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Bayesian Repetitive Deferred Sampling Plan Indexed Through Relative Slopes

K.K. Suresh, S. Umamaheswari and K. Pradeepa Veerakumari

Department of Statistics, Bharathiar University,
Coimbatore, Tamilnadu, India.

Abstract

This paper deals with designing of Bayesian Repetitive Deferred Sampling Plan (BRDS) indexed through incoming and outgoing quality levels with their relative slopes on the OC curve. The Repetitive Deferred Sampling (RDS) Plan has been developed by Shankar and Mohapatra (1991) and this plan is an extension of the Multiple Deferred Sampling Plan MDS - (c_1 , c_2) , which was proposed by Rambert Vaerst (1981).

Keywords: Bayesian Sampling Plan, Acceptable Quality Level (AQL), Limiting Quality Level (LQL), Indifference Quality Level (IQL), Relative Slopes, Gamma-Poisson Distribution.

1. INTRODUCTION

Acceptance sampling uses sampling procedure to determine whether to accept or reject a product or process. It has been a common quality control technique that used in industry and particularly in military for contracts and procurement of products. It is usually done as products that leave the factory, or in some cases even within the factory. Most often a producer supplies number of items to consumer and decision to accept or reject the lot is made through determining the number of defective items in a sample from that lot. The lot is accepted, if the number of defectives falls below the acceptance number or otherwise, the lot is rejected. Acceptance sampling by attributes, each item is tested and classified as conforming or non-conforming. A sample is taken and contains too many non-conforming items, then the batch is rejected, otherwise it is accepted. For this method to be effective, batches containing some non-conforming items must be acceptable. If the only acceptable percentage of non-conforming items is zero, this can only be achieved through examing every item and removing the item which are non-conforming. This is known as 100% inspection. Effective acceptance sampling involves effective selection and the application of specific rules for lot inspection. The acceptance-sampling plan applied on a lot-by-lot basis becomes an element in the overall approach to maximize quality at minimum cost. Since different sampling plans may be statistically valid at different times during the process, therefore all sampling plans should be periodically reviewed.

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The general problem for process control is one towards maintaining a production process in such a state that the output from the process confirms to design specifications. As the process operates it will be subject to change, which causes the quality of the output to deteriorate. Some amount of deterioration can be tolerated but at some point it becomes less costly to stop and overhaul the process. The problem of establishing control procedures to minimize long-run expected costs has been approached by several researchers through Bayesian decision theory.

Classical analysis is directed towards the use of sample information. In addition to the sample information, two other types of information are typically relevant. The first is the knowledge of the possible consequences of the decision and the second source of non sample information is prior information. Suppose a process a series of lots is supplying product. Due to random fluctuations these lots will be differing quality, even though the process is stable and incontrol. These fluctuations can be separated into within lot variation of individual units and between lot variations.

2. BAYESIAN ACCEPTANCE SAMPLING

Bayesian Acceptance Sampling approach is associated with utilization of prior process history for the selection of distributions (viz., Gamma Poisson, Beta Binomial) to describe the random fluctuations involved in Acceptance Sampling. Bayesian sampling plans requires the user to specify explicitly the distribution of defectives from lot to lot. The prior distribution is the expected distribution of a lot quality on which the sampling plan is going to operate. The distribution is called prior because it is formulated prior to the taking of samples. The combination of prior knowledge, represented with the prior distribution, and the empirical knowledge based on the sample which leads to the decision on the lot.

A complete statistical model for Bayesian sampling inspection contains three components:

1. The prior distribution (i.e.) the expected distribution of submitted lots according to quality.
2. The cost of sampling inspection, acceptance and rejection.
3. A class of sampling plans that usually defined by means of a restriction designed to give a protection against accepting lots of poor quality.

Risk-based sampling plans are traditional in nature, drawing upon producer and consumer type of risks as depicted by the OC curve. Economically based sampling plans explicitly consider certain factors as cost of inspection, accepting a non-conforming unit and rejection a conforming unit, in an attempt to design a cost-effective plan. Bayesian plan design procedures take into account the past history of similar lots submitted previously for the inspection purposes. Non-Bayesian plan design methodology is not explicitly based upon the past history.

Case and Keats (1982) have examined the relationship between defectives in the sample and defectives in the remaining lot for each of the five prior distributions. They observe that the use of a binomial prior renders sampling useless and inappropriate. These serve to make the designers and users of Bayesian sampling plans more aware of the consequence associated with selection of particular prior distribution.

Calvin (1984) has provided procedures and tables for implementing Bayesian Sampling Plans. A set of tables presented by Oliver and Springer(1972)which are based on assumption of Beta prior distribution with specific posterior risk to achieve minimum sample size, which avoids the problem of estimating cost parameters. It is generally true that Bayesian Plan requires a smaller sample size than a conventional sampling plan with the same producer and consumer risk. Scafer (1967) discusses single sampling by attributes using three prior distributions for lot quality. Hald (1965) has given a rather complete tabulation and discussed the properties of a system of single sampling attribute plans obtained by minimizing average costs, under the assumptions that the costs are linear with fraction defective p and that the distribution of the quality is a double binomial distribution. The optimum sampling plan (n, c) depends on six parameters namely N, p_r, p_s, p_1, p_2 and w_2 where N is the lot size, p_r, p_s are normalized cost parameters and p_1, p_2, w_2 are the parameters of prior distribution. It may be shown, however that the weights combine with the p's is such a way that only five independent parameters are left out.

The Repetitive Deferred Sampling Plan has been developed by Shankar and Mohapatra (1991) and this plan is essentially an extension of the Multiple Deferred Sampling Plan MDS - (c_1, c_2) which was proposed by Rambert Vaerst (1981). In this plan the acceptance or rejection of a lot in deferred state is dependent on the inspection results of the preceding or succeeding lots under Repetitive Group Sampling (RGS) inspection. RGS is a particular case of RDS plan. Vaerst (1981) has modified the operating procedure of the MDS plan of Wortham and Baker (1976) and designed as MDS-1.Wortham and Baker (1976) have developed Multiple Deferred State Sampling (MDS) Plans and also provided tables for construction of plans. Suresh (1993) has proposed procedures to select Multiple Deferred State Plan of type MDS and MDS-1 indexed through producer and consumer quality levels considering filter and incentive effects.

Vedaldi (1986) has studied the two principal effects of sampling inspection which are filter and incentive effect for attribute Single Sampling Plan and also proposed a new criterion based on the (AQL, $1-\alpha$) point of the OC curve and an incentive index. Lilly Christina (1995) has given the procedure for the selection of RDS plan with given acceptable quality levels and also compared RDS plan with RGS plan with respect to operating ratio (OR) and ASN curve. Suresh and Pradeepa Veerakumari (2007) have studied the construction and evaluation of performance measures for Bayesian Chain Sampling Plan (BChSP-1). Suresh and Saminathan (2010) have studied the selection of Repetitive Deferred Sampling Plan through acceptable and limiting quality levels. Suresh and Latha (2001) have studied Bayesian Single Sampling Plan through Average Probability of Acceptance involving Gamma-Poisson model.

The operating ratio was first proposed by Peach (1947) for measuring quantitatively the relative discrimination power of sampling plans. Hamaker (1950) has studied the selection of Single Sampling Plan assuming that the quality characteristics follow Poisson model such that the OC curve passes through indifference quality level and the relative slope of OC curve at that quality level.

This paper related to Bayesian Repetitive Deferred Sampling Plan for Average Probability of Acceptance function for consumer's and producer's quality levels and relative slopes.

3. CONDITIONS FOR APPLICATION OF RDS PLAN

1. Production is steady so that result of past, current and future lots are broadly indicative of a continuing process.
2. Lots are submitted substantially in the order of their production.
3. A fixed sample size, n from each lot is assumed.
4 Inspection is by attributes with quality defined as fraction non-conforming.

4. OPERATING PROCEDURE FOR RDS PLAN

1. Draw a random sample of size n from the lot and determine the number of defectives (d) found therein.
2. Accept the lot if $d \leq c_{1,}$ Reject the lot if $d > c_2$.
3. If $c_1 < d < c_2$, Accept the lot provided i proceeding or succeeding lots are accepted under RGS inspection plan, otherwise reject the lot.

Here c_1 and c_2 are acceptance numbers such that $c_1 < c_2$. When i=1 this plan reduces to RGS plan.

The operating characteristic function P_a *(p)* for Repetitive Deferred Sampling Plan is derived by Shankar and Mohapatra (1991) using the Poisson Model as

$$p_a(p) = \frac{p_a(1-p_c)^i + p_c p_a^i}{(1-p_c)^i} \tag{4.1}$$

Where

$$p_a = p[d \le c_1] = \sum_{r=0}^{c_i} \frac{e^{-x} x^r}{r!} \tag{4.2}$$

$$p_c = p[c_1 < d < c_2] = \sum_{r=0}^{c_2} \frac{e^{-x} x^r}{r!} - \sum_{r=0}^{c_1} \frac{e^{-x} x^r}{r!} \tag{4.3}$$

Where $x = np$

The probability density function for the Gamma distribution with parameters α and β is

$$\Gamma(p / \alpha, \beta) = \begin{cases} \frac{e^{-p\beta} p^{\alpha-1} \beta^{\alpha}}{\Gamma\alpha}, & p \ge 0, \alpha \ge 0, \beta \ge 0 \\ 0, & otherwise \end{cases} \tag{4.4}$$

Suppose that the defects per unit in the submitted lots p can be modeled with Gamma distribution having parameters α and β.

Let p has a prior distribution with density function given as

$$w\,(p) = \frac{e^{-pt} p^{s-1} t^s}{\Gamma(s)}, \quad s,\ t > 0 \ and \ p > 0 \tag{4.5}$$

With parameters s and t and mean, $\overline{p} = \frac{s}{t} = \mu$ (say)

The Average Probability of Acceptance (APA) is given as

$$\overline{p} = \int_0^{\infty} p_a(p)\, w(p)\, dp \tag{4.6}$$

$$= \sum_{r_1=0}^{c_1} \frac{n^{r_1} t^s}{(r_1!)\,\Gamma(s)} * \frac{\Gamma(s+r_1)}{(n+t)^{s+r_1}} + \sum_{r_1=0}^{c_1} \sum_{r_1=c_1+1}^{c_2} \frac{n^{r_1 i + r_2} t^s}{(r_1!)^i (r_1!)\Gamma s} * \frac{\Gamma(s+r_1 i+r_2)}{(n+ni+t)^{s+r_1 i+r_2}}$$
$$- \sum_{r_1=0}^{c_2} \frac{in^{r_1+r_1 i} t^s}{(r_1!)^{1+i}\Gamma s} * \frac{\Gamma(s+r_1+r_1 i)}{(n+ni+t)^{s+r_1+r_1 i}} + \sum_{r_1=0}^{c_1} \sum_{r_2=c_1+1}^{c_2} \frac{in^{r_1 i+2r_2} t^s}{(r_1!)^i (r_2!)^2 \Gamma s}$$
$$* \frac{\Gamma(s+r_1 i+2r_2)}{(2n+ni+t)^{s+r_1 i+2r_2}} - \sum_{r_1=0}^{c_1} \frac{in^{2r_1+r_1 i} t^s}{(r_1!)^{2+i}\Gamma(s)} * \frac{\Gamma(s+2r_1+r_1 i)}{(2n+ni+t)^{s+2r_1+r_1 i}} + \cdots \tag{4.7}$$

In particular, the average probability of acceptance for $c_1 = 0$, $c_2 = 1$ is obtained as follows:

$$\bar{P} = \frac{s^s}{(n\mu+s)^s} + \frac{n\mu(s)^{s+1}}{(n\mu+in\mu+s)^{s+1}} + \frac{i(n\mu)^2 (s)^{s+1}(s+1)}{(2n\mu+in\mu+s)^{s+2}}$$
$$+ \frac{i(1+i)(n\mu)^3 (s)^{s+1}(s+1)(s+2)}{2(3n\mu+in\mu+s)^{s+3}} \tag{4.8}$$

5. DESIGNING PLANS FOR GIVEN AQL, LQL, α AND β

Tables 1(a) and 1(b) are used to design Bayesian Repetitive Deferred Sampling Plan ($c_1 = 0$, $c_2 = 1$) for given AQL, LQL, α and β.

The steps utilized for selecting Bayesian Repetitive Deferred Sampling Plan (BRDS) are as follows:

1. To design a plan for given (AQL, 1-α) and (LQL, β) first calculate the operating ratio μ_2/μ_1.
2. Find the value in Table 1(b) under the column for the appropriate α and β, which is closest to the desired ratio.
3. Corresponding to the located value of μ_2/μ_1 the value of s, i can be obtained.
4. The sample size can be obtained as $n\mu_1/\mu_1$ where $n\mu_1$ can be obtained against the located value μ_2/μ_1.

5.1. EXTAMPLE

Suppose the value for μ_1 is assumed as 0.004 and value for μ_2 is assumed as 0.065 then the operating ratio is calculated as 16.25. Now the integer approximately equal to this calculated operating ratio and their corresponding parametric values are observed from the table 1(b). The actual $n\mu_1 = 0.2731$ and $n\mu_2 = 2.9667$ at ($\alpha = 0.05$ and $\beta = 0.10$),

Table 1(a): Certain nμ values for specified values of P (μ)

s	***i***	**Probability of Acceptance**						
		0.99	**0.95**	**0.90**	**0.50**	**0.10**	**0.05**	**0.01**
1	1	0.0911	0.2185	0.3277	1.6381	12.4671	25.9351	133.7519
	2	0.0729	0.1821	0.2913	1.4197	11.0657	23.0959	119.2829
	3	0.0729	0.1639	0.2549	1.3105	10.4287	21.8037	112.8765
	4	0.0547	0.1457	0.2367	1.2559	10.0647	21.0757	109.2547
	5	0.0547	0.1457	0.2185	1.2013	9.8281	20.6207	106.9251
3	1	0.1093	0.2549	0.3823	1.2741	4.2043	6.0061	12.2305
	2	0.0911	0.2185	0.3277	1.0921	3.7857	5.4965	11.3933
	3	0.0911	0.2003	0.2913	1.0011	3.6401	5.3145	11.1385
	4	0.0911	0.1821	0.2367	0.9465	3.5673	5.2417	11.0293
	5	0.0729	0.1639	0.2185	0.9101	3.5309	5.2053	10.9929
5	1	0.1093	0.2731	0.3823	1.2013	3.4217	4.6047	8.0627
	2	0.1093	0.2367	0.3277	1.0375	3.1123	4.2589	7.6623
	3	0.0911	0.2003	0.2913	0.9465	3.0031	4.1497	7.5895
	4	0.0911	0.1821	0.2731	0.8919	2.9667	4.1315	7.5713
	5	0.0911	0.1821	0.2549	0.8373	2.9303	4.1133	7.5531
7	1	0.1275	0.2731	0.4005	1.1831	3.1305	4.1133	6.8251
	2	0.1093	0.2367	0.3459	1.0193	2.8575	3.8221	6.5521
	3	0.1093	0.2185	0.3095	0.9101	2.7665	3.7675	6.5157
	4	0.0911	0.2003	0.2731	0.8555	2.7483	3.7493	6.5157
	5	0.0911	0.1821	0.2549	0.8191	2.7301	3.7311	6.5157
9	1	0.1275	0.2731	0.4005	1.1649	2.9849	3.8585	6.2245
	2	0.1093	0.2367	0.3459	1.0011	2.7301	3.6219	6.0243
	3	0.1093	0.2185	0.3095	0.9101	2.6573	3.5673	6.0061
	4	0.0911	0.2003	0.2731	0.8555	2.6209	3.5491	6.0061
	5	0.0911	0.2549	0.2185	0.8009	2.6209	3.5491	6.0061

Table 1(b): Values of μ_2/μ_1 tabulated against s and i for given α and β for Bayesian Repetitive Deferred Sampling Plan

s	*i*	μ_2/μ_1 *for* α*=0.05* β*=0.10*	μ_2/μ_1 *for* α*=0.05* β*=0.05*	μ_2/μ_1 *for* α*=0.05* β*=0.01*	μ_2/μ_1 *for* α*=0.01* β*=0.10*	μ_2/μ_1 *for* α*=0.01* β*=0.05*	μ_2/μ_1 *for* α*=0.01* β*=0.01*
1	1	57.05767	118.6961	612.1368	136.8507	284.6883	1468.188
	2	60.76716	126.8309	655.0406	151.7929	316.8162	1636.254
	3	63.62843	133.0305	688.6913	143.0549	299.0905	1548.374
	4	69.07824	144.6513	749.8607	183.9982	385.2962	1997.344
	5	67.45436	141.5285	733.8717	179.6728	376.9781	1954.755
3	1	16.49392	23.56257	47.98156	38.46569	54.95059	111.8984
	2	17.32586	25.15561	52.14325	41.55543	60.33480	125.0637
	3	18.17324	26.53270	55.60909	39.95719	58.33699	122.2667
	4	19.58979	28.78473	60.56727	39.15807	57.53787	121.0681
	5	21.54301	31.75900	67.07077	48.43484	71.40329	150.7942
5	1	12.52911	16.86086	29.52289	31.30558	42.12900	73.76670
	2	13.14871	17.99282	32.37136	28.47484	38.96523	70.10339
	3	14.99301	20.71742	37.89066	32.96487	45.55104	83.30955
	4	16.29160	22.68808	41.57770	32.56531	45.35126	83.10977
	5	16.09171	22.58814	41.47776	32.16575	45.15148	82.90999
7	1	11.46283	15.06152	24.99121	24.55294	32.26118	53.5302
	2	12.07224	16.14744	27.68103	26.14364	34.96889	59.94602
	3	12.66133	17.24256	29.82014	25.31107	34.46935	59.61299
	4	13.72092	18.71842	32.52971	30.16795	41.15587	71.52250
	5	14.99231	20.48929	35.78089	29.96817	40.95609	71.52250
9	1	10.92970	14.12852	22.79202	23.41098	30.26275	48.81961
	2	11.53401	15.30165	25.45120	24.97804	33.13724	55.11711
	3	12.16156	16.32632	27.48787	24.31199	32.63769	54.95059
	4	13.08487	17.71892	29.98552	28.76948	38.95829	65.92865
	5	10.28207	13.92350	23.56257	28.76948	38.95829	65.92865

Table 1(c): Values of tabulated $n\mu_0$, $n\mu_1$, $n\mu_2$ and $n\mu_2/n\mu_1$ against s and i for given $P(\mu)$ for Bayesian Repetitive Deferred Sampling Plan.

s	i	$n\mu_0$	$n\mu_1$	$n\mu_2$	OR
1	1	1.6381	0.3277	12.4671	38.0442
	2	1.4197	0.2913	11.0657	37.9873
	3	1.3105	0.2549	10.4287	40.9129
	4	1.2559	0.2367	10.0647	42.5209
	5	1.2013	0.2185	9.8281	44.9798
3	1	1.2741	0.3823	4.2043	10.9974
	2	1.0921	0.3277	3.7857	11.5523
	3	1.0011	0.2913	3.6401	12.4960
	4	0.9465	0.2367	3.5673	15.0709
	5	0.9101	0.2185	3.5309	16.1597
5	1	1.2013	0.3823	3.4217	8.9503
	2	1.0375	0.3277	3.1123	9.4974
	3	0.9465	0.2913	3.0031	10.3093
	4	0.8919	0.2731	2.9667	10.8630
	5	0.8373	0.2549	2.9303	11.4959
7	1	1.1831	0.4005	3.1305	7.8165
	2	1.0193	0.3459	2.8575	8.2610
	3	0.9101	0.3095	2.7665	8.9386
	4	0.8555	0.2731	2.7483	10.0633
	5	0.8191	0.2549	2.7301	10.7105
9	1	1.1649	0.4005	2.9849	7.4529
	2	1.0011	0.3459	2.7301	7.8927
	3	0.9101	0.3095	2.6573	8.5858
	4	0.8555	0.2731	2.6209	9.5968
	5	0.8009	0.2185	2.6209	11.9949

Now $n = \frac{n\mu_1}{\mu_1} = \frac{0.2731}{0.004} = 68.275 \approx 68$

The parameters required for the plan is $n = 68$ with $i = 4$ and $s = 5$.

6. DESIGNING OF BAYESIAN REPETITIVE DEFERRED SAMPLING PLAN (BRDS) INDEXED WITH RELATIVE SLOPES OF ACCEPTABLE AND LIMITING QUALITY LEVELS

6.1. Selection of Parameters With Relative Slope h_1 At The Acceptable Quality Level

Table 2(a) is used to select the parameters for Bayesian Repetitive Deferred Sampling Plan indexed with μ_1 and h_1.For example, for given $\mu_1 = 0.01$ and $h_1 = 0.07$ from Table 2(a) under the column headed h_1, locate the value is equal to or just greater than the desired value h_1. Corresponding to this h_1, the values of parameters associated with the relative slopes are $n\mu_1 = 0.1457$, $s = 1$ and $i = 5$. From this one can obtain the sample size as $n = n\mu_1/\mu_1 \approx 14.57$.Thus the parameters are $n = 15$, $s = 1$ and $i = 5$

6.2. Selection of Parameters with Relative Slope h_2 at The Limiting Quality Level

Table 2(a) is used to select the parameters for Bayesian Repetitive Deferred Sampling Plan indexed with μ_2 and h_2.For example, for given $\mu_2 = 0.2$ and $h_2 = 1.7$ from Table 2(a) under the column headed h_2 , locate the value is equal to or just greater than the desired value h_2. Corresponding to this h_2, the values of parameters associated with the relative slopes are $n\mu_2 = 3.5673$, $s = 3$ and $i = 4$. From this one can obtain the sample size as $n = n\mu_2/\mu_2 \approx 17.8365$. Thus the parameters are $n = 18$, $s = 3$ and $i = 4$.

6.3. Selection of Parameters with Relative Slope h_0 at The Inflection Point

Table 2(a) is used to select the parameters for Bayesian Repetitive Deferred Sampling Plan indexed with μ_0 and h_0.For example, for given $\mu_0 = 0.05$ and $h_0 = 0.86$ from Table 2(a) under the column headed h_0, locate the value is equal to or just greater than the desired value h_0. Corresponding to this h_0, the values of parameters associated with the relative slopes are $n\mu_0 = 0.8373$, $s = 5$ and $i = 5$. From this one can obtain the sample size as $n = n\mu_0/\mu_0 \approx 16.746$ .Thus the parameters are $n = 17$, $s = 5$ and $i = 5$.

Table 2(a): Relative slopes for Acceptable, Indifference and Limiting Quality Levels

s	i	$n\mu_0$	$n\mu_1$	$n\mu_2$	h_0	h_1	h_2	h_2/h_1	h_2/h_0	h_0/h_1
1	1	1.6381	0.2185	12.4671	0.6037	0.0809	0.9409	11.6254	1.5586	7.4587
	2	1.4197	0.1821	11.0657	0.5983	0.0745	0.9431	12.6577	1.5764	8.0297
	3	1.3105	0.1639	10.4287	0.5902	0.0736	0.9402	12.7740	1.5930	8.0189
	4	1.2559	0.1457	10.0647	0.5846	0.0688	0.9360	13.5966	1.6010	8.4924
	5	1.2013	0.1457	9.8281	0.5751	0.0779	0.9317	11.9514	1.6202	7.3764
3	1	1.2741	0.2549	4.2043	0.8909	0.0847	1.8561	21.9144	2.0833	10.5189
	2	1.0921	0.2185	3.7857	0.8771	0.0794	1.7955	22.6193	2.0471	11.0492
	3	1.0011	0.2003	3.6401	0.8541	0.0813	1.7426	21.4223	2.0402	10.4999
	4	0.9465	0.1821	3.5673	0.8297	0.0794	1.7033	21.4465	2.0530	10.4465
	5	0.9101	0.1639	3.5309	0.8055	0.0739	1.6766	22.6759	2.0814	10.8948
5	1	1.2013	0.2731	3.4217	0.9839	0.0894	2.2183	24.8004	2.2545	11.0001
	2	1.0375	0.2367	3.1123	0.9814	0.0851	2.0910	24.5859	2.1307	11.5391
	3	0.9465	0.2003	3.0031	0.9505	0.0718	1.9920	27.7597	2.0958	13.2451
	4	0.8919	0.1821	2.9667	0.9162	0.0696	1.9352	27.7880	2.1123	13.1555
	5	0.8373	0.1821	2.9303	0.8674	0.0847	1.8944	22.3598	2.1839	10.2384
7	1	1.1831	0.2731	3.1305	1.0445	0.0856	2.4031	28.0691	2.3008	12.1997
	2	1.0193	0.2367	2.8575	1.0433	0.0803	2.2235	27.7054	2.1313	12.9996
	3	0.9101	0.2185	2.7665	0.9894	0.0834	2.0971	25.1357	2.1195	11.8591
	4	0.8555	0.2003	2.7483	0.9505	0.0832	2.0365	24.4782	2.1426	11.4244
	5	0.8191	0.1821	2.7301	0.9099	0.0793	1.9994	25.2084	2.1973	11.4726
9	1	1.1649	0.2731	2.9849	1.0736	0.0833	2.5168	30.2087	2.3443	12.8860
	2	1.0011	0.2367	2.7301	1.0734	0.0773	2.2977	29.7083	2.1406	13.8787
	3	0.9101	0.2185	2.6573	1.0358	0.0801	2.1587	26.9458	2.0840	12.9296
	4	0.8555	0.2003	2.6209	0.9915	0.0797	2.0839	26.1311	2.1017	12.4333
	5	0.8009	0.2549	2.6209	0.9293	0.1785	2.0567	11.5202	2.2131	5.2054

7. CONSTRUCTION OF TABLES:

The expression for APA function for Bayesian Repetitive Deferred Sampling Plan $\bar{p}$ is given in equation

$$\bar{p} = \int p_a \, w\,(p)\, dp$$

$$\bar{p} = \frac{s^s}{(n\mu+s)^s} + \frac{n\mu(s)^{s+1}}{(n\mu+in\mu+s)^{s+1}} + \frac{i(n\mu)^2(s)^{s+1}(s+1)}{(2n\mu+in\mu+s)^{s+2}} + \frac{i(1+i)(n\mu)^3(s)^{s+1}(s+1)(s+2)}{2(3n\mu+in\mu+s)^{s+3}} \qquad (7.1)$$

Where $\mu = s/t$, is mean value of the product quality p.
Differentiating the APA function with respect to μ gives

$$\frac{d\,\bar{p}}{d\mu} = \frac{-n(s)^{s+1}}{(s+n\mu)^{s+1}} + \frac{n(s)^{s+2}(1-n\mu-in\mu)}{(s+n\mu+in\mu)^{s+2}} + \frac{ni(s)^{s+2}(s+1)(n\mu)(2-2(n\mu)-i(n\mu))}{(s+2n\mu+in\mu)^{s+3}} + \frac{ni(1+i)(s)^{s+2}(s+1)(s+2)(n\mu)^2(3-3(n\mu)-i(n\mu))}{(s+3n\mu+in\mu)^{s+4}} \qquad (7.2)$$

The relative slope h at μ is,

$$h = \frac{-\mu}{\bar{p}} \frac{d\,\bar{p}}{d\mu}$$

Differentiating the APA function with respect to μ and equating at μ we get various values of (s, i) and their corresponding $n\mu_{1,}$ $n\mu_{0,}$ $n\mu_2$ values are substituted in the equation and the relative slopes at $\mu = \mu_{0,}\ \mu_{1\ ,}\ \mu_2$ the values h_0 $_{,}\ h_{1\ ,}\ h_2$ are obtained and tabulated in Table 2(a).

8. CONCLUSION

There are many ways to determine an appropriate sampling plan. However, all of them are either settled on a non- economic basis or do not take into consideration the producer's and consumer's quality and risk requirements.

Using the Bayesian Sampling Attribute plan without a cost function for a prior distribution can reduce the sample size, while if producer's risk and consumer's risk are appropriate. The work presented in this paper mainly relates to the procedure for designing Bayesian Repetitive Deferred Sampling Plan indexed with relative slopes at Acceptable, Limiting and Indifference Quality Levels. Tables are provided here which are tailor-made, handy and ready-made uses to the industrial shop-floor conditions.

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Prof. Dr. K.K. Suresh, Professor and Head, Department of Statistics, Bharathiar University, Coimbatore-641 046.

Ms. S. Uma Maheswari, Research Scholar, Department of Statistics, Bharathiar University, Coimbatore-641 046.

Dr. K. Pradeepa Veerakumari, Assistant Professor, Department of Statistics, Bharathiar University, Coimbatore-641 046.