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Secured and Unsecured Loans in the Financing Activities of an Entrepreneur

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Abstract

This paper considers an entrepreneur who potentially sets up a monopolistic firm but faces the risk of a demand shock. The entrepreneur has two possible choices for financing: he can use the capital good component of his production as collateral for a low interest secured loan or he can obtain funds through an unsecured loan that does not require collateral but charges a high interest rate. Through his cost minimization problem, the choice of financial contracts determines the marginal costs of production and the inputs of factors of production. The entrepreneur's choice of financial loan, therefore, has a significant effect on the output market and may be detrimental to social welfare in some cases.

Keywords: Secured and Unsecured Loans; Borrowing Constraint; Collateral;
JEL Classifications: G21; G31; L12

1 Introduction

Entrepreneurs who do not have enough funds to set up a firm must raise funds from a financial institution or capital investor. Berger and Udell (1998) investigate the relationship between the financial growth cycle of firms and the resources available for financing. Their study finds that small and medium-size enterprises (SMEs), which have a potential for growth but a limited track record, obtain financing from private equity and debt markets (for example, angel finance and venture capital) or loans from financial institutions (banks) rather than from public markets (public equity and commercial paper markets). Why do some start-up firms raise funds from banks and others from venture capital? Financial transactions are usually associated with asymmetric information and uncertainty. In particular, asymmetric information is a serious problem for young firms that have a limited track record. It is well known that the existence of various types of financial contracts and transactions reduce problems stemming from asymmetric information between the borrower and lender. A number of studies have investigated the types of financing available to entrepreneurs and SMEs from the standpoint of the asymmetric information literature: Ueda (2004), Dessi (2005), Order (2006), Inderst et.al. (2007), Hvide and Leite (2008), and Winton and Yerramilli (2008).

In contrast, our paper considers different types of financial transactions with a focus on the risk management of financial institution; the essential difference between bank finance and venture capital, or angel finance, is the attitude toward risk. Banks usually avoid taking risk and typically demand collateral with a loan.[1] On the other hand, venture capitalists take a share of ownership in the SME through equity participation or capital subscription. Venture capitalists earn profit through the sale of equity after the SME has achieved success; venture capital is rewarded for taking risk. Focusing on the difference between bank finance and venture capital, our model explores two types of financial contract: a secured loan with guaranteed collateral and an unsecured loan.

In this paper, we consider how the existence of two types of financial constraint

[1] Contracts guaranteed with collateral have mainly been studied in the context of asymmetric information and incentive problems. It is well known that the inefficiency stemming from asymmetric information in financial transactions is improved by a self-selection mechanism through the choice of financial contract (Bester 1985). Collateral also reduces moral hazard by influencing the borrower's incentive (Boot et. al. 1991, Bester 1994 Berger and Udell 1998). In addition, the lender can save monitoring costs with contracts backed by collateral. For example, Rajan and Winston (1995) show that banks always demand collateral without monitoring.

affects the behavior of the entrepreneur as borrower of funds and the output market where the entrepreneur sells his product. If the bank demands collateral with a loan, the entrepreneur faces a borrowing constraint that limits the size of the loan to the value of the collateral. In many cases the asset that the entrepreneur provides as collateral is the equipment and infrastructure required for production, for example, real estate as building of factory or office, automobiles, and/or machinery. Suppose a standard production technology that requires capital goods and labor as inputs. Only capital goods can be used as collateral for a loan. If the entrepreneur expands the scale of production, he must increase the value of the collateral. This implies that increasing the input of capital goods alone leads to a distortion in factor inputs although a secured loan that is guaranteed with collateral generally demands a low interest rate. On the other hand, because venture capital takes a large risk when financing the entrepreneur's business, the promised rate of return for venture capital must reflect a risk premium. The high rate of return implies a large burden of interest payments for the entrepreneur. Through the cost minimization problem of the entrepreneur, the high interest rate pushes up the entrepreneur's marginal costs of production. Therefore, the choice of financial contract changes the cost function of the entrepreneur and should significantly influence the output market.

This paper assumes an entrepreneur who potentially sets up a monopolistic firm but faces the risk of a demand shock.[2] We study the effect of different financial contracts on the output market and social welfare through the behavior of the entrepreneur. We show that characteristic features for the product technology and the market, i.e., capital intensity, market size and risk of a demand shock, crucially affect the entrepreneur's choice of financial contract. We conclude that while an entrepreneur with a large market prefers an unsecured loan, and an entrepreneur who faces a large risk of a demand shock prefers a secured loan backed with collateral.[3] Moreover, we show that the entrepreneur's choice of financial loan may be undesirable for consumers.

The remainder of this paper is organized as follows. Section 2 describes the basic set-up of the economy. Section 3 derives cost functions for the entrepreneur under

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[2] Similar to our model, there exists a literature that analyzes the interaction between the financing activities of firms and imperfect competitive output markets, for instance, Brander and Lewis (1986), Fulghieri and Nagarajan (1992), and Glazer (1994). Chapter 7 of Tirol (2006) provides a survey of this literature.

[3] The letter result is consistent with Berger and Udell (1990)'s empirical finding and the result of Chen (2006) although this considers asymmetric information problem.

each type of financial contract. Section 4 shows equilibrium in the output market and the entrepreneur's optimal choice of financial contract, and considers social welfare. Section 5 summarizes the conclusions of this paper.

2 Basic Set-up

Consider an entrepreneur who has invented a new good and potentially sets up a monopolistic firm. When the entrepreneur starts his business, the market size is uncertain, he faces the risk of the demand shock, and because production takes time, he must set up his firm before the market size is determined. There are two factors of production, capital goods and labor. However, as production takes time and revenue is zero until the product is sold, the entrepreneur needs operating funds to hire the capital and labor required for production. While he has some personal funds, these are not sufficient to cover all operating costs. Thus, the entrepreneur must borrow funds from a financial institution. There are two types of loans. Under the first type, the entrepreneur uses capital goods as collateral for a low interest secured loan. Under the second type he obtain funds through an unsecured loan that does not require collateral but charges a higher interest rate than the secured loan. We respectively call the former loan type *contract C* and the latter *contract N*.

The inverse demand function of the output market is given as

$$P = A - x, \tag{1}$$

where P and x are respectively price and quantity. A represents market size and is a probability variable that reflects a demand shock; During production A becomes α with probability ρ or zero with probability $1-\rho$.

The entrepreneur has the following production technology:

$$x = k^{\sigma} l^{1-\sigma}, \tag{2}$$

where k and l, respectively, represent the capital good and labor. For simplicity, we assume that the value of the capital good is zero once production is finished. The operating funds required to obtain the capital good and employ labor are

$$C = k + wl, \tag{3}$$

where the capital good is set as the model numeraire and w is the wage.

The entrepreneur finances operating funds using personal funds and a loan obtained from a financial institution under either contract C or contract N. Therefore, we have $C = b_i + f$, where $b_i\,(i = C, N)$ denotes borrowed funds with the subscript indicating the type of financial contract, and f denotes personal funds.

From (3), the borrowing requirement for the entrepreneur is

$$b_i = k_i + wl_i - f \,. \tag{4}$$

There are number of identical financial institutions from which the entrepreneur can obtain a loan. Because of competition among financial institutions, the profit that financial institutions earn from loans is zero and financial institutions offer identical contracts in equilibrium. While contract N does not require any collateral for the loan, there is a collateral requirement for contract C that limits the value of funds the entrepreneur can borrow to the value of the collateral. We assume that before the firm is established the entrepreneur has no assets, other than his personal funds, that the financial institution will accept as collateral.[4] Of course, labor cannot be used as collateral, and therefore, if the entrepreneur chooses contract C, he must collateralize his capital goods. When the entrepreneur enters contract C, he faces the borrowing constraint:

$$b_C \le \delta_C k_C \,. \tag{5}$$

Note that capital good is the numeraire. $\delta_C \in (0,1]$ is the loan-to-value ratio, which will be determined by the depreciation of the capital good, and the creditor's cost of repossessing the collateral and selling the associated capital good.

The entrepreneur faces a demand shock, i.e., if the probability variable A equals zero, the revenue of the firm is zero even once production has been completed. When $A = 0$ occurs, the entrepreneur has no incentive to continue production, and as the entrepreneur pays wages before the market size A is known, the only asset the financial institution can claim is the capital good. Therefore, when the market size associated with a contract is zero, the financial institution forces the entrepreneur to cease production activity and seizes the capital goods, regardless of whether the financial contract is secured or not. In the case of contract C, because the financial institution acquires the collateral, the contract is risk-free for the financial institution. For simplicity, we assume that the interest rate of the risk-free asset is zero, so the interest rate on contract C equals zero, i.e., $r_C = 0$. Similarly, a financial institution that enters contract N can also seize the capital goods. Considering the cost of seizing and reselling the capital good, the collectable value is $\delta_N k_N$, where $\delta_N \in (0,1)$. Because contract N is not secured with collateral, however, the financial institution will incur additional legal costs over and above the costs associated with contract C. Therefore, we assume that

$$\delta_N < \delta_C \,. \tag{6}$$

4 Our results hold if we assume that the entrepreneur has another bankable asset. For detail, see footnote 6.

For simplicity, $\delta_C = 1$ is assumed for the remainder of the paper. In appendix A, the assumption of $\delta_C = 1$ does not lead to qualitative difference from the general case of $\delta_C \leq 1$.

With contract N, it is possible that the debt of the entrepreneur exceeds the value of collectable capital goods, i.e., $\delta_N b_N > k_N$. When A=0, the financial institution cannot collect the full principal of the loan and must incur a loss. In equilibrium, from the no-arbitrage condition, the expected return on contract N must equal the return on investment in a safe asset. Thus, the interest rate of contract N, r_N, is determined by the following no arbitrage condition:

$$\rho(1 + r_N)b_N + (1 - \rho)\delta_N k_N = b_N, \tag{7}$$

where the right-hand side (R.H.S) represents the return on a safe investment (the interest rate for safe assets is zero). The second term of left-hand side (L.H.S.) represents the expected value of the seized capital good when A=0. From (7), we have $\delta_N b_N > k_N \Leftrightarrow r_N > 0$; the interest rate of contract N is greater than that of the safe asset due to the risk premium.

For financial institutions, the difference between each type of contract is the method used to manage risk. When $A = 0$ is realized, the financial institution seizes the capital good, and for contract C, the financial institution manages default risk using collateral. For contract N, on the other hand, the financial institution incurs a loss with $A = 0$, but obtains high interest revenue with $A = \alpha$; the financial institution manages risk by charging a risk premium on the loan. In an effective financial market, the financial institution's expected profit from contract N is zero.

(Figure 1 is around here.)

Figure 1 describes the profits of the entrepreneur and the order of events. Before the productive activity, financial institutions offer an interest rate of r_N for contract N and the entrepreneur decides whether to enter into contract N or contract C. The entrepreneur then purchases capital goods and employs labor using operating funds that have been financed using personal funds f and a loan b_i. If $A = \alpha$ is realized, the entrepreneur obtains revenue $P \cdot x_i$ and repays the principal and interest, $(1 + r_i)b_i$, to the financial institution. After production, the value of the capital good is zero. In this case, the profit of the entrepreneur is $\pi_i = P \cdot x_i - (1 + r_i)b_i$. Alternatively, if $A = 0$, the output market disappears, the entrepreneur has profit of zero and is bankrupt. Therefore, the entrepreneur's expected profit is $E\pi_i = \rho \cdot \pi_i$. In order to guarantee an incentive for the entrepreneur to establish a firm, at least one of following participation constraints must be satisfied:

$$\rho\pi_N \geq f \quad \text{or} \quad \rho\pi_C \geq f. \tag{8}$$

3 Cost Functions under Financial Contracts

We consider the cost minimization problem of the entrepreneur for each type of financial contract. As production costs are financed using borrowed and personal funds, the entrepreneur's total cost TC_i equals the repayment of the principal and interest. From (4), we have

$$TC_i = (1+r_i)b_i = (1+r_i)k_i + (1+r_i)wl_i - (1+r_i)f. \tag{9}$$

If the entrepreneur has a small amount of personal funds, he must borrow a large amount of funds from a financial institution, which leads to large interest payments. The third term of the R.H.S. represents the opportunity benefit of personal funds, which is the benefit from reduced interest payment due to the use of personal funds.

First, we consider contract C. The entrepreneur faces the borrowing constraint (5) and using (4), and $\delta_C = 1$, this borrowing constraint can be rewritten as

$$w \cdot l_C \leq f. \tag{10}$$

When the borrowing constraint is binding, (10) implies that all wages are paid from the entrepreneur's personal funds. In other words, all capital goods are purchased using a loan from a financial institution and then the capital good is pledged as collateral.[5] [6] Under the borrowing constraint (10) and the technology (2), the entrepreneur minimizes (9). Given that $r_C = 0$, the cost minimization problem is

$$\begin{aligned} &\min_{k_C, l_C}\{k_C + wl_C - f\} \\ &s.t. \quad x_C = k_C^{\sigma} l_C^{1-\sigma}. \\ &\qquad\ w \cdot l_C \leq f \end{aligned}$$

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5 For example, suppose that the entrepreneur purchases machines and infrastructure for his factory or office using a mortgage from a bank. Of cause, this property comes from the assumption of $\delta_C = 1$. In the general case of $\delta_C \in (0,1)$, the rewritten borrowing constraint is given by (A.1) in appendix A.

6 As mentioned in footnote 4, suppose the entrepreneur has a bankable asset, for example real estate as his house. Define the value of the additional asset as h. In this case, the participation constraints (8) are rewritten as $\rho\pi_C \geq f + h$, (5) is replaced by $b_C \leq \delta_C k_C + h$, and (10) becomes $w \cdot l_C \leq f + h$. Because these changes are the same as an increase in f, they do not qualitatively affect our results.

For the entrepreneur with has large amount of personal funds f, the borrowing constraint (10) is not binding, and the cost function is

$$TC=\sigma^{-\sigma}(1-\sigma)^{-(1-\sigma)}w^{1-\sigma}x-f. \tag{11}$$

From the cost minimization problem, we derive the condition under which the borrowing constraint (10) binds as follows;

$$x>\tilde{x}_C,\ \text{with}\ \ \tilde{x}_C\equiv\left(\frac{\sigma}{1-\sigma}\right)w^{-(1-\sigma)}f. \tag{12}$$

This condition implies that an entrepreneur who plans for a large production scale relative to his available personal funds faces a borrowing constraint due to a large loan requirement. If (12) is satisfied, the capital good and labor input are determined by two constraints (2) and (10), and the total cost function under contract C is

$$TC_C=b_C=\left(\frac{w}{f}\right)^{\frac{1-\sigma}{\sigma}}(x_C)^{\frac{1}{\sigma}},\ \text{if}\ \ \tilde{x}_C<x. \tag{13}$$

From (13), the marginal cost under contract C is an increasing function due to $\sigma<1$. From the borrowing constraint (10), the entrepreneur who chooses contract C pays wages from his personal funds. Thus, in order to expand the level of production, he must increase the input capital goods because the labor input is fixed. That is, the expansion of production leads to a distortion in factor inputs.

Next, we consider contract N. When the entrepreneur chooses contract N with interest rate r_N, the cost minimization problem is as follows:

$$\min_{k_N,l_N}\{(1+r_N)k_N+(1+r_N)wl_N-(1+r_N)f\}$$
$$s.t.\quad x_N=k_N{}^{\sigma}l_N{}^{1-\sigma}$$

From (7), $r_N\geq 0$ holds if and only if $b_N\geq\delta_N k_N$. From (4), the condition is rewritten as

$$(1-\delta_N)k_N+wl_N\geq f. \tag{14}$$

If inequality (14) is violated, the entrepreneur minimizes $TC_N=k+wl-f$ subject to (2). In this case, the cost function is same as (11) because $r_N=0$. Factor demand functions for capital goods and labor are respectively $k=\sigma^{1-\sigma}(1-\sigma)^{-(1-\sigma)}w^{1-\sigma}x$ and $l=\sigma^{-\sigma}(1-\sigma)^{\sigma}w^{-\sigma}x$. Substituting these factor demand functions into (14), we have

$$x\geq\tilde{x}_N\equiv\sigma^{\sigma}(1-\sigma)^{1-\sigma}(1-\sigma\delta_N)^{-1}w^{-(1-\sigma)}f. \tag{15}$$

Note that $\tilde{x}_N<\tilde{x}_C$. If (15) holds, i.e., $r_N\geq 0$, the no arbitrage condition (7) is rewritten as $(1+r_N)=\rho^{-1}\left[1-(1-\rho)\delta_N k_N b_N^{-1}\right]$. Substituting this into (9), the total cost is rewritten as $TC_N=\rho^{-1}\left[\{1-(1-\rho)\delta_N\}k_N+wl_N-f\right]$. In the case, the cost minimization problem is given as follows;

$$\min_{k_N,l_N}\{(1-(1-\rho)\delta_N)\rho^{-1}k_N+w\rho^{-1}l_N-f\rho^{-1}\}$$
$$s.t.\quad x_N=k_N{}^{\sigma}l_N{}^{1-\sigma}$$

Therefore, the cost function for contract N is

$$TC_N=\frac{(1-(1-\rho)\delta_N)^{\sigma}w^{1-\sigma}}{\rho\sigma^{\sigma}(1-\sigma)^{1-\sigma}}x_N-\frac{f}{\rho},\ \text{if}\ \tilde{x}_N<x_N. \tag{16}$$

From (16), the marginal cost is constant as $\dfrac{(1-(1-\rho)\delta_N)^{\sigma}w^{1-\sigma}}{\rho\sigma^{\sigma}(1-\sigma)^{1-\sigma}}$ under the Cobb-Douglas technology.

Therefore, the entrepreneur who can choose the type of financial contract faces the following cost function.

Proposition 1

(13) and (16) indicate a threshold $\hat{x}$; $TC_C<TC_N$ if $\tilde{x}_N<x<\hat{x}$, while $TC_N<TC_C$ if $\hat{x}<x$. That is, the entrepreneur faces the following cost function:

$$TC=\begin{cases}\sigma^{-\sigma}(1-\sigma)^{-(1-\sigma)}w^{1-\sigma}x-f & if\quad x<\tilde{x}_C\\ (w/f)^{\frac{1-\sigma}{\sigma}}x_C{}^{\frac{1}{\sigma}} & ,\ if\quad \tilde{x}_C<x<\hat{x}.\\ \{1-(1-\rho)\delta_N\}^{\sigma}w^{1-\sigma}\rho^{-1}\sigma^{-\sigma}(1-\sigma)^{-(1-\sigma)}x_N-f\rho^{-1} & if\quad \hat{x}<x\end{cases} \tag{17}$$

Proof: See Appendix B.

(Figure 2 is around here.)

Figure 2 shows the cost functions (17). If the production level is smaller than $\tilde{x}_N$, a loan to the entrepreneur does not involve any risk for the financial institutions, the entrepreneur does not face a borrowing constraint and the interest rate is zero. Thus, the entrepreneur is indifferent between the type of financial contract and the cost function is given by (11). This case is not interesting as we would like to focus on the difference between the cost functions associated with each type of contract. Hence, we assume (12).

4 Equilibrium

In this section we derive the market equilibrium. First, given contracts N and C, we derive the output and expected profit of the entrepreneur. The entrepreneur chooses the contract that generates larger expected profit. If A=0, his firm goes bankrupt and his profit is zero. Therefore, from (1), the expected profit under each contract is

$$E\pi_i = \rho\{(\alpha - x_i)x_i - (1+r_i)b_i\} + (1-\rho)\times 0. \tag{18}$$

Now we define the tie-break rule. That is, if the expect profits from each contract are the same, we assume that the entrepreneur chooses contract C. Thus, he chooses contract N if and only if $E\pi_N(x_N^*) > E\pi_C(x_C^*)$, where x_i^* represents the equilibrium output of the firm. From (18), this inequality is equivalent to $\pi_N(x_N^*) > \pi_C(x_C^*)$, so, for simplicity, we consider the profit maximization problem using definite value rather than expected value.

We consider the entrepreneur's optimal output under each financial contract. From the inverse demand function (1), the marginal revenue is $MR = \alpha - 2x$. From (17), marginal costs under each contract are respectively $MC_N = \sigma^{-\sigma}(1-\sigma)^{-(1-\sigma)}(1+r_N)w^{1-\sigma}$ and $MC_C = \frac{1}{\sigma}(1+r_C)\left(\frac{w}{f}\right)^{\frac{1-\sigma}{\sigma}} x^{\frac{1-\sigma}{\sigma}}$. From the first order condition for profit maximization, $MR = MC$, the entrepreneur's optimal output $x_i^*(i = N, C)$ must satisfy the following conditions:[7]

$$\alpha - 2x_N^* = \frac{(1-(1-\rho)\delta_N)^\sigma w^{1-\sigma}}{\rho\sigma^\sigma(1-\sigma)^{1-\sigma}}, \tag{19}$$

$$\alpha - 2x_C^* = \frac{1}{\sigma}(1+r_C)\left(\frac{w}{f}\right)^{\frac{1-\sigma}{\sigma}} (x_C^*)^{\frac{1-\sigma}{\sigma}}. \tag{20}$$

From (19) and (20), we have the equilibrium outputs $x_N^* = x_N^*(\alpha,\sigma,w,\rho,\delta_N)$ and $x_C^* = x_C^*(\alpha,\sigma,w,f)$. In addition, from (19) and (20), we have the entrepreneur's profit functions as follows:

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7 The second order conditions for profit maximization are satisfied.

$$\pi_N^* = Px_N^* - TC_N = (x_N^*)^2 + \frac{f}{\rho}, \tag{21}$$

$$\pi_C^* = Px_C^* - TC_C = \left[x_C^* + \frac{1-\sigma}{\sigma}\left(\frac{w}{f}\right)^{\frac{1-\sigma}{\sigma}} (x_C^*)^{\frac{1-\sigma}{\sigma}} \right] x_C^* . \tag{22}$$

The entrepreneur compares the profits of (21) to (22) and chooses the financial contract which guarantee the largest expected profit.

Here, we analyze the properties of the optimal financial contract for the entrepreneur. Figure 3 and 4 provide these market equilibria. MC_N and MC_C have an intersection at

$$\bar{x} = \left(\frac{1}{\rho}\right)^{\frac{\sigma}{1-\sigma}} (1-(1-\rho)\delta_N)^{\frac{\sigma^2}{1-\sigma}} \left(\frac{\sigma}{1-\sigma}\right)^{\sigma} \left(\frac{1+r_N}{1+r_C}\right)^{\frac{\sigma}{1-\sigma}} w^{-(1-\sigma)} f . \tag{23}$$

MR is a negatively sloped straight line and shifts vertically with changes in α. We define $\bar{\alpha}$ as the market size for which *MR* passes through the intersection of MC_N and MC_C:

$$\bar{\alpha} = \frac{(1-(1-\rho)\delta_N)^{\sigma} w^{1-\sigma}}{\rho\sigma^{\sigma}(1-\sigma)^{1-\sigma}} + 2\left(\frac{1}{\rho}\right)^{\frac{\sigma}{1-\sigma}} (1-(1-\rho)\delta_N)^{\frac{\sigma^2}{1-\sigma}} \left(\frac{\sigma}{1-\sigma}\right)^{\sigma} \frac{f}{w^{1-\sigma}} . \tag{24}$$

Figures 3 and 4 respectively show the cases of $\alpha < \bar{\alpha}$ and $\alpha > \bar{\alpha}$. From these figures and (24), we have the following relationship between the optimal outputs x_i^* and the market size α.

(Figure 3 and 4 are around here.)

Proposition 2

If $\alpha < \bar{\alpha}$ ($\alpha > \bar{\alpha}$), $x_N^* < x_C^*$ ($x_N^* > x_C^*$) holds. Moreover, $\frac{\partial \bar{\alpha}}{\partial \rho} < 0$, $\frac{\partial \bar{\alpha}}{\partial \delta_N} > 0$, and $\frac{\partial \bar{\alpha}}{\partial f} > 0$.

The marginal cost function associated with contract N is horizontal while the marginal cost for contract C is upward-sloping. Thus, the optimal financial contract for the entrepreneur depends on the market size α. Moreover, a large risk of demand shock (small ρ), and large costs for the seizure and resale of the capital good in contract N (large δ_N) raise the interest rate of contract N, r_N. Thus, a decrease in ρ and an increase in δ_N push up the marginal cost of contract N, and then expand the region where the contract N is undesirable for the entrepreneur. On the other hand, a large amount of personal funds (large f) loosens the borrowing constraint and reduces the marginal cost of contract C.

From the inverse demand function (1), the consumer surplus when $A=0$ is given by $CS_i = \frac{1}{2}(x_i^*)^2$. Therefore, we have the following proposition.

Proposition 3
If $\alpha < \bar{\alpha}$ $(\alpha > \bar{\alpha})$, $CS_N < CS_C$ $(CS_N > CS_C)$ holds.

Under the given production technology (2), we cannot solve for the equilibrium output explicitly. But, Proposition 2 gives conditions that determine which type of financial contract is desirable for consumers in the monopolistic market. A large level of production leads to a large consumer surplus. Thus, the desirable financial contract for consumers depends on market size.

On the other hand, large output does not always lead to large profit. We define the difference between (21) and (22) as

$$G \equiv \pi_N^* - \pi_C^* = (x_N^*)^2 - \left[x_C^* + \frac{1-\sigma}{\sigma}(1+r_C)\left(\frac{w}{f}\right)^{\frac{1-\sigma}{\sigma}}(x_C^*)^{\frac{1-\sigma}{\sigma}} \right] x_C^* + \frac{f}{\rho}. \quad (25)$$

From (25) and Proposition 3, we have the following result.

Proposition 4
If $G > 0$ $(G \leq 0)$, the entrepreneur selects contract N (contract C). However, the choice of the entrepreneur is not always desirable for consumers.

From (25), the entrepreneur's choice does necessarily maximize the consumer surplus for the following two reasons: (i) the difference between the marginal cost functions and (ii) the different signs for the constant terms of the cost function (17). If the cost functions have no constant terms and both marginal cost functions are horizontal, then the profit function for each financial contract is $(x_i^*)^2$ $(i = C, N)$. In this case, the financial contract with higher profit leads to larger consumer surplus in the second best economy. But, this is not true in general because of the difference in the shapes of the cost functions. As the producer surplus equals total revenue minus variable cost, it can be rewritten as profit plus the constant term of the cost function.[8] That is, the difference between the first and second terms in (25) indicates the difference between the producer surpluses of contracts N and C. In Figures 3 and 4, the producer surplus of contract N is equal to area $dge_N\, p_N^*$. On the other hand, the producer surplus of contract C is the domain bounded by $Ohe_C\, p_C^*$. The magnitude of these areas depends on parameters. This is the first reason why the entrepreneur's choice of financial contract is not always desirable for consumers. Moreover, in contract N, there is the benefit from interest payments saved through the use of personal funds. This term is positive, and hence the entrepreneur tends to prefer contract N. This is the second reason why the entrepreneur's choice of financial contract is not always desirable for consumers.

Now, we define welfare. In our model, financial institutions earn zero expected profits due to perfect competition in the financial market. Therefore, the social welfare is composed of the consumer surplus and the profits of entrepreneur, i.e., $W_i = CS_i + \pi_i$. Moreover, we define the difference between the social welfare of each type of financial contract as

$$H \equiv W_N - W_C = \frac{1}{2}\left[(x_N^*) - (x_C^*)\right] + G. \tag{26}$$

Thus, if $H > 0$ $(H < 0)$ holds, contract N (contract C) is associated with greater social welfare in the monopolistic market.

We have done our analysis under unrestricted capital intensity σ. In our model, however, the first order condition (20) cannot be explicitly solved with general $\sigma \in (0,1)$. Here, we provide an example by assuming $\sigma = \frac{1}{3}$. In the case, from (19)

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[8] From (17) and (21), the producer surplus of contract N is
$PS_N = P \cdot x_N^* - \frac{(1-(1-\rho)\delta_N)^\sigma w^{1-\sigma}}{\rho\sigma^\sigma (1-\sigma)^{1-\sigma}} x_N = \pi_N^* - \frac{f}{\rho} = (x_N^*)^2$, which is the first term on the R.H.S. of (25). On the other hand, from (17) and (22), the producer surplus of contract C is

$$PS_C = P \cdot x_C^* - \left(\frac{w}{f}\right)^{\frac{1-\sigma}{\sigma}} (x_C^*)^{\frac{1}{\sigma}} = \pi_C^* = \left(x_C^* + \frac{1-\sigma}{\sigma}(1+r_C)\left(\frac{w}{f}\right)^{\frac{1-\sigma}{\sigma}} (x_C^*)^{\frac{1-\sigma}{\sigma}} \right)(x_C^*),$$ which

is the second term of (25).

and (20), we have equilibrium outputs of each financial contract $x_N^* = \frac{\alpha - 3\cdot 2^{-\frac{2}{3}}\left(1-(1-\rho)\delta_N\right)^{\frac{1}{3}} w^{\frac{2}{3}} \rho^{-1}}{2}$ and $x_C^* = \frac{-1+\sqrt{1+3(w/f)^2\alpha}}{3(w/f)^2}$. Substituting them into (25) and (26) respectively and substituting $\sigma = \frac{1}{3}$ into (24), we have figure 5, where other parameters are set as $w = f = 1$ and $\delta_N = 0.8$. Since (25) denotes the entrepreneur's choice of financial contract, the upper (lower) region of (25) represents contract N (contract C) occurs in equilibrium. The upper region of (24) and (26) are that contract N (contract C) is desirable for consumers and the society respectively. This figure implies that large risk from demand shock (small ρ) leads to the advantage of contract C because (24), (25), and (26) are downward-sloping. The reason is why high risk for lending requires large risk premium (large r_N) and then it leads to high marginal cost for production. On the other hand, if the entrepreneur increases the scale of production under contract C, the distortion on factor inputs expands. Therefore, the large market size (large α) increases the advantage of contract N. In addition, (25) does not coincide (24) and (26). This implies that the entrepreneur's decision of financial contract is not always desirable for consumers as mentioned in proposition 4. In the region bounded by (24) and (25), the entrepreneur's choice of financial contract is not desirable for consumers. Moreover, in the region bounded by (25) and (26), the entrepreneur's decision does not maximize the second best social welfare.

6 Summaries

We have analyzed secured and unsecured loans through the financing activities of an entrepreneur who faces the risk of a demand shock. If he chooses the secured loan by collateralizing the capital good component of his production, he enjoys low interest payment but faces the borrowing constraint. If he closes the unsecured contract, he can borrow larger funds, but he must bear high interest payment. Through his cost minimization problem, the choice of financial contracts changes the marginal costs of production. The entrepreneur's choice of financial loan, therefore, has a significant effect on the output market and the social welfare. Moreover, we show that the product technology, wage rate, frictions of financial transaction, size of output market, and risk from demand shock affect the entrepreneur's choice of financial contract. An entrepreneur entering in a large market prefers unsecured loan while he faced large risk selects secured loan. Finally, we show the case that the entrepreneur's choice of financial loan is detrimental to the social welfare.

Appendix

Appendix A: Cost Function with $\delta_C < 1$

In this appendix, we derive the cost function of contract C when $\delta_C < 1$. From (4) and (5), the borrowing constraint (10) is replaced to

$$(1-\delta_C)k_C + wl_C \le f \qquad \text{. (A.1)}$$

In the case, the cost minimization problem is following;

$$\min_{k_C, l_C}\{k_C + wl_C - f\}$$
$$s.t. \quad x_C = k_C^{\sigma} l_C^{1-\sigma} \quad ,$$
$$(1-\delta_C)k_C + w \cdot l_C \le f$$

which is illustrated by figure A. If the borrowing constraint (A.1) is not binding, from the production function (2), the total cost (9), and $r_C = 1$, the optimal capital-labor ratio is given by the ray OS; $\frac{k_C}{l_C} = \frac{\sigma}{1-\sigma} w$. At point S, $k = \frac{\sigma}{1-\delta\sigma} f$ and $l = \frac{1-\sigma}{1-\delta\sigma}\frac{f}{w}$ hold. Substituting them into (2), the threshold production level for binding borrowing constraint (A.1) is given as follows;

$$\tilde{x}_C \equiv \frac{\sigma^{\sigma}(1-\sigma)^{1-\sigma}}{w^{1-\sigma}} \frac{f}{1-\delta\sigma}. \qquad \text{(A.2)}$$

If $x \le \tilde{x}_C$, the cost function is same as (11). On the other hand, if $x > \tilde{x}_C$, the borrowing constraint is binding. The production is done on the range between S and T. Point T is the upper limit of production scale under contract C; the entrepreneur cannot achieve production scales more than $\hat{x}_C$ under contract C.

(Figure A is around here.)

Now, we consider the cost function if $\tilde{x}_C < x < \hat{x}_C$ while it cannot be derived explicitly. By totally differentiating (2) and (A.1), we have

$$\begin{bmatrix} \sigma k_C^{-(1-\sigma)} l_C^{1-\sigma} & (1-\sigma)k_C^{\sigma} l_C^{-\sigma} \\ 1-\delta_C & w \end{bmatrix} \begin{bmatrix} dk_C \\ dl_C \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} dx_C + \begin{bmatrix} 0 \\ 1 \end{bmatrix} df + \begin{bmatrix} 0 \\ k_C \end{bmatrix} d\delta_C. \qquad \text{(A.3)}$$

We define the determinant as Δ. On the range between S and T in figure A, we have $-\frac{dk_C}{dl_C}=\frac{x_{l_C}}{x_{k_C}}=\frac{\sigma}{(1-\sigma)}\frac{l_C}{k_C}<\frac{w}{1-\delta_C}$, which is the slope of (A.1). Therefore, we have

$$\Delta \equiv k_C^{-(1-\sigma)} l_C^{-\sigma}\left[\sigma w l_C-(1-\delta_C)(1-\sigma)k_C\right]>0. \tag{A.4}$$

From (A.3) and (A.4), we have $\frac{dk_C}{dx_C}=-\frac{w}{\Delta}>0$, $\frac{dl_C}{dx_C}=-\frac{1-\delta_C}{\Delta}>0$,

$\frac{dk_C}{df}=-\frac{(1-\sigma)k_C^{\sigma}l_C^{-\sigma}}{\Delta}<0$, $\frac{dl_C}{df}=-\frac{\sigma k_C^{-(1-\sigma)}l_C^{1-\sigma}}{\Delta}>0$, $\frac{dk_C}{d\delta_C}=-\frac{(1-\sigma)k_C^{1+\sigma}l_C^{-\sigma}}{\Delta}<0$,

and $\frac{dl_C}{d\delta_C}=-\frac{\sigma k_C^{\sigma}l_C^{1-\sigma}}{\Delta}>0$. Because $TC_C=k_C+wl_C-f$, we have marginal cost of contract C as

$$\frac{\partial(TC_C)}{\partial x_C}=\frac{dk_C}{dx_C}+w\frac{dl_C}{dx_C}=\frac{w\delta_C}{\Delta}>0. \tag{A.5}$$

Therefore, we can conclude that the assumption of $\delta_C=1$ in section 2 does not lead to qualitative difference result from the general case of $\delta_C\le 1$ except the existence of the upper limit of production scale $\hat{x}_C$. Here, we provide an example when $\sigma=\frac{1}{2}$. From (2), (A.1), and TC_C, if $\tilde{x}_C<x<\hat{x}_C$, the cost function is given by

$$TC_C=\frac{\delta_C f}{2(1-\delta_C)}-\sqrt{\left(\frac{f}{2(1-\delta_C)}\right)^2-\frac{w}{1-\delta_C}x^2}+\sqrt{\left(\frac{f}{2}\right)^2-(1-\delta_C)wx^2}-f. \tag{A.6}$$

Appendix B: Proof of Proposition 1

First of all, we show that (13) and (16) have an intersection of $\hat{x}$. We define the difference between (13) and (16) as

$$F(x)\equiv TC_N-TC_C=\frac{(1-(1-\rho)\delta_N)^{\sigma}w^{1-\sigma}}{\rho\sigma^{\sigma}(1-\sigma)^{1-\sigma}}x_N-\frac{f}{\rho}-\left(\frac{w}{f}\right)^{\frac{1-\sigma}{\sigma}}(x_C)^{\frac{1}{\sigma}}. \tag{B.1}$$

From (B.1), we have

$$F'(x)=\frac{(1-(1-\rho)\delta_N)^{\sigma}w^{1-\sigma}}{\rho\sigma^{\sigma}(1-\sigma)^{1-\sigma}}-\frac{1}{\sigma}\left(\frac{w}{f}\right)^{\frac{1-\sigma}{\sigma}}(x_C)^{\frac{1-\sigma}{\sigma}}, \quad \text{(B.2)}$$

$$F''(x)=-\frac{1-\sigma}{\sigma^2}\left(\frac{w}{f}\right)^{\frac{1-\sigma}{\sigma}}x^{\frac{1-2\sigma}{\sigma}}<0, \quad \text{(B.3)}$$

$F(0)=-(1+r_N)f<0$, and $F(\infty)=F'(\infty)=-\infty$ due to l'Hospital's Rule. Thus, $F(x)$ is a concave function with a unique maximum point, which is defined as $\bar{x}$. Calculating $F'(\bar{x})=0$, we have $\bar{x}=\left(\frac{1}{\rho}\right)^{\frac{\sigma}{1-\sigma}}(1-(1-\rho)\delta_N)^{\frac{\sigma^2}{1-\sigma}}\left(\frac{\sigma}{1-\sigma}\right)^{\sigma}\left(\frac{1+r_N}{1+r_C}\right)^{\frac{\sigma}{1-\sigma}}\frac{f}{w^{1-\sigma}}$, which is, of course, the same as (24). Moreover, we have $F(\bar{x})=\frac{f}{\rho}\left[\left(\frac{1-(1-\rho)\delta_N}{\rho}\right)^{\frac{\sigma}{1-\sigma}}-1\right]>0$ and $\tilde{x}_C<\bar{x}$. Therefore, there exists $\hat{x}$ for which $F(\hat{x})=0$ and $\bar{x}<\hat{x}$. The cost function (17) is illustrated as Figure 2.

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Figure 1 : Profits of the Entrepreneur and the Order of Events

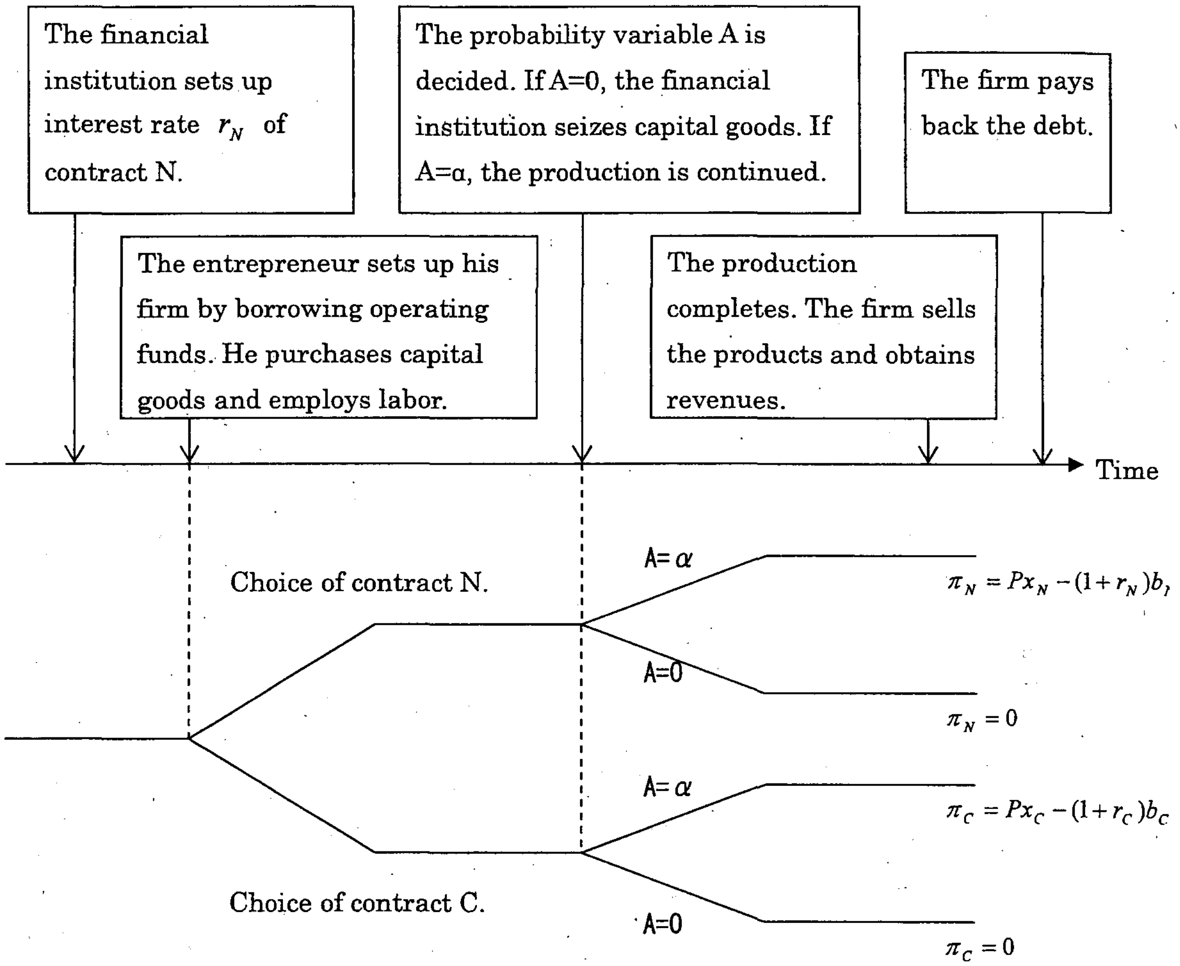

Figure 2: Cost Function

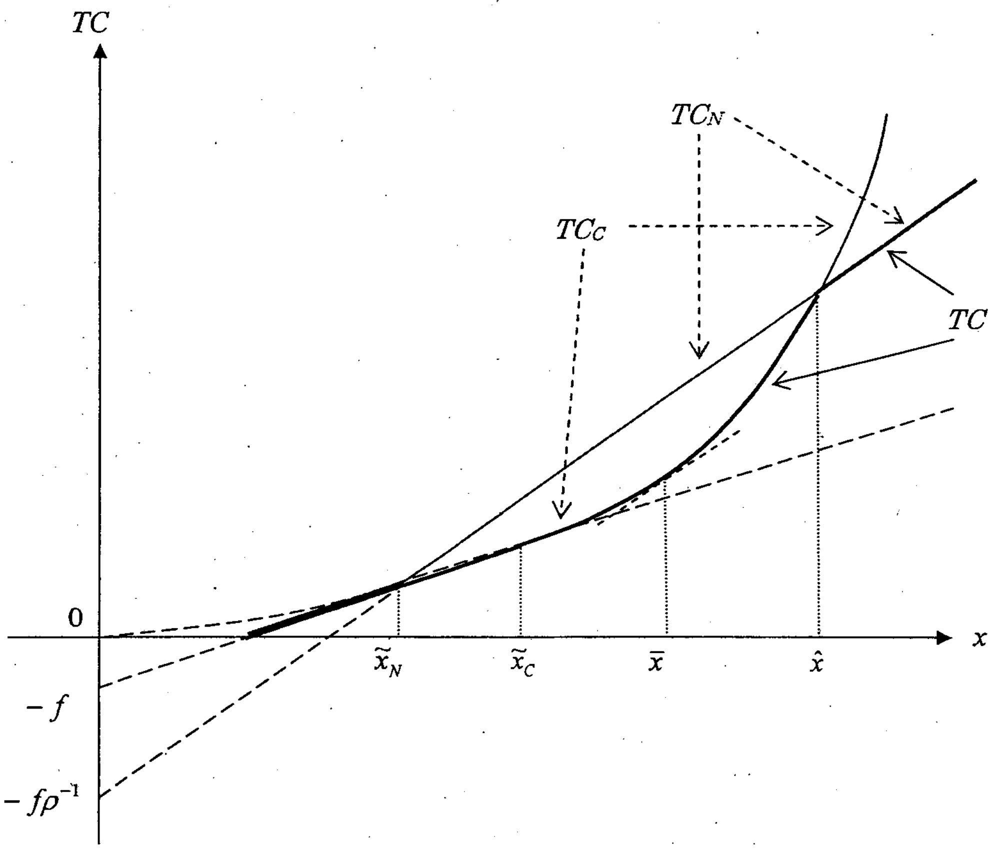

Figure 3: The case of $\alpha < \overline{\alpha}$

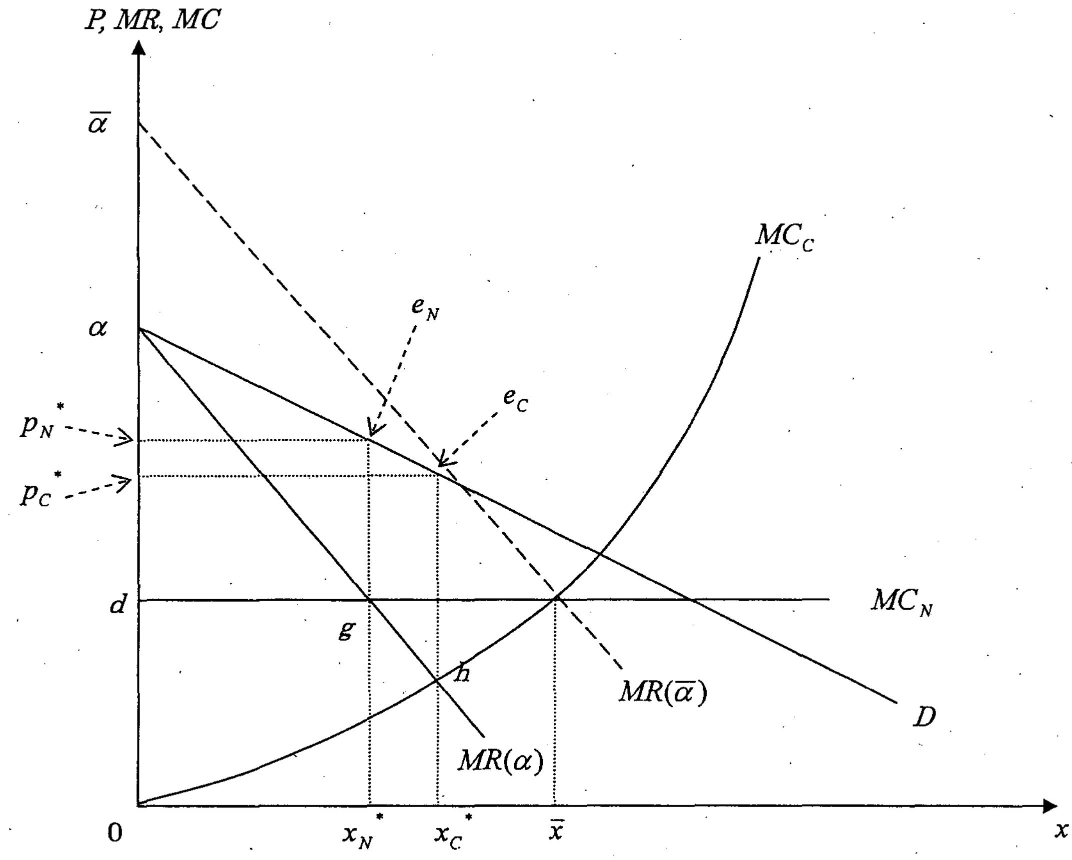

Figure 4: The case of $\alpha > \overline{\alpha}$

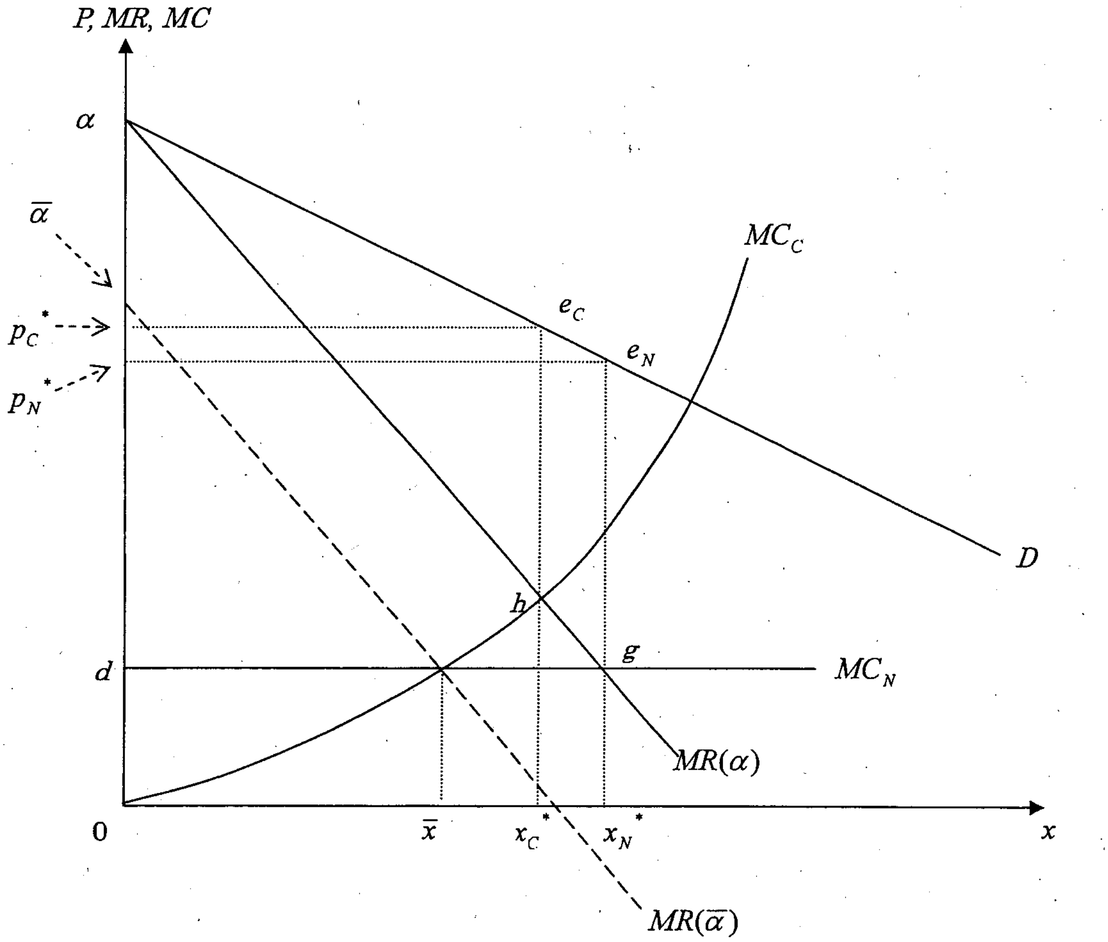

Figure 5: The case of $\sigma = \frac{1}{3}$

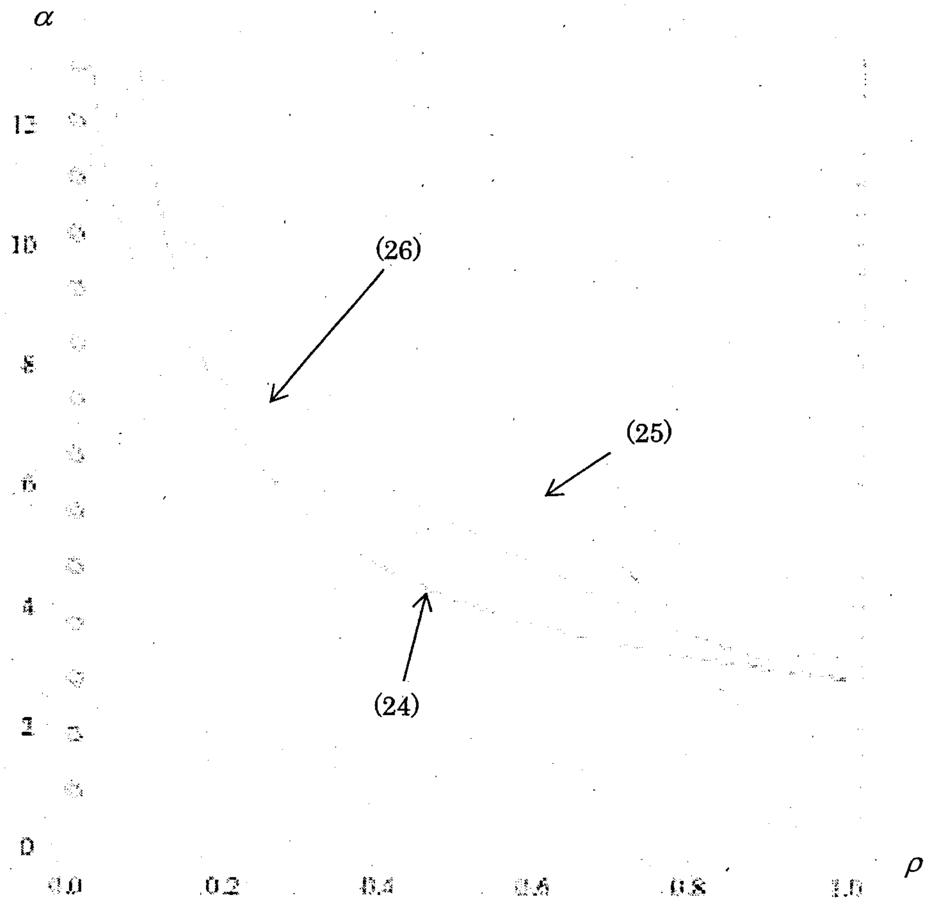

Figure A: The cost minimization problem under contract C with $\delta_C < 1$.