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Nonlinear Stochastic Evolution Equations: Analysis and Numerics

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Nonlinear Stochastic Evolution Equations: Analysis and Numerics

Technische Universität Berlin
November 3 - 5, 2016

Organizers:
Etienne Emmrich (Berlin), Raphael Kruse (Berlin)
Petra Wittbold (Duisburg-Essen)

http://www.tu-berlin.de/?spde2016

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Outline

This workshop brings together experts on nonlinear stochastic evolution equations to discuss recent developments in this field of research.

Since the 1970s the analysis of stochastic partial differential equations (SPDEs) is one of the most active research fields within mathematics, which has led to many new and fruitful collaborations of several mathematical branches such as differential equations, stochastic analysis, numerics or functional analysis. Nowadays, SPDEs are often used to model complex phenomena, which are influenced by uncertainties or stochastic perturbations. Typically, the presence of noise leads to a loss of regularity which prevents the application of traditional (deterministic) solution theory or approximation methods.

The goal of this workshop is to discuss open problems and recent progress made in the mathematical treatment of nonlinear stochastic evolution equations and to offer a fruitful environment to foster the exchange of ideas.

We wish you all an inspiring workshop and a pleasant stay in Berlin.

Etienne Emmrich
Raphael Kruse
Petra Wittbold

Venue: Thursday, Nov 3, 2016

- Room: **ACK 484**
- Address: TU Berlin, Campus Wedding,
  Ackerstraße 76, 13355 Berlin
- Travel options:
  - Closest S-Bahn: Nordbahnhof (S1, S2, S25)
  - Closest U-Bahn: Voltastraße (U8)

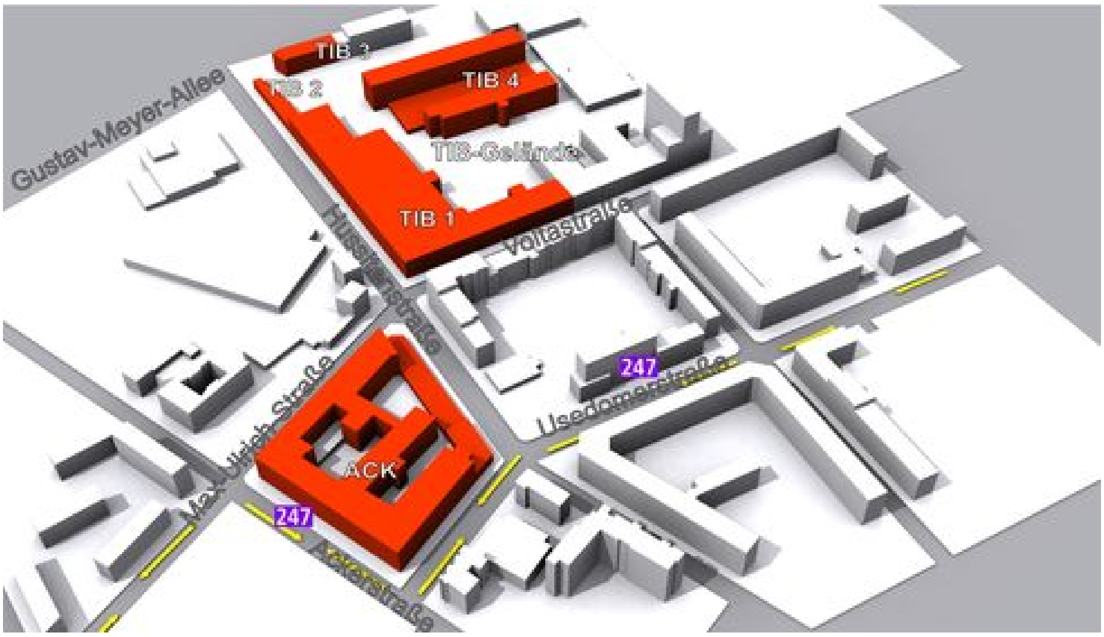
Bus transfers:

Free of charge bus transfer to Campus Wedding. Entry possibilities:

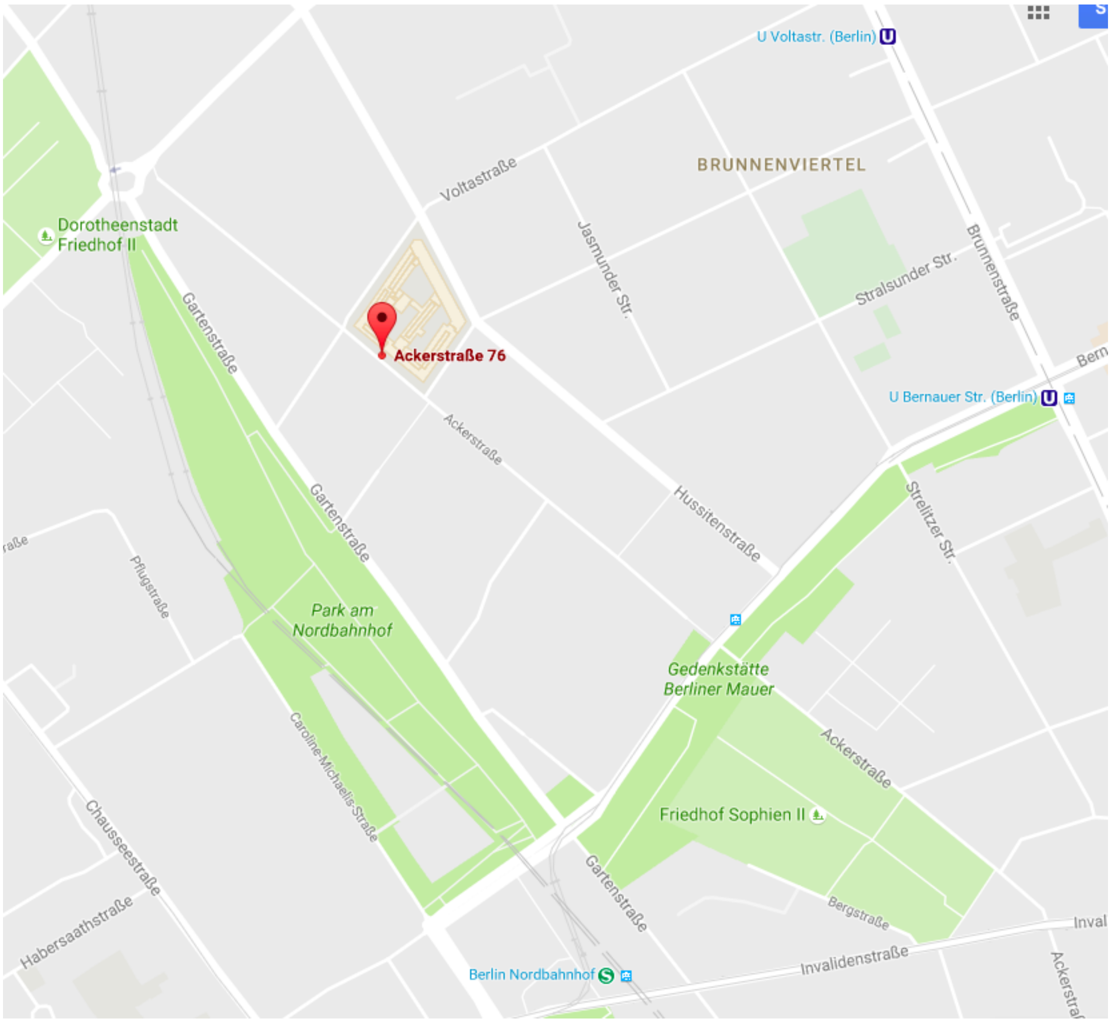
- **Thu, 09:00** at Hotel Carmer 16, Carmerstr. 16, 10623 Berlin
- **Thu, 09:15** in front of the TU Berlin main building,
  Straße des 17. Juni 135, 10623 Berlin

Free of charge bus transfer to the conference dinner. Entry possibility:

- **Thu, 18:30** in front of the ACK building,
  Ackerstraße 76, 13355 Berlin



Campus Wedding



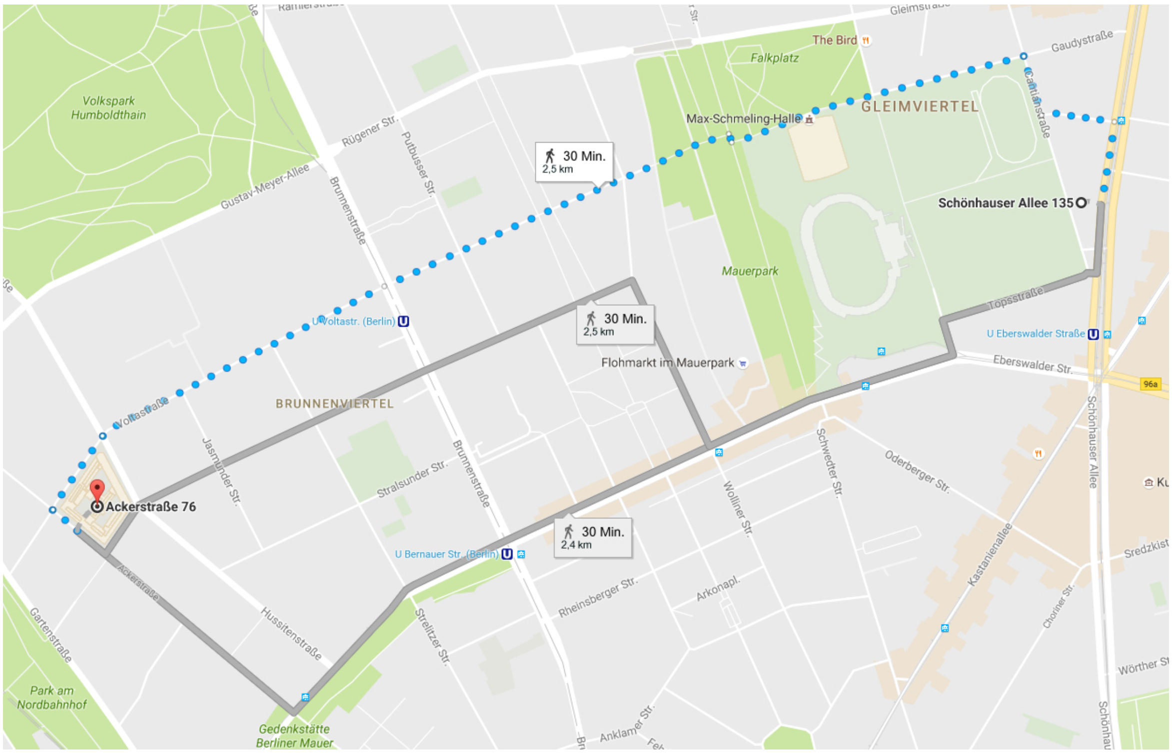
Lunch options:

- Cafeteria TU Ackerstraße
- Several Cafés and Restaurants close to
  - Nordbahnhof
  - Gedenkstätte Berliner Mauer
  - Zionskirchplatz
  - Naturkundemuseum
  - Crossing Invalidenstr. / Ackerstr.

Conference Dinner

- **Thu, 19:00** in the restaurant **Neugrüns Köche**,
  Schönhauser Allee 135, 10437 Berlin

Conference Dinner is within walking distance (2.5 km) or take the bus transfer.

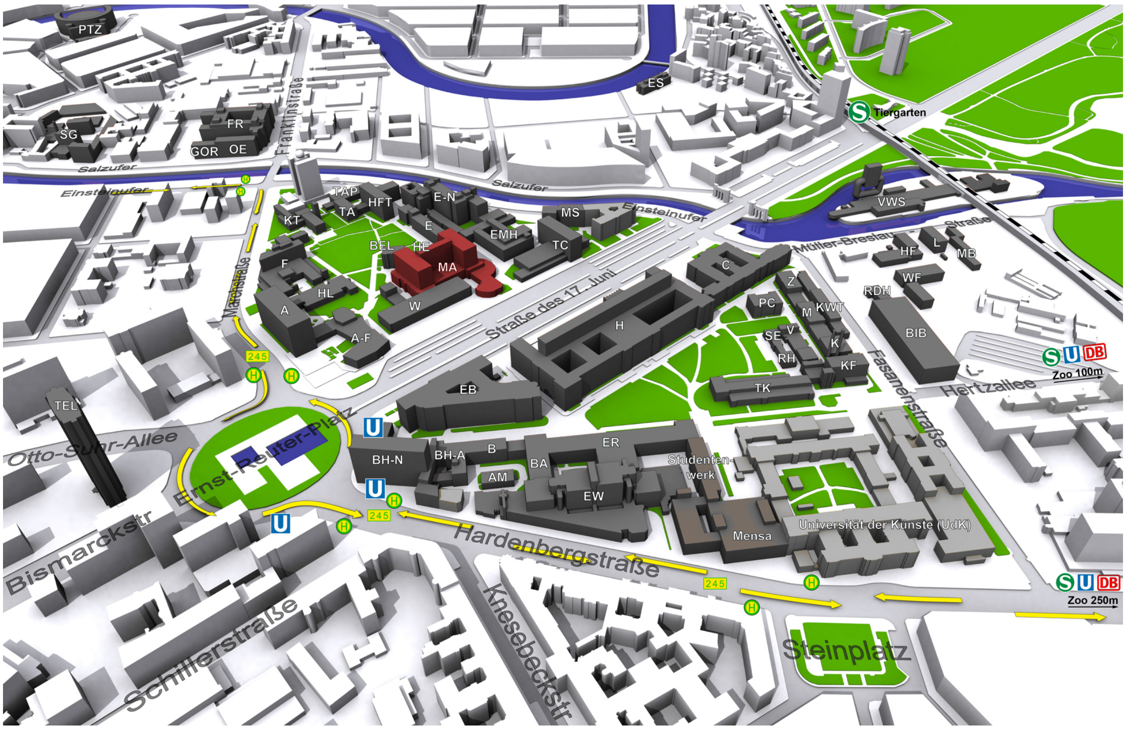


A larger version of this map is included in your conference bag.

Venue: Friday & Saturday, Nov 4 - 5

- Room: **MA 415**
- Address: TU Berlin, Math Building, Campus Charlottenburg, Straße des 17.Juni 136, 10623 Berlin
- Travel options:
  - Closest S-Bahn: Tiergarten (S5, S7, S75)
  - Closest U-Bahn: Ernst-Reuter Platz (U2)

Campus Charlottenburg



	Thursday	Friday	Saturday
Venue	**Room: ACK 484, Ackerstraße 76**	**Room: MA 415, Straße des 17. Juni 136**	**Room: MA 415, Straße des 17. Juni 136**
10:00–10:45	*A new regularity result for the Kolmogorov equation for SPDEs with multiplicative noises* Arnaud DEBUSSCHE	*Some stability results for stochastic conservation laws* Guy VALLET	*Well-posedness and regularization by noise for nonlinear PDE* Benjamin GESS
10:45–11:30	*Simulating weak convergence rates for SPDE approximations* Annika LANG	*Commutator-free Magnus integrators and their areas of application* Mechthild THALHAMMER	*Convergence in Hölder norms with applications to Galerkin approximations and Monte-Carlo methods* Sonja COX
11:30–12:15	*— Coffee Break —*		
12:15–13:00	*Global solutions to random 3D vorticity equations for small initial data* Michael RÖCKNER	*Convergence of approximate solutions to stochastic first-order scalar conservation laws, convergence of the approximation by the Finite Volume method* Julien VOVELLE	*Rough Gronwall Lemma and weak solutions to RPDEs* Martina HOFMANOVÁ
13:00–14:30	*— Lunch Break —*		
14:30–15:15	*On stochastic optimal control in ferromagnetism* Andreas PROHL	*Simulation of infinite-dimensional Lévy - Processes* Andrea BARTH	*Numerical approximation of the stochastic Cahn-Hilliard equation near the sharp interface limit* L'ubomír BAŇAS
15:15–16:00	*Invariant measures for stochastic Navier-Stokes Equations in unbounded domains* Zdzisław BRZEŹNIAK	*Inhomogeneities and temperature effects in Bose-Einstein condensates* Anne DE BOUARD	*Quasilinear SPDEs via rough paths* Hendrik WEBER
16:00–16:45	*— Coffee Break —*		
16:45–17:30	*Coercivity condition for higher order moments of nonlinear SPDEs and existence of solution under local monotonicity* David ŠIŠKA	*Strong convergence of a fully discrete finite element approximation of the stochastic Cahn–Hilliard equations* Stig LARSSON	
17:30–18:15	*Compressible fluid flows driven by stochastic forcing* Eduard FEIREISL		
19:00	*—Conference Dinner—*		

Abstracts

L'ubomír BAŇAS

Numerical approximation of the stochastic Cahn-Hilliard equation near the sharp interface limit

We present an implicit semi-discrete (in time) numerical scheme for the stochastic Cahn-Hilliard equation with (asymptotically small) multiplicative noise. For the proposed scheme we derive strong convergence rates which depend polynomially on the interfacial thickness parameter. In addition, we perform numerical studies to show asymptotic behavior of the model.

This is a joint work with D. Antonopoulou and A. Prohl.

Andrea BARTH

Simulation of infinite-dimensional Lévy-Processes

In various applications, stochastic partial differential equations are not driven by Gaussian noise but rather by one whose marginal distributions have heavier tails. Unlike the case of an infinite-dimensional Gaussian process, a general infinite-dimensional Lévy process cannot be built from independent, one-dimensional Lévy processes and still admit these one-dimensional distributions as its marginals. In this talk I introduce an approach to construct time-dependent random fields that have marginal distributions which follow certain Lévy measures. I show convergence of the method and wrap up with some numerical examples.

This is joint work with Andreas Stein (University of Stuttgart).

Zdzisław BRZEŹNIAK

Invariant measures for stochastic Navier-Stokes Equations in unbounded domains

Sonja COX

Convergence in Hölder norms with applications to Galerkin approximations and Monte-Carlo methods

In a recent article we demonstrated that if a sequence of piecewise affine linear pro- cesses converges in the strong sense with a positive rate to a stochastic process which is strongly Hölder continuous in time, then this sequence converges in the strong sense even with respect to much stronger Hölder norms and the convergence rate is essentially reduced by the Hölder exponent. This principle has a number of applications: it may be used to derive strong convergence rates of multilevel Monte Carlo approximations of expectations of Banach space valued stochastic processes. Another application is to obtain pathwise convergence rates of spectral Galerkin ap- proximations of non-linear stochastic partial differential equations. This in turn can be used to extend regularity results for SODEs to SPDEs.

Joint work with Martin Hutzenthaler, Arnulf Jentzen, Jan van Neerven, Timo Welti.

Anne DE BOUARD

Inhomogeneities and temperature effects in Bose-Einstein condensates

We will review in this talk some mathematical results concerning stochastic models used by physicist to describe BEC in the presence of fluctuations (that may arise from inhomogeneities in the confinement parameters), or BEC at finite temperature. The results describe the effect of those fluctuations on the structures — e.g. vortices — which are present in the deterministic model, or the convergence to equilibrium in the models at finite temperature. We will also describe the numerical methods which have been developed for those models.

These are joint works with Reika Fukuizumi, Arnaud Debussche, and Romain Poncet.

Arnaud DEBUSSCHE

A new regularity result for the Kolmogorov equation for SPDEs with multiplicative noise

In this talk we present a new regularity result on the solution of the Kolmogorov equation associated to a stochastic parabolic PDE of the type:

$$du = (\partial_{xx} u + f(u))dt + \sigma(u)dW$$

with boundary conditions and initial data. We prove a new estimate on the second differential of the solution of this Kolmogorov equation. This is crucial for the study of the weak order of numerical scheme and we apply this to a time discretization of the equation. The main tool is a formula for the second differential which is obtained thanks to Malliavin calculus.

This is a joint work with C.E. Bréhier.

Eduard FEIREISL

Compressible fluid flows driven by stochastic forcing

We discuss several problems arising in the mathematical theory of compressible viscous fluids driven by stochastic forcing. We focus on the basic problems of well-posedness and qualitative properties of solutions including the long-time behavior. We introduce the concept of relative energy and show how this can be used in the analysis.

Joint work with D.Breit (Edinburgh), M.Hofmanová (TU Berlin).

Benjamin GESS

Well-posedness and regularization by noise for nonlinear PDE

In this talk we will revisit regularizing effects of noise for nonlinear SPDE. In this regard we are interested in phenomena where the inclusion of stochastic perturbations leads to increased regularity of solutions as compared to the unperturbed, deterministic case. Closely related, we study effects of production of uniqueness of solutions by noise, i.e. instances of nonlinear SPDE having a unique solution, while non-uniqueness holds for the deterministic counterparts. The talk will concentrate on these effects in the case of nonlinear scalar conservation laws and stochastic porous media equations.

Martina HOFMANOVÁ

Rough Gronwall Lemma and weak solutions to RPDEs

In this talk, I will present recent results that provide the framework for the study of rough path driven PDEs in the context of weak solutions. The main tool is a new rough Gronwall Lemma argument whose application is rather wide: among others, it allows to derive the basic energy estimates leading to the proof of existence. Besides, we develop a suitable tensorization method which is the key for establishing uniqueness.

The talk is based on a joint work with Aurelien Deya, Massimiliano Gubinelli and Samy Tindel.

Annika LANG

Simulating weak convergence rates for SPDE approximations

The finding of weak convergence rates for approximations of solutions is one of the current and still partly open problems in the numerical analysis of stochastic partial differential equations. The confirmation of the existing theory as well as of conjectured rates with simulations has hardly been done and has not been successful for equations driven by multiplicative noise so far. In this talk it is discussed why the standard methods fail even for toy examples to recover theoretical rates and new error estimators are introduced that allow for simulations of weak convergence rates for SPDEs driven by multiplicative noise.

This is joint work with Andreas Petersson.

Stig LARSSON

Strong convergence of a fully discrete finite element approximation of the stochastic Cahn–Hilliard equations

We consider the the stochastic Cahn–Hilliard equation (Cahn–Hilliard–Cook equation) driven by smooth additive Gaussian noise in a spatial domain in dimension $d \leq 3$. We discretize the equation using a standard finite element method is space and a fully implicit backward Euler method in time. By proving optimal error estimates on subsets of the probability space with arbitrarily large probability and uniform-in-time moment bounds we show that the numerical method converges strongly to the solution as the discretization parameters vanish.

This is a joint work with Daisuke Furihata, Mihaly Kovacs, and Fredrik Lindgren.

Andreas PROHL

On stochastic optimal control in ferromagnetism

A model is proposed to control domain wall motion in ferromagnets in the presence of thermal fluctuations, and the existence of an optimal stochastic control process is proved. The convergence of a Galerkin approximation of the problem is shown in 1D, which then allows to apply Pontryagin's maximum principle for this finite dimensional setting. The resulting coupled system of forward-backward SDE's is numerically solved by means of the stochastic gradient method and the least squares MC method to enable practical simulations.

This is joint work with A.K. Majee, T. Dunst (U Tübingen), and G. Vallet (U Pau).

Michael RÖCKNER

Global solutions to random 3D vorticity equations for small initial data

One proves the existence and uniqueness in $(\mathcal{L}^p(\mathbb{R}^3))^3$, $\frac{3}{2} < p < 2$, of a global mild solution to random vorticity equations associated to stochastic 3D Navier-Stokes equations with linear multiplicative Gaussian noise of convolution type, for su ciently small initial vorticity. This resembles some earlier deterministic results of T. Kato and are obtained by treating the equation in vorticity form and reducing the latter to a random nonlinear parabolic equation. The solution has maximal regularity in the spatial variables and is weakly continuous in $(\mathcal{L}^3 \cap \mathcal{L}^{\frac{3p}{4p-6}})^3$ with respect to the time variable. Furthermore, we obtain the pathwise continuous dependence of solutions with respect to the initial data. This is joint work with Viorel Barbu.

David ŠIŠKA

Coercivity condition for higher order moments of nonlinear SPDEs and existence of solution under local monotonicity

Abstract: Higher order moment estimates for solutions to nonlinear SPDEs governed by locally-monotone operators are obtained under appropriate coercivity condition. These are then used to extend known existence and uniqueness results for nonlinear SPDEs under local monotonicity conditions to allow derivatives in the operator acting on the solution under the stochastic integral.

Joint work with Neelima (School of Mathematics, University of Edinburgh and Ramjas College, University of Delhi).

Mechthild THALHAMMER

Commutator-free Magnus integrators and their areas of application

In this talk, I shall introduce the class of commutator-free Magnus integrators for non-autonomous linear evolution equations and identify different areas of application. Commutator-free Magnus integrators are (formally) given by a composition of several exponentials that comprise certain linear combinations of the values of the defining operator at specified nodes. Avoiding the costly evaluation of commutators, they provide a favourable alternative to standard Magnus integrators, in particular for large-scale applications. Non-autonomous linear evolution equations also arise as a part of more complex problems, for instance in connection with nonlinear evolution equations of the form $u'(t) = A(t)u(t) + B(u(t))$. A natural approach is thus to apply operator splitting methods combined with commutator-free Magnus integrators. Relevant applications include Schrödinger equations with space-time-dependent potential describing Bose-Einstein condensation or diffusion-reaction systems with additional multiplicative noise modelling pattern formation.

Guy VALLET

Some stability results for stochastic conservation laws

Our aim in this communication is to present a result of well-posedness and some stability results for the solution of some stochastic conservation laws and some error estimates of their approximation.

Julien VOVELLE

Convergence of approximate solutions to stochastic first-order scalar conservation laws, convergence of the approximation by the Finite Volume method

We use the kinetic formulation of stochastic first-order scalar conservation laws to give a general framework to the study of the convergence of approximate solutions. We apply the result to the convergence of the approximation by the Finite Volume method, in the case of compactly supported multiplicative noise. The case of additive noise is also discussed.

This is a joint work with Sylvain Dotti (Aix-Marseille University).

Hendrik WEBER

Quasilinear SPDEs via rough paths

In this talk I will present a new approach to solve singular stochastic PDE which extends directly Gubinelli's notion of controlled rough paths and is also closely related to Hairer's theory of regularity structures. The approach is implemented for the variable-coefficient uniformly parabolic PDE

$$\partial_2 u - a(u)\partial_1^2 u - \sigma(u) f = 0,$$

where f is an irregular random distribution. The assumptions allow, for example, for an f which is white in time and only mildly coloured in space.

The key result is a deterministic stability result (in the spirit of the Lyons-Itô map) for solutions of this equation with respect to f but also the products vf and $v\partial_1^v$, with v solving the constant-coefficient equation $\partial_2 v - a_0 \partial_1^2 v = f$. On the stochastic side it is shown how these (renormalised) products can be constructed for a random f.

This talk is based on joint work with F. Otto.

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