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著者	Tomoda Yasunobu, Okamura Makoto
journal or publication title	Kobe city university of foreign studies working paper series
number	48
page range	1-19
year	2014-04-01
URL	http://id.nii.ac.jp/1085/00001691/

Kobe City University of Foreign Studies Working Paper Series

48

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April 2014

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Cost Function and Market Equilibrium: Trade-off between Borrowing Constraint and Risk Premium

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Abstract

We consider the cost function and market equilibrium when a representative entrepreneur chooses secured or unsecured loans. The interest rate of the former is relatively low because of its collateral, but it comes with a borrowing constraint. The latter has a high interest rate stemming from risk premium. Owing to the cost minimization problem, these loans affect the cost function of the entrepreneur and the output market. We show that a short-run equilibrium generally does not exist. Moreover, if we assume a free entry condition in the long run, we obtain a unique equilibrium in which all entrepreneurs choose secured loans.

Keywords: Secured and Unsecured Loans; Borrowing Constraint; Collateral
JEL Classifications: G21; G31; L12

1 Introduction

In this study, we examine how financial loans influence the cost functions of entrepreneurs and equilibrium of the output market. We consider two types of loans, one secured and the other unsecured. For financial institutions, the difference between the loan contracts is the measure of risk management. For secured loans, the default risk is managed by collateral. For unsecured loans, the risk is managed by charging a risk premium. We find that the properties of these loans affect the cost function of the borrowers through the cost minimization problem and lead to a non-continuous supply curve.

For the entrepreneur, a secured loan comes with a borrowing constraint: the entrepreneur's borrowing is limited to below the value of the collateral. In many cases, entrepreneurs can collateralize their assets such as the land and building of the factory or office, automobiles, and/or the machinery used for production. If we consider the standard Cobb–Douglas production technology using capital goods and labor, these collateralized production factors are considered capital goods. With regard to the cost minimization problem, the entrepreneur faces the borrowing constraint as well as production technology. The new constraint leads to a distortion in the input combination, although the loan generally has a low interest rate. On the other hand, the risk premium of the unsecured loan increases the production cost of the entrepreneur. In spite of the cost minimization problem, the high interest rate raises the entrepreneur's marginal cost of production. Thus, financial loans change the cost function of the entrepreneur and significantly influence the output market.

Our main results are as follows. We consider a demand shock such that the output market vanishes. The representative entrepreneur must set up her business under uncertainty. We do not consider any asymmetric information problems. If the entrepreneur chooses a secured loan, she has an increasing cost function. If she chooses an unsecured loan, she faces a constant and relatively high marginal cost. Thus, she would choose a secured loan if her production level is relatively small and an unsecured loan otherwise. In the short run, if the output market has a relatively small demand, she would choose a secured loan. However, we show that if the demand is relatively large, the entrepreneur has no incentive to increase her production level and the excess demand remains unresolved. In the long run, assuming a free entry condition, there is a unique equilibrium at which all the entrepreneurs choose secured loans and maintain their production at the minimum level.

We classify the related literature into two groups. The first group focuses on the asymmetric information and incentive problems in the field of financial economics; we know that various types of financial transactions and contracts reduce the problems arising from the asymmetric information between borrowers and lenders through improvement in the incentive problems. In particular, these are serious issues for young firms that have a limited track record. For example, Berger and Udell (1998) investigate the relationship between the financial growth cycle of firms and resources of finance. A considerable amount of literature has studied the various forms of financing entrepreneurs and small and medium size enterprises; for example, Ueda (2004), Dessi (2005), Order (2006), Inderst et al. (2007), Hvide and Leite (2008), and Winton and Yerramilli (2008). The studies that investigate the financial contracts based on collaterals are in particular closely related to our study. Bester (1985) found that the inefficiency from asymmetric information on financial transactions can be reduced by self-selection through the choice of financial contracts. Collaterals also reduce the moral hazard problem by controlling the borrower's incentives (Berger and Udell 1990, Boot et al. 1991, Bester 1994). In addition, the lender can save on monitoring costs by contract back collaterals. For example, Rajan and Winton (1995) show the equilibrium that banks always demand collaterals without monitoring.

In contrast, we consider two types of financial loans from the point of demand uncertainty, not asymmetric information. Especially, we focus on the equilibrium of perfectly competitive markets stemming from demand shock and the two types of loans, not on the incentive problem. Our main interest is the activity of the entrepreneur based on the cost minimization problem and equilibrium of the output market rather than the loan market.

The second group of studies considers the relationship between borrowing constraints and macroeconomic dynamics. Barro et al. (1995) consider a neo-classical growth model with physical and human capital, assuming that physical capital is financed from abroad but human capital must be financed by domestic savings because it cannot be collateralized. Furthermore, Marquez (1985), De Gregorio (1996), Birchenall (2008), and Fishman and Krausz (2010) investigate the macroeconomic dynamics under borrowing constraints. In particular, Faig and Gagnon (2008) show that if the access to a secured loan is restricted by a borrowing constraint, individuals over-invest in capital goods that can be collateralized and consume and invest inefficiently at low levels. This paper is closely related to our study in that it focuses on the possibility of over-investment on capital goods that can be collateralized. However, our main interest is the existence of market equilibrium in a static environment, so that our study is more fundamental than the above literature.

The remainder of this paper is organized as follows. In section 2, we introduce our basic model. Section 3 examines the cost function when the entrepreneur borrows secured or unsecured loans. Section 4 considers the equilibrium of the output market in the short run and long run. Section 5 presents our brief concluding remarks.

2 The Model

We assume a representative entrepreneur who faces the risk of demand shock. She must commence her production before the market size is known. Production takes time, and the entrepreneur does not receive cash earnings until she sells the products. While she has some personal funds, it would not be sufficient to cover all her operating fund needs. Thus, she will have to borrow funds from a financial institution. She needs operating funds to invest on two types of production factors, distinguished according to whether they are acceptable as collateral or not; we call these production factors capital goods and labor, respectively.[1] The entrepreneur has two possible resources of finance. She can pledge her capital goods as collateral for a low interest *secured loan* in case she does not have other assets that the financial institution would accept as collateral, or she can raise funds through an *unsecured loan,* which does not require collaterals but charges a high interest rate. We call these as *loan S* and *loan U,* respectively.

The demand of the output market is uncertain owing to the demand shock. For simplicity, we assume two possibilities: the market will be realized with a probability of ρ, and the market will vanish with a probability of $1-\rho$. In

[1] Assume that "capital goods" include all the factors of production that financial institutions accept as security, for example, machine tools, heavy equipment, commercial premise, building structure, and land.

the former case, the demand function of the output market is given as

$$D = D(P), \tag{1}$$

where P and D are, respectively, the market price and the quantity of demand. We assume that $D'(P) < 0$, $\lim_{P \to 0} D(P) = \infty$ and $\lim_{P \to \infty} D(P) = 0$.

The entrepreneur has the following Cobb–Douglas production technology:

$$x = Bk^{\sigma} l^{1-\sigma}, \tag{2}$$

where $\sigma \in (0,1)$, and k and l are the capital good and labor input, respectively. For simplicity, we assume that the value of the capital good is zero after the production process is over. The operating funds required for the capital good and labor can be shown as

$$C = k + wl, \tag{3}$$

where the capital good is set as numeraire and w stands for wage. The entrepreneur finances her operating funds from her personal funds and a loan, either loan S or loan U, borrowed from a financial institution. Therefore, we have

$$C = b_i + f, \tag{4}$$

where $b_i \ (i = S, U)$ represents the borrowed funds, the sub-script denoting the financial contracts, and f is the entrepreneur's personal funds.

The economy has a number of identical financial institutions. Owing to competition, the profit of all the financial institutions from loans is zero, and all of them offer identical financial contracts in equilibrium. For loan U, the entrepreneur does not require any collateral, but for loan S, the entrepreneur is required to provide collateral and her borrowing is limited to below the value of the collateral. We assume that before the entrepreneur sets up her firm, she has no asset that the financial institution would accept as collateral except her personal funds. Thus, when the entrepreneur chooses loan S, she faces the borrowing constraint

$$b_S \le k_S. \tag{5}$$

Note that the capital goods are numeraire.

The representative entrepreneur is risk neutral and therefore maximizes her expected profit. Even if the output market vanishes, she will have to pay wages. Therefore, what the financial institution can recover is only the capital goods. The financial institution stops the production of the firm and seizes the capital goods. With regard to loan S, the entrepreneur has provided collateral and the loan is risk-free for the financial institution. For simplicity, we assume that the interest rate of a risk-free asset is zero in the economy; that is, $r_S = 0$. Similarly, the financial institution granting loan U can seize the capital goods if the market vanishes. However, when we consider the cost of seizing the capital goods, the recoverable value is $\delta_U k_U$, where $\delta_U \in (0,1)$. This is because loan U does not have collateral, and the financial institution cannot recover all its claims and may further have to incur additional legal costs. With regard to loan U, the debt of the entrepreneur can exceed the value of the recoverable capital goods; that is, $b_U > \delta_U k_U$. When the output market vanishes, the financial institution cannot recover the full principal of the loan and hence would incur a loss. In equilibrium, from the no-arbitrage condition, the expected returns on contract U must equal the returns on a safe investment. Thus, the interest rate of contract U, r_U, is determined by the following no-arbitrage condition:

$$\rho(1+r_U)b_U + (1-\rho)\delta_U k_U = b_U, \quad (6)$$

where the right-hand side represents the returns on a safe investment (the interest rate for safe assets is zero) and the second term on the left-hand side represents the value of the seized capital goods. From (6), we have $b_U > \delta_U k_U \Leftrightarrow r_U > 0$; the interest rate of loan U is greater than that of the safe investment owing to risk premium.

For financial institutions, the difference between the contracts is the measure of risk management. When a demand shock occurs, the financial institution seizes the capital goods. In loan S, the financial institution manages its default risk through collateral. In loan U, the financial institution faces the risk of loss but obtains high interest revenue in case the risk does not materialize; that is, the financial institution manages the risk by charging a risk premium. Therefore, the entrepreneur faces a trade-off between the borrowing constraint (5) and a high interest rate.

Here, we summarize the order of events and profit generation of the entrepreneur. At the commencement of production, the entrepreneur borrows funds b_i. She uses her personal funds f and borrowed funds b_i to purchase capital goods and employ labor. If the expected output market is realized, the entrepreneur gains a revenue of $P \cdot x_i$ and repays the principal and interest, $(1+r_i)b_i$, to the financial institution. After the production period, the value of the capital goods becomes zero. From (3) and (4), the cash flow at that point of time will be $\pi_i = P \cdot x_i - (1+r_i)b_i$. On the other hand, if the market vanishes and the entrepreneur becomes bankrupt, the capital goods are seized and her profit will be zero. If she had not set up her firm, she would have had her personal funds f, which is the opportunity cost for starting a business. Therefore, from (3) and (4), the expected profit of the entrepreneur is

$$E\pi_i = \rho \cdot P \cdot x_i - ETC_i, \tag{7}$$

where $ETC_i \equiv \rho(1+r_i)(k_i + wl_i - f) + f$ represents the expected total cost of the entrepreneur. In the long-run market equilibrium, even when $E\pi_i > 0$, other entrepreneurs will start production and the free entry condition will be as follows:

$$\max\{E\pi_S, E\pi_U\} = 0. \tag{8}$$

3 Cost Functions under Financial Contracts

We consider the cost minimization problem of the representative entrepreneur. When the entrepreneur chooses loan S, she faces the borrowing constraint (5). From (3) and (4), we can rewrite (5) as

$$w \cdot l_S \le f. \tag{9}$$

When the borrowing constraint is binding, (9) implies that all the wages have been paid with only the entrepreneur's personal funds. In other words, all the capital goods are purchased with the loan availed from the financial institution and the capital goods are then pledged as collateral.[2] Under the rewritten borrowing constraint (9) and the technology (2), the entrepreneur minimizes her expected total cost ETC_S, (7). Noting that the interest rate of loan S is zero, we have the following lemma.

Lemma 1

For borrowing loan S, the expected cost function is

$$ETC_S = \begin{cases} \rho B^{-1} \sigma^{-\sigma} (1-\sigma)^{-(1-\sigma)} w^{1-\sigma} x_S + (1-\rho) f & \text{if } \; x_S \le \tilde{x}_S & (10.a) \\ \rho B^{-\frac{1}{\sigma}} w^{\frac{1-\sigma}{\sigma}} f^{-\frac{1-\sigma}{\sigma}} x_S^{\frac{1}{\sigma}} + f & \text{if } \; \tilde{x}_S \le x_S & (10.b) \end{cases},$$

where $\tilde{x}_S \equiv B\sigma^{\sigma}(1-\sigma)^{-\sigma} w^{-(1-\sigma)} f$.

Proof: The cost minimization problem of the entrepreneur is

2 This implies that the entrepreneur purchases the land and building of a factory or an office, automobiles, and/or the machinery by mortgaging them to a bank.

$$\min_{k_S, l_S}\{\rho(k_S + wl_S) + (1-\rho)f\}$$

$$s.t.\quad x_U = Bk_U{}^{\sigma} l_U{}^{1-\sigma}$$

$$w \cdot l_S \le f .$$

If the borrowing constraint (9) is not binding, the demand functions of the capital goods and labor are, respectively, $k_S = B^{-1}\sigma^{1-\sigma}(1-\sigma)^{-(1-\sigma)}w^{1-\sigma}x_S$ and $l_S = B^{-1}\sigma^{-\sigma}(1-\sigma)^{-\sigma}w^{-\sigma}x_S$. Thus, we have (10.a). If the borrowing constraint (9) is binding, the labor demand becomes $l_S = f \cdot w^{-1}$. Substituting the labor demand into (2), we have $k_S = B^{\frac{-1}{\sigma}} w^{\frac{1-\sigma}{\sigma}} f^{-\frac{1-\sigma}{\sigma}} x_S^{\frac{1}{\sigma}}$. From these factors of demand functions, we have (10.a). The threshold that (9) is binding is $\tilde{x}_S$. Q.E.D.

If the firm's production scale x_S is small relative to the entrepreneur's personal funds *f*, the borrowing constraint (9) will not be binding, there will be no inefficiency in factor inputs, and the financial institution will face no risk. On the other hand, if the entrepreneur plans to raise the production scale to more than $\tilde{x}_S$, the borrowing constraint (9) will be binding, and since the labor input is based on (9), the entrepreneur will have to bring in more collateralized capital goods to raise her production scale, and the marginal cost will increase to (10.b).

Next, we consider the case when the entrepreneur chooses to avail loan U. She minimizes her expected total cost ETC_U subject to production technology (2) under the given borrowing interest rate r_U. Therefore, the factor demand functions and the total expected cost function will be as follows:

$$k_U = B^{-1}\sigma^{1-\sigma}(1-\sigma)^{-(1-\sigma)}w^{1-\sigma}x_U \text{ and } l_U = B^{-1}\sigma^{-\sigma}(1-\sigma)^{-\sigma}w^{-\sigma}x_U . \quad (11)$$

$$ETC_U = (1+r_U)\rho B^{-1}\sigma^{-\sigma}(1-\sigma)^{-(1-\sigma)}w^{1-\sigma}x_U + [1-\rho(1+r_U)]f . \quad (12)$$

Obviously, if $r_U = 0$, (12) is equivalent to (10.a). This is because a small borrowing amount will be charged a low interest rate from (6). $r_U = 0$ implies a risk-free loan for financial institutions. However, if the borrowing amount increases, the financial institution will face the risk that all its claims cannot be recovered; it will require a risk premium $r_U > 0$. From (3), (4), (6), and (11), the gross interest rate will be as follows:

$$R_U(x_U) \equiv (1+r_U) = \frac{[1-(1-\rho)\delta_U\sigma]B^{-1}\sigma^{-\sigma}(1-\sigma)^{-(1-\sigma)}w^{1-\sigma}x_U - f}{\rho[B^{-1}\sigma^{-\sigma}(1-\sigma)^{-(1-\sigma)}w^{1-\sigma}x_U - f]}. \quad (13)$$

If the output market vanishes, the financial institution will be able to seize the firm's capital goods in the fraction of δ_U. In the most extreme case of $\delta_U = 0$, we have $R_U = \rho^{-1}$; the per unit risk premium is not associated with the scale of lending and production. Otherwise, the interest rate will be an increasing function of the production scale, as the following lemma shows.

Lemma 2

The following properties hold on the gross interest rate:

$$\frac{\partial R_U}{\partial x_U} = \frac{(1-\rho)\delta_U B^{-1}\sigma^{1-\sigma}(1-\sigma)^{-(1-\sigma)}w^{1-\sigma}f}{\rho[B^{-1}\sigma^{-\sigma}(1-\sigma)^{-(1-\sigma)}w^{1-\sigma}x_U - f]^2} > 0,$$

$$\frac{\partial^2 R_U}{\partial x_U{}^2} = -\frac{2(1-\rho)\delta_U B^{-1}\sigma^{1-2\sigma}(1-\sigma)^{-2(1-\sigma)}w^{2(1-\sigma)}f}{\rho[B^{-1}\sigma^{-\sigma}(1-\sigma)^{-(1-\sigma)}w^{1-\sigma}x_U - f]^3} < 0, \text{ and}$$

$$\lim_{x_U\to\infty} R_U = \rho^{-1}[1-(1-\rho)\delta_U\sigma].$$

$R_U \geq 1$ if and only if $x_U \geq \tilde{x}_U$, where $\tilde{x}_U \equiv \dfrac{B\sigma^{\sigma}(1-\sigma)^{1-\sigma}f}{(1-\delta_U\sigma)w^{1-\sigma}}$.

From (12), (13), and lemma 2, we have the cost function as the following lemma:

Lemma 3

$$ETC_U = \begin{cases} \rho B^{-1}\sigma^{-\sigma}(1-\sigma)^{-(1-\sigma)}w^{1-\sigma}x_U + (1-\rho)f & \text{if } x_U \leq \tilde{x}_U \quad (14.a) \\ [1-(1-\rho)\delta_U\sigma]B^{-1}\sigma^{-\sigma}(1-\sigma)^{-(1-\sigma)}w^{1-\sigma}x_U & \text{if } \tilde{x}_U \leq x_U \quad (14.b) \end{cases}.$$

Note that (14) is continuous when $x_U = \tilde{x}_U$. From lemmas 1 and 2, we have $\tilde{x}_U < \tilde{x}_S$ whenever $\delta_U < 1$. If a demand shock occurs, the capital goods seized under loan U will be less than loan S. Thus, the financial institution granting loan U will need a risk premium even if the borrowing amount is relatively small.

Here, we consider the decision of financial contracts. The entrepreneur minimizes her expected total cost by choosing financial loans based on her production scale. From lemmas 1 and 3, we have the following proposition:

Proposition 1

If we define $\hat{x}$ as $ETC_S(\hat{x}) = ETC_U(\hat{x})$ in the range of $\tilde{x}_U < \hat{x}$, we have the unique $\hat{x} \in (\tilde{x}_U, \infty)$, and the expected total cost function will be

$$ETC = \begin{cases} \rho B^{-1} \sigma^{-\sigma} (1-\sigma)^{-(1-\sigma)} w^{1-\sigma} \cdot x + (1-\rho) f & if \quad x \le \tilde{x}_S & (15.a) \\ \rho B^{-\frac{1}{\sigma}} w^{\frac{1-\sigma}{\sigma}} f^{-\frac{1-\sigma}{\sigma}} \cdot x^{\frac{1}{\sigma}} + f & if \quad \tilde{x}_S \le x \le \hat{x}\,. & (15.b) \\ [1-(1-\rho)\delta_U \sigma] B^{-1} \sigma^{-\sigma} (1-\sigma)^{-(1-\sigma)} w^{1-\sigma} \cdot x & if \quad \hat{x} \le x & (15.c) \end{cases}$$

Proof: From (10) and (14), we set $F(x) \equiv ETC_U(x) - ETC_S(x)$. Note that $F(x) = 0$ even if $x \le \tilde{x}_U$. Furthermore, we have

$$F'(x) \frac{[1-(1-\rho)\delta_U \sigma] w^{1-\sigma}}{B \sigma^{\sigma} (1-\sigma)^{1-\sigma}} - \frac{\rho}{\sigma} B^{-\frac{1}{\sigma}} \left(\frac{w}{f}\right)^{\frac{1-\sigma}{\sigma}} x^{\frac{1-\sigma}{\sigma}},$$

$$F''(x) = -\frac{\rho(1-\sigma)}{\sigma^2 (1-\sigma)^{1-\sigma}} B^{-\frac{1}{\sigma}} \left(\frac{w}{f}\right)^{\frac{1-\sigma}{\sigma}} x^{\frac{1-2\sigma}{\sigma}} < 0.$$

Defining $\bar{x}$ as $F'(\bar{x}) = 0$, we have $\bar{x} = \rho^{-\frac{\sigma}{1-\sigma}} [1-(1-\rho)\delta_U \sigma]^{\frac{\sigma}{1-\sigma}} B \sigma^{\sigma} (1-\sigma)^{-(1-\sigma)} w^{-(1-\sigma)} f$. Note that $\tilde{x}_S < \bar{x} \Leftrightarrow \delta_U \sigma < 1$ and $F(\bar{x}) = [1-(1-\rho)\delta_U \sigma]^{\frac{1}{1-\sigma}} \rho^{-\frac{\sigma}{1-\sigma}} f - f$. $F(\bar{x}) > 0$ is equivalent to $1 - \rho^{\sigma} > (1-\rho)\delta_U \sigma$, which holds because of $\rho \in (0,1)$, $\delta_U \in [0,1)$, and $\sigma \in (0,1)$. Moreover, we have $\lim_{x \to \infty} F(x) = \lim_{x \to \infty} F'(x) = -\infty$ from the l'Hospital's Rule. Thus, we have the unique $\hat{x} \in (\bar{x}, \infty)$ such that $F(\bar{x}) = 0$. Eventually, $\tilde{x}_U < \tilde{x}_S < \bar{x} < \hat{x}$ holds. Therefore, we have the total expected cost function (15).

Q.E.D.

The expected total cost function (15) is shown in figure 1. When the range is $x \in (0, \tilde{x}_U)$, each loan leads to the same cost functions. When the production scale is more than $\tilde{x}_U$, the financial institution granting loan U will be able to seize only a part of the capital goods in case a demand shock occurs. Thus, the financial institution faces a default risk. On the other hand, loan S collateralizes all the capital goods and the borrowing constraint does not bind in the range of $x \in (\tilde{x}_U, \tilde{x}_S)$. The entrepreneur prefers loan S because the additional interest rate as risk premium can be saved. When the production scale exceeds $\tilde{x}_S$, the marginal cost under loan S rises because of the binding borrowing constraint. On the other hand, the marginal cost under loan U is constant. Thus, there is a threshold $\hat{x}$ at which the cost of each type of loan reverses. Eventually, loan U will be advantageous for a large production scale.

The expected total cost (15) directly leads to the expected marginal cost (EMC) and expected average cost (EAC) functions as follows:

$$EMC = \begin{cases} \rho B^{-1}\sigma^{-\sigma}(1-\sigma)^{-(1-\sigma)} w^{1-\sigma} & if \;\; x \le \tilde{x}_S & (16.a) \\ \rho\sigma^{-1} B^{-\frac{1}{\sigma}} w^{\frac{1-\sigma}{\sigma}} f^{-\frac{1-\sigma}{\sigma}} \cdot x^{\frac{1-\sigma}{\sigma}} & if \;\; \tilde{x}_S \le x \le \hat{x}. & (16.b) \\ [1-(1-\rho)\delta_U\sigma] B^{-1}\sigma^{-\sigma}(1-\sigma)^{-(1-\sigma)} w^{1-\sigma} & if \;\; \hat{x} \le x & (16.c) \end{cases}$$

$$EAC = \begin{cases} \rho B^{-1}\sigma^{-\sigma}(1-\sigma)^{-(1-\sigma)} w^{1-\sigma} + (1-\rho) f \cdot x^{-1} & if \;\; x \le \tilde{x}_S & (17.a) \\ \rho\sigma^{-1} B^{-\frac{1}{\sigma}} w^{\frac{1-\sigma}{\sigma}} f^{-\frac{1-\sigma}{\sigma}} \cdot x^{\frac{1-\sigma}{\sigma}} + f \cdot x^{-1} & if \;\; \tilde{x}_S \le x \le \hat{x}. & (17.b) \\ [1-(1-\rho)\delta_U\sigma] B^{-1}\sigma^{-\sigma}(1-\sigma)^{-(1-\sigma)} w^{1-\sigma} & if \;\; \hat{x} \le x & (17.c) \end{cases}$$

We set $\underline{x}$ such that it leads to the minimum point of the expected average cost; from (17.b), we have

$$\underline{x} = \rho^{-\sigma} B \sigma^{\sigma}(1-\sigma)^{-(1-\sigma)} w^{-(1-\sigma)} f, \text{ and } \tilde{x}_S < \underline{x} < \bar{x} \qquad (18)$$

See figure 2. From figure 1, it is obvious that $\underline{x}$ exists between $\tilde{x}_S$ and $\bar{x}$.

4 Market Equilibrium

4.1 Short-Run Equilibrium

First, we consider a short-run equilibrium in a perfect competitive market with no free entry condition (8). From (18), the smallest average cost is $EAC(\underline{x}) = \rho^{\sigma} B^{-1}\sigma^{-\sigma}(1-\sigma)^{-(1-\sigma)}$. If the market price is lower than $\underline{P} \equiv EAC(\underline{x})$, the expected profit is negative. In this case, the entrepreneur does not supply any output. From (16.c) and (17.c), we find that by availing loan U, her average cost and marginal cost are same. We define the price that equals them as $\overline{P} \equiv [1-(1-\rho)\delta_U \sigma]B^{-1}\sigma^{-\sigma}(1-\sigma)^{-(1-\sigma)}w^{1-\sigma}$. Thus, if $\overline{P} < P$, the entrepreneur can make her profit rise to infinity by setting $x \to \infty$. If the price is $P \in [\underline{P}, \overline{P}]$, the entrepreneur chooses loan S and sets the output level that yields the price equals to the marginal cost. From (16.b), we have the supply curve $x = \rho^{\frac{\sigma}{1-\sigma}} \sigma^{\frac{\sigma}{1-\sigma}} B^{\frac{1}{1-\sigma}} w^{-1} f \cdot P^{\frac{\sigma}{1-\sigma}}$. In particular, we should note the case of $P = \overline{P}$. If the entrepreneur chooses loan U, her profit is zero because the price equals the average cost. If she chooses loan S, she can have a positive profit by setting $x = \overline{x}$. Thus, when $P = \overline{P}$, the entrepreneur chooses loan S and does not expand her production level.

Thus, we have the following lemma.

Lemma 4

The supply curve of the entrepreneur is

$$x = \begin{cases} 0 & \text{if } P < \underline{P} & (19.a) \\ \rho^{\frac{\sigma}{1-\sigma}} \sigma^{\frac{\sigma}{1-\sigma}} B^{\frac{1}{1-\sigma}} w^{-1} f \cdot P^{\frac{\sigma}{1-\sigma}} & \text{if } \underline{P} \le P \le \overline{P}. & (19.b) \\ \infty & \text{if } \overline{P} < P & (19.c) \end{cases}$$

This is shown in figure 3. The red solid lines and black dashed lines denote, respectively, the supply curve (19) and marginal cost (16). Note that (19) is not continuous in the ranges of $x \in (0, \underline{x})$ and $x \in (\overline{x}, \infty)$. In the former case, the expected profit is negative because the production scale is too small. In the latter case, the entrepreneur has no incentive to produce at that level. This is because the optimal choices of the entrepreneur are to set $x = \overline{x}$ if $P = \overline{P}$ and $x \to \infty$ if $\overline{P} < P$.

Since the supply curve is not continuous, the market equilibrium does not exist when the demand curve does not locate in the range of (19.b). Thus, lemma 4 yields the following proposition directly.

Proposition 2

If $D(\underline{P}) < \underline{x}$, the market equilibrium does not exist. If $\underline{x} < D(\underline{P})$ and $D(\overline{P}) < \overline{x}$, there is a unique market equilibrium. If $\overline{x} < D(\overline{P})$, the market equilibrium. Loan U is not selected in equilibrium.

In the case of $D(\underline{P}) < \underline{x}$, the market is too small and the entrepreneur does not start production because she will not be able to cover the fixed opportunity cost f. If $\underline{x} < D(\underline{P})$ and $D(\overline{P}) < \overline{x}$, there is a unique market equilibrium that is Warlasian stable. In the case of $\overline{x} < D(\overline{P})$, the representative entrepreneur has no incentive to produce more than $\overline{x}$ when $P = \overline{P}$ because $\overline{x}$ is the unique product level that yields the maximum profit. The excess demand $D(\overline{P}) - \overline{x}$ remains unresolved in the market. However, if the price rises from $\overline{P}$, the optimal activity of the entrepreneur will be to produce infinitely. However, since the demand is finite, such a case does not arise. Eventually, we conclude that a short-run equilibrium does not exist if $\overline{x} < D(\overline{P})$. Note that the supply curve (19.b) is derived from loan S. This implies that the entrepreneur chooses loan S in an equilibrium and that no entrepreneur will choose loan U in such an economic environment.

4.2 Long-Run Equilibrium

Next, we consider a long-run equilibrium, assuming the free entry condition (8). If the price is $P \in (\underline{P}, \overline{P}]$, the profit of the entrepreneur is positive, and this leads to the entry of other entrepreneurs. In particular, even if $P = \overline{P}$, which leads to an excess demand in the short run, a positive profit leads to new entries. Since the long-run equilibrium requires zero profit, we have the following proposition.

Proposition 3

In the unique long-run market equilibrium where the price is $\underline{P}$ and the production level per capita is $\underline{x}$, all the entrepreneurs choose loan S.

A long-run market equilibrium is characterized by the intersection of the marginal cost curve and the average cost curve; that is, $(\underline{x}, \underline{P})$ in figure 3. In this unique equilibrium, all the entrepreneurs choose loan S. If an entrepreneur chooses loan U, the risk premium raise her marginal cost. Since the competition stemming from free entry is very severe, the entrepreneurs availing loan U find it difficult to survive. On the other hand, the disadvantage of availing loan S is the borrowing constraint. The larger the production scale, the more is the distortion of factor inputs. In a long-run equilibrium, every entrepreneur produces at the minimum level $\underline{x}$, and so the distortion from the borrowing constraint is not large in the long-run equilibrium.

5 Summary

We investigated the cost function and market equilibrium when a representative entrepreneur chooses secured or unsecured loans. We considered a representative entrepreneur who faces a demand shock. Secured loans manage their default risk through collateral security, while unsecured loans do so by charging a risk premium. When the borrower enters into a secured contract, she enjoys a low interest rate but faces a borrowing constraint. This leads to a distortion in production and is a serious issue when the production level is relatively large. Therefore, the representative entrepreneur who uses a Cobb–Douglas technology with a constant returns to scale decides on financial contracts according to her production scale; for a small (large) production, she would decide on a secured (unsecured) loan.

The existence of the short-run equilibrium depends on demand size. If the demand is small, the entrepreneur does not supply any good because the revenue does not cover her fixed opportunity cost. If the demand size is medium, there is unique equilibrium that is Warlasian stable. However, even if the market size is large, the entrepreneur has no incentive to raise her production scale and any excess demand remains unresolved in the market; a market equilibrium does not exist in this case.

In the long-run market equilibrium, a positive profit leads to the entry of new rival entrepreneurs. The competition is strict, and every entrepreneur operates at the production level that results in the minimum average cost. Therefore, the entrepreneurs choose only the secured loan because it is advantageous for a small production scale.

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Figure 1 : The expected total cost

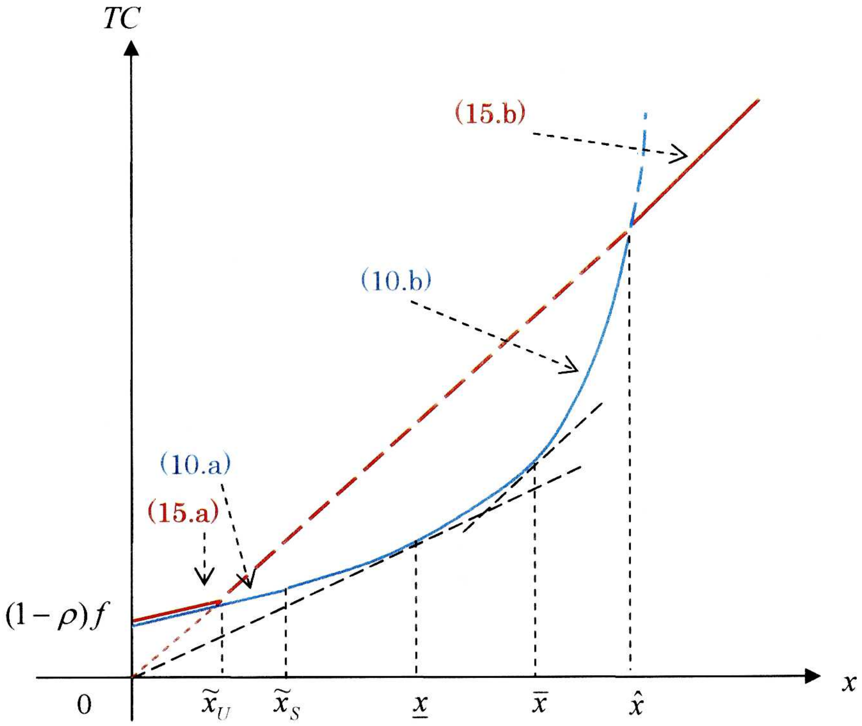

Figure 2 : The expected marginal cost and average costs

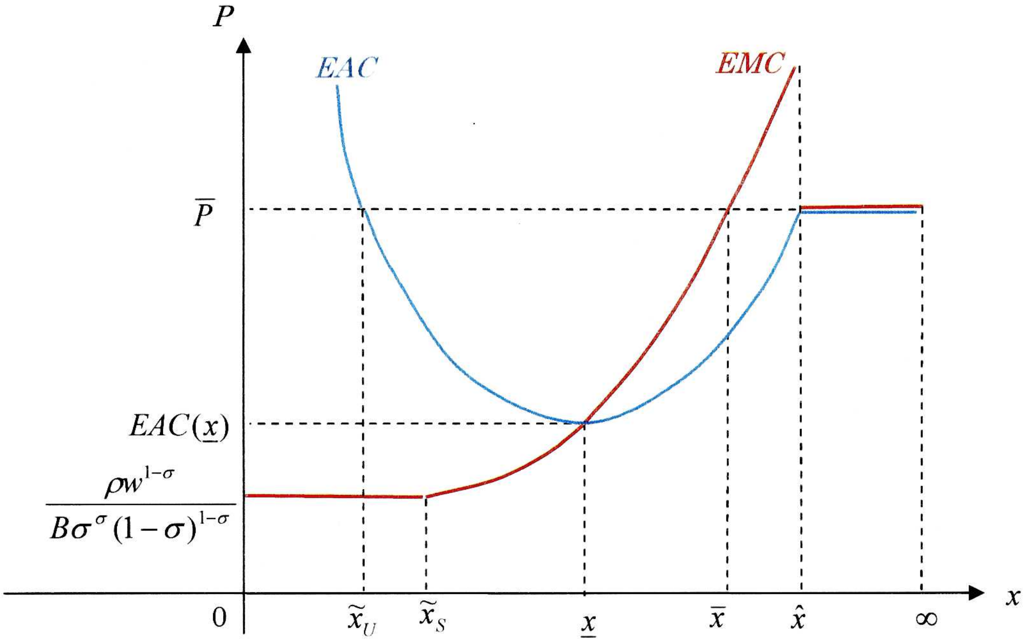

Figure 3 : Supply curve