An Inventory Model with Stochastic Leadtime, Partial Shipment, and Delivery Information Delay

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摘要

我們研究的是一個含有運輸信息延遲的有限離散庫存繫統. 此繫統含有三種不同的運輸模式和兩種連續的運輸時間. 對于最便宜的隨機運輸模式, 我們用一個隨機比例因子來刻畫它的隨機運輸時間和部分運輸. 緊急運輸模式和正常的運輸模式擁有確定的運輸時間和全部運輸. 如果實現的比例因子比較小, 隻有隨機運輸模式和緊急運輸模式是有啲地運輸模式; 如果實現的比例因子比較閤適, 隻有隨機運輸模式是有啲地運輸模式; 如果實現的比例因子比較大, 隻有隨機運輸模式和正常運輸模式是有啲地運輸模式. 除去一些特殊的比例因子的值, 對于隨機運輸模式來說, S 存儲策略不再是最優的策略. 但是依賴當前狀態的 S 存儲策略依然是緊急運輸模式和正常運輸模式的最優策略. 我們同樣分析瞭當前狀態對于最優值的影響. 並且證明運輸信息的延遲會帶來更多的費用.

Abstract

This paper is concerned with a finite periodic review inventory system with delivery information delay, two consecutive leadtimes, and three delivery modes. The stochastic delivery mode is the cheapest mode with a stochastic leadtime and partial shipments that are generated by a random proportional factor, whereas the emergency delivery mode and the regular delivery mode are more expensive with deterministic leadtimes and full shipment. If the realized proportional factor is low, then only the stochastic delivery mode and the emergency delivery mode are efficient; if the realized proportional factor is medium, then only the stochastic delivery mode is efficient and if the realized proportional factor is high, then only the stochastic delivery mode and the regular delivery mode are efficient. Except for extreme values of the proportional factor, a base-stock policy is not optimal for the stochastic delivery mode, but a state-dependent base-stock policy is still optimal for either the emergency delivery mode or the regular delivery mode. We also investigate the impact of current states on optimal levels, and prove that delivery information delay can incur more costs than when there is no delivery information delay.

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Chapter 1

Introduction

Most companies serve customers with different degrees of demand variability, and deal with suppliers with varying levels of supply uncertainty. Companies with a superior ability to forecast both demand and supply uncertainty can afford to produce and deliver in inexpensive productions and logistics services. On the contrary, companies with an inferior ability to forecast supply and demand ability have to pay by using emergency production and logistics to respond to unexpected surges in demand and shortages in supply.

Many companies have learned the importance of managing a portfolio of supplies, and have recognized the value of learning about customer demand in advance. In addition, the advances in manufacturing technologies, logistics services, and globalization have made it possible for companies to satisfy the needs of their customers from sources that have different prices, leadtimes, and service qualities. The cost of products and logistics services increases when shorter leadtimes and a more accurate delivery schedule are required, and it is therefore critical for companies to manage a portfolio of production and logistics technologies that balances the tradeoff between the quality of demand and supply information and the cost of production and logistics services.

For example, Hewlett-Packard's MOD0 boxes are assembled in its Singapore factory, but the factory allows Hewlett-Packard's distribution centers to choose ocean shipment or emergency air delivery (Beyer and Ward(2000)).

1.1 Related Literature

There is a large body of related research that deals with multiple delivery modes and leadtime uncertainties.

The first work on inventory models with multiple delivery modes is that of Fukuda (1964), who allows both expedited and regular orders to be placed simultaneously and includes a fixed cost for ordering in a multi-period inventory system. The most general modeling work is that of Whittmore and Saunders (1977), who construct a multi-period dynamic model and allow both the long and short leadtimes to be of arbitrary lengths. Whittmore and Saunders also derive two conditions under which one of the delivery modes is useless and only the other delivery mode is useful throughout the planning period. Scheller-Wolf and Tayur (1998) study a Markovian production/inventory model with two consecutive delivery modes, and prove the optimality of a base-stock policy, when the base-stock levels depend on the current state of the underlying Markov chain.

In connecting the forecast updating process with two and three delivery modes, Sethi, Yan and Zhang model the process to be one that is analogous to peeling away the layers of an onion (Sethi, Yan and Zhang 2001 and 2003), in that is the information in any given period which is hidden in the core of the onion, has a number of sources of uncertainties that are resolved successively. With this process of demand information updating, Sethi, Yan and Zhang investigate the optimal inventory decision, and find that the optimal policy for the model with two consecutive delivery modes is a base-stock type. In follow up papers by Feng et al. (2003a, 2003b) It is demonstrated that the base-stock policy is not optimal for systems with more than two delivery modes. We refer the reader to a recent book by Sethi, Yan and Zhang (2005) for more detailed study of inventory models with deterministic multiple-delivery modes and demand information updates.

Another related area of research is two echelon inventory models with expediting deliveries. Lawson and Porteus (2000) study the classical serial multistage model with expediting. By reformulating the Clark-Scarf model, they add the new stage of high holding cost into the system so that the normal leadtime between stages is exactly one period. At each stage, a unit is shipped downstream by either the regular delivery mode or the emergency delivery mode in each period. Lawson and Porteus (2000) show that a top-down base-stock policy is optimal in each period. This model is generalized by Muharremoglu and Tsitsiklis (2003), but their model differs from the additive cost structure of Lawson and Porteus, by allowing a more general cost structure. In Muharremoglu and Tsitsiklis (2003), the extended echelon base-stock policies are shown to be optimal.

The problem becomes much more difficult with a stochastic delivery mode than when delivery times are deterministic. Most of the work on this problem focuses on modeling the stochastic leadtime with the assumptions of no-order crossing, no partial shipment, and independence of supply and demand. Kaplan (1970) introduces a stochastic supply process, and focuses on a finite periodic review system that proves the optimality of the (s, S) policy. Ehrhardt (1984) extends the optimality to an infinite system with the objectives of minimizing the discounted cost and average cost.

Song and Zipkin (1996) generalize these stochastic-leadtime models and model an exogenous supply process as a queue, which is described as a discrete time Markov process. Using the notion of currently complete supply information, they demonstrate that the optimal policy is a state-dependent base-stock policy and obtain the interesting result that the base-stock level need not increase with the leadtime. Chen and Song (2001) consider a multi-stage serial inventory system with Markov-modulated demand, in which the Markov-modulated leadtime can be regarded as Markov-modulated demand. They show that the optimal policy is an echelon base-stock policy with state-dependent order up to levels.

Recently, Chen and Yu (2004) model the supply process with a finite-state Markov chain. Although it can be considered to be a special case of the process of Song and Zipkin (1996), it can be implemented efficiently. Chen and Yu attempt to quantify the potential value of the delivery information, and using numerical examples show that the difference in cost can be as much as 41 percents.

Yet another related area is that of single stage models with multiple suppliers.

Gerchak and Parlar (1990) discuss the issue of two suppliers with random yield in EOQ setting. Parlar and Wang (1993) introduce two suppliers into a single-period model with both random supply and demand. Ramasesh et al (1991) analyze the dual sourcing problem for an (s, Q) system with random leadtimes. Ramasesh et al assume that both suppliers are identical, and demonstrate that significant benefits cause accrued from dual sourcing when leadtimes are random. Anupindi and Akella (1993) discuss three inventory models with two stochastic suppliers. The first assumes that each supplier with a given probability either supplies the full order quantity immediately or supplies nothing. In the second model, each supplier delivers a random proportional order quantity, and the portion of the order that is unfilled is canceled. The third model is the same as the second model, except that the remaining quantity is delivered in the next period. All decisions are made before the decision maker knows the exact delivery information. Anupindi and Akella also show that the optimal policy includes two critical numbers: when the initial inventory exceeds an upper threshold, do not order; when it is between the lower and upper thresholds, order from the less reliable (that with a more certain leadtime) but cheaper supplier; and when it is below the lower threshold, order from both suppliers.

In this paper, we study a finite periodic review inventory system with three delivery modes: a stochastic delivery mode of leadtime k and $k+1$ with a probability of p and $1-p$, respectively; a regular delivery mode with a leadtime of $k+1$; and an emergency delivery mode with a leadtime of k. We model the system with these three delivery modes for cases with and without delivery information delay. We denote the scenario in which the stochastic leadtime is known before the ordering decision is made as the case of no delivery information delay; and denote the scenario in which the stochastic leadtime is known after the order decision is made as the case of delivery information delay. For the cases both with and without information delay we also allow partial shipments for the stochastic delivery mode. We use dynamic programming to derive the form of optimal policy that minimizes the expected cost function with respect to the ordering decision over three sources of supplies. We demonstrate that, both for the case of information delay and that of no-information delay, the optimal policy remains a base-stock type if partial delivery is not allowed. We also characterize the structure property of the optimal policy, and classify the conditions under which the regular and emergency delivery modes are most effective. Moreover, we study the cost structure for a number of specialized cases. The ultimate purpose of this paper is to characterize the form of the optimal policy.

In this paper, we only study the stochastic delivery mode with two possible leadtimes, i.e., k and $k+1$. For cases in which there is a stochastic leadtime with more that two outcomes, we demonstrate the optimal policy is no longer a base-stock type using a counter example. As distinct from the deterministic multiple-delivery modes, we allow a stochastic leadtime, and partial deliveries., and in contrast to the modes of stochastic leadtimes, we focus on the characterization of optimal policies with information delay. Our model differs from models of random yield in that we study a multiple period, dynamic inventory system.

The rest of this paper is organized as follows. The next section presents our notations and model formulation. In section 3, we show our detailed analysis and derive the main results. Section 4 illustrates the nonoptimality of a base-stock policy using numerical examples. Finally, we summarize our contributions and end with a brief description of future work.

Chapter 2

Notations and Model Formulation

We consider a finite stochastic periodic review inventory model with stochastic leadtimes, partial shipments, and delivery information delay, as is shown in Figure 1. The product can be ordered from three sources with different costs and different leadtimes, and the delivery time can be either stochastic or deterministic. For deterministic delivery modes, the leadtime is known to the decision maker before a replenishment decision is made, and the order will be fulfilled in one batch. In the stochastic delivery mode, in contrast, the partial shipment is allowed, and one order can also be fulfilled by two deliveries in two consecutive periods. The proportional factor can also be random. Moreover, the information about the delivery schedule, including delivery time and quantities, may not be known to the decision maker before a replenishment decision is made. We regard this situation as case of the delivery information delay. We assume that the customer demand is random and that if the realized demand is unsatisfied, then the negative amount will be fulfilled in the next period. For each period,

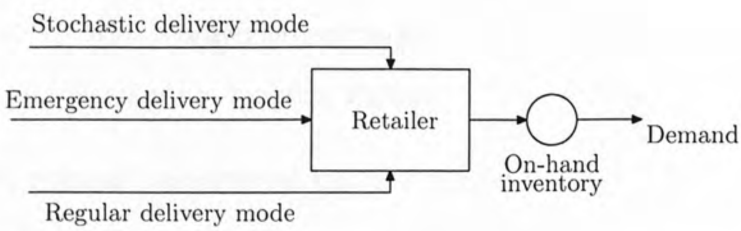

Figure 2.1: The inventory model with three delivery modes

the holding/shortage cost is evaluated. Our objectives are to find an optimal replenishment policy that minimizes the expected total costs and to derive its structural characteristics.

For convenience and simplicity, we use the following notations in this paper.

- $< 1, N >$: Time horizon, $< 1, N >= \{1, 2, \cdots, N\}$.
- D_t: The demand of period t with the density $f_t(\cdot)$ and distribution function $F_t(\cdot)$.
- $D[t, t+L]$: The total demand over periods t, $t+1$, ..., $t+L$.
- k: The leadtime of the emergency delivery mode.
- $k+1$: The leadtime of the regular delivery mode.
- c_s: The unit purchase cost for the stochastic delivery mode.
- c_e: The unit purchase cost for the emergency delivery mode with leadtime k.
- c_r: The unit purchase cost for the regular delivery mode with leadtime $k+1$.

- $R_t \in [0, 1]$: The random proportional factor in period t with the density $f_{R_t}(\cdot)$ and distribution function $F_{R_t}(\cdot)$.

- r_t: The realized proportional factor in period t.

- q_t^s: The order quantity of the stochastic delivery mode in period t. Random partial shipment $R_t q_t$ (or realized partial shipment $r_t q_t$) will arrive at period $t + k$, and the remaining part $q_t - R_t q_t$ (or the remaining realized partial shipment $q_t - r_t q_t$) will arrive at period $t + k + 1$.

- q_t^e: The order quantity from the emergency delivery mode in period t.

- q_t^r: The order quantity from the regular delivery mode in period t.

- x_t: The on-hand inventory at the beginning of period t.

- x_{N+1}: The on-hand inventory at the end of the last period N.

- I_t: The in-delivery order quantities at the beginning of period t.

- $\xi_t = x_t + I_t$: The inventory position at the beginning of period t, where $\xi_1 = x_1$ is the initial inventory position in the first period.

- y_t^s: The inventory position immediately after an order is made from the stochastic delivery mode in period t.

- y_t^e: The inventory position immediately after an order is made from the emergency delivery mode in period t.

- y_t^r: The inventory position immediately after an order is made from the regular delivery mode in period t.

- $H(\cdot)$: The holding/shortage cost and it is a twice differential convex function.
- $V_t(\cdot|ID)$: The optimal cost function over periods $[t+k, N]$ for the case of delivery information delay.
- $V_t(\cdot|NID)$: The optimal cost function over periods $[t+k, N]$ for the case of no delivery information delay.
- $V_{N+1}(x_{N+1})$: The terminal cost function and it is a twice differential convex function.

In our model, there are two deterministic delivery modes and one stochastic delivery mode. One of the deterministic delivery modes is the emergency delivery mode with the leadtime k, and the other is the regular delivery mode with the leadtime $k+1$. The stochastic delivery mode has a random proportional factor R_t, which belongs to $[0, 1]$. If the order quantity is q_t at period t, then $R_t q_t$ will arrive at period $t+k$ and the remaining portion of the order will arrive at period $t+k+1$.

The random variable R_t can be both deterministic and discrete as well. For example, if $R_t = 1$ with a probability 1, then the leadtime of the stochastic delivery mode is k, and if $R_t = 0$ with a probability 1, then the leadtime of the stochastic delivery mode is $k+1$. These are both deterministic cases. We now look at a discrete case. Assume $R_t = 0$ with a probability 1/2 and $R_t = 1$ with a probability 1/2. There is then a 1/2 probability that the order will be delivered in one batch with a leadtime k or a leadtime $k+1$.

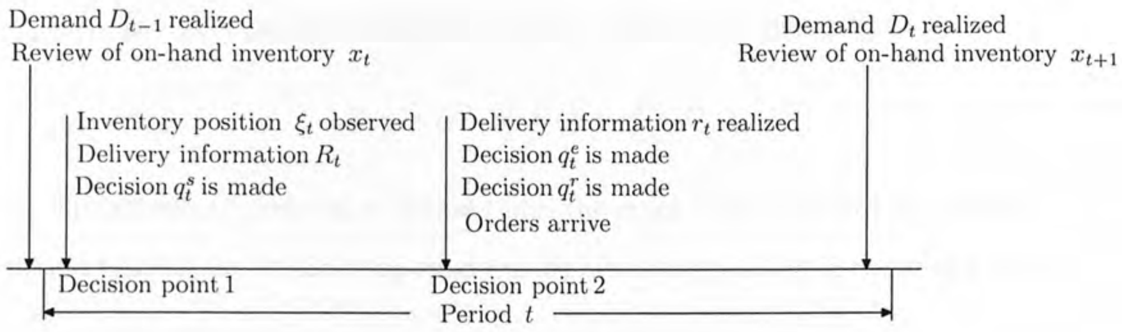

Figure 2.2: Event sequences in the delivery information delay model

Assume that the stochastic delivery mode is the cheapest of the three delivery modes. If the random proportional factor is $R_t = 0$ with a probability 1 or $R_t = 1$ with a probability 1, then the stochastic delivery mode replaces the regular delivery mode or the emergency delivery mode, respectively. The emergency delivery mode must be more expensive than the regular delivery mode because of its shorter leadtime, Otherwise the regular delivery mode is useless. Thus, we need the assumption $c_e > c_r > c_s$.

The sequence of events in period t is as follows. There are two decision points in each period. At the first decision point, the retailer learns the distribution and density functions of the stochastic delivery mode, and then places an order from the stochastic delivery mode. At the second decision-point, the random proportional factor is realized, and the retailer places orders from other deterministic delivery modes. The orders then arrive, and at the end of each period, demand occurs and the holding/shortage cost is evaluated. A timeline of the system dynamics and the ordering decisions is illustrated in Figure 2.

The inventory position dynamic equation can then be defined as

$$\xi_{t+1} = \xi_t + q_t^r + q_t^e + q_t^s - D_t.$$

The inventory position at period t plus the order quantities that are placed at period t minus the demand of period t is the inventory position in the next period.

Optimal Cost Function: Our first task is to choose a sequence of order quantities from the three delivery modes over time so as to minimize the total expected costs for the whole planning horizon. We use the dynamic programming approach to study the problem. For any period $t \in \{1, \cdots, N-k\}$, we are able to obtain the optimal cost function $V_t(\xi_t|ID)$ from period $t+k$ to period N. Note that the shortest leadtime is k, and therefore orders that are placed in period t will be delivered after period $t+k-1$. As there are two decision points in each period, we start with the second decision point, for which the proportional factor is realized as r_t. Let $G_t(q_t^s, \xi_t|r_t)$ denote the optimal cost function at the second decision point.

Thus, when $1 \leq t \leq N-k$,

$$\begin{aligned} G_t(q_t^s, \xi_t|r_t) &= \min_{\substack{y_t^e \geq r_t q_t^s + \xi_t \\ y_t^r \geq y_t^e + (1-r_t)q_t^s}} \{c_e(y_t^e - r_t q_t^s - \xi_t) + c_r(y_t^r - y_t^e - (1-r_t)q_t^s) \\ &\quad + EH(y_t^e - D[t, t+k]) + E_{D_t} V_{t+1}(y_t^r - D_t|ID)\}. \end{aligned} \quad (2.1)$$

On the right-hand side of the equation, the first two items are the ordering costs from the emergency delivery mode and the regular delivery mode, respectively. $EH(y_t^e - D[t, t+k])$ is the expected holding/shortage cost of period $t+k$. $E_{D_t} V_{t+1}(y_t^r - D_t|ID)$ is the dynamic part, which denotes the expected optimal cost function of future periods. We then go back to the first decision point.

Only the distribution and density functions of the random proportional factor are observed, and the optimal cost function can then be defined as

$$V_t(\xi_t|ID) = \min_{q_t^s \geq 0}\{cq_t^s + \int_0^1 G_t(q_t^s, \xi_t|r_t) f_{R_t}(r_t) dr_t\}. \tag{2.2}$$

By combining (2) and the terminal cost function, we can obtain the total optimal cost function

$$V_0(\xi_1|ID) = \sum_{t=1}^{k} H(x_t - D_t) + V_1(\xi_1|ID).$$

Assume that there are no external inputs in any of the period, which means that the in-delivery order quantity at each period contains only what was ordered starting from the first period, and the first item is the summation of the holding/shortage costs of the first k periods. The second term is the optimal cost function from period $k+1$ to period N.

Chapter 3

The Optimal Replenishment Policy

3.1 Preliminaries

Before making an analysis, we introduce the preliminary results. These results will be used throughout the paper.

Lemma 3.1 *$H(y)$ is a twice differential convex function. $D(\in [0, +\infty))$and $R(\in [0, 1])$ are random variables with density functions $f_D(\cdot)$ and $f_R(\cdot)$, respectively. $EH(y - D)$, $EH(Ry)$, and $E_R E_D H(Ry - D)$ are thus convex on y.*

Lemma 3.2 *(Feng et al. 2003a) Let $g(\cdot)$and $h(\cdot)$ be convex functions with $\widetilde{x}$ and $\widetilde{z}$ as their respective unconstrained minima. For a given $b \geq 0$, let $\widehat{a}$ minimize $g(x) + h(x + b)$. Then, for any a,*

$$\min_{x \geq a, z \geq x+b} [g(x) + h(z)] = \begin{cases} g(a \bigvee \widetilde{x}) + h(\widetilde{z} \bigvee (a + b)), & \text{if } \widetilde{z} \geq \widetilde{x} + b, \\ g(\widehat{a} \bigvee a) + h((\widehat{a} \bigvee a) + b) & \text{if } \widetilde{z} < \widetilde{x} + b\ . \end{cases}$$

There is $(x^*(b), z^*(b)) \in \mathcal{S} = \{(x,z)|x \geq a, z \geq x+b\}$, *which is independent of* a *but depends on* b, *that minimizes* $g(x) + h(z)$.

$$(x^*(b), z^*(b)) = \begin{cases} (\widetilde{x}, \widetilde{z}), & \textit{if } \widetilde{z} \geq \widetilde{x} + b, \\ (\widehat{a}, \widehat{a} + b), & \textit{if } \widetilde{z} < \widetilde{x} + b. \end{cases}$$

In addition, when b is increasing, $x^*(b)$ is decreasing and $z^*(b)$ is increasing.

Proof: The Lemma is cited from Feng et al. (2003), and the reader is directed to this work for the proof and details.△

$g(x) + h(x+b)$ is also joint convex on x and b. Therefore, the function $\min_{x \geq a, z \geq x+b}[g(x) + h(z)] = g(x^*(b)) + h(z^*(b))$ is convex on b.

Lemma 3.3 *Suppose that* $g(\cdot)$*and* $h(\cdot)$ *are convex functions and* a *and* b *are positive constants.* r *is a given value that belongs to the interval* $(0,1)$. *Let* $f(x,y) = ay + bx + g(y + rx) + h(y + x)$. *Then, the following are true.*

(1) $f(x,y)$ *is joint convex on* x *and* y.

(2) Assume that the optimal values are $x^*(r)$ *and* $y^*(r)$, *which minimize* $f(x,y)$ *and depend on* r. *When* r *is increasing,* $x^*(r)$ *is increasing and* $y^*(r)$ *is decreasing.*

(3) Consider the case in which y *is given and* $f(x,y)$ *is convex on* x. *Assume that the minimum point of* $f(x,y)$ *is* $x^*(y,r)$. *Given* r, *as* y *is increasing,* $x^*(y,r)$ *is decreasing. Given* y, *as* r *is increasing,* $x^*(y,r)$ *is also decreasing, and* $x^*(y,r) + y$ *depends on* y.

Proof: See the Appendix.△

To derive the optimal replenishment policy in each period of the model, we consider the optimal replenishment policy at the second decision point and the first decision point of each period in sequence, and then combine them to form the optimal replenishment policy for each period.

3.2 The optimal replenishment policy at the second decision point

Lemma 3.4 *Assume that $V_{t+1}(\cdot|ID)$ is a convex function. At the second decision point of period t, the random proportional factor R_t is realized as r_t and q_t is given. The optimal policies for the emergency delivery and the regular delivery modes are both of base-stock type. We denote the optimal base-stock levels as $S_t^{e*}((1-r_t)q_t^s)$ for the emergency delivery mode and $S_t^{r*}((1-r_t)q_t^s)$ for the regular delivery modes.*

Proof: We rewrite (1) as follows.

$$\begin{aligned} G_t(q_t^s, \xi_t|r_t) &= \min_{\substack{y_t^e \geq r_t q_t^s + \xi_t \\ y_t^r \geq y_t^e + (1-r_t)q_t^s}} \{[c_e(y_t^e - r_t q_t^s - \xi_t) + EH(y_t^e - D[t, t+k])] \\ &\quad + [c_r(y_t^r - y_t^e - (1-r_t)q_t^s) + E_{D_t}V_{t+1}(y_t^r - D_t|ID)]\}. \quad (3.1) \end{aligned}$$

As $V_{t+1}(\cdot|ID)$ is a convex function, $c_r(y_t^r - y_t^e - (1-r_t)q_t^s) + E_{D_t}V_{t+1}(y_t^r - D_t|ID)$ is still convex on y_t^r and y_t^e. However, $c_e(y_t^e - r_t q_t^s - \xi_t) + EH(y_t^e - D[t, t+k])$ is also convex on y_t^e and thus according to Lemma 3.2 (Feng et al. 2003a), the optimal policies for the emergency delivery mode and the regular delivery mode are base-stock policies.$\triangle$

We have proved that the optimal policy for the emergency and the regular delivery modes are are base-stock policies. From (3) the respective optimal base-stock levels are clearly functions of the proportional factor and the order quantity from the stochastic delivery mode. The following proposition establishes a monotonic property with respect to the proportional factor and the stochastic order quantity.

Proposition 3.1 *(1) For a given order quantity q_t^s from the stochastic delivery mode, the optimal base-stock level of the emergency delivery mode $S_t^{e*}((1-r_t)q_t^s)$ is increasing and the optimal base-stock level of the regular delivery mode $S_t^{r*}((1-r_t)q_t^s)$ is decreasing with respect to the proportional factor r_t.*

(2) For a given proportional factor r_t, the optimal base-stock level for the emergency delivery mode $S_t^{e*}((1-r_t)q_t^s)$ is decreasing and the optimal base-stock level for the regular delivery mode $S_t^{r*}((1-r_t)q_t^s)$ is increasing with respect to the order quantity q_t^s of the stochastic delivery mode.

Proof: (1) Given q_t^s, when r_t is increasing, $(1-r_t)q_t^s$ is decreasing. According to Lemma 3.2, $S_t^{e*}((1-r_t)q_t^s)$ is increasing and $S_t^{r*}((1-r_t)q_t^s)$ is decreasing. The proof is similar to that of (2).$\triangle$

At period t, an increase of $(1-r_t)q_t^s$ means that more units are delivered that affect the optimal cost function of future periods (dynamic part). Therefore, the emergency delivery mode plays a less important role in decreasing the cost function of future periods, which leads to a smaller optimal base-stock level for this mode. At the same time, an increase of $(1-r_t)q_t^s$ leads to a higher optimal base-stock level for the regular delivery mode.

3.3 The optimal replenishment policy at the first decision point

Using the results that were developed in the previous subsection, we can deal with the optimal decision making process for the stochastic delivery mode. We first present the existence property.

Lemma 3.5 *Assume that $V_{t+1}(\cdot|ID)$ is a convex function and that there is a unique optimal order quantity $q_t^{s*}(\xi_t|ID)$ from the stochastic delivery mode.*

Proof: For a given proportional factor r_t, Lemma 3.2(Feng et al. 2003a) indicates that $G_t(q_t^s, \xi_t|r_t)$ is still convex in q_t^s and ξ_t. Lemma 3.1 further indicates that $\int_0^1 G_t(q_t^s, \xi_t|r_t) f_{R_t}(r_t) dr_t$ preserves the convexity and thus that the optimal order quantity is $q_t^{s*}(\xi_t|ID)$ from the stochastic delivery mode. In addition, $V_t(\xi_t|ID)$ is also a convex function.$\triangle$

We summarize our characterization of the optimal policy in the following theorem.

Theorem 3.1 *(1) For any period t, $V_t(\cdot|ID)$ is a convex function.*

(2) At any period t, the optimal replenishment policy is as follows. At the first decision-point, optimal order quantity $q_t^{s*}(\xi_t|ID)$ is placed from the stochastic delivery mode. At the second decision-point, the random proportional factor is realized as r_t. Order up to the optimal base-stock level $S_t^{e*}((1-r_t)q_t^{s*}(\xi_t|ID))$ from the emergency delivery mode and order up to the optimal base-stock level $S_t^{r*}((1-r_t)q_t^{s*}(\xi_t|ID))$ from the regular delivery mode.

Proof:(1) As we mentioned in the proof of Lemma 3.5, if $V_{t+1}(\cdot|ID)$ is a convex function then we can also obtain $V_t(\cdot|ID)$ as a convex function. Then, by induction, we can complete our proof.

(2) is obvious from two previous lemmas.$\triangle$

We can prove that an optimal order quantity for the stochastic delivery mode exists, but whether it is subject to a base-stock policy is still unknown. After studying the following no delivery information delay model, we will be able to demonstrate that a base-stock policy is not optimal for the stochastic delivery mode.

Chapter 4

Specialized Case: No Delivery Information Delay Model

We first consider a special case in which delivery information from the stochastic delivery mode is known before the decision is made. When delivery information is present, that is, when the proportional factor is known, the decision maker knows the leadtime and delivery schedule, and the decision on how much to order from these sources is made jointly. The results that are derived from this case allow us to further investigate the impact of delivery information delay.

Specially, there are three sources of supply: the emergency delivery mode with a leadtime of k; the regular delivery mode with a leadtime of $k+1$; and the stochastic delivery mode with $r_t q_t^s$ and $(1-r_t)q_t^s$ for leadtimes of k and $k+1$, respectively.

As the realized proportional factor is known, the sequence of events is simpler. At the beginning of each period, the stochastic proportional factor is known, and

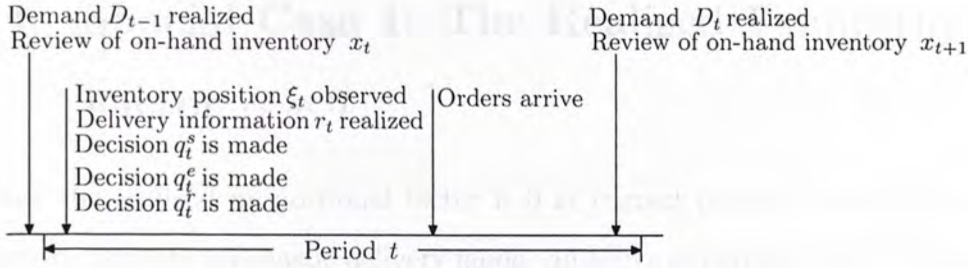

Figure 4.1: Event sequences in the no delivery information delay model

the retailer then places orders from all delivery sources. The orders arrive, and at the end of each period, demand occurs and the holding/shortage cost is evaluated. The sequence of events is depicted in Figure 3.

The assumption $c_e > c_r > c_s$ is still required, and in addition we call a delivery mode inefficient in the current period if it is not used under the optimal replenishment policy, because of its high cost or long leadtime compared to other delivery modes.

In particular, if the proportional factor r_t is binary, i.e., $r_t = 0$ or 1, then an order from the stochastic delivery mode will be delivered in a complete batch with leadtime k or leadtime k+1, respectively. As we have indicated in section 2, this case corresponds to the case in which there is no partial shipment, which has been studied as an inventory model with two-consecutive leadtimes in the literature. Base-stock policies are still optimal for the two efficient delivery modes. In the subsequent sections, we summarize these results.

4.1 Special Case 1: The Realized Proportional Factor $r_t = 0$

When the realized proportional factor is 0 at current period t, then full order quantity from the stochastic delivery mode will arrive at period $t+k+1$. The regular delivery mode is inefficient because the stochastic delivery mode is cheaper, although the two modes have the same leadtime $k+1$.

Theorem 4.1 *(1) Assume that $V_{t+1}(\cdot|NID)$ is a convex function. When $r_t = 0$, the regular delivery mode is inefficient. The optimal policies for the stochastic delivery mode and the emergency delivery mode are base-stock policies, and there are two optimal base stock levels $S_t^{s*}(r_t)|_{r_t=0} = S_t^{s*}(0)$ and $S_t^{e*}(r_t)|_{r_t=0} = S_t^{e*}(0)$. Furthermore, $S_t^{s*}(0) \geq S_t^{e*}(0)$.*

(2) The optimal replenishment policy is as follows. When $\xi_t < S_t^{e*}(0)$, order the emergency delivery mode up to $S_t^{e*}(0)$ and the stochastic delivery mode up to $S_t^{s*}(0)$. When $S_t^{s*}(0) \geq \xi_t \geq S_t^{e*}(0)$, order the stochastic delivery mode up to $S_t^{s*}(0)$. Otherwise, place no order.

Proof: The optimality of a base-stock policy can be demonstrated by using Lemma 3.2 (Feng et al. 2003a).$\triangle$

4.2 Special Case 2: The Realized Proportional Factor $r_t = 1$

When the realized proportional factor is 1 at current period t, then full order quantity from the stochastic delivery mode will arrive at period $t+k$. The emergency delivery mode with the same leadtime k is thus inefficient because of its high cost.

Theorem 4.2 *(1) Assume that $V_{t+1}(\cdot|NID)$ is a convex function. When $r_t = 1$, the emergency delivery mode is inefficient. The optimal policies for the stochastic delivery mode and the regular delivery mode are base-stock policies. There are two optimal base-stock levels $S_t^{s*}(r_t)|_{r_t=1} = S_t^{s*}(1)$ and $S_t^{r*}(r_t)|_{r_t=1} = S_t^{r*}(1)$. Furthermore, $S_t^{r*}(1) \geq S_t^{s*}(1)$.*

(2) The optimal replenishment policy is as follows. When $\xi_t < S_t^{s*}(1)$, order the stochastic delivery mode up to $S_t^{s*}(1)$ and the regular delivery mode up to $S_t^{r*}(1)$. When $S_t^{r*}(1) \geq \xi_t \geq S_t^{s*}(1)$, order the regular delivery mode up to $S_t^{r*}(1)$. Otherwise, make no order.

Proof: The optimality of a base-stock policy can be demonstrated by using Lemma 3.2 (Feng et al. 2003a).△

4.3 Comparison of the Two Special Cases

We list the optimal base-stock levels of the two special cases in Table 4.1. Factually, from another point of view, these two special cases can be a stochastic-leadtime inventory model. In this case, the stochastic delivery mode has the leadtime k with one probability and the leadtime $k+1$ with the other probability, and it would thus be interesting to explore the order relationships among these four optimal base-stock levels. We know that if the full order quantity from the stochastic delivery mode is delivered at period $t+k+1$, i.e., $r_t = 0$, then the optimal level of the stochastic delivery mode is greater than the optimal level of the emergency delivery mode, i.e., $S_t^{s*}(0) \geq S_t^{e*}(0)$; If the full order quantity

	$r_t = 1$	$r_t = 0$
The regular delivery mode	$S_t^{r*}(1)$	
The stochastic delivery mode	$S_t^{s*}(1)$	$S_t^{s*}(0)$
The emergency delivery mode		$S_t^{e*}(0)$

Table 4.1: Optimal base-stock levels in two special cases

from the stochastic delivery mode is delivered at period $t+k$, i.e., $r_t = 1$, then the optimal level of the regular delivery mode is greater than the optimal level of the stochastic delivery mode, i.e., $S_t^{r*}(1) \geq S_t^{s*}(1)$.

Proposition 4.1 *(1) Assume that $V_{t+1}(\cdot|NID)$ is a convex function. The optimal base-stock level of the stochastic delivery mode in case $r_t = 1$ is larger than the optimal base-stock level of the emergency delivery mode in case $r_t = 0$, i.e., $S_t^{s*}(1) \geq S_t^{e*}(0)$.*

(2) If $\frac{\partial EH(y-D[t,t+k])}{\partial y} > 0$ and $\frac{\partial E_{R_{t+1}} E_{D_t} V_{t+1}(y-D_t, R_{t+1}|NID)}{\partial y} < -c_r$ at point $S_t^{s*}(1)$, then $S_t^{s*}(0) > S_t^{r*}(1)$. If $\frac{\partial EH(y-D[t,t+k])}{\partial y} > 0$ and $\frac{\partial E_{R_{t+1}} E_{D_t} V_{t+1}(y-D_t, R_{t+1}|NID)}{\partial y} < -c_s$ at point $S_t^{s*}(1)$, then $S_t^{s*}(0) > S_t^{s*}(1)$. However, if both $\frac{\partial EH(y-D[t,t+k]}{\partial y}$ and $\frac{\partial E_{R_{t+1}} E_{D_t} V_{t+1}(y-D_t, R_{t+1}|NID)}{\partial y}$ at point $S_t^{s*}(1)$ are less than zero, then $S_t^{s*}(0) < S_t^{s*}(1)$.

(3). If the optimal base-stock level of the stochastic delivery mode in case $r_t = 0$ is less than the optimal base-stock level of the stochastic delivery mode in case $r_t = 1$, i.e, $S_t^{s*}(0) < S_t^{s*}(1)$, then the optimal cost function of case $r_t = 1$ is less than the optimal cost function of case $r_t = 0$, i.e., $V_t(\xi_t, 1|NID) < V_t(\xi_t, 0|NID)$.

Proof: See the Appendix.△

When $r_t = 1$, the stochastic delivery mode primarily influences the expected holding/shortage cost $EH(y - D[t, t+k])$. When $r_t = 0$, the stochastic delivery mode primarily influences $E_{R_{t+1}} E_{D_t} V_{t+1}(y - D_t, R_{t+1}|NID)$, which is the optimal cost function in future periods. $\frac{\partial EH(y-D[t,t+k])}{\partial y} > 0$ and $\frac{\partial E_{R_{t+1}} E_{D_t} V_{t+1}(y-D_t,R_{t+1}|NID)}{\partial y} < -c_s$ at point $S_t^{s*}(1)$ mean that compared to case $r_t = 1$, the order from the stochastic delivery mode in case $r_t = 0$ mainly influences the future periods, and we can still order more quantities from the stochastic delivery mode to decrease the optimal cost function of these future periods. If $\frac{\partial EH(y-D[t,t+k])}{\partial y} > 0$ and $\frac{\partial E_{R_{t+1}} E_{D_t} V_{t+1}(y-D_t,R_{t+1}|NID)}{\partial y} < -c_r$ at point $S_t^{s*}(1)$, in case $r_t = 0$, then more quantities from the regular delivery mode can decrease the optimal cost function of future periods. Therefore regardless of which event occurs, the optimal base-stock level of the stochastic delivery mode in case $r_t = 0$ is greater than the optimal base-stock level of the stochastic delivery mode in case $r_t = 1$, i.e, $S_t^{s*}(0) > S_t^{s*}(1)$. However, if both $\frac{\partial EH(y-D[t,t+k]}{\partial y}$ and $\frac{\partial E_{R_{t+1}} E_{D_t} V_{t+1}(y-D_t,R_{t+1}|NID)}{\partial y}$ at point $S_t^{s*}(1)$ are less than zero, then the delivery mode with the leadtime k that affects the expected holding/shortage cost and optimal cost function of future periods plays a crucial role. Thus, $S_t^{s*}(0) < S_t^{s*}(1)$.

Only two delivery modes are efficient for both special cases, but they are for each case different. More interesting results will be derived in the analysis of the case with partial delivery.

4.4 The Case with Partial Delivery ($r_t \in (0, 1)$)

When the proportional factor r_t is between 0 and 1, then partial shipment is allowed. Partial shipment affects the inventory position of periods k and $k+1$.

As the goods that are ordered from the emergency delivery mode are delivered at period $t+k$, the expected holding/shortage cost of period $t+k$ is directly affected. Thus, the decision on the stochastic delivery mode should be made based on the decision on the emergency delivery mode. Decisions should be made as the emergency delivery mode first, then the stochastic delivery mode, and finally the regular delivery mode.

When the realized proportional factor r_t belongs to the interval $(0,1)$, it is not possible for all three delivery modes to be efficient at the same time. We have observed that only two delivery modes are efficient in each special case, and this property is extended to the case with partial delivery.

Lemma 4.1 *When $r_t \in (0,1)$, the stochastic delivery mode is always efficient. However, the emergency delivery mode and the regular delivery mode can not be efficient at the same time, and thus we can only order a positive amount from either the emergency delivery mode or the regular delivery mode.*

Proof: If all three delivery modes are efficient, then assume that the optimal quantities are $q_t^{s*}(r_t, \xi_t)$, $q_t^{e*}(r_t, \xi_t)$, and $q_t^{r*}(r_t, \xi_t)$. Then, order a little more quantity from the stochastic delivery mode, say $q_t^{s*}(r_t, \xi_t) + \Delta$, where $\Delta(> 0)$ is small enough. Order $q_t^{e*}(r_t, \xi_t) - r_t\Delta$ from the emergency delivery mode and $q_t^{r*}(r_t, \xi_t) - (1 - r_t)\Delta$ from the regular delivery mode. The ordering costs will decrease by $c_e r_t \Delta + c_r(1 - r_t)\Delta - c_s\Delta$, but at the same time the expected holding/shortage cost and optimal cost function of future periods remain the same, which is a contradicts the optimality of $q_t^{s*}(r_t, \xi_t)$, $q_t^{e*}(r_t, \xi_t)$, and $q_t^{r*}(r_t, \xi_t)$. Hence, only two delivery modes can be efficient in each period.

Here, we demonstrate that the stochastic delivery mode is always efficient.

Otherwise, if there is a realized proportional factor r_t that makes only the emergency delivery mode and the regular delivery mode efficient, then we suppose that their optimal order quantities are $q_t^{e*}(r_t, \xi_t)$ and $q_t^{r*}(r_t, \xi_t)$. If $q_t^{e*}(r_t, \xi_t) > \frac{q_t^{r*}(r_t,\xi_t)}{1-r_t}$, then we order $\frac{q_t^{r*}(r_t,\xi_t)}{1-r_t}$ from the stochastic delivery mode instead of the order quantity $\frac{q_t^{r*}(r_t,\xi_t)}{1-r_t}$ from the regular delivery mode. This means that the ordering cost is less and the other costs are the same, which contradicts the optimality of $q_t^{e*}(r_t, \xi_t)$ and $q_t^{r*}(r_t, \xi_t)$. If $q_t^{e*}(r_t, \xi_t) \leq \frac{q_t^{r*}(r_t,\xi_t)}{1-r_t}$, then similar actions and results can be taken for the emergency delivery mode, and therefore the stochastic delivery mode is always efficient.$\triangle$

Apparently, there is a situation in which only the emergency delivery mode is efficient, which means that we order nothing from the stochastic delivery mode. We can regard this situation as a special case of the situation in which the emergency delivery mode and the stochastic delivery mode are efficient. A similar analysis can also be applied to the regular delivery mode. Thus, we have three different situations in our model: One in which the emergency delivery mode and the stochastic delivery mode are efficient; the one in which the stochastic delivery mode is efficient; and the one in which the regular delivery mode and the stochastic delivery mode are efficient.

Lemma 4.2 *There is $[r_t^{1*}, r_t^{2*}] \subseteq [0,1]$ for which only the stochastic delivery mode is efficient. When $r_t \in [0, r_t^{1*})$, only the emergency delivery mode and the stochastic delivery mode are efficient. When $r_t \in (r_t^{2*}, 1]$, only the regular delivery mode and the stochastic delivery mode are efficient. Meanwhile, $V_t(\xi_t, r_t|NID)$ is continuous on ξ_t and r_t.*

Proof: See the proof of Proposition 4.2.$\triangle$

As the realized proportional factor r_t increase from 0 to 1, the main influence of the stochastic delivery mode moves from the cost function of future periods to the expected holding/shortage cost. When the stochastic delivery mode pays more attention to the cost function of future periods, r_t is low and the emergency delivery mode is efficient as a primary force that affects the expected holding/shortage cost. When the stochastic delivery mode pays more attention to the expected holding/shortage cost, r_t is large and the regular delivery mode is efficient as a primary force that affects the cost function of future periods.

If only the stochastic delivery mode is efficient, then we can regard this situation as a special case of the other two situations simultaneously. We then rewrite the optimal cost function for the case with partial delivery for two possible situations. The first is the situation in which the stochastic delivery mode and the emergency delivery mode are efficient. The corresponding optimal cost function is given by

$$\begin{aligned} V_t(\xi_t, r_t|NID) &= \min_{q_t^s \geq 0, y_t^e \geq \xi_t} \{c_s q_t^s + c_e(y_t^e - \xi_t) + EH(y_t^e + r_t q_t^s - D[t, t+k]) \\ &+ E_{R_{t+1}} E_{D_t} V_{t+1}(y_t^e + q_t^s - D_t, R_{t+1}|NID)\}. \end{aligned} \tag{4.1}$$

The other situation is that in which the stochastic delivery mode and the regular delivery mode are efficient. The corresponding optimal cost function is given by

$$\begin{aligned} V_t(\xi_t, r_t|NID) &= \min_{q_t^s \geq 0, y_t^r \geq \xi_t + q_t} \{c_s q_t^s + c_r(y_t^r - \xi_t - q_t^s) + EH(r_t q_t^s - D[t, t+k]) \\ &+ E_{R_{t+1}} E_{D_t} V_{t+1}(y_t^r - D_t, R_{t+1}|NID)\}. \end{aligned} \tag{4.2}$$

Using these two optimal cost functions, we can investigate the optimal replenishment policies for these two optimal cost functions. Lemma 4.2 tells us the kind of proportional factors lead to efficiency of the emergency delivery mode and the stochastic delivery mode (or the regular delivery mode and the stochastic delivery mode).

Theorem 4.3 *(1) For each period t and any r_t, $V_t(\xi_t, r_t|NID)$ is convex on the inventory position ξ_t.*

(2) When $r_t \in [0, r_t^{1*})$, the emergency delivery mode and the stochastic delivery mode are efficient, and there are two optimal values $S_t^{e*}(r_t)$ and $q_t^{s*}(r_t, \xi_t)$. The optimal policy is as follows. When $\xi_t \leq S_t^{e*}(r_t)$, order the emergency delivery mode up to $S_t^{e*}(r_t)$, then order $q_t^{s*}(r_t, \xi_t)$ from the stochastic delivery mode. When $\xi_t > S_t^{e*}(r_t)$, order nothing from the emergency delivery mode, then order $q_t^{s*}(r_t, \xi_t)$ from the stochastic delivery mode. Otherwise, order nothing.

(3) When $r_t \in [r_t^{1*}, r_t^{2*}]$, only the stochastic delivery mode is efficient. There exist one optimal value $q_t^{s*}(r_t, \xi_t)$. The optimal policy is that order $q_t^{s*}(r_t, \xi_t)$ from the stochastic delivery mode.

(4) When $r_t \in (r_t^{2*}, 1]$, the regular delivery mode and the stochastic delivery mode are efficient, and there are two optimal values S_t^{r*} and $q_t^{s*}(r_t, \xi_t)$. The optimal policy is to order $q_t^{s*}(r_t, \xi_t)$ from the stochastic delivery mode. When $S_t^{r*} \geq q_t^{s*}(r_t, \xi_t) + \xi_t$, order the regular delivery mode up to S_t^{r*}. Otherwise, order nothing.

Proof: The results can be derived from Lemma 4.2 directly. We need to use the method of induction for (1).$\triangle$

Whatever the value of r_t, only two delivery modes can be efficient, one of which must be the stochastic delivery mode. In spite of the uncertainty of the delivery schedule, the low cost of the stochastic delivery mode makes it efficient all the time. A state-dependent base-stock policy is still optimal for the emergency delivery mode and a base-stock policy is optimal for the regular delivery mode. As can be seen, the optimal order quantity $q_t^{s*}(r_t, \xi_t)$ of the stochastic delivery mode depends on the current states r_t and ξ_t, and the optimal base-stock level of the emergency delivery mode also depends on the realized proportional factor r_t. The impact of these states on the optimal levels will be studied.

Proposition 4.2 *(1) Given ξ_t, when r_t is increasing in the interval $[0, r_t^{1*})$, $q_t^{s*}(r_t, \xi_t)$ is increasing and $S_t^{e*}(r_t)$ is decreasing; When r_t is increasing in the interval $[r_t^{1*}, r_t^{2*}]$, $q_t^{s*}(r_t, \xi_t)$ is decreasing, and when r_t is increasing in the interval $(r_t^{2*}, 1]$, $q_t^{s*}(r_t, \xi_t)$ is decreasing.*

(2) Given $r_t \in [0, r_t^{1*})$, when ξ_t increases to $S_t^{e*}(r_t)$, $q_t^{s*}(r_t, \xi_t)$ is independent of ξ_t. When ξ_t increases after $S_t^{e*}(r_t)$, $q_t^{s*}(r_t, \xi_t)$ decreases to 0 and $q_t^{s*}(r_t, \xi_t) + \xi_t$ depends on ξ_t. Given $r_t \in [r_t^{1*}, 1]$, $q_t^{s*}(r_t, \xi_t)$ is decreasing and $q_t^{s*}(r_t, \xi_t) + \xi_t$ also depends on ξ_t. Hence, a base-stock policy is not optimal for the stochastic delivery mode.

Proof: See the Appendix.△

Moreover, when r_t increases from 0 to r_t^{1*}, the influence of the emergency delivery mode on the expected holding/shortage cost becomes smaller, and its optimal base-stock level decreases. In contrast, the role of the stochastic delivery mode becomes more important, although its proportional factor is rather low. Thus, the order quantity from the stochastic delivery mode is increasing. and when r_t increases from r_t^{1*} to r_t^{2*}, the stochastic delivery mode is the only ef-

Given ξ_t	$r_t \in [0, r_t^{1*})$	$r_t \in [r_t^{1*}, r_t^{2*}]$	$r_t \in (r_t^{2*}, 1]$
Optimal level of regular mode	0	0	S_t^{r*}
Order quantity of stochastic mode	$q_t^{s*}(r_t, \xi_t) \uparrow$	$q_t^{s*}(r_t, \xi_t) \downarrow$	$q_t^{s*}(r_t, \xi_t) \downarrow$
Optimal level of emergency mode	$S_t^{e*}(r_t) \downarrow$	0	0

Table 4.2: Impacts of the proportional factor on the optimal levels

ficient delivery mode. An increasing percentage of the order quantity from the stochastic delivery mode is distributed to affect the expected holding/shortage cost, and the larger proportional factor leads to a lower order quantity. When r_t inceases from r_t^{1*} to 1, quite a large percentage of the order quantity from the stochastic delivery mode still results in a decrease in the order quantity. However, as the stochastic delivery mode primarily influences the expected holding/shortage cost, the cost function of future periods will be affected by the regular delivery mode, and thus the regular mode is the efficient delivery mode.

Proposition 4.2 also demonstrates that the optimal policy of the stochastic delivery mode is not a base-stock policy in the general case, because the partial shipments do not make the order quantity plus the inventory position appear as a whole item. The optimality of a base stock policy is lost even in the model with two consecutive leadtimes.

4.5 Comparison of the Delivery Information Delay Model and the No Delivery Information Delay Model

Our final objective is to quantify the effects of delivery information delay, which we can carry out by comparing the delivery information delay model with the no delivery information delay model. A new model is introduced as follows to continue our comparison.

At period t, assume that there is no delivery information delay in current period only, and that there is a delivery information delay in each future period. The optimal cost function of future periods is therefore the same as in the delivery information delay model. At current period t, given a realized proportional factor r_t, suppose that the optimal cost function is $VD_t(\xi_t, r_t)$. Using a similar proof to that of Theorem 4.3 and Proposition 4.3, we can obtain results. Assume that the optimal levels are $qD_t^{s*}(r_t, \xi_t)$, $SD_t^{e*}(r_t)$, and SD_t^{r*}. There is $[rD_t^{1*}, rD_t^{2*}] \subseteq [0, 1]$. Given ξ_t, when r_t is increasing in interval $[0, rD_t^{1*})$, $qD_t^{s*}(r_t, \xi_t)$ is increasing and $SD_t^{e*}(r_t)$ is decreasing and when r_t is increasing in interval $[rD_t^{1*}, rD_t^{2*}]$, $qD_t^{s*}(r_t, \xi_t)$ is decreasing; when r_t is increasing in interval $(rD_t^{2*}, 1]$, $qD_t^{s*}(r_t, \xi_t)$ is decreasing. Moreover, $qD_t^{s*}(r_t, \xi_t)$ is decreasing, as r_t is given and ξ_t is increasing. However, $qD_t^{s*}(r_t, \xi_t) + \xi_t$ depends on ξ_t and r_t.

We now compare the delivery information delay model with this new model. We can place an upper bound and a lower bound on the optimal order quantity from the stochastic delivery mode in the delivery information delay model.

	Delivery ID model	New model	No delivery ID model
Current period t	Information delay	No delay	No information delay
Future periods	Information delay	Delay	No information delay
Notation	$V_t(\xi_t\|ID)$	$VD_t(\xi_t, r_t)$	$V_t(\xi_t, r_t\|NID)$
	$q_t^{s*}(\xi_t\|ID)$	$qD_t^{s*}(r_t, \xi_t)$	$q_t^{s*}(r_t, \xi_t)$
	$S_t^{e*}(r_t, q_t^{s*}(\xi_t\|ID))$	$SD_t^{e*}(r_t)$	$S_t^{e*}(r_t)$
	$S_t^{r*}(r_t, q_t^{s*}(\xi_t\|ID))$	SD_t^{r*}	S_t^{r*}

Table 4.3: Comparison of the three models (ID = Information delay)

The order relationship of the optimal cost functions among the three previously mentioned models is shown in Theorem 4.5.

Theorem 4.4 *(1) The optimal order quantity* $q_t^{s*}(\xi_t|ID)$ *is less than* $qD_t^{s*}(rD_t^{1*}, \xi_t)$*, but is greater than* $\min\{qD_t^{s*}(0,\xi_t), qD_t^{s*}(1,\xi_t)\}$. $q_t^{s*}(\xi_t|ID)$ *is decreasing as* ξ_t *is increasing, but the optimal policy of the stochastic delivery mode in the delivery information delay model is not a base-stock policy, because* $q_t^{s*}(\xi_t|ID)+\xi_t$ *depends on* ξ_t.

(2) For any given inventory position ξ_t, $V_t(\xi_t|ID) > VD_t(\xi_t, r_t) > V_t(\xi_t, r_t|NID)$.

Proof: See the Appendix. $\triangle$

The theorem proves that a base-stock policy is not optimal for the stochastic delivery mode in either the no delivery information delay model or the delivery information delay model.

The cost difference $V_t(x_t|ID) - V_t(x_t, r_t|NID)$ is called the delivery information cost. Intuitively, delivery information delay incurs greater costs, and from another point of view, also complicates the decision setting. We need to consider each possible case and balance all these cases, which leads to more unexpected order quantities and generates a delivery information cost. Hence, sharing delivery information definitely decreases the total cost of the system.

Chapter 5

Nonoptimality of a Base-Stock Policy in Numerical Examples

In the previous analysis, we focus on an inventory model with two consecutive leadtimes and demonstrate that when the stochastic delivery mode has two partial shipments, i.e., $r_t \neq 0$ and 1, a base-stock policy is not optimal. However, when r_t is an extreme value, such as 0 or 1, a base stock policy is still optimal for the two efficient delivery modes. In this section, we illustrate this phenomenon using a numerical example.

The numerical example is a stationary inventory system with a planning horizon of two periods. Only the stochastic delivery mode exists, which has a realized proportional factor r. The ordering cost for the stochastic delivery mode is $c = 5$, and the holding/shortage cost function is $H(x) = x^2$. The demand D is 10 with a probability 1/2 and 5 with a probability 1/2. The initial inventory is x_1.

The optimal cost function of the system is as follows.

$$\begin{aligned} V_1(x_1) &= \min_{q_1^s \geq 0}\{5q_1^s + 1/2(x_1 + rq_1^s - 5)^2 + 1/2(x_1 + rq_1^s - 10)^2 \\ &\quad +1/2[1/2(x_1 + q_1^s - 10)^2 + 1/2(x_1 + q_1^s - 15)^2] \\ &\quad +1/2[1/2(x_1 + q_1^s - 15)^2 + 1/2(x_1 + q_1^s - 20)^2]\}. \end{aligned} \tag{5.1}$$

The optimal order quantity is then $q_1^{s*} = \frac{5(5+3r)}{2(r^2+1)} - \frac{r+1}{r^2+1}x_1$. If a base-stock policy is optimal, then $\frac{r+1}{r^2+1}$ should be 1, which means that $r = 0$ or 1. If it is not the case, then the optimality of a base-stock policy is lost.

The explanation is as follows. The proportional factor leads to two partial shipments. We can not regard the order quantity and the inventory position as a whole, as in other studied models in the literature, because only partial shipment of the order and the inventory position affect the corresponding expected holding/shortage cost. Therefore, a base-stock policy is no longer optimal.

Moreover, if we allow the stochastic delivery mode to have three consecutive leadtimes, then we will find that a base-stock policy is no longer optimal, even if the realized proportional factors are extreme values. Feng et al. (2003) give a counter example with three periods to illustrate this problem.

The main reason for the nonoptimality of a base-stock policy is that the Markovian property is lost from the decision process. In other models in which a base-stock policy is optimal, there are no additional decisions and inputs after the decision period and before the order arrival period. However, in an inventory model with three periods, there are three deterministic delivery modes with three different leadtimes (fast, medium, and slow). Suppose that we place an order from the slow delivery mode with a leadtime 3 in the first period. We can still make an additional order from the fast delivery mode with a leadtime 1 in period 2, which arrives immediately. The decision for the slow delivery mode with a leadtime 3 in period 1 can not be made based on the state in period 1, because it is influenced by the decisions of period 2. Therefore the Markovian property is lost.

Chapter 6

Conclusion and Future Work

6.1 Conclusion

We consider finite periodic review inventory systems with delivery information delay, two consecutive leadtimes, and three delivery modes, one of which is stochastic and the other two deterministic. The stochastic delivery mode is the cheapest with a stochastic leadtime and partial shipments that are generated by a random proportional factor. The deterministic delivery modes are more expensive with deterministic leadtimes and full shipment. A state-dependent base-stock policy is optimal for the deterministic delivery modes, but not for the stochastic delivery mode. For the special case of the no delivery information delay model, at most two delivery modes are efficient for each realized proportional factor. In the range of proportional factor $[0, 1]$, there are three intervals $[0, r_t^{1*})$, $[r_t^{1*}, r_t^{2*}]$, and $(r_t^{2*}, 1]$. In $[0, r_t^{1*})$, only the stochastic delivery mode and the emergency delivery mode are efficient. In $[r_t^{1*}, r_t^{2*}]$, only the stochastic delivery mode is efficient. In $(r_t^{2*}, 1]$, only the stochastic delivery mode and the regular delivery mode are efficient. If the realized proportional factor is an extreme value, such as 0 or 1, then the optimal replenishment policy is a base-stock policy, but if that is not the case, then a base-stock policy is no longer optimal for the stochastic delivery mode. However, a state-dependent base-stock policy is still optimal for either the emergency delivery mode or the regular delivery mode.

We also investigate the impact of current states on the optimal levels. When the proportional factor increases, the optimal order quantity for the stochastic delivery mode at first increases and then decreases. Hence, we can place an upper bound and a lower bound on the optimal order quantity from the stochastic delivery mode in the delivery information delay model. The optimal order quantity from the stochastic delivery mode depends on the inventory position ξ_t, and as ξ_t is increasing, it is decreasing. In addition, the two corresponding changed amounts are different because of the nonoptimality of a base-stock policy. We prove that delivery information delay incurs greater costs than when there is no delivery information delay. Finally, numerical examples in the section 5 tells us that the optimality of a base-stock policy is indeed lost when the realized proportional factor is not an extreme value.

6.2 Future Work

We can also handle the stochastic delivery mode in a non-proportional way. Assume that there is a stochastic number S that is generated after the placement of an order from the stochastic mode at each period. If the order quantity q_t is less than S , then the full shipment can be delivered; otherwise, only S can be delivered at first and the remaining quantity will arrive $k+1$ periods later. What does the optimal policy look like? Assume that the holding/shortage cost $H(\cdot)$ is a convex function, then the partial shipment $\min\{S, q_t\}$ allows $EH(\min\{S, q_t\})$ to have the same minimum point as $H(\cdot)$. Although $EH(\min\{S, q_t\})$ can still be convex on the left of the minimum point, it loses convexity on the right of the minimum point and maintains the increasing property. If this extension is applied to in a new model, then the partial shipment $\min\{S, q_t\}$ will appear in holding/shortage cost $H(\cdot)$, which means that the convexity on the right of the minimum point of $EH(\cdot)$ will be lost, and will become a unimodal function. It is a well known result that we can not keep the property of unimodal except under some special conditions, such as special demand distribution functions. Thus, it should be a challenge to study such a model for general demand distribution.

There are other directions that future work could taste. Naturally, we could extend our models to infinite inventory systems, and another extension would be to consider a more general stochastic process, such as that in Song and Zipkin (1996). Unfortunately, even in models with deterministic delivery modes and three consecutive leadtimes, the optimal policy is not a base-stock policy, but we can build up some special situations to expand our studied models. Similar work has been carried out by Gross and Soriano (1972) and Chiang and Gutierrez (1996, 1998). We could also look at the quantification of the performance under a non-optimal but implementable policy, as Tagaras and Vlachos (2001) have undertaken in a model with deterministic delivery modes.

Chapter 7

Appendix

Proof of Lemma 3.3: (1) To prove the joint convexity of $f(x,y)$, we simply need to check wether its Hessian is positive semidefinite.

$$\partial^2 f(x,y)/\partial x^2 = r^2 g''(y+rx) + h''(x+y)$$

$$\partial^2 f(x,y)/\partial x \partial y = rg''(y+rx) + h''(x+y)$$

$$\partial^2 f(x,y)/\partial y \partial x = rg''(y+rx) + h''(x+y) = \partial^2 f(x,y)/\partial x \partial y$$

$$\partial^2 f(x,y)/\partial y^2 = g''(y+rx) + h''(x+y).$$

The Hessian can then be written as:

$$H = \begin{pmatrix} \partial^2 f(x,y)/\partial x^2 & \partial^2 f(x,y)/\partial y \partial x \\ \partial^2 f(x,y)/\partial x \partial y & \partial^2 f(x,y)/\partial x^2 \end{pmatrix}.$$

Apparently, $detH > 0$, and it is easy to verify the Hessian H is positive semidefinite. Therefore, $f(x,y)$ is joint convex on x and y. Assume that the optimal values are $x^*(r)$ and $y^*(r)$, which are functions of r.

(2) As $x^*(r)$ and $y^*(r)$ minimize $f(x,y)$, $\partial f(x^*(r), y^*(r))/\partial x = 0$ and $\partial f(x^*(r), y^*(r))/\partial y = 0$, equally,

$$b + rg'(rx^*(r) + y^*(r)) + h'(x^*(r) + y^*(r)) = 0$$

and

$$a + g'(rx^*(r) + y^*(r)) + h'(x^*(r) + y^*(r)) = 0.$$

Thus,

$$g'(rx^*(r) + y^*(r)) = -\frac{a-b}{1-r}$$

and

$$h'(x^*(r) + y^*(r)) = \frac{a-b}{1-r} - a.$$

When r is increasing, $-\frac{a-b}{1-r}$ is decreasing, and $rx^*(r) + y^*(r)$ is decreasing. At the same time, $\frac{a-b}{1-r} - a$ is increasing, and $x^*(r) + y^*(r)$ is increasing. From these two observations, we can derive that $x^*(r)$ is increasing and $y^*(r)$ is decreasing.

(3) Given y, $f(x,y) = ay + bx + g(y + rx) + h(y + x)$ is convex on x. Thus, an optimal value $x^*(y,r)$ exists, which makes $\partial f(x,y)/\partial x = 0$. Then $b + r^2 g'(y + rx^*(y,r)) + h'(x^*(y,r) + y) = 0$. Given r, as y is increasing, $x^*(y,r)$ is decreasing. Given y, as r is increasing, $x^*(y,r)$ is also decreasing. Rewrite $b + r^2 g'(y + rx^*(y,r)) + h'(x^*(y,r) + y) = 0$ in the following way:

$$b + r^2 g'((1-r)y + r(y + x^*(y,r))) + h'(x^*(y,r) + y) = 0.$$

Apparently, $x^*(y,r) + y$ depends on y and r.△

Proof of Proposition 4.1:(1) For any period t, when the proportional factor $r_t = 0$, $S_t^{s*}(0) \geq S_t^{e*}(0)$. If $S_t^{s*}(0) = S_t^{e*}(0)$, then there is no order from the stochastic delivery mode and only the emergency delivery mode is efficient. In

Given ξ_t	Case $r_t = 0$	The new model	Case $r_t = 1$
The emergency delivery mode	$S_t^{e*}(0)$	Y_t^{e*}	0
The stochastic delivery mode	$S_t^{s*}(0)$	0	$S_t^{s*}(1)$
The regular delivery mode	0	0	$S_t^{r*}(1)$

Table 7.1: Comparison of the three cases

this situation, the result can also be verified by the following proof. Assume $S_t^{s*}(0) > S_t^{e*}(0)$. It is not easy to prove $S_t^{s*}(1) > S_t^{e*}(0)$ directly.

First, we need to introduce a new model to help us complete the proof. The new model is as follows. At current period t, assume that we can only place an order from the emergency delivery mode, which means that only the emergency delivery mode is efficient. The other features of the model are the same as our studied model, for example, the cost function of future periods is the same. Apparently, it is optimal to adopt a base-stock policy for the emergency delivery mode in period t. Suppose that the optimal base-stock level is Y_t^{e*}.

Looking at Table 7.1, it can be seen that the process of finishing the proof is as follows. It is obvious that Y_t^{e*} is smaller than $S_t^{s*}(1)$. If we can prove that $Y_t^{e*} \geq S_t^{e*}(0)$, then $S_t^{s*}(1) > S_t^{e*}(0)$ and the proof is finished.

Assume that $S_t^{e*}(0) > Y_t^{e*}$. We then take $(Y_t^{e*}, S_t^{s*}(0))$ instead of the optimal level $(S_t^{e*}(0), S_t^{s*}(0))$. Compare the cost functions that are generated by these two pairs. $(c_e - c_s)(S_t^{e*}(0) - Y_t^{e*})$ is the additional ordering cost for the optimal case. According to the definition of optimality, $EH(Y_t^{e*} - D[t, t+k]) - EH(S_t^{e*}(0) - D[t, t+k])$ should be greater than $(c_e - c_s)(S_t^{e*}(0) - Y_t^{e*})$. In interval $[Y_t^{e*}, S_t^{e*}(0)]$, $\frac{\partial EV_{t+1}(y - D_t|NID)}{\partial y} < -c_s$, otherwise, $S_t^{s*}(0) \leq S_t^{e*}(0)$. This contradicts our assumption that $S_t^{s*}(0) > S_t^{e*}(0)$, and thus $EV_{t+1}(Y_t^{e*} - D_t|NID) - EV_{t+1}(S_t^{e*}(0) -$

$D_t|NID) > c_s(S_t^{e*}(0) - Y_t^{e*})$. Then,

$$\begin{aligned} &EH(Y_t^{e*} - D[t,t+k]) + EV_{t+1}(Y_t^{e*} - D_t|NID) \\ &-[EH(S_t^{e*}(0) - D[t,t+k]) + EV_{t+1}(S_t^{e*}(0) - D_t|NID)] \\ &> c_e(S_t^{e*}(0) - Y_t^{e*}). \end{aligned}$$

In the new model, only the emergency delivery mode is efficient at period t, and its optimal policy is a base-stock policy. The inequality affirms that the base-stock level $S_t^{e*}(0)$ leads to the better performance of the new model than the optimal level Y_t^{e*} (our assumption), which is Obviously a contradiction. Hence $S_t^{e*}(0) \leq Y_t^{e*}$, and $S_t^{s*}(1) > S_t^{e*}(0)$.

(2) is obvious. (3) If $S_t^{s*}(1) > S_t^{s*}(0) \geq S_t^{e*}(0)$, then both $\frac{\partial EH(y-D[t,t+k])}{\partial y}$ and $\frac{\partial E_{D_t} E_{R_{t+1}} V_{t+1}(y-D_t,R_{t+1}|NID)}{\partial y}$ at point $S_t^{s*}(1)$ are less than zero. Of course, $V_t(\xi_t, 1|NID) < V_t(\xi_t, 0|NID)$.$\triangle$

Proof of Proposition 4.2: For simplicity and convenience, we take $EH(y - D[t,t+k]) = L(y)$ and $E_{R_{t+1}} E_{D_t} V_{t+1}(y - D_t, R_{t+1}|NID) = G(y)$ in this proof.

Lemma 4.7 and (1): The proof consists of four steps. In Table 7.2, we divide the cost function into three parts: ordering cost, the expected holding/shortage cost, and the cost function of future periods to enable us to compare each part directly.

Assume that there are r_t^1 and $r_t^2 = r_t^1 + \Delta$, which belong to $(0,1)$ and $\Delta > 0$, where Δ is small enough.

- Step 1: The optimal cost function $V_t(\xi_t, r_t|NID)$ is continuous on r_t.

  If at r_t^1, then the regular delivery mode and the stochastic delivery mode are

	Given ξ_t
Ordering cost	$cq_t^s + c_e q_t^e + c_r q_t^r$
The expected holding/shortage cost	$L(y_t)$
The cost function of future periods	$G(y)$

Table 7.2: Cost structure of the cost function at period t

efficient, and the optimal levels are $q_t^{s*}(r_t^1, \xi_t)$ and S_t^{r*}. However, in case r_t^2, suppose that we can only make orders from the regular delivery mode and the stochastic delivery mode. Let us take $q_t^s(r_t^2, \xi_t) = \frac{r_t^1}{r_t^2} q_t^{s*}(r_t^1, \xi_t)$. Without loss of generality, if $V_t(\xi_t, r_t^1|NID) \leq V_t(\xi_t, r_t^2|NID)$, then $|V_t(\xi_t, r_t^1|NID) - V_t(\xi_t, r_t^2|NID)| < (c_r - c_s)\frac{r_t^2 - r_t^1}{r_t^2} q_t^{s*}(r_t^1, \xi_t) = (c_r - c_s)\frac{\Delta}{r_t^2} q_t^{s*}(r_t^1, \xi_t)$. If at r_t^2, then the emergency delivery mode and the stochastic delivery mode are efficient, and the optimal levels are $q_t^{s*}(r_t^2, \xi_t)$ and $S_t^{e*}(r_t^2)$. However, in case r_t^1, suppose that we can only place orders from the emergency delivery mode and the stochastic delivery mode. Let us take $q_t^s(r_t^1, \xi_t) = \frac{1-r_t^2}{1-r_t^1} q_t^{s*}(r_t^2, \xi_t)$ and $S_t^e(r_t^1) = S_t^{e*}(r_t^2)$. Without loss of generality, if $V_t(\xi_t, r_t^1|NID) \geq V_t(\xi_t, r_t^2|NID)$, then $|V_t(\xi_t, r_t^2|NID) - V_t(\xi_t, r_t^1|NID)| < (c_e - c_s)\frac{r_t^2 - r_t^1}{1-r_t^1} q_t^{s*}(r_t^2, \xi_t) = (c_e - c_s)\frac{\Delta}{1-r_t^1} q_t^{s*}(r_t^2, \xi_t)$. Remember that Δ is small enough, and thus the optimal cost function is continuous on r_t.

- Step 2: If only the stochastic delivery mode is efficient at a given interval, then $q_t^{s*}(r_t)$ (given ξ_t, let $q_t^{s*}(\xi_t, r_t) = q_t^{s*}(r_t)$) is decreasing as r_t is increasing.

  If at both r_t^1 and r_t^2, then only the stochastic delivery mode is efficient. Assume that $q_t^{s*}(r_t^1)$ and $q_t^{s*}(r_t^2)$ are optimal order quantities. To compare these two optimal order quantities, suppose that we still order $q_t^{s*}(r_t^1)$ from the stochastic delivery mode as in case r_t^2, i.e., $q_t^s(r_t^2) = q_t^{s*}(r_t^1)$. The ordering cost is the same, that is, $G(\xi_t + q_t^s(r_t^2)) = G(\xi_t + q_t^{s*}(r_t^1))$ and $L(r_t^2 q_t^s(r_t^2) + \xi_t) = L(r_t^1 q_t^{s*}(r_t^1) + \xi_t + \Delta q_t^{s*}(r_t^1))$. Assume that we order less from the stochastic delivery mode in case r_t^1, say $q_t^{s*}(r_t^1) - \epsilon$, where ϵ is a positive infinitesimal. As the changes in the ordering cost and the cost function of future periods are the same, we focus our attention on the changes in the expected holding/shortage cost. If $L'(r_t^1 q_t^{s*}(r_t^1) + \xi_t)$ is negative, then $L'(r_t^1 q_t^{s*}(r_t^1) + \xi_t + \Delta q_t^{s*}(r_t^1)) < L'(r_t^1 q_t^{s*}(r_t^1) + \xi_t)$. Because of the optimality of $q_t^{s*}(r_t^1)$ in case r_t^1, $L(r_t^1 q_t^{s*}(r_t^1) + \xi_t - r_t^1\epsilon) - L(r_t^1 q_t^{s*}(r_t^1) + \xi_t) > 0$. However,

$$L(r_t^1 q_t^{s*}(r_t^1) + \xi_t + \Delta q_t^{s*}(r_t^1) - r_t^1\epsilon) - L(r_t^1 q_t^{s*}(r_t^1) + \xi_t + \Delta q_t^{s*}(r_t^1)) <$$
$$L(r_t^1 q_t^{s*}(r_t^1) + \xi_t - r_t^1\epsilon) - L(r_t^1 q_t^{s*}(r_t^1) + \xi_t).$$

Therefore, $q_t^{s*}(r_t^2) \leq q_t^s(r_t^2) = q_t^{s*}(r_t^1)$. If $L'(r_t^1 q_t^{s*}(r_t^1) + \xi_t)$ is positive, and $L'(r_t^1 q_t^{s*}(r_t^1) + \xi_t + \Delta q_t^{s*}(r_t^1)) > L'(r_t^1 q_t^{s*}(r_t^1) + \xi_t)$. Because of the optimality of $q_t^{s*}(r_t^1)$ in case r_t^1, $L(r_t^1 q_t^{s*}(r_t^1) + \xi_t - r_t^1\epsilon) - L(r_t^1 q_t^{s*}(r_t^1) + \xi_t) < 0$. On the other hand,

$$L(r_t^1 q_t^{s*}(r_t^1) + \xi_t + \Delta q_t^{s*}(r_t^1) - r_t^1\epsilon) - L(r_t^1 q_t^{s*}(r_t^1) + \xi_t + \Delta q_t^{s*}(r_t^1)) <$$
$$L(r_t^1 q_t^{s*}(r_t^1) + \xi_t - r_t^1\epsilon) - L(r_t^1 q_t^{s*}(r_t^1) + \xi_t).$$

Therefore, $q_t^{s*}(r_t^2) \leq q_t^s(r_t^2) = q_t^{s*}(r_t^1)$. Hence, when the stochastic delivery mode is the only efficient delivery mode, $q_t^{s*}(r_t)$ is decreasing as r_t increasing.

- Step 3: If the stochastic delivery mode and the regular delivery mode are efficient at a given interval, then $q_t^{s*}(r_t)$ (given ξ_t, let $q_t^{s*}(\xi_t, r_t) = q_t^{s*}(r_t)$) is decreasing as r_t is increasing.

  If at both r_t^1 and r_t^2, then the stochastic delivery mode and the regular delivery mode are efficient, and $q_t^{s*}(r_t^1)$ and S_t^{r*} are optimal levels for case r_t^1.

Factually, as a supplementary delivery mode, the decision on the regular delivery mode is always made last, and its optimal base-stock level is determined by its unit purchase cost c_r and the cost function of future periods $G(y)$. Therefore, S_t^{r*} is independent of r_t^1, and $q_t^{s*}(r_t^2)$ and S_t^{r*} are optimal levels for case r_t^2. As in the previous analysis, $q_t^{s*}(r_t^1)$ is decreasing as r_t is increasing.

- Step 4: If the stochastic delivery mode and the emergency delivery mode are efficient at a given interval, $q_t^{s*}(r_t)$ (given ξ_t, let $q_t^{s*}(\xi_t, r_t) = q_t^{s*}(r_t)$) is increasing and $S_t^{e*}(r_t)$ is decreasing as r_t is increasing.

  Use Lemma 4.3.

Based on the results of the four steps, if the emergency delivery mode and the stochastic delivery mode (alternatively the stochastic delivery mode or the regular delivery mode and the stochastic delivery mode) are efficient at r_t^1 and r_t^2, then they are efficient in the interval $[r_t^1, r_t^2]$. Furthermore, there is a $[r_t^{1*}, r_t^{2*}] \subseteq [0, 1]$ where only the stochastic delivery mode is efficient, and when $r_t \in [0, r_t^{1*})$, only the emergency delivery mode and the stochastic delivery mode are efficient. When $r_t \in (r_t^{2*}, 1]$, only the regular delivery mode and the stochastic delivery mode are efficient. Lemma 4.7 and (1) are thus proved.

(2) can be derived from Lemma 4.3.△

Proof of Theorem 4.4: Compare the delivery information delay model with the new model. As the dynamic part is the same, we focus on the current period t. If there is no delivery information delay, then all decisions are made under full information. If there is delivery information delay, then we evaluate each case with a given proportional factor r_t and integrate all of them. Thus, we can only obtain an optimal quantity of the stochastic delivery mode under the expectation of proportional factor R_t. However, the optimal order quantity $q_t^{s*}(\xi_t|ID)$ is between the upper bound and lower bound of the new model, which are that $q_t^{s*}(\xi_t|ID)$ is less than $qD_t^{s*}(rD_t^{1*}, \xi_t)\}$ and is greater than $\min\{qD_t^{s*}(0, \xi_t), qD_t^{s*}(1, \xi_t)\}$. For any case, $q_t^{s*}(\xi_t|ID)$ is generally not optimal, and thus the optimal cost function $V_t(\xi_t|ID)$ is greater than $VD_t(\xi_t, r_t)$.

We can prove that $VD_t(\xi_t, r_t) > V_t(\xi_t, r_t|NID)$ by induction.

- Step 1: At period $N-k-2$, according to previous analysis, we know that

$$VD_{N-k-2}(\xi_{N-k-2}, r_{N-k-2}) > V_{N-k-2}(\xi_{N-k-2}, r_{N-k-2}|NID).$$

- Step 2: At period t, assume that $VD_{t+1}(\xi_{t+1}, r_{t+1}) > V_{t+1}(\xi_{t+1}, r_{t+1}|NID)$. If $VD_t(\xi_t, r_t) \leq V_t(\xi_t, r_t|NID)$, then we can simply take the policy for the new model to be the optimal policy for the no delivery information delay model. The optimal cost function should then be less than $V_{t+1}(\xi_{t+1}, r_{t+1}|NID)$, which contradicts the optimality of $VD_{t+1}(\xi_{t+1}, r_{t+1})$, and thus $VD_t(\xi_t, r_t) > V_t(\xi_t, r_t|NID)$.

- Step 3: For any period t, we have $VD_t(\xi_t, r_t) > V_t(\xi_t, r_t|NID)$.$\triangle$

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