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Effect of Deformation on Semi–infinite Viscothermoelastic Cylinder Based on Five Theories of Generalized Thermoelasticity

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Abstract: We consider a dynamical problem for semi-infinite viscothermoelastic semi infinite cylinder loaded mechanically and thermally and investigated the behaviour of variations of displacements, temperatures and stresses. The problem has been investigated with the help of five theories of the generalized viscothermoelasticity by using the Kelvin – Voigt model. Laplace transformations and Hankel transformations are applied to equations of constituent relations, equations of motion and heat conduction to obtain exact equations in transformed domain. Hankel transformed equations are inverted analytically and for the inversion of Laplace transformation we apply numerical technique to obtain field functions. In order to obtain field functions i.e. displacements, temperature and stresses numerically we apply MATLAB software tools. Numerically analyzed results for the temperature, displacements and stresses are shown graphically.

Keywords: Kelvin–Voigt model; Mechanical and Thermal loads; Green and Naghdi Theory; Hankel transformation; Field functions.

1. INTRODUCTION

Nowacki [1] has thoroughly established the theory of elasticity, classical coupled thermoelasticity, non–classical generalized thermoelasticity and waves in solids of thermoelasticity. Biot [2] has studied the theory of coupled thermoelasticity of classical one and to remove the inconsistency in the

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classical uncoupled theory that there may have no effect on the temperature to change elastic outcome. In classical theory, it was arrived to reality that a part of the result of the energy equation tends to infinity (large) when homogeneous elastic medium subjected to thermal or mechanical instability, this results that the temperature, stresses and displacement fields are felt at an infinite (large) distance from the source of instability instantaneously. This comes to result that the solution has an infinite velocity of proliferation, which is impossible physically. To overcome this problem of infinite velocity of proliferation Lord and Shulman [3] have made many attempts to amend the Fourier law of heat conduction to acquire a hyperbolic differential equation of heat conduction. Green and Lindsay [4] adapted entropy production inequality derived from constituent relations for stresses and generalized by introducing two unlike relaxation times parameters constrained by inequalities $t_1 \geq t_0 \geq 0$. The supposition of thermoelasticity without energy dissipation (TWED), projected by Green and Naghdi [5] which have been illustrated by system of field equations, in which rate of heat flux and temperature gradient (change) relation is compared to classical coupled thermoelasticity theory, which replaces the Fourier's law of heat conduction. Chandrasekharaiah [6] and Tzou [7] have been proposed a model in which an extension of the Fourier's law is changed by an estimation to a adapted Fourier's law with two unusual time conversions, a phase lag of temperature gradient τ_θ and a phase lag of heat flux τ_q. Sherif and El-Maghraby [8] investigated a dynamic problem of penny shaped crack of infinite thermoelastic solid focused on recommended temperature and stress distribution. Dhaliwal and Singh [9] and Graff [10] have given much attention to such type of problems for classical and non classical theories of thermoelasticity. Othman and Singh [11] studied the effect of five theories under the rotation of generalized micropolar thermoelastic half space. Sharma et al. [12] have considered the interruption due to normal and thermal point loads acting on half space boundary. Mukhopadhyay [13] examined the effects of thermal relaxation on viscothermoelastic interactions with a spherical cavity of periodical loading. The viscothermoelastic relations in Kelvin – Voigt model type Rayleigh-Lamb wave viscoelastic plate have been studied by Sharma [14] to achieve amplitudes of displacements and temperature.

The theory of Linear Viscoelasticity was thoroughly established by Bland [15]. A lot of mathematical models based on viscoelastic problems have been used by Hunter [16] and Flugge [17] that could be managed the dissipation of energy in vibrating solids and it was observed that internal friction produces decrease of energy and diffusion. Tripathi et al. [18] studied the displacements, temperature and stress distribution in a semi infinite cylinder. Sharma et al. [19] investigated the generalized homogenous isotropic magneto viscothermoelastic solid half space with strip loading. Sharma et al. [20] studied the deformation of mechanical and thermal loadings of semi infinite cylinder in the framework of coupled theory of viscothermoelasticity. Samia [21] had explored the cause of initial stress due to hydrostatic and the gravity field on a fiber-reinforced medium of thermoelasticity with an internal heat source.

As per knowledge of the authors no exact and systematic learning on the outcome of mechanical and thermal variations on two and three dimensional vibrations of conducting viscoelastic cylindrical structures is available in the literature. Keeping in view the above facts, the present paper is devoted to study the homogenous and isotropic semi infinite viscothermoelastic cylinder subjected to five theories of generalized thermoelasticity to present the variations of displacements, temperature change and stresses in considered boundary conditions of mechanical forces and heat sources. The partial differential equations have been transformed into ordinary differential equations by applying combination of Laplace and Hankel transforms. Hankel transformations are inverted analytically and inverse Laplace transformations are determined numerically. Analytical results have been computed numerically in MATLAB software tools and presented graphically for field functions i.e. stresses, temperature change and displacements.

2. FORMULATION OF PROBLEM

We have considered a viscothermoelastic homogenous isotropic, semi infinite cylinder of thick plate of height $2h$ and radius r defined as $-h \le z \le h$ and $0 \le r < \infty$ and at initial temperature T_0 in undisturbed state. We consider the axis of symmetry as z–axis and the origin of the coordinate system between lower and upper faces of the plate. The material is described by the Kelvin – Voigt model of the linear viscoelasticity. We consider the cylindrical coordinate system (r, ϑ, z) and all the considered quantities of semi infinite cylinder are independent of the coordinate ϑ. The displacement vector $\boldsymbol{u}(r, z, t) = (u, 0, w)$ and temperature $T(r, Z, t)$ have been considered for analysis. All the considered functions i.e. displacements, temperatures and stresses will depend on r, z, t only. The constitutive relations and governing equations for viscothermoelastic semi–infinite cylinder in the framework of five theories i.e. (CT, LS, GL, GN and C–T) of generalized thermoelasticity are given by (Nowacky [2] and Dhaliwal and Singh [9]) as

Stress –strain temperature relations

$$\sigma_{rr} = 2\mu^* \frac{\partial u}{\partial r} + \lambda^* e - \beta^* \left(T + t_1 \delta_{2k} \frac{\partial T}{\partial t} \right) \tag{1}$$

$$\sigma_{zz} = 2\mu^{*}\frac{\partial w}{\partial z} + \lambda^{*} e - \beta^{*}\left(T + t_1 \delta_{2k}\frac{\partial T}{\partial t}\right) \tag{2}$$

$$\sigma_{rz} = \mu^{*}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right) \tag{3}$$

The basic governing equations of motion and heat conduction for semi–infinite generalized viscothermoelastic cylinder in the absence of body forces along with heat sources are given by (Nowacky [2] and Dhaliwal and Singh [9]) as

$$\mu^{*}\left(\nabla^{2}u - \frac{1}{r^{2}}u\right) + (\lambda^{*} + \mu^{*})\frac{\partial e}{\partial r} - {}^{2\,*}\left(1 + \delta_{2k} t_1 \frac{\dot{\partial}}{\partial t}\right)\frac{\partial \dot{T}}{\partial r} = \rho\frac{\partial^{2} u}{\partial t^{2}} \tag{4}$$

$$\mu^{*}\nabla^{2}w + (\lambda^{*} + \mu^{*})\frac{\partial e}{\partial z} - \beta^{*}\left(1 + \delta_{2k} t_1 \frac{\partial}{\partial t}\right)\frac{\partial T}{\partial z} = \rho\frac{\partial^{2} w}{\partial t^{2}} \tag{5}$$

$$K\left(n_2 + v_0\frac{\partial}{\partial t}\right)\nabla^{2}T - \rho C_e\left(n_1\frac{\partial}{\partial t} + t_0\frac{\partial^{2}}{\partial t^{2}}\right)T = \beta^{*}T_0\left(n_1\frac{\partial}{\partial t} + n_0 t_0 \delta_{1k}\frac{\partial^{2}}{\partial t^{2}}\right)e \tag{6}$$

where

$$\lambda^{*} = \lambda\left(1 + \alpha_0\frac{\partial}{\partial \mathrm{t}}\right), \mu^{*} = \mu\left(1 + \alpha_1\frac{\partial}{\partial \mathrm{t}}\right), \beta^{*} = \beta_{\mathrm{e}}\left(1 + \beta_0\frac{\partial}{\partial \mathrm{t}}\right), \beta_{\mathrm{e}} = (3\lambda + 2\mu)\alpha_{\mathrm{T}} \tag{7}$$

$$\beta_0 = \frac{(3\lambda\alpha_0 + 2\mu\alpha_1)\alpha_T}{\beta_{\mathrm{e}}}, \nabla^{2} = \frac{\partial^{2}}{\partial \mathrm{z}^{2}} + \frac{\partial^{2}}{\partial \mathrm{r}^{2}} + \frac{1}{\mathrm{r}}\frac{\partial}{\partial \mathrm{r}}, \mathrm{e} = \frac{\partial \mathrm{w}}{\partial \mathrm{z}} + \frac{\partial \mathrm{u}}{\partial \mathrm{r}} + \frac{\mathrm{u}}{\mathrm{r}}$$

Here, $T(r, z, t)$ is absolute temperature, e is cubical dilatation, ρ is density mass, α_0, α_1 are viscoelastic relaxation times, λ and μ are Lame's constants, $\lambda^{*}$and $\mu^{*}$are viscoelastic parameters, α_T is the coefficient of linear thermal expansion, C_0 is specific heat at constant strain, K is thermal conductivity, $(\delta_{ik};\ i = 1, 2)$ is Kronecker delta. v_0, t_0, t_1 are relaxation times, n_0, n_1, n_2 are dimensional parameters.

The equations (4)–(6) are the field equations for semi-infinite generalized viscothermalelastic cylinder valid to classical and non classical coupled theories, and their generalizations are given below:

1. The coupled viscothermalelasticity (CT theory) for semi-infinite cylinder under conditions:

$$n_1 = n_2 = 1, v_0 = t_0 = t_1 = 0, n_0 = 0 \tag{8}$$

The equation (4) to (6) becomes

$$\mu^* \nabla^2 u - \frac{\mu^*}{r^2} u + (\lambda^* + \mu^*) \frac{\partial e}{\partial r} - \beta^* \frac{\partial T}{\partial r} = \rho \frac{\partial^2 u}{\partial t^2} \tag{9}$$

$$\mu^* \nabla^2 w + (\lambda^* + \mu^*) \frac{\partial e}{\partial z} - \beta^* \frac{\partial T}{\partial z} = \rho \frac{\partial^2 w}{\partial t^2} \tag{10}$$

$$K \nabla^2 T - \rho C_e \frac{\partial T}{\partial t} = \beta^* T_0 \frac{\partial e}{\partial t} \tag{11}$$

2. Lord-Shulman (LS theory) for semi-infinite cylinder under conditions:

$$n_0 = n_1 = n_2 = 1, v_0 = t_1 = 0, t_0 > 0 \tag{12}$$

where t_0 is relaxation time

Equations (4) and (5) will remain same as equations (9) and (10) but equation (6) becomes:

$$K \nabla^2 T - \rho C_e \left(\frac{\partial}{\partial t} + t_0 \frac{\partial^2}{\partial t^2} \right) T = \beta^* T_0 \left(\frac{\partial}{\partial t} + t_0 \delta_{1k} \frac{\partial^2}{\partial t^2} \right) e \tag{13}$$

3. Green – Lindsay (GL theory) for semi-infinite cylinder under conditions:

$$n_0 = 0, n_1 = n_2 = 1, v_0 = 0, t_1 \geq t_0 > 0 \tag{14}$$

Equations (4) and (5) will remain same and equation (6) becomes,

$$K \nabla^2 T - \rho C_e \left(\frac{\partial}{\partial t} + t_0 \frac{\partial^2}{\partial t^2} \right) T = \beta^* T_0 \left(\frac{\partial e}{\partial t} \right) \tag{15}$$

4. The equation of generalized semi-infinite viscothermoelastic cylinder without energy dissipation, Green–Nagdhi (GN theory) under conditions

$$n_0 = t_0 = 1, n_1 = v_0 = t_1 = 0, n_2 > 0 \tag{16}$$

Equations (4) and (5) will remain same as equations (9) and (10) and equation (6) becomes,

$$K(n_2) \nabla^2 T - \rho C_e \frac{\partial^2 T}{\partial t^2} = \beta^* T_0 \delta_{1k} \frac{\partial^2 e}{\partial t^2} \tag{17}$$

5. Chandrasekharaiah–Tzou theory (C–T theory) is modified classical viscothermoelasticity model in which Fourier's law of heat conduction is specified by the equation if

$$n_0 = n_1 = n_2 = 1, v_0 = \tau_\theta > 0, t_0 = \tau_q > 0, 0 < \tau_\theta < \tau_q, t_1 = 0 \tag{18}$$

here, τ_θ, τ_q are phase lag of temperature gradient and phase lag of heat flux respectively. Chandrasekharaiah–Tzou (C–T) theory is extension of LS model in sense that both τ_θ and τ_q that have arrived into the formulation of an initial boundary value problem depend on the Taylor series approximation of a modified Fourier law of heat conduction.

Equation (4) and (5) will remain same as (9) and (10) and equation (6) becomes,

$$K\left(1 + \ddot{A}\frac{\partial}{\partial t}\right)\nabla^2 T = \left(1 + \ddot{A}_q\frac{\partial}{\partial t}\right)\left(T_0\beta^*\delta_{1k}\frac{\partial e}{\partial t} + \rho C_e\frac{\partial T}{\partial t}\right) \tag{19}$$

3. INITIAL AND BOUNDARY CONDITIONS

Initially the medium is assumed to be at rest and undisturbed, both mechanically and thermally, so that the initial conditions are given as:

$$u(r,z,0) = 0 = \frac{\partial}{\partial t}u(r,z,0), w(r,z,0) = 0 = \frac{\partial}{\partial t}w(r,z,0)$$

$$T(r,z,0) = 0 = \frac{\partial}{\partial t}T(r,z,0); -h \le z \le h, r \ge 0 \tag{20}$$

The surface of semi-infinite viscothermoelastic cylinder subjected to mechanical load acting on the origin, mathematically this provides us

$$\sigma_{zz} = \sigma_0 f(r,z,t); 0 \le r \le a \text{ and } -h \le z \le h \tag{21}$$

$$\sigma_{rz} = 0; 0 \le r < \infty \text{ and } -h \le z \le h$$

$$\mathrm{T} = 0; 0 \le r < \infty \text{ and } -h \le z \le h$$

The surface of semi-infinite viscothermoelastic cylinder subjected to thermal load acting on the origin, mathematically this provides us

$$\sigma_{zz} = 0; 0 \le r < \infty \text{ and } -h \le z \le h \tag{22}$$

$$\sigma_{rz} = 0; 0 \le r < \infty \text{ and } -h \le z \le h$$

$$\frac{\partial T}{\partial z} = \theta_0 f(r,z,t); 0 \le r \le a \text{ and } -h \le z \le h$$

where σ_0 and θ_0 are constants.

4. SOLUTION OF PROBLEM

We define following non-dimensional quantities to remove the complexity of the problem so that the equations become handy to solve

$$R = \frac{r}{c_1}, Z = \frac{z}{c_1}, \theta = \frac{T}{T_0}, U = \frac{\rho c_1 u}{\beta_e T_0}, W = \frac{\rho c_1 w}{\beta_e T_0}, \tau_{ij} = \frac{\sigma_{ij}}{\beta_e T_0}$$

$$\tau = \frac{t}{c_1}, \delta_0 = 2\delta^2(\hat{\alpha}_1 - \hat{\alpha}_0) + \hat{\alpha}_0,\ \varepsilon = \frac{T_0 \beta_e^2}{\rho C_e(\lambda + 2\mu)},\ \omega^* = \frac{C_e(\lambda + 2\mu)}{K} \quad (23)$$

$$\delta^2 = \frac{c_2^2}{c_1^2}, \hat{\alpha}_0 = \frac{\alpha_0}{c_1}, \hat{\alpha}_1 = \frac{\alpha_1}{c_1}, \hat{\beta}_0 = \frac{\beta_e}{c_1}, c_1^2 = \frac{\lambda + 2\mu}{\rho}, c_2^2 = \frac{\mu}{\rho}$$

$$v_0' = \frac{v_0}{c_1}, t_0' = \frac{t_0}{c_1}, t_1' = \frac{t_1}{c_1}, n_0' = \frac{n_0}{c_1}, n_1' = \frac{n_1}{c_1}, n_2' = \frac{n_2}{c_1}$$

Here for our convenience the primes have been suppressed. On substituting the above non – dimensional quantities of equations (23) in equations (1)–(6) via equation (7) we obtain the following equations:

$$\tau_{ZZ} = \left(1 + \delta_0 \frac{\partial}{\partial \tau}\right)\frac{\partial W}{\partial Z} + (1 - 2\delta^2)\left(1 + \hat{\alpha}_0 \frac{\partial}{\partial \tau}\right)\left(\frac{\partial U}{\partial R} + \frac{U}{R}\right) - \left(1 + \hat{\beta}_0 \frac{\partial}{\partial \tau}\right)\left(1 + t_1 \delta_{2k} \frac{\partial}{\partial \tau}\right)\theta \quad (24)$$

$$\tau_{RZ} = \delta^2 \left(1 + \hat{\alpha}_1 \frac{\partial}{\partial \tau}\right)\left(\frac{\partial U}{\partial Z} + \frac{\partial W}{\partial R}\right) \quad (25)$$

$$\left(1+\delta_0\frac{\partial}{\partial\tau}\right)\left(\frac{\partial^2 U}{\partial R^2}+\frac{1}{R}\frac{\partial U}{\partial R}-\frac{U}{R^2}\right)+\delta^2\left(1+\hat{\alpha}_1\frac{\partial}{\partial\tau}\right)\frac{\partial^2 U}{\partial Z^2}+ \left((1-\delta^2)+(\delta_0-\hat{\alpha}_1\delta^2)\frac{\partial}{\partial\tau}\right)\frac{\partial^2 W}{\partial R\partial Z}-\left(1+\hat{\beta}_0\frac{\partial}{\partial\tau}\right)\left(1+t_1\delta_{2k}\frac{\partial}{\partial\tau}\right)\frac{\partial\theta}{\partial R}=\frac{\partial^2 U}{\partial\tau^2} \tag{26}$$

$$\left((1+\delta^2)+(\delta_0-\hat{\alpha}_1\delta^2)\frac{\partial}{\partial\tau}\right)\left(\frac{\partial^2 U}{\partial R\partial Z}+\frac{1}{R}\frac{\partial U}{\partial Z}\right)+\left(1+\delta_0\frac{\partial}{\partial\tau}\right)\frac{\partial^2 W}{\partial Z^2}+ \delta^2\left(1+\hat{\alpha}_1\frac{\partial}{\partial\tau}\right)\left(\frac{\partial^2 W}{\partial R^2}+\frac{1}{R}\frac{\partial W}{\partial R}\right)-\left(1+\hat{\beta}_0\frac{\partial}{\partial\tau}\right)\left(1+t_1\delta_{2k}\frac{\partial}{\partial\tau}\right)\frac{\partial\theta}{\partial Z}=\frac{\partial^2 W}{\partial\tau^2} \tag{27}$$

$$\left(n_2+v_0\frac{\partial}{\partial\tau}\right)\left(\frac{\partial^2}{\partial R^2}+\frac{1}{R}\frac{\partial}{\partial R}+\frac{\partial^2}{\partial Z^2}\right)\theta-\omega^*\left(n_1\frac{\partial}{\partial\tau}+t_0\frac{\partial^2}{\partial\tau^2}\right)\theta =\varepsilon\omega^*\left(1+\hat{\beta}_0\frac{\partial}{\partial\tau}\right)\left(n_1\frac{\partial}{\partial\tau}+n_0t_0\delta_{1k}\frac{\partial^2}{\partial\tau^2}\right)\left(\frac{\partial W}{\partial Z}+\frac{\partial U}{\partial R}+\frac{U}{R}\right) \tag{28}$$

We classify Laplace transformation and Hankel transformation by the equations given below:

$$L\{f(R,Z,\tau)\}=\int_0^\infty f(R,Z,\tau)e^{-p\tau}d\tau=\bar{f}(R,Z,p) \tag{29}$$

$$H(\bar{f}(R,Z,p))=\int_0^\infty RJ_n(\alpha R)\bar{f}(R,Z,\tau)dR=\tilde{f}(\alpha,Z,p) \tag{30}$$

Upon using above two equations of transformation from equations (29) and (30) to equations (24) – (28), the non–vanishing solution of resulting equations are

$$\tilde{\tau}_{ZZ}=\alpha_0^*\alpha p(1-2\delta^2)\tilde{U}+p\delta_0^* D\tilde{W}-\beta_0^* p^2\tau_1\tilde{\theta} \tag{31}$$

$$\tilde{\tau}_{RZ}=\delta^2\alpha_0^*\mathrm{p}\left(D\tilde{U}-\alpha\tilde{W}\right) \tag{32}$$

$$\begin{bmatrix}(D^2-\gamma_{11}) & -\gamma_{12}D & \gamma_{13}\\ \gamma_{21}D & (D^2-\gamma_{22}) & -\gamma_{23}D\\ \gamma_{31} & \gamma_{32}D & -(D^2-\gamma_{33})\end{bmatrix}\begin{bmatrix}\tilde{U}\\ \tilde{W}\\ \tilde{\theta}\end{bmatrix}=0 \tag{33}$$

where

$$\gamma_{11}=\left(\frac{\alpha^2}{c_1^*}+\frac{p}{c_1^*\delta_0^*}\right),\gamma_{12}=\frac{c_2^*\alpha}{c_1^*},\gamma_{13}=\frac{pc_3^*}{c_1^*},\gamma_{21}=c_2^*\alpha,\gamma_{22}=c_1^*\alpha^2+\frac{p}{\delta_0^*},$$

$$\gamma_{23}=c_3^*p,\gamma_{31}=\frac{\varepsilon\omega^*\beta_0^*p^2\tau_0'n_1\alpha}{n_2v_0^*},\gamma_{32}=\frac{\varepsilon\omega^*\beta_0^*p^2\tau_0'n_1}{n_2v_0^*},\gamma_{33}=\alpha^2-\frac{\omega^*p\tau_0n_1}{n_2v_0^*} \tag{34}$$

$$c_1^*=\frac{\delta^2\alpha_1^*}{\delta_0^*},c_2^*=\frac{(1-\delta^2)\delta^*}{\delta_0^*},c_3^*=\frac{\beta_0^*\tau_1}{\delta_0^*},D=\frac{d}{dx}$$

$$(\delta_0+p^{-1})=\delta_0^*,(\hat{\alpha}_1+p^{-1})=\alpha_1^*,(\hat{\alpha}_0+p^{-1})=\alpha_0^*,(\hat{\beta}_0+p^{-1})=\beta_0^*,(t_1\delta_{2k}+p^{-1})=\tau_1;$$

$$\left(\frac{\delta_0-\hat{\alpha}_1\delta^2}{1-\delta^2}+p^{-1}\right)=\delta^*,\left(\frac{v_0}{n_2}+p^{-1}\right)=v_0^*,\left(\frac{t_0}{n_1}+p^{-1}\right)=\tau_0,\left(\frac{n_0}{n_1}t_0\delta_{1k}+p^{-1}\right)=\tau_0'$$

Here p is Laplace transform parameter and α is Hankel transform parameter. On solving equation (33) we get

$$\left\{D^6-AD^4+BD^2-C\right\}\tilde{U}(R,Z,\tau)=0 \tag{35}$$

where

$$A=\gamma_{11}+\gamma_{22}+\gamma_{33}-\gamma_{12}\gamma_{21}-\gamma_{32}\gamma_{23} \tag{36}$$

$$B=\gamma_{11}\gamma_{22}+\gamma_{22}\gamma_{33}+\gamma_{33}\gamma_{11}-\gamma_{12}\gamma_{23}\gamma_{31}-\gamma_{21}\gamma_{32}\gamma_{13}+\gamma_{11}\gamma_{22}\gamma_{32}+\gamma_{13}\gamma_{31} \tag{37}$$

$$C=\gamma_{11}\gamma_{22}\gamma_{33}+\gamma_{13}\gamma_{31}\gamma_{22} \tag{38}$$

Similarly we can have the solution for $\tilde{W}$ and $\tilde{\theta}$.

$$\left(D^6-AD^4+BD^2-C\right)\left\{\tilde{W}(\alpha,Z,\tau),\tilde{\theta}(\alpha,Z,p)\right\}=0 \tag{39}$$

Factors of equation (35) are given by:

$$\left((D^2-m_1^2)(D^2-m_2^2)(D^2-m_3^2)\right)\tilde{U}(R,Z,p)=0 \tag{40}$$

The solution of the equations (40) is given by

$$\tilde{U} = \sum_{j=1}^{3} A_j \cosh m_j Z; j = 1,2,3 \tag{41}$$

Similarly we can have the solutions for $\tilde{W}$ and $\tilde{\theta}$.'

$$\tilde{W} = \sum_{j=1}^{3} A_j V_j \sinh m_j Z; j = 1,2,3 \tag{42}$$

$$\tilde{\theta} = \sum_{j=1}^{3} A_j S_j \cosh m_j Z; j = 1,2,3 \tag{43}$$

where

$m_j = \pm m_j; (j = 1,2,3)$ are roots of equation (40) and $A_j; (j = 1,2,3)$ are arbitrary constants.

$$V_j = \frac{\gamma_{21} m_j^2 - (\gamma_{33}\gamma_{21} + \gamma_{31}\gamma_{23})}{-m_j^4 + m_j^2(\gamma_{33} + \gamma_{22} + \gamma_{32}\gamma_{23}) - \gamma_{22}\gamma_{33}}; j = 1,2,3 \tag{44}$$

$$S_j = \frac{m_j^2(\gamma_{32}\gamma_{21} - \gamma_{31}) + \gamma_{31}\gamma_{22}}{-m_j^4 + m_j^2(\gamma_{33} + \gamma_{22} + \gamma_{32}\gamma_{23}) - \gamma_{22}\gamma_{33}}; j = 1,2,3 \tag{45}$$

Using equations (41)–(43) in equations (31)–(32), the stresses are given as

$$\tilde{\tau}_{ZZ} = \left(\gamma_{41} + \gamma_{42} m_j V_j + \gamma_{43} S_j\right).\sum_{j=1}^{3} A_j \cosh m_j z \tag{46}$$

$$\tilde{\tau}_{RZ} = \gamma_{44} \sum_{j=1}^{3} A_j \sinh m_j z \left(1 - \alpha V_j\right) \tag{47}$$

where $\gamma_{41} = p\alpha_0^* \alpha(1 - 2\delta^2), \gamma_{42} = \delta_0^* p, \gamma_{43} = \beta_0^* \tau_1 p, \gamma_{44} = \delta^2 \alpha_1^* p$ (48)

Taking inverse Hankel transformation of equations (41)–(43) and (46)–(47) we obtain

$$\bar{U} = \int_0^\infty \left[\sum_{j=1}^{3} A_j \cosh m_j Z\right] J_1(\alpha R)\, d\alpha \tag{49}$$

$$\bar{W} = \int_0^\infty \left[\sum_{j=1}^{3} A_j V_j \sinh m_j Z \right] \alpha J_0(\alpha R)\, d\alpha \tag{50}$$

$$\bar{\theta} = \int_0^\infty \left[\sum_{j=1}^{3} A_j S_j \cosh m_j Z \right] \alpha J_0(\alpha R)\, d\alpha \tag{51}$$

$$\bar{\tau}_{ZZ} = \int_0^\infty \left[\left(\gamma_{41} + \gamma_{42} m_j V_j + \gamma_{43} S_j \right) . \sum_{j=1}^{3} A_j \cosh m_j z \right] \alpha J_0(\alpha R)\, d\alpha \tag{52}$$

$$\bar{\tau}_{RZ} = \gamma_{44} \int_0^\infty \left[\sum_{j=1}^{3} A_j \sinh m_j z \left(1 - \alpha V_j \right) \right] J_1(\alpha R)\, d\alpha \tag{53}$$

where $A_j, j = 1,2,3$ are unknowns to be determined.

Upon applying the inverse Hankel transform to the boundary conditions given in equations (21) and (22) via (51) to (53) by using lengthy and straight forward calculations, we get equations as:

Case I: Mechanical load applied on the surface of Semi Infinite Cylinder

$$\begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{pmatrix}^{-1} \begin{pmatrix} \sigma_0 \bar{f}(\alpha, Z, p) \\ 0 \\ 0 \end{pmatrix}; i, j = 1,2,3. \tag{54}$$

Case II: Thermal load applied on the surface of Semi Infinite Cylinder

$$\begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M'_{31} & M'_{32} & M'_{33} \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 0 \\ \theta_0 \bar{f}(\alpha, Z, p \end{pmatrix}; i, j = 1,2,3. \tag{55}$$

where $M_{11} = \left((\gamma_{41} + \gamma_{42} m_1 V_1 + \gamma_{43} S_1) \cosh m_1 Z \right) \alpha;$

$M_{21} = \sinh m_1 Z (1 - \alpha V_1);$

$M_{31} = \alpha S_1 \cosh m_1 Z;$

$$M'_{31} = \alpha S_1 m_1 \sinh m_1 Z \tag{56}$$

The elements $M_{12}, M_{13}, M_{22}, M_{23}, M_{32}, M_{33}, M'_{32}, M'_{33}$ can be written from $M_{11}, M_{21}, M_{31}, M'_{31}$ by replacing $M_{11}, M_{21}, M_{31}, M'_{31}$ and m_1, V_1, S_1 with .

5. INVERSE OF DOUBLE TRANSFORMS

In order to obtain the solution in physical domain, we have to make inverse transformations of equations (54) to (55). To remove the complication of the solution in the Laplace transformation in physical domain we find the inversion of Laplace transformation which is obtained by using the Gaver – Stehfast algorithm [22, 23]. The work completed which have been done by Widder [24] who extended an inversion operator for Laplace transform. Gaver–Stehfast [22, 23] modified this operator and derived the formula

$$f(t) = \frac{\ln 2}{\tau} \sum_{j=1}^{k} D(j,k) F\left(j \frac{\ln 2}{\tau} \right) \tag{57}$$

with

$$D(j,k) = (-1)^{j+M} \sum_{n=m}^{\min(j\,,\,M)} \frac{n^M (2n)!}{(M-n)!n!(n-1)!(j-n)!(2n-j)!} \tag{58}$$

where k is an even integer, whose value depends on the word length the computer used. $M = \dfrac{k}{2}$ and m is the integer part of the $\dfrac{(j+1)}{2}$. The optimal value of k was chosen as described in Stehfast algorithm [23], for the rapid convergence of results with the desired exactness. The technique of numerical integration Press et al. [25] has been used with the help of variable step size used to evaluate the integrals engaged in related equations. Computer analyzed and simulated results have been obtained by the use of MATLAB software tools.

6. NUMERICAL RESULTS AND DISCUSSION

With the view to demonstrate and compare the analytical results which are achieved in the previous sections in the context of the coupled theory (CT), Lord – Shulman theory (LS), Green – and Lindsay theory (GL), Green – Naghdi theory (GN) and Chandrasekharaiah – Tzou theory (C–T) theories of visothermoelasticity, we propose some numerical computations. For mechanical and thermal loading we are supposed to take

$$f(R,Z,\tau) = \begin{cases} \sigma_0 F(a-R)\, F(\tau) \text{ for mechanical loads} \\ \theta_0 H(a-R)\, H(\tau) \text{ for thermal loads} \end{cases} \tag{59}$$

Here σ_0 and θ_0 are constants. On taking Laplace and Hankel transformations in equation (62) we get:

$$\tilde{f}(\alpha, Z, p) = \begin{cases} \dfrac{\sigma_0\, a J_1(\alpha, a)}{\alpha p} \text{ for mechanical loads} \\ \dfrac{\theta_0\, a J_1(\alpha, a)}{\alpha p} \text{ for thermal loads} \end{cases} \tag{60}$$

We can use the relation $J_1(Z) = -\dfrac{d}{dZ} J_0(Z)$ in equation (60) for similarity of stresses and temperature. In order to express the problem, the physical data of copper material has been taken for the computation purpose given by (Mukhopadhyay [13])

$\lambda = 8.2 \times 10^{10}\ Nm^{-2},\ \mu = 4.2 \times 10^{10}\ Nm^{-2},\ \rho = 8.950 \times 10^{10}\ kg\ m^{-3},$

$\varepsilon_T = 0.00265, \alpha_T = 1.0 \times 10^{-8} / K, K = 1.13 \times 10^2\, Cal\, m^{-1}\, s^{-1}\, K^{-1}, Z = h = 1, a = 1$

$\alpha_0 = \alpha_1 = 6.8831 \times 10^{-13}\ s\ ,\ \omega^* = 1.11 \times 10^{11}\ s^{-1}, T_0 = 300 K\ ,\ \sigma_0 = \theta_0 = 1,$

The computer analyzed results of field functions i.e. for mechanical load and thermal loads in the context of the CT, LS, GL, GN and C–T theories of visothermoelasticity. Fig. 1 and Fig. 2 are drawn for displacements versus non dimensional radius, Fig. 3 to Fig. 5 for mechanical loads (i.e. stresses and temperature versus radius) and Fig. 6 to Fig. 8 for thermal loads (i.e. stresses and temperature change versus radius). It is observed from Fig. 1 that initially the variation of radial displacement is high and with increase in the value of (R) the vibrations of variation go on decreasing and die out. It is also revealed from Fig. 2 that vibrations of variation are reverse of Fig. 1. The variations in Fig. 2 are very low from mean position at ($R = 0$) and with increase in radius from origin the variations go on increasing and die out.

Figs. 3 to 5 have been plotted for temperature and stresses versus non–dimensional radius (R) for different theories of generalized viscothermoelasticity for mechanical loadings. It is inferred from the Fig. 3 that the variation of temperature is largest at ($R = 0$) and with increasing value of (R) the variation of temperature go on decreasing with increase in the value of (R). Fig. 4 has been plotted for radial stress versus non – dimensional radius (R). It is examined from Fig. 4 that the initially variation is highest at origin and lowest at ($R = 2.8$) and increases up to $R = 3.5$, after that with increase in the value of (R), the variations go on decreasing and die out. Fig. 5 has been plotted for axial stress versus radius (R). Fig. 5 revealed that initially the variations are low below mean position decreases up to ($R = 1.5$), and increases with increasing value of radius (R) up to ($R = 2.2$), after that the variation of vibrations go on decreasing and die out. Figs. 6 to 8 have been plotted for temperature change and stresses for thermal loadings. It can be noted from Fig. 6 that initially the variations of temperature change is high and with increase in value of (R) the variations go on decreasing. It is revealed from Fig. 7 that the variations of radial stress (τ_{RZ}) is highest at ($R = 0$) and with increase in the value of (R)

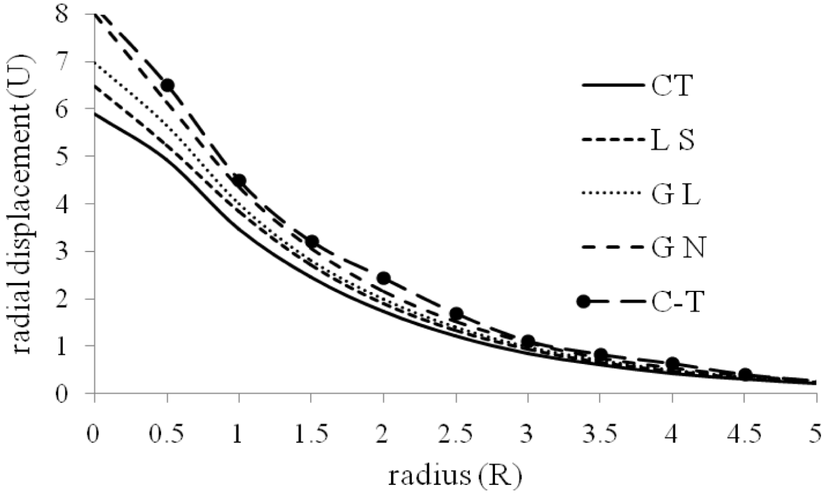

Figure 1: Radial displacement against radius (R) for different theories.

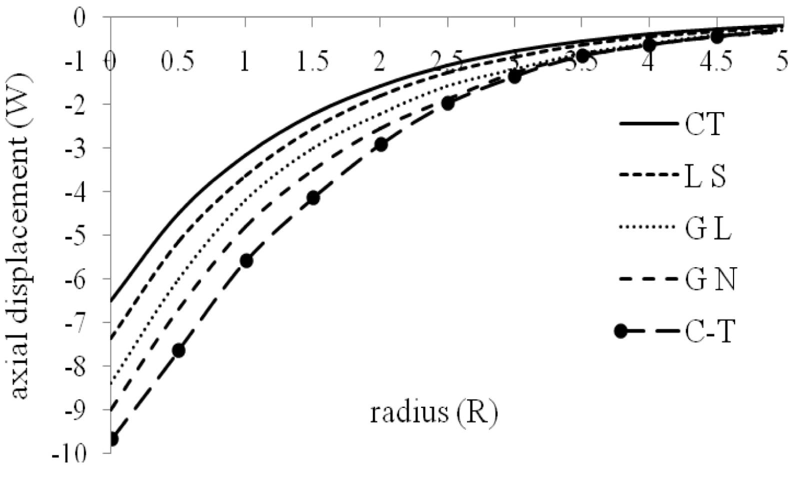

Figure 2: Axial displacement against radius (R) for different theories.

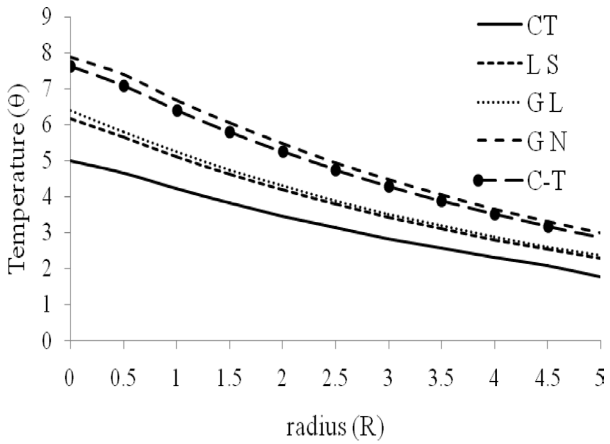

Figure 3: Temperature against radius (R) for mechanical loads.

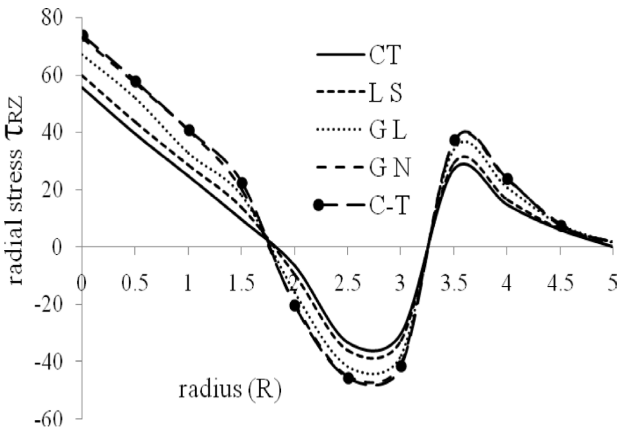

Figure 4: Radial stress (τ_{RZ}) against radius (R) for mechanical loads.

the variation go on decreasing. It can be inferred from Fig. 8 that the initially variation of axial stress (τ_{ZZ}) is lowest below mean position and with increase in the value of (R) the variation go on increasing and die out at $(R = 4.5)$.

From the trends of variations of displacements, temperature and stresses, it is noticed that the variations are largest in case of (GN) and (C – T) theories of generalized thermoelasticity in all the figures. The variations in of (GN) and (C – T) theories of generalized thermoelasticity are close to each other. The nature of relaxation parameters of generalized thermoelasticity significantly affect characteristics and behavior of vibrations of variation of displacements, temperatures and stresses in contrast to coupled theory of thermoelasticity.

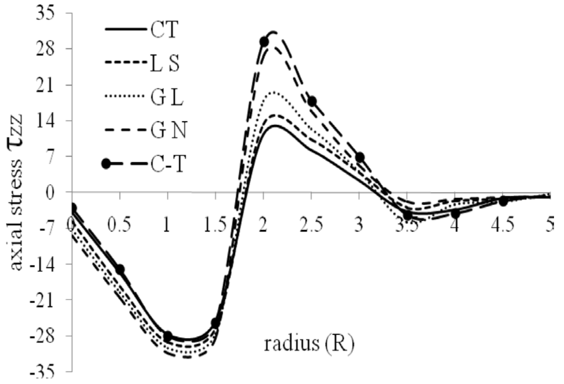

Figure 5: Axial stress (τ_{ZZ}) against radius (R) for mechanical loads.

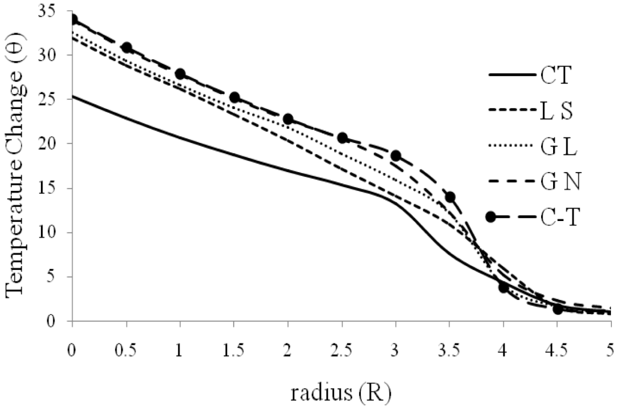

Figure 6: Temperature change (θ) against radius (R) for thermal loads.

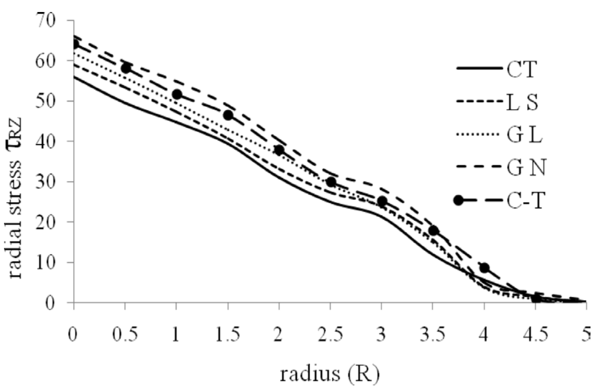

Figure 7: Radial stress (τ_{RZ}) against radius (R) for thermal loads.

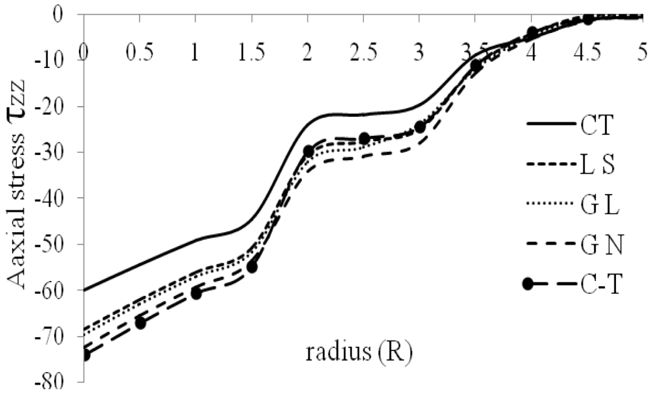

Figure 8: Axial stress (τ_{zz}) against radius (R) for thermal loads.

7. CONCLUDING REMARKS:

1. With the help of non dimensional quantities the simplified the system of governing equations of motion and heat conduction of semi infinite viscothermoelastic cylinder have been solved with the help of combination of Laplace and Hankel transformations.
2. The problem has been investigated with the help of five theories (CT, LS, GL, GN and C–T) of generalized viscothermoelasticity.
3. The solutions have been directly calculated without using potential formation technique.
4. The comparison of five theories i.e. CT, LS, GL, GN and C–T of viscothermoelasticity have been made in all the figures and noticed that variations in case of Green and Naghdhi (GN) and Chandrasekharaiah and Tzou (C–T) are largest and close to each other due to effect of relaxation parameters of generalized viscothermoelasticities.
5. The inversion numerical methods used in the text are very rapid and correct in contrast to any other method.
6. The study may be useful in studying the behaviour of mechanical and thermal characteristics of various types of loading problems in solving engineering problems using five theories of viscothermoelasticity and verifying finite speeds of wave propagation. The presnt analytical and computational results may provide information for experimental scientists / researchers / seismologists working in this subject and field. Study may also find useful applications in the drawing and creation of sensors and other sound waves in addition to bio industries.

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