The Hong Kong Logistics Industry and A Study of Inventory Management Models with Advance Ordering

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"*Trust in the Lord with all your heart and lean not on your own understanding; in all your ways acknowledge him, and he will make your paths straight.*" Proverbs 3:5-6

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The Hong Kong Logistics Industry and

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This thesis is divided into two parts: Part A: Logistics in Hong Kong – Overview and Prospects and Part B: Inventory Management with Advance Ordering.

In Part A, we conduct a study about the logistics industry in Hong Kong. We review the current situation and development of logistics-related sectors in both Hong Kong (mainly) and China, and also the Hong Kong government's policy for developing the logistics industry. With data we compiled and our knowledge on the field, we have done a SWOT analysis and predicted some future trends on the logistics industry in Hong Kong.

In Part B, we consider a model in which a retailer faces financial incentives for placing definite advance orders. The longer the window size for placing advance orders, the deeper the discounts would be given. We develop a model for finding an optimal replenishment policy for the retailer. An optimal ordering policy is found for advance ordering with window size 1. In addition, we have run several simulations to study advance ordering with larger window size, investigate the costs trade-offs in the inventory problem and the value of committing earlier, and also, do comparisons between different heuristic policies for advance ordering.

摘要

這篇論文包括兩個部份: 甲部 香港物流 - 概覽與前瞻 及 乙部 可預先訂貨制度之下的庫倉管理。

在甲部，我們進行了香港物流業的研習。我們回顧有關物流業的營商機構在香港（主要部份）和中國的現況和發展，以及香港政府鼓勵物流業的政策。透過資料重新編纂和藉著我們對物流業的認識，我們作了一個強弱優劣分析（SWOT Analysis）及預測香港物流業的發展趨勢。

在乙部，我們考慮關於一個零售商在面對財務優惠時，應如何決定預先訂貨（Advance Ordering）。此財務優惠是預訂期（Window Size）越長，所給的購物折扣就會越大。我們建立了一個零售商的模型，嘗試找出最優化的補充存貨政策（Optimal Replenishment Policy）。在有一周期預先訂貨的情況下，我們已找出最優化的補充存貨政策。另外，我們進行了一系列的模擬（Simulations），分別研習有多周期預先訂貨的情況，探討各成本的平衡（Trade-offs）和預先訂貨的價值，以及進行不同的補充存貨政策的啓發性模擬（Heuristics）的比較。

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Chapter 0
Introduction

This thesis is a study on logistics issues. It is divided into two parts: Part A: Logistics in Hong Kong – Overview and Prospects and Part B: Inventory Management with Advance Ordering.

In Part A, we have conducted an industry study, which aims to provide a general understanding of the current situation of Hong Kong and China's logistics industry and its related sectors, including government organizations. We analyse the strengths and weaknesses of Hong Kong, and the opportunities and threats that Hong Kong is facing. In addition, we make some predictions on the future trends of the logistics industry.

After we have a general view of what logistics is about, we go to a more specific aspect of logistics management that we are interested in, that is inventory management. In Part B, we study the optimal ordering policy for a retailer over a finite horizon if advance ordering is allowed. A general mathematical model is developed. Although we cannot solve it to optimality for the general case, we obtain some partial solutions and insights. We ran several simulations to compare different ordering policy heuristics and analyze the numerical results.

PART A

Logistics in HK - Overview and Prospects

A.1 Study Objectives

Logistics is a hot topic in Hong Kong nowadays. In this chapter, we present an industry study, with the goal of providing:

- a general understanding of the current situation of Hong Kong's logistics industry and its sub-sectors,
- a general understanding of the current situation of China's logistics industry and its sub-sectors,
- a general understanding of the Hong Kong Government's policies and initiatives for supporting, promoting and improving the logistics industry,
- analysis of the strengths and weaknesses of Hong Kong in the field of logistics,
- investigations into the challenges and opportunities faced by the logistics industry in Hong Kong. (We are especially interested in diagnosing the relationship and competition of logistics industry between China and Hong Kong.),
- predictions on the development and trends of Hong Kong logistics industry in the future, and
- suggestions of research questions for further study.

We hope this is an informative and helpful report that can provide readers clear knowledge and be a valuable reference about the logistics industry in Hong Kong.

A.2 Methodology

Our study is based mainly on secondary data from the following sources:

i) Industry studies by different organizations,

ii) Trade and professional association publications,

iii) Government publications, HKSAR[1]

iv) Trade magazines and journals,

v) Newspaper and magazines,

vi) Consultancy reports,

vii) Conferences and seminars proceedings,

viii) Books, and

ix) Online sources.

A.3 What is Logistics?

In order to effectively study the logistics industry, the first question we need to answer is what logistics is and what it is about. The Council of Logistics Management (CLM), in USA gives the following definition: ([70])

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[1] Hong Kong Special Administrative Region

"Logistics is that part of the supply chain process that plans, implements, and controls the efficient, effective forward and reverse flow and storage of goods, services, and related information between the point of origin and the point of consumption in order to meet customers' requirements."

A logistics process has the following characteristics:

- It concerns the movement of goods, information and funds.
- Its services may span sea, air and land transport, and involve various policy areas. ([24])

The key to success in logistics is to achieve integration and synchronization of the supply chain. ([31]) It is about multi-enterprise and multi-modal connectivity and real-time visibility. ([37])

The importance of logistics is highlighted by the fact that:

- Total logistics costs (packaging, storage, transport, inventories, administration and management) are estimated to reach up to 20% of total production costs in OECD[2] countries, while freight costs alone (transport and insurance) can make up to 40% of values of exports for several African landlocked countries. ([76])
- The costs of logistics services account for 10 – 13 percent of gross domestic product (GDP) in most developed countries and regions around the world. These costs include transportation, inventory holding as well as administrative charges related to inventory and transportation. ([34])
- Importance of logistics and transport in total manufacturing costs is increased. ([75]) In particular, transport usually accounts for a quarter of total logistics

[2] Organization for Economic Co-operation and Development (OECD). Its member countries list: http://www.oecd.org. China is not included.

costs in OECD countries, storage for a fifth, and inventories for a sixth. ([76])

A.3.1 Major Trends

We observed that logistics is a topic hot in Hong Kong now and the rest of the world. Some main drivers in the growing awareness of logistics issues are listed below:

- ¤ Globalization of world trade ([6][24][28][75])

  In order to cut down costs, companies now would utilize the efficiency of transportation to source raw materials and manufacture their products from different countries, where the costs are low enough. At the same time, the logistics involved affects our global transport policy; there is more government attention to the improvement of transport infrastructure. Borderless transport management, real-time cargo monitoring and information systems are needed. Moreover, the current hub-and-spoke systems may possibly change to more direct origin-to-destination deliveries and movements through smaller terminals in the future.

- ¤ The advent of the Internet and E-commerce and advance of IT ([6][24][28];[33], p.31;[37])

  They improve the communication between different business parties. Through better information connectivity, customers can research products and companies, compare features, choose global suppliers and so influence price and delivery options. They are the prime drivers for the new economy.

  E-commerce may lead to a different way of doing business, with outsourcing of the logistics function. More and more companies would like to outsource the logistics activities to third party logistics providers (3PL), so that they can concentrate on their core business and reduce logistics costs and inventory. A 3PL has dedicated handling and management skills and can help to build up professional logistics systems. It especially benefits those companies that are involved in high-value products that need special handling.

¤ Accession of China into the WTO ([6][24])

China joined the WTO at the end of 2001. It is expected that the trade volume of China will rise a lot with its open policy. Hong Kong, as a gateway to China, probably can get some of the market share of it by developing logistics.

¤ Fast changing practices in the supply chain and the logistics sector ([6][10][24][28][33][37])

Products have shorter product life cycles than before. Quick reaction to the market is needed. Players and roles in distribution channels also have been changed from the past. Collaboration in logistics helps speed-to-market. Manufacturers increasingly adopt mass customization and just-in-time manufacturing. Retailers have changed ordering practice from traditional pre-season orders to speed sourcing (last minute ordering) and more replenishment orders (within season ordering). These changes lead to great concern on coordination of logistics flow. They try to use EDI and bar-coding to connect with other parties in the logistics chain.

¤ Increasing customer service requirements ([28][37])

Customers nowadays are not only concerned with the quality of a product, but they are also concerned with whether they can get the product in the right time, right quantity, right place and right price. Moreover, they would ask for after-sale service, which involves so-called reverse logistics.

¤ Changing material handling and transport technologies ([37])

The improvement of technologies in material handling and transport changes the cost tradeoffs and encourages people to re-think their logistics processes to cut down costs and improve customer services.

¤ Competitive pressures ([28][37])

Under the bad economy, there are more pressures for improved financial performance and inventory reduction, and the competition between companies is keener than before. Logistics is one of the ways for reducing cost and inventory. This forces the companies to develop supply chain vision and co-operation.

A.4 Key Features of the Logistics Industry in Hong Kong and China

A.4.1 China Industry

In China, the understanding and implementation of the integrated logistics concept is very weak. Only very few companies provide the whole logistics flow services, although there are many companies participating in logistics operations as carriers, shippers or freight forwarders. The logistics industry is now only in an infancy stage of development. ([33])

Freight Forwarding

There were 1,542 international freight forwarding agencies in China in mid-1999. 444 of them are Sino-foreign joint ventures and about 100 of these are the offices of Hong Kong firms. ([33], p.43)

Most foreign companies are only authorized to conduct business in international freight forwarding, not domestic freight forwarding. They need to co-operate with local forwarders to conduct business by ([33], p.20):

- Setting up a minority-owned joint venture with a Chinese international forwarder or a foreign trade enterprise with annual import/export turnover value of US$50 million.
- Appointing a Chinese forwarder as an agent and set up a local representative office.

Applying related licenses is very time consuming. The approval steps are unclear and subject to different interpretation of local authorities, even worst is the change of policy from central government. ([33])

From the Chinese government's view, since many foreign companies didn't commit to transfer their technologies and invest in the long-term, their applications have been turned down. ([33])

Shipping

This sector is open for wholly-owned foreign participation. ([33])

In the past, China's shipping trade was hampered by outdated small ships, isolated shippers, unimproved water channels, more competitive road and air transport modes and regional recession. This situation has changed. According to a 1998 survey by the US-China Business Council, 39% of respondent firms used ship transport for transporting goods within China. ([33], p.63)

Ports in Shanghai serve as a gateway between the sea and inland provinces near Yangtze River. Compared to all Chinese regions, the heaviest port traffic occurs in South China, which has ports of Guangzhou, Shantou, Panyu, Zhongshan, Zhuhai and Shenzhen. ([33], p.14) (See Appendix I)

Since 1949, because direct trade between Taiwan and the Mainland has been prohibited, Taiwan-Mainland trade can only be routed via Hong Kong. It currently accounts for 1 million TEU or 6% of Hong Kong's total container throughput. ([3], p.7)

The Chinese authorities have devoted much effort to facilitate the shipping industry. For example, they have:

- Established 15 free trade zones, which have three basic functions: export-oriented processing, transshipment trading and free customs duties warehousing. Foreign companies can invest logistics facilities inside them. ([33], p.7)
- Improved port management systems and executed several policies in support of foreign funded enterprises (together with lower inland transportation costs and increase of retail activities) in Shenzhen, which has contributed to Shenzhen ports' rising throughput. For example, the Guanlan Container Warehousing Centre was set up in Shenzhen. ([33])
- Developed central and western China which has increased cargo movement and port throughput (e.g. Chongqing). ([33])

Mainland Ports are at a serious disadvantage compared to Hong Kong in terms of customs clearance, government regulations, and the quality of related services. The inefficiency of customs clearance and other regulatory procedures is well known, even though the various ports try to improve efficiency by providing a one-stop service for customs clearance. ([3][25])

<u>Air</u>

According to a 1998 survey by the US-China Business Council, 45% of respondent firms used air transport for transporting goods within China. ([33], p.63) International freight traffic in China has increased from 81,000 tonnes in 1990 to 315,000 tonnes in 1998, an average annual increase of 18%. ([32], p.27)

Foreign participation is most restricted in the air freight sector. Seven domestic airlines have dominated regional operations – China Southern, China Southwest, China Eastern, China Northern, China Northeast, China Northwest and Air China. Designated foreign carriers are given a maximum entitlement of 50 tonnes per week. In order to increase their authorized cargo volume, foreign airlines are keen to tie up with domestic airlines. Federal Express is currently the only dedicated US cargo carrier flying into China. ([33])

Recently, the industry is gradually opening up for foreign participation under multilateral conferences between China and Western countries, such as the United States and European countries. Many airports in China have begun developing and improving their infrastructure and operations. The flight connections between China and the other foreign countries are increasing and become more frequent.

Regarding to the development of air industry in China, there are a number of new airports in southern China near Hong Kong, including Baiyun (Guangzhou), Huangtian (Shenzhen), Zhuhai and Macau. They are relatively small compared with the Hong Kong International Airport (HKIA) and have sparse route networks. ([32], p.26)

Railway

According to a 1998 survey by the US-China Business Council, 59% of respondent firms used rail transport for transporting goods within China. ([33], p.63)

China's railways are heavily used. Bookings need to be made as much as 30 to 40 days in advance, because rail priority is given to the State Development Planning Commission rather than the free market. ([33], p.11)

Although foreign participation is most restricted in this sector, Chinese authority is always looking for foreign participation in rail freight related business opportunities. For instance, OOCL cooperates with the Ministry of Railways in providing Reefer-on-Rail services. ([33])

Road

According to a 1998 survey by the US-China Business Council, 82% of respondent firms used road transport for transporting goods within China. ([33], p.63) Many companies prefer to use road rather than rail because the railway containers are not compatible with those used by shipping companies. ([33])

Since 1980s, economic and urban development in Shenzhen Special Economic Zone immediately north of our border and later in other parts of the Pearl River Delta

(PRD) has led to the intensification of cross-border traffic, including both the movements of personnel and freight. ([8])

Being a key element in the growth of the PRD is the 302km Guangzhou-Shenzhen-Zhuhai Superhighway built in 1993. It is a dual three-lane highway with a design speed of 120 km per hour. It directly benefits Hong Kong firms that have relocated their manufacturing operations in Guangdong Province. ([4])

Warehouse

Warehouses act as intermediate collection points within a logistics flow. Since the mid-1980s, state-owned enterprises have been developing their warehousing capacity from meeting their own warehousing needs into commercial operating units. ([33], p.34)

However, the locations of warehouses are often not well-planned and the system does not operate as an efficient distribution network. The capacities of the warehouses are limited and their service and quality are poor. They are simply used as places for storing stock and no special handling equipment and services are available. ([33])

A.4.2 National developments in China

There are mainly five developments that we have to be concerned with, as they will have important impact on the freight forwarding and logistics industry:

1. In 1999, China announced the policy to develop the western part of the country. Better linkages would be established between coastal cities and interior cities. An efficient and large transportation network will improve cargoes flowing throughout China. ([33])
2. Xiamen will be a rapidly growth market for freight forwarding and logistics industry, since its location is important for trading with Korea, Japan and especially Taiwan. ([33], p.18)
3. The Pearl River Delta (PRD), the Yangtze River Corridor and Shanghai are places with heavy concentration of light industries. The economic growth and cargo movement in these regions will drive the growth of freight forwarding and the logistics industry. ([33])
4. (Domestics and global) retail chain stores become more and more popular in China. They would rely on freight forwarding and logistics services to overcome distribution and inventory problems. ([33])
5. WTO Accession

   China entered WTO at the end of 2001. ([43]) It is anticipated that China will liberalize the distribution service and related sectors for foreign participation. Intense competition will occur in the freight forwarding and logistics industry. ([33]) (See Appendix III)

   The relaxation of regulations in the logistics industry will allow foreign service suppliers to set up wholly-owned subsidiaries in some areas and conduct their business in the domestic market. Large amount of foreign investment will flow into the China. Companies in the China market can enjoy direct trade with other WTO members. ([33])

   China's exports are expected to rise by an additional 2.4% per year over the first five years of WTO accession, while the effect on imports could be higher.

([33], p.37) Based on the Mainland's and US's estimation, Hong Kong's re-exports to and from the Mainland will be 4-6% higher by the year 2005 than it would otherwise be if China did not enter the WTO. ([32], p.40)

Prospects

There are growing markets for high tech and high-value products in China. 3PL will be gradually accepted by China enterprises. In order to match the need of logistics development, warehouses will be modernized and upgraded to support multiple functions, such as packaging and goods categorization. They will be strategically located in efficient distribution networks. ([33])

Parties within a supply chain will be more aware of the importance of integration, which will improve the efficiency of the sourcing process. EDI technologies will be utilized to connect freight forwarders, carriers, customs, port authorities, buyers, manufacturers, banks and warehouses/distribution centers. ([10])

Foreign companies will continue to operate with domestic companies in China as joint ventures. They themselves have the advantage of access to international freight forwarding networks and customers but the local companies are more familiar with the Mainland culture and market. They are able to do low-cost operations in China and especially good at dealing with Chinese authorities. ([33]) If foreign companies don't need domestic partners anymore by the effect of WTO, the local companies would face a very keen competition with the foreign companies, due to the lack of advanced technologies, global transportation network and expertise in the logistics market.

A.4.3 Hong Kong Industry

Freight Forwarding

There are some 2700 freight forwarding companies in Hong Kong. ([6]) On the Mainland, Hong Kong firms currently account for around one-fourth of "foreign" freight forwarders. They have moved to Shenzhen and further into Guangdong Province for the following reasons. Firstly, Hong Kong's manufacturing industry has moved northward. Secondly, mainland ports are being heavily used. Thirdly, China already has developed its own market for freight forwarding. So, freight forwarders follow their customers and entered the China market. ([32], p.25; [33])

Hong Kong is facing increasing competition in export trade, which is the result from the free-trade agreement (e.g. NAFTA[3]) and mainly customers' need for speed sourcing and more replenishment services. ([23]) - Free-trade agreements lead to regionalization of economic. The member countries would prefer trading with each other due to the benefits in such free-trade zone agreements. If Hong Kong is left out of such agreements, fewer countries would use Hong Kong's export services. Besides, the trend of speed sourcing and frequent replenishment induces countries to import products or raw materials from nearby countries rather than Hong Kong.

Shipping

Goods that passed through the Hong Kong port accounted for close to 90% of all freight movement by mode (i.e., ocean, river, rail, road and air). ([25]) Hong Kong

[3] North American Free Trade Agreement. Website: http://www.ustr.gov/naftareport./intro.htm

is served by over 80 international shipping lines with 380 weekly container line services to over 500 destinations. ([6])

In 1999, Hong Kong handled a total of 16.2 million TEU to become the busiest container port of the world. The throughput recorded represents an 11.2% increase as of 1998. Singapore came second in 1999, with a throughput of 15.9 million TEU. ([25]) Hong Kong has a growth of 11.6% to 18.1 million TEU in 2000, however, no growth in first three quarters of 2001 (but the total annual volume is expected to be not far off from 18 million TEU, and Hong Kong can maintain its position of being the world's busiest container port). ([65])

In the past, port facilities were underdeveloped in China, while Hong Kong was a deepwater port and it had a strategic location, superior facilities, especially those for container ships, and supporting services. So, Hong Kong was predominant in handling China's foreign trade. ([25])

Nowadays, more than half of containers shipped through Hong Kong are transshipment cargoes from China. Among these a large proportion come from PRD, where has been the Hong Kong production base. A few of them come from outside of provinces and even as far as to central and northwest places of China. ([40]) In 1999, about 68% of Hong Kong's ocean container traffic are related to Guangdong and South China. ([17])

Port and Terminals

Port ownership and management relies heavily on private participation in Hong Kong, while in other countries, ports are always in the form of state-subsidized or state-owned. ([7]) Currently, there are 4 terminal operators that own and manage 8 container terminals, comprising 18 container vessel berths: ([41])

(i) Hongkong International Terminals (HIT) which operates Terminals 4, 6 and 7

(ii) COSCO-HIT Terminals which operates Terminal 8 East;

(iii) CSX World Terminals which operates Terminal 3

(iv) Modern Terminals which operates Terminals 1, 2, 5 and 8 West.

Hong Kong has a Hong Kong Port and Maritime Board (PMB)[4]. The board comprises of the Port Development Committee (PDC) and a new Shipping Committee. The PDC deals with port development, while the Shipping Committee looks at all aspects of improving shipping services in Hong Kong. The two committees will advise and assist the Government so it can co-ordinate its resources and those of the maritime sector to promote shipping, logistics services development, support Hong Kong's economic development and create job opportunities. The PMB will also assist the shipping industry in Hong Kong by building up a library and information center, concentrating on gathering more detailed statistics and data on shipping activities. ([9], p.49)

The PMB stresses not making Hong Kong to compete with any other country; but to develop new markets and opportunities by providing shipping firms with information regarding services available in the HKSAR[5]. ([9])

Mid-stream

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[4] Before it was reorganised and renamed the Hong Kong Port and Maritime Board (PMB) on 1 June 1998, it is called the Port Development Board (PDB), which was formed in April, 1990

[5] Hong Kong Special Administrative Region.

In 2000, midstream throughput grew by 6.83% to 3.03 million TEUs. ([60]) Mid-stream operation is important in Hong Kong (handles more than 1/6 of port's container throughput), but it has a constraint on the growth of its throughput. It is because there is insufficient backup land to store containers before they are picked up by trucks or loaded onto ships. To solve this, the Government has leased two permanent mid-stream sites on Stonecutters Island comprising 6.7 hectares with some 460 metres of quay length. It is available since 1998 for use by mid-stream operators. ([7])

River trade

In 2000, river trade grew by nearly 14% to 4.4 million TEUs. ([60]) To some extent, river trade emerges because there are inadequate road transports, particularly in the Pearl River Delta region. For cargoes near the waterways, river trade is clearly more cost effective than overland hauling, because a river craft can move more containers than a tractor-trailer. It seems that river trade can reduce pressure on the territory's road system, however, many shippers still prefer moving cargoes overland. ([25]) It may be because overland hauling is faster and more efficient in providing door-to-door transport services.

River trade increases the cargo sources for the terminals in Kwai Chung. Some major players in the container handling industry see it as a worthwhile direction of diversification. ([25]) For example, Hutchison Port Holdings owns River Trade Terminal at Tuen Mun.

Port Development

For port planning strategy, the PMB uses a "trigger point mechanism"[6] to determine the need for new terminals. Despite the slow-down of the growth of ocean container throughput compared to the last decade, Hong Kong needs additional container terminals in order to meet the forecasted demand for the territory's container handling services. ([25])

The new Container Terminal 9 (CT9), on Tsing Yi Island opposite the eight existing terminals at Kwai Chung, will occupy an area of 70 hectares and consist of four deep sea berths and two feeder berths. The latter are designed to cater for the interchange of containers between large ocean-going vessels and regional carriers. The first berth of CT 9 will come on stream in mid 2002, and, when fully developed, in 2004, the terminal will provide an additional capacity of at least 2.6 million TEUs. ([7][18])

Air

At the Hong Kong International airport, 65 international airlines operate some 3800 flights between Hong Kong and nearly 130 destinations worldwide. The airport also has several airport logistics facilities: Asia Terminal Logistics Center (ATL), Hong Kong International Distribution Center (HIDC), Hong Kong Air Cargo Terminal Limited (HACTL) (Super Terminal One and Express Centre), Asia Airfreight Terminal (AAT) and TradePort Hong Kong Limited. ([6][19][30][77])

In 1999, the Hong Kong International airport (HKIA) at Chek Lap Kok handled 1.97

[6] The "trigger point mechanism" is central to the planning of the number of berths for Hong Kong's container terminals. According to the PMB, "new berths were not triggered until forecast throughput equaled working capacity of existing and planned berths. This caused berth supply to lag one berth behind demand." (PDB 1992, p.23) ([25], p.4)

million tonnes of cargo. In terms of international cargoes, Hong Kong ranked first in the world. Hong Kong was the world's second busiest cargo airport after Memphis (2.4 million tonnes). In 2000, the airport handled over 2.2 million tones of cargo and 178,000 aircraft. While air cargo accounted only for 1% of the total cargo tonnage handled in Hong Kong, it already constitutes 24% or HK$664.3 billion of Hong Kong's trade value. About 83% of HKIA's air cargo is origin/destination type cargo, of which 70% is Mainland-related. ([12])

The HKIA is well situated. It lies on the Pacific Rim and is with less than 5 hours flying time to most of the population of the world. It has strong air route networks, frequency flight schedule and high quality of air cargo handling and supporting services. It is strongly believed that the network is the main factor to attract people to utilize the airport and efforts are always made to expand it. ([51]) The opening of the second runway at the new airport and the reduction in landing charges have encouraged DHL Worldwide Express to move its regional air freight center to Hong Kong. The aim of HKIA is not only to focus on increasing air cargo throughput, but also higher quality and additional value, like providing express services. ([49])

In 1997, the government approved HK$175m to set up a new Air Cargo Clearance System (ACCS) – a dedicated government-owned computer system – at the airport. The implementation of the ACCS provides a fair, equal and open customs clearance environment for the air cargo industry, without any individual cargo terminal operator dependent on competitors. The ACCS is linked to six cargo operators. There are not many countries in the world with computers that are comparable to those of the ACCS. ([9])

In 2000, to better utilize the airport's location at the mouth of the PRD, the Airport Authority has awarded a license for the development, operation, management and maintenance of a marine cargo terminal. Located at the northeast quay of the airport island alongside existing passenger ferry terminals, this HK$43 million facility will have an annual design capacity of 300,000 tonnes. Starting from March 2001, it accommodates both high-speed ferries and conventional river trade vessels. It is now linking the airport with at least 20 ports in the PRD. This marine transportation of air cargo would be more efficient in terms of travel times and costs than the ground access alternatives. ([12])

Besides, such as mentioned in the first paragraph, logistics facilities are developed in the commercial districts of the airport island to complement the cargo activities nearby. Such facilities allow local and foreign companies to provide value-added services such as packaging, labeling, sourcing, storage, distribution, quality control and product testing. ([5][12])

There are several airports in the PRD all aiming at the international market. Although it is believed that the HKIA will remain as the leading airport for many years to come, given the special arrangement under the Basic Law and its past performance in the international air transport market, coordination with other airports is a major concern in recent years. But up to now, the present trend of strong cooperation among large airlines teaming up to provide a world wide network of services does not appear to be common among airlines utilizing airports in the PRD Region. ([7])

Express Cargo

Two key needs of global manufacturing and sourcing are time-definiteness and door-to-door services. Shippers are no longer satisfied with the current express services that rely on passenger aircraft to transport their cargo. They urgently look for an Asian express hub. ([7][45])

So, in order to strength Hong Kong's position as a hub, even replacing Subic Bay in Philippines for FeDEX, Hong Kong Airport Authority will use an open-bid policy for the operation of an Express Cargo Center by one international express company. It is located inside the airport, and it will open in 2003. However, this plan is restricted by the air service agreements, in particular the 'Fifth freedoms'[7]. It is controversial to liberalize the air right ([39]) but the industry is optimistic that Hong Kong government will open it to foreign companies soon. ([49])

Railway

Hong Kong's Kowloon Canton Railway Corporation (KCR) is the first railway company in the world to obtain ISO9001 certification ([19]). Its East Rail operates freight services as a multi-modal system between Hong Kong and major freight distribution centers in the Mainland including Xian, Wuhan, Chengdu and Changsha. (See Appendix II) It has a simple and speedy cross-boundary procedure of less than 60 seconds. ([19]) However, the freight development is constrained by keen

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[7] Following the precedent established by the US and UK at the "Bermuda I" conference in 1946, the Hong Kong government negotiates for various air service "freedoms" on behalf of its airlines:

- *First freedoms* are rights to fly over a country without landing.
- *Second freedoms* are rights to land in a country strictly for technical seasons, such as refueling.
- *Third freedoms* are rights to pick up cargo in a carrier's home country and transport it to a foreign country.
- *Fouth freedoms* are rights to pick up cargo in a foreign country and transport it to the carrier's home country.
- *Fifth freedoms* are rights to carry cargo between two foreign countries. ([22])

competition from the other freight operators and by the lack of a direct access to the container port at Kwai Chung. At present, transshipment by rail from the Mainland has to rely on trucks to perform the last leg of their journey to the container port at Kwai Chung. ([7], p.63)

To overcome the handicap of over-reliance on trucks, KCR has put forward a proposal to build a freight tunnel from Tai Wai to the container port at Kwai Chung, which will enable re-exports from the Mainland to reach the portside directly. ([7], p.63)

To complement this, KCR is also looking into the feasibility of participating in the development of a freight storage and distribution center in Pinghu, 15km north of Shenzhen. Implementation of this proposal will enable exports from the inner provinces to route via Pinghu to Hong Kong. ([7], p.63)

Road

Heavy container trucks to and from our ports over the years have created serious traffic congestion in Hong Kong, at the border crossing points of Man Kam To (opened 1979), Sha Tau Kok (1985) and Lok Ma Chau (1990), as well as in Shenzhen City when passing through its urban area to other towns in the PRD Region. Such traffic congestion may be viewed as negative economic externalities generated by our port expansion programmes. ([7][8])

The introduction of the Deep Bay Bridge and the Zhuhai Bridge are recent proposals, which could lead to a significant structural transformation of HKSAR, can help on reducing the traffic congestion and diffusing our business influence to a wider market across the country. ([32])

A.5 Growth Trends and Statistics for Hong Kong

a) Freight

After the Asian financial crisis of 1998, freight activities start to gradually recover and increase. In recent years, from 1998 to 2000, the increasing rate is very slow and the total freight amount seems to be constant. (See Figure 1)

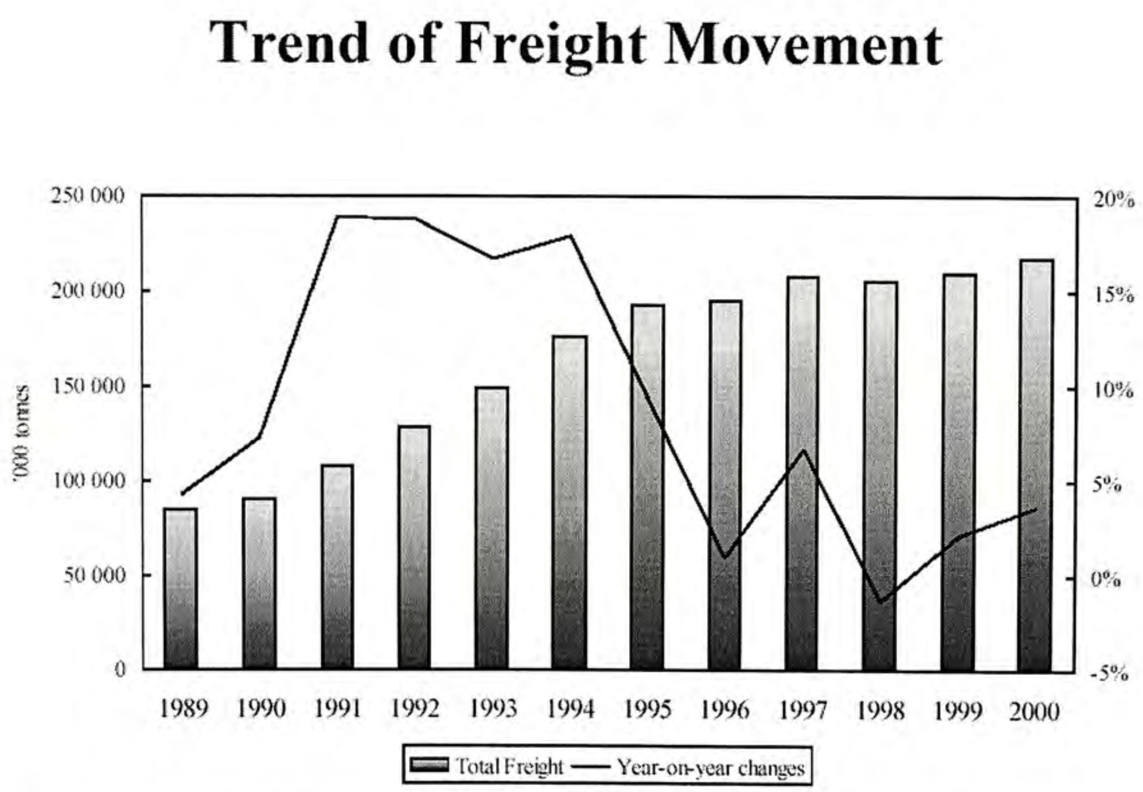
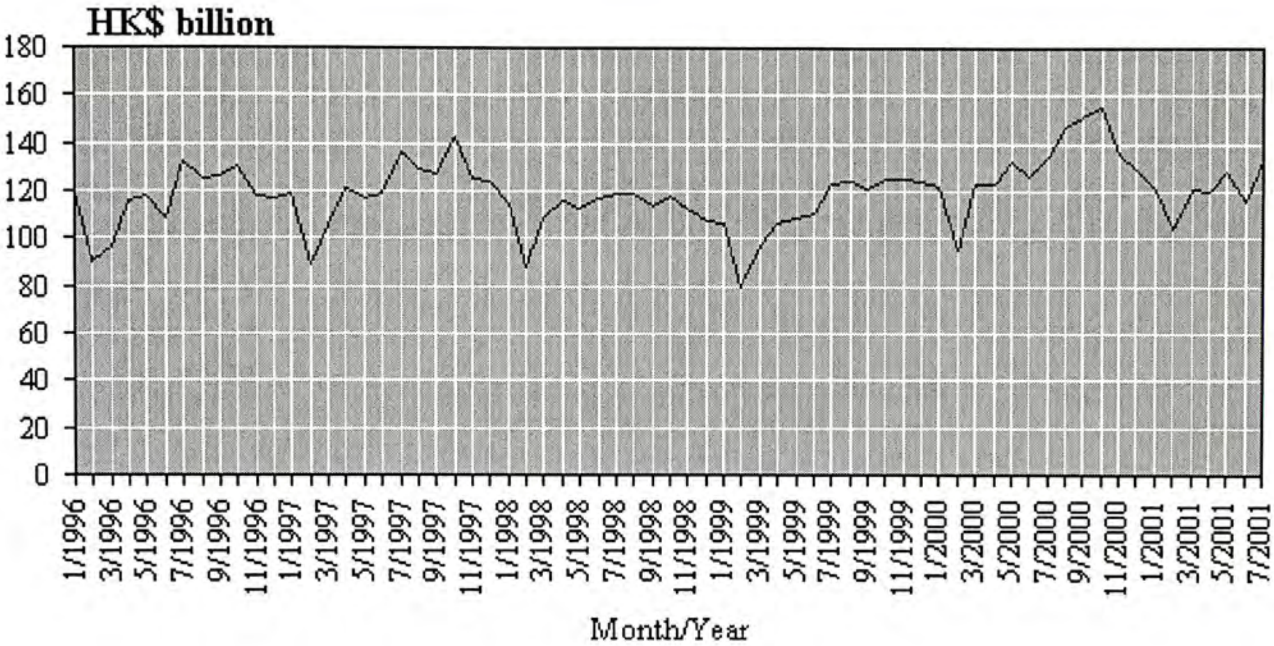


Figure 1: Trend of Freight Movement (1989 – 2000)

Source: Hong Kong Port and Maritime Board

There are several reasons for this phenomenon. One is the global economic recession. No industry can avoid being affected by it. It seems that the situation will still last for a certain period. We expect that the China WTO accession will stimulate the freight industry and economy in the near future.

Total Exports



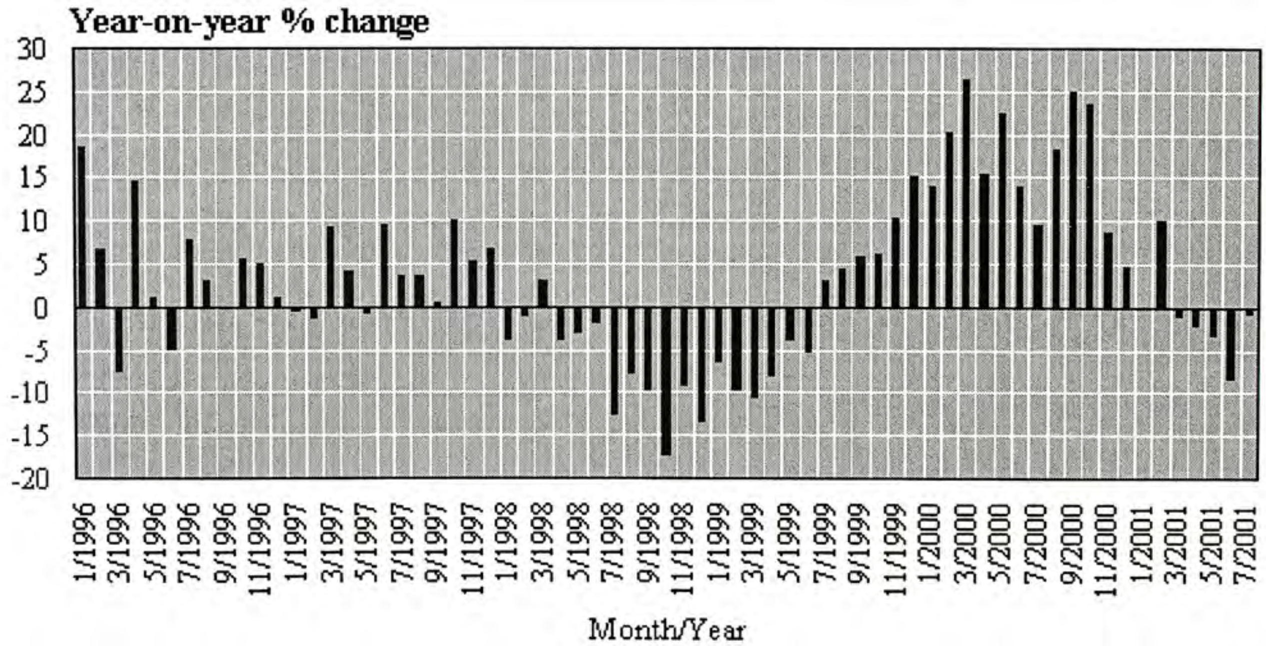


Figure 2: External Trade Aggregate Figure – Total Exports (Jan 1996 – Jul 2001)

Source:Census and Statistics Department

In examing total export figures (See Figure 2), there is a cyclical structure around a certain level in the graph of total export value (upper), although many fluctuations occur in the year-on-year % change graph (lower). In July 2001, the negative % change has decreased sharply. It is possibly due to the cycle in the upper graph start to enter a peak. From the relationship between the cycle and year-on-year % change, only when a upward trend appears in the total export value graph, the year-on-year change graph will show positive increases.

See Figure 3 below, Hong Kong has trade with many countries but the trade between the Mainland and Hong Kong dominates. 43.1% of imports, 29.9% of domestic exports and 35.1% of re-exports of Hong Kong are from/to China. China contributes most in imports and re-exports, but just follows USA in domestic exports. Since the price Index is lower in China than in Hong Kong, so Hong Kong mainly do sourcing from China (HK$715 billion in 2000), rather than transfer materials to China (HK$54.2 billion in 2000).

From the data in domestic exports, we know that Hong Kong is no longer a production city. The value of domestic exports is decreased in 2000 (HK$181 billion) comparing in 1995 (HK$231.7 billion). Moreover, it is very small when compared to the value of imports (HK$1658 billion in 2000) and re-exports (HK$1391.7 billion in 2000).

Trade by Main Country/Territory			*HK$ billion*
Type of trade/ Main country/Territory	1995	1999	2000
Imports	1,491.1	1,392.7	1,658.0
	(+19.2)	(-2.5)	(+19.0)
The mainland of China	539.5	607.5	715.0
Japan	221.3	162.7	199.0
Taiwan	129.3	100.4	124.2
USA	115.1	98.6	112.8
Republic of Korea	73.3	65.4	80.6
Asia-Pacific Economic Co-operation	1,260.2	1,200.7	1,436.9
European Union	160.4	127.2	144.3
Domestic Exports	231.7	170.6	181.0
	(+4.3)	(-9.5)	(+6.1)
USA	61.3	51.4	54.4
The mainland of China	63.6	50.4	54.2
United Kingdom	10.9	10.4	10.7
Germany	12.2	8.5	9.3
Taiwan	8.0	5.1	6.1
Asia-Pacific Economic Co-operation	177.7	129.5	139.6
European Union	40.1	32.8	32.9
Re-exports	1,112.5	1,178.4	1,391.7
	(+17.4)	(+1.7)	(+18.1)
The mainland of China	384.0	399.2	488.8
USA	231.0	269.4	311.0
Japan	70.1	67.5	82.1
United Kingdom	32.3	45.5	52.4
Germany	45.8	44.1	50.6
Asia-Pacific Economic Co-operation	845.3	899.0	1,078.9
European Union	160.7	184.1	206.9

Figure 3: Trade by Main Country/Territory

Source:Census and Statistics Department

Inward and Outward Movements of Cargo			*'000 tonnes*
	1995	1999	2000
Discharged			
By air	685	841	952#
By water	101,770	106,305	108,900#
By ocean	87,048	88,621	90,000#
By river	14,723	17,684	18,800#
By road	16,198	20,500*	22,142
By rail (1)	997	293	318
Total	119,651	127,940*	132,312#
Loaded			
By air	772	1,133	1,287#
By water	54,136	62,533	66,400#
By ocean	40,127	39,601	42,800#
By river	14,009	22,932	23,600#
By road	18,092	17,915*	17,791
By rail	316	173	133
Total	73,316	81,754*	85,611#

Notes: (1) Figures exclude livestock

Provisional figures/estimates

* Revised figures

Figure 4: Inward and Outward Movements of Cargo

Source:Census and Statistics Department

Mode of Transport	Transport Volume (Million Tons)	Turnover Volume (Billion Ton/km)
Expressway	65.441	25.63
Road	12.24	0.59
Water	101.654	179.93
Air	0.3178	0.54
Pipeline	24.018	4.57
Total	203.6708	211.28

Figure 5: Transport Volume of Guangdong Province in 1999

Source:Census and Statistics Department

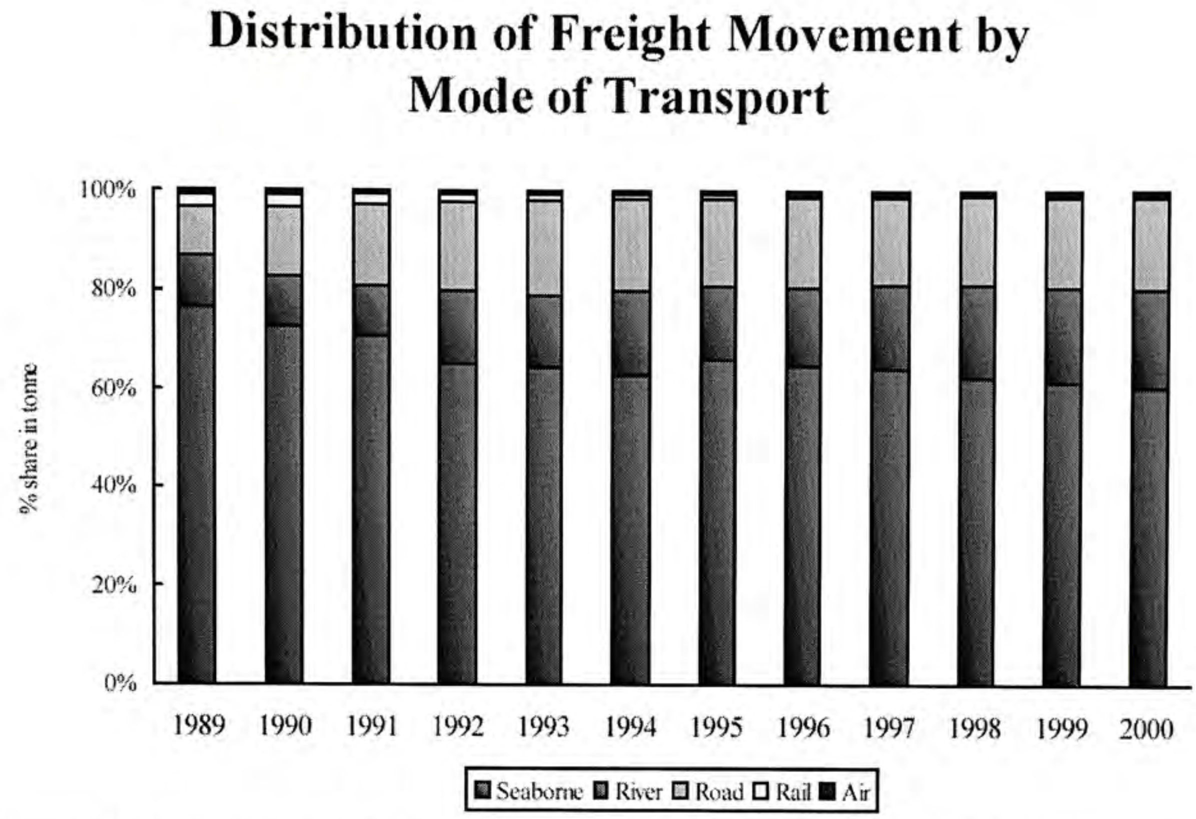


Figure 6: Distribution of Freight Movement by Mode of Transport (1989 – 2000)

Source: Hong Kong Port and Maritime Board

In terms of cargo discharged amounts, from 1995 to 2000, cargo transported by air has increased by 40%, ocean has increased by 3.4%, river has increased by 27.7%, road has increased by 36.7% but rail has decreased by 68.1%. (See Figure 4 above)

In terms of cargo loaded amounts, from 1995 to 2000, cargo transported by air has increased by 66.7%, ocean has increased by 6.7%, river has increased by 68.5% but road has decreased by 1.7% and rail has decreased by 57.9%. (See Figure 4 above)

The trend is that freight movements by air and river become more important and occupies increasingly large proportion of the total freight, but the importance of rail declines a lot. There is a slow increase for using road and river. (See Figure 6 above)

It should be noted that the pie for freight industry keep enlarging. Except rail, all kinds of transport have shown increased amount of cargo handled. What we can conclude from the figures is the shift of the use of different transportation mode and the relative importance among them.

Water is the most important mode of transport in Guangdong Province. It takes up the 49.9% of the total transport volume in 1999. We are certain that it has contributed to the increase of the Hong Kong freight volume on water transport. (See Figure 5)

In contrast, air is the least important mode of transport in Guangdong Province. It is just the 0.2% of the total transport volume in 1999. This reflects that flights from Hong Kong may always go to the inner part of China and other foreign countries, but not for express services between Guangdong and Hong Kong.

b) Shipping

Statistical Awareness:

Some of the cargoes shipped through the port are not recorded as the territory's trade. Such cargoes are transshipments that pass through the territory without their ownership ever being transferred to any entity in Hong Kong. Thus, Hong Kong trade statistics do not fully reflect the importance of Hong Kong, which operates the busiest container port in the world. In other words, transshipment cargoes are not recorded as its imports, exports or re-exports.

The other point is that the recent global economic recession, financial crisis in Asia and the existing port development programmes in South China outside the HKSAR may force us to consider technical adjustment in Hong Kong's growth estimates, and subsequent reassessment of HK's development programmes.

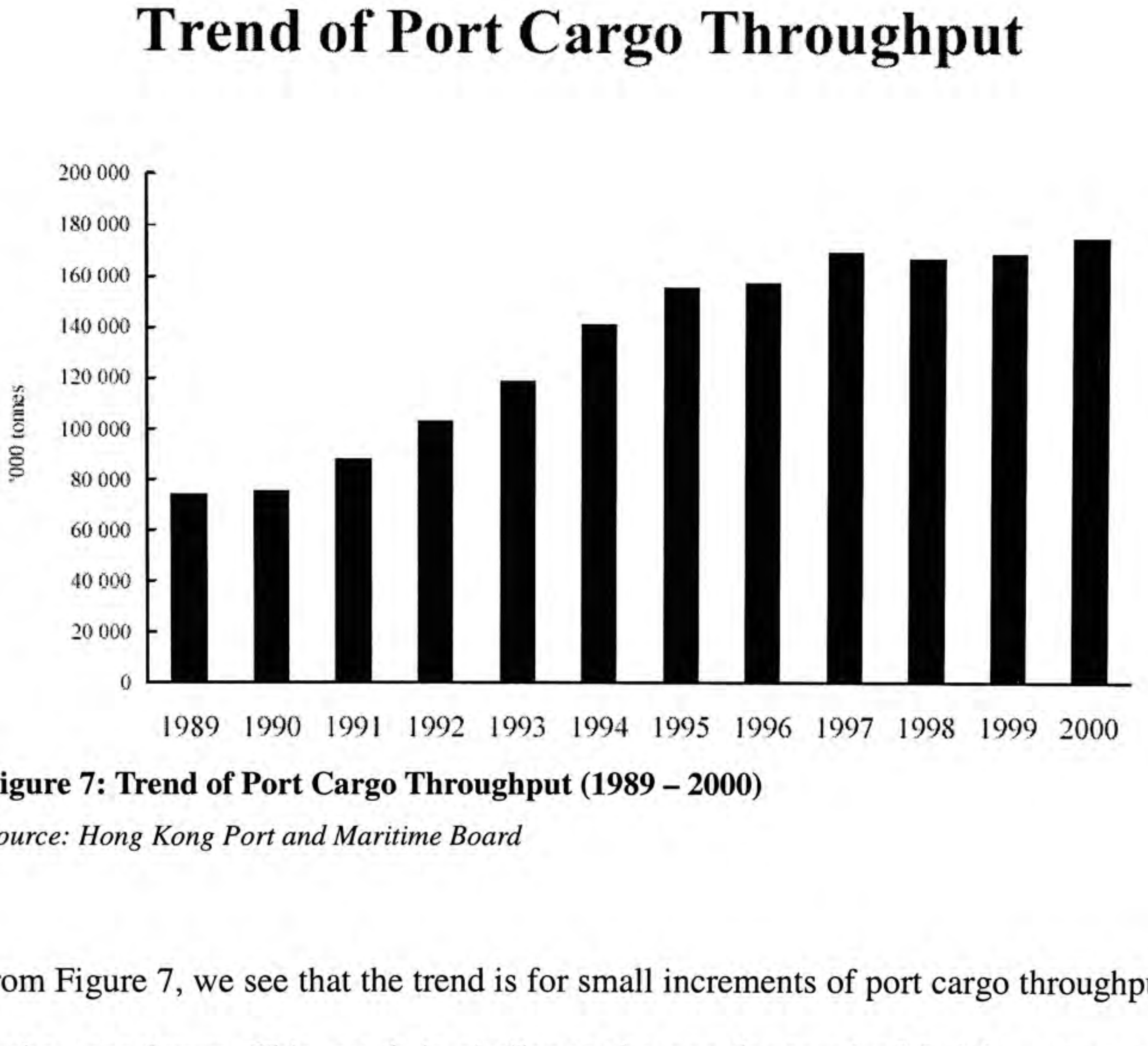


Figure 7: Trend of Port Cargo Throughput (1989 – 2000)

Source: Hong Kong Port and Maritime Board

From Figure 7, we see that the trend is for small increments of port cargo throughput in the near future. This graph is similar to the one for trend of freight movement (Figure 1), since very large proportion of freight is transported by water.

From Figure 8, we see that river trade[8] handles more and more containers while the percentage share of mid-stream[9] in container throughput is lowering. The average

[8] "River trade vessels which make use of the numerous waterways in the Pearl River Delta area to transport cargoes to and from Guangdong.", from HKPMB Annual Report 1998. ([1])

[9] "Mid-stream operations involve the loading and unloading of cargoes from ships moored at buoys or anchorages in the harbour. Cargoes are taken from ships to shore by lighters which have their own

growth of river trade within 1996-1999 is 25%. It may mean that people who used mid-stream has changed to use Kwai Chung Terminals. (See Figure 9) Moreover, increasing trade with China increases the use of river trade, which is a very economic and environmentally friendly alternative to the increasingly congested road system. ([1])

As container throughput is increasing, although Kwai Chung Terminals' percentage share in TEUs stays at a constant level as shown in Figure 8, it still means they are handling increasing amount of cargoes (Figure 10). Based on the forecast and analysis of PMB, additional Container Terminals need to be built for meeting the demand. The coming new one is Container Terminal 9.

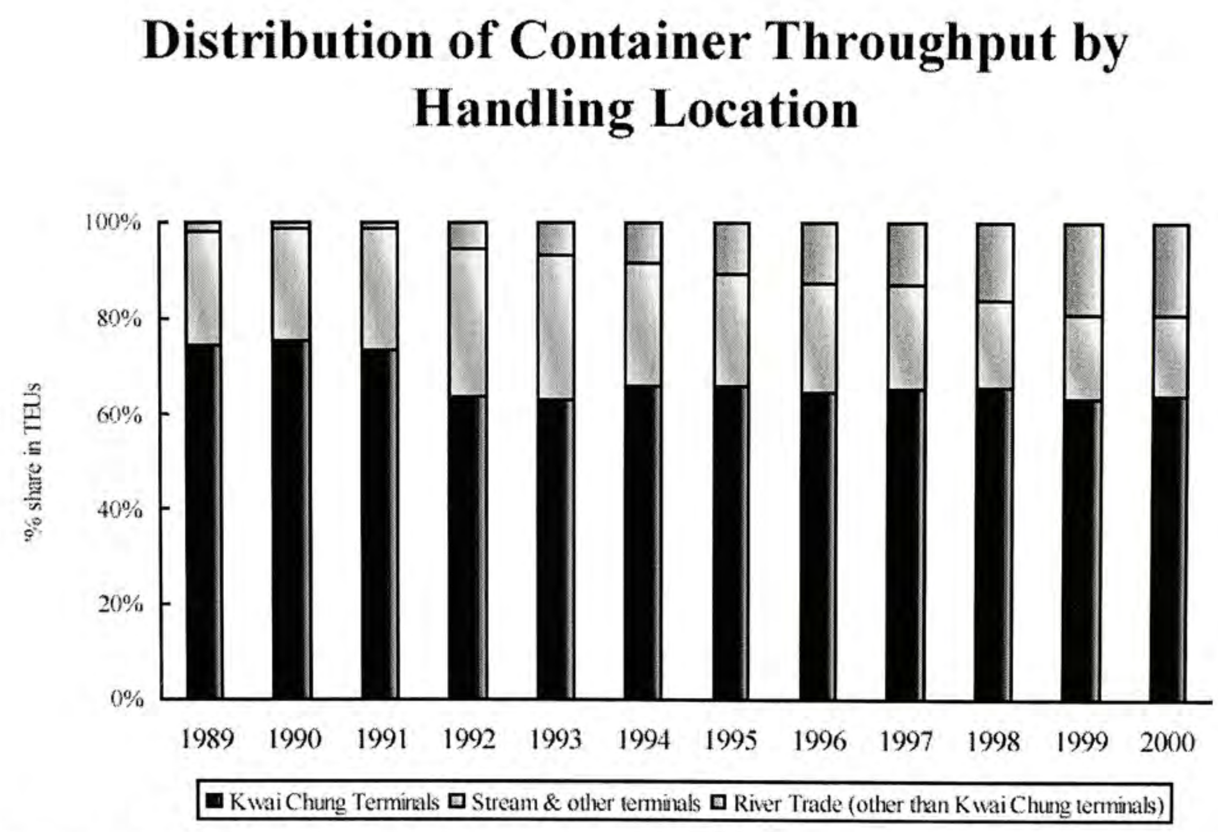


Figure 8: Distribution of Container Throughput by Handling Location (1989 – 2000)

Source: Hong Kong Port and Maritime Board

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derricks.", from HKPMB Annual Report 1998. ([1])

From Figure 9, we see that the growth rate of midstream has dropped 22.8%, because facilities at midstream is mainly for handling bulk cargo, not containerised cargo, and the world trend is turning more and more bulk cargoes into containers (Figure 10), as they are easier to be managed. Even if some midstream operators can handle container, it is available only for small vessels. The Kwai Chung container ports are well equiped and can attract a lot of usage, although they cost more than midstream operations.

From Figure 9, we also see that the development of Shenzhen ports is astonishing. The average growth of feeder/barge traffic (1996-1999) at Shenzhen ports is 91.4%. The average growth of containerized port traffic by direct, tran-shipment and river trades (1996-1999) is 80.2%, while that of Hong Kong is just 6.6%. Although the actual volume of containers that Shenzhen ports handled is small (2,986,000 TEU in 1999) compared with Hong Kong (16,211,000 TEU in 1999), the development rate is so fast that we cannot be slow to react to this threat.

The average growth (1996-1999) of containerized port traffic is 10.6% in the regions of Shenzhen and Hong Kong. The trend is anticipated to continue to rise.

PORT OF HONG KONG ONLY						
	1995	1996	1997	1998	1999	Average Growth 1996-1999 (% p.a.)
Direct Shipment of Hong Kong and China cargo	8,432	8,600	9,573	9,101	9,851	
Growth (%)		2.0	11.3	-4.9	8.2	4.0
Trans-shipment						
- at Kwai Chung	2,253	2,442	2,437	2,320	2,398	1.6
- in Midstream	284	221	120	89	101	-22.8
- Total	2,537	2,663	2,557	2,409	2,499	
Growth(%)		5.0	-4.0	-5.8	3.7	-0.4
River trade	1,581	2,197	2,437	3,072	3,861	
Growth(%)		39.0	10.9	26.1	25.7	25
Total, Hong Kong	**12,550**	**13,460**	**14,567**	**14,582**	**16,211**	**6.6**
PORTS OF HONG KONG AND SHENZHEN						
	1995	1996	1997	1998	1999	
Direct Shipment of Hong Kong and China cargo	8,630	8,967	10,360	10,233	11,697	
Growth (%)		3.9	15.5	-1.2	14.3	7.9
Trans-shipment						
- at Kwai Chung	2,253	2,442	2,437	2,320	2,398	1.6
- in Midstream	284	221	120	89	101	-22.8
- Total	2,537	2,663	2,557	2,409	2,499	
Growth (%)		5.0	-4.0	-5.8	3.7	-0.4
River trade	1,581	2,197	2,437	3,072	3,861	
Growth (%)		39.0	10.9	26.1	25.7	25.0
Feeder/Barge traffic at Shenzhen ports	85	221	447	820	1,140	91.4
Total						
- Hong Kong	**12,550**	**13,460**	**14,567**	**14,582**	**16,211**	**6.6**
- Shenzhen Ports	**283**	**588**	**1,145**	**1,952**	**2,986**	**80.2**
Total Hong Kong and Shenzhen Ports	**12,833**	**14,048**	**15,712**	**16,534**	**19,197**	
Growth (%)		**9.5**	**11.8**	**5.2**	**16.1**	**10.6**

Figure 9: Containerised Port Traffic by Direct, Trans-shipment and River Trades (1995-1999) (000 TEU)

Source: Hong Kong Port and Maritime Board

From Figure 10, we see that container throughput of Hong Kong is increasing within 1989-2000, but the recent growth rate is lower than the growth rate before 1996. For container throughput of Shenzhen, the growth rate is fluctuated during 1989 – 1999.

Sometimes it is more than 100%, like in 1991, 1992 and 1996, but sometimes in between there are low growth rates, like in 1990, 1993 and 1999. (See Figure 12)

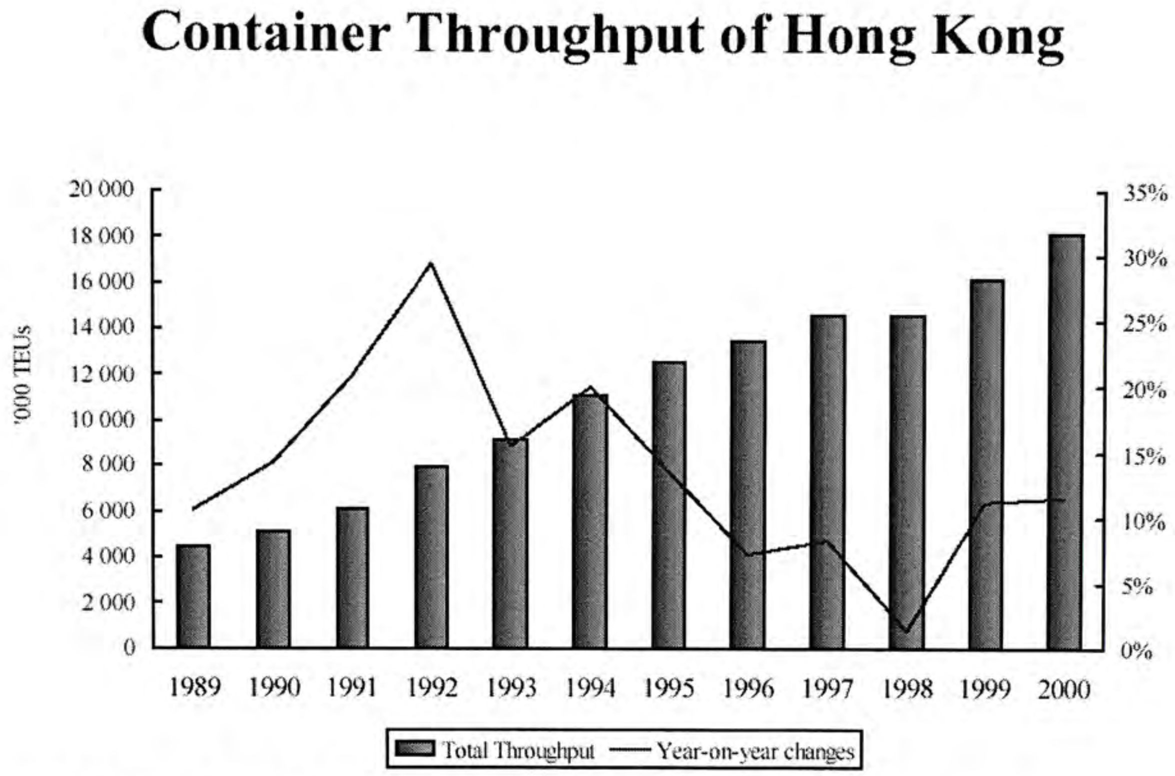


Figure 10: Container Throughput of Hong Kong (1989 – 2000)

Source: Hong Kong Port and Maritime Board

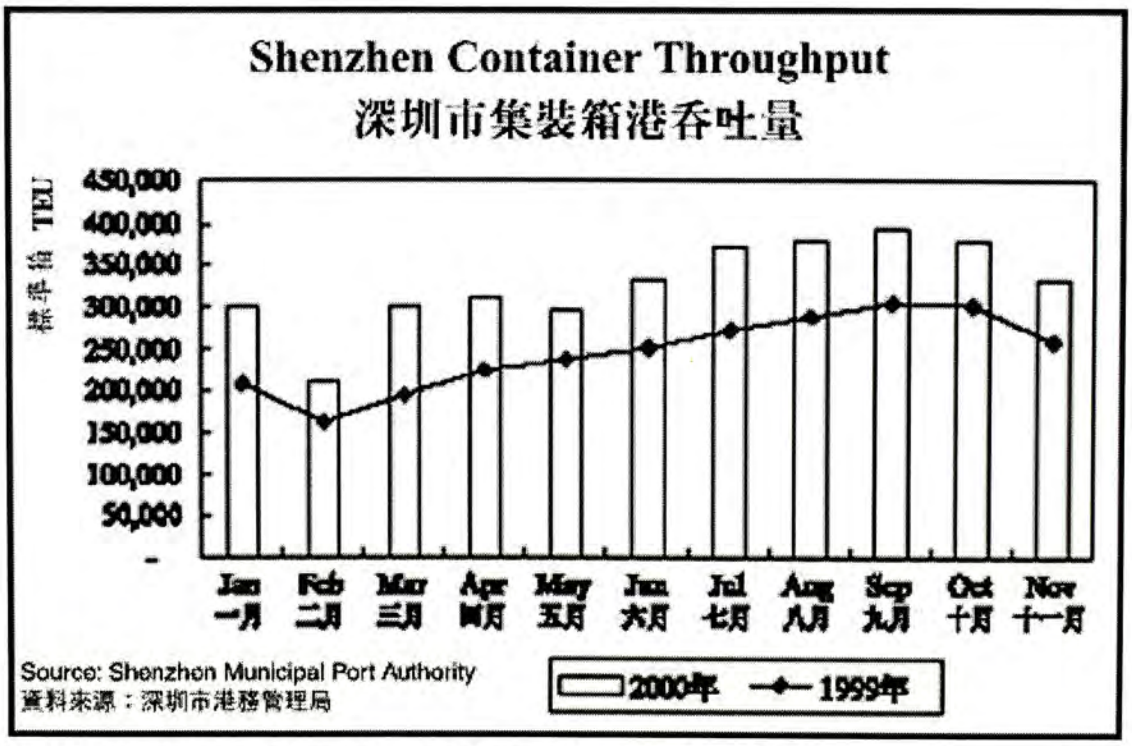


Figure 11: Container Throughput of Shenzhen (1999 – 2000)

Source: Shenzhen Municipal Port Authority

Form Figure 11, we see that container throughput of Shenzhen is increasing when we compare the number of TEUs in 2000 to the number in 1999. There appears to be a seasonal cycle. The lowest container throughput occurs in every February and the highest container throughput occurs in every September. For the former case, it may be due to the holiday of Lunar New Year.

年份	增長率（%）	總吞吐量	蛇口招港碼頭	蛇口集裝箱碼頭	赤灣集裝箱碼頭	赤灣凱豐碼頭	鹽田國際碼頭	機場港務碼頭	海星港口碼頭
1989	82.2	1.84	1.84						
1990	-2.7	1.79	1.79						
1991	182.7	5.06	2.96		2.1				
1992	101.2	10.18	2.33	4.9	2.95				
1993	26.1	12.84	2.6	6.7	3.3		0.18	0.06	
1994	38.5	17.79	3.16	8.71	3.68	0.91	1.33		
1995	59.4	28.36	2.32	8.99	3.34	3.14	10.57		
1996	107.7	58.9	4.78	8.99	4.82	4.96	35.35		
1997	94.8	114.73	7.48	21.48	6	15.02	63.84	0.69	
1998	70.1	195.17	14.2	46.31	7	20.33	103.8	0.81	2.72
1999	53	298.61	27.6	57.41	13.14	35.01	158.81	3.02	3.61

Figure 12: Shenzhen Ports Container Throughput (1989 – 1999) (0,000 TEU)

Source: Shippers Today

We need to pay special attention to the growth of two Shenzhen ports, Shekou and Yantian ports. They have the greatest potential to develop into successful ports like the one in Hong Kong. In 1999, Yantian port has the greatest container throughput among Shenzhen ports, that is 1,588,100 TEUs or 53.2%. Shekou port has second large container throughput among Shenzhen ports, that is 850,100 TEUs or 28.5%. (See Figure 12)

Year	Containerised Cargo	Break Bulk Cargo	Dry Bulk Cargo	Liquid Bulk Cargo	Total
	Throughput (000 tonnes)				
1999 Baseline Position	110,097	18,332	18,548	21,861	168,838
2000	120,518	19,466	18,917	23,258	182,159
2005	166,635	24,987	20,915	31,040	243,577
2010	199,232	30,362	21,375	36,308	287,277
2015	242,773	36,054	23,834	41,705	344,366
2020	273,999	39,784	26,270	46,259	386,312
	Average Annual Growth Rate (%)				
2000-2005	6.7%	5.1%	2.0%	5.9%	6.0%
2005-2010	3.6%	4.0%	0.4%	3.2%	3.4%
2010-2015	4.0%	3.5%	2.2%	2.8%	3.7%
2015-2020	2.4%	2.0%	2.0%	2.1%	2.3%
1999-2020	4.4%	3.8%	1.7%	3.6%	4.0%

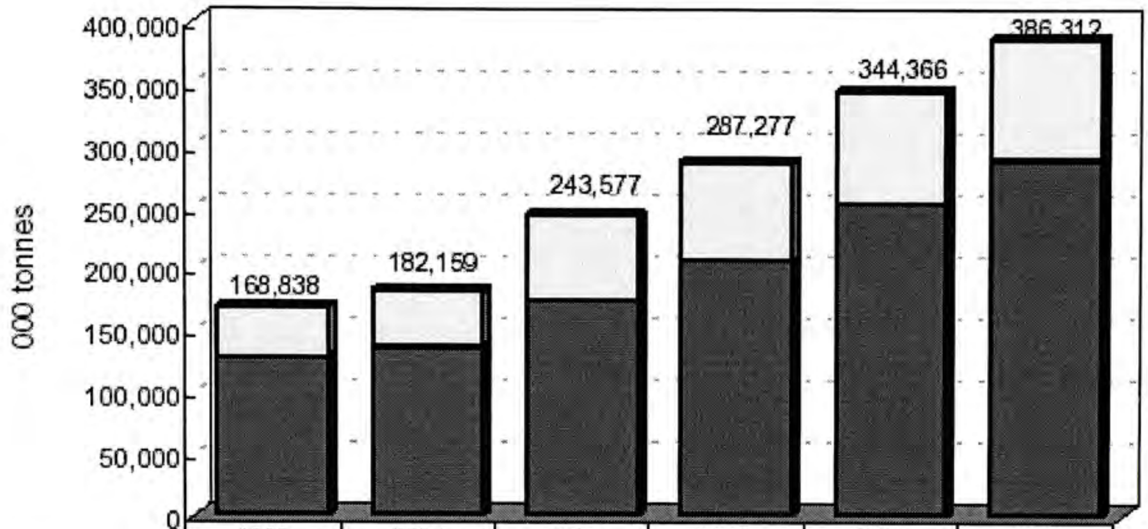
Figure 13: Port Cargo Forecasts by Type of Cargo (1999 – 2020)

Source: Hong Kong Port and Maritime Board

According to the Hong Kong Port and Maritime Board, the average annual growth for container traffic for Hong Kong as a whole – ocean and river – is estimated to be 4.5% (1999-2020). The average annual growth rate over 1999-2005 is expected to be 7.4% as compared with 6.6% for 1995-1999.As containerised cargo accounts for over 60% of the port traffic, the overall growth pattern is dominated by the trend of container throughput. (See Figure 13)

With regard to non-containerised cargo, break bulk is expected to expand at a rate of approximately 3.8% per annum between 1999 and 2020, reaching 39.8 million tonnes in 2020. Dry bulk is projected to increase at 1.7% per annum over 1999-2020, reaching approximately 26.3 million tonnes in 2020. Liquid bulk is forecast to increase at 3.6% per annum, hitting 46.3 million tonnes by 2020. (See Figure 13)

The accession of China into the World Trade Organisation will boost up cargo throughput in Hong Kong very much in the beginning 5 years, 2000 – 2005, according to a report by the HK PMB.



	1999	2000	2005	2010	2015	2020
River Cargo	40,615	45,235	69,814	80,471	90,938	96,989
Ocean Cargo	128,223	136,924	173,763	206,806	253,428	289,323

Figure 14: Port Cargo Forecasts by Ocean and River (1999-2020)

Source: Hong Kong Port and Maritime Board

Overall port traffic is forecast to increase at 4.0% per annum over 1999-2020 when the total throughput reaches 386.3 million tonnes. Ocean cargo is projected to increase at 4.0% per annum over the forecast period reaching 289.3 million tones by 2020, and river cargo is projected to increase at 4.2% per annum, reaching 97.0 million tones by 2020. River cargo's growth rate is larger than ocean cargo's growth rate, because the close relationship and increasing trade between China and Hong Kong. WTO accession of China will particularly boost up the total cargo throughput in 2000 - 2005. (See Figure 14)

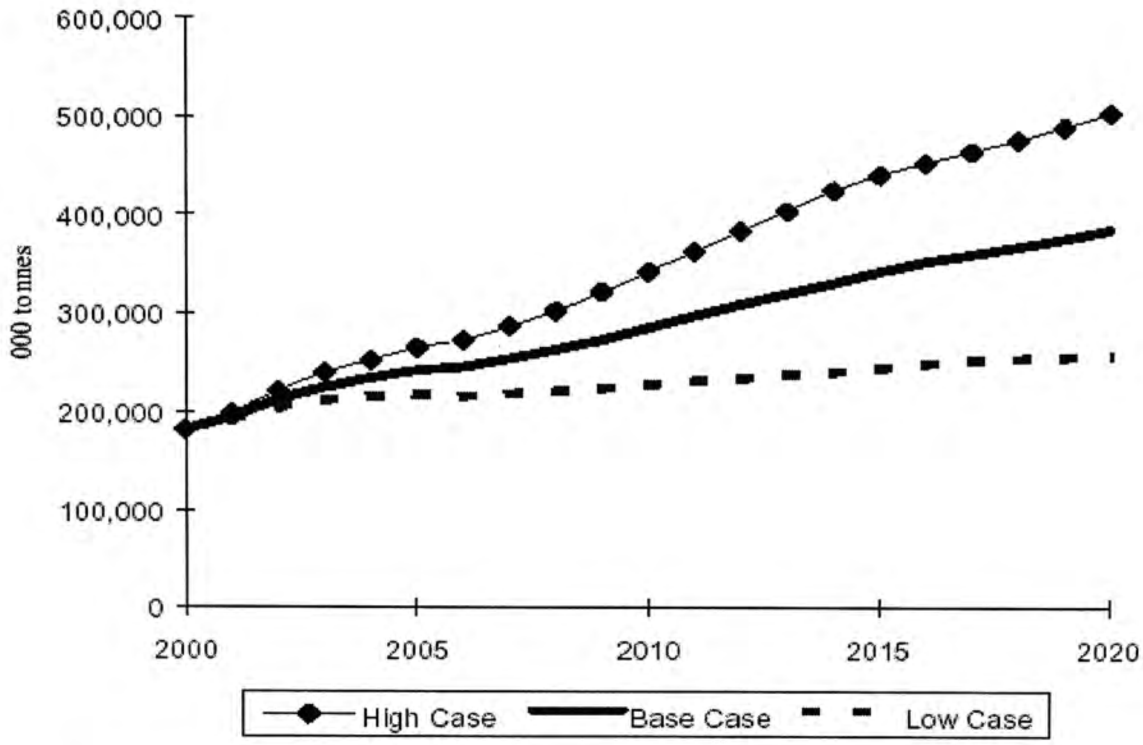


Figure 15: Projected Trend of Overall Port Traffic – High, Base and Low Cases (2000 -2020)

Source: Hong Kong Port and Maritime Board

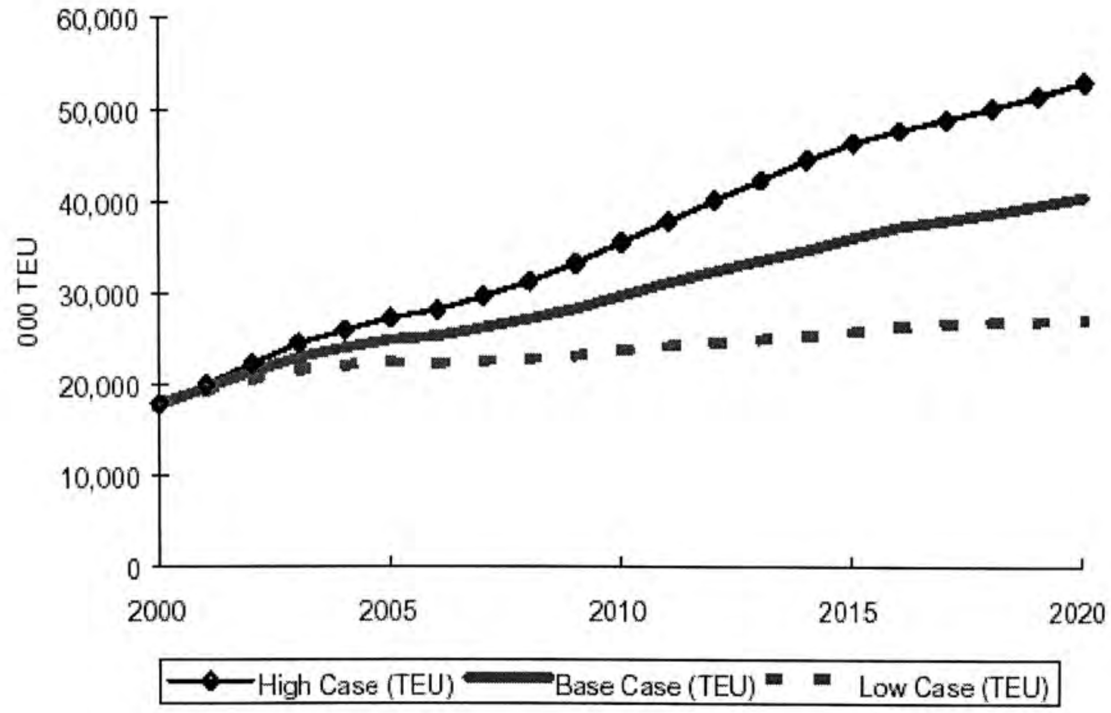


Figure 16: Projected Trend of Port Container Throughput – High, Base and Low Cases (2000 -2020)

Source: Hong Kong Port and Maritime Board

The high and low forecasts provide for comparison of a more optimistic and a more pessimistic view of the future in terms of overall traffic growth and also the performance of Hong Kong's port relative to its competitors. A total volume of 505.0 million tonnes throughput are predicted for 2020 in the high-volume scenario, and a lower volume of 258.9 million tones are projected for 2020 in the low-volume scenario. (See Figure 15 and Figure 16)

According to the PMB, Hong Kong is forecasted to enjoy continued strong growth in traffic over the next 20 years. Some important points are as the following:

- The average annual growth rate of the cargo pool (South China including Hong Kong) is forecast to be 8.6% (1999-2020).
- Overall port traffic is projected to increase at 4.0% p.a. over 1999-2020 when the total throughput reaches 386.3 million tones.
- Ocean cargo is estimated to increase at 4.0% per annum over the forecast period reaching 289.3 million tones by 2020. River cargo is forecast to increase at 4.2% per annum, reaching 97.0 million tones by2020.
- Average annual growth for container traffic as a whole – ocean and river – is estimated to be 4.5% (1999-2020). Throughput in 2020 is projected to be two and a half times the 1999 level. The average annual growth rate over 1999-2005 is expected to be 7.4% as compared with 6.6% for 1995-1999.
- Trans-shipment's share of Hong Kong's ocean container cargo base is forecast to fall from 22% in 1999 to 8% by 2020.

c) Air

The total amount of air cargo throughput has slightly increased in 2000. Tonnage of air cargo export is always higher than that of import. It is expected that the overall volume will grow more because logistics center and express center will open soon and better services can be provided.

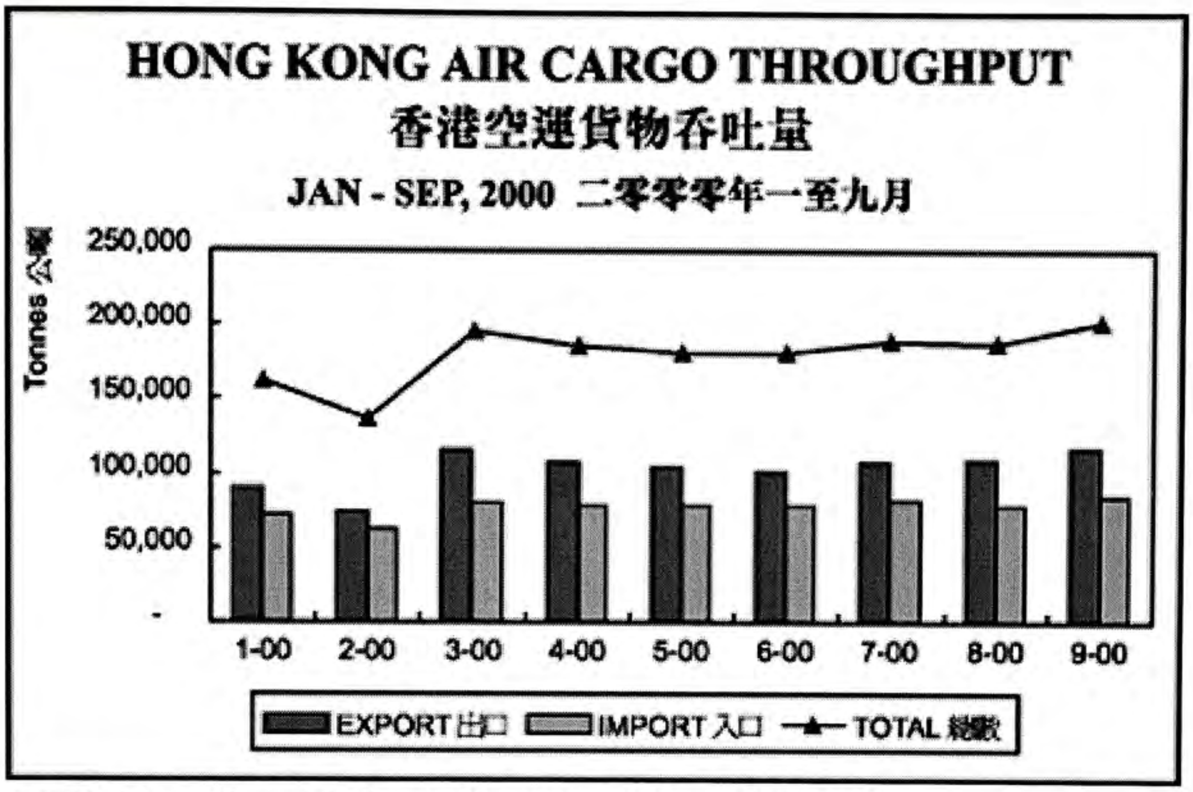


Figure 17: Hong Kong Air Cargo Throughput (Jan - Sep 2000)

Source: Airport Authority

d) Railway

Figure 18 only shows tonnage handled by railway from January to April in 2001. Comparing to the same period in last year, the % growth is –21.23%. We believe that railway will take less and less an important role in freight industry, because railway is only connected to Mainland. Its distribution coverage is limited and speed is not fast enough for current customer needs. Moreover, competition arises from other mode of transportation, especially river trade, which is cheap in price and convenient.

Hong Kong External Trade By Rail, 2001						
	Tonnage Discharged	% growth over last year	Tonnage Loaded	% growth over last year	Total	% growth over last year
Jan	18	-28.00%	5	-64.29%	23	-41.03%
Feb	19	46.15%	6	-25.00%	25	19.05%
March	25	-19.35%	9	-25.00%	34	-20.93%
April	26	-16.13%	7	-41.67%	33	-23.26%
Grand Total : Jan - Apr, 2001						
	Tonnage Discharged	% growth over last year	Tonnage Loaded	% growth over last year	Total	% growth over last year
	88	-12.00%	27	-41.30%	115	-21.23%

Figure 18: Hong Kong External Tade By Rail (Jan – Apr 2001)

Source : Hong Kong Monthly Digest of Statistics

e) Road

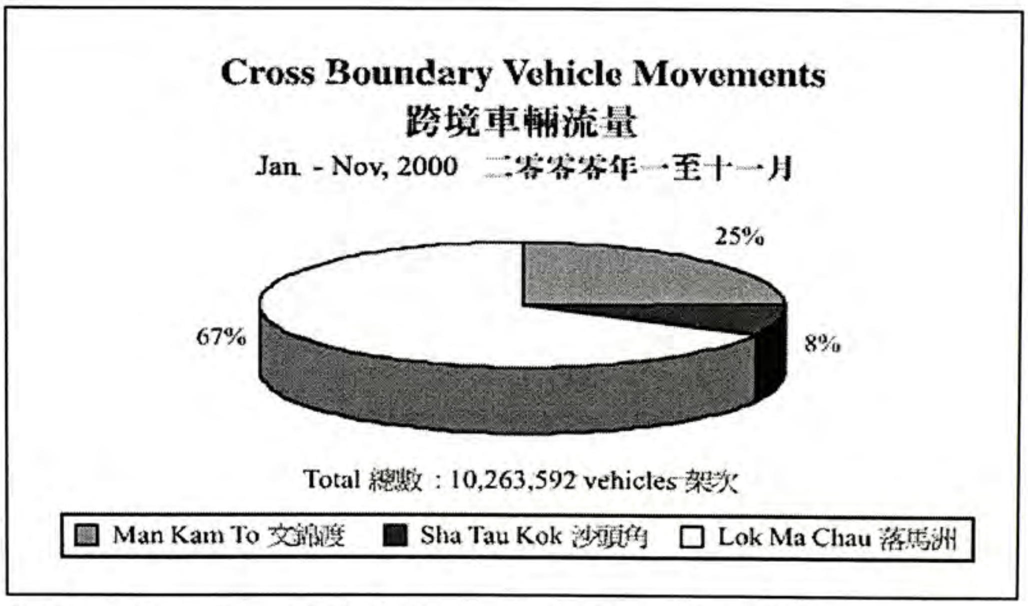


Figure 19: Cross Boundary Vehicle Movements (Jan – Nov 2000)

Source: Hong Kong Customs and Excise Department

Lok Ma Chau is the busiest border crossing point. Its vehicle throughput increased by 4.1% from Jan-Jun 2000 to Jan-Jun 2001. Great concern should be given to this point, otherwise, the congestion problem will become more serious.

The second busy border crossing point is Man Kam To. It has decreasing use rate, which is 17% when compare vehicle throughput in Jan-Jun 2001 with Jan-Jun 2000. The least busy border crossing point is Sha Tau Kok. It also has decreasing use rate, which is 9.8% when compare vehicle throughput in Jan-Jun 2001 with Jan-Jun 2000.

Control Point	6/2001	6/2000	6/2001 6/2000 (% change)	1-6/2001	1-6/2000	(Number) 1-6/2001 1-6/2000 (% change)
Man Kam To						
Goods Vehicles	184 898	217 680	-15.1%	1 044 864	1 259 245	-17.0%
Passenger Vehicles	3 691	2 374	55.5%	23 109	16 013	44.3%
Private Cars	19 995	18 406	8.6%	116 822	106 920	9.3%
Total	208 584	238 460	-12.5%	1 184 795	1 382 178	-14.3%
Sha Tau Kok						
Goods Vehicles	46 805	55 213	-15.2%	275 142	304 961	-9.8%
Passenger Vehicles	6 594	5 155	27.9%	41 347	31 835	29.9%
Private Cars	16 039	15 916	0.8%	93 421	91 722	1.9%
Total	69 438	76 284	-9.0%	409 910	428 518	-4.3%
Lok Ma Chau						
Goods Vehicles	547 545	531 709	3.0%	3 018 345	2 900 810	4.1%
Passenger Vehicles	34 892	27 547	26.7%	193 607	164 369	17.8%
Private Cars	94 727	79 719	18.8%	554 123	427 484	29.6%
Total	677 164	638 975	6.0%	3 766 075	3 492 663	7.8%
Overall						
Goods Vehicles	779 248	804 602	-3.2%	4 338 351	4 465 016	-2.8%
Passenger Vehicles	45 177	35 076	28.8%	258 063	212 217	21.6%
Private Cars	130 761	114 041	14.7%	764 366	626 126	22.1%
Total	955 186	953 719	0.2%	5 360 780	5 303 359	1.1%

Figure 20: Cross Boundary Vehicle Movements

Source: Hong Kong Customs and Excise Department

The total number of vehicles, crossing the HK-Shenzhen boundary, dropped by 2.8% (comparing vehicle throughput in Jan-Jun 2001 with Jan-Jun 2000). It may be due to the bad economy in Hong Kong, which lead to lower freight movement. (See Figure 19 and Figure 20)

A.6 Competitive Analysis for Hong Kong as a Logistics Hub

A.6.1 Current Industry Strengths

❍ The natural advantage ([6][19][28])

Hong Kong is at the crossroad of international shipping. It is at a strategic and ideal geographic location, where is in the heart of Asia, midway between Japan in the north and Singapore and Malaysia in the south. Half of the world's population can be reached within 5 hours of flight. It acts as Mainland China's gateway to the rest of the world.

❍ World-class freight center ([19][28])

One of the world's leading shipping and aviation hubs, Hong Kong offers transportation services which are speedy and efficient, safe and reliable, and secure and precise. Hong Kong set high service standards for itself. Moreover, it has efficient customs clearance procedures and shippers can do some customs free operations[10].

❍ Leading edge infrastructure ([19][28])

Hong Kong has sophisticated infrastructure. In the city, there are well-established transportation network with road and rail direct to China, modern airport and (container) port, and advanced aviation and cargo facilities.

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[10] "Hong Kong's free port status enables operators to accept containers as late as three hours before sailing, so shippers may pull out cargo for reworking or alternative delivery and may carry out cargo quality inspection as well as pick and pack services within the terminals." ([19])

○ Winning business environment ([6][19][25])

Efficient and reliable supporting business services are available in Hong Kong. For instance, they are financial, insurance, legal, arbitration and other related services.

Some people said Hong Kong is a hub whereas ports in China are spokes. Hong Kong, as a hub, is enjoying concentration of activities and scale economy that 'spoke' ports cannot enjoy to the same degree.

A.6.2 Current Industry Weaknesses

⊕ High costs in shipping ([3], p.5, [54])

Hong Kong has higher tariffs to shipping lines and terminal handling charges than Shenzhen ports, and relatively high road haulage costs to and from Mainland.

⊕ Problem channels ([6][54])

There are always congestions at cross-border checkpoints and the roads to the container port.

⊕ Lack of 3PL services ([21])

By a study conducted by Hong Kong's Lingnan University, Hong Kong is behind Singapore in the provision of 3PL services. It is because the private sector does not have adequate incentives to invest in the 3PL industry as the investment rate of return (IRR) is not attractive.

⌗ Lack of integrated services ([54])

As logistics become popular, many transportation companies or warehouse companies just turn their company names to include the word 'logistics'. In fact, they are still providing only part of the logistics services, instead of integrated services. This is not logistics and cannot help the service required customers.

⌗ Lack of logistics expertise ([54])

The subject of logistics is just recently set up in Universities. Now, we have very few logisticians who have knowledge and experience in the field.

A.6.3 Competitiveness Challenges

➢ Integration in Asia ([34])

The Asia-Pacific region is a very complex market because its three billion people live in more than 20 countries, speak 25 languages and more than 700 dialects, and are located on 24,000 separate land masses, many of which are small islands. It is difficult for Hong Kong to build an integrated supply chain network with China.

➢ Promotion of e-logistics

After experiencing the burst of the IT bubble, companies have lost confidence in e-commerce and many have over-reacted to investments in e-technology. In fact, many companies have insufficient income to implement the technology. ([37]) Besides, it is difficult to develop an e-logistics platform for the Hong Kong logistics industry, ([20]) because firstly, we are lack of local e-logistics expertise ([37]) and secondly, companies are afraid of sharing information with other parties within a supply chain.

➢ Competition from Mainland ports

Hong Kong faces increasing competition from neighboring ports. As major ports are restricted to deepwater sites – a condition in favor of Hong Kong – Huangpu, Shantou and Xiamen are likely to remain as feeder ports. But Shenzhen's ports – Mawan, Chiwan and Shekou (the dredging of the Tonggu channel allows Shekou and Chiwan to become genuine deep-water ports probably by around 2007-2009.) ([3], p.7), particularly, Yantian – could grow to challenge Hong Kong for the right to be the gateway to south China. ([29], p.87) We provide more description of Yantian, Shekou and Chiwan ports as follows:

Yantian Port

It is located at the eastern coast of Shenzhen. Although it is separated from the PRD waterways by Hong Kong, Yantian, like the other Mainland ports of Dalian, Ningpo and Fuzhou, has the advantage of a deepwater approach and enjoys an advantage in that it receives cargoes from Fujian, northern Guangdong, and even east China by barges. It also plans to start feeder services to the PRD to overcome its geographical disadvantage. ([25])

Yantian has an ample supply of backup land for developing an in-dock rail terminal to connect to Mainland's rail network and serve the central and western inner provinces. The number of calls at Yantian has increased and the port has been expanded to 5 container berths. ([3])

ShekouPort

Shekou is located at the southwest of Shenzhen. It is supported by rail, road and water services. Shekou Freight Station has more than 200 modern terminal operation machines and provides services including stevedoring of bulk and containerized cargo, warehousing, logistics support, etc. Shekou Container Terminals specializes in the handling of containers supported by modern container handling equipment and state-of-the-art terminal management computer systems. ([4], p.52;[25])

Chiwan Port

Chiwan lies next to the slightly larger Shekou Port, at the western part of Shenzhen. It is the biggest transshipment center in South China for bulk chemical fertilizers, the volume of which accounts for about 10% of the country's relevant imports. ([4], p.53)

➢ Competition from Taiwan

According to a recent PMB report, it is expected that Hong Kong maintains most of its trans-shipment traffic until Taiwan-Mainland shipping links are fully liberalized, at which point some 50% of traffic will be routed via other ports. There will be little immediate change in trading relations between Mainland China and Taiwan, with liberalization of shipping and trade links occurring around 2002-04. By 2010, it is assumed 100% of North/Central China-Taiwan cargo which was routed via Hong Kong will ship via Mainland China ports. 65% of south China-Taiwan cargo which might previously have routed via Hong Kong is also assumed to switch to other Chinese ports by 2010. ([3], p.7;[7])

Kaohsiung then will present a more serious challenge to Hong Kong than do nearby

Mainland ports, since it is not only closer to Hong Kong but also more developed than ports in China as a world-class container port. ([25])

Assuming Taiwan's WTO accession will closely follow that of the Mainland (possibly in 2002), ([43]) the prospect of cross-strait trade normalization also means that Hong Kong's intermediary function will diminish significantly. ([25])

➢ Possible decline in use of Mid-stream operations

Experts in the industry predict that an increasing amount of the container throughput will come from intra-Asia cargoes carried by relatively small ships. If this is true, the demand for terminal services will decline, whereas the demand for mid-stream services will continue to grow. However, if larger vessels are adopted when the volume of intra-Asia trade expands, the relative demand for mid-stream services might decline, unless mid-stream operators can handle increasingly larger ships. ([25])

➢ Possible decline in use of River Trade

River trade is expected to continue its upward trend, however, its share of cross-boundary traffic is anticipated to reach equilibrium around 2006. Its share of traffic is likely to eventually decline because of new road infrastructure and more competitive trucking markets. ([3], p.17)

➢ Competition from China airport

Many people discuss the use of either Xian or Chengdu as a hub in China to cater for air traffic between Asia and Europe. Although both of them are at present not of the similar scale of Chek Lap Kok, the completion of the massive airport development and expansion programmes, which involve a total of not fewer than a hundred airports in the country, may make us re-examine our future development strategies. ([7])

➢ Competition from Taipei airport

Hitherto, the major competitor of air industry for Hong Kong in the medium/long term is suggested to be Taipei, which offers an immense local market, liberal air service agreements, and available airport capacity, although it does not have Hong Kong's convenient ground access to the Mainland. ([7], p.85)

A.6.4 Future Opportunities

➭ Freight Forwarding:

Shanghai and Guangdong seems are ideal places for setting up a logistics business, since the regions are well-developed, well-located and well-equipped in terms of business infrastructure and operational networks, and have a well-educated labor force. However, the registered capital[11] need to be around US$12 million and it is difficult to find qualified partner due to keen competition there. ([33], p.55)

On the other hand, inland provinces are good candidates to be considered. Economic policy in the Ninth and the Tenth Five-year Plan put focus on developing them. As a result, these second-tier areas will have increasing throughput of cargo and provide greater choice of appropriate partners and easier license approval procedures. ([33],

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[11] "Registered capital" refers to the investor's legal liability to the society measured by its investment amount. Website: http://cntw2000.com/Eng/qa/index.asp?Item=contents&num=83

p.56)

Hong Kong freight forwarders already have a strong client base in China, particularly the large pool of Hong Kong manufacturers. ([32]) Once China's domestic distribution market opened, Hong Kong operators should consider providing domestic, plus international freight forwarding services, which include more customized and sophisticated services like warehousing and distribution, trucking and consolidation, etc. It is one of the ways to compete with China firms, as Hong Kong's operating costs is relatively high. ([33])

At one side, Hong Kong companies need to compete with foreign competitors, multinational firms. Those firms have the advantage of owning an international transshipment network, advanced IT and management systems, but Hong Kong firms have international exposure and experienced personnel, who have also accumulated market knowledge and investment in China. ([33])

In short, as direct links that will be established between China and the outside world diminish the intermediate role of Hong Kong, Hong Kong companies need to find niche markets and innovative, value-added and integrating components of the supply chain, in order to survive in the rigorous competition of freight forwarding and logistics market in China. It is likely that some of the new foreign investors will form partnership with Hong Kong companies to access the benefits of their extensive experience. ([33])

- Shipping

To serve and support the need of Southern, Central and Western China's economic growth, container terminal operators are encouraged to step up liaison and coordination with river trade terminals in the PRD area, and integrate with Yantian, Shekou and Chiwan into a mutually supportive port system. ([25])

Today cooperation among rival container terminal operators in the same port is a trend. Moreover, there is an increase in operational and ownership integration among ports and port operators. We foresee that more Hong Kong's terminal operators will form shipping alliances among themselves or beyond the territorial boundary, with ports in PRD. ([25])

↬ Air

The industry in Asia-Pacific has been developing in the direction of code sharing, global alliances and global multilateral agreements. Airports in China and Hong Kong especially need to cooperate to avoid repetition in services and in resources allocation. ([23])

Recently, Five airports: Guangzhou Baiyun Airport, Shenzhen Airport, Zhuhai Airport, Macao Airport and Hong Kong Airport, have preliminary concluded two multilateral cooperation agreement. First, when the weather is bad, Hong Kong can transfer passengers to other airports. Second, the five airports will promote themselves as a whole. ([50])

While the advantage of Hong Kong Airport is having the good international network and frequent international freight schedule, the advantage of airports in the PRD is having the good domestic network and frequent domestic freight schedule. Both sides can cooperate can bring out some synergy. ([48])

↬ Railway

Because of the continuous open policy of China, the Ministry of Railways will decentralize the management system and begin to seek private investment and cooperation. We expect many opportunities will be offered for foreign companies. ([33])

↬ Help from tourism promotion

We want to point out that as Hong Kong continually develops and promotes tourism, which would bring more tourists to Hong Kong, this could in turn benefit the air freight industry, since about more than half of air cargoes are transported by passenger aircraft. This can reduce the use of full freight and more flexible service can be provided. ([48])

↬ Gain of Supply Chain Management (SCM):

Although SCM has become a hot topic in Hong Kong, only a few of manufacturers have implemented it. We should realize that if Hong Kong's export community takes a fully proactive supply chain management approach towards speed sourcing and replenishment, income gains in the new practices of sourcing would be achievable. ([23])

A.7 Changing Conditions and Infrastructure Needs

A.7.1 Trade

The globalization of the manufacturing process, formation of economic trading regions, demand for high quality shipping services are affecting cargo throughput of ports globally. ([40], p.259)

It seems that the shipping cargo volume of the world has entered a saturation period ([40], p.261), partly because of a slow-down of trade in the world, and due in particular to a realization that the Asian dynamic economies have shown signs of "maturing". Fortunately, Mainland China's economic growth, with high export growth, has remained strong throughout most of the last 20 years. ([3])

The Guangdong cargo base now accounts for about two-thirds of Hong Kong's ocean cargo. Guangdong economy continues look healthy, moreover, China's WTO accession and the possible rationalization of customs procedures and tariffs could encourage direct trade to South China ports (especially Shenzhen) and expected to boost economic growth by 10%. ([3])

According to the Boeing report ([2]), the growth of air cargo in Intra-Asia is expected to be 8.6%[12] per annum from 1999 to 2019. It has also projected that the express service market will constitute about 9.2% of the international air cargo demand in 1999 to 31% in 2019. ([7])

With China joining the WTO, together with a gradual shift in the composition of China's trade towards high value/low volume goods and demand for "just-in-time"

[12] as measured in revenue tonne-kilometers [RTK]

manufacturing in global trend, it is believed that China's international air freight will continue to grow rapidly. ([32], p.27)

A.7.2 Technology

We believe a mature logistics service can be provided only if there is an adequate related IT infrastructure support. ([54]) As we promote efficiency in logistics, we need to promote the effective use of IT.

There is much available commercial software for accomplishing the tasks within a supply chain, such as Warehouse Management System (WMS) for inventory control, Transportation Management System (TMS) for delivery control and Enterprise Resource Planning System (ERP) for resource allocation.

A.7.3 Investment

According to the fifth annual Policy Address of HKSAR Chief Executive, Hong Kong would invest up to $2 billion to construct a new exhibition center at Chek Lap Kok in conjunction with the Airport Authority. The government and the two railway corporations would invest $600 billion over the next 15 years in projects such as railways and roads and on improving links with the PRD. These plans included an express railway service that would cut traveling time between Hong Kong and Guangzhou to one hour. ([52])

A report, entitled "Economic Impacts of Logistics Centre Development in Hong Kong" (2000) by Hong Kong Lingnan University, pointed out that Hong Kong would need to invest HK$4.7 billion (US$609 million) per year over five years just to catch up with logistics hubs such as Singapore. ([21])

We expect there would be more investment put on logistics infrastructure and technology by both Hong Kong government and logistics related companies in the near future.

A.7.4 Human Resources

Logistics concerns the integration of services. The kind of people that the industry wants are those who have wide knowledge about different functions within a supply chain and understand the concept of coordination. This practice is new for Hong Kong and there is much need to promote the education and professional development of logistics.

Recently, several universities in Hong Kong have just started up degree and diploma programmes in logistics to provide the needed training. Some of them are listed in Table 1. ([72])

Table 1: Academic institutions providing logistics study

Academic Institutions	Course Title
The Hong Kong Baptist University	Professional Diploma in Logistics Management
The University of Hong Kong	Professional Certificate in Logistics Studies
The Hong Kong University of Science and Technology	Diploma in Logistics Management; Master of Technology Management in Global Logistics Management
The Hong Kong Polytechnic University	Bachelor of Arts (Hons) in Logistics and Supply Chain Management
City University of Hong Kong	Continuing Education Diploma in Shipping and Logistics Management
Lingnan Institute of Further Education	Professional Diploma in Logistics Management

A.7.5 Government and Regulation

In the 2001 Policy Address ([16]), the Chief Executive has stated that,

> "With Hong Kong's excellent transportation facilities and the PRD's high productivity, together we can develop into a logistics hub to link the Mainland with the world. We can promote the development of an inter-modal system and consider other supporting facilities to speed up the flow of goods and information. The provision of integrated services will also strengthen Hong Kong's competitive advantage as a supply-chain base."

It shows that the Hong Kong government realizes the importance of the logistics industry to Hong Kong and the government's firm and resolute attitude on promoting the industry and its sub-sectors. So, the Economic Services Bureau has set up its Policy Objectives:

- to attract the flow of cargoes through the port and airport of Hong Kong

([6][24])

- to facilitate the development of our infrastructure to enable Hong Kong to take advantage of opportunities in "virtual logistics" ([6][24])
- to facilitate Hong Kong companies in providing logistics services under the demand and supply chain that operates through or from Hong Kong. ([6][24])
- to develop our airport and port and promote Hong Kong as a major international and regional transportation and logistics hub. ([13])

A committee on Logistics Services Development (CLSD) was set up under the Port and Maritime Board (PMB) in May 2000 to consider how the various segments in the supply chain can work more closely together to promote the development of logistics services of Hong Kong. ([24])

Later in December 2000, PMB commissioned the McClier Corporation to carry out a "Study to Strength Hong Kong's Role as the Preferred International and Regional Transportation and Logistics Hub". ([24]) Please refer to [26] for its executive summary and the initial three-year phase is shown in Table 2. This is a very important report that gives guidelines to Hong Kong government to purse the targets mentioned above.

Table 2: Three-year phase in "Study to Strength Hong Kong's Role as the Preferred International and Regional Transportation and Logistics Hub

Target Projects (Three-year phase)
i) The development of an Integrated Operational Plan
ii) The establishment of an Intra-Asian Integrator Hub
iii) The creation of Value Added Logistics Parks

iv) The establishment of a PRD Road – Fast Track Pipeline
v) The establishment of a PRD High Speed Boat – Fast Track Pipelime
vi) The establishment of an Inland Logistics Rail Pipeline

Practically, the government has set up cornerstones and organizational support of "Logistics Hong Kong". See Table 3. Several organizations have been set up for support the development and promotion of Hong Kong logistics industry. See Table 4. 2 'C's and 4 'L's are the cornerstones for "Logistics Hong Kong" and strengthening of them is the focus work in 2002. ([24]) The LOGSCOUNCIL has agreed to set up the five project groups, P-, E-, H-, M-, S-logistics Project Groups. More detail of their tasks can refer to [44].

Table 3: Organizations arrangement

Establish Date	Name	Function	Remark
2001 ([19])	Steering Committee on Logistics Development (LOGSCOM)	- (Policy Steer) provide the policy steer and accelerate measures to take forward "Logistics Hong Kong"	▪ it is chaired by the Financial Secretary
11 Dec 2001 ([19] [42][47])	Hong Kong Logistics Development Council (LOGSCOUNCIL)	- (Facilitation) implement directives from the LOGSCOM and provide a forum for the public and private sector stakeholders to discuss and co-ordinate matters concerning the industry and to carry out joint projects	▪ it is chaired by the Secretary for Economic Services
2001 ([19])	Port, Maritime and Logistics Development Unit	- (Administrative Support) support the development of policy initiatives and coordinate and integrate actions and programmes emanating from LOGSCOM and LOGSCOUNCIL	▪ it is restructured from the PMB division in ESB

Table 4: 2 'C's and 4 'L's [24][44]

2 'C's	
Connectivity	***Collaboration***
- strengthening of connectivity between the different modes of transport links between Hong Kong and our cargo sources	- facilitation of collaboration amongst the players in the chain under the four pillars of "Logistics Hong Kong" (4 'L's)
P-logistics – strengthen the infrastructure and inter-modal transportation links under the demand and supply chain that operates through Hong Kong.	E-logistics – enhance collaboration and develop the IT links to better integrate and enhance the efficiency of service providers in the demand and supply chain that operates through or from Hong Kong
	H-logistics – collaborate with stakeholders for a more integrated short term and long term manpower development programme to support "Logistics Hong Kong"
Regulatory infrastructure – improve and expedite customs and immigration clearance	M-logistics – develop marketing synergies amongst the promotional programmes of the AAHK, HKPMB, HKTDC[13] and Invest hong Kong to create a more effective promotional campaign for "Logistics Hong Kong"
	S-logistics – review how assistance to SMEs can be strengthened so as to encourage them to embrace the concepts and practices of 4 'L's
4 'L's	
physical, electronic, human resources, marketing logistics	

A.8 Recommendations

✦ Shipping

In order to maintain Hong Kong as an important hub in sea cargo handling, it is suggested to lower the terminal tariffs by ([25]):

[13] Hong Kong Trade Development Council

- increasing competition in Hong Kong's container handling industry by adding independent operators, or
- setting up legislation against unfair and anti-competitive practices

Moreover, Hong Kong needs to implement better infrastructure, longer boundary crossing hours and simpler customs procedures, as well as to improve treatment of empty container and do more external promotion. ([32], p.26)

As charges in Hong Kong are always higher than those in Mainland, but Hong Kong can render more complete and reliable related functions and supporting services, it suggests that much of the administrative, commercial and financial activity associated to the movement of container cargoes could remain in Hong Kong even though the actual loading, unloading, staking, consolidation and warehousing of containers are done more cost-effectively in the other ports. ([25])

✦ Air

Guangdong Province is expanding to become one of the world's premier manufacturing districts, so, Hong Kong's policy must give air cargo providers enough flexibility for expansion and the improvement of services. Various modes of transport should be established as efficient direct travel links to induce more direct transfer of cargoes between places in PRD and HKIA. ([7])

Regarding to the express center, it is suggested to run express services by hub-and-spoke systems. The advantages are ([7], p.73;[22], p.20):

- By consolidating freight en route to many destinations onto a single plane, it enables airlines to achieve an economy of density by using larger, more efficient aircraft.

- It can improve utilization of aircraft capacity through more predictable traffic flows.

✦ Railway

For future development, it is proposed the need for close coordination between Shenzhen and Hong Kong authorities. Since, for example, units for statistics of railway cargo throughput are different in Shenzhen subway plan and Hong Kong West Rail planning, i.e. Hong Kong uses TEU, while Mainland uses tonnage or full truckload. The different views can cause difficulties in forecasting and planning. ([8], p.3)

✦ Road

In order to enhance the efficiency of land traffic links between Hong Kong and cities in the PRD Region, it may be necessary to establish an expressway extension network linking Hong Kong's highways directly with expressways from various PRD cities. ([32])

Besides, HKSAR government and Chinese authorities will have to coordinate to improve the efficiency of customs clearing at the border and lower the costs of cargo movement between Hong Kong and the Mainland. ([8][32])

✦ Boost cargo traffic in Hong Kong

Many people suggest that Hong Kong government should streamline custom regulations, simplify re-export procedures, reduce aircraft landing fees and provide seamless integrated transportation services ([28])

✦ Provision of value-added services

Since the more value we add, the more we will be able to develop opportunities for our logistics sector. This can help to better serve customers and create more job opportunities. So, we should try to perform more and different value-added services. ([6])

A.9 Conclusions

To summarize, Hong Kong has strategic geographical location and outstanding facilities. It has higher port productivity and frequency of callings (over 440 per week). It provides speedy and reliable delivery. Hong Kong also enjoys developed and efficient logistics services allied to straightforward and transparent customs. This is combined with world-class banking and financial institution and experience in international trade practices, securing timely payments and document processing. ([3], p.6)

At the same time, Hong Kong has the problems of high shipping cost, congested roads, and lack of 3PL and integrated services, logistics expertise and IT common platform.

Regarding to the capacities of ports in China, we want to point out that unless the ports can attract enough cargoes to feed them, an increase in their physical capacities does not necessarily imply a one-to-one reduction in Hong Kong's required facilities. ([25])

We believe that although there is erosion of Hong Kong's dominance and the other regional ports' increasing competitiveness, this does not mean an absolute decline in cargo throughput in Hong Kong. Hong Kong will still be a hub port but it will handle a diminishing share of the region's total port cargoes. ([25])

A.10 Further Work

Since for this study, time is limited, we can only utilise second-hand or published data, which are not the most updated information. We suggest an in-depth survey and interview with the real players should be conducted in order to get the primary industry data.

Moreover, in the process of collecting data, we find that there are very limited sources of statistics for logistics industry. They only provide statistics about the related sub-sectors. We hope the survey can give us more figures about the logistics industry, so that we can do analysis on more accurate data.

This study hasn't covered all of the sub-sectors in a logistics industry in detail, such as warehousing, 3PL and e-logistics software firms. It is worthwhile to explore them also in order to compile a complete industry report and make thorough analysis.

At last, since China has entered WTO recently, further study may investigate the real settlement and execution of the logistics-related WTO agreements in China. We can also study the transition of China and Hong Kong's logistics industries.

PART B:

Inventory Management with Advance Ordering

Chapter B.1

Introduction

B.1.1 Overview

Recent advances in information technology have enabled companies to obtain more advanced demand/order information that was not possible to acquire before. Substantial research has been devoted to effectively utilizing and quantifying the benefits of advance ordering for suppliers (Hariharan and Zipkin [29]; Gallego and Ozer [23]; Gilbert and Ballou [24]).

However, little has been reported on how the supplier in the upper stage of a supply chain can induce the firm in the lower stage to place advance orders. Since the company will reduce its flexibility by placing advance orders, it is important to have proper incentive schemes so that it will trade off the loss in flexibility with the cost savings.

Figure 1: A Supply Chain

Consider a simple supply chain depicted in Figure 1. In this supply chain, the retailer is able to receive advance orders from her customers. She is able to do so, perhaps, because some of her customers have regular planned activities such as scheduled maintenance that consume a specific quantity of certain items, or on-going projects such as construction that need certain items at pre-determined timeframe. Among the total demand of the retailer in a period, a part of it was known and became firm order K periods ago, some $(K - 1)$ periods ago... some 1 period ago, and the rest just arrives during the period. Previous research effort has focused on effectively using and quantifying the benefits of this information for the retailer (as compared with the scenario without the advance order information). Several studies report that the retailer can save up to 13% in inventory-related costs when comparing the cases with/without advance ordering information (e.g., Ozer [43]).

Typically the two players, the manufacturer and retailer, in the above supply chain operate independently (or in a decentralized setting). Firstly, we would like to ask if the manufacturer can also benefit from advance orders from the retailer. In addition, we have another question: how should the manufacturer design financial incentives so that the retailer places advance orders to the manufacturer? One possibility is for the manufacturer to provide a price discount scheme for advance ordering. For instance, when an order is placed for an immediately delivery, he offers a per-unit price c, which is "given" in the market, while an order for a delivery of one period later, he offers a discount; furthermore, the longer the delivery time, the deeper the discount.

For the first question above, we would expect the manufacturer can get benefit from the retailer placing advance orders, as the situation is very similar to the retailer benefiting from advance information from her customers. In this paper, we put our focus on the second question: how does the manufacturer design an incentive scheme to attract the retailer placing advance orders. We will study the effects of a particular kind of incentive scheme: a price discount scheme. In particular, we examine the behavior of the retailer under such a scheme.

In the following chapters, we investigate the optimal ordering policy for a retailer if a price discount scheme is given. We study the different alternative ways that the retailer may respond to the manufacturer, in order to investigate whether such a price discount scheme is attractive to the retailer to place advance orders. Moreover, this study can give the manufacturer insights on how to set a price discount scheme.

We define 'window size' to be the maximum number of time periods (days) that an order can be placed in advance.[1] For example, if the window size is 1, the retailer can place an order today for delivery tomorrow, i.e. 1-day in advance. Note that window size 0 means the ordering and delivery occur in the same day only.

The remainder of Part B in the thesis is organized as follows. In Chapter B.2, we introduce the notation and general model of advance ordering with window size K. In Chapter B.3, we present a proof of the optimal policy and structural results for ordering with window size 0. In Chapter B.4, we describe the analysis on finding optimal policy for advance ordering with window size 1. In Chapter B.5 and Chapter B.6, we report on our simulations studies comparing several heuristic ordering policies for the retailer as well as the near optimal policy derived from Chapter B.4. Chapter B.5 addresses the case of window size 1 and Chapter B.6 the case of window size K respectively. These simulation studies let us gain insights into the problem structure and the magnitude of the cost-benefit tradeoff. In Chapter 7, we draw some conclusions and point out possible future research directions.

[1] It is called 'demand leadtime' in Hariharan and Zipkin (1995)

B.1.2 Literature Review

Advance demand information (ADI) has aroused much interest recently. It is because the developments in advanced enterprise information systems and the Internet have enabled information to be readily accessible to multiple members of a supply chain. These facilitated much more effective supply chain management, as pointed out by Lee [36].

With ADI, we can update observed demand to have better forecasting, as suggested by Ozer [43]. Xie and Shugan [62] say ADI can give sellers more control over advance selling by decreasing arbitrage.

We can investigate the value of information through studying the effects of ADI in a supply chain. Ozer [43] and Chen [13] find that the advance information can lower the inventory cost, in other words, it can substitute inventory. Gilbert and Ballou [24] agree with this and state that the earlier the order placed, the lower the raw material inventory carried by a supplier and the lower the cost of excess capacity. Moreover, according to Harihanran and Zipkin [29], the effect of a demand leadtime[1] on overall system performance is precisely the same as a corresponding reduction in the supply leadtime. Xie and Shugan [62] find that the profit from advance selling to early orders can be up to twice the profit from only spot selling.

Concerning how to get ADI, Chen [13], Xie and Shugan [62] think that some customers are willing to place an order with delayed delivery because different

---

[1] Demand leadtime has the same meaning as window size. "the time from a customer's order until the due date" Hariharan and Zipkin [16]

customers have different degrees of aversion to order delays. So, as Ozer [43] says, we can have a portfolio of different lead times. Xie and Shugan [62] suggest if we have capacity that would not be available in the immediate period, we can also get ADI. Furthermore, Gallego and Ozer [23] tell us that risk-averse people would provide their demand information in advance if they want to minimize the risk of delivery delays, supply chain partners would provide ADI to each others and the e-commerce of customized products such as personal computers would provide ADI for the product components. Hariharan and Zipkin [29] point out that buyers of custom products typically order in advance of their actual needs, for example, Chinese moon cakes.

However, if the above situations don't exist, an incentive scheme is needed in order to induce the ADI and get the benefits. Price discount scheme is the most popular method since it has been suggested by Gilbert and Ballou [24], Xie and Shugan [62], and Gallego and Ozer [23], who tell us that price sensitive customers would place orders in advance of their needs. Chen [13] suggests another incentive scheme is to provide priority service to customers who order early.

When dealing with models involving ADI, many researchers use concepts of dynamic programming to formulate the total cost function, like the work of Fukuda [22], Xie and Shugan [62] and Ozer [43]; stochastic process to describe the demand process, like the work of Hariharan and Zipkin [29], Chen [37] and Fisher and Raman [21]; updating demand forecast, like the work of Christopher et al [18] [28] and Choi et al [15][16]; and Game Theory, like the work of Donohue [20].

Our work is actually motivated by the work of Gilbert and Ballou [24]. They have proposed a conceptual approach to pricing policy, which needs the estimation of responsiveness of customers to various levels of discounts and lengths of commitment. However, they didn't establish the actual function from this relation. Our work provides some information by examining how a retailer places orders when advance ordering is allowed.

The price discount scheme we discuss gives the retailer a series of discounts and hence decreases purchasing prices as delivery lead time increases. Wang [60] studied ordering policies for systems with decreasing purchasing prices also, but his paper doesn't involve any advance information and ordering. The series of prices are stochastic.

Our work can be seen as similar to the work of Fukuda [22], since both of us present our models in similar dynamic programming and cost setting, although his model is in the context of ordering policies with lead time rather than described as an advance ordering situation. (Essentially Fukuda only addresses the case of window size 1) More discussion would be found in Section B.4.5.

Chapter B.2

Model Formulation

B.2.1 Introduction

We consider a finite time horizon from period 1 to N. A retailer can place definite orders K periods in advance for delivery in period n and she can add new orders until period n. A definite order specifies not only the quantity to buy, but also the period to be shipped by the supplier.

In order to entice the retailer to commit earlier, the supplier (manufacturer) has provided a financial incentive scheme to her. Specifically, if a definite order is placed for immediate delivery, it will be charged a unit price, c, while the charge for delivery one period later is $\beta_1 c$, ..., K periods later, is $\beta_K c$, etc., where $0 < \beta_{i+1} < \beta_i < 1$ are discount factors, $i = 1,\ldots,K-1$. Suppose we take the perspective of a retailer who is given such a price discount scheme and has to decide on her ordering quantities for the current and up to next K periods. We denote the definite order placed in period $(n - i)$ for delivery in period n by y_{n-i}^{n}, $i = 0,\ldots,K$. Demand faced by the retailer for period n, D_n with $n = 1,\ldots,N$, are serially independent (i.e., time-independent) and identically distributed from $J(x)$, $x \geq 0$.

Let us consider the N-period problem without order setup cost. Now it is at the beginning of period n and the retailer reviews her initial inventory x_n. Then she needs to decide on how much to order for the current period n, period (n + 1), up to period (n + K), given that she previously issued some definite orders for these periods. That is, for period n, we have committed orders [$y_{n-K}^{n}, y_{n-K+1}^{n}, \ldots, y_{n-1}^{n}$]; for period ($n$ + 1), we have committed orders [$y_{n-K+1}^{n+1}, y_{n-K+2}^{n+1}, \ldots, y_{n-1}^{n+1}$]; ...; for ($n - K + 1$) period, we have a committed order y_{n-1}^{n+K-1}; for period (n + K), there is no definite order yet. For notational need, let $y_{n-K-i}^{n} = 0$ if $i > 0$. After she has decided on $y_n^n, y_n^{n+1}, \ldots, y_n^{n+K}$, orders for delivery in periods $n, n+1, \ldots, n+K$, then items ordered for the current period n arrive. Then, demand for period n is realized and fulfilled from on-hand stock as much as possible; unfilled demand is backlogged and incurs shortage cost q per unit. Inventory left at the end of the period incurs holding cost h per unit.

Any backorders from period N would not be satisfied.

We let $g_{n-i}(n+j) = \sum_{k=n+j-K}^{n-i} y_k^{n+j}$ for $i = 1, \ldots K - j$ and $j = 0, \ldots K-1$. That is, $g_{n-i}(n+j)$ is the total quantity committed for delivery in period $(n+j)$ by period $(n-i)$. Clearly, only these $g_{n-i}(n+j)$ matter in deciding on $y_n^n, y_n^{n+1}, \ldots, y_n^{n-i+K}$, at the beginning of n.

We have made the following assumptions in this problem:

i) $q > c$, otherwise, it would never be optimal to buy new stock at the last period and possibly in earlier periods.

ii) order setup cost is zero.

iii) lead time is zero.

iv) the supplier has unlimited capacity.

B.2.2 Mathematical Model

The framework of the formulation is a dynamic program (*DP*). (See Figure 1)

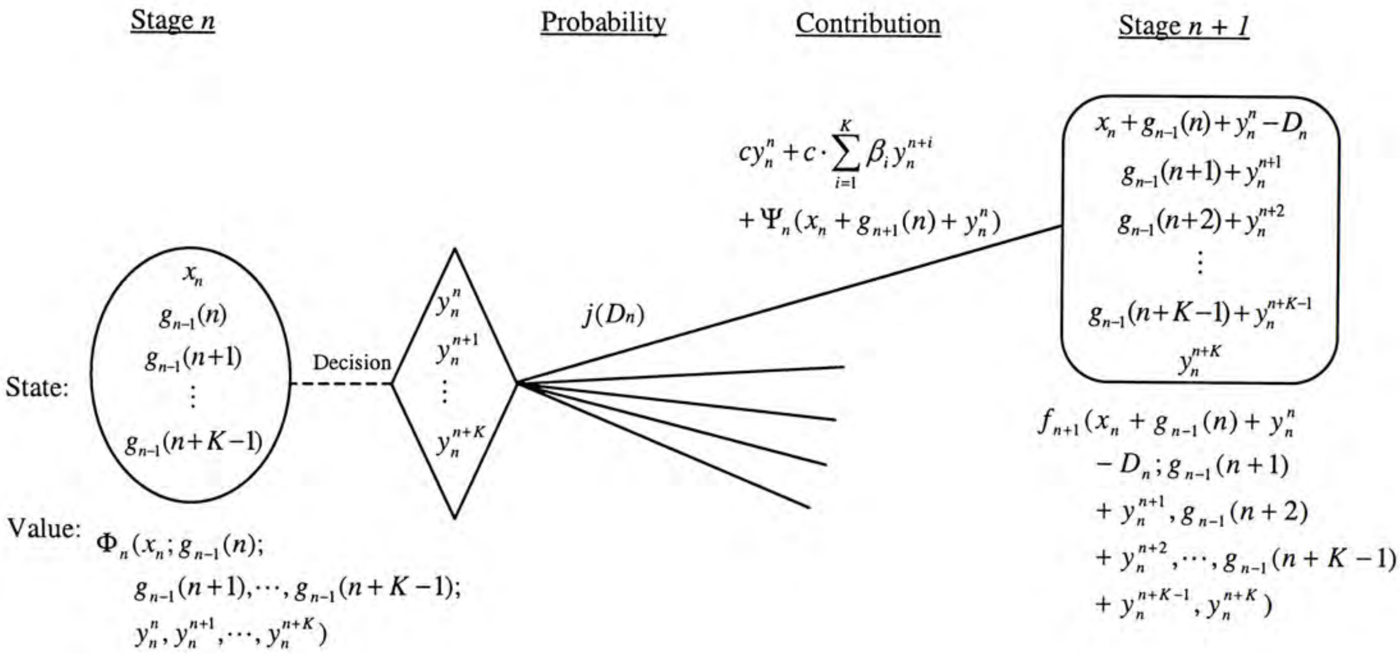

Figure 1: Probabilistic Dynamic Programming

At the beginning of period *n*, we assume inventory level is x_n, cumulative committed order quantity for delivery in period *n* is $g_{n-1}(n)$ and for period $(n+i)$

is $g_{n-1}(n+i)$ for $i=1,\ldots,K-1$, and an order of quantity y_n^n for the current period and y_n^{n+i} for the next *i* period (where $i=1,\ldots,K-1$) will be placed.

The loss cost function gives the expected cost due to holding or shortage of items for period *n*:

$$
\begin{aligned}
&\Psi_n(x_n + g_{n-1}(n) + y_n^n) \\
&= hE[\max(0, x_n + g_{n-1}(n) + y_n^n - D_n)] + qE[\max(0, D_n - x_n - g_{n-1}(n) - y_n^n] \\
&= h\int_0^{x_n+g_{n-1}(n)+y_n^n} (x_n + g_{n-1}(n) + y_n^n - D_n)\, dJ(D_n) \qquad \text{--- (1)} \\
&\quad + q\int_{x_n+g_{n-1}(n)+y_n^n}^{\infty} (D_n - x_n - g_{n-1}(n) - y_n^n)\, dJ(D_n)
\end{aligned}
$$

and the inventory balance equation gives the initial inventory of the next period:

$$x_{n+1} = x_n + g_{n-1}(n) + y_n^n - D_n \qquad \text{--- (2)}$$

where D_n is the customer demand in period *n*, $n=1,\ldots,N$.

If the retailer orders y_n^n for the current period, y_n^{n+1} for the period *n + 1*,…, and y_n^{n+K} for the period (*n + K*), then we have the cost-to-go function, which is the optimal total expected cost from period *n* to the last period *N*:

$$
\begin{aligned}
&f_n(x_n; g_{n-1}(n), g_{n-1}(n+1), \ldots, g_{n-1}(n+K-1)) \\
&= \min_{y_n^n \ge 0, y_n^{n+1} \ge 0, \ldots, y_n^{n+K} \ge 0} \{\Phi_n(x_n; g_{n-1}(n), g_{n-1}(n+1), \ldots, g_{n-1}(n+K-1); y_n^n, y_n^{n+1}, \ldots, y_n^{n+K})\}
\end{aligned}
\qquad \text{--- (3)}
$$

where

$$
\begin{aligned}
&\Phi_n(x_n; g_{n-1}(n), g_{n-1}(n+1), \ldots, g_{n-1}(n+K-1); y_n^n, y_n^{n+1}, \ldots, y_n^{n+K}) \\
&= cy_n^n + c\sum_{i=1}^{K} \beta_i y_n^{n+i} + \Psi_n(x_n + g_{n-1}(n) + y_n^n) \\
&\quad + Ef_{n+1}(x_n + g_{n-1}(n) + y_n^n - D_n; g_{n-1}(n+1) + y_n^{n+1}, \ldots, g_{n-1}(n+K-1) + y_n^{n+K-1}, y_n^{n+K})\}
\end{aligned}
\qquad \text{--- (4)}
$$

with $n = 1,\ldots,N$ and $0 \le K \le N-1$.

B.2.3 Preliminaries

Before we go to any proofs, we need Lemma 1 and Lemma 2 below.

Lemma 1

The loss cost function $\Psi_n(z)$, as defined by (1), is convex in z.

Proof

From Eq.(1), we have

$$\begin{aligned}\Psi_n(z) &= E\left[q\,[\max(0, D_n - z)] + h\,[\max(0, z - D_n)]\right] \\ &= \int_0^{\infty} q\,[\max(0, D_n - z)] + h\,[\max(0, z - D_n)]\; j(D_n)\; dD_n\end{aligned} \qquad \text{--- (5)}$$

where $z \in \Re$.

Let

$$I(z, D_n) = q\,[\max(0, D_n - z)] + h\,[\max(0, z - D_n)]\; j(D_n) \qquad \text{--- (6)}$$

If $f(z)$ and $g(z)$ are convex functions, then so is $h(z) = \max(f(z), g(z))$. Therefore, $[\max(0, D_n - z)]$ and $[\max(0, z - D_n)]$ are convex functions in z for each fixed D_n. Since summation and scalar multiplication of convex functions yields convex functions, for each fixed D_n, we know $I(z, D_n)$ is convex in z.

Moreover, we know the convexity of $I(z, D_n)$ is preserved after taking expectation on D_n and so convexity of $\Psi_n(z)$ is proved.

Lemma 2

The derivative (slope) of the loss cost function $\Psi_n(z)$, as defined by (1)

(i) tends to h, as $z \to \infty$.

(ii) tends to $-q$, as $z \to -\infty$.

Proof

$$\Psi_n(z) = h\int_0^z (z-D)\,dJ(D) + q\int_z^\infty (D-z)\,dJ(D)$$

$$\frac{\partial \Psi_n(z)}{\partial z} = h\int_0^z dJ(D) - q\int_z^\infty dJ(D)$$

$$= (h+q)J(z) - q$$

(i) As $z \to \infty$,

$J(z) \to 1$

Therefore, $\dfrac{d\,\Psi(z)}{d\,z} \to h + q - q = h$

(ii) As $z \to -\infty$,

$J(z) \to 0$

Therefore, $\dfrac{d\,\Psi(z)}{d\,z} \to -q$

B.2.4 Table of Variables

Table 1 below lists all the variables that appear in this and following chapters with their definitions.

Table 1: variable definitions

Variable	Definition
K	window size; the maximum number of days an order can be placed before the delivery day
D_n	customer demand in period n, $n = 1,\ldots,N$, *i.i.d.*, with a common distribution function $J(x)$, $x \geq 0$
β_i	cost discount factor given to placing an order n periods in advance (window size n), where $0 < \beta_{i+1} < \beta_i < 1$, $i = 1,\ldots,N-1$
y_{n-i}^{n}	definite order placed in period $(n - i)$ for delivery in period n, where $i = 0$ (for the current period),$\ldots, K$
$g_{n-i}(n+j)$	the cumulative committed order quantity for delivery in period (n + j) by period $(n - i)$, where $g_{n-i}(n+j) = \sum_{k=n+j-K}^{n-i} y_k^{n+j}$ for $i = 1,\ldots,K-j$ and $j = 0,\ldots,K-1$
c	unit purchasing cost
h	unit holding cost
q	unit shortage cost
x_n	inventory level at the beginning of period n
N	last period, the horizon ranges from 1 to N

Chapter B.3

Study of Window Size 0

B.3.1 Introduction

The case for window size 0 is a special case where the retailer can only place order for delivery for the current period; no advance order is allowed. There are already well-known results for this problem. For completeness, and as foundation for further investigations for non-zero window sizes, we present the proofs of these results. We want to point out that although the results are published before (Hillier [31]), details of the proofs are not given in most textbooks. The framework of our proofs follows Bertsekas [10].

B.3.2 Mathematical Model

The total expected cost for all N periods, which consists of total purchasing, holding and shortage costs, is:

$$E_{D_n, n=1,\ldots,N}\{\sum_{n=1}^{N} cy_n^n + \Psi_n(x_n + y_n^n)\} \qquad \text{--- (1)}$$

We minimize this expected cost by applying the Dynamic Programming (*DP*), framework shown in Section B.2.2. Here we specialize to the case where the window size is zero, i.e. $K = 0$.

Since $K = 0$, the general *DP* recursion Eq.2.2.(3) and Eq.2.2.(4) are much simplified.

Again, at the beginning of period *n*, we assume inventory level is x_n and an order of quantity y_n^n for the current period will be placed. However, as this case has no advance ordering, there is no committed order for delivery in period *n*, i.e., $g_{n-1}(n) = 0$. So, we would dropped the committed order argument, $g_{n-1}(n)$, from the original formulations, as shown below.

Then, the loss cost function, Eq.2.2.(1), is reduced to

$$\Psi_n(x_n + y_n^n)$$
$$= h\int_0^{x_n+y_n^n}(x_n + y_n^n - D_n)\,dJ(D_n) + q\int_{x_n+y_n^n}^{\infty}(D_n - x_n - y_n^n)\,dJ(D_n) \qquad \text{--- (2)}$$

and the inventory balance equation, Eq.2.2.(2) is reduced to

$$x_{n+1} = x_n + y_n^n - D_n \qquad \text{--- (3)}$$

The retailer orders y_n^n for the current period, then we have the cost-to-go function, Eq.2.2.(3) and Eq.2.2.(4), reduced to

$$f_n(x_n) = \min_{y_n^n \geq 0}[cy_n^n + \Psi_n(x_n + y_n^n) + E\{f_{n+1}(x_n + y_n^n - D_n)\}] \qquad \text{--- (4)}$$

where $n = 1,\ldots,N$.

B.3.3 Proof of Window Size 0

B.3.3.1 Proof of the optimal ordering policy

Firstly, let

$$f_{N+1}(x_{N+1}) = 0 \qquad \text{--- (5)}$$

We introduce the variable, $z_n = x_n + y_n^n$, and Eq.(4) becomes

$$f_n(x_n) = \min_{z_n \ge x_n}[cz_n + \Psi_n(z_n) + E\{f_{n+1}(z_n - D_n)\}] - cx_n \qquad \text{--- (6)}$$

Let

$$G_n(z) = cz + \Psi_n(z) + E\{f_{n+1}(z - D_n)\} \qquad \text{--- (7)}$$

cz is a linear, hence convex, function in z. $\Psi_n(z)$ is a convex function in z by Lemma 1. Now, we *assume* f_{n+1} is convex in z. Thus, $E[f_{n+1}(z - D_n)]$ is convex in z. Hence, G_n is convex in z.

Moreover, we *assume* $\lim_{|z|\to\infty} G_n(z) = \infty$, so, G_n has an unconstrained minimum w.r.t. z, denoted by S_n, which is also defined as the order-up-to level at period n. Then in the view of the constraint $z_n \ge x_n$, we can see that the R.H.S. of Eq.(7) is minimized at:

$$\tilde{z}_n = \begin{cases} S_n & \text{if} \quad x_n < S_n \\ x_n & \text{if} \quad x_n \ge S_n \end{cases} \qquad \text{--- (8)}$$

Using the reverse transformation, $y_n = z_n - x_n$, we can see that the minimum in the *DP* Eq.(4) is attained at

$$y_n^n = \begin{cases} S_n - x_n & \text{if} \quad x_n < S_n \\ 0 & \text{if} \quad x_n \ge S_n \end{cases} \qquad \text{--- (9)}$$

Thus, an optimal policy is determined by a sequence of scalars $\{S_1, S_2, \ldots, S_{N-1}, S_N\}$ and has the form

$$\tilde{y}_n^n(x_n) = \begin{cases} S_n - x_n & \text{if} \quad x_n < S_n \\ 0 & \text{if} \quad x_n \geq S_n \end{cases} \qquad \text{--- (10)}$$

(where for each n, the scalar S_n minimizes the function $G_n(z_n)$),

provided that we can show that

(1) the functions G_n are convex.

(2) $\lim_{|z| \to \infty} G_n(z) = \infty$, so that scalars S_n exists.

We proceed to show the above properties inductively, in the course of which we need to prove

(3) the cost-to-go functions f_n are convex.

(4) $\lim_{|z| \to \infty} f_n(z) = \infty$.

☙ Firstly, we prove the optimality of the policy in period N, in which Eq.(7) becomes

$$G_N(z) = cz + \Psi_N(z) \qquad \text{--- (11)}$$

For (1):

We have that f_{N+1} is the zero function, so it is convex. Hence, G_N is convex.

For (2):

By Lemma 2 and $q > c$, we can see that $G_N(z)$ has a derivative (slope) that becomes negative as $z \to -\infty$ and becomes positive as $z \to \infty$:

$\because \quad \lim_{z \to -\infty} \frac{d\,G_N(z)}{d\,z} = c - q < 0 \quad$ and $\quad \lim_{z \to \infty} \frac{d\,G_N(z)}{d\,z} = c + h > 0$

$\therefore \qquad \lim_{|z| \to \infty} G_N(z) = \infty$

For (3):

As shown above, an optimal policy at time N is given by

$$\tilde{y}_N^N(x_N) = \begin{cases} S_N - x_N & \text{if} \quad x_N < S_N \\ 0 & \text{if} \quad x_N \geq S_N \end{cases} \qquad \text{--- (12)}$$

Furthermore, with the *DP* Eq.(5) and Eq.(6), we have

$$f_N(x_N) = \begin{cases} c(S_N - x_N) + \Psi_N(S_N) & \text{if} \quad x_N < S_N \\ \Psi_N(x_N) & \text{if} \quad x_N \geq S_N \end{cases} \qquad \text{--- (13)}$$

which is a convex function because Ψ is convex and S_N minimizes $cz + \Psi_N(z)$. (See Figure 1 below) Thus, given the convexity of f_{N+1}, we were able to prove the convexity of f_N.

For (4):

We can see that $f_N(z)$ has a derivative (slope) that tends to $-c$ as $z \to -\infty$,

$\because \qquad z \to -\infty, \quad z < S_N$

$$\frac{df_N(z)}{dz} = \frac{d[c(S_N - z) + \Psi_N(S_N)]}{dz} = -c$$

and the slope of $f_N(z)$ tends to h as $z \to \infty$.

$\because \qquad z \to \infty, \quad z > S_N$

$$\frac{df_N(z)}{dz} = \frac{d[\Psi_N(z)]}{dz} = h \qquad \text{by Lemma 2}$$

$$\therefore \qquad \lim_{|z| \to \infty} f_N(z) = \infty$$

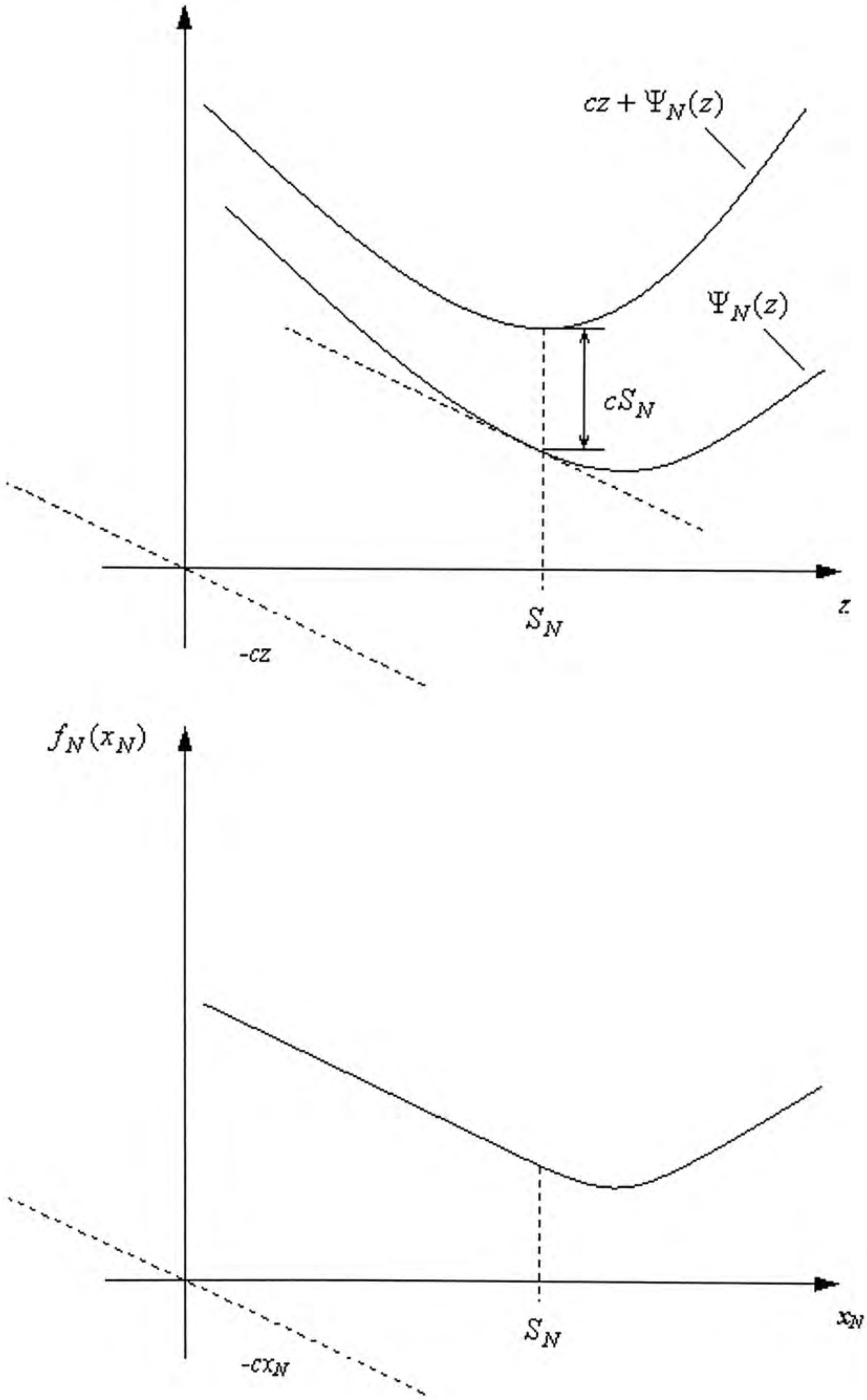

Figure 1: Structure of the cost-to-go function, $\mathbf{f_N(x_N)}$. Note that $y = -\mathbf{cx_N}$ is parallel to the tangent line of $\mathbf{\Psi_N(z)}$ at $z = S_N$, since $\dfrac{\mathbf{d[cz + \Psi_N(z)]}}{\mathbf{dz}} = \mathbf{0}$ at $z = S_N$, which implies $\mathbf{\Psi_N'(S_N) = -c}$.

We have proved that (1) to (4) for period N is true; that is if f_{N+1} is convex, it is true that G_N is convex and $\lim_{|z|\to\infty} G_N(z) = \infty$. Furthermore, f_N is convex and $\lim_{|z|\to\infty} f_N(z) = \infty$.

☙ In general case, we *assume* (1) to (4) holds for period $n \geq k$; that is f_{k+1} is convex and $\lim_{|z|\to\infty} f_{k+1}(z) = \infty$, G_k is convex and $\lim_{|z|\to\infty} G_k(z) = \infty$, f_k is convex and $\lim_{|z|\to\infty} f_k(z) = \infty$.

☙ Now, we examine period (k – 1), in which Eq.(7) becomes

$$G_{k-1}(z) = cz + \Psi_{k-1}(z) + Ef_k(z - D_{k-1}) \qquad \text{--- } (14)$$

For (1):

By the assumption of general case, f_k is convex in z. Hence, by properties of convex functions, $E[f_k(z - D_{k-1})]$ is convex in z. Moreover, cz is a linear function in z and so can be seen as a convex function in z. Also, $\Psi_{k-1}(z)$ is a convex function in z by Lemma 1. Hence, G_{k-1} is convex in z.

For (2):

The derivative of $cz + \Psi_{k-1}(z)$ becomes $c - q < 0$ (by assumption) as $z \to -\infty$ and the derivative of $cz + \Psi_{k-1}(z)$ becomes $c + h > 0$ as $z \to \infty$.

$$\therefore \qquad \lim_{|z|\to\infty} cz + \Psi_{k-1}(z) = \infty$$

Consider the last part of $G_{k-1}(z)$. We *assume* $E[D] = C$, where C is a constant, $0 \leq C < \infty$.

For $z \to \infty$,

$$\lim_{z\to\infty} E[f_k(z - D_{k-1})] \geq \lim_{z\to\infty} f_k[E(z - D_{k-1})]$$
$$(\because \quad \textit{Jensen's Inequality})$$
$$= \lim_{z\to\infty} f_k[z - E(D_{k-1})]$$
$$= \lim_{z\to\infty} f_k[z - C]$$
$$= \infty$$
$$(\because \quad \textit{by inductive hypothesis } (4))$$

so, $\lim_{z\to\infty} E[f_k(z - D_{k-1})] = \infty$.

For $z \to -\infty$,

$$\lim_{z\to-\infty} E[f_k(z - D_{k-1})] \geq \lim_{z\to-\infty} f_k[E(z - D_{k-1})]$$
$$(\because \quad \textit{Jensen's Inequality})$$
$$= \lim_{z\to-\infty} f_k[z - E(D_{k-1})]$$
$$= \lim_{z\to-\infty} f_k[z - C]$$
$$= \infty$$
$$(\because \quad \textit{by inductive hypothesis } (4))$$

so, $\lim_{z\to-\infty} E[f_k(z - D_{k-1})] = \infty$.

$$\therefore \qquad \lim_{|z|\to\infty} E[f_k(z - D_{k-1})] = \infty$$

Then we combine the above results.

$$\lim_{|z|\to\infty} G_{k-1}(z) = \lim_{|z|\to\infty}[cz + \Psi_{k-1}(z) + Ef_k(z - D_{k-1})]$$
$$= \lim_{|z|\to\infty}[cz + \Psi_{k-1}(z)] + \lim_{|z|\to\infty}[Ef_k(z - D_{k-1})]$$
$$= \infty + \infty$$
$$= \infty$$

For (3):

With (1) and (2) for $(K-1)$, we can conclude that $G_{K-1}(z)$ has an unconstrained minimum at S_{K-1}. Hence, the optimal ordering policy at time $(k-1)$ is given by

$$\tilde{y}_{k-1}^{k-1}(x_{k-1}) = \begin{cases} S_{k-1} - x_{k-1} & \textit{if} \quad x_{k-1} < S_{k-1} \\ 0 & \textit{if} \quad x_{k-1} \geq S_{k-1} \end{cases}$$

Furthermore, with reference to the *DP* Eq.(6), we have

$$f_{k-1}(x_{k-1}) = \begin{cases} c(S_{k-1} - x_{k-1}) + \Psi_{k-1}(S_{k-1}) + E\{f_k(S_{k-1} - D_{k-1})\} & \text{if} \quad x_{k-1} < S_{k-1} \\ \Psi_{k-1}(x_{k-1}) + E\{f_k(x_{k-1} - D_{k-1})\} & \text{if} \quad x_{k-1} \geq S_{k-1} \end{cases}$$

which is a convex function because Ψ and $E[f(x-D)]$ are convex and S_{k-1} minimizes $cz + \Psi_{k-1}(z) + E\{f_k(z - D_{k-1})\}$. (See Figure 2 below) Thus, given the convexity of f_k, we were able to prove the convexity of f_{k-1}.

For (4):

We can see that $f_{k-1}(z)$ has a derivative (slope) that tends to $-c$ as $z \to -\infty$,

$\because \qquad z \to -\infty, \quad z < S_{k-1}$

$$\frac{df_{k-1}(x_{k-1})}{dx_{k-1}} = \frac{d[c(S_{k-1} - x_{k-1}) + \Psi_{k-1}(S_{k-1}) + E\{f_k(S_{k-1} - D_{k-1})\}]}{dx_{k-1}} = -c$$

and the value of $f_{k-1}(z)$ approaches ∞ as $z \to \infty$,

$\because \qquad z \to \infty, \quad z > S_{k-1}$

$$\begin{aligned} \lim_{z\to\infty} f_{k-1}(z) &= \lim_{z\to\infty} \Psi_{k-1}(z) + E\{f_k(z - D_{k-1})\} \\ &= \infty + \infty \\ &= \infty \end{aligned}$$

$$\therefore \qquad \lim_{|z|\to\infty} f_{k-1}(z) = \infty$$

We have proved that (1) to (4) also hold for period (k – 1); that is if f_k is convex, it is true that G_{k-1} is convex and $\lim_{|z|\to\infty} G_{k-1}(z) = \infty$. Furthermore, f_{k-1} is convex and $\lim_{|z|\to\infty} f_{k-1}(z) = \infty$.

Thus, we have proved the optimality of the order-up-to policy, described by (10), for the case of window size 0.

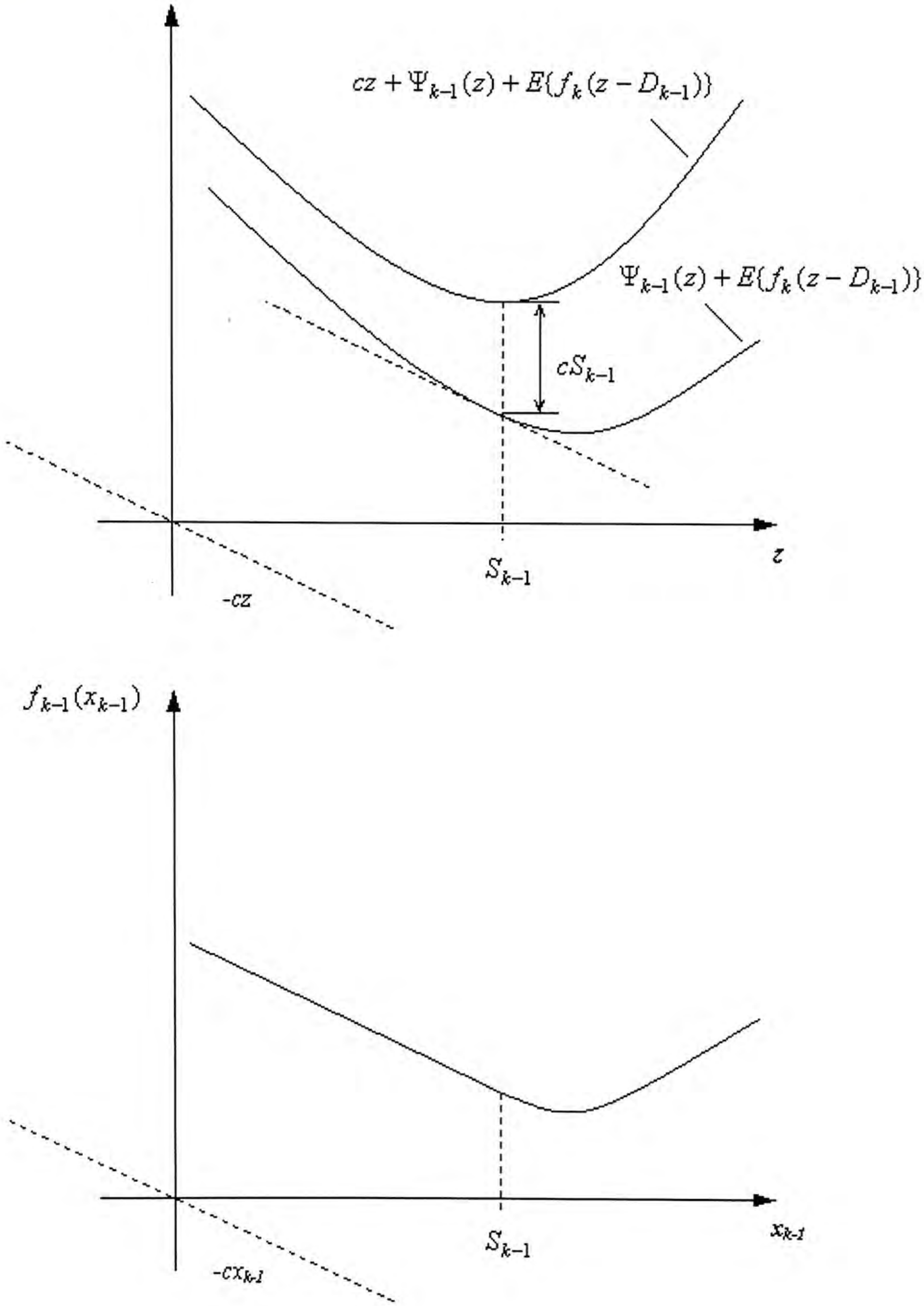

Figure 2: Structure of the cost-to-go function, $\mathbf{f_{k-1}(x_{k-1})}$. Note that $y = -\mathbf{c}\mathbf{x}_{k-1}$ is parallel to the tangent line of $\mathbf{\Psi_{k-1}(S_{k-1}) + E[f_k(S_{k-1} - D_{k-1})]}$ at $z = S_{k-1}$, since $\mathbf{\frac{d[cz + \Psi_{k-1}(z) + E[f_k(z - D_{k-1})]}{dz} = 0}$ at $z = S_{k-1}$, which implies $\mathbf{\Psi_{k-1}'(S_{k-1}) + E[f_k'(S_{k-1} - D_{k-1}) = -c}$.

B.3.3.2 Finding optimal order-up-to level

In the previous subsection, we have proved that the optimal ordering policy is an order-up-to policy. In the following, we would try to find out the optimal order-up-to levels, which are very important ingredients that make the optimal ordering policy implementable.

Analysis for the Last Period

Here, we derive the optimal order-up-to level for the last period *N* of the horizon. We *assume* the period begins with inventory level of x_N and cumulative committed order quantity $g_{N-1}(N)$.

By definition,

$$G_N(z) = cz + h\int_0^z (z-s)\,dJ(s) + q\int_z^\infty (s-z)\,dJ(s) \qquad \text{--- (15)}$$

From the convexity of $G_N(z)$ in z and by the first-order differentiation, we get:

$$\frac{\partial G_N(z)}{\partial z} \qquad \text{--- (16)}$$

by using Leibniz's Formula[1]

$$= h\int_0^z dJ(t) - q\int_z^\infty dJ(t) + c$$
$$= hJ(z) - q[1-J(z)] + c \quad ;$$
$$= (h+q)J(z) - q + c$$

$$\frac{\partial^2 G_N(z)}{\partial z^2} = (h+q)\,j(z) > 0 \qquad \text{--- (17)}$$

[1] see Appendix III

Thus, $G_N(z)$ is minimal when

$$z = J^{-1}(\frac{q-c}{q+h}) \qquad \text{--- (18)}$$

Thus, the order-up-to level at period *N*, is $S_N = J^{-1}(\frac{q-c}{q+h})$.

Since $z = x_N + y_N^N$, the optimal order quantity y_N^N is given by

$$\tilde{y}_N^N = \begin{cases} S_N - x_N & \text{if} \quad x_N < S_N \\ 0 & \text{if} \quad x_N \geq S_N \end{cases} \qquad \text{--- (19)}$$

Analysis for the Second Last Period

To start, given the inventory level x_{N-1}, we have the cost-to-go function for period (*N* - 1) is:

$$f_{N-1}(x_{N-1}) = \min_{y_{N-1}^{N-1} \geq 0} [cy_{N-1}^{N-1} + \Psi_{N-1}(x_{N-1} + y_{N-1}^{N-1}) + E\{f_N(x_{N-1} + y_{N-1}^{N-1} - D_{N-1})\}] \qquad \text{--- (20)}$$

By Eq.(7), we have

$$G_{N-1}(z_{N-1}) = cz_{N-1} + \Psi_{N-1}(z_{N-1}) + E\{f_N(z_{N-1} - D_{N-1})\} \qquad \text{--- (21)}$$

☙ Firstly, consider the loss cost function Ψ_{N-1} in Eq.(21):

[let $t = D_{N-1}$]

$$\Psi_{N-1}(z_{N-1}) = h\int_0^{z_{N-1}} (z_{N-1} - t)\, dJ(t) + q\int_{z_{N-1}}^{\infty} (t - z_{N-1})\, dJ(t) \qquad \text{--- (22)}$$

We differentiate it w.r.t. z_{N-1}:

$$\frac{\partial \Psi_{N-1}(z_{N-1})}{\partial z_{N-1}}$$

by using Leibniz's Formula[2]

$$
\begin{aligned}
&= h\int_0^{z_{N-1}} dJ(t) - q\int_{z_{N-1}}^{\infty} dJ(t) \\
&= h\,J(z_{N-1}) - q[1 - J(z_{N-1})] \\
&= (h+q)\,J(z_{N-1}) - q
\end{aligned}
\qquad \text{--- (23)}
$$

❧ Next, we consider $E[f_N]$ in Eq.(21):

At period *N*, we have shown that (See Eq.(13))

$$
f_N(x_N) = \begin{cases} c(S_N - x_N) + \Psi_N(S_N) & \text{if} \quad x_N < S_N \\ \Psi_N(x_N) & \text{if} \quad x_N \geq S_N \end{cases}
$$

where from Eq.(18),

$$
S_N = J^{-1}\left(\frac{q-c}{q+h}\right) \qquad \text{--- (24)}
$$

By Eq.(3), the initial inventory is,

$$
x_N = x_{N-1} + y_{N-1}^{N-1} - D_{N-1} = z_{N-1} - D_{N-1}
$$

So,

$$
\begin{aligned}
& x_N < S_N \\
\Leftrightarrow\ & z_{N-1} - D_{N-1} < S_N \\
\Leftrightarrow\ & z_{N-1} - S_N < D_{N-1} \\
\Leftrightarrow\ & z_{N-1} - J^{-1}\left(\frac{q-c}{q+h}\right) < D_{N-1}
\end{aligned}
\qquad \text{--- (25)}
$$

that is, $x_N < S_N$ corresponds to the case when $D_{N-1} > z_{N-1} - S_N$, and similarly,

$$
\begin{aligned}
& x_N \geq S_N \\
\Leftrightarrow\ & z_{N-1} - D_{N-1} \geq S_N \\
\Leftrightarrow\ & z_{N-1} - S_N \geq D_{N-1} \\
\Leftrightarrow\ & z_{N-1} - J^{-1}\left(\frac{q-c}{q+h}\right) \geq D_{N-1}
\end{aligned}
\qquad \text{--- (26)}
$$

Therefore,

---

[2] see Appendix III

$$E f_N(x_N) = \int_{z_{N-1}-S_N}^{\infty} f_N(x_N)\, dJ(t) + \int_0^{z_{N-1}-S_N} f_N(x_N)\, dJ(t)$$
$$= \int_{z_{N-1}-S_N}^{\infty} f_N(z_{N-1}-t)\, dJ(t) + \int_0^{z_{N-1}-S_N} f_N(z_{N-1}-t)\, dJ(t)$$
$$= \int_{z_{N-1}-S_N}^{\infty} \{c(S_N - z_{N-1}+t) + \Psi_N(S_N)\} dJ(t) + \int_0^{z_{N-1}-S_N} \Psi_N(z_{N-1}-t)\, dJ(t)$$
$$= \int_{z_{N-1}-J^{-1}(\frac{q-c}{q+h})}^{\infty} \{c(J^{-1}(\frac{q-c}{q+h}) - z_{N-1}+t) + \Psi_N(S_N)\} dJ(t)$$
$$+ \int_0^{z_{N-1}-J^{-1}(\frac{q-c}{q+h})} \Psi_N(z_{N-1}-t)\, dJ(t)$$

--- (27)

We differentiate it w.r.t. z_{N-1}:

$$\frac{\partial Ef_N(z_{N-1}-D_{N-1})}{\partial z_{N-1}} = \int_0^{z_{N-1}-J^{-1}(\frac{q-c}{q+h})} \frac{\partial \Psi_N(z_{N-1}-t)}{\partial z_{N-1}}\, dJ(t) + \int_{z_{N-1}-J^{-1}(\frac{q-c}{q+h})}^{\infty} \{-c\}\, dJ(t)$$

by proof of Lemma 2 and Leibniz's Formula,

$$= \int_0^{z_{N-1}-J^{-1}(\frac{q-c}{q+h})} \{(h+q)J(z_{N-1}-t)\} dJ(t)$$
$$- qJ(z_{N-1} - J^{-1}(\frac{q-c}{q+h})) + \int_{z_{N-1}-J^{-1}(\frac{q-c}{q+h})}^{\infty} \{-c\} dJ(t) \qquad \text{--- (28)}$$
$$= (h+q)\int_0^{z_{N-1}-S_N} J(z_{N-1}-t)\, dJ(t) - qJ(z_{N-1}-S_N) - c\,[1 - J(z_{N-1}-S_N)]$$

☙ Then, we have

$$\frac{\partial G_{N-1}(z_{N-1})}{\partial z_{N-1}} = c + \frac{\partial \Psi_{N-1}(z_{N-1})}{\partial z_{N-1}} + \frac{\partial E f_N(z_{N-1}-D_{N-1})}{\partial z_{N-1}}$$
$$= (h+q)\,J(z_{N-1}) - q + (c-q)\,J(z_{N-1}-D_{N-1}) + (h+q)\int_0^{z_{N-1}-S_N} J(z_{N-1}-t)\, d(t)$$

--- (29)

Thus, by setting Eq.(29) equal to 0, we have $\tilde{z}_{N-1}$ is a solution of the equation

$$(h+q)J(z_{N-1})-q+(c-q)J(z_{N-1}-D_{N-1})+(h+q)\int_0^{z_{N-1}-S_N} J(z_{N-1}-t)\,d(t)=0$$

--- (30)

and $S_{N-1}=\tilde{z}_{N-1}$.

Up to now, we have found the optimal order-up-to levels of period *N* and (implicitly) of period (*N* - 1), denoted S_N and S_{N-1} respectively. For the order-up-to levels of earlier periods, $S_{N-2}, S_{N-3}, \ldots S_2, S_1$, we cannot present their expressions at this moment, since the *DP* becomes very complicated with more stages involved.

Chapter B.4

Study of Window Size 1

B.4.1 Introduction

If the window size is equal to 1, placing orders for the current day and next day is allowed. In Section B.4.3.1, we prove there is an optimal order-up-to policy for 2 periods, that is $N = 2$. In Section B.4.3.2, we calculate the corresponding optimal order-up-to levels.

In another view, our work in this section is actually describing the last 2 periods within a finite horizon. So, we hope the proofs and calculations below may give you insights to find the optimal ordering policy of window size 1 with multiple periods, $N > 2$. Moreover, with this concept, we will use notation period N as the last period and period (N - 1) as the second last period, instead of period 2 and period 1, respectively.

Note that all the formulations before Section B.4.4 are with general demand distribution. In Section B.4.4, we would study a special case of window size 1 – with uniformly distributed demand. As mentioned in literature review (Section B.1.2), our model is similar to that of Fukuda [22]. We would discuss it in more detail and do comparison between both works in Section B.4.5.

B.4.2 Mathematical Model

Again, at the beginning of period *n*, we assume inventory level is x_n, cumulative committed order quantity for delivery in period *n* is $g_{n-1}(n)$, and an order of quantity y_n^n for the current period will be placed, and an order quantity y_n^{n+1} will also be placed for the next period.

For $K=1$, the cost-to-go function reduces to:

$$f_n(x_n; g_{n-1}(n)) = \min_{y_n^n \geq 0, y_n^{n+1} \geq 0} \{\Phi_n(x_n; g_{n-1}(n); y_n^n, y_n^{n+1})\} \quad \text{--- (1)}$$

where

$$\Phi_n(x_n; g_{n-1}(n); y_n^n, y_n^{n+1})$$
$$= cy_n^n + c\beta_1 y_n^{n+1} + \Psi_n(x_n + g_{n-1}(n) + y_n^n) + Ef_{n+1}(x_n + g_{n-1}(n) + y_n^n - D_n; y_n^{n+1})\}$$

--- (2)

Note that $g_{n-1}(n) = y_{n-1}^n$, since only 1-day advance order can be placed in each period, the committed order would just contain one order made the previous day.

B.4.3 Optimal Ordering Policy for Window Size 1

B.4.3.1 Proof of the optimal ordering policy

Firstly, let

$$y_N^{N+1} = 0 \qquad f_{N+1}(x_{N+1}, g_N(N+1)) = 0 \qquad \text{--- (3)}$$

and the recursion equation be

$$\begin{aligned} f_n(x_n, g_{n-1}(n)) = \min_{y_n^n \geq 0, y_n^{n+1} \geq 0} [cy_n^n + c\beta_1 y_n^{n+1} + \Psi_n(x_n + g_{n-1}(n) + y_n^n) \\ + E\{f_{n+1}(x_n + g_{n-1}(n) + y_n^n - D_n, y_n^{n+1})\}] \end{aligned} \qquad \text{--- (4)}$$

where $n = (N-1), N$.

Analysis for the Last Period

This case is similar to the case of window size 0, in Section B.3.2. The committed order, $g_{N-1}(N)$, is included and affects the optimal order-up-to level, S_N and order quantity, $\tilde{y}_N^N$. Figure 1 shows the relationship between them. The results we got are as follows:

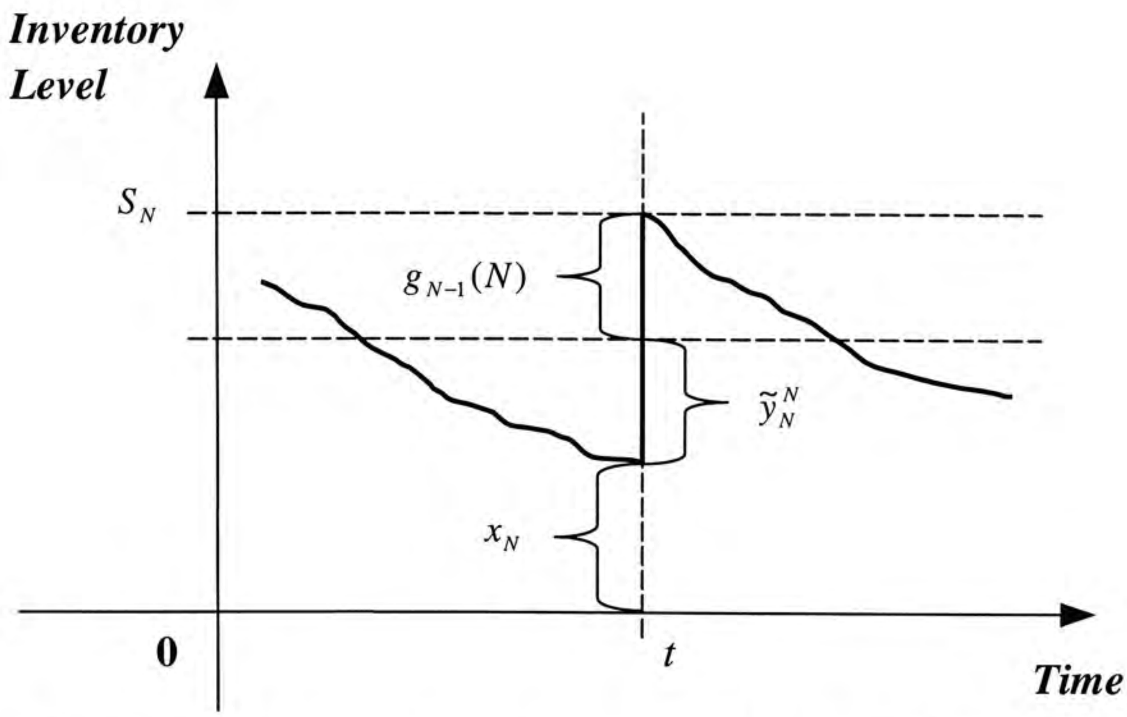

Figure 1: Relationship between optimal inventory level, order-up-to level and order quantity, committed order and initial inventory level.

For period N, we have shown that the order-up-to level S_N is:

$$J^{-1}(\frac{q-c}{q+h}) \qquad \text{--- (5)}$$

Hence, the optimal order quantity is the order-up-to level less the initial inventory and committed orders in period N, i.e.,

$$\tilde{y}_N^N = J^{-1}(\frac{q-c}{q+h}) - x_N - g_{N-1}(N) \qquad \text{--- (6)}$$

The optimal policy for period N is an order-up-to policy, i.e.,

$$\tilde{y}_N^N(x_N, g_{N-1}(N)) = \begin{cases} S_N - x_N - g_{N-1}(N) & \text{if} \quad x_N + g_{N-1}(N) < S_N \\ 0 & \text{if} \quad x_N + g_{N-1}(N) \geq S_N \end{cases} \qquad \text{--- (7)}$$

Furthermore, with reference to the *DP* Eq.(3) and Eq.(4), f_N becomes

$$f_N(x_N, g_{N-1}(N)) = \begin{cases} c(S_N - x_N - g_{N-1}(N)) + \Psi_N(S_N) & \text{if} \quad x_N + g_{N-1}(N) < S_N \\ \Psi_N(x_N + g_{N-1}(N)) & \text{if} \quad x_N + g_{N-1}(N) \geq S_N \end{cases}$$

--- (8)

Analysis for the Last Two Periods

Now, you stay at period (N – 1) and decide orders for periods (N – 1) and N.

To start, given the inventory level x_{N-1} and cumulative committed order quantities $g_{N-2}(N-1)$ for period (N – *1*), and with reference to Eq.(3) and Eq.(4), we have the cost-to-go function for period (N - *1*):

$$\begin{aligned} f_{N-1}(x_{N-1}; g_{N-2}(N-1)) &= \min_{y_{N-1}^{N-1} \geq 0, y_{N-1}^{N} \geq 0} \Phi_{N-1}(x_{N-1}; g_{N-2}(N-1); y_{N-1}^{N-1}, y_{N-1}^{N}) \\ &= \min_{y_{N-1}^{N-1} \geq 0, y_{N-1}^{N} \geq 0} \{ c y_{N-1}^{N-1} + c\beta_1 y_{N-1}^{N} \\ &\quad + \Psi_{N-1}(x_{N-1} + g_{N-2}(N-1) + y_{N-1}^{N-1}) \\ &\quad + E f_N(x_{N-1} + g_{N-2}(N-1) + y_{N-1}^{N-1} - D_{N-1}; y_{N-1}^{N}) \} \end{aligned} \qquad \text{--- (9)}$$

The optimal policy covers two periods will be developed by examining the Hessian matrix of Φ_N:

$$\begin{pmatrix} \dfrac{\partial^2 \Phi_{N-1}}{\partial (y_{N-1}^{N-1})^2} & \dfrac{\partial^2 \Phi_{N-1}}{\partial (y_{N-1}^{N-1}) \partial (y_{N-1}^{N})} \\ \dfrac{\partial^2 \Phi_{N-1}}{\partial (y_{N-1}^{N}) \partial (y_{N-1}^{N-1})} & \dfrac{\partial^2 \Phi_{N-1}}{\partial (y_{N-1}^{N})^2} \end{pmatrix}$$

--- *(10)*

We are going to show that it is positive semi-definite, so Φ_N is a jointly convex function w.r.t. y_{N-1}^{N-1} and y_{N-1}^{N} and thus, we can get the optimal values of them.[1]

☙ Firstly, we have the loss cost function Ψ_{N-1} in Eq.(9):

[let $t = D_{N-1}$]

$$\begin{aligned} &\Psi_{N-1}(x_{N-1} + g_{N-2}(N-1) + y_{N-1}^{N-1}) \\ &= h \int_0^{x_{N-1}+g_{N-2}(N-1)+y_{N-1}^{N-1}} (x_{N-1} + g_{N-2}(N-1) + y_{N-1}^{N-1} - t)\, dJ(t) \\ &\quad + q \int_{x_{N-1}+g_{N-2}(N-1)+y_{N-1}^{N-1}}^{\infty} (t - x_{N-1} - g_{N-2}(N-1) - y_{N-1}^{N-1})\, dJ(t) \end{aligned}$$

--- *(11)*

We differentiate it w.r.t. y_{N-1}^{N-1} and y_{N-1}^{N} respectively:

$$\frac{\partial \Psi_{N-1}(x_{N-1} + g_{N-2}(N-1) + y_{N-1}^{N-1})}{\partial y_{N-1}^{N-1}}$$

by using Leibniz's Formula[2]

$$\begin{aligned} &= h \int_0^{x_{N-1}+g_{N-2}(N-1)+y_{N-1}^{N-1}} dJ(t) - q \int_{x_{N-1}+g_{N-2}(N-1)+y_{N-1}^{N-1}}^{\infty} dJ(t) \\ &= h\, J(x_{N-1} + g_{N-2}(N-1) + y_{N-1}^{N-1}) - q[1 - J(x_{N-1} + g_{N-2}(N-1) + y_{N-1}^{N-1})] \\ &= (h+q)\, J(x_{N-1} + g_{N-2}(N-1) + y_{N-1}^{N-1}) - q \end{aligned}$$

--- *(12)*

[1] see Appendix II
[2] see Appendix III

$$\frac{\partial\, \Psi_{N-1}(x_{N-1} + g_{N-2}(N-1) + y_{N-1}^{N-1})}{\partial\, y_{N-1}^{N}} \qquad \text{--- (13)}$$

$$= 0$$

Next, we consider $E[f_N]$ in Eq.(9):

At period *N*, we have shown that (See Eq.(8))

$$f_N(x_N, g_{N-1}(N)) = \begin{cases} c(S_N - x_N - g_{N-1}(N)) + \Psi_N(S_N) & \text{if} \quad x_N + g_{N-1}(N) < S_N \\ \Psi_N(x_N + g_{N-1}(N)) & \text{if} \quad x_N + g_{N-1}(N) \geq S_N \end{cases}$$

where from Eq.(5),

$$S_N = J^{-1}\left(\frac{q-c}{q+h}\right) \qquad \text{--- (14)}$$

By Eq.2.2.(2), the initial inventory is

$$x_N = x_{N-1} + g_{N-2}(N-1) + y_{N-1}^{N-1} - D_{N-1}$$

So,

$$\begin{aligned} & x_N + g_{N-1}(N) < S_N \\ \Leftrightarrow\ & x_{N-1} + g_{N-2}(N-1) + y_{N-1}^{N-1} - D_{N-1} + g_{N-1}(N) < S_N \\ \Leftrightarrow\ & x_{N-1} + g_{N-2}(N-1) + y_{N-1}^{N-1} + g_{N-1}(N) - S_N < D_{N-1} \\ \Leftrightarrow\ & x_{N-1} + g_{N-2}(N-1) + y_{N-1}^{N-1} + y_{N-1}^{N} - J^{-1}\left(\frac{q-c}{q+h}\right) < D_{N-1} \end{aligned} \qquad \text{--- (15)}$$

and

$$\begin{aligned} & x_N + g_{N-1}(N) \geq S_N \\ \Leftrightarrow\ & x_{N-1} + g_{N-2}(N-1) + y_{N-1}^{N-1} - D_{N-1} + g_{N-1}(N) \geq S_N \\ \Leftrightarrow\ & x_{N-1} + g_{N-2}(N-1) + y_{N-1}^{N-1} + g_{N-1}(N) - S_N \geq D_{N-1} \\ \Leftrightarrow\ & x_{N-1} + g_{N-2}(N-1) + y_{N-1}^{N-1} + y_{N-1}^{N} - J^{-1}\left(\frac{q-c}{q+h}\right) \geq D_{N-1} \end{aligned} \qquad \text{--- (16)}$$

Thus, when $x_N + g_{N-1}(N) < S_N$,

$$\int_{x_{N-1}+g_{N-2}(N-1)+y_{N-1}^{N-1}+g_{N-1}(N)-S_N}^{\infty} f_N(x_N, y_{N-1}^N)\, dJ(t)$$

$$= \int_{x_{N-1}+g_{N-2}(N-1)+y_{N-1}^{N-1}+g_{N-1}(N)-S_N}^{\infty} f_N(x_{N-1} + g_{N-2}(N-1) + y_{N-1}^{N-1} - t, y_{N-1}^N)\, dJ(t)$$

$$= \int_{x_{N-1}+g_{N-2}(N-1)+y_{N-1}^{N-1}+g_{N-1}(N)-S_N}^{\infty} \{c(S_N - x_{N-1} - g_{N-2}(N-1) - y_{N-1}^{N-1} + t - y_{N-1}^N) + \Psi_N(S_N)\} dJ(t) \qquad \text{--- (17)}$$

$$= \int_{x_{N-1}+g_{N-2}(N-1)+y_{N-1}^{N-1}+y_{N-1}^N-J^{-1}(\frac{q-c}{q+h})}^{\infty} \{c(J^{-1}(\frac{q-c}{q+h}) - x_{N-1} - g_{N-2}(N-1) - y_{N-1}^{N-1} + t - y_{N-1}^N) + \Psi_N(S_N)\}\, dJ(t)$$

and when $x_N + g_{N-1}(N) \geq S_N$,

$$\int_0^{x_{N-1}+g_{N-2}(N-1)+y_{N-1}^{N-1}+g_{N-1}(N)-S_N} f_N(x_N, y_{N-1}^N)\, dJ(t)$$

$$= \int_0^{x_{N-1}+g_{N-2}(N-1)+y_{N-1}^{N-1}+g_{N-1}(N)-S_N} f_N(x_{N-1} + g_{N-2}(N-1) + y_{N-1}^{N-1} - t, y_{N-1}^N)\, dJ(t)$$

$$= \int_0^{x_{N-1}+g_{N-2}(N-1)+y_{N-1}^{N-1}+g_{N-1}(N)-S_N} \Psi_N(x_{N-1} + g_{N-2}(N-1) + y_{N-1}^{N-1} - t + y_{N-1}^N)\, dJ(t) \qquad \text{--- (18)}$$

$$= \int_0^{x_{N-1}+g_{N-2}(N-1)+y_{N-1}^{N-1}+y_{N-1}^N-J^{-1}(\frac{q-c}{q+h})} dJ(t) \{h\int_0^{x_{N-1}+g_{N-2}(N-1)+y_{N-1}^{N-1}-t+y_{N-1}^N} (x_{N-1} + g_{N-2}(N-1) + y_{N-1}^{N-1} - t + y_{N-1}^N - s)dJ(s) + q\int_{x_{N-1}+g_{N-2}(N-1)+y_{N-1}^{N-1}-t+y_{N-1}^N}^{\infty} (s - x_{N-1} - g_{N-2}(N-1) - y_{N-1}^{N-1} + t - y_{N-1}^N)dJ(s)\}$$

Therefore, by combining (17) and (18), we have $E[f_N]$ in Eq.(9):

$$Ef_N(x_{N-1} + g_{N-2}(N-1) + y_{N-1}^{N-1} - D_{N-1}; y_{N-1}^N)$$

$$= \int_0^{x_{N-1}+g_{N-2}(N-1)+y_{N-1}^{N-1}+y_{N-1}^N-J^{-1}(\frac{q-c}{q+h})} dJ(t) \{h\int_0^{x_{N-1}+g_{N-2}(N-1)+y_{N-1}^{N-1}-t+y_{N-1}^N} (x_{N-1} + g_{N-2}(N-1) + y_{N-1}^{N-1} - t + y_{N-1}^N - s)dJ(s) + q\int_{x_{N-1}+g_{N-2}(N-1)+y_{N-1}^{N-1}-t+y_{N-1}^N}^{\infty} (s - x_{N-1} - g_{N-2}(N-1) - y_{N-1}^{N-1} + t - y_{N-1}^N)dJ(s)\} \qquad \text{--- (19)}$$

$$+ \int_{x_{N-1}+g_{N-2}(N-1)+y_{N-1}^{N-1}+y_{N-1}^N-J^{-1}(\frac{q-c}{q+h})}^{\infty} \{c(J^{-1}(\frac{q-c}{q+h}) - x_{N-1} - g_{N-2}(N-1) - y_{N-1}^{N-1} + t - y_{N-1}^N) + \Psi_N(S_N)\}\, dJ(t)$$

We differentiate it w.r.t. y_{N-1}^{N-1} and y_{N-1}^N, and find they have the same result:

$$\frac{\partial Ef_N(x_{N-1}+g_{N-2}(N-1)+y_{N-1}^{N-1}-D_{N-1};y_{N-1}^{N})}{\partial y_{N-1}^{N-1}}$$
$$=\frac{\partial Ef_N(x_{N-1}+g_{N-2}(N-1)+y_{N-1}^{N-1}-D_{N-1};y_{N-1}^{N})}{\partial y_{N-1}^{N}}$$

by using Leibniz's Formula,

$$=\int_0^{x_{N-1}+g_{N-2}(N-1)+y_{N-1}^{N-1}+y_{N-1}^{N}-J^{-1}(\frac{q-c}{q+h})} dJ(t)$$
$$\{(h+q)J(x_{N-1}+g_{N-2}(N-1)+y_{N-1}^{N-1}+y_{N-1}^{N}-t)\}$$
$$-qJ(x_{N-1}+g_{N-2}(N-1)+y_{N-1}^{N-1}+y_{N-1}^{N}-J^{-1}(\frac{q-c}{q+h})) \qquad \text{--- (20)}$$
$$+\int_{x_{N-1}+g_{N-2}(N-1)+y_{N-1}^{N-1}+y_{N-1}^{N}-J^{-1}(\frac{q-c}{q+h})}^{\infty}\{-c\}\,dJ(t)$$

Now, with formulae Eq.(12), Eq.(13) and Eq.(20), we are able to calculate the components of the Hessian Matrix (10).

(A) Find $\dfrac{\partial^2\Phi_{N-1}}{\partial(y_{N-1}^{N-1})^2}$

We have the first and second derivatives of $\Phi_{N-1}(x_{N-1};g_{N-2}(N-1);y_{N-1}^{N-1},y_{N-1}^{N})$ in y_{N-1}^{N-1} as follows:

$$\frac{\partial\Phi_{N-1}(x_{N-1};g_{N-2}(N-1);y_{N-1}^{N-1},y_{N-1}^{N})}{\partial y_{N-1}^{N-1}}$$
$$=\frac{\partial c y_{N-1}^{N-1}}{\partial y_{N-1}^{N-1}}+\frac{\partial\Psi_{N-1}}{\partial y_{N-1}^{N-1}}+\frac{\partial E[f_N]}{\partial y_{N-1}^{N-1}}$$
$$=(h+q)J(x_{N-1}+g_{N-2}(N-1)+y_{N-1}^{N-1})-q \qquad \text{--- (21)}$$
$$+(c-q)J(x_{N-1}+g_{N-2}(N-1)+y_{N-1}^{N-1}+y_{N-1}^{N}-J^{-1}(\frac{q-c}{q+h}))$$
$$+(h+q)\int_0^{x_{N-1}+g_{N-2}(N-1)+y_{N-1}^{N-1}+y_{N-1}^{N}-J^{-1}(\frac{q-c}{q+h})} dJ(t)$$
$$\cdot J(x_{N-1}+g_{N-2}(N-1)+y_{N-1}^{N-1}+y_{N-1}^{N}-t)$$

$$\frac{\partial^2 \Phi_{N-1}(x_{N-1}; g_{N-2}(N-1); y_{N-1}^{N-1}, y_{N-1}^{N})}{\partial (y_{N-1}^{N-1})^2}$$
$$= (h+q) j(x_{N-1} + g_{N-2}(N-1) + y_{N-1}^{N-1})$$
$$+ (h+q) \int_0^{x_{N-1}+g_{N-2}(N-1)+y_{N-1}^{N-1}+y_{N-1}^{N}-J^{-1}(\frac{q-c}{q+h})} dJ(t) \qquad \text{--- (22)}$$
$$\cdot j(x_{N-1} + g_{N-2}(N-1) + y_{N-1}^{N-1} + y_{N-1}^{N} - t)$$
$$\geq 0$$

(B) Find $\dfrac{\partial^2 \Phi_{N-1}}{\partial (y_{N-1}^{N})^2}$

Similarly, we have the first and second derivatives of $\Phi_{N-1}(x_{N-1}; g_{N-2}(N-1); y_{N-1}^{N-1}, y_{N-1}^{N})$ in y_{N-1}^{N} as follows:

$$\frac{\partial \Phi_{N-1}(x_{N-1}; g_{N-2}(N-1); y_{N-1}^{N-1}, y_{N-1}^{N})}{\partial y_{N-1}^{N}}$$
$$= \frac{\partial c\beta_1 y_{N-1}^{N}}{\partial y_{N-1}^{N}} + \frac{\partial E[f_N]}{\partial y_{N-1}^{N}}$$
$$= c(\beta_1 - 1) \qquad \text{--- (23)}$$
$$+ (c-q) J(x_{N-1} + g_{N-2}(N-1) + y_{N-1}^{N-1} + y_{N-1}^{N} - J^{-1}(\frac{q-c}{q+h}))$$
$$+ (h+q) \int_0^{x_{N-1}+g_{N-2}(N-1)+y_{N-1}^{N-1}+y_{N-1}^{N}-J^{-1}(\frac{q-c}{q+h})} dJ(t)$$
$$\cdot J(x_{N-1} + g_{N-2}(N-1) + y_{N-1}^{N-1} + y_{N-1}^{N} - t)$$

$$\frac{\partial^2 \Phi_{N-1}(x_{N-1}; g_{N-2}(N-1); y_{N-1}^{N-1}, y_{N-1}^{N})}{\partial (y_{N-1}^{N})^2}$$
$$= (h+q) \int_0^{x_{N-1}+g_{N-2}(N-1)+y_{N-1}^{N-1}+y_{N-1}^{N}-J^{-1}(\frac{q-c}{q+h})} dJ(t) \qquad \text{--- (24)}$$
$$\cdot j(x_{N-1} + g_{N-2}(N-1) + y_{N-1}^{N-1} + y_{N-1}^{N} - t)$$
$$\geq 0$$

(C) Find $\dfrac{\partial^2 \Phi_{N-1}}{\partial (y_{N-1}^{N-1}) \partial (y_{N-1}^{N})}$ and $\dfrac{\partial^2 \Phi_{N-1}}{\partial (y_{N-1}^{N}) \partial (y_{N-1}^{N-1})}$

The anti-diagonal terms of the Hessian are identical, i.e.,

$$\frac{\partial^2\Phi_{N-1}(x_{N-1};g_{N-2}(N-1);y_{N-1}^{N-1},y_{N-1}^{N})}{\partial(y_{N-1}^{N-1})\partial(y_{N-1}^{N})}=\frac{\partial^2\Phi_{N-1}(x_{N-1};g_{N-2}(N-1);y_{N-1}^{N-1},y_{N-1}^{N})}{\partial(y_{N-1}^{N})\partial(y_{N-1}^{N-1})}$$

$$=(h+q)\int_0^{x_{N-1}+g_{N-2}(N-1)+y_{N-1}^{N-1}+y_{N-1}^{N}-J^{-1}(\frac{q-c}{q+h})}dJ(t)$$

$$\cdot\, j(x_{N-1}+g_{N-2}(N-1)+y_{N-1}^{N-1}+y_{N-1}^{N}-t)$$

$$\geq 0$$

--- *(25)*

In order to conduct the convexity test[3], we calculate

$$\left(\frac{\partial^2\Phi_{N-1}(x_{N-1};g_{N-2}(N-1);y_{N-1}^{N-1},y_{N-1}^{N})}{\partial(y_{N-1}^{N-1})^2}\right)\cdot\left(\frac{\partial^2\Phi_{N-1}(x_{N-1};g_{N-2}(N-1);y_{N-1}^{N-1},y_{N-1}^{N})}{\partial(y_{N-1}^{N})^2}\right)$$

$$-\left(\frac{\partial^2\Phi_{N-1}(x_{N-1};g_{N-2}(N-1);y_{N-1}^{N-1},y_{N-1}^{N})}{\partial(y_{N-1}^{N-1})\partial(y_{N-1}^{N})}\right)^2$$

$$=(h+q)^2\, j(x_{N-1}+g_{N-2}(N-1)+y_{N-1}^{N-1})$$

$$\cdot\int_0^{x_{N-1}+g_{N-2}(N-1)+y_{N-1}^{N-1}+y_{N-1}^{N}-J^{-1}(\frac{q-c}{q+h})} j(x_{N-1}+g_{N-2}(N-1)+y_{N-1}^{N-1}+y_{N-1}^{N}-t)\,dJ(t)$$

$$\geq 0$$

--- *(26)*

Finally, from Eq.(22), Eq.(24) and Eq.(26), we know the Hessian Matrix of $\Phi_{N-1}(x_{N-1};g_{N-2}(N-1);y_{N-1}^{N-1},y_{N-1}^{N})$ is positive semi-definite and therefore $\Phi_{N-1}(x_{N-1};g_{N-2}(N-1);y_{N-1}^{N-1},y_{N-1}^{N})$ is jointly convex with y_{N-1}^{N-1} and y_{N-1}^{N}.[4] Thus, there exists $\tilde{y}_{N-1}^{N-1}$ and $\tilde{y}_{N-1}^{N}$ such that $\Phi_{N-1}(x_{N-1};g_{N-2}(N-1);y_{N-1}^{N-1},y_{N-1}^{N})$ attains its minimum.

B.4.3.2 Finding optimal order-up-to level

We try to find the closed form solution for the optimal solution $(\tilde{y}_{N-1}^{N-1},\tilde{y}_{N-1}^{N})$.

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[3] see Appendix II

[4] see Appendix II

Theorem 1

$$\tilde{y}_{N-1}^{N-1} = [S_{N-1} - x_{N-1} - g_{N-2}(N-1)]^{+} \quad \text{--- (27)}$$

If $x_{N-1} + g_{N-2}(N-1) < S_{N-1}$, $\tilde{y}_{N-1}^{N}$ is the solution of equation:

$$c(\beta_1 - 1) + (c-q)J(w + y_{N-1}^{N}) + (h+q)\int_0^{w+y_{N-1}^{N}} J(w + J^{-1}(\frac{q-c}{q+h}) + y_{N-1}^{N} - t)\,dJ(t) = 0$$

--- (28)

Then if $\tilde{y}_{N-1}^{N} < 0$, $\tilde{y}_{N-1}^{N} = 0$ is set and $\tilde{y}_{N-1}^{N-1}$ is changed to be the solution of equation:

$$\begin{aligned}&(h+q)\,J(w' + J^{-1}(\frac{q-c}{q+h}) + y_{N-1}^{N-1}) - q + (c-q)\,J(w' + y_{N-1}^{N-1}) \\ &+ (h+q)\int_0^{w'+y_{N-1}^{N-1}} J(w' + J^{-1}(\frac{q-c}{q+h}) + y_{N-1}^{N-1} - t)\,dJ(t) = 0\end{aligned} \quad \text{--- (29)}$$

Then if $\tilde{y}_{N-1}^{N-1} < 0$, $\tilde{y}_{N-1}^{N} = 0$ is set.

If $x_{N-1} + g_{N-2}(N-1) \geq S_{N-1}$, $\tilde{y}_{N-1}^{N}$ is a solution of the equation:

$$c(\beta_1 - 1) + (c-q)J(w' + y_{N-1}^{N}) + (h+q)\int_0^{w'+y_{N-1}^{N}} J(w' + J^{-1}(\frac{q-c}{q+h}) + y_{N-1}^{N} - t)\,dJ(t) = 0$$

--- (30)

Then if $\tilde{y}_{N-1}^{N} < 0$, $\tilde{y}_{N-1}^{N} = 0$ is set.

Note:

$$S_{N-1} = x_{N-1} + g_{N-2}(N-1) + y_{N-1}^{N-1} = J^{-1}(\frac{q-(1-\beta_1)c}{q+h}) \quad \text{--- (31)}$$

$$w = J^{-1}(\frac{q-(1-\beta_1)c}{q+h}) - J^{-1}(\frac{q-c}{q+h}) \quad \text{--- (32)}$$

and

$$w' = x_{N-1} + g_{N-2}(N-1) - J^{-1}(\frac{q-c}{q+h}) \quad \text{--- (33)}$$

Proof

By setting $\frac{\partial \Phi_{N-1}}{\partial y_{N-1}^{N-1}}$ and $\frac{\partial \Phi_{N-1}}{\partial y_{N-1}^{N}}$ equal to 0 and re-arranging the terms, we have:

From Eq.(21),

$$\frac{\partial \Phi_{N-1}}{\partial y_{N-1}^{N-1}} = 0 \quad \Rightarrow$$

$$\begin{aligned}
&(h+q)J(x_{N-1} + g_{N-2}(N-1) + \tilde{y}_{N-1}^{N-1}) - q = \\
&\quad (q-c)J(x_{N-1} + g_{N-2}(N-1) + \tilde{y}_{N-1}^{N-1} + \tilde{y}_{N-1}^{N} - J^{-1}(\frac{q-c}{q+h})) \\
&\quad - (h+q)\int_0^{x_{N-1}+g_{N-2}(N-1)+\tilde{y}_{N-1}^{N-1}+\tilde{y}_{N-1}^{N}-J^{-1}(\frac{q-c}{q+h})} dJ(t) \\
&\quad \cdot J(x_{N-1} + g_{N-2}(N-1) + \tilde{y}_{N-1}^{N-1} + \tilde{y}_{N-1}^{N} - t)
\end{aligned} \quad \text{--- (34)}$$

Form Eq.(23),

$$\frac{\partial \Phi_{N-1}}{\partial y_{N-1}^{N}} = 0 \quad \Rightarrow$$

$$\begin{aligned}
&c(\beta_1 - 1) = \\
&\quad + (q-c)J(x_{N-1} + g_{N-2}(N-1) + \tilde{y}_{N-1}^{N-1} + \tilde{y}_{N-1}^{N} - J^{-1}(\frac{q-c}{q+h})) \\
&\quad - (h+q)\int_0^{x_{N-1}+g_{N-2}(N-1)+\tilde{y}_{N-1}^{N-1}+\tilde{y}_{N-1}^{N}-J^{-1}(\frac{q-c}{q+h})} dJ(t) \\
&\quad \cdot J(x_{N-1} + g_{N-2}(N-1) + \tilde{y}_{N-1}^{N-1} + \tilde{y}_{N-1}^{N} - t)
\end{aligned} \quad \text{--- (35)}$$

Note that the R.H.S. of Eq.(34) and Eq.(35) are the same. Then we substitute Eq.(35) into Eq.(34) and get

$$\begin{aligned}
&(h+q)J(x_{N-1} + g_{N-2}(N-1) + \tilde{y}_{N-1}^{N-1}) - q + c(1-\beta_1) = 0 \\
&\tilde{y}_{N-1}^{N-1} = J^{-1}(\frac{q-(1-\beta_1)c}{q+h}) - x_{N-1} - g_{N-2}(N-1) \\
&\tilde{y}_{N-1}^{N-1} = S_{N-1} - x_{N-1} - g_{N-2}(N-1)
\end{aligned} \quad \text{--- (36)}$$

where $S_{N-1} = x_{N-1} + g_{N-2}(N-1) + \tilde{y}_{N-1}^{N-1} = J^{-1}(\frac{q-(1-\beta_1)c}{q+h})$

Now, we have the policy for $\tilde{y}_{N-1}^{N-1}$:

$$\tilde{y}_{N-1}^{N-1}(x_{N-1}, g_{N-2}(N-1)) = \begin{cases} S_{N-1} - x_{N-1} - g_{N-2}(N-1) & if \quad x_{N-1} + g_{N-2}(N-1) < S_{N-1} \\ 0 & if \quad x_{N-1} + g_{N-2}(N-1) \geq S_{N-1} \end{cases}$$

--- *(37)*

To find $\tilde{y}_{N-1}^{N}$:

- If $x_{N-1} + g_{N-2}(N-1) < S_{N-1}$, we substitute $\tilde{y}_{N-1}^{N-1} = S_{N-1} - x_{N-1} - g_{N-2}(N-1)$ into Eq.(35) and get:

$$c(\beta_1 - 1) + (c-q)J(x_{N-1} + g_{N-2}(N-1) + (S_{N-1} - x_{N-1} - g_{N-2}(N-1)) + \tilde{y}_{N-1}^{N} - J^{-1}(\frac{q-c}{q+h}))$$
$$+ (h+q)\int_0^{x_{N-1}+g_{N-2}(N-1)+(S_{N-1}-x_{N-1}-g_{N-2}(N-1))+\tilde{y}_{N-1}^{N}-J^{-1}(\frac{q-c}{q+h})} dJ(t)$$
$$\cdot J(x_{N-1} + g_{N-2}(N-1) + (S_{N-1} - x_{N-1} - g_{N-2}(N-1)) + \tilde{y}_{N-1}^{N} - t) = 0$$

or

$$c(\beta_1 - 1) + (c-q)J(w + y_{N-1}^{N}) + (h+q)\int_0^{w+y_{N-1}^{N}} J(w + J^{-1}(\frac{q-c}{q+h}) + y_{N-1}^{N} - t)\, dJ(t) = 0$$

where $w = J^{-1}(\frac{q-(1-\beta_1)c}{q+h}) - J^{-1}(\frac{q-c}{q+h})$

However, if $\tilde{y}_{N-1}^{N} < 0$, we set $\tilde{y}_{N-1}^{N} = 0$ as $\tilde{y}_{N-1}^{N}$ must be non-negative, and we need to substitute it into Eq.(9) to find the corresponding $\tilde{y}_{N-1}^{N-1}$.

To find $\tilde{y}_{N-1}^{N-1}$, for this case, we substitute $\tilde{y}_{N-1}^{N} = 0$ into Eq.(21) and set it equal to zero:

$$\frac{\partial f_{N-1}(x_{N-1}; g_{N-2}(N-1))}{\partial y_{N-1}^{N-1}}$$
$$= (h+q)\, J(x_{N-1} + g_{N-2}(N-1) + y_{N-1}^{N-1}) - q$$
$$+ (c-q)\, J(x_{N-1} + g_{N-2}(N-1) + y_{N-1}^{N-1} - J^{-1}(\frac{q-c}{q+h}))$$
$$+ (h+q)\int_0^{x_{N-1}+g_{N-2}(N-1)+y_{N-1}^{N-1}-J^{-1}(\frac{q-c}{q+h})} J(x_{N-1} + g_{N-2}(N-1) + y_{N-1}^{N-1} - t)\, dJ(t)$$
$$= 0$$

or

$$(h+q)\ J(w'+J^{-1}(\frac{q-c}{q+h})+y_{N-1}^{N-1})-q+(c-q)\ J(w'+y_{N-1}^{N-1})$$

$$+(h+q)\int_0^{w'+y_{N-1}^{N-1}} J(w'+J^{-1}(\frac{q-c}{q+h})+y_{N-1}^{N-1}-t)\ dJ(t)=0$$

where $w'=x_{N-1}+g_{N-2}(N-1)-J^{-1}(\frac{q-c}{q+h})$

However, if $\tilde{y}_{N-1}^{N-1}<0$, we set $\tilde{y}_{N-1}^{N-1}=0$ as $\tilde{y}_{N-1}^{N-1}$ must be non-negative.

☙ If $x_{N-1}+g_{N-2}(N-1)\geq S_{N-1}$, we substitute $\tilde{y}_{N-1}^{N-1}=0$ into Eq.(35) and get:

$$c(\beta_1-1)+(c-q)J(x_{N-1}+g_{N-2}(N-1)+\tilde{y}_{N-1}^{N}-J^{-1}(\frac{q-c}{q+h}))$$

$$+(h+q)\int_0^{x_{N-1}+g_{N-2}(N-1)+\tilde{y}_{N-1}^{N}-J^{-1}(\frac{q-c}{q+h})} dJ(t)$$

$$\cdot J(x_{N-1}+g_{N-2}(N-1)+\tilde{y}_{N-1}^{N}-t)=0$$

or

$$c(\beta_1-1)+(c-q)J(w'+y_{N-1}^{N})+(h+q)\int_0^{w'+y_{N-1}^{N}} J(w'+J^{-1}(\frac{q-c}{q+h})+y_{N-1}^{N}-t)\,dJ(t)=0$$

where $w'=x_{N-1}+g_{N-2}(N-1)-J^{-1}(\frac{q-c}{q+h})$

However, if $\tilde{y}_{N-1}^{N}<0$, we set $\tilde{y}_{N-1}^{N}=0$ as $\tilde{y}_{N-1}^{N}$ must be non-negative.

Theorem 1 is summarized in Figure 2 on p.119.

Remarks:

1) For the case of $x_{N-1}+g_{N-2}(N-1)<S_{N-1}$, at the beginning of period $(N-1)$, the optimal quantity of the order for period N and the optimal quantity of the order for period $(N-1)$ are independent of each other.

- The order quantity for period $(N-1)$ ($\tilde{y}_{N-1}^{N-1}$) depends on the inventory level x_{N-1}, the cumulative committed order quantity $g_{N-2}(N-1)$ at the beginning of period $(N-1)$ and the fixed costs.
- The order quantity for period N ($\tilde{y}_{N-1}^{N}$) only depends on the fixed costs.

2) If $x_{N-1}+g_{N-2}(N-1)<S_{N-1}$, $x_N = J^{-1}(\frac{q-(1-\beta_1)c}{q+h}) - D_{N-1}$ and $g_{N-1}(N)=\tilde{y}_{N-1}^{N}$, which is calculated from Eq.(28). We find that $(x_N, g_{N-1}(N))$ both are independent of the initial inventory level x_{N-1} and the cumulative committed order quantity $g_{N-2}(N-1)$ in period $(N-1)$, i.e. $(x_{N-1}, g_{N-2}(N-1))$. Thus, if $x_{N-1}+g_{N-2}(N-1)<S_{N-1}$, the arguments of f_{N-1} and f_N are independent.

3) The optimal order-up-to level for period $(N-1)$, $J^{-1}(\frac{q-(1-\beta_1)c}{q+h})$, shown in Eq.(31), has a nice Newsboy interpretation if we consider:

(i) $C_u = (q-c)+\beta_1 c$

If we order less than demand, it would incur shortage cost (q), save purchase cost (c), but we 'can' advance order at cost $\beta_1 c$ to fill backorder.

(ii) $C_o = (c+h)-\beta_1 c$

If we order more than demand, it would incur purchase cost (c) and holding cost (h), but since inventory is carried to the next period, we can order one fewer for the next period and thus save $\beta_1 c$.

4) Properties of y_{N-1}^{N} in Eq.(28) and Eq.(30):

The L.H.S. of Eq.(28) and Eq.(30) are continuous monotonic increasing functions with y_{N-1}^{N}, since the slope of y_{N-1}^{N} is increasing in the jointly convex function, Φ_{N-1}. By differentiating the L.H.S. of Eq.(28), w.r.t. y_{N-1}^{N} using Leibniz's formula, we get:

$$(c-q)j(w+y_{N-1}^{N})+(h+q)\int_{0}^{w+y_{N-1}^{N}} j(w+J^{-1}(\frac{q-c}{q+h})+y_{N-1}^{N}-t)\,dJ(t)$$
$$+(h+q)\,J(J^{-1}(\frac{q-c}{q+h}))\,j(w+y_{N-1}^{N})$$
$$=(h+q)\int_{0}^{w+y_{N-1}^{N}} j(w+J^{-1}(\frac{q-c}{q+h})+y_{N-1}^{N}-t)\,dJ(t)\geq 0$$

Differentiating the L.H.S. of Eq.(30), w.r.t. y_{N-1}^{N}, we get:

$$(c-q)\,j(w'+y_{N-1}^{N})+(h+q)\int_{0}^{w'+y_{N-1}^{N}} j(w'+J^{-1}(\frac{q-c}{q+h})+y_{N-1}^{N}-t)\,dJ(t)$$
$$+(h+q)\,J(J^{-1}(\frac{q-c}{q+h}))\,j(w'+y_{N-1}^{N})$$
$$=(h+q)\int_{0}^{w'+y_{N-1}^{N}} j(w'+J^{-1}(\frac{q-c}{q+h})+y_{N-1}^{N}-t)\,dJ(t)\geq 0$$

B.4.4 Special Case of Uniformly Distributed Demand

In the previous sections, we haven't specified any kind of demand distribution. In order to understand more about the calculated results and the application of them in the case of window size 1, we focus on the special case with Uniform Distribution as the demand distribution.

U (0, 1) Demand Distribution

Firstly, we study our problem results with U (0, 1) demand distribution. We get $\tilde{y}_{N-1}^{N}$ by substituting the solution formulae presented in Theorem 1 with U (0,1). Then, Eq.(31) becomes

$$S_{N-1} = \left(\frac{q-(1-\beta_1)c}{q+h} \right) - g_{N-2}(N-1) \qquad \text{--- (38)}$$

Eq.(28) becomes

$$c(\beta_1 - 1) + (c-q)(w + y_{N-1}^{N}) + (h+q)\int_0^{w+y_{N-1}^{N}} (w + (\frac{q-c}{q+h}) + y_{N-1}^{N} - s)\, ds = 0 \qquad \text{--- (39)}$$

where from Eq.(32),

$$w = (\frac{q-(1-\beta_1)c}{q+h}) - (\frac{q-c}{q+h}) = \frac{\beta c}{q+h} \qquad \text{--- (40)}$$

Eq.(30) becomes

$$c(\beta_1 - 1) + (c-q)(w' + y_{N-1}^{N}) + (h+q)\int_0^{w+y_{N-1}^{N}} (w' + (\frac{q-c}{q+h}) + y_{N-1}^{N} - s)\, ds = 0 \qquad \text{--- (41)}$$

where from Eq.(33),

$$w' = x_{N-1} + g_{N-2}(N-1) - (\frac{q-c}{q+h}) \qquad \text{--- (42)}$$

With our by hand calculation and the help of Mathematica, we get the following solutions:

$$\tilde{y}_{N-1}^{N-1} = [S_{N-1} - x_{N-1}]^{+} \qquad \text{--- (43)}$$

where S_{N-1} as Eq.(38).

If $x_{N-1} + g_{N-2}(N-1) < S_{N-1}$, by using Eq.(39) and Eq.(40),

$$\tilde{y}_{N-1}^{N} = \sqrt{\frac{2c(1-\beta_1)}{q+h}} - \frac{c\beta_1}{q+h} \qquad \text{--- (44)}$$

otherwise, if $x_{N-1} + g_{N-2}(N-1) \geq S_{N-1}$, by using Eq.(41) and Eq.(42),

$$\tilde{y}_{N-1}^{N} = \sqrt{\frac{2c(1-\beta_1)}{q+h}} + \frac{q-c}{q+h} - g_{N-2}(N-1) - x_{N-1} \qquad \text{--- (45)}$$

Remark:

$\tilde{y}_{N-1}^{N}$ in Eq.(44) is non-negative if $\sqrt{\frac{2c(1-\beta_1)}{q+h}} \geq \frac{c\beta_1}{q+h}$ or

$$c\beta_1^{\,2} + 2(h+q)\beta_1 - 2(h+q) \leq 0 \qquad \text{--- (46)}$$

Then we differentiate the L.H.S. of Eq.(46) w.r.t. β_1 once and twice, and we get $2c\beta_1 + 2(h+q)$ and $2c$, respectively.

Since the second derivative is positive, we conclude that the L.H.S. of Eq.(46) is strictly convex in β_1 and $\tilde{y}_{N-1}^{N}$ in Eq.(44) is non-negative if

$$0 \geq \beta_1 \geq -\frac{h+q+\sqrt{h+q}\sqrt{2c+h+q}}{c} \qquad \text{--- (47)}$$

and

$$0 \leq \beta_1 \leq -\frac{h+q-\sqrt{h+q}\sqrt{2c+h+q}}{c} \qquad \text{--- (48)}$$

The case in (47) is rejected, since by definition, $\beta_1 \geq 0$.

We are interested in Eq.(48) as we want to know how large the β_1 we set will prohibit a retailer to place an advance order. We try to put real numbers from Table 5.3.(1) in it and we get β_1 need to be no greater than 0.854. Someone may ask why it is not near to 1 as if a discount is given, no matter how much, placing advance order can benefit from purchasing cost saving. We suggest this may due to the variance of the demand distribution and the costs trade-off involved. Such kind of discussion will be presented in detail in the simulation study, Chapter B.5.

U (a, b) and U (0, b) Demand Distribution

We also study our problem results with U (a, b) and U (0, b) demand distributions. All the results are summarized as below:

U (a, b) Demand Distribution

$\tilde{y}_{N-1}^{N-1} = [S_{N-1} - x_{N-1}]^+$

where

$$S_{N-1} = (b-a)\left(\frac{q-(1-\beta)\,c}{q+h}\right) + a - g_{N-2}(N-1) \qquad \text{--- (49)}$$

If $\tilde{y}_{N-1}^{N-1} = 0$,

$$\tilde{y}_{N-1}^{N} = \frac{1}{q+h}\left(\begin{array}{l}(1+a-b)\,q + c\left(-1+b+a\,(-1+\beta)-b\beta\right) - \\ \sqrt{\begin{array}{l}(1+a-b)^2 c^2 + q\left(a^2 q + (-1+b)^2 q - 2a\,(h+bq)\right) - 2c\,(b^2 q - ah\beta \\ + q\,(1+2a+a^2-a\beta) + b\left(h\,(-1+\beta) + q\,(-3-2a+\beta)\right)\end{array}}\end{array}\right)$$

--- (50)

or

$$\tilde{y}_{N-1}^{N} = \frac{1}{q+h}\left(\begin{array}{l}(1+a-b)\,q + c\left(-1+b+a\,(-1+\beta)-b\beta\right) + \\ \sqrt{\begin{array}{l}(1+a-b)^2 c^2 + q\left(a^2 q + (-1+b)^2 q - 2a\,(h+bq)\right) - 2c\,(b^2 q - ah\beta \\ + q\,(1+2a+a^2-a\beta) + b\left(h\,(-1+\beta) + q\,(-3-2a+\beta)\right)\end{array}}\end{array}\right)$$

--- (51)

otherwise, $\tilde{y}_{N-1}^{N-1} > 0$,

$$\tilde{y}_{N-1}^{N} = -\frac{1}{q+h}\left(\begin{array}{l} c + gh - q + gq - a\,(h+q) + hx + qx - \\ \sqrt{\begin{array}{l}(1+a-b)^2 c^2 + q\left(a^2 q + (-1+b)^2 q - 2a\,(h+bq)\right) - 2c\,(b^2 q - ah\beta \\ + q\,(1+2a+a^2 - a\beta) + b\left(h\,(-1+\beta) + q\,(-3-2a+\beta)\right)\end{array}} \end{array}\right)$$

--- (52)

or

$$\tilde{y}_{N-1}^{N} = -\frac{1}{q+h}\left(\begin{array}{l} c + gh - q + gq - a\,(h+q) + hx + qx + \\ \sqrt{\begin{array}{l}(1+a-b)^2 c^2 + q\left(a^2 q + (-1+b)^2 q - 2a\,(h+bq)\right) - 2c\,(b^2 q - ah\beta \\ + q\,(1+2a+a^2 - a\beta) + b\left(h\,(-1+\beta) + q\,(-3-2a+\beta)\right)\end{array}} \end{array}\right)$$

--- (53)

☙ U (0, b) Demand Distribution

$\tilde{y}_{N-1}^{N-1} = [S_{N-1} - x_{N-1}]^{+}$

where

$$S_{N-1} = b\left(\frac{q-(1-\beta)\,c}{q+h}\right) - g_{N-2}(N-1) \qquad \text{--- (54)}$$

If $\tilde{y}_{N-1}^{N-1} = 0$,

$$\tilde{y}_{N-1}^{N} = \frac{1}{q+h}\left(\begin{array}{l}(1-b)(q-c) - cb\beta - \\ \sqrt{(1-b)^2(c^2+q^2) - 2c\,(q\,(b^2+1) + b\left((q+h)\,(\beta-1) - 2qb\right)}\end{array}\right) \qquad \text{--- (55)}$$

or

$$\tilde{y}_{N-1}^{N} = \frac{1}{q+h}\left(\begin{array}{l}(1-b)(q-c) - cb\beta + \\ \sqrt{(1-b)^2(c^2+q^2) - 2c\,(q\,(b^2+1) + b\left((q+h)\,(\beta-1) - 2qb\right)}\end{array}\right) \qquad \text{--- (56)}$$

otherwise, $\tilde{y}_{N-1}^{N-1} > 0$,

$$\tilde{y}_{N-1}^{N} = -\frac{1}{q+h}\left(\begin{array}{l}(c-q) + (q+h)(x+g) - \\ \sqrt{(1-b)^2(c^2+q^2) - 2c\,(q\,(b^2+1) + b\left((q+h)\,(\beta-1) - 2qb\right)}\end{array}\right) \qquad \text{--- (57)}$$

or

$$\tilde{y}_{N-1}^{N} = -\frac{1}{q+h}\left(\begin{array}{l}(c-q)+(q+h)(x+g)+ \\ \sqrt{(1-b)^2(c^2+q^2)-2c\left(q\left(b^2+1\right)+b\left((q+h)(\beta-1)-2qb\right)\right)}\end{array} \right) \quad \text{---}(58)$$

B.4.5 Discussion of Fukuda's paper

As mentioned before, our work is similar to the work of Fukuda [22] about the study of the advance ordering policy. In this section, we would like to perform some comparisons, and point out the differences between Fukuda's work and ours.[5]

B.4.4.1 Introduction of Fukuda's paper

Fukuda used *DP* to study the inventory problem in which amounts of stock ordered at unit prices c_k and c_{k+1} $(c_k > c_{k+1})$ are delivered, respectively, k and $(k + 1)$ periods later. If ordering decisions being made in every other period is assumed, analogous results to the results of this problem are obtained for the case in which amounts of stock ordered at unit prices c_k, c_{k+1} and c_{k+2} $(c_k > c_{k+1} > c_{k+2})$ are delivered, respectively, k and $(k + 1)$ and $(k + 2)$ periods later.

B.4.4.2 Comparison and Discussion

- *Problem Formulation*

Both models are *DP* problems and the recursion equations are similar.

Fukuda's dynamic programming recursion formula is:

[5] Please note that we have reported Fukuda's results using our notations.

$$f_{N-1}(r_{N-1}) = \min_{y_{N-1}^{N-1}>0,\, y_{N-1}^{N}>0} \{cy_{N-1}^{N-1} + c\beta_1 y_{N-1}^{N} + \Psi_{N-1}(r_{N-1} + y_{N-1}^{N-1}) + \alpha Ef_N(r_{N-1} + y_{N-1}^{N-1} + y_{N-1}^{N} - D_{N-1})\} \quad \text{--- (59)}$$

where $r_{N-1} = x_{N=1} + g_{N-2}(N-1)$

Our dynamic programming recursion formula (recall from Eq.(9)) is:

$$f_{N-1}(x_{N-1}; g_{N-2}(N-1)) = \min_{y_{N-1}^{N-1}>0,\, y_{N-1}^{N}>0} \{cy_{N-1}^{N-1} + c\beta_1 y_{N-1}^{N} + \Psi_{N-1}(x_{N-1} + g_{N-2}(N-1) + y_{N-1}^{N-1}) + Ef_N(x_{N-1} + g_{N-2}(N-1) + y_{N-1}^{N-1} - D_{N-1}; y_{N-1}^{N})\}$$

The main differences are:

i) We haven't included a discount factor for future costs, while Fukuda has α as the time discount factor.

ii) Fukuda uses r_{N-1} to represent the total inventory before replenishment at period (N - 1). This makes the formula simpler and collapses the problem from 2-Dimension (x, g) to 1-Dimension (x).

Both of us use the same Newsboy policy at the last period N.

▫ *Order-up-to levels and optimal orders*

Fukuda's results:

In his paper, Fukuda hasn't explicitly calculated the amounts of the order-up-to levels and optimal orders, but we could follow his logic and get the formulations as the following. (Please refer to his paper for the detail proof of the optimal ordering policy.)

The last period optimal order-up-to level:

$$S_N = J^{-1}(\frac{q-c}{q+h})$$

The optimal order:

$$\tilde{y}_N^N = [S_N - r_N]^+$$

To find the second last period optimal order-up-to levels:

We let

$$\hat{S} = J^{-1}(\frac{q-(1-\beta_1)c}{q+h}), \text{ where } \hat{S} > S_N$$

$$\begin{aligned} \tilde{\Psi}(r) &= c(\hat{S}-r)+\Psi(\hat{S}) \qquad & r < \hat{S} \\ &= \Psi(r) & r \geq \hat{S} \end{aligned} \qquad \text{--- (60)}$$

$$\begin{aligned} \Lambda(v) &= c(v-\hat{S})+\Psi(v)-\Psi(\hat{S}) \qquad & v < \hat{S} \\ &= 0 & v \geq \hat{S} \end{aligned} \qquad \text{--- (61)}$$

From Eq.(59), Eq.(60) and Eq.(61), and with re-arrangements, we have the cost-to-go function for period (N - 1):

$$f_{N-1}(r) = \tilde{\Psi}(r) + \min_{v \geq r}\{c\beta_1(v-r)+\Lambda(v)+\alpha\int_0^\infty f_N(r-t)f(t)\,dt\} \qquad \text{--- (62)}$$

Then we let $F_2(v)$, (from the minimization part of Eq.(62))

$$F_2(v) = c\beta_1 v + \Lambda(v) + \alpha\int_0^\infty f_N(v-t)f(t)\,dt$$

Taking first derivative w.r.t. v and then substitute $v = \hat{S}$

$$F_2'(\hat{S}) = c\beta_1 + \alpha\int_0^\infty f_N'(\hat{S}-t)f(t)dt$$

Then we have two cases depending on the sign of $F_2'(\hat{S})$:

Case (a): $F_2'(\hat{S}) < 0$, ($S_{N-1} > \hat{S} > S_N$)

S_{N-1} is given by the equation:

$$F_2'(S_{N-1}) = 0 = c\beta_1 + \alpha\int_0^\infty f_N'(S_{N-1}-t)f(t)\,dt$$

$$\tilde{y}_{N-1}^{N-1} = \begin{cases} \hat{S} - r_{N-1} & \text{if} \quad r_{N-1} < \hat{S} \\ 0 & \text{if} \quad \hat{S} \le r_{N-1} < S_{N-1} \\ 0 & \text{if} \quad r_{N-1} \ge S_{N-1} \end{cases}$$

$$\tilde{y}_{N-1}^{N} = \begin{cases} S_{N-1} - \hat{S} & \text{if} \quad r_{N-1} < \hat{S} \\ S_{N-1} - r_{N-1} & \text{if} \quad \hat{S} \le r_{N-1} < S_{N-1} \\ 0 & \text{if} \quad r_{N-1} \ge S_{N-1} \end{cases}$$

or

(a1) for $r_{N-1} < \hat{S}$, order amount $\hat{S} - r_{N-1}$ at *c* and amount $S_{N-1} - \hat{S}$ at $c\beta$;

(a2) for $\hat{S} \le r_{N-1} < S_{N-1}$, order amount $S_{N-1} - r_{N-1}$ at $c\beta$;

(a3) for $r_{N-1} \ge S_{N-1}$, order none.

<u>Case (b)</u>: $F_2'(\hat{S}) \ge 0$, ($\hat{S} \ge S_{N-1} > S_N$)

S_{N-1} is given by the equation:

$$F_2'(S_{N-1}) = 0 = c + \Psi'(S_{N-1}) + \alpha\int_0^\infty f_N'(S_{N-1}-t)f(t)\,dt$$

$$\tilde{y}_{N-1}^{N-1} = \begin{cases} S_{N-1} - r_{N-1} & \text{if} \quad r_{N-1} < S_{N-1} \\ 0 & \text{if} \quad r_{N-1} \ge S_{N-1} \end{cases}$$

$$\tilde{y}_{N-1}^{N} = 0$$

or

(b1) for $r_{N-1} < S_{N-1}$, order amount $S_{N-1} - r_{N-1}$ at *c*;

(b2) for $r_{N-1} \ge S_{N-1}$, order none.

Our results:

Please refer to Theorem 1 in the previous section.

Both works have the same solution space, as shown in Figure 2 below.

- *Work Objective*

Fukuda provided the structure of the optimal solution for the case of window size 1. He also obtained the result for the case of window size 2 when orders can only be made in every other period (thus decoupling the ordering decisions). But in our general model, we are trying to study the situation when ordering is allowed in every period.

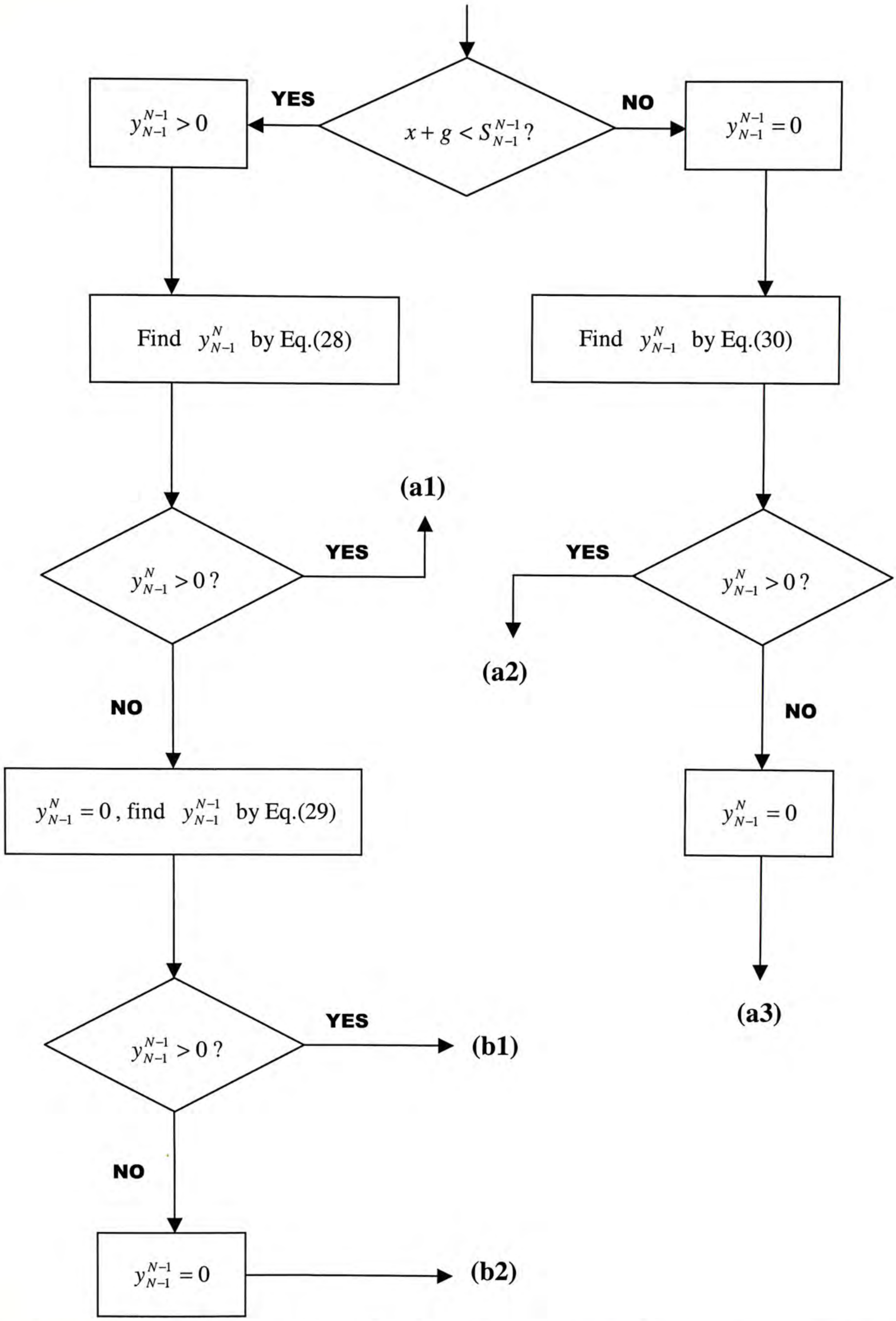


Figure 2: Summary of Theorem 1 with paths matching to the cases in Fukuda's work. (p. 117)

Chapter B.5

Simulation Study of Window Size 1

B.5.1 Simulation Models

Although we cannot find out the optimal ordering policy for window size 1 over a finite horizon in Chapter B.4, here we propose an ordering heuristic policy that related to our work in Section B.4.3. In order to see how efficient it works, we try to run the simulation of it (*NOOP*) and do comparisons with 4 heuristics (*NOHP*, *NOHPa*, *NOHPb* and *LOHP*) and 3 benchmark policies: mean of demand advanced ordering (*MAOP*), no advanced ordering (*NAOP*) and deterministic demand policy (*DOP*), assuming all demands are known in advance. We are interested in comparing the ordering patterns and cost savings.

Simulation model (1): Near Optimal Order Policy (*NOOP*)

In Section B.4.3, we have solved for the optimal ordering policy in a two-period inventory problem. Since the solutions are complicated (See Theorem 1), we propose a heuristic for the simulation study. Whenever $\tilde{y}_t^{t+1} < 0$, the heuristic just simply set $y_t^{t+1} = 0$. We would apply this ordering pattern on a rolling basis to a finite horizon, that is for day t, we place orders for "today" ($\tilde{y}_t^t$) and "tomorrow" ($\tilde{y}_t^{t+1}$).

Policy Formulation:

$$\tilde{y}_t^t = [S_t - x_t - g_{t-1}(t)]^+ \qquad \text{---} (1)$$

If $x_t + g_{t-1}(t) < S_t$, $\tilde{y}_t^{t+1}$ is $\max[0, \bar{y}_t^{t+1}]$ where $\bar{y}_t^{t+1}$ is a solution of the equation:

$$c(\beta_1 - 1) + (c-q)J(w + y_t^{t+1}) + (h+q)\int_0^{w+y_t^{t+1}} J(w + J^{-1}(\frac{q-c}{q+h}) + y_t^{t+1} - t)\, dJ(t) = 0$$

--- (2)

If $x_t + g_{t-1}(t) \geq S_t$, $\tilde{y}_t^{t+1}$ is $\max[0, \bar{y}_t^{t+1}]$ where $\bar{y}_t^{t+1}$ is a solution of the equation:

$$c(\beta_1 - 1) + (c-q)J(w' + y_t^{t+1}) + (h+q)\int_0^{w'+y_t^{t+1}} J(w' + J^{-1}(\frac{q-c}{q+h}) + y_t^{t+1} - t)\, dJ(t) = 0$$

--- (3)

where

$$S_t = x_t + g_{t-1}(t) + y_t^t = J^{-1}(\frac{q-(1-\beta_1)c}{q+h}) \qquad \text{---} (4)$$

$$w = J^{-1}(\frac{q-(1-\beta_1)c}{q+h}) - J^{-1}(\frac{q-c}{q+h}) \qquad \text{---} (5)$$

and

$$w' = x_t + g_{t-1}(t) - J^{-1}(\frac{q-c}{q+h}) \qquad \text{---} (6)$$

Simulation model (2): Newsboy Order Heuristic Policy (*NOHP*)

This policy places a Newsboy order everyday, whilst considering the expected residual inventory from the day before. The derivation of the policy is given in Appendix IV.

Policy Formulation:

$$\tilde{y}_t^t = [J^{-1}(\frac{q-c}{q+h}) - x_t - g_{t-1}(t)]^+ \qquad \text{--- (7)}$$

$$\tilde{y}_t^{t+1} = \begin{cases} [J^{-1}(\frac{q-c}{q+h}) - (x_t + g_{t-1}(t) - \mu)]^+ & if \quad \tilde{y}_t^t = 0 \\ \mu & if \quad \tilde{y}_t^t > 0 \end{cases} \qquad \text{--- (8)}$$

Simulation model (3): Newsboy Order Heuristic Policy - a (*NOHPa*)

This policy is come from modifying *NOHP*. When we consider the Newsboy level of the next day, we adjust the unit purchasing cost by the given discount.

Policy Formulation:

$$\tilde{y}_t^t = [J^{-1}(\frac{q-c}{q+h}) - x_t - g_{t-1}(t)]^+ \qquad \text{--- (9)}$$

$$\tilde{y}_t^{t+1} = \begin{cases} [J^{-1}(\frac{q-c\beta_1}{q+h}) - (x_t + g_{t-1}(t) - \mu)]^+ & if \quad \tilde{y}_t^t = 0 \\ [J^{-1}(\frac{q-c\beta_1}{q+h}) - (J^{-1}(\frac{q-c}{q+h}) - \mu)]^+ & if \quad \tilde{y}_t^t > 0 \end{cases} \qquad \text{--- (10)}$$

Simulation model (4): Newsboy Order Heuristic Policy - b (*NOHPb*)

This policy is come from modifying *NOHP* and referencing on *NOOP*. When we consider the Newsboy level of the next day, we set it equal to S_t. (See Eq.(4) above)

Policy Formulation:

$$\tilde{y}_t^t = [J^{-1}(\frac{q-c}{q+h}) - x_t - g_{t-1}(t)]^+ \qquad \text{--- (11)}$$

$$\tilde{y}_t^{t+1} = \begin{cases} [J^{-1}(\frac{q-c\,(1-\beta_1)}{q+h}) - (x_t + g_{t-1}(t) - \mu)]^+ & if \quad \tilde{y}_t^t = 0 \\ [J^{-1}(\frac{q-c\,(1-\beta_1)}{q+h}) - (J^{-1}(\frac{q-c}{q+h}) - \mu)]^+ & if \quad \tilde{y}_t^t > 0 \end{cases} \qquad \text{--- (12)}$$

Remark:

NOHPb is like the 'converse' of *NOHPa*, since the main difference between them is *NOHPa* using β_1 while *NOHPb* using $(1-\beta_1)$ in their formulations. (See Eq.(10) and Eq.(12))

Simulation model (5): Lead Order Heuristic Policy (*LOHP*)

In this policy, we also order the Newsboy amount for the current day, but we see the 1-day advance order as a 1-day lead time delivery. So, to determine the optimal amount of the advance order, we calculate the safety stock (*ss*) and get the Order-Up-to-Level (*OUL*) for the next day. Since this conventional calculation assumed that there is no other deliveries will arrive during the lead time *(L)* and the review time *(T)*, we will deduct the *OUL* by the current day order to obtain the needed *OUL* ($\tilde{y}_t^{t+1}$). The derivation of the policy is given in Appendix V.

Policy Formulation:

$$\tilde{y}_t^t = [J^{-1}(\frac{q-c}{q+h}) - x_t - g_{t-1}(t)]^+ \qquad \text{--- (13)}$$

$$\tilde{y}_t^{t+1} = [\min\{OUL - x_t - g_{t-1}(t) - \tilde{y}_t^t, \tilde{y}_t^t\}]^+ \qquad \text{--- (14)}$$

Remark:

If $q < c$, we allow the "safety stock" to be negative. This can make sure the service level is according to CSL[1].

Simulation model (6): Mean Advance Order Policy (*MAOP*)

This is a simple policy where only the mean of demand will be ordered each day.

Policy Formulation:

$$y_t^t = E(D_t) = \mu \qquad \text{--- (15)}$$

$$y_t^{t+i} = E(D_{t+i}) = \mu, \qquad \forall i = 1,2,\ldots \qquad \text{--- (16)}$$

Simulation model (7): No Advance Order Policy (*NAOP*)

This is a simple and common known policy. Everyday only the Newsboy amount will be ordered. As there will be no advance order placed, this policy provides the "worst-case" benchmark where no discount benefit can be obtained.

Policy Formulation:

$$y_t^t = J^{-1}\left(\frac{q-c}{h+q}\right) - x_t \qquad \text{--- (17)}$$

$$y_t^{t+i} = 0, \qquad \forall i = 1,2,\ldots \qquad \text{--- (18)}$$

[1] Cycle Service Level

Simulation model (8): Deterministic Order Policy (*DOP*)

In this policy, we assume the demand is known, so we will place the exact order always in advance to get the discount benefit. This is the optimal policy to minimize the cost. If $q < c$, we would prefer backlogging rather than meeting the demand in the last period(s).

Policy Formulation:

In the first day, *DOP* starts by

$$\tilde{y}_1^1 = E(D_1) \qquad \text{--- (19)}$$

$$\tilde{y}_1^2 = E(D_2) \qquad \text{--- (20)}$$

For $t > 1$,

$$\tilde{y}_t^t = 0 \qquad \text{--- (21)}$$

If $c\beta_1 < (N-t)\,q$,

$$\tilde{y}_t^{t+1} = E(D_{t+1}) \qquad \text{--- (22)}$$

otherwise,

$$\tilde{y}_t^{t+1} = 0 \qquad \text{--- (23)}$$

It is contrast to *NAOP*, which is always placing orders in the current day. *DOP* acts as a base case among the policies in the following comparisons.

Note:

In all policies, $\tilde{y}_N^{N+1} = 0$.

B.5.2 Simulation Program Structure

B.5.2.1 Program Flow

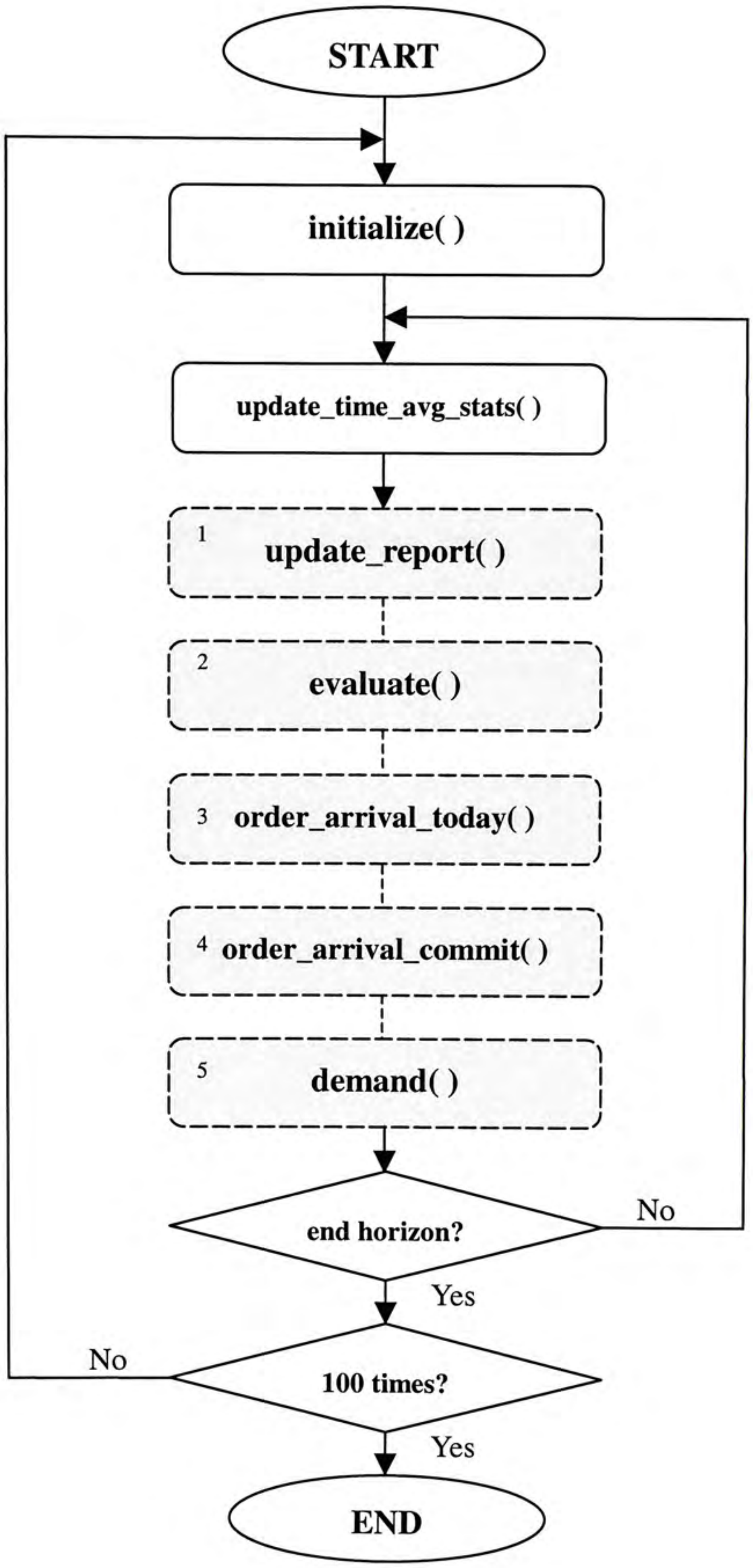


Figure 1: Simulation Program Framework

Figure 1 shows the framework and the flow of the simulation programs. In each simulation run (the outside loop), we input 100 sets of demand, which are generated from a Normal Distribution with mean and standard deviation given. Each set of demand contains N numbers and is used in each N-day simulation (the inside loop). The outputs of an N-day simulation run are the total cost, the purchasing costs of the current orders and the advance orders, the holding cost and the shortage cost. The numerical results and analysis we presented later are based on the average outputs after a 100 N-day simulation run.

Figure 1 shows the main functions in our simulation program. The colored boxes are the 5 events we have to simulate. The event details are presented in Table 1 and their tasks will be presented in next subsection.

Table 1: Description of events

Event type	Event description
1	End of the simulation after N days and generate a report
2	Inventory evaluation (and possible ordering) at the beginning of a day
3	Arrival of a current order to the retailer from the supplier
4	Arrival of a committed order to the retailer from the supplier
5	Demand for the product from a customer

Now let us look into the general flow of the simulation programs. Firstly, we need to initialize the simulation clock and output variables before going through the N-day simulation. Starting from day 1, **update_time_avg_stats( )** would update all the statistics. Then each of the 5 events would occur in sequence according to the time of the embedded simulation clock. After the event is done, if it is not the end of the horizon, the flow will go back to **update_time_avg_stats( )** to update all the statistics again, so on and so forth. The general sequence of the events to occur is (2) → (3) → (4) → (5) within a day. If it comes to the end of the horizon, event (1) would occur to make a report that summarizes the results of that *N*-day simulation. The above is about the inside loop. With the control of the outside loop, the simulation program will end after a 100 run of the inside loop and we can get the average results.

B.5.2.2 Program Functions

We would briefly explain the tasks of each of the functions shown in Figure 1 in the following. The tasks listed are not necessarily in their execution order.

initialize( )

- initialize the simulation clock
- initialize the inventory levels
- initialize the statistical counters (output variables)
- set the timing of the events to occur

update_time_avg_stats( )

- determine the status of the inventory level during the previous interval
- update the (holding) inventory counter or the backorder counter

(Event 1) **update_report( )**

- compute the purchasing costs, holding cost, shortage cost and total cost
- generate the cumulative day by day performance report

(Event 2) **evaluate( )**

- determine the amount of the current order and the advance order according to the specific policy
- update the order costs of the current order and the advance order
- determine the order arrival times

(Event 3) **order_arrival_today( )**

- get the amount of today's order, which is delivered immediately
- increase the inventory level

(Event 4) **order_arrival_commit( )**

- get the amount of committed order, which is ordered in advance for delivery on the current day
- increase the inventory level

(Event 5) **demand( )**

- get the amount of customer demand
- decrease the inventory level

B.5.2.3 Program Implementation Note

1) In our implementation, the shortage cost does not include the purchasing cost for buying back the items for backorders. If there is one backorder (before the end of horizon), one unit purchasing cost and one unit shortage cost are counted

separately.

2) For the last period, if backorder occurs, the program will only update the shortage cost but not the purchasing cost. This means we will not buy additional items to fulfill the backorder demand after reaching the end of horizon.

3) Although we have several different policies, we can use the same framework as shown in Figure 1 to implement all of them. What we need is to modify the function, evaluate( ), for the individual policy.

4) For finding $\tilde{y}_t^{t+1}$ by *NOOP*, we need to solve Eq.4.3.(28), Eq.4.3.(29) or Eq.4.3.(30). Since the demand distribution is in Normal, there are no closed-form expressions for both of them. We use numerical integration to evaluate the definite integrals inside the formulae. The method we chosen is called *Simpson one-third rule*[1].

[1] It gets the approximated integral value by finding the approximate area under the graph of the function within the upper and lower bounds of the integral. It takes two adjacent strips and joining the three points on the function curve with a parabola. The equation is:

$$I \approx \frac{1}{3} \sum_{i=1,3,5,\ldots}^{n} (y_i + 4y_{i+1} + y_{i+2})\Delta x$$

where I is the approximated value of the integral, y_i is a point on the curve, Δx is the width of a strip, i increases with step 2 and n is the number of strips. This rule requires that there be an even number of equally spaced strips. We have arbitrarily chosen n to be 1000.

B.5.3 Simulation Numerical Analysis

B.5.3.1 Simulation Settings

We assume the demand has a Normal Distribution. Its mean and *s.d.* are 100 and 30 respectively, so the demand will realize within about [0, 200].[1] The horizon is 20 days, i.e., $N = 20$; unit purchasing cost (*c*) is \$10; unit holding cost (*h*) is \$0.0055 and unit shortage cost (*q*) is \$25.[2] The discount rate (β_1) is 0.8. The optimal Newsboy ratio is 0.6[3]. The results shown below are the averages over 100 simulation runs. This is the base setting for the following comparisons; that means, in each of the cases below, we only change one of the variables and keep the others as the values in this base case.

Table 1: general setting

μ	*s.d.*	*N*	β_1	*c*	*h*	*q*	*Newsboy Ratio*
100	30	20	0.8	10	0.0055	25	0.6

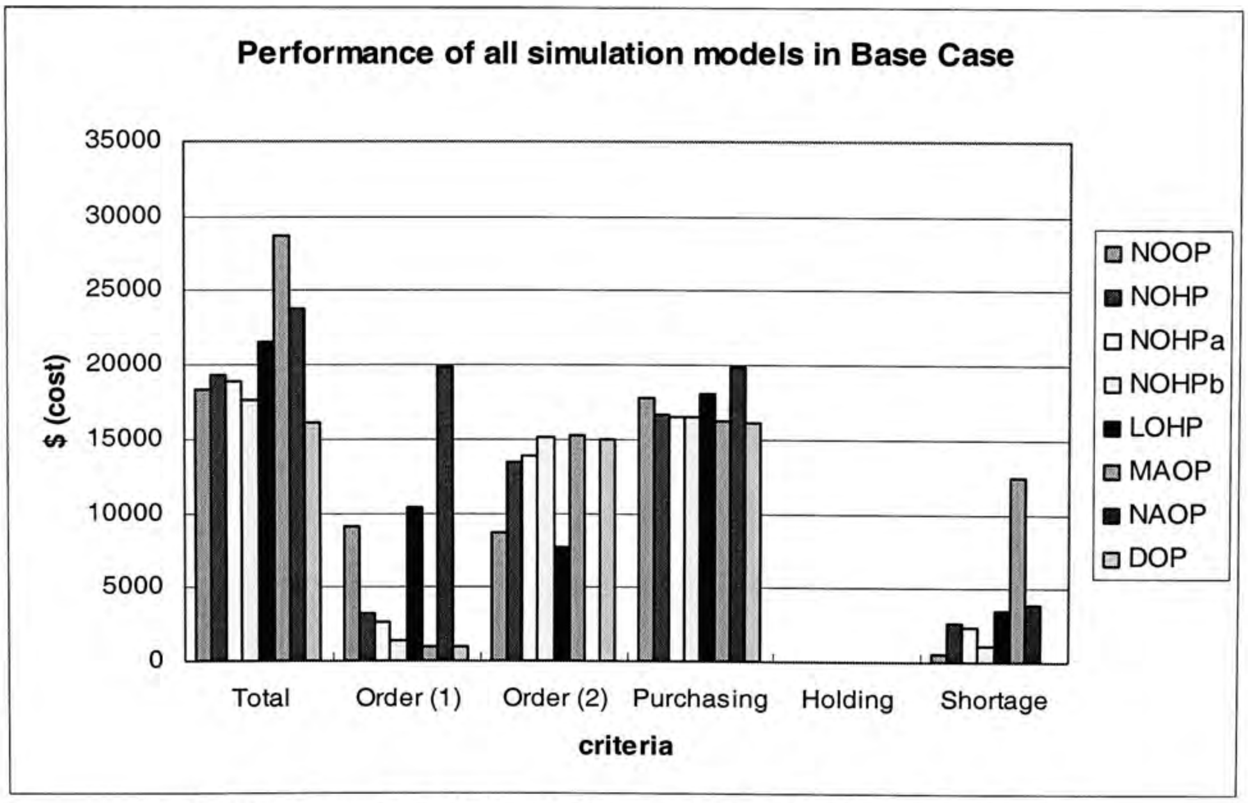


Figure 1: cost of all simulation models with different criteria

[1] *Empirical Rule*: 99.7% of the observations fall within **3 standard deviations** of the **mean.**
[2] *The cost setting is reference on Nahmias, Production and Operations Analysis (3rd), p.287.*
[3] Newsboy ratio = $\left(\dfrac{q-c}{q+h}\right)$

Table 2: definition of criteria

Criteria	*Definition*
Total	total cost, which is equal to the sum of total purchasing cost, total holding cost and total shortage cost
Order (1)	total number of items purchased by current orders
Order (2)	total number of items purchased by advance orders
Purchasing	total purchasing cost
Holding	total holding cost
Shortage	total shortage cost

From Figure 1, we can see that *NOHPb* does the best with the lowest total cost among the stochastic heuristics (so excluding *DOP*). *NOOP* does the second best and then *NOHPa*, *NOHP*, *LOHP*, *NAOP* and the worst is *MAOP*. The definitions of the quantities in the graph please refer to Table 2.

B.5.3.2 Simulation Result Analysis:

In the simulation study, we would focus on testing the sensitivity of the solution and the performance of the model to the relative values of the parameters, such as the unit purchasing cost, unit shortage cost and unit holding cost. The ordering behavior and pattern are also our major concerns in the simulation study.

Case I: Comparison between all simulation models by varying the standard deviation of demand (*s.d.*)

In this case, we would like to see the change of the heuristics' performance by varying the *s.d.* of the normal demand distribution. The range of *s.d.* is from 5 to 35 in steps of 5. So, the demand fluctuation ranges from small to large.

Table 3: ***s.d.*** **setting**

s.d.	5	10	15	20	25	30	35

From Figure 2 below, we can see the Order (1) is increasing and Order (2) is decreasing with the increasing *s.d.*, but the total order is keep constantly at about 2000.

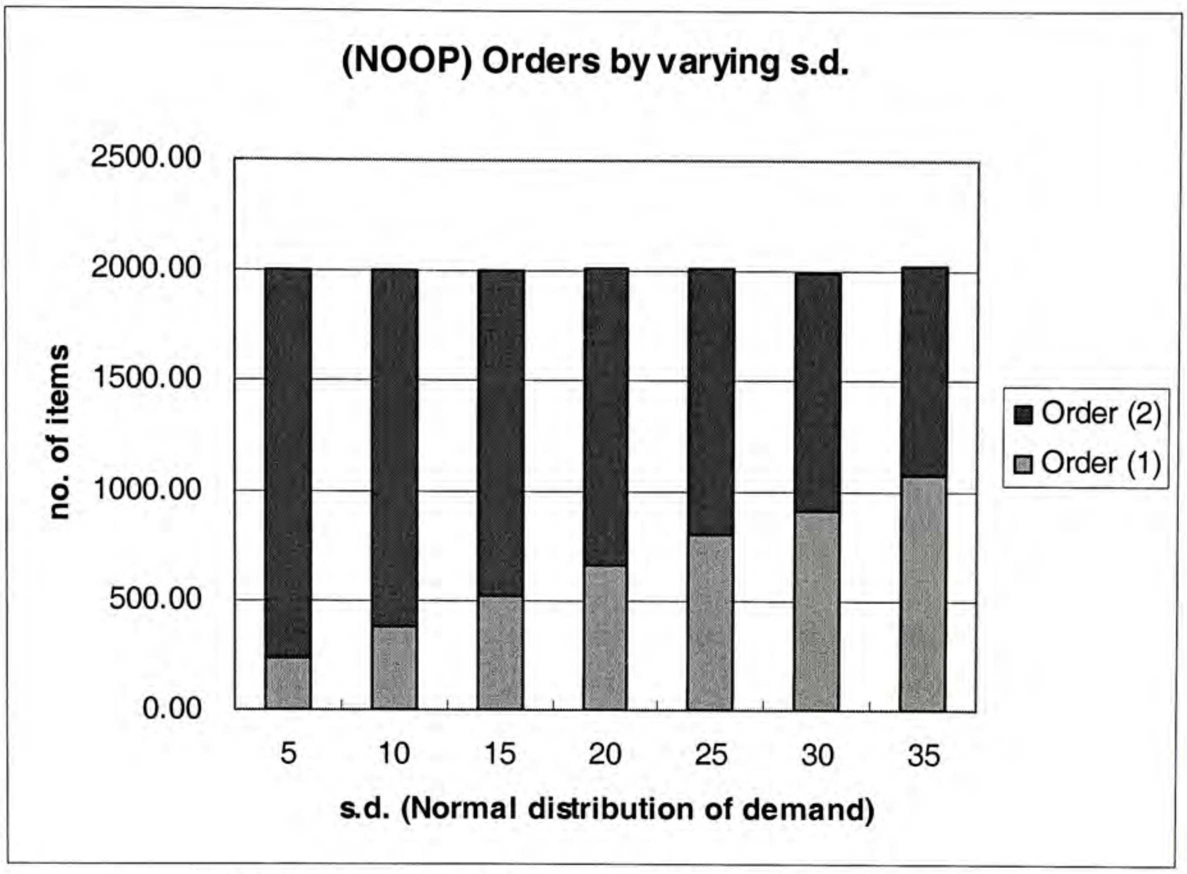


Figure 2: order pattern of *NOOP* with varying *s.d.*

We see that as the variation of demand (*s.d.*) increases, it is preferable for *NOOP* to place orders as late as possible in order to get more updated information about the inventory level and have a greater chance to meet the actual demand, although we can get a discount when place order in advance. So, in such a highly fluctuating demand situation, we place more orders for the current day and fewer orders for the future and sometimes, it results in lower total orders.

In Figure 3, the current day orders for *NAOP* and *LOHP* are staying at about 2000 and 1000 respectively. Those for *NOOP*, *NOHP*, *NOHPa* and *NOHPb* are increasing linear with increasing *s.d.*, with that for *NOOP* has a higher increasing rate than the other three policies. The current day orders for *DOP* and *MAOP* are not zero because there are orders for day 1.

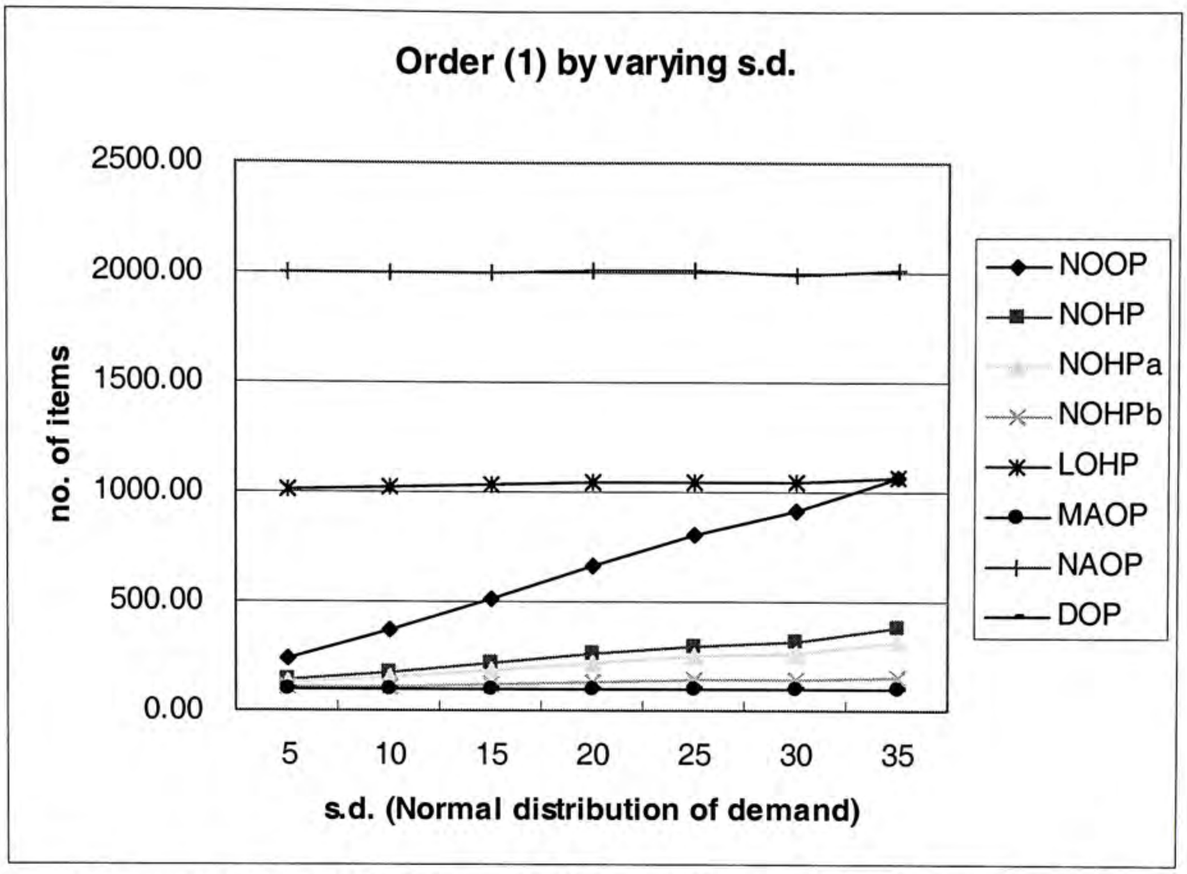


Figure 3: Order (1) patterns of all simulation models with varying *s.d.*

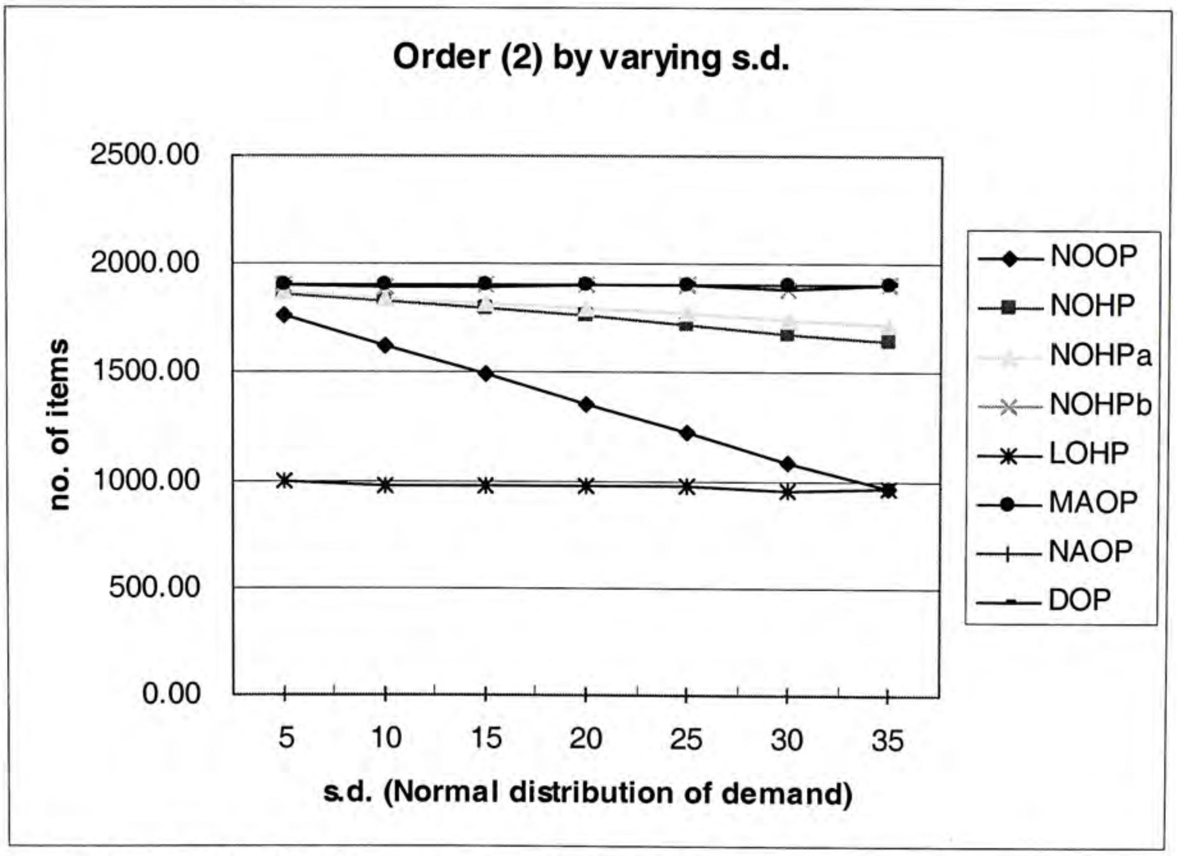


Figure 4: Order (2) patterns of all simulation models with varying *s.d.*

In Figure 4, the advance orders for *LOHP* is also approximately constant at about 1000. Advanced orders for *NOOP*, *NOHP*, *NOHPa* and *NOHPb* are decreasing approximately linear with increasing *s.d.*, with *NOOP* having a higher decreasing rate than *NOHP*. Advanced orders for *MAOP* and *DOP* are approximately constant at 1900. There are no advance orders for *NAOP* as it is a no advance order policy.

As the standard deviation increases, *NOHP* order more for the current day and place fewer orders for the next day. We can explain this phenomenon by referring to Eq.5.1.(7) and Eq.5.1.(8). For the case $\tilde{y}_t^t > 0$, $\tilde{y}_t^{t+1} = \mu$ is independent of the change of *s.d.*, so we consider the case of $\tilde{y}_t^t = 0$. We find that when *s.d.* equal to 5, T^*[4] is equal to 101.27 and when *s.d.* equal to 30, T^* is equal to 108.86. The increase of T^* is small compared to the increase of *s.d.*.[5] Also, the demand fluctuates a lot, so does the inventory on hand. As a result, $[T^* - (x_t + g_{t-1}(t) - \mu)]^+$ has more chances to be zero compared to the case when the *s.d.* is small and the advance orders will be lower in amount. Since T^* is higher and the advance orders are fewer, the current orders are higher.

The reason that for *NOHPa* and *NOHPb*, Order (1) increases and Order (2) decreases with increasing *s.d.* can be explained similarly as for *NOHP*. The rate of increase/decrease is lower than *NOHP* because $J^{-1}(\frac{q-c(1-\beta)}{q+h}) > J^{-1}(\frac{q-c\beta}{q+h}) > T^*$ which makes the inventory level generally higher than *NOHP*. (See Eq.5.1.(8), Eq.5.1.(10) and Eq.5.1.(12))

Both the amount of current order and advance order of *LOHP* are almost the same.

The sum of the two orders is shown in Figure 5. All the simulation models have their curves in a similar shape. The problem implements a fully backorder mechanism (100% demand is fulfilled, except the last period and *DOP*) and so total orders is very close to the total demand. From Table 4[6], we can see the differences between the total orders of different policies and the total actual demand. (Excluding *DOP*) *MAOP* has the lowest average difference, then *NAOP*, *LOHP*, *NOOP*, *NOHP*,

---

[4] Recall that T^* is the optimal order-up-to level according to the Newsboy model: $T^* = J^{-1}\left(\frac{q-c}{q+h}\right)$

[5] The change depends on the Newsboy ratio: $T^* = \mu + (Newsboy\ ratio)\sigma$.

[6] We compare: $\sum_t order_t - \sum_t demand_t$

NOHPa and *NOHPb* has the highest. Note that according to *MAOP*, each order is exactly the mean of demand; this leads to the total order amount being closest to the real demand in average, although varying for different *s.d.* values. The difference between total orders and demand of *NOHPb* increases with increasing *s.d.*, but not for the others.

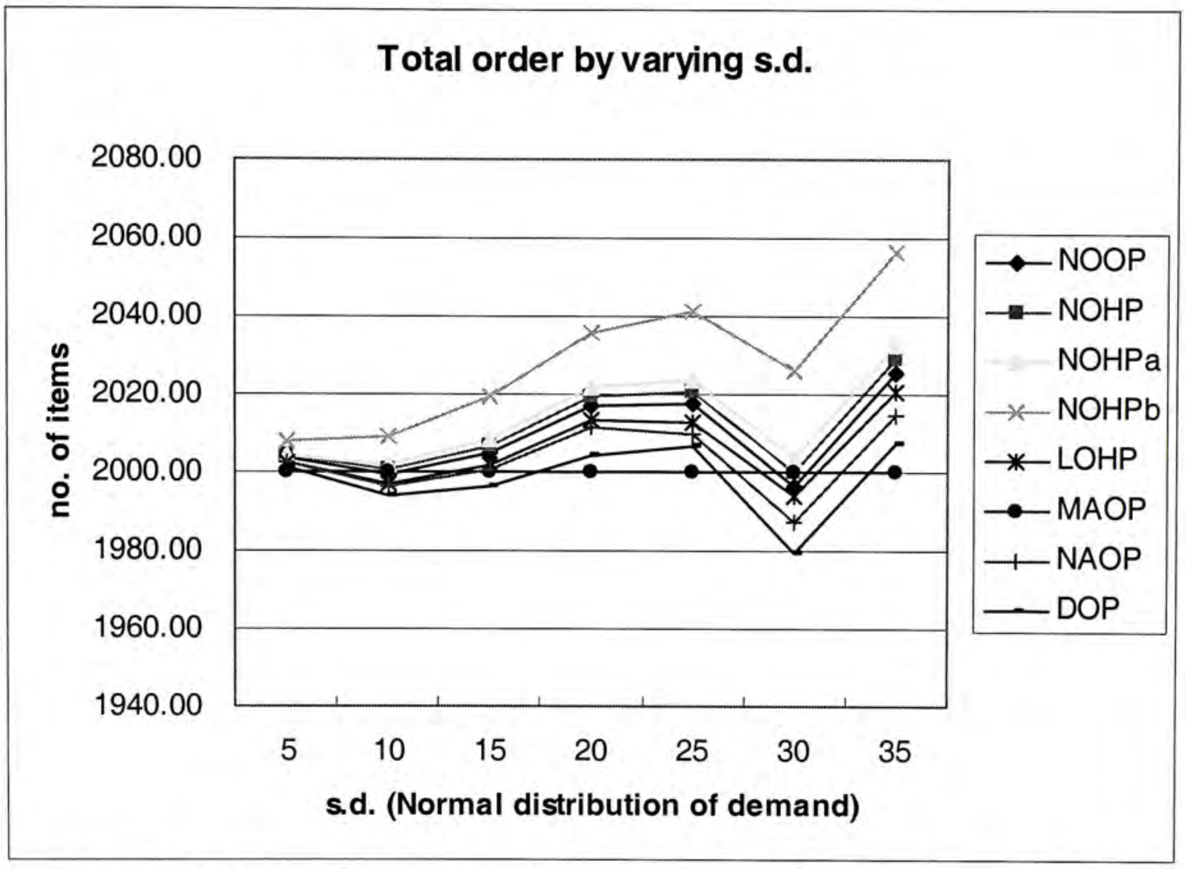


Figure 5: total order pattern of all simulation models with varying *s.d.*

Table 4: difference between total order and total demand of all simulation models

s.d.	*NOOP*	*NOHP*	*NOHPa*	*NOHPb*	*LOHP*	*MAOP*	*NAOP*
5	2.10	2.55	3.03	6.53	1.26	-1.46	1.09
10	5.68	6.86	8.10	15.27	3.16	6.31	2.48
15	8.56	10.37	12.18	22.87	5.73	3.67	4.28
20	12.93	15.33	17.76	31.79	9.38	-3.94	7.33
25	10.52	13.77	16.88	34.10	5.70	-6.91	2.77
30	17.18	20.53	24.04	46.64	14.26	20.59	8.01
35	18.51	22.06	25.77	49.11	13.41	-7.08	7.59
Average	10.78	13.07	15.39	29.47	7.56	1.60	4.79

Within the total orders of *NOHP* and *NOHPa*, the advance order proportion is much larger than the current order proportion, so the policies can get a lot of discount benefit, which is greater than what *NOOP* gets. The total orders placed by them exceed the real demand by quite a lot, by referring to *DOP* in Figure 5, but the shortage costs incurred are still higher than *NOOP* and holding costs are lower than *NOOP*. (See Figure 7 and Figure 6 respectively) Finally, the performances of them are close to *NOOP* (See Figure 8) and we would conclude that large amount of *NOHP* and *NOHPa*'s total orders are for fulfilling the backorders.

Because *NAOP* only considers the cost minimization for a one-day horizon, it is conservative in placing orders and the total orders placed are fewer compared to the other policies.

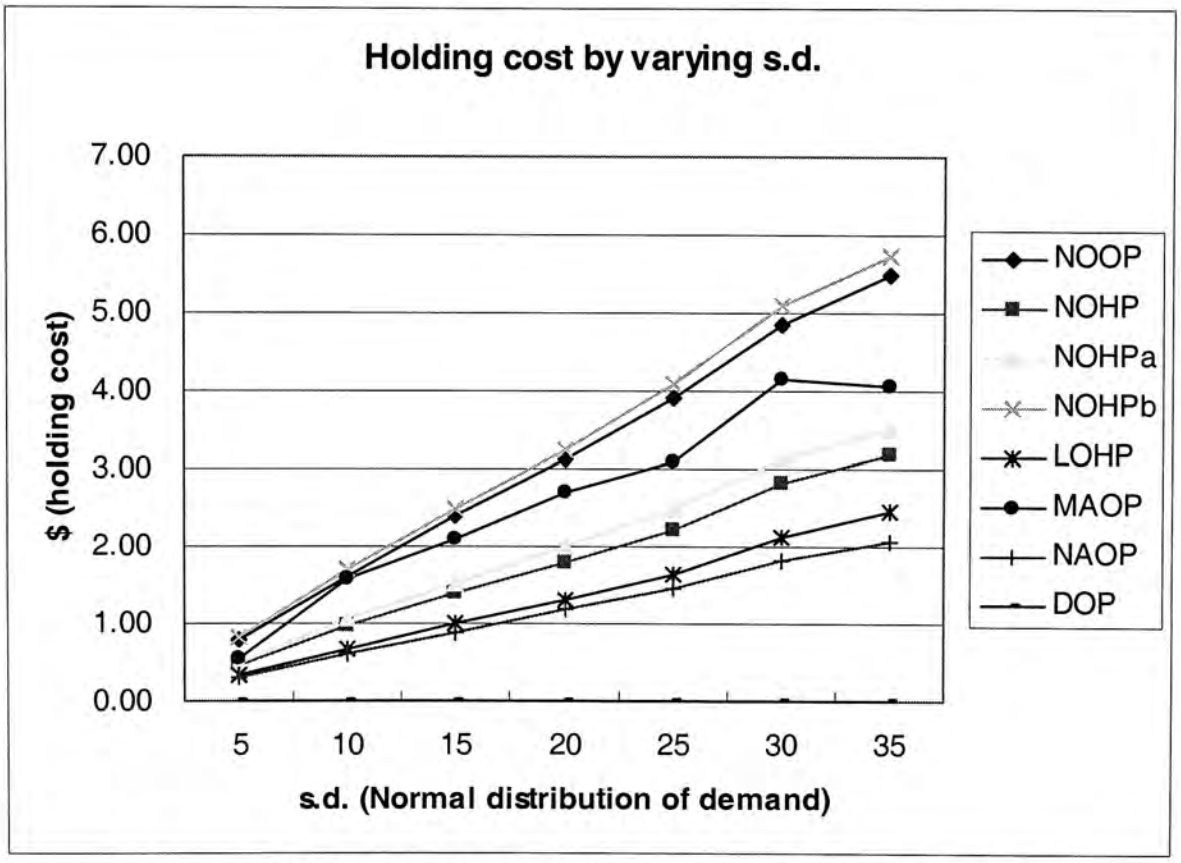


Figure 6: holding cost of all simulation models with varying *s.d.*

In Figure 6 and Figure 7, we can see the holding costs and shortage costs of all the simulation models are increasing with the increasing *s.d.* quite linearly, except *MAOP*. Moreover, if the holding cost increases at a high rate (as shown in Figure 6), then the rate of increase of the shortage cost will generally be low (See Figure 7). *DOP* is $0 at both holding and shortage cost since we know the exact demand.

The practice of *NOOP*, placing more orders for the current day when *s.d.* larger, would make the retailer holds more inventory and get fewer backorders (than if placing less orders for the current day). It is reasonable to do this since the unit shortage cost is much larger than the unit holding cost. Finally, for this policy, the holding cost and shortage cost are comparable. We see that *NOOP* balances well between these two costs and so has the second lowest total cost than the other policies. (See Figure 8)

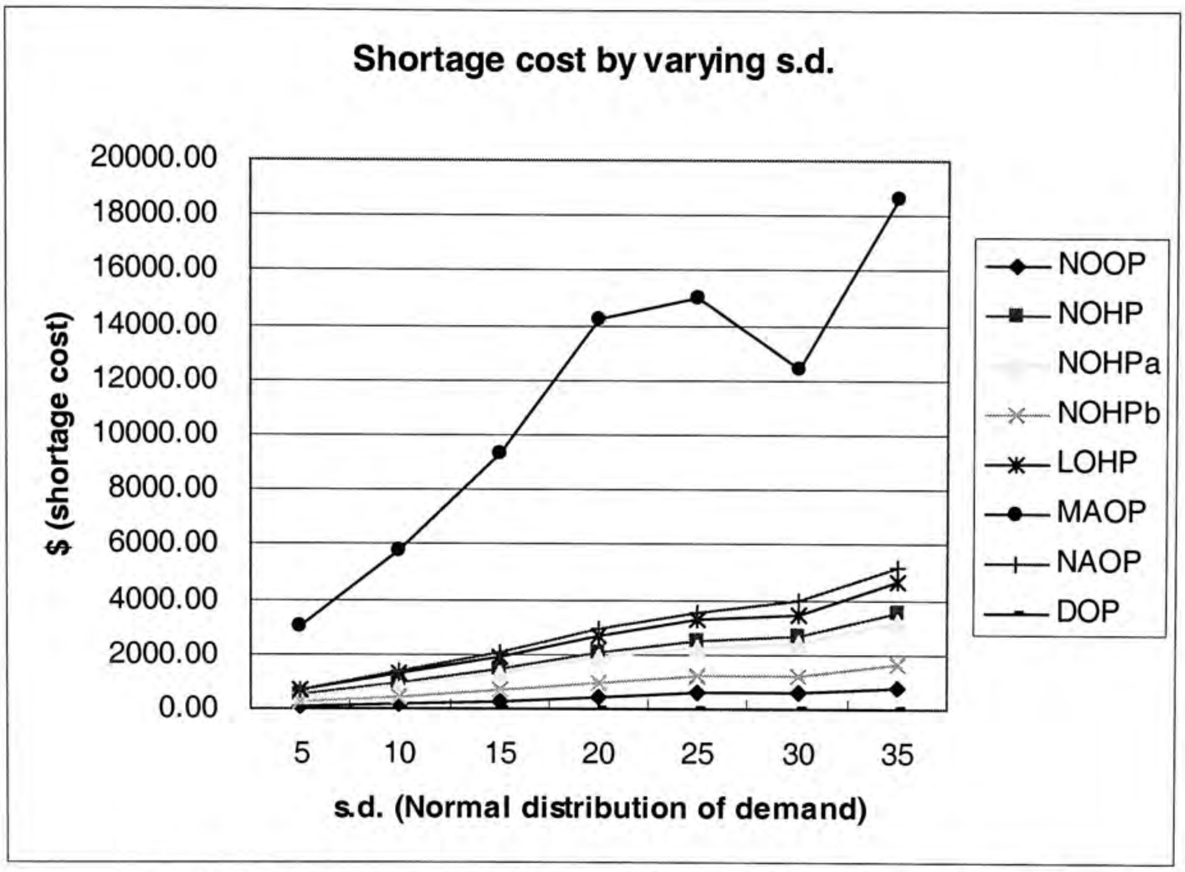


Figure 7: shortage cost of all simulation models with varying *s.d.*

NAOP has the lowest holding cost and the second highest shortage cost. As the shortage cost dominates and no discount benefit can be got by the policy, it has the second highest total cost, which is less than *MAOP*. (See Figure 8)

Figure 8 shows the total costs resulting from the 8 simulation models with varying *s.d.*. Excluding *MAOP*, they all have the similarly shaped curves, with total costs increasing with increasing *s.d.*.

DOP incurs the least cost, which is about $16000. Other policies deviate from this base case as the *s.d.* becomes larger. After *DOP*, *NOHPb* has the lowest total cost, then *NOOP*, *NOHPa*, *NOHP*, *NOHP*, *LOHP*, *NAOP* and *MAOP* has the highest total cost.

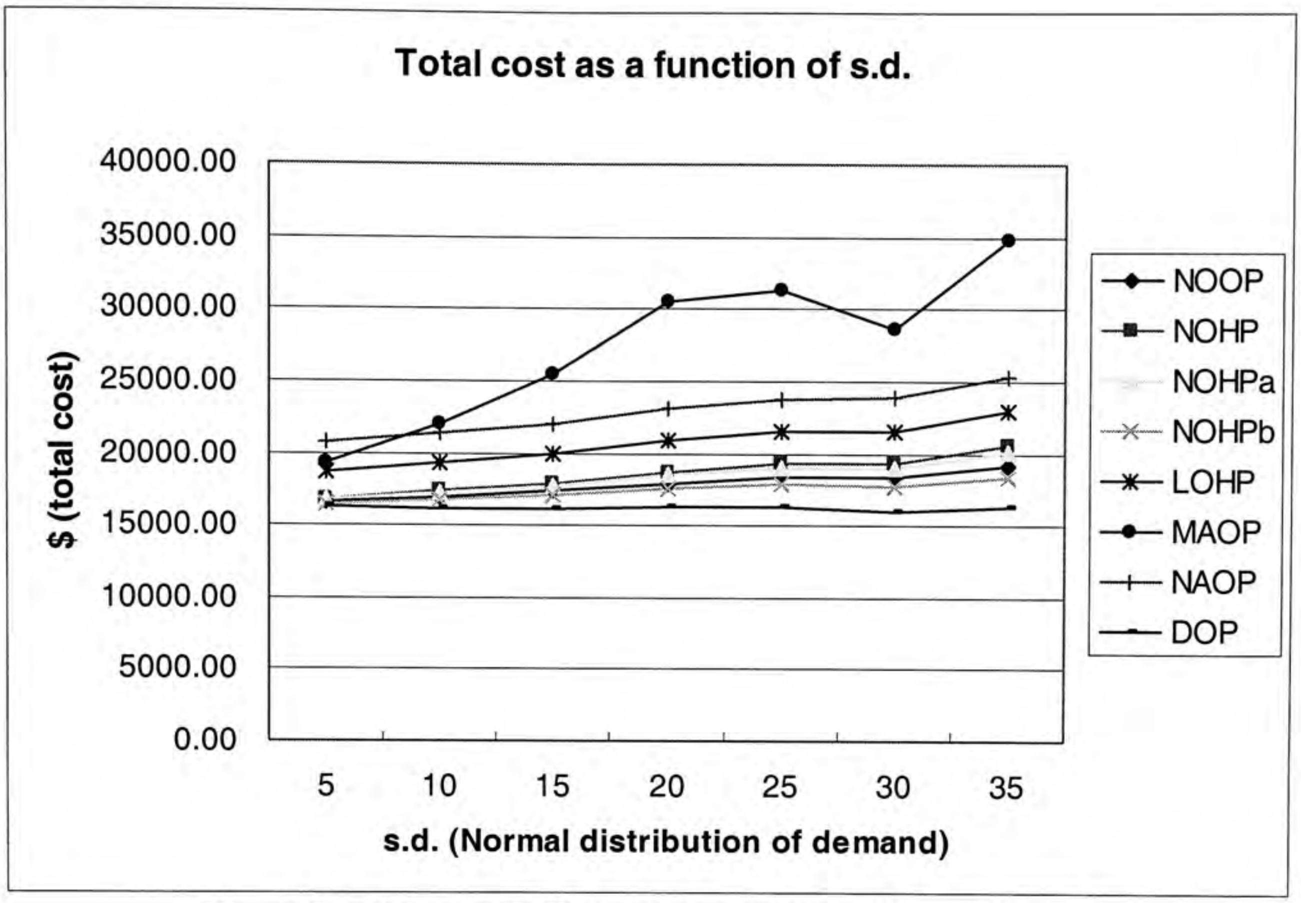


Figure 8: total cost of all simulation models with varying *s.d.*

Since in *NOOP*, *NOHP*, *NOHPa* and *NOHPb*, we have taken expectation on the customer demand, if the demand is fluctuating a lot, this policy would do worse than when the demand is stable. (See Figure 8) Moreover, the amount of total order would deviates more from the total actual demand (See Table 4), in an environment where demand is fluctuating.

It is interesting that although *MAOP* has the order amount closest to the real demand in average, (See Table 4) the average total cost of it is the largest. Only when the *s.d.* is small, this policy is better than *NAOP*, since the real demand is really close to the mean.

Table 5 shows precisely how each policy performs comparing to *DOP*, in terms of total cost saving, which is calculated by

[(total cost of specific policy - total cost of *DOP*) / total cost of *DOP*] x 100%

--- *(1)*

We can see *NOHPb* does better than all the policies with stochastic demand, but more and more badly than *DOP* as *s.d.* increases. Please note that, in the base case:

s.d. equal to 30, *NAOP* incurs about 49% more than the total cost of *DOP*, compared with *NOHPb* which incurs only 10% more cost than *DOP*. This shows no advance ordering cost much more than placing advance orders.

Table 5: total cost saving of each policy compared to *DOP*, with varying *s.d.*

s.d.	*NOOP*	*NOHP*	*NOHPa*	*NOHPb*	*LOHP*	*MAOP*	*NAOP*
5	0.02	0.04	0.03	0.02	0.16	0.19	0.28
10	0.05	0.07	0.07	0.04	0.19	0.36	0.32
15	0.07	0.11	0.10	0.05	0.23	0.58	0.36
20	0.10	0.15	0.14	0.08	0.29	0.88	0.42
25	0.13	0.18	0.16	0.10	0.32	0.92	0.46
30	0.15	0.20	0.18	0.10	0.34	0.79	0.49
35	0.18	0.27	0.24	0.13	0.41	1.14	0.56
Average	0.10	0.15	0.13	0.07	0.28	0.69	0.41

Case II: Comparison between all simulation models by varying the discount rate (β_1)

In this case, we would like to see the change of the heuristics' performance by varying the discount rate, β_1, which is given to the advance order. The range of β_1 is explored from 0 to 1 in steps of 0.2.

Table 6: β_1 setting

β_1	0.0	0.2	0.4	0.6	0.8	1.0

In Figure 9, we observe that Order (1) is increasing and Order (2) is decreasing nonlinearly with the increasing β_1 (They are also clearly shown in Figure 10 and Figure 11), but the total order is kept at around 2000. For the case of 0 unit purchasing cost, *NOOP* has ordered totally 18999.96 items in advance and 114.85 items only for day 1. This advance order seems too high compared to a daily average demand of 100 items.

If a lower discount rate is given to advance order, *NOOP* would place more advance order and reduce the order for the current day, so as to get more discount benefit. This increase amount of advance order is not linear but exponential. The reason is that it wants to prevent the high shortage cost. In addition, the unit purchasing cost is discounted because of advanced ordering.

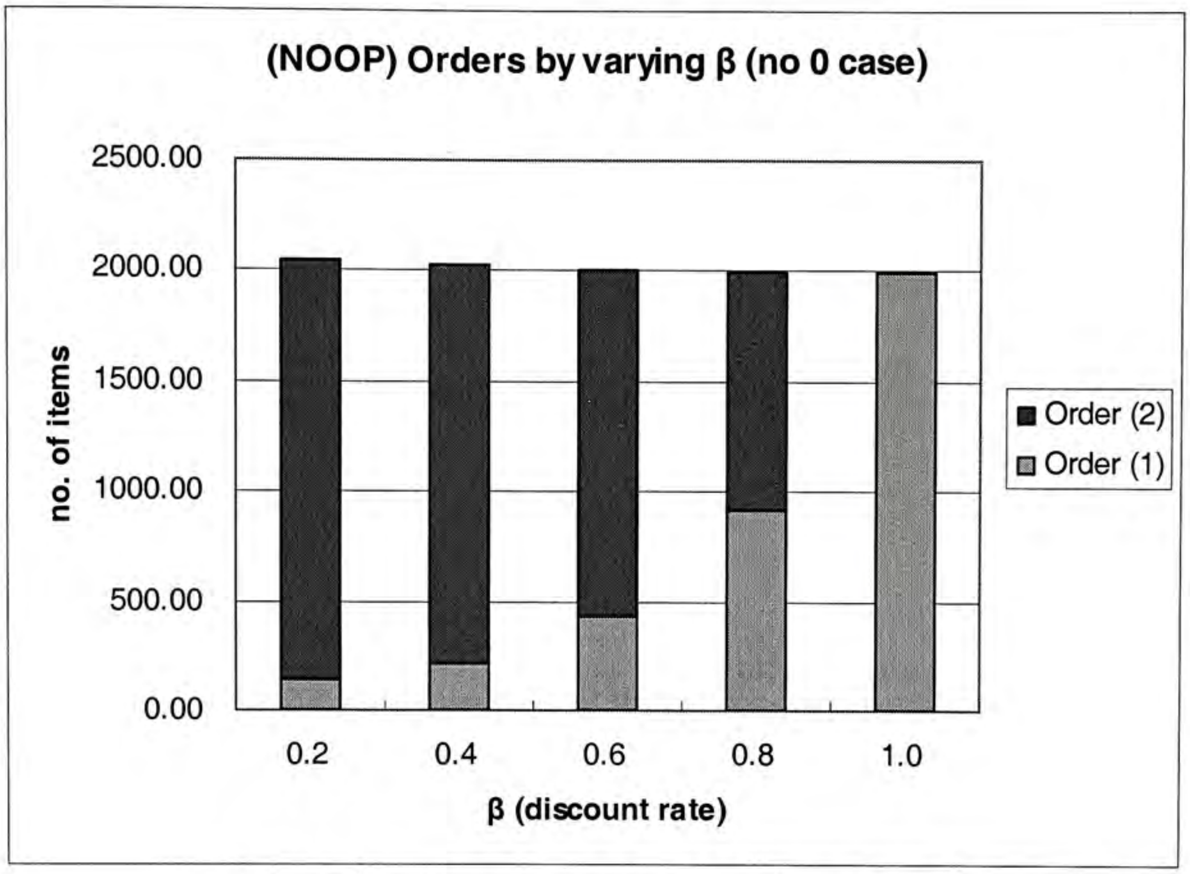


Figure 9[7]: order pattern of *NOOP* with varying β_1

In Figure 10, we see that the Order (1) for *NAOP, NOHP, LOHP*, *MAOP* and *DOP* are about 1987, 319, 1039, 100 and 98, respectively for all values of β_1 since these policies do not depend on β_1. (There is no β_1 involved in their formulations) For the same reason, in Figure 11, we see that Order (2) of *NAOP, NOHP, LOHP*, *MAOP* and *DOP* are constant at about 0, 1681, 955, 1900 and 1881, respectively. So, the sum of both orders is around 2000. (See Figure 12 in page 143)

Order (1) of *NOOP* is increasing convexly, from around 115 to 1999, in Figure 10 and Order (2) is decreasing concavely, from around 19000 to 0 in Figure 11. The resulting total order is decreasing convexly from around 19115 to 1999 in Figure 12. Excluding the case of zero discount rate, the total order of *NOOP* is about 2000, so the concave curve in Figure 11 is a flip of the convex curve in Figure 10.

[7] A detailed figure is in Appendix VI

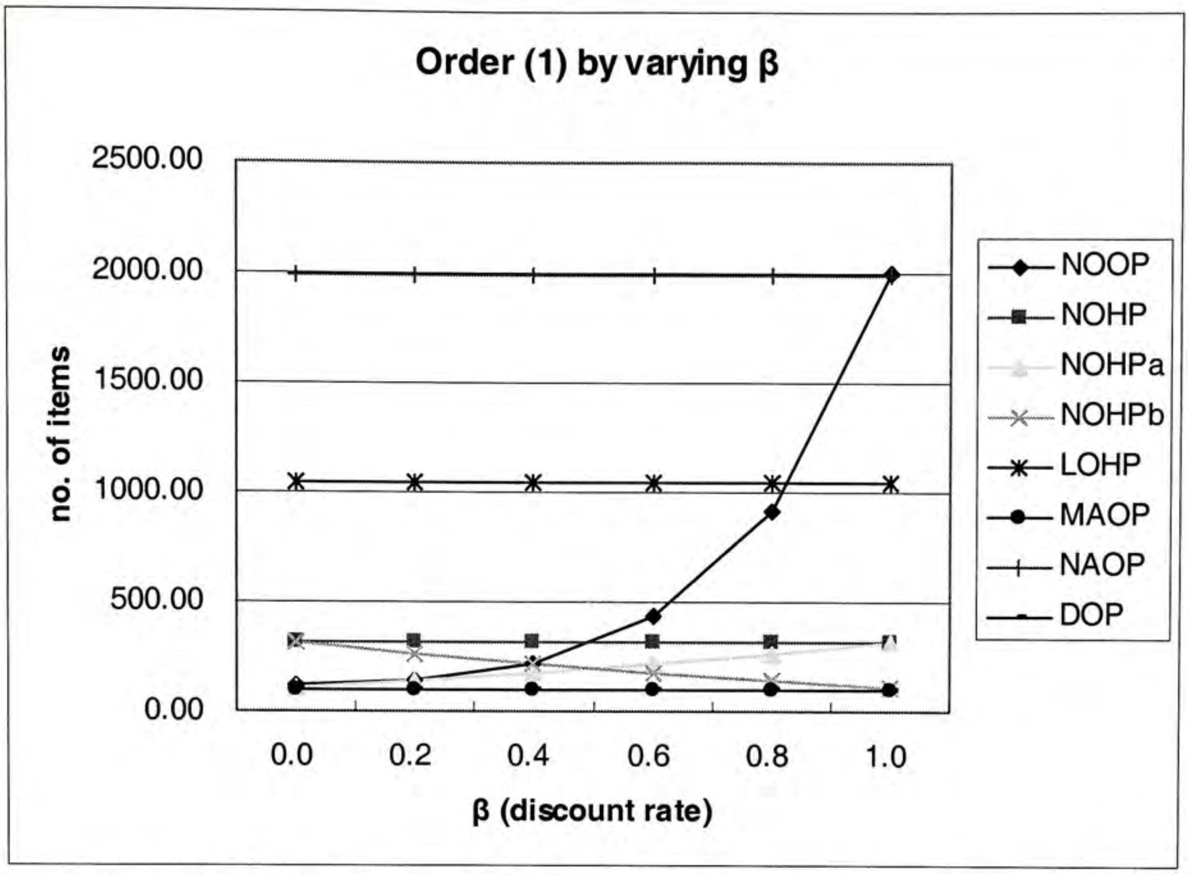


Figure 10: Order (1) patterns of all simulation models with varying β_1

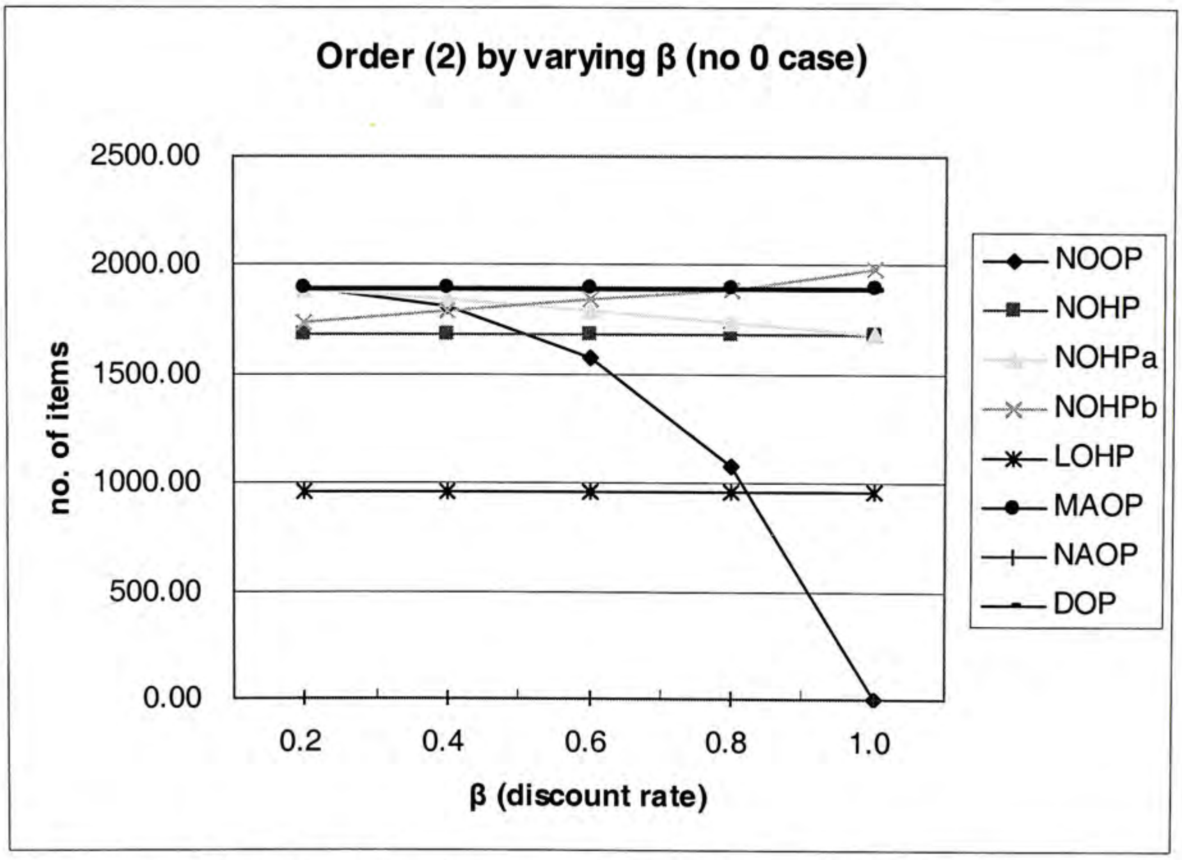


Figure 11[7]: Order (2) patterns of all simulation models with varying β_1

Order (1) of *NOHPa* (*NOHPb*) is increasing (decreasing) convexly, from around 108 to 319, in Figure 10 and decreasing (increasing) concavely, from around 1981 to

1681 in Figure 11. The resulting total order is decreasing (increasing) and slightly convexly from around 2089 to 2000 in Figure 12.

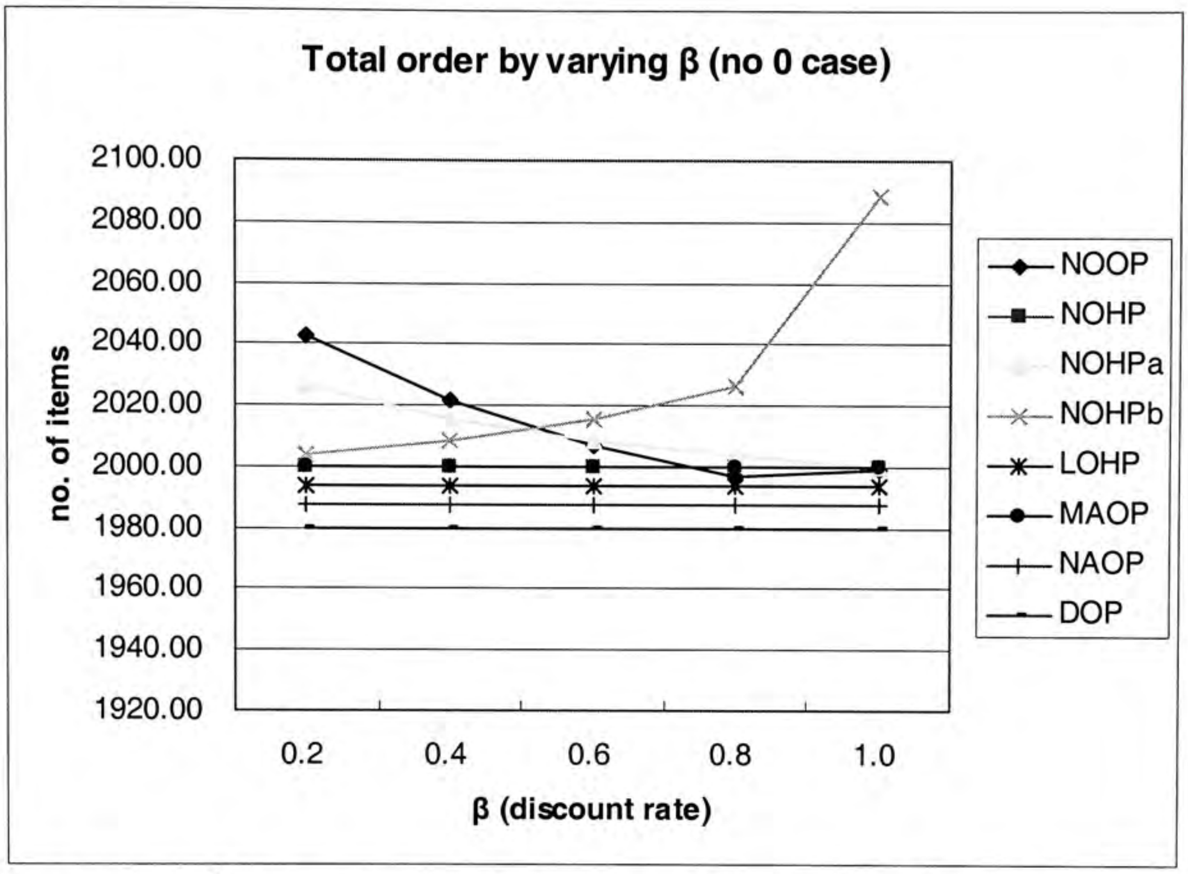


Figure 12[7]: total order patterns of all simulation models with varying β_1

The total order of *NOOP* at a lower discount rate is much higher than at higher discount rate; but excluding the case of zero unit purchasing cost, we find that the opportunity of advance ordering increases the size of total orders only very slightly, by less than 2.2%.[8]

In Figure 13, we find that the total holding costs of the policies *NAOP, NOHP, LOHP* and *MAOP* are staying at levels \$2, \$3, \$2 and \$4 respectively.

In Figure 14, we find that the total shortage costs of the policies *NAOP, NOHP, LOHP* and *MAOP* are staying at levels \$3955, \$2670, \$3471 and \$12442 respectively.

[8] When $\beta_1 = 0.2$, the total order is 2042.93; when $\beta_1 = 1$, the total order is 1998.89.

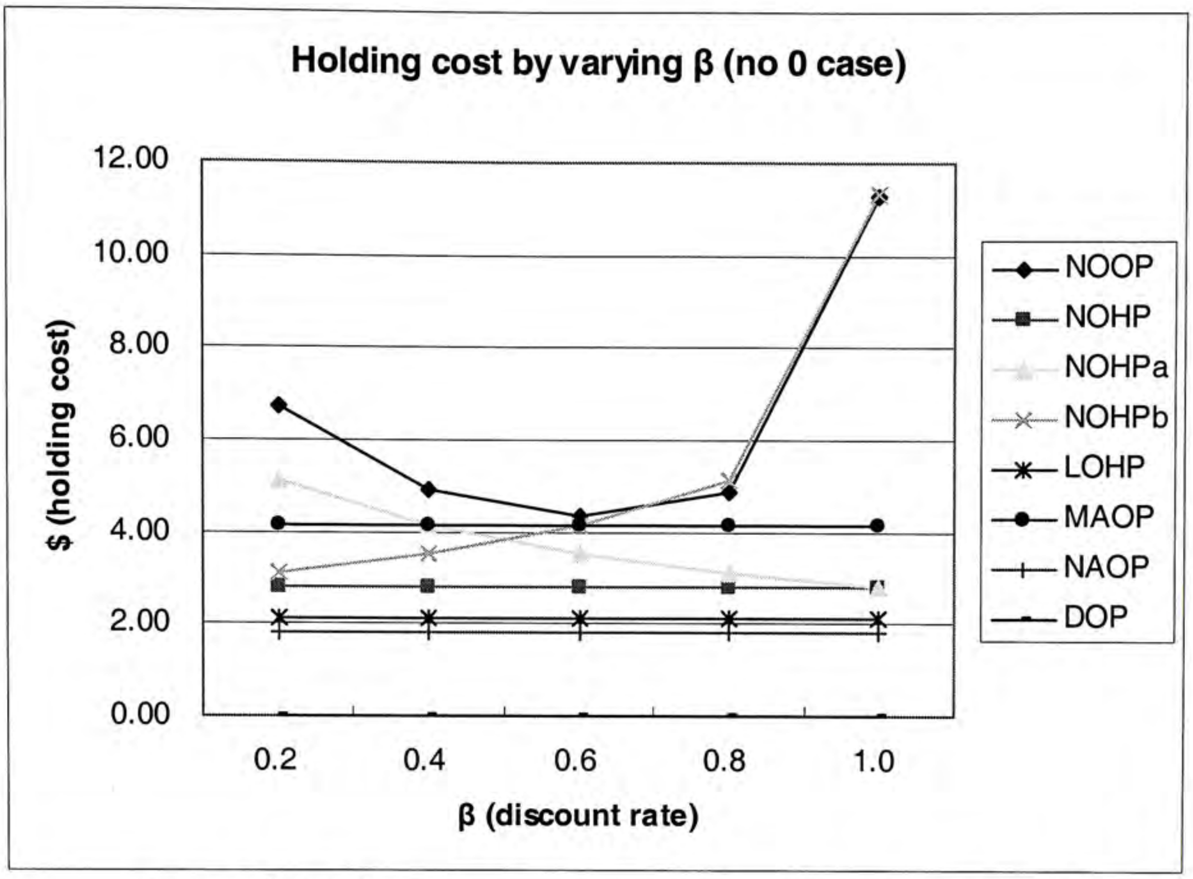


Figure 13[7]: holding cost of all simulation models with varying β_1

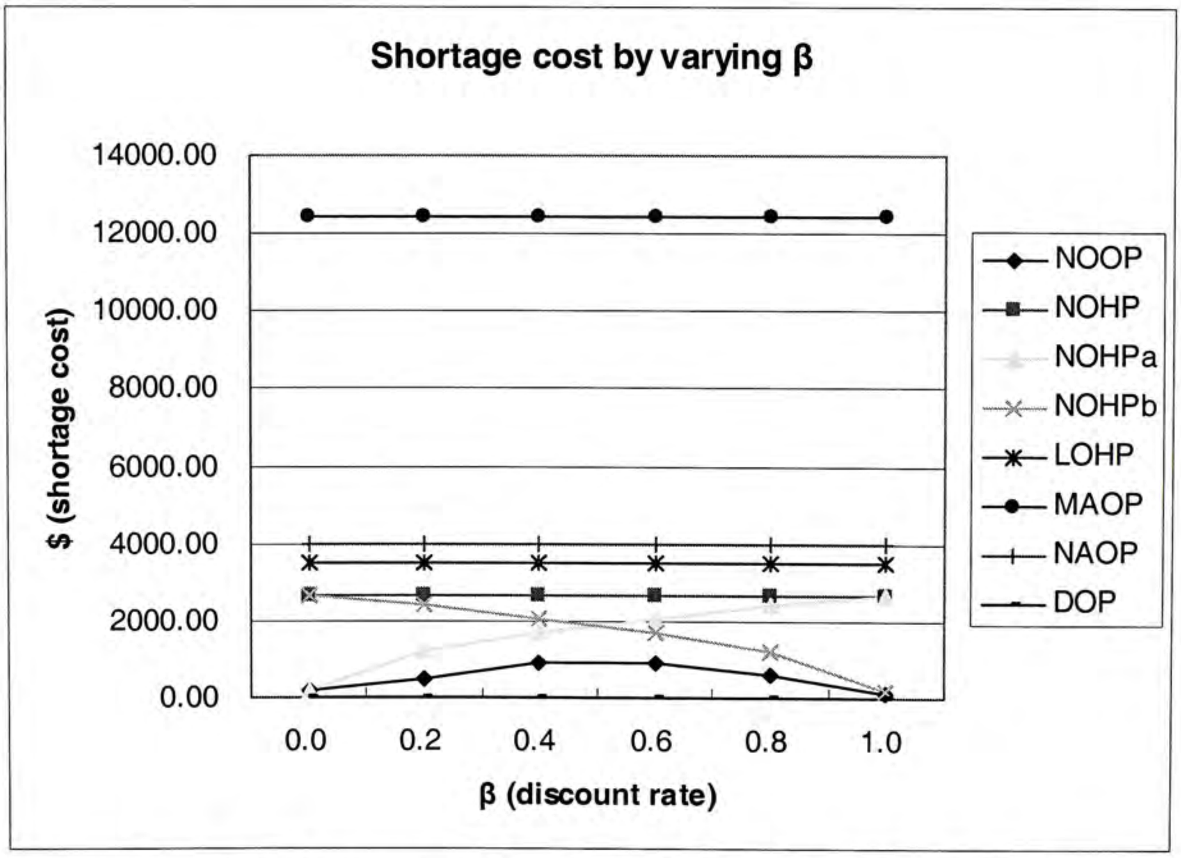


Figure 14: shortage cost of all simulation models with varying β_1

It is interesting to note that for *NOOP*, *NOHPa* and *NOHPb*, the holding cost are convex curves (*NOOP* with a minimum at $\beta_1 = 0.6$, valued \$4.33) and the shortage costs are concave curves (*NOOP* with a maximum at $\beta_1 = 0.4$, valued \$894.38); but the total cost increases quite linearly with the discount rate. (See Figure 15)

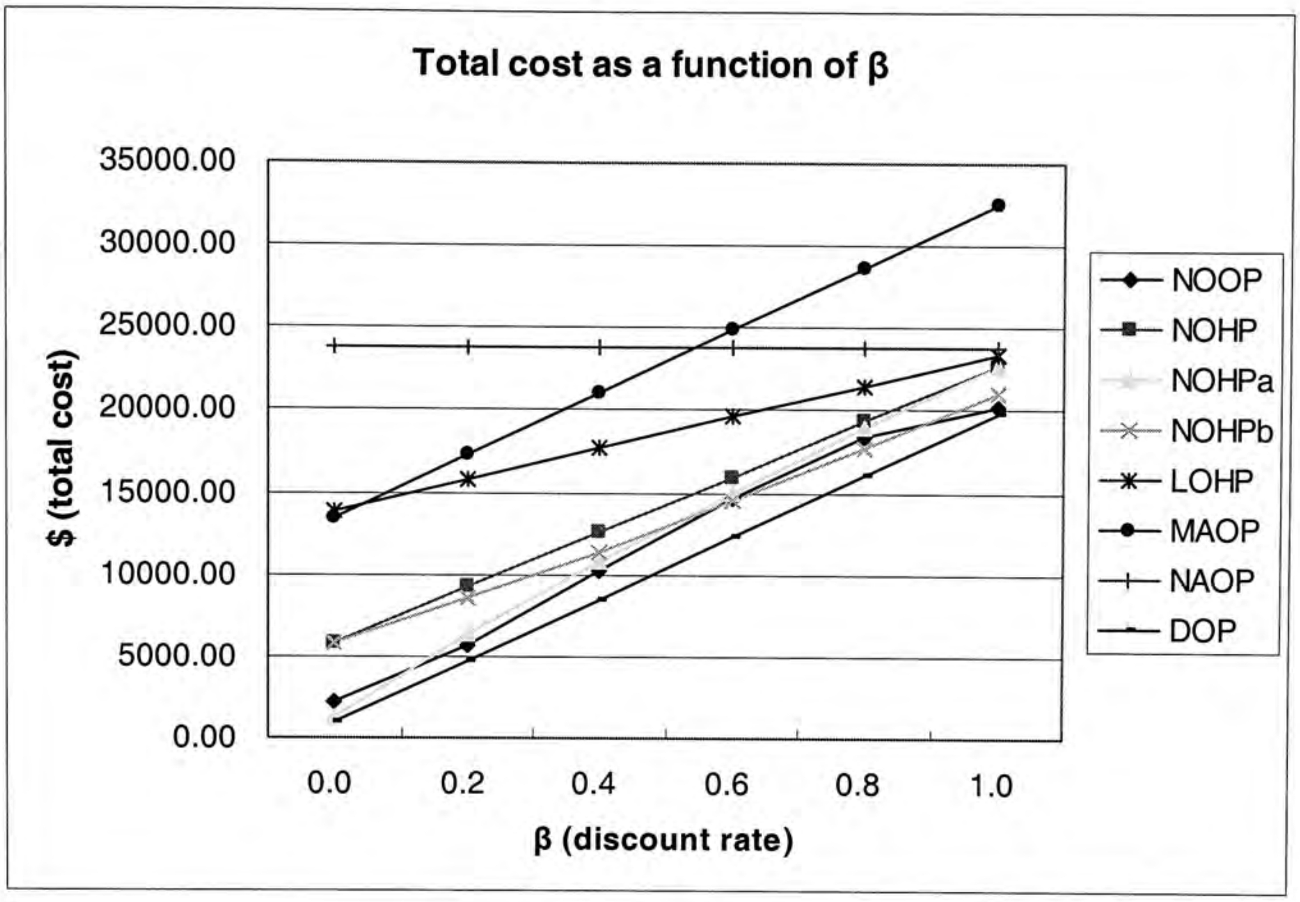


Figure 15: total cost of all simulation models with varying β_1

The total costs of different simulation models are shown in Figure 15. *NAOP* is constant at about \$23831 since this policy is not changed by changing discount rates. On the other hand, as *DOP* always can place the exact demand order in advance and get the benefit of discount, it gets the lowest total cost. *NOOP* has less total cost than *NOHP* except for the slight fluctuation at high values of β_1. *NOHP* has less total cost than *LOHP*; costs for both increase linearly with the increasing β_1.

If the discount rate is less than 0.5, *NOHPb* is doing better than *NOHPa*; vice versa, if the discount rate is larger than 0.5 (including the base case setting). This can be explained by the remark relating to *NOHPb* in Section B.5.1.

Please note that when there is no discount for advance order, that is $\beta_1 = 1$, *NOOP* will not place order in advance, but it is still doing better than *NAOP* in terms of total cost, since *NOOP* has looking ahead one day for the total cost minimization while *NAOP* just consider the cost minimization of the current day.

Since *NAOP* only places orders for the current day, it is independent of the discount rate. Thus, the number of orders, the holding cost, the shortage cost and the total cost are constant for all values of discount rates.

As the amount ordered by *NOHP* and *LOHP* are constant, the holding costs and the shortage costs are also constant, but since the purchasing costs increase with discount rate, the total costs increase. When there is no discount rate ($\beta_1 = 1$), there is not much difference between *NOHP*, *LOHP* and *NAOP*.

MAOP can sometimes do better than *NAOP* if the discount rate given is small enough (in our simulation, it need to be less than about 0.5) in order that its inflexible ordering practice, (i.e., placing orders without review on-hand inventory level) does not have significant impact.

Table 7 shows numerically how each policy performs comparing to the *DOP*, in terms of total cost saving, defined as Eq.(1) on page 139. In average, we can see *NOHPa* does better than all the other policies with stochastic demand. The savings of all the policies with small β_1 are much larger than those with large β_1, especially for *NAOP*.

Table 7: total cost saving of each policy compared to *DOP*, with varying β_1

β_1	*NOOP*	*NOHP*	*NOHPa*	*NOHPb*	*LOHP*	*MAOP*	*NAOP*
0.0	1.31	4.97	0.31	4.97	13.10	12.68	23.25
0.2	0.21	0.94	0.34	0.80	2.32	2.63	4.02
0.4	0.21	0.48	0.27	0.34	1.08	1.47	1.80
0.6	0.20	0.30	0.22	0.18	0.60	1.03	0.94
0.8	0.15	0.20	0.18	0.10	0.34	0.79	0.49
1.0	0.02	0.15	0.15	0.07	0.18	0.64	0.20
Average	0.35	1.17	0.24	1.08	2.94	3.21	5.12

From Table 7, we note that *NOHPb* is a very reasonable heuristic over a wide realistic range of β_1, 0.5 - 1.0, while *NOHPa* is a very reasonable heuristic over the range of β_1, 0.1 - 0.4.

Case III: Comparison between all simulation models by varying the unit purchasing cost (c)

In this case, we would like to see the change of the heuristics' performance by varying the purchasing cost, c. The range of c is from 0.0055 to 100 as shown in Table 8.

We choose 2, 50 and 100 because we want to see the effect when c is very small (10 divided by 5), large (10 times 5) and very large (10 times 10), respectively. We set c equal to 0.0055 and 25, because we also want to know the effect when the unit purchasing cost is equal to the unit holding cost and the unit shortage cost, respectively. The value of 12.5 is chosen since it makes the Newsboy ratio equal to 0.5.

Table 8: *c* setting

c	0.0055	2	10	12.5	25[9]	50[9]	100[9]

Figure 16 shows that before $c = 50$, the pattern of Order (1) has a convex shape and Order (2) behaviors conversely that it shows a concave shape. If $c = 0.0055$, *NOOP* has ordered 18999.96 items in advance and only 203.64 for current day. The total order is very high compared to a daily average demand of 100 items. For $50 \le c \le 100$, Order (1) is about 1890 and Order (2) is equal to zero. Generally, total orders gradually decline with increasing c. (See Figure 19)

If the purchasing cost is very small, *NOOP* would order a lot of items for the current day to prevent backorder. The low shortage cost is shown in Figure 21. If the unit purchasing cost is very large (larger than the shortage cost), *NOOP* places current

[9] Note that this case violates the assumption $q \ge c$.

orders only and no advance order is placed. It would like to buy the exact amount of items when they are really needed. That means the demand will be backlogged first and fulfilled in the next day. If the unit purchasing cost is less than $12.5 (where Newsboy ratio = 0.5), *NOOP* would place more advance order than current order.

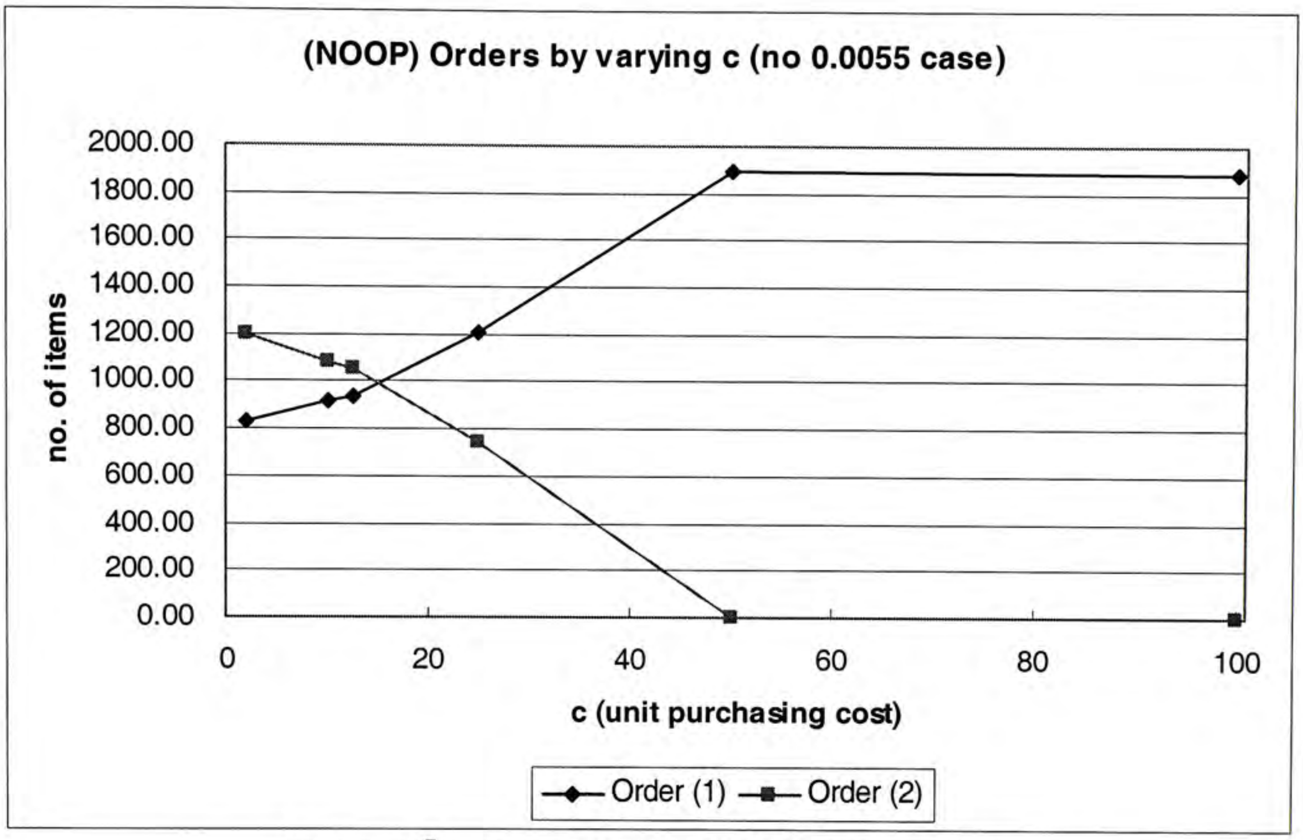


Figure 16[7]: order pattern of *NOOP* with varying *c*

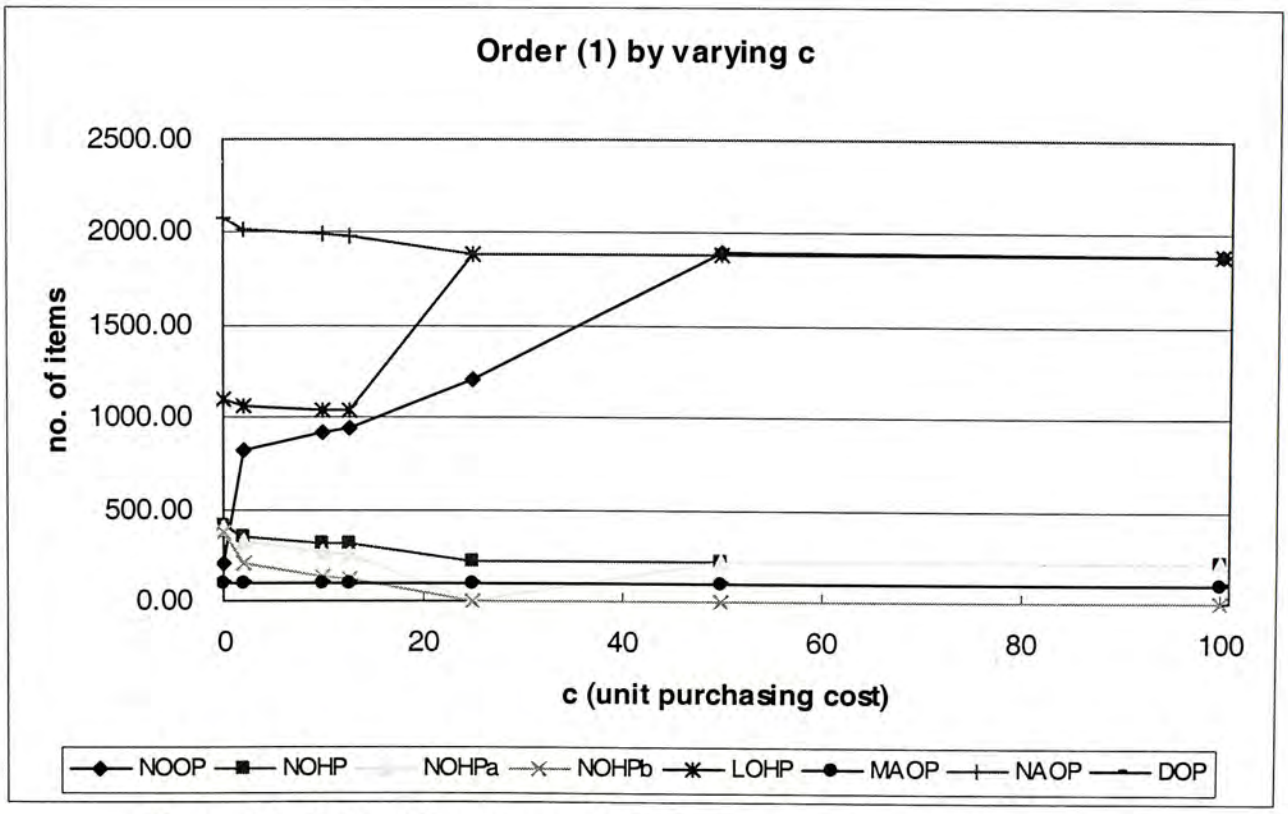


Figure 17: Order (1) patterns of all simulation models with varying *c*

Figure 17 shows the total current day orders for all the heuristics. Only for *MAOP* and *DOP* do the total current day orders stay constant at 100 and 98 respectively. The total current orders for *NAOP* and *NOHP* gradually decreases and stays constant for $c \geq 25$. The current day orders for *NOHPa* and *NOHPb* are approximately convex functions of *c* with minima at 25.

From Figure 17, we see that for both *NOHPa* and *NOHPb*, current orders decrease as the unit purchasing cost increases. (Except when the unit purchasing cost is equal to the shortage cost) This is because the increase of unit purchasing cost lowers the Newsboy critical fractile which is involved in their formulae for deciding y_t^t .

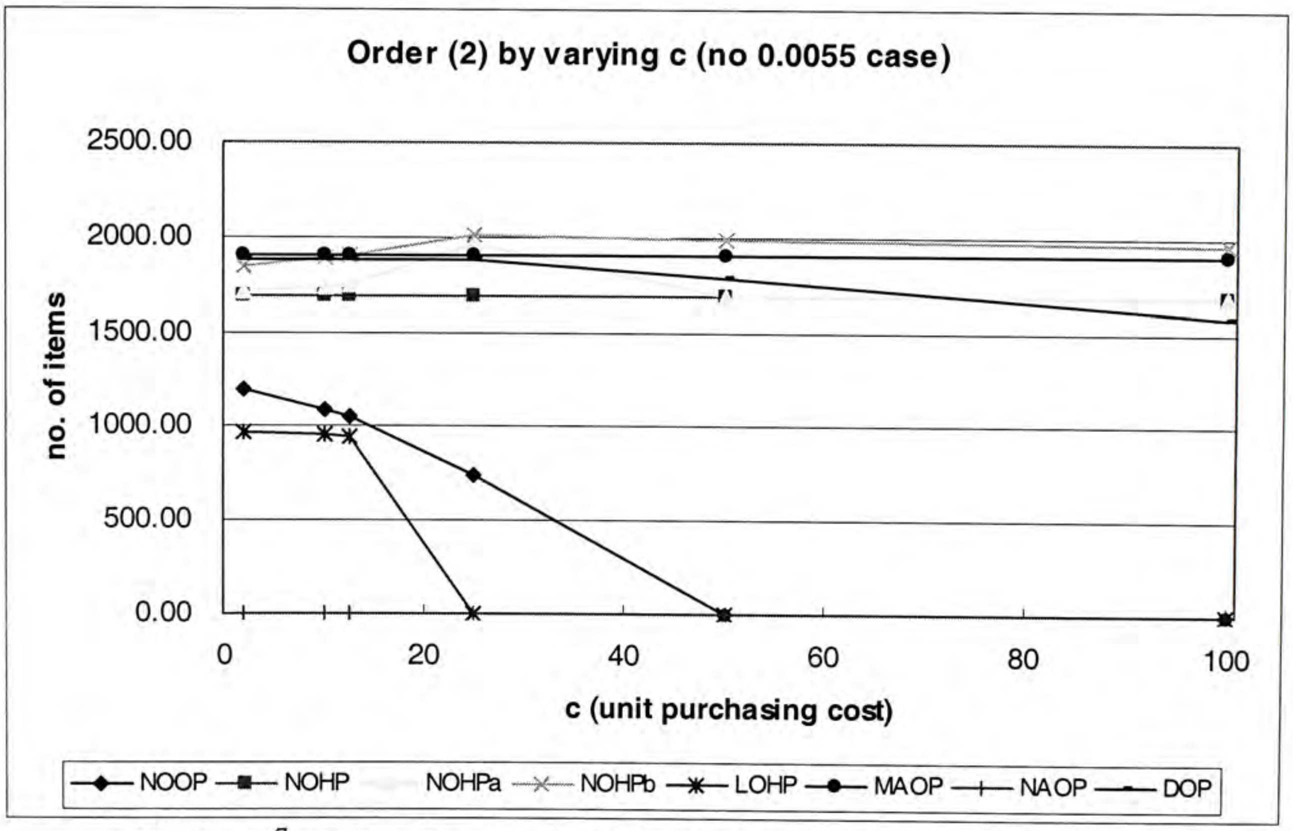


Figure 18[7]: Order (2) patterns of all simulation models with varying *c*

Figure 18 shows that the advance orders of *NOHP* and *MAOP* stays constant at 1681 and 1900, respectively for all values of *c*. That for *DOP* decreases gradually from 1881 when the unit purchasing cost is larger than the unit shortage cost, since it is better to backlogging rather than to fulfilling the orders by buying the very expensive items. For *NOOP*, *NOHPa*, *NOHPb* and *LOHP*, Order (2) as a function of *c* appears to be an upside-down flip of in Figure 17, thus resulting in total orders staying constant at about 2000. (See Figure 19)

For *NOHP*, when the unit purchasing cost is increasing, the orders for the current day become less but the advance orders are kept constant. Current orders decrease because the increase of the unit purchasing cost lowers the Newsboy critical fractile T^*. As a result, the total order is decreasing.

For *NOHP*, *NOHPa* and *NOHPb*, in particular, when the unit purchasing cost equals to the unit shortage cost, we only buy very few items in current day. By violating the assumption that the purchasing cost is less than the shortage cost, then $T^* = 0$ which means that it is more economic to backorder demand rather than to fulfill the demand immediately by purchasing the very high price items. At the time, all demand is backlogged and the policies would just place advance orders to try to fulfill it (Except the last period's demand, which are even left unfulfilled). In the following day, only the backorders that cannot be fulfilled by the committed order would be filled by the current order. As the policies would only fulfill demand in next day, only few items can be left from committed orders and after fulfilling the current demand. Thus, although the shortage costs and total costs increase, a lot of purchasing and holding costs can be saved.

As the unit purchasing cost increases, the order *NAOP* placed decreases to save the purchasing cost. However, as mentioned before, since *NAOP* only considers the optimization of 1-day, it will not order much to prevent the backorder and hold the inventory for the next day. So, it has the lowest holding cost and the highest shortage cost and results in the highest total cost. In particular, the unit purchasing cost equal to the unit shortage cost is the threshold to stop buying any items as the optimal inventory level is found to be zero (Newsboy level equals to zero). The policy will backorder all the demand in the current day and fulfill them in the next day. The situation is similar to *NOOP*. Please refer to the explanation in the corresponding paragraph above.

For *LOHP*, firstly both the current order and the advance order are decreasing to save purchasing cost, until the critical point that unit purchasing cost equals to the unit shortage cost, all the orders are then placed by the current day and the advance order drops to zero. Thus, the total orders are decreasing with the increasing unit purchasing cost up to the critical point, after when the total orders keep at certain level, to save the total purchasing cost. This can be explained by Eq.5.1.(13) and Eq.5.1.(14). We note that when the unit purchasing cost is not less than the unit shortage cost, the Newsboy ratio and *OUL* drop to zero. *OUL* forces the advance order to zero, so all the demand needs to be filled only by current orders. This is just as the same as the case of *NAOP* and you can see both of their curves overlap after $c = q$.

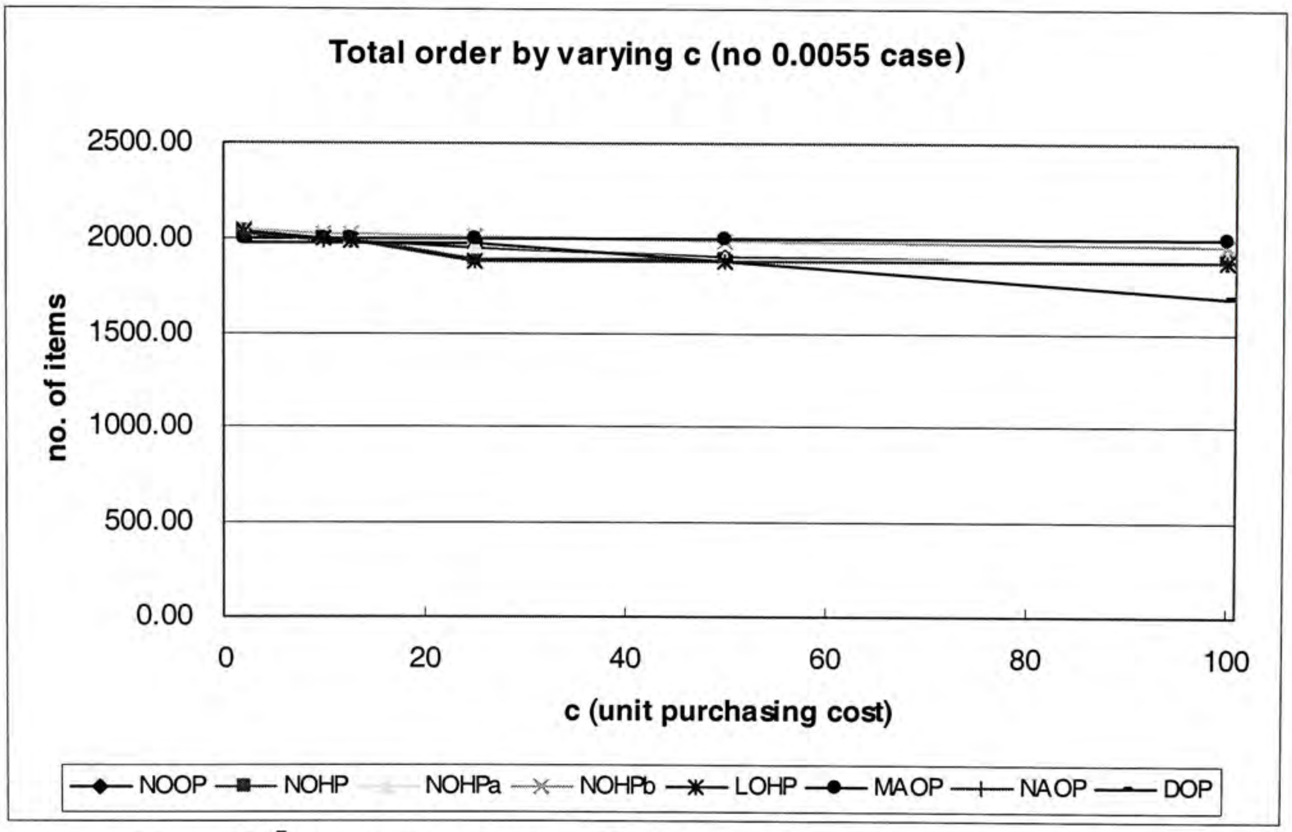


Figure 19[7]: total order pattern of all simulation models with varying *c*

In Figure 19, except *MAOP* staying at the level 2000, all the total order curves of different simulation models are decreasing with the increasing *c*. The total orders for *NOHP*, *NOHPa*, *LOHP* and *NAOP* are very similar and they stay constant after the unit purchasing cost equal to 25.

For *NOOP*, the total order placed decreases when the unit purchasing cost is high, so to save the purchasing cost. This is easy to understand when we think about in the last few periods, we would prefer backorder rather than fulfill the demand.

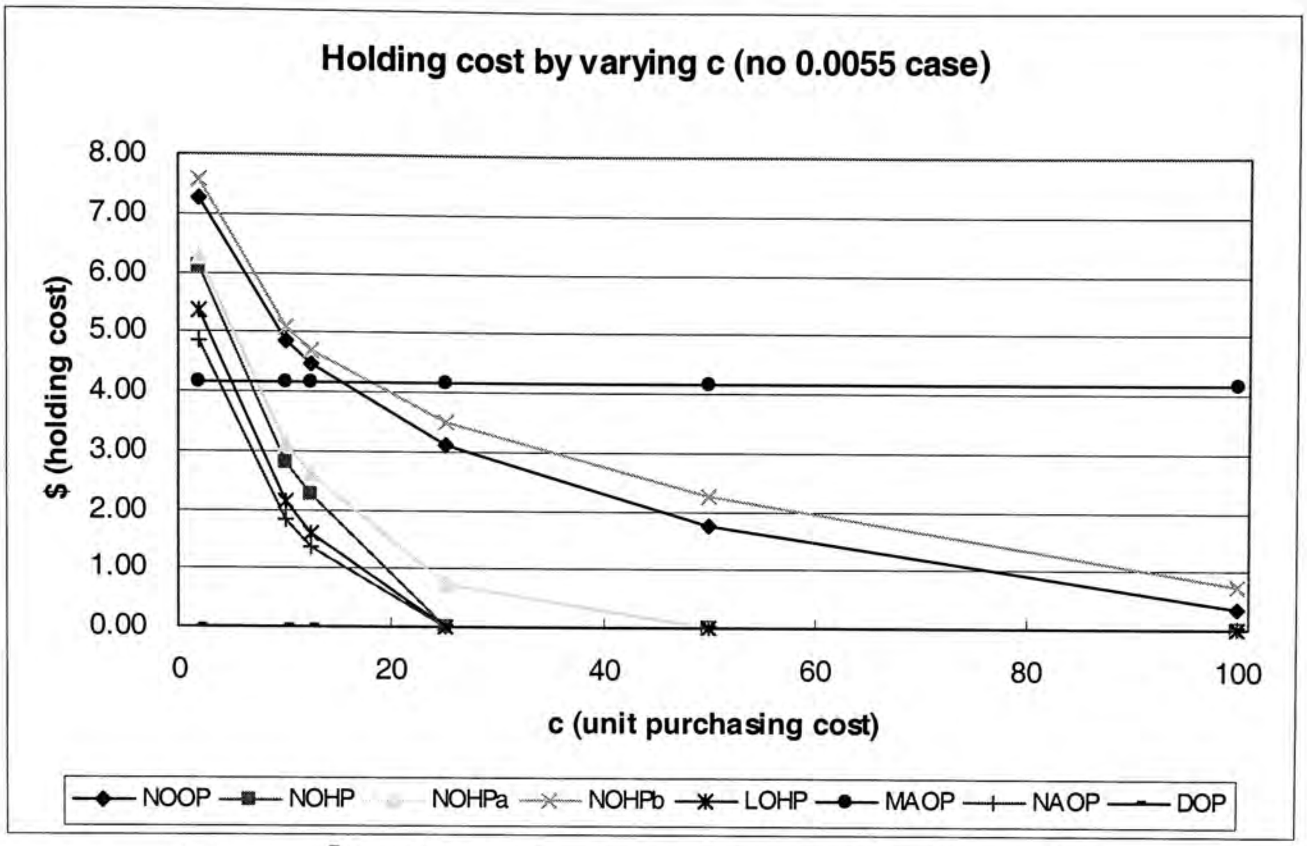


Figure 20[7]: holding cost of all simulation models with varying *c*

In Figure 20, the holding costs of all the simulation heuristics decrease convexly with increasing *c*, except *MAOP*. *NOHPb* performs very closely to *NOOP*. The shape of the curves of *NOHP*, *NOHPa*, *LOHP* and *NAOP* are similar. As *NOOP* places very large advance orders when *c* equal to 0.0055, it incurs very large holding cost.

For *LOHP* and *NAOP*, only current orders are placed when $c \geq q$; these policies are backlogging all the demand and fulfilling the backorders in the next day with current day orders. This is confirmed by Figure 20 which shows these policies hold zero inventories and by Figure 21 which shows very high shortage costs for these 2 policies.

Figure 21 shows that the shortage costs for *NOHP*, *NOHPa*, *NAOP* and *LOHP* have similarly profiles. For both *NAOP* and *LOHP*, the shortages cost stay constant at 49485 for $c \geq 25$. For *NOHP*, the shortage cost stays constant at 43741 for $c \geq 25$, whereas *NOHP* takes the same shortage cost for $c \geq 50$. The shortage costs for *NOOP*, *NOHPb* and *DOP* gradually increase with *c*.

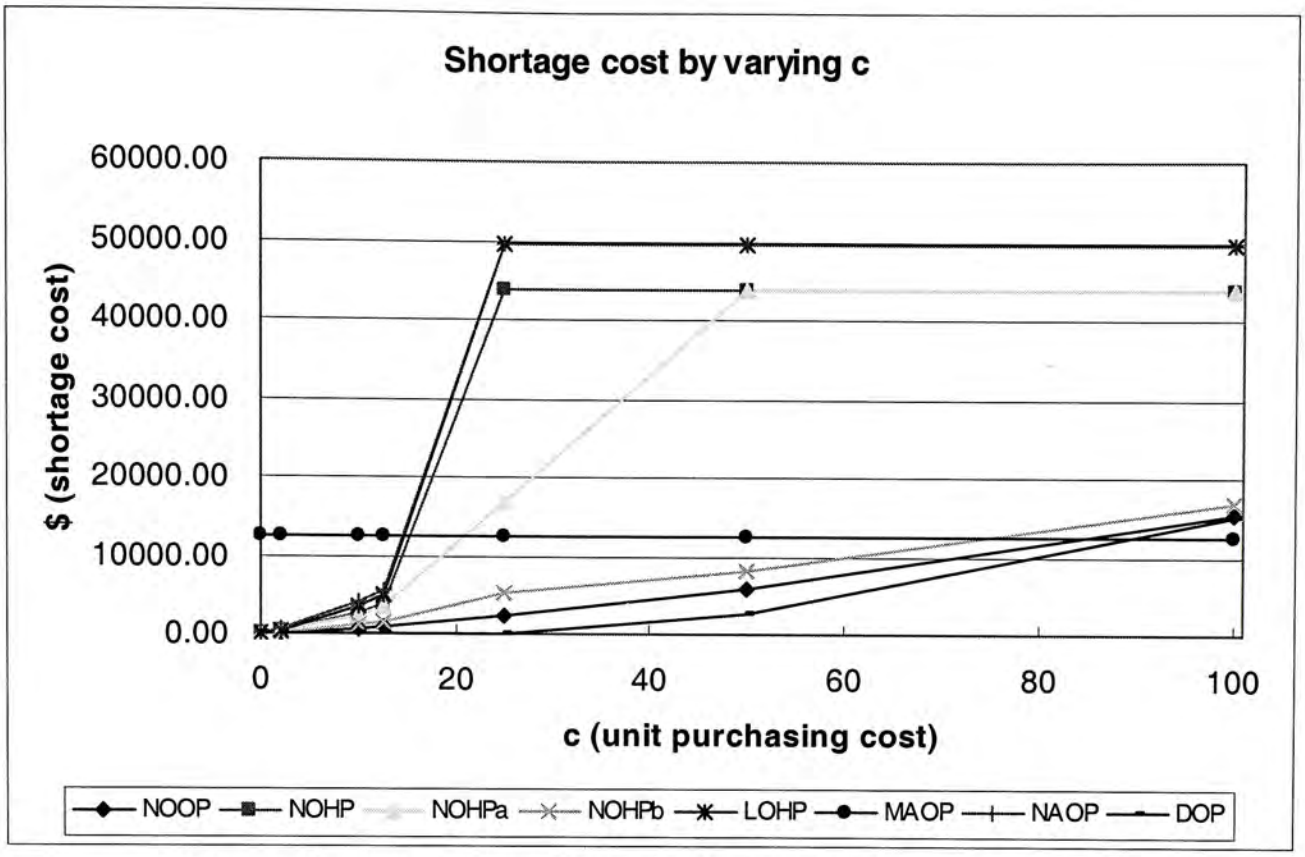


Figure 21: shortage cost of all simulation models with varying *c*

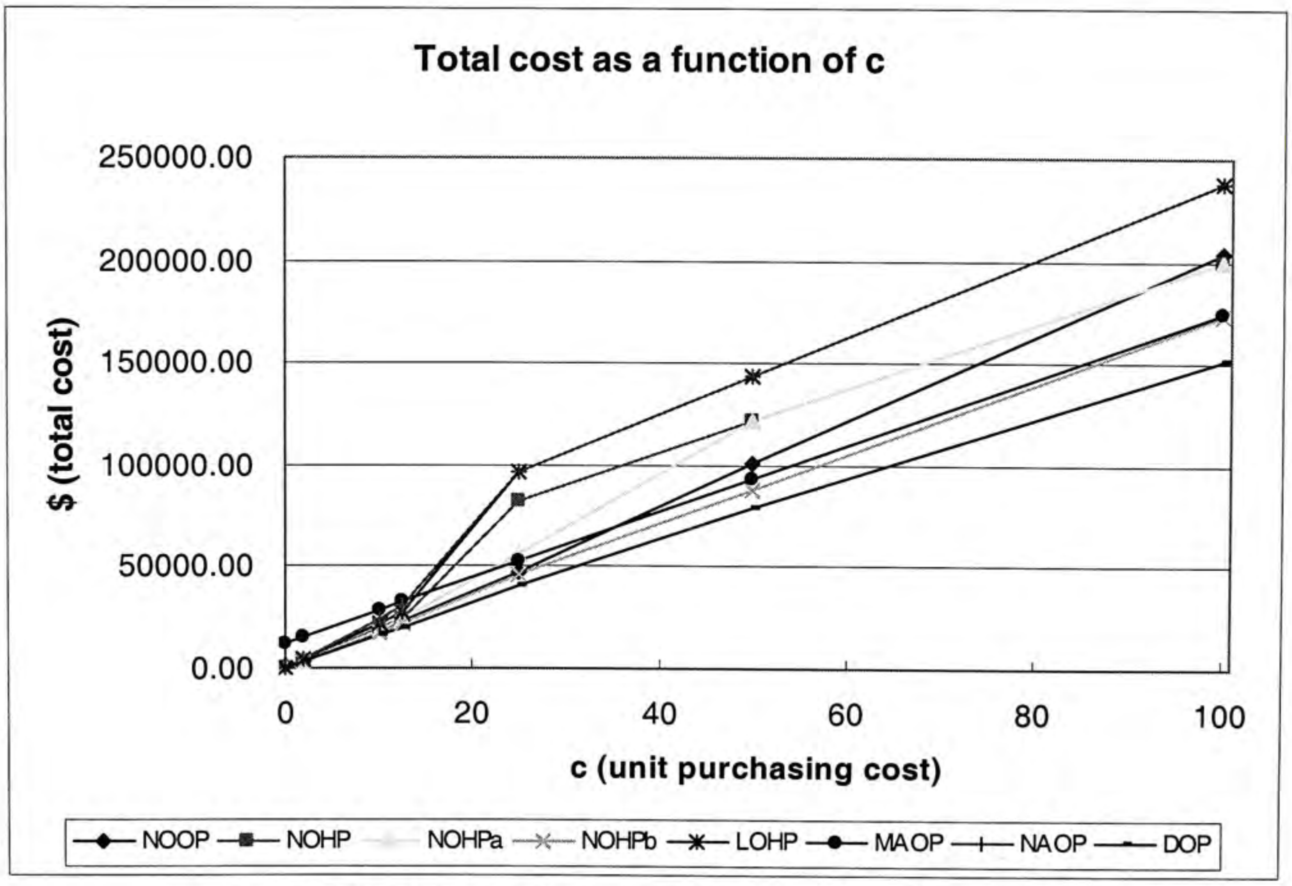


Figure 22: total cost of all simulation models with varying *c*

In Figure 22, we see that the total costs for all the heuristics are all increasing with c. Excluding *DOP*, *NOHPb* performs the best. Surprisingly, *MAOP* performs the next best for $c \geq 25$. *LOHP* is always very close to *NAOP* and both perform the worst after $c = 25$. *NOOP*'s performance is approximately the average of all the policies. Note that *NOOP, NOHPb, MAOP* and *DOP* increase linearly with *c*.

For *NOOP*, we have made the assumption, $q > c$, in our calculation. So, in cases when the unit purchasing cost and shortage cost settings violate this assumption, we find that it does much worse than when $q > c$ and has higher total cost than several policies.

In general, *NOHP*, *LOHP* and *NAOP*'s low-buying behavior (order less when the unit purchasing cost is high) have made the holding cost even drop to the zero level and the shortage cost increase to very high. The total cost is worse than *NOOP*. But this result is also under the assumption that the purchasing cost is less than the shortage cost, otherwise, *NOHP* (and *NOHPa*) has lower total cost than *NOOP*, as shown in Figure 22.

Table 9 shows numerically how each policy performs comparing to the *DOP*, in terms of total cost saving, defined as Eq.(1) on page 139. We can see *NOHPb* does better than all the policies with stochastic demand, if $c < q$, except when *c* equals to 0.0055, where *NOHP* and *NOHPa* do the best.

Table 9: total cost saving of each policy compared to *DOP*, with varying *c*

c	*NOOP*	*NOHP*	*NOHPa*	*NOHPb*	*LOHP*	*MAOP*	*NAOP*
0.0055	116.70	1.50	1.50	1.52	1.51	1411.13	1.55
2	0.14	0.15	0.14	0.10	0.27	3.89	0.41
10	0.15	0.20	0.18	0.10	0.34	0.79	0.49
12.5	0.15	0.23	0.20	0.10	0.37	0.63	0.51
25	0.18	1.06	0.40	0.14	1.41	0.32	1.41
50	0.28	0.55	0.55	0.11	0.82	0.19	0.82
100	0.35	0.32	0.32	0.15	0.57	0.15	0.57
Average	16.85	0.57	0.47	0.32	0.76	202.44	0.82

From Table 9, we observe that *NOHP*, *LOHP* and *NAOP* cost much higher than *NOOP* when the unit purchasing cost equals to the unit shortage cost, because at this point, the Newsboy ratio drops to zero. However, since this is not a single period problem and fully backlogging is allowed, large amount of shortage cost is incurred to the retailer. When the unit purchasing cost is larger than the unit shortage cost, the total cost of the 3 policies and *NOOP* would be close because the 3 policies would save a lot of money by delaying the purchase of expensive items.

Case IV: Comparison between all simulation models by varying the unit holding cost (*h*)

In this case, we would like to see the change of the heuristics' performance by varying the holding cost, *h*. The range of *h* is from 0.0011 to 25 as shown in Table 10.

We choose 0.0011, 0.0275 and 0.055 because we want to see the effect when *h* is very small (0.0055 divided by 5), large (0.0055 times 5) and very large (0.0055 times 10), respectively. We set *h* equal to 10 and 25, because we also want to know the effect when the unit holding cost is equal to the unit purchasing cost and the unit shortage cost, respectively. The value of 5 is chosen since it makes the Newsboy ratio equal to 0.5.

Table 10: *h* setting

h	0.0011	0.0055	0.0275	0.055	5	10	25

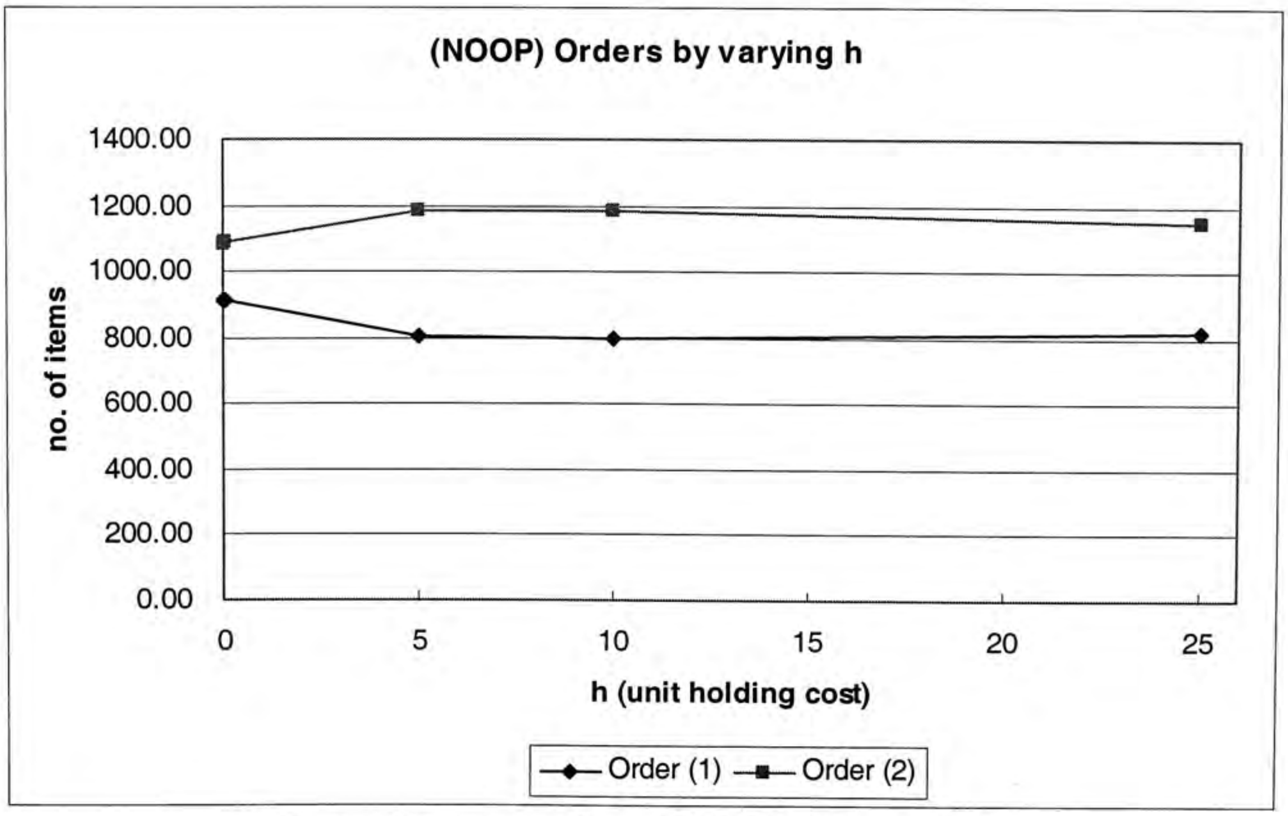


Figure 23: order pattern of *NOOP* with varying *h*

In Figure 23, we see that Order (2) is always larger than Order (1) for *NOOP*. Order (1) as a function of *h* shows a slightly convex shape with minimum at 10, and Order (2) also shows concave property with maximum at 10. The total order is gradually decreasing with increasing *h*.

Generally, the change of unit holding cost doesn't affect the policy's ordering behavior much, comparatively to the change of other parameters. Please note that if the unit holding cost is as the same as the unit purchasing cost, *NOOP* places the largest advance order and the smallest current order.

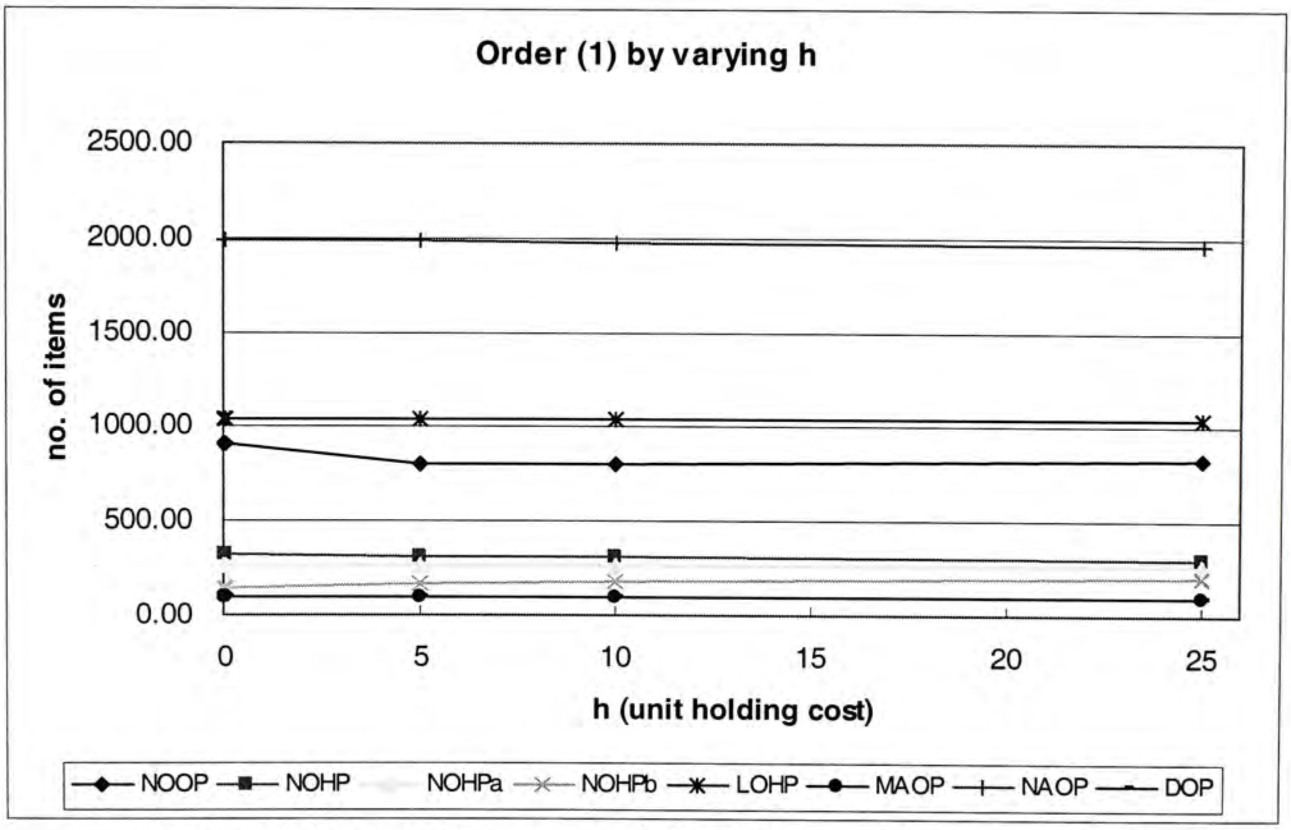


Figure 24: Order (1) patterns of all simulation models with varying *h*

Figure 24 shows that the sizes of the current order for *NAOP, NOHP* and *LOHP* are very slowly decreasing with increasing *h*; this is because increasing of unit holding cost reduces the critical fractile of the Newsboy model gradually. The size of the current order for *NOHPa* fluctuates about 266, while that for *NOHPb* increases with increasing *h*.

Figure 25 shows that the size of the advance order for *NOHP* is approximately constant at 1681, while those for *LOHP*, *NOHPa* and *NOHPb* are slowly decreasing with increasing *h*.

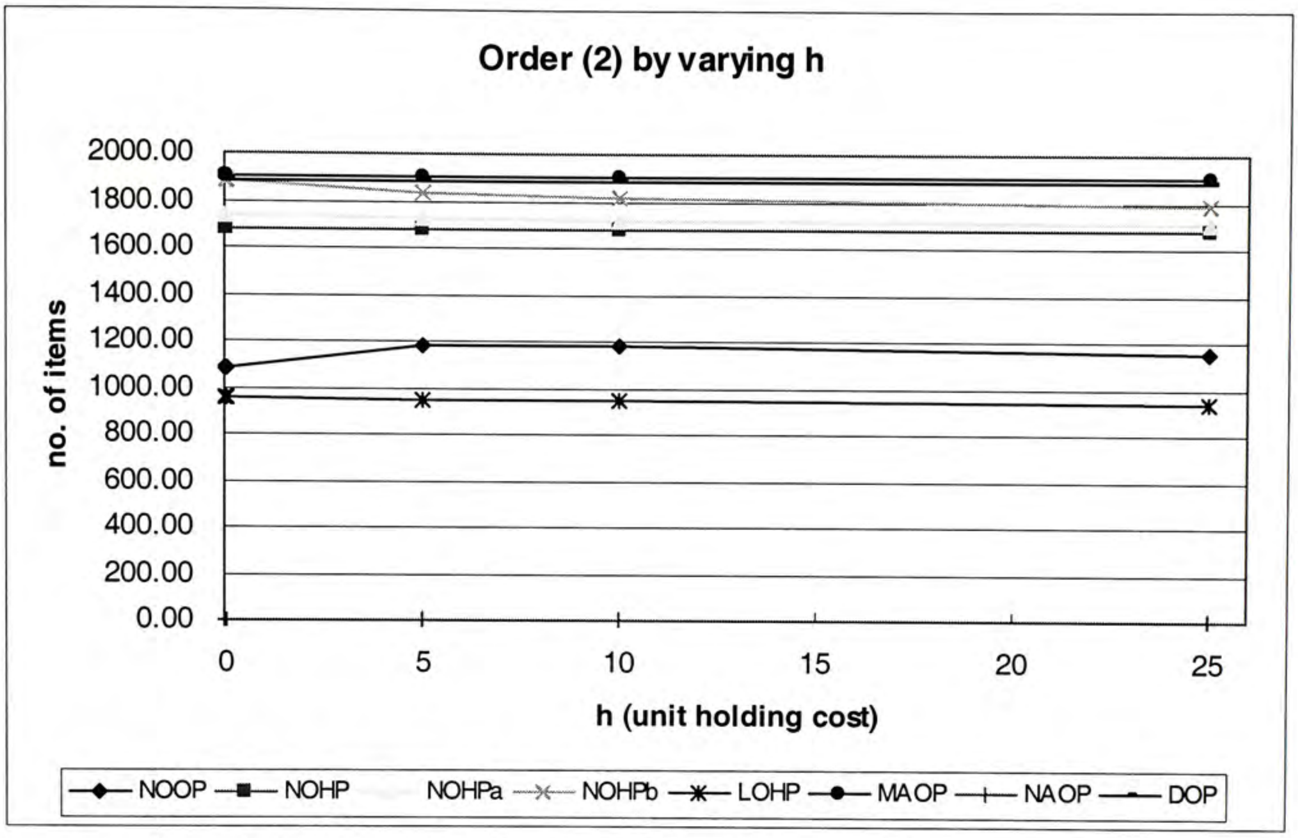


Figure 25: Order (2) patterns of all simulation models with varying *h*

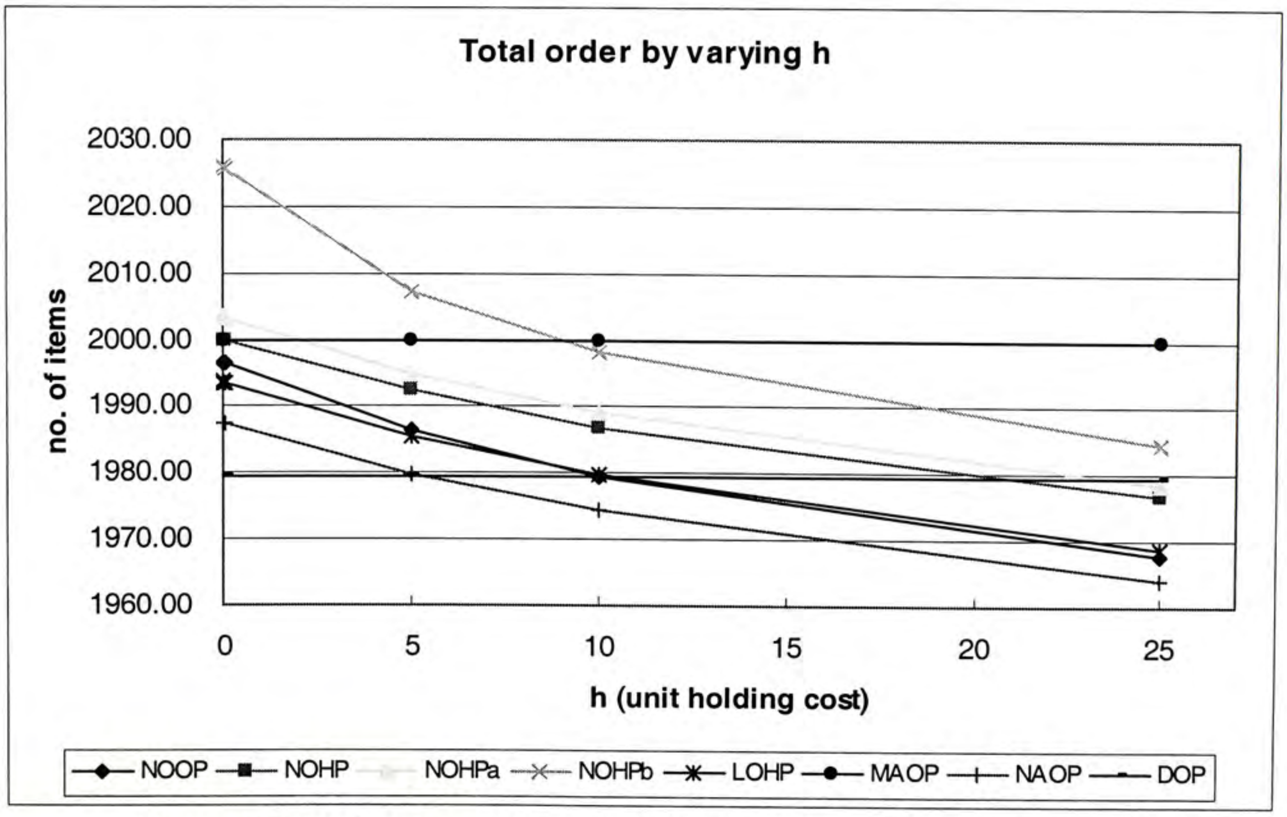


Figure 26: total order pattern of all simulation models with varying *h*

Figure 26 shows that only for *DOP* and *MAOP* do the size of the total orders stay constant, at 1979 and 2000 respectively. For other heuristics, the size of the total order decreases convexly with increasing *h*. *NOHPb* places the most orders, followed by *NOHPa*, *NOHP* and *NOOP*; *LOHP* and *NAOP* places the least orders.

For *LOHP*, because the Newsboy ratio (and hence *OUL*) are lowered by the increased unit holding cost, both the current order and advance order and hence total order are about the same and decrease gradually with increasing unit holding cost. The total order of *LOHP* is similar to that of *NOOP*. It tries to buy less and store less to save holding cost.

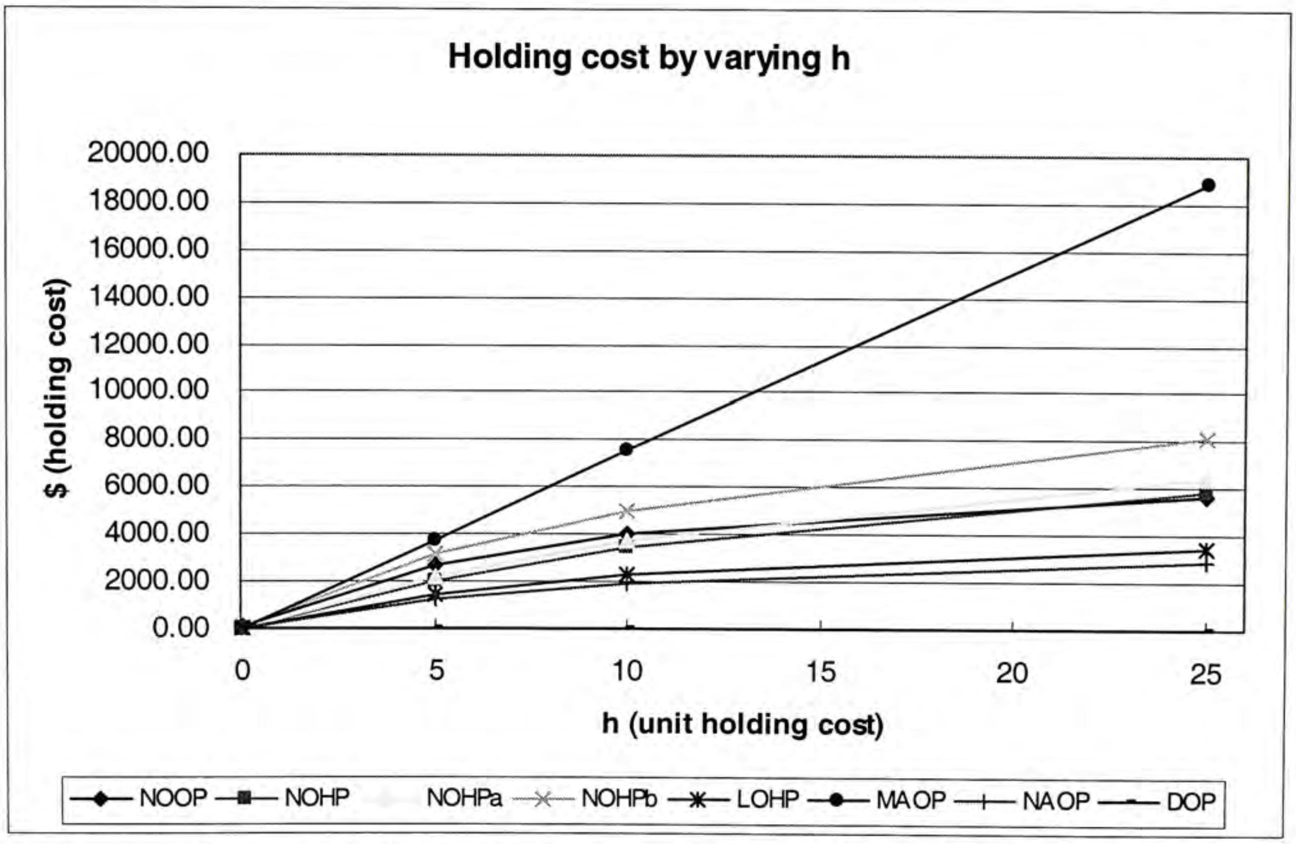


Figure 27: holding cost of all simulation models with varying *h*

In both Figure 27 and Figure 28, excluding *DOP* and *MAOP*, the costs for all the heuristics increase concavely with *h*. However, from our simulation data, we note that the inventory they actually held is decreasing with increasing *h*, but as *h* becomes really large, it makes the total holding cost still increasing. The holding cost of *MAOP* increases linearly and its shortage cost stays constant at 12441 with increasing *h*, since it behaves the same no matter what value of *h* (or any other parameters) is.

A high holding cost does not seem to induce the heuristics *NOHP*, *NOHPa* and *NOHPb* to buy fewer items, which places the highest total orders among the policies. As shown in Figure 27 above, their holding costs are very large. However, there is a good 'side effect' with this; the shortage cost is comparatively lower by holding more inventories. (See Figure 28)

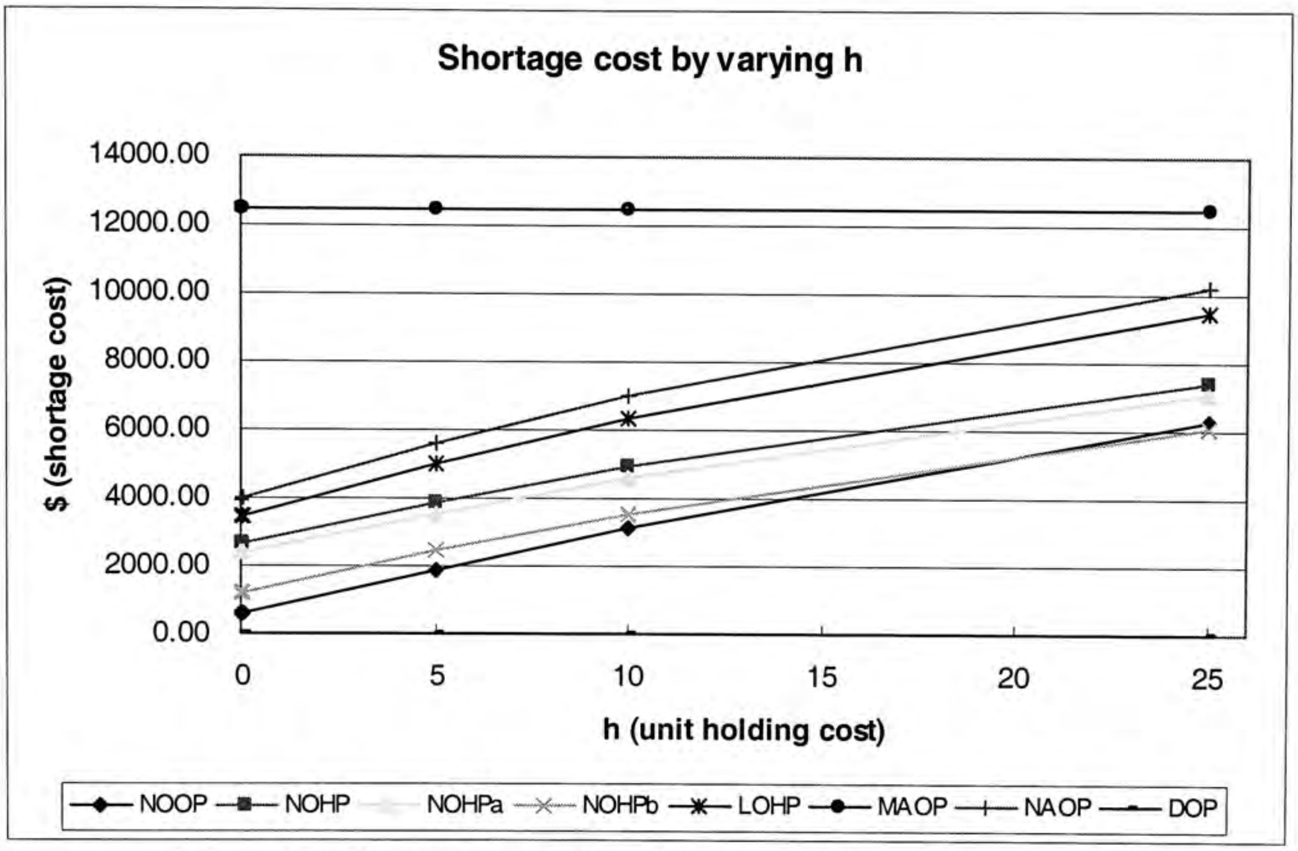


Figure 28: shortage cost of all simulation models with varying *h*

It is interesting that for *MAOP*, both holding costs and shortage costs are the highest, compared to those of other heuristics. It may be because the orders are placed in poor timing and of inappropriate amounts.

If the unit holding cost is high, *NOOP* would order less and less in total, so as to prevent any items left at the end of the horizon being charged a holding cost, although the chance of shortage will be larger. After the balance of the holding cost and shortage cost, *NOOP* gets the lowest total cost among the policies, except the case of $h \leq 5$, where *NOOP* gets the second lowest total cost.

Total costs of different simulation models are summarized in Figure 29. Total cost for *DOP* is constant at \$16032. Others are increasing concavely with increasing *h*.

MAOP has the highest total cost, then *NAOP* and *LOHP*. *NOOP*, *NOHP*, *NOHPa* and *NOHPb*.

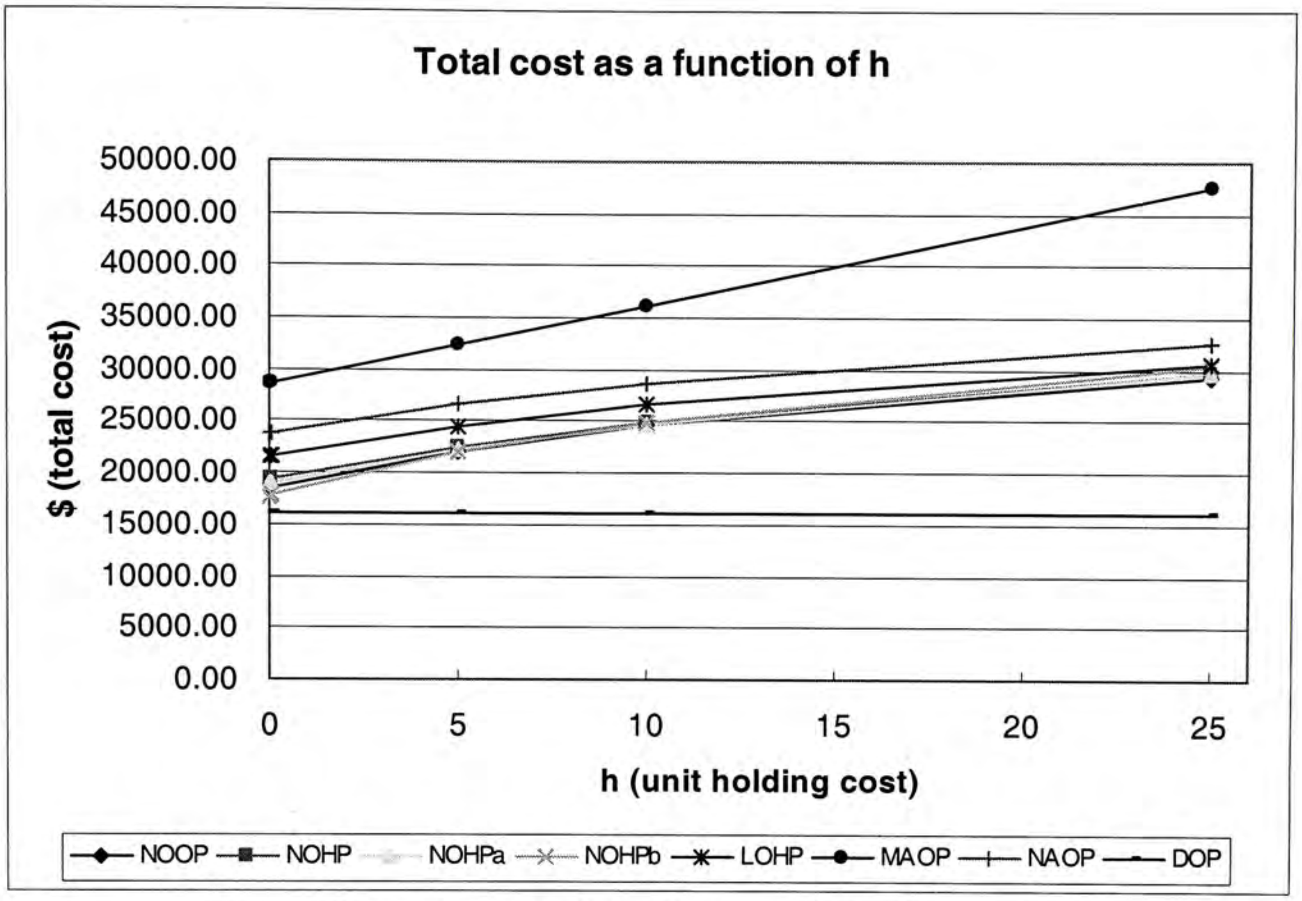


Figure 29: total cost of all simulation models with varying *h*

Orders placed by *NAOP* decrease slightly as the unit holding cost increases. (See Figure 26) Thus, *NAOP* holds fewer inventories and more backorders would occur as *h* increases. So, *NAOP* has the second highest shortage cost and finally, it has the second highest total cost.

Table 11: total cost saving of each policy compared to *DOP*, with varying *h*

h	*NOOP*	*NOHP*	*NOHPa*	*NOHPb*	*LOHP*	*MAOP*	*NAOP*
0.0011	0.15	0.20	0.18	0.10	0.34	0.79	0.49
0.0055	0.15	0.20	0.18	0.10	0.34	0.79	0.49
0.0275	0.15	0.21	0.18	0.10	0.34	0.79	0.49
0.055	0.15	0.21	0.18	0.11	0.34	0.79	0.49
5	0.38	0.40	0.39	0.37	0.52	1.02	0.66
10	0.53	0.55	0.55	0.55	0.66	1.26	0.79
25	0.82	0.85	0.86	0.90	0.92	1.96	1.04
Average	0.33	0.37	0.36	0.32	0.49	1.06	0.63

Table 11 shows precisely how each policy performs compared to *DOP*, in terms of total cost saving, as defined by Eq.(1) on page 139. We can see *NOHPb* does better than all the policies with stochastic demand in average. If h is large, the relative savings are larger for *MAOP* and *NAOP*. Compared to *DOP*, all the policies cost more and more if h increases.

Case V: Comparison between all simulation models by varying the unit shortage cost (q)

In this case, we would like to see the change of the heuristics' performance by varying the shortage cost, q. The range of q is from 0.0055 to 250 as shown in Table 12.

We choose 5, 125 and 250 because we want to see the effect when h is very small (25 divided by 5), large (25 times 5) and very large (25 times 10), respectively. We set q equal to 0.0055 and 10, because we also want to know the effect when the unit shortage cost is equal to the unit holding cost and the unit purchasing cost, respectively. The value of 20 is chosen since it makes the Newsboy ratio equal to 0.5.

Table 12: *q* setting

q	0.0055[9]	5[9]	10[9]	20	25	125	250

In Figure 30, we find that sometimes Order (1) is larger than Order (2) and sometimes this reverses. The turning point seems in between $q = 10$ and $q = 20$. When the shortage cost is 5, only Order (1) is placed, i.e. Order (2) is zero. The total orders are around 2000.

When the shortage cost is high, the total order of *NOOP* will be higher, with greater proportion to the advance order than the current order, since higher inventory on hand certainly can help to prevent backorders and the discount benefit can be obtained from advance ordering.

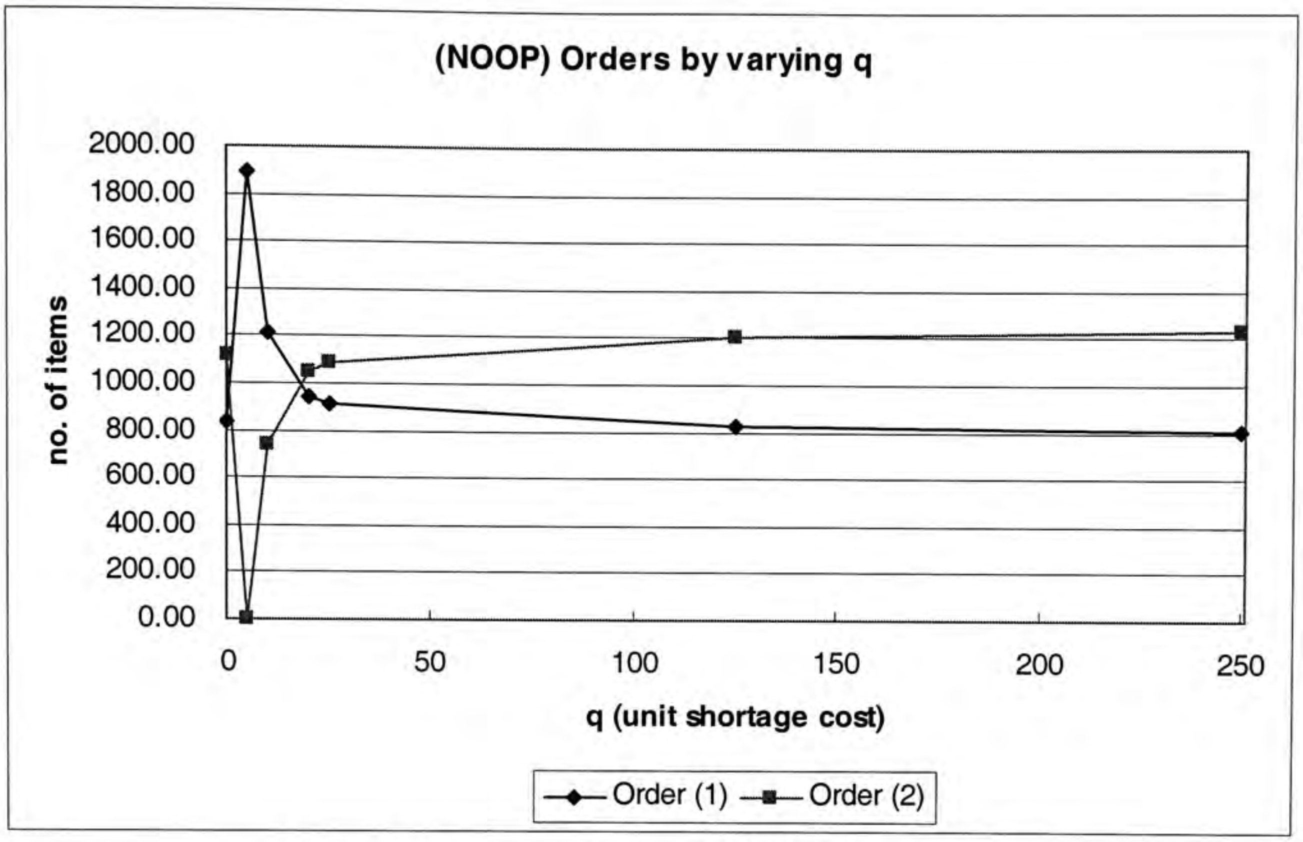


Figure 30: order pattern of *NOOP* with varying *q*

If the shortage cost is equal to 5, *NOOP* would only place order for the current day and no order is for the next day. The reason is that the total cost would be not much different to fulfill demand by holding items from yesterday or backorder the demand yesterday and buy the items back today. Since the closer to the time that the demand realized, the more information (the inventory level) we can get to predict the demand more accurately, *NOOP* would rather backorder all the demands and place order at the time when customer needed. This probably can save the 'spare' item purchasing cost and holding cost.

This argument is illustrated by several figures. At q equal to 5, we can see that in Figure 33, the total order placed by *NOOP* is the least among the cases of other values of q; From Figure 34, we find that there are few inventories held and low holding cost; In Figure 35, the shortage cost of *NOOP* is the highest among the cases of other values of q. However, this case, $q = 5$, violates our assumption, $q > c$, so that the total cost is not the lowest among the policies in Figure 36.

But note that if the unit holding cost and unit shortage cost are the same, intuitively, it is optimal to place advance orders to have the discount benefit. Thus, the solution based on *NOOP* may still not be very close to the real optimal solution.

In Figure 31, we see that the orders for immediate delivery for *NAOP* and *NOHP* are gradually increasing with q. For *NOOP*, the maximum current order size occurs at $q = 5$ and *LOHP* has its maximum between 0.005 and 10, both valued about 1880, which coincide with the minimum of *NAOP*. The maximum size of the immediate orders for *NAOPa* and *NAOPb* occurs at $q = 10$ and $5 \le q \le 10$ respectively.

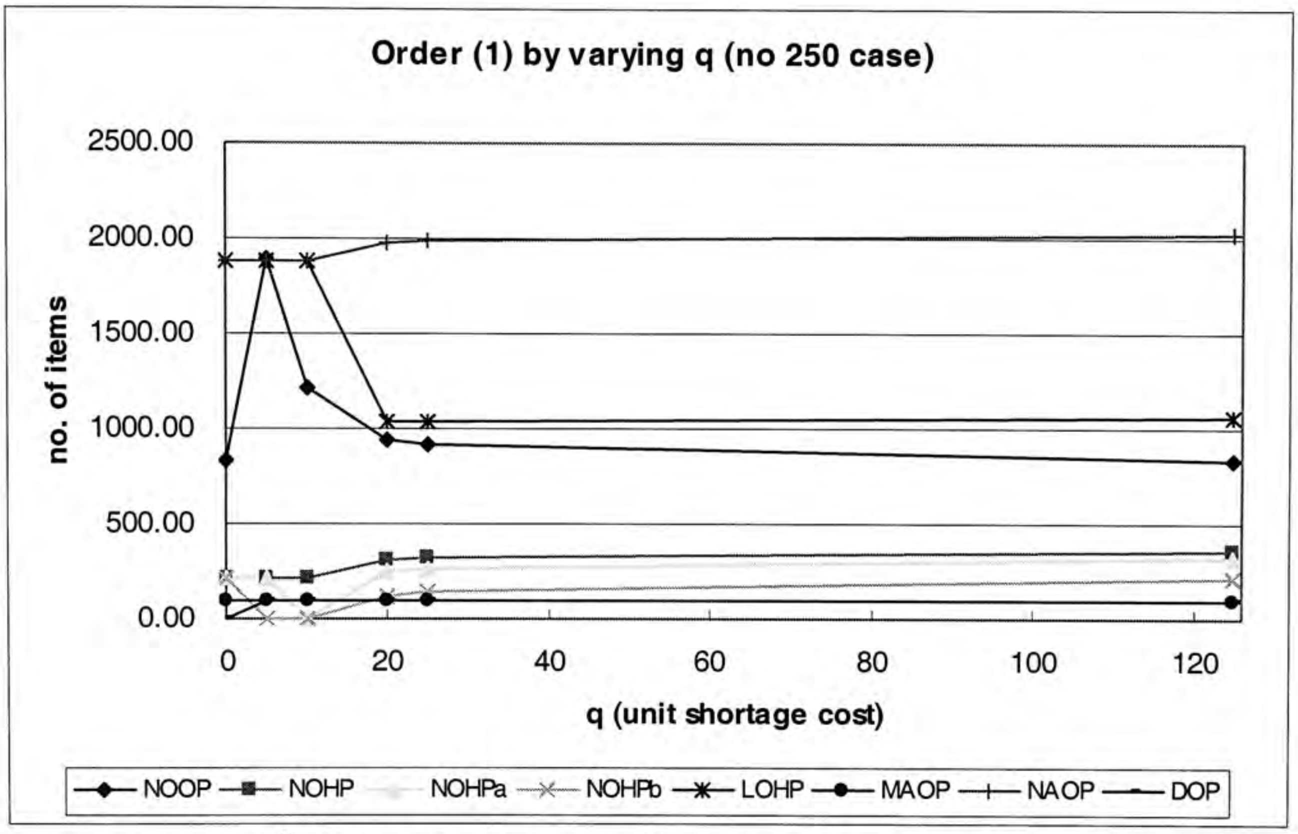


Figure 31[7]: Order (1) patterns of all simulation models with varying q

In Figure 32, we find that the advance orders for *DOP* and *NOHP* are constant at levels 1900 and 1700. For *NOOP*, the minimum advance order size occurs at $q = 5$ and *LOHP* has its minimum at $0.0055 \le q \le 10$, both valued 0. *DOP* has the largest Order (2), followed by *NOHP, NOOP, LOHP*. Finally *NAOP* has the least; its Order (2) is zero.

NOHP keeps its advance order constant and increases its current order as the unit shortage cost becomes higher. The effect on the current orders is due to the increase of unit shortage cost making T^*, the Newsboy critical fractile, increases gradually. Moreover, when the unit shortage cost is less than 10, the current orders of *NOHP* are at a constant level. The explanation is just as described in "Change of unit purchasing cost" in page 150. Since demand is backlogged and fulfilled by committed orders, there is few holding cost and the shortage cost is the highest at $q = c$.

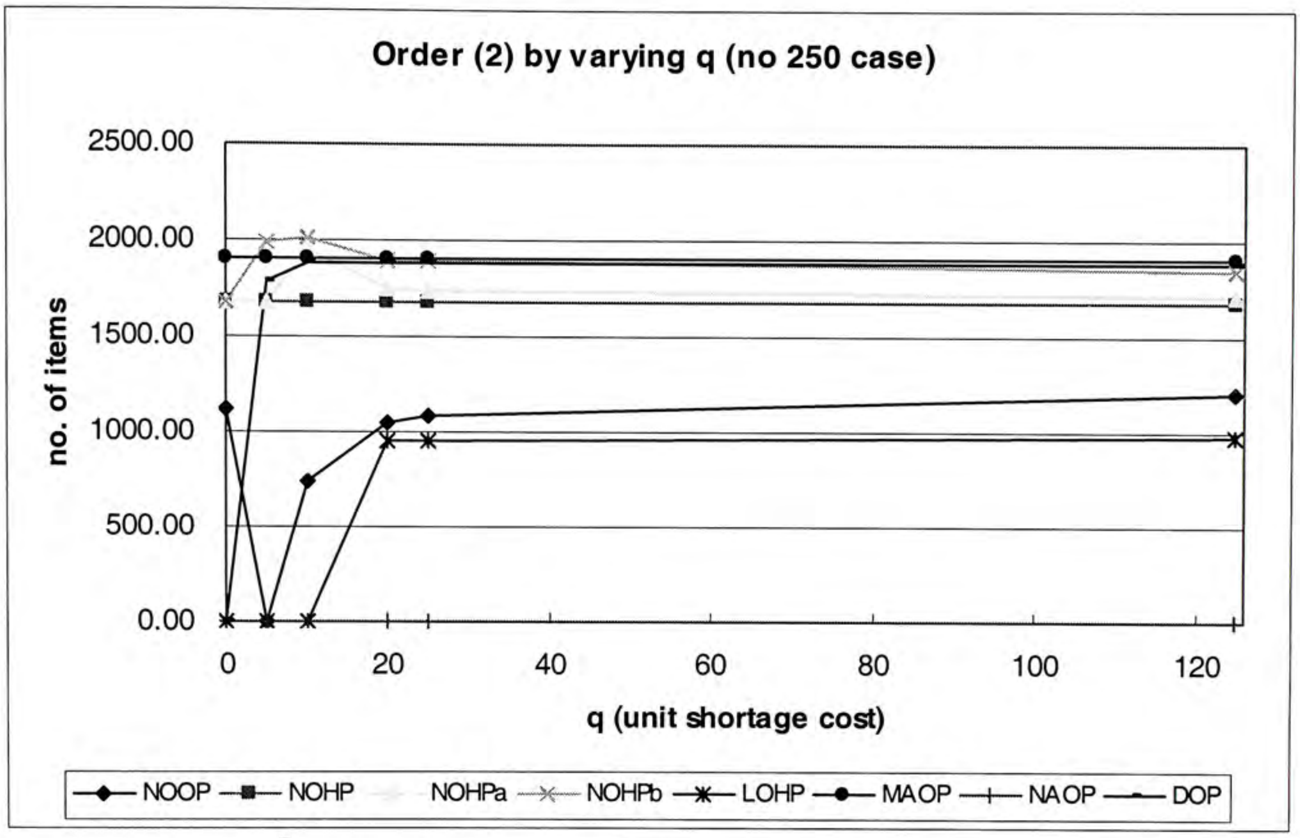


Figure 32[7]: Order (2) patterns of all simulation models with varying *q*

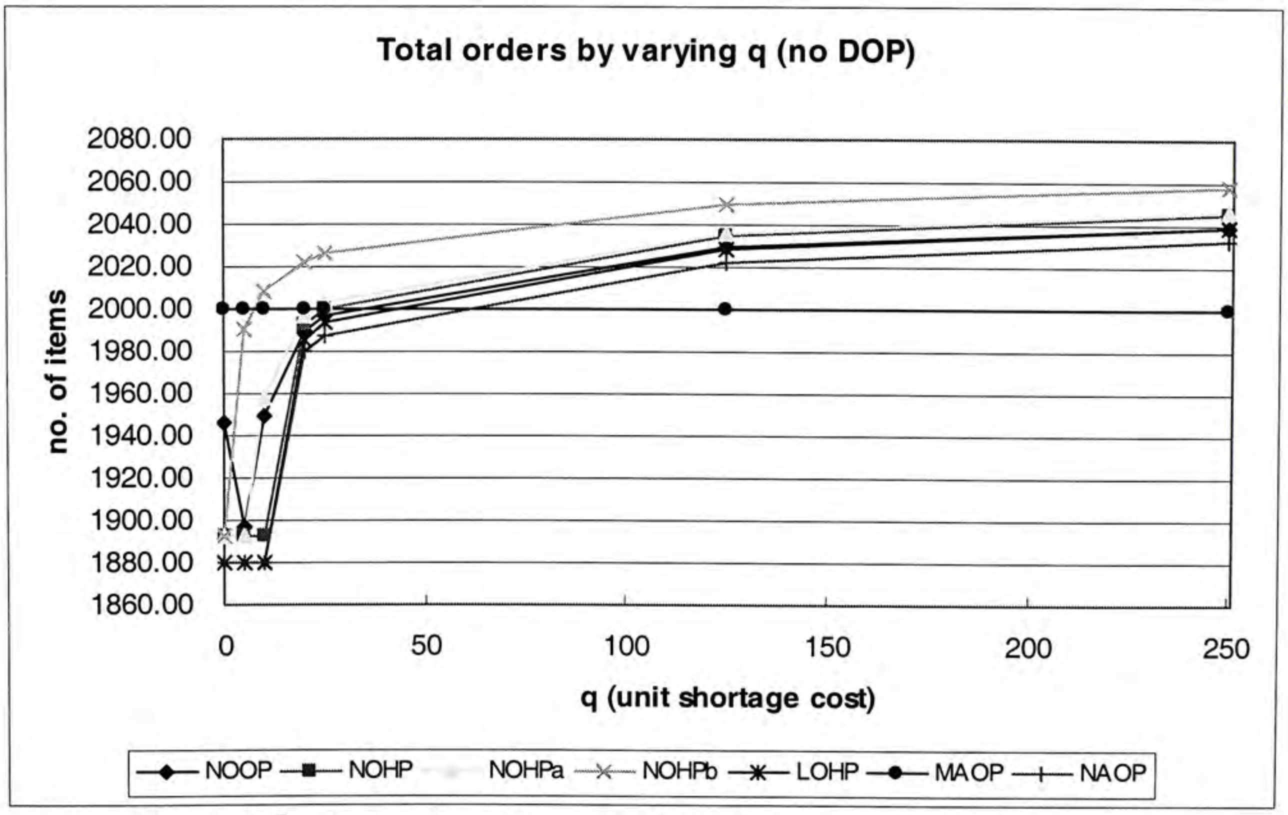


Figure 33[7]: total order pattern of all simulation models with varying *q*

Figure 33 shows that the total order for *MAOP* is constant at 2000 as q varies. The shape of the total cost curves of *NOHP*, *NOHPa*, *LOHP* and *NAOP* are very similar, all increasing for $q > 10$. The shape of the total cost curve of *NOOP* is close to that of *NOHPa* but not that of *NOHPb*.

For *LOHP*, this case is just the inverse of the case, "Change of the unit purchasing cost" in page 150. When $q \leq 10$, all orders are placed for the current day and no order is for the next day, just like *NAOP* and *NOOP*. When q is larger than this threshold value, the current order and the advance order are about the same level and slowly increasing with increasing unit shortage cost, as is the total order, when more inventories are held to prevent shortages.

For the case of the unit shortage cost very small (0.0055), *DOP* just backlogged all the demand without placing any orders.

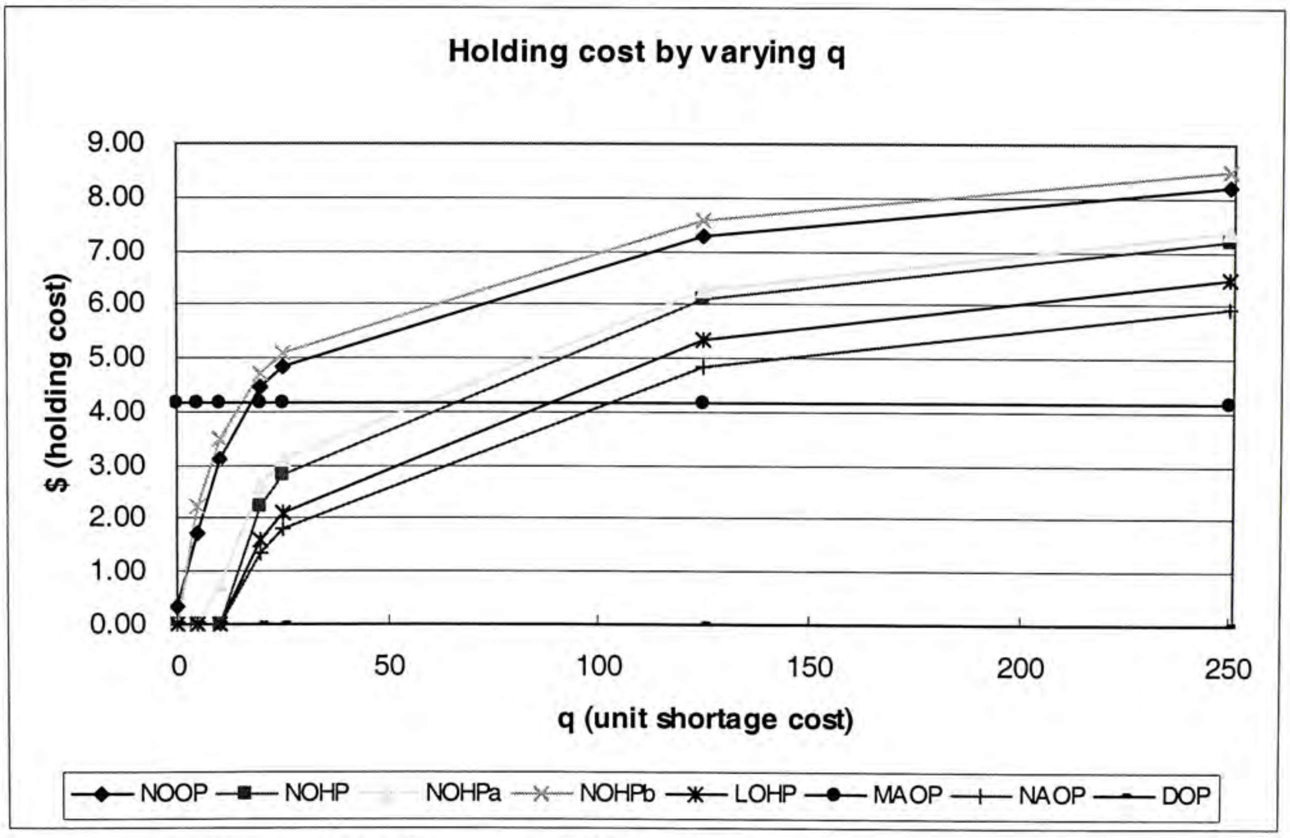


Figure 34: holding cost of all simulation models with varying *q*

In Figure 34, we see that the shape of the holding cost as a function of q for *NOHP*, *NOHPa*, *LOHP* and *NAOP* is very similar; the beginning is flat and then increases concavely with q. The holding cost of *NOHPb* is close to that of *NOOP*, as their curves increase concavely from $q = 0$. The curve for *NOHPa* is very close to the curve for *NOHP*.

From Figure 35, we see that the shortage cost as a function of q for all the heuristics has a shape that firstly goes to peak and then decreases with increasing q. (Except *MAOP*, which curve is linearly increasing with q) For *NOOP*, *NOHPa* and *DOP*, the maximum occurs at $q = 5$, while the others' maxima occurs at $q = 10$. *MAOP* has the highest shortage cost due to its insensitivity to inventory level. The shortage costs for *NOHP*, *NAOP* and *LOHP* are similar, having the highest shortage costs. *NOOP* has the lowest shortage cost (excluding *DOP*).

From our simulation data, we note that when the curves in Figure 35 go to a peak, the amount of backorders of the heuristics actually is decreasing or keeping at constant, but as q becomes large, it makes the total shortage cost still increasing.

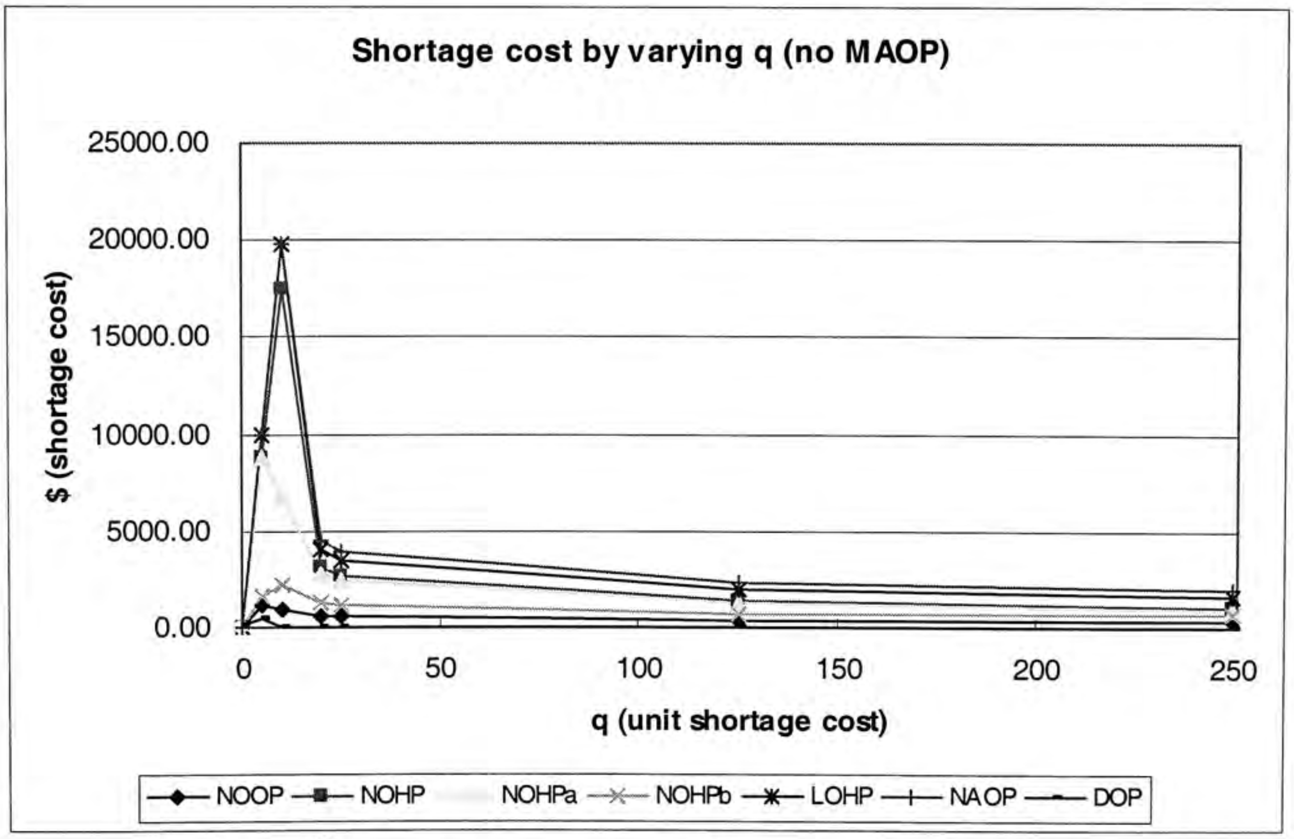


Figure 35[7]: shortage cost of all simulation models with varying *q*

For *NAOP*, the amount of current order placed and inventory held increase as the unit shortage cost increases (Newsboy level increases), but the amount of them are the least among the heuristics (See Figure 33 and Figure 34). So, it may be because generally *NAOP* does not order enough to fulfill demand, we find its shortage cost is the highest among the heuristics. (See Figure 35) Finally, it has the highest total cost.

For *NOHP* and *NOHPa*, in general, within the assumption range that $q > c$, the policies have increasing amount of total order and inventory held to prevent shortage as the unit shortage cost increases. So, the holding cost is increasing and the shortage cost is decreasing. But as you may see in Figure 35, the shortage costs of these policies are still high. This may be because the items they ordered are not enough, in comparison to *NOOP* and *NOHPb*.

When the value of q is below the critical value of $q = c$, *LOHP* behaves similarly to *NAOP*. When $q > c$, *LOHP* has higher holding costs and lower shortage cost than those of *NAOP*. So, the total cost of *LOHP* is the same as *NAOP* when $q < c$; when $q > c$, *LOHP* is doing better than *NAOP*, since *LOHP* have gained the discount benefit.

In Figure 33, Figure 34, Figure 35 and Figure 36, the curves of *LOHP*, *NOHP* and *NAOP* are very similar. From this performance, we induce they have similar ordering behaviors in this case, although *NAOP* does not allow advance orders while *LOHP* and *NOHP* allow.

Figure 36 summarizes the total costs incurred by all the heuristics. Only *DOP* has constant total cost of about $16032 for $q \geq 10$. When $q \leq 10$, *DOP* prefers backlogging rather than fulfilling demand as the shortage cost is small. The total costs as a function of q for *NAOP, LOHP* and *NOHP* have a similar shape; with the local maxima for all three attained at $q = 10$ (where also is the local maximum of *NOOPb*). *NOOP* and *NOOPa* have the same local maximum at $q = 5$. *MAOP* always has the highest total cost and *NOHPb* has the lowest total cost.

Table 13 shows numerically how each policy performs comparing to *DOP*, in terms of total cost saving, defined as Eq.(1) on page 139. We can see *NOHPb* consistently does better than all the policies with stochastic demand, if $q > c$. *NOOP*, *NOHP*, *NOHPa*, *LOHP* and *NAOP* cost less and less with increasing q.

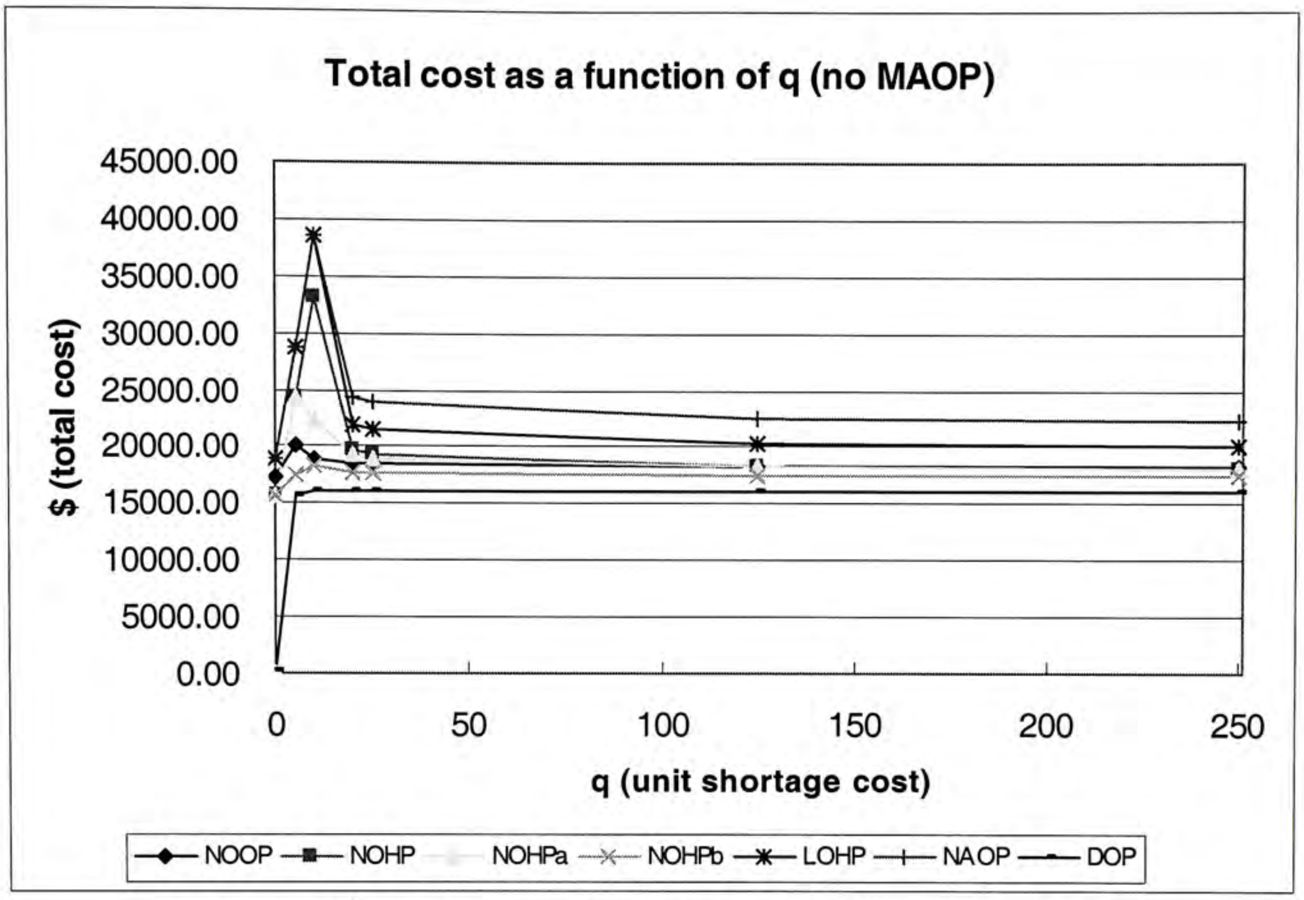


Figure 36[7]: total cost of all simulation models with varying *q*

Table 13: total cost saving of each policy compared to *DOP*, with varying *q*

q	*NOOP*	*NOHP*	*NOHPa*	*NOHPb*	*LOHP*	*MAOP*	*NAOP*
0.0055	150.09	135.50	135.50	135.50	163.88	141.07	163.88
5	0.28	0.55	0.55	0.12	0.82	0.19	0.82
10	0.18	1.06	0.40	0.14	1.41	0.32	1.41
20	0.15	0.23	0.20	0.10	0.37	0.63	0.51
25	0.15	0.20	0.18	0.10	0.34	0.79	0.49
125	0.14	0.14	0.13	0.09	0.26	3.89	0.41
250	0.14	0.13	0.12	0.10	0.25	7.77	0.39
Average	21.59	19.69	19.58	19.45	23.91	22.09	23.99

B.5.3.3 Conclusion

Through the analysis in the previous subsection, the observations and interpretations of the policies have been presented. The summary of the performance and characteristics of each of the policies are given below.

Please note that when we are talking about the best or the worst case among the policies, we will ignore the deterministic case, *DOP*, as being fair to the stochastic cases, which we are more concerned about.

<u>Simulation model (1): Near Optimal Order Policy (*NOOP*)</u>

NOOP well balances the holding cost and the shortage cost. It performs the second best for different setting of parameters, which means it has the second lowest total cost, under the assumption that the purchasing cost is less than the shortage cost. The weaknesses of *NOOP* are that its implementation is complicated and the computation time is long.

<u>Simulation model (2): Newsboy Order Heuristic Policy (*NOHP*)</u>

NOHP performs worse than *NOOP*, *NOHPa* and *NOHPb* but they have very similar results when there are changes of standard deviation and unit holding cost. *NOHP* places a lot of orders in advance to save much money. Change of unit purchasing cost, unit holding cost and unit shortage cost don't affect the amount of advance orders, as the current order is generally not equal to 0 and the advance order will be the mean of the demand according to Eq.5.1.(7) and Eq.5.1.(8).

<u>Simulation model (3): Newsboy Order Heuristic Policy - a (*NOHPa*)</u>

NOHPa is similar to *NOHPb* as shown in the discussion on "Change of discount rate". For discount rate less than 0.5, *NOHPa* performs better than *NOHPb*; otherwise, vice versa. In terms of total cost, *NOHPa* is always higher than *NOOP* but lower than *NOHP*. The advantage of this policy is its easy implementation.

<u>Simulation model (4): Newsboy Order Heuristic Policy - b (*NOHPb*)</u>

NOHPb is similar to *NOHPa* as shown in the discussion on "Change of discount rate". For discount rate larger than 0.5, *NOHPb* performs better than *NOHPa*; otherwise, vice versa.. The curves of the total costs of *NOHPb* in most of the cases have the similar shape with those of *NOOP*. The advantage of this policy is its easy implementation

<u>Simulation model (5): Lead Order Heuristic Policy (*LOHP*)</u>

Although the amount of advance order is bounded by the amount of current order, the advance orders actually are always less than the current orders and both of them are very close. Generally, except the cases "Change of unit purchasing cost" and "Change of unit shortage cost", *LOHP* has lower current orders, higher advance orders, higher total orders, higher holding cost, lower shortage cost and lower total cost than *NAOP*. *LOHP* acts very closely to *NAOP* and it is just doing better than *NAOP* and is in between *NAOP* and *NOHP*, which can be clearly seen in the savings tables in Section B.5.3.2.

<u>Simulation model (6): Mean Advance Order Policy (*MAOP*)</u>

Because of *MAOP* placing orders without reviewing the current inventory level, *MAOP* always incur the highest holding cost and shortage cost. But as it utilizes the discount benefit by placing orders in advance, the total cost is lower than that of *NAOP* under small demand fluctuation. In high demand fluctuating situation, *MAOP* always does the worst. Except the case "Change of standard derivation of demand", the total cost of *MAOP* is linearly increasing with any increasing parameter.

<u>Simulation model (7): No Advance Order Policy (*NAOP*)</u>

Since there is no advance order placed, the total order is equal to the sum of the current orders and it can get none of the discount benefit. Moreover, as *NAOP* just do the minimization by considering the current day only, it always does the worse among the policies.

<u>Analysis of Deterministic Order Policy (*DOP*)</u>

DOP is a special case. Since the demand is known, the changes of *s.d.* of the demand distribution and unit holding cost have no effect on the performance of *DOP*.

DOP places the current order only for the first day demand of the horizon and places advance orders for all the other known demand to get the full discount benefit. The change of discount rate greatly affects the total cost that can be saved.

As a result, *DOP* gets the most discount benefit and incurs no shortage cost and holding cost, the total cost would be very low. Moreover, if the unit purchasing cost is greater than the unit shortage cost, it would backlog the demand rather than fulfill it. Thus, the total cost incurred by *DOP* must be the minimum.

Chapter B.6

Simulation Study of Window Size K

B.6.1 Simulation Models

We are interested in studying the effect of different window size on advance ordering policy. However, even for the cases of window size 1, as presented in Chapter B.4, the mathematical proofs are quite complicated and only very few analytical closed-form formulae for the optimal solutions can be obtained. So, we propose some heuristic policies and do simulations on them to see the effect. The heuristics are motivated by some of the policies in Section B.5.1 in the previous chapter.

Simulation (1): Residual Order Heuristic Policy with Window K (*NOHP-K*)

This policy is generalized from *NOHP* in Section B.5.1. For the current day, we place an order to bring the current inventory up to the "Newsboy" level (if current inventory is below). For future days, we suppose there is a mean demand each day. We assume the stock we have today will decrease gradually by the mean demand until the stock level drops below the Newsboy optimal level, where we will start to place orders to bring the inventory level back to the Newsboy optimal level and thereafter. This situation is depicted in Figure 1.

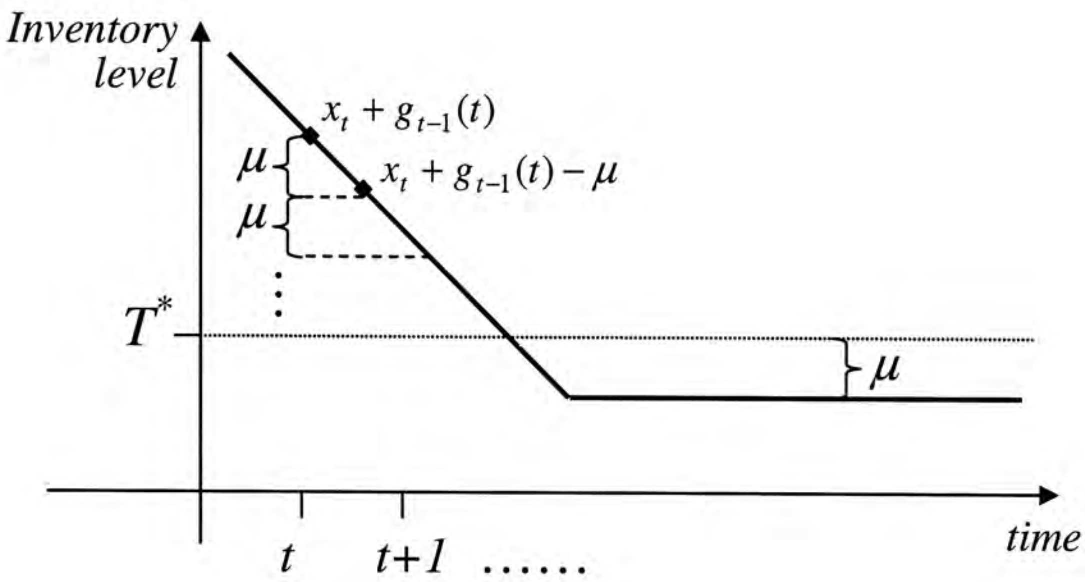


Figure 1: the expected inventory level at the end of each day

Let T^* be the optimal order-up-to level according to the Newsboy model, i.e., $T^* = J^{-1}\left(\frac{q-c}{q+h}\right)$.

The heuristic rule for the orders placed up to K days in advance is given below:

$$\begin{aligned}
\tilde{y}_t^t &= [T^* - x_t - g_{t-1}(t)]^+ \\
\tilde{y}_t^{t+1} &= [T^* - g_{t-1}(t+1) - \max\{x_t + g_{t-1}(t) - \mu, T^* - \mu\}]^+ \\
&\vdots \\
\tilde{y}_t^{t+K-1} &= [T^* - g_{t-1}(t+K-1) - \max\{x_t + g_{t-1}(t) - (K-1)\mu, T^* - \mu\}]^+ \\
\tilde{y}_t^{t+K} &= [T^* - \max\{x_t + g_{t-1}(t) - K\mu, T^* - \mu\}]^+
\end{aligned} \qquad \text{--- (1)}$$

Remarks:

1) Note that the formulation of *NOHP-K* looks like the formulation of the Newsboy model (1-period problem):

$$\tilde{y}(x) = \begin{cases} T^* - x & \text{if} \quad x < T^* \\ 0 & \text{if} \quad x \geq T^* \end{cases}$$

2) The general idea of *NOHP-K* is to bring the inventory level to T^* for each day in the horizon, and place orders as early as possible.

Simulation (2): Residual Order Heuristic Policy with Window K - a (*NOHPa-K*)

This policy is generalized from *NOHPa* in Section B.5.1 and faces the similar situation in Figure 1.

Let T^0 be the optimal order-up-to level in the current day according to the Newsboy model, i.e., $T^0 = J^{-1}\left(\frac{q-c}{q+h}\right)$.

Let T^i be the optimal order-up-to level in the *i*th advance day according to the Newsboy model, i.e., $T^i = J^{-1}\left(\frac{q-c\beta_i}{q+h}\right)$

The heuristic rule for the orders placed up to *K* days in advance is given below:

$$\begin{aligned} \tilde{y}_t^t &= [T^0 - x_t - g_{t-1}(t)]^+ \\ \tilde{y}_t^{t+1} &= [T^1 - g_{t-1}(t+1) - \max\{x_t + g_{t-1}(t) - \mu, T^0 - \mu\}]^+ \\ &\vdots \\ \tilde{y}_t^{t+K-1} &= [T^{K-1} - g_{t-1}(t+K-1) - \max\{x_t + g_{t-1}(t) - (K-1)\mu, T^{K-2} - \mu\}]^+ \\ \tilde{y}_t^{t+K} &= [T^K - \max\{x_t + g_{t-1}(t) - K\mu, T^{K-1} - \mu\}]^+ \end{aligned} \quad \text{--- (2)}$$

Simulation (3): Residual Order Heuristic Policy with Window K - b (*NOHPb-K*)

This policy is generalized from *NOHPb* in Section B.5.1 and faces the similar situation in Figure 1.

Let T^0 be the optimal order-up-to level in the current day according to the Newsboy model, i.e., $T^0 = J^{-1}\left(\frac{q-c}{q+h}\right)$.

Let T^i be the optimal order-up-to level in the *i*th advance day according to the Newsboy model, i.e., $T^i = J^{-1}\left(\frac{q-c(1-\beta_i)}{q+h}\right)$

The heuristic rule for the orders placed up to *K* days in advance is given below: (same as Equation Set (2) above)

$$\begin{aligned}
\tilde{y}_t^t &= [T^0 - x_t - g_{t-1}(t)]^+ \\
\tilde{y}_t^{t+1} &= [T^1 - g_{t-1}(t+1) - \max\{x_t + g_{t-1}(t) - \mu, T^0 - \mu\}]^+ \\
&\vdots \\
\tilde{y}_t^{t+K-1} &= [T^{K-1} - g_{t-1}(t+K-1) - \max\{x_t + g_{t-1}(t) - (K-1)\mu, T^{K-2} - \mu\}]^+ \\
\tilde{y}_t^{t+K} &= [T^K - \max\{x_t + g_{t-1}(t) - K\mu, T^{K-1} - \mu\}]^+
\end{aligned} \qquad \text{--- } (3)$$

Simulation (4): Lead Order Heuristic Policy with Window K (*LOHP-K*)

This policy is generalized from *LOHP* in Section B.5.1. (Please refer to Section B.5.1 for the definitions of *L*, *T* and *OUL* used below) We also place the Newsboy order for the current day. For future orders, we see them as the *k*-day lead time deliveries, so *L* increases as the number of days in advance increases and we get the different *OUL*. Again, since the conventional calculation of *OUL* assumes that there is no order arrival during the lead time and the review time, we deduct all the former orders and get the needed *OUL*. However, we should notice that as the lead time is longer, the safety stock and the *OUL* are larger that will have a lot of deviation from the real demand, so we would select the smaller one between the *OUL* and the previous order. We believe that the prediction of the previous order, which is nearer to realization, would be more accurate.

For the brief review of *OUL* calculation, please refer to Appendix V.

The formulations we have:

$$\tilde{y}_t^t = [OUL_t - x_t - g_{t-1}(t)]^+ \qquad \text{where } OUL_t = Newsboy = J^{-1}(\frac{q-c}{q+h})$$

$$\tilde{y}_t^{t+1} = [\min\{OUL_{t+1} - x_t - g_{t-1}(t) - \tilde{y}_t^t, \tilde{y}_t^t\}]^+ \qquad \text{where } OUL_{t+1} = R_{1+1} + ss_{1+1}$$

$$\vdots$$

$$\tilde{y}_t^{t+K} = [\min\{OUL_{t+K} - x_t - \tilde{y}_t^{t+K-1}, \tilde{y}_t^{t+K-1}\}]^+ \qquad \text{where } OUL_{t+K} = R_{1+K} + ss_{1+K}$$

--- *(4)*

Simulation (5): Mean Advance Order Policy with Window K (*MAOP-K*)

This policy is generalized from *MAOP* in Section B.5.1. We will place the mean of demand as advance orders to get the discount benefit.

Policy Formulation:

On the first day, we place up to $K + 1$ orders as the following:

$$\begin{aligned} \tilde{y}_1^1 &= E(D_1) = \mu \\ \tilde{y}_1^2 &= E(D_2) = \mu \\ &\vdots \\ \tilde{y}_1^{1+K} &= E(D_{1+K}) = \mu \end{aligned} \qquad \text{--- (5)}$$

Starting from the second day, $t > 1$, we only place the *K*th advance order as the following:

$$\begin{aligned} \tilde{y}_t^t &= 0 \\ &\vdots \\ \tilde{y}_t^{t+K-1} &= 0 \\ \tilde{y}_t^{t+K} &= E(D_{t+K}) = \mu \end{aligned} \qquad \text{--- (6)}$$

<u>Simulation (6): Deterministic Order Policy with Window K (*DOP-K*)</u>

This policy is generalized from *DOP* in Section B.5.1. Since we know the demand in advance, we can order this exact amount as early as possible to get the greatest discount benefit. This is the optimal policy to minimize the cost. If $(N-i)q < c\beta$, we would prefer backlogging rather than meeting the demand in the $(N-i)$ th period, for $i = 0, \ldots, N-1$.

Policy Formulation:

On the first day, we place up to $K + 1$ orders as the following:

$$\begin{aligned} \tilde{y}_1^1 &= E(D_1) \\ \tilde{y}_1^2 &= E(D_2) \\ &\vdots \\ \tilde{y}_1^{K+1} &= E(D_{K+1}) \end{aligned} \qquad \text{--- (7)}$$

Starting from the second day, we only place the *K*-advance order as the following:

For $t > 1$,

$$\tilde{y}_t^t = 0 \qquad \text{--- (8)}$$

If $c\beta_i < (N - t - i + 1)q$,

$$\tilde{y}_t^{t+i} = E(D_{t+1}) \qquad \text{--- (9)}$$

otherwise,

$$\tilde{y}_t^{t+i} = 0 \qquad \text{--- (10)}$$

where $i = 1, \ldots, K$.

Remarks:

If the discount rates, β_t, $t = 1, \ldots, K$ is not monotonic, then it could affect the formulation of *DOP*, in which we would place the greatest amount of order at the time of the greatest discount given.

Note:

1) Discount rates, β_t, $t = 1, \ldots, K$ affect the ordering amount of *NOOP* (see Theorem 1). Likewise, the discount rates affect the ordering amount of *NOHPa-K*, *NOHPb-K* and *DOP-K*, since β_t is involved in their formulae.

2) In all policies, $\tilde{y}_N^{N+1} = 0$.

B.6.2 Simulation Program Structure

B.6.2.1 Program Flow

The program flow is the same as shown in Figure 5.2.(1).

B.6.2.2 Program Functions

The program functions are the same as those presented in Section B.5.2.2, except the function **evaluate( )**. It now has the tasks:

- determine the amount of the current order and the *K* advance orders according to the specific policy
- update the order costs of the current order and the advance orders
- determine the arrival times of the current order and the nearest advance order

B.6.2.3 Program Implementation Note

1) Please see the remarks in Section B.5.2.3. They are also applicable in this window size *K* case.

2) Window size 1 is just a special case of window size *K* in our programming. We have only written 8 programs for each of the policies: *NOOP*, *NOHP-K*, *NOHPa-K*, *NOHPb-K*, *LOHP-K*, *MAOP-K*, *NAOP* and *DOP-K*. We run the

simulations in Chapter B.5 (those are of window size 1) by passing an argument *K* equal to 1 to the programs of *NOHP-K*, *NOHPa-K*, *NOHPb-K*, *LOHP-K*, *MAOP-K* and *DOP-K*. Then we can get the results of *NOHP*, *NOHPa*, *NOHPb*, *LOHP*, *MAOP* and *DOP* from them, respectively.

3) To build up the mechanism of placing advance orders, we have utilized a simple concept. We assume for each day, there is a bin for collecting the committed orders. We have constructed an array of bins to do this. So, for example, if the window size is 2, the retailer firstly review her on-hand inventory, i.e. x_t, and review today's bin to see how much the total committed order is for today, that is $g_{t-1}(t)$, and according to the specific ordering policy, if necessary, she places the current order for today for immediate delivery and updates committed orders to the corresponding days' bins. As a result, each bin belongs to one day within the horizon, say *t*, and it collects all the committed orders that are placed in different days in advance for delivery on the same day *t*.

B.6.3 Simulation Numerical Analysis

B.6.3.1 Simulation Settings – Base Case

We assume the demand has a Normal Distribution. Its mean and *s.d.* are 100 and 30 respectively, so the demand will realize within about [0, 200].[1] The horizon is 20 days, i.e., $N = 20$; unit purchasing cost (*c*) is $10; unit holding cost (*h*) is $0.0055 and unit shortage cost (*q*) is $25.[2] The discount rate (β_1) is 0.95. The optimal Newsboy ratio is 0.6[3]. The results shown below are the average over 100 simulation runs. This is the base setting for the following comparisons; that means, in the case below, we only change the window size and keep the other variable as the values in this base case.

Table 1: base settings

μ	*s.d.*	*N*	*c*	*h*	*q*	*Newsboy Ratio*
100	30	20	10	2	25	0.6

In the following simulation runs, we choose the discount rates in a convexly decreasing series. The beta values are shown in Table 2 and the convex shape is displayed in Figure 1 below. The reasons for using this series are:

1) It seems to be more common in real life than a concavely decreasing series. If the retailer places orders very early, say *N* days in advance, the benefit of the manufacturer can get will be not much different between orders for *N* and (*N* + 1),

---

1 *Empirical Rule*: 99.7% of the observations fall within **3 standard deviations** of the **mean.**

2 *The cost setting is reference on Nahmias, Production and Operations Analysis (3rd), p.287.*

3 Newsboy ratio = $\left(\dfrac{q-c}{q+h}\right)$

so the discount given for these kind of orders is the same.

Table 2: setting of β_k in convexly decreasing series

Day in advance, *k*	beta	Day in advnace, *k*	beta
1	0.95	11	0.50
2	0.90	12	0.50
3	0.85	13	0.50
4	0.80	14	0.50
5	0.75	15	0.50
6	0.70	16	0.50
7	0.65	17	0.50
8	0.60	18	0.50
9	0.55	19	0.50
10	0.50		

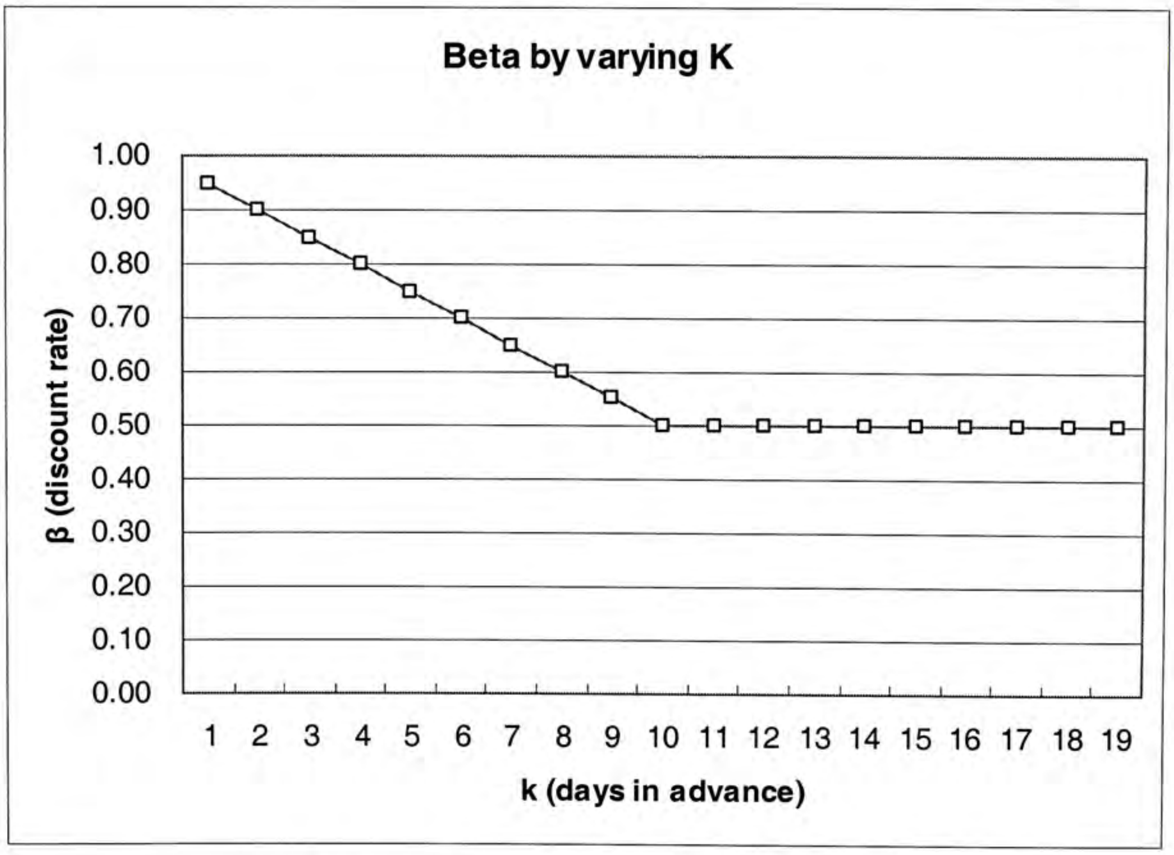


Figure 1: convexly decreasing of beta

2) For *NOHP-K*, *LOHP-K* and *MAOP-K*, how the discount rates decrease affects the total purchasing cost only. As mentioned in the remark of Section B.6.1, these

three ordering policies are not affected by β_k, we would expect the results of total holding cost and shortage cost are also not affected by β_k, no matter in what way β_k is decreasing. But β_k does reduce the unit purchasing cost for advance orders. Thus, when we see the change of total cost with varying β_k, it will be due to the change of total purchasing cost only.

B.6.4.2 Simulation Result Analysis

In the simulation study, we will concentrate on the study of *NOHP-K*, *NOHPa-K*, *NOHPb-K*, *LOHP-K*, *MAOP-K*, *NAOP* and *DOP-K*, since we cannot get closed-form analytic solutions to extend *NOOP* to a policy for window size *K*. So, when we say 'all policies' or 'all heuristics' in the following paragraphs, we are referring to the above seven heuristics.

Besides, we would focus on testing the sensitivity of changes of window size and find out how it will affect the performance of the heuristic models, such as the purchasing cost shortage cost, holding cost and total cost. The ordering behavior and pattern of the policies are also our major concerns in the analysis.

<u>Comparison between all simulation models by varying the window size (*K*)</u>

In this case, we would like to see the change of heuristics' performance by varying the window size of advance ordering, *K*. The range of *K* is from 1 to 19. Please note that when *K* equal to 19, it means we can place the current order and the advance orders which cover the whole horizon in day 1. In the context below, Order (*i*) means the total number of items purchased by placing orders $(i-1)$ days in advance.

Table 3: K setting

K	1	3	5	7	9	14	19

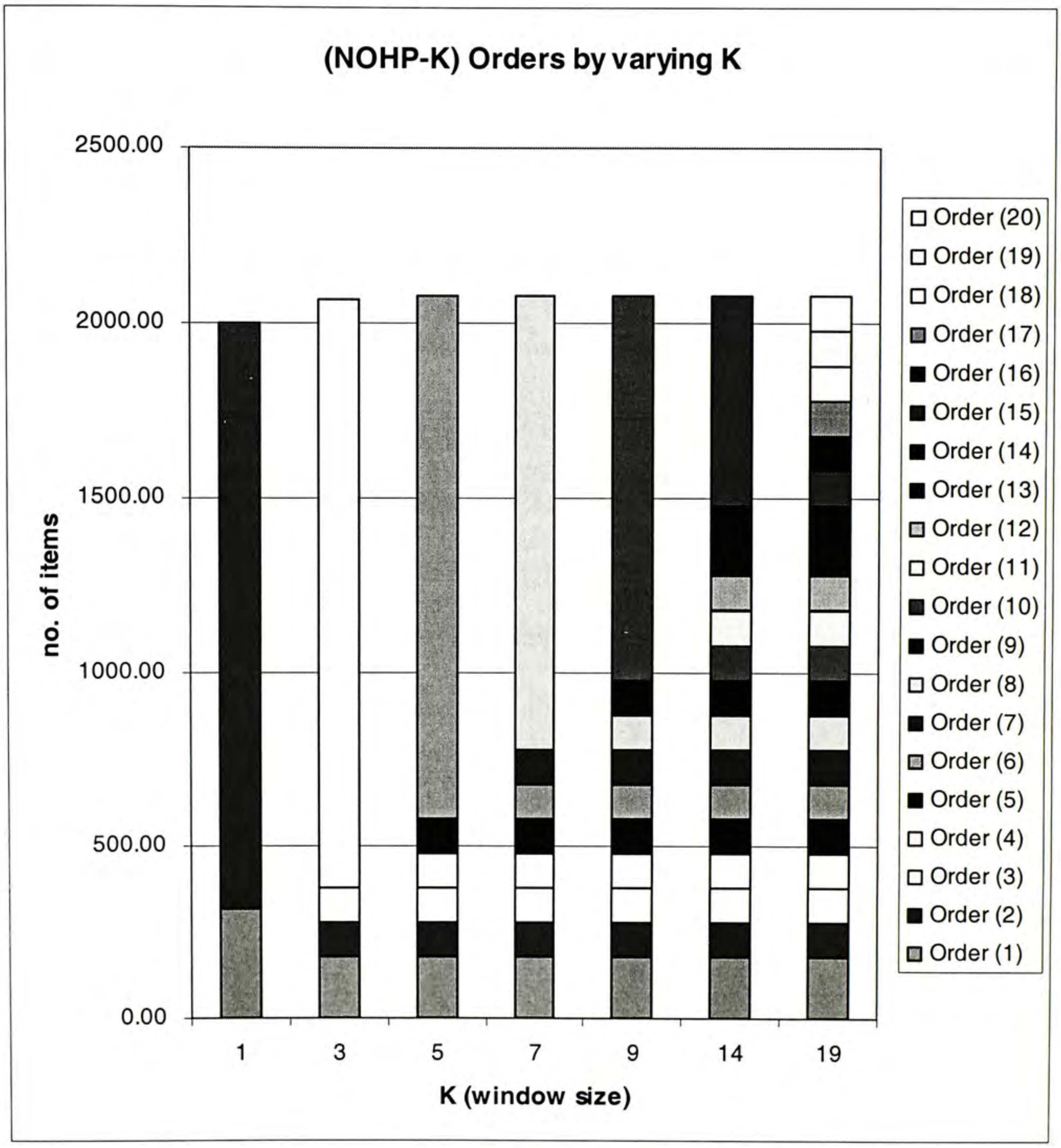


Figure 2: order pattern of *NOHP-K* with varying *K*

In Figure 2, we see that the total order is increasing and reaches a constant level about 2075.12 starting from *K* equal to 5. *NOHP-K* places the largest proportion of total order as Order (*K* + 1), except the case of *K* equal to 19. For the cases other than

K equal to 1, Order (1) keeps at 175.12 and the advance orders in between Order (1) and Order (K + 1) are 100 (including Order (K + 1) for the case of K equal to 19), which is the mean of the demand distribution for each period.

So, we see that *NOHP-K* just places the advance orders equal to the mean of demand (μ), except the current order and Order (K + 1). From the simulation output data, we also get that Order (K + 1) is always multiples of 100 (μ) and has the relation with the horizon N:

$$(N-K)*\mu\text{, where } K<N \qquad \text{--- (1)}$$

The explanation of the above observations is as follows: At day 1, *NOHP* would look ahead the inventory levels up to day (K + 1). Once a period's inventory level dropped below T^*[4], we place advance order to bring it up to T^*, and thereafter place advance order amount μ. (since we expect the demand in each day is μ). At day 2, we would just place order for Order (K + 1) equal to μ, since the inventory levels in the days before day K are already filled up to T^* (by the advance orders in day 1). Then we will get the ordering pattern like Figure 2, and you would realize that we get Eq.(1) because (N - K) is the number of times to repeat placing advance orders, Order (2) to Order (K + 1).

With this ordering pattern, it turns out the greatest proportion of the total orders are ordered by placing Order (K + 1). Consequently, *NOHP-K* saves a lot of purchasing cost with the discounts given and performing better than *LOHP-K*, which does not

[4] Recall that T^* is the optimal order-up-to level according to the Newsboy model: $T^* = J^{-1}\left(\frac{q-c}{q+h}\right)$

well utilize Order ($K + 1$). (See Figure 5 and Figure 14)

Besides, this ordering pattern is very similar to that of *DOP-K*. They both place K advance orders at day 1 and thereafter place one advance order, Order ($K + 1$) at each day. But *NOHP-K* needs to place current orders to adjust the inventory level, so that it can meet the better forecasted demand on that day; while *DOP-K* has no need to place any current orders except day 1.

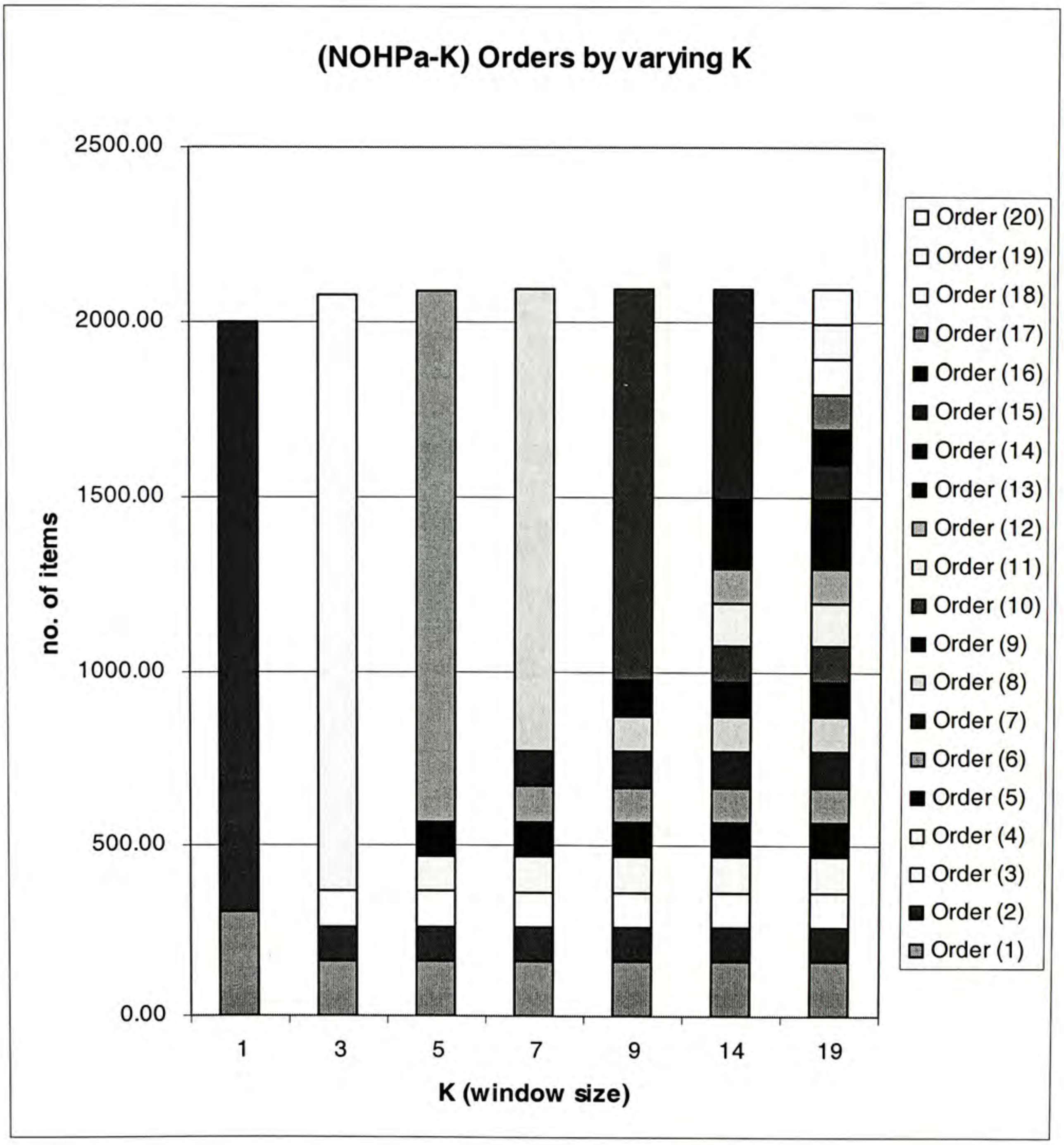


Figure 3: order pattern of *NOHPa-K* with varying *K*

In Figure 3, we see that the total order is increasing and reaches a constant level about 2095.53 starting from K equal to 14. *NOHPa-K* like *NOHP-K* places the largest proportion of total order as Order (K + 1), except the case of K equal to 19. For the cases other than K equal to 1, Order (1) keeps at 160 and the advance orders in between Order (1) and Order (K + 1) are about 100 (including Order (K + 1) for the case of K equal to 19), which is the mean of the demand distribution for each period.

We note that from Eq.6.1.(2), the Newsboy level of *NOHPa-K* is slightly higher than that of *NOHP-K* as the unit purchasing cost having a discount factor. So, the advance orders placed by *NOHPa-K* generally are slightly larger than those placed by *NOHP-K*.

The ordering pattern of *NOHPa-K* is similar to that of *NOHP-K* and *DOP-K*. Please see the description about *NOHP-K* similar to *DOP-K* in the above.

In Figure 4, we see that the total order is increasing from 2044.81 to 2118.56. *NOHPb-K* places the largest proportion of total order as Order (K + 1), except the case of $K = 19$. For the cases other than $K = 1$, Order (1) is approximately 109 and the advance orders in between Order (2) and Order (K + 1) are about 100 which is the mean of the demand distribution for each period.

It is interesting that Order (2) is particularly larger than the other advance orders and even Order (1). Order (2) is decreasing with the increasing window size.

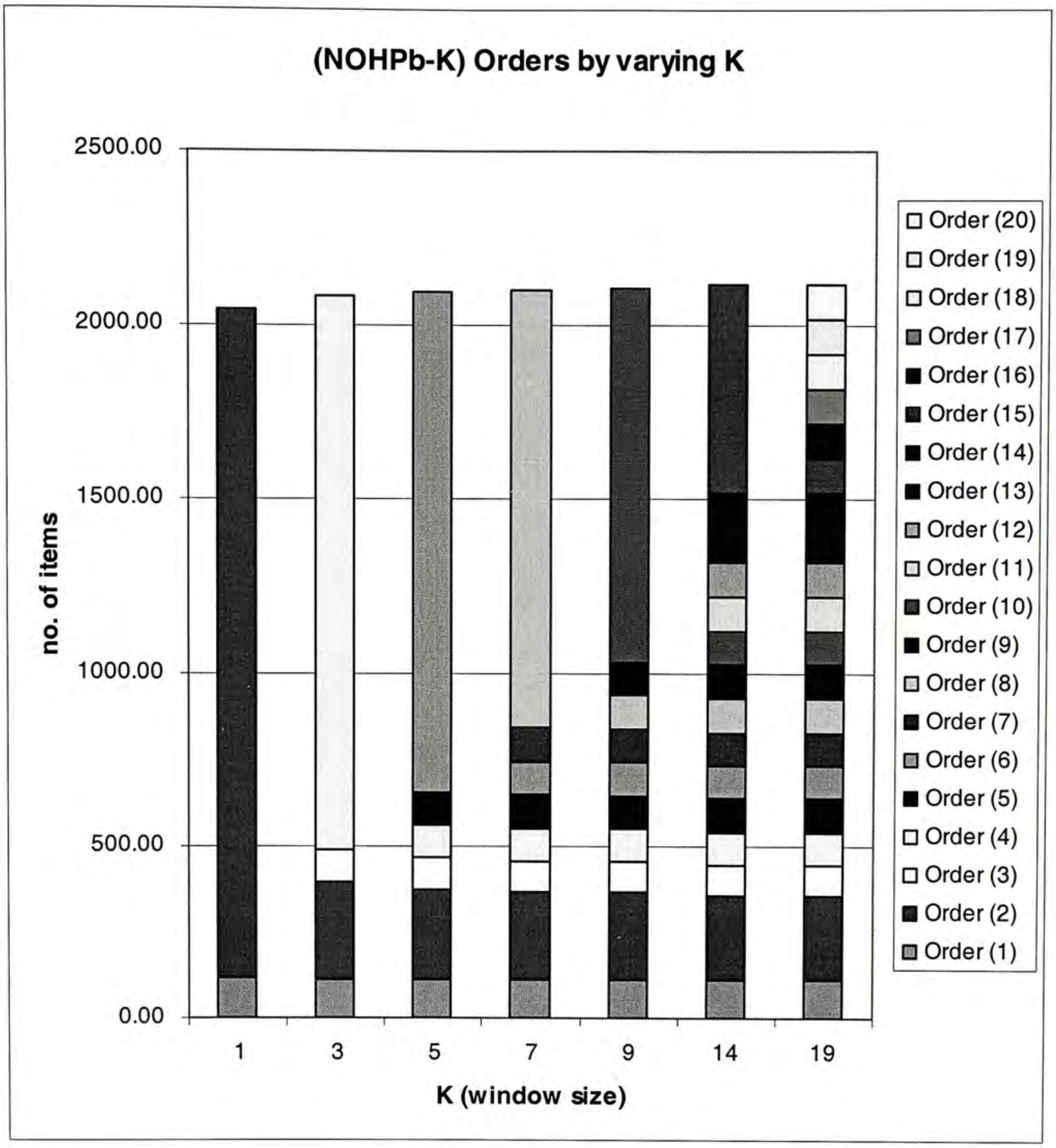


Figure 4: order pattern of *NOHPb-K* with varying *K*

In Figure 5, we see that the total order placed by *LOHP-K* fluctuates about 2000. Order (1) always occupies the largest proportion of the total order. We observe that Order (k) > Order (k + 1), for fixed K and $1 \le k \le K$. It is resulted from Eq.6.1.(4), which takes the minimization between Order (k) and Order (k + 1). We also find that as K increases, Order (k) decreases or being equal, for fixed k and $1 \le k \le K$. At the case K equal to 19, from Order (2) to Order (19) are the amounts slightly above 100, which is the mean of the demand distribution for each period.

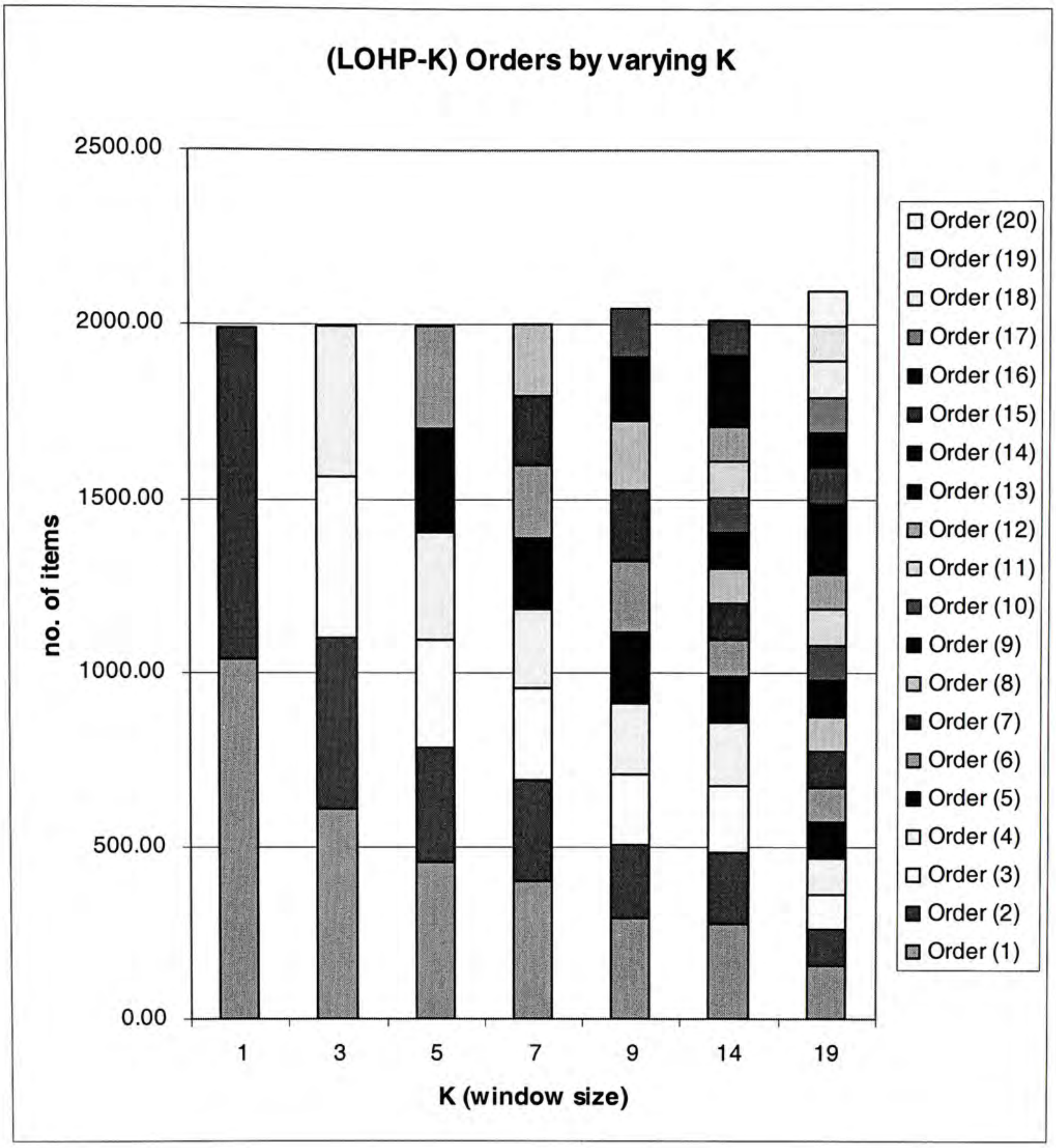


Figure 5: order pattern of *LOHP-K* with varying *K*

As we know the Order (1) of *LOHP-K* must be the greatest proportion of the total order, the policy hasn't well utilized the discounts given by placing more advance orders. The total cost of it is the highest among the policies, excluding *MAOP-K*. (See Figure 14)

Moreover, we see that the Order (1) to Order (K + 1) are distributed quite evenly. When the window size equal to 19, Order (2) to Order (K + 1) are about 100, which is the mean of the demand. This pattern is very similar to *NOHP-K* in Figure 2 and Figure 10. For this reason, they have similar amount of total cost incurred. (See Figure 14)

From Figure 8, Figure 9 and Figure 10, we see that as the window size becomes large, the difference between the ordering patterns of *LOHP-K* and *DOP-K* becomes very small. So, it can approximate *DOP-K* only when the window size is large enough.

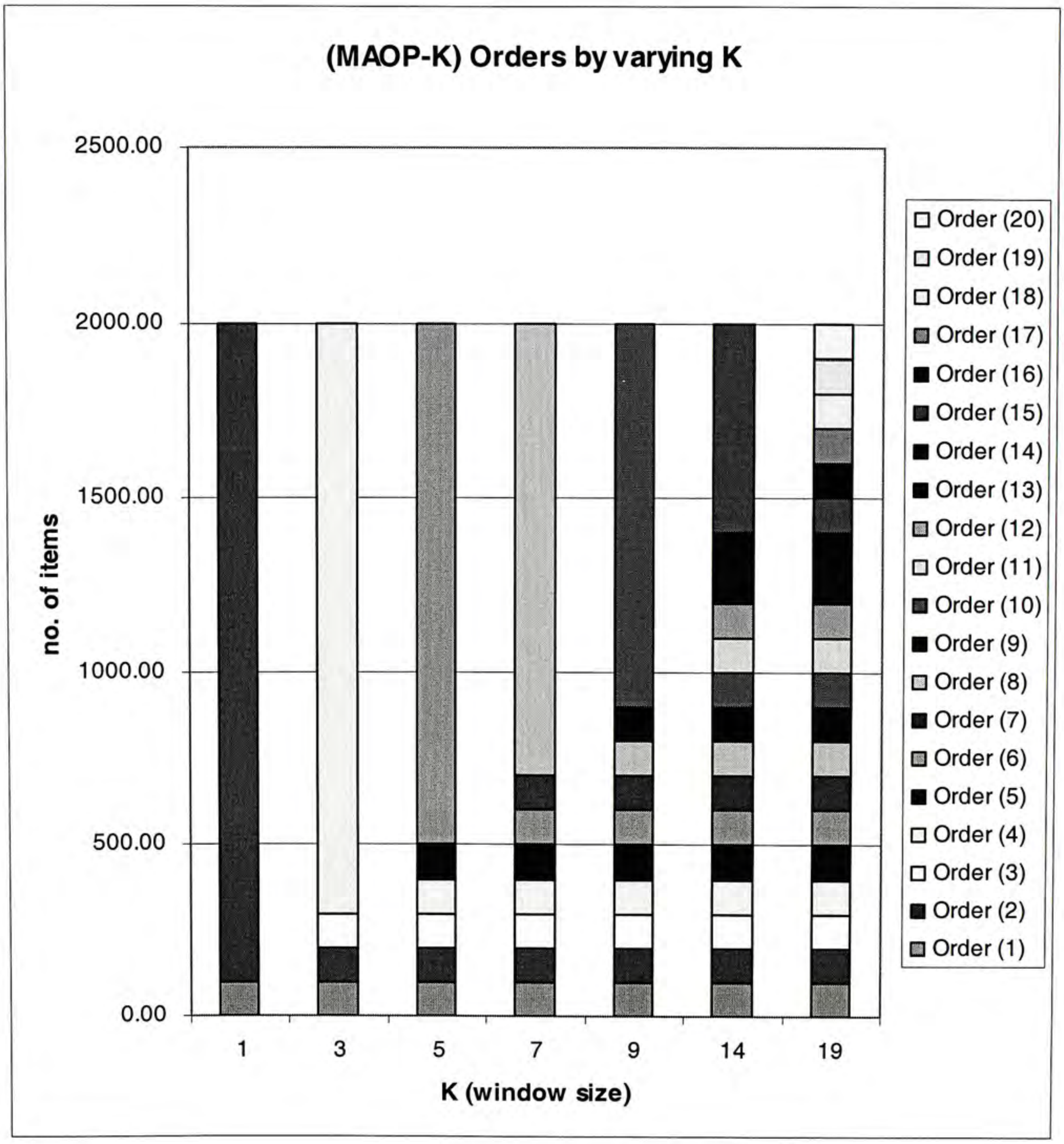


Figure 6: order pattern of *MAOP-K* with varying *K*

According to Eq.6.1.(5) and Eq.6.1.(6), *MAOP-K* just orders the mean of demand for day 1 and future days. Therefore Order (1) to Order (*K*), as shown in Figure 6, are exactly 100, and Order ($K + 1$) is a multiple of 100, according to Eq.(1) on page 185. The total order is exactly 2000.

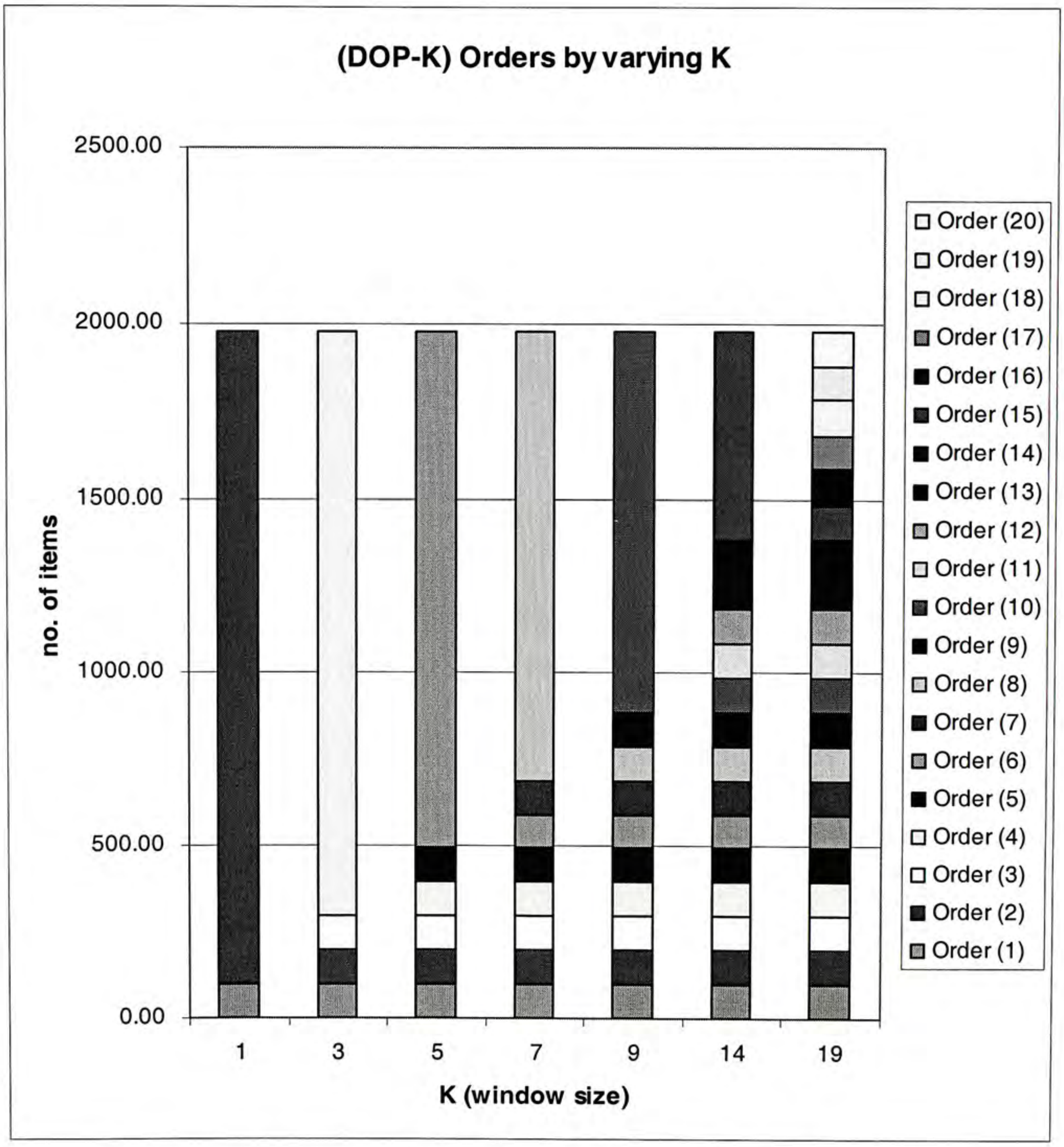


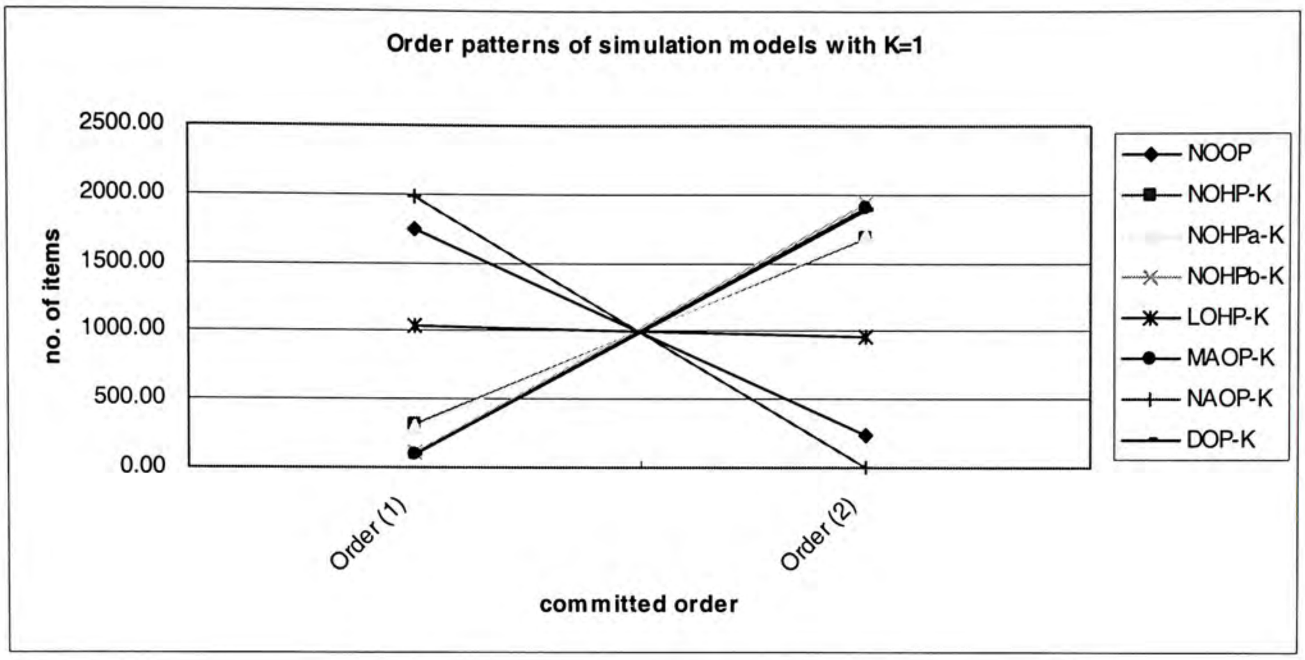
Figure 7: order pattern of *DOP-K* with varying *K*

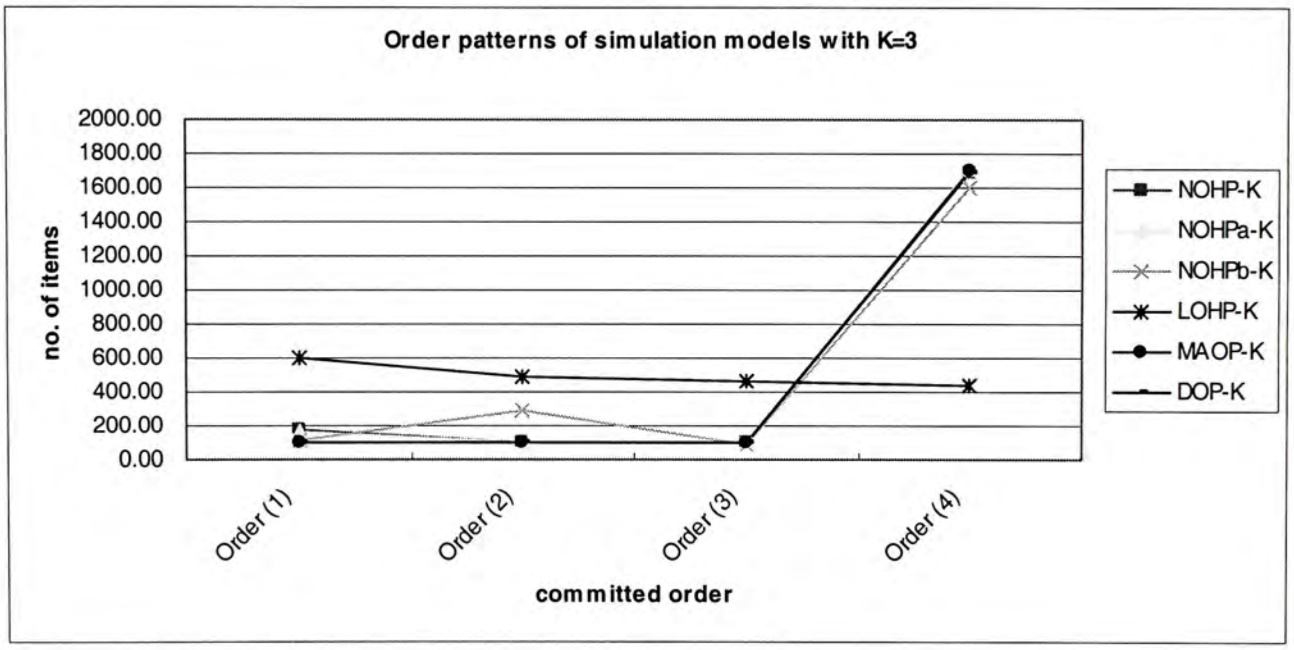
From Figure 7, we can know the actual demand is 1979.41, since the total demand is equal to the total order in this deterministic case of window size 1. As stated in the introduction of *DOP-K* in Section B.6.1, *DOP-K* places orders as early as possible since the demand is known. So, Figure 7 shows the largest proportion of the total order is Order (*K*). In the case of *K* equal to 19, all the Order (1) to Order (19) are approximately 100, which is the mean of the demand distribution for each period.

Since *DOP-K* is the deterministic case, from its ordering amounts, we can know the actual average demands[5] generated by our simulation program. From Eq.6.1.(7) to Eq.6.1.(10), we see that Order (1) to Order (*K*) are the average demands from day 1 to day (*K*). The particular interesting case is at window size equal to 19, in which the Order (1) to Order (*K* + 1) are all the 20 average demands over the whole horizon. This knowledge of the actual situation can help us to make more precise analysis on the heuristics.

From Table 4 on page 200, as the window size increases, *NOHPb-K*, *LOHP-K* and *MAOP-K* have less and less total cost savings compared to *DOP-K*. *DOP-K* can do much better than the other policies with the increasing window size because it can places the orders with the exact amount of demand in an earlier time as the window size becomes larger. This makes the most of the discount benefits. But *NOHP-K*, *NOHPa-K* and *LOHP-K* always need to use the current order to adjust the inventory level to meet the current periods' demand, which has no discount benefit. In Figure 10, you can see Order (1) of *NOHP-K*, *NOHPa-K* and *LOHP-K* are larger than *DOP-K* and this has increased their inventory costs.

[5] average over 100 simulation runs





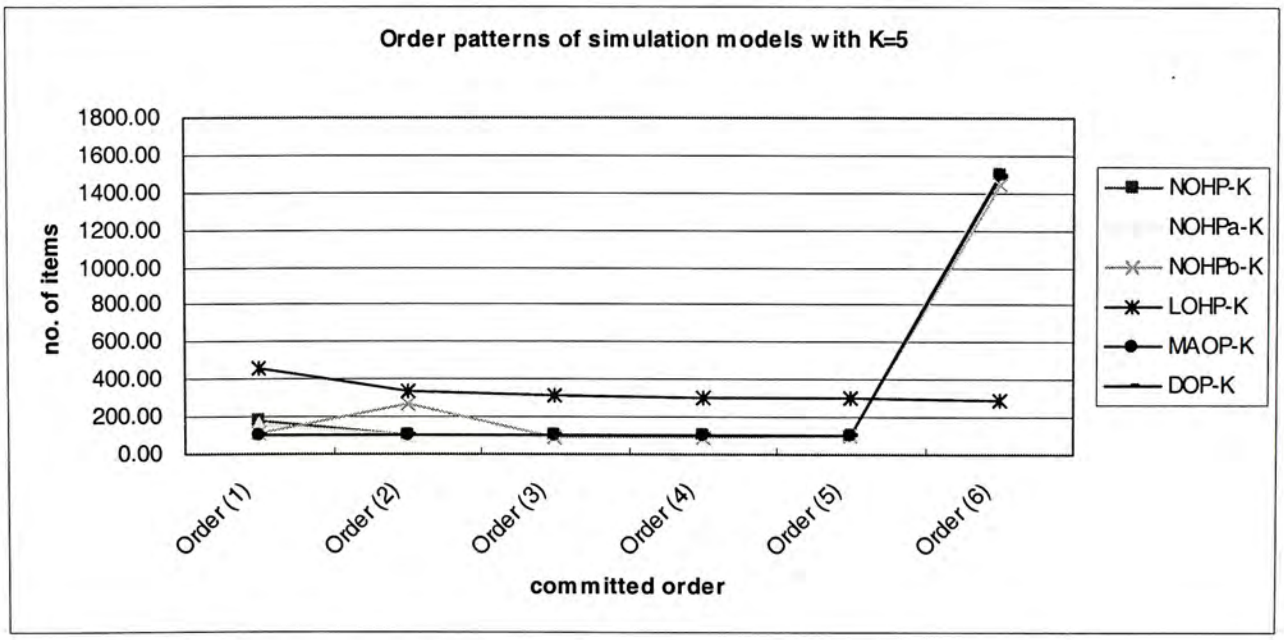


Figure 8: order patterns of simulation models with $K = 1, 3, 5$

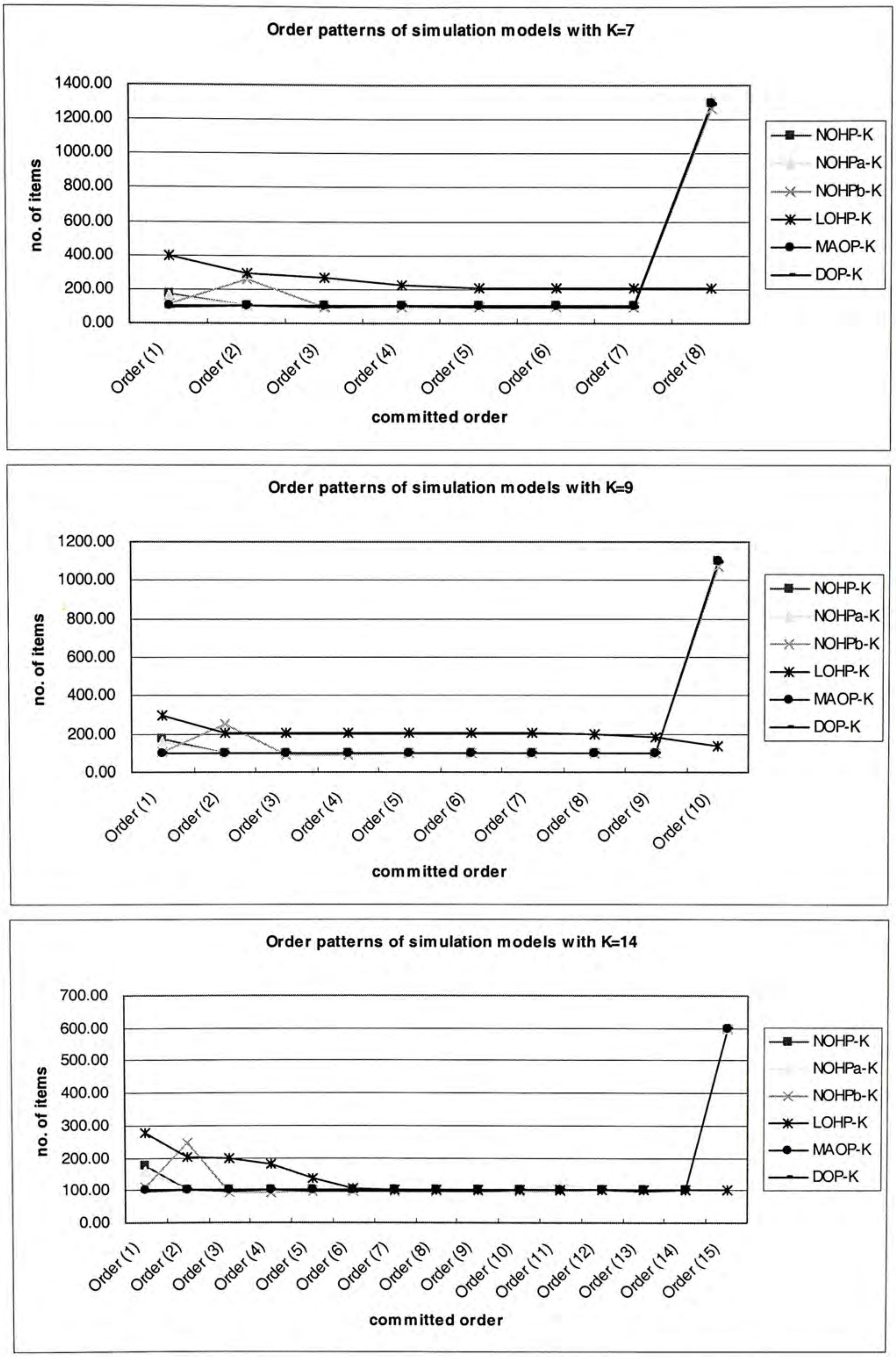


Figure 9: order patterns of simulation models with K = 7, 9, 14

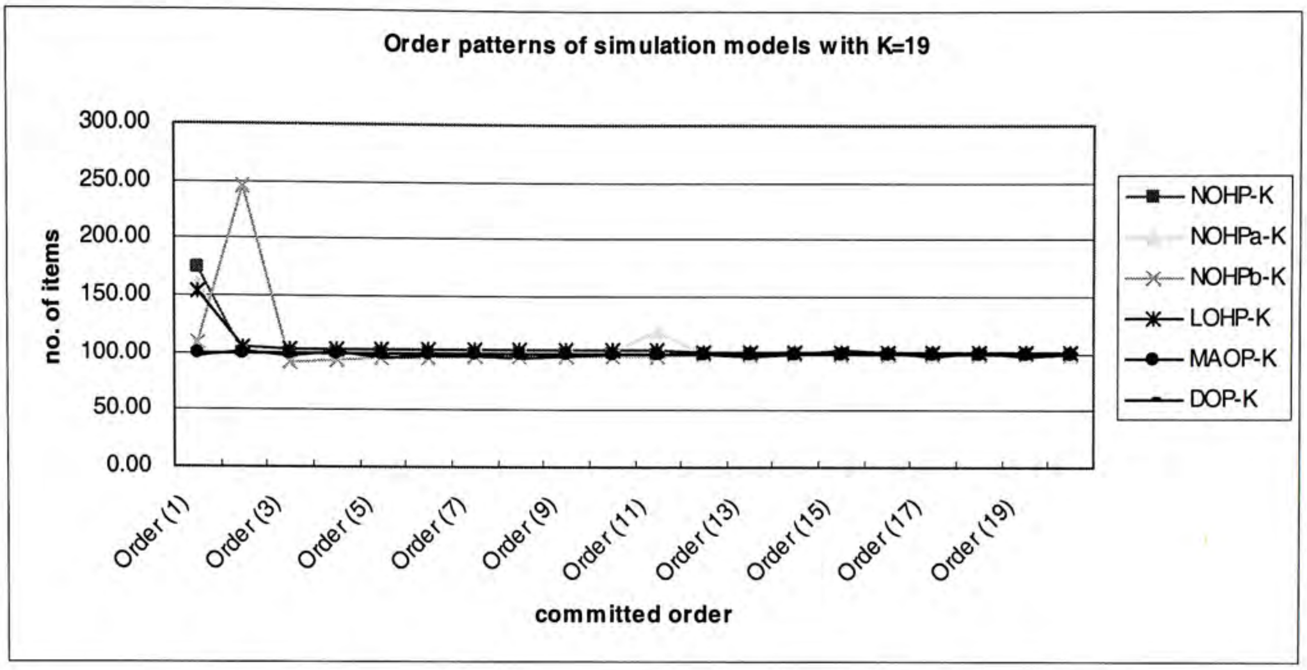


Figure 10: order patterns of simulation models with *K* = 19

From Figure 8, Figure 9 and Figure 10 above, they clearly show the differences of the ordering patterns of different heuristics with different values of *K*. Figure 11 summarizes the total orders placed by different heuristics with different window size.

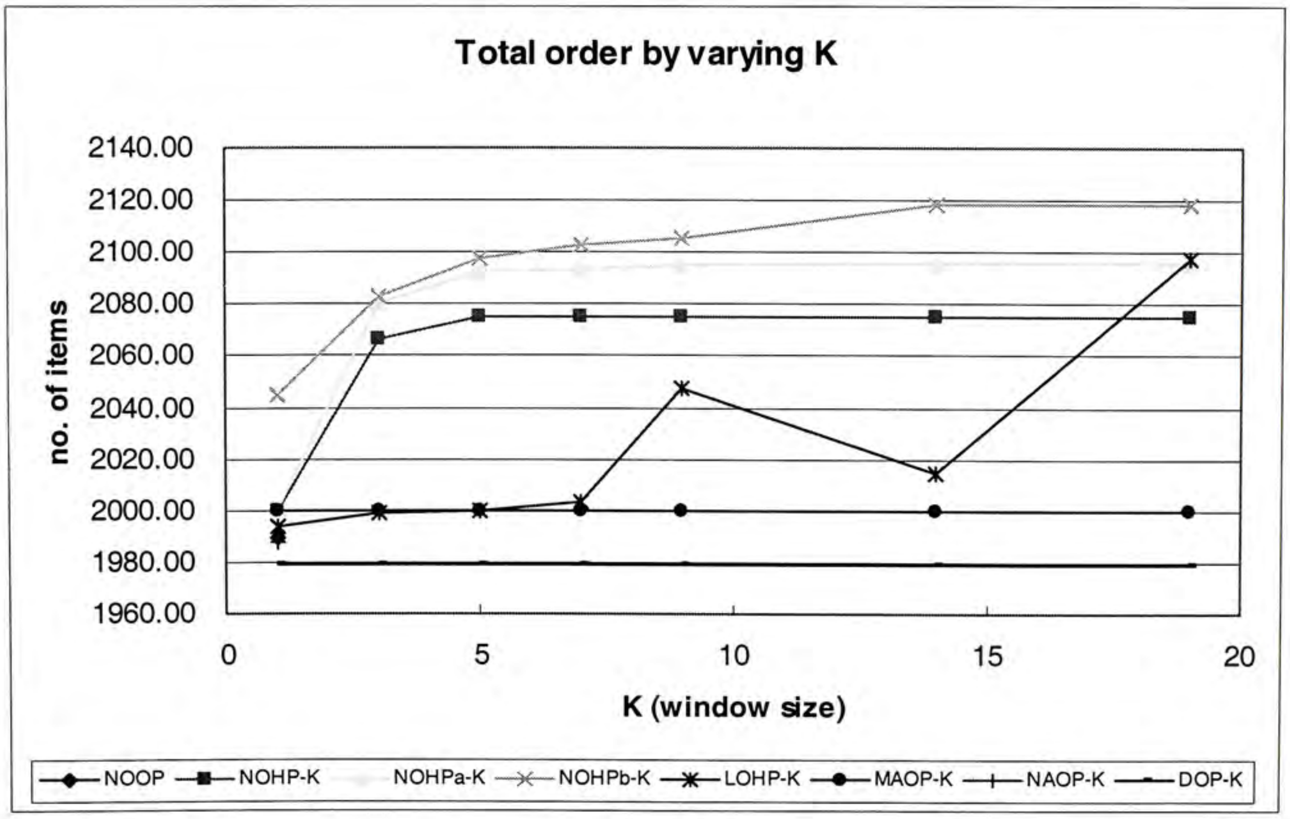


Figure 11: total order pattern of all simulation models with varying *K*.

Generally, *NOHP-K* and *NOHPa-K* act similarly to *DOP-K*. They have similar amounts of Order (2) to Order (*K*), which is about 100, the mean of the demand distribution, and Order (K + 1), which is especially larger than the other Orders. But Order (1) of *NOHP-K* is always larger than that of *DOP-K*.

The Orders of *LOHP-K* are usually larger than the other policies. However, as *K* increases, the amounts of the *LOHP-K* orders become closer and closer to those of the *DOP-K* orders. The number of orders of *LOHP-K* is high at the beginning of the advance order window and become constant towards the end of the window. This is very different from other policies. As *K* increases, we can see that the amounts of Order (1) and Order (K + 1) become less and less. When *K* reaches 19, the lines of *NOHP-K* and *LOHP-K* are very close, and they are close to the line of *DOP-K*.

When $K \geq 9$, the Order (2) of *NOHPb-K* is even larger than any Order of *LOHP-K*, although the Order (2) itself is decreasing with the increasing *K*.

Figure 12 shows the holding cost of *LOHP-K* increases quite linearly and the holding cost of *NOHPb-K* increases concavely with increasing *K*. The holding cost for *NOHP-K* (*NOHPa-K*) starts from 2.81 (2.88) at *K* equal to 1 and stays at about the level 7.14 (8.19) starting from *K* equal to 5 (14). The holding cost of *NOHP-K* is higher than that of *LOHP-K* for $K \leq 14$. Note that the shape of the curves in Figure 12 is very similar to that in Figure 11.

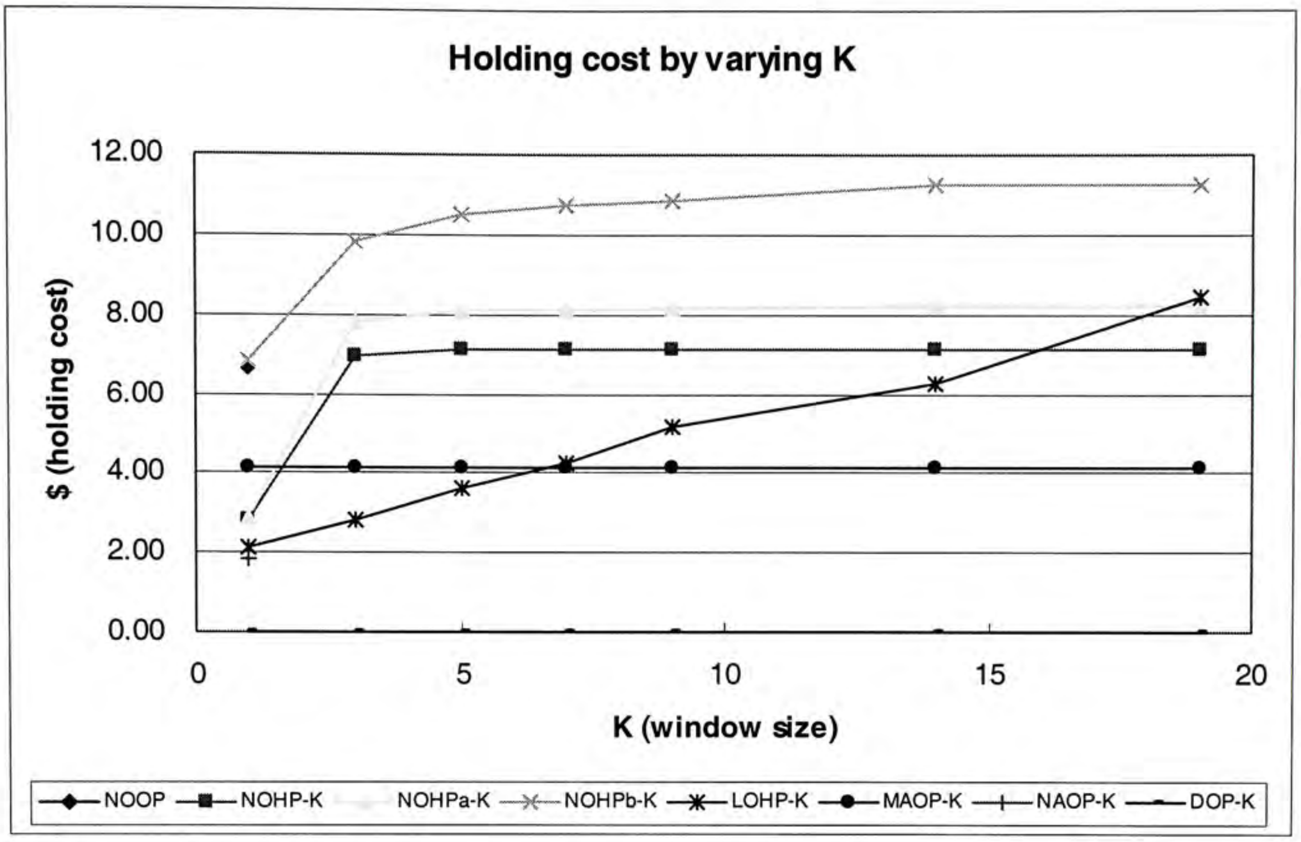


Figure 12: holding cost of all simulation models with varying *K*

The holding cost of *NOHPb-K* is always higher than *NOHPa-K*, which in turn is higher than *NOHP-K* because the total order placed by *NOHPb-K* is higher than *NOHPa-K*, which in turn is higher than *NOHP-K*. However, note that *NOHP-K* places more current orders, Order (1), than *NOHPa-K* and *NOHPa-K* places them more than *NOHPb-K*. This means *NOHPb-K* enjoys the most of discount benefit by placing large proportion of its orders in advance. Thus, the total cost of *NOHPb-K* is the lowest among the three policies.

Figure 13 is like an up-down flip of Figure 12. The shortage costs of *LOHP-K* and *NOHP-K* are decreasing convexly with increasing *K*. *NOHP-K* (*NOHPa-K*) starts from 2669.77 (2601.91) at *K* equal to 1 and stays at the level 1140.16 (961.39) starting from *K* equal to 5 (14). The shortage cost of *NOHP-K* is lower than that of *LOHP-K* for $K \leq 14$.

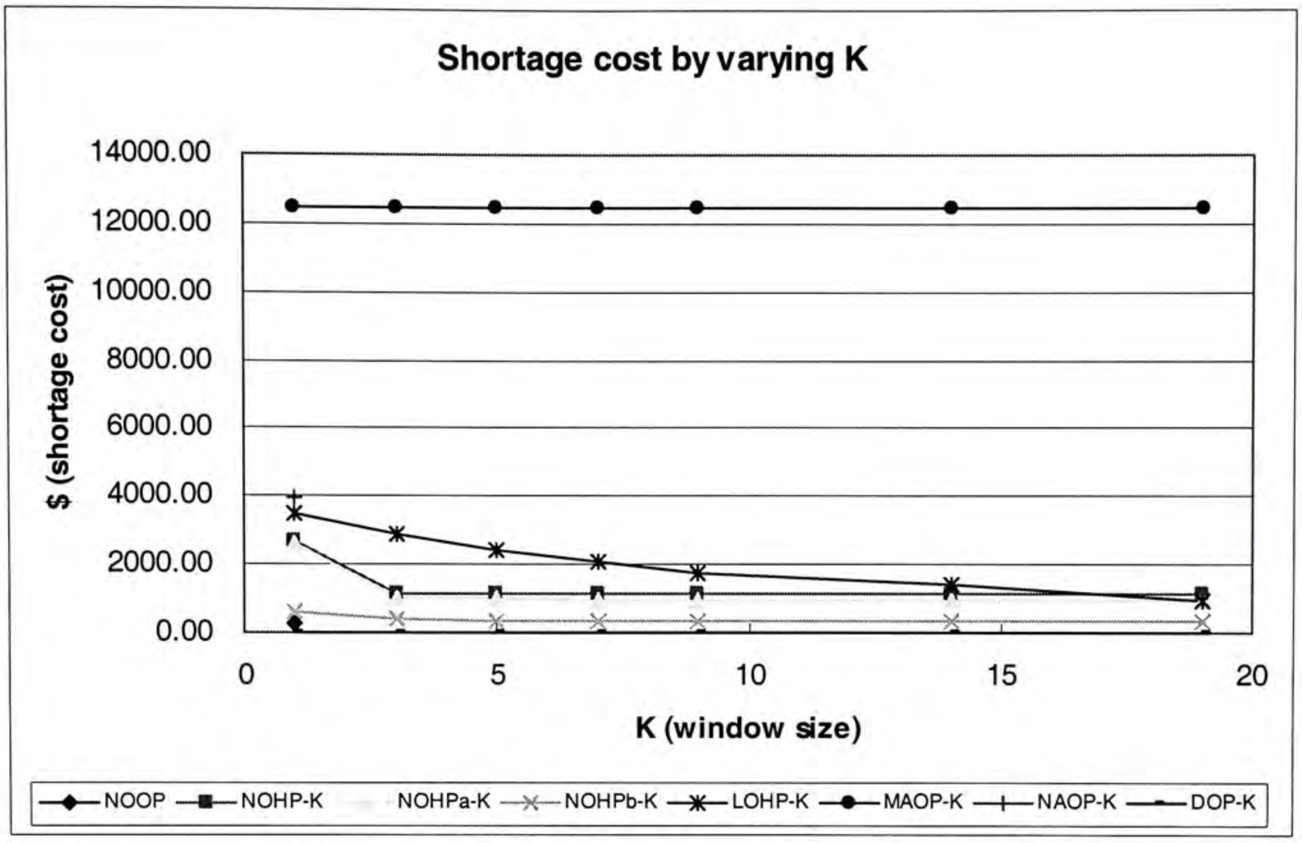


Figure 13: shortage cost of all simulation models with varying *K*

We notice that the total order of *NOHP-K*, *NOHPa-K*, *NOHPb-K* and *MAOP-K* are always higher than the actual demand, by comparing to the total demand of *DOP-K* in Figure 11. We would expect the holding cost of them is usually high.

It is interesting that the holding cost and shortage cost of *NOHP-K* stays approximately constant for $K \geq 5$. We think this is because a window size of 5 is large enough, to de-couple the early placed orders from the current situation of demand. Thus *NOHP-K* would give similar amount of advance orders. So, in every day except day 1, a similar amount of committed order arrive which tries to bring the inventory level up to the Newsboy level T^*. Then in general, *NOHP-K* has the same amount of inventory to face the demand in each day, and it faces the same amount of backorders and inventory left at the end of the day, so as the same shortage and holding costs incurred.

The larger window size is, the more holding cost we need to pay for *LOHP-K*. The reason is that the larger the window size is, the longer time in between the ordering day and the delivery day. Then we make poor forecasting and place too much advance orders by *LOHP-K* - each of them at least equal to the current order $\tilde{y}_t'$ (From Eq.6.1.(4)) and the larger the window size is, the larger the *OUL* would be.

In the previous paragraph, we have mentioned that when window size becomes large, the holding cost also becomes large, which means we are generally holding too much inventory. So, backorders would occur less frequently and the shortage costs would be reduced as window size increases.

For *DOP-K*, since we know the demand before ordering, its holding and shortage costs are zero.

In Figure 14, under the setting of the convexly decreasing β_k, it shows the total costs of the heuristics with varying *K*. If we have window size 1, *NOHPb-K* performs the best which has the lowest total cost. *NOHP-K* and *NOHPa-K* are always doing better than *LOHP-K* and has similar shape to the curve of *DOP-K*. For *K* equal to 19, *LOHP-K* is doing only worse than *NOHPb-K*. The trends of all the curves are decreasing with increasing *K* and have a clear convex shape.

Table 4 shows precisely how each policy performs comparing to *DOP*, in terms of total cost saving, which is calculated by

[(total cost of specific policy - total cost of *DOP*) / total cost of *DOP*] x 100%

--- *(2)*

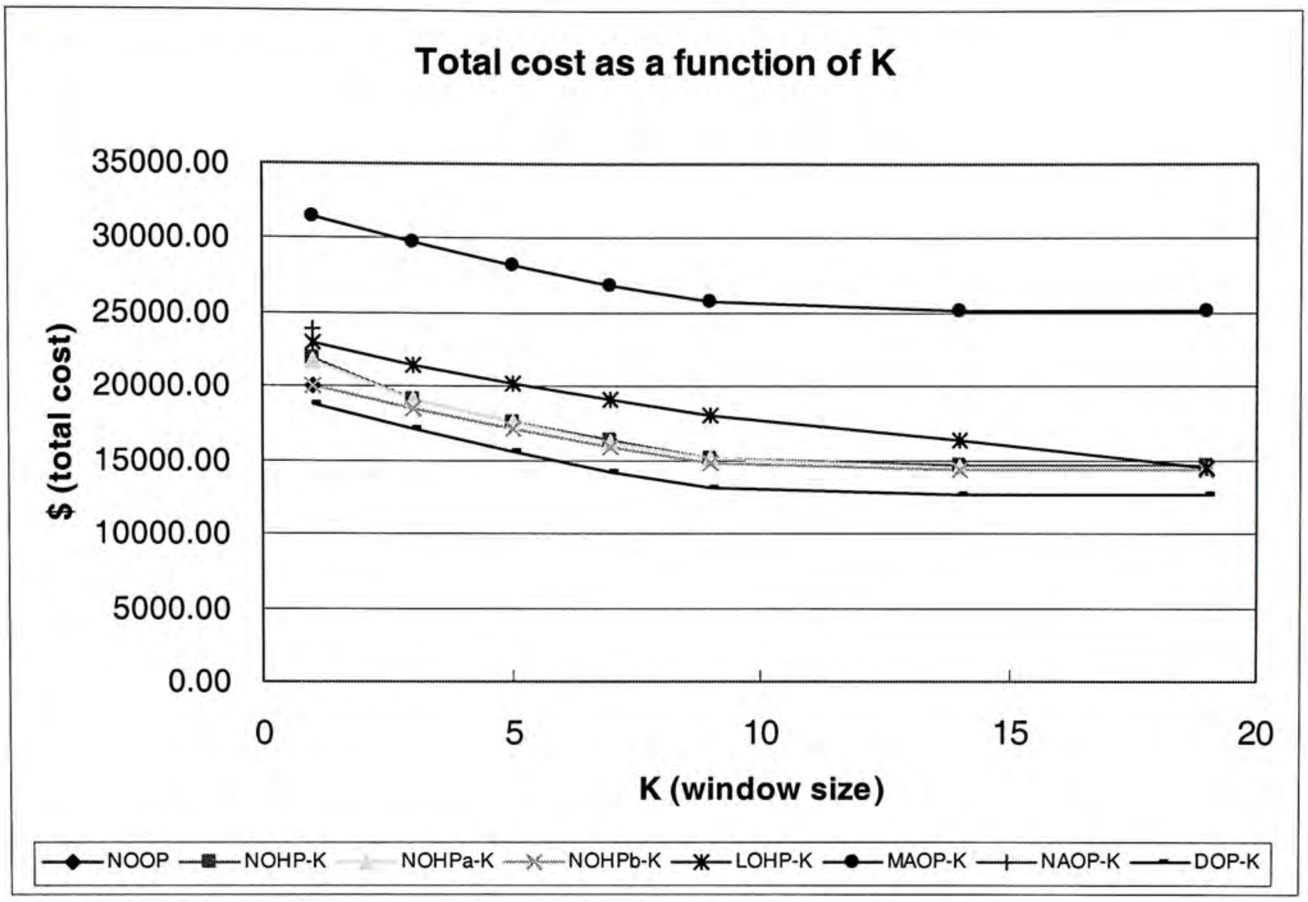


Figure 14: total cost of all simulation models with varying *K*

While *K* increases, the deviation of all the policies from *DOP-K* largely increased, especially *MAOP-K*. *NOHPb-K* closely approximates *DOP-K* for all values of *K*.

Table 4: total cost saving of each policy compared to *DOP-K*, with varying *K*

K	*NOOP*	*NOHP-K*	*NOHPa-K*	*NOHPb-K*	*LOHP-K*	*MAOP-K*	*NAOP*
1	0.07	0.16	0.15	0.07	0.22	0.67	0.26
3	-	0.12	0.11	0.08	0.25	0.74	-
5	-	0.13	0.13	0.10	0.29	0.81	-
7	-	0.14	0.14	0.11	0.34	0.88	-
9	-	0.16	0.15	0.13	0.38	0.96	-
14	-	0.16	0.15	0.13	0.30	1.00	-
19	-	0.16	0.15	0.13	0.15	1.00	-
Average	-0.85	0.15	0.14	0.11	0.28	0.87	-0.82

Chapter B.7

Conclusion and Further Studies

Much research has been done on optimal ordering policy for retailers under many different settings. Recently, as technology enables us to have advance demand information, advance ordering becomes a hot topic in the field of inventory management. From our literature review and also our research, we can see that advance ordering is very important both for the manufacturer and retailer to save cost.

In my research, I focus on how a retailer should place orders when faced with a price discount scheme, which provides incentives to induce advance orders. We develop a dynamic programming model for this problem and propose a policy based on the optimal solution to the model. Our simulation results indicate that by using a heuristic ordering policy, the retailer can save up to 39% inventory cost (including the purchase, holding and shortage costs), compared to no advance ordering policy. For some other policies that we proposed, we also find that they are still better than the policy without advance ordering. Thus, a retailer is usually better to place advance orders in such a price discount scheme. On the other hand, the price discount scheme is very effective to attractive advance orders.

Our research is of significance also because it can help the manufacturer (parties in the upper stages of supply chain) design incentives to induce advance ordering from the retailer (parties in the lower stages of supply chain). In particular, the analysis of our simulation study can quantify the value of advance information and is useful for the manufacturer to justify the discounts given in the scheme.

In addition, we have proposed a heuristic ordering policy *NOHPb* which is a simple (compared to *NOHP*) and sensitive to discount rate(s) policy (better than *NOHP*). It performs very close to *NOOP* in many cases of the simulation study, so it can be used to approximate the real optimal ordering policy. The most important features of *NOHPb* are that it applies to a general window size *K* (*NOHPb-K*), its fast computation time and it is easily implementable in practice.

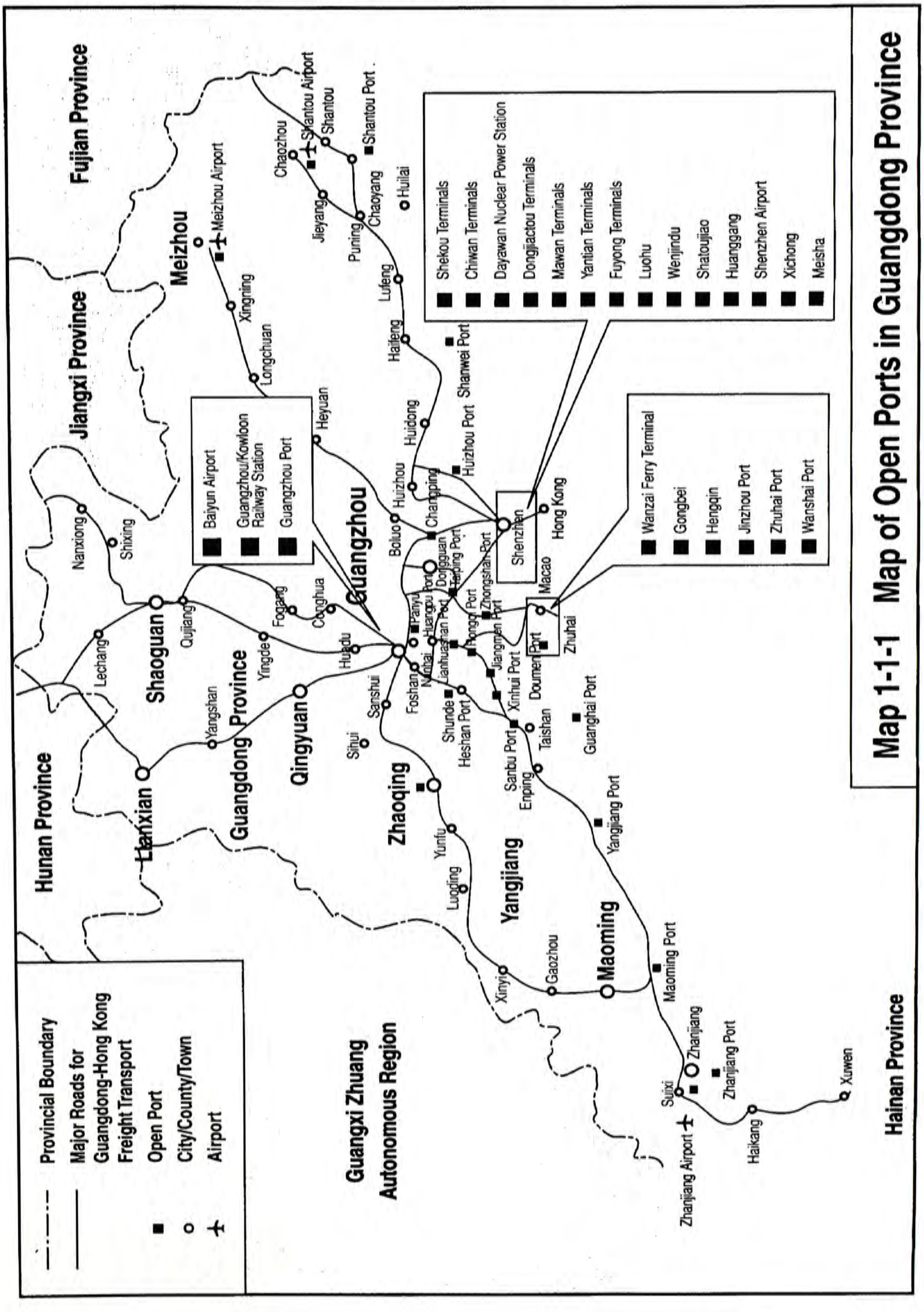
There are some further studies that can be done to extend the results in this thesis. Firstly, we can modify the 'description' of demand. In our simulation study, we have assumed that the demand has the Uniform or Normal distribution respectively and that they are *i.i.d.* by period. Trying some other distributions, e.g. Poisson distribution, may give us more insights into the problem and apply our model in other domains (rather than the manufacturer-retailer relation). We can also consider demands in different periods are Markovian, i.e., each the demand distribution of a given period depends on the demand of the previous period.

Secondly, for the study on the general model (window size *K*), we have only run several simulations and have not solved it analytically.

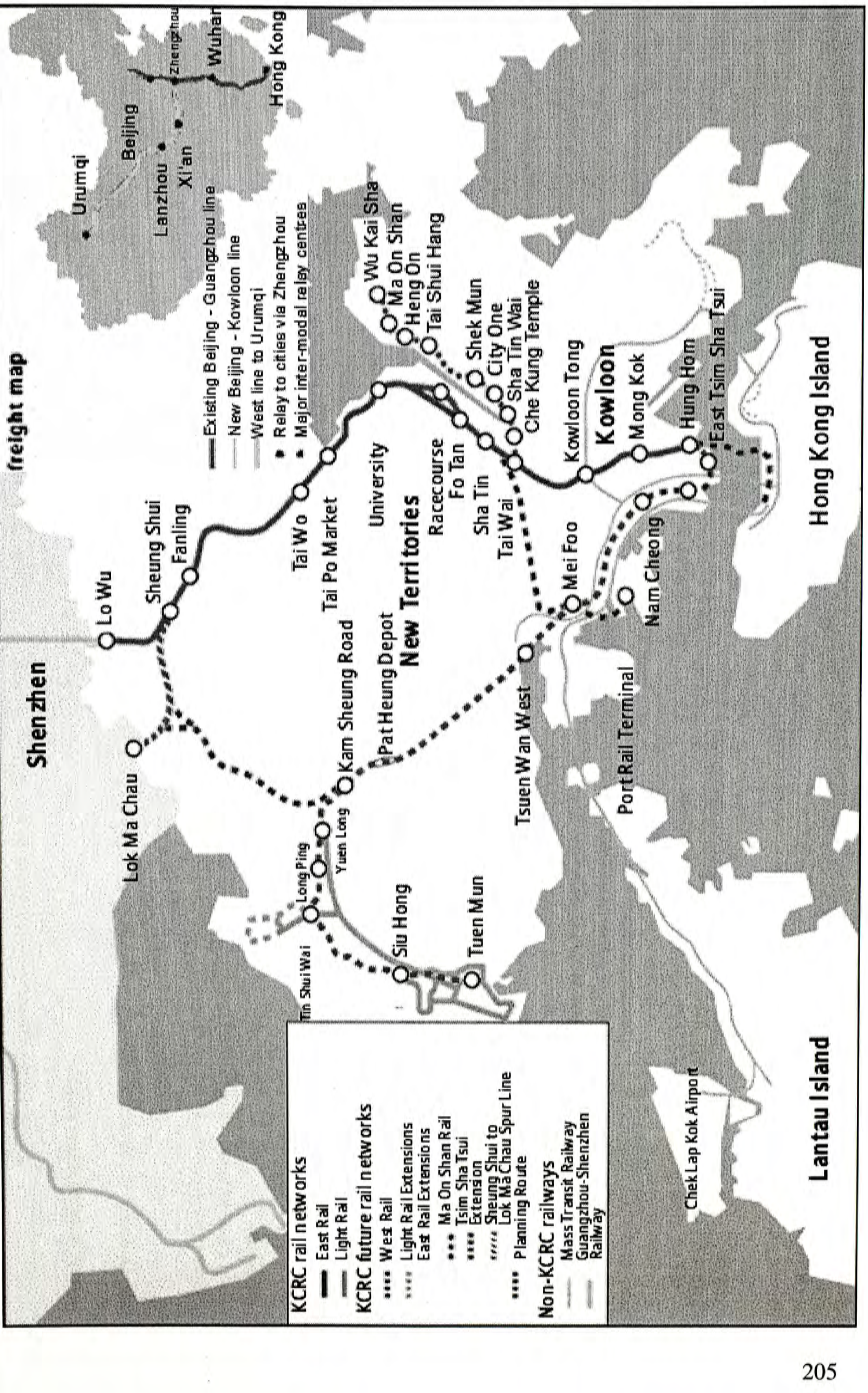
Finally, it is natural to extend this research, to ask the question: what the optimal discount scheme for the manufacturer? In addition, we note that a general supply chain can have many stages and in each stage, there may be many multiple parties. It is interesting to develop the incentive schemes available for the supply chain to encourage all parties in the lower stage to place advance orders, especially if demand distributions for different retailers may be different.

Appendix (Part A)

I. <u>Map of Open Ports in Guangdong Province</u>



II. <u>Map of Railway (Kowloon-Canton Railway Corporation, KCRC)</u>



III. Market Liberalisation under the China-US WTO Agreements

Existing Rules on Foreign Participation	Terms of China-US WTO Agreement
Trading & Distribution Rights	
Pilot JV trading companies in Pudong and Shenzhen have import-export rights.	Will provide trading rights to foreign companies, to be progressively phased in over three years.
Foreign-invested manufacturers and pilot JV retail enterprises have import-export rights for their own business, but cannot act as agents for other enterprises.	Full distribution rights to foreign firms to be phased in. Comprehensive commitments on wholesaling, retailing, franchising and direct sales.
China generally prohibits foreign firms from distributing products other than those they make in China, or from controlling their own distribution networks.	Will open up related services such as maintenance and repair, storage and warehousing, freight forwarding, trucking and air express services.
Wholesale	
Except for JV trading companies in Pudong and Shenzhen and JV retailing companies in certain pilot cities, foreign companies are not allowed to distribute products produced overseas.	Except for salt and tobacco, JVs can distribute most products five years after accession.
Except for the wholesaling of their own products produced in China, foreign companies are prohibited from being involved in the wholesaling of other products.	Majority ownership in JVs allowed two years after accession; no geographic or quantitative restriction by then. No restriction on equity/form of establishment three years after accession.
Except for investing in one minority-owned JV wholesale enterprise in each of the four municipalities, foreign firms are prohibited from being involved in wholesaling.	Foreign-invested enterprises can distribute their products manufactured in China and provide subordinate services upon accession.
Foreign firms are not allowed to own and manage distribution networks, wholesaling outlets, or warehouses in China.	Foreign service suppliers can provide the full range of related subordinated services, including after sales services, for the products they distribute.

Existing Rules on Foreign Participation	Terms of China-US WTO Agreement
Retailing	
JVs are allowed in the capital cities of all provinces and autonomous regions, central administered municipalities, independent planning cities with provincial status and SEZs.	JVs are allowed in five SEZs and eight major cities upon accession.
For JV retail enterprises with less than three outlets, the foreign partner can own up to 65% of the share. Except specifically exempted by the State Council, those with more than three outlets should be majority-owned by the Chinese partner.	In Beijing and Shanghai, no more than four JVs are allowed. No more than two JVs are allowed in other localities. Two JVs in Beijing can set up branches within Beijing.
Approved JV retail enterprises may expand into wholesale business.	Majority ownership in JVs allowed two years after accession. All provincial capitals, Chongqing and Ningbo will be opened to JVs by then.
	No geographic, quantitative restriction, equity/form of establishment restriction three years after accession. However, department stores of over 20,000 square metres and chain stores with more than 30 stores will continue to be limited to minority-owned JVs only.*
	Except for tobacco, retailing of all products will be allowed five years after accession.
Freight Forwarding	
Foreign freight forwarders can have no more than a 50% share in JVs and require an investment of no less than US$ 1 million. Foreign partners have to be in business for a minimum of three years to quality for a first JV.	Majority ownership in JVs allowed one year after accession.
	Wholly-owned subsidiaries allowed four years after accession.
Required to observe a five-year waiting period for forming a second JVe, and a one-year waiting period for establishing branches. An additional investment of US$ 120,000 is required for each additional branch.	JVs are not limited to conduct international freight forwarding business only
The business of JVs is limited to certain geographical areas.	
Very few JVs are allowed to handle domestic freight forwarding	

Appendix (Part B)

I. Derivation of the Newsboy Model

(Wagner [58], p.809)

We define variables:

c	unit purchase cost
h	unit holding cost
q	unit shortage cost
z	total inventory level
S	optimal inventory level
$L(z)$	loss cost function,

$$L(z) = \begin{cases} \int_0^z h(z-m)f(m)\,dm + \int_z^\infty q(m-z)j(m)\,dm & \textit{for} \quad z \geq 0 \\ \int_0^\infty q(m-z)j(m)\,dm & \textit{for} \quad z < 0 \end{cases}$$

Please note that there is x amount of initial inventory.

If $\frac{dL(z)}{dz} = L'(z)$ exists, $z = S$ that minimizes $[cz + L(z)]$ must satisfy $c + L'(z) = 0$.

By advanced calculus, $z \geq 0$,

$$L'(z) = \int_0^z hf(m)\,dm + \int_z^\infty qj(m)\,dm$$
$$= hJ(z) - q(1 - J(z))$$

So,

$$c + L'(S) = 0$$
$$c + hJ(S) - q(1 - J(S)) = 0$$
$$J(S) = \frac{q-c}{h+q}$$

II. Hessian Matrix (Hillier [31], p.955; Anton [3], p.479)

For a two-dimensional matrix,

$$\begin{pmatrix} \dfrac{\partial^2 f(x_1,x_2)}{\partial x_1^{\,2}} & \dfrac{\partial^2 f(x_1,x_2)}{\partial x_1\, \partial x_2} \\ \dfrac{\partial^2 f(x_1,x_2)}{\partial x_1\, \partial x_2} & \dfrac{\partial^2 f(x_1,x_2)}{\partial x_2^{\,2}} \end{pmatrix}$$

We can check the jointly convexity of $f(x_1,x_2)$ w.r.t. x_1 and x_2 by the following table:

Table 1: convexity test for a function of two variables

Quantity	Convex	Strictly Convex	Concave	Strictly Concave
$\frac{\partial^2 f(x_1,x_2)}{\partial x_1'} \cdot \frac{\partial^2 f(x_1,x_2)}{\partial x_2'} - \left[\frac{\partial^2 f(x_1,x_2)}{\partial x_1 \partial x_2}\right]^2$	≥ 0	>0	≥ 0	>0
$\frac{\partial^2 f(x_1,x_2)}{\partial x_1^2}$	≥ 0	>0	≤ 0	<0
$\frac{\partial^2 f(x_1,x_2)}{\partial x_2^2}$	≥ 0	>0	≤ 0	<0

III. Useful Formula

Leibniz's Formula

$$\frac{d}{dx}\int_{L(x)}^{U(x)} f(x,t)\,dt = \int_{L(x)}^{U(x)} \frac{f(x,t)}{dx}\,dt + f(x,U(x))\frac{dU(x)}{dx} - f(x,L(x))\frac{dL(x)}{dx}$$

IV. <u>Derivation of Newsboy Order Heuristic Policy (*NOHP*)</u>

For completeness, we summarize the case for $K = 0$, i.e. no advanced ordering.

We only consider the optimization of period N. The cost function is:

$$f_N(x_N; g_{N-1}(N)) = \min_{y_N^N \geq 0}\{cy_N^N + \Psi(x_N + g_{N-1}(N) + y_N^N)\} \qquad \text{--- (1)}$$

To solve the $\tilde{y}_N^N$, we solve a canonical Newsboy problem to give:

$$J(x_N + g_{N-1}(N) + y_N) = \frac{q-c}{q+h}$$

So the optimal policy is:

$$\tilde{y}_N^N = [J^{-1}(\frac{q-c}{q+h}) - x_N - g_{N-1}(N)]^+ \qquad \text{--- (2)}$$

Note:

Let T^* be the optimal order-up-to level according to the Newsboy model, i.e., $T^* = J^{-1}(\frac{q-c}{q+h})$, and we define functions:

$$\Psi^*(r) = h\int_0^{J^{-1}(r)} (J^{-1}(r) - s)dJ(s) + q\int_{J^{-1}(r)}^{\infty} (s - J^{-1}(r))dJ(s) \qquad \text{--- (3)}$$

$$\tilde{\Psi}(v) = h\int_0^{v} (v - s)dJ(s) + q\int_{v}^{\infty} (s - v)dJ(s) \qquad \text{--- (4)}$$

Aside:

$$\begin{aligned}
\frac{\partial}{\partial r}\Psi^*(r) &= h\int_0^{J^{-1}(r)} \frac{\partial}{\partial r}J^{-1}(r)dJ(s) + q\int_{J^{-1}(r)}^{\infty} -\frac{\partial}{\partial r}J^{-1}(r)dJ(s) \\
&= \frac{\partial}{\partial r}J^{-1}(r)[hr - q(1-r)] \\
&= \frac{\partial}{\partial r}J^{-1}(r)[(h+q)r - q] \\
&\geq 0 \qquad \text{if} \quad r \geq \frac{q}{q+h}
\end{aligned}$$

So, $\Psi^*(r)$ is an increasing function in *r*.

By Eq.(2), we will have two cases:

➢ If $x_N + g_{N-1}(N) \le T^*$, then $\tilde{y}_N^N = T^* - x_N - g_{N-1}(N)$, and

$$f_N(x_N; g_{N-1}(N)) = c(T^* - x_N - g_{N-1}(N)) + h\int_0^{T^*}(T^* - s)dJ(s) + q\int_{T^*}^{\infty}(s - T^*)dJ(s)$$

We have the "optimal loss", $\Psi^*(\frac{q-c}{q+h}) = h\int_0^{T^*}(T^* - s)dJ(s) + q\int_{T^*}^{\infty}(s - T^*)dJ(s)$

Also, the expected residual inventory is:

$$E_N(x_N + g_{N-1}(N) + y_N^N - D_N) = \int_0^{\infty}(T^* - s)dJ(s) = T^* - E(D_N) = T^* - \mu \quad \text{--- (5)}$$

➢ If $x_N + g_{N-1}(N) > T^*$, then $\tilde{y}_N^N = 0$, and

$$\begin{aligned} f_N(x_N; g_{N-1}(N)) &= h\int_0^{x_N + g_{N-1}(N)}(x_N + g_{N-1}(N) - s)dJ(s) \\ &\quad + q\int_{x_N + g_{N-1}(N)}^{\infty}(s - x_N + g_{N-1}(N))dJ(s) \\ &= \tilde{\Psi}(x_N + g_{N-1}(N)) \\ &> \Psi^*(\frac{q-c}{q+h}) + c[T^* - (x_N + g_{N-1}(N))]^+ \end{aligned}$$

In this case, the expected residual inventory is:

$$E_N(x_N + g_{N-1}(N) - D_N) = x_N + g_{N-1}(N) - \mu \quad \text{--- (6)}$$

Now, we go to the case $K = 1$. The cost-to-go function is:

$$f_{N-1}(x_{N-1}; g_{N-2}(N-1)) = \min_{y_{N-1}^{N-1} \ge 0, y_{N-1}^N \ge 0}\{cy_{N-1}^{N-1} + c\beta_1 y_{N-1}^N + \Psi(x_{N-1} + g_{N-2}(N-1) + y_{N-1}^{N-1}) + Ef_N(x_{N-1} + g_{N-2}(N-1) + y_{N-1}^{N-1} - D_{N-1}, y_{N-1}^N)\}$$

Now, we consider a "lower" approximation.

$$
\begin{aligned}
f_{N-1}(x_{N-1}; g_{N-2}(N-1)) \geq & \min_{y_N^N \geq 0, y_N^{N+1} \geq 0} \{c y_{N-1}^{N-1} + c\beta_1 y_{N-1}^N + \Psi(x_{N-1} + g_{N-2}(N-1) + y_{N-1}^{N-1}) \\
& + E_{N-1}\{\Psi^*(\frac{q-c}{q+h}) \\
& + c[T^* - (x_{N-1} + g_{N-2}(N-1)) + y_{N-1}^{N-1} + y_{N-1}^N - D_{N-1})]^+\} \\
\geq & \min_{y_N^N \geq 0, y_N^{N+1} \geq 0} \{c y_{N-1}^{N-1} + c\beta_1 y_{N-1}^N + \Psi(x_{N-1} + g_{N-2}(N-1) + y_{N-1}^{N-1}) \\
& + \Psi^*(\frac{q-c}{q+h}) \\
& + c[T^* - (x_{N-1} + g_{N-2}(N-1) + y_{N-1}^{N-1} + y_{N-1}^N - \mu)]^+\} \\
& (\because E(X^+) \geq E(X))
\end{aligned}
$$

--- *(7)*

We claim that a near optimal heuristic policy is:

$$\tilde{y}_{N-1}^{N-1} = [T^* - x_{N-1} - g_{N-1}(N)]^+ \qquad \text{--- (8)}$$

$$
\begin{aligned}
\tilde{y}_{N-1}^{N} &= [T^* - x_{N-1} - g_{N-1}(N) - \tilde{y}_{N-1}^{N-1} + \mu]^+ \\
&= \begin{cases} T^* - (x_{N-1} + g_{N-2}(N-1) - \mu)]^+ & if \quad \tilde{y}_{N-1}^{N-1} = 0 \\ \mu & if \quad \tilde{y}_{N-1}^{N-1} > 0 \end{cases} \qquad \text{--- (9)}
\end{aligned}
$$

The rationale is as the following. Consider the formula from Eq.(7):

$$
\begin{aligned}
& \min_{y_N^N \geq 0, y_N^{N+1} \geq 0} \{c y_{N-1}^{N-1} + c\beta_1 y_{N-1}^N + \Psi(x_{N-1} + g_{N-2}(N-1) + y_{N-1}^{N-1}) + \Psi^*(\frac{q-c}{q+h}) \\
& \qquad + c[T^* - (x_{N-1} + g_{N-2}(N-1) + y_{N-1}^{N-1} + y_{N-1}^N - \mu)]^+\}
\end{aligned} \qquad \text{--- (10)}
$$

(i) Suppose $[T^* - (x_{N-1} + g_{N-2}(N-1) + y_{N-1}^{N-1} + y_{N-1}^N - \mu)]^+ = 0$,

then an obvious choice is to choose y_{N-1}^N as small as possible and choose $y_{N-1}^{N-1} = [T^* - x_{N-1} - g_{N-2}(N-1)]^+$, but $$[T^* - (x_{N-1} + g_{N-2}(N-1) + y_{N-1}^{N-1} + y_{N-1}^{N} - \mu)]^+ = 0 \quad \Rightarrow$$

$$T^* - x_{N-1} - g_{N-2}(N-1) - y_{N-1}^{N-1} - y_{N-1}^{N} + \mu \le 0 \quad \text{or}$$

$$y_{N-1}^{N} \ge T^* - x_{N-1} - g_{N-2}(N-1) - y_{N-1}^{N-1} + \mu$$

, hence the definition of $\tilde{y}_{N-1}^{N}$.

(ii) Suppose $[T^* - (x_{N-1} + g_{N-2}(N-1) + y_{N-1}^{N-1} + y_{N-1}^{N} - \mu)]^+ > 0$,

then coefficient of $y_{N-1}^{N} = \beta_1 c - c < 0$, so we want to make it as large as possible,

but $$[T^* - (x_{N-1} + g_{N-2}(N-1) + y_{N-1}^{N-1} + y_{N-1}^{N} - \mu)]^+ > 0 \quad \Rightarrow$$

$$y_{N-1}^{N} < T^* - x_{N-1} - g_{N-2}(N-1) - y_{N-1}^{N-1} + \mu .$$

For y_{N-1}^{N-1}, the optimal choice may seem to be $[J^{-1}(\frac{q}{q+h}) - x_{N-1} - g_{N-2}(N-1)]^+$,

but since $\frac{\partial}{\partial r}\Psi^*(r) \ge 0$ when $r \ge \frac{q}{q+h}$,

and comparing the value of $[T^* - (x_{N-1} + g_{N-2}(N-1) + y_{N-1}^{N-1} + y_{N-1}^{N} - \mu)]^+$ when

$y_{N-1}^{N-1} = [T^* - x_{N-1} - g_{N-2}(N-1)]^+$ and $y_{N-1}^{N-1} = [J^{-1}(\frac{q}{q+h}) - x_{N-1} - g_{N-2}(N-1)]^+$,

we see that $\tilde{y}_{N-1}^{N-1} = [T^* - x_{N-1} - g_{N-2}(N-1)]^+$ is a reasonable choice.

In conclusion,

$$\tilde{y}_t^t = [T^* - x_t - g_{t-1}(t)]^+ \quad \text{--- (11)}$$

$$\tilde{y}_t^{t+1} = \begin{cases} [T^* - (x_t + g_{t-1}(t) - \mu)]^+ & \text{if} \quad \tilde{y}_t^t = 0 \\ \mu & \text{if} \quad \tilde{y}_t^t > 0 \end{cases} \quad \text{--- (12)}$$

Please pay attention to Eq.(12), which can be re-written to the following:

$$\tilde{y}_t^{t+1} = \begin{cases} [T^* - (x_t + g_{t-1}(t) - \mu)]^+ & \text{if} \quad \tilde{y}_t^t = 0 \\ [T^* - (T^* - \mu)]^+ & \text{if} \quad \tilde{y}_t^t > 0 \end{cases} \qquad \text{--- (13)}$$

As presented in Eq.(5), $(T^* - \mu)$ is the expected residual inventory if an order is placed for the current day, and in Eq.(6), $(x_t - g_{t-1}(t) - \mu)$ is the expected residual inventory if no order is placed for the current day.

So, when we consider tomorrow's order, y_t^{t+1}, we actually consider what will be left today and order the amount that can bring back the inventory level to the Newsboy optimal amount. If no order is placed for the current day, that means today's inventory level is higher than the Newsboy level, we will place fewer orders for the next day; otherwise, we will place more advance orders.

V. Derivation of Lead Order Heuristic Policy (*LOHP*)

A brief review of *OUL* calculation:

To understand the safety inventory requirement, we need to investigate the sequence of events over time as the retailer places orders. Chopra [17] has a clear description on it: The retailer places the first order at time 0 such that the lot size ordered and the inventory on hand sum to the *OUL*. Once an order is placed, the replenishment lot arrives after the lead time *L*. the next review period is time *T* when the retailer places the second order, which arrives at time *T+L*. The *OUL* represents the inventory available to meet all demand that arises between periods 0 and *T+L*.

Formulations:

Probability (demand during $L+T \leq OUL$) = CSL[1]

Mean demand during $T+L$ periods, $R_{T+L} = (T+L)\,R$

s.d. of demand during $T+L$ periods, $\theta_{T+L} = \sqrt{T+L}\,\sigma_R$

Safety stock, $ss_{T+L} = F_s^{-1}(CSL) \times \theta_{T+L}$

Order-Up-to-Level, $OUL = R_{T+L} + ss_{T+L}$

We have made the following modifications for the policy *LOHP*:

We set *CSL* equal to the Newsboy ratio, $CSL = \frac{q-c}{q+h}$, so to make the comparisons with other policies fair.

Since evaluation of inventory level is done on daily basis, we set $T = 1$;

Since 1-day advance order is delivered on next day of the ordering day. The situation will be like there is 1-day lead time, we set $L = 1$.

We know that the lead time between the order day and the delivery day will incur the policy to hold much safety stock. So, we make the current order as a bound for the advance order. We believe the order for today is reliable to reflect the current inventory and demand situation. So, $\tilde{y}_t^t \geq \tilde{y}_t^{t+1}$.

Remark:

If $q < c$, we allow ss_{T+L} to be negative. This can make sure the service level is according to *CSL*.

---

[1] Cycle Service Level

VI. Additional Graphics of Window Size 1 Simulation Study

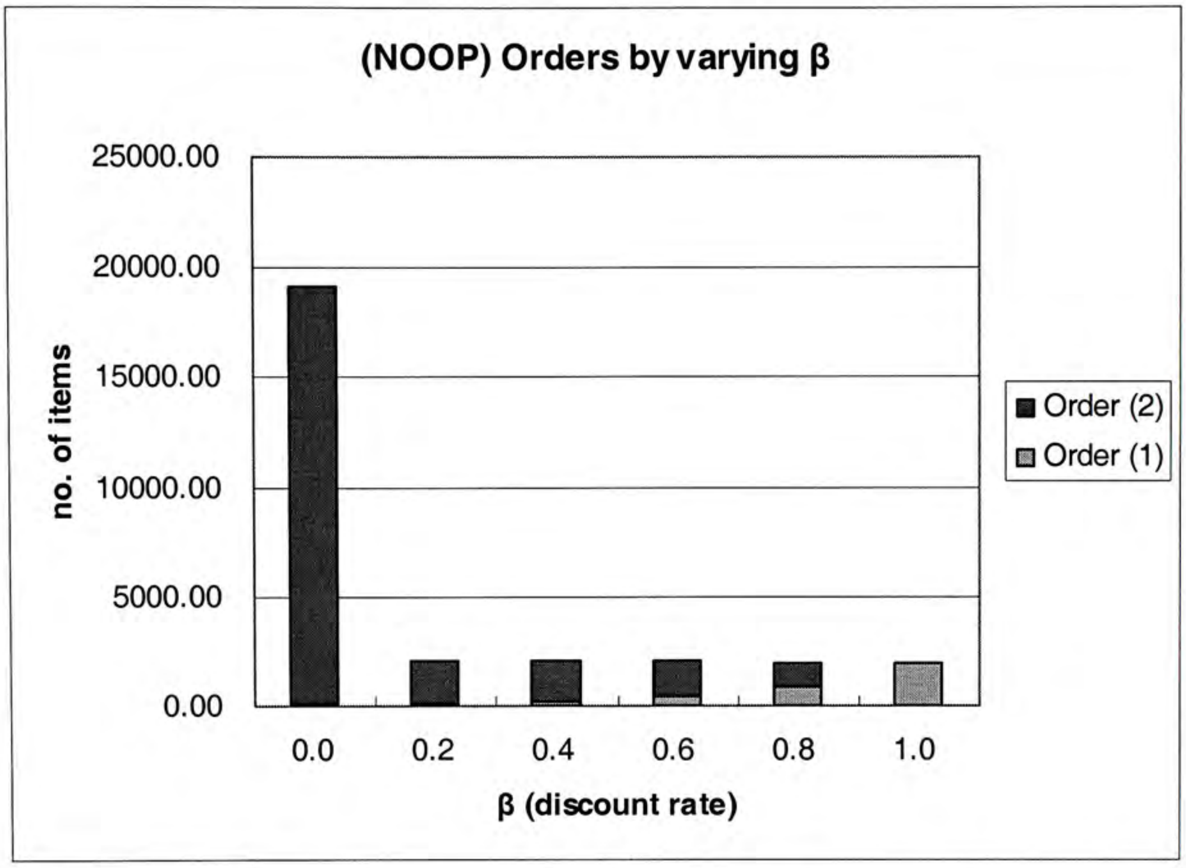


Figure 1 : order pattern of NOOP with varying β

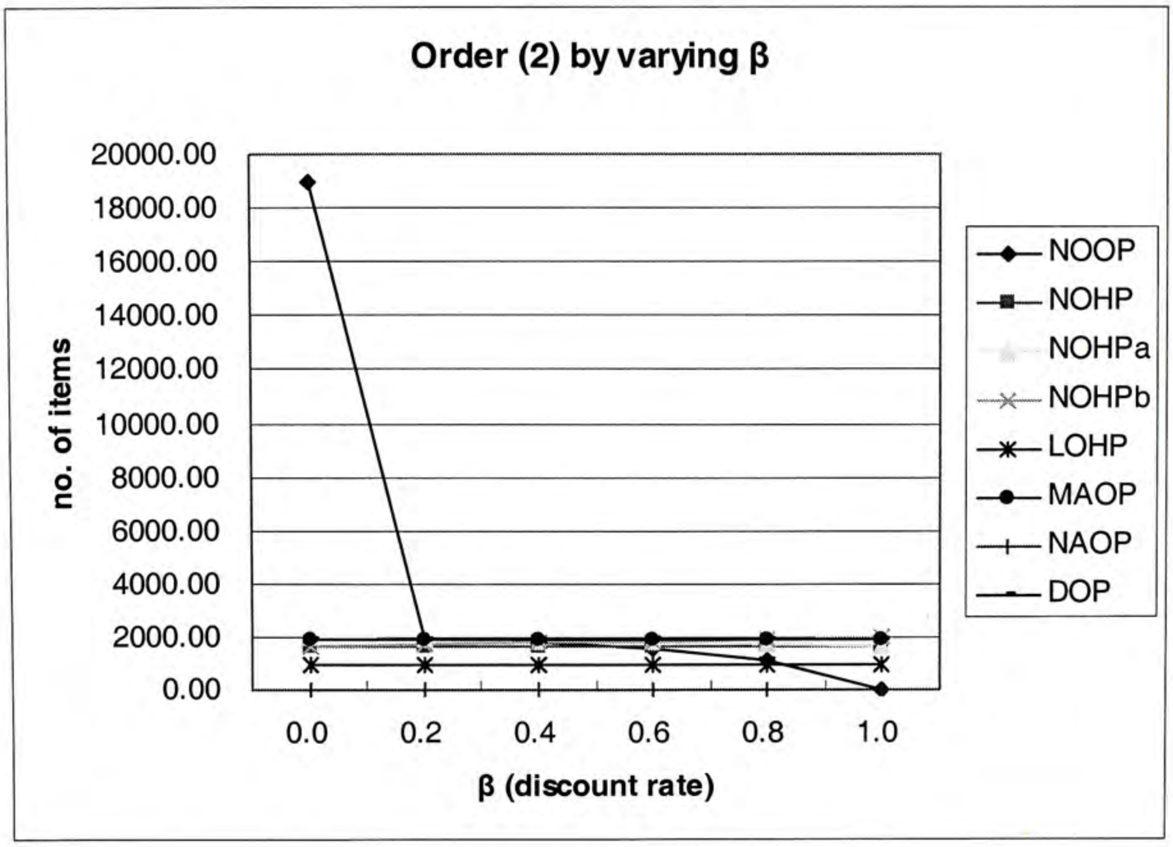


Figure 2: Order (2) patterns of all simulation models with varying β

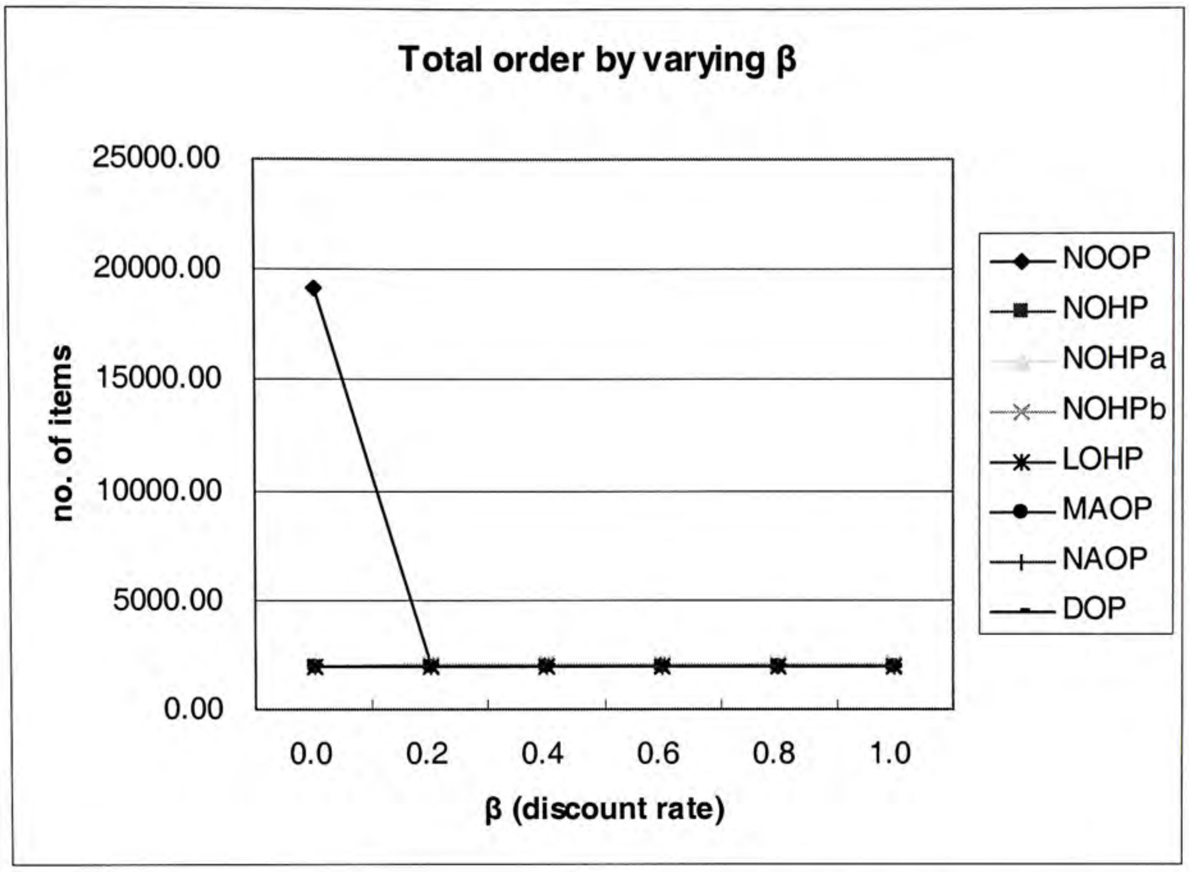


Figure 3: total order patterns of all simulation models with varying β_1

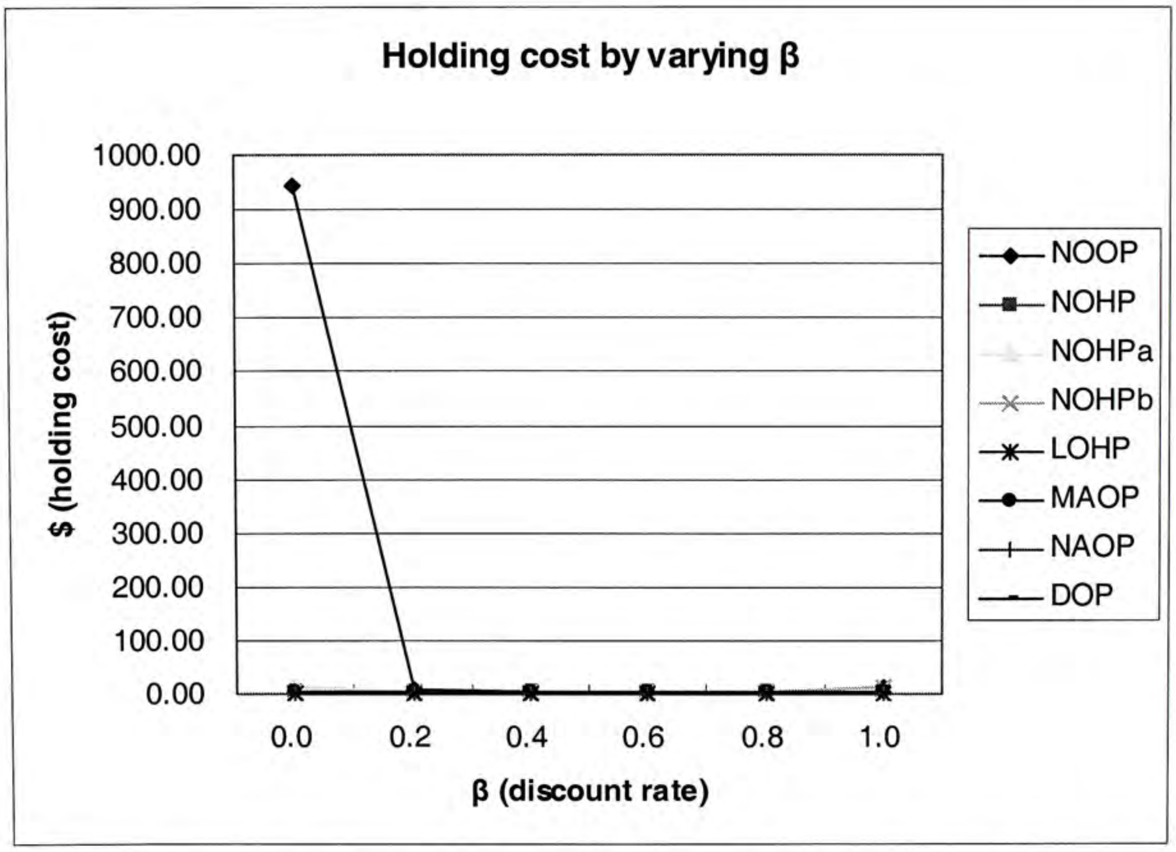


Figure 4: holding cost of all simulation models with varying β

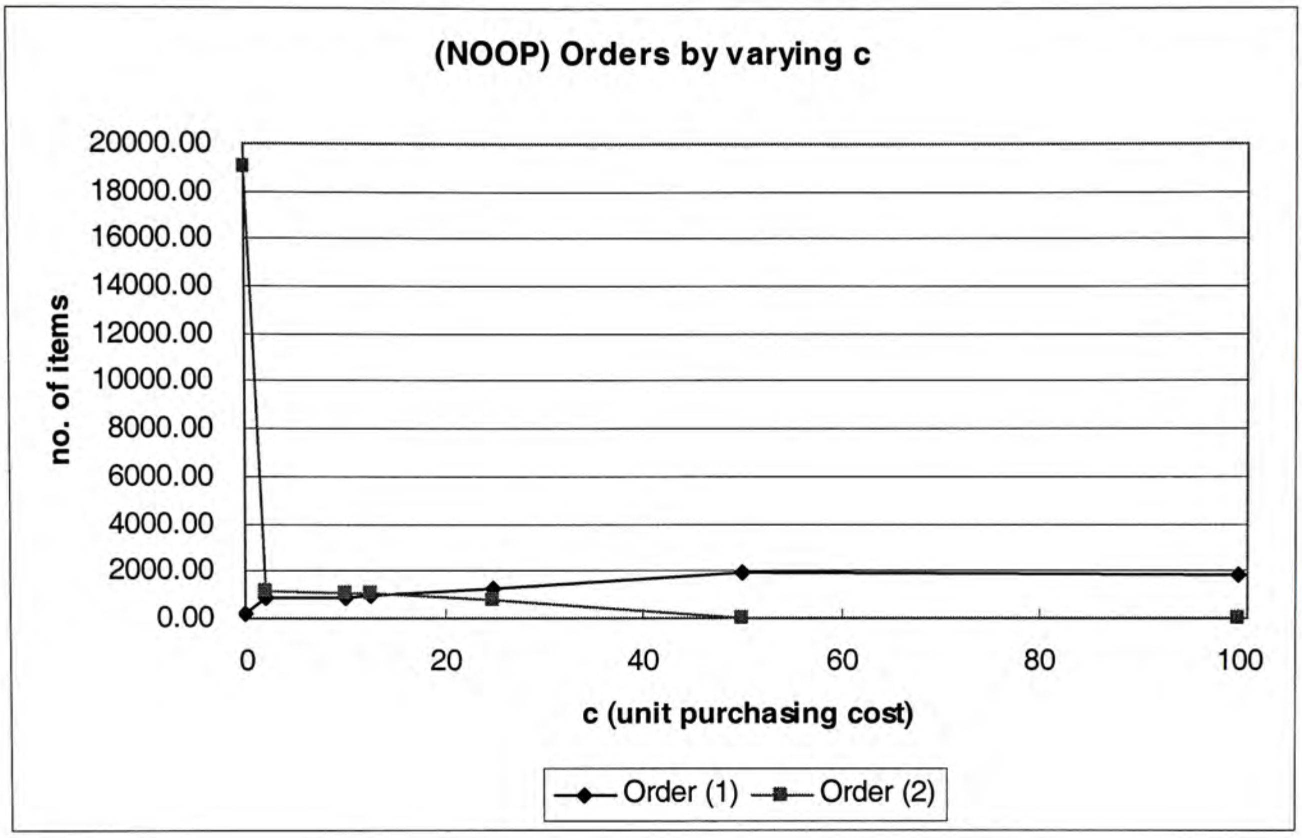


Figure 5: order pattern of *NOOP* with varying *c*

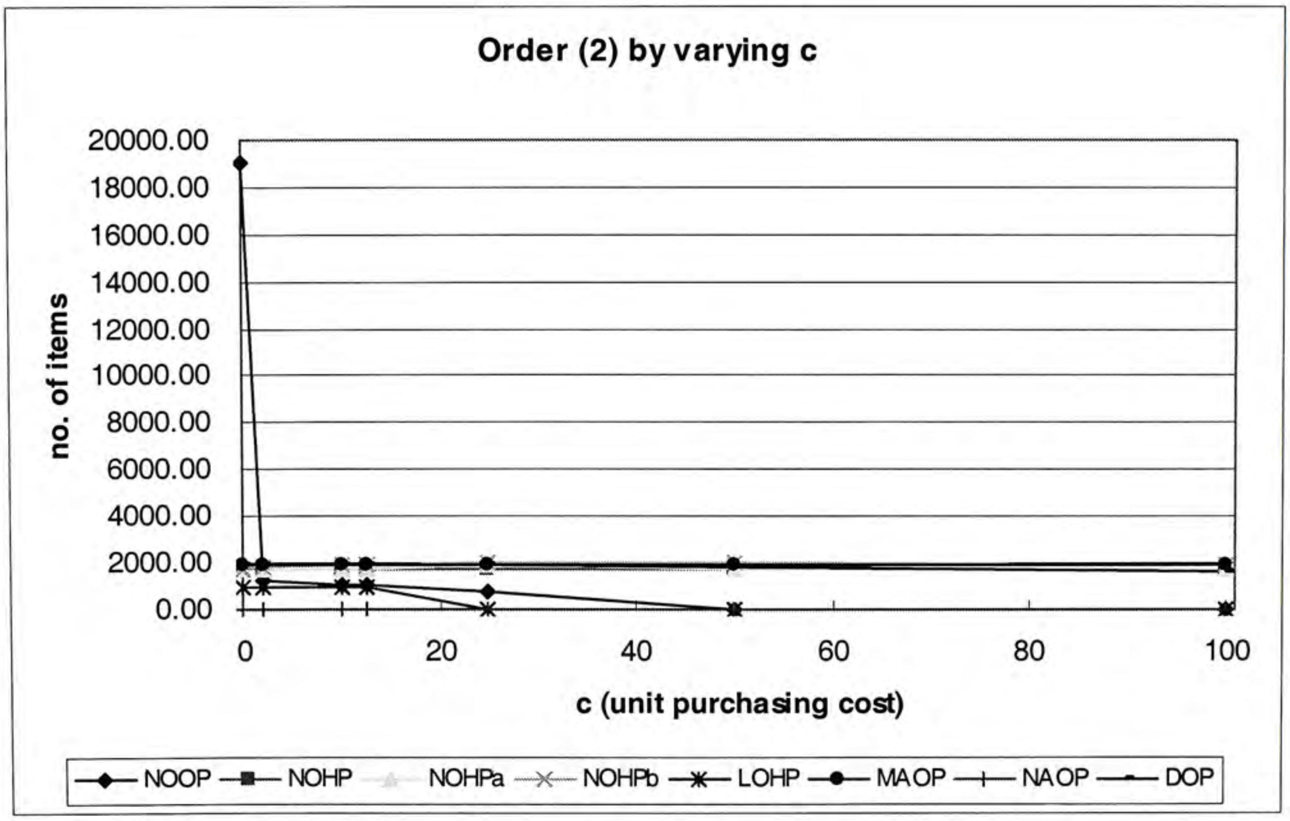


Figure 6: Order (2) patterns of all simulation models with varying *c*

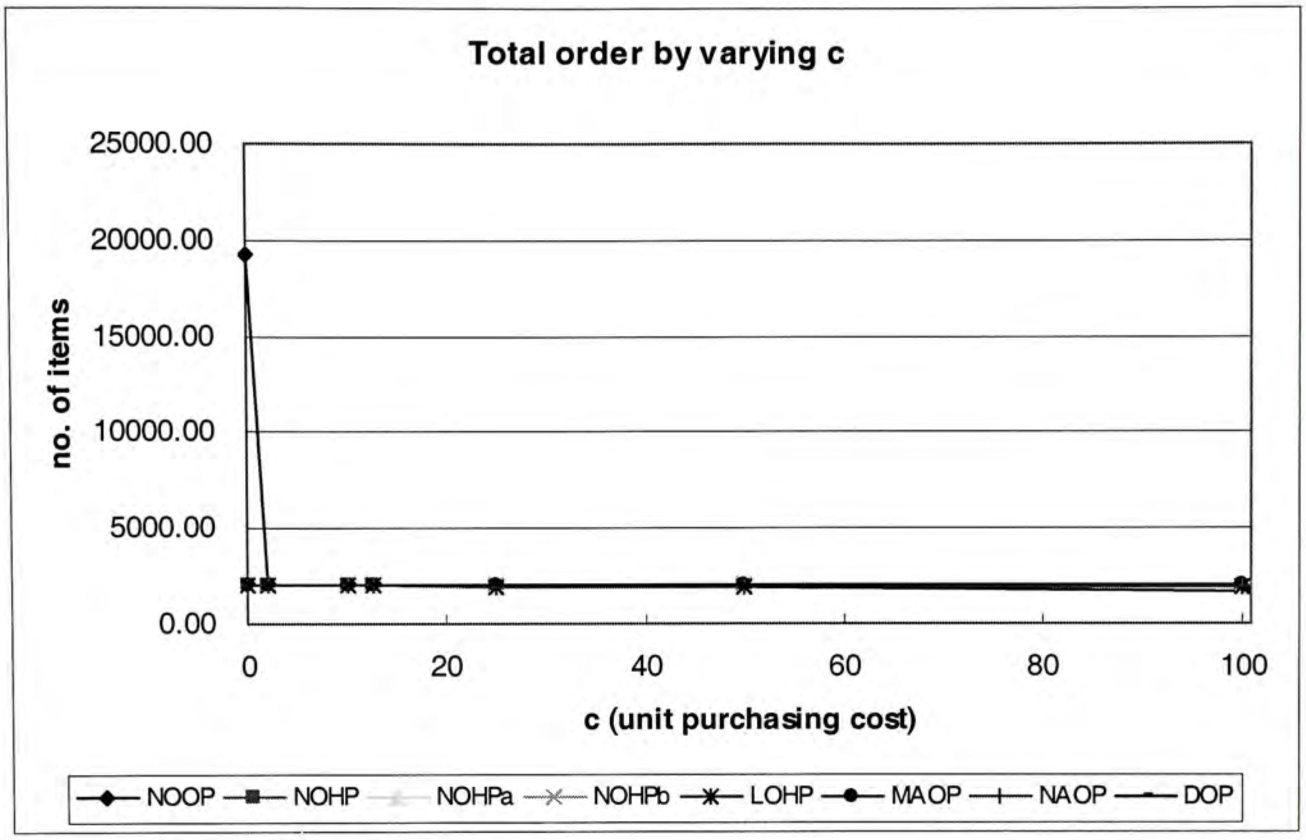


Figure 7: total order pattern of all simulation models with varying *c*

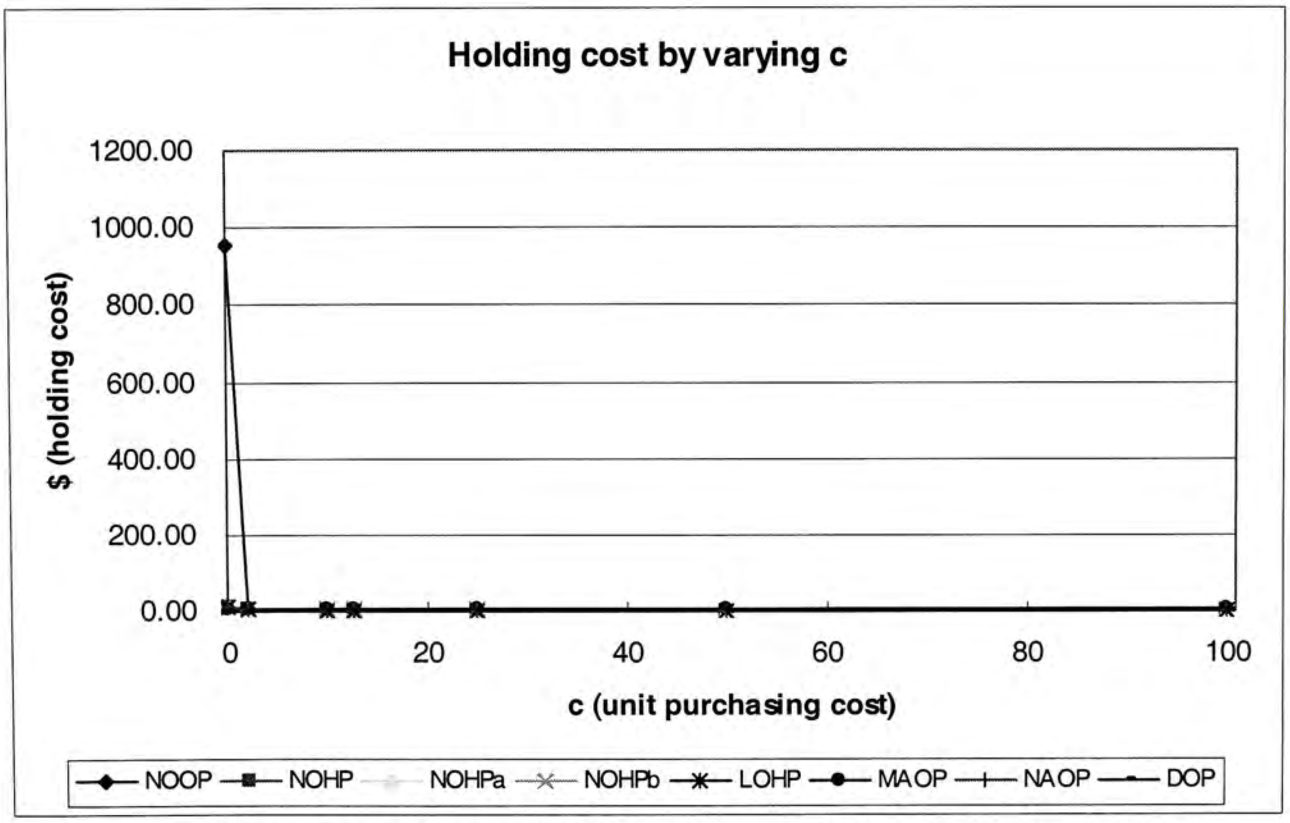


Figure 8: holding cost of all simulation models with varying *c*

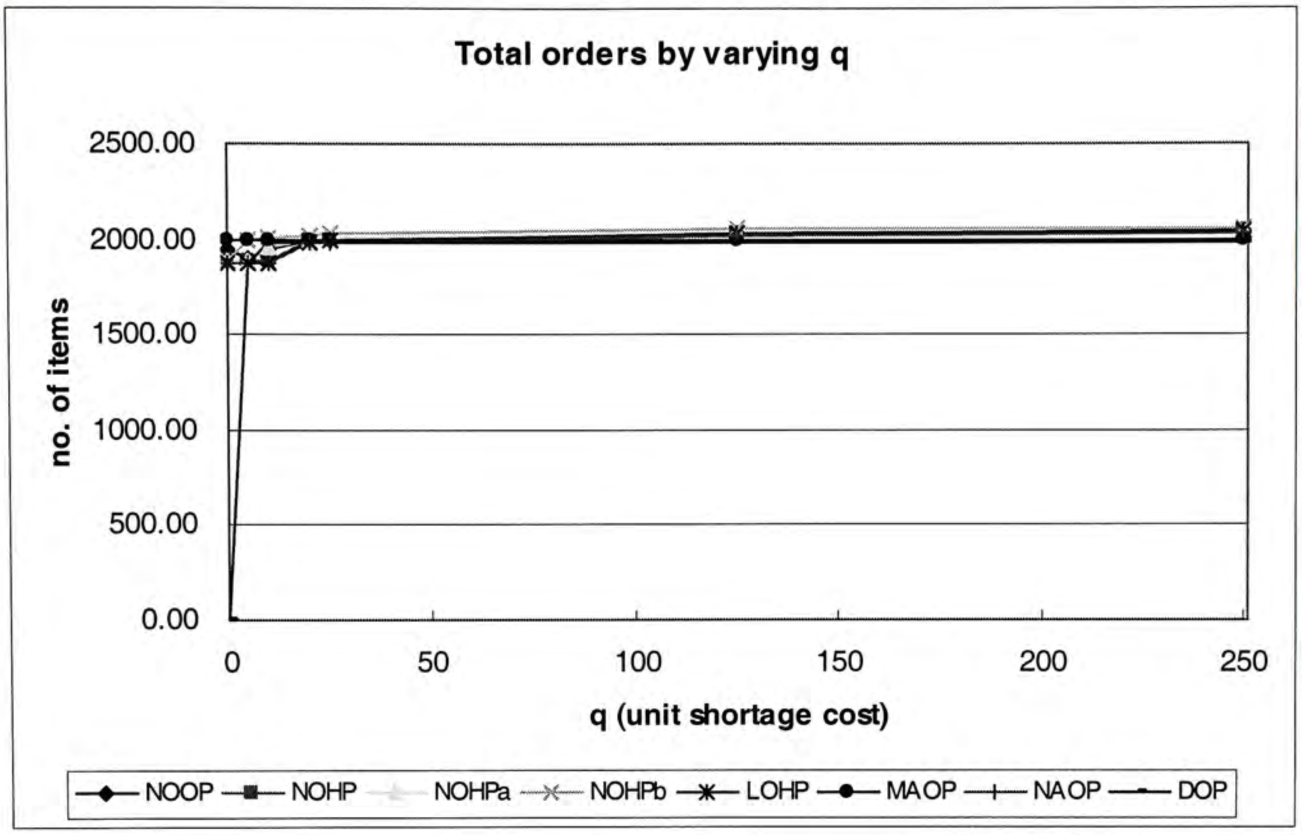


Figure 9: total order pattern of all simulation models with varying *q*

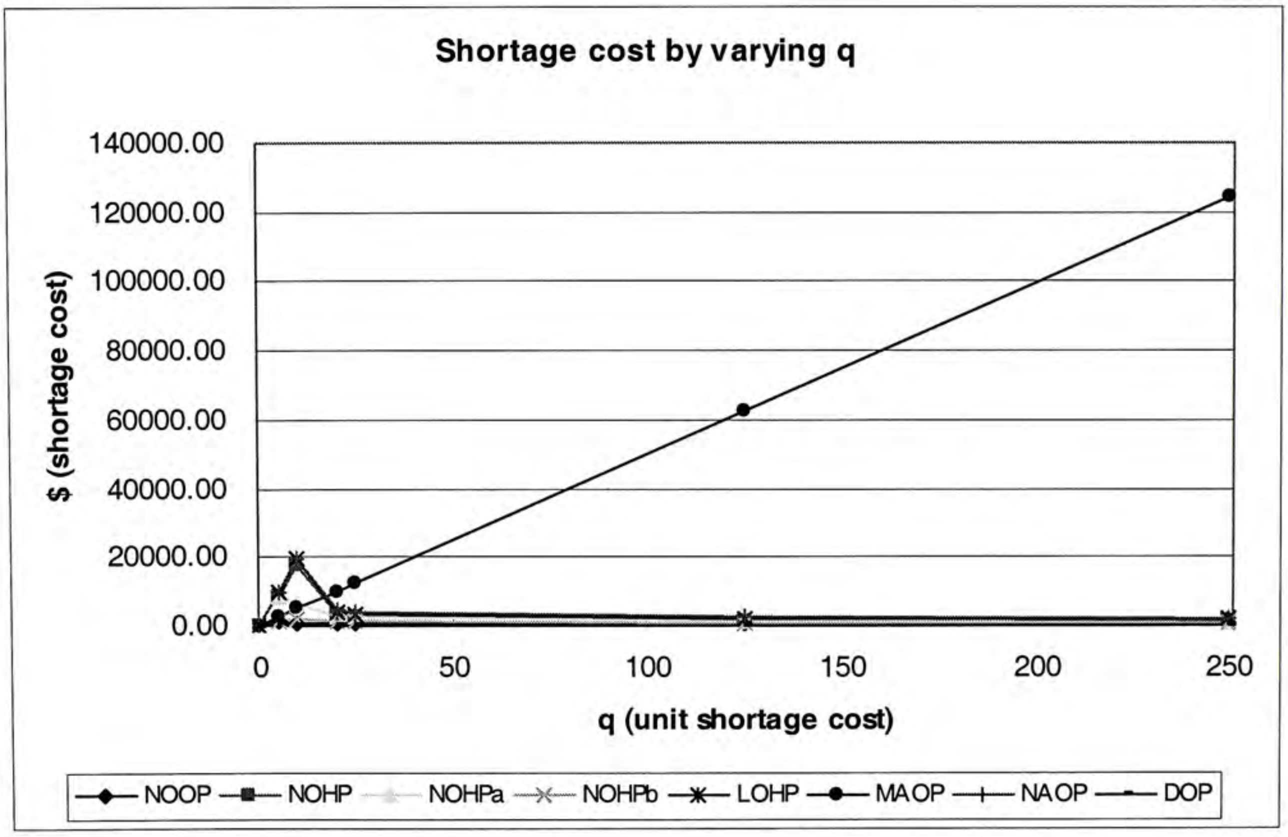


Figure 10: shortage cost of all simulation models with varying *q*

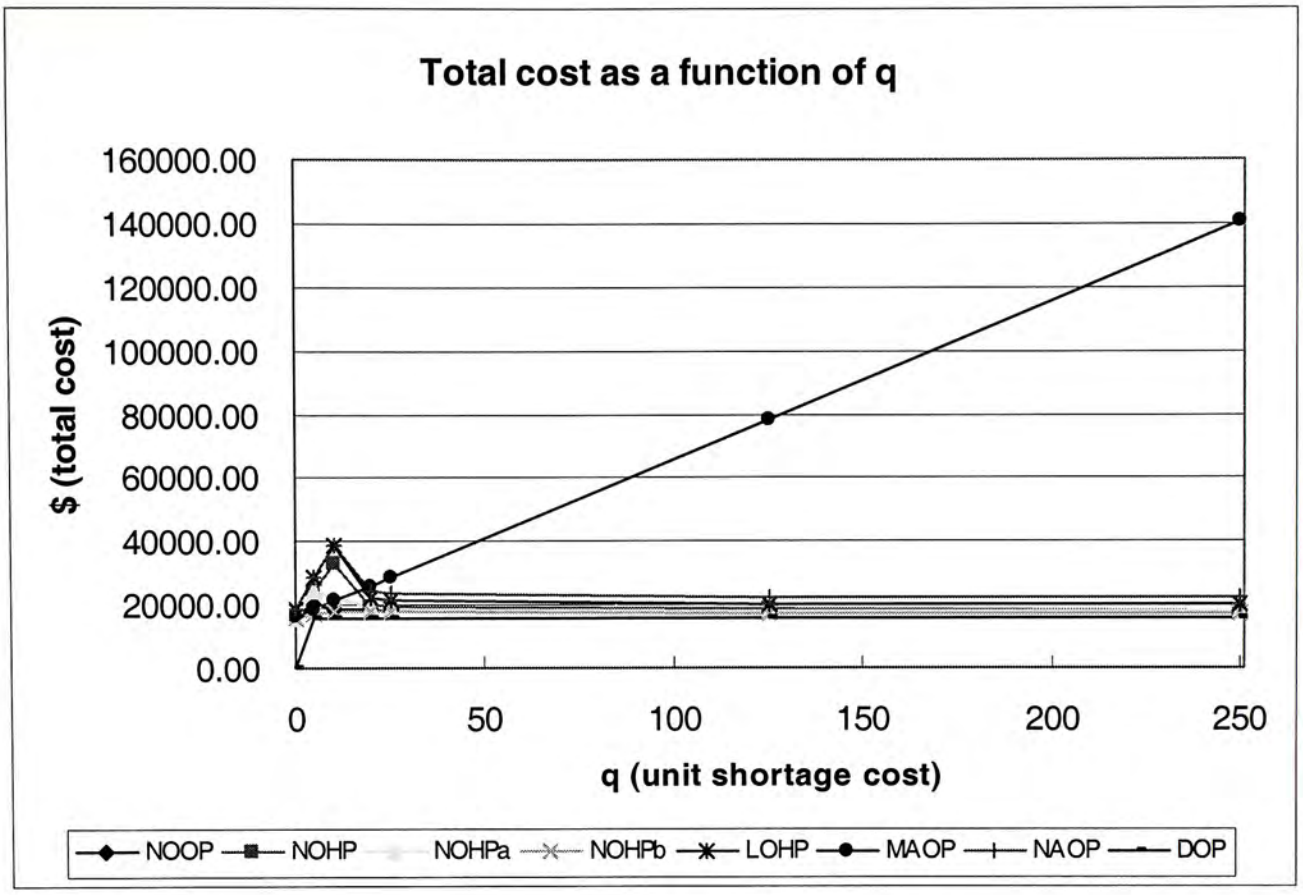


Figure 11: total cost of all simulation models with varying *q*

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