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Absolute Mean Graceful Labeling in Path Union of Various Graphs

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ABSTRACT

Present paper aims to focus on absolute mean graceful labeling in path union of various graphs. We proved path union of graphs like tree, path P_n, cycle C_n, complete bipartite graph $K_{m,n}$, grid graph $P_m \times P_n$, step grid graph St_n and double step grid graph DSt_n are absolute mean graceful graphs.

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1. Introduction

Throughout present paper, we shall acknowledge $G = (p, q)$, a finite, simple and undirected graph with $V(G)$-vertex set having p vertices and $E(G)$ -edge set having q edges. For a graph $G = (V, E)$, a function with domain V or E or $V \cup E$ is known as a graph labeling for G. Graceful labeling of a graph G is popular concept firstly established by Alexander Rosa [1]. The name graceful labeling was given by Solomon Golomb [2] which was earlier familiar as β-valuation. Kaneria, Makadia and Meghapara [3] proved graceful labeling for grid related graph. Kaneria and Makadia [4] prooved graceful labeling for double step grid graph. All path graphs P_n, cycle C_n and complete bipartite graph $K_{m,n}$ were proved graceful graphs in the early researches in study of graceful lageling. Kaneria and Chudasama [5] introduced absolute mean graceful labeling and proved that it holds true for this new labeling. Current paper is to study the same labeling for path union of finite number of copies of above mentioned graphs and enhances wide scope of operations on such graphs consisting absolute mean graceful labeling. For comprehensive learning of graph labeling, we refereed Gallian [6].

Take path $P_n, P_n, P_{n-1}, \ldots, P_3, P_2$ and put them vertically. A graph made by joining horizontal vertices of paths $P_n, P_n, P_{n-1}, \ldots, P_3, P_2$ is defined as step grid graph and denoted by St_n, $n \geq 3$. It is obvious that $|V(St_n)| = \frac{1}{2}(n^2 + 3n - 2)$ and $|E(St_n)| = n + n - 2$. Similarly, by arranging $P_2, P_3, \ldots, P_n, P_n, P_{n-1}, \ldots, P_3, P_2$ and then joining vertices horizontally, we get double step grid graph DSt_n. A function f is said to be an absolute mean graceful labeling of a graph G, if f: $V(G) \to \{0, \pm1, \pm2, \ldots, \pm|E|\}$ is injective and edge labeling function f^* : $E(G) \to \{1; 2; :::; E\}$ defined as $f^*(e) = \left\lceil \frac{|f(u) - f(v)|}{2} \right\rceil$ is bijective, $\forall$ $e = (u; v) \in E(G)$. A graph which admits such labeling is called absolute mean graceful graph. In the previous work of this labeling, many graphs has been proved as absolute mean graceful graphs such as all path graph P_n, cycle graph C_n, complete bipartite graph $K_{m,n}$, grid graph $P_m \times P_n$, step grid graph St_n and double step grid graph DSt_n. For a graph G, if $G_1, G_2, \ldots, G_t$ $(t \geq 2)$ are t copies of G, then graph made by adding an edge from G_i to G_{i+1} (for $i = 1, 2, \ldots, t - 1$) is said to be path union of G which is denoted as $P(G_1, G_2, \ldots, G_t)$ or $P(t \cdot G)$.

2. Main Results

Theorem I : Every $P(t \cdot P_n)$ is absolute mean graceful graph.

Proof : Let P_n be absolute mean graceful graph having p and q numbers of vertices and edges respectively. So that $p = n$ and $q = n - 1$.

Since P_n is absolute mean graceful graph proved by Kaneria and Chudasama [5], there exists absolute mean

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graceful labeling $f: V(P_n) \rightarrow \{0, \pm 1, \pm 2, ..., \pm |E|\}$ and edge labeling function $f^*: : E(P_n) \rightarrow \{1, 2, ..., |E|\}$ defined as $f^*(e) = \left\lceil \frac{|f(u) - f(v)|}{2} \right\rceil$, which are injective and bijective respectively, $\forall\, e = (u, v) \in E(P_n)$.

Let $\upsilon_{i,1}, \upsilon_{i,2}, ..., \upsilon_{i,n}$ be vertices of $P(t \cdot P_n)$ made up with t copies of path graph P_n by joining $\upsilon_{i,k}$ and $\upsilon_{i+1,k}$, where $k = \left\lceil \frac{n+1}{2} \right\rceil$. Here $\upsilon_{i,j}$ denotes j^{th} vertex of i^{th} copy of P_n in $P(t \cdot P_n)$, $\forall\, i = 1, 2, ..., t$ and $\forall\, j = 1, 2, ..., n$. So that $P' = |V(P(t \cdot P_n))| = tq$ and $Q' = |E(P(t \cdot P_n))| = t(q+1) - 1$. Let us define vertex labeling function $g : V(P(t \cdot P_n)) \rightarrow \{0, \pm 1, \pm 2, ..., \pm Q'\}$ as :

$$g(\upsilon_{i,j}) = \begin{cases} (-1)i(i-1)n, & \forall i = 1, 2, \ldots, t \text{ and } j = 1 \\ (-1)^{i+j+1}\left[\left|g(\upsilon_{i,j-1})\right| + 1\right], & \forall i = 1, 2, \ldots, t;\ \forall j = 2, 3, \ldots, n. \end{cases}$$

Which is an injective function for vertex labeling of $P(t \cdot P_n)$. It is easy to check that an induced edge labeling function g^*, defined as per the definition of absolute mean graceful labeling is bijective. Therefore, $P(t \cdot P_n)$ is absolute mean graceful graph.

Theorem II : Every $P(t \cdot C_n)$, for n - an even number is absolute mean graceful graph.

Proof : Let C_n, for n - an even number is absolute mean graceful graph having p and q number of vertices and edges respectively. So that $p = n$ and $q = n$.

Since C_n is absolute mean graceful graph proved by Kaneria and Chudasama [5], let $f: V(C_n) \rightarrow \{0, \pm 1, \pm 2, ..., \pm |E|\}$ and $f^*: E(C_n) \rightarrow \{1, 2,..., |E|\}$ be vertex and edge labelings.

Let $V(C_n) = \{\upsilon_1, u_2, ..., u_n\}$. Let $V(P(t \cdot C_n)) = \{u_{i,1}, u_{i,2}, ..., v_{i,n}\}$ be vertex set of path union graph G made up with t copies of cycle graph C_n by joining $u_{i,1}$ and $u_{i+1,k}$, where $k = \frac{n+2}{2}$, $\forall\, i = 1, 2, ..., t-1$, where $u_{i,j}$ denotes j^{th} vertex of i^{th} copy of C_n in G, $\forall\, i = 1, 2, ..., t$ and $\forall\, j = 1, 2, ..., n$. So that $P' = |V(P(t{\cdot}C_n))| = tn$ and $Q' = |E(P(t{\cdot}C_n))| = t(n+1) - 1$.

Let us define vertex labeling function $g : V(G) \rightarrow \{0, \pm 1, \pm 2, ..., \pm Q\}$ as :

Case I: When $\frac{n}{2}$ is an even number

$$g(\upsilon_{i,j}) = \begin{cases} f(\upsilon_j), & i = 1 \text{ and } \forall j = 1, 2, \ldots, n \\ (-1)^{j+1}\left[\left|g(\upsilon_{i-1,j})\right| + q + 1\right], & \forall i = 3, 5, 7, \ldots, t-1 \text{ or } t \\ & \forall j = 1, 2, \ldots, n \\ (-1)^{j}\left[\left|g(\upsilon_{i-1,j})\right| + q + 1\right], & \forall i = 2, 4, 6, \ldots, t-1 \text{ or } t \\ & \forall j = 1, 2, \ldots, n. \end{cases}$$

Case II: When $\frac{n}{2}$ is an odd number

$$g(\upsilon_{i,j}) = \begin{cases} f(\upsilon_j), & \forall i = 2, 3, \ldots, t \\ (-1)^{j+1}\left[\left|g(\upsilon_{i-1,j})\right| + q + 1\right], & \forall j = 1, 2, \ldots, n. \end{cases}$$

Which is an injective vertex labeling function for $P(t{\cdot}C_n)$. It is easy to check that edge labeling function g^* is bijective. Therefore, $P(t{\cdot}C_n)$ is an absolute mean graceful graph.

Illustration 1. Absolute mean graceful labeling in path union of 4 copies of cycle C_6.

Theorem III : Every $P(t{\cdot}K_{m,n})$ is absolute mean graceful graph.

Proof : Let $K_{m,n}$ be absolute mean graceful graph with p and q number of vertices and edges. So that $p = m + n$ and $q = mn$.

Since $K_{m,n}$ is absolute mean graceful graph proved by Kaneria and Chudasama [5], there exists absolute mean graceful labeling $f: V(K_{m,n}) \rightarrow \{0, \pm 1, \pm 2, ..., \pm q\}$ and an edge labeling function $f^*: E(K_{m,n}) \rightarrow \{1, 2, ..., |E|\}$ which are injective and bijective respectively.

Let $V(K_{m,n}) = \{u_1, u_2, ..., u_m\} \cup \{u_1, u_2, ..., u_n\}$. Let $V(P(t{\cdot}K_{m,n})) = M \cup N = \{u_{i,1}, u_{i,2}, ..., u_{i,j}\} \cup \{u_{i,1}, u_{i,2}..., u_{i,k}\}$ be vertex set of path union graph $P(t \cdot K_{m,n})$ made up with t copies of complete bipartite graph $K_{m,n}$ by joining

(1) vertex $u_{i;k}$, where $k = \left\lceil \frac{m+1}{2} \right\rceil$ and vertex $u_{i+1;1}$, if m is an even number;

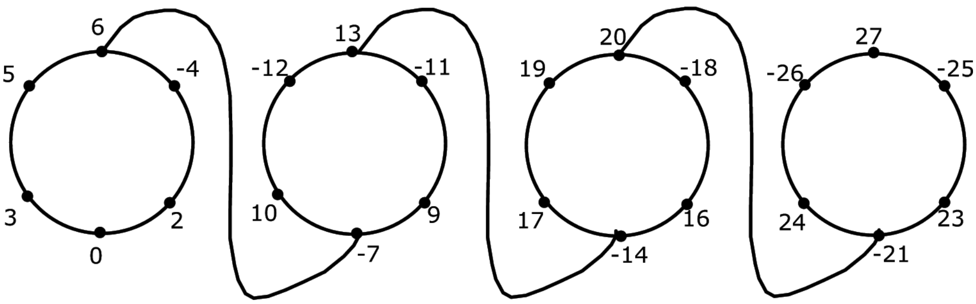

Figure 1. Absolute mean graceful labeling for path union of 4 copies of cycle C_6 with $|V(G)| = 24$ and $|E(G)| = 27$.

(2) vertex $u_{i;\,k}$, where $k = \left\lceil \frac{m+1}{2} \right\rceil$ and vertex $u_{i+1;\,l}$, where $l = \left\lceil \frac{n+1}{2} \right\rceil$, if m is an odd number. Which holds for $\forall\, i = 1, 2, \ldots, t-1$.

It is obvious that $u_{i,j}$ and $u_{i,k}$ denotes j^{th} and k^{th} vertices of M and N parts in i^{th} copy of $K_{m,n}$ in $P(t \cdot K_{m,n})$, $\forall\, i = 1, 2, \ldots, t$, $\forall\, j = 1, 2, \ldots, m$ and $\forall\, k = 1, 2, \ldots, n$. Clearly, $P' = |V(P(t \cdot K_{m,n}))| = t(m+n)$ and $Q' = |E(P(t \cdot K_{m,n}))| = tmn + t - 1$. Let us define vertex labeling function g: $V(P(t \cdot K_{m,n})) \rightarrow \{0, \pm 1, \pm 2, \ldots, \pm Q\}$ as :

$$g(u_{i,j}) = \begin{cases} f(u_j), & i = 1 \text{ and } \forall\, j = 1, 2, \ldots, m \\ g(u_{i-1,j}) + q + 1, & \forall\, i = 2, 3, \ldots, t,\ \forall\, j = 1, 2, \ldots, m. \end{cases}$$

$$g(v_{i,j}) = \begin{cases} f(v_k), & i = 1 \text{ and } \forall\, k = 1, 2, \ldots, n \\ g(v_{i-1,k}) - q - 1, & \forall\, i = 2, 3, \ldots, t,\ \forall\, k = 1, 2, \ldots, n \end{cases}$$

Which gives an injective vertex labeling of $P(t \cdot K_{m,n})$. It is easy to verify that an edge labeling function g^* is bijective function. Therefore, $P(t \cdot K_{m,n})$ is an absolute mean graceful graph.

Illustration 2: Absolute mean graceful labeling in 5 copies of $K_{3,4}$.

Theorem IV : Every $P(t \cdot P_m \times P_n)$ is absolute mean graceful graph.

Proof : Let $P_m \times P_n$ be absolute mean graceful graph with p and q number of vertices and edges respectively. So that $p = mn$ and $q = 2mn - m - n$.

Since $P_m \times P_n$ is absolute mean graceful graph proved by Kaneria and Chudasama [5], let f^*: $V(P_m \times P_n) \rightarrow \{0, \pm 1, \pm 2, \ldots, \pm q\}$ and f^*: $E(P_m \times P_n) \rightarrow \{1, 2, \ldots, q\}$ be vertex labeling injective and edge labeling bijective functions respectively.

Let $u_{j,k}$ be vertices of $P_m \times P_n$, $\forall\, i = 1, 2, \ldots, m$, $\forall\, k = 1, 2, \ldots, n$. Let $u_{i,j,k}$ be the $u_{j,k}$ located vertex of i^{th} copy of P_m

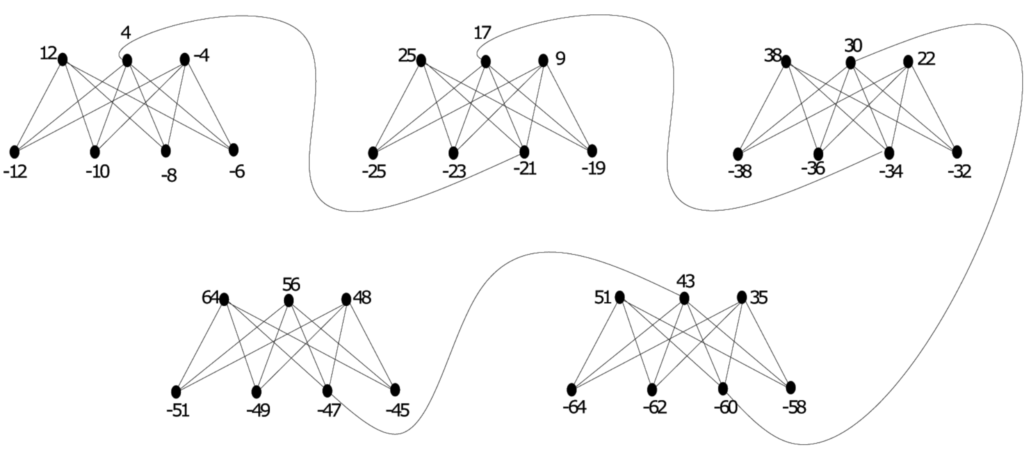

Figure 2. Absolute mean graceful labeling for 5 copies of $K_{3,4}$ with $|V(G)| = 35$ and $|E(G)| = 64$.

$\times P_n$ of $P(t \cdot P_m \times P_n)$ which is made up with t copies of grid graph $P_m \times P_n$, by joining

(1) vertex $u_{i,1,1}$ and vertex $u_{i+1,m,n}$, if q is an odd number or

(2) vertex $u_{i,1,1}$ and vertex $u_{i+1,m,n1}$, if q is an even number,

which holds true for $\forall\, i = 1, 2, \ldots, t-1$; $\forall\, j = 1, 2, \ldots, m$; $\forall\, k = 1, 2, \ldots, n$. It is obvious that $P' = |V(P(t \cdot P_m \times P_n))| = tmn$ and $Q' = |E(P(t \times P_m \times P_n))| = t\,[2mn - m - n + 1] - 1$.

Define vertex labeling $g : V(P(t \cdot P_m \times P_n)) \to \{0, \pm 1, \pm 2, \ldots, \pm Q'\}$ as:

$$g\left(v_{i,j,k}\right) = \begin{cases} f\left(v_{j,k}\right), & i = 1 \text{ and } \forall\, j = 1, 2, \ldots, m,\ \forall\, k = 1, 2, \ldots, n \\ g\left(v_{i-1,j,k}\right) + \left[(-1)^{i+1}(q+1)\right], & \forall\, i = 2, 3, \ldots, t; \\ & \forall\, j = 1, 2, \ldots, m \\ & \forall\, k = 1, 2, \ldots, n. \end{cases}$$

Which gives an injective vertex labeling for $P(t \cdot P_m \times P_n)$ to verify that edge labeling function g^* is bijective. Therefore, $P(t \cdot P_m \times P_n)$ is an absolute mean graceful graph.

Theorem V : Every $P(t \cdot St_n)$ is absolute mean graceful graph.

Proof: Let St_n be absolute mean graceful graph with p and q number of vertices and edges respectively. So that

$$p = \frac{n^2 + 3n - 2}{2} \text{ and } q = n_2 + n - 2.$$

Since St_n is absolute mean graceful graph proved by Kaneria and Chudasama [5], let $f: V(St_n) \to \{0, \pm 1, \pm 2, \ldots, \pm q\}$ and $f^*: E(St_n) \to \{1, 2, \ldots, q\}$ be injective vertex labeling and bijective edge labeling functions respectively.

Let $u_{1,k}$ $(1 \le k \le n)$ be vertices in first row, $u_{2,k}$ $(1 \le k \le n)$ be vertices in second row, $u_{3,k}$ $(2 \le k \le n)$ be vertices in third row, $u_{j,k}$ $(j - 1 \le k \le n)$ be vertices in j^{th} row and $u_{n,k}$ $(n - 1 \le k \le n)$ be vertices in n^{th} row. Let $u_{i,j,k}$ be $u_{j,k}$ – located vertex of j^{th} copy of St_n of G which is made up with t copies of St_n by joining vertices $u_{i,2,1}$ and $u_{i+1,}$ $t - 1$; $\forall\, j = 1, 2, \ldots, n$; $\forall\, k = 1, 2, \ldots, n$. So that $P' = |(G)| = \frac{t}{2}\left(n^2 + 3n - 2\right)$ and $Q' = |E(G)| = t\,(n^2 + n - 1) - 1$.

Let us define vertex labeling function $g: V(P(t \times St_n)) \to \{0, \pm 1, \pm 2, \ldots, \pm Q'\}$ as :

$$g\left(v_{i,j,k}\right) = \begin{cases} f\left(v_{j,k}\right), & i = 1 \text{ and } \forall\, j,\ k = 1, 2, \ldots, n \\ g\left(v_{i-1,j,k}\right) + q + 1, & if\ g\left(v_{i-j,j,k}\right) \ge 0, \quad \forall i = 2, 3, \ldots, t; \\ & \forall\, j,\ k = 1, 2, \ldots, n \\ g\left(v_{i-1,j,k}\right) - q - 1, & if\ g\left(v_{i-1,j,k}\right) < 0, \quad \forall i = 2, 3, \ldots, t; \\ & \forall\, j,\ k = 1, 2, \ldots, n. \end{cases}$$

Which is an injective function for vertex labeling of G. It is clear that induced edge labeling function g^* can be defined as per the definition of absolute mean graceful labeling which is bijective. Therefore, $P(t \times St_n)$ holds absolute mean graceful labeling and hence it is absolute mean graceful graph.

Illustration 3. Absolute mean graceful labeling in path union of 3 copies of step grid graph St_n.

Theorem VI : Every $P(t \cdot DSt_n)$ is absolute mean graceful graph.

Proof : Let DSt_n be absolute mean graceful graph with p and q are number of vertices and edges respectively. So that

$$p = \frac{n}{4}(n + 6) \text{ and } q = \frac{n^2 + 3n - 2}{2}.$$

Since DSt_n is absolute mean graceful graph proved by Kaneria and Chudasama [5], let $f: V(DSt_n) \to \{0, \pm 1, \pm 2, \ldots, \pm q\}$ and $f^*: E(DSt_n) \to \{1, 2, \ldots, q\}$ be injective vertex labeling and bijective edge labeling functions respectively.

We mention each vertices of first row are like $u_{1,k}$ $(1 \le k \le n)$, second row like $u_{2,k}$ $(1 \le k \le n)$, third row like $u_{3,k}$ $(1 \le k \le n - 2)$ and fourth like $v_{4,k}$ $(1 \le k \le n\ \ 4)$. Similarly, last row is like $u_{l,k}$ $(1 \le k \le 2)$, where $l = \frac{n+2}{2}$. Let $u_{i,j,k}$ be $u_{j,k}$ located vertex of i^{th} copy of DSt_n of $P(t \cdot DSt_n)$ which is made up with t copies of DSt_n by joining vertices $u_{i,1,1}$ and $u_{i+1,l,1}$, where $l = \frac{n+2}{2}$ and $\forall\, i = 1, 2, \ldots, t\ \ 1$. It is obvious that $P' = |V(P(t \cdot DSt_n))| = \frac{tn}{4}(n + 6)$ and $Q' = |E(P(t \cdot DSt_n))| = t\left(\frac{n^2 + 3n}{2}\right) - 1$.

Let us define vertex labeling function $g : V(P(t \cdot DSt_n)) \to \{0, \pm 1, \pm 2, \ldots, \pm Q'\}$ and $\forall\, j = 1, 2, \ldots, \frac{n+2}{2}$ and $\forall\, k = 1, 2, \ldots, n$

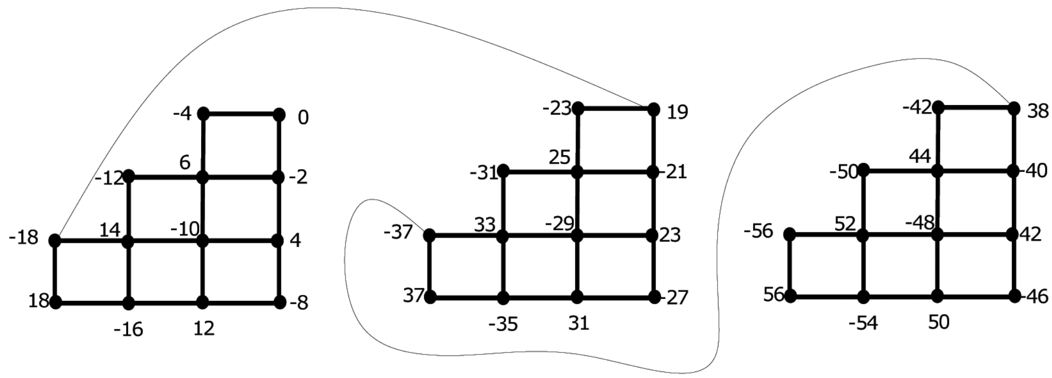

Figure 3. Absolute mean graceful labeling for path union of 3 copies of step grid graph St_n with $|V(T)| = 13$ and $|E(G)| = 18$.

$$g\left(\upsilon_{i,j,k}\right)=\begin{cases} f\left(\upsilon_{j,k}\right), & i=1 \\ g\left(\upsilon_{i-1,j,k}\right)-q-1, & if\ g\left(\upsilon_{i-1,j,k}\right)\le -2, \quad \forall i=2,3,\ldots,t; \\ -q-1, & if\ g\left(\upsilon_{i-1,j,k}\right)=0,\ \forall\ i=2,3,\ldots,t; \\ q+1, & if\ g\left(\upsilon_{i-1,j,k}\right)=-1,\ \forall\ i=2,3,\ldots,t; \\ g\left(\upsilon_{i-1,j,k}\right)+q+1, & if\ g\left(\upsilon_{i-1,j,k}\right)\ge 2,\ \forall\ i=2,3,\ldots,t; \end{cases}$$

Which is an injective function for vertex labeling of $P(t{\cdot}DSt_n)$. It is clear that induced edge labeling function g^* can be defined as per the definition of absolute mean graceful labeling which is bijective function. Therefore, $P(t{\cdot}DSt_n)$ holds absolute mean graceful labeling and hence it is absolute mean graceful graph.

Theorem VII : Every $P(t\times T)$is absolute mean graceful graph, where T consists absolute mean graceful labeling.

Proof : Let T be absolute mean graceful tree with $p = |V(T)|$ and $q = |E(T)|$. It is clear that $q = p - 1$.

Since T is absolute mean graceful tree, there exists absolute mean graceful labeling $f : V(T) \rightarrow \{0, \pm 1, \pm 2, \ldots, \pm q\}$ which is an injective function and the induced edge labeling function $f^* : E(T) \rightarrow \{1, 2, \ldots, q\}$ defined as $f^*(e)=\left\lceil\left|\frac{|f(u)-f(\upsilon)|}{2}\right|\right\rceil$ is bijective for every edge $e = (u, \upsilon) \in E(T)$.

Let $V(T) = \{u_1, u_2, \ldots, u_p\}$. Let $u_{i,1}, u_{i,2}, \ldots, u_{i,p}$ be vertices of path union graph $P(t\times T)$ made up with t copies of tree graph T by joining $u_{i,p}$ and $u_{i+1,1}$, for $\forall i = 1, 2, \ldots, t-1$ and $\forall j = 1, 2, \ldots, p$, where $u_{i,j}$ denotes j^{th} vertex of i^{th} copy of T in $P(t\times T)$. So that $P' = |V(P(t\times T))| = tp$ and $Q' = |E(P(t\times T))| = tp - 1$. By defining vertex labeling function $g: V(P(t\times T)) \rightarrow \{0, \pm 1, \pm 2, \ldots, \pm Q\}$ as :

$$g\left(\upsilon_{i,j}\right)=\begin{cases} f\left(\upsilon_{j}\right), & i=1\ and\ \forall\ j=1,2,\ldots,p \\ g\left(\upsilon_{i-1,j}\right)-q-1, & if\ g\left(\upsilon_{i-j,j}\right)\le 0, \quad \forall i=2,3,\ldots,t; \\ & \forall\ j=1,2,\ldots,p \\ g\left(\upsilon_{i-1,j}\right)+q+1, & if\ g\left(\upsilon_{i-1,j}\right)> 0, \quad \forall i=2,3,\ldots,t; \\ & \forall\ j=1,2,\ldots,p. \end{cases}$$

Which is an injective function for vertex labeling of $P(t\times T)$. It is clear that induced edge labeling function g^* can be defined as per definition of absolute mean graceful labeling which is bijective. Hence, $P(t\times T)$ holds absolute mean graceful labeling it is absolute mean graceful graph.

Illustration 4. Absolute mean graceful labeling in path union of 3 copies of tree T.

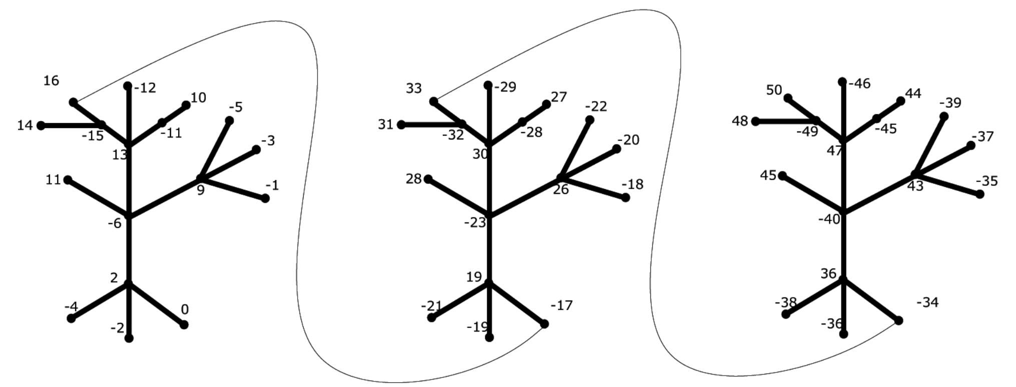

Figure 4. Absolute mean graceful labeling for path union of 3 copies of tree T with $|V(T)| = 17$ and $|E(G)| = 16$.

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