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Prime Coloring of Crossing Number Zero Graphs

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ABSTRACT

In this paper, prime coloring and its chromatic number of some crossing number zero graphs are depicted and its results are vali–dated with few theorems. Prime Coloring is defined as G be a loop less and Without multiple edges with n distinct Vertices on Color class C={c_1,c_2,c_3,…..c_n} a bijection ψ :V {c_1,c_2,c_3,…..c_n} if for each edge e = c_ic_j, i≠j , gcd{ ψ (c_i), ψ (c_j)}=1, ψ (c_i) and ψ (c_j) receive distinct Colors. The Chromatic number of Prime coloring is minimum cardinality taken by all the Prime colors. It is denoted by $\eta(G)$.

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1. Introduction

We use the terminology as Bondy.J.A. and Murty U.S.R (1976). Let G= (V, E) be simple graph with crossing number zero i.e (Cr(G)=0) and here order of V be| V |= n. In this paper, we discuss Prime graphs and Coloring. The Prime labeling emanate with Entringer *et al.* 1980. Be the graph G = (V, E) with vertex set V is said to have a Prime labeling if there exists a bijection f: V(G) →Z^+ –{0}, Z^+ is the set of positive integers such that for each edge $xy \in E(G)$,gcd {f(x),f(y)}=1. It begin to be an interesting problem to investigate the Prime coloring as possible in a graph with n vertices. Moreover, the general graph G is proper colorable but not prime graph forever. The Coloring of a graph is explained as pair wise adjacent vertices receive conflict colors.

2. Prime Coloring of Crossing Number Zero Graphs

In this entire paper, we consider ψ is the bijective function and ψ' is the Chromatic number of G as well as η is also chromatic number of prime graph.

Theorem 2.1
Every path P_n graph is Prime if it is Prime colorable graph.

Proof:
Let the path P_n with n distinct vertices, the bijection ψ :V(P_n)→{ c_1,c_2,c_3,…..c_n } graph if for each edge e = c_ic_j , gcd{ ψ (c_i), ψ (c_j)}=1, i≠j,
ψ (c_i) =c_i
ψ (c_j)=c_j , receive distinct colors. Then the path P_n Graph is Prime colorable graph.

Corollary 2.1.1: If path P_n graph is prime then
$\eta(P_n) = 2$.

Example 2.1: P_n is Prime colorable graph

Cr(G)=0, gcd { ψ (c_i), ψ (c_j)}=1, $\eta(P_n) = 2$.

Figure 1

Theorem 2.2: Every Cycle C_n, n≥4 if it is Prime colorable graph.

Proof: Assume that Cycle C_n on n≥4 vertices with the bijection ψ :V(C_n) → {c_1,c_2,c_3,…..c_n} graph if for each edge e = c_ic_j, gcd{ψ (c_i),ψ (c_j)}=1,i≠j ,

ψ (c_i) =c_i
ψ (c_j)=c_j

receive distinct colors on cycle C_n on n≥4 .

Corollary 2.2.1: If Cycle graph C_n, n≥4 is Prime,

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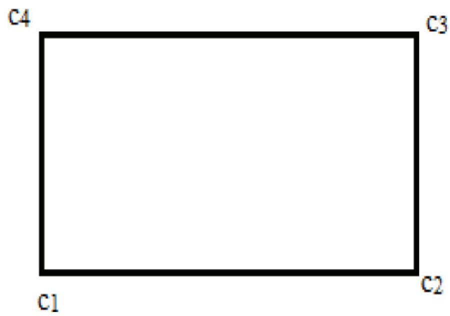
then $\eta(C_n) = \begin{cases} 2, n \ \ is\, even \\ 3, n \ \ is\, odd \end{cases}$

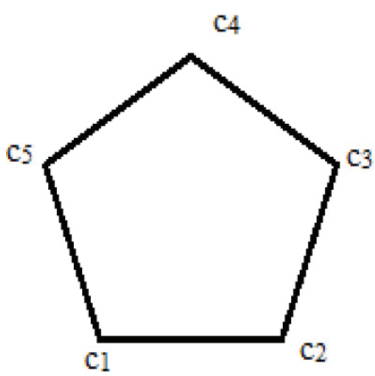
Example 2.2: Cycle C_4



Cr(G)=0, gcd {ψ (c_i), ψ (c_j)}=1, $\eta(C_4) = 2$

Figure 2

Cycle C_5



Cr(G)=0,gcd {ψ (c_i), ψ (c_j)}=1, $\eta(C_5) = 3$

Figure 3

Theorem 2.3 If a Complete graph K_n, n≤3 is Prime graph, then K_n is Prime colorable graph with n≤3.

Proof

Proof by induction n=1 obviously holds.

n=2 then the graph is a path by the theorem (2.1) holds.

n=3 then the graph is triangle every gcd {ψ (c_i), ψ (c_j)} = 1, i≠j then the graph is prime and also receive distinct colors.

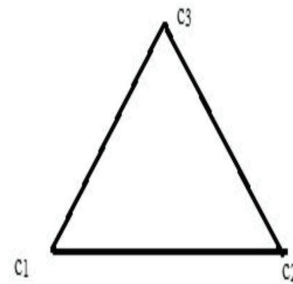
n=4 then the vertices assigns distinct colors but gcd {ψ (c_i), ψ (c_j)} ≠ 1. But e = $c_i c_j$, then the graph is not a prime. Therefore n>3, K_n is not a prime colorable graph.

Corollary 2.3.1 Every Complete graph K_n is Proper Colorable but not Prime colorable.

Consider K_n is complete graph pair wise adjacent vertices are connected by the definition of graph coloring it has to assign distinct colors, but the labeled graph K_n with {1,2,….n} ,gcd (1,n) not equal to 1 for n ≥3, K_n is not Prime graph. Thus K_n is not a Prime colorable

Example 2.3

The complete graph K_3 is Prime Cloring graph.



Cr(G)=0,gcd {ψ (c_i), ψ (c_j)}=1, $\eta(K_3) = 3$.

Figure 4

Complete graph K_4

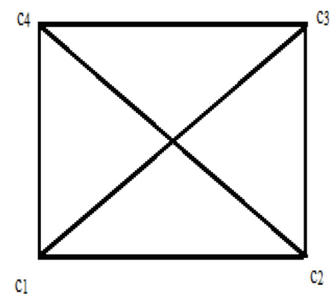


Figure 5

Here Cr(G)≠0 ,
gcd {ψ (c_1), ψ (c_2)}=1, gcd {ψ (c_1), ψ (c_4)}=1,
gcd {ψ (c_1), ψ (c_3)}=1,But gcd{ψ (c_2), ψ (c_4)}≠1.
consider the general K4 is proper colorable with Chromatic number $\psi'(K_4) = 4$,But $\eta(K_4)$ not defined.
Complete graph K_5

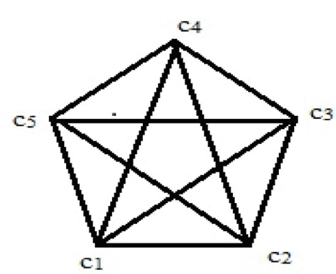


Figure 6

Here, Cr(G)≠0
gcd {ψ (c_1), ψ (c_2)}=1,

gcd $\{\psi(c_1), \psi(c_4)\}=1$,
gcd $\{\psi(c_1), \psi(c_3)\}=1$,
gcd $\{\psi(c_1), \psi(c_5)\}=1$,
gcd $\{\psi(c_3), \psi(c_5)\}=1$.

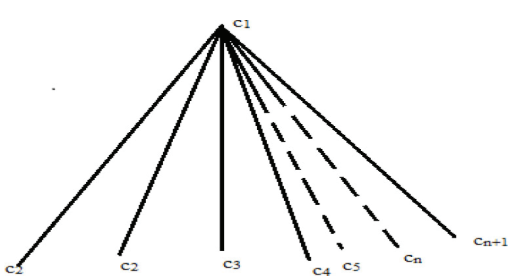
But gcd $\{\psi(c_2), \psi(c_4)\}\neq 1$.consider the General K_5 is proper colorable with chromatic number $\psi'(K_5)=5$. But $\eta(K_5)$ not defined.

Theorem 2.4 If Star graphs $K_{1,n}$ is Prime graph then $\eta(K_{1,n})=2$.

Proof

Let $V=\{c_1,c_2,c_3\ldots\ldots c_{n+1}\}$ vertices of Star graph $K_{1,n}$. The graph labeled as well as colored, by the bijection $\psi:V(K_{1,n}) \to \{c_1,c_2,c_3,\ldots..c_{n+1}\}$ graph if for each edge e= c_ic_j ,gcd$\{\psi(c_i), \psi(c_j)\}=1$, $i\neq j$ $\psi(c_i)$ and $\psi(c_j)$ receive distinct colors .Therefore the coloring pairs of are the distinct. The coloring pairs are at most 2 and also every gcd (i,j)=1 is a prime graph.
Thus,
$\psi(c_i)=c_i$
$\psi(c_j)=c_j$, for $1\leq i,j\leq n+1$ then every $i\neq j$ is Prime colorable. Therefore minimum coloring needed for the Star graph is $\eta(K_{1,n})=2$.

Example 2.4: Star graph $K_{1,n}$



Cr(G)=0,gcd $\{\psi(c), \psi(c_j)\}=1$, $\eta(K_{1,n})=2$.

Figure 7

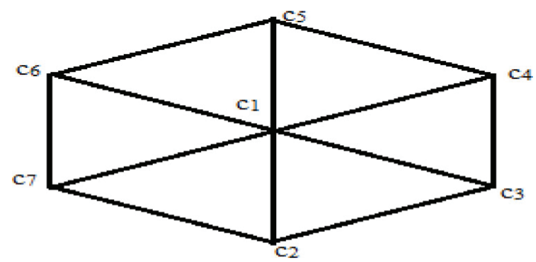
Theorem 2.5 If Wheel graph W_n, $n\geq 4$ is Prime graph, then $\eta(W_n)=4$. when n is even.

Proof

Let V= $\{c_1,c_2,c_3\ldots\ldots c_{k+1}\}$ vertices of wheel graph W_n. The graph labeled as well as colored.
By the bijection $\psi:V(W_n) \to \{c_1,c_2,c_3,\ldots..c_{k+1}\}$graph if for each edge e = c_ic_j , gcd$\{\psi(c_i),(c_j)\}=1$, $i\neq j$,$\psi(c_i)$ and $\psi(c_j)$ receive distinct colors. Therefore the coloring pairs of are the distinct. Therefore the coloring pairs are at most four and also gcd(i,j)=1 is a prime graph.
Thus,$\psi(c_i)=c_i$
$\psi(c_j)=c_j$ for $1\leq i,j\leq k+1$

Then every $i\neq j$ is prime colorable. Therefore minimum coloring needed for the Wheel graph is $\eta(K_{1,n})=2$.

Example 2.5 Wheel graph W_5



Cr(G)=0,gcd$\{\psi(c_i), \psi(c_j)\}=1$ $\eta(W_n)=4$.

Figure 8

3. Main Results

Lemma 3.1: G is Prime graph $\Leftrightarrow$ G is Prime colorable graph.

Proof

Let us assume G is prime label with the bijection $\psi:V(G) \to \{c_1,c_2,c_3,\ldots..c_n\}$graph if for each edge w = c_ic_j, gcd $\{\psi(c_i), \psi(c_j)\}=1$, $i\neq j$,$\psi(c_i)$ & $\psi(c_j)$ receives distinct colors. Now coloring the vertices is proper coloring. Then the resulting graph with different colors. Conversely G is Prime coloring by the definition of graph coloring holds it is a prime graph.

Corollary 3.1.1 If G is Prime Graph then $\psi'(G)=\eta(G)$

Corollary 3.1.2 Every spanning sub graph of prime graph is prime colorable graph.
Seoud and yousse et.al (1999) proved every spanning sub graph is prime graph. Now coloring proper by the graph become an prime colorable gcd$\{\psi(c_i), \psi(c_j)\}=1$, $i\neq j$, $\psi(c_i)$ and $\psi(c_j)$ receive distinct colors. We observe that every spanning sub graph is also prime colorable.

Corollary 3.1.3: Every tree is prime colorable graph.
Every tree is prime is proved by Entringer $n\leq 50$ vertices . The tree have been properly colored as the color class C=$\{c_1,c_2,c_3,\ldots..c_n\}$. Tree is a spanning sub graph of the general graph G by the above corollary **(3.1.1)** holds, there for every tree is prime colorable.

Corollary 3.1.4: Every prime graph is proper colorable graph.

Corollary 3.1.5: Every graph G is proper Colorable but not prime color

Lemma 3.2
If $\psi'(P_n)=\eta(P_n)=2$, then the following are equivalent

i. P_n is Path.
ii. P_n is Prime graph.
iii. P_n is Prime colorable graph.

Proof

(i) $\Rightarrow$ (ii) let the path P_n with n distinct vertices then the chromatic number of the graph $\psi'(P_n)=2$ we have to prove P_n is a prime graph with gcd{ ψ (c$_i$), ψ (c$_j$)}=1, i≠j ,then P_n is a prime graph.

(ii) $\Rightarrow$ (iii)

By the theorem (2.1) refers to every prime graph P_n which assigned the distinct colors and by the definition of a prime color then $\eta(P_n) = 2$.

(iii) $\Rightarrow$ (i) from the proof given prime colorable graph $\psi'(P_n)=\eta(P_n) = 2$

Then n distinct vertices are connected as a path.

Lemma 3.3

If $\psi'(C_n)=\eta(C_n) = \begin{cases} 2, \text{ n is even} \\ 3, \text{ n is odd} \end{cases}$, then the following are equivalent

i. C_n is Cycle
ii. C_n is Prime graph.
iii. C_n is Prime colorable graph.

Proof

Case (i) n is odd:

(i) $\Rightarrow$ (ii)

Consider C_n be the cycle with distinct n vertices. If Cycle C_n, n>3 is an odd cycle with 2n−1 vertices (Samir K.Vaidya, Udayan M. Prajapati) (2012) *et. al.*, by the Fusion, switching, duplication of the vertices than the graph is the prime graph.

(ii) $\Rightarrow$ (iii)

Consider C_n is prime graph when n is odd, be the fusion, switching, duplication is allowed then the graph is prime by the definition of prime coloring C_n is prime colorable graph.

(iii) $\Rightarrow$ (i)

By the definition prime coloring , all the vertices are distinct when n is odd i.e $c_1 = c_{2n-1}$, n>3 and holds $\psi'(C_n)=\eta(C_n)=3$.

Case (ii) n is even

(i) $\Rightarrow$ (ii) Clearly the even number of vertices on the cycle is prime and gcd { ψ (c$_i$), ψ (c$_j$)}=1, i ≠ j

(ii) $\Rightarrow$ (iii)

By the theorem (2.2) the graph gcd { ψ (c$_i$), ψ (c$_j$)}=1, i≠j, then $\psi'(C_n)=\eta(C_n)=2$ pair wise adjacent vertices receive distinct colors.

(iii) $\Rightarrow$ (i)

$\psi'(C_n)=\eta(C_n)=2$ with the Cycle C_n, n>3 we get $c_1 = c_{2n-2}, i \neq j$ vertices are distinct and even vertices of C_n is cycle.

Lemma 3.4 If G is the prime graph and Δ is the maximum degree of the graph then $\eta(G) \leq \Delta + 1$

Conclusion

This article discussed & absorbed the upshot of graph coloring technique which associated with the prime labeled graph. We elucidated the coloring techniques of some crossing number zero graphs and also evinced the some interesting results obtained by Prime Coloring graphs.

References

Bondy, J. A., Murty U.S.R.: Graph Theory with Applications, Macmillan, London (1976).

Galian, J. A.: A Dynamic Survey of Graph Labeling. Electronic Journal of Combinatorics, **16**, #DS6 (2009).

Lee, S.M., Wui, J., Yeh, I.: On the Amalgamation Of Prime Graphs. Bull, Malaysian, Math.Soc. (Second Series), **11**, 59–67 (1988).

Oleg, P.: Trees are Almost Prime. Discrete Mathematics Elsevier **307**, 1455–1462 (2007). https://doi.org/10.1016/j.disc.2005.11.083

Samir K. V., Udayan, M. P.: Some New Results on Prime Graphs. Open Journal Of Discrete Mathematics, **2**, 99-104 (2012). http://dx.doi.org/10.4236/ojdm.2012.23019

Thomas, K.: Elementary Number Theory with Applications, Elsevier, 103–115 (2005).

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