 2.4, users transmit with power level $\boldsymbol{p}(n)$ under SDLA-I. In contrast, under pure SDLA-II, users transmit with power level $\tilde{\boldsymbol{p}}(n)$ which is generated by averaging $\boldsymbol{p}(n)$. The mixed SDLA-II in Fig. 2.4 represents a transmission scenario, where users transmit with power level $\boldsymbol{p}(n)$ at the first 100 iterations, and afterwards transmit with power level $\tilde{\boldsymbol{p}}(n)$ generated by averaging $\boldsymbol{p}(n)$ from the 101-th iteration. As expected, the iterate averaging $\tilde{\boldsymbol{p}}(n)$ starts to work after $\boldsymbol{p}(n)$ is near to the NE $\boldsymbol{p}^*$.

□ **End of chapter.**

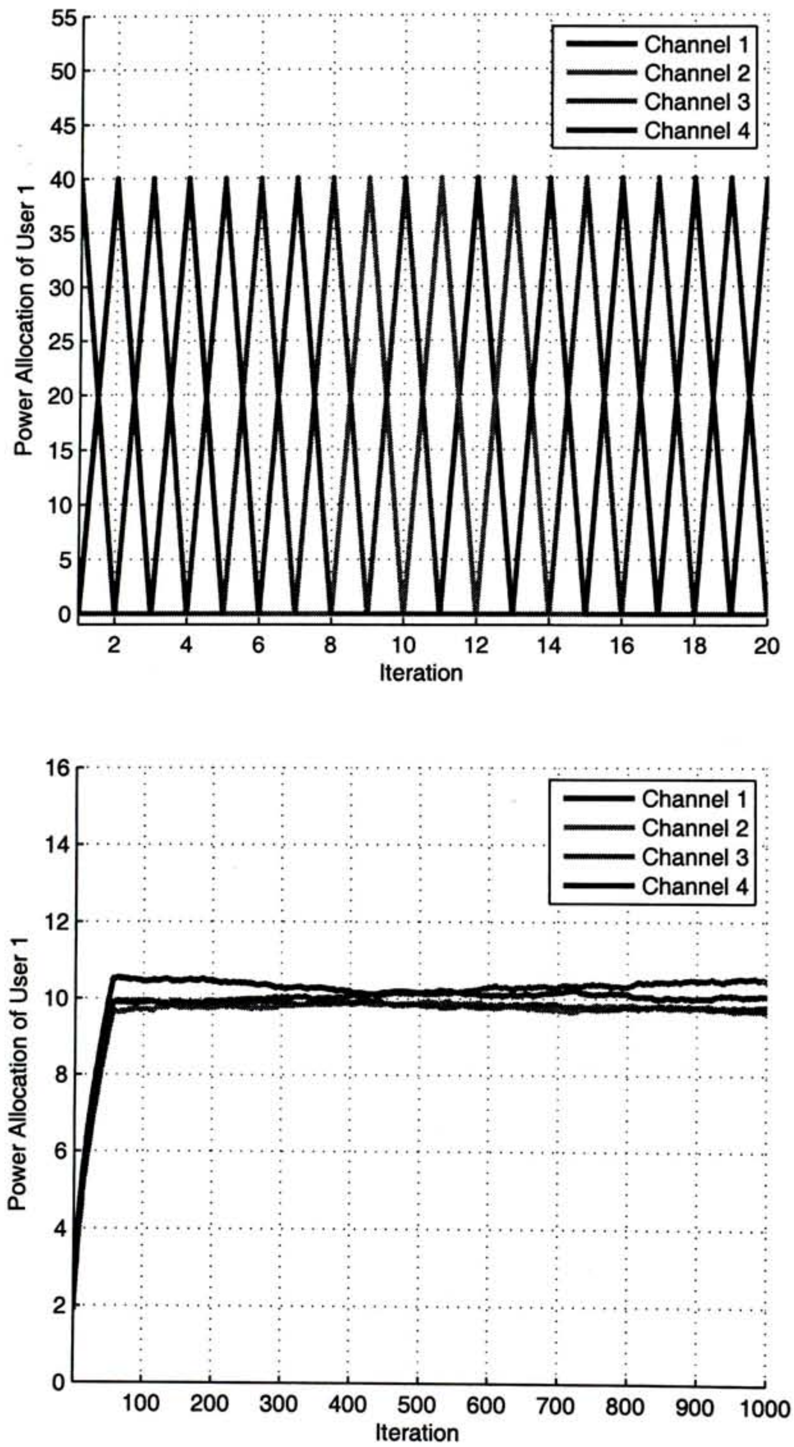

Figure 2.1: Comparison of IWFA and SDLA-I: In SDLA-I, $\upsilon = 20\%$ and $a_n = 0.5$.

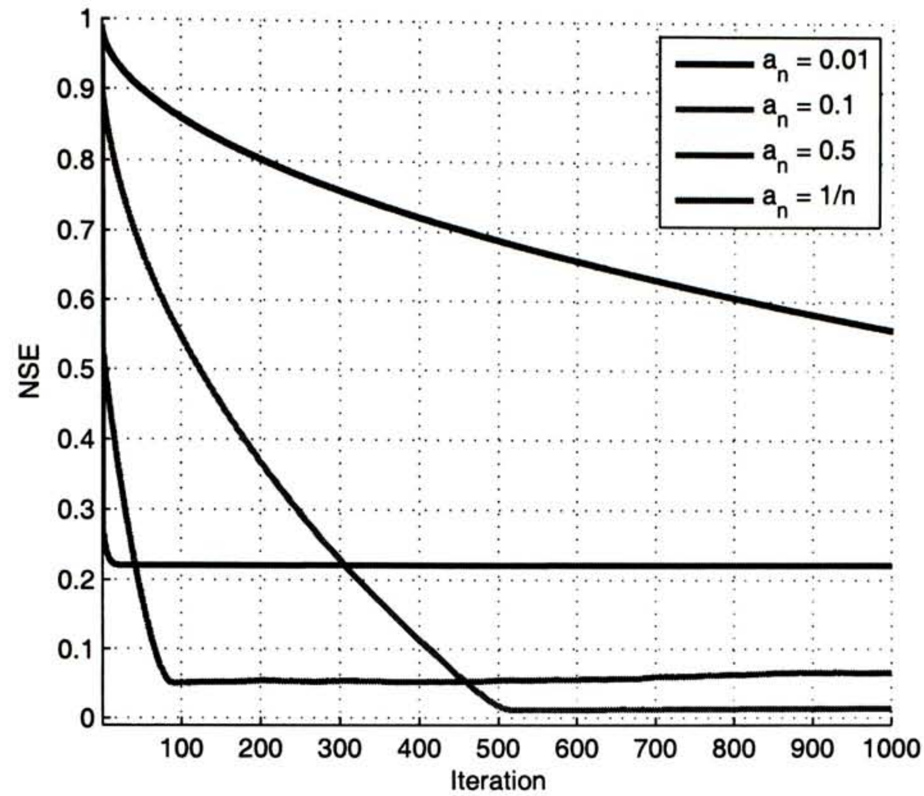

Figure 2.2: Impact of Step Sizes: $\upsilon = 20\%$.

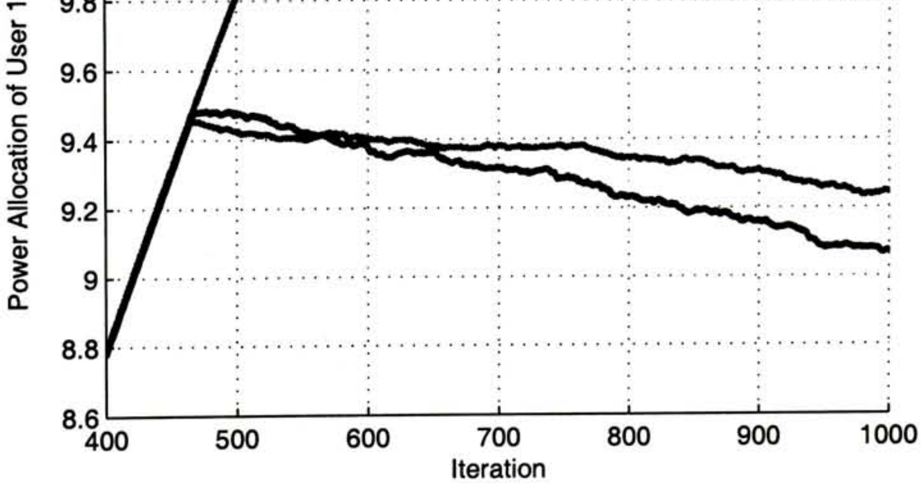

Figure 2.3: Impact of Time-Varying Rate: $a_n = 0.1$.

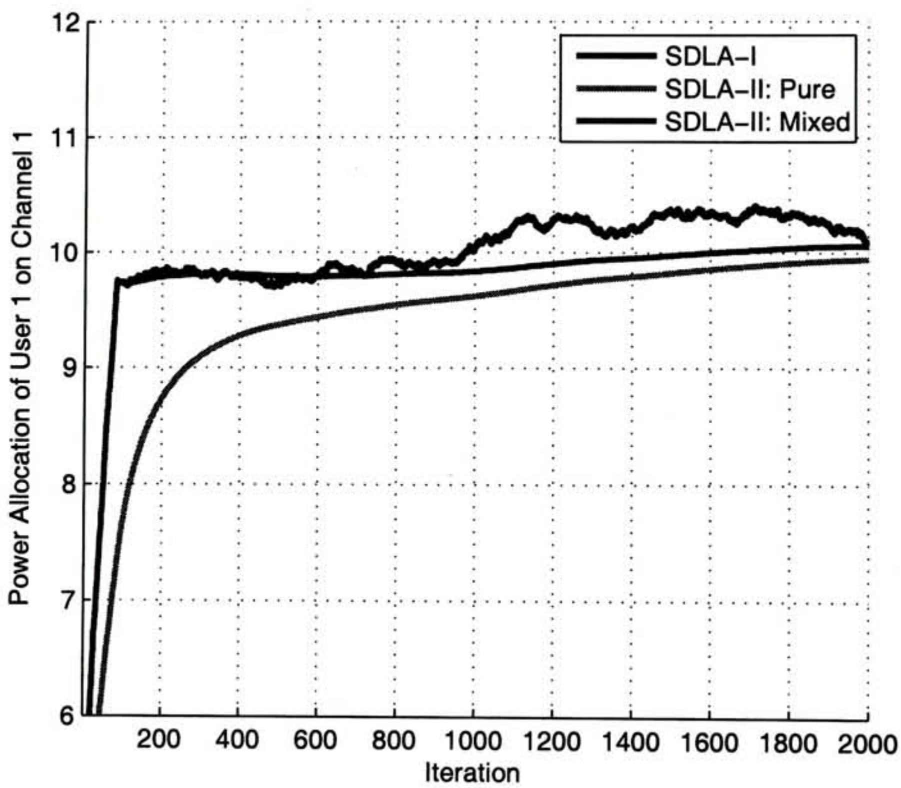

Figure 2.4: Learning NE with Averaging - SDLA-II: $\upsilon = 30\%$ and $a_n = 0.5$.

Chapter 3

Power Control for One-to-Many Transmissions

Punishment is justice for the unjust.

—— *Saint Augustine*

One-to-many transmission is a basic component of many communication networks [66]. Indeed, it can be traced back to the original study of broadcast channels in 1970s [16]. Due to its significance, substantial research works have been carried out on resource allocation for broadcast channels [37]. Nevertheless, superposition coding, whose complexity can be high for practical implementation, is required to achieve the capacity of broadcast channels [16]. Therefore, low complexity schemes, like code division multiple access (CDMA), were proposed and investigated [66]. Indeed, receivers in CDMA networks simply treat other users' signals as noises. In such schemes, distributed transmit power control has been extensively investigated in the past two decades [14].

When there are multiple sources in the network, the distributed power control for one-to-many transmissions can be regarded as a generalization of distributed power control in Gaussian interference channels, where one source only transmits to one destination. In spite of its significance, the capacity region of Gaussian interference channels is still unknown [9]. Nevertheless, distributed power control in Gaussian interference channels recently draws great interests (see, e.g., [39] and the references therein). Indeed, noncooperative game theory has been vastly applied, and much progress has been made on distributed power control in Gaussian interference channels [54] [64] [26] [58][57] [43]. However, the framework of standard Nash equilibrium problem (NEP) used in these works cannot be generalized to the one-to-many transmissions considered in this chapter.

In particular, we investigate the distributed power control problem from the receivers' point of view. More precisely, each receiver in our model individually decides its required transmit power from its associated transmitter. This approach is justified especially in large wireless networks where distributiveness is significantly preferred. Such scheme is also expected in future user-centric wireless networks. Indeed, adopting this approach, the receivers can alleviate the computation load imposed on their transmitters. For receiver i, its achievable rate also depends on other receivers' strategies which cause interference at receiver i. Note that each receiver experiences two kinds of interference. One is endogenous interference caused by the transmission from its source to other receivers. The other one is extraneous interference caused by other sources' transmission. This is different from existing works on distributed power control in Gaussian interference channels where only extraneous interference exists [39][58][57][74].

Another more fundamental issue in our problem is that those receivers share the same source need to competitively access their source's power resource, resulting in coupled strategy spaces among receivers that have the same source. In contrast, the standard Nash equilibrium (NE) based approach assumes uncoupled strategy spaces among decision makers [46]. Indeed, our problem turns out to be a generalized Nash equilibrium problem (GNEP) [19]. Though there are severe analytical difficulties in GNEPs, variational inequality (VI) theory plays an important role in solving GNEPs. Resorting to VI theory, we obtain several interesting theoretical results. Moreover, a penalty-based distributed algorithm IP^{2}JA is proposed.

Note that a VI approach has also been adopted in [48] to design cognitive radio systems under temperature-interference constraints. Besides the obvious difference in the application scenarios, [48] and our work differ significantly in the design of distributed algorithms. In particular, those distributed algorithms proposed in [48] adopt a Lagrangian pricing scheme and involve solving exactly a standard Nash subgame during every update of Lagrangian parameters. In contrast, we adopt a penalty-based scheme in our proposed distributed algorithm IP^{2}JA, which does not have the restriction mentioned.

The rest of this chapter is organized as follows. Section 3.1 describes the specific system model. In section 3.2, we present a GNEP formulation along with several theoretical results. The penalty-based distributed algorithm IP^{2}JA is described in section 3.3. Section 3.4 presents some numerical results for performance evaluation.

3.1 System Model

We consider a general one-to-many transmission network consisting of multiple sources and multiple destinations, as shown in Fig.3.1. Specifically, there are a set of transmitters $\mathbb{T} = \{S_1, ..., S_M\}$, each of which wishes to simultaneously transmit information to several receivers. For a particular source S_i, we denote by $\mathbb{R}_i = \{D_1^{(i)}, ..., D_{N_i}^{(i)}\}$ the set of N_i destinations that source S_i transmits information to. Without loss of generality, we assume sources are

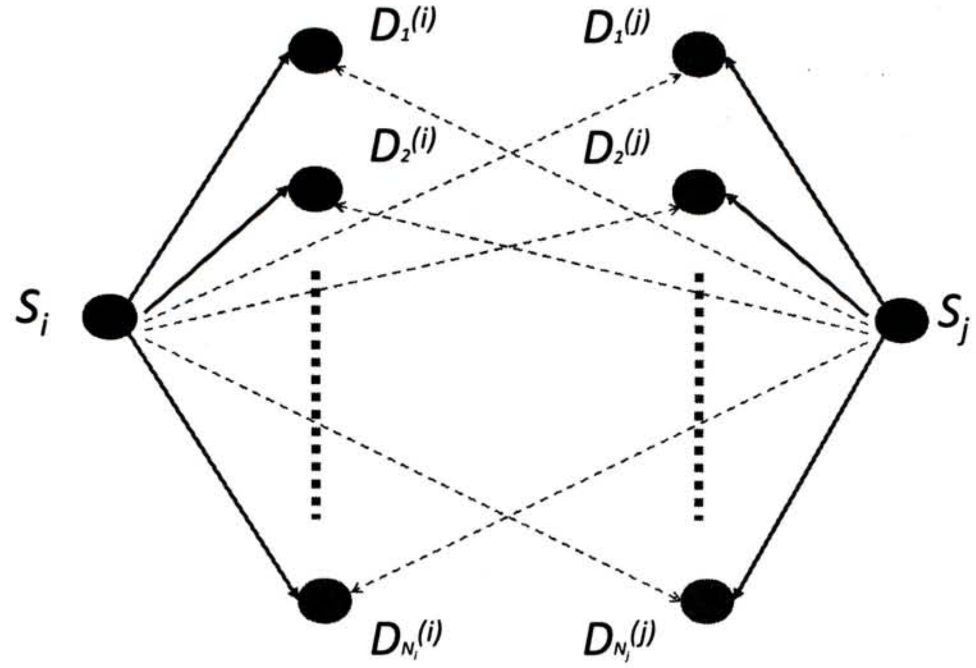

Figure 3.1: System Model

associated with distinct receiver sets, i.e., $\mathbb{R}_i \cap \mathbb{R}_j = \emptyset, \forall i \neq j$. Note that conceptually distinct receivers in the previous assumption might be the same physical receiver in reality. We further denote by $\mathbb{M} = \{1, 2, ..., M\}$ the index set of transmitters and $\mathbb{M}_i = \{1, 2, ..., N_i\}$ the index set of receivers associated with transmitter S_i.

The channels between different network nodes are modeled as frequency-flat Gaussian interference channels. Extending current work to frequency-selective scenario is straightforward but with a cost of increased notational complexity. Denote by $n_j^{(i)}$ the spectral density of the zero-mean additive white Gaussian noise (AWGN) at receiver $D_j^{(i)}$. Without loss of generality, the channel bandwidth is normalized to 1. Denote the channel power gain for the $S_i - D_k^{(j)}$ link by $g_{i,k}^{(j)}$. As for the availability of channel state information (CSI), receiver $D_j^{(i)}$ is only aware of its local CSI, i.e., $g_{i,j}^{(i)}$, which can be obtained through training sequences from transmitter S_i. Note that many existing distributed power allocation schemes (see, e.g., [58][57]) require CSI to be available at the transmitters, which is often achieved through feedback from the receivers, resulting in certain extra communication overhead. In contrast, transmitters need not know the CSI in our scheme since the receivers are the decision makers. Besides, we also assume throughout this thesis that the channel gains are bounded, i.e., $|g_{i,k}^{(j)}| \leq G, \forall k \in \mathbb{M}_j, \forall i, j \in \mathbb{M}$.

The power $p_j^{(i)}$ used by source S_i for transmission to receiver $D_j^{(i)}$ needs to satisfy spectral constraints, i.e., $\underline{p}_j^{(i)} \leq p_j^{(i)} \leq \bar{p}_j^{(i)}$. Meanwhile, the maximal power available at transmitter S_i is denoted by $P_{\max}^{(i)}$. In order to avoid trivial cases, we assume that $\sum_{j \in \mathbb{M}_i} \bar{p}_j^{(i)} \geq P_{\max}^{(i)}$,

$\sum_{j\in\mathbb{M}_i} \underline{p}_j^{(i)} \leq P_{\max}^{(i)}$, and $\bar{p}_j^{(i)} \leq P_{\max}^{(i)}$. We then group the power allocation of transmitter S_i into a column vector $\boldsymbol{p^{(i)}} \in \mathcal{R}^{N_i}$, i.e., $\boldsymbol{p^{(i)}} := [p_1^{(i)}, p_2^{(i)}, ..., p_{N_i}^{(i)}]^T$. We further group all the transmit power into a column vector $\boldsymbol{p} \in \mathcal{R}^N$ where $N = N_1 + N_2 + ... + N_M$, i.e., $\boldsymbol{p} := [\boldsymbol{p^{(1)}}^T, \boldsymbol{p^{(2)}}^T, ..., \boldsymbol{p^{(M)}}^T]^T$.

We assume that each receiver is only interested in its own signal and treats interferences as noises. So the signal-to-interference-plus-noise-ratio (SINR) at receiver $D_j^{(i)}$ is given by

$$\gamma_j^{(i)} = \frac{K_j^{(i)} g_{i,j}^{(i)} p_j^{(i)}}{\alpha_j^{(i)} g_{i,j}^{(i)} \sum_{k\neq j, k\in\mathbb{M}_i} p_k^{(i)} + \sum_{k\neq i, k\in\mathbb{M}} g_{k,j}^{(i)} \sum_{l\in\mathbb{M}_k} p_l^{(k)} + n_j^{(i)}}, \tag{3.1}$$

where $K_j^{(i)}$ is a system design parameter. For instance, $K_j^{(i)}$ can represent the processing gain in a wireless network that applies direct sequence spread spectrum (DSSS) technique. Parameter $\alpha_j^{(i)} \in [0, 1]$ can be used to model the level of endogenous interference. To be more specific, let us consider again a wireless network that applies DSSS technique. If source S_i uses orthogonal sequences for transmission to its different receivers, then there will be no endogenous interference in ideal scenarios and thus $\alpha_j^{(i)} = 0, \forall j \in \mathbb{M}_i$. However, even orthogonal sequences in reality could become nonorthogonal at the receivers due to various factors such as fading [66]. As a result, endogenous interference may be present. In an extreme case, $\alpha_j^{(i)} = 1$ indicates the endogenous interference is as serious as extraneous interference for receiver $D_j^{(i)}$. Since including the effect of $\alpha_j^{(i)}$ in our scheme is straightforward, for ease of exposition, we in the sequel only consider the worst case, i.e., $\alpha_j^{(i)} = 1, \forall j \in \mathbb{M}_i, \forall i \in \mathbb{M}$. Similar model was also adopted by [75] to design power control scheme for stochastic wireless networks.

We also denote the interference-plus-noise at receiver $D_j^{(i)}$ by

$$\mathcal{I}_j^{(i)}(\boldsymbol{p_{-j}^{(i)}}) = g_{i,j}^{(i)} \sum_{k\neq j, k\in\mathbb{M}_i} p_k^{(i)} + \sum_{k\neq i, k\in\mathbb{M}} g_{k,j}^{(i)} \sum_{l\in\mathbb{M}_k} p_l^{(k)} + n_j^{(i)}, \tag{3.2}$$

where $\boldsymbol{p_{-j}^{(i)}}$ is the vector formed by all the transmit powers except that of transmitter S_i for transmission to receiver $D_j^{(i)}$. Then the maximum achievable rate for receiver $D_j^{(i)}$ is given by

$$R_j^{(i)}(\boldsymbol{p}) = R_j^{(i)}(p_j^{(i)}, \boldsymbol{p_{-j}^{(i)}}) = \ln(1 + \frac{K_j^{(i)} g_{i,j}^{(i)} p_j^{(i)}}{\mathcal{I}_j^{(i)}(\boldsymbol{p_{-j}^{(i)}})}), \tag{3.3}$$

where we denote $\boldsymbol{p}$ by $(p_j^{(i)}, \boldsymbol{p_{-j}^{(i)}})$ (and sometimes in the sequel) to emphasize the role of $p_j^{(i)}$.

3.2 A GNEP Approach

3.2.1 Problem Formulation

We formulate the power control problem as a noncooperative game in this section. We assume each receiver is a player of game $\mathbb{G}$ (defined below). Each player chooses its decision individually. More specifically, receiver $D_j^{(i)}$ is associated with a utility function given by

$$J_j^{(i)}(p_j^{(i)}, \boldsymbol{p}_{-\boldsymbol{j}}^{(\boldsymbol{i})}) = w_j^{(i)} R_j^{(i)}(p_j^{(i)}, \boldsymbol{p}_{-\boldsymbol{j}}^{(\boldsymbol{i})}), \tag{3.4}$$

where $w_j^{(i)}$ is a nonnegative weight factor measuring the relative importance of receiver $D_j^{(i)}$'s rate in the network. Denoting $g^{(i)}(\boldsymbol{p}^{(\boldsymbol{i})}) = \sum_{j\in\mathbb{M}_i} p_j^{(i)} - P_{\max}^{(i)}$, the strategy set of player $D_j^{(i)}$ is given by

$$\boldsymbol{\Phi}_j^{(i)}(\boldsymbol{p}_{-\boldsymbol{j}}^{(\boldsymbol{i})}) = \{p_j^{(i)} : g^{(i)}(p_j^{(i)}, \boldsymbol{p}_{-\boldsymbol{j}}^{(\boldsymbol{i})}) \leq 0, \underline{p}_j^{(i)} \leq p_j^{(i)} \leq \bar{p}_j^{(i)}\}. \tag{3.5}$$

For later use, we also define the set value mapping

$$\boldsymbol{\Phi}(\boldsymbol{p}) = \prod_{i\in\mathbb{M}} \prod_{j\in\mathbb{M}_i} \boldsymbol{\Phi}_j^{(i)}(\boldsymbol{p}_{-\boldsymbol{j}}^{(\boldsymbol{i})}), \tag{3.6}$$

and the global power strategy set

$$\boldsymbol{\Phi} = \{\boldsymbol{p} \in \mathbb{R}^N : \boldsymbol{g}(\boldsymbol{p}) \preceq \boldsymbol{0}, \underline{\boldsymbol{p}} \preceq \boldsymbol{p} \preceq \bar{\boldsymbol{p}}\}, \tag{3.7}$$

where $\boldsymbol{g}(\boldsymbol{p}) = [g^{(1)}(\boldsymbol{p}^{(\boldsymbol{1})}), ..., g^{(M)}(\boldsymbol{p}^{(\boldsymbol{M})})]^T$, $\underline{\boldsymbol{p}} = [\underline{p}_1^{(1)}, ..., \underline{p}_{N_M}^{(M)}]^T$, $\bar{\boldsymbol{p}} = [\bar{p}_1^{(1)}, ..., \bar{p}_{N_M}^{(M)}]^T$, and $\boldsymbol{0}$ denotes the N-dimensional zero vector. For player $D_j^{(i)}$, treating the strategies $\boldsymbol{p}_{-\boldsymbol{j}}^{(\boldsymbol{i})}$ of other players as exogenous variables, it aims at solving the following optimization problem P-1:

$$\text{P-1:} \qquad \text{maximize}_{p_j^{(i)}\in\boldsymbol{\Phi}_j^{(i)}(\boldsymbol{p}_{-\boldsymbol{j}}^{(\boldsymbol{i})})} \quad J_j^{(i)}(p_j^{(i)}, \boldsymbol{p}_{-\boldsymbol{j}}^{(\boldsymbol{i})}), \tag{3.8}$$

whose optimal solution set is denoted by $B_j^{(i)}(\boldsymbol{p}_{-\boldsymbol{j}}^{(\boldsymbol{i})})$. Now we formally define game $\mathbb{G}$ as follows:

$$\mathbb{G} = \{\mathbb{N}, \{\boldsymbol{\Phi}_j^{(i)}(\boldsymbol{p}_{-\boldsymbol{j}}^{(\boldsymbol{i})})\}_{j\in\mathbb{M}_i, i\in\mathbb{M}}, \{J_j^{(i)}(p_j^{(i)}, \boldsymbol{p}_{-\boldsymbol{j}}^{(\boldsymbol{i})})\}_{j\in\mathbb{M}_i, i\in\mathbb{M}}\}, \tag{3.9}$$

where $\mathbb{N} = \bigcup_{j\in\mathbb{M}} \mathbb{M}_j$ is the set of players, $J_j^{(i)}(p_j^{(i)}, \boldsymbol{p}_{-\boldsymbol{j}}^{(\boldsymbol{i})})$ is player $D_j^{(i)}$'s utility function, and $\boldsymbol{\Phi}_j^{(i)}(\boldsymbol{p}_{-\boldsymbol{j}}^{(\boldsymbol{i})})$ is player $D_j^{(i)}$'s strategy space. Note that player $D_j^{(i)}$'s strategy $p_j^{(i)}$ in power control game $\mathbb{G}$ is constrained in two aspects. In particular, $p_j^{(i)}$ is constrained by both coupling constraint that depends on other players' strategies, i.e., $g^{(i)}(\boldsymbol{p}^{(\boldsymbol{i})}) \leq 0$, and individual constraints that are independent of other players' strategies, i.e., $\underline{p}_j^{(i)} \leq p_j^{(i)} \leq \bar{p}_j^{(i)}$. This makes the problem investigated in this thesis differs from recent works, which involve only uncoupled constraints in the noncooperative game formulated for distributed power control in Gaussian interference channels (see, e.g., [39] [58][57]).

Indeed, the distributed power control problem that we investigate is a GNEP. Research on GNEP has gained momentum recently, especially from the operation research community. However, unlike NEP, a systematic approach to GNEP is still lacking. We refer interested readers to the survey paper [19]. Here we only recall several basic concepts that are used throughout this thesis.

Definition 3.1 *A power allocation tuple $\boldsymbol{p}^* \in \boldsymbol{\Phi}(\boldsymbol{p}^*)$ is called a generalized Nash equilibrium (GNE) of game $\mathbb{G}$ if and only if $p_j^{(i)*} \in B_j^{(i)}(\boldsymbol{p}_{-j}^{(i)*}), \forall j \in \mathbb{M}_i, \forall i \in \mathbb{M}$.*

Accordingly, the problem of finding a GNE $\boldsymbol{p}^*$ in game $\mathbb{G}$ is called a GNEP. At a GNE no player can increase its utility by unilaterally changing its power allocation strategy.

Before ending this section, we stress that players in game $\mathbb{G}$ are just normative. They may or may not be the real decision makers. Indeed, if a central decision maker exists, it can compute a GNE and implement the GNE in the network.

3.2.2 Preliminary Results

Several issues exist in game $\mathbb{G}$. Firstly, a basic question is the existence of solution(s), since the lack of GNE indicates an unstable distributed system. Another classical issue is the uniqueness of the solution to game $\mathbb{G}$. A unique GNE is obviously desirable for operators to predict and control network behaviors. Besides, how to arrive at a GNE from initially nonequilibria states is of practical interest. We address the first two issues in this subsection and leave the last one to the next section.

GNEPs in general are extremely hard problems [19]. A current popular approach to a GNEP is to apply the relatively well developed theory of VI. We first define a mapping $\boldsymbol{F} : \mathbb{R}^N \mapsto \mathbb{R}^N$ as $\boldsymbol{F}(\boldsymbol{p}) := [F_1^{(1)}(p_1^{(1)}, \boldsymbol{p}_{-1}^{(1)}), ..., F_{N_M}^{(M)}(p_{N_M}^{(M)}, \boldsymbol{p}_{-N_M}^{(M)})]^T$, where

$$F_j^{(i)}(p_j^{(i)}, \boldsymbol{p}_{-j}^{(i)}) = -\nabla_{p_j^{(i)}} J_j^{(i)}(p_j^{(i)}, \boldsymbol{p}_{-j}^{(i)}) = -\frac{w_j^{(i)} K_j^{(i)} g_{i,j}^{(i)}}{K_j^{(i)} g_{i,j}^{(i)} p_j^{(i)} + \mathcal{I}_j^{(i)}(\boldsymbol{p}_{-j}^{(i)})}. \tag{3.10}$$

Now we state the connection between the GNEP in power control game $\mathbb{G}$ and a VI problem $VI(\boldsymbol{\Phi}, \boldsymbol{F})$, the aim of which is to find a vector $\boldsymbol{p}^* \in \boldsymbol{\Phi}$ such that $(\boldsymbol{p} - \boldsymbol{p}^*)^T \boldsymbol{F}(\boldsymbol{p}^*) \geq \boldsymbol{0}, \forall \boldsymbol{p} \in \boldsymbol{\Phi}$.

Lemma 3.1 *Every solution of $VI(\boldsymbol{\Phi}, \boldsymbol{F})$ is a GNE of the power control game $\mathbb{G}$.*

Proof Suppose $\boldsymbol{p}^* = (p_j^{(i)*}, \boldsymbol{p}_{-j}^{(i)*})$ is an arbitrary solution to $VI(\boldsymbol{\Phi}, \boldsymbol{F})$. Then we have $p_j^{(i)*} \in \boldsymbol{\Phi}_j^{(i)}(\boldsymbol{p}_{-j}^{(i)*}), \forall j \in \mathbb{M}_i, \forall i \in \mathbb{M}$, implying that $\boldsymbol{p}^* \in \boldsymbol{\Phi}(\boldsymbol{p}^*)$. Now consider any player $D_j^{(i)}$. Let $\boldsymbol{q} = [p_1^{(1)*}, ..., q_j^{(i)}, ..., p_{N_M}^{(M)*}]^T$, where $q_j^{(i)} \in \boldsymbol{\Phi}_j^{(i)}(\boldsymbol{p}_{-j}^{(i)*})$ is arbitrary. Clearly, $\boldsymbol{q} \in \boldsymbol{\Phi}$. Then, $(\boldsymbol{q} - \boldsymbol{p}^*)^T \boldsymbol{F}(\boldsymbol{p}^*) = (q_j^{(i)} - p_j^{(i)*}) F_j^{(i)}(p_j^{(i)*}, \boldsymbol{p}_{-j}^{(i)*}) \geq 0$, where the inequality follows from the fact that $\boldsymbol{p}^*$ is a solution to $VI(\boldsymbol{\Phi}, \boldsymbol{F})$. Note that $J_j^{(i)}$ is concave in $p_j^{(i)}$ on the set $\boldsymbol{\Phi}_j^{(i)}(\boldsymbol{p}_{-\boldsymbol{j}}^{(\boldsymbol{i})*})$. We conclude that $p_j^{(i)*}$ maximizes $J_j^{(i)}(p_j^{(i)}, \boldsymbol{p}_{-\boldsymbol{j}}^{(\boldsymbol{i})})$ over $\boldsymbol{\Phi}_j^{(i)}(\boldsymbol{p}_{-\boldsymbol{j}}^{(\boldsymbol{i})*})$, based on the optimality condition [Prop. 3.1, Section 3] in [7]. That is, $p_j^{(i)*} \in B_j^{(i)}(\boldsymbol{p}_{-\boldsymbol{j}}^{(\boldsymbol{i})*}), \forall j \in \mathbb{M}_i, \forall i \in \mathbb{M}$. Thus $\boldsymbol{p}^*$ is a GNE of game $\mathbb{G}$.

Lemma 4.1 implies that we can compute a solution to $VI(\boldsymbol{\Phi}, \boldsymbol{F})$ to find a GNE in game $\mathbb{G}$. However, the reverse is not true. Nevertheless, it is fine just to obtain one GNE (if any) in game $\mathbb{G}$ through solving $VI(\boldsymbol{\Phi}, \boldsymbol{F})$ for practical consideration.

Observing the special characteristic of GNE $\boldsymbol{p}^*$ in game $\mathbb{G}$ which is also a solution to $VI(\boldsymbol{\Phi}, \boldsymbol{F})$, such GNE $\boldsymbol{p}^*$ will be referred to as variational equilibrium (VE) in the rest of this thesis [19]. More formally, denoted by $SOL(\boldsymbol{\Phi}, \boldsymbol{F})$ the solution set of $VI(\boldsymbol{\Phi}, \boldsymbol{F})$, we have the following definition.

Definition 3.2 *A GNE* $\mathbf{p}^*$ *in game* $\mathbb{G}$ *is called variational equilibrium if* $\mathbf{p}^* \in SOL(\boldsymbol{\Phi}, \boldsymbol{F})$.

Now we address the issue of existence of GNE(s) in game $\mathbb{G}$ in the following proposition.

Proposition 3.1 *At least one GNE exists in the power control game* $\mathbb{G}$.

Proof It is obvious that $\boldsymbol{\Phi}$ is a convex and compact set. Besides, the mapping $\boldsymbol{F} : \boldsymbol{\Phi} \to \mathbb{R}^N$ is continuous. Then, based on Corollary 2.2.5 in [21], we conclude that $SOL(\boldsymbol{\Phi}, \boldsymbol{F})$ is nonempty and compact. The existence of GNE in game $\mathbb{G}$ thus follows by Lemma 4.1.

Though the existence of GNE(s) in game $\mathbb{G}$ has been confirmed by Proposition 4.1, it is generally more involved to establish the uniqueness of GNE. In fact, it is recognized by practitioners that GNEPs tend to have nonunique GNEs. This is also true in game $\mathbb{G}$. Instead, we provide sufficient conditions to establish the uniqueness of VE in game $\mathbb{G}$.

Proposition 3.2 *A unique VE* $\mathbf{p}^*$ *exists in game* $\mathbb{G}$ *if the following conditions hold:*

$$(K_j^{(i)} + 1)g_{i,j}^{(i)} > \sum_{k \in \mathbb{M}} N_k g_{k,j}^{(i)} \tag{3.11}$$

$$(K_j^{(i)} + 1)g_{i,j}^{(i)} > \sum_{k \in \mathbb{M}} \sum_{l \in \mathbb{M}_k} g_{i,l}^{(k)} \tag{3.12}$$

for all $j \in \mathbb{M}_i$ *and* $i \in \mathbb{M}$.

Proof The entry $\nabla_{p_l^{(k)}} F_j^{(i)}(\boldsymbol{p})$ of Jacobian matrix $\nabla \boldsymbol{F}(\boldsymbol{p})$ is given by

$$\nabla_{p_l^{(k)}} F_j^{(i)}(\boldsymbol{p}) = \frac{w_j^{(i)} K_j^{(i)} g_{i,j}^{(i)} A_l^{(k)}}{(K_j^{(i)} g_{i,j}^{(i)} p_j^{(i)} + \mathcal{I}_j^{(i)}(\boldsymbol{p}_{-\boldsymbol{j}}^{(\boldsymbol{i})}))^2} \tag{3.13}$$

where

$$A_l^{(k)} = \begin{cases} K_j^{(i)} g_{i,j}^{(i)} & \text{if } k = i,\ l = j; \\ g_{i,j}^{(i)} & \text{if } k = i,\ l \neq j; \\ g_{k,j}^{(i)} & \text{if } k \neq i. \end{cases}$$

If the conditions (4.11) and (4.12) are satisfied, both $\nabla \boldsymbol{F}(\boldsymbol{p})$ and $(\nabla \boldsymbol{F}(\boldsymbol{p}))^T$ are strictly diagonally dominant. Noting further all diagonal elements of $\nabla \boldsymbol{F}(\boldsymbol{p})$ are positive, it follows that $\nabla \boldsymbol{F}(\boldsymbol{p})$ is positive definite and thus $\boldsymbol{F}(\boldsymbol{p})$ is strictly monotone. Furthermore, $\boldsymbol{\Phi}$ is convex and compact and $\boldsymbol{F}(\boldsymbol{p})$ is continuous, based on Theorem 2.3.3 in [21], $SOL(\boldsymbol{\Phi}, \boldsymbol{F}(\boldsymbol{p}))$ has at most one element. Besides, from the proof of Proposition 1, we know that $SOL(\boldsymbol{\Phi}, \boldsymbol{F}(\boldsymbol{p}))$ is not empty. We conclude that a unique VE $\boldsymbol{p}^*$ exists in game $\mathbb{G}$.

Proposition 4.2 provides sufficient conditions to guarantee the uniqueness of VE in game $\mathbb{G}$. A careful thinking reveals that sufficient conditions (4.11) and (4.12) are indeed intuitive. On the one hand, from the game theory point of view, these conditions imply that, though coupled with other players in both utility functions and constraint spaces, player $D_j^{(i)}$ has a more significant influence on its utility value than other players do. From the communication point of view, these conditions impose upper bounds on the interference received by receiver $D_j^{(i)}$. Under mild interference conditions, receiver $D_j^{(i)}$'s transmission rate is not heavily influenced by other receivers.

We further point out that conditions (4.11) and (4.12) are not stringent for real applications. For example, if $K_j^{(i)}$ represents the processing gain in universal mobile telecommunications system (UMTS) - frequency-division duplexing (FDD) networks, then typical value of $K_j^{(i)}$ is 256. Besides, the channel gains in wireless networks usually satisfy that $g_{i,j}^{(i)} \gg g_{k,j}^{(i)}, k \neq i$. Then by appropriately control the number N_k of target receivers of transmitter $S_k, \forall k \in \mathbb{M}$, conditions (4.11) and (4.12) can be guaranteed. Note that it is necessary to apply certain signal processing techniques (spread spectrum, for one) demonstrated in system parameter $K_j^{(i)}$. Otherwise, conditions (4.11) and (4.12) cannot be satisfied since the receiver $D_j^{(i)}$ is confronted with a high interference level mainly caused by signals for transmitter S_i's other receivers.

Proposition 4.2 does not imply the uniqueness of GNE in game $\mathbb{G}$ though it does establish the uniqueness of VE in game $\mathbb{G}$. Nevertheless, VEs are more socially stable than other GNEs [11]. This further justifies our interests in VE. Another justification is given in the following proposition.

Proposition 3.3 *Denote by* $\boldsymbol{w} = [w_1^{(1)}, ..., w_{N_M}^{(M)}]^T$ *the weighting vector on network users' rates.*

(i) The set of GNEs in game $\mathbb{G}$ *is unchanged with different* $\boldsymbol{w}$*'s.*

(ii) The set of VEs in game $\mathbb{G}$ *can be changed with different* $\boldsymbol{w}$*'s.*

Proof (i). Let $\bar{\boldsymbol{w}}$ and $\tilde{\boldsymbol{w}}$ be any two weighting vectors such that $\bar{\boldsymbol{w}} \neq \tilde{\boldsymbol{w}}$. The corresponding sets of GNEs are denoted by $GNE(\bar{\boldsymbol{w}})$ and $GNE(\tilde{\boldsymbol{w}})$, respectively. Then $\exists j \in \mathbb{M}_i, i \in \mathbb{M}$, $\bar{w}_j^{(i)} \neq \tilde{w}_j^{(i)}$. Let $\bar{w}_j^{(i)}$ and $\tilde{w}_j^{(i)}$ be the first different elements in $\bar{\boldsymbol{w}}$ and $\tilde{\boldsymbol{w}}$, and $\hat{\boldsymbol{w}} = [\bar{w}_1^{(1)}, ..., \bar{w}_{j-1}^{(i)}, \tilde{w}_j^{(i)}, \bar{w}_{j+1}^{(i)}, ..., \bar{w}_{N_M}^{(M)}]^T$. For any $\boldsymbol{p}^* = (p_j^{(i)*}, \boldsymbol{p}_{-j}^{(i)*}) \in GNE(\bar{\boldsymbol{w}})$, it can be shown that $\boldsymbol{p}^* = (p_j^{(i)*}, \boldsymbol{p}_{-j}^{(i)*}) \in GNE(\hat{\boldsymbol{w}})$. The reverse argument is also true. Hence, $GNE(\bar{\boldsymbol{w}}) = GNE(\hat{\boldsymbol{w}})$. Following a similar argument, we can construct a finite sequence $(\hat{\boldsymbol{w}} =)\hat{\boldsymbol{w}}_1$, $\hat{\boldsymbol{w}}_2$,...,$\hat{\boldsymbol{w}}_n (= \tilde{\boldsymbol{w}})$, $n \leq N$, such that $GNE(\hat{\boldsymbol{w}}_1) = GNE(\hat{\boldsymbol{w}}_2) = ... = GNE(\hat{\boldsymbol{w}}_n)$. Hence, we conclude that $GNE(\bar{\boldsymbol{w}}) = GNE(\tilde{\boldsymbol{w}})$.

(ii). Denote by $\boldsymbol{\Phi}^{\boldsymbol{0}}$ and $\boldsymbol{\partial\Phi}$ the set of interior points and boundary points of $\boldsymbol{\Phi}$, respectively. *Case 1:* If a VE $\boldsymbol{p}^* \in \boldsymbol{\Phi}^{\boldsymbol{0}}$, then $\boldsymbol{F}(\boldsymbol{p}^*, \boldsymbol{w}) = \boldsymbol{0}$ since $(\boldsymbol{p} - \boldsymbol{p}^*)^T \boldsymbol{F}(\boldsymbol{p}^*, \boldsymbol{w}) \geq 0, \forall \boldsymbol{p} \in \boldsymbol{\Phi}$. By the definition of $\boldsymbol{F}$, $\boldsymbol{F}(\boldsymbol{p}^*, \boldsymbol{w}) = \boldsymbol{0}$ holds if and only if $\boldsymbol{w} = \boldsymbol{0}$. Then the result follows trivially.

Case 2: If a VE $\boldsymbol{p}^* \in \boldsymbol{\partial\Phi}$, we assume that its associated weighting vector $\bar{\boldsymbol{w}} \neq \boldsymbol{0}$ to avoid trivial cases. We next show that there exist $\bar{\boldsymbol{w}}$ and $\tilde{\boldsymbol{w}}$ such that $VE(\bar{\boldsymbol{w}}) \neq VE(\tilde{\boldsymbol{w}})$, where $VE(\boldsymbol{w})$ denotes the set of VEs associated with $\boldsymbol{w}$. Consider $\bar{\boldsymbol{w}} = [1, 0, ..., 0]^T$ and $\tilde{\boldsymbol{w}} = [0, 0, ..., 1]^T$. We have

$$0 \leq (\boldsymbol{p} - \boldsymbol{p}^*)^T \boldsymbol{F}(\boldsymbol{p}^*, \bar{\boldsymbol{w}}) = (p_1^{(1)} - p_1^{(1)*}) F_1^{(1)}(\boldsymbol{p}^*, \bar{\boldsymbol{w}}) = -\frac{(p_1^{(1)} - p_1^{(1)*}) K_1^{(1)} g_{1,1}^{(1)}}{K_1^{(1)} g_{1,1}^{(1)} p_1^{(1)} + \mathcal{I}_1^{(1)}(\boldsymbol{p}_{-1}^{(1)})}, \forall \boldsymbol{p} \in \boldsymbol{\Phi},$$

which implies that $p_1^{(1)} - p_1^{(1)*} \leq 0$. Thus, it is clear that $\boldsymbol{p}^* = [\bar{p}_1^{(1)}, \underline{p}_2^{(1)}, ..., \underline{p}_{N_M}^{(M)}]^T \in VE(\bar{\boldsymbol{w}})$ and $\boldsymbol{p}^* \notin VE(\tilde{\boldsymbol{w}})$. Similarly, $\tilde{\boldsymbol{p}}^* = [\underline{p}_1^{(1)}, ..., \underline{p}_{N_M-1}^{(M)}, \bar{p}_{N_M}^{(M)}]^T \in VE(\tilde{\boldsymbol{w}})$ and $\tilde{\boldsymbol{p}}^* \notin VE(\bar{\boldsymbol{w}})$. Hence, we conclude that $VE(\bar{\boldsymbol{w}}) \neq VE(\tilde{\boldsymbol{w}})$.

Proposition 4.3 implies that different weighting vector $\boldsymbol{w}$'s may correspond to different VE $\boldsymbol{p}^*$'s. This is an intuitive result. From communication engineering point of view, the associated desirable network power operation points are expected to vary after network operators make certain adjustment in weighting vectors. Nevertheless, it is interesting to note that the set of GNEs in game $\mathbb{G}$ is independent of weighting vector $\boldsymbol{w}$. Hence, in a certain sense, the relationship between GNEs and VEs in game $\mathbb{G}$ is analogous to that between Pareto optimal boundary and certain pareto optimal points in the capacity region of multiple access channels [67].

3.3 Algorithm Design

In this section, we propose an iterative partially penalized Jacobi algorithm (IP^{2}JA) to compute a VE of game $\mathbb{G}$ by appropriately modifying the penalty methods proposed in [20]. First, note that, compared to standard NEP, the main difficulty in the GNEP of game $\mathbb{G}$ arises from the coupling power resource constraints at transmitters, i.e., $g^{(i)}(\boldsymbol{p}^{(i)}) \leq 0$. To

Step 1: Initialization:
Receiver $D_j^{(i)}, \forall j \in \mathbb{M}_i, \forall i \in \mathbb{M}$, starts with an arbitrarily feasible power allocation, i.e., $p_j^{(i)}(0) \in \boldsymbol{\Psi}_j^{(i)}$. Initialize $k > 1$ and $\rho_j^{(i)} > 0$. Set $t := 0$ and $n := 0$.

Step 2: Jacobi Iteration:
Receiver $D_j^{(i)}, \forall j \in \mathbb{M}_i, \forall i \in \mathbb{M}$, updates its power allocation $p_j^{(i)}(n+1,t)$ by

$$p_j^{(i)}(n+1,t) = [p_j^{(i)}(n,t) + a(t)\nabla_{p_j^{(i)}}\tilde{J}_j^{(i)}(\boldsymbol{p}(n,t))]^+_{\boldsymbol{\Psi}_j^{(i)}}, \tag{3.15}$$

where $(a(t))_{t=1}^{+\infty}$ are positive step sizes, and $[x]^+_{\boldsymbol{\Psi}_j^{(i)}}$ denotes the projection of x onto set $\boldsymbol{\Psi}_j^{(i)}$.
Set $n := n+1$. If $n \leq N_t$ and $\boldsymbol{p}(n,t)$ has not converged, repeat Step 2; otherwise, go to Step 3.

Step 3: Penalty Updating:
Set $\boldsymbol{p}(t+1) = \boldsymbol{p}(n,t)$. If $\boldsymbol{p}(t+1) \notin \boldsymbol{\Phi}$, and for all receivers,

$$|\nabla_{p_j^{(i)}} J_j^{(i)}(\boldsymbol{p}(t+1))| \geq \rho_j^{(i)}|\nabla_{p_j^{(i)}} g_+^{(i)}(\boldsymbol{p^{(i)}}(t+1))|, \tag{3.16}$$

then set $\rho_j^{(i)} := k\rho_j^{(i)}, \forall j \in \mathbb{M}_i, \forall i \in \mathbb{M}$. Set $n := 0$ and $t := t+1$.

Step 4: Convergence Verification:
If $\boldsymbol{p}(t)$ has converged, then stop; otherwise, go to Step 2.

Table 3.1: Detail steps of IP^{2}JA

resolve this difficulty, we consider a penalized version of receiver $D_j^{(i)}$'s utility maximization problem:

$$\text{P-2:} \quad \text{maximize}_{p_j^{(i)} \in \boldsymbol{\Psi}_j^{(i)}} \quad \tilde{J}_j^{(i)}(p_j^{(i)}, \boldsymbol{p_{-j}^{(i)}}) := J_j^{(i)}(p_j^{(i)}, \boldsymbol{p_{-j}^{(i)}}) - \rho_j^{(i)}((g_+^{(i)}(\boldsymbol{p^{(i)}}))^\gamma + \epsilon)^{\frac{1}{\gamma}} \tag{3.14}$$

where $\rho_j^{(i)} \in \mathbb{R}_{++}$ is penalty parameter, γ is a positive integer such that $\gamma \geq 3$, $\epsilon \in \mathbb{R}_{++}$ is a small value, $\boldsymbol{\Psi}_j^{(i)} = \{p_j^{(i)} \in \mathbb{R} : \underline{p}_j^{(i)} \leq p_j^{(i)} \leq \bar{p}_j^{(i)}\}$, and $g_+^{(i)}(\boldsymbol{p^{(i)}}) = \max(0, g^{(i)}(\boldsymbol{p^{(i)}}))$. We formally describe IP^{2}JA in Algorithm 3.1.

The basic idea behind IP^{2}JA is simple. By introducing penalty parameter $\boldsymbol{\rho}$, we obtain a penalized NEP in a new game $\mathbb{G}^\dagger(\boldsymbol{\rho})$. In particular, given penalty parameter $\boldsymbol{\rho}$, game $\mathbb{G}^\dagger(\boldsymbol{\rho})$ has the same player set as game $\mathbb{G}$ but different utility function $\tilde{J}_j^{(i)}(p_j^{(i)}, \boldsymbol{p_{-j}^{(i)}})$, and decoupled strategy space $\boldsymbol{\Psi} = \prod_{j \in \mathbb{M}_i, i \in \mathbb{M}} \boldsymbol{\Psi}_j^{(i)}$. Then we inexactly solve a sequence of games $\mathbb{G}^\dagger(\boldsymbol{\rho})$ to approach the original VE of game $\mathbb{G}$. More precisely, if $\epsilon = 0$ in IP^{2}JA, an NE (if any) of game $\mathbb{G}^\dagger(\boldsymbol{\rho})$ is clearly a VE of game $\mathbb{G}$. If current solution $\boldsymbol{p}(t)$ is not feasible and criteria (3.16) are satisfied, penalty parameter $\boldsymbol{\rho}$ will be increased in the next iteration to force feasibility. The philosophy behind updating criteria (3.16) is that the penalty parameter

$\boldsymbol{\rho}$ would be increased when $\rho_j^{(i)}|\nabla_{p_j^{(i)}} g_+^{(i)}(\boldsymbol{p}^{(i)}(t+1))|$ becomes relatively small compared to $|\nabla_{p_j^{(i)}} J_j^{(i)}(\boldsymbol{p}(t+1))|$. It should be pointed out that criteria (3.16) need to be satisfied for all $j \in \mathbb{M}_i, i \in \mathbb{M}$ before updating the penalty parameter $\boldsymbol{\rho}$. In other words, we need not update penalty parameter $\boldsymbol{\rho}$ when only a subset of players violate feasibility. This reduces the frequency of penalty control and thus communication overhead in the network.

Before we delve into the theoretical analysis for IP^{2}JA, we make several remarks as follows.

Remark 3.1 *Note that $g_+^{(i)}(\boldsymbol{p}^{(i)})$ is nonsmooth and not continuously differentiable. So we use $\gamma \geq 3$ and $\epsilon \in \mathbb{R}_{++}$ to make $\tilde{J}_j^{(i)}(p_j^{(i)}, \boldsymbol{p}_{-j}^{(i)})$ smooth and twice continuously differentiable. Otherwise, we would also encounter several unnecessary technical difficulties when dealing with game $\mathbb{G}^\dagger(\boldsymbol{\rho})$ even though it was a standard NEP. Due to the existence of ϵ, the final solution obtained by IP^{2}JA is not an exact VE of game $\mathbb{G}$. Nevertheless, as shown later, we can make the approximate VE as exact as possible by selecting a small enough ϵ.*

Remark 3.2 *Recent approaches in communication field to GNEPs need to solve exactly a standard NEP (and obtain one associated NE) in every iteration of updating dual pricing parameters (see, e.g., [74], [48], and [49]). In contrast, we do not have such restriction in IP^{2}JA. In particular, IP^{2}JA can simply check and update penalty parameter $\boldsymbol{\rho}$ after N_t iterations in solving game $\mathbb{G}^\dagger(\boldsymbol{\rho})$ without considering whether or not the associated NE has been reached. In an extreme case, only one single iteration, i.e., $N_t = 1$, need to be performed before penalty updating in IP^{2}JA.*

The following lemma facilitates the analysis of the convergent behavior of IP^{2}JA.

Lemma 3.2 *Given $b \in \mathbb{R}$ and an integer $\gamma \geq 3$, the function $f : \mathbb{R} \mapsto \mathbb{R}_+$ defined by $f(x) = (\max(0, x+b))^\gamma$ is twice continuously differentiable, convex, and*

$$f'(x) = \gamma(\max(0, x+b))^{\gamma-1}, \textit{and } f''(x) = \gamma(\gamma-1)(\max(0, x+b))^{\gamma-2}. \tag{3.17}$$

Proof We first compute the following:

$$\frac{f(x+t) - f(x)}{t} = \frac{(\max(0, x+t+b) - (\max(0, x+b))^\gamma)^\gamma}{t}$$

$$= \begin{cases} \frac{(x+t+b)^\gamma - (x+b)^\gamma}{t} & \text{if } x > -b \text{ and } -|x+b| \leq t \leq |x+b|; \\ 0 & \text{if } x < -b \text{ and } -|x+b| \leq t \leq |x+b|; \\ \frac{(\max(0,t))^\gamma}{t} & \text{if } x = -b. \end{cases}$$

Then it can be readily verified that

$$\lim_{t \to 0^+} \frac{f(x+t) - f(x)}{t} = \lim_{t \to 0^-} \frac{f(x+t) - f(x)}{t} = \begin{cases} \gamma(x+b)^{\gamma-1} & \text{if } x > -b; \\ 0 & \text{if } x \leq -b. \end{cases}$$

Hence, $f'(x) = \gamma(\max(0, x+b))^{\gamma-1}$ which is obviously continuous. A similar argument also applies to $f''(x)$. Noting that $\forall x \in \mathbb{R}, f''(x) \geq 0$, we conclude that $f(x)$ is a convex function. This completes the proof.

We continue our analysis by investigating the properties of the penalized game $\mathbb{G}^{\dagger}(\boldsymbol{\rho})$ since, as shown later, it plays an important role in justifying the proposed algorithm IP^{2}JA. In particular, we summarize the main results in the following proposition.

Proposition 3.4 *Define a mapping* $\tilde{\boldsymbol{F}} : \mathbb{R}^N \mapsto \mathbb{R}^N$ *given by*

$$\tilde{\boldsymbol{F}}(\boldsymbol{p}) := [\tilde{F}_1^{(1)}(p_1^{(1)}, \boldsymbol{p}_{-\boldsymbol{1}}^{(\boldsymbol{1})}), ..., \tilde{F}_{N_M}^{(M)}(p_{N_M}^{(M)}, \boldsymbol{p}_{-\boldsymbol{N_M}}^{(\boldsymbol{M})})]^T, \tag{3.18}$$

where $\tilde{F}_j^{(i)}(p_j^{(i)}, \boldsymbol{p}_{-\boldsymbol{j}}^{(\boldsymbol{i})}) = -\nabla_{p_j^{(i)}} \tilde{J}_j^{(i)}(\boldsymbol{p})$. *Then given penalty parameters* $\boldsymbol{\rho}$ *and integer* $\gamma \geq 3$, *game* $\mathbb{G}^{\dagger}(\boldsymbol{\rho})$ *is characterized by the following properties:*

(i) The set of NEs in game $\mathbb{G}^{\dagger}(\boldsymbol{\rho})$ *is the same as the solution set* $SOL(\boldsymbol{\Psi}, \tilde{\boldsymbol{F}})$ *of* $VI(\boldsymbol{\Psi}, \tilde{\boldsymbol{F}})$.

(ii) At least one NE $\tilde{\boldsymbol{p}}^*$ *exists in game* $\mathbb{G}^{\dagger}(\boldsymbol{\rho})$.

Furthermore, if conditions (11) and (12) in Proposition 2 are satisfied, then $\exists \delta > 0$ *such that if* ϵ *is chosen to satisfy*

$$(\sum_{j \in \mathbb{M}_i, k \neq j} \rho_k^{(i)} - \rho_j^{(i)}) B^{(i)} \leq \delta, \forall j \in \mathbb{M}_i, \forall i \in \mathbb{M}, \tag{3.19}$$

where $B^{(i)}$ *is given by*

$$B^{(i)} = \epsilon(\gamma - 1)((g_+^{(i)}(\boldsymbol{p}^{(\boldsymbol{i})})^{\gamma} + \epsilon)^{\frac{1-2\gamma}{\gamma}} (g_+^{(i)}(\boldsymbol{p}^{(\boldsymbol{i})}))^{\gamma-2}, \tag{3.20}$$

then game $\mathbb{G}^{\dagger}(\boldsymbol{\rho})$ *further has the following properties:*

(iii) There exists a unique NE $\tilde{\boldsymbol{p}}^*$ *in game* $\mathbb{G}^{\dagger}(\boldsymbol{\rho})$.

(iv) If $a(t)$ *is a small enough positive step size, for any initial value* $\tilde{\boldsymbol{p}}(0, t) \in \boldsymbol{\Psi}$, *the sequence* $(\tilde{\boldsymbol{p}}(n, t))_{n=1}^{N_t}$ *generated by Jacobi iteration in IP*^{2}*JA converges to the unique NE* $\tilde{\boldsymbol{p}}^*$ *in game* $\mathbb{G}^{\dagger}(\boldsymbol{\rho})$ *when* $N_t \to \infty$.

Proof See Appendix D.

By applying Proposition 4.4, we are now in a position to state the convergent property of IP^{2}JA.

Proposition 3.5 *Let* $(\boldsymbol{p}^{\epsilon}(t))_{t=1}^{+\infty}$ *be the sequence generated by IP*^{2}*JA. If Proposition 4(.4iv) holds,* *(i) every limit point* $\bar{\boldsymbol{p}}^{\epsilon}$ *of* $(\boldsymbol{p}^{\epsilon}(t))_{t=1}^{+\infty}$ *is a GNE of game* $\mathbb{G}^{\epsilon}$ *defined as*

$$\mathbb{G}^{\epsilon} = \{\mathbb{N}, \{\boldsymbol{\Phi}_j^{(i)}(\boldsymbol{p}_{-j}^{(i)})\}_{j\in\mathbb{M}_i, i\in\mathbb{M}}, \{J_j^{\epsilon(i)}(p_j^{(i)}, \boldsymbol{p}_{-j}^{(i)})\}_{j\in\mathbb{M}_i, i\in\mathbb{M}}\}, \tag{3.21}$$

where $\mathbb{N}$ *and* $\boldsymbol{\Phi}_j^{(i)}(\boldsymbol{p}_{-j}^{(i)})$ *are the same as those of game* $\mathbb{G}$ *but* $J_j^{\epsilon(i)}(p_j^{(i)}, \boldsymbol{p}_{-j}^{(i)})$ *is given by*

$$J_j^{\epsilon(i)}(p_j^{(i)}, \boldsymbol{p}_{-j}^{(i)}) = J_j^{(i)}(p_j^{(i)}, \boldsymbol{p}_{-j}^{(i)}) - \rho_j^{(i)}\epsilon^{\frac{1}{\gamma}}; \tag{3.22}$$

(ii) if $\epsilon \to 0$*, the limit point* $\bar{\boldsymbol{p}}^{\epsilon}$ *approaches to* $\boldsymbol{p}^*$ *which is a GNE of game* $\mathbb{G}$.

Proof The sequence $(\boldsymbol{p}^{\epsilon}(t))_{t=1}^{+\infty}$ generated by IP^{2}JA is bounded due to the box constraint $\boldsymbol{\Psi}$. Furthermore, the constraints $\boldsymbol{g}(\boldsymbol{p}) \leq \boldsymbol{0}$ are linear. Then based on Theorem 2.12 in [20], the penalty parameters $\boldsymbol{\rho}$ are updated a finite times. If Proposition 4.4(iv) holds, the first part of Proposition 4.5 follows based on Theorem 2.5 in [20]. Since penalty parameters $\boldsymbol{\rho}$ are only updated a finite times and thus finite, $\rho_j^{(i)}\epsilon^{\frac{1}{\gamma}} \to 0$ if $\epsilon \to 0$. With a slight abuse of notation, it follows that $\mathbb{G}^{\epsilon} \to \mathbb{G}$ if $\epsilon \to 0$. Hence, the corresponding GNEs satisfy that $\bar{\boldsymbol{p}}^{\epsilon} \to \boldsymbol{p}^*$ if $\epsilon \to 0$.

We remark that IP^{2}JA does not generate an exact GNE to the original game $\mathbb{G}$. Clearly, the inexactness of the generated GNE comes from the introduction of parameter ϵ. However, parameter ϵ makes the objective function $\tilde{J}_j^{(i)}(p_j^{(i)}, \boldsymbol{p}_{-j}^{(i)})$ in problem P-2 smooth and twice continuously differentiable, which facilitates theoretical analysis and numerical computation. Besides, the generated GNE becomes exact when ϵ goes to 0. Hence, we can trade off the exactness of the generated GNE and numerical computation convenience by choosing different ϵ's.

3.4 Numerical Results

Numerical results are provided in this section for illustration purposes. We choose simulation parameters as follows unless specified otherwise. The number of transmitters is 4 and each transmitter is associated with 4 receivers. The weight vector $\boldsymbol{w}$ is set to be the unit vector. For clarity, receivers have the same power constraints and design parameters, i.e., $\underline{p}_j^{(i)} = 0.1, \bar{p}_j^{(i)} = 6, K_j^{(i)} = 32, \forall i, j$. The maximal power of each transmitter is 10. All the background noise powers are assumed to be 0.1. As for the parameters in IP^{2}JA, we set $\gamma = 4$, $\epsilon = 0.1$, $k = 1.5$, $N_t = 1$. The initial penalty vector $\boldsymbol{\rho}$ is randomly generated from the interval $[0.5, 1]$. We set the mean $u_{kj}^{(i)}$ of exponentially distributed random variable $g_{kj}^{(i)}$ to be I if $k = i$, and 1 otherwise. Thus, the network can have different transmission node densities by tuning parameter I. In particular, larger I implies sparser transmission node density. We set $I = 10$ unless specified otherwise.

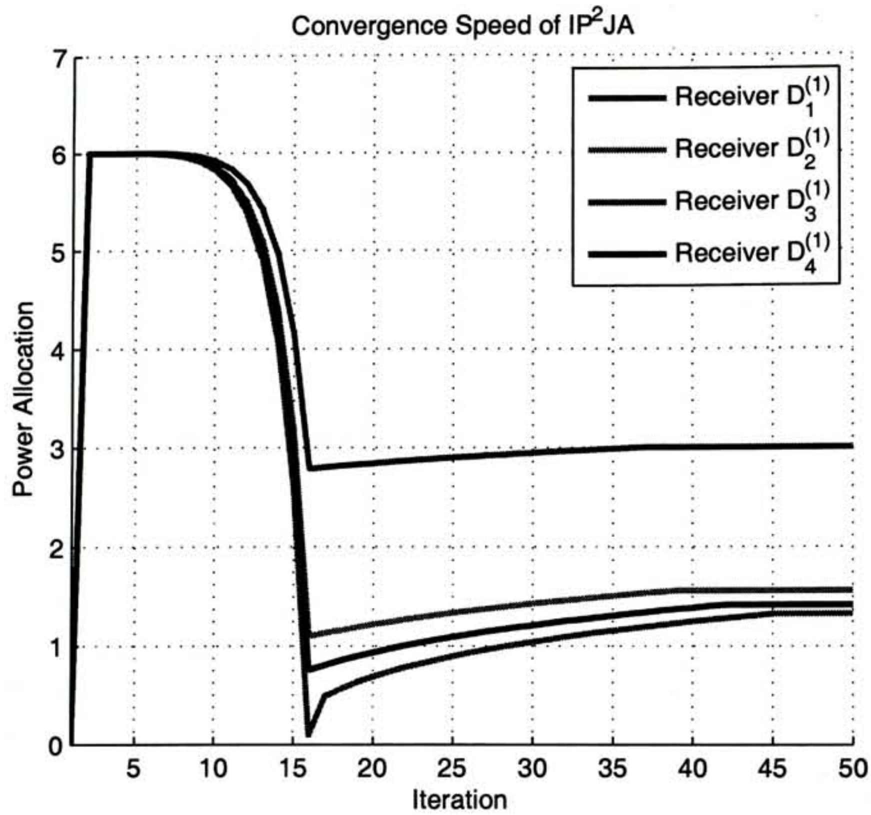

Figure 3.2: Convergence behaviors of IP^2JA

In Fig.3.2, we study the convergence behaviors of IP^2JA. Fig.3.2 shows the power allocation of receivers versus iteration number. For clarity, we only show the power evolution of the receivers associated with transmitter S_1. We can see that IP^2JA converges relatively fast. This fast convergent property is desirable for practical implementation.

We next compare our proposed one-to-many transmission scheme with the one-to-one opportunistic transmission scheme in Fig.3.3. Only one receiver, whose channel to the corresponding transmitter is the best in terms of channel power gains, is scheduled for receiving information in opportunistic transmission scheme [66]. As illustrated, one-to-many transmission scheme performs better than opportunistic transmission scheme. Nevertheless, the sum rates in both schemes nearly stop increasing when the power resource $P_{\max}$ grows beyond $10dB$. Indeed, the interference becomes a dominated factor in such scenarios. This implies that network overall interference cannot be overcome by simply increasing transmit power and thus more advanced approaches for handling interference are expected in interference-limited wireless networks.

Fig.3.4 shows the average rate versus iteration number under 4 different choices of numbers of receivers associated with each transmitter. It is shown that the average rate decreases as each transmitter is associated with more receivers. This is an intuitive result since the power resource of each transmitter is limited. Thus, the power resource required and obtained by the corresponding individual receiver becomes less when the number of receivers

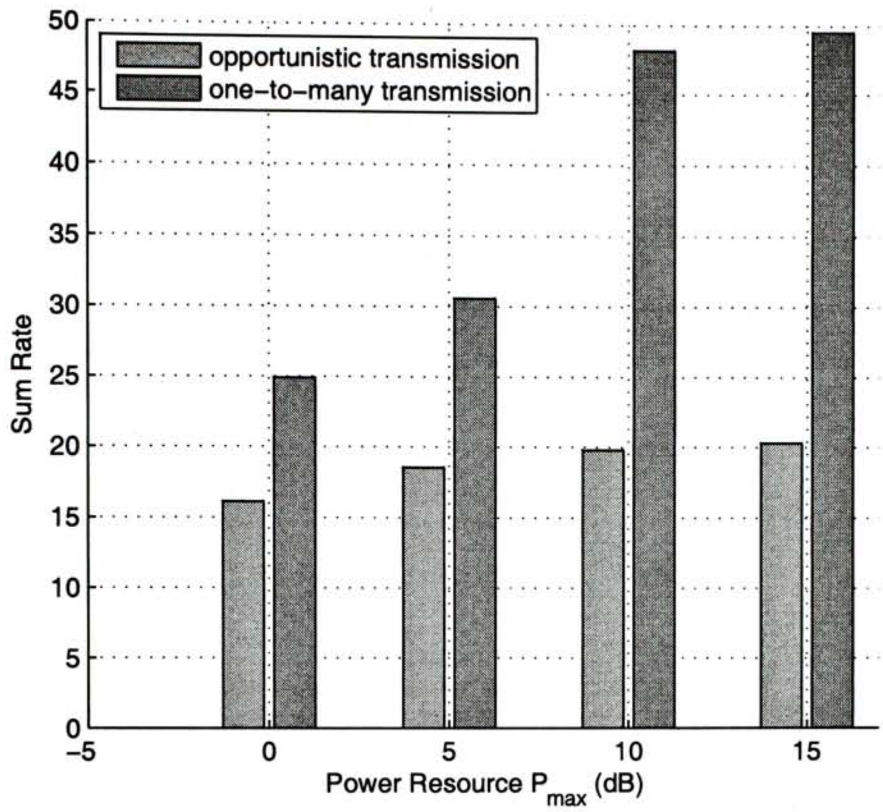

Figure 3.3: Comparison of one-to-many and opportunistic transmissions

associated with each transmitter grows. Another interesting observation in Fig.3.4 is that the average rate converges after only 2 iterations. This implies that the desired network performance (i.e., average rate) can be obtained in a few iterations even though the power allocation in IP^2JA has not converged.

Finally, we study the impacts of weight vector on the utilities of individual user and network, respectively. The weight vector is chosen as follows. The studied user $D_j^{(i)}$ is associated with weight $x \in \{0, 0.2, 0.4, 0.8\}$ while the remaining users are allocated with equal weight $\frac{1-x}{N-1}$. The numerical results are given in Table 3.2. The two studied users 1 and 16 represent two classes of receivers. More specifically, the channel gain between user 1 and the corresponding transmitter is better than the mean channel gain. In contrast, the channel gain between user 16 and the corresponding transmitter is worse than the mean channel gain. As shown in Table 3.2, both user 1 and 16 have higher utilities when their corresponding weight factors increase. However, the network utility deteriorates when the network weighs user 16 more. The converse is true for user 1. Indeed, as pointed out in Proposition 4.3, the network operation points can be varied by tuning the weighting vector.

□ **End of chapter.**

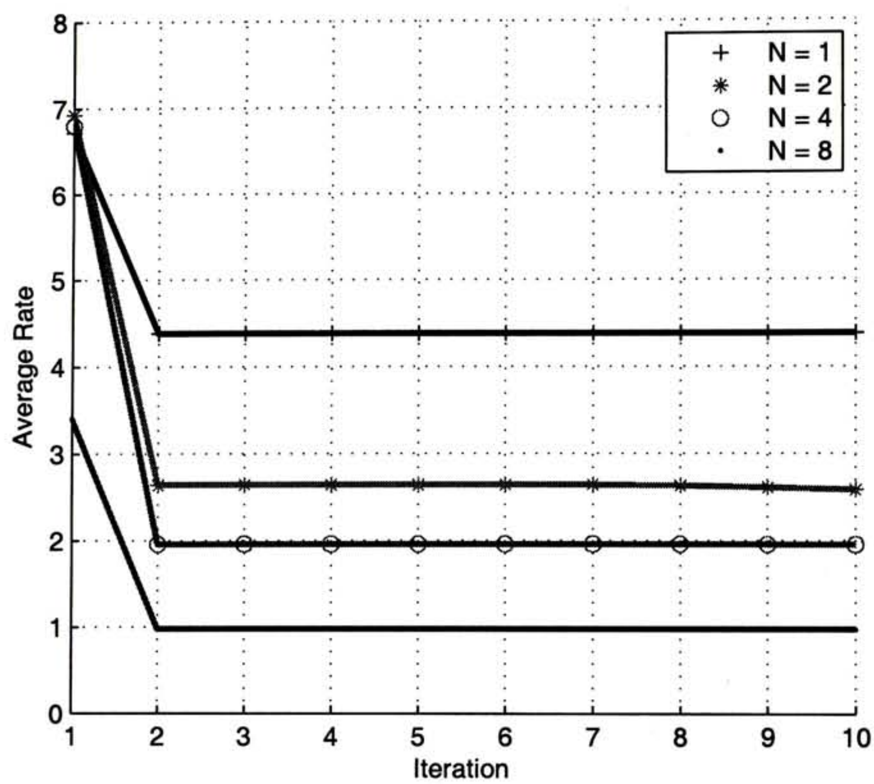

Figure 3.4: Impacts of the number of associated receivers

Weight	0	0.2	0.4	0.6	0.8
User 1	0	1.0332	1.5684	2.3272	3.3072
Network	2.7295	2.8708	2.9957	3.3345	3.8455
User 16	0	0.2262	0.7900	1.2154	2.0981
Network	2.6691	2.4955	2.3878	2.2656	2.1358

Table 3.2: Impacts of Weight Vector

Chapter 4

Flow Allocation in Multiple Access Networks

Never follow the beaten track, it leads only where others have been before.

—— *Alexander Graham Bell*

During the past two decades, we have witnessed an ever increasing demand of high data rate services in wireless communications. An end user (EU) is normally just associated with one access point (AP) in today's wireless networks such as Wireless Local Area Networks (WLANs) to access the Internet. However, researchers realize that the performance of this classic scheme may be unsatisfactory to meet the demand of high data rate services from EUs. Besides, this single-AP based scheme may be prone to suffer from fading due to the single link between EU and the corresponding AP [66]. As a result, more flexible WLANs, where EUs can be associated with multiple APs to get access to the Internet, are drawing increasingly interests from both academia and industry (see, e.g., [82] and references therein).

Another practical motivation comes from the recent interests in femtocell networks, in which consumers can install home base stations (BSs) for better indoor wireless voice and data communications [12]. However, an EU in femtocell networks tends to suffer from low throughput due to the limited capacity of backhaul connection to legacy cellular networks. Therefore, allowing the EUs to simultaneously access different home BSs in femtocell networks becomes a natural solution to aggregate sum rates of different backhaul links and thus avoid traffic bottleneck [12] [29]. For simplicity, we also refer to this multiple home BSs access scheme in femtocell networks as multi-AP based scheme.

In spite of the potentials of multi-AP based scheme, the resource allocation problem in such scenarios is challenging. In particular, how the resources at APs can be used efficiently while satisfying the rate requirements from EUs? On the one hand, each EU can be connected to multiple APs and each AP may also need to serve multiple EUs, making the resource allocation problem (especially distributed implementation) seemingly prohibitively difficult. On the other hand, interference is a severe issue in multi-AP based scheme due to the simultaneous transmission from multiple APs to EUs, which often makes the resource allocation problem non-convex and thus hard to solve [27].

In [29], the authors formulated a general game-based framework for the multi-AP based scheme to develop self-organizing femtocell networks. Specifically, the APs send independently coded information to multiple EUs over orthogonal channels. In such a scenario, they focused on how individual AP independently decides its transmit power over several orthogonal channels, each of which has been allocated to one EU, to maximize its own data rate.

However, there exist several issues in the approach adopted in [29]. The first problem lies in the non-convexity nature of their formulation. And it is well known that this non-convexity issue of maximizing the total achievable rate over the available orthogonal channels in multi-user scenario makes the problem generally NP-hard [39]. Thus, globally optimal performance cannot be obtained by [29]. Moreover, [29] assumed an EU simply treats all the other APs' signals as noises when decoding a particular AP's signal. In spite of the low complexity, the achievable rate region by this decoding scheme is a strict subset of the capacity region of multiple access channel (MAC) [16]. This further degrades the achievable date rate of the framework proposed in [29].

In contrast, as it will become clear later, we adopt a reverse approach in this chapter which avoids the difficulty caused by the non-convexity. In particular, unlike [29], we study the power allocation problem from the EUs' point of view, though we also adopt a game-based approach. In our model, each EU individually decides its flow rate distribution from different APs to minimize the total power it consumes while guaranteeing its own quality-of-service (QoS) requirement (in terms of aggregate flow rate). Note that EUs need to competitively access the power resources of APs, resulting in a coupled strategy space among EUs. Furthermore, rather than treating the interference from other APs as noises when decoding some AP's signal, we adopt successive interference cancellation (SIC) at the EUs (decoders) to avoid performance loss (or equivalently to achieve the capacity) [66]. Practical implementation scheme of SIC in MAC has already been proposed (see, e.g., [72]). Besides, to make our approach more generally applicable, we impose additional interference temperature constraints. These constraints are of practical interests. For example, they can be applied in cellular networks so that the wireless signals generated in a particular cell would not cause too much interference to adjacent cells. Also, interference temperature constraints are commonly adopted by cognitive radio (CR) networks [10]. For convenience, we will refer to these interference temperature constraints as they are applied in CR networks in this chapter.

Interestingly, this reverse approach helps us investigate the resource allocation problem in question as a convex one. As a result, we are able to show that the Nash equilibrium (NE) of our formulated resource allocation game not only exists but also is unique. What is more, the unique NE in our formulation also turns out to be the globally optimal solution though NE is often known as an inefficient operating point [4]. Hence, the two proposed distributed algorithms can converge not only to the unique NE but in fact a globally optimal solution.

The rest of this chapter is organized as follows. Section 4.1 describes the specific system model and presents a game formulation for the problem in question. In section 4.2, we characterize the relevant properties of NE in the studied game. We propose two distributed algorithms which can satisfy different requirements in real implementation in section 4.3. Section 4.4 presents extensive numerical results for performance evaluation.

4.1 System Model and Problem Formulation

4.1.1 System Model

We consider a radio network as shown in Fig.4.1, where N APs denoted by $\mathbb{N} = \{1, 2, ..., N\}$ simultaneously transmit information to I EUs denoted by $\mathbb{I} = \{1, 2, ..., I\}$ over I orthogonal channels. Each EU receives signals over its pre-assigned channel which is orthogonal to other EUs' channels. We assume all the channels have equal bandwidth W but note that extension of our work to unequal bandwidth case is straightforward. Without loss of generality, we assume channel i is assigned to EU i. Besides, there are K monitoring devices (MDs) denoted by $\mathbb{K} = \{1, 2, ..., K\}$, which regulates the interference caused by transmission from APs to EUs. The maximum interference level that MS k can tolerate over channel i is denoted by $T_{k,i}$.

We denote by g_i^n the channel power gain from AP n to EU i, and $g_{k,i}^n$ the channel power gain from AP n to MD k over channel i. The power AP n allocates for transmission to EU i is denoted by p_i^n. AP n has a total power constraint $\bar{p}^n$, i.e., $\sum_{i\in\mathbb{I}} p_i^n \leq \bar{p}^n$. Besides, due to the interference temperature constraints regulated by MDs, the transmit powers of APs also need to satisfy $\sum_{n\in\mathbb{N}} g_{k,i}^n p_i^n \leq T_{k,i}, \forall i \in \mathbb{I}, \forall k \in \mathbb{K}$.

Unlike most previous works (see, e.g., [77], [58], [74]) which applied low complexity decoding scheme that simply treats other signals as noises when decoding a particular signal, we adopt SIC at the EUs to achieve the capacity of Gaussian multiple access channels. Besides, it is assumed that the zero-mean additive white Gaussian noise (AWGN) at EU i has spectral density N_i.

As for the availability of channel state information (CSI), network nodes can obtain their desired information through training sequences and channel feedback. In particular, EU i is assumed to be aware of its local CSI, i.e., g_i^n, $\forall n \in \mathbb{N}$, by measuring the received power of the training sequences, and $g_{k,i}^n$, $\forall n \in \mathbb{N}, \forall k \in \mathbb{K}$, by feedback from MDs. However, EU i may or may not know other EUs' CSI.

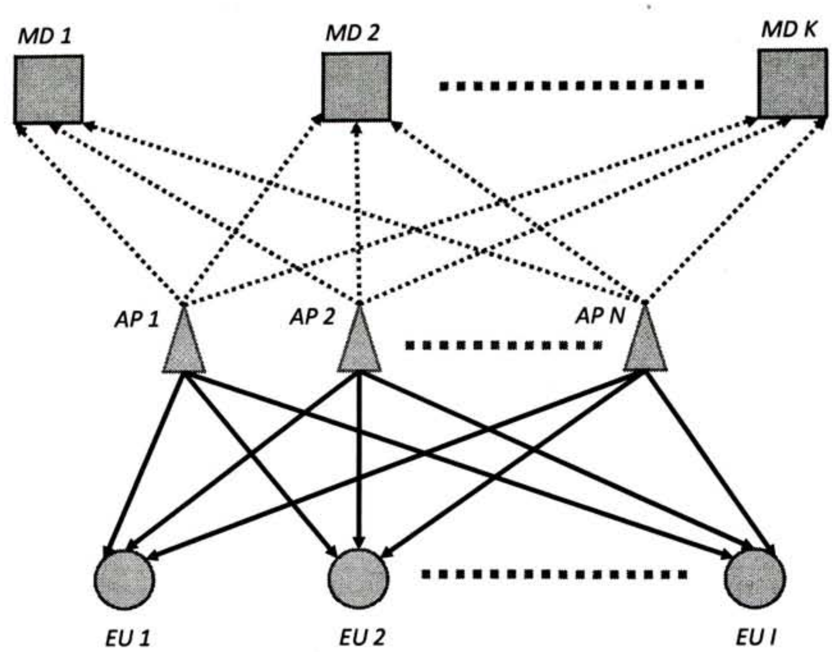

Figure 4.1: System Model

4.1.2 Problem Formulation

Before we formally formulate our problem, we define for EU i the following self-mapping function

$$\pi_i : \{1, 2, ..., N\} \rightarrow \{1, 2, ..., N\} \tag{4.1}$$

such that $g_i^{\pi_i(1)} > g_i^{\pi_i(2)} > ... > g_i^{\pi_i(N)}$. In other words, "$\pi_i(n) = m$" implies that the channel power gain between EU i and AP m is the n-th largest of all the links connecting EU i to APs. We denote by $\mathcal{S}_i = \frac{p_i}{N_i W}$ the transmit signal-to-noise ratio (SNR) of EU i where $p_i = \sum_{n \in \mathbb{N}} p_i^n$ is the total power of APs use for EU i's information flow. We further denote by $R_i^{\pi_i(n)}$ the information flow rate from AP $\pi_i(n)$ to EU i. We then group the information flow rates of EU i into a column vector $\boldsymbol{R_i}$, i.e., $\boldsymbol{R_i} = [R_i^{\pi_i(1)}, R_i^{\pi_i(2)}, ..., R_i^{\pi_i(N)}]^T$. Then we have the following lemma.

Lemma 4.1 *Given the information flow rate vector $\boldsymbol{R_i}$, the minimum required $\mathcal{S}_i$ is given by*

$$\mathcal{S}_i(\boldsymbol{R_i}) = \sum_{n \in \mathbb{N}} S_i^{\pi_i(n)}(\boldsymbol{R_i}) = \sum_{n \in \mathbb{N}} \frac{1}{g_i^{\pi_i(n)}} \cdot \exp\left(\sum_{m=n+1}^{N} R_i^{\pi_i(m)}\right) \cdot \left(\exp(R_i^{\pi_i(n)}) - 1\right), \tag{4.2}$$

where $S_i^{\pi_i(n)}$ is the transmit SNR of the link between AP $\pi_i(n)$ and EU i.

Lemma 1 can be derived by studying the capacity region $\mathcal{C}$ of Gaussian multiple access channels. In particular, $\mathcal{C}$ can be characterized by [16]:

$$\mathcal{C} = \{\boldsymbol{R_i} : \sum_{n\in\mathbb{A}} R_i^n \leq \ln(1 + \sum_{n\in\mathbb{A}} g_i^n S_i^n), \forall \mathbb{A} \subset \mathbb{N}\}. \tag{4.3}$$

To obtain the minimum required $\mathcal{S}_i$ for rate vector $\boldsymbol{R_i}$, we can consider a linear programming (LP) problem:

$$\begin{aligned} \text{minimize} \quad & \sum_{n\in\mathbb{N}} S_i^n \\ \text{subject to} \quad & \sum_{n\in\mathbb{A}} g_i^n S_i^n + 1 - \exp(\sum_{n\in\mathbb{A}} R_i^n) \geq 0, \forall \mathbb{A} \subset \mathbb{N} \\ & S_i^n \geq 0, \forall n \in \mathbb{N}. \end{aligned} \tag{4.4}$$

Then we can show that the optimal value of the above LP problem is given by Lemma 1. For a more detail derivation of Lemma 1, we refer interested readers to [65].

Lemma 1 implies that EU i would require more power resource from APs that have better links to EU i. Clearly, this flexible multi-AP based scheme allows the power resource to be used more efficiently by taking advantage of the multi-user diversity in the networks.

Now we model EU i's QoS requirement in terms of minimum rate requirement R_i^{min}. In particular, EU i's total flow rate R_i should satisfy

$$R_i = \sum_{n\in\mathbb{N}} R_i^{\pi_i(n)} = \sum_{n\in\mathbb{N}} [\mathcal{S}_i^{(-1)}(\boldsymbol{R_i})]_n \geq R_i^{min}, \tag{4.5}$$

where $\mathcal{S}_i^{(-1)}$ denotes the inverse function of $\mathcal{S}_i(\boldsymbol{R_i})$ defined in (4.2), i.e., $\mathcal{S}_i^{(-1)}(\boldsymbol{R_i}) = \boldsymbol{R_i}$, and $[\mathcal{S}_i(\boldsymbol{R_i})]_n$ denotes the n-th coordinate of vector $\mathcal{S}_i^{(-1)}(\boldsymbol{R_i})$, i.e., $[\mathcal{S}_i(\boldsymbol{R_i})]_n = R_i^{\pi_i(n)}$.

To appreciate the EUs' flow distributions, let us now consider a special scenario, where only one EU exists and interference constraints are relaxed. Under this setting, we have the following proposition.

Proposition 4.1 *Suppose only EU i exists and interference constraints are relaxed in the network. Without loss of generality, let n_0 be the index such that $R_i^{\pi_i(n_0)*}$ is the first zero element in a rate vector $\boldsymbol{R_i^*}$. If $\boldsymbol{R_i^*}$ can satisfy EU i's QoS, i.e., $\sum_{n\in\mathbb{N}} R_i^{\pi_i(n)*} \geq R_i^{min}$, then $\boldsymbol{R_i^*}$ minimizes its required transmit SNR $\mathcal{S}_i$ (c.f. equation (4.2)) if and only if*

$$R_i^{\pi_i(n)*} = \begin{cases} \ln(1 + \frac{g_i^{\pi_i(n)} \bar{p}^{\pi_i(n)}}{N_i W}) & \text{if } n < n_0 - 1; \\ R_i^{min} - \sum_{n<n_0-1} R_i^{\pi_i(n)*} & \text{if } n = n_0 - 1; \\ 0 & \text{if } n > n_0 - 1, \end{cases}$$

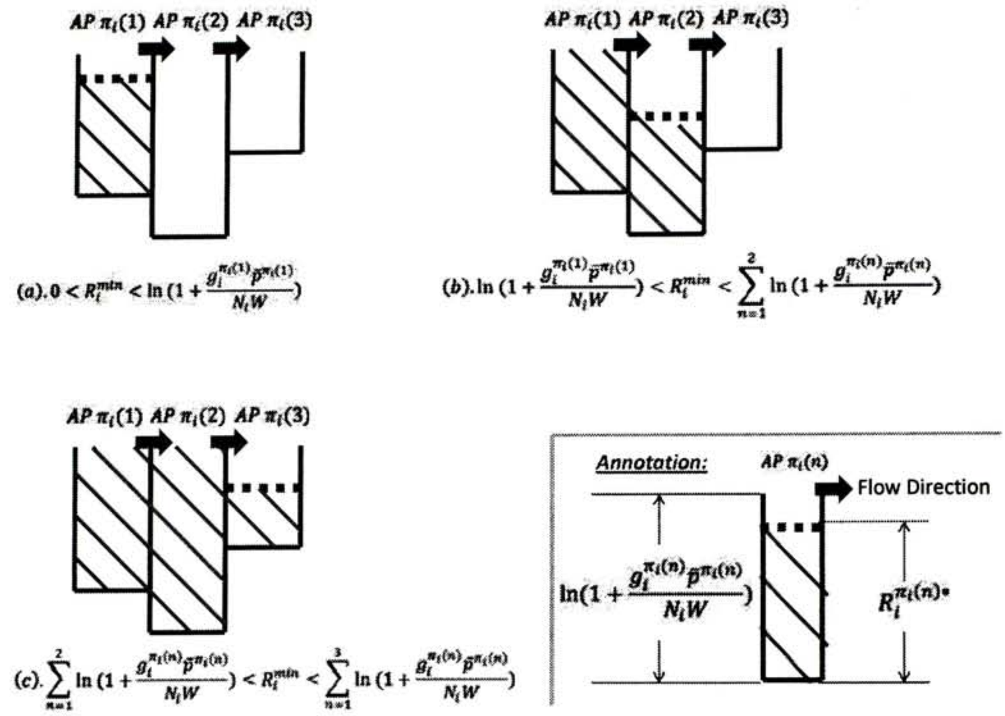

Figure 4.2: Illustration of Pseudo-Waterfilling Strategy

where $R_i^{min} - \sum_{n<n_0-1} R_i^{\pi_i(n)*} \leq \ln(1 + \frac{g_i^{\pi_i(n_0-1)}\bar{p}^{\pi_i(n_0-1)}}{N_i W})$.

Proof See Appendix C.

Proposition 3.1 describes how an EU should allocate its flow requirement over different APs. Roughly speaking, an EU always first tries to seek flows from the AP whose link to that EU is the best for the moment. If the best AP fails to satisfy that EU's QoS requirement, it continues seeking flows from its second best AP. This process continues until the EU's QoS is satisfied. Interestingly, this process resembles (but is not the same as) the well known waterfilling power allocation strategy [66]. Hence, we regard it as a pseudo-waterfilling strategy. We further illustrate this pseudo-waterfilling process in Fig.4.2, as well as by numerical results provided in Section 4.4.

Nevertheless, due to the limited power resource of each AP and interference constraints, EUs compete with each other to meet their respective minimum rate requirements. Thus, Proposition 1 is not sufficient to characterize collective behaviors of EUs. Instead, we further resort to game theory to analyze the competitive behaviors between EUs. Toward this end, EU i is associated with a utility function given by

$$J_i(\boldsymbol{R_i}) = -\mathcal{S}_i(\boldsymbol{R_i}), \forall i \in \mathbb{I}. \tag{4.6}$$

Note that EU i with utility function $J_i(\boldsymbol{R_i})$ aims to minimize its total required transmit SNR from APs. This formulation is not directly equivalent to minimizing the power resource EU

i consumes. Indeed, the total power resource p_i required by EU i equals $N_iW\mathcal{S}_i$. Hence, our formulation can be viewed as a weighted power minimization problem, where EU $i \in \mathbb{I}$ is weighted by $\frac{1}{N_iW}$.

The strategy set of EU i is given by

$$\boldsymbol{\Phi_i}(\boldsymbol{R_{-i}}) = \{\boldsymbol{R_i} \in \mathcal{R}_+^N : \sum_{n\in\mathbb{N}} R_i^n \geq R_i^{min}, \sum_{n\in\mathbb{N}} g_{k,i}^n N_iWS_i^n(\boldsymbol{R_i}) \leq T_{k,i}, \forall k \in \mathbb{K}, \\ N_iWS_i^n(\boldsymbol{R_i}) + \sum_{j\neq i, j\in\mathbb{I}} N_jWS_j^n(\boldsymbol{R_j}) \leq \bar{p}^n, \forall n \in \mathbb{N}\}, \tag{4.7}$$

where $\mathcal{R}_+^N$ denotes the non-negative orthant of N-dimensional Euclidean space, $\boldsymbol{R_{-i}}$ represents the rate allocation of all EUs except EU i. In (4.7), the first constraint implies the minimum rate required by EU i, the second set of constraints denotes the maximum interference levels regulated by each MD over every channel, and the last set of constraints imposes a total power constraint to each AP. For later use, we also define the set value mapping

$$\boldsymbol{\Phi}(\boldsymbol{R}) = \prod_{i\in\mathbb{I}} \boldsymbol{\Phi_i}(\boldsymbol{R_{-i}}), \tag{4.8}$$

and the global flow rate allocation strategy set

$$\boldsymbol{\Phi} = \{\boldsymbol{R} = [\boldsymbol{R_1}, \boldsymbol{R_2}, ..., \boldsymbol{R_N}] : \sum_{n\in\mathbb{N}} R_i^n \geq R_i^{min}, \forall i \in \mathbb{I}, \\ \sum_{n\in\mathbb{N}} g_{k,i}^n N_iWS_i^n(\boldsymbol{R_i}) \leq T_{k,i}, \forall i \in \mathbb{I}, \forall k \in \mathbb{K}, \sum_{i\in\mathbb{I}} N_iWS_i^n(\boldsymbol{R_i}) \leq \bar{p}^n, \forall n \in \mathbb{N}\}, \tag{4.9}$$

For a particular EU i, given the flow rate vectors of other EUs, i.e., $\boldsymbol{R_{-i}}$, it aims at solving the following optimization problem to decide its own flow rate allocation vector $\boldsymbol{R_i}$:

$$\begin{aligned} &\text{maximize} \;\; J_i(\boldsymbol{R_i}) \\ &\text{subject to} \;\; \boldsymbol{R_i} \in \boldsymbol{\Phi}_i(\boldsymbol{R_{-i}}), \end{aligned} \tag{4.10}$$

whose optimal solution set is denoted by $\mathcal{B}(\boldsymbol{R_{-i}})$, i.e., EU i's best response function.

Now we are in a position to formulate the following non-cooperative game to characterize the interaction between EUs:

$$\mathbb{G} = \{\mathbb{I}, \{\boldsymbol{\Phi_i}(\boldsymbol{R_{-i}})\}_{i\in\mathbb{I}}, \{J_i(\boldsymbol{R_i})\}_{i\in\mathbb{I}}\}, \tag{4.11}$$

where $\mathbb{I}$ is the set of players, i.e., EUs, $\boldsymbol{\Phi_i}(\boldsymbol{R_{-i}})$ is EU i's strategy space, and $J_i(\boldsymbol{R_i})$ is EU i's utility function. Clearly, the concept of Nash equilibrium (NE) plays a fundamental role in characterizing the non-cooperative game $\mathbb{G}$ [46]. In particular, no EU can increase its utility by unilaterally changing its flow rate allocation strategy at an NE state. We formally define the NE in game $\mathbb{G}$ as follows.

Definition 4.1 *Flow rate allocation $\boldsymbol{R}^* = [\boldsymbol{R}_1^*, \boldsymbol{R}_2^*, ..., \boldsymbol{R}_N^*]$ is called Nash equilibrium of the non-cooperative game $\mathbb{G}$ if and only if, for any EU $i \in \mathbb{I}$, the following condition holds:*

$$J_i(\boldsymbol{R}_i^*) \geq J_i(\boldsymbol{R}_i), \forall \boldsymbol{R}_i \in \boldsymbol{\Phi}_i(\boldsymbol{R}_{-i}^*). \tag{4.12}$$

It should be pointed out that game $\mathbb{G}$ differs from many conventional non-cooperative game models where players' utilities couple with each other but strategy spaces are independent (see, e.g., [45]). In contrast, it is interesting to note that players in game $\mathbb{G}$ have coupled strategy spaces but their associated utilities are independent. Therefore, the approach used in [45] cannot be directly applied in this thesis. Instead, we resort to other approaches to tackle game $\mathbb{G}$, especially in distributed algorithms design, as shown in the forthcoming sections.

A careful reader may be concerned with the increasing hardware complexity of EUs' equipments since we formulate the flow allocation game $\mathbb{G}$ from EUs' point of view. We would like to stress that players in game $\mathbb{G}$ are just normative. They may or may not be the real decision makers. Indeed, if a central decision maker exists, it can compute the NE and then implements the NE in the network. Likewise, the APs in our model can also serve as decision makers. So our proposed scheme will not necessarily increase the EUs' hardware complexity. Besides, wireless networks in the future might be user-centric. That is, EUs with enhanced hardware in future wireless networks would be intelligent and cognitive. Then EUs in game $\mathbb{G}$ could be the true decision makers without difficulties even if they were incapable of implementing our proposed scheme for the time being.

4.2 Characterization of NE

The NE state represents a steady state which is central to the understanding of distributed wireless information flow allocation in this thesis. The first question arising in game $\mathbb{G}$ is the existence of NE since lack of such equilibrium implies the instability of a distributed system. Moreover, the uniqueness of NE is also desirable for network operators to predict the distribution of wireless information flow and thus adjust the network parameters accordingly. Besides, it is of great significance to develop (possibly distributed) algorithms to reach the NE from initially non-equilibria states [4]. We will address these issues in this and next section.

4.2.1 Feasibility Assumption

Note that the strategy space $\boldsymbol{\Phi}$ may be empty. As an extreme case, MDs set $T_{k,i} = 0$, $\forall i \in \mathbb{I}, \forall k \in \mathbb{K}$. Then it cannot be guaranteed that EUs can meet their minimum flow rate requirements. Therefore, determining the non-emptiness of strategy space $\boldsymbol{\Phi}$ is also of interest. An admission control scheme aiming at identifying EUs who require infeasible minimum flow rate requirements may also be needed. This feasibility identification problem is interesting and will be our future work. In this thesis, we make the following assumption.

Assumption 4.1 *There exists a feasible information flow rate allocation* $\boldsymbol{R^0} = [\boldsymbol{R^0_1}, \boldsymbol{R^0_2}, ..., \boldsymbol{R^0_N}] \in \boldsymbol{\Phi}$ *such that* $\sum_{n\in\mathbb{N}} g^n_{k,i} N_i W S^n_i(\boldsymbol{R^0_i}) < T_{k,i}, \forall i \in \mathbb{I}, \forall k \in \mathbb{K}$, *and* $\sum_{i\in\mathbb{I}} N_i W S^n_i(\boldsymbol{R^0_i}) < \bar{p}^n, \forall n \in \mathbb{N}$.

We remark that Assumption 1 should not be regarded as a stringent one. In fact, it is just the notable Slater condition with minor modification which is commonly assumed and required in optimization problems [63]. Note that we do not require $\sum_{n\in\mathbb{N}} R^n_i \geq R^{min}_i, \forall i \in \mathbb{I}$, to be inactive at $\boldsymbol{R^0}$ while Slater condition requires all inequality constraints to be inactive at some point. This is not a contradiction since $\sum_{n\in\mathbb{N}} R^n_i \geq R^{min}_i, \forall i \in \mathbb{I}$, can be recast as equality constraints without any performance loss as demonstrated in the following proposition.

Proposition 4.2 *Suppose the NE set* $\mathbb{R}$ *of game* $\mathbb{G}$ *is not empty. Then any* $\boldsymbol{R^*} = [\boldsymbol{R^*_1}, \boldsymbol{R^*_2}, ..., \boldsymbol{R^*_N}] \in \mathbb{R}$ *satisfies that*

$$\sum_{n\in\mathbb{N}} R^{\pi_i(n)*}_i = R^{min}_i, \forall i \in \mathbb{I}. \tag{4.13}$$

In other words, for any EU $i \in \mathbb{I}$, *we can replace the "*$\geq$*" in* $\sum_{n\in\mathbb{N}} R^n_i \geq R^{min}_i$ *by "*$=$*" while retaining the same NE set* $\mathbb{R}$.

Proof For any $\boldsymbol{R^*} \in \mathbb{R}$, suppose there exists some $i \in \mathbb{I}$ such that $\sum_{n\in\mathbb{N}} R^{\pi_i(n)*}_i > R^{min}_i$. Given $\boldsymbol{R^*_{-i}}$, it can be readily checked that EU i's utility $J_i(\boldsymbol{R_i})$ is a strictly monotone decreasing function of $\boldsymbol{R_i}$. Thus if $\sum_{n\in\mathbb{N}} R^{\pi_i(n)*}_i > R^{min}_i$, EU i can choose another feasible flow rate allocation strategy $\boldsymbol{R^\dagger_i}$ such that

$$\boldsymbol{R^\dagger_i} \preceq \boldsymbol{R^*_i}, \boldsymbol{R^\dagger_i} \neq \boldsymbol{R^*_i}, \text{and} \sum_{n\in\mathbb{N}} R^{\pi_i(n)\dagger}_i \geq R^{min}_i, \tag{4.14}$$

with the corresponding utility $J_i(\boldsymbol{R^\dagger_i}) > J_i(\boldsymbol{R^*_i})$. This contradicts the assumption that $\boldsymbol{R^*}$ is an NE. In fact, we can repeat this argument until (4.13) holds. This completes the proof.

4.2.2 Existence and Uniqueness of NE

In this subsection, we investigate the existence and uniqueness of NE in game $\mathbb{G}$. Before we state the main results in this subsection, we remark that, though the existence of NE in a game can often be readily verified, it is generally more involved to establish the uniqueness of NE. In fact, many realistic models do not possess the uniqueness property, motivating the investigation of the sufficient conditions for the uniqueness of NE case by case [4]. Nevertheless, by identifying game $\mathbb{G}$ as a best-response potential game, we show that a unique NE exists in game $\mathbb{G}$ [71]. We summarize this result in the following proposition.

Proposition 4.3 *Game* $\mathbb{G} = \{\mathbb{I}, \{\boldsymbol{\Phi}_i(\boldsymbol{R}_{-i})\}_{i\in\mathbb{I}}, \{J_i(\boldsymbol{R}_i)\}_{i\in\mathbb{I}}\}$ *possesses a unique NE* $\boldsymbol{R}^*$.

Proof We first claim that the function $\mathcal{P}(\boldsymbol{R}) : \boldsymbol{\Phi} \to \mathcal{R}$ given by

$$\mathcal{P}(\boldsymbol{R}) = \sum_{j\in\mathbb{I}} J_j(\boldsymbol{R}_j) \tag{4.15}$$

is a potential function of game $\mathbb{G}$. Indeed, we have

$$\begin{aligned} arg \max_{\boldsymbol{R}_i\in\boldsymbol{\Phi}_i(\boldsymbol{R}_{-i})} J_i(\boldsymbol{R}_i) =& arg \max_{\boldsymbol{R}_i\in\boldsymbol{\Phi}_i(\boldsymbol{R}_{-i})} -\mathcal{S}_i(\boldsymbol{R}_i) \\ =& arg \max_{\boldsymbol{R}_i\in\boldsymbol{\Phi}_i(\boldsymbol{R}_{-i})} (-\mathcal{S}_i(\boldsymbol{R}_i) - \sum_{j\neq i, j\in\mathbb{I}} \mathcal{S}_j(\boldsymbol{R}_j)) \\ =& arg \max_{\boldsymbol{R}_i\in\boldsymbol{\Phi}_i(\boldsymbol{R}_{-i})} \sum_{j\in\mathbb{I}} J_j(\boldsymbol{R}_j) \\ =& arg \max_{\boldsymbol{R}_i\in\boldsymbol{\Phi}_i(\boldsymbol{R}_{-i})} \mathcal{P}(\boldsymbol{R}_i, \boldsymbol{R}_{-i}), \forall \boldsymbol{R}_{-i} \in \boldsymbol{\Phi}_{-i}(\boldsymbol{R}_i), \end{aligned} \tag{4.16}$$

where $\boldsymbol{\Phi}_{-i}(\boldsymbol{R}_i)$ denotes the global strategy space given EU i's flow rate allocation $\boldsymbol{R}_i$. Since (4.16) holds for any EU $i \in \mathbb{I}$, game $\mathbb{G}$ by definition is a best-response potential game [71]. In other words, we have an associated coordination game $\hat{\mathbb{G}} = \{\mathbb{I}, \{\boldsymbol{\Phi}_i(\boldsymbol{R}_{-i})\}_{i\in\mathbb{I}}, \{\mathcal{P}(\boldsymbol{R}_i, \boldsymbol{R}_{-i})\}_{i\in\mathbb{I}}\}$, where all players share a common utility $\mathcal{P}(\boldsymbol{R})$ such that the best response $\mathcal{B}_i(\boldsymbol{R}_{-i})$ of each player $i \in \mathbb{I}$ in game $\mathbb{G}$ is the same as its best response in game $\hat{\mathbb{G}}$.

We denote by $\hat{\mathbb{R}}$ the set of maxima of $\mathcal{P}(\boldsymbol{R})$ on the domain $\boldsymbol{\Phi}$ which is nonempty by Assumption 1. Since $\mathcal{P}(\boldsymbol{R})$ is a real-valued continuous function on a nonempty compact (i.e., closed and bounded) set $\boldsymbol{\Phi}$, $\hat{\mathbb{R}}$ is always nonempty according to the Weierstrass Theorem [63]. Besides, $\boldsymbol{\Phi}$ is a convex set, and $\mathcal{P}(\boldsymbol{R})$ is continuously differentiable on the interior of $\boldsymbol{\Phi}$ and strictly concave on $\boldsymbol{\Phi}$. Then, based on Theorem 3 in [55], we conclude that the NE of game $\mathbb{G}$ is unique.

Note that NE in general is inefficient and the price of anarchy can even be unbounded (see, e.g., [17]). Nevertheless, from the proof of Proposition 3, it is interesting to note that the unique NE $\boldsymbol{R}^*$ in fact maximizes $\sum_{i\in\mathbb{I}} J_i(\boldsymbol{R}_i)$. This is a very desirable result which implies that the social optimum can be obtained if we can find a scheme to reach the unique NE $\boldsymbol{R}^*$ by playing game $\mathbb{G}$, which is the very topic of the next section.

4.3 Distributed Algorithms Design

In this section, we propose two distributed algorithms for EUs to reach NE along with the corresponding theoretical convergence analysis. The design idea of the first algorithm is directly based on sequential best-response path (referred to as D-SBRA) [30]. However, as shown later, D-SBRA may be inconvenient due to its inherent drawbacks. Hence, we propose another algorithm to resolve the difficulties in D-SBRA by further resorting to partial dual decomposition (referred to as P-SBRA) [49]. Nevertheless, it will become clear later that P-SBRA requires the participation of APs while interaction only occurs among EUs in D-SBRA. Therefore, network operators can choose either D-SBRA or P-SBRA in practical implementation according to specific network situations.

4.3.1 D-SBRA

To begin with, we recite the relevant concepts of sequential best-response path from [30]. In particular, a sequence $(\boldsymbol{R}^t)_{t=0}^{\infty}$ in strategy space $\boldsymbol{\Phi}$ is a sequential best-response path if EUs response one by one according to best-response strategy. A sequential best-response path is admissible if all I EUs have taken their best-response strategies at least once whenever I successive periods have passed. We now state the favorable convergent property of game $\mathbb{G}$ in the following proposition which also justifies the convergence of D-SBRA.

Proposition 4.4 *Every admissible sequential best-response path played in game $\mathbb{G}$ converges to the unique NE $\boldsymbol{R}^*$.*

Proof We need the following lemma that follows directly from Theorem 2 in [30].

Lemma 4.2 *If game $\mathbb{G}$ has continuous best-response functions, compact strategy sets, and a unique NE $\boldsymbol{R}^*$, $\mathbb{G}$ is a best-response potential game if and only if every admissible sequential best-response path converges to $\boldsymbol{R}^*$.*

Clearly, best-response function $\mathcal{B}_i(\boldsymbol{R}_{-i}), \forall i \in \mathbb{I}$, is continuous on the strategy set $\boldsymbol{\Phi}_i(\boldsymbol{R}_{-i})$. Besides, $\boldsymbol{\Phi}_i(\boldsymbol{R}_{-i}), \forall i \in \mathbb{I}$, is a compact strategy set. Note that we have shown the existence and uniqueness of NE in game $\mathbb{G}$ in Proposition 3.3. Hence, Proposition 3.4 follows by Lemma 3.2. This completes the proof.

Proposition 4 implies that there exist infinite ways to reach the NE in game $\mathbb{G}$ as long as the sequential best-response path is admissible. However, we only provide an ordered-version for D-SBRA here due to limited space. We formally summarize D-SBRA for the distributed wireless information flow allocation problem in Tabel 4.1.

Note that each EU needs to know the global CSI and other EUs' strategies in each response to compute its own strategy space and obtain best response in D-SBRA. This

Step 1: Initialization:
EUs exchange channel power gain information and start with an arbitrarily feasible flow rate allocation, i.e., $\boldsymbol{R}(0) \in \boldsymbol{\Phi}$. Set $t := 0; i := 1$.

Step 2: Computation:
EU i calculates its strategy space:

$$\boldsymbol{\Phi_i}(t) = \boldsymbol{\Phi_i}(\boldsymbol{R_1}(t+1), ..., \boldsymbol{R_{i-1}}(t+1), \boldsymbol{R_{i+1}}(t), ..., \boldsymbol{R_I}(t)). \tag{4.17}$$

EU i computes its current flow rate allocation $\boldsymbol{R_i}(t+1)$ by solving the following best response problem:

$$\boldsymbol{R_i}(t+1) = arg \max_{\boldsymbol{R_i} \in \boldsymbol{\Phi}_i(t)} J_i(\boldsymbol{R_i}). \tag{4.18}$$

EU i broadcasts its new flow rate allocation $\boldsymbol{R_i}(t+1)$. Set $i := i+1$. If $i \leq I$, go to Step 2; otherwise, go to Step 3.

Step 3: Convergence Verification:
If stopping criteria are satisfied, then stop; otherwise, set $t := t+1; i := 1$, and go to Step 2.

Table 4.1: Detail steps of D-SBRA

requirement may cause unacceptable level of communication overhead for some wireless networks. Worse still, some EUs may take advantage of other EUs by telling false information. In this scenario, a mechanism guaranteeing truth-telling is the dominant strategy for every EU may be required to achieve the global optimum. Therefore, we propose P-SBRA to overcome these drawbacks of D-SBRA.

4.3.2 P-SBRA

Recall that EU i's strategy space $\boldsymbol{\Phi_i}(\boldsymbol{R_{-i}})$ are constrained in two aspects. In particular, $\boldsymbol{R_i}$ is constrained by both individual constraints that are independent of other EUs' strategies, i.e., the first two set of constraints in (4.7), and coupling constraints that depend on other EUs' strategies, i.e., the last set of constraints in (4.7). The main cause for the inconveniences in D-SBRA comes from the coupling issue among EUs' strategy spaces. To resolve this problem, we relax the coupling constraints by resorting to partial dual decomposition approach [49].

To begin with, let us introduce some notations for ease of exposition. In particular, we denote $\boldsymbol{\Psi} = \prod_{i \in \mathbb{I}} \boldsymbol{\Psi_i}$, where $\boldsymbol{\Psi_i}$ is EU i's own independent strategy space given by

$$\boldsymbol{\Psi_i} = \{\boldsymbol{R_i} \in \mathcal{R}_+^N : \sum_{n \in \mathbb{N}} R_i^n \geq R_i^{min}, \sum_{n \in \mathbb{N}} g_{k,i}^n N_i W S_i^n(\boldsymbol{R_i}) \leq T_{k,i}, \forall k \in \mathbb{K}\}. \tag{4.19}$$

We further define

$$f^l(\boldsymbol{R}) = \sum_{i\in\mathbb{I}} N_i W S_i^l(\boldsymbol{R_i}) - \bar{p}^l = \sum_{i\in\mathbb{I}} \sum_{n\in\mathbb{N}:\pi_i(n)=l} N_i W S_i^{\pi_i(n)}(\boldsymbol{R_i}) - \bar{p}^l, \forall l \in \mathbb{N}. \tag{4.20}$$

We group all the $f^l(\boldsymbol{R})$ into a column vector $\boldsymbol{f}(\boldsymbol{R})$, i.e., $\boldsymbol{f}(\boldsymbol{R}) = [f^1(\boldsymbol{R}), f^2(\boldsymbol{R}), ..., f^N(\boldsymbol{R})]^T$.

Now let us introduce the Nash game (NG) (denoted by $\mathbb{G}^\dagger$) cost function $\mathcal{U} : \boldsymbol{\Psi} \times \boldsymbol{\Psi} \rightarrow \Re$ defined as [49]:

$$\mathcal{U}(\boldsymbol{R}; \boldsymbol{x}) = \sum_{i\in\mathbb{I}} \mathcal{U}_i(\boldsymbol{R_{-i}}, \boldsymbol{x_i}) = \sum_{i\in\mathbb{I}} J_i(\boldsymbol{x_i}). \tag{4.21}$$

Here note that we write $\mathcal{U}_i$ as a function of $\boldsymbol{R_{-i}}$ and $\boldsymbol{x_i}$ for consistency though it in fact does not depend on $\boldsymbol{R_{-i}}$. Since a unique NE $\boldsymbol{R}^*$ exists in game $\mathbb{G}$ by Proposition 3.3, based on Lemma 1 in [49], $\boldsymbol{R}^*$ satisfies

$$\mathcal{U}(\boldsymbol{R}^*; \boldsymbol{R}^*) \geq \mathcal{U}(\boldsymbol{R}^*; \boldsymbol{x}), \forall \boldsymbol{x} \in \boldsymbol{\Psi}, \text{ and } f^l(\boldsymbol{R}^*_{-i}, \boldsymbol{x_i}) \leq 0, \forall l \in \mathbb{N}. \tag{4.22}$$

Construct the following constrained optimization problem

$$\begin{aligned} \text{minimize } & \mathcal{U}(\boldsymbol{R}^*; \boldsymbol{x}) \\ \text{subject to } & \tilde{f}^l(\boldsymbol{R}^*; \boldsymbol{x}) \leq 0, \forall l \in \mathbb{N}, \\ & \boldsymbol{x} \in \boldsymbol{\Psi}, \end{aligned} \tag{4.23}$$

where $\tilde{f}^l(\boldsymbol{R}^*; \boldsymbol{x}) = \sum_{i\in\mathbb{I}} f^l(\boldsymbol{R}^*_{-i}, \boldsymbol{x_i})$. Then it is clear that $\boldsymbol{R}^*$ is a solution to (4.23) with optimal NG cost $\mathcal{U}^* = \mathcal{U}(\boldsymbol{R}^*; \boldsymbol{R}^*)$. That is, we can solve (4.23) to obtain the NE of game $\mathbb{G}$. Toward this end, we relax the coupled constraints and obtain the corresponding partial Lagrangian function given by

$$\begin{aligned} \mathcal{L}(\boldsymbol{R}^*; \boldsymbol{x}; \boldsymbol{\lambda}) =& \mathcal{U}(\boldsymbol{R}^*; \boldsymbol{x}) - \boldsymbol{\lambda}^T \tilde{\boldsymbol{f}}(\boldsymbol{R}^*; \boldsymbol{x}) \\ =& \sum_{i\in\mathbb{I}} \mathcal{U}_i(\boldsymbol{R}^*_{-i}, \boldsymbol{x}_i) - \sum_{l\in\mathbb{N}} \lambda_l \tilde{f}^l(\boldsymbol{R}^*; \boldsymbol{x}) \\ =& \sum_{i\in\mathbb{I}} \mathcal{U}_i(\boldsymbol{R}^*_{-i}, \boldsymbol{x}_i) - \sum_{l\in\mathbb{N}} \lambda_l \sum_{i\in\mathbb{I}} f^l(\boldsymbol{R}^*_{-i}, \boldsymbol{x}_i), \end{aligned} \tag{4.24}$$

where $\tilde{\boldsymbol{f}}(\boldsymbol{R}^*; \boldsymbol{x}) = [\tilde{f}^1(\boldsymbol{R}^*; \boldsymbol{x}), \tilde{f}^2(\boldsymbol{R}^*; \boldsymbol{x}), ..., \tilde{f}^N(\boldsymbol{R}^*; \boldsymbol{x})]^T$, and $\boldsymbol{\lambda}$ is the corresponding Lagrangian multiplier vector. Then the following proposition that follows directly from Theorem 2 in [49] comes into handy.

Proposition 4.5 $\boldsymbol{R}^*$ *is the NE of game* $\mathbb{G}$ *if only if there exists corresponding dual variable*

$\boldsymbol{\lambda}^*$ *such that*

$$\boldsymbol{R}^* = arg\{[\max_{\boldsymbol{x}\in\boldsymbol{\Psi}} \mathcal{U}(\boldsymbol{R};\boldsymbol{x};\boldsymbol{\lambda}^*)]|_{\boldsymbol{x}=\boldsymbol{R}}\}; \tag{4.25}$$

$$\boldsymbol{R}^* \in \boldsymbol{\Psi}, \tilde{\boldsymbol{f}}(\boldsymbol{R}^*;\boldsymbol{R}^*) \preceq \boldsymbol{0}; \tag{4.26}$$

$$\lambda_l \tilde{f}^l(\boldsymbol{R}^*;\boldsymbol{R}^*) = 0, \forall l \in \mathbb{N}; \tag{4.27}$$

$$\lambda_l \geq 0, \forall l \in \mathbb{N}. \tag{4.28}$$

We then resort to a decomposition approach to solve the above set of equations to achieve the NE $\boldsymbol{R}^*$. Consider the associated dual function given by

$$\begin{aligned} g(\boldsymbol{\lambda}) &= [\max_{\boldsymbol{x}\in\boldsymbol{\Psi}} \quad \mathcal{L}(\boldsymbol{R};\boldsymbol{x};\boldsymbol{\lambda})]|_{\boldsymbol{x}=\boldsymbol{R}} \\ &= \sum_{i\in\mathbb{I}} \max_{\boldsymbol{x_i}\in\boldsymbol{\Psi}_i} (\mathcal{U}_i(\boldsymbol{R_{-i}},\boldsymbol{x_i}) - \sum_{l\in\mathbb{N}} \lambda_l f^l(\boldsymbol{R_{-i}},\boldsymbol{x_i})) \\ &= \sum_{i\in\mathbb{I}} (J_i(\boldsymbol{R_i^\dagger}) - \sum_{l\in\mathbb{N}} \lambda_l \sum_{n\in\mathbb{N}:\pi_i(n)=l} N_i W S_i^{\pi_i(n)}(\boldsymbol{R_i^\dagger})) + \sum_{i\in\mathbb{I}}\sum_{l\in\mathbb{N}} \lambda_l \bar{p}^l \\ &= \sum_{i\in\mathbb{I}} \mathcal{L}_i(\boldsymbol{R_{-i}^\dagger};\boldsymbol{R_i^\dagger};\boldsymbol{\lambda}) + \sum_{i\in\mathbb{I}}\sum_{l\in\mathbb{N}} \lambda_l \frac{p_{max}^l}{N_0 W}, \end{aligned} \tag{4.29}$$

where $\boldsymbol{R}^\dagger = [\boldsymbol{R_1^\dagger}, \boldsymbol{R_2^\dagger}, ..., \boldsymbol{R_I^\dagger}]$ is an NE of NG $\mathbb{G}^\dagger(\boldsymbol{\lambda})$ with the same player set as game $\mathbb{G}$ but different utility function $\mathcal{L}_i(\boldsymbol{R_{-i}};\boldsymbol{R_i};\boldsymbol{\lambda}) = J_i(\boldsymbol{R_i}) - \sum_{l\in\mathbb{N}} \lambda_l \sum_{n\in\mathbb{N}:\pi_i(n)=l} N_i W S_i^{\pi_i(n)}(\boldsymbol{R_i})$, $\forall i \in \mathbb{I}$, and decoupled strategy space $\boldsymbol{\Psi_i}, \forall i \in \mathbb{I}$. That is, we can solve game $\mathbb{G}^\dagger(\boldsymbol{\lambda})$ to obtain $g(\boldsymbol{\lambda})$. In fact, this new game $\mathbb{G}^\dagger(\boldsymbol{\lambda})$ is trivial since there are no coupling issues in both the utilities and strategy spaces among players. In other words, given $\boldsymbol{\lambda}$, EU $i, \forall i \in \mathbb{I}$, only needs to solve the following convex optimization problem:

$$\mathcal{B}_i^\dagger(\boldsymbol{\lambda}) = arg \max_{\boldsymbol{R_i}\in\boldsymbol{\Psi}_i} \; J_i(\boldsymbol{R_i}) - \sum_{l\in\mathbb{N}} \lambda_l \sum_{n\in\mathbb{N}:\pi_i(n)=l} N_i W S_i^{\pi_i(n)}(\boldsymbol{R_i}). \tag{4.30}$$

Besides, the best-response $\mathcal{B}_i^\dagger(\boldsymbol{\lambda})$ in (4.30) is unique since $\boldsymbol{\Psi}_i$ is a nonempty bounded convex set and $\mathcal{L}_i$ is strictly concave with respect to $\boldsymbol{R_i}$ on $\boldsymbol{\Psi}_i$. Hence, given $\boldsymbol{\lambda}$, as long as all the EUs take their associated best response once, we can obtain the trivial NE $\boldsymbol{R}^\dagger(\boldsymbol{\lambda})$ of game $\mathbb{G}^\dagger(\boldsymbol{\lambda})$.

The next key step is to update $\boldsymbol{\lambda}$ iteratively, making NE $\boldsymbol{R}^\dagger(\boldsymbol{\lambda})$ of game $\mathbb{G}^\dagger(\boldsymbol{\lambda})$ converge to the unique NE $\boldsymbol{R}^*$ of the original game $\mathbb{G}$. Indeed, the dual variable $\boldsymbol{\lambda}$ in the dual problem

$$\min_{\boldsymbol{\lambda}\succeq\boldsymbol{0}} \quad g(\boldsymbol{\lambda}) \tag{4.31}$$

Step 1: Initialization:
EUs start with an arbitrarily feasible flow rate allocation, i.e., $\boldsymbol{R}(0) \in \boldsymbol{\Phi}$. Set $\boldsymbol{\lambda} := \boldsymbol{0}; t := 1$.

Step 2: Computation:
Each EU $i \in \mathbb{I}$ computes its current flow rate allocation $\boldsymbol{R_i}(t+1)$ by solving the following problem:

$$\boldsymbol{R_i}(t+1) = arg \max_{\boldsymbol{R_i} \in \boldsymbol{\Psi}_i} J_i(\boldsymbol{R_i}) - \sum_{l \in \mathbb{N}} \lambda_l(t) \sum_{n \in \mathbb{N}:\pi_i(n)=l} N_i W S_i^{\pi_i(n)}(\boldsymbol{R_i}) \quad (4.33)$$

and obtains the associated required $S_i^{\pi_i(n)}, \forall n \in \mathbb{N}$, according to (4.2).
Each EU $i \in \mathbb{I}$ notifies AP $\pi_i(n), \forall n \in \mathbb{N}$, its required SNR $S_i^{\pi_i(n)}$.

Step 3: Subgradient Update:
Each AP $l \in \mathbb{N}$ updates its dual variable $\lambda_l(t+1)$ as follows:

$$\lambda_l(t+1) = [\lambda_l(t) - \beta(t)(\bar{p}^l - \sum_{i \in \mathbb{I}} N_i W S_i^l)]^+. \quad (4.34)$$

Step 4: Convergence Verification:
If stopping criteria are satisfied, then stop; otherwise, set $t := t+1$, and go to Step 2.

Table 4.2: Detail steps of P-SBRA

can be updated by applying iterative subgradient method, i.e.,

$$\lambda_l(t+1) = [\lambda_l(t) - \beta(t)(\bar{p}^l - \sum_{i \in \mathbb{I}} N_i W S_i^l)]^+, \forall l \in \mathbb{N}, \quad (4.32)$$

where t is the iteration index, $\beta(t)$ is the positive iteration step size, and $[\cdot]^+$ denotes the projection onto the set of non-negative numbers. Now we are in a position to summarize P-SBRA in Table 4.2.

By Proposition 3.5, we note that the iteration process in P-SBRA continues until the following complementary conditions are satisfied:

$$\lambda_l(\bar{p}^l - \sum_{i \in \mathbb{I}} N_i W S_i^l) = 0, \forall l \in \mathbb{N}. \quad (4.35)$$

Besides, the convergence of subgradient updating process in P-SBRA2 can be guaranteed by certain choices of step sizes, such as $\beta(t) = \frac{\beta_0}{t}, \beta_0 > 0$, which satisfies the diminishing step size rule [63].

Furthermore, given the Lagrangian variable $\boldsymbol{\lambda}$, each EU only needs to know its local CSI to carry out its best response in P-SBRA, resulting in less communication overhead than that of D-SBRA. However, this advantage comes with the requirement of the participation of APs. Therefore, D-SBRA is desirable when APs may not be able to play such a coordination role.

Before ending this subsection, we point out that the Lagrangian variable $\boldsymbol{\lambda}$ in P-SBRA has a nice economic interpretation. In fact, we can view $\lambda_l, \forall l \in \mathbb{N}$, as a price that EUs need to pay for the violation of AP l's total power constraint. In particular, if $\sum_{i\in\mathbb{I}} N_i W S_i^l > \bar{p}^l$, then λ_l in the next run will be increased according to (4.32). Then EUs will experience a higher price when asking for flows from AP l and may try to seek flows from other APs. As a result, the violation of AP l's total power constraint may be alleviated in the next run. This dynamic process continues until the unique NE $\boldsymbol{R}^*$ is reached. However, it should be noted that no real payment needs to be carried out in the implementation of P-SBRA.

4.3.3 Best Response and Layered Structure

In this subsection, we study how the best responses (4.18) in D-SBRA and (4.33) in P-SBRA can be obtained. Note that best response problem (4.18) and (4.33) are both convex and thus can be solved very efficiently using standard convex optimization methods (interior point method, for one) [63]. Nevertheless, we here resort to subgradient method to help us further appreciate the implicit layered structure in our studied problem. Since best response problem (4.18) and (4.33) are similar, we only discuss problem (4.33) and the layered structure of P-SBRA here for brevity. D-SBRA can be analyzed in a similar fashion.

To begin with, we denote the objective function in problem (4.33) by $\mathcal{U}_i(\boldsymbol{R_i}, \boldsymbol{\lambda})$, which can be explicitly written as

$$\mathcal{U}_i(\boldsymbol{R_i}, \boldsymbol{\lambda}) = -\sum_{n\in\mathbb{N}}(1+\lambda_{\pi_i(n)}N_iW)S_i^{\pi_i(n)}(\boldsymbol{R_i}), \forall i \in \mathbb{I}. \tag{4.36}$$

Then consider the Lagrangian function for EU i:

$$\begin{aligned}
&\mathcal{L}_i(\boldsymbol{R_i}, \boldsymbol{\lambda}, \boldsymbol{u_i}, v_i) \\
=& -\sum_{n\in\mathbb{N}}(1+\lambda_{\pi_i(n)}N_iW)S_i^{\pi_i(n)}(\boldsymbol{R_i}) + v_i\Big(\sum_{n\in\mathbb{N}} R_i^{\pi_i(n)} - R_i^{min}\Big) \\
&+\sum_{k\in\mathbb{K}} u_{k,i}\Big(T_{k,i} - \sum_{n\in\mathbb{N}} g_{k,i}^{\pi_i(n)} N_i W S_i^{\pi_i(n)}(\boldsymbol{R_i})\Big) \\
=& -\sum_{n\in\mathbb{N}}\Big(1+\lambda_{\pi_i(n)}N_iW + \sum_{k\in\mathbb{K}} u_{k,i} g_{k,i}^{\pi_i(n)} N_i W\Big)S_i^{\pi_i(n)}(\boldsymbol{R_i}) + v_i\sum_{n\in\mathbb{N}} R_i^{\pi_i(n)} + \sum_{k\in\mathbb{K}} u_{k,i}T_{k,i} - v_iR_i^{min},
\end{aligned} \tag{4.37}$$

where $\boldsymbol{u_i} = [u_{1,i}, ..., u_{K,i}]$ and v_i are the associated non-negative Lagrange multipliers. We then resort to subgradient method to obtain the optimal solution $\boldsymbol{R_i^*}$. Toward this end, let us further consider the associated dual function given by

$$g_i(\boldsymbol{u_i}, v_i) = \max_{\boldsymbol{R_i}\succeq\boldsymbol{0}} \ \mathcal{L}_i(\boldsymbol{R_i}, \boldsymbol{\lambda}, \boldsymbol{u_i}, v_i). \tag{4.38}$$

The corresponding dual problem

$$\min_{(\boldsymbol{u_i}, v_i) \succeq \mathbf{0}} \quad g_i(\boldsymbol{u_i}, v_i) \tag{4.39}$$

can be solved via subgradient method. In particular, with initial feasible $(\boldsymbol{u_i}(0), v_i(0))$, the sequence $(\boldsymbol{u_i}(s), v_i(s))_{s=0}^{\infty}$ obtained from the subgradient method is given by

$$u_{k,i}(s+1) = [u_{k,i}(s) - \alpha_i(s)(T_{k,i} - \sum_{n \in \mathbb{N}} g_{k,i}^{\pi_i(n)} N_i W S_i^{\pi_i(n)}(s))]^+, \forall k \in \mathbb{K}, \tag{4.40}$$

$$v_i(s+1) = [v_i(s) - \alpha_i(s)(\sum_{n \in \mathbb{N}} R_i^{\pi_i(n)}(s) - R_i^{min})]^+, \tag{4.41}$$

where s is the iteration index, $\alpha_i(s)$ is the positive iteration step size. Then the primal solution $\boldsymbol{R_i^*}(s)$ during the s-th iteration is given by

$$\boldsymbol{R_i^*}(s) = arg \max_{\boldsymbol{R_i} \succeq \mathbf{0}} \quad \mathcal{L}_i(\boldsymbol{R_i}, \boldsymbol{\lambda}, \boldsymbol{u_i}(s), v_i(s)). \tag{4.42}$$

Now we are in a position to describe the inherent layered structure of P-SBRA as shown in Fig. 4.3. In particular, this layered structure can be viewed as a Stackelberg game with APs being the leaders and EUs being the followers [4]. On the leader side, based on the current SNR requirement $\boldsymbol{S}$ from the EUs, the APs set the corresponding power price $\boldsymbol{\lambda}$ to guarantee that their power resources are not over utilized. Then on the follower side, given current $\boldsymbol{\lambda}$, each EU $i \in \mathbb{I}$ chooses its flow rate allocation vector $\boldsymbol{R_i}$ to maximize its utility $\mathcal{U}_i$.

Note that, given $\boldsymbol{\lambda}$, there also exists internal layered structure of EU $i \in \mathbb{I}$. Specifically, $\boldsymbol{u_i}$ denotes interference price at the physical layer that EU i needs to pay for its violation of the interference constraints set by the MDs. Meanwhile, v_i denotes QoS guarantee price at the transport layer that EU i needs to pay if it cannot satisfy the corresponding QoS request. Given $\boldsymbol{u_i}$ and v_i, EU i adjusts its flow rate allocation $\boldsymbol{R_i}$ accordingly, and vice versa. After all EUs decide their flow rate allocation strategies, the corresponding required SNR $\boldsymbol{S}$ can be fed back to the leaders, i.e., APs. Then APs can update their power price $\boldsymbol{\lambda}$ accordingly, initiating a new round adjustment. This dynamic process continues until convergence or stopping criteria are satisfied.

We remark that the layered structure of P-SBRA differs from those revealed in the context of cross layer optimization, in which layered structure exists only in individual user's protocol stack (see, e.g., [15], [23], and references therein). In particular, the layered structure of P-SBRA includes not only the internal layered structure in individual user's protocol stack but also an external layer due to the coordination of APs.

So far, we can see that SIC plays a fundamental role in our proposed scheme. A careful reader might be concerned with practical issues in applying SIC in wireless networks. Indeed, current receivers generally treat interference as noise though substantial research works have

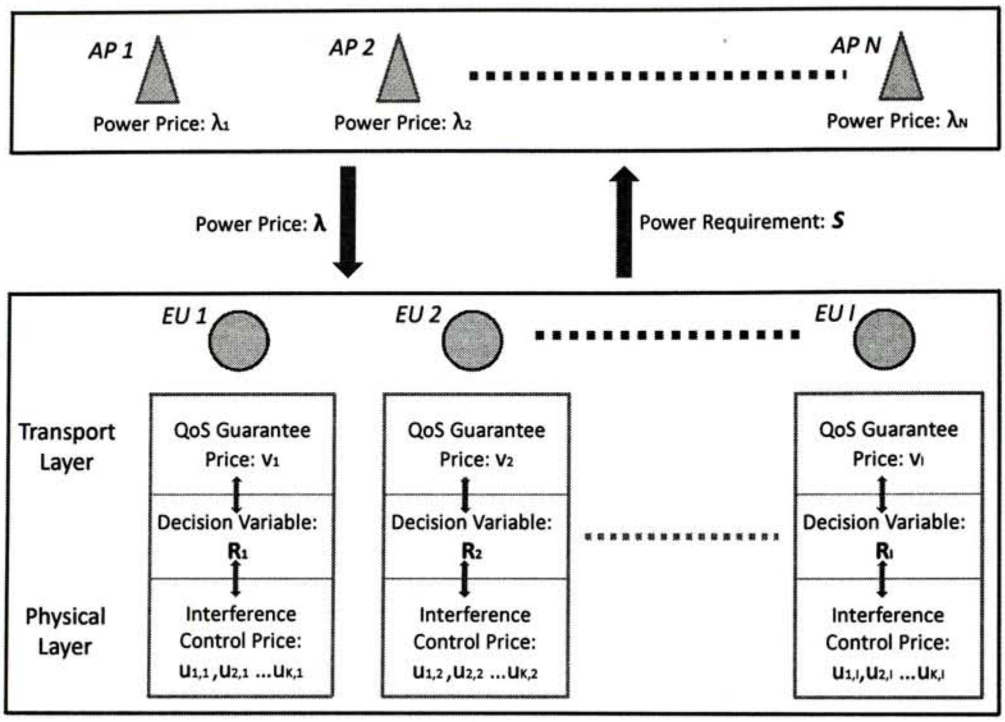

Figure 4.3: Layered Structure of P-SBRA

been carried out on interference-aware receivers. Nevertheless, it is widely accepted that current adopted receiving technique is increasingly suboptimal when the number of interferers grows. Therefore, it is believed that the application of interference cancellation including SIC, which brings dramatic capacity gain (also shown in the next section), will become popular in the future interference-limited wireless networks [6]. Another interesting issue is how inexact SIC affects the performance of our proposed scheme. A satisfactory answer to this question requires a careful modeling of the inexactness of SIC, which is beyond the scope of this thesis. Nevertheless, we are positive about the potentials of SIC. Indeed, [5] shows that, compared to no interference cancellation, SIC still doubles the system capacity even with 50% channel estimation error which causes inexact SIC.

4.4 Performance Evaluation

4.4.1 Protocol Evaluation

We investigate the gain obtained by multi-AP based scheme compared to single-AP based scheme in this subsection. In particular, we use outage probability, which is defined as the probability that the minimum required power[1] by EUs to guarantee the QoS is greater than a given power threshold of APs, as the performance metric [45].

[1]For simplicity, we normalize N_i and W throughout all the simulation results. Thus, our scheme aiming at minimizing the total transmit SNR also minimizes the total power consumption in this scenario.

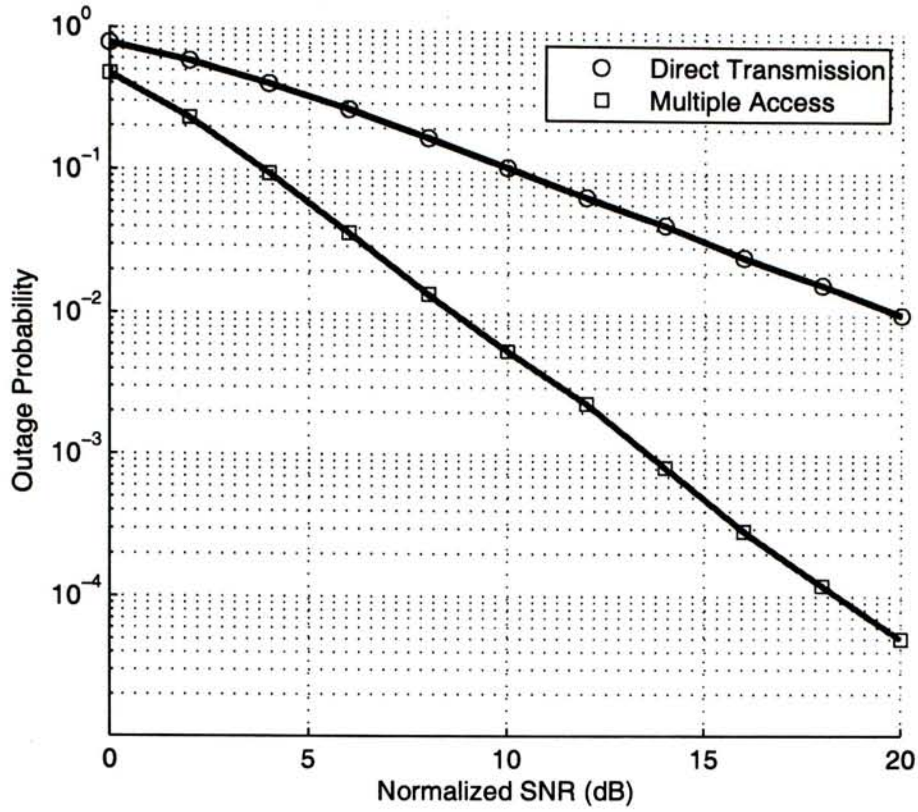

Figure 4.4: Comparison of single-AP and multi-AP schemes: 2-AP 2-EU case

For clarity, we assume a uniform QoS requirement, i.e., $R_i^{min} = R, \forall i \in \mathbb{I}$. Without loss of generality, we set $R = 1$. To compare the outage performance in scenarios with different number of network nodes, we normalize the total SNR as $\bar{\mathcal{S}} = \frac{\sum_{i \in \mathbb{I}} S_i}{Ig^{-1}(\exp(R)-1)}$, which can be interpreted as total additional SNR required to combat against fading compared to the AWGN channel with channel gain g and the same rate requirement R. We set $g = 1$ here. Accordingly, all the channel gains in our model are simulated as experiencing Rayleigh fading and thus follow exponential distribution, the mean of which is set to be 1.

We set the number of APs and EUs to be the same, i.e., $I = N$, so that each EU can be associated with an AP exactly in single-AP based scheme. We plot the results for 2-AP 2-EU scenario and 3-AP 3-EU scenario in Fig. 4.4 and Fig. 4.5, respectively. Here, direct transmission denotes the single-AP based scheme, and multiple access denotes the multi-AP based scheme. As expected, multiple access transmission brings a diversity order of 2 in Fig. 4.3 and 3 in Fig. 4.5. In contrast, the diversity order of direct transmission is only 1 in both scenarios. This evidently demonstrates the benefits brought by multi-AP based scheme over that of single-AP based scheme.

Note that we do not impose peak power constraint at the APs and interference temperature constraint at the MDs in this simulation since it is not clear how we should define the outage probability while incorporating these additional constraints. Instead, we just impose a total power threshold for the APs in this simulation. Nevertheless, if we could find an

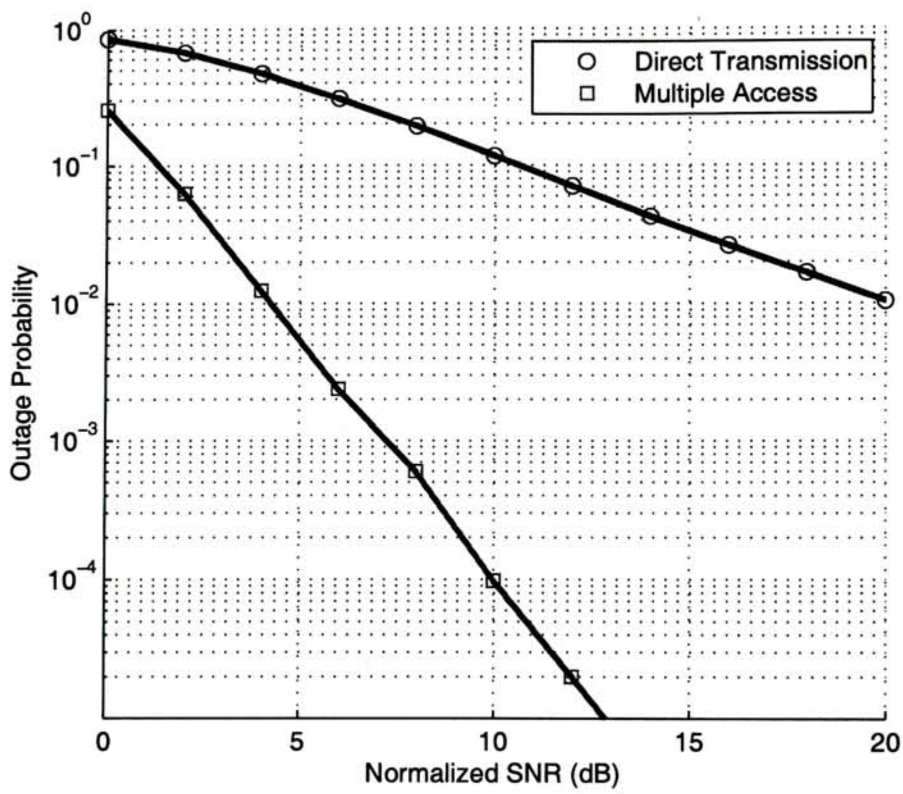

Figure 4.5: Comparison of single-AP and multi-AP schemes: 3-AP 3-EU case

appropriate definition for the outage probability to take into account these additional constraints, we argue that the extra diversity order obtained by multiple access scheme will be maintained though a deteriorated outage performance might be observed.

4.4.2 Convergence and Performance

We provide some numerical results in this subsection to illustrate the convergence behaviors of the two proposed algorithms. Simulation parameters are chosen as follows. The number of EUs, the number of APs, and the number of MDs are set to be 16, 8, and 4, respectively. Note that all the channel power gains are exponentially distributed. We set the mean of g_i^n to be 1 , $\forall n \in \mathbb{N}, \forall i \in \mathbb{I}$, while the mean of $g_{k,i}^n$ is 0.2, $\forall n \in \mathbb{N}, \forall k \in \mathbb{K}, \forall i \in \mathbb{I}$. For clarity, we set the peak power constraint at each AP to be the same, i.e., $\bar{p}^n = 8, \forall n \in \mathbb{N}$, and also a uniform interference temperature constraint at each MD over every channel, i.e., $T_{k,i} = 8, \forall k \in \mathbb{K}, \forall i \in \mathbb{I}$, and a uniform QoS requirement, i.e., $R_i^{min} = R = 1, \forall i \in \mathbb{I}$.

We plot the results in Fig.4.6 which shows the evolutions of power allocation of APs associated with D-SBRA and P-SBRA as a function of iteration index. For clarity, we only show the evolutions of power allocation of AP1, AP4, and AP8 in Fig.4.6. The initial price vector $\boldsymbol{\lambda}$ for P-SBRA is randomly generated. We can see both algorithms converge relatively fast. Similar fast convergent behaviors of D-SBRA and P-SBRA can be observed with

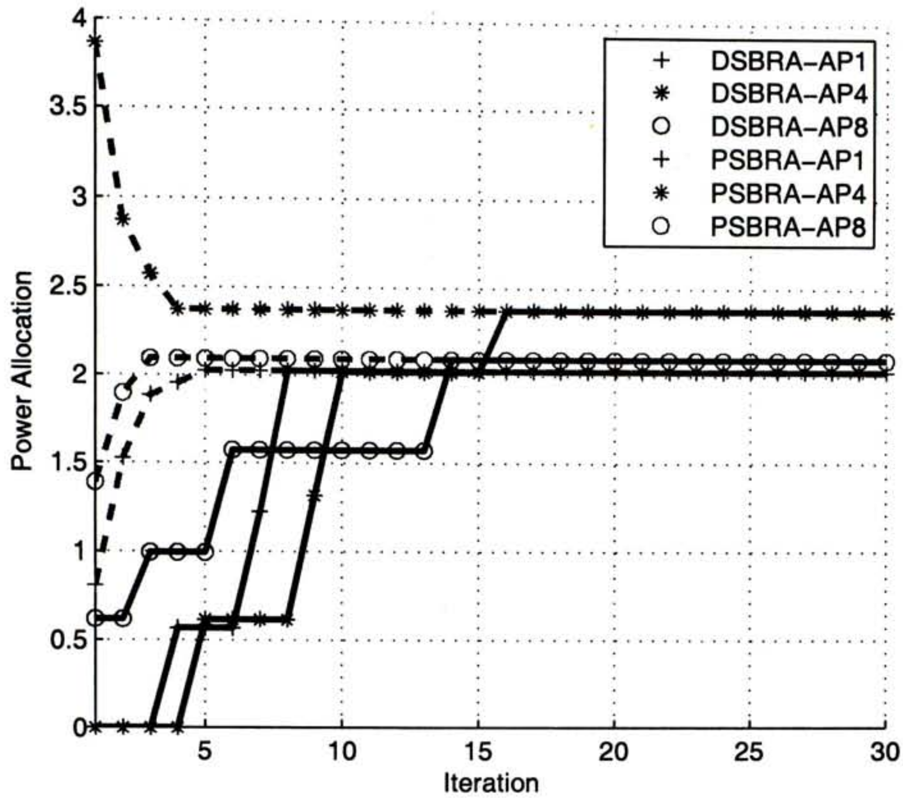

Figure 4.6: Convergence behaviors of D-SBRA and P-SBRA

other simulation parameters. The fast convergence behaviors of two proposed algorithms are desirable for practical implementation. Moreover, as expected, the convergent speed of P-SBRA is greater than that of D-SBRA. This is understandable since each EU under D-SBRA needs to wait for the responses of all the other EUs before updating its own flow allocation while EUs under P-SBRA can update their responses simultaneously.

To further appreciate the performance, we compare our proposed scheme with the popular iterative water-filling algorithm (IWFA) which treats interference as noise when decoding. We choose IWFA as it has been extensively studied and advocated by many researchers for distributed resource allocation in wireless networks (see, e.g., [29] [58] [74] and references therein). For fair comparison, we relax the interference constraint for the time being. There are 5 APs with peak power $\bar{p}^n = 100$ which ensures feasibility. The performance metric we use is the ratio of the total energy consumption to the sum flow rates, which represents the amount of energy needed for each unit flow rate. The numerical results are shown in Fig.4.7 in which SICA denotes our proposed SIC-based algorithms. As illustrated, SICA has a remarkable performance improvement over IWFA. These performance gains are due to two unique features in SICA. One the one hand, the SIC technique used in our scheme archives the capacity of multiple access channels while the performance of IWFA is suboptimal since IWFA simply treats interference as noises when decoding. On the other hand, APs in our scheme use their powers more wisely than those in IWFA. Indeed, APs in IWFA are selfish

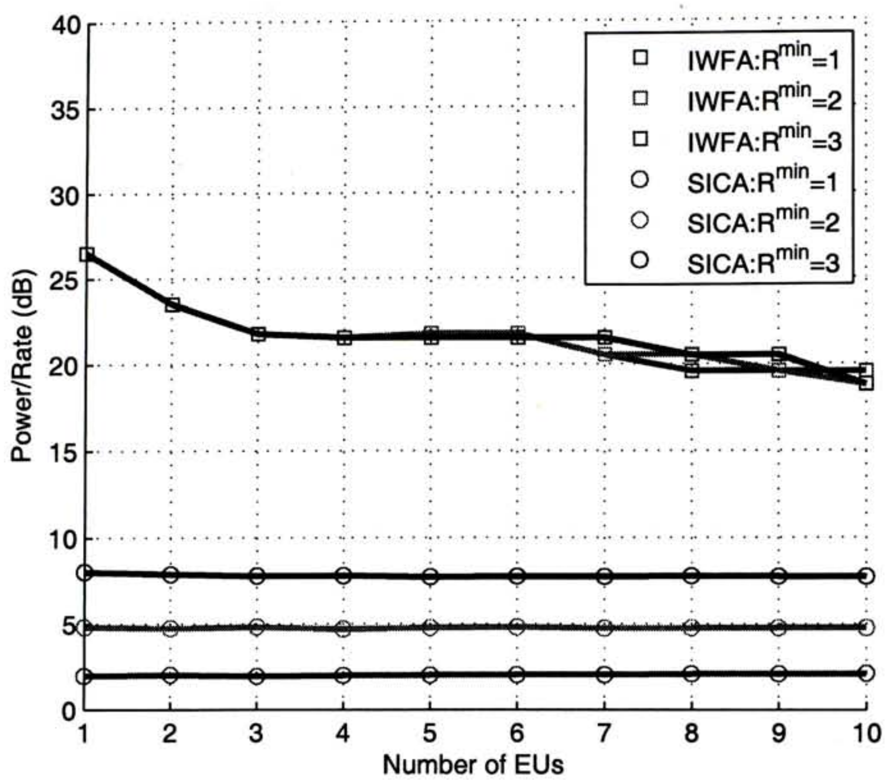

Figure 4.7: Comparison of IWFA and SICA

and only care about maximizing their own flow rates to EUs, resulting in excessive interference and unnecessary power waste. In contrast, APs in our scheme use their powers only when EUs require flows from them. Thus, network powers are wisely used and interference are kept to the minimum in our rate-on-demand (RoD) scheme.

4.4.3 Flow Distribution

To better understand the pseudo-waterfilling strategy in Proposition 1, we provide some numerical results in a scenario with just one EU present, i.e., $I = 1$. Common simulation parameters are chosen as follows. The number of APs, and the number of MDs are 8 and 2, respectively. The channel power gains and the peak power constraint at each AP are the same as those in previous subsection.

We plot the results in Table 4.3 where $T_k = 10, \forall k \in \mathbb{K}$. In fact, with such a high interference tolerance level, the unique EU can choose flows from any APs while still satisfying the interference constraint. In particular, as shown in the table, the EU can satisfy its flow rate requirement 1 served by its "best" AP7. However, as flow rate requirement R^{min} increases to 2.5, AP7 cannot satisfy the EU's QoS even when its peak power 8 is used. As a result, the EU seeks flows from its second "best" AP2. Similar behaviors can be observed when R^{min} further increases to 3 and 3.5, respectively.

R_{min}	AP1	AP2	AP3	AP4	AP5	AP6	AP7	AP8
1	0	0	0	0	0	0	1.336	0
2.5	0	0.947	0	0	0	0	8.000	0
3	0	8.000	1.474	0	0	0	8.000	0
3.5	0	8.000	8.000	0	8.000	0	8.000	6.442

Table 4.3: Transmit Power Distribution with 1 EU and $T_k = 10$

R_{min}	AP1	AP2	AP3	AP4	AP5	AP6	AP7	AP8
1	0	0	0	0	0	0	1.336	0
2.5	0	0.947	0	0	0	0	8.000	0
3	0	8.000	0.316	0	1.757	0	8.000	0
3.5	NA	NA	NA	NA	NA	NA	NA	NA

Table 4.4: Transmit Power Distribution with 1 EU and $T_k = 2$

However, this flow seeking behavior largely depends on the simulation parameters and might not always be true. In particular, we plot the results in Table 4.4 where $T_k = 2, \forall k \in \mathbb{K}$. In fact, with such a low interference tolerance level, the EU cannot freely choose flows from any APs since now the transmission behaviors of APs are regulated and may not be able to transmit at their peak powers. This situation can be observed in Table 4.4. In the previous simulation results shown in Table 4.3, the flow rate requirement $R^{min} = 3$ can be satisfied by AP7, AP2, AP3 with power 8, 8, 1.474, respectively. However, this flow rate allocation violates the interference constraint in this simulation. In fact, as shown in Table 4.4, the flow rate requirement $R^{min} = 3$ now needs to be satisfied by AP7, AP2, AP5, AP3 with power 8, 8, 1.757, 0.316, respectively. Note that the flow allocation problem becomes infeasible when R^{min} increases to 3.5 shown in Table 4.4.

It is also of interest to see the network flow distribution from the APs' perspective. Additional numerical results are provided in Table 4.5 and 4.6 with two EUs present for this purpose. In these two tables, the entry (x, y) at the intersection of EU i's row and AP j's column indicates that AP j which is EU i's y-th best AP allocates x units of power for the transmission between AP j and EU i. Table 4.5 where $T_k = 10$ shows that AP6, EU1's best AP, can satisfy EU1's flow rate requirement 1. The same is true for EU2's best AP, i.e., AP8. However, as flow rate requirement R^{min} increases to 3, AP6 cannot satisfy EU1's QoS, and so does AP8. As a result, AP3, which happens to be the second best AP of both EU1 and EU2, splits its power resource and provides the additional required flows for EU1 and EU2, respectively.

The network flow distribution changes in Table 4.6 where $T_k = 2$. In particular, when the flow rate requirement $R^{min} = 3$, AP3 now cannot transmit with power 1.875 to EU 2 as in Table 4.5 due to the low interference tolerance level. In this scenario, additional AP is needed to satisfy EU2's QoS. Indeed, AP3 now just transmits with power 1.166 to EU2. The remaining flows are provided by EU2's 4-th best AP, i.e., AP7, with power 5.875. Interestingly, AP6, EU2's third best AP, remains silent in this scenario.

R_{min}		AP1	AP2	AP3	AP4	AP5	AP6	AP7	AP8
1	EU 1	(0, 6)	(0, 7)	(0, 2)	(0, 8)	(0, 5)	(0.738, 1)	(0, 3)	(0, 4)
	EU 2	(0, 6)	(0, 7)	(0, 2)	(0, 8)	(0, 5)	(0, 3)	(0, 4)	(0.880, 1)
3	EU 1	(0, 6)	(0, 7)	(0.226, 2)	(0, 8)	(0, 5)	(8.000, 1)	(0, 3)	(0, 4)
	EU 2	(0, 6)	(0, 7)	(1.875, 2)	(0, 8)	(0, 5)	(0, 3)	(0, 4)	(8.000, 1)

Table 4.5: Transmit Power Distribution with 2 EUs and $T_k = 10$

R_{min}		AP1	AP2	AP3	AP4	AP5	AP6	AP7	AP8
1	EU 1	(0, 6)	(0, 7)	(0, 2)	(0, 8)	(0, 5)	(0.738, 1)	(0, 3)	(0, 4)
	EU 2	(0, 6)	(0, 7)	(0, 2)	(0, 8)	(0, 5)	(0, 3)	(0, 4)	(0.880, 1)
3	EU 1	(0, 6)	(0, 7)	(0.226, 2)	(0, 8)	(0, 5)	(8.000, 1)	(0, 3)	(0, 4)
	EU 2	(0, 6)	(0, 7)	(1.166, 2)	(0, 8)	(0, 5)	(0, 3)	(5.875, 4)	(8.000, 1)

Table 4.6: Transmit Power Distribution with 2 EUs and $T_k = 2$

4.4.4 A Grid Network Simulation

To further verify the various arguments mentioned above, we carry out simulation for a 9×9 grid network as shown in Fig. 4.8, where 30 EUs denoted by green dots randomly locate within the network, 9 APs denoted by magenta boxes locate at $(1.5 \cdot i, 1.5 \cdot j)$ where $i, j \in \{1, 2, 3\}$, and 4 MDs denoted by red stars locate at the four corners, respectively. The channel gain $g_{ij} = d_{ij}^{-2}$ where d_{ij} denotes the Euclidean distance between node i and node j. In such a network, we are interested in how the random distributed EUs allocate their information flows based on the proposed algorithms.

We plot the simulation results in Fig. 4.8 where an information flow exists if there is a line between an EU and an AP. To be more specific, if an EU are connected to more than one AP, then the connection is denoted by a red line; otherwise, the corresponding connection is denoted by a blue line. The three subfigures are associated with either different peak power constraint at APs or different interference temperature constraints at MDs. However, we assume a uniform rate requirement $R^{min} = 1$ which remains the same in all the three subfigures.

In the first subfigure, the peak power $\bar{p}^n = 100, \forall n \in \mathbb{N}$ and the interference temperature $T_{k,i} = 10, \forall k \in \mathbb{K}, \forall i \in \mathbb{I}$. Under such abundant power resource at every AP and high interference tolerance level at every MD over each channel, every EU can satisfy its flow rate requirement by just seeking flow from its "best" AP. However, this is not true in the second subfigure where the peak power $\bar{p}^n$ is the same but interference temperature $T_{k,i}$ is set to be 1. Although the power resource is still abundant, some EUs may need to seek flows from more than one AP since their corresponding "best" APs cannot transmit at a high power level which violates the interference constraint. As expected, there are 4 EUs seeking flows from more than one AP shown in the second subfigure. Similar outcome can be observed in the third subfigure where the peak power $\bar{p}^n$ is only 20 and the interference temperature $T_{k,i}$ is still 10. Now though MDs have a high interference tolerance level, some EUs may need to seek flows from more than one AP since their corresponding "best" APs may not have enough power resources to satisfy the QoS requirements. This situation is shown in the third figure where 5 EUs seek flows from more than one AP.

Nevertheless, we can see only a few (0, 4, and 5 in the three subfigures, respectively) EUs out of total 30 EUs seek flows from more than one AP. As mentioned, this result is favorable. On the one hand, only a few EUs in the network need to carry out SIC in decoding, which reduces the average decoding complexity in a network setting. On the other hand, it also reduces the implementation complexity of flow splitting[2] in the wired network which is carried out for only a few EUs. This implies that the multi-AP based transmission scheme using the proposed algorithms brings considerable gains with a moderate cost in complexity.

□ **End of chapter.**

[2]For the flow splitting technique, we refer readers to [32] and references therein.

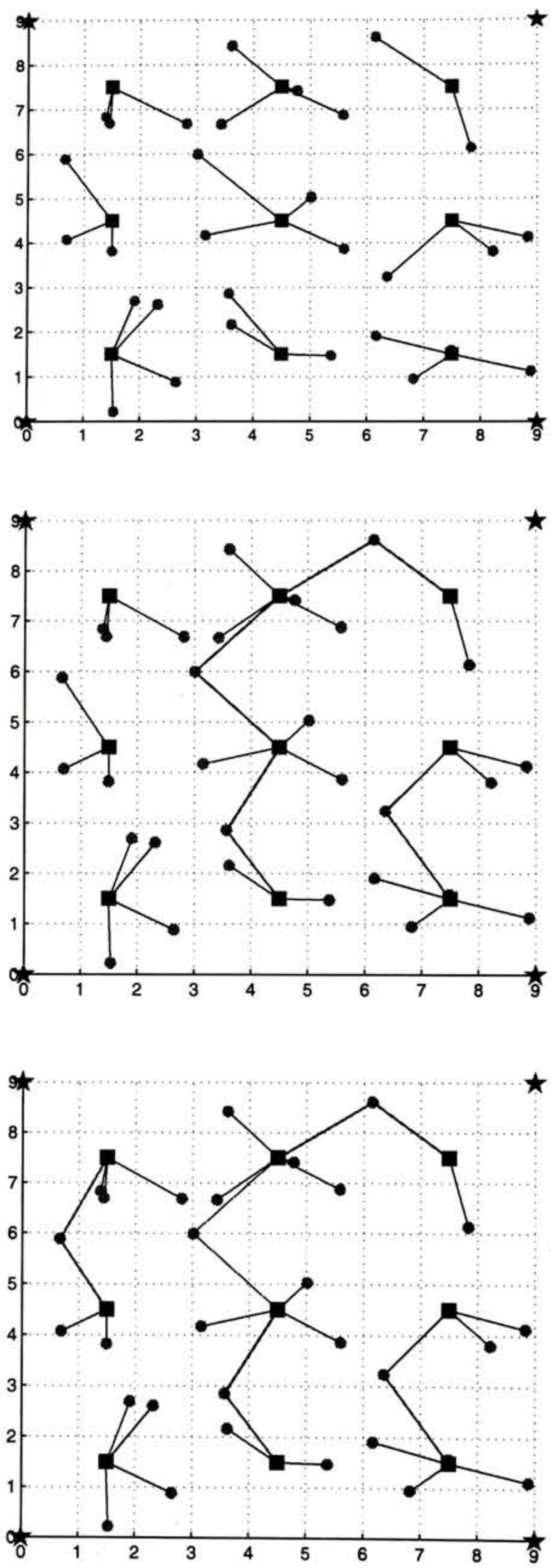

Figure 4.8: 9 × 9 Grid Network: The first subfigure: $\bar{p}^n = 100$, $T_{k,i} = 10$; The second subfigure: $\bar{p}^n = 100$, $T_{k,i} = 1$; The third subfigure: $\bar{p}^n = 20$, $T_{k,i} = 10$.

Chapter 5

Relay Assignment in Cooperative Networks

A journey of a thousand miles begins with a single step.

— *Lao-tzu*

It is believed that smarter techniques are needed in every detail of the design of the future wireless networks to meet the increasing demand of high data rate services. As a promising solution, cooperative radio has recently attracted much attention from both academia and industry. Indeed, cooperative communication (CC) technique can improve transmission diversity and capacity by allowing single-antenna users to exploit other users' antennas, generating a virtual multiple-input and multiple-output (MIMO) system [36].

Relay assignment plays an important role in reaping the benefits brought by CC. Substantial research works have been carried out on relay assignment for a single source-destination pair. In [31], the diversity performance of many single-relay assignment schemes in the literature were revisited and a signal-to-noise ratio (SNR) based multiple relay selection scheme was proposed. An interesting result on relay selection is that choosing a single "best" relay to assist a transmission pair is sufficient for achieving full diversity even with the presence of multiple relays [81].

Despite the many proposed relay selection protocols for single user scenario, extending relay assignment to wireless networks environment remains challenging. In [61], a polynomial-time centralized algorithm was proposed for such extension under an implicit assumption that there existed an infinite number of orthogonal channels in the networks. However, this assumption may not hold since the available spectrum for users in wireless networks is usually quite limited. Moreover, users may have QoS requests in some wireless services (online interactive games, for one). Therefore, given limited spectrum resource, an admission control scheme is required to ensure admitted user meets its QoS request [41]. Meanwhile, appropriate spectrum allocation scheme is also needed to maximize the number of admitted users. One such spectrum allocation algorithm using bipartite matching can be found in [80]. However, this centralized algorithm may not be practical due to its complexity. Thus, other practical algorithms are expected and distributed ones are even more desirable.

In this chapter, given the limited amount of spectrum resource for multiple users in a wireless network, we jointly perform relay assignment and admission control to carefully exploit the scarce spectrum. In particular, each user has a QoS request in the form of minimal data rate requirement. Our objectives are to maximize the number of admitted users while minimizing the spectrum consumption. Noting that these two objectives may conflict with one another, we treat them unequally by giving higher priority to the former one. Hence, we look for the best solution with respect to maximizing the number of admitted users, and, in case of more than one such solution, the best with respect to minimizing the spectrum consumption.

The outline of the rest of this chapter is as follows. Section 5.1 describes the system model and problem formulation. In section 5.2, an appropriate decomposition approach is presented to tackle the problem in question. A distributed algorithm is proposed in section 5.3. Section 5.4 presents some numerical results.

5.1 System Model and Problem Formulation

5.1.1 Three-Node Relay Model

Let us consider a classic three-node relay model which works in a time-division way. In particular, the communication between the source and the destination is carried out in two time slots. In the first time slot, the source transmits a signal to the destination which is overheard by the relay as well. Then in the second time slot the relay forwards its overheard signal to the destination based on some cooperative protocol. Based on how the relay functions during the cooperative transmission, two basic cooperative protocols, amplify-and-forward (AF) and decode-and-forward (DF), were proposed in [36] by Laneman *et al.* The detailed description of these two protocols are omitted here. Instead, we directly give the capacity expressions for AF, DF and direct transmission (DT) as follows.

1. AF

$$I_{AF} = \frac{1}{2}log_2\left(1 + \lambda_{sd} + \frac{\lambda_{sr}\lambda_{rd}}{\lambda_{sr} + \lambda_{rd} + 1}\right). \tag{5.1}$$

2. DF

$$I_{DF} = \frac{1}{2}\min\{log_2(1 + \lambda_{sr}), log_2(1 + \lambda_{sd} + \lambda_{rd})\}. \tag{5.2}$$

3. DT

$$I_{DT} = log_2(1 + \lambda_{sd}). \tag{5.3}$$

In (5.1), (5.2), and (5.3), λ_{sd}, λ_{sr} and λ_{rd} denote the SNR of the source-destination, the source-relay and the relay-destination channels, respectively. These capacity expressions are directly used as performance metric in this thesis without considering specific communication techniques.

5.1.2 Network Model

We consider an N-node CRN with each node acting as either a source, a potential relay, or a destination. Specifically, denote the set of transmission pairs (users) by $\mathbb{N}_{\mathbb{P}} = \{p_1, p_2, ..., p_{N_P}\}$ where $p_i = (s_i, d_i)$, and the set of relays by $\mathbb{N}_{\mathbb{R}} = \{r_1, r_2, ..., r_{N_R}\}$. We assume each node transmits with the same power P for ease of presentation. The results in this thesis can be easily extended for the unequal power case.

Orthogonal Frequency-Division Multiple Access (OFDMA) is applied to avoid interference. We also assume that the locations of nodes are known a priori and the channel gain h_{ij} between node i and node j only includes the effect of path loss. So the SNR $\lambda_{i,j}$ of the transmission from node i to node j equals $\frac{P}{N_0} d_{i,j}^{-v}$, where $d_{i,j}$ is the distance between node i and node j, N_0 is the noise power (assumed to be equal at all the receiving nodes), and v is the path loss factor.

Besides, each user has a QoS request, i.e., minimal data rate requirement R_i^{min}. For simplicity, we assume $R_i^{min} = R^{min}, \forall i$. Moreover, each user is allowed to cooperate with at most one relay. Note that a user may not always be equipped with a relay for either insufficient number of relays exist in the network or DT outperforms cooperating with a relay for that user. Last but not least, denote by W the bandwidth of a subchannel and each subchannel is allocated to a user. The total available bandwidth is W^{max}.

We now discuss our objective functions in this setting. First and foremost, we try to maximize the number of admitted users since it is impossible to serve all the users at their desired QoS requests due to limited spectrum resource. In case of more than one optimal solution, the one with respect to minimizing the spectrum consumption is the most desirable solution.

5.1.3 Problem Formulation

In this section, we present an integer programming formulation for our problem. We firstly introduce a binary variable x_i to specify whether or not user i is admitted, i.e.,

$$x_i = \begin{cases} 1, & \text{if user } i \text{ is admitted} \\ 0, & \text{otherwise} \end{cases} . \tag{5.4}$$

We introduce another binary variable $y_{i,j}$ to indicate whether or not relay j is assigned to user i, i.e.,

$$y_{i,j} = \begin{cases} 1, & \text{if relay } j \text{ is assigned to user } i \\ 0, & \text{otherwise} \end{cases}. \tag{5.5}$$

Besides, denote by w_i the number of subchannels allocated to user i. If user i chooses to cooperate with relay j, the transmission data rate per subchannel of user i is given by $R_{i,j} = WI_C(\lambda_{s_i,d_i}, \lambda_{s_i,r_j}, \lambda_{r_j,d_i})$ where $I_C(\cdot) = I_{AF}(\cdot)$ for AF and $I_C(\cdot) = I_{DF}(\cdot)$ for DF. If user i chooses DT (denoted by $\emptyset$), then the corresponding data rate per subchannel $R_{i,\emptyset} = WI_{DT}(\lambda_{s_i,d_i})$. In a word, R_i can be written as

$$R_i = x_i w_i [\sum_{j=1}^{N_R} y_{i,j} R_{i,j} + (1 - \sum_{j=1}^{N_R} y_{i,j}) R_{i,\emptyset}]. \tag{5.6}$$

We now present a preliminary form P_I of our problem.

$$\text{maximize} \quad \sum_{i=1}^{N_P} x_i \tag{5.7}$$

$$\text{subject to} \quad \sum_{i=1}^{N_P} x_i w_i W \leq W^{max}, \tag{5.8}$$

$$R_i \geq x_i R^{min}, \forall i, \tag{5.9}$$

$$\sum_{i=1}^{N_P} y_{i,j} \leq 1, \forall j, \tag{5.10}$$

$$\sum_{j=1}^{N_R} y_{i,j} \leq 1, \forall i, \tag{5.11}$$

$$x_i \in \{0,1\}, \forall i, y_{i,j} \in \{0,1\}, \forall i, \forall j, w_i \in Z^+, \forall i. \tag{5.12}$$

Constraint (5.8) implies the total spectrum limit. Constrain (5.9) applies minimum data rate requirement to each user. Constraint (5.10) indicates that each relay can be assigned to at most one user. Constraint (5.11) determines that each user can cooperate with at most one relay. Note that problem P_I is extremely hard due to its complicated combinatorial nature in relay assignment, subchannel allocation, and admission control. Worse still, there may exist more than one optimal solution to P_I. As a result, we need to find all the optimal solutions and then pick up the one that can minimize the total spectrum consumption. The following proposition, the proof of which is similar to that in [42], comes in handy.

Proposition 5.1 *The optimal solution to the following optimization problem P_{II} is the one* *to P_I that minimizes total spectrum consumption:*

$$\text{maximize} \quad \epsilon \sum_{i=1}^{N_P} x_i - (1-\epsilon) \sum_{i=1}^{N_P} w_i W \tag{5.13}$$

$$\text{subject to} \quad (5.8), (5.9), (5.10), (5.11), (5.12). \tag{5.14}$$

where ϵ is a constant such that $\frac{W^{max}}{W^{max}+1} < \epsilon < 1$.

5.2 Centralized Scheme

Noting that problem P_{II} is still NP-hard, a further careful study on the structure of P_{II} is clearly needed in order to make it solvable for practical implementation. Toward this end, we decompose P_{II} into two successive subproblems: relay assignment and admission control. Specifically, the algorithm first tries to find a feasible solution to P_{II}, and then maintains the feasibility of the solution at each step while attempts to improve it iteratively. Though this approach is theoretically suboptimal, its effectiveness is verified by numerical results.

5.2.1 Generalized Relay Assignment

To start with, a careful observation indicates that the optimal solution to P_{II} has a property given by the following proposition of which the proof is omitted for brevity. This result is a cornerstone for the subsequent analysis.

Proposition 5.2 *Given solution $x_i^*, \forall i$, and $y_{i,j}^*, \forall i,j$, the optimal number of subchannels allocated to user i is given by*

$$w_i^* = x_i^* \left\lceil \frac{R_{min}}{R_i^*} \right\rceil, \tag{5.15}$$

where $R_i^* = \sum_{j=1}^{N_R} y_{i,j}^* R_{i,j} + (1 - \sum_{j=1}^{N_R} y_{i,j}^*) R_{i,\emptyset}$, and $\lceil \alpha \rceil$ is the smallest integer greater than or equal to α.

Proof We show this proposition by enumerating all the possible cases.

Case 1: If $x_i^* = 0$, then $w_i^* = 0$. If not, i.e., $w_i^* > 0$, we can always increase the objective value in P_{II} by decreasing a bit value ϵ from w_i while all the constraints are still held. We can repeat this process until $w_i^* = 0$.

Case 2: If $x_i^* = 1$ and $\sum_{j=1}^{N_R} y_{i,j} = 0$, then w_i should satisfy $w_i^* R_{i,\emptyset} \geq R_{min}$. Considering the constraint $w_i \in \mathbb{Z}^+$, we have $w_i^* \geq \lceil R_{min}/R_{i,\emptyset} \rceil \geq R_{min}/R_{i,\emptyset}$. By the same argument used in **Case 1:**, we claim that the optimal solution attained at equality, i.e., $w_i^* = \lceil R_{min}/R_{i,\emptyset} \rceil$.

Case 3: If $x_i^* = 1$ and $\sum_{j=1}^{N_R} y_{i,j} \neq 0$, then there exist exactly one $j \in \{1, 2, ..., N_R\}$ such that $y_{i,j} = 1$ and $y_{i,k} = 0$ for $k \neq j, k \in \{1, 2, ..., N_R\}$. In this case, we attain the optimal solution $w_i^* = \lceil R_{min}/R_{i,j} \rceil$ following a similar argument as in previous two cases. This completes the proof.

Based on Proposition 5.2, we are now able to give a network graph model for further discussions. Consider a graph $G = (V_1, V_2, E)$ with weights $w_{i,j}$, where V_1 is the set of N_P transmission pairs, V_2 is the set of N_R potential relays, E is the set of links, i.e., $< i, j > \in E$, for $i \in V_1, j \in V_2$, and $w_{i,j}$ is the number of subchannels needed for user i by cooperating with relay j. Now it seems that the optimal relay assignment with respect to graph G can be treated as a minimum weighted bipartite graph matching problem that saturates every vertex in V_1. The problem is described in three aspects.

1. Neglect of DT: It is reasonable for user i to choose not to cooperate when DT outperforms CC. Moreover, the backoff of user i from relay assignment can leave more potential relays to other needed users.

2. Much redundant information: Node $i \in V_1$ has links to all the nodes in V_2 in graph G. Consequently, the storage and computational load can be high which actually can be mitigated by better formulation of the graph G.

3. Imperfect matching with respect to V_1: It is possible that there does not exist a matching that saturates every vertex in V_1 for the graph $G = (V_1, V_2, E)$. Hence, we cannot associate each user with a transmission strategy directly if imperfect matching happens.

To resolve the first aspect, fictitious vertex is introduced into the graph formulation. Specifically, introduce a fictitious relay r_{0i} to V_2 for user $i, \forall i$, to represent its DT mode. Besides, fictitious vertex r_{0i} has a unique link to vertex $(s_i, d_i) \in V_1$. To resolve the second aspect, the following proposition comes into handy.

Proposition 5.3 *Given the transmit SNR $\lambda = P/N_0$, relay j is preferred to a transmission pair $\{s_i, d_i\}$ if only if their geographical locations satisfy the following condition (5.16) for AF and (5.17) for DF, respectively.*

$$d_1^v > \frac{1}{2\lambda}(A + \sqrt{A^2 + 4A\lambda^2}), \tag{5.16}$$

$$d_1^v > \max\{d_2^v + \sqrt{d_2^{2v} + \lambda d_2^v}, \frac{1}{2}(d_3^v + \sqrt{d_3^{2v} + 4\lambda d_3^v})\}, \tag{5.17}$$

where $A = \lambda d_2^v + \lambda d_3^v + d_2^v d_3^v$, d_1, d_2, and d_3 denote the distance of s_i-d_i, s_i-r_j, and r_j-d_i, respectively.

Proof Here we only consider DF, and conditions for AF can be derived in a similar way. User i employs relay j if and only if the capacity of relay transmission is larger than that of direct transmission, i.e.,

$$\frac{1}{2}min\{log_2(1+\lambda_{sr}), log_2(1+\lambda_{sd}+\lambda_{rd})\} > log_2(1+\lambda_{sd})$$

which implies the following two conditions

$$log_2(1+\lambda_{sr}) \geq log_2(1+\lambda_{sd}) \tag{5.18}$$

$$log_2(1+\lambda_{sd}+\lambda_{rd}) \geq log_2(1+\lambda_{sd}) \tag{5.19}$$

should hold simultaneously. Substituting $\lambda d_{s,d}^{v}$ for λ_{sd}, $\lambda d_{s,r}^{v}$ for λ_{sr}, and $\lambda d_{r,d}^{v}$ for λ_{rd}, and then solve the equation set of (5.18) and (5.19), we obtain the conditions under which DF is more favored than DT.

Now we are able to delete unnecessary links in graph G. In particular, for each vertex $(s_i, d_i) \in V_1$ and vertex $r_j \in V_2$, check whether their geographical positions satisfy the conditions given by Proposition 5.3. If not, delete the link $< (s_i, d_i), r_j >$. Continue this process for all possible combinations of i and j.

After introducing fictitious vertices and deleting unnecessary links, we attain a modified graph $\tilde{G} = (\tilde{V_1}, \tilde{V_2}, \tilde{E})$ with weights $\tilde{w}_{i,j}$, where $\tilde{V_1}$ is the same as the original V_1, $\tilde{V_2}$ is the set of relays consisting of both real and fictitious relays, i.e., $\tilde{V_2} = \{r_{01}, r_{02}, ..., r_{0N_P}, r_1, r_2, ..., r_{N_R}\}$, $\tilde{E}$ is the set of essential links attained from E by adding fictitious links and deleting useless links in the original graph G, and $\tilde{w}_{i,j}$ is the number of subchannels needed for user i by cooperating with (real or fictitious) relay $\tilde{j}$. We now claim the following proposition holds which implies we have also resolved the third aspect of the problem caused by the original graph G.

Proposition 5.4 *Modified graph $\tilde{G} = (\tilde{V_1}, \tilde{V_2}, \tilde{E})$ contains a matching that saturates every vertex in $\tilde{V_1}$.*

Proof To begin with, for $S \subseteq \tilde{V_1}$, denote by $\eta(S)$ the neighbor set of S in modified graph $\tilde{G}$ which is defined as the set of all vertices adjacent to verices in S, i.e.,

$$\eta(S) = \{v \in \tilde{V_1} : \exists < u, v > \in \tilde{E}, u \in S\}.$$

Now let us consider an $S^{\dagger} \subseteq \tilde{V_1}$. Note that, by construction of the modified graph $\tilde{G}$, there always exists a unique edge between vertex (s_i, d_i), i.e., user i, and vertex r_{0i}, $\forall i \in S^{\dagger}$. So the cardinality of $\eta(S^{\dagger})$ is at least as large as that of $S^{\dagger}$, i.e.,

$$|\eta(S^{\dagger})| \geq |S^{\dagger}|. \tag{5.20}$$

Then by Hall's theorem [1], condition (5.20) is both necessary and sufficient for graph $\tilde{G} = (\tilde{V_1}, \tilde{V_2}, \tilde{E})$ to contain a matching that saturates every vertex in $\tilde{V_1}$. This completes the proof.

We are now in a position to treat the problem in question as a minimum weight bipartite matching problem with respect to graph $\tilde{G}$ that can be solved efficiently using existing algorithms, such as Hungarian algorithm [28]. By solving this subproblem, we obtain the answers to three questions: to cooperate or not, whom to cooperate with, and how the spectrum should be allocated. Hence, we regard it as generalized relay assignment problem.

5.2.2 Admission Control

In this subsection, we consider another subproblem: admission control. Suppose the number of subchannels allocated to user i is w_i^* obtained in the generalized relay assignment process. Then the remaining admission control problem can be formulated as

$$z = \text{maximize} \quad \sum_{i=1}^{N_P} (\epsilon - (1-\epsilon) w_i^* W) x_i \tag{5.21}$$

$$\text{subject to} \quad \sum_{i=1}^{N_P} w_i^* x_i \le \frac{W^{max}}{W}, \tag{5.22}$$

$$x_i \in \{0,1\}, \ \forall i. \tag{5.23}$$

Denote this problem by P_{III}. Note that P_{III} can be categorized as 0-1 Knapsack integer problem which is generally NP-hard. Anyway, P_{III} can be solved efficiently by resorting to dynamic programming (DP) as long as $\frac{W^{max}}{W}$ is not too large. DP is an approach whereby an optimal solution for a problem in question is derived recursively from solving some other slightly different problems of which the size is smaller than that of the original problem.

Next we briefly summarize the idea of this approach. A more detail discussion on DP can be found in [73]. Firstly, let us define a problem $P_\alpha(\beta)$:

$$f_\alpha(\beta) = \text{maximize} \quad \sum_{i=1}^{\alpha} (\epsilon - (1-\epsilon) w_i^* W) x_i \tag{5.24}$$

$$\text{subject to} \quad \sum_{i=1}^{\alpha} w_i^* x_i \le \beta, \tag{5.25}$$

$$x_i \in \{0,1\}, \ \forall i. \tag{5.26}$$

Note that $z = f_{N_P}(\frac{W^{max}}{W})$. A further careful observation indicates the following recursion:

$$f_\alpha(\beta) = max\{f_{\alpha-1}(\beta), \epsilon - (1-\epsilon) w_i^* W + f_{\alpha-1}(\beta - w_i^*)\}. \tag{5.27}$$

Then starting with $f_0(\beta) = 0$ for $\beta \ge 0$, we use recursion (5.27) to derive $f_{N_P}(\frac{W^{max}}{W})$.

Now we iterate back from $f_{N_P}(\frac{W^{max}}{W})$ to obtain the associated optimal solution $x_i^*, \forall i$. Toward this end, we define an indicator

$$F_r(\lambda) = \begin{cases} 0, & \text{if } f_r(\lambda) = f_{r-1}(\lambda) \\ 1, & \text{otherwise} \end{cases}. \tag{5.28}$$

Then if $F_{N_P}(\frac{W^{max}}{W}) = 0$, we set $x_{N_P} = 0$ as $f_{N_P}(\frac{W^{max}}{W}) = f_{N_P-1}(\frac{W^{max}}{W})$ and continue this process for $f_{N_P-1}(\frac{W^{max}}{W})$. Otherwise, we set $x_{N_P} = 1$ as $f_{N_P}(\frac{W^{max}}{W}) = (\epsilon - (1-\epsilon)w_i^* W) + f_{N_P-1}(\frac{W^{max}}{W} - w_{N_P}^*)$ and continue this process for $f_{N_P-1}(\frac{W^{max}}{W} - w_{N_P}^*)$. Clearly, we can obtain the associated optimal solution to the admission control problem after iterating back N_P times. Now we obtain the answer to the fundamental question of our problem: which users should be admitted.

5.2.3 Iteration Algorithm and Some Remarks

After obtaining a feasible initial solution by the above procedures, we describe how to iteratively upgrade this solution in this section. We firstly introduce some other notations to facilitate the description. Denote the set of current admitted nodes by $\tilde{V}_1^A$ and their corresponding selected (generalized) relay set by $\tilde{V}_2^A$. Now we further modify graph $\tilde{G}$ by deleting all the unadmitted nodes and their incident edges and denote the residual graph by $\tilde{G}^*$. We now apply Hungarian algorithm to graph $\tilde{G}^*$ again and obtain a new selected relay set $\tilde{V}_2^{A*}$. Substitute $\tilde{V}_2^{A*}$ for $\tilde{V}_2^A$ and update the remaining spectrum and remove all the admitted nodes and their incident edges in graph $\tilde{G}$, producing a new graph $\tilde{G}^-$. Then restart generalized relay assignment and admission control subalgorithms for $\tilde{G}^-$ to check whether or not more users can be admitted. Repeat this procedure until no more user can be admitted any more.

We have now obtained a good feasible solution to P_{II}. Before ending this section, it should be pointed out that the complexity of Hungarian algorithm is $\mathcal{O}((N_R + N_P)^3)$ and the complexity of DP is $\mathcal{O}((N_P W^{max})/W)$. Besides, the algorithm terminates within at most N_P iterations. Hence, compared with the complexity of exhaustive search, it is a workable and efficient approach and its effectiveness will be verified by numerical results.

5.3 A Simple Distributed Algorithm

Note that the decomposition approach proposed in previous section is a centralized scheme. A simple distributed algorithms are proposed in this section to overcome the drawbacks of the centralized one. And in the sequel, we make a more practical assumption that each user

i only has geographical information of the set of relays given by

$$\mathbb{A}_i = \{r_j : d_{s_i,r_j} \leq d_{s_i,d_i}, d_{r_j,d_i} \leq d_{s_i,d_i}, r_j \in N_R\}, \tag{5.29}$$

as well as that of its destination in the sequel. Under such limited information, the challenge here is how to perform relay assignment and admission control in a distributed fashion with as good performance as possible. Toward this end, we exploit timers and assume that the network is synchronized. Note that timers are commonly used in networking medium access control (MAC) protocols and a timer-based relay selection scheme is also proposed for a single isolated source-destination pair setting [8]. In our problem, the tricky part is to design good timer schemes so that the performance of distributed algorithms is satisfactory.

To begin with, user i calculates its needed number of subchannels $w_{i,r_\emptyset}$ for DT that can satisfy its QoS request. Then users take turns to broadcast their needed number of subchannels, i.e., $w_{i,r_\emptyset}, \forall i$. Based on these information, each user can find the average needed number of subchannels by simple calculation, i.e., $\bar{w} = \frac{1}{N_P}\sum_{i=1}^{N_P} w_{i,r_\emptyset}$. Then user i initiates its first timer T_i^1 such that

$$T_i^1 \propto |w_{i,r_\emptyset}(\bar{w} - w_{i,r_\emptyset})|. \tag{5.30}$$

Suppose timer T_k^1 expires first, then user k can either pick up its best relay $r_j \in \mathbb{A}_k$, or chooses not to cooperate if $w_{k,r_\emptyset} \leq w_{k,i}, \forall r_i \in \mathbb{A}_k$. If the former event occurs, user k should broadcast the selected relay r_j to prevent other users from selecting r_j afterwards. This process continues until all users have decided their transmission strategies.

Then users compete for admission in the following stage. In particular, each user maintains a register value W^- indicating the remaining available spectrum in the networks. And user i initiates its second timer T_i^2 such that

$$T_i^2 \propto \frac{\epsilon - (1-\epsilon)w_{i,r_i}W}{w_{i,r_i}W}, \tag{5.31}$$

where $r_i \in \{r_\emptyset\} \cup \mathbb{A}_i$. Suppose the timer T_k^2 expires first, then user k succeeds in competing for admission and broadcasts its needed spectrum resource $w_{i,r_i}W$. All the other users update W^- and user checks whether W^- is still enough for its Qos request. If not, user i quits from competing for admission. Otherwise user i will continue competing in the next run. This process continues until no user competes for admission any more, which also completes the whole process.

The design philosophy of T_i^1 is that users with median $w_{(\cdot),r_\emptyset}$ deserve higher priority in relay selection process from a statistical point of view since whether or not these users can be admitted hinges heavily on the cooperation of their best relays. The design philosophy of T_i^2 is more straightforward. The idea is just to construct an admission set from scratch: the user brings the best immediate reward $((\epsilon - 1 - \epsilon w_{i,r_i}W)/w_{i,r_i}W)$ is admitted in each step, which is similar to the idea of renowned greedy algorithm but carried out in a distributed fashion through the application of timers.

5.4 Numerical Results

For illustration purposes, we provide simulation results under the following simulation parameters. In particular, we study a disk area with radius 200. All sorts of nodes are distributed according to spatial Poisson point process. The densities are 0.001 for source nodes and destination nodes, and 0.07 for relay nodes. Effective number of transmission pairs is the minimum number of source nodes and destination nodes generated. And the source set and destination set are paired randomly. The path loss exponent is 4. The minimum data rate for each user is $200Kbps$. The total available bandwidth is $3GHz$ and bandwidth of each subchannel is $100KHz$.

Fig.5.1 compares the number of admitted users under different transmission strategies. Here, direct transmission scheme (DTS) means all the users transmit directly to their receivers and the number of admitted users are calculated using the admission control algorithm; invariable cooperation scheme (ICS) means every user always transmit with the help of a properly selected relay using the proposed relay assignment algorithm; adaptive cooperation scheme (ACS) is the proposed transmission protocol in this thesis. It is shown in Fig.5.1 that ACS remarkably outperforms DTS. And we can obtain about $5dB$ gain in this scenario. Note that ICS performs slightly worse than ACS due to its neglect of DT. This gap would be larger if the quality of direct channels is improved where CC is less favored.

Fig.5.2 compares the performance of the distributed scheme to that of centralized scheme for AF-based cooperative network. It is shown that the maximum number of admitted users of the simple distributed scheme is remarkably close to that of centralized one with only a negligible gap under our network where nodes distribute according to spatial Poisson point process, which justifies the design philosophy. However, it should be noted that the performance of this simple scheme largely depends on network distribution.

□ **End of chapter.**

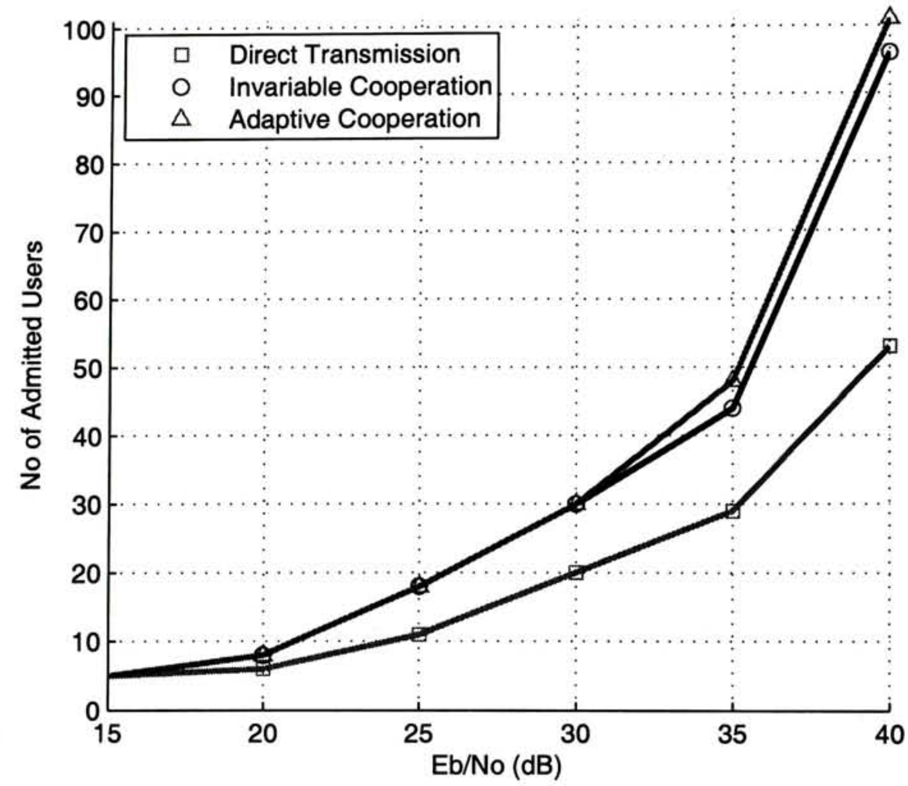

Figure 5.1: Comparison of DTS, ICS, and ACS

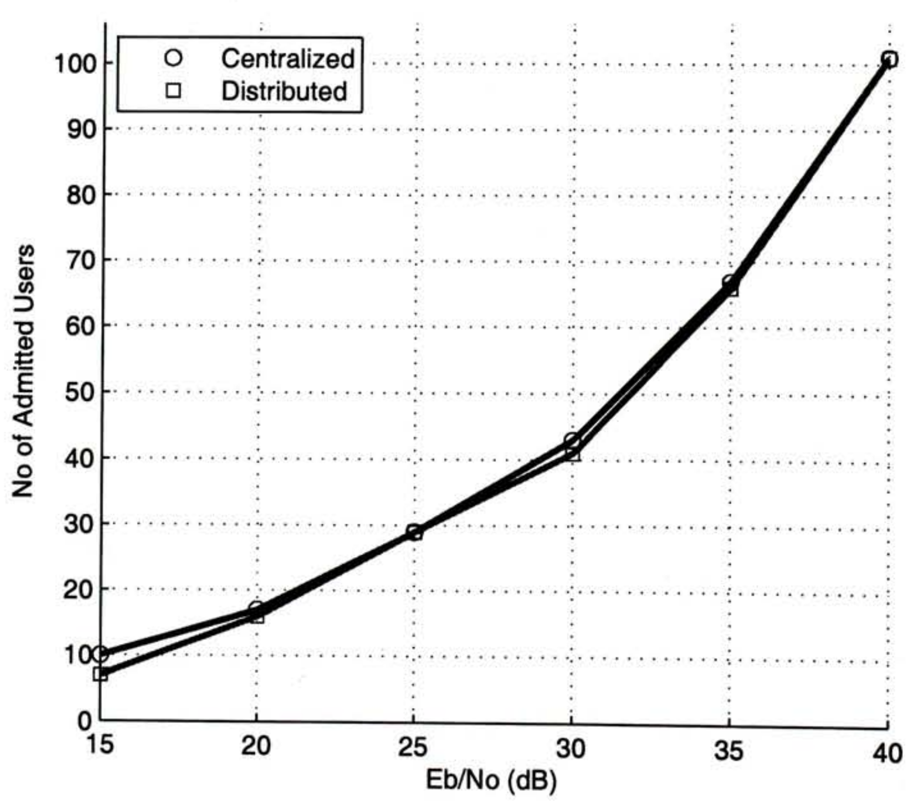

Figure 5.2: Comparison of Centralized and Distributed Algorithms

Chapter 6

Conclusions and Future Work

It is a mistake to try to look too far ahead. The chain of destiny can only be grasped one link at a time.

—— *Sir Winston Churchill*

6.1 Conclusions

This thesis investigates several resource allocation problems for typical wireless transmission scenarios. In particular, the thesis illustrates the roles of learning, competition, and coordination in multiuser communication systems. Centralized and/or distributed wireless resource allocation schemes are developed for the various resource allocation problems considered. Here we briefly summarize the main results in this thesis as follows.

Chapter 2 illustrates the value of learning in communication scenarios with stochastic and limited information, by investigating the distributed power control problem for stochastic parallel Gaussian interference channels. To the best of our knowledge, all the existing works only considered deterministic transmission scenarios. Our work fills this gap by studying the distributed power control problem in stochastic transmission scenarios. Indeed, existing schemes including the popular IWFA cannot work for stochastic communication scenarios. We instead resort to learning theory and propose distributed learning algorithms to solve this problem. We provide sufficient conditions to guarantee the convergence of the proposed algorithms. Using projected dynamical systems theory, we show that the algorithmic convergence speed is "exponential" in some sense.

Chapter 3 further considers one-to-many transmission scenarios, extended from the one-to-one cases. Unlike many existing works formulating the distributed power control problem as NEPs, the problem we consider is a GNEP. Resorting to VI theory, we show the existence of GNE in the formulated noncooperative power control game. Besides, we identify the VE as our desired network operation point. Sufficient conditions for the uniqueness of VE are also investigated. Then we propose a penalty-based distributed algorithm IP^2JA along with convergence analysis. IP^2JA can converge without the need for solving exactly an NEP in every iteration and is thus desirable for practical implementation. Indeed, this chapter well illustrates the roles of competition and coordination in wireless resource allocation designs.

Chapter 4 presents a general framework for the distributed wireless information flow allocation problem in multiple access networks, where the end users can seek wireless flows from multiple access points. Unlike Chapter 2 and 3, Chapter 4 takes a reverse approach. Interestingly, this reverse approach helps us investigate the resource allocation problem in question as a convex one. As a result, we are able to show that the unique NE of our formulated resource allocation game turns out to be the globally optimal solution. Hence, the two proposed distributed algorithms can converge not only to the unique NE but in fact a globally optimal solution. The roles of competition and coordination are also demonstrated in this flow allocation problem.

Chapter 5 illustrates the value of coordination when it comes to the inherently complicated resource allocation problem. Indeed, the joint relay assignment and admission control for cooperative networks is NP-hard. We mainly take a centralized approach. In particular, we decompose the problem into two subproblems. A good final solution can be obtained by iteratively solving the two subproblems. We also propose a simple heuristic algorithm to solve the problem in a distributed fashion.

In a word, the main contributions of the thesis are as follows.

1. Through illustrating the roles of learning, competition, and coordination in several typical wireless resource allocation problems, we shed some lights on the basic principles and techniques for wireless network design.

2. We obtain certain new results for the studied communication scenarios, most of which are well-established models in the field of communication.

6.2 Future Work

Though several resource allocation problems for typical wireless transmission scenarios have been studied carefully in the previous chapters, this thesis is by no means comprehensive as wireless resource allocation is such a broad field. In this section, we discuss some open problems in the previous chapters which deserve further study, respectively.

1. Learning for Parallel Gaussian Interference Channels

   Recall the assumption (i) in Theorem 2.1, which guarantees the existence of a unique equilibrium solution (almost surely) and is also the major requirement for the convergence of the proposed learning algorithms. Note that assumption (i) is not necessary for the uniqueness of NE. Moreover, assumption (i) may never be met in some special scenarios such as the parallel multiple access channels. Thus, investigating the necessary conditions for the uniqueness issue is of interest. Meanwhile, it is also interesting to study weaker conditions than assumption (i) but still guaranteeing the convergence of the proposed learning algorithms.

Also recall the assumption (ii) in Theorem 2.1, which requires the step size sequences to be diminishing. It might be inconvenient to implement diminishing step size sequences in practical systems. In contrast, the choice of constant step size may be more desirable. Indeed, step size in practice is typically kept away from zero in order to allow "tracking". Besides, it is simple to implement constant step size in practice. However, the convergence analysis in Chapter 2 is not directly applicable to the constant step size case. Thus, how the choice of constant step size affects the schemes proposed in Chapter 2 deserves further study.

Furthermore, it is of interest to study how the network operator should schedule the transmission links in a way such that the assumption (i) in Theorem 2.1 can be satisfied in practical implementation. Clearly, transmission links that are too close to each other should not be scheduled to be active simultaneously. Otherwise, the assumption (i) in Theorem 2.1 which essentially requires the interference in the network should be weak cannot be satisfied. Hence, only a fraction of the transmission links can be active at a given time interval. This naturally raises a scheduling question. That is, which fraction of the transmission links should be scheduled to be active? This is a challenging combinatorial problem. One even more challenging objective is that whether and how the combinatorial scheduling problem can be implemented in a distributed fashion.

Besides, the randomness of the transmission environment only affects the utilities of each player while the players' strategy space is deterministic in the stochastic game formulation in Chapter 2. Extending existing formulation to the scenarios where randomness also exists in the players' strategy space is of practical interest. For example, distributed users have coupled interference constraints in cognitive radio networks (see, e.g., [48]). These interference constraints involve the channel gains from the secondary networks to the primary networks. Compared to traditional networks (cellular networks, for one), it is even harder to obtain the exact information about the channel gains from the secondary networks to the primary networks. Consequently, stochastic issues naturally arise in these interference constraints designed for cognitive radio networks.

2. Power Control for One-to-Many Transmissions

   We in this problem identify the VE as our desired network operation point and provide sufficient conditions for the uniqueness of the VE. However, the uniqueness issue of the

GNE is still open. In fact, we lose "most" of the GNEs if we restrict our attention to the VEs. Thus, it would certainly be desirable to provide sufficient conditions for the uniqueness of the GNE as well, if possible. Otherwise, a careful characterization of the set of the GNEs might be pursued.

If the answer to the uniqueness issue of the GNE is negative, one has to decide which GNE the network should be operated at. If the decision has to be made online, some protocols need to be designed such that the network can switch between different equilibrium states. Hopefully, under these switching protocols, the network can operate at the most "efficient" GNE for most of the times by switching between different GNEs.

Recall that we assume each transmitter can send information to all its targeted destinations in Chapter 3. However, the number of destinations that a transmitter can send data to in practice is often bounded due to hardware constraint. Besides, the connection overhead costs can be too high if a transmitter attempts to communicate with too many destinations. Thus, it is desirable to limit the number of simultaneous transmissions from one transmitter. This again raises a challenging combinatorial problem: which fraction of the destinations should be scheduled to receive information from a particular transmitter? Distributed protocols that allow the combinatorial destinations selection problem to be solved in a distributed manner is preferable but more challenging to design.

Besides, the formulation of the power control problem for one-to-many transmissions in Chapter 3 is deterministic. That is, we do not take into account the randomness of the communication environment and estimation errors of the feedback information. However, as argued in Chapter 2, randomness and errors are inevitable in practice. So it is also desirable to investigate the problem in question in the presence of stochastic factors. Perhaps the analysis can be facilitated by the results in Chapter 2 for power allocation in parallel Gaussian interference channels.

3. Flow Allocation in Multiple Access Networks

We simply assume the feasibility of this problem in Chapter 4. Therefore, determining whether or not the problem in question is feasible deserves further study. If the problem is not feasible, an admission control scheme aiming at identifying end users who require infeasible QoS requirement may be needed.

Besides, successive interference cancellation plays a fundamental role in our proposed flow allocation scheme. An unexplored but interesting issue here is how inexact successive interference cancellation would affect the performance of the various proposed schemes.

A relevant problem is flow allocation in broadcast networks. In particular, each transmitter uses superposition coding to simultaneously send information flows to the end users which adopt successive interference cancellation to decode their received signals. This broadcast scenario is essentially a downlink system while the multiple access scenario studied in Chapter 4 is an uplink system. It is known that there is a "duality" between uplink system and downlink system for power control problem in traditional cellular networks. Nevertheless, it is not clear whether there exist similar duality results for the two flow allocation problems in question. This is an interesting problem that deserves further study.

Another research direction is to investigate the flow allocation problem in multiple access networks where the network nodes are equipped with multiple antennas. Since MIMO can provide extra spatial degrees of freedom, the performance of the proposed schemes in Chapter 4 may be further boosted with an increase in the system complexity. Similarly, one can also explore whether any duality results exist between flow allocation in multiple access networks and flow allocation in broadcast networks when MIMO technique is adopted in the system.

4. Relay Assignment in Cooperative Networks

   The joint relay assignment and admission control is an NP-hard problem. The algorithms we currently propose are largely heuristic. Thus, it requires further study to find bounds on the NP-hard optimal solutions. Furthermore, characterizing the performance gap between the NP-hard optimal solutions and our proposed schemes is clearly of practical interest. Nevertheless, we should point out that a complete answer to these issues may be too ambitious to pursue directly. Instead, toward this end, we suggest that one narrow down the general case to some special but important cases such as uplink and dowlink systems.

□ **End of chapter.**

Appendix A

Proof of Theorem 2.1

We first derive a recursion inequality characterizing the relationship between $\| \boldsymbol{p}(n) - \boldsymbol{p}^* \|$ and $\| \boldsymbol{p}(n+1) - \boldsymbol{p}^* \|$ in the following lemma.

Lemma A.1 *The sequence $(\boldsymbol{p}(n))_{n=0}^{\infty}$ generated by iteration (2.9) satisfies*

$$\begin{aligned} \| \boldsymbol{p}(n+1) - \boldsymbol{p}^* \|^2 \leq & \| \boldsymbol{p}(n) - \boldsymbol{p}^* \|^2 + 5C^2 a^2(n) \\ & + 2 \sum_{i \in \mathcal{N}} a_i(n) \epsilon_i(n) - 2 \sum_{i \in \mathcal{N}} a_i(n) \boldsymbol{s}_i(n+1)^T (\boldsymbol{p}_i^* - \boldsymbol{p}_i(n)), \end{aligned} \tag{A.1}$$

where $\boldsymbol{p}^* \in \Phi$ is any NE, C is some large enough constant, and $a(n) = (\sum_{i \in \mathcal{N}} a_i^2(n))^{\frac{1}{2}}$.

Proof This proof is inspired by constructions from [22] [3]. Consider a fixed trajectory $(\boldsymbol{g}(n))_{n=0}^{\infty}$. Recall that $\boldsymbol{q}(n) = \boldsymbol{p}(n) + D(n) \frac{\hat{\boldsymbol{f}}(n)}{\boldsymbol{p}(n)}$ and $\boldsymbol{p}(n+1) = \mathcal{P}_{\Phi}[\boldsymbol{q}(n)]$. We first have

$$\begin{aligned} & \| \boldsymbol{p}(n+1) - \boldsymbol{p}(n) \| \leq \| \boldsymbol{q}(n) - \boldsymbol{p}(n) \| = \| \boldsymbol{p}(n) + D(n) \frac{\hat{\boldsymbol{f}}(n)}{\boldsymbol{p}(n)} - \boldsymbol{p}(n) \| \\ = & \| D(n) \hat{\boldsymbol{s}}(n) \| = (\sum_{i \in \mathcal{N}} a_i^2(n) \| \hat{\boldsymbol{s}}_i(n) \|^2)^{\frac{1}{2}} < C (\sum_{i \in \mathcal{N}} a_i^2(n))^{\frac{1}{2}} = C a(n), \end{aligned} \tag{A.2}$$

where the first inequality follows from Lemma 2.1(ii) and C is some large enough constant.

The existence of C is guaranteed by the boundedness of $\hat{\boldsymbol{s}}$. We proceed by deriving that

$$\begin{aligned}
&C^2a^2(n)+\parallel \boldsymbol{p}(n)-\boldsymbol{p}^*\parallel^2-\parallel \boldsymbol{p}(n+1)-\boldsymbol{p}^*\parallel^2\\
\geq&\parallel \boldsymbol{p}(n+1)-\boldsymbol{p}(n)\parallel^2+\parallel \boldsymbol{p}(n)-\boldsymbol{p}^*\parallel^2-\parallel \boldsymbol{p}(n+1)-\boldsymbol{p}^*\parallel^2\\
=&2(\boldsymbol{p}(n+1)-\boldsymbol{p}(n))^T(\boldsymbol{p}^*-\boldsymbol{p}(n))\\
=&2(\boldsymbol{p}(n+1)-\boldsymbol{q}(n))^T(\boldsymbol{p}^*-\boldsymbol{p}(n))+2(D(n)\hat{\boldsymbol{s}}(n))^T(\boldsymbol{p}^*-\boldsymbol{p}(n))\\
=&2(\boldsymbol{p}(n+1)-\boldsymbol{q}(n))^T(\boldsymbol{p}^*-\boldsymbol{q}(n))+2(\boldsymbol{p}(n+1)-\boldsymbol{q}(n))^T(\boldsymbol{q}(n)-\boldsymbol{p}(n))\\
&+2(D(n)\hat{\boldsymbol{s}}(n))^T(\boldsymbol{p}^*-\boldsymbol{p}(n))\\
\geq&2(\boldsymbol{p}(n+1)-\boldsymbol{q}(n))^T(\boldsymbol{q}(n)-\boldsymbol{p}(n))+2(D(n)\hat{\boldsymbol{s}}(n))^T(\boldsymbol{p}^*-\boldsymbol{p}(n))\\
=&2(\boldsymbol{p}(n+1)-\boldsymbol{p}(n))^T(\boldsymbol{q}(n)-\boldsymbol{p}(n))+2(\boldsymbol{p}(n)-\boldsymbol{q}(n))^T(\boldsymbol{q}(n)-\boldsymbol{p}(n))\\
&+2(D(n)\hat{\boldsymbol{s}}(n))^T(\boldsymbol{p}^*-\boldsymbol{p}(n))\\
\geq&-2\parallel \boldsymbol{p}(n+1)-\boldsymbol{p}(n)\parallel\parallel \boldsymbol{q}(n)-\boldsymbol{p}(n)\parallel-2\parallel \boldsymbol{p}(n)-\boldsymbol{q}(n)\parallel^2+2(D(n)\hat{\boldsymbol{s}}(n))^T(\boldsymbol{p}^*-\boldsymbol{p}(n))\\
\geq&-4C^2a^2(n)+2(D(n)\hat{\boldsymbol{s}}(n))^T(\boldsymbol{p}^*-\boldsymbol{p}(n))
\end{aligned}\tag{A.3}$$

where the first inequality follows from (A.2), the second inequality follows from Lemma 2.1(iii), and the last inequality also follows from (A.2).

Rearranging terms in (A.3), we proceed as follows:

$$\begin{aligned}
\parallel \boldsymbol{p}(n+1)-\boldsymbol{p}^*\parallel^2\leq&\parallel \boldsymbol{p}(n)-\boldsymbol{p}^*\parallel^2+5C^2a^2(n)-2(D(n)\hat{\boldsymbol{s}}(n))^T(\boldsymbol{p}^*-\boldsymbol{p}(n))\\
=&\parallel \boldsymbol{p}(n)-\boldsymbol{p}^*\parallel^2+5C^2a^2(n)-2\sum_{i\in\mathcal{N}}(a_i(n)\hat{\boldsymbol{s}}_{\boldsymbol{i}}(n))^T(\boldsymbol{p}_{\boldsymbol{i}}^*-\boldsymbol{p}_{\boldsymbol{i}}(n))\\
\leq&\parallel \boldsymbol{p}(n)-\boldsymbol{p}^*\parallel^2+5C^2a^2(n)+2\sum_{i\in\mathcal{N}}a_i(n)\epsilon_i(n)\\
&+2\sum_{i\in\mathcal{N}}a_i(n)(R_i(\boldsymbol{p}_{\boldsymbol{i}}(n),\boldsymbol{p}_{-\boldsymbol{i}}(n)|\boldsymbol{g}(n+1))-R_i(\boldsymbol{p}_{\boldsymbol{i}}^*,\boldsymbol{p}_{-\boldsymbol{i}}(n)|\boldsymbol{g}(n+1)))\\
\leq&\parallel \boldsymbol{p}(n)-\boldsymbol{p}^*\parallel^2+5C^2a^2(n)+2\sum_{i\in\mathcal{N}}a_i(n)\epsilon_i(n)\\
&+2\sum_{i\in\mathcal{N}}a_i(n)\boldsymbol{s}_{\boldsymbol{i}}(n+1)^T(\boldsymbol{p}_{\boldsymbol{i}}(n)-\boldsymbol{p}_{\boldsymbol{i}}^*)
\end{aligned}\tag{A.4}$$

where the second inequality follows from (2.10), and the last inequality follows from the concavity of $R_i(\cdot,\cdot|\boldsymbol{g})$ with respect to the first argument. This completes the proof.

We further need the following well-known lemma in stochastic approximation theory [52].

Lemma A.2 *Let $\{\mathcal{F}_n\}$ be an increasing sequence of σ-algebras and $e_n,\alpha_n,\beta_n,\eta_n$ be finite, nonnegative, $\mathcal{F}_n$-measurable random variables. If it holds almost surely that $\sum_{n=0}^{\infty}\alpha_n<\infty,\sum_{n=0}^{\infty}\beta_n<\infty$, and*

$$\mathbb{E}(e_{n+1}|\mathcal{F}_n)\leq(1+\alpha_n)e_n+\beta_n-\eta_n, \tag{A.5}$$

then $(e_n)_{n=0}^{\infty}$ *converges and* $\sum_{n=0}^{\infty} \eta_n < \infty$ *almost surely.*

Now taking the conditional expectation $\mathbb{E}[\cdot|\mathcal{F}_n]$ of both sides in (A.1) yields

$$\begin{aligned}
&\mathbb{E}[\| \boldsymbol{p}(n+1) - \boldsymbol{p}^* \|^2 |\mathcal{F}_n] \\
\leq &\mathbb{E}[\| \boldsymbol{p}(n) - \boldsymbol{p}^* \|^2 |\mathcal{F}_n] + \mathbb{E}[5C^2a^2(n) + 2\sum_{i\in\mathcal{N}} a_i(n)\epsilon_i(n)|\mathcal{F}_n] \\
&- \mathbb{E}[2\sum_{i\in\mathcal{N}} a_i(n)\boldsymbol{s_i}(n+1)^T(\boldsymbol{p_i^*} - \boldsymbol{p_i}(n))|\mathcal{F}_n] \\
= &\| \boldsymbol{p}(n) - \boldsymbol{p}^* \|^2 + \mathbb{E}[5C^2a^2(n) + 2\sum_{i\in\mathcal{N}} a_i(n)\epsilon_i(n)|\mathcal{F}_n] \\
&- \mathbb{E}[2\sum_{i\in\mathcal{N}} a_i(n)\boldsymbol{s_i}(n+1)^T(\boldsymbol{p_i^*} - \boldsymbol{p_i}(n))|\mathcal{F}_n],
\end{aligned} \tag{A.6}$$

We see (A.5) is satisfied by substituting $e_n = \| \boldsymbol{p}(n) - \boldsymbol{p}^* \|^2$, $\alpha_n = 0$, $\beta_n = \mathbb{E}[5C^2a^2(n) + 2\sum_{i\in\mathcal{N}} a_i(n)\epsilon_i(n)|\mathcal{F}_n]$, and $\eta_n = \mathbb{E}[2\sum_{i\in\mathcal{N}} a_i(n)\boldsymbol{s_i}(n+1)^T(\boldsymbol{p_i^*} - \boldsymbol{p_i}(n))|\mathcal{F}_n]$.

Clearly, e_n, α_n, and β_n are finite, nonnegative, $\mathcal{F}_n$-measurable, and $\sum_{n=0}^{\infty} \alpha_n = 0 < \infty$. $\sum_{n=0}^{\infty} \beta_n < \infty$ follows from assumption (ii) and (iii) in Theorem 2.1. η_n is also obviously finite, and $\mathcal{F}_n$-measurable. The nonnegativeness of η_n follows from Lemma 2.2 and assumption (i) in Theorem 2.1. Thus, all the conditions in Lemma A.2 are satisfied. We conclude that $e_n = \| \boldsymbol{p}(n) - \boldsymbol{p}^* \|^2$ converges almost surely, and that

$$\sum_{n=0}^{\infty} \eta_n = \sum_{n=0}^{\infty} \mathbb{E}[2\sum_{i\in\mathcal{N}} a_i(n)\boldsymbol{s_i}(n+1)^T(\boldsymbol{p_i^*} - \boldsymbol{p_i}(n))|\mathcal{F}_n] \tag{A.7}$$

is finite almost surely.

We still need to show $e_n = \| \boldsymbol{p}(n) - \boldsymbol{p}^* \|^2$ converges to 0 almost surely. If this is not true, the event $A = \{w : \lim_{n\to\infty} e_n(w) = e(w) > 0\}$ has nonzero probability where w is a trajectory on the associated probability space. Then for any $w \in A$, there exists a large enough $N(w)$ such that

$$\begin{aligned}
\sum_{n=N(w)}^{\infty} 2\sum_{i\in\mathcal{N}} a_i(n)\boldsymbol{s_i}(n+1)^T(\boldsymbol{p_i^*} - \boldsymbol{p_i}(n)) &\geq 2\sum_{n=N(w)}^{\infty} \min_{i\in\mathcal{N}} a_i(n) \min_{\boldsymbol{s}} \tau(\boldsymbol{s}) \| \boldsymbol{p}^* - \boldsymbol{p}(n) \|^2 \\
&\geq 2\sum_{n=N(w)}^{\infty} \min_{i\in\mathcal{N}} a_i(n) \min_{\boldsymbol{s}} \tau(\boldsymbol{s})\, e(w) = +\infty
\end{aligned} \tag{A.8}$$

where the first inequality follows from Lemma 2.2 and the last inequality follows from assumption (ii) in Theorem 2.1. Since (A.8) happens with nonzero probability, the random sum $\sum_{n=0}^{\infty} \eta_n$ in (A.7) cannot be finite almost surely, resulting in a contradiction. Hence, we conclude that $(\boldsymbol{p}(n))_{n=0}^{\infty}$ converges to $\boldsymbol{p}^*$ almost surely. This completes the proof.

□ **End of chapter.**

Appendix B

Proof of Theorem 2.2

The proof essentially follows the same arguments as the proof of Theorem 2.1. Specifically, we first observe that

$$
\begin{aligned}
&\mathbb{E}[\| \boldsymbol{p}(n+1) - \boldsymbol{p}^* \|^2 |\mathcal{F}_n] \\
=&\mathbb{E}[\| \mathcal{P}_\Phi(\boldsymbol{p}(n) + D(n)\hat{\boldsymbol{s}}(n)) - \mathcal{P}_\Phi(\boldsymbol{p}^* + D(n)\bar{\boldsymbol{s}}(\boldsymbol{p}^*)) \|^2 |\mathcal{F}_n] \\
\leq&\mathbb{E}[\| \boldsymbol{p}(n) - \boldsymbol{p}^* + D(n)(\hat{\boldsymbol{s}}(n) - \bar{\boldsymbol{s}}(\boldsymbol{p}^*)) \|^2 |\mathcal{F}_n] \\
=&\mathbb{E}[\| \boldsymbol{p}(n) - \boldsymbol{p}^* + D(n)(\bar{\boldsymbol{s}}(\boldsymbol{p}(n)) + \boldsymbol{\theta}(n) - \bar{\boldsymbol{s}}(\boldsymbol{p}^*)) \|^2 |\mathcal{F}_n] \\
=& \| \boldsymbol{p}(n) - \boldsymbol{p}^* \|^2 + \sum_{i\in\mathcal{N}} a_i^2(n) \| \bar{\boldsymbol{s}}_i(\boldsymbol{p}(n)) - \bar{\boldsymbol{s}}_i(\boldsymbol{p}^*) \|^2 \\
&+ 2\sum_{i\in\mathcal{N}} a_i(n)(\bar{\boldsymbol{s}}_i(\boldsymbol{p}(n)) - \bar{\boldsymbol{s}}_i(\boldsymbol{p}^*))^T(\boldsymbol{p}_i(n) - \boldsymbol{p}_i^*) \\
&+ \mathbb{E}[\| D(n)\boldsymbol{\theta}(n) \|^2 |\mathcal{F}_n] + 2(\boldsymbol{p}(n) - \boldsymbol{p}^*)^T\mathbb{E}[D(n)\boldsymbol{\theta}(n)|\mathcal{F}_n] \\
&+ 2(\bar{\boldsymbol{s}}(\boldsymbol{p}(n)) - \bar{\boldsymbol{s}}(\boldsymbol{p}^*))^T\mathbb{E}[D^2(n)\boldsymbol{\theta}(n)|\mathcal{F}_n] \\
=& \| \boldsymbol{p}(n) - \boldsymbol{p}^* \|^2 + \mathbb{E}[\| D(n)\boldsymbol{\theta}(n) \|^2 |\mathcal{F}_n] + \sum_{i\in\mathcal{N}} a_i^2(n) \| \bar{\boldsymbol{s}}_i(\boldsymbol{p}(n)) - \bar{\boldsymbol{s}}_i(\boldsymbol{p}^*) \|^2 \\
&+ 2\sum_{i\in\mathcal{N}} a_i(n)(\bar{\boldsymbol{s}}_i(\boldsymbol{p}(n)) - \bar{\boldsymbol{s}}_i(\boldsymbol{p}^*))^T(\boldsymbol{p}_i(n) - \boldsymbol{p}_i^*) \\
\leq& \| \boldsymbol{p}(n) - \boldsymbol{p}^* \|^2 + \mathbb{E}[\| D(n)\boldsymbol{\theta}(n) \|^2 |\mathcal{F}_n] + \max_{i\in\mathcal{N}} a_i^2(n) \sum_{i\in\mathcal{N}} \| \bar{\boldsymbol{s}}_i(\boldsymbol{p}(n)) - \bar{\boldsymbol{s}}_i(\boldsymbol{p}^*) \|^2 \\
&+ 2\sum_{i\in\mathcal{N}} a_i(n)(\bar{\boldsymbol{s}}_i(\boldsymbol{p}(n)) - \bar{\boldsymbol{s}}_i(\boldsymbol{p}^*))^T(\boldsymbol{p}_i(n) - \boldsymbol{p}_i^*) \\
\leq& \| \boldsymbol{p}(n) - \boldsymbol{p}^* \|^2 + \mathbb{E}[\| D(n)\boldsymbol{\theta}(n) \|^2 |\mathcal{F}_n] + L^2 \max_{i\in\mathcal{N}} a_i^2(n) \sum_{i\in\mathcal{N}} \| (\boldsymbol{p}_i(n) - \boldsymbol{p}_i^*) \|^2 \\
&- 2\min_{i\in\mathcal{N}} a_i(n)\tau(\bar{\boldsymbol{s}}) \| \boldsymbol{p}(n) - \boldsymbol{p}^* \|^2 \\
=& \| \boldsymbol{p}(n) - \boldsymbol{p}^* \|^2 + \sum_{i\in\mathcal{N}} a_i^2(n)\mathbb{E}[\| \boldsymbol{\theta}_i(n) \|^2 |\mathcal{F}_n] - (2\tau(\bar{\boldsymbol{s}}) \min_{i\in\mathcal{N}} a_i(n) \\
&- L^2 \max_{i\in\mathcal{N}} a_i^2(n)) \| \boldsymbol{p}(n) - \boldsymbol{p}^* \|^2 .
\end{aligned} \tag{B.1}
$$

Here the first equality follows from (2.16). The first inequality follows from (2.17). The fourth equality follows from assumption $\mathbb{E}[\boldsymbol{\theta}(n)|\mathcal{F}_n] = \mathbf{0}$. The last inequality follows from Lemma 2.3, assumption (i) in Theorem 2.2, and Lemma 2.2.

Substitute $e_n = \| \boldsymbol{p}(n) - \boldsymbol{p}^* \|^2$, $\alpha_n = 0$, $\beta_n = \sum_{i\in\mathcal{N}} a_i^2(n)\mathbb{E}[\| \boldsymbol{\theta_i}(n) \|^2 |\mathcal{F}_n]$, and $\eta_n = (2\tau(\bar{\boldsymbol{s}}) \min_{i\in\mathcal{N}} a_i(n) - L^2 \max_{i\in\mathcal{N}} a_i^2(n)) \| \boldsymbol{p}(n) - \boldsymbol{p}^* \|^2$. It is straightforward to verify that all the assumptions in Lemma A.2 are satisfied. We conclude that $e_n = \| \boldsymbol{p}(n) - \boldsymbol{p}^* \|^2$ converges almost surely, and that

$$\sum_{n=0}^{\infty} \eta_n = \sum_{n=0}^{\infty} (2\tau(\bar{\boldsymbol{s}}) \min_{i\in\mathcal{N}} a_i(n) - L^2 \max_{i\in\mathcal{N}} a_i^2(n)) \| \boldsymbol{p}(n) - \boldsymbol{p}^* \|^2 \tag{B.2}$$

is finite almost surely.

We further claim that $e_n = \| \boldsymbol{p}(n) - \boldsymbol{p}^* \|^2$ converges to 0 almost surely. Observe that $2\tau(\bar{\boldsymbol{s}}) \min_{i\in\mathcal{N}} a_i(n) - L^2 \max_{i\in\mathcal{N}} a_i^2(n)$ is bounded away from 0 by assumption (ii), this claim holds by following a similar argument by contrapositive as that of the proof for Theorem 2.1.

□ **End of chapter.**

Appendix C

Proof of Proposition 3.1

It is equivalent to show that the rate vector $\boldsymbol{R_i^*}$ is the optimal solution to the following optimization problem:

$$\begin{aligned} \text{minimize} \quad & S_i(\boldsymbol{R_i}) \\ \text{subject to} \quad & R_i^{\pi_i(n)} \leq \ln(1+\frac{g_i^{\pi_i(n)}\bar{p}^{\pi_i(n)}}{N_i W}), \forall n \in \mathbb{N}, \\ & \sum_{n\in\mathbb{N}} R_i^{\pi_i(n)} \geq R_i^{min}, \\ & R_i^{\pi_i(n)} \geq 0, \forall n \in \mathbb{N}. \end{aligned} \tag{C.1}$$

We prove the *only if* part first. Toward this end, we note that the necessary condition for optimality is that $\boldsymbol{R_i^*}$ should satisfy the Karush-Kuhn-Tucker (KKT) conditions [63]. Consider the Lagrangian:

$$\begin{aligned} & L_i(\boldsymbol{R_i}, \boldsymbol{\gamma}, \boldsymbol{\eta}, \zeta) \\ & = S_i(\boldsymbol{R_i}) + \sum_{n\in\mathbb{N}} \gamma^{\pi_i(n)}(R_i^{\pi_i(n)} - \ln(1+\frac{g_i^{\pi_i(n)}\bar{p}^{\pi_i(n)}}{N_i W})) - \sum_{n\in\mathbb{N}} \eta^{\pi_i(n)} R_i^{\pi_i(n)} + \zeta(R_i^{min} - \sum_{n\in\mathbb{N}} R_i^{\pi_i(n)}), \end{aligned} \tag{C.2}$$

where $\boldsymbol{\gamma}, \boldsymbol{\eta}$, and ζ are the corresponding Lagrange multipliers. Then the KKT conditions are given by

$$\frac{\partial S_i}{\partial R_i^{\pi_i(n)}}(\boldsymbol{R_i}) + \gamma_i^{\pi_i(n)} - \eta_i^{\pi_i(n)} - \zeta = 0, \forall n \in \mathbb{N} \tag{C.3}$$

$$\gamma_i^{\pi_i(n)} \geq 0,\ R_i^{\pi_i(n)} \leq \ln(1+\frac{g_i^{\pi_i(n)}\bar{p}^{\pi_i(n)}}{N_iW}),\ \gamma_i^{\pi_i(n)}(R_i^{\pi_i(n)} - \ln(1+\frac{g_i^{\pi_i(n)}\bar{p}^{\pi_i(n)}}{N_iW})) = 0, \forall n \in \mathbb{N} \tag{C.4}$$

$$\eta_i^{\pi_i(n)} \geq 0,\ R_i^{\pi_i(n)} \geq 0,\ \eta_i^{\pi_i(n)} R_i^{\pi_i(n)} = 0, \forall n \in \mathbb{N} \tag{C.5}$$

$$\zeta \geq 0, \sum_{n\in\mathbb{N}} R_i^{\pi_i(n)} \geq R_i^{min}, \zeta(\sum_{n\in\mathbb{N}} R_i^{\pi_i(n)} - R_i^{min}) = 0, \tag{C.6}$$

where

$$\frac{\partial S_i}{\partial R_i^{\pi_i(n)}}(\boldsymbol{R_i}) = \frac{1}{g_i^{\pi_i(n)}}\exp(\sum_{l=n}^{N} R_i^{\pi_i(l)}) + \sum_{m=1}^{n-1}\frac{1}{g_i^{\pi_i(m)}}\exp(\sum_{l=m+1}^{N} R_i^{\pi_i(l)})\cdot(\exp(R_i^{\pi_i(m)})-1). \tag{C.7}$$

Now we claim that $R_i^{\pi_i(n)*} = \ln(1+\frac{g_i^{\pi_i(n)}\bar{p}^{\pi_i(n)}}{N_iW}), \forall n < n_0 - 1$. If this claim is not true, $\exists \tilde{n} < n_0 - 1$ such that $0 < R_i^{\pi_i(\tilde{n})*} < \ln(1+\frac{g_i^{\pi_i(\tilde{n})}\bar{p}^{\pi_i(\tilde{n})}}{N_iW})$. Then it follows that the corresponding Lagrangian multiplier $\gamma_i^{\pi_i(\tilde{n})} = 0$ and $\eta_i^{\pi_i(\tilde{n})} = 0$ by complementary slackness conditions in (C.4) and (C.5), respectively. Thus,

$$\begin{aligned}
0 =& \frac{\partial S_i}{\partial R_i^{\pi_i(\tilde{n}+1)}}(\boldsymbol{R_i^*}) + \gamma_i^{\pi_i(\tilde{n}+1)} - \eta_i^{\pi_i(\tilde{n}+1)} - \zeta \\
=& \frac{\partial S_i}{\partial R_i^{\pi_i(\tilde{n}+1)}}(\boldsymbol{R_i^*}) + \gamma_i^{\pi_i(\tilde{n}+1)} - \eta_i^{\pi_i(\tilde{n}+1)} - (\frac{\partial S_i}{\partial R_i^{\pi_i(\tilde{n})}}(\boldsymbol{R_i^*}) + \gamma_i^{\pi_i(\tilde{n})} - \eta_i^{\pi_i(\tilde{n})}) \\
=& \frac{\partial S_i}{\partial R_i^{\pi_i(\tilde{n}+1)}}(\boldsymbol{R_i^*}) + \gamma_i^{\pi_i(\tilde{n}+1)} - \frac{\partial S_i}{\partial R_i^{\pi_i(\tilde{n})}}(\boldsymbol{R_i^*}) \\
=& (\frac{1}{g_i^{\pi_i(\tilde{n}+1)}} - \frac{1}{g_i^{\pi_i(\tilde{n})}})\exp(R_i^{\pi_i(\tilde{n})*}) + \gamma_i^{\pi_i(\tilde{n}+1)} \\
>& 0,
\end{aligned} \tag{C.8}$$

where the first two equalities follow from (C.3), the third equality follows from the facts that $R_i^{\pi_i(\tilde{n}+1)*} > 0$ and thus $\eta_i^{\pi_i(\tilde{n}+1)} = 0$ by complementary slackness conditions in (C.4) and $\gamma_i^{\pi_i(\tilde{n})} = 0$ and $\eta_i^{\pi_i(\tilde{n})} = 0$, the fourth equality follows from (C.7), and the inequality follows from the assumption $g_i^{\pi_i(\tilde{n})} > g_i^{\pi_i(\tilde{n}+1)}$ and the non-negativeness of $\gamma_i^{\pi_i(\tilde{n}+1)}$ in (C.4). Clearly, (C.8) gives us the desired contradiction.

We further claim that $R_i^{\pi_i(n)*} = 0, \forall n > n_0 - 1$. If this claim is not true, $\exists \hat{n} > n_0 - 1$ such that $0 < R_i^{\pi_i(\hat{n})*} \leq \ln(1 + \frac{g_i^{\pi_i(\hat{n})} \bar{p}^{\pi_i(\hat{n})}}{N_i W})$. It follows that $\eta_i^{\pi_i(\hat{n})} = 0$ by complementary slackness conditions in (44). We also have

$$\begin{aligned}
\sum_{n \in \mathbb{N}} R_i^{\pi_i(n)*} &= \sum_{n < n_0 - 1} R_i^{\pi_i(n)*} + R_i^{\pi_i(n_0-1)*} + \sum_{n > n_0 - 1} R_i^{\pi_i(n)*} \\
&= \sum_{n < n_0 - 1} R_i^{\pi_i(n)*} + R_i^{min} - \sum_{n < n_0 - 1} R_i^{\pi_i(n)*} + \sum_{n > n_0 - 1} R_i^{\pi_i(n)*} \\
&\geq R_i^{min} + R_i^{\pi_i(\hat{n})*} > 0,
\end{aligned} \tag{C.9}$$

which implies that $\zeta = 0$ by complementary slackness conditions in (C.6). Thus,

$$0 = \frac{\partial S_i}{\partial R_i^{\pi_i(\hat{n})}}(\boldsymbol{R_i^*}) + \gamma_i^{\pi_i(\hat{n})} - \eta_i^{\pi_i(\hat{n})} - \zeta = \frac{\partial S_i}{\partial R_i^{\pi_i(\hat{n})}}(\boldsymbol{R_i^*}) + \gamma_i^{\pi_i(\hat{n})} > 0, \tag{C.10}$$

resulting in a contradiction.

Hence, we conclude that $\boldsymbol{R_i^*}$ is the unique solution satisfying the KKT conditions given at the beginning of this proof. This completes the proof of *only if* part.

Note that problem (C.1) is a convex optimization problem since it can be readily checked that the associated objective function and the inequality constraints are continuously differentiable convex functions. Then it follows that KKT conditions are also sufficient for the optimality [63]. This completes the proof of *if* part.

□ **End of chapter.**

Appendix D

Proof of Proposition 4.4

The existence of $\tilde{\boldsymbol{F}}$ is justified by Lemma 4.2. We can further compute $\tilde{F}_j^{(i)}(p_j^{(i)}, \boldsymbol{p}_{-j}^{(i)})$ as

$$\tilde{F}_j^{(i)}(p_j^{(i)}, \boldsymbol{p}_{-j}^{(i)}) = -\frac{w_j^{(i)} K_j^{(i)} g_{i,j}^{(i)}}{K_j^{(i)} g_{i,j}^{(i)} p_j^{(i)} + \mathcal{I}_j^{(i)}(\boldsymbol{p}_{-j}^{(i)})} + \rho_j^{(i)}((g_+^{(i)}(\boldsymbol{p}^{(i)}))^\gamma + \epsilon)^{\frac{1-\gamma}{\gamma}}(g_+^{(i)}(\boldsymbol{p}^{(i)}))^{\gamma-1}. \quad \text{(D.1)}$$

(i). Consider an arbitrary NE $\tilde{\boldsymbol{p}}^*$. Given $\tilde{\boldsymbol{p}}_{-j}^{(i)*}$, for any receiver $D_j^{(i)}$, we know that $\tilde{J}_j^{(i)}$ is concave in $\tilde{p}_j^{(i)}$ on the set $\boldsymbol{\Psi}_j^{(i)}$ by Lemma 4.2. Then we have $(\tilde{p}_j^{(i)} - \tilde{p}_j^{(i)*})\tilde{F}_j^{(i)}(\tilde{p}_j^{(i)*}, \tilde{\boldsymbol{p}}_{-j}^{(i)*}) \geq 0, \forall \tilde{p}_j^{(i)} \in \boldsymbol{\Psi}_j^{(i)}$, which follows from the optimality condition [Prop. 3.1, Section 3] in [7]. Summing over $j \in \mathbb{M}_i, i \in \mathbb{M}$, we conclude that $(\tilde{\boldsymbol{p}} - \tilde{\boldsymbol{p}}^*)^T \tilde{\boldsymbol{F}}(\tilde{\boldsymbol{p}}^*) \geq 0, \forall \tilde{\boldsymbol{p}} \in \boldsymbol{\Psi}$, implying that $\tilde{\boldsymbol{p}}^*$ is a solution to $VI(\boldsymbol{\Psi}, \tilde{\boldsymbol{F}})$. Conversely, for any $\tilde{\boldsymbol{p}}^* \in SOL(\boldsymbol{\Psi}, \tilde{\boldsymbol{F}})$, we have $(\tilde{\boldsymbol{p}} - \tilde{\boldsymbol{p}}^*)^T \tilde{\boldsymbol{F}}(\tilde{\boldsymbol{p}}^*) \geq 0, \forall \tilde{\boldsymbol{p}} \in \boldsymbol{\Psi}$. Then for $\forall j \in \mathbb{M}_i, \forall i \in \mathbb{M}, \forall \tilde{\boldsymbol{p}} = (\tilde{p}_j^{(i)}, \tilde{\boldsymbol{p}}_{-j}^{(i)*}) \in \boldsymbol{\Psi}$, $(\tilde{p}_j^{(i)} - \tilde{p}_j^{(i)*})\tilde{F}_j^{(i)}(\tilde{p}_j^{(i)*}, \tilde{\boldsymbol{p}}_{-j}^{(i)*}) \geq 0$, i.e., $p_j^{(i)*} \in B_j^{(i)}(\boldsymbol{p}_{-j}^{(i)*})$. Hence, $\tilde{\boldsymbol{p}}^*$ is also an NE of game $\mathbb{G}^\dagger(\boldsymbol{\rho})$.

(ii). By Lemma 4.2 and similar arguments in the proof of Proposition 4.1, $SOL(\boldsymbol{\Psi}, \tilde{\boldsymbol{F}})$ is nonempty and compact. Combining the result in (i), the existence of NE in game $\mathbb{G}^\dagger(\boldsymbol{\rho})$ follows.

(iii). By Lemma 4.2, we can further compute $\nabla_{p_l^{(k)}} \tilde{F}_j^{(i)}(\boldsymbol{p})$ given by

$$\nabla_{p_l^{(k)}} \tilde{F}_j^{(i)}(\boldsymbol{p}) = \frac{w_j^{(i)} K_j^{(i)} g_{i,j}^{(i)} A_l^{(k)}}{(K_j^{(i)} g_{i,j}^{(i)} p_j^{(i)} + \mathcal{I}_j^{(i)}(\boldsymbol{p}_{-j}^{(i)}))^2} + \rho_j^{(i)} B^{(k)} \delta_{ki}, \quad \text{(D.2)}$$

where $B^{(i)}$ is given in (3.20), and $\delta_{ki} = 1$ if $k = i$ and 0 otherwise. When conditions (4.11) and (4.12) are satisfied, a sufficient condition for Jacobian matrix $\nabla \tilde{\boldsymbol{F}}(\boldsymbol{p})$ to be positive definite is

$$B^{(i)}\Big(\sum_{j \in \mathbb{M}_i, k \neq j} \rho_k^{(i)} - \rho_j^{(i)}\Big) \leq \delta, \forall j \in \mathbb{M}_i, \forall i \in \mathbb{M}, \quad \text{(D.3)}$$

where δ is some sufficiently small positive real number. Note that the existence of δ is guaranteed by the density of the reals. Now if $\exists i \in \mathbb{M}$ such that $g_+^{(i)}(\boldsymbol{p}^{(i)}) = 0$, then $B^{(i)} = 0$ and $(D.3)$ follows trivially. If $\exists i \in \mathbb{M}$ such that $g_+^{(i)}(\boldsymbol{p}^{(i)}) \neq 0$, then

$$\begin{aligned} B^{(i)}(\sum_{j\in\mathbb{M}_i,k\neq j} \rho_k^{(i)} - \rho_j^{(i)}) =& \epsilon(\sum_{j\in\mathbb{M}_i,k\neq j} \rho_k^{(i)} - \rho_j^{(i)})(\gamma-1)((g_+^{(i)}(\boldsymbol{p}^{(i)}))^{\gamma}+\epsilon)^{\frac{1-2\gamma}{\gamma}}(g_+^{(i)}(\boldsymbol{p}^{(i)}))^{\gamma-2} \\ \leq& \epsilon(\sum_{j\in\mathbb{M}_i,k\neq j} \rho_k^{(i)} - \rho_j^{(i)})(\gamma-1)(g_+^{(i)}(\boldsymbol{p}^{(i)}))^{-1-\gamma} \leq \delta. \end{aligned} \tag{D.4}$$

Clearly, if we select ϵ such that $\epsilon \leq \min_{j\in\mathbb{M}_i,i\in\mathbb{M}} \delta(\sum_{j\in\mathbb{M}_i,k\neq j} \rho_k^{(i)} - \rho_j^{(i)})^{-1}(\gamma-1)^{-1}(g_+^{(i)}(\boldsymbol{p}^{(i)}))^{1+\gamma}$, then $(D.3)$ holds. Thus, the existence of a unique NE $\tilde{\boldsymbol{p}}^*$ in game $\mathbb{G}^{\dagger}(\boldsymbol{\rho})$ follows from a similar argument in the proof of Proposition 4.2.

(iv). Define mapping $\mathcal{R}_j^{(i)} : \mathbb{R}^N \mapsto \mathbb{R}$ as $\mathcal{R}_j^{(i)}(\boldsymbol{p}) = p_j^{(i)} - \alpha \tilde{F}_j^{(i)}(p_j^{(i)}, \boldsymbol{p}_{-j}^{(i)})$, and mapping $\mathcal{T}_j^{(i)} : \mathbb{R}^N \mapsto \mathbb{R}$ as $\mathcal{T}_j^{(i)}(\boldsymbol{p}) = [\mathcal{R}_j^{(i)}(\boldsymbol{p})]^+_{\boldsymbol{\Psi}_j^{(i)}}$, where $[x]^+_{\boldsymbol{\Psi}_j^{(i)}}$ denotes the projection of x on $\boldsymbol{\Psi}_j^{(i)}$. We further let $\mathcal{R}(\boldsymbol{p}) = [\mathcal{R}_1^{(1)}(\boldsymbol{p}), ..., \mathcal{R}_{N_M}^{(M)}(\boldsymbol{p}))]^T$ and $\mathcal{T}(\boldsymbol{p}) = [\mathcal{T}_1^{(1)}(\boldsymbol{p}), ..., \mathcal{T}_{N_M}^{(M)}(\boldsymbol{p}))]^T$. Then $\forall \boldsymbol{p}, \boldsymbol{q} \in \boldsymbol{\Psi}$,

$$\begin{aligned} &\| \mathcal{T}(\boldsymbol{p}) - \mathcal{T}(\boldsymbol{q}) \|_2^2 = \| [\mathcal{R}(\boldsymbol{p})]^+ - [\mathcal{R}(\boldsymbol{q})]^+ \|_2^2 \leq \| \mathcal{R}(\boldsymbol{p}) - \mathcal{R}(\boldsymbol{q}) \|_2^2 = \| \boldsymbol{p} - \alpha \tilde{\boldsymbol{F}}(\boldsymbol{p}) \\ &- (\boldsymbol{q} - \alpha \tilde{\boldsymbol{F}}(\boldsymbol{q})) \|_2^2 = \| \boldsymbol{p} - \boldsymbol{q} \|_2^2 + \alpha^2 \| \tilde{\boldsymbol{F}}(\boldsymbol{p}) - \tilde{\boldsymbol{F}}(\boldsymbol{q}) \|_2^2 - 2\alpha(\tilde{\boldsymbol{F}}(\boldsymbol{p}) - \tilde{\boldsymbol{F}}(\boldsymbol{q}))^T(\boldsymbol{p} - \boldsymbol{q}) \\ &\leq \| \boldsymbol{p} - \boldsymbol{q} \|_2^2 + (\alpha L)^2 \| \boldsymbol{p} - \boldsymbol{q} \|_2^2 - 2\alpha\beta \| \boldsymbol{p} - \boldsymbol{q} \|_2^2 = (L^2\alpha^2 - 2\beta\alpha + 1) \| \boldsymbol{p} - \boldsymbol{q} \|_2^2 . \end{aligned}$$

Here, the first equality follows from the box constraint $\boldsymbol{\Psi}$ which naturally results in decomposition of projection. The first inequality follows from the nonexpansive property of projection (see, e.g., [Prop. 3.2, Section 3] in [7]). Note that by our assumption of the boundedness of channel gains, we can show that $\tilde{\boldsymbol{F}}(\boldsymbol{p})$ is Lipschitz continuous, i.e., $\| \tilde{\boldsymbol{F}}(\boldsymbol{p}) - \tilde{\boldsymbol{F}}(\boldsymbol{q}) \|_2 \leq L \| \boldsymbol{p} - \boldsymbol{q} \|_2$ for some large enough $L > 0$. Also, the positive definite property of Jacobian matrix $\nabla \tilde{\boldsymbol{F}}(\boldsymbol{p})$ implies $\tilde{\boldsymbol{F}}(\boldsymbol{p})$ is strongly monotone, i.e., $(\tilde{\boldsymbol{F}}(\boldsymbol{p}) - \tilde{\boldsymbol{F}}(\boldsymbol{q}))^T(\boldsymbol{p} - \boldsymbol{q}) \geq \beta \| \boldsymbol{p} - \boldsymbol{q} \|_2^2$ where $\beta > 0$. Thus, the second inequality holds. Clearly, there exists $\alpha > 0$ such that $\bar{\alpha} = L^2\alpha^2 - 2\beta\alpha + 1$ lies in $(0, 1)$. Then it follows that $\mathcal{T}(\boldsymbol{p})$ is a contraction mapping by choosing small enough α. Then by [Prop. 5.4, Section 3] in [7], the Jacobi iteration in IP^{2}JA is well defined and its convergence follows.

□ **End of chapter.**

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