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Commutativity of Prime Ring with Orthogonal Symmetric Biderivations

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ABSTRACT

The intention of the present research article is to generalize the performance of prime rings (commutativity) with certain algebraic identities using jordan ideals. Familiar results characterizing commutativity of prime ring with orthogonal biderivations have been discussed here with Jordan ideals. Whenever some biderivations of prime ring satisfying certain commutator relations $\left[B_1(u,v),B_2(v,w)\right]=[u,w]$, $B_3(v,w)B_1(w,u)-B_2(w,u)B_3(v,w)=[u,v]$, for all $u,v,w\in J$ then that ring is commutative.

1. Introduction

More than a few authors, investigated the structure prime ring & semiprime rings (commutativity) accepting the derivations, generalized derivations etc. The notion of derivations of prime rings was originated by (Posner 1957), jordan derivations of prime rings was originated by (cusack 1975). These derivations was extended by (Bell and Daif 1995) for commutativity of prime rings. Later on (Bresar 1993) used centralizing concept using derivations. These generalizations was done in the article derivations using semiprime rings with results are commutative by (Daif 1998). The concept of symmetric biderivations on prime and semiprime rings was introduced by (Vukman 1989).The notation and terminology in this paper follows (Vukman 1990 and Oukhtite 2011). Many authors have their contribution to orthogonality of derivations on semiprime as well as prime rings. The idea of orthogonality of derivations on semiprime as well as prime rings was developed by (Vukman and Bresar 1989). (Argac 2004) studied orthogonality conditions for generalized derivations. (Ashraf 2010) obtained the orthogonality conditions for a pair of derivations in gamma rings. with their results (Jaya Subba Reddy *et. al.* 2016) obtained the essential and sufficient conditions of biderivations to be orthogonal. (Oukhtite *et. al.* 2014) proved the commtativity results of prime rings with derivations using jordan ideals. In this current study it was extended the results of commutativity of prime rings with orthogonal biderivations using Jordan ideals. In the present article we studied some theorems related to commutativity of prime rings using commutator identities satisfied by biderivations with Jordan ideals. We established the following theorems as follows.

Theorem 1: Any two biderivations B and B_2 satisfies the condition $\left[B_1(u,v),B_2(v,w)\right]=[u,w]$, where $u,v,w\in J$ then B_1 and B_2 are orthogonal and thus R is commutative.

Theorem 2: Any three nonzero biderivations B_1, B_2 and B_3 of R satisfies one of the following

(i) $B_3(v,w)B_1(w,u)=B_2(w,u)B_3(v,w)$,

(ii) $B_3(v,w)B_1(w,u)-B_2(w,u)B_3(v,w)=[u,v]$, for all $u,v,w\in J$ then is commutative and $B_1=B_2$.

2. Preliminaries

In each part of this article all rings assumed to be associative and possesses an identity. As a well-known the commutator $(uv - vu)$ will be symbolized as $[u, v]$. We are wellknown that R is a prime ring if $uRv = 0 \Rightarrow u = 0$ or $v = 0$ and is semiprime if $uRu = 0 \Rightarrow u = 0$. If $D(uv) = D(u)v + vD(u)$, for any $u,v\in R$ then we call this additive map $D:R\to R$ is a derivation. We Defined, biadditive mapping $B(.,.):R\times R\to R$ as a symmetric biderivation if $B(uv,r)=B(u,r)v+uB(v,r)$, for any $u,v,r\in R$. Clearly, in next case also $B(u,vr)=B(u,v)r+vB(u,r)$, for every $u,v,r\in R$. Any pair (d, g) of derivations

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are orthogonal if d(u)Rg(v) = 0 = g(v)Rd(u) for any $u, v \in R$ (Vukman and Bresar 1989). Likewise, any pair (*B, D*) of biderivations are said to be orthogonal if $B(u,v)RD(v,r) = (0) = D(v,r)RB(u,v)$ for all $u, v, r \in R$. If $u \cdot x \in J$, for any $u \in J, x \in R$, then we say J is a Jordan ideal of R. Note that B(*x*) means *B(x,m)* means for some $m \in J$.

In the entire paper R act as a prime ring with 2-torson free & $J \neq 0$ is a jordan ideal of R

Following are known results to the readers

Res 1: If $[a, u^2] = 0$ for any $u \in J$, then is in center of R.

Res 2: If an additive subgroup is a subset of Z(R), then R is commutative ring.

Res 3: a non commutative ring R satisfies $a[u, vw]b = 0$, for every $v, w \in J$, $u \in R$, then a = 0 or b = 0.

We studied the following lemmas for proving the main theorems

Lemma 2.1(Reddy C.J.S and Reddy B.R 2016)

A semiprime ring *R* of characteristic not two, a pair of biderivations B_1 and B_2 are to be orthogonal ⇔ the following results are equivalent:

(1) $B_1B_2 = 0$

(2) $B_1(u,v)B_2(v,w) = 0$ or $B_2(u,v)B(v,w) = 0$.

(3) B_1B_2 is a Biderivation.

(4) $B_1(u,v)B_2(v,w) + B_2(v,w)B_1(u,v) = 0$, for every u,v,w. $u \in R$

Lemma 2.2

Any two biderivations B_1 and B_2 satisfies the condition $B_1(B_2(u,v) - u) = 0$, for every pair $u, v \in J$, then orthogonality of B_1 and B_2 are satisfied, also either $B_1 = 0$ or $B_2 = 0$.

Proof: Consider $B_2(u,v) \neq 0$

We have $B_1B_2(u,v) = 0$, for any $u, v \in J$ (1)

By using lemma 2.1 B_1, B_2, are orthogonal, that is

$B_1(u,m)B_2(v,w) + B_2(u,w)B_1(v,m) = 0$, for any $m \in J$. (2)

Put [s, pq]y instead of *v*, for any p,q $\in J, s \in R$ in the equation (2) and use (2), to get

$$B_1(u,m)B_2([s,pq]v,w) + B_2(u,w)B_1([s,pq]v,m) = 0 \quad (3)$$

$$B_1(u,m)[s,pq]B_2(v,w) + B_2(u,w)[s,pq]B_1(v,m) = 0$$

Replace *v* by *vt* for some $t \in J$ in the equation (3), to obtain (4)

$$B_1(u,m)[s,pq]vB_2(t,w) + B_2(u,w)[s,pq]qB_1(t,m) = 0 \quad (4)$$

Writing B_2 *(u, v)* instead of t in the equation (4), to get the following

$$B_1(u,m)[s,pq]vB_2^{\,2}(u,v) = 0 \quad (5)$$

With the use of Res 3, since $B_2(u,v) \neq 0$, either R is commutative or $B_1(u,m) = 0$.

Assume that R is commutative, then replacing u by u^2 in the equation (1), we get

$$2B_2(u,v)B_1(u,w) = 0. \quad (6)$$

Thus either $B_2(u,v) = 0$ or $B_1(u,w) = 0$. Therefore in all the cases $B_1 = 0$.

Lemma 2.3

Any two biderivations B_1 and B_2 satisfies the condition $B_1(B_2(u,v) - u) = 0$, for every pairs $u, v \in J$, then orthogonality of B_1 and B_2 are satisfied and also B_1 is zero.

Proof: consider for every $u, v \in J$, $B_1(B_2(u,v) - u) = 0$. (7)

If R is commutative, substitute u by u^2 in equation (7), we get

$$B_1(u,m)B_2(u,v) = 0. \quad (8)$$

Already a well known result, from the definition, a prime ring itself an integral domain, so the equation (8) reduces to $B_1(u,m) = 0$ or $B_2(u,v) = 0$.

If $B_2(u,v) \neq 0$ then $B_1(u,m) = 0$ and let *R* is non commutative and $B_2(u,v) \neq 0$. Put *u* by *up* in the equation (7), where $p \in J$, find that

$$B_1(u,w)B_2(p,v) + B_2(u,v)B_1(p,w) = 0. \quad (9)$$

From lemma 2.1, B_1 and B_2 are orthogonal.

One can see that the equation (9) is same as compared with equation (2) so using the above lemma, we conclude $B_1(u,v) = 0$.

Lemma 2.4

Any two biderivations B_1 and B_2 satisfies the condition $B_1(u,v)B_2(v,w) = [u,w]$, for any $u, v, w \in J$, then either $B_1 = 0$ or $B_2 = 0$, and also R is commutative.

Proof: Consider for every $u, v, w \in J, B_2(u,v) \neq 0$, we have

$$B_1(u,v)B_2(v,w) = [u,w]. \quad (10)$$

Replacing *w* by *wm*, for any $m \in J$, in the equation (10), to get

$$B_1(u,v)wB_2(v,m) = w[u,m]. \quad (11)$$

Replacing w by $[s,t]w$ for some $[s,t]w \in J$, where $s,t \in R$ in the equation (11), to get.

$$B_1(u,v)[s,t]wB_2(v,m)=[s,t]w[u,m] \quad (12)$$

Left multiplication of (11) by [*s, t*], to get

$$[s,t]B_1(u,v)wB_2(v,m)=[s,t]w[u,m]. \quad (13)$$

From equation (12) and equation (13), we get $[B_1(u,v),[s,t]]wB_2(v,m)=0$. Since $B_2(v,m)\neq 0$, the primeness of R, implies that

$$[B_1(u,v),[s,t]]=0\cdot \quad (14)$$

So $B_1(u,v)$ is commuting, then Bresar (Bresar 1993), gives that *R* is commutative and equation (10) becomes $B_1(u,v)B_2(v,w)=0$. And also B_1 and B_2 are orthogonal biderivations. (15)

Because of $B_2(v,w)\neq 0$, leads to $B_1(u,v)=0$

Using (10), we conclude that $[u,w]=0$, therefore R is commutative.

3. Main Theorems

Theorem 3.1

Any two biderivations B_1 and B_2 satisfies the condition $[B_1(u,v),B_2(v,w)]=[u,w]$, for every $u,v,w\in J$ then B_1 and B_2 are orthogonal thus R is commutative.

Proof: If $B_1=0$ or $B_2=0$, then the given condition becomes $[u,w]=0$, for any $u,w\in J$, so *R* is commutative. Next consider B_1 and B_2 are nonzero biderivations implies

$$[B_1(u,v),B_2(v,w)]=[u,w], \text{ for all } u,v,w\in J. \quad (16)$$

By replacing *w* with *wm*, $m\in J$ in the Eq. (16), to get

$$B_2(v,w)[B_1(u,v),m]+[B_1(u,v),w]B_2(v,m)=0. \quad (17)$$

By replacing *w* with *w*[*s, pq*] in the equation (17), to obtain

$$B_2(v,w)[s,pq][B_1(u,v),m]+[B_1(u,v),w][s,pq]B_2(v,m)=0,$$

for every $p,q\in J, s\in R$ (18)

By replacing *w* with tw for some $t\in R$, in (18), we get

$$B_2(v,t)w[s,pq][B_1(u,v),m] + [B_1(u,v),t]w[s,pq]B_2(v,m)=0. \quad (19)$$

By replacing t with $B_1(u,v)t$ in the equation (19), we get

$$B_2B_1(u,v)tw[s,pq][B_1(u,v),m]=0,$$

$$B_2B_1(u,v)R[s,pq][B_1(u,v),m]=0\cdot \quad (20)$$

Using the primeness of R, either $B_2B_1=0$ which implies that orthogonality of biderivations B_1 and B_2 are satisfied using lemma 2.1. Now from the Res 3 and using lemma 2.2, either $B_1=0$ or $B_2=0$ then R is commutative. Otherwise $[s,pq][B_1(u,v),m]=0$. In such case also is commutative.

Corollary 3.1

Any two biderivations $B_1(u,v)$ and B_2 (v, w) satisfies the condition $[B_1(u,v),B_2(v,w)]=0$, for every $u,v,w\in J$ then R is commutative.

Proof: Given condition that $[B_1(u,v),B_2(v,w)]=0$, for every $u,v,w\in J$. (21)

By replacing *w* with *wm* in the equation (21), we get

$$[B_1(u,v),B_2(v,wm)]=0$$

$$[B_1(u,v),w]B_2(v,m)+B_2(v,w)[B_1(u,v),m]=0 \quad (22)$$

We observed that the equation (22) and equation (17) are identical, continuing the procedure as we done in the theorem 3.1, it is clear that R is commutative.

Theorem 3.2

For any three nonzero biderivations B_1, B_2 and B_3 of R satisfies one of the conditions (i) $B_3(v,w)B_1(w,u)=B_2(w,u)B_3(v,w)$, (ii) $B_3(v,w)B_1(w,u)-B_2(w,u)B_3(v,w)=[u,v]$, for every $u,v,w\in J$ then R is commutative and $B_1=B_2$.

Proof: (i) Let us consider the condition

$$B_3(v,w)B_1(w,u)-B_2(w,u)B_3(v,w)=0\cdot \quad (23)$$

Replacing *u*m² in place of *u* in the Eq. (23), to get

$$B_3(v,w)B_1(w,u)m^2+B_3(v,w)u(w,m^2) \\ -uB_2(w,m^2)B_3(v,w) \\ -B_2(w,u)m^2B_3(v,w)=0$$

By using the equation (23), we get

$$B_2(w,u)[B_3(v,w),m^2] \\ +[B_3(v,w),u]B_1(w,m^2)=0 \quad (24)$$

Replacing *u* by *u*[*s, pq*] in the equation (24), we get

$$B_2(w,u)[s,pq][B_3(v,w),m^2] \\ +[B_3(v,w),u][s,pq]B_1(w,m^2)=0. \quad (25)$$

Replacing *u* with tu² in the equation (25),to get

$$B_2(w,t)u^2[s,pq][B_3(v,w),m^2] \\ +[B_3(v,w),t]u^2[s,pq]B_1(w,m^2)=0. \quad (26)$$

Replacing t with B_3(v, w)t in equation (26), to get

$$B_2B_3(v,w)Ru^2[s,pq][B_3(v,w),m^2]=0. \quad (27)$$

It is clear that (27) and (20) are identical, hence we conclude that R is commutative then equation (23) becomes

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$$B_3(v,w)R[B_1(w,u)-B_2(w,u)]=0 \quad (28)$$

Since B_3(q, r) is non zero, $B_1(w,u)=B_2(w,u)$.

(ii) Consider B_1 and B_2 are non zero biderivations such that $B_3(v,w)B_1(w,u)-B_2(w,u)B_3(v,w)=[u,v]$. (29)

Replacing u with um² in the equation (29), we get

$$B_3(v,w)B_1(w,um^2)-B_2(r,um^2)B_3(v,w)=[um^2,v]$$

$$B_2(w,u)[B_3(v,w),m^2]+[B_3(v,w),u]B_1(w,m^2)=0. \quad (30)$$

It is clear that the equation (30) and the equation (24) are identical, proceeding the same procedure as in (i) it is clear that R is commutative, then condition (ii) becomes

$$B_3(v,w)R[B_1(w,u)-B_2(w,u)]=0$$

Since $B_3(v,w)\neq 0$, leads to $B_1=B_2$. (31)

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