Grasp Planning in Discrete Domain

by

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ABSTRACT

This dissertation presents a complete and efficient algorithm for frictionless/frictional form-closure grasp synthesis on 3-D objects represented by discrete points. The algorithm combines a local search process with a recursive decomposition of the problem into sub-problems when the local search encounters a local minimum. First, an initial grasp is selected randomly in the given point set. The ray-shooting based check algorithm is employed to test the form-closure property of the grasp. If the selected grasp does not satisfy form-closure, the local search procedure is executed iteratively in the direction of reducing the distance between the centroid of the convex hull corresponding to the grasp and the origin of the wrench space until a form-closure grasp is found or a local minimum is encountered. When a local minimum is encountered, the algorithm decomposes the problem into a few sub-problems in subsets of the points according to existence conditions of form-closure grasps. A search tree whose root represents the original problem is empolyed to perform the searching process. The sub-problems are represented as children of the root node and the same procedure is recursively applied to a child selected based on a heuristics until a form-closure grasp is obtained. The algorithm can be revised to obtain a local optimal form-closure grasp and ensure kinematic feasibility of robot fingers. The proposed algorithm is implemented and its efficiency is illustrated by numerical examples.

摘要

本論文提供一個完整及高效率的規則系統，在一個由不連續點所組成的無摩擦或有摩擦的三維物件上，搜尋機械人多指手形封閉抓握。這規則系統結合一個局部搜尋程序及一個遞歸問題分解程序；每當局部搜尋程序遇到一個局部極小值，這個遞歸問題分解程序便會將原問題分解爲多個子問題。首先，在已給予的不連續點中隨機選取一個初始抓握。我們利用一個以射線投射原理爲基礎的驗算規則系統來測試抓握的形封閉性能。如已選取之抓握不符合形封閉，局部搜尋程序會反覆地執行，以縮短抓握對應的凸殼距心及力組空間的原點之間的距離，直至得到一形封閉抓握或遇到一個局部極小值。若遇到一個局部極小值時，規則系統會基於現時的存在條件將原問題分解爲多個的子問題，以限定於不同子集上的點來合成抓握。如此，我們可得出一搜索樹形圖。原問題是樹形圖的根，子問題便是其子枝。規則系統會以一智能指數來決定先選取那個子問題，而同樣的程序便會遞歸地應用於被選取之子問題上，直至得到一形封閉抓握爲止。本論文亦提及如何將規則系統修訂爲獲取最優形封閉抓握，以及確保機械人多指手的運動學可行性。我們以電腦程式仿真這規則系統，並以多個數值例子闡明其效能。

LIST OF VARIABLES

G	a grasp
n	number of fingers per grasp
m	number of sides of polyhedral convex cone in approximating friction cone
n_p	number of primitive contact wrenches per grasp
Ω	set of surface points of grasped object
N	number of points in set Ω
r_j	position vectors w.r.t. object's coordinate frame with origin at the center of mass
n_i	surface normal at point r_i
f_i	grasp forces of finger i
τ_i	moment corresponds to grasp forces f_i
w_i	primitive contact wrench of finger i in frictionless case
w_{ij}	primitive contact wrench corresponds to j-th side of polyhedral friction cone of finger i in frictional case
W	wrench matrix of a grasp
w_{ext}	external wrench
μ	friction coefficient
θ_μ	friction cone angle
s_{ij}	j-th edge vector of polyhedral convex cone of grasp force f_i
$H(W)$	convex hull of W

CONTENTS

Chapter 1

Introduction

Form-closure is a fundamental property in multifingered robotic grasps and fixture layout design of workpieces. Under a form-closure grasp, any external wrench applied on the grasped object can be balanced by grasp forces of the robot hand. This dissertation proposes an approach to compute form-closure grasps on 3-D objects represented by discrete points.

Much research effort has been directed to testing form-closure property, which is known as the forward problem, of a given grasp. Salisbury and Roth [Salisbury82] have shown that a necessary and sufficient condition for form-closure is that the primitive contact wrenches resulted by contact forces at the contact points positively span the entire wrench space. This condition is further proven to be equivalent to that the origin of the wrench space lies strictly inside the convex hull of the primitive contact wrenches [Mishra87], [Montana91], [Murray94]. Nguyen [Nguyen86] proposed a simple test algorithm for 2-finger form-closure grasps. Trinkle [Trinkle92] provided a formulation of quantitative test for detecting form-closure grasp as a linear programming problem. In [Liu99], Liu has developed a qualitative test algorithm of form-closure grasps by transforming the problem to a ray-shooting problem of a convex hull. This qualitative test algorithm is employed in this work to check the form-closure property of the candidate grasps.

Another important issue concerning grasp stability is the sufficient and necessary conditions for the number of fingers needed to achieve form-closure grasp. Reulaux [Reulaux76] has shown that in frictionless case at least 4 and 7 fingers are required to achieve 2-D and 3-D form-closure grasps, respectively. Mishra *et al.* [Mishra87] found that 6 (resp. 12) fingers are sufficient for 2-D (resp. 3-D) frictionless form-closure grasps. Markenscoff *et al.* [Markenscoff90] further tightened the results by proving that 4 and 7 fingers are sufficient to achieve 2-D and 3-D form-closure grasps, respectively. When Coulomb friction is taken into account, 2 and 3 fingers are, respectively, sufficient in 2-D and 3-D cases.

This dissertation deals with the problem of grasp synthesis, which is also an important research aspect. Grasp synthesis is known as the reverse problem of form-closure. It concerns the problem of placing contacts on an object with given geometry to prevent object motions. Much works have been directed in tackling polyhedral/polygonal objects. Mishra, Schwartz and Sharir [Mishra87] have presented an algorithm for computing positive grips for polyhedral/polygonal objects in time linear to the number of faces/sides. In [Nguyen88], Nguyen presented a geometric approach to find maximal contact regions where two fingers can be positioned independently while maintaining a stable grip on a polygon. Liu [Liu00] has provided an algorithm for calculating planar form-closure grasps of n fingers on polygonal objects based on a new sufficient and necessary condition for form-closure. Ponce *et al.* [Ponce95] presented an approach in computing 3-finger stable grasps on planar polygonal objects with a projective algorithm based on linear programming and variable elimination among linear constraints. In [Ponce97] they further extended their approach to 3-D 4-finger grasps. Wang [Wang00] has presented an algorithm for optimal design for 3-D fixture synthesis in a point-set domain. We have recently developed algorithms [Lam01] for computing form-closure grasps on 3-D curved objects by iteratively moving the convex hull of the primitive contact wrenches towards the origin of the wrench space. This algorithm is known as the local search algorithm in this dissertation. However, the algorithm is not complete as the local search occasionally traps at a local minimum. In [Lam02], we revise the local search algorithm and provide a complete algorithm of frictionless grasp synthesis using a divide-and-conquer technique. This algorithm and its extension to frictional grasps are described in dissertation.

The contributions of this dissertation lie in three aspects:

First, we formulate the problem of form-closure synthesis as a local discrete search problem. We provide a performance index which efficiently leads the centroid of the convex hull corresponds to the grasp move towards the origin of the wrench space, i.e. enhancing the form-closure property of the grasp.

Second, we provide a complete algorithm to search for a form-closure grasp, i.e. the algorithm is always possible to locate a form-closure grasp (if exists) from a given set of points. To the best of our knowledge, this is the first complete algorithm that heuristically searches for a form-closure grasp in a discrete domain. The other complete algorithm is the combinatory approach which combines all possible points in the point set and checks exhaustively form-closure property of all the combinations.

Third, the proposed algorithm can be widely used in manipulating curved objects whereas most existing form-closure synthesis algorithms deal with polygonal/polyhedral objects only. Besides, the proposed algorithm can take kinematic constraints of robotic fingers into account, which greatly enhances its practicability.

This dissertation consists of six chapters. In Chapter 2, we provide some mathematical preliminaries of form-closure grasps and the problem definition of this dissertation. In Chapter 3, we describe the qualitative test developed by Liu [Liu99] which is employed extensively in this work for checking the form-closure property of the candidate grasps. Then, we describe a local search algorithm for finding a form-closure grasp. The local search algorithm has been presented in [Lam01]. The local search utilizes a quantitative measure developed from the qualitative test algorithm to estimate the grasp selection. We also introduce the feature of kinematic feasibility check for the algorithm. As long as the inverse kinematic model of the robotic hand is built, the algorithm can discard any candidate grasp with grip points not reachable by the corresponding finger. Chapter 4 describes a divide-and-conquer strategy which solves the local minimum problem and makes our algorithm complete. The detailed processes of problem decomposition for frictionless grasps and frictional grasps are described. In Chapter 5, the implementation of the proposed algorithm is described. A number of numerical examples are shown to demonstrate the effectiveness of the algorithm. As the algorithm is applicable for both frictionless and frictional grasps, examples in both kinds of contact models are given. We also provided examples of kinematic feasibility assurance in the later part of Chapter 5. The concluding remarks are given in Chapter 6.

Chapter 2

Mathematical Preliminaries and Problem Definition

In this chapter, the discrete approach of grasp synthesis is discussed and the assumptions made in the algorithms to be proposed are given. We also provide the preliminary materials concerning the grasp models and form-closure property of robotic grasps. Two kinds of contact models, namely *frictionless point contact* and *frictional point contact*, used in this work are studied. The definitions of form-closure based on the two contact models are to be given. The problem definition is then provided in the later part of this chapter.

This dissertation aims at locating a number of contact locations on an object with given geometry to prevent all motions of the object by applying forces through the contacts. The property of such kind of grasps or fixtures in resisting object motions is known as *form-closure*. The analysis of form-closure is intrinsically geometric and it does not consider the kinematics of the robot hand. The detailed definitions of form-closure grasps will be given in section 2.3 and 2.4. Now, let's study the grasp synthesis in point set domain.

2.1 Grasp Synthesis in Discrete Domain

In a typical grasp or fixture synthesis problem, continuous surfaces of the grasped body are often assumed to be available for contacts. However, the nonlinear constraints imposed by curved objects are always not manageable for algorithms tackling continuous surface with optimization approaches. For that reason, most research works have been done on grasping or fixturing polyhedral objects. Moreover, the functional or manufacturing requirements of the part may sometimes impose a point set constraint on the problem. In some situations, the robot fingers or fixture elements are allowed to contact with the object only at a set of discrete point locations instead of continuous surface of locations. Besides, for objects defined by mesh points instead of continuous surfaces in their CAD models, investigations in continuous approach are sometimes not achievable.

Turbine airfoils are good examples of 3-D workpieces with complex geometry. Fig. 2.1 shows a turbine airfoil model provided by author in [Wang01]. The geometric shape of airfoil is primarily defined by its aerodynamics. However, the geometric representation in a CAD system is usually approximated by parametric surfaces such as B-splines. Only a dense set of points of the surfaces are defined exactly as calculated in the aerodynamic analysis. In order to minimize the effects of the geometric approximation, the airfoil is required to be fixtured or grasped at some of these precise surface locations in its manufacture and inspection. Therefore, the point-set constraint is imposed by the practical conditions related to the functional and/or manufacturing requirements.

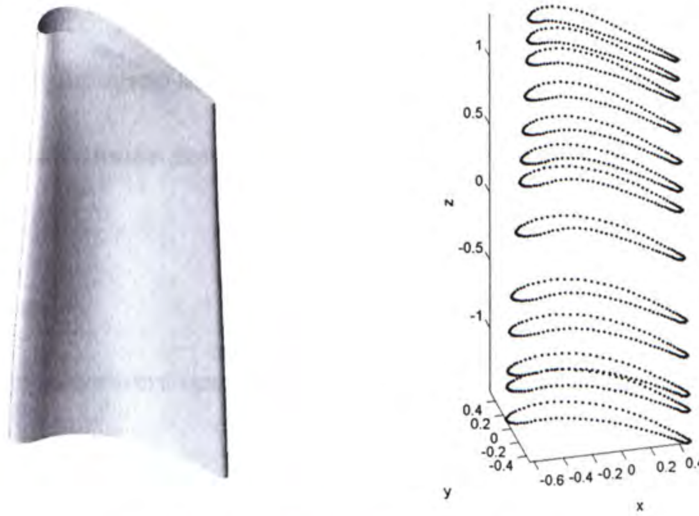


Fig. 2.1. An airfoil model with 1546 surface points.

Objects in point set domain are also easy to prepare for those originally defined with parametric equations. Fig 2.2 shows an example of object model generated by revolving a spline about a fixed axis with variable radius. The object is discretized into 5130 surface points by varying the two parameters involved.

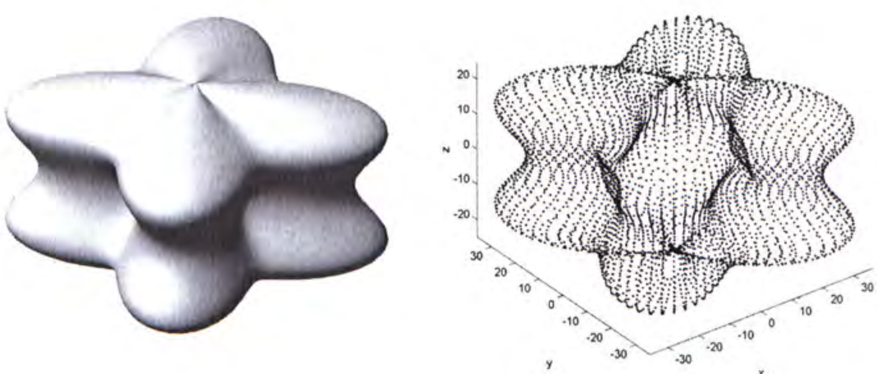


Fig. 2.2. A curved object defined with parametric equations being discretized into 5130 contact points.

The main advantage of tackling grasp objects in discrete domain lies in two aspects. First, objects with arbitrary geometry can be handled, as long as the object's surface points can be precisely defined, where the computational complexity is not influenced by the complexity of the object geometry. Second, the kinematic constraints of robot fingers can be considered during the fingertip placement, which enhance the practicality of the algorithm. We will see in the next chapter that kinematic constraint check has been implemented in this work.

2.2 Assumptions

There are two assumptions made throughout this work to allow precise analysis, which are:

1) All contacts are point-to-point hard contacts.
2) A collection of points (w.r.t. the object frame with origin at the center of mass) representing the exterior surface of the object are available and they are the candidate contact locations. The corresponding surface normal of each of these points is well-defined and obtainable.

As stated in the second assumption, the point set should contain surface point vectors and surface normal vectors both w.r.t. the object's center of mass. This turns out to assume that the center of mass of the manipulated object is given or obtainable.

Besides, there is predefined condition for the point set. Each candidate contact is connected with four neighboring points, which also belong to the point set and are geometrically near to the corresponding point. This provides a connective relation between adjacent points and this allows local motions of fingers. The number of neighboring points is fixed to be four because it is suitable in representing forward and backward motions on the 2-D surface. If the grasped object is originally defined in parametric equations, the connective relation between adjacent points can be defined according to the parameters. If the object is obtained from CAD models, the connective relation between adjacent points can be defined according to mesh data. In Chapter 3, the local search algorithm, which probes for form-closure grasp based on local motions of fingers, will be introduced.

Form-closure property concerns the capability of the grasp to completely constrain the motions of the grasped object through the contacts. The combination of force and moment at a contact is called *wrench*. In [Trinkle92], Trinkle has given a definition of form-closure: a fixed set of contacts on a rigid body is said to exhibit form-closure if the body's equilibrium is maintained despite the application of any possible externally applied wrench. The following two sections study the conditions of frictionless and frictional form-closure.

2.3 Frictionless Form-Closure Grasp

A frictionless point contact is obtained when there is no friction between the fingertip and the object. Frictionless contacts almost never occur in a practical situation, but they are useful when the friction between the finger and the object is low or unknown. Since a frictionless contact cannot exert forces except in the normal direction, modeling a contact as frictionless insures that we do not rely on frictional forces when we manipulate the object.

It is known that seven frictionless contacts are necessary [Mishra87] to hold a 3-D object in form-closure and sufficient [Markenscoff90] for a 3-D object without rotational symmetries. In this work, the number of fingers n to grasp objects with frictionless contacts is fixed to be seven ($n = 7$). Suppose seven hard fingers are to grasp a 3-D rigid body without rotational symmetries in the absence of friction at the contact points. Denote a grasp by $G = \{ r_i \}$, $i = 1, 2, \ldots, 7$, where r_i denotes the position vector of the i-th grasp point w.r.t. the object coordinate frame origined at the center of mass.

To hold the object and balance any external forces and torques, each finger must apply to the object a force f_i called *grasp force*. In the absence of friction, grasp forces f_i are in the normal directions of surface of the object at the contact points. The force and moment τ_i, corresponding to grasp force f_i, applied at the center of mass of the object is given by

$$\begin{pmatrix} f_i \\ \tau_i \end{pmatrix} = \begin{pmatrix} f_i \\ r_i \times f_i \end{pmatrix} = \alpha_i \underbrace{\begin{pmatrix} n_i \\ r_i \times n_i \end{pmatrix}}_{w_i} \tag{2.1}$$

where n_i is the surface normal vector at the i-th contact point and α_i is a non-negative constant representing the magnitude of the grasp force. The combination w_i of the force f_i and moment τ_i is called *wrench*.

The essential requirement for form-closure is the total restraint of the grasped object, where the contact forces are sufficient to balance any external forces. Therefore,

DEFINITION 1: FRICTIONLESS FORM-CLOSURE GRASP

$W=(w_1, w_2, \ldots, w_7) \in R^{6x7}$ is the *wrench matrix* of a 7-finger frictionless grasp.

For any external wrench $w_{ext} \in R^6$ applied at the object, if it is always possible to find an $\alpha \in R^7$ with $\alpha_i \geq 0$ such that

$$W\alpha + w_{ext} = 0, \tag{2.2}$$

the grasp is said to be form-closure.

2.4 Frictional Form-Closure Grasp

A frictional point contact model is used when friction exists between the fingertip and the object. In this work, the frictional model is referred as the *Coulomb friction* model. It is known that three fingers are sufficient to hold an object in 3-D frictional cases. Suppose that $n \geq 3$ hard fingers are to grasp a rigid object in a 3-D workspace. Denote a grasp by $G = \{ r_i \}$, $i = 1, 2, \ldots, n$, where r_i denotes the position vector of the i-th grasp point w.r.t. the object coordinate frame origined at the center of mass. Assume that the Coulomb friction with friction coefficient μ exists at the contact points. To ensure non-slipping at the contact point, the grasp force f_i must satisfy

$$f_{ix}^2 + f_{iy}^2 \leq \mu^2 f_{iz}^2, \tag{2.3}$$

where (f_{ix}, f_{iy}, f_{iz}) denotes x, y and z components of the grasp force f_i w.r.t. the object coordination frame.

The nonlinear constraints in (2.3) geometrically define a cone called *friction cone*. The angle of the cone with respect to the normal is given by

$$\theta_{\mu} = \tan^{-1} \mu . \tag{2.4}$$

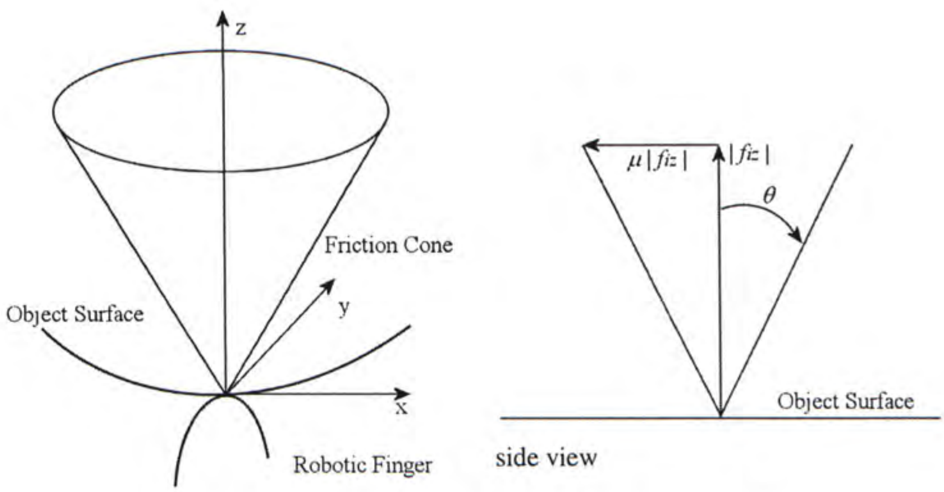


Fig. 2.3. Geometric interpretation of friction cone.

Table 2.1 lists a number of friction coefficients for common materials. Typical values of μ are less than 1, and hence the friction cone angle θ_{μ} is typically less than 45.

Steel on steel	0.58	Wood on wood	0.25 – 0.5
Polyethylene on steel	0.3 – 0.35	Wood on metals	0.2 – 0.6
Polyethylene on self	0.5	Wood on leather	0.3 – 0.4
Rubber on solids	1-4	Leather on metal	0.6

Table. 2.1. Static friction coefficients for some common materials
(Source: CRC Handbook of Chemistry and Physics)

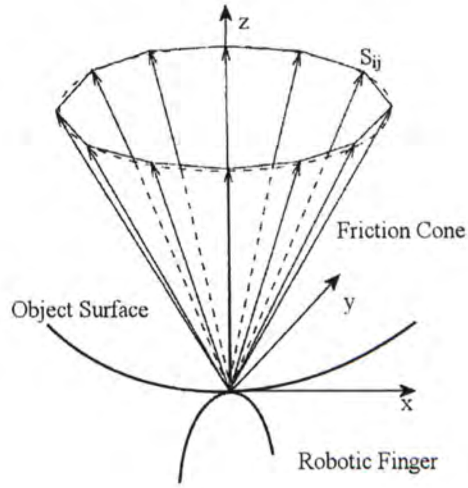


Fig. 2.4. Linearization of friction cone by a polyhedral convex cone.

In the present of friction, forces can be exerted in any direction that is within the friction cone for the contact. To simplify the problem, the friction cone is linearized by a polyhedral convex cone with m sides (Fig 2.1). Under this approximation, the grasp force f_i can be represented as

$$f_i = \sum_{j=1}^{m} \lambda_{ij} s_{ij}, \quad \lambda_{ij} \geq 0 \tag{2.5}$$

where s_{ij} represents the j-th edge vector of the polyhedral convex cone for the grasp force f_i. Coefficients λ_{ij} are nonnegative constants. The force and torque, corresponding to the grasp force f_i, applied at the center of mass of the object is given by

$$w_i = \begin{pmatrix} f_i \\ \tau_i \end{pmatrix} = \begin{pmatrix} f_i \\ r_i \times f_i \end{pmatrix}. \tag{2.6}$$

Substituting (2.5) into (2.6) derives

$$w_i = \sum_{j=1}^{m} \lambda_{ij} u_{ij} \tag{2.7}$$

where

$$u_{ij} = \begin{pmatrix} s_{ij} \\ r_i \times s_{ij} \end{pmatrix}. \tag{2.8}$$

The vectors u_{ij} is normalized as follows:

$$w_{ij} = \frac{1}{\| u_{ij} \|} u_{ij}. \tag{2.9}$$

The term $\|u_{ij}\|$ denotes the L_2 norm of vector u_{ij}. Vectors w_{ij} are called *primitive contact wrenches*. The norms of the primitive contact wrenches w_{ij} are equal to one. Let

$$\alpha_{ij} = \lambda_{ij} \| u_{ij} \|. \tag{2.10}$$

The net wrench applied at the object by the n fingers is

$$\begin{aligned} w_{net} &= \sum_{i=1}^{n}\sum_{j=1}^{m}\alpha_{ij}w_{ij} \\ &= W\alpha \end{aligned} \tag{2.11}$$

where W and α are given by

$$\begin{aligned} W &= (w_{11}, w_{12}, \cdots, w_{1m}, \cdots, w_{n1}, w_{n2}, \cdots, w_{nm}), \\ \alpha &= (\alpha_{11}, \alpha_{12}, \cdots, \alpha_{1m}, \cdots, \alpha_{n1}, \alpha_{n2}, \cdots, \alpha_{nm}). \end{aligned} \tag{2.12}$$

W is a $6 \times nm$ wrench matrix and its column vectors are the primitive contact wrenches. For convenience, w_i with a single subscript i is used, instead of w_{ij}, to denote the i-th column vector of grasp matrix W, and α_i is used to represent the i-th component of vector α. Let $n_p = nm$ be the number of the primitive contact wrenches of a grasp.

DEFINITION 2: FRICTIONAL FORM-CLOSURE GRASP

Suppose that an n-finger frictional grasp is given. If any external wrench $w_{ext} \in R^6$ applied at the object can be balanced by the grasp forces f_i of the fingers, the grasp is said to be form-closure.

According to the definition of frictional form-closure grasp and the linear approximation of friction cone, we obviously have the following proposition.

PROPOSITION 1:

Assume that m segments are used to approximate each friction cone. For any external wrench w_{ext} applied at the object, if it is always possible to find an α with $\alpha_{ij} \geq 0$ such that

$$W\alpha + w_{ext} = 0\,, \tag{2.13}$$

the grasp is form-closure.

2.5 Problem Statement

Denote the point set containing the N permissible surface points $r_i = (x_i, y_i, z_i)^T$, i = 1, 2, …, N, of the object by Ω. Surface points r_i are the position vectors w.r.t. the object coordinate frame with origin at the center of mass. Denote the surface normal at contact point r_i by n_i. Then, we have the following problem definition:

PROBLEM 1:

Given a set Ω of surface points and their corresponding surface normals representing a 3-D object of arbitrary geometry, find a n-finger form-closure grasp in the point set Ω.

Chapter 3

A Qualitative Test Algorithm and a Local Search Algorithm

In this chapter, a qualitative test algorithm for checking form-closure grasps is reviewed. The test algorithm plays an important role in the form-closure grasp searching algorithms developed in my work. Then, a local search algorithm will be proposed for finding form-closure grasps. In the last section, the issue of kinematic feasibility of robot fingers will be discussed.

3.1 Qualitative Test Algorithm

An inevitable step in the grasp planning algorithms is qualitative test of a form-closure grasp. The qualitative test algorithm developed by Liu [Liu99] is employed. The testing algorithm works on the basis of a ray-shooting process, which is formulated as a linear programming problem based on the duality between convex hulls and convex polytopes. It can be applied to both frictionless and frictional grasps as it does not use the special properties of frictionless/frictional grasps. The idea of the test algorithm is briefed below and the details can be referred to [Liu99].

As discussed before, [Salisbury82] have proved that a necessary and sufficient condition for form-closure is that the primitive contact wrenches resulted by contact forces at the contact points positively span the entire wrench space. This condition is equivalent to that the origin of the wrench space lies strictly inside the convex hull of the primitive contact [Mishra87], [Montana91], [Murray94]. This condition gives rise to the following theorem:

THEOREM 1:

Suppose that an n-finger grasp is given. Denote the convex hull of the contact wrenches w_i by $H(W)$. Assume that point P is an interior point of $H(W)$. The ray from point P to the origin O of the wrench space R^6 intersects $H(W)$ in a point Q only. A form-closure grasp is equivalent to that the distance $\|PQ\|$ between points P and Q is strictly larger than the distance $\|PO\|$ between points P and O.

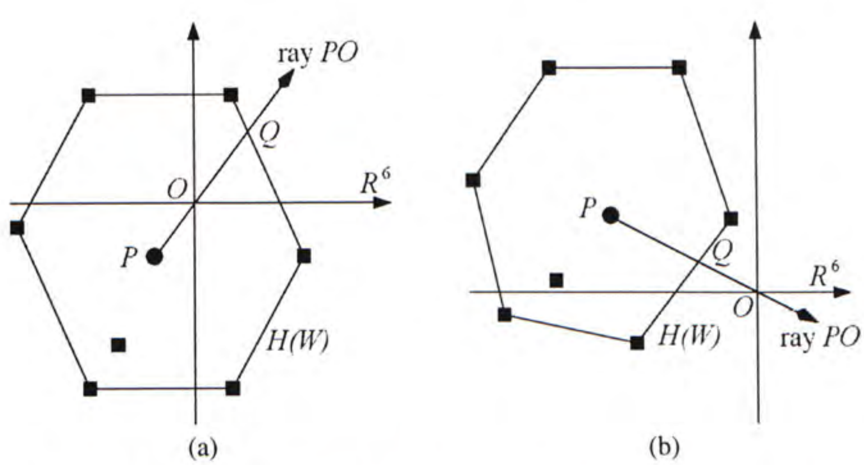


Fig. 3.1. (a) A form-closure grasp and (b) a non-form-closure grasp.

The proof of this theorem can be found in [Liu99]. This theorem is illustrated in Fig. 3.1. In Fig. 3.1(a), the origin O is on the segment PQ and hence the grasp is form-closure. In Fig. 3.1(b), the origin O does not lie on segment PQ and thus the grasp is not a form-closure grasp. It should be emphasized that one interior point P is sufficient to check whether the origin O is inside the convex hull. The point P cannot lie on the boundary of $H(W)$ because the check requires the origin located strictly inside the convex hull. As shown in Fig. 3.2, the origin is not strictly contained by $H(W)$ even if $\|PQ\|$ is strictly larger than $\|PO\|$ when point P and the origin O are on the same facet.

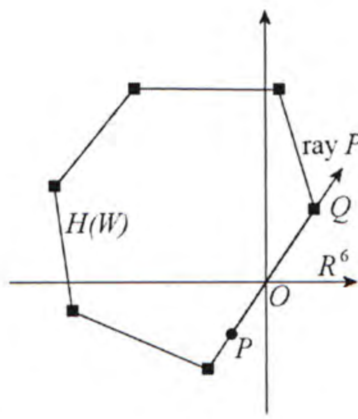


Fig. 3.2. The point P cannot be on the boundary of $H(W)$.

It is obvious that a strictly positive combination of the contact wrenches must be an interior point of the convex hull $H(W)$. Therefore, we can choose point P as the centroid of the n_p points (n_p is the number of contact wrenches, $n_p = n = 7$ in frictionless case and $n_p = nm$ in frictional case):

$$P = \frac{1}{n_p} \sum_{i=1}^{n_p} w_i \, . \tag{3.1}$$

To locate the intersection point Q of $H(W)$ with the ray from the interior point P to the origin O of the wrench space R^6, we first detect the facet E of $H(W)$

intersected by the ray PO, and then calculate the intersection of the facet with the ray PO . The problem of detecting the facet is closely related to the ray-shooting problem of a convex hull defined as follows:

DEFINITION 3: RAY-SHOOTING PROBLEM

Let M be a given set of points in R^d. Assume that the convex hull $H(M)$ contains the origin. Given a ray emanating from the origin, find the facet of $H(M)$ intersected by this ray.

It is well known in Computational Geometry that the ray-shooting problem can be transformed to a linear programming problem based on the duality between convex hull and convex polytope. Note that a ray-shooting problem assumes that the convex hull contains the origin. By applying a coordinate translation -P of on points in R^6, we readily change the origin point to point P, which lies exactly inside the convex hull. After the coordinate translation, the convex hull $H(W)$ is dual to the convex polytope

$$(w_i - P)^T x \leq 1 \qquad i = 1,2,\cdots,n_p . \tag{3.2}$$

Denote the direction vector of the ray PO by t. Based on the duality between convex hull and convex polytope, the ray-shooting problem is equivalent to a problem of maximizing the objective function:

$$z = t^T x \tag{3.3}$$

subject to the constraints in (3.2). Suppose that the optimal solution of the linear programming problem is $\vec{e}=(e_1, e_2, \cdots, e_6)$. According to the duality, the facet E of $H(W)$ intersected by the ray PO is

$$e_1x_1 + e_2x_2 + \cdots + e_6x_6 = 1. \tag{3.4}$$

Then, the intersection point Q of $H(W)$ with the ray PO is the intersection of the hyperplane defined by (3.4) with the ray PO.

SUMMARY OF THE QUALITATIVE TEST ALGORITHM

Step 1: Calculate all the primitive contact wrenches w_i.

Step 2: Use (3.1) to calculate an interior point P of the convex hull $H(W)$.

Step 3: By the linear programming, calculate the optimal point $\vec{e}=(e_1, e_2, \cdots, e_6)$ that maximizes function z in (3.3) subject to the constraints in (3.2). The hyperplane $e_1x_1 + e_2x_2 + \cdots + e_6x_6 = 1$ corresponding to the optimal point $\vec{e}$ is the facet E of $H(W)$ intersected by the ray PO.

Step 4: Calculate the intersection Q of the ray PO with the hyperplane $e_1x_1 + e_2x_2 + \cdots + e_6x_6 = 1$. The point Q is the intersection of $H(W)$ with the ray PO.

Step 5: If the distance $\|PQ\|$ is larger than the distance $\|PO\|$, the grasp is form-closure; otherwise, it is not.

Step 6: The algorithm ends.

This chapter presents an efficient local search algorithm which heuristically searches for a form-closure grasp on 3-D objects represented by discrete points. The algorithm has been presented in [Miu01] and employed in [Miu02]. The local search starts from a random selection of an initial grasp. The algorithm searches for a form-closure grasp in the direction of reducing the distance between the convex hull of the primitive wrenches and the origin of the wrench space. The key idea here is to move the convex hull gradually closer and closer to the origin of the wrench space (Fig 3.3).

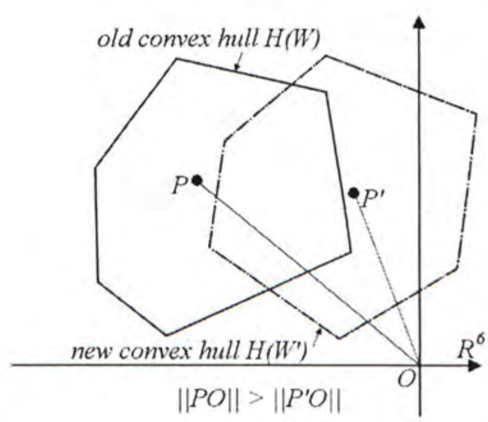


Fig. 3.3. The local search moves the centroid of the convex hull closer to the origin of the wrench space.

3.2 Local Search Algorithm

Here, a method is proposed to acquire the motion of the grasping positions to search for a form-closure grasp. As mentioned in Chapter 2, each of the given points in set Ω is connected with four neighboring points. Therefore, at every grasping position $r_i \in R^3$, four possible directions of motions are defined by the four adjacent points of $\zeta_{i_l} \in \Omega$, $i = 1, 2, \ldots, n$ is the finger index, $l = 1, 2, 3, 4$ is the index of the four adjacent points. To simplify the problem, it assumes that at an iteration only one finger can be moved to its neighboring position.

Consequently, for a grasp with n contacts there are totally $4n$ candidate grasps $CG_{i_l} \in R^{3\times n}$ where $CG_{i_l} = (r_1, r_2, \ldots, r_{i-1}, \zeta_{i_l}, r_{i+1}, \ldots, r_n)$ at an iteration.

Observed from the qualitative test, it is desirable to move the convex hull $H(W)$ towards the origin O of the wrench space R^6 until the origin O completely lies inside the convex hull. Therefore, the distance $\|PO\|$ is considered as the heuristics.

One may notice that the heuristics $\|PO\|$ employed in the local search algorithm can be replaced by the distance difference $\|PO\|-\|PQ\|$. The use of $\|PO\|-\|PQ\|$ as the heuristics gives a similar but different meaning to the algorithm as $\|PO\|$ does. The distance difference $\|PO\|-\|PQ\|$ can be used to measure how far is a grasp from being form-closure. Both approaches have been implemented [Miu01], [Miu02] and they both give desirable results. Therefore, either $\|PO\|$ and $\|PO\|-\|PQ\|$ can be employed as the heuristics in the local search.

A candidate grasp that has the minimum value of distance $\|PO\|$ among all the $4n$ candidate grasps should be adopted as the new grasp. Denote $\|PO\|$ obtained in the k-th iteration with $\|PO^k\|$. If the minimum value of the distance $\|PO\|$ is not strictly smaller than $\|PO^k\|$, it means none of the possible next motions can enhance the form-closure property. In this case, a local minimum exists in the heuristics function $\|PO\|$ and the search should be terminated. A form-closure grasp is obtained if it is possible to perform the search iteratively until $\|PO\| < \|PQ\|$. The local search algorithm is described in detail as follows:

SUMMARY OF THE LOCAL SEARCH ALGORITHM

Step 1: $k = 1$; arbitrarily choose an initial grasp $G^k = \{ r_i \}$, for $i = 1, 2, \ldots, n$, where all $r_i \in \Omega$. Calculate the primitive contact wrenches w_i^k.

Step 2: Check whether G^k is form-closure using the qualitative test algorithm; if so, return G^k.

Step 3: For each of the grasping position r_i, locate the four adjacent positions $\zeta_{i_l} \in \Omega$, for $l = 1, 2, 3, 4$; generate all the $4n$ combinations of candidate grasps CG_{i_l}.

Step 4: For each candidate grasp CG_{i_l}, calculate the corresponding $\|PO(CG_{i_l})\|$.

Step 5: Find the candidate grasp CG^* with the minimum value of $\|PO\|$. If $\|PO(CG^*)\|$ is not strictly smaller than $\|PO^k\|$, report that local minimum is encountered; return G^k. Otherwise, $k = k + 1$; update $G^k = CG^*$, $w_i^k(CG^*)$, $\|PO^k\| = \|PO(CG^*)\|$; go back to *Step 2*.

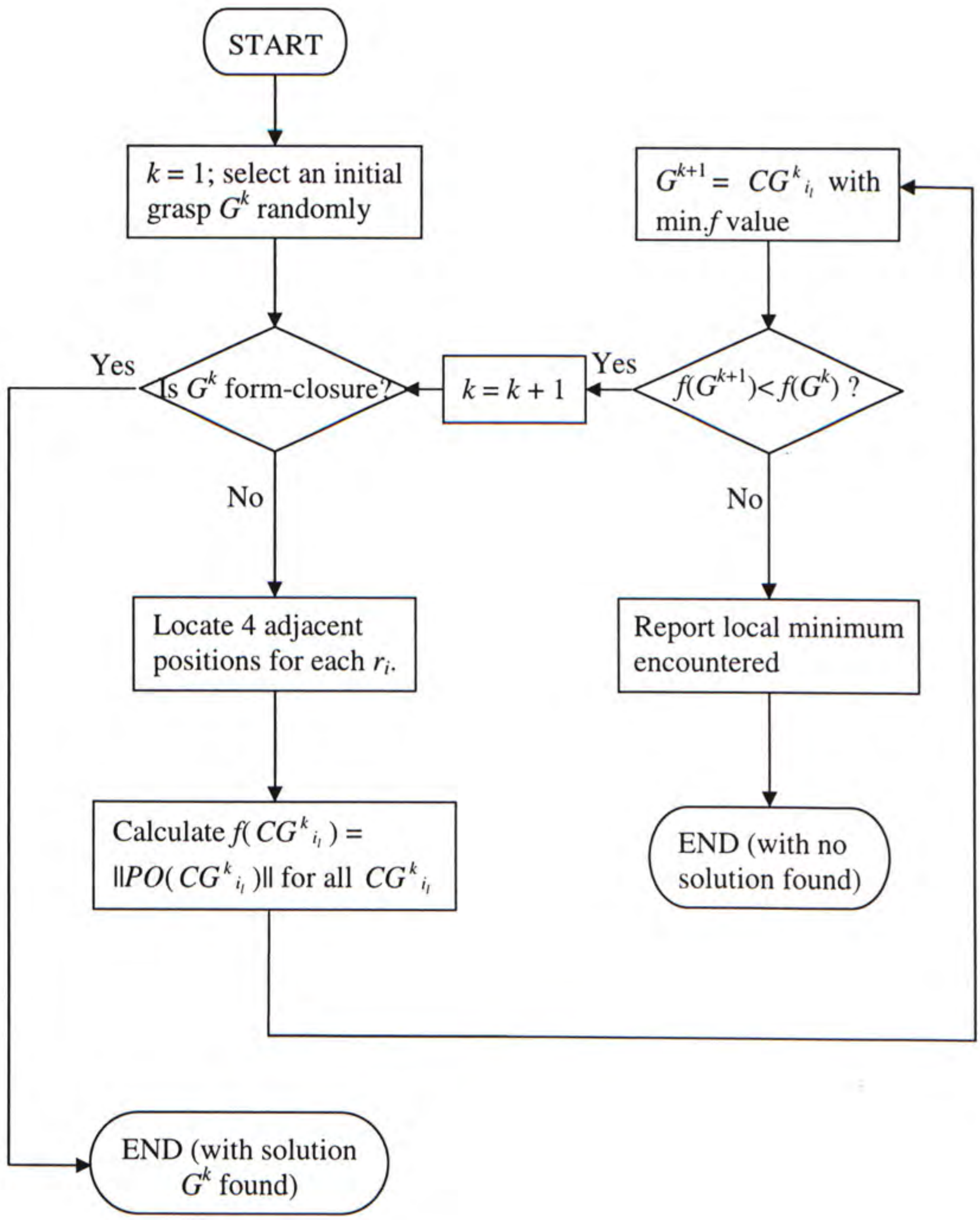


Fig. 3.4. The block diagram of the local search algorithm

The above algorithm can be revised to obtain a local optimal grasp. It can be done by discarding the termination instruction in *Step 2*. Then, the iteration process will end only when $\|PO\|$ is not strictly decreasing.

Note that the local search terminates under two situations: when a form-closure grasp is obtained in *Step 2* or when a local minimum is encountered in *Step 5*. In case a local minimum is encountered, the algorithm needs to select another initial grasp and start the local search again. The existence of local minima in the performance index $\|PO\|$ depends on the object's geometry. The probability of encountering a local minimum increases with the complexity of the object's geometry, while, in normal situations, a convex object is free from local minima. Random re-selection of initial grasp is not a good way to solve the local minimum problem, as it does not guarantee a solution. In the next chapter, a divide-and-conquer strategy is provided to divide the problem into sub-problems with less points so as to overcome the local minimum problem and at the same time to improve the computational efficiency.

3.3 Grasp planning under kinematic constraints

An important feature of the proposed algorithm is that it ensures kinematic feasibility of robot fingers. As long as the inverse kinematic model of each finger is developed, the algorithm can discard any candidate grasp with one or more candidate grip point not reachable by the corresponding finger.

In ordinary study of form-closure, kinematic constraints are not taken into consideration. The analysis of form-closure is intrinsically geometric, in so far as it does not consider the kinematics of the grasping mechanism or the magnitude of contact forces. Most literatures regards the kinematic analysis of grippers or robotic fingers belongs to the area of *force-closure*.

In this dissertation, the only kinematic concern is the reachability of the robotic fingers. When the local search algorithm makes decision on the grasp choice, it is important to ensure that the selected candidate grasp is feasible for the robot hand.

The problem of developing the inverse kinematic model of a given hand has been extensively investigated. As long as the inverse kinematic model is built, we can acquire the set of joint angles corresponds to any grasp. For any candidate grasp that incurs joint angles beyond the joint limits, we can properly discard that candidate grasp.

Chapter 4

A Divide-and-Conquer Technique

To cope with the local minimum problem, a divide-and-conquer strategy is employed to divide the problem in the original point set Ω into sub-problems in subsets. The division is based on an important observation that *the primitive contact wrenches corresponding to the contact points must not all lie on one side of any hyperplane passing through O to achieve a form-closure grasp*. Having this observation, we can drastically reduce the number of eligible candidate grasps by neglecting those grasps with all the wrenches on one side of a specific hyperplane called *separating hyperplane*. The determination of the separating hyperplane will be discussed in the first section of this chapter. The problem division process is different for frictionless and frictional models due to the difference in the properties of the primitive contact wrenches of the two cases. Therefore, the divide-and-conquer technique for frictionless and frictional grasps will be discussed separately in Sections 4.2 and 4.3.

4.1 Determining a Separating Hyperplane

As discussed before, a separating hyperplane in the wrench space R^6 should be obtained when a local minimum of $\|PO\|$ is encountered during the local search process. The selected separating hyperplane should be easily obtainable and able to lead the algorithm to escape from the local minimum.

Recall, in the qualitative test, that the hyperplane defined in (3.17) is the facet E of $H(W)$ intersected by the ray PO. Here, a hyperplane Y that contains the origin of the wrench space and is parallel to the facet E obtained in the local minimum iteration is selected as the separating hyperplane (Fig 4.1):

$$e_1x_1 + e_2x_2 + \cdots + e_6x_6 = 0. \tag{4.1}$$

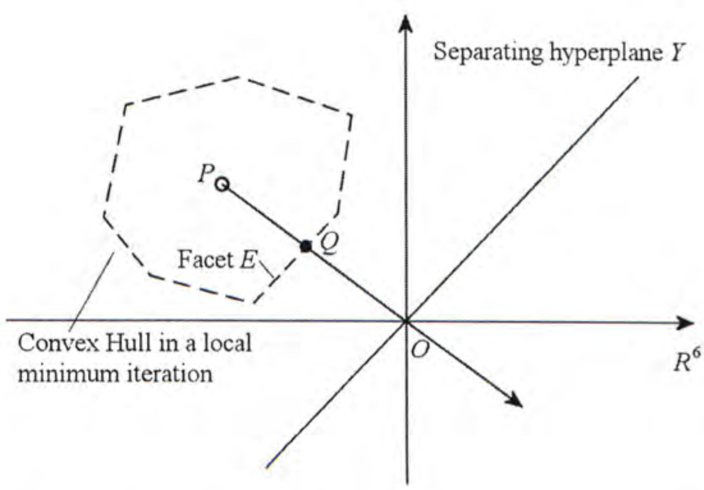


Fig. 4.1. Separating hyperplane Y defined in wrench space R^6 parallel to facet E.

As facet E is readily obtainable from the qualitative test algorithm, no extra computational effort has to be made in yielding separating hyperplane Y. Besides, hyperplane Y can lead the algorithm to a new initial grasp that avoids the local minimum just encountered.

The separating hyperplane divides the wrench space into two half-spaces, which are denoted by Y^+ and Y^-, respectively. Y^+ is the half-space:

$e_1x_1 + e_2x_2 + \cdots + e_6x_6 \geq 0$; and Y^- denotes the half-space: $e_1x_1 + e_2x_2 + \cdots + e_6x_6 < 0$.

4.2 Divide-and-Conquer in Frictionless Case

In frictionless contact model, forces can be only exerted in the normal direction of the contact point. Therefore, each contact point corresponds to one primitive contact wrench only. The original point set Ω is divided into two subsets according to which side of the separating hyperplane that the corresponding primitive contact wrench located. As shown in Fig 4.2, the separating hyperplane Y divides the original point set Ω into two subsets:

$$\begin{aligned} \Omega(Y^-) &= \{r_j \in \Omega \mid w_j \in Y^-, j = 1,2,\cdots,N\}, \\ \Omega(Y^+) &= \{r_j \in \Omega \mid w_j \in Y^+, j = 1,2,\cdots,N\}. \end{aligned} \tag{4.2}$$

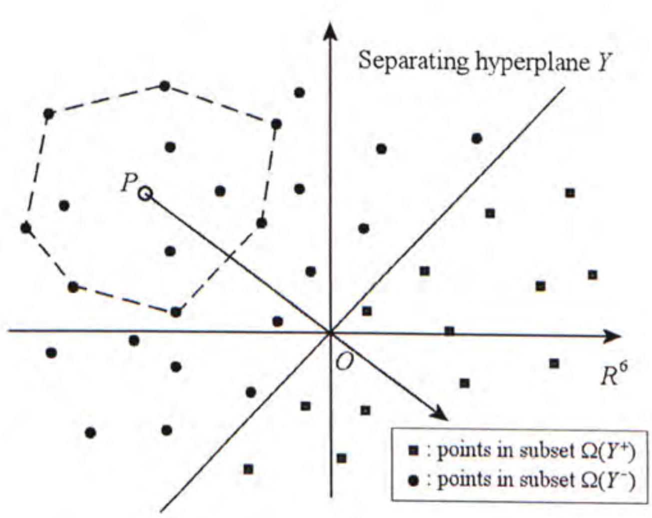


Fig. 4.2. Separating hyperplane Y divides point set Ω into subsets $\Omega\,(Y^+)$ and $\Omega(Y^-)$.

Since in a form-closure grasp the seven points should not all belong to subset $\Omega(Y^-)$ or subset $\Omega(Y^+)$, points of form-closure grasps must be selected from both of these subsets. This gives rise to a division of the problem in the original set into problems in the subsets based on *existence conditions* of form-closure grasps. Six problems in the subsets need to be considered, which are grasps consisting of

- one grasp point from subset $\Omega(Y^-)$ and six grasp points from subset $\Omega(Y^+)$,
- two grasp points from subset $\Omega(Y^-)$ and five grasp points from subset $\Omega(Y^+)$,
- three grasp points from subset $\Omega(Y^-)$ and four grasp points from subset $\Omega(Y^+)$,
- four grasp points from subset $\Omega(Y^-)$ and three grasp points from subset $\Omega(Y^+)$,
- five grasp points from subset $\Omega(Y^-)$ and two grasp points from subset $\Omega(Y^+)$, and
- six grasp points from subset $\Omega(Y^-)$ and one grasp point from subset $\Omega(Y^+)$.

The sub-problems obtained are represented as child nodes of the search tree whose root represents the problem in the original set (Fig 4.3). The *(i,j)* in the node denotes that i and j grasp points must be selected from subsets $\Omega(Y^-)$ and $\Omega(Y^+)$, respectively, in order to obtain a form-closure grasp in the corresponding subsets of points.

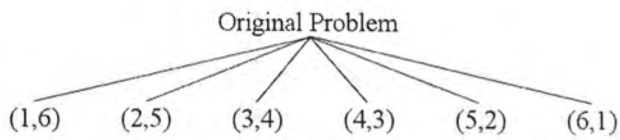


Fig. 4.3. The search tree generated by decomposition of the problems for 7-finger frictionless grasps.

From the six child nodes, one of them is selected and the local search of a form-closure grasp is carried out within the subset of points of the selected child node. When a new local minimum is encountered, a new separating hyperplane defined in the same way is employed to further divide the subset into three or four subsets. Based on the existence condition, a new set of sub-problems are generated and represented as child nodes of the node selected. This process is performed recursively until a form-closure grasp is found or all the nodes have been explored. It should be noted that no child will be generated for a node if none of the combination of the subsets divided by the separating hyperplane satisfies the existence condition of form-closure grasps.

To determine which child should be traversed first, a node with eligible contact points more evenly distributed over the point sets is considered as having a higher chance to contain form-closure grasps. Therefore, a heuristic function h is defined as follows:

$$h(< a_1 \; a_2 \cdots \; a_c >) = \sum_{i=1}^{c} \left(\frac{7}{c} - a_i \right)^2 , \tag{4.3}$$

where is c the total number of subsets and a_i is the number of points to be selected from subset Ω_i. A child node with a smaller h value will be traversed prior to the one with a larger h value.

Note that not all nodes on the search tree will be traversed. As long as a form-closure grasp is obtained, the algorithm will stop the search. If there does not exist any form-closure grasp in the given point sets, the algorithm will traverse all the nodes in the search tree and reports that no solution is obtained.

Although the algorithm employs heuristics to guide the local search and selection of the sub-problems, it is a complete one, i.e., it always finds a form-closure grasp or reports no solution after all the nodes in the search tree have been visited. To the best of our knowledge, this is the first complete algorithm that heuristically searches for a form-closure grasp in a discrete domain. The other complete algorithm is the combinatory approach which combines all possible points in the point set and checks exhaustively form-closure property of all the combinations. The computational complexity of the exhaustive search is $O(N^7)$, where N denotes the number of points in the point set. Compared to the exhaustive search algorithm, our algorithm is more efficient because a large number of candidates have been eliminated whenever a problem is decomposed into sub-problems by the separating hyperplane. For example, suppose that the separating hyperplane corresponding to the first local minimum divides the original point set into two subsets with equal number of points. If we carry out exhaustive search on the six problems in the sub-sets, the number of all possible grasps is equal to $6(N/2)^7$, which is much smaller than N^7. Therefore, wherever a problem is decomposed at a local minimum, the combinatory number will be reduced significantly. Furthermore, for each problem in a subset, the algorithm does not exhaustively check all possible combinations but employs the local search algorithm whose complexity is proportional to the number of the points in the subset. Therefore, the efficiency of the proposed algorithm can be concluded. However, a rigorous analysis on the computation complexity is difficult but necessary.

4.3 Divide-and-Conquer in Frictional Case

In frictional contact model, forces can be exerted in any direction within the friction cone. As stated in Chapter 2, the friction cone is linearized by a polyhedral convex cone with m sides. Therefore, each contact point corresponds to m primitive contact wrenches. As shown in Fig 4.4, the original point set Ω is divided into *three* subsets according to how the separating hyperplane cuts the collection of m primitive contact wrenches for a contact point. The three subsets are as follows:

$$\begin{aligned}
&\Omega(Y^-) = \{r_i \in \Omega \mid w_{ij} \in Y^-, i = 1,2,\cdots,S, j = 1,2,\cdots,m\}, \\
&\Omega(Y^+) = \{r_i \in \Omega \mid w_{ij} \in Y^+, i = 1,2,\cdots,S, j = 1,2,\cdots,m\}, \\
&\Omega(Y^0) = \{r_i \in \Omega \mid r_i \notin \Omega(Y^-)\}, r_i \notin \Omega(Y^+)\}.
\end{aligned} \tag{4.2}$$

$\Omega(Y^0)$ denotes the subset containing contact points with polyhedral frictional cone located in both sides of the separating hyperplane.

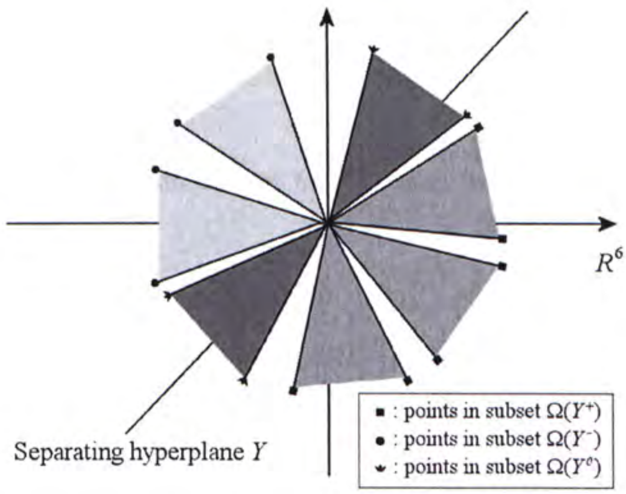


Fig. 4.4. Separating hyperplane Y divides point set Ω into subsets $\Omega(Y^+)$, $\Omega(Y^-)$ and $\Omega(Y^0)$.

Since in a form-closure grasp the n points should not all belong to subset $\Omega(Y^-)$ or subset $\Omega(Y^+)$, points of form-closure grasps must be selected from the subset(s) as stated in any one of the following combinations:

- $\Omega(Y^-)$, $\Omega(Y^+)$ and $\Omega(Y^0)$,
- $\Omega(Y^-)$ and $\Omega(Y^+)$ only,
- $\Omega(Y^-)$ and $\Omega(Y^0)$ only,
- $\Omega(Y^+)$ and $\Omega(Y^0)$ only, and
- $\Omega(Y^0)$ only.

Similarly, a number of existence conditions of form-closure grasps is defined to divide the problem in the original set into problems in the subsets. To illustrate the idea, let's consider a 3-finger frictional grasp. In such a case, eight problems in the subsets need to be considered, which are grasps consisting of

- one grasp point from subset $\Omega(Y^-)$, one grasp point from subset $\Omega(Y^+)$ and one grasp point from subset $\Omega(Y^0)$.

- two grasp points from subset $\Omega(Y^-)$ and one grasp point from subset $\Omega(Y^+)$,
- one grasp point from subset $\Omega(Y^-)$ and two grasp points from subset $\Omega(Y^+)$,
- two grasp points from subset $\Omega(Y^-)$ and one grasp point from subset $\Omega(Y^0)$,
- one grasp point from subset $\Omega(Y^-)$ and two grasp points from subset $\Omega(Y^0)$,
- two grasp points from subset $\Omega(Y^+)$ and one grasp point from subset $\Omega(Y^0)$,
- one grasp point from subset $\Omega(Y^+)$ and two grasp points from subset $\Omega(Y^0)$, and
- all the three grasp points from subset $\Omega(Y^0)$.

The sub-problems obtained are represented as child nodes of the search tree whose root represents the problem in the original set (Fig. 4.5). The *(i,j,k)* in the node denotes that *i*, *j* and *k* grasp points must be selected from subsets $\Omega(Y^-)$, $\Omega(Y^+)$ and $\Omega(Y^0)$, respectively, in order to obtain a form-closure grasp in the corresponding subsets of points. Following this fashion, the mechanism of generating the existence conditions and the division of original problem into sub-problems for a general *n*-finger grasp can be done.

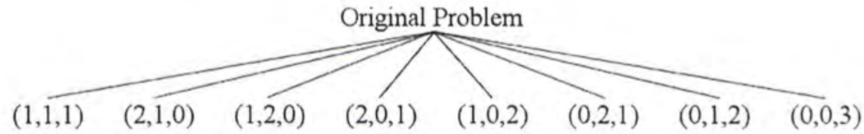


Fig. 4.5. The search tree generated by decomposition of the problems for 3-finger frictional grasps.

Then, most what have to be performed is just similar to the frictionless case. From all the child nodes, one of them is selected and the local search of a form-closure grasp is carried out within the subset of points of the selected child node. When a new local minimum is encountered, a new separating hyperplane defined in the same way is employed to further divide the subset into nine or less subsets. Based on the existence condition, a new set of sub-problems are generated and represented as child nodes of the node selected. This process is performed recursively until a form-closure grasp is found or all the nodes have been explored. It should be noted that no child will be generated for a node if none of the combination of the subsets divided by the separating hyperplane satisfies the existence condition of form-closure grasps.

Again, a node with eligible contact points more evenly distributed over the point sets is considered as having a higher chance to contain form-closure grasps. Therefore, the heuristic function h in frictional case to determine which child should be traversed first is defined as follows:

$$h(< a_1\ a_2 \cdots\ a_c >) = \sum_{i=1}^{c} \left(\frac{n}{c} - a_i \right)^2 , \tag{4.3}$$

where is c the total number of subsets and a_i is the number of points to be selected from subset Ω_i. A child node with a smaller h value will be traversed prior to the one with a larger h value.

Chapter 5

Implementation and Examples

The algorithms for frictionless and frictional form-closure grasp synthesis developed in this work are implemented with MatLab to verify their practicality. In this chapter, a number of simulation examples will be shown. In section 5.1, examples of frictionless grasps will be shown, whereas a 7-finger robotic hand is to grasp a number of 3-D objects. In section 5.2, examples of frictional grasps on varies objects will be shown. The friction cones are all linearized by polyhedral convex cone with 20 segments. The number of contacts is three to five. It should be noted that most simulations we have done do not contain any local minimum. The examples with local minimum are shown just for the purpose of illustrating the efficiency of the divide-and-conquer strategy in the algorithm. In the last section, examples of form-closure grasp synthesis with consideration of kinematics constraints will be given.

5.1 Examples of Frictionless Grasps

In this section, all contacts in the examples are assumed to be frictionless point contacts. Two different objects represented by point sets will be used and they are without rotational symmetries. Seven contact locations in yielding form-closure grasp are to be found in each example.

Example 1

In the first example, the 3-D curved object shown in Chapter 2 is used (Fig. 5.1). The object is formed by revolving an equation about the z-axis whereas the radius of revolution varies. There are 5130 predetermined contact points. Fig. 5.2 shows the selected initial grasp, which is not form-closure. Then the algorithm performs the local search process to iteratively improve the grasp until a local minimum is encountered in the 43rd iteration (Fig. 5.4). Then, the search tree for the problem is generated and the child (3,4) is adopted. A new initial grasp is selected based on the existence condition and the local search is carried out again. Finally, a form-closure grasp (Fig. 5.3) is obtained. The total number of iterations involved is 84.

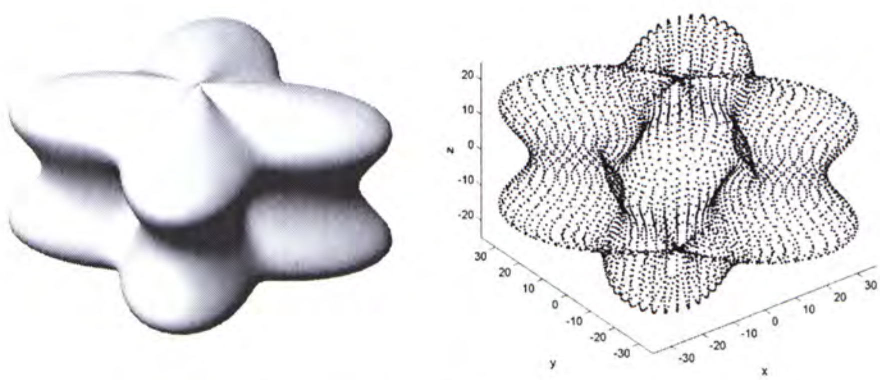


Fig. 5.1. A curved object with 5130 given candidate contact locations.

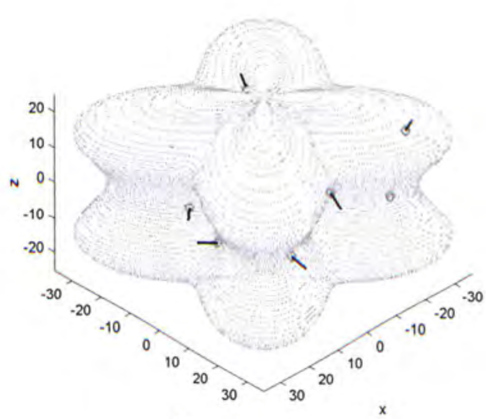


Fig. 5.2. Example 1: Initial grasp (non-form-closure).

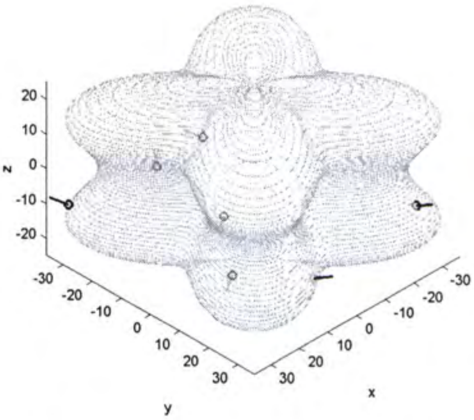


Fig. 5.3. Example 1: Final grasp (form-closure).

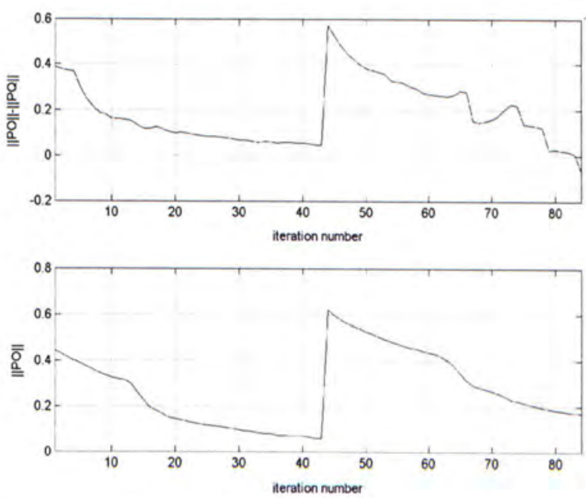


Fig. 5.4. Example 1: The distance $\|PO\|$ encounters a local minimum at 43[rd] iteration and the distance difference $\|PO\| - \|PQ\| < 0$ (form-closure obtained) at last.

Example 2

The second example used the same object as Example 1. However, only 1066 candidate locations are given. These candidate contact points are located on the gridded region as shown in Fig. 5.5. It is obvious that these candidate contact locations cannot form any form-closure grasp. This example shows the efficiency of the algorithm in discarding a large number of combination of grasps which must not be form-closure. Fig. 5.6 shows the initial contact positions on the object. The performance index $\|PO\|$ obtains its first local minimum value in the 48^{th} iteration (Fig. 5.8). Then, the algorithm generates all the eligible child states and perform local search on each of them in a best-first order, namely (3,4), (4,3), (2,5), (5,2), (1,6) and (6,1). Finally, the algorithm finishes search on all the child states and reports that no solution found. The final grasp shown in Fig. 5.7 is non-form-closure. 311 iterations are involved in total. This number is greatly smaller than ${}_{1066}C_7$.

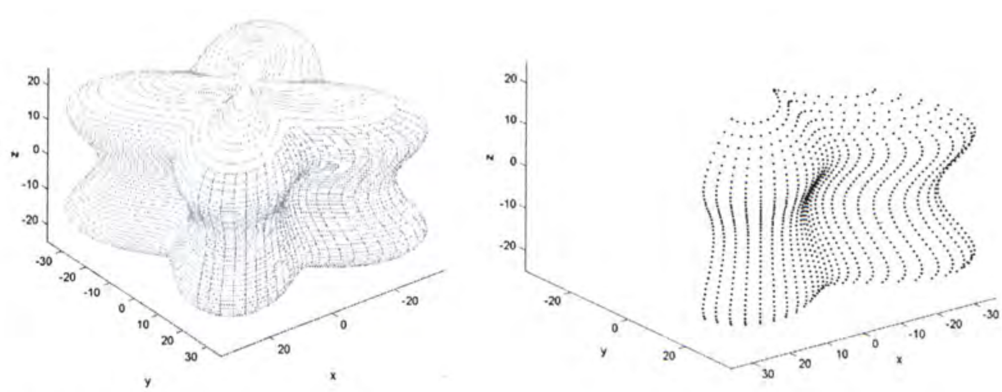


Fig. 5.5. The curved object with 1066 given points all located in the gridded region, which is impossible for yielding a form-closure grasp.

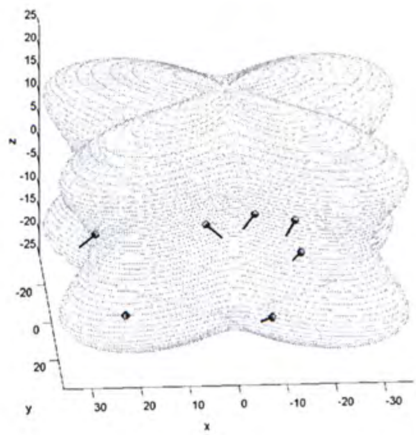


Fig. 5.6. Example 2: Initial grasp (non-form-closure).

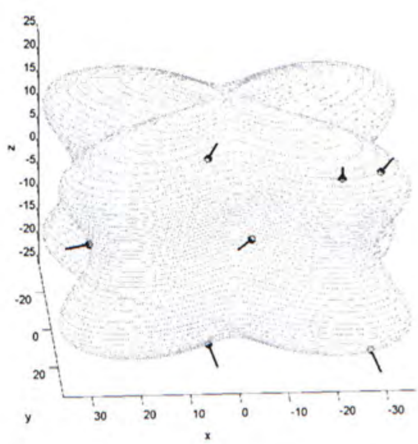


Fig. 5.7. Example 2: Final grasp (non-form-closure).

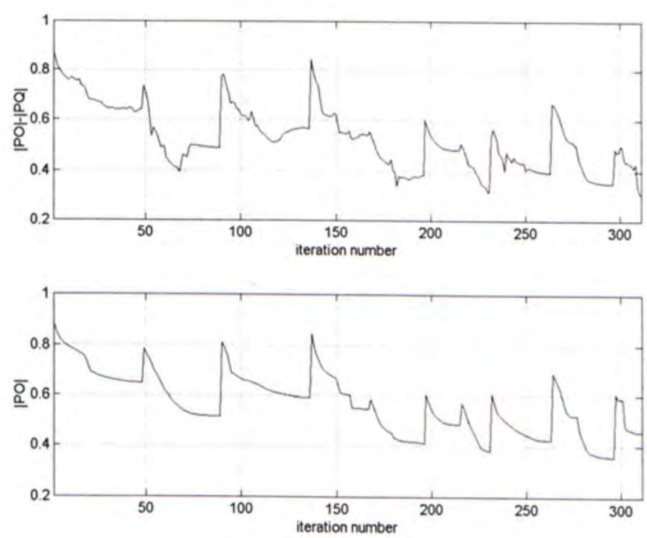


Fig. 5.8. Example 2: The distance ||*PO*|| encounters local minima for six times and the distance difference ||*PO*|| - ||*PQ*|| > 0 (no form-closure obtained) at last.

Example 3

The model used in both Examples 3 and 4 is an airfoil (Fig. 5.9) with 1546 predetermined candidate locations and surface normals. The model has been shown in Chapter 2.

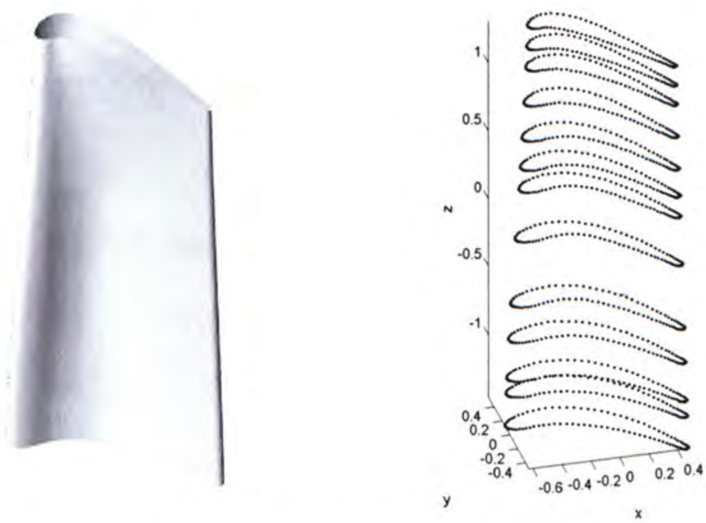


Fig. 5.9. An airfoil model with 1546 surface points.

In Example 3, the algorithm takes 68 iterations in total to obtain a form-closure grasp. The initial grasp shown in Fig. 5.10 is non-form-closure. After 62 iterations, the algorithm encounters a local minimum in the performance measure $\|PO\|$. A separating hyperplane is defined to divide the candidate contact points into two subsets and the algorithm performs the local search process in the state (3,4) in the search tree. Then, a form-closure grasp (Fig. 5.11) is found after 6 more iterations. Fig. 5.12 plots the distance difference $\|PO\|$ - $\|PQ\|$ and the distance $\|PO\|$ against the iteration number.

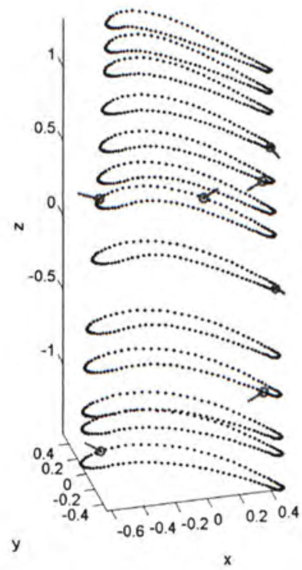


Fig. 5.10. Example 3: Initial grasp (non-form-closure).

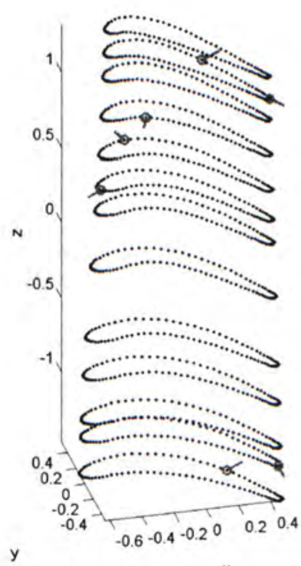


Fig. 5.11. Example 3: Final grasp (form-closure).

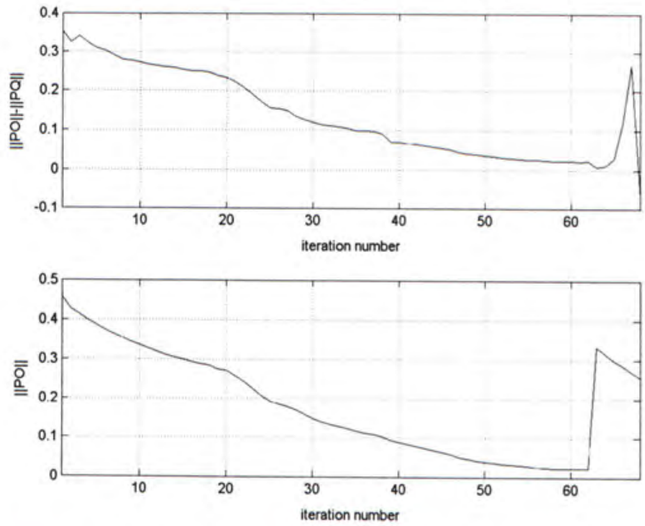


Fig. 5.12. Example 3: The distance ||*PO*|| encounters a local minimum at 62nd iteration and the distance difference ||*PO*|| - ||*PQ*|| < 0 (form-closure obtained) at last.

Example 4

The fourth example contains 171 iterations in total. In Fig. 5.13, seven initial contact locations are selected randomly. As the selected grasp is non-form-closure, the algorithm searches locally for a form-closure grasp. Two local minima are encountered in the 90th and 126th iteration respectively. Finally, the algorithm finds a form-closure grasp (Fig. 5.14). Fig. 5.15 plots the distance difference $\|PO\|$ - $\|PQ\|$ and the distance $\|PO\|$ against the iteration number.

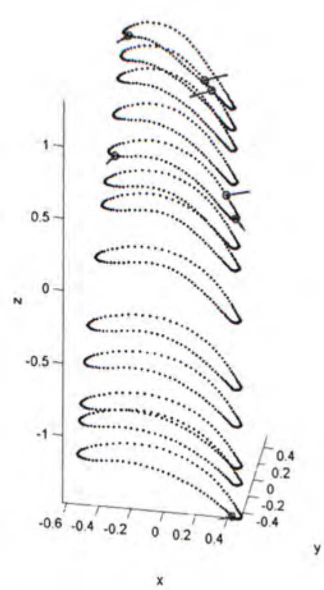


Fig. 5.13. Example 4: Initial grasp (non-form-closure).

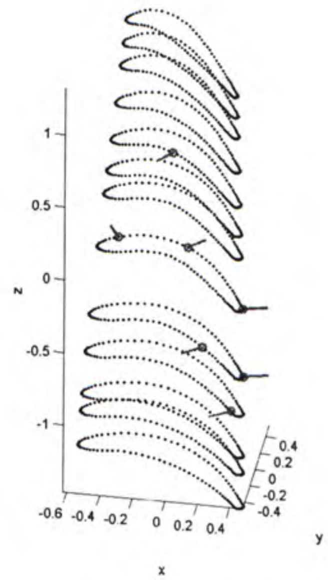


Fig. 5.14. Example 4: Final grasp (form-closure).

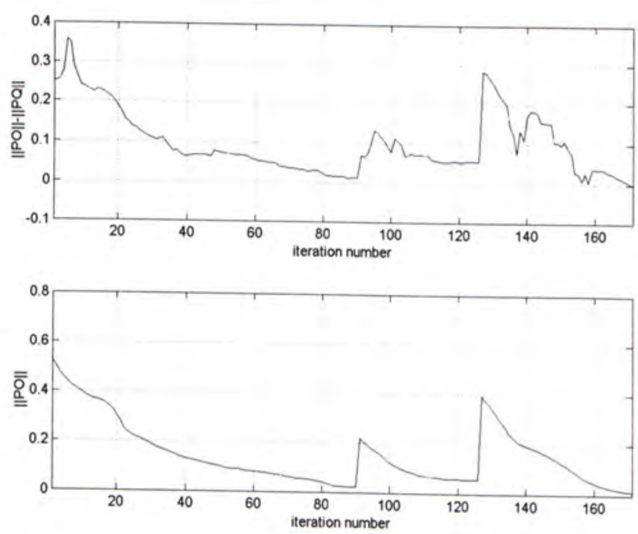


Fig. 5.15. Example 4: The distance $\|PO\|$ encounters two local minima and the distance difference $\|PO\| - \|PQ\| < 0$ (form-closure obtained) at last.

5.2 Examples of Frictional Grasps

Three examples of frictional grasp with different number of fingers are shown in this section. In all these examples, the friction cones are linearized with polyhedral convex cone with 20 segments (i.e. $m = 20$).

Example 5

In this example, the algorithm is required to obtain a 4-finger form-closure grasp on the airfoil model in the present of friction. A friction coefficient $\mu = 0.2$ exists between the object and each finger. The initial grasp upon random selection is non-form-closure (Fig. 5.16). The algorithm performs local search in obtaining trajectories of finger motions in the direction to decrease $\|PO\|$. In the 62nd iteration, a local minimum is encountered (Fig. 5.18). The search tree for the problem is generated and the child (1,1,2) is adopted. Then, a new grasp is selected based on the existence conditions of node (1,1,2). Finally, a form-closure grasp is obtained (Fig. 5.17) in the 71st iteration.

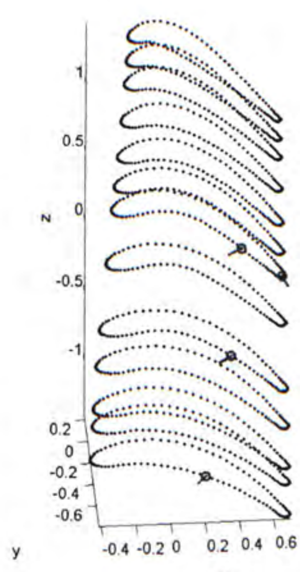


Fig. 5.16. Example 5: Initial grasp (non-form-closure).

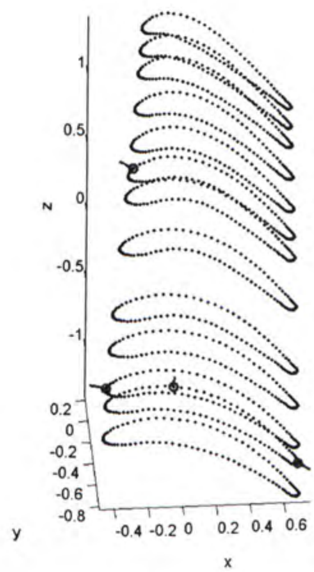


Fig. 5.17. Example 5: Final grasp (form-closure).

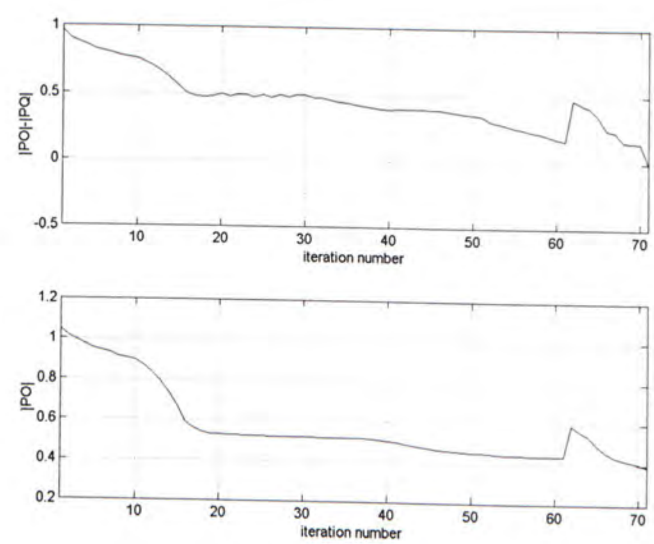


Fig. 5.18. Example 5: The distance $\|PO\|$ encounters a local minimum at 62nd iteration and the distance difference $\|PO\| - \|PQ\| < 0$ (form-closure obtained) at last.

Example 6

It is known that three fingers are sufficient to grasp object in 3-D in the present of friction. In the sixth example, a 3-finger frictional form-closure grasp on the airfoil model is to be found. The friction coefficient μ is again fixed to be 0.2. The initial grasp upon random selection is non-form-closure (Fig. 5.19). The algorithm performs local search in obtaining trajectories of finger motions in the direction to decrease $\|PO\|$. As shown in Fig. 5.19 the distance $\|PO\|$ decreases strictly and no local minimum is encountered in this example. Finally, a form-closure grasp is obtained (Fig. 5.17) after 43 iterations of computation.

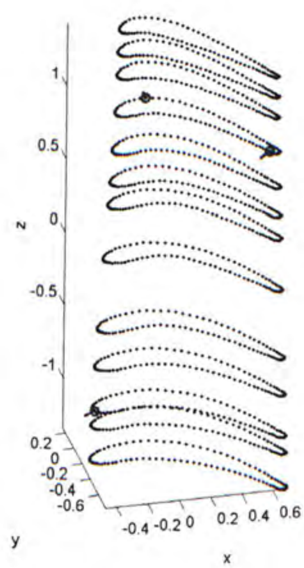


Fig. 5.19. Example 6: Initial grasp (non-form-closure).

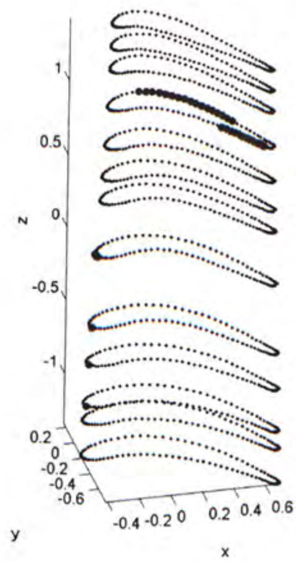


Fig. 5.20. Example 6: Trajectories of the three contact locations during the local search.

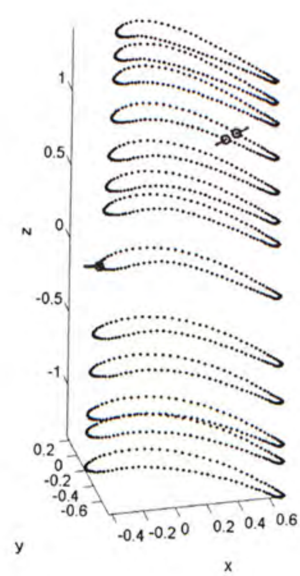


Fig. 5.20. Example 6: Final grasp (form-closure).

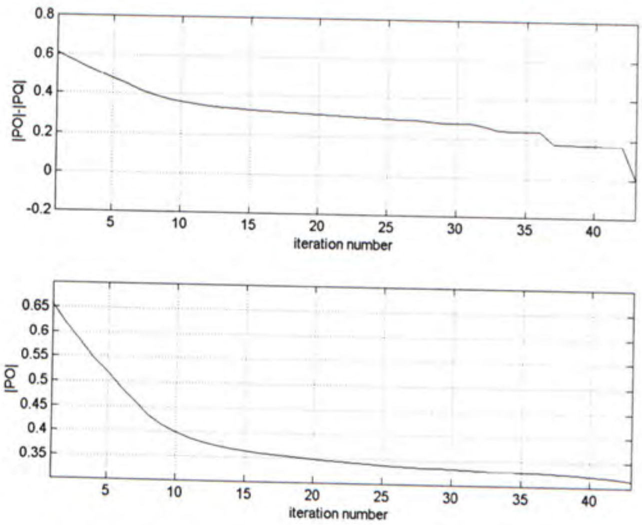


Fig. 5.21. Example 6: The distance $\|PO\|$ strictly decreases throughout the process until the distance difference $\|PO\| - \|PQ\| < 0$ (form-closure obtained) at last.

Example 7

As stated in Chapter 4, the distance difference $\|PO\|-\|PQ\|$ can also be used as the performance index of the local search. The statement is examined in this example. This example uses a spherical object and five contact locations on the object surface are to be found. The friction coefficient μ is defined to be 0.3. In Fig. 5.22, as all the contact points of the initial grasp are located at the lower part of the sphere, the contacts cannot resist forces acting upwards and so the grasp is not form-closure, even in the present of friction. After performing the local search for 23 iterations, a form-closure grasp shown Fig. 5.23 is obtained. As shown in Fig. 5.24, $\|PO\|-\|PQ\|$ decreases strictly throughout the search, i.e. no local minimum is found during the search.

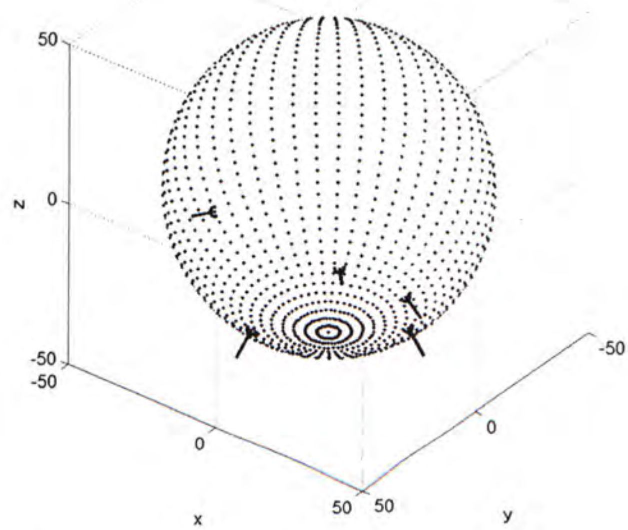


Fig. 5.22. Example 7: Initial grasp (non-form-closure) on a spherical object.

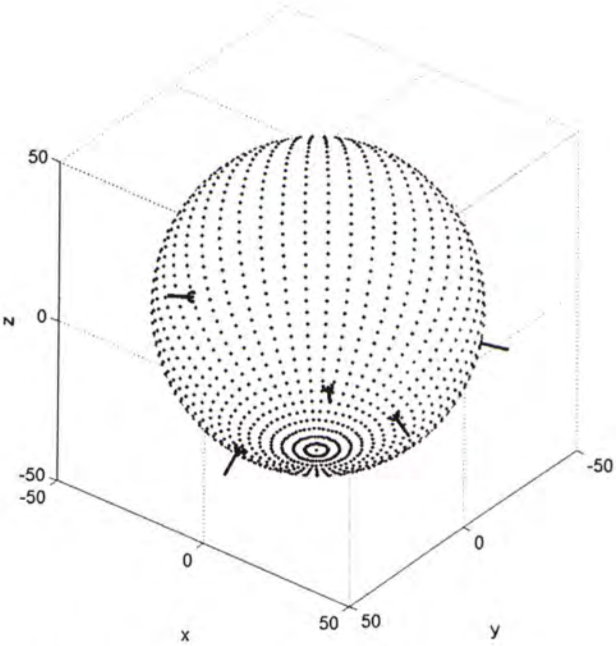


Fig. 5.23. Example 7: Final grasp (form-closure).

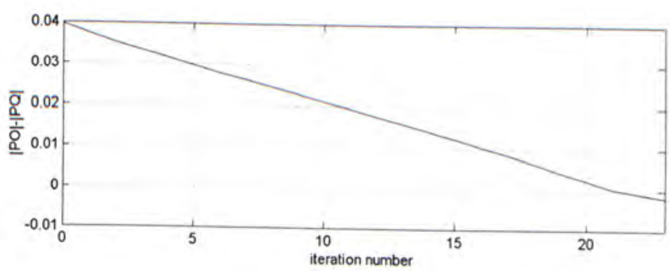


Fig. 5.24. Example 7: Distance $\|PO\|$ - $\|PQ\|$, instead of $\|PO\|$, used as the performance index in local search, form-closure yielded after 23 iterations.

The local search algorithm can be revised to obtain a local optimal grasp. It is done by neglecting the termination instruction of *Step 2* of the local search algorithm. Then, the iteration process will end only when $\|PO\|$ is not strictly decreasing. Fig. 5.24 shows the trajectories of the five contact locations from the initial grasp to a final grasp that is a local optimal form-closure grasp. We can see that the final grasp obtained by the revised algorithm holds the sphere much more stable than the original final grasp does. Fig. 5.25 shows the convergence behavior of $\|PO\|$-$\|PQ\|$.

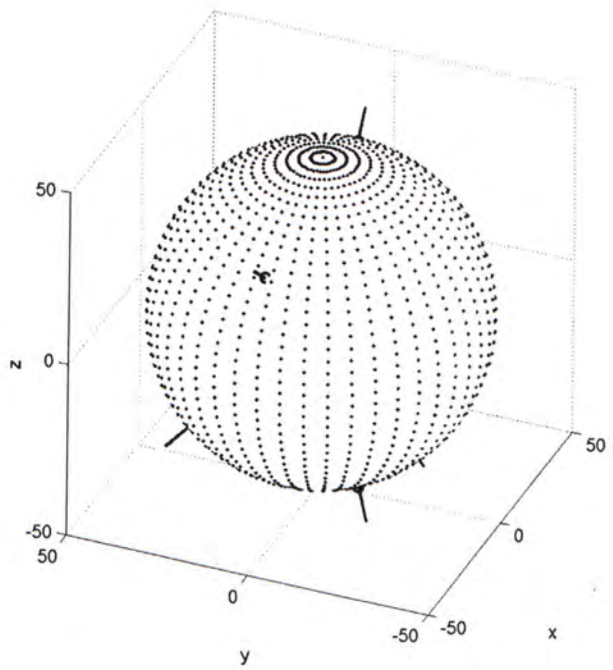


Fig. 5.25. Revised algorithm for Example 7: local optimal grasp obtained.

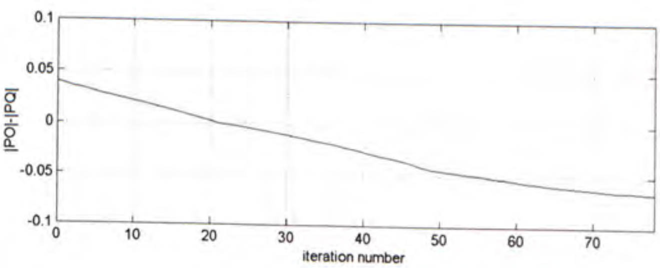


Fig. 5.26. Revised algorithm for Example 7: form-closure yielded after 77 iterations.

5.3 Examples of Grasps under Kinematic Constraints

In this section, two examples will be used to demonstrate the capability of the algorithm in computing form-closure grasps when kinematic constraints are taken into consideration. The robotic hand model used in the simulations is a *Yaskawa Hand* which contains five 3-DOF fingers. Each finger has links $l_1 = 22.75mm$, $l_2 = 43.00mm$, $l_3 = 51.25mm$ and $l_4 = 50.35mm$. The joint limits are defined to be $\theta_1 \in [-\pi/4, \pi/4]$ *rad*, $\theta_2 \in [-\pi/2, \pi/2]$ *rad*, $\theta_3 \in [-5\pi/6, 0]$ *rad*. Fig. 5.27 shows the geometric shape of the reachable workspace of each finger. The inverse kinematic model is developed, which calculates the joint angles of the five fingers from a given set of grasp points. Therefore, for any candidate grasp that incurs joint angles beyond the joint limits, we can properly discard that candidate grasp.

For each of the two examples, the friction coefficient μ is defined to be 0.3. The hand palm frame ΣH is of the same orientation as the object frame ΣO with a pure translation in - z direction.

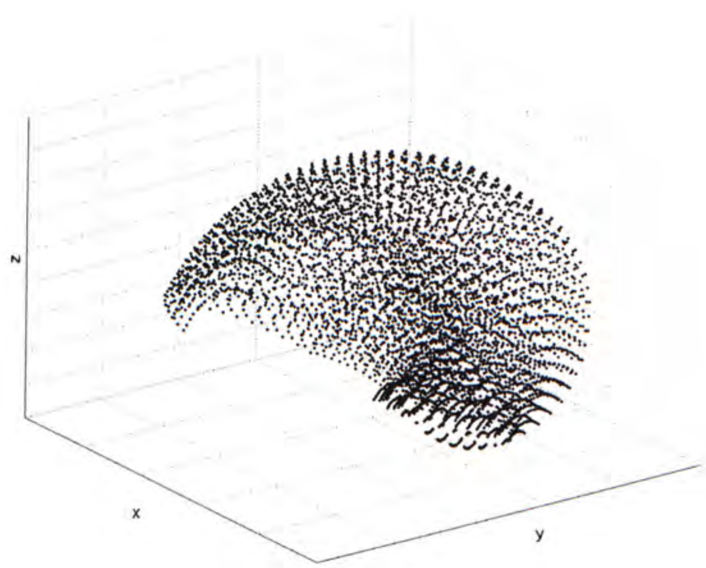
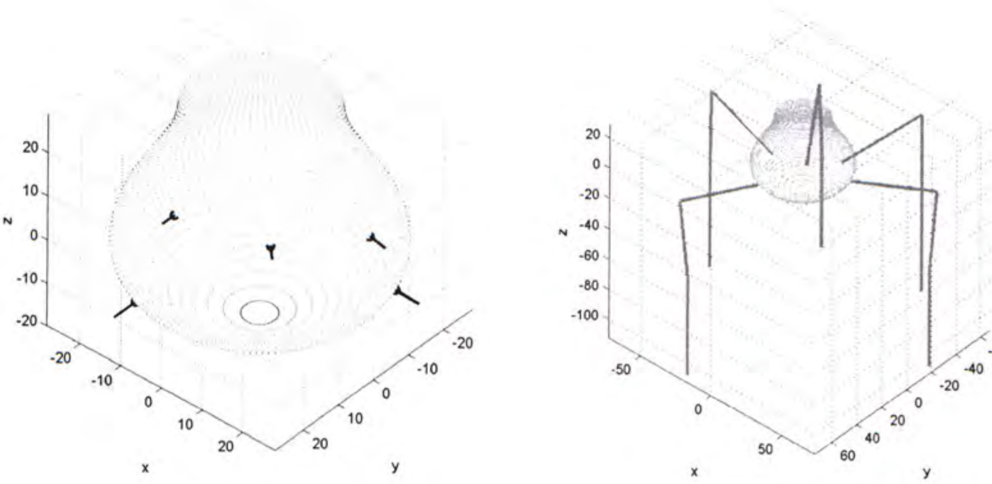


Fig. 5.27. Geometric shape of the reachable workspace of each finger.

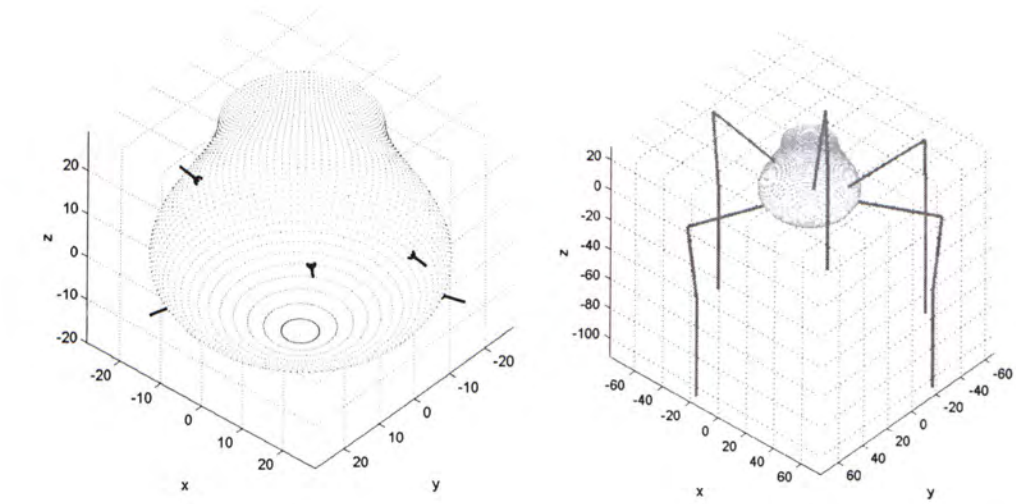
Example 8

Fig. 5.28(a) shows the object of light bulb shape used in this example. A kinematically feasible initial grasp, which is non-form-closure, is chosen randomly on the object. Fig. 5.28(b) shows the configuration of the 5-finger robot hand. The algorithm searches for form-closure grasp and, at the same time, check for the kinematic feasibility of each of the candidate contact locations. Fig. 5.29 shows the form-closure grasp obtained in the end and the corresponding finger configurations.



(a) (b)

Fig. 5.28. Example 8: Initial grasp (non-form-closure)
(a) contact locations and (b) configuration of robot hand.

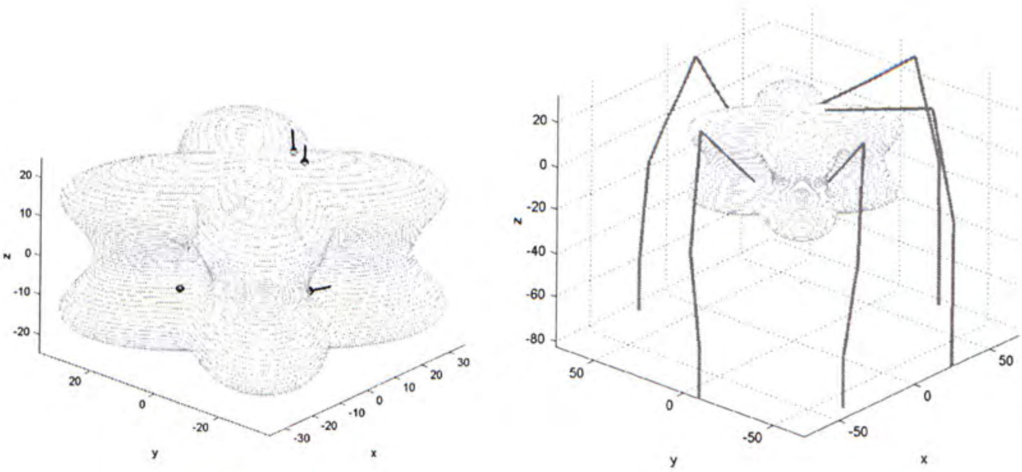


(a) (b)

Fig. 5.29. Example 8: Final grasp (form-closure)
(a) contact locations and (b) configuration of robot hand.

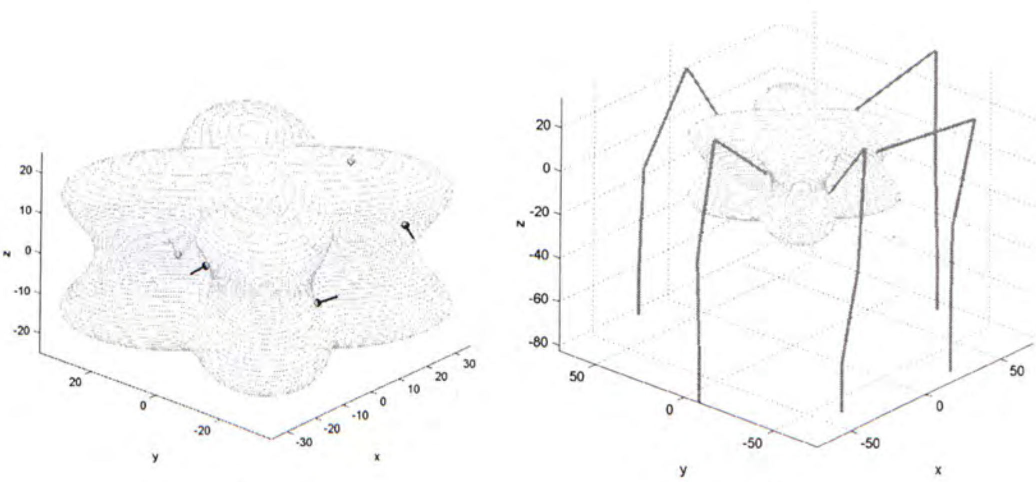
Example 9

This is another example showing the capability of the algorithm in computing form-closure grasp under kinematic constraints. Fig. 5.30 shows the initial grasp and its finger configurations and Fig. 5.31 shows the final grasp obtained.



(a) (b)

Fig. 5.30. Example 9: Initial grasp (non-form-closure)
(a) contact locations and (b) configuration of robot hand.



(a) (b)

Fig. 5.31. Example 9: Final grasp (form-closure)
(a) contact locations and (b) configuration of robot hand.

Chapter 6

Conclusions

In this dissertation, we address the problem of form-closure grasp synthesis in point set domain. A simple and efficient algorithm for locally searching a form-closure grasp is developed. The algorithm starts with a randomized initial grasp and searches for trajectories of fingertip positions to enhance form-closure. We have introduced a quantitative index to measure a grasp how far a grasp is from being closure. The heuristic measure effectively leads the convex hull *H(W)* of the primitive contact wrenches to move toward the origin of the wrench space R^6. The proposed algorithm is applicable for both frictionless and frictional models with any number of contacts. The algorithm can handle objects with arbitrary geometry while the complexity of the object does not influent the computational time of the local search. It also ensures the kinematic feasibility of robotic fingers. The algorithm can be widely used in grasp planning of multifingered robot hands and fixture layout design of workpieces with arbitrary geometry.

We propose a complete and efficient heuristic algorithm for searching for a form-closure grasp in a discrete point set. The complete algorithm solves the local minimum problem incurred by the local search and it always guarantees a solution as long as it exists. We have defined a separating plane to divide the point set into subsets. Based on an important observation of form-closure condition in the wrench space, the original problem is decomposed into sub-problems while the problem size is reduced significantly. To the best of our knowledge, no other complete algorithm, except for an algorithm which exhaustively tests form-closure property of possible combinations, is available for such a problem.

We have implemented the algorithm and examined its performance with simulation study. Nine numerical examples are given in Chapter 6 to demonstrate the results of the algorithm in tackling frictionless and frictional grasps and kinematic check.

There are some possible future extensions of this work. One important extension is the derivate of the complexity of the search algorithm in terms of the best, the average, and the worst scenarios. Another possible future work is the generalization of the work to other contact models such as soft-finger, line, and planar contacts.

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