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Exponential Weighted Moving Average (EWMA) Chart Under The Assumption of Moderateness And Its 3△ Control Limits

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Abstract Moderate distribution is a very good alternative of normal distribution proposed by Naik V.D and Desai J.M. [4], which has mean deviation as scale parameter rather than the standard deviation. Mean deviation (δ) is a very good alternative of standard deviation (σ) as mean deviation is considered to be the most intuitively and rationally defined measure of dispersion. This fact can be very useful in the field of quality control to construct the control limits of the control charts. On the basis of this fact Naik V.D. and Tailor K.S. [5] have proposed 3δ control limits. In 3δ control limits, the upper and lower control limits are set at 3δ distance from the central line where δ is the mean deviation of sampling distribution of the statistic being used for constructing the control chart. In this paper it has been assumed that the underlying distribution of the variable of interest follows moderate distribution proposed by Naik V.D and Desai J.M. [4] and 3δ control limits of exponential weighted moving average chart are derived. Also an empirical study is carried out to illustrate the use these charts.

Keywords Mean deviation, Moderate distribution, Exponential weighted moving average, 3δ control limits.

1. INTRODUCTION

A fundamental assumption in the development of control charts for variables is that the underlying distribution of the concerned quality characteristic is normal. The normal distribution is one of the most important distributions in the statistical inference in which mean (μ) and standard deviation (σ) are

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the parameters. Naik V.D and Desai J.M. [4] have suggested an alternative of normal distribution, which is called moderate distribution. In moderate distribution mean (μ) and mean deviation (δ) are the pivotal parameters and, they have properties similar to normal distribution.

Naik V.D. and Tailor K.S. [5] have proposed the concept of 3δ control limits on the basis of moderate distribution. Under this rule, the upper and lower control limits are set at 3δ distance from the central line where δ is the mean deviation of sampling distribution of the statistic being used for constructing the control chart. Thus in the proposed control charts, under the moderateness assumption, the control limits for any statistic T should be determined as follows.

$$\text{Central line (CL)} = \text{Expected value of T} = \mu$$

$$\text{Lower Control Limit (LCL)} = \text{Mean of } T - 3\delta_T = \mu - 3\delta_T$$

$$\text{Upper Control Limit (UCL)} = \text{Mean of } T + 3\delta_T = \mu + 3\delta_T$$

Where μ is mean of T and δ_T is the mean deviation of T.

It is found that since δ provides exact average distance from mean and σ provides only an approximate average distance, 3δ limits can be considered to be more rational as compared to 3σ limits.

Naik and Tailor have derived 3δ control limits of $\overline{X}$-chart, R-chart, s (sample standard deviation) chart and d (sample mean deviation) chart. Tailor has also suggested 3δ control limits of moving average and moving range charts.

Thus, in this paper it is assumed that the underlying distribution of the concerned variable follows moderate distribution and 3δ control limits for exponential weighted moving average is derived. An empirical study is also carried out to illustrate the use of the chart.

2. EXPONENTIAL WEIGHTED MOVING AVERAGE (EWMA) CHART

The concept of EWMA chart was introduced by Roberts S.W [7]. The exponentially weighted moving average chart is a type of moving mean chart in which an 'exponentially weighted mean' is calculated each time a new result becomes available. The EWMA control chart is a very good alternative to the Shewhart chart, when we are interested in detecting small shifts.

New weighted mean $= (\propto \times \text{new result}) + ((1-\propto) \times \text{previous result})$, where $\propto$ is the 'smoothing constant'. It has a value between 0 and 1, many statistician use $\propto = 0.2$, but choice of $\propto$ has to be left to the judgment of the quality control specialist, the smaller the value of $\propto$, the greater the influence of the historical data.

The EWMA chart is much more effective than moving average chart for detecting small shifts. If it is important to recognize small shifts early in the process, then the value of $\propto$ should be small. If $\propto = 1$, the EWMA chart reduces itself to the usual $\bar{X}$-chart. This has been used by some organizations, particularly in the process industries, as the basis of new 'control (performance) chart' systems. Great care must be taken when using these systems since they do not show changes in variability very well and the basis for weighting data is often either questionable or arbitrary.

3. (3δ) CONTROL LIMITS FOR EXPONENTIAL WEIGHTED MOVING AVERAGE (EWMA) CHART

Suppose a measurable quality characteristic of the product is denoted by X. Suppose that m samples, each of size n, are drawn at more or less regular interval of time from the production processes. These samples are known as subgroups, and for each of these subgroups the values of exponential weighted moving mean $\bar{X}_t$ are obtained. Let the distribution of the variable X be moderate with mean μ and mean deviation δ, then, as proved by Naik V.D and Desai J.M. [4], the distribution of $\bar{X}$ is also moderate with mean μ and mean deviation $\frac{\delta}{\sqrt{n}}$. Further, if the distribution of X is not moderate, and the number of units in each subgroup is 4 or more, then on the basis of central limit theorem for moderate distribution, it can be said that $\bar{X}$ follows moderate distribution.

The EWMA function is defined as,

$Z_t = \propto x_i + (1-\propto)Z_{i-1}$, where $0 < \propto \leq 1$

If the individual $\bar{X}$ are independent random variables with variance $\frac{\sigma^2}{n}$, then the variance of Z_t is defined as

$$\sigma_t^{\,2} = \frac{\sigma^2}{n}\left(\frac{\propto}{2-\propto}\right)\left[1-(1-\alpha)^{2t}\right]$$

Therefore
$$\sigma_t = \left[\frac{\sigma^2}{n}\left(\frac{\propto}{2-\propto}\right)\left[1-(1-\alpha)^{2t}\right]\right]^{\frac{1}{2}}$$
$$= \frac{\sigma}{\sqrt{n}}\left[\left(\frac{\propto}{2-\propto}\right)\left[1-(1-\alpha)^{2t}\right]\right]^{\frac{1}{2}} \quad (1)$$

Since we are assuming moderateness, the mean error of Z_t is defined as

$$\delta_t = \sqrt{\frac{\pi}{2}}\frac{\delta}{\sqrt{n}}\left[\left(\frac{\propto}{2-\propto}\right)\left[1-(1-\alpha)^{2t}\right]\right]^{\frac{1}{2}} \quad (2)$$

Thus, the 3δ- control limits of EWMA chart can be determined as follows
Central line (C.L) $= \mathrm{E}\left(\bar{X}_t\right)$

$$\overline{\overline{X}}_t \quad (3)$$

Lower control limit (L.C.L) $= E\left(\bar{X}_t\right) - 3\delta_t$

$$= \overline{\overline{X}}_t - 3\sqrt{\frac{\pi}{2}}\frac{\delta}{\sqrt{n}}\left[\left(\frac{\propto}{2-\propto}\right)\left[1-(1-\alpha)^{2t}\right]\right]^{\frac{1}{2}}$$
$$= \overline{\overline{X}}_t - 5.3184\frac{\delta}{\sqrt{n}}\left[\left(\frac{\propto}{2-\propto}\right)\left[1-(1-\alpha)^{2t}\right]\right]^{\frac{1}{2}} \quad (4)$$

Upper control limit (U.C.L) $= \mathrm{E}\left(\bar{X}_t\right) + 3\delta_t$

$$= \overline{\overline{X}}_t + 3\sqrt{\frac{\pi}{2}}\frac{\delta}{\sqrt{n}}\left[\left(\frac{\propto}{2-\propto}\right)\left[1-(1-\alpha)^{2t}\right]\right]^{\frac{1}{2}}$$
$$= \overline{\overline{X}}_t + 5.3184\frac{\delta}{\sqrt{n}}\left[\left(\frac{\propto}{2-\propto}\right)\left[1-(1-\alpha)^{2t}\right]\right]^{\frac{1}{2}} \quad (5)$$

Where $\overline{\overline{X}}_t$ and δ_t are typically estimated from preliminary data as sample mean and sample mean deviation.

As α is small and if t increases, the effect of starting value soon dissipates and the mean error converges to its asymptotic value.

i.e $\delta_t = \sqrt{\frac{\pi}{2}} \frac{\delta}{\sqrt{n}} \left[\left(\frac{\propto}{2-\propto} \right) \right]^{\frac{1}{2}}$

The control limits for EWMA chart are usually based on the asymptotic mean deviation of the statistic. Hence asymptotic 3δ-control limits for this chart can be derived as following way,

Central line (C.L) = E ($\overline{\overline{X_t}}$)

$$= \overline{\overline{X}}_t \tag{6}$$

Lower control limit (L.C.L) $= \mathrm{E}\left(\bar{X}_t\right) - 3\delta_t$

$$\begin{aligned} &= \overline{\overline{X_t}} - 3\sqrt{\frac{\pi}{2}} \frac{\delta}{\sqrt{n}} \left[\left(\frac{\propto}{2-\propto} \right) \right]^{\frac{1}{2}} \\ &= \overline{\overline{X_t}} - 5.3184 \frac{\delta}{\sqrt{n}} \left[\left(\frac{\propto}{2-\propto} \right) \right]^{\frac{1}{2}} \end{aligned} \tag{7}$$

Upper control limit (U.C.L) $= \mathrm{E}\left(\bar{X}_t\right) + 3\delta_t$

$$\begin{aligned} &= \overline{\overline{X_t}} + 3\sqrt{\frac{\pi}{2}} \frac{\delta}{\sqrt{n}} \left[\left(\frac{\propto}{2-\propto} \right) \right]^{\frac{1}{2}} \\ &= \overline{\overline{X_t}} + 5.3184 \frac{\delta}{\sqrt{n}} \left[\left(\frac{\propto}{2-\propto} \right) \right]^{\frac{1}{2}} \end{aligned} \tag{8}$$

4. AN EMPIRICAL STUDY FOR EWMA CHART:

To illustrate the use of EWMA control scheme, we use a set of simulated observations taken from Lucas J. M and Crosier R.B [2]. The data, together with the corresponding EWMA values, are shown in Table 1. The target value is taken to be 0, so the process is in control for the first 10 observations. The mean level was shifted upward by approximately one standard deviation for the last nine observations. The parameters of the EWMA are chosen to be α = 0.25, δ = 1, n = 1
Asymptotic 3-control limits for EWMA chart are obtained by

Table 1:

t	Observed value	EWMA = + (1−)
0	–	0.0
1	1.0	0.25
2	–0.5	0.063
3	0.0	0.047
4	–0.8	–0.165
5	–0.8	–0.324
6	–1.2	–0.543
7	1.5	–0.032
8	–0.6	–0.174
9	1.0	0.119
10	–0.9	–0.135
11	1.2	0.198
12	0.5	0.274
13	2.6	0.855
14	0.7	0.817
15	1.1	0.887
16	2.0	1.166
17	1.4	1.224
18	1.9	1.393
19	0.8	1.245

LCL = −2.010, CL = 0 and UCL = 2.010
Similarly, asymptotic 3σ-control limits for EWMA chart are obtained by
LCL = −1.134, CL = 0 and UCL = 1.134
Now, EWMA chart for moderateness and normality assumptions can be constructed as follows.

From figure 1, it can be seen that under moderateness assumption with 3δ-control limits, EWMA chart is under the statistical control, while under normality assumption with 3σ-control limits, it is out of control as one sample point falls outside the UCL. The point which shows out of control situation in EWMA chart under normality assumption shows under control situation in EWMA chart under moderateness assumption.

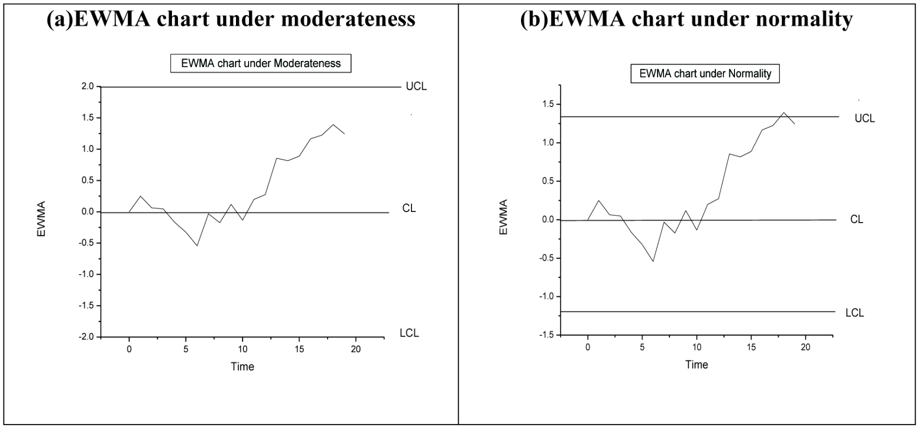

Figure 1:

It can also be seen from figure 1, that using asymptotic control limits rather than the time varying limits, makes the EWMA chart much less sensitive to process shifts in the first few observations. This could be a significant problem if a large shift occurs early, or if after an out of control condition the process is not properly reset.

5. SUMMARY

This paper derives the EWMA chart under the assumption of moderateness. The 3δ - control limits are derived for the EWMA chart with time –varying control limits. Also the asymptotic 3δ - control limits are derived for the same chart. An empirical study is carried out to illustrate this chart. A comparative study is carried out for the EWMA chart under moderateness assumption and under normality assumption and it is found that EWMA chart under moderateness assumption perform better than EWMA chart under normality assumption.

Hence, it is suggested that EWMA chart under moderateness assumption with 3δ - control limits should be used for effective performance of the chart.

6. APPENDIX

(a) Moderate Distribution

Suppose the p.d.f. of a distribution of a random variable X is defined as,

$$f(x) = \frac{1}{\pi\delta} e^{-\frac{1}{\pi}\left(\frac{X-\mu}{\delta}\right)^2}, -\infty < X < \infty, \delta > 0$$

Then, the random variable X may be said to be following moderate distribution with parameters μ and δ and may be denoted as $X \sim M(\mu, \delta)$. It can be proved that,

(i) $\int_{-\infty}^{\infty} f(x) = 1$

(ii) Mean = E(x) = μ

(iii) Mean deviation $= E\left[\left|X - \pi\right|\right] = \delta$

(vi) Standard deviation $= \sqrt{\frac{\pi}{2}}\delta$

(v) M.G.F $= M_x^{(t)} = e^{\pi t + \frac{\pi}{4}\delta^2 t^2}$

(vi) $f(\mu - X) = f(\mu + x)$

(b) 3σ-control limits of EWMA chart

C.L. $= \overline{\overline{X}}$

$$\text{U.C.L.} = \overline{\overline{X}} + 3\sigma\left[\left(\frac{\propto}{2-\propto}\right)\left[1-(1-\alpha)^{2t}\right]\right]^{\frac{1}{2}}$$

$$\text{L.C.L.} = \overline{\overline{X}} - 3\sigma\left[\left(\frac{\propto}{2-\propto}\right)\left[1-(1-\alpha)^{2t}\right]\right]^{\frac{1}{2}}$$

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