Technische Universität Berlin
Institut für Mathematik

Some Fundamental Considerations for the Application of Macroscopic Models in the Field of Pedestrian Crowd Simulation

Frank Huth Günter Bärwolff Hartmut Schwandt

Preprint 2012/16

Preprint-Reihe des Instituts für Mathematik
Technische Universität Berlin

Report 2012/16 July 2012

Some Fundamental Considerations for the Application of Macroscopic Models in the Field of Pedestrian Crowd Simulation

Frank Huth, Günter Bärwolff, and Hartmut Schwandt

1 The Need to Model Pedestrian Flow

The last century has experienced an enormous growth of cities all over the world. This continuing process is accompanied by the growth of urban facilities. So, it is convenient to be able to do a great deal of every day tasks in limited space or to use complex nodes in the sector of public transport to be able to flexibly change between trains and/or buses or to enjoy the pleasure of participating in mass events. In the wake of this tendencies, more and more complex facilities are created and more and more complex pedestrian streams might arise.

The chance to assist the planning of such facilities by simulation of the pedestrian streams to be expected, gets an ever larger importance, because some points that inhibit or even prohibit real life trials with pedestrians are:

- they are expensive because large numbers of people are to be involved,
- they might be beyond the limits of ethics (especially for emergency situations)
- the need to know which pedestrian streams to expect arises in the (architectural) planning phase of facilities, so the site to be assessed does not already exist.

With respect to realism, simulations as well as real life trials carry problems. Trials are artificial situations usually with a nonrepresentative sample of participants. Simulations on the other hand, depend on an appropriate set of rules to generate flow-behavior and patterns as close to reality as possible.

The major paths to simulate pedestrian flows are:

microscopic   by simulating individual agents,
mesoscopic/statistical   to simulate the effects in the field linking mircoscopic and macroscopic approaches and

Technische Universität Berlin
Institut für Mathematik
Sekr. MA 6-4
Straße des 17.Juni 136
10623 Berlin, Germany
e-mail: huth@math.tu-berlin.de, baerwolf@math.tu-berlin.de, schwandt@math.tu-berlin.de,
WWW home page: `http://www.math.tu-berlin.de`

macroscopic perceiving a pedestrian flow as a flow of continuous matter, which is the subject of this article.

2 Applying Macroscopic Models for Pedestrian Flow

The field of the application of partial differential equations (referred to as PDEs further on) to pedestrian flow simulations in the framework of a macroscopic model is not very well developed and scarce information is available. This is especially true for multi-species simulations, where the interaction of crossing and facing streams of pedestrians is considered.

When it comes to modeling pedestrian movement by PDEs, fundamental questions arise. These questions have already been comprehensively addressed in e.g. [2], where it has been made very clear, that and why simple fluid dynamics shouldn't be the way to approach this topic. So we restrict our reasoning to some supplementary aspects.

In the following sections, we try to pin down some of the problems that are present, when one tries to apply the tools of macroscopic pedestrian crowd simulation. And we try to present an idea how to deal with the emerging questions — if we have one.

2.1 Pedestrians as Moving Matter

One fact is, that a pedestrian usually has the desire to move in one peace from one location to another. The other fact is, that a basic assumption for the applicability of PDEs is, that they deal with matter that is considered to be a continuum that may be divided into ever smaller (up to infinitesimally small) peaces without loosing their macroscopically defined properties. This clearly is a contradiction to be dealt with.

This very question arises in fluid dynamics too, where — under the right conditions — PDEs are used too, to describe the motion of particles. Due to the assumption, that these fluid particles stupidly interact by direct collisions, statistical analysis shows that a good approximation of a flow by PDEs can be expected for sufficiently densely packed particles.

This property is measured by the Knudsen Number Kn, which is defined by $\mathrm{Kn} = \frac{\lambda}{L}$. There λ is the mean free (without collision) path length of the traveling particles and L is a characteristic length. This characteristic length is usually taken to be the diameter of the interacting particles. The condition $\mathrm{Kn} \leq 1$ is usually considered sufficient to apply PDEs giving a good approximation.

If this condition were applied to pedestrian flows in this manner, the normal case of no one is bumping into each other, obstacles and walls were not computable. Further, a "packing" that dense of pedestrians would usually be perceived as inconvenient or even threatening.

A way to approach this problem could be to state:

Assumption 1 *Pedestrian movement is determined by a field of influences that results in their walking direction and speed. In that model every considered aspect (like planed path to a target, obstacles, walls, other pedestrians and so on) has a foot print in the environment of the pedestrians generating an interaction field with distant forces or rather effects, that can be approximated continuously in space. The informational peaces of this field can be expressed by PDEs.*

This assumption provides a basis for the decision making of pedestrians, whether they are perceived as unstructured moving matter, or as individually acting particles.

Now we review the point from above that the “matter” that PDEs deal with, is considered to be continuous and arbitrarily fine dividable. In the case that this “matter” consists of pedestrians, this assumption should be considered to be applicable with restrictions only and is directly linked to the scale of the simulation (see section 2.2).

A further question directly connected to the preceding one, is if the properties carried by the samples of the pedestrians, that we consider, are sufficiently homogeneous to use PDEs. This question is judged in fluid dynamics by seeking the answer to the question, if a so called representative elementary volume (REV) exists. Roughly speaking this means, that the REV has to hold a sufficiently large number of particles to dampen out the fluctuations due to the fact, that influencing and observable properties are carried by particles individually. On the other hand the the REV shall be sufficiently small to be able to consider it arbitrarily small in the context of PDEs or at least in the context of the methods for the numerical approximation of their solution. This duality draws the limits of the parameter-space and scale in which the validity of macroscopic models can be expected and proven.

2.2 Scale of Simulated Settings

As we have argued, the scale is a vital question when it comes to the applicability of macroscopic models. If for instance pedestrians escape slowly, but surely through a keyhole sized bottleneck, the simulated results have to be received with doubt. Spatial refinement should be applied with care and a uniform cell size is of importance because of the obvious dependency of the model from the scale of the simulation as we will see in the following sections.

The considerations concerning scale can be fixed at two extreme points. First, a large scale point of view and last a small scale point of view.

2.2.1 Large Scale Point of View

If we assume REV sample-sizes of pedestrians, that are large enough to let averages rule the scene, we are in tune with the notion of fluid dynamics. So mass division should be no problem within the bounds of the accuracy needed. Hence figure 1 could be considered a valid simulation in this respect.

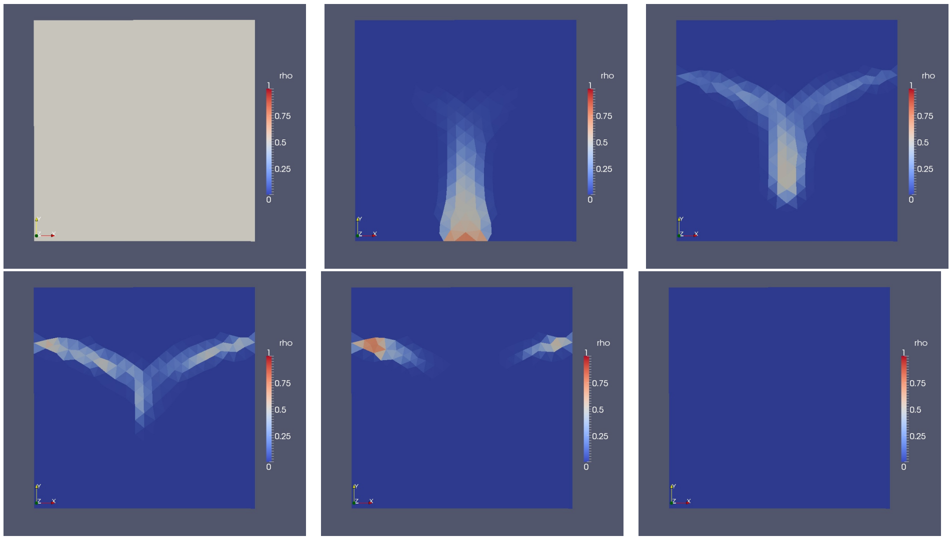

Fig. 1 Simple One Entry, Two Exits Simulation (Time Steps $0-6$) with $\rho|_{\text{Entry}} = 1$ for $t \in [0, 1]$.

The question, that remains unanswered at this point, is how large a sample size it takes to provide for a sufficiently good approximation by PDEs. To our knowledge no scientific data are available providing a data-basis to answer this question.

So, if we take the sample size for instance to be a hundred (a ten by ten block of) pedestrians — how ever densely packed — and assume lane-formation with a width of up to five pedestrians in a head-on encounter of two pedestrian streams, we shouldn't see any lanes in our simulation, because the lanes are too small to be resolved by our spatial resolution. The same observation should be true for similar pattern-forming like lumping and spontaneous roundabout formations. These effects all have to be modeled by a first order model (see [1] for instance) applying an appropriate fundamental diagram. But sufficiently general fundamental diagrams are not available to our knowledge at the current time.

Remark 1. Considering the condition for the existence of an REV: if $\rho_{(i)} \approx 0$ for some $x \in \Omega$ (Ω being the inner of the computational domain) the precondition for the existence of an REV is violated because the volume of the REV would tend to be infinitely large. From this point of view this is the point at which a model produces unreliable results if it doesn't break down at all. On the other hand, if densities are very low, interaction between pedestrians is rare and so the predefined direction to a target will probably be followed by the pedestrians in a very well predictable way.

So, there is good reason to apply such a model even if pedestrian densities around zero are encountered.

It seems to be not quite clear, if the methods to introduce an-isometry by defining an interaction neighborhood into the models as discussed in [2], does make sense at this scale because these neighborhoods were swallowed completely by single REVs.

2.2.2 Intermediate Scale Point Of View

At this scale, assumption 1 still holds, but the divisibility of mass as well as the representativeness of the cells in the sense of REVs surely can't be assumed with sufficient accuracy.

Interpreting figure 1 might provide a realistic simulation to a certain degree. But let's assume to use cell-sizes to hold up to 10 pedestrians each and an area of 10 continuous cells that hold 1% of its maximum density each. This means, that a tenth of a person is held by each cell and so one person is spread over an area potentially holding a hundred persons. The question is where this person really is. This question can be viewed in the light of the probability, that pedestrians of a certain kind are present in a certain area. But it is not clear, in which way the overall interaction can be faithfully simulated that way, given the nonlinear nature of fundamental diagrams.

Further, if we consider figure 1, we see that the mass disperses along the way the pedestrians take (this effect is connected to the choice (sole availability) of inappropriate fundamental diagrams.

Hybridization with some kind of microscopic model like has been implemented in [10] or some other particle-tracing model could be used to implement mass-contraction at points, where a modeled person is to be assumed.

An alternative way to tackle this question could be that the way that a certain portion of the pedestrian mass takes, can be interpreted as the realization of a certain amount of likelihood, that a prototype-person would take this route in a certain situation. So the proportion of the pedestrian mass simulated, would give the overall proportion of a flux to be expected on that route. The fairly recent paper [11] approaches this question in the context of moving measures, but with the restriction to one pedestrian species only.

This implies, that validation-methods have to be appropriate in that sense. In our example this would mean, that for a sufficiently high number of separately moving pedestrians in reality the likelihood to take one path or the other should be matched to the flux proportions in the simulation.

At the scale considered in this section [7, 8, 2] provide a very promising approach to the topic. There (an)isotropic fields of vision (or perception) are introduced to better reflect the informational basis on which pedestrians operate. The drawback of the methods presented there is, that they:

- are developed to a point, where only regular cells are used but we need real life triangulations,
- introduce finite-stenciled functions, that probably are computationally expensive to use and
- seem not to respect a maximum density for multiphase fluxes, which is a vital part in such simulations.

2.2.3 Small Scale Point Of View

The problems discussed in section 2.2.2 are even more vivid in this setting. Consider figure 1 as being the entrance and passage of a singe person through our simulated domain. At this resolution, a single individual spreads over several cells, which makes it necessary to apply completely different approaches concerning fundamental diagrams, interpretation of mass-distribution, flux and so on. To deal with such a scale is surely beyond the scope of a model considered in this article.

2.3 Pedestrians as Acting Particles

Now we turn our attention to the nature of the pedestrians as "intelligent particles" (cf. [2]) interacting in the simulation.

Encounters of large groups of pedestrians generate complex situations, that the individuals have to deal with. The complexity surely is high enough to prohibit tractability and is uncertain to a degree, that robustness of the decisions in the sense considered in [3] is an issue and decisions have to be taken fast (in real-time).

So how deal pedestrians with this situation — since they are not omniscient creatures? The answer is, that there are very strong indications, that the decision-making process of pedestrians is based on the application of heuristics (see e.g. [2, 12, 6, 3]), that are more or less complex, not perfect (misjudgments are made and corrected) but obviously sufficiently successful in usual circumstances. These heuristics are used like a handymans toolbox (take the hammer in this situation and the screwdriver in another). An example of this is default (lazy) behavior: don't object to a set standard, do what the others do, because it seems to have been successful in the past and hopefully will be now. So, seeking Co-directionally moving pedestrians may serve two purposes. First personal convenience due to seeking areas with less conflicts and second stay out of the way of pedestrians, that do not share his or her walking speed and direction. This way a win-win strategy is used aside from the sheer existence of the will to cooperate.

Concerning the complexity of the rules in charge they are certainly expected to be of a much lesser complexity than for instance a chess game, or optimal robot control. For instance an optimum choice route finding approach (as in e.g. [5]) seems to neglect the cost for making that choice.

According to [3] the information basis for the application of heuristics is filtered. This happens on the one hand due to the limited "processing power" at hand but more importantly it filters out irrelevant information. This rather than the mere configuration of the human visual field (as claimed in e.g. [6, 2]) leads to anisotropic perception. This configuration is the result of the adaptation to the needs in the course of human evolution, so it *did* make sense at least. On the other hand information is provided by hearing too and an individual is able to allot a certain amount of awareness to information coming from other directions in a situation of being chased or just moving in a group of related individuals. But having said that, anisotropic perception ideally *is* a part of a pedestrian simulation.

The idea to add simple "random" components to human behavior seems not to be the way to model the effects of individual decision-making, because cooperative or competing behavior does shift the overall outcome into a certain direction, where simple "random" behavior implies no direction at all. And further, it likely introduces "freezing by heating" effects (cf. [4, 9]) into normal-behavior simulations.

Assumption 2 *The information base, that is processed by individual pedestrians to make decisions, is not purely factual, but a perception or even a (re)constructed picture (based on the experiences of these individuals) of the reality. In the case of non-collision driven flow, the velocity and walking direction is a product of a heuristics-based decision-making process by individual pedestrians.*

The amount of information processed by individual pedestrians may surely vary very much form pedestrian to pedestrian and beyond that from situation to situation for each pedestrian (e.g. the mobile rings, involvement in other activities like chatting with a fellow-pedestrian and the like). So in reality there isn't a prototype situation with a prototype pedestrian in it. Simplifications are unavoidable and the quest is to find a simple (manageable) set of rules to hopefully reproduce crucial effects in pedestrian movement. This should be possible, if we assume a common set of rules which pedestrians adhere to. The effects of such rules can be observed in every day life. Examples of that are queuing, lane formation and similar effects.

3 Conclusion and a Possible Model Outline

As has been seen, the specific needs of macroscopic models limit their sensible field of application and the expectations should be modest in fields, where vital assumptions for the application of macroscopic means are not met.

So, what can be expected? For very densely packed crowds, the parameters needed to apply a macroscopic model with sufficient accuracy, may be approached. There a first order model should likely be used, because individual behavior can't be resolved.

What shouldn't be expected at the other hand? Surely an approach, that suites all scales equally well. Further it can't be expected, that untypical behavior of a single individual — even if having large scale effects (due to the fact, that the system is a chaotic dynamic system) — can be caught by a macroscopic simulation.

At the limits, where the application of macroscopic models is approximately possible, the resolution most likely is too coarse to make effects like lane and roundabout formation visible.

If the resolution of the model approaches the limits, where the structures mentioned above, should be visible, special modifications should be introduced to catch such effects too.

How could the sketch of a macroscopic model look like in the light of the found necessities?

If we perceive pedestrian flow as a transport problem, we start with the governing equation

$$\frac{\partial \rho_i}{\partial \vartheta} + \nabla \cdot (\rho_i v_i) = 0$$

of a transport problem, that describes the mass flow. Here ϑ denotes the time, $i \in \{1, \ldots, n\}$ and n is the number of pedestrian "types" or "species" distinguished by certain properties of which a desired walking direction and speed should be the most obvious ones. Further ρ_i is the current density and v_i the current speed of a species in a given computational domain. The perception of what ρ_i as a measure of pedestrians shall mean, is a separate topic.

This model surely has to be closed by a model for the v_i, which certainly is the core task of the model development.

At scales, where structures like lanes, clusters roundabouts and the like are not resolvable, a simple split of the pedestrian motion into a planed (aimed at a target in the computational domain or a part of its boundary) and a local reaction to local inconveniences like high densities could be applied. In this model a computational domain geometry induced potential could be used to generate the planed velocity field, that's magnitude depends on the local density.

When it comes to the limit of spatial refinement, where structures like lane formation and the like should be recognizable, local "indentation" of the potential field to generate a planed or local v_i field could be used to promote clustering of same-species pedestrian mass and separation of different-species pedestrian mass.

4 Acknowledgment

The authors gratefully acknowledge support of the present work by the German people, that generously funds the DFG to finance the project SCHW548/5-1 + BA1189/4-1 this way.

References

1. Nicola Bellomo and Christian Dogbé. On the modeling of traffic and crowds: A survey of models, speculations, and perspectives. *SIAM Review*, 53(3):409–463, 2011.
2. Emiliano Cristiani, Benedetto Piccoli, and Andrea Tosin. Modeling self-organization in pedestrians and animal groups from macroscopic and microscopic viewpoints. In Nicola Bellomo, Giovanni Naldi, Lorenzo Pareschi, and Giuseppe Toscani, editors, *Mathematical Modeling of Collective Behavior in Socio-Economic and Life Sciences*, Modeling and Simulation in Science, Engineering and Technology, pages 337–364. Birkhäuser Boston, 2010.
3. G. Gigerenzer. Why heuristics work. *Perspectives on Psychological Science*, 3:20–29, 2008.
4. Dirk Helbing, Illés J. Farkas, and Tamás Vicsek. Freezing by heating in a driven mesoscopic system. *Phys. Rev. Lett.*, 84(6):1240–1243, Feb 2000.
5. S.P. Hoogendoorn, P.H.L. Bovy, and W. Daamen. Microscopic pedestrian wayfinding and dynamics modelling. In M. Schreckenberg and S.D. Sharma, editors, *Pedestrian and Evacuation Dynamics*, pages 123–154, Berlin, 2002. Springer.
6. Mehdi Moussaïd, Dirk Helbing, and Guy Theraulaz. How simple rules determine pedestrian behavior and crowd disasters. *PNAS*, 108(17):6884–6888, April 2011.
7. Benedetto Piccoli and Andrea Tosin. Pedestrian flows in bounded domains with obstacles. *Continuum Mechanics and Thermodynamics*, 21:85–107, 2009.
8. Benedetto Piccoli and Andrea Tosin. Time-evolving measures and macroscopic modeling of pedestrian flow. *Archive for Rational Mechanics and Analysis*, 199:707–738, 2011. 10.1007/s00205-010-0366-y.
9. Leo Radzihovsky and Noel A. Clark. Comment on "Freezing by heating in a driven mesoscopic system". *Phys. Rev. Lett.*, 90(18):189603, May 2003.
10. Thomas Slawig. Hybrid macro-microscopic simulation of pedestrian flow. Technical report, Institut f. Mathematik, TU Berlin, 2007.
11. Andrea Tosin and Paolo Frasca. Existence and approximation of probability measure solutions to models of collective behaviors. *Networks and Heterogeneous Media (NHM)*, 6, 2011.
12. Ulrich Weidmann. Transporttechnik der Fußgänger – transporttechnische Eigenschaften des Fußgängerverkehrs (Literaturstudie). *Schriftenreihe der IVT*, 90, March 1993. in German.