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Abstract

We consider the R&D investments competition of the two duopolistic firms in a weakly complementary technologies economy. By "*the weakly complementary technologies*", we mean that each firm *can* produce goods *without both of the two technologies* but it incurs more redundant costs than that in the case each or both of the technologies may be available for it. By "*the strongly complementary technologies,*" we mean that the firm *cannot* produce the goods *at all without the use of both of them.* We derive the investments competition equilibria in R&D of the two weakly complementary technologies with and without the (cross-) licensing system. By comparing of the R&D investment levels in the two equilibria, we show that the (cross-) licensing system promotes the R&D investments when the duopolistic firms can produce goods by using of the two weakly complementary technologies.

JEL Classification Numbers:

Key Words: complementary technologies, (cross-) licensing system, duopoly, R&D investment

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1. Introduction

Technological innovations are supported by the technological inventions, which have the following two features. First, technological inventions have the characteristic of public goods. Since they are a kind of information, the discoverer of such an invention cannot exclude other persons from using it. Also many people can use it simultaneously. These features of technological inventions induce non-discoverers' (non-inventors') imitation of them, and discourage researchers or inventors from creating them. Secondly, technological inventions are associated with other technologies. When the discoverers or inventors of plural distinct technological inventions differ, how does the coordination make among the discoverers affect the interest of each discoverer or inventor, and also affect their R&D incentives of them.

Over the past decade of the last century, a number of studies have been discussed on the effects of the relations among technological inventions upon the R&D activities, the licensing or the patent systems.

The technological inventions of the type of cumulative relations in which one (some) of them is (are) developed or worked out on the basis of other preceding one(s). Under such cumulative technological innovations, Green and Scotcher(1995), Chang(1995) show(s) that the externality due to the lack of the coordination among the discoverers of plural distinct technological inventions decreases their incentives to create or develop them.

Another feature of the relations among the technological inventions is the complementary relation among them. As we typically have seen in the IT (Information Technologies) industries, technological innovations occur on the basis of plural distinct technological inventions invented (developed) in different systems of technologies. In this environment, distinct technologies are technologically complementary for the product produced by making use of them. Of course, in such a complementary technological innovation, the externality problem due to the lack of the coordination among the discoverers of plural complementary technologies does occur. Heller and Eisenberg(1998) point out that the existence of such externality brings the result known as "the tragedy of anti-commons:" When the property rights of plural distinct technologies are authorized to different agents (firms), the externality brings excessive exercises of the exclusive rights, and under-utilization of these technologies, and discourages incentives for R&D activities of agents(firms).

In the case where a product is produced by utilizing such complementary inventions, a (cross-) licensing for them has strategic importance. If two different firms have one of the two distinct technological inventions with a strongly (perfect) complementarity, then they cannot produce a product at all without a (cross-) licensing for them. In practice, as the positive analyses in the appliance and IC industries conducted by Grindley and Teece(1997) and Hall and Ziedonis(2001) show us, the conditions of the firms' (cross-) licensing for the product have a great effect on their incentives for R&D activities in such industries where strongly complementary technological inventions are indispensable for producing the product[1]. In economic literatures, however, there have been made few theoretical studies on how the conditions of the firms' (cross-) licensing for the product affect firms' incentives for R&D activities. Fershtman and Kamien(1992) does not analyze how the difference between a cross-licensing and unilateral licensing affect firms' incentives for R&D

[1] A great number of distinct technological inventions with a strongly (perfect) complementarity, for example, IC technologies, software technologies, LCD(Liquid Crystal Display) technologies and so on, are indispensable for a appliance producer producing and selling a cellular phone.

activities at all, since they focus on a dynamic prospect of technological innovation in their seminar study in such a context.

Therefore, in this paper, we make a theoretical analysis on how the difference between a cross-licensing and a unilateral licensing affects firms' incentives for R&D activities in a Cournot duoply. When firms invest in R&D of two distinct technologies with strong complementarity, both technologies are indispensable for producing products, so note that the only licensing that may occur is a cross-licensing. Therefore, in our analysis, we focus on the case where each duopolistic firm can invest on R&D for the two distinct technological inventions *with weak complementarity* with each other. But we conduct our analysis by a static model with respect to each firm's R&D investment since we does not treat dynamic aspects of technological innovations.

In section 2, we describe our model, and analyze the problem of R&D in Cournot duopoly with weakly complementary technological innovations without licensing as a benchmark in section 3. In section 4, we examine the conditions under which (cross-) licensings occur. However we explore which (cross-) licensing may occur at every state of nature in the Appendix. Extending our analysis to the case with a (cross-) licensing, we make a theoretical analysis on how the difference between a cross-licensing and unilateral licensing affects firms' incentives for R&D activities in a Cournot duoply in section 5. In the final section, we present our concluding remarks.

2. The Model

Let us consider a duopolistic market in which a homogeneous product is produced and sold by two firms: Firm x and Firm y. At the first stage, each firm invests in R&D for the two distinct but weakly complementary technologies, A and B

simultaneously. By "*the weakly complementary technologies*", we mean that each firm *can* produce the goods *without both of the two technologies* but it incurs more redundant costs than that in the case with each or both of the technologies[2]. Denote by $x_A, x_B (\geq 0)$ and $y_A, y_B (\geq 0)$, the investment levels for the technologies A, B of firm x and the investment levels for the technologies A, B of firm y, respectively. If it succeeds in the development of at least one of these technologies, a process innovation brings the reduction of its marginal cost of production. Assume that each firm has constant returns to scale production technology, so it has a linear cost function,

$$C_i(q_i) = c_i q_i = (c_A + c_B) q_i, \qquad i = x, y, \tag{1}$$

where c_A, c_B are marginal costs correspondence to the technology A and B respectively and $c_A = \underline{c_A} + c = c_B = \underline{c_B} + c$. That is, $\underline{c_k}, k = A, B$ is the marginal cost of production without succeeding in the development of the technology k, and $c(>0)$ is the cost reduction level due to the development of the technology.

That is, we can look upon $c(>0)$ as the measure of the degree of technological innovation. Without loss of generality, we can assume that $\underline{c_A} = \underline{c_B} = 0$. Under this setting, if each firm succeeds in the development of the technology A, it can reduce its marginal cost of production from $2c$ to c. If it succeeds in the development of both technologies, the firm can reduce its marginal cost of production from $2c$ to 0. At the end of the first stage, "nature" chooses whether each firm succeeds in the development of the technologies or not. Suppose that each firm succeeds in the development of the technology j with probability

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2 That is, we distinguish "the weakly complementary technologies" from "*the strongly complementary technologies*", in that the firm *cannot* produce the goods *at all without the use of both of them*. For the similar analysis of in the case of the strongly complementary technologies, see Okamura, Shinkai and Tanaka (2002).

$p_k(x_k)=1-e^{-x_k}, k=A,B$ and $p_k(y_k)=1-e^{-y_k}, k=A,B$. This probabilities function is well defined since we have,

$p_k'(\cdot)>0, p_k''(\cdot)<0, p_k'(0)\to\infty$ and $p_k(0)=0, p_k(\infty)=1, k=A,B$ [3]. We also assume that $p_k(\cdot)$ are identically and independently distributed.

The inverse market demand function of the product is given by

$$p=a-Q \tag{2}$$

where p is the market price and Q is the aggregate output of production in the market, that is $Q=q_x+q_y$. We assume that $\frac{1}{4}a>c$, and this assumption guarantees that even the firm with the highest cost can produce the positive quantity of output. At the beginning of the second stage, each firm knows all successes or failures of the both firms' developments of the technologies. At the second stage, if a (cross-) licensing system is available, then each firm bargains to its rival and agrees a (cross-) licensing contract, and they divide the total profit due to the licensing according to the Nash bargaining solution. If the (cross-) licensing system is not available, then the game proceeds to the third stage. At the third stage, each firm's cost of production realized and each firm chooses its quantity of output simultaneously, that is, Cournot competition occurs. Finally, the final profit of each firm is realized and the game is over. The timing of the game is illustrated in Figure 1.

Before our main analysis, we conduct some preliminary works. Denote by $\pi_i(q_i,q_j)$, Firm i's profit. Then $\pi_i(q_i,q_j)$ is given by

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3 In this paper, we assume the effect of the R&D activity on a process innovation as a static one. These properties of the success probability function are similar ones for the dynamic "memoryless" or "Poisson" patent race model associated with Reiganum(1982). In his model, the research technology is characterized by the assumption that a firm's probability of making a discovery and obtaining a patent at a point of time depends only on this firm's current R&D investment level and not on its past R&D experience. For Introductory illustration to the dynamic patent race

$$\pi_i(q_i,q_j) = (a - q_i - q_j - c_i)q_i \quad (i \neq j, i = x, y). \tag{3}$$

Since Firm i chooses its output so as to maximize (3) in Cournot equilibrium, we easily see that the equilibrium output of Firm i can be obtained as a function of c_i and c_j.

$$q_i^*(c_i,c_j) = \frac{(a - 2c_i + c_j)}{3} \quad i,j = x,y, i \neq j \tag{4}$$

Substituting (4) into (3) yields Firm i's Cournot equilibrium profit,

$$\pi_i(c_i,c_j) \equiv \pi_i(q_i^*(c_i,c_j), q_j^*(c_j,c_i)) = \left(\frac{a - 2c_i + c_j}{3}\right)^2. \tag{5}$$

(The first stage)	(The Second Stage)	(The third stage)
R&D competition Decision on R&D investment level of the firms x, y	Bargaining for licensing Choice of the license fee	Decision on quantity of outputs

Cournot competition

Nature's choise on

Success or failure

of the development

Figure 1. Timing of the game

In the next section, we analyze the problem of R&D in a Cournot duopoly with weakly complementary technological innovation without licensing as a benchmark.

3. R&D investment without (cross-) licensings-------A benchmark-----

In our model, all possible states of nature are 16 ways. Denote by $\{X,Y\}$, the a

model, see section 10.2 of Chap.10 in Tirole(1989).

state of nature in that the state of nature Firm x faces and the one Firm y does are X and Y, respectively: Where $X, Y \in \{AB, A, B, \phi\}$ and "AB", "A", "B" and "ϕ" implies that each firm successes in development of technologies A and B, A,B and nothing, respectively. Hence, all possible states of nature that we have are give by {AB, AB}, {AB, A}, {AB, B}, {AB, ϕ}, {A, AB}, {A, A}, {A, B}, {A, ϕ}, {B, AB}, {B, A}, {B, B}, {B, ϕ}, {ϕ, AB}, {ϕ, A}, {ϕ, B} and {ϕ, ϕ}. From the model setting in the proceeding section, the corresponding realized Cournot duopoly equilibrium profit of Firm x are $\pi_x(0,0), \pi_x(0,c), \pi_x(0,c), \pi_x(0,2c)$, $\pi_x(c,0), \pi_x(c,c)$, $,\pi_x(c,c), \pi_x(c,2c)$, $\pi_x(c,0), \pi_x(c,c), \pi_x(c,c), \pi_x(c,2c)$, $\pi_x(2c,0), \pi_x(2c,c), \pi_x(2c,c)$ and $\pi_x(2c,2c)$. We also have the corresponding realized Cournot duopoly equilibrium profit of Firm y are $\pi_y(0,0), \pi_y(c,0), \pi_y(c,0), \pi_y(2c,0)$, $\pi_y(0,c), \pi_y(c,c)$,

$,\pi_y(c,c), \pi_y(2c,c)$, $\pi_y(0,c), \pi_y(c,c), \pi_y(c,c), \pi_y(2c,c)$, $\pi_y(0,2c), \pi_y(c,2c), \pi_y(c,2c)$ and $\pi_y(2c,2c)$. Then, the expected profit of Firm x without (cross-) licensing is given by,

$$\prod_x \equiv E\Pi_x(x_A, x_B, y_A, y_B) = (1-e^{-x_A})(1-e^{-x_B})H_1 + (1-e^{-x_A})e^{-x_B}H_2 + e^{-x_A}(1-e^{-x_B})H_3 + e^{-x_A}e^{-x_B}H_4 - x_A - x_B, \tag{6}$$

where we have

$$H_1 = (1-e^{-y_A})(1-e^{-y_B})\pi_i(0,0) + (1-e^{-y_A})e^{-y_B}\pi_i(0,c) + e^{-y_A}(1-e^{-y_B})\pi_i(0,c) + e^{-y_A}e^{-y_{B}B}\pi_i(0,2c), \tag{7a}$$

$$H_2 = (1-e^{-y_A})(1-e^{-y_B})\pi_i(c,0) + (1-e^{-y_A})e^{-y_B}\pi_i(c,c) + e^{-y_A}(1-e^{-y_B})\pi_i(c,c) + e^{-y_A}e^{-y_B}\pi_i(c,2c), \tag{7b}$$

$$H_3 = (1-e^{-y_A})(1-e^{-y_B})\pi_i(c,0) + (1-e^{-y_A})e^{-y_B} + e^{-y_A}(1-e^{-y_B})\pi_i(c,c)$$

$$+ \ e^{-y_A}e^{-y_B}\pi_i(c,2c) \tag{7c}$$

and

$$H_4 = (1-e^{-y_A})(1-e^{-y_B})\pi_i(2c,0) + (1-e^{-y_A})e^{-y_B}\pi_i(2c,c) + \ e^{-y_A}(1-e^{-y_B})\pi_i(2c,c)$$

$$+ \ e^{-y_A}e^{-y_B}\pi_i(2c,2c). \tag{7d}$$

The first order condition is given by [3]

$$\frac{\partial\Pi_x(x_A,x_B,y_A,y_B)}{\partial x_A} = e^{-x_A}(1-e^{-x_B})H_1 + \ e^{-x_A}e^{-x_B}H_2 - e^{-x_A}(1-e^{-x_B})H_3 -$$

$$e^{-x_A}e^{-x_B}H_4 - 1$$

$$= e^{-x_A}\{(1-e^{-x_B})H_1 + \ (2e^{-x_B}-1)H_2 - \ e^{-x_B}H_4\} - 1 = 0. \tag{8}$$

From (7b) and (7c), we see that $H_2 = H_3$, so by (8) we can obtain

$$e^{-x_A}\{(1-e^{-x_B})H_1 + \ (2e^{-x_B}-1)H_2 - \ e^{-x_B}H_4\} - 1 = 0 \tag{9}$$

Noticing the symmetry of Firm x and y in a Cournot duopoly, we focus on the symmetric equilibrium hereafter throughout this paper. Then, we can denote the probability of the failure for the development of each technology, by $s = e^{-x_A} = e^{-x_B} = e^{-y_A} = e^{-y_B}$. Rewriting the equations (7a),(7b),(7c) and (7d), we obtain

$$H_1 = (1-s)^2\pi_i(0,0) + 2s(1-s)\pi_i(0,c) + \ s^2\pi_i(0,2c) \tag{10a}$$

$$H_2 = H_3 = (1-s)^2\pi_i(c,0) + 2s(1-s)\pi_i(c,c) + \ s^2\pi_i(c,2c) \tag{10b}$$

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[3] The second order condition at the equilibrium we derive is that $1-\left[\frac{s}{1-s}-\frac{s^2}{1-s}(H_2-H_4)\right]^2 > 0$ holds. If $0 < s \le 1/2$ holds, then we can easily show that this inequality holds.

$$H_4 = (1-s)^2 \pi_i(2c, 0) + 2s(1-s)\pi_i(2c, c) + s^2 \pi_i(2c, 2c). \tag{10c}$$

Therefore the first order condition (9) is expressed by

$$\phi(s) = \frac{\partial \Pi_x(x_A, x_B, y_A, y_B)}{\partial x_A} = sV(s) - 1$$

$$= s\{(1-s)(H_1 - H_2) + s(H_2 - H_4)\} - 1 = 0,$$

$$\text{where } V(s) \equiv (1-s)(H_1 - H_2) + s(H_2 - H_4). \tag{11}$$

From (10a), (10b) and (10c), we see that

$$H_1 - H_2 = (1-s)^2 [\pi_i(0, 0) - \pi_i(c, 0)] + 2s(1-s)[\pi_i(0, c) - \pi_i(c, c)]$$

$$+ s^2 [\pi_i(0, 2c) - \pi_i(c, 2c)] \tag{12a}$$

$$H_2 - H_4 = (1-s)^2 [\pi_i(c, 0) - \pi_i(2c, 0)] + 2s(1-s)[\pi_i(c, c) - \pi_i(2c, c)]$$

$$+ s^2 [\pi_i(c, 2c) - \pi_i(2c, 2c)] \tag{12b}$$

By noticing the above fact, $H_1 - H_2$ and $H_2 - H_4$ can be rewritten into

$$H_1 - H_2 = (n_1 - 2n_2 + n_3)s^2 + (2n_2 - 2n_1)s + n_1 \tag{13a}$$

$$H_2 - H_4 = (n_4 - 2n_5 + n_6)s^2 + (2n_5 - 2n_4)s + n_4. \tag{13b}$$

Where we have

$$n_1 = \pi_i(0, 0) - \pi_i(c, 0), \quad n_2 = \pi_i(0, c) - \pi_i(c, c)$$

$$n_3 = \pi_i(0, 2c) - \pi_i(c, 2c), \quad n_4 = \pi_i(c, 0) - \pi_i(2c, 0)$$

$$n_5 = \pi_i(c, c) - \pi_i(2c, c) \quad n_6 = \pi_i(c, 2c) - \pi_i(2c, 2c). \tag{14}$$

Substituting the equation (5) into $n_t{}'s, t = 1, \cdots, 6$ in the (11), we obtain $n_t{}'s, t = 1, \cdots, 6$ as follows:

$$n_1 = \pi_i(0, 0) - \pi_i(c, 0) = \frac{4}{9}c(a-c) \tag{15a}$$

$$n_2 = \pi_i(0, c) - \pi_i(c, c) = \frac{4}{9}ac \quad (15b)$$

$$n_3 = \pi_i(0, 2c) - \pi_i(c, 2c) = \frac{4}{9}c(a+c) \quad (15c)$$

$$n_4 = \pi_i(c, 0) - \pi_i(2c, 0) = \frac{4}{9}c(a-3c) \quad (15d)$$

$$n_5 = \pi_i(c, c) - \pi_i(2c, c) = \frac{4}{9}c(a-2c) \quad (15e)$$

$$n_6 = \pi_i(c, 2c) - \pi_i(2c, 2c) = \frac{4}{9}c(a-c). \quad (15f)$$

From the above equations, we can easily notice that $n_1 = n_6$, $n_1 + n_3 = 2n_2$ and $n_4 + n_6 = 2n_5$. Therefore, $V(s)$ in the first order condition (11) can be simplified to

$$V(s) = [(2n_5 - 2n_4) - (2n_2 - 2n_1)]s^2 + [(2n_2 - 2n_1) - n_1 + n_4]s + n_1. \quad (16)$$

Furthermore, by equation (12), noticing the fact that $n_5 - n_4 = n_2 - n_1$, $2(n_2 - n_1) - (n_1 - n_4) = 0$, we can finally see that

$$V(s) = n_1.$$

So the first order condition (11) yields

$$\phi(s) = n_1 s - 1 = 0. \quad (17)$$

If $\phi(1) = n_1 - 1 = \pi_i(0,0) - \pi_i(c,0) > 0$, then there exists a unique equilibrium solution $s^* = \frac{1}{n_1} (0 < s^* \le 1)$ that solves the equation (17). Since $s^* = e^{-x^*}$, the R&D investment expenditure at the symmetric equilibrium is obtained:

$$x^* = \ln\frac{4}{9}c(a-c) = \ln\ [\pi_i(0, 0) - \pi_i(c, 0)]. \quad (18)$$

That is we obtain the following proposition.

Proposition 1

Suppose that $a \ge 3\sqrt{2}$ and $\frac{1}{4}a > c$. If $\frac{a - \sqrt{a^2 - 9}}{2} < c < \frac{1}{4}a$, then there exists a unique positive symmetric equilibrium R&D investment expenditure x^* in our model without (cross-) licensing,

$$x^* = \ln\frac{4}{9}c(a-c).$$

Hence, $\frac{\partial x^*}{\partial a} > 0$, $\frac{\partial x^*}{\partial c} > 0$.

Proof: From the preceding argument of the proposition, if $\phi(1) = n_1 - 1 = \pi_i(0,0) - \pi_i(c,0) - 1 > 0$, then x^* hence $s^* = e^{-x^*}$ is a unique solution of the first order condition (18). Solving the quadratic inequality with respect to $c, \phi(1) = n_1 - 1 = \pi_i(0,0) - \pi_i(c,0) - 1 = \frac{4}{9}c(a-c) - 1 > 0$, we obtain

$$\frac{a-\sqrt{a^2-9}}{2} < c < \frac{a+\sqrt{a^2-9}}{2} \text{ and } a \geq 3. \tag{19}$$

Furthermore, since $s^* = e^{-x^*}$ has to satisfy the second order condition given in footnote 3, $s^* = \frac{1}{n_1} \leq 1/2$. The last inequality is equivalent to the inequality $-2c^2 + 2ac - 9 \geq 0$. Solving this quadratic inequality with respect to c, we obtain

$$\frac{a-\sqrt{a^2-18}}{2} \leq c \leq \frac{a+\sqrt{a^2-18}}{2} \text{ and } a \geq 3\sqrt{2}. \tag{20}$$

We can show that any $c \in \{c \,|\, \frac{a-\sqrt{a^2-9}}{2} < c < \frac{1}{4}a\}$ satisfies both (19) and (20).

We can easily show that $\frac{\partial x^*}{\partial a} > 0$ and $\frac{\partial x^*}{\partial c} > 0$ if $\frac{a-\sqrt{a^2-9}}{2} < c < \frac{1}{4}a$.

Q.E.D.

One of the above conditions is the one that guarantees the firm with the highest cost to produce the product, $\pi_i(2c,0) > 0 \Leftrightarrow \frac{1}{4}a > c$. In words, this condition expresses the feature of weakly complementary technological inventions. Another condition,

$\phi(1) = n_1 - 1 = \pi_i(0,0) - \pi_i(c,0) - 1 \Leftrightarrow \frac{4}{9}c(a-c) - 1 > 0$ implies that the increment of benefit of a firm with one technology due to becoming available for another technology by unilateral licensing from its rival with two technologies exceeds its marginal cost of R&D investment.

4. The Conditions under which (cross-) licensings may occur

There are 16 states of nature since in our model the firms faces to develop the two weakly complementary technologies as we have seen in the preceding section. That is, $\{AB, AB\}$, $\{AB, A\}$, $\{AB, B\}$, $\{AB, \phi\}$, $\{A, AB\}$, $\{A, A\}$, $\{A, B\}$, $\{A, \phi\}$, $\{B, AB\}$, $\{B, A\}$, $\{B, B\}$, $\{B, \phi\}$, $\{\phi, AB\}$, $\{\phi, A\}$, $\{\phi, B\}$ and $\{\phi, \phi\}$. Notice that there are 12 cases where a (cross-) licensing may occurs, except for $\{AB, AB\}$, $\{A, A\}$, $\{B, B\}$ and $\{\phi, \phi\}$. In our model, firms and their R&D investments at the equilibrium are symmetry. Therefore we can classify these 12 states of nature into the next four categories.

Ⅰ. The cases in which each firm succeeds in the development of one of the different technologies: $\{A, B\}$ and $\{B, A\}$. In the case included in this category, a cross-licensing may occur.

Ⅱ. The cases in which any one of the firms succeeds in the development of only one of the two technologies, and the other firm fails the developments of both of the two: $\{A, \phi\}$, $\{B, \phi\}$, $\{\phi, A\}$ and $\{\phi, B\}$.

Ⅲ. The cases in which one of the firms succeeds in the development of one of the two technologies, and the other firm does both of the two: $\{AB, A\}$, $\{AB, B\}$, $\{A, AB\}$ and $\{B, AB\}$.

Ⅳ. The cases in which any one of the firms succeeds in the development of the two technologies, and the other firm fails the developments of both: $\{AB, \phi\}$ and $\{\phi, AB\}$.

In the following, we present the conditions under which a (cross-) licensing occurs and the correspondent licensing fee in the cases included in each of the above four categories. For the concrete derivation of the licensing conditions and the licensing fees in the case contained the four categories, see Appendix.

Ⅰ. In the cases included in this category, the cross-licensing fee is given by

$$F_{\mathrm{I}}=0. \tag{21}$$

We have the cross-licensing conditions under which the cross-licensing occurs:

$$2a > c \tag{22}$$

Ⅱ. In the cases included in this category, the unilateral licensing fee is given by

$$F_{\mathrm{II}}=\frac{1}{2}[\pi_i(c,2c)-\pi_j(2c,c)]=\frac{c(2a-3c)}{6}>0. \tag{23}$$

The unilateral licensing conditions under which the unilateral licensing occurs:

$$\frac{2}{7}a > c \tag{24}$$

Ⅲ. In the cases included in this category, the unilateral licensing fee is given by

$$F_{\mathrm{III}}= \frac{1}{2}[\pi_i(0,c)-\pi_j(c,0)] =\frac{c(2a-c)}{6}>0. \tag{25}$$

The unilateral licensing conditions under which the unilateral licensing occurs:

$$\frac{2}{5}a > c \tag{26}$$

Ⅳ. In the cases included in this category, there are two type of cases in which the unilateral licensing occurs. In one type of the cases, the unilateral licensing of only one technology occurs. In the other type of the cases, the unilateral licensing of both of the two technologies occurs. As we show in the Appendix, the unilateral licensing strategy of both of the two technologies dominates that of only one technology for the licenser firm. At last, the unilateral licensing fee in the cases included in this category is given by

$$F_{\mathrm{IV}}= \frac{1}{2}[\pi_i(0,2c)-\pi_j(2c,0)]=\frac{2c(a-c)}{3}>0, \tag{27}$$

that is the fee of the unilateral licensing of both of the two technologies.

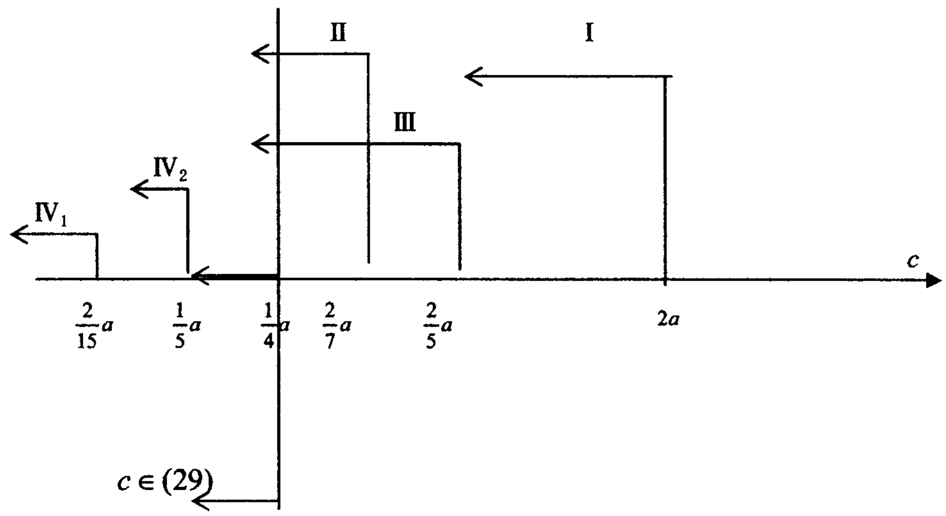
The unilateral licensing conditions under which the unilateral licensing occurs:

$$\frac{1}{5}a>c \tag{28}$$

From the assumption presented in section 2 that guarantees even the Cournot firm with the highest cost can produce,

$$\frac{1}{4}a>c. \tag{29}$$

So we see that all the four licensing fees are strictly positive by (29). From the above argument, as can see in Figure 2 below, under the assumption (29), the conditions we have to examine in which any (cross-) licensing occurs are three:



I. The cases in which each firm succeeds in the development of one of the different technologies:

II. The cases in which any one of the firms succeeds in the development of only one of the two technologies, and the other firm fails the developments of both of the two:

III. The cases in which one of the firms succeeds in the development of one of the two technologies, and the other firm does both of the two:

IV. The cases in which any one of the firms succeeds in the development of the two technologies, and the other firm fails the developments of both: IV_2. In this category, a unilateral licensing of the two technologies occur. IV_2. In this category, a unilateral licensing of only one technology occurs.

Figure 2 The conditions under that (cross-) licensing occur

The first one is the case, $\frac{1}{5}a \le c \le \frac{1}{4}a$, within this area, (cross-) licensings included in the three categories I , II and III stated above. The second one is $\frac{1}{15}a \le c \le \frac{1}{5}a$, within this area, a (cross-) licensing occurs in the case included in the four categories I , II , III and IV_2. The third one is $c \le \frac{2}{15}a$, within this area IV_1, the case in which the unilateral licensing of only one technology that may occur. As a result, any (cross-) licensing occurs in the case included in the five categories I , II, III , IV_2 and IV_1. However, as we discuss earlier, the unilateral licensing strategy of both of the two technologies dominates that of only one technology for the licenser firm. At last, all the conditions with (cross-) licensings we have to examine are the two cases, $\frac{1}{5}a \le c \le \frac{1}{4}a$ and $c \le \frac{1}{5}a$.

5. R&D investment with (cross-) licensings

In our model, as we find in the preceding section, which form of licensing occurs in a cross-licensing and a unilateral licensing, depends on the relation between the potential size of demand a and the measure of the degree of technological innovation c. One side, a cross-licensing is a kind of trade of exchange of its own technology with the other's one. The other side, a single licensing is the unilateral offer of the firm which succeeds in the development of the technology to the firms which fails to develop the technology. Since these two types of trades differ from each other, we distinguish these two types of trades from each other. We pay attention to the area, $c \leq \frac{1}{5}a$ in which both a cross-licensing and a unilateral licensing occur in this section. When a unilateral licensing occurs, there exists a difference in technical superiority between the two firms due to the outcome of their developments. In our model, however, the state of nature which realizes in consequence of the two firm's R&D can be expressed by a pair of the number of the technologies of each firm that it succeeds in the development, since the measure of the degree of the technical innovation, "c" is same for the two firms and they are also symmetric. Denote such state of nature by (m_i, m_j), where m_i and m_j are the number of the technologies of Firm i that it succeeds in the development, and the one of Firm j that succeeds in the R&D, respectively. Then, when we pay attention to the case where only a unilateral licensing occurs, the states we have to examine are (2, 1),(2, 0),(1, 0),(1, 2),(0, 2) and (0, 1). In preparation for the following derivation of the equilibrium, we write down the part of firm i's expected profit in these states below.

(4) The part of firm i's expected profit in (2, 1)

Since the threat point of firm j in this state is $\pi_j(c,0)$, the unilateral licensing results in the ex-post profit of firm i, $\pi_i(0,0)+F_{\text{III}}$, where F_{III} is given by (25).

Hence the part of firm i's expected profit in the state (2, 1), Π_{21} can be give by

$$\Pi_{21} = (1-e^{-x_A})(1-e^{-x_B})\{e^{-y_A}+e^{-y_B}-2e^{-y_A}e^{-y_B}\}[\pi_i(0,0)+F_{\text{III}}]. \tag{30}$$

(5) The part of firm i's expected profit in (2, 0)

Since the threat point of firm i in this state is $\pi_i(0,2c)$, the unilateral licensing results in the ex-post profit of firm $i, \pi_i(0,0)+F_{\mathrm{IV}}$, where F_{IV} is given by (27)

Hence the part of firm i's expected profit in the state (2, 0), Π_{20} can be give by

$$\Pi_{20}=(1-e^{-x_A})(1-e^{-x_B})\{e^{-y_A}+e^{-y_B}-2e^{-y_A}e^{-y_B}\}[\pi_i(0,0)+F_{\mathrm{IV}}]. \quad (31)$$

(6) The part of firm i's expected profit in (1, 0)

Since the threat point of firm i in this state is $\pi_i(c,2c)$, the unilateral licensing results in the ex-post profit of firm $i, \pi_i(c,c)+F_{\mathrm{II}}$, where F_{II} is given by (23).

Hence the part of firm i's expected profit in the state (1, 0), Π_{10} can be give by

$$\Pi_{10}=[e^{-x_A}+e^{-x_B}-2e^{-x_A}e^{-x_B}]e^{-y_A}e^{-y_B}[\pi_i(c,c)+F_{\mathrm{II}}]. \quad (32)$$

(7) The part of firm i's expected profit in (1, 2)

Since the threat point of firm i in this state is $\pi_i(2c,c)$, the unilateral licensing results in the ex-post profit of firm $i, \pi_i(0,0)$ - F_{III}, where F_{III} is given by (25).

Hence the part of firm i's expected profit in the state (1, 2), Π_{12} can be give by

$$\Pi_{12}=[e^{-x_A}+e^{-x_B}-2e^{-x_A}e^{-x_B}](1-e^{-y_A})(1-e^{-y_B})\ [\pi_i(0,0)-F_{\mathrm{III}}]. \quad (33)$$

(8) The part of firm i's expected profit in (0, 2)

Since the threat point of firm i in this state is $\pi_i(2c,0)$, the unilateral licensing results in the ex-post profit of firm $i, \pi_i(0,0)$ - F_{IV}, where F_{IV} is given by (27).

Hence the part of firm i's expected profit in the state (0, 2), Π_{02} can be give by

$$\Pi_{02}=e^{-x_A}e^{-x_B}\ (1-e^{-y_A})(1-e^{-y_B})\ [\pi_i(c,c)-F_{\mathrm{IV}}]. \quad (34)$$

(9) The part of firm i's expected profit in (0, 1)

Since the threat point of firm i in this state is $\pi_i(2c,c)$, the unilateral licensing results in the ex-post profit of firm $i, \pi_i(c,c)$ - F_{II}, where F_{II} is given by (23).

Hence the part of firm i's expected profit in the state (0, 1), Π_{02} can be give by

$$\Pi_{01}=e^{-x_A}e^{-x_B}\,(1-e^{-y_A})(1-e^{-y_B})\;\;[\pi_i(c,c)-F_{\text{II}}]. \tag{35}$$

Denoting by $\tilde{\Pi}_i$, Firm i's expected profit with any licensing, it can be expressed by the sum of Firm i's expected profit without licensing, Π_i presented in the preceding section, the increment part of Firm i's expected profit with only a cross-licensing Π_i^{CR} and the increment part of Firm i's expected profit with a unilateral licensing. That is, we have

$$\tilde{\Pi}_i=\Pi_i+\Pi_i^{CR}+(1-e^{-x_A})(1-e^{-x_B})[AF_{\text{III}}+BF_{\text{IV}}]+[e^{-x_A}+e^{-x_B}-2e^{-x_A}e^{-x_B}][BF_{\text{III}}-CF_{\text{III}}]$$

$$-e^{-x_A}e^{-x_B}[CF_{\text{IV}}+AF_{\text{II}}] \tag{36}$$

where, $A = e^{-y_A}+e^{-y_B}-2e^{-y_A}e^{-y_B}$, $B = e^{-y_A}e^{-y_B}$, $C = (1-e^{-y_A})(1-e^{-y_B})$ (37).

The increment part of Firm i's expected profit with only a cross-licensing Π_i^{CR} is given by,

$$\Pi_i^{CR} = (1-e^{-x_A})e^{-x_B}(1-e^{-y_A})e^{-u_B}h+e^{-x_A}(1-e^{-x_B})(1-e^{-y_A})e^{-y_B}h, \tag{38}$$

where $h=\pi_i(0,0)-\pi_i(c,c)$.

Therefore, in the case with any licensings, one of the first order conditions is the partial derivative of $\tilde{\Pi}_i$ with respect to x_A is equal to 0. Taking consideration the symmetry of the two firms and using that $s=e^{-x_A}=e^{-x_B}=e^{-y_A}=e^{-y_B}$, we can write the first order condition as follows:

$$\frac{\partial\tilde{\Pi}}{\partial x_A} = \frac{\partial\Pi}{\partial x_A}+\frac{\partial\Pi^{CR}}{\partial x_A}+\;s(1-s)[AF_{\text{III}}+BF_{\text{IV}}]+s(2s-1)[BF_{\text{II}}-CF_{\text{III}}]$$

$$+s^2[CF_{\text{IV}}+AF_{\text{II}}]=0. \tag{39}$$

Now, in (38), we let

$$N(s) = s\{(1-s)[AF_{III} + BF_{IV}] + (2s-1)[BF_{II} - CF_{III}] + s[CF_{IV} + sF_{II}]\} = s\psi(s) \quad (40)$$

where, $\psi(s) = (1-s)[AF_{III} + BF_{IV}] + (2s-1)[BF_{II} - CF_{III}] + s[CF_{IV} + sF_{II}]$

$$= \{(1-s)A - (2A-1)C\}F_{III} + \{(1-s)B - sC\}F_{IV} + \{(2s-1)B + sA\}F_{II}.$$

From (37), we see that $A=2s\ (1-s)$, $B = s^2$, $C = (1-s)^2$ holds. So the coefficients of F_{III}, F_{IV}, F_{II} in $\psi(s)$ are given by

$$(1-s)A - (2A-1)C = (1-s)^2 \quad (41a)$$

$$(1-s)B - sC = s(1-s) \quad (41b)$$

$$(2s-1)B + sA = s^2 \quad . \quad (41c)$$

Therefore, we have

$$\begin{aligned}\psi(s) &= (1-s)^2 F_{III} + s(1-s)F_{IV} + s^2 F_{II} \\ &= (F_{III} - F_{IV} + F_{II})s^2 + (F_{IV} - 2F_{III})s + F_{III} \\ &= (F_{IV} - 2F_{III})s + F_{III}. \quad (42)\end{aligned}$$

where the last equality holds since $F_{III} - F_{IV} + F_{II} = 0$. Substituting (42) into (40), we have

$$N(s) = s \cdot \psi(s) = \{(F_{IV} - 2F_{III})s + F_{III}\}s. \quad (43)$$

Derive the partial derivative of (38) w.r.t. x_A and using the symmetry of firms,

$$\begin{aligned}\frac{\partial \Pi^{CR}}{\partial x_A} &= e^{-x_A}e^{-x_B}(1-e^{-y_B})e^{-y_A}h - e^{-x_A}(1-e^{-x_B})(1-e^{-y_A})e^{-y_B}h \\ &= s^2(2s-1)(1-s)h\end{aligned} \quad (44)$$

where $h = \pi_i(0,0) - \pi_i(c,c) > 0$. Let $g(s) = \dfrac{\partial \tilde{\Pi}}{\partial x_A}$. Then form (11),(39),(40), (43) and (44), the first condition is reduced to

$$g(s) = \frac{\partial \tilde{\Pi}}{\partial x_A} = \frac{\partial \Pi}{\partial x_A} + \frac{\partial \Pi^{CR}}{\partial x_A} + N(s) = \phi(s) + s^2(2s-1)(1-s)h + N(s) \quad (45)$$

So we see that

$$g(0) = \phi(0) = -1 < 0$$

and

$$g(\tfrac{1}{2})=\phi(\tfrac{1}{2})-1+N\left(\frac{1}{2}\right)=\phi(\tfrac{1}{2})-1+\frac{1}{4}F_{IV}$$
$$=\frac{4c(a-c)-9}{9}+\frac{c(a-c)}{6}=\frac{11c(a-c)-9}{18}>0$$

and

$$0<\phi(1)<g(1)=\phi(1)+F_{IV}-F_{III}=\left(\frac{4c(a-c)}{9}-1\right)+\frac{2c(a-c)}{3}-\frac{c(2a-c)}{6}$$
$$=\left(\frac{4c(a-c)}{9}-1\right)+\frac{c(2a-3c)}{6}$$

(46)

where the second inequality holds if $\frac{a-\sqrt{a^2-9}}{2}<c<\frac{1}{4}a$, and the last inequality holds since $\frac{c(2a-3c)}{6}>0$ holds from (29).

From (45) and that $\phi(s)=n_1 s-1$, we have

$$g'(s)=\phi'(s)+s(-8s^2+9s-2)h+N'(s)$$

From (43), we see that $N'(s)=\psi(s)+(F_{IV}-2F_{III})s$

Here put $B(s)\equiv s(-8s^2+9s-2)h$. Then, we have,

$$g(s)=\ \phi(s)+B(s)+N(s)\ \text{ and}$$
$$g'(s)=\ \phi'(s)+B'(s)+N'(s)$$
$$=\ n_1+B'(s)+N'(s) \tag{47}$$

$$g'(0)=n_1+F_{III}>\phi'(0)=n_1>0\quad,\quad g'(1)=n_1+2F_{IV}-3F_{III}-h=\frac{c(10a-21c)}{18}>0$$

(48). Since we get

$$B'(s)+N'(s)=s(-8s^2+9s-2)h+\frac{2c(a-(1+s)c)}{3}=s(-8s^2+9s-2)\frac{c(2a-c)}{9}+\frac{2c(a-(1+s)c)}{3}$$

$$B'(0)+N'(0)=\frac{2c(a-c)}{3}>0. \tag{49}$$

$$B''(s)+N''(s)=-\frac{8c(2a-c)}{3}s^2+2c(2a-c)s-\frac{4c(a+c)}{9}. \tag{50}$$

Therefore we can conclude that if

$$0<s<\frac{3c(2a-c)+3\sqrt{c^2(2a-c)^2+\dfrac{32c^2(2a-c)(a+c)}{27}}}{8c(2a-c)}<\frac{3c(2a-c)(1+\sqrt{59/27})}{8c(2a-c)}<1$$

, then $B''(s)+N''(s)>0$ from (50).

However, we can easily show that

$$\frac{1}{2}<\frac{3c(2a-c)+3\sqrt{c^2(2a-c)^2+\dfrac{32c^2(2a-c)(a+c)}{27}}}{8c(2a-c)}.$$

Also we know that $s^*\le\frac{1}{2}$ from Proposition 1 in section 3. And we see that if $0<s<\frac{1}{2}$, then $B''(s)+N''(s)>0$ and $g'(s^*)$ = $\phi'(s)+B'(s)+N'(s)>\phi'(s)=n_1>0$. Therefore, since $B''(s^*)+N''(s^*)>0$, taking into account the fact (47), (48), we conclude that

$$g'(s^*)=\ \phi'(s^*)+B'(s^*)+N'(s^*)>\phi'(s^*)=n_1 \qquad (51)$$

The fact that (48) and (51) holds simultaneously implies that $g(s)$ has to intersect the horizontal axis at the left side of the point s^*. Furthermore, from (46), if $\frac{a-\sqrt{a^2-9}}{2}<c<\frac{1}{4}a$, then

$g\left(\frac{1}{2}\right)=\phi\left(\frac{1}{2}\right)+N\left(\frac{1}{2}\right)>\phi\left(\frac{1}{2}\right)>0,\because N\left(\frac{1}{2}\right)=\frac{1}{4}F_{IV}>0$. Note that

$B\left(\frac{1}{2}\right)+\phi\left(\frac{1}{2}\right)=0+\phi\left(\frac{1}{2}\right)=\phi\left(\frac{1}{2}\right)$. The argument above is illustrated in Figure 4 below. Then, we obtain the following proposition without proof.

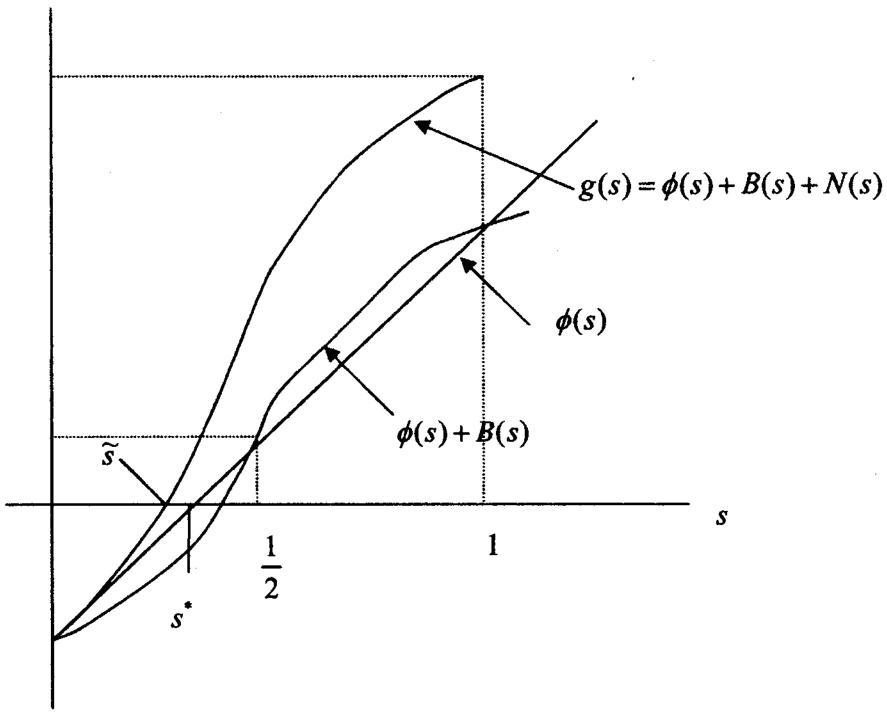


Figure 4. The equilibrium in the case with licensing $\tilde{s} < s^* (< 1/2)$

Proposition 2

If Proposition 1 holds, and hence there exists the equliribrium probability of failure $s^* = \dfrac{1}{n_1}(0 < s^* \le 1/2)$ in the model without licensing. Then, there exists $\tilde{s}(< s^* \le 1/2)$ the equliribrium probability of failure in the model with unilateral licensing. Therefore, the equilibrium R&D expenditure in the model of licensing is greater than that in the model without licensing: $\tilde{x} = -\ln \tilde{s} > -\ln s^* = x^*$.

This proposition shows that a (cross-) licensing system promotes R&D

investment when the duopolistic firms can produce goods by using the two weakly complementary technologies.

In Okamura, Shinkai and Tanaka(2002), the existence of a cross-licensing system discourages firm's R&D investments, when the duopolistic firms can produce goods by using the two strongly complementary technologies, where any unilateral licensing cannot occur since firms needs both of the two technologies for their production of a product. The existence of a cross-licensing system decreases firms' incentives for R&D through the chance of the exchanges of their technologies. A unilateral licensing, however, encourages firms' incentives for R&D through the chance of their receiving (paying) of the licensing fee. When complementary technological innovation occurs, the effect of a cross-licensing system on firms' incentives for R&D works in the opposite direction to that of a unilateral licensing system does.

5. Concluding Remarks

In this paper, we explored the incentives for R&D investments of the duopolistic firms facing technological innovations in weakly complementary technologies by analyzing a simple static innovation model. The first result we obtain is that effect of a cross licensing as the exchange trade of technologies upon the incentives for R&D differs from the effect of the licensing as a unilateral trade upon them. The effects due to the difference between a cross licensing and a unilateral licensing systems upon the incentives for R&D changes the relations among technologies such as substitutability or complementarity. Therefore, the above discussion suggests us the importance of noticing the relations among technologies, to analyze how firms determine the incentives for R&D under complex technological innovations.

There remain many problems for future researches. First, in this paper, we focus on the symmetric equilibrium in order to make our analysis easy. In practice, however, firms *cannot be symmetric* in the industries where the complementary technologies are indispensable for the production of goods. In addition, the role of R&D ventures who do not produce products but concentrate on R&D, increases its importance in such industries. Secondly, in this paper, since we concentrate on the effect on the incentives for R&D, we did not examine the effects on the social welfare at the equilibrium. If we clarify how governments should plan and exercise the policy for technologies under complementary technological innovations, it is important for us to explore the implications on the economic welfare at the equilibrium in our model.

References

Chang, H.F. (1995), "Patent Scope, Antitrust Policy, and Cumulative Innovation," *Rand Journal of Economics*, vol.26: pp.34-57.

Cohen, W.M., Nelson, R.R. & J.P. Walsh. (2000), "Protecting Their Intellectual Assets: Appropriability Conditions and Why U.S. Manufacturing Firms Patent (or Not)," NBER Working Paper, No.7552.

Eswaran, M. (1993), "Cross-Licensing of Competing Patent as Facilitating Device," *Canadian Journal of Economics*, vol.27: pp.689-708.

Fershtman, C. & M.I Kamien. (1992), "Cross Licensing of Complementary Technologies," *International Journal of Industrial Organization*, vol.10: pp.329-348.

Green, J.R. & S. Scotchmer. (1995), "On the Division of Profit in Sequential

Innovation," *Rand Journal of Economics*, vol.26: pp.20-33.

Grindley, P.C. & D.J. Teece. (1997), "Managing Intellectual Capital: Lisensing and Cross-Licensing in Semiconductors and Electronics," *California Management Review*, vol.39: pp.8-41.

Hall, B.H. & R.H. Ziedonis. (2001), "The Patent Paradox Revisited: An Empirical Study of Patenting in the US Semiconductor Industry, 1979-1995," *Rand Journal of Economics*, vol.32: pp.101-128.

Heller, M.A. & R.S.Eisenberg. (1998), "Can Patent Deter Innovation? The Anticommons in Biomedial Research," *Science*, vol.280: pp.698-701.

Okamura, M., T. Shinkai. & S. Tanaka. (2002), "R & D Competition and Cross-Licensing in the Complementary Technological Innovation," (mimeo), forthcoming in Working Paper, Kobe City University of Foreign Studies.

Scotchmer, S. (1991), "Standing on the Shoulders of Giants: Cumulative Research and the Patent Law," *Journal of Economic Perspectives*, vol. 5: pp.29-41.

Tirole, J.(1989), The Theory of Industrial Organization, MIT Press, Cambridge