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Persistent Inequality in a Simple Two Sector Model*

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Abstract

This paper proposes a theory for the persistence of income inequality using a simple overlapping generations model with two consumption goods and two production sectors. There are two classes of workers, high and low skilled, each with a comparative advantage in a different type of production. Our model has a parameter range in which workers permanently specialize in the production type associated with their respective comparative advantages. This pattern of specialization leads to persistent income inequality, between skill classes, that is determined by the structure of demand for the two consumption goods. When the model is extended to a multi-sector framework, persistent inequality occurs under natural conditions on sector specific productivities. Finally this paper analyzes several egalitarian policies.

Keywords: Persistent Income Inequality; Elasticity of output to human capital; Comparative Advantage; Complete Specialization; Egalitarian Policy
JEL Classifications: J24; O15

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1. Introduction

The main purpose of this paper is to examine the new implication for the persistence of income inequality through human capital accumulation. A considerable literature has explained the occurrences of persistent inequality by the result from the difference of investment activities for human capital between rich and poor individuals in long-term equilibrium. On the other hand, we propose an alternative model in which all individuals have a same marginal propensity of human capital investment. In our model, there are two kinds of consumption goods and two classes of workers distinguished by amount of human capital. Individuals specialize in the production type associated with their respective comparative advantages. We prove that a natural pattern of such specialization leads to the gap of the amount of human capital in long-term although all workers make a same investment to human capital of their children.

In seminal paper, Galor and Zeira (1993) show that inequality in the distribution of income persistence in the long run under credit market imperfections and indivisible investment technology. The indivisibility requires an amount of funds for investment to human capital. Since the financial market is imperfect, only relative rich individuals can invest for the technology while poor ones cannot. The difference of the investment activities leads to persistent income inequality. A number of studies, including Banerjee and Newman (1993), Freeman (1996), Aghion and Bolton (1997), Piketty (1997), Ghatak and Jiang (2002), Mookhejee and Ray (2002, 2003), and Sakuragawa and Mitsui (2002), also conclude that financial market imperfections and indivisible technology imply an absence of opportunities for poor individuals to access high earning investment technologies.

Another stream of research on persistent inequality is based on the segregation or segmentation of society as descried by Benabou (1994, 1996) and Durlauf (1996). Wealthy families have an incentive to isolate themselves from the rest of society in order to maintain a high level of education for their children. The gap in education levels and a positive externality between social classes leads to persistent inequality in income.

Chakraborty and Das (2005) and Castello-Climent and Domenech (2008) show that the gap in private health investment between the rich and poor is a cause of persistent inequality. In their model, individuals can extend their life expectancy by investment for own health. But, less wealthy individuals are unable to afford to care their health; the life expectancy is short. Thus, poor individuals have little incentive to invest their human capital. Hence, the difference for investment in education and health care between wealthy and less wealthy individuals leads to persistent inequality.

De la Crolix and Doepke (2003) and Moav (2005) present a model in which fertility and education decisions are interdependent. Families with less human capital decide to have more children and invest less education. The difference of the investment for human capital yields a large income inequality in the next generation.

Mani (2001) and Das (2007) explore another source of persistent income inequality on the demand side; non-homothetic (hierarchic) preferences lead to persistent income inequality. Mani (2001) presents a model in which there are a basic good and two kinds of luxury (medium and high skilled) goods. Because productions of these goods vary in skill requirements, workers must make an amount of invest for human capital in order to produce luxury goods. When income inequality is high, the low and medium skilled classes face low demand for their products. Because the relative price of their products is low, this implies low returns to human capital investment for medium skilled labor; they do not afford higher education for their children.

Above literature provides persuasive explanations for the mechanisms that lead to persistent inequality in various economies and societies; the indivisibility of investment technology, the segmented society, the gap on health care, the difference of fertility, or hierarchic preference yield the difference of investment for human capital and, in result, perpetuating income inequality. In contrast, we present a model in which every individual has a same marginal propensity of human capital investment (education for children) by assuming homothetic preference. Moreover, the education technology is dividable, and labor and goods markets are perfect competitive. We present a difference mechanism which leads to persistent inequality from the production technologies with different elasticities of output to human capital, competitively job selection, and the adjustment of goods price and wage.

Our model is developed from Glomm and Ravikumar (1992) by introducing the construction of the classical Ricardian framework. Our overlapping generations model has two kinds of consumption goods and two production sectors. Technologies in each sector differ in the elasticity of output to human capital. There are two types of workers who are classified as high skilled and low skilled according to their endowments of human capital. While high skilled workers have an absolute advantage in both types of production, each type of worker has a comparative advantage in a different type of production. Hence, similar to the Ricardian model of international trade, three kinds of market equilibria are possible: a complete specialization case and two incomplete specialization cases. Since production technologies differ between sectors, the human capital accumulation functions vary across the household dynasties of each skill class.

As such the speed of human capital accumulation depends on the pattern of specialization. Thus, the transition of the gap between human capital endowments leads to changes in the pattern of specialization.

We show that this model leads to an equilibrium with persistent income inequality when complete specialization occurs in the steady state. High skilled workers have a comparative advantage in the more productive sector, and thus high skilled dynasties accumulate human capital at a greater rate than low skilled dynasties. This difference in accumulation rates leads to an increase in income inequality. In contrast, the rapid human capital accumulation of high skilled workers improves the productivity of the good in which they have a comparative advantage thereby increasing production and supply and reducing the price for this good. Because wage equals the market value of workers' marginal production in perfectly competitive markets, the human capital accumulation of high skilled workers reduces the high skilled wage. This effect softens income inequality. As a result, these two effects decide the intensity of income inequality, and persistent inequality occurs in the economy.

Economies in the real world have many kinds of goods and production sectors. We extend the two sector model to multi-sector framework and show that a perfectly competitive economy naturally leads to persistent inequality under natural parameter conditions on sector specific productivities. In not only the two sector model but also in the multi-sector model, the degree of income inequality is determined by the relative market sizes for the consumption goods, but is independent of productivities parameters. However, an increase in productivity leads to the expansion of income inequality in long run.

Finally, we consider three policies which have equalization effects. An income redistribution policy, which is a combination of a proportional income tax and an income transfer, changes the transition function of human capital inequality. We show that even if the income redistribution is very small, this policy prevents income inequality from exploding. Moreover, we consider two other policies: a purchase of goods by government and commodity taxes. We show that these have egalitarian effects through the demand-side of the good markets. These effects have over-looked in one sector models.

The remainder of this paper is organized as follows. Section 2 describes the basic set-up of the economy. Subsection 3.1 considers short-run equilibrium and the relationship between the human capital gap and income inequality. Subsection 3.2 shows long-run equilibrium and the existence of a parameter range that leads to persistent income inequality. In section 4, we consider egalitarian policies. Subsections

4.1 and 4.2 respectively show that purchases of goods by government and changes in the commodity tax rate have equalization effects. Section 5 provides the conclusions of this paper.

2. The Model

We consider an overlapping generations economy in which individuals live for two periods, young and old. Time is denoted as $t = 0,1,2,\cdots$. Each individual has a single child. While individuals have identical preferences, they are classified into two types, low skilled (l) and high skilled (h), based on their endowments of human capital at period $t = 0$. An individual's stock of human capital in period t is denoted as z_t^i ($i = l,h$) with the assumption that $z_0^l < z_0^h$. The economy is populated by a unitary mass of individuals, and the low and high skilled population shares are L and H, respectively, i.e., $L + H = 1$. This economy produces two goods, X and Y, in sectors X and Y, respectively. Imagine that good X is basic goods like food, while good Y is more sophisticated items like books or computers. We set good X as the model numeraire and denote the price of good Y as P_t. All workers are employed in one of the production sectors and supply their labor (human capital) inelastically. Firms enter freely into each sector; both the product and labor markets are perfectly competitive.

2.1 Technologies and the comparative advantage

In the X and Y sectors, goods are produced using only labor (human capital). The difference in human capital endowments for type l and h workers is not qualitative, but rather only reflects the amount of human capital. Both l and h type individuals can potentially work in either sector X or sector Y. The outputs of a worker with a human capital endowment of z and employed in sector X or sector Y are respectively given by the following technologies: $x_t = f_X(z_t^i)$ and $y_t = f_Y(z_t^i)$, $i = l,h$.[1] We assume that

[1] Because we consider the Ricardian framework, the production function for good X is a linear technology with respect to the number of employed workers. If employed number of type l and type h in sector X are respectively n_t^l and n_t^h, the total amount of good X produced is $X = f_X(z_t^l) \cdot n_t^l + f_X(z_t^h) \cdot n_t^h$. Thus, the labor required type i for

$$f_X(z_t^i) = A \cdot (z_t^i)^a, \quad f_Y(z_t^i) = B \cdot (z_t^i)^b, \tag{1}$$

where *a, b, A,* and *B* are positive constants. a and b represent the elasticities of output to human capital and we assume that the elasticity in sector *Y* is greater than in sector *X.* [2] We permit not only $0 < a < b < 1$ but also $0 < a < 1 < b$ and $1 < a < b$.

When $z_t^l < z_t^h$ holds, from (1) we have the following inequality:

$$\frac{f_X(z_t^h)}{f_Y(z_t^h)} < \frac{f_X(z_t^l)}{f_Y(z_t^l)}. \tag{2}$$

From the Ricardian theory of comparative advantage, this inequality implies that a type *l* (resp. *h*) worker has a comparative advantage in the production of good *X* (resp. *Y*) while a high skilled worker has an absolute advantage in both goods. Because of the perfectly competitive nature of product markets, three kinds of market equilibrium are possible, depending on the relative price of good *Y* to good *X*: (i) $P_t = \frac{f_X(z_t^h)}{f_Y(z_t^h)} < \frac{f_X(z_t^l)}{f_Y(z_t^l)}$, (ii) $\frac{f_X(z_t^h)}{f_Y(z_t^h)} < P_t < \frac{f_X(z_t^l)}{f_Y(z_t^l)}$, and (iii) $\frac{f_X(z_t^h)}{f_Y(z_t^h)} < \frac{f_X(z_t^l)}{f_Y(z_t^l)} = P_t$. The first case implies that low skilled workers specialize completely in the production of good *X* and high skilled workers engage in the production of both goods. In the second case type *l* (resp. type *h*) workers specialize completely in producing good *X* (resp. good *Y*). In the third case all type *h* workers specialize completely in the production of good *Y* and type *l* workers turn out both goods. We refer to these market equilibria respectively as *regimes (i), (ii), and (iii).* In regime (ii), each type of labor specializes completely according to its comparative advantage. On the other hand, incomplete specialization occurs in regimes (i) and (iii). We define the number of type *h* workers who are employed in sector *X* in regime (i) as $m_t^{h,X} \in (0, H)$. Similarly, the number of type *l* workers who are employed in the sector *Y* in regime (iii) is denoted as $m_t^{l,Y} \in (0, L)$.

2.2 Preferences, education, and demands for each good

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production of one unit of good *j*, $j = X, Y$, is $\frac{1}{f_j(z_t^i)}$.

[2] This assumption reflects that the higher human capitals of workers in the production of books, the greater outputs rather than foods.

Next, we consider the lifespan of households. In the young period, each individual obtains human capital through education, and all tuition fees are covered by funds from the individual's parents. For simplicity, we assume that individuals take no economic decisions in young periods. Only in their old periods, individuals decide their economic activity; they obtain employment with a firm in either sector *X* or sector *Y*. The income derived from this employment is used to consume goods X and *Y* and to bring up the child.

We assume that the bringing up a child with health bodies and minds requires sufficient nutrition and education in childhood.[3] We do not consider the borrowing and lending for education funds. Developing the framework of Glomm and Ravikumar (1992), the human capital of the *t+1* generation, z_{t+1}^i, is formed according to the following function;

$$z_{t+1}^i = \theta (e_{x,t}^i)^{\beta} (e_{y,t}^i)^{1-\beta} (z_t^i)^{\gamma}, \tag{3}$$

where θ is a positive constant and $\beta, \gamma \in (0,1)$. $e_{x,t}^i$ and $e_{y,t}^i$ are consumptions of goods *X* and *Y* in childhood and their parents buy them for the child. z_t^i captures the positive externality from the parent's human capital.

Individuals obtain the utility from the consumptions in the old period and the altruism for their children. All individuals have the identical preference as follows: $U = \delta \log c_{x,t} + \eta \log c_{y,t} + \sigma \log z_{t+1}$, where $\delta, \eta, \sigma \in (0,1)$, and $\delta + \eta + \sigma = 1$. $c_{x,t}$ and $c_{y,t}$ are, respectively, consumptions of good *X* and *Y* in the old period. The third term captures the parents' utility which they feel from their children grows. Substituting (3) into this preference, we have

$$U = \delta \log c_{x,t} + \eta \log c_{y,t} + \sigma\beta \log e_{x,t} + \sigma(1-\beta) \log e_{y,t} + \sigma\gamma \log z_t + \log\theta. \tag{4}$$

Individuals supply their human capital inelastically in the old period, and all workers with the same stock of human capital earn the same wage, denoted by $w_t^i = w(z_t^i)$, due to the competitive nature of the labor market. Since individuals manage for

[3] Most of studies, including Glomm and Ravikumar, assume one kind of consumption good. In contrast, our model has two kinds of goods, so we assume equation (3).

consumptions of themselves in the old period and their children, the budget constraint is

$$w_t^i = c_{x,t}^i + P_t c_{y,t}^i + e_{x,t}^i + P_t e_{y,t}^i . \tag{5}$$

(4) and (5) yield each demand function as follows;

$$c_{x,t}^i = \delta w_t^i ,\ c_{y,t} = \frac{\eta w_t^i}{P_t}\ e_{x,t} = \sigma\beta w_t^i \text{, and } \ e_{y,t} = \frac{\sigma(1-\beta)w_t^i}{P_t} \tag{6}$$

Because the populations of type *l* and type *h* workers are respectively *L* and *H*, the national income in period *t* is $I_t = L \cdot w_t^l + H \cdot w_t^h$, the total demands for good *X* and *Y* are

$$X_t^D = (\delta + \sigma\beta)I_t ,\ \text{ and } \ Y_t^D = \frac{(\eta + \sigma(1-\beta))I_t}{P_t} . \tag{7}$$

3. Market Equilibrium

3.1 Short-run equilibrium

We begin our analysis by focusing on only one period (we omit the time subscript *t* in this section). From (7), the market equilibrium conditions for good *X* and good *Y* in regimes (i), (ii) and (iii) are, respectively,

$$\begin{cases} (\delta + \sigma\beta)I = f_X(z^l)\cdot L + f_X(z^h)\cdot m^{h,X} \\ \dfrac{(\eta + \sigma(1-\beta))I}{P^{(i)}} = f_Y(z^h)\cdot(H - m^{h,X}) \end{cases}, \tag{8}$$

$$\begin{cases} (\delta + \sigma\beta)I = f_X(z^l)\cdot L \\ \dfrac{(\eta + \sigma(1-\beta))I}{P^{(ii)}} = f_Y(z^h)\cdot H \end{cases}, \tag{9}$$

$$\begin{cases} (\delta + \sigma\beta)I = f_X(z^l)\cdot(L - m^{l,Y}) \\ \dfrac{(\delta + \sigma(1-\beta))I}{P^{(iii)}} = f_Y(z^l)\cdot m^{l,Y} + f_Y(z^h)\cdot H \end{cases}, \tag{10}$$

where the superscript on the price of good *Y* denotes the type of regime.

As both the product and labor markets are perfectly competitive, firms' profits must be zero in every sector in accordance with the free entry condition. In regime (i) there are three kinds of firms: firms in sector *X* that employ workers of type *l*, firms in sector *X* that employ workers of type *h*, and firms in sector *Y* that employ workers of type *h*. The profits of these firms are, respectively, $\pi_X = f_X(z^l) - w_t^{l,(i)} = 0$,

$\pi_X = f_X(z^h) - w_t^{h,(i)} = 0$, and $\pi_Y = P^{(i)} \cdot f_Y(z^h) - w_t^{h,(i)} = 0$. In equilibrium, all high skilled workers must receive the same wage as they are employed in both sectors. Therefore, the wage rates in regime (i) are

$$\begin{cases} w^{l,(i)} = f_X(z^l) \\ w^{h,(i)} = f_X(z^h) = P^{(i)} \cdot f_Y(z^h) \end{cases}. \tag{11}$$

In the same manner, the wage rates in regimes (ii) and (iii) are, respectively,

$$\begin{cases} w^{l,(ii)} = f_X(z^l) \\ w^{h,(ii)} = P^{(ii)} \cdot f_Y(z^h) \end{cases}, \tag{12}$$

$$\begin{cases} w^{l,(iii)} = f_X(z^l) = P^{(iii)} \cdot f_Y(z^l) \\ w^{h,(iii)} = P^{(iii)} \cdot f_Y(z^h) \end{cases}. \tag{13}$$

These equations imply that workers' wages must equal the values of their products. The second equation of (11) and the first equation of (13), combined with (1), respectively determine the relative price of good Y in regimes (i) and (iii):

$$P^{(i)} = \frac{f_X(z^h)}{f_Y(z^h)} = \frac{A}{B}(z^h)^{a-b}, \quad P^{(iii)} = \frac{f_X(z^l)}{f_Y(z^l)} = \frac{A}{B}(z^l)^{a-b}. \tag{14}$$

Because $z^l < z^h$ and $a < b$, we have $P^{(i)} < P^{(iii)}$. Moreover, (9) combined with (1) yields

$$P^{(ii)} = \Phi \frac{A(z^l)^a}{B(z^h)^b}, \text{ with } \Phi \equiv \frac{\eta + \sigma(1-\beta)}{\delta + \sigma\beta} \frac{L}{H}. \tag{15}$$

Note that, because of $\delta + \eta + \sigma = 1$ and $L + H = 1$, a small β and η implies a large demand for good Y compared with the demand for good X. And a small H implies a small supply of good Y in regime (ii). Thus, Φ represents the relative scarcity of good Y. Throughout this paper, we assume that $\Phi > 1$. Furthermore, we define the human capital gap as

$$\lambda = \frac{z^h}{z^l}. \tag{16}$$

Using λ, (8) and (14) yield $m_t^{h,x} = (\delta + \sigma\beta)H - \dfrac{(\eta + \sigma(1-\beta))L}{\lambda^a}$ and (10) and (14) yield

$m_t^{l,Y} = (\eta + \sigma(1-\beta))L - (\delta + \sigma\beta)H\lambda^b$.

Figure 1 illustrates the supply function for good Y. If this supply function intersects the demand function (7) for good Y within the range of the lower horizontal line, regime

(i) holds. The vertical line corresponds to regime (ii) and the upper horizontal line represents regime (iii). The following proposition claims that the regime is determined by the relationship between the human capital ratio and the relative scarcity of good *Y*.

Proposition 1

If $\Phi^{a^{-1}} < \lambda$, regime (i) holds. If $\Phi^{b^{-1}} \leq \lambda \leq \Phi^{a^{-1}}$, regime (ii) holds. If $\lambda < \Phi^{b^{-1}}$, regime (iii) holds. That is, $\lambda^{(i)} > \lambda^{(ii)} > \lambda^{(iii)}$ holds.
Proof: See appendix 1.

The type of regime that occurs in market equilibrium depends on the relationship between the relative productivity of each sector and the relative expenditure on each good. Proposition 1 states that regime (i) (resp. regime (iii)) tends to hold when the demand for good *X* (resp. *Y*) is large, the population share of type *h* (resp. type *l*) workers is large, and/or the economy has a large (resp. small) human capital gap between type *l* and *h* workers; these conditions lead to a low relative price for good *Y* and regime (i) occurs.

Next we compare the degree of income inequality among regimes. (11)-(15) and (1) yield the equilibrium wage rates for each regime as follows:

$$\begin{cases} w^{l,(i)} = A\cdot(z^l)^a \\ w^{h,(i)} = A\cdot(z^h)^a \end{cases}, \quad \begin{cases} w^{l,(ii)} = A\cdot(z^l)^a \\ w^{h,(ii)} = \Phi A\cdot(z^l)^a \end{cases}, \quad \begin{cases} w^{l,(iii)} = A\cdot(z^l)^a \\ w^{h,(iii)} = A\lambda^b(z^l)^a \end{cases}. \tag{17}$$

We define a measure of income inequality by considering the ratio of wage income for type *h* and *l* workers: $\Lambda \equiv \dfrac{w(z^h)}{w(z^l)}$. From (17) and proposition 1, we have

$$\Lambda = \begin{cases} \lambda^a, & if\ \Phi^{a^{-1}} < \lambda \\ \Phi, & if\ \Phi^{b^{-1}} < \lambda < \Phi^{a^{-1}} \\ \lambda^b, & if\ \lambda < \Phi^{b^{-1}} \end{cases}. \tag{18}$$

Now we compare the intensity of income inequality between each regime. Since regime (i) (resp. regime (iii)) is valid when $\Phi^{a^{-1}} < \lambda^{(i)}$ (resp. $\lambda^{(iii)} < \Phi^{b^{-1}}$), we have $(\lambda^{(iii)})^b < \Phi < (\lambda^{(i)})^a$ although $a < b$. From (18), we have the magnitude correlation of Λ in each regime.

Proposition 2

$$\Lambda^{(i)} > \Lambda^{(ii)} > \Lambda^{(iii)}.$$

Proposition 2 indicates the order of the intensity of inequality. Because regime (ii) is the complete specialization case, national income is allocated to each class of workers according to the expenditure ratio for each good. From (7), the low skilled class receives $(\delta+\sigma\alpha)I$ as their wages and then per capita income is $\dfrac{(\delta+\sigma\beta)I}{L}$. As same way, per capita income for high skilled workers is $\dfrac{(\eta+\sigma(1-\beta))I}{H}$. Thus, the income inequality is Φ. In regime (i), because some high skilled workers engage in the production of good X, the share of national income earned by high skilled workers is greater in regime (i) than in regime (ii).

3.2. Long-run equilibrium

In this subsection, we analyze the transition of human capital and income inequality. As the education expenditure for children is funded from their parents' income and the wage distribution is affected by each skilled worker's endowment of human capital, the parent generation's level of human capital influences the human capital accumulation of their children. Substituting (6) and (15) into (3), we have the following equations for the transitions of human capital in regimes (i)-(iii):

$$\begin{cases} z_{t+1}^{l\ (i)} = \theta\sigma\beta^{\beta}(1-\beta)^{1-\beta}A^{\beta}B^{1-\beta}(z_t^h)^{(1-\beta)(b-a)}(z_t^l)^{a+\gamma} \\ z_{t+1}^{h\ (i)} = \theta\sigma\beta^{\beta}(1-\beta)^{1-\beta}A^{\beta}B^{1-\beta}(z_t^h)^{a+(1-\beta)(b-a)+\gamma} \end{cases}, \tag{20}$$

$$\begin{cases} z_{t+1}^{l\ (ii)} = \theta\sigma\beta^{\alpha}(1-\beta)^{1-\beta}A^{\beta}B^{1-\beta}\Phi^{-(1-\beta)}(z_t^h)^{(1-\beta)b}(z_t^l)^{a\beta+\gamma} \\ z_{t+1}^{h\ (ii)} = \theta\sigma\beta^{\beta}(1-\beta)^{1-\beta}A^{\beta}B^{1-\beta}\Phi^{\beta}(z_t^h)^{(1-\beta)b}(z_t^l)^{a\beta} \end{cases}, \tag{21}$$

$$\begin{cases} z_{t+1}^{l\ (iii)} = \theta\sigma\beta^{\beta}(1-\beta)^{1-\beta}A^{\beta}B^{1-\beta}(z_t^l)^{a+(1-\beta)(b-a)+\gamma} \\ z_{t+1}^{h\ (iii)} = \theta\sigma\beta^{\beta}(1-\beta)^{1-\beta}A^{\beta}B^{1-\beta}(z_t^h)^{b+\gamma}(z_t^l)^{(a-b)\beta} \end{cases}. \tag{22}$$

Proposition 1 provides the ranges of parameters for each regime. Hence, (17)-(19) can be rewritten using λ as follows:

$$\lambda_{t+1} = \begin{cases} (\lambda_t)^{a+\gamma}, & if\ \Phi^{a^{-1}} < \lambda_t \\ \Phi(\lambda_t)^{\gamma}, & if\ \Phi^{b^{-1}} \le \lambda_t \le \Phi^{a^{-1}} \\ (\lambda_t)^{b+\gamma}, & if\ \lambda_t < \Phi^{b^{-1}} \end{cases}. \tag{23}$$

The dynamic behavior of (23) is classified into three cases according to the parameter ranges: $a+\gamma<b+\gamma<1$, $a+\gamma<1<b+\gamma$, and $1<a+\gamma<b+\gamma$, which are, respectively,

described by figures 2, 3, and 4.[4] In the case of $a+\gamma<b+\gamma<1$, figure 2 shows that even if the economy starts from regime (i) or (ii), it moves to regime (iii) within a finite number of periods and the human capital gap λ_t converges to one. In the case of $a+\gamma<1<b+\gamma$, figure 3 illustrates that regime (ii) occurs regardless of the regime in the first period. In this case, λ_t converges to $\Phi^{\frac{\delta}{1-\delta}}>1$; the human capital gap remains in the steady state. In the case of $1<a+\gamma<b+\gamma$, figure 4 shows that the economy moves to regime (i) within a finite number of periods even if the economy was in regime (ii) or (iii) in the first period. The results indicated by these figures are summarized as proposition 3.

Proposition 3

If $a+\gamma<b+\gamma<1$, λ_t convergences to one. If $a+\gamma<1<b+\gamma$, λ_t convergences to $\Phi^{\frac{\delta}{1-\sigma}}>1$. If $1<a+\gamma<b+\gamma$, λ_t increases over time.

Glomm and Ravikumar (1992) show that the shape of the accumulation function for human capital determines the transition of the human capital gap (and income inequality) in a one sector model; i.e., income inequality converges to one if the accumulation function is concave, and diverges if the accumulation function is convex. In contrast, if $a+\gamma<1<b+\gamma$ in our model, the dynamic behavior of our model is completely different from that of Glomm and Ravikumar (1992). Suppose the economy is in regime (iii), in which the transition of λ is convex and type 1 works for both sectors. According to (22) the speed of human capital accumulation for high skilled dynasties is faster than that for low skilled dynasties, and the human capital gap and income inequality expand. In this case, since the productivity of high skilled workers increases rapidly, the employment of low skilled workers in sector *Y* continues to decline over time. Hence, the economy moves into regime (ii) within a finite number of periods. In this regime, the human capital endowment of type h workers grows faster than that of type *l* workers. The improvement in the productivity of sector *Y* leads to a large increase in the output of good *Y*. But, since the demand for good *Y* does not grow at the same speed as the production of good *Y*, the price of good *Y* decreases by (14). Therefore, the human capital of high skilled workers no longer grows explosively; λ converges to a

[4] Figures 2-4 do not consider the cases where parameter values are determined by $a+\gamma=1$ and $b+\gamma=1$.

constant values of $\Phi^{\frac{1}{1-\gamma}}$. In this result, the adjustment of the relative price between good X and Y, which is not observed in one sector models, leads to the stability of the economy.[5]

Next, we focus on income inequality Λ_t. (23) and proposition 3 directly yield the following corollary.

Corollary 1

If $a\delta+\sigma<b\delta+\sigma<1$, Λ_t convergences to one. If $a+\gamma<1<b+\gamma$, Λ_t equals Φ after the period when the economy moves to regime (ii). If $1<a+\gamma<b+\gamma$, Λ_t increases over time.

In one sector models with single kind of good, the value of human capital coincides with wage income. In our two sector model, in contrast, the human capital gap does not necessary correspond to income inequality. In particular, if the economy is in regime (ii), income inequality is independent of the gap of human capital and it is determined by the relative scarcity of good Y.

4. Egalitarian Policies

4.1 Purchase of goods by government

In this subsection, we show that government purchases of the low skill intensive good reduces income inequality. We assume that government purchases are financed by a proportional income tax as in the previous subsection. The government's budget is given by $\tau_w I_t = G_t^X + P_t G_t^Y$, where G_t^X and G_t^Y are, respectively, the amounts of goods purchased by the government. We set the expenditure ratio between good X and good Y as $\eta \equiv \frac{G_t^X}{P_t G_t^Y}$. Thus, the demands from the government for each good are

[5] Here we prove that human capitals z_t^i do not diverge in regime (ii). $a+\gamma<1<b+\gamma$ yields $\lambda=\frac{z^h}{z^l}=\Phi^{\frac{1}{1-\gamma}}$ in steady state. Substituting it into (21), we have

$z_{t+1}^h=\theta\sigma\alpha^\alpha\beta^\beta A^\alpha B^\beta\Phi^{\alpha\left(\frac{1+a-\gamma}{1-\gamma}\right)}(z_t^h)^{a\alpha+b\beta+\gamma}$. $a+\gamma<1$ implies $a\alpha+b\beta+\gamma<1$. Therefore, z^h does not explode.

$$G_t^X = \frac{\eta}{1+\eta}\tau_w I_t, \quad G_t^Y = \frac{1}{1+\eta}\tau_w \frac{I_t}{P_t}. \tag{33}$$

Therefore, the total demands from households and the government are

$$\overline{X}_t^D = \overline{\alpha} \cdot I_t, \quad \overline{Y}_t^D = \overline{\beta} \cdot \frac{I_t}{P_t}, \tag{34}$$

where $\overline{\alpha} \equiv (1-\tau_w)(\alpha+\gamma) + \frac{\eta}{1+\eta}\tau_w$ and $\overline{\beta} \equiv (1-\tau_w)\beta + \frac{1}{1+\eta}\tau_w$. $\overline{\alpha}$ and $\overline{\beta}$ imply the sums of expenditure shares from household and the government. Note that $\overline{\alpha} + \overline{\beta} = 1$, $\frac{\partial \overline{\alpha}}{\partial \eta} > 0$, and $\frac{\partial \overline{\beta}}{\partial \eta} < 0$. Moreover, we set $\overline{\Phi} \equiv \frac{\overline{\alpha}}{\overline{\beta}}\frac{L}{H}$ and then we have $\frac{\partial \overline{\Phi}}{\partial \eta} < 0$. The total supply is same as in figure 1. Therefore, proposition 1 yields the ranges for each regime, i.e., regime (i) holds if $\overline{\Phi}^{a^{-1}} < \lambda$, regime (ii) holds if $\overline{\Phi}^{b^{-1}} \le \lambda \le \overline{\Phi}^{a^{-1}}$, and regime (iii) holds if $\lambda < \overline{\Phi}^{b^{-1}}$. From $\frac{\partial \overline{\Phi}}{\partial \eta} < 0$, increasing η expands regime (i) and contracts regime (iii). And, because we have $\frac{\partial(\overline{\Phi}^{a^{-1}-b^{-1}})}{\partial \eta} = \left(\frac{1}{a} - \frac{1}{b}\right)\overline{\Phi}^{a^{-1}-b^{-1}-1}\frac{\partial \overline{\Phi}}{\partial \eta} < 0$, increasing η reduces the range of regime (ii).

From (15), we obtain the degree of income inequality as follows:

$$\Lambda_t = \begin{cases} \lambda_t^{\,a}, & if\ \overline{\Phi}^{a^{-1}} < \lambda_t \\ \overline{\Phi}, & if\ \overline{\Phi}^{b^{-1}} < \lambda_t < \overline{\Phi}^{a^{-1}} \\ \lambda_t^{\,b}, & if\ \lambda_t < \overline{\Phi}^{b^{-1}} \end{cases}. \tag{35}$$

Thus, an increase in government purchases of good X only correct income inequality in regime (ii).

From (17)-(19), the transition of the human capital gap is similar to (20) with the exception of the ranges for each regime, i.e.,

$$\lambda_{t+1} = \begin{cases} (\lambda_t)^{a\delta+\sigma}, & if\ \overline{\Phi}^{a^{-1}} < \lambda_t \\ \overline{\Phi}^{\delta} \cdot (\lambda_t)^{\sigma}, & if\ \overline{\Phi}^{b^{-1}} \le \lambda_t \le \overline{\Phi}^{a^{-1}} \\ (\lambda_t)^{b\delta+\sigma}, & if\ \lambda_t < \overline{\Phi}^{b^{-1}} \end{cases}. \tag{36}$$

Figure 6 illustrates (36) and the effect of increasing η when $a\delta + \sigma < 1 < b\delta + \sigma$ holds. Persistent inequality occurs in regime (ii) alone. Therefore, the purchase of good X by the government corrects the degree of persistent income inequality.

Results from the purchase of goods by the government are summarized in the following proposition.

Proposition 4

If the government increases its purchases of good *X*, regime (i) expands and regimes (ii) and (iii) contract. If the economy is regime (ii), the policy reduces income inequality.

Government purchases of goods influence the demands for goods although this policy does not affect the supply-side of the economy. In regimes (i) and (iii), production technologies and human capital endowments, which are decided in the previous period, determine the prices of goods as (13). In other words, when incomplete specialization occurs, the supply side determines the income inequality as (16). Thus, government purchases have no effect on wages and the degree of inequality. In contrast, in the case of complete specialization, the total demand for each good determines the relative price of goods and income inequality. Therefore, government purchases affect the degree of income inequality through a change of the relative prices of goods. In other words, if the government purchases good *X*, in which low skilled labor has a comparative advantage, the increase in good *X*'s price raises the value of low skilled labor. Because this egalitarian effect comes from a change in the relative price, it does not appear in one sector models.

Government expenditure is usually directed towards public works; this government activity should affect relative prices. For example, the main part of government expenditure designed to stimulate the economy is usual public construction. Therefore, proposition 8 implies that if the technology of construction industry is low skilled labor intensive, the fiscal policy has an egalitarian effect.

4.2 Commodity taxes

In this subsection, we consider government imposed commodity taxes and define the tax rates on good *X* and good *Y*, respectively, as τ_X and τ_Y. Assume that the government requires a financial resource G and spends the tax revenue.[6] The budget of the government is

$$G = \tau_X \cdot (X_t + E_t) + \tau_Y P_t Y_t . \tag{37}$$

The prices of each good that households face are, respectively, $(1+\tau_X)$ and $(1+\tau_Y)P_t$.

6 Suppose that the financial resource is, for example, a transfer for a foreign country. As we examined in subsections 5.1 and 5.2, both a transfer policy for households and government purchases work as egalitarian policies. In order to focus on the effect of commodity taxes, we assume that the government's financial revenue from the commodity taxes disappears from our model.

Thus, from (3) and (4), the total demands for good X, good Y, and total expenditure for education are, respectively,

$$X_t^D = \frac{\alpha}{1+\tau_X} I_t, \; Y_t^D = \frac{\beta}{1+\tau_Y} \frac{I_t}{P_t}, \; E_t^D = \frac{\gamma}{1+\tau_X} I_t. \tag{38}$$

The supplies of both goods are same as in subsection 3.1. We define $\tilde{\Phi} = \frac{1+\tau_X}{1+\tau_Y} \frac{\beta}{\alpha+\gamma} \frac{L}{H}$.

By replacing Φ with $\tilde{\Phi}$ in subsection 3.1, we have a characterization of the short-run equilibrium. From a comparison of $\tilde{\Phi}$ with $\overline{\Phi}$ in subsection 5.2, we find that decreasing τ_X and the increasing τ_Y has an effect that is identical to an increase in η. Therefore, from proposition 8, we have the following corollary.

Corollary 2

If the government decreases the tax rate of good X and increases the tax rate of good Y, regime (i) expands and regimes (ii) and (iii) contract. If the economy is in regime (ii), the policy reduces income inequality.

Because commodity taxes change the relative price of goods, a reduction of the tax rate for good X has an egalitarian effect by the same mechanism as government purchases of good X. Under the value added tax (VAT) in the EU, several goods are non-taxable. For example, most foods are non-taxable because they are regarded as necessities. Thus, VAT is usually considered to benefit low income households because they have a high Engel's coefficient. In contrast, our model has homothetic preferences; the Engel's coefficient is constant. However, corollary 3 claims that if the production of food is low skill intensive, decreasing the tax rate on food raises the wages of low skilled workers. Therefore, we conclude that VAT corrects income inequality through a change in the relative price on good markets.

5. Conclusions

In a simple model with two consumption goods, two production sectors, and two skill classes, we have studied persistent income inequality that stems from comparative advantages for the skill classes in the labor market. Our model has three possible market regimes. In the complete specialization case, the relative expenditure shares of the consumption goods determine the income shares of each skill. Furthermore, the relative population shares for each skill class also affect their respective income shares. Therefore, education expenditure and then the transition of human capital accumulation differ across skill classes. The accumulation function of human capital differs from regime to regime. If the accumulation function is concave in one regime and convex in the other regimes, the economy moves into the complete specialization regime. In this case, rapid growth of high skilled human capital pushes down the price of the good that is high skill intensive, the human capital gap and income inequality converge to constants, rather than exploding. As a result, persistent income inequality occurs in the economy.

The main point of persist inequality is that a large human capital gap does not necessarily correspond with a large income gap. Even when some workers have large human capital endowments and produce the good in which they have a comparative advantage, their large output is supplied to the market, and through competition the price of the good decreases. This price effect reduces income inequality and has been over-looked in one sector models. We conclude that complete specialization and the price mechanism lead to persistent income inequality.

We have extended the two sector model to the multi-sector framework. An economy with both high and low productivity sectors leads to complete specialization and then persistent inequality. This is a natural description of the real world. By analogical inference from our result, persistent inequality is universal even when markets are perfectly competitive and there are no frictions or no regulations. Moreover, we have concluded that an expansion of productivity leads to persistent inequality.

Our model does not have any distortions or frictions with transactions; the labor and product markets are both perfectly competitive, and workers chose their jobs freely based on their comparative advantages. However, natural parameter conditions for sector specific productivity in the multi-sector model lead to persistent inequality. High skilled workers have a comparative advantage in highly productive industries. Thus, job selection the human capital accumulation of children and income inequality is perpetual.

If inequality expands significantly, it is possible that the government may adopt some policy to promote income equalization. We have studied three egalitarian policies. An income redistribution policy through a combination of proportional income taxes and transfers has two equalization effects: a direct transfer of income in the short run and human capital accumulation in long run. However, because persistent inequality implies complete specialization, income inequality is determined only from the structure of the goods market and the population ratio between skill classes. Therefore, in a completely specialized economy, this policy only has a direct income transfer effect in one period.

Finally, we have shown that government purchases and commodity taxes have egalitarian effects through the demand-side of the good markets. In a completely specialized economy, if the government buys the low skill intensive good directly, the policy raises the low skilled wage. A reduction of the commodity tax on the low skill intensive good has the same effect for income equalization. These effects have been over-looked in one sector models.

Appendix 1: Proof of proposition 1.

The intersection of the demand curve (5) with the vertical supply curve $Y = H \cdot f_Y(z^h)$ is given by (14) as $P = \Phi \cdot \frac{A \cdot (z^l)^a}{B \cdot (z^h)^b}$. Since the demand curve is a monotonic decreasing function, we can find which regime holds by investigating the location of the intersection of the demand curve with the vertical line. We have that regime (ii) is valid if $P^{(i)} \leq P = \Phi \cdot \frac{A \cdot (z^l)^a}{B \cdot (z^h)^b} \leq P^{(iii)}$. Thus, we have three ranges of parameters: regime (i) holds if $\Phi \cdot \frac{A \cdot (z^l)^a}{B \cdot (z^h)^b} < P^{(i)} = \frac{A}{B}(z^h)^{a-b}$ from (13), regime (ii) holds if $P^{(i)} = \frac{A}{B}(z^h)^{a-b} \leq \Phi \cdot \frac{A \cdot (z^l)^a}{B \cdot (z^h)^b} \leq P^{(iii)} = \frac{A}{B}(z^l)^{a-b}$ from (14), regime (iii) holds if $P^{(iii)} = \frac{A}{B}(z^l)^{a-b} < \Phi \cdot \frac{A \cdot (z^l)^a}{B \cdot (z^h)^b}$ from (13). Rewriting these inequalities, we have proposition 1. Q.E.D.

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Figure 1: Supply function for market Y

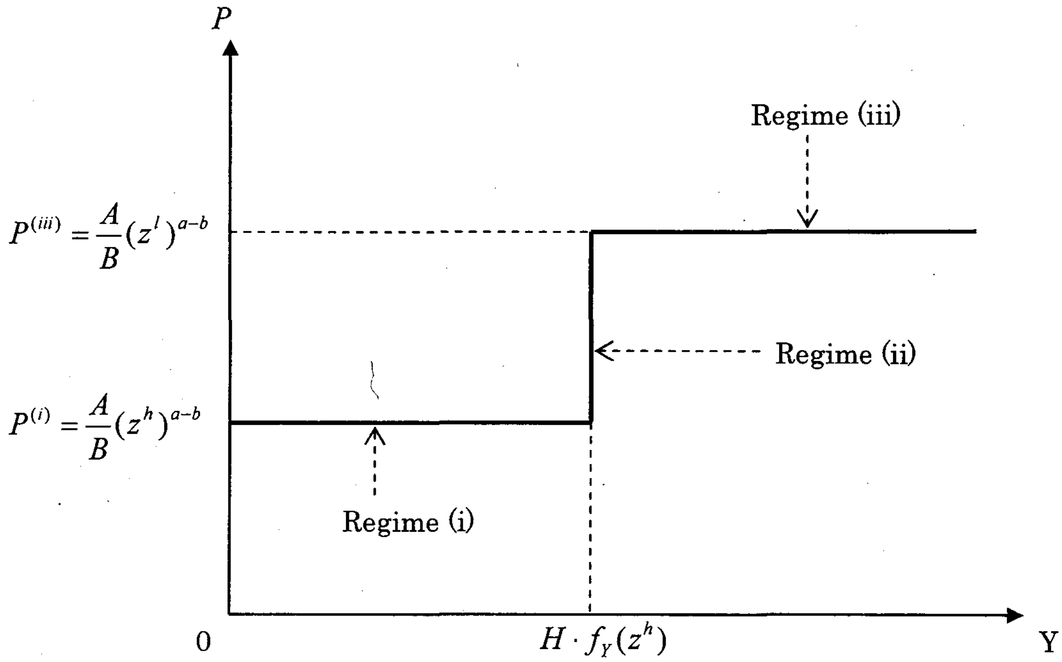

Figure 2: The transition of λ_t in the case of $a\delta+\sigma<b\delta+\sigma<1$

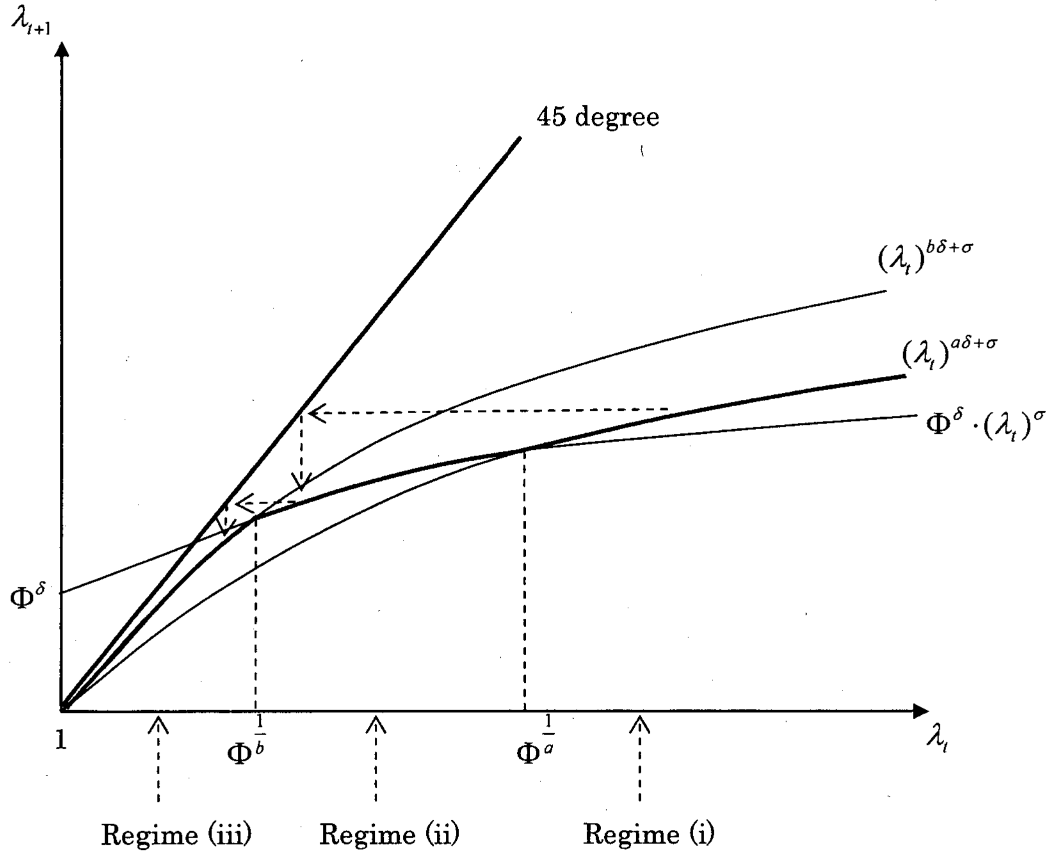

Figure 3: The transition of λ_t in the case of $a\delta+\sigma<1<b\delta+\sigma$

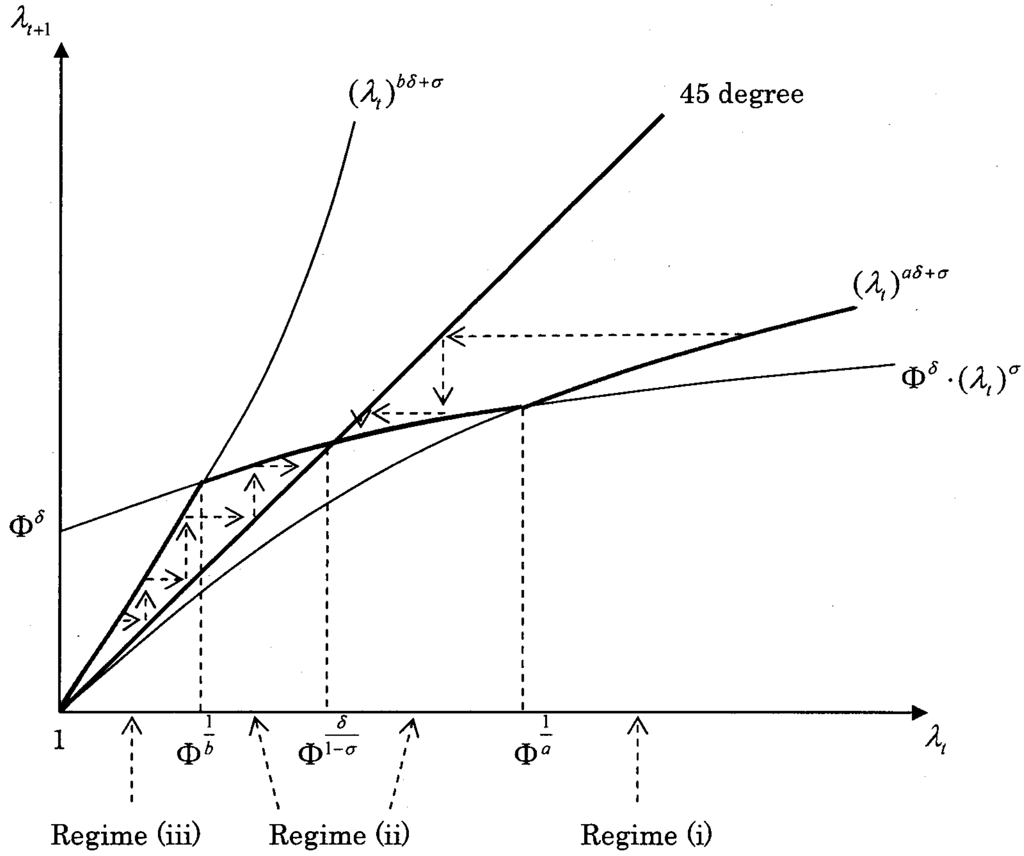

Figure 4: The transition of λ_t in the case of $1 < a\delta + \sigma < b\delta + \sigma$

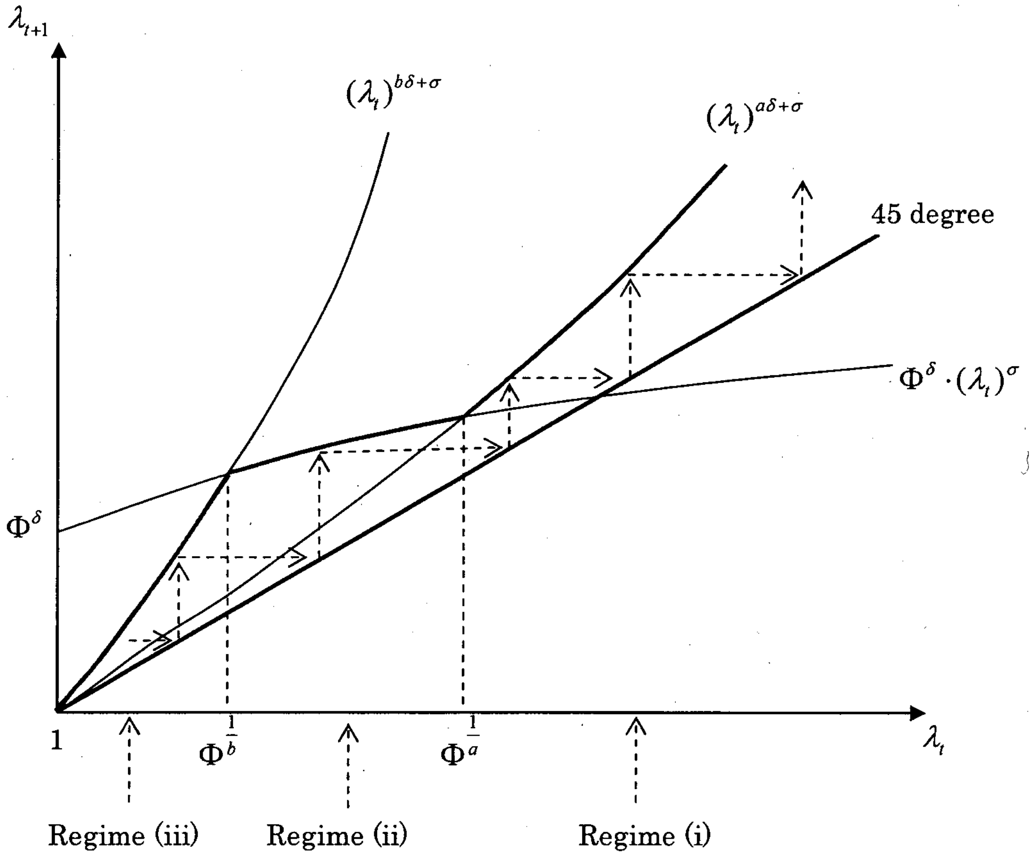

Figure 5: The effect of an increase in η