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# INFORMATION ON MIPLIB'S `TIMETAB`-INSTANCES

by

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**Abstract**

This report provides information for the `timetab`-instances of the new MIPLIB. This includes data for both, the underlying real-world application and the resulting graph problem.

## 1 Origin

In the `timetab`-instances, we want to solve the Cyclic Railway Timetabling Problem. In this problem, we are given information about a railway network, consisting of a graph representing its infrastructure and different traffic lines. Each of the traffic lines is operated every $T$ time units. We want to determine periodic departure times within the basic interval $[0, T)$ at every stop of every line.

The constraints of our instances have various sources. Besides elementary conditions on running and stopping times, there are also more advanced topics included, e.g. conditions on single tracks.

In our objective function, we penalize the passenger waiting times that occur along change activities and along the stopping activities. Moreover, the vehicle waiting times that occur along the stopping activities and during the turnover activities are penalized. This models the piecewise constant nature of vehicle costs, i.e. precisely the number of vehicles can be minimized.

The real-world problems, which are the base for our slightly perturbed data sets, have the properties that are shown in Table 1. Notice that the number of turnover activities is strictly greater than twice the number of pairs of traffic lines. This is, because we do not restrict vehicles always to serve the same line. Rather, a train may continue on every line which starts at the station where its last trip finished, given that the lines share the same type of trains. Certainly, an additional potential for optimization is made accessible, but the minimal number of vehicles required when allowing such line changes is no more guaranteed to be met[LP02a].

| Quantity | `timetab1` | `timetab2` |
|---|---|---|
| Pairs of traffic lines | 11 | 13 |
| Change activities | 55 | 143 |
| Stopping activities with extension of minimal stopping time allowed | 44 | 72 |
| Number of pairs of lines synchronized to a frequency of $\frac{T}{2}$ | 10 | 10 |
| Turnover activities | 38 | 44 |

Table 1: Classification numbers of the real-world problems

# 2 Graph Model (PESP)

The Cyclic Railway Timetabling Problem is usually modeled as a Periodic Event Scheduling Problem[SU89]. This model is based on so-called *events*. With an event $i$, we associate either the arrival or the departure of a given directed traffic line at a certain station. A *periodic timetable* $\pi$ then assigns to each of the events a point of time within the basic interval $[0, T)$. Constraints are of the form

$$\ell_{ij} \leq (\pi_j - \pi_i - \ell_{ij}) \bmod T + \ell_{ij} \leq u_{ij}, \tag{1}$$

where $\ell_{ij}$ and $u_{ij}$ are input parameters, which require the difference $\pi_j - \pi_i$ to reside in the interval $[\ell_{ij}, u_{ij}]$, modulo the period time $T$. Due to this special structure, events and constraints can be interpreted as vertices and arcs of a directed graph.

The PESP with period time $T$ is $\mathcal{NP}$-complete. In particular, it is at least as hard as finding out whether a graph has a coloring with at most $T$ colors[Odi97].

When analyzing data that is obtained by some train network planning and analysis software, such as VISUM[Vis03], there are many redundancies in the resulting digraph associated with the PESP instance. These can be eliminated in a preprocessing phase that "contracts" the graph. For example, nodes with degree at most one as well as arcs with *span* $d_{ij} := u_{ij} - \ell_{ij}$ equal to zero can be contracted. Notice that the size of the initial digraphs essentially depends on how safety arcs are generated. They are needed to ensure a safety distance between two consecutive trains. If two trains share five consecutive tracks, this could be translated into five safety arcs, as well. However, our preprocessing method only creates one single safety arc in this case.

# 3 MIP formulation

Based on an approach by Nachtigall[Nac98], the MIP is formulated in terms of *periodic tension* variables $x_{ij}$, instead of *node potential* variables $\pi_i$. For a given node potential $\pi$, the corresponding periodic tension $x$ is defined as

$$x_{ij} := (\pi_j - \pi_i - \ell_{ij}) \bmod T + \ell_{ij}, \ \forall (i,j) \in A. \tag{2}$$

| Quantity | timetab1 | timetab2 |
|---|---|---|
| Original Digraph | | |
| Nodes | 4604 | 5344 |
| Arcs | 5053 | 5859 |
| Run/stop arcs | 4582 | 5318 |
| safety arcs | 225 | 265 |
| Contracted Digraph | | |
| Nodes | 56 | 88 |
| Arcs | 226 | 381 |
| – with $d_{ij} = T - 1$ | 72 | 164 |
| – with $d_{ij} \geq 0.9 \cdot T$ | 153 | 253 |
| – with $d_{ij} \leq 0.1 \cdot T$ | 41 | 70 |
| average span | 77.76% | 79.19% |

Table 2: Classification numbers of the digraphs

Notice that not every vector $x \in \mathbb{Q}^{|A|}$ can be derived from a node potential in this way. So, in order that the tensions obtained by a MIP-solver actually generate a periodic timetable $\pi$, they must satisfy some additional constraints which involve additional artificial integer variables $p$, see Liebchen and Peeters[LP02b]. We will make this precise below.

We are given a PESP digraph $D = (V, A)$ with period time $T$ and for every arc $a$ a lower bound $\ell_a$, an upper bound $u_a$, and a coefficient $c_a$.

Let $\{a_1, \ldots, a_{n-1}\}$ be the arcs of a spanning tree $H$ of the underlying undirected graph of $D$. Define the cycle-arc incidence matrix $\Gamma = (\gamma_{ij})_{(m-n+1)\times m}$ generated by $H$ as:

$$\gamma_{ij} := \begin{cases} 0, & \text{if } a_j \text{ is not part of the cycle induced by } a_j, \\ 1, & \text{if } a_j \text{ is part of the cycle induced by } a_j \\ & \text{and } a_j \text{ is traversed in the same direction as } a_i, \\ -1, & \text{else.} \end{cases} \tag{3}$$

The mixed integer linear program to be solved is:

$$\begin{array}{ll} \min & cx \\ \text{s.t.} & \Gamma x = pT \\ & \ell \leq x \leq u \\ & \underline{p} \leq p \leq \overline{p} \\ & x \in \mathbb{Q}^m \\ & p \in \mathbb{Z}^{m-n+1}. \end{array} \tag{4}$$

The box-constraints $\underline{p} \leq p \leq \overline{p}$ are obtained by valid inequalities that have been introduced by Odijk[Odi97]. They heavily depend on the choice of the spanning tree $H$.

We may classify trees by their *width*

$$\prod_{i=1}^{m-n+1} (\overline{p} - \underline{p} + 1). \tag{5}$$

Liebchen[Lie03] observed a significant correlation between the width and the solution time of CPLEX©. Hence, spanning trees with a small width are preferable. But minimizing a linearized variant of this objective over spanning trees is MAX-SNP-hard[GA03].

In Deo et. al.[DKP95], several heuristics for constructing a spanning tree with small width are proposed, among them `UV` (unexplored vertices) and `NT` (non-tree edges). For `timetab1`, we obtained the smallest width by applying a weighted variant of `UV`, for `timetab2`, we obtainted the smallest width by applying `NT`. The widths and the distributions of the number of possible values for the integer variables $p$ are reported in Table 3.

| Quantity | `timetab1` | `timetab2` |
|---|---|---|
| Width | $1.24111 \cdot 10^{73}$ | $3.46383 \cdot 10^{127}$ |
| $n$ | ♯ variables with $n$ integer values | |
| 1 | 13 | 17 |
| 2 | 64 | 113 |
| 3 | 38 | 74 |
| 4 | 38 | 54 |
| 5 | 15 | 30 |
| 6 | 3 | 5 |
| 7 | 0 | 1 |

Table 3: Properties of the cycle bases

# 4 Solution Behavior with CPLEX©

The optimal value for `timetab1` is 764772. It has been obtained and proven after two consecutive runs of CPLEX© 8.0 on an Athlon XP 1500+ with 512MB on a formulation refined by 200 valid inequalities in the root node of the branch and bound tree. The two runs took less than six hours in total.

For `timetab2`, the best solution we know has an objective value of 1129073. The value of 1020507 has been identified by CPLEX© 8.0 as a lower bound, implying a gap of about ten percent. These values have been attained only within several days, by alternated runs of CPLEX© 8.0 on a formulation refined by up to 1000 valid inequalities, and a genetic algorithm[NV96].

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