Technische Universität Berlin
Institut für Mathematik

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Frank Huth Günter Bärwolff Hartmut Schwandt

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1 Introduction

Transport equations of the form:

$$\frac{\partial \rho_i}{\partial \vartheta} + \nabla \cdot (\rho_i v_i) = 0\,, \tag{1}$$

that describe mass flow, may be applied in the field of macroscopic pedestrian flux simulations. In this equation ϑ denotes the time, $i \in \{1, \ldots, n\}$ where n is the number of pedestrian "types" or "species" distinguished by certain properties of which a desired walking direction and speed should be the most obvious ones. Further ρ_i is the current density and v_i the current velocity of a species in a given computational domain. The applicability of such models in this field is considered in [3].

Equation (1) is defined on a certain open domain Ω in space for $\vartheta > 0$ and has to be supplemented by appropriate initial and boundary conditions.

The key question that one faces to close (1), is to find a sensible $v_i = v_i(\Omega, \partial\Omega; \rho_1, \ldots, \rho_n)$. One approach decomposes v_i like this:

$$v_i = a_i V d_i \tag{2}$$

with:

V $\in [0, 1]$ is a normalized speed chosen by a fundamental diagram (which is the focus of this paper).
d_i is a unit vector field giving an intentionally chosen direction.
a_i is a constant.

Technische Universität Berlin
Institut für Mathematik
Sekr. MA 6-4
Straße des 17.Juni 136
10623 Berlin, Germany
e-mail: huth@math.tu-berlin.de, baerwolf@math.tu-berlin.de, schwandt@math.tu-berlin.de,
WWW home page: http://www.math.tu-berlin.de

This model may be perceived as a generalization of a model by Bick and Nevell which in turn has been based on the model presented by Lighthill, Whitham and Richards.

The central part $V = V(\rho = \sum_{i=1}^{n} \rho_i)$ reflects the ability to follow a certain direction according to the local conditions. In [3] we made the following

Assumption 1 *The information base, that is processed by individual pedestrians to make decisions, is not purely factual, but a perception or even a (re-)constructed image (based on the experiences of these individuals) of the reality. In the case of a non-collision driven flow, the velocity and walking direction is a product of a heuristics-based decision-making process by individual pedestrians.*

With V and d_i being part of the decision-making process, they are expected to be subject to and hence being modeled by a heuristics approach (see e.g., [2]).

2 Scale

In [3], we consider the question of spatial refinement. There, we name cases with rather

large spatial cells "large scale",
medium spatial cells "intermediate scale",
small spatial cells "small scale".

Further, we argue there that a rather uniform cell size should be applied, because of the implications that the cell size has for certain modeling aspects.

For large scales, we should not expect to see effects like clustering, lane formation, roundabouts and similar effects, because the spatial resolution does not permit to resolve such structures.

For intermediate scales, we may well expect to see effects like them metioned above.

For small scale effects, we have resolutions in the vicinity applied in cellular automata models or even beyond that. These scales are certainly not accessible by classical macroscopic modelling and new approaches like have been presented in [4, 5, 1] are likely a better choice, if a macroscopic model is considered to fit at all.

3 Velocity-Magnitude and the Fundamental Diagram.

For a one-directional one-species flux $J = \rho V(\rho) d$, a fundamental diagram can be drawn displaying the dependencies between the three quantities J, V and ρ.

The advantage of the application of fundamental diagrams contrary to trying to model pedestrian behavior from "force"-equilibria is, that laws which are very complicated to be modeled by the equilibrium laws might be expressed and applied efficiently by a fundamental diagram.

The disadvantage is that it may well be an oversimplification in many situations. The known fundamental diagrams are primarily derived from one-directional or at least 180°-encounter flows. Further, several measurements of the fundamental diagram have been carried out giving diverging information in terms of what the real dependencies are. According to [6], the given values for the maximum pedestrian density, where movement is possible at all, vary from 3.8 pedestrians/m^2 – 10 pedestrians/m^2 and the dependency of V from the fact, if the movement is unidirectional or multi-directional is discussed controversially.

A promising approach to gain some insight into the governing laws from the experimental point of view for yet too simple configurations might be provided by the methods used in [7, 10]. But a problem (aside from the simplicity of the setting) with this method is, that it does not measure the perception of the pedestrian. This perception is expected to be anisotropic with respect to his/her walking direction and a pedestrian waiting for clearance of a jam in front with nobody behind him/her, may sense a pretty low density without the real chance to move in the intended direction with measurements done this way.

All experimental measurements known to the authors so far, are done in settings of chanel-flows and bottlenecks. This is probably the case, because the experiments are well controllable and relevant for evacuation simulations (which are of primary importance). Our intention is to aim beyond that and so the question thats left open so far is what happens in situations, that are not confined in this way.

What were needed at least, is a fundamental diagram, that gives:

$$V = V(\rho_1, \ldots, \rho_n, d_1, \ldots d_n)$$

which holds a $[0,1]^n \times [0,\pi]^n$ cube of dimension $2n$ of information, with n the number of pedestrian species considered. It might be expected, that several symmetries condense the necessary information by a certain degree.

To get an impression of the influence of different fundamental diagrams, concerning quantity and quality of the solutions, we ran simulations applying the following dependencies:

$$V(\rho) = 1 - \rho \tag{3}$$

$$V(\rho) = (1 - \rho)^2 \tag{4}$$

$$V(\rho) = 1 - \rho^2 \tag{5}$$

$$V(\rho) = 1 - \exp(-1.913/5.4(1/\rho - 1)) \tag{6}$$

Here (6) (in comparison to Fig. 1) is adapted to fit the normalized $V \in [0,1]$ and $\rho \in [0,1]$ conditions. The results differ to a degree to indicate the need for a good approximation in this respect.

A very interesting remark made in [9, p. 65] concerning the cause of the better flux for a counter flow with a 50% to 50% split with respect to a 90% to 10% split, leads to an easy conclusion. The statement made there is that the likely cause for that is the establishment of a stable lane formation in the former split and no such formation in the later. So generalizing this idea one could deduce, that the key question is, if the establishment of stable patterns is possible or not. We will discus this idea in Sect. 5 a bit more in detail.

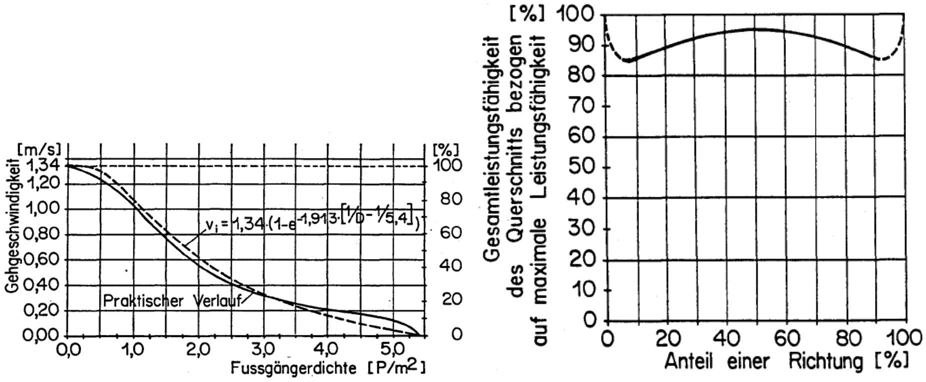

Fig. 1 Left: Empirical Relation Between Density and Velocity versus its Approximation by the Term Given by Kladek (taken from [9, p. 62]). Right: One-Directional Capacity Percentage with Counter Flow of a Given Split Percentage (taken from [9, p. 65]).

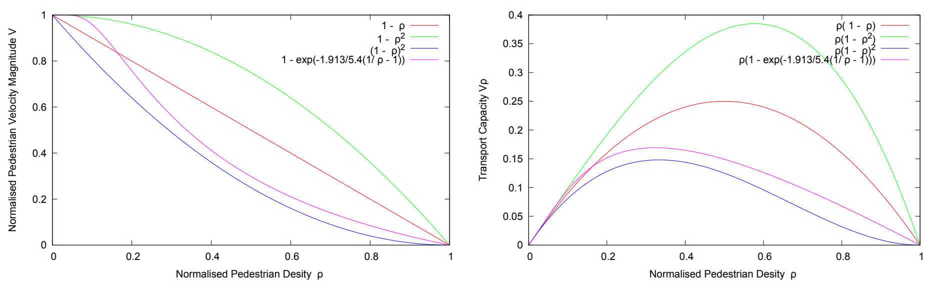

Fig. 2 Left: V-Profiles of (3) – (6) in Comparison; Right: Transport Capacities for the V-Profiles Given

3.1 Fundamental Diagrams as a Matter of Scale.

The deciding factor for the walking speed of a pedestrian is his/her *perception* (see Assumption 1) of the density. So for large scales the perceived and the material density are approximately the same. But the smaller the scale gets, the larger is the proportion that the pedestrian, who measures the density him/herself, takes him/herself of the total density (see [3]). The consequences are twofold. At the one hand the perception-relevant density is overestimated and at the other hand anisotropic effects underestimated. This implies a limit to the applicability of a macroscopic model of a classical design. The same considerations in their context have been done in the recent paper [8], where a division into "test agents" and "field agents" has been introduced.

An extreme example is the following. If we approach a spatial refinement, where one person fits one cell (as in cellular automata), or even beyond that, we get the curious effect, that the person can't move because it "feels" its own sole presence as "too crowded to be able to move" because $\rho_{(i)} = 1$ in at least one cell. This in turn leads to the disintegration of the person into a veil of mass with $\rho_{(i)} < 1$ moving through the computational domain (if it does not stay in a frozen state at all). A way to resolve this problem for such scales could be to measure the density by the introduction of perceptional anisotropic-stenciled measures (see [1] for instance). But the use of this technique is confined to appropriately small scales (see Sect. 2).

An implementational trick is applied to introduce a certain amount of anisotropy (appropriate for sufficiently large scale simulations), by using downstream interpolation for V with respect to d_i. This way to introduce anisotropy possibly underestimates this factor for intermediate scale and surely does so for small scale simulations (see Sect. 2).

4 Simplified Flux-Capacity Consideration.

To get a first estimate for the possible flux-capacity of a certain area, we do the following simplified considerations.

Taking the estimates in Fig. 2, and assuming, that the direction of encounter is irrelevant, we see that the maximum flux is to be expected at about a quarter of the maximum density. This is about 1.35 pedestrians/m^2. Every value higher than that leads to congestion. So for a setting of n pedestrian species facing in a confined area, the per pedestrian species density might be at most $1.35/n$ pedestrians/m^2.

5 Improving the Fundamental Diagram.

Due to the fact, that the acquisition of a measured fundamental diagram that is more realistic with respect to angles of crossing directions of pedestrians is not to be expected in the near future, heuristic approaches could be investigated.

5.1 Angles of Encounter.

So, here we take up the thread from Sect. 3 concerning the effects shown in Fig. 1. When considering the question if there is a possibility of the formation of stable patterns, the answer (for straight walking directions in two dimensions) is, that for any other angle, than $k\pi$ ($k \in \mathbb{Z}$) such patterns can't exist. So the superficially counter-intuitive result is, that the worst case is an encounter with an angle of $(k + 1/2)\pi$. The question if the law is expected to be

$$\delta_{ij} = |\sin \angle^{d_i}_{d_j}| \tag{7}$$

or rather

$$\delta_{ij} = 1 - |\cos \angle^{d_i}_{d_j}| \tag{8}$$

or something even more or less complex, remains to be investigated. Our suspicion is, that it should be rather (8), than (7). This factor could be introduced by a factor to get a pedestrian-specific corrected velocity V_i^c by

$$V_i^{\mathrm{c}} = \left[\prod_{j \in \{1,\ldots,n\},\, j \neq i} (1 - l\delta_{ij} \frac{\rho_j}{\rho}) \right] V$$

where $l \in [0, 1]$ and could suspectedly be approximately 0.5.

5.2 Influence of Pedestrian Mixture.

The next part of this consideration is the ρ_i to ρ_j mixture. This should be expected to be asymmetric in a way, that the weaker flux is subject to stronger obstruction due to the impact of sheer mass of the counter-flux. A simple law could be to introduce a factor $m + (1 - m)\frac{\rho_i}{\rho}$, where $m \in [0, 1]$ and probably not less than 0.5.

This considerations lead to a definition of a pedestrian-dependant velocity magnitude:

$$\tilde{V}_i = \tilde{V}_i(\rho_i, \ldots, \rho_n, d_1, \ldots, d_n) = (m + (1 - m)\frac{\rho_i}{\rho})V_i^{\mathrm{c}}$$

5.3 Structure-Emerging Relaxation Consideration.

The idea, that structures take time to emerge and take effect at the one hand and may break down instantaneously at the other, could be expressed by:

$$\frac{\partial}{\partial \vartheta} \hat{V}_i(x) + \beta(\hat{V}_i - \tilde{V}_i) = 0$$

with $\beta > 0$. To account for the asymmetric behavior with respect to the breakdown of structures a definition of:

$$V_i(x) := \min\{\hat{V}_i(x), \tilde{V}_i(x)\} \tag{9}$$

seems to be prudent.

5.4 Influence of Scales.

A further open question is the better adaption of the fundamental diagram to smaller scales (see Sect. 2), where the approach of e.g. [1] fits better.

6 Conclusion

The proposed heuristics-based adaptations of the fundamental diagram to multi-directional pedestrian streams should be subject to real life experiment validation and parameter adaptation.

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