Gamma Photon Exposure Buildup Factors for Some Spin Ice Compounds Using G-P Fitting Method

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Abstract: Exposure buildup factors(EBF) for some spin ice compounds such as Dysprosium Titanate ($Dy_2Ti_2O_7$), Dysprosium Stannate ($Dy_2Sn_2O_7$), Holmium Titanate ($Ho_2Ti_2O_7$) and Holmium Stannate ($Ho_2Ti_2O_7$) useful in nuclear engineering have been computed for photon energy range 0.015 to 15 MeV upto penetration depth of 40 mean free path. The EBF for these compounds were found to be the largest at photon energy 15 MeV excepts in low energy. The EBF for the compounds containing tin were found to be the largest as well as shown a peak at 30 keV photon energy.

Keywords: spin ice; gamma; exposure; shielding

1. INTRODUCTION

The intensity of gamma-ray beam through a medium follows Lambert's Beer law ($I=I_0 e^{-\mu t}$) for narrow, monochromatic gamma-ray for thin absorbing material, where I and I_0 are transmitted and initial photon densities, μ is linear attenuation coefficient and t is the thickness of medium. In case of broad beam, polychromatic gamma-ray entering into thick medium, the law is made applicable by introducing a correction factor called as "buildup factor B". Now the modified equation is $I= B \times I_0 e^{-\mu t}$ including the buildup factors, B. The B is 1 when all the above conditions are satisfied. The buildup is defined as the ratio of total value of specified radiation quantity at any point to the contribution to that value from radiation reaching to the point without having undergone a collision.

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Dysprosium Titanate ($Dy_2Ti_2O_7$), Dysprosium Stannate ($Dy_2Sn_2O_7$), Holmium Titanate ($Ho_2Ti_2O_7$) and Holmium Stannate ($Ho_2Sn_2O_7$) are geometrically frustrated magnetic materials called as "Spin ice compounds" due to incompatibility between the lattice symmetry and spin-spin interactions [1]. Geometrical frustrations in the spin ice compound occur when the spins on its lattice are not capable of simultaneous minimizing their pair-wise energies. The long-range magnetic dipolar interactions are responsible for spin ice behavior in the Isingpyrochlore magnets [2]. The magnetic rare-earth ions in these materials are situated on the lattice of corner-sharing tetrahedral, where their spins are constrained by crystal field interactions to point either directly toward or directly away from the centers of the tetrahedral. The neutron scattering measurements of spin correlations show that the magnetic Coulomb force acts between the magnetic monopoles by fluctuating the magnetic monopoles between high and low density states close to the critical point [3,4].

The spin ice compounds have high neutron absorption cross section and hence these materials are useful for research and power reactors for controlling the nuclear fission processes. The $Dy_2Ti_2O_7$ and Dy_2TiO_5 are being used as a neutron absorber material in the form of control rods in VVER types Russian reactors [5,6]. On the other hand, $Dy_2Ti_2O_7$exhibits n-type semiconductor properties for single crystal with band-gap of 2.4 eV [7]. The existence of magnetic monopoles in the spin ice compounds provides new era of material properties of condensed matter. It is interesting to note that the spin ice compound exhibit the magnetic monopole at low temperature which become thrust area in condensed matter physics. Therefore, it is interesting to study the buildup of gamma-ray in these materials whereas the photon interaction parameters for some spin ice compounds has been already reported [8]. Although various investigators studied exposure buildup factors for composite materials such as soils, fly-ash, building materials, glasses, detectors, etc elsewhere. However,exposure buildup factors for spin ice compounds are not found in the literature. This has encouraged us to study the exposure buildup factors for spin ice compounds. In view of above, the exposure buildup factors, EBFfor some spin ice compounds such as $Dy_2Ti_2O_7$ (DTO), $Dy_2Sn_2O_7$ (DSO), $Ho_2Ti_2O_7$ (HTO) and $Ho_2Sn_2O_7$(HSO) were calculated using G-P fitting method for photon energy range 0.015 to 15 MeV up to penetration depth 40 mfp.The calculation of buildup factor is explained below.

2. THEORETICAL BACKGROUND FOR EXPOSURE BUILDUP FACTORS

The compilation for buildup factors by various codes is reported by American Nuclear Society(ANS) [9].The data in the report covers energy range 0.015-15

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MeV up to penetration depth of 40 mean free path (mfp). The buildup factors in the ANS report [9] are for 23 elements (Z=4-92). Harima et al., [10] developed a fitting formula, called Geometric Progression (G-P) which gives buildup factors of the good agreement with the ANS [9]. Harima [11] reviewed extensively and reported the current gamma-ray buildup factors. Various researchers investigated gamma-ray buildup factors for different materials; soil [12] fly-ash [13], building materials [14] borosilicate glass [15], and neutron detectors [16] which shows that the G-P fitting is a useful method for estimation of the buildup factors. The EBF by American Nuclear Society [9], G-P fitting method and MCNP-5 [17] for water for photon energy range 0.015-15 MeV up to 40 mfp are standardized [13,15]. It was found that the calculation of buildup factor by present work and ANS [9] agrees within a few percentages. Therefore, G-P fitting method was chosen in the present investigation for calculation of buildup factor for spin ice compounds. The computational work of the EBF for spin ice compounds is done in three steps as;

1. Calculation of equivalent atomic number [18,19]
2. Calculation of the G-P fitting parameters [18,19]
3. Calculation of the exposure buildup factors [10,11]

The buildup of photons is mainly due to multiple scattering events by Compton scattering, so that equivalent atomic number, Z_{eq} is derived from the Compton scattering interaction process. The Z_{eq} for individual compound is estimated by the ratio of $(\mu/\rho)_{Compton} / (\mu/\rho)_{Total}$, at a specific energy with the corresponding of an element at same energy. Thus first the Compton partial mass attenuation coefficient, $(\mu/\rho)_{Compton}$ and the total mass attenuation coefficients, $(\mu/\rho)_{Total}$ are obtained using WinXcom program [20]. The Z_{eq} for a compound is calculated by logarithmic interpolation method [18,19] using formulaas;

$$Z_{eq} = \frac{Z_1(\log R_2 - \log R) + Z_2(\log R - \log R_1)}{\log R_2 - \log R_1} \qquad (1)$$

where Z_1 and Z_2 are the atomic numbers for elements corresponding to the ratios R_1 and R_2 respectively and R is the ratio for the compound at a specific energy. The G-P fitting parameters are calculated in a similar fashion of logarithmic interpolation procedure for Z_{eq}.

Finally buildup factors are calculated using G-P fitting parameters (b, c, a, X_k and d) in the following equations given below;

$$B(E,x) = 1 + \frac{(b-1)(K^x - 1)}{K-1} \quad \text{for} \quad K \neq 1 \qquad (2)$$

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Table 1: G-P fitting parameters for exposure buildup factors and equivalent atomic numbers for Dysprosium Titanate.

E (MeV)	Dysprosium Titanate					
	b	c	a	X_k	d	Z_{eq}
0.015	1.0038	0.4726	0.1412	17.5594	-0.0853	54.0117
0.02	1.0051	0.4059	0.1785	16.1795	-0.0633	54.6299
0.03	1.1389	0.3922	0.2006	12.9133	-0.0586	29.4912
0.04	1.2557	0.3294	0.2279	15.1075	-0.0915	29.8026
0.05	1.2701	0.3222	0.1883	12.7991	-0.1215	30.0687
0.06	2.5696	0.4150	0.0431	8.8073	-0.0879	52.2446
0.08	1.9233	0.1107	0.3618	15.8968	-0.0474	53.2488
0.1	1.4373	0.0465	0.6729	14.1635	-0.2353	53.7892
0.15	1.2024	0.2662	0.3335	13.8105	-0.1891	54.5460
0.2	1.2126	0.4639	0.1883	14.3167	-0.1009	54.9626
0.3	1.3285	0.5942	0.1257	13.9224	-0.0597	55.4439
0.4	1.4359	0.7260	0.0832	14.1235	-0.0491	55.7198
0.5	1.5062	0.8131	0.0575	14.0940	-0.0392	55.8980
0.6	1.5487	0.8743	0.0388	13.8378	-0.0297	56.0205
0.8	1.5990	0.9415	0.0213	13.7096	-0.0228	56.1452
1	1.6109	0.9799	0.0126	13.2603	-0.0216	56.2063
1.5	1.5461	1.0847	-0.0120	13.8071	-0.0092	55.1397
2	1.5550	1.0858	-0.0097	13.0930	-0.0135	52.3293
3	1.5318	1.0598	0.0030	12.9511	-0.0319	47.7906
4	1.4884	1.0287	0.0160	13.3932	-0.0438	45.6202
5	1.5095	0.9469	0.0460	13.6227	-0.0711	44.4616
6	1.4927	0.9239	0.0575	13.8438	-0.0807	43.7796
8	1.5303	0.8823	0.0795	14.1512	-0.0985	42.9015
10	1.5021	0.9687	0.0584	14.2084	-0.0780	42.3980
15	1.5731	1.1185	0.0349	14.1585	-0.0599	42.0452

$$B(E,x) = 1 + (b-1)x \quad \text{for} \quad K = 1, \tag{3}$$

where,

$$K(E,x) = cx^a + d\frac{\tanh(x/X_K - 2) - \tanh(-2)}{1 - \tanh(-2)}, \tag{4}$$
$$\text{for penetration depth (X)} \leq 40 \text{ mfp}$$

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Table 2: G-P fitting parameters for exposure buildup factors and equivalent atomic numbers for Dysprosium Stannate.

E (MeV)	Dysprosium Stannate					
	b	c	a	X_k	d	Z_{eq}
0.015	1.0037	0.4780	0.1387	16.4327	-0.0765	54.5644
0.02	1.0054	0.4071	0.1775	16.3207	-0.0628	54.9871
0.03	3.1580	0.9567	0.1267	28.4222	-0.1869	39.2244
0.04	3.4746	0.3233	0.1123	21.9350	-0.0424	39.5706
0.05	2.8139	0.0836	-0.2359	12.0947	0.0273	39.8069
0.06	2.2308	0.5783	0.1072	12.2439	-0.1169	54.3571
0.08	1.9811	0.1576	0.1367	16.1207	0.0053	54.9555
0.1	1.4526	0.0430	0.6674	14.3022	-0.2031	55.2681
0.15	1.1984	0.2518	0.3466	13.7769	-0.1972	55.7035
0.2	1.2035	0.4571	0.1916	14.2598	-0.1026	55.9456
0.3	1.3180	0.5869	0.1284	13.8802	-0.0610	56.2361
0.4	1.4252	0.7190	0.0852	14.1216	-0.0499	56.4081
0.5	1.4959	0.8074	0.0589	14.0974	-0.0396	56.5187
0.6	1.5400	0.8688	0.0401	13.8213	-0.0301	56.6006
0.8	1.5921	0.9368	0.0224	13.6786	-0.0232	56.6786
1	1.6051	0.9761	0.0135	13.2471	-0.0220	56.7216
1.5	1.5421	1.0784	-0.0105	13.7239	-0.0100	56.0583
2	1.5541	1.0670	-0.0045	13.0872	-0.0175	54.3747
3	1.5284	1.0448	0.0080	13.0977	-0.0362	51.7961
4	1.4736	1.0305	0.0174	13.4513	-0.0460	50.4516
5	1.4911	0.9564	0.0460	13.6863	-0.0728	49.6781
6	1.4806	0.9319	0.0588	13.9112	-0.0838	49.2358
8	1.5259	0.9077	0.0766	14.1585	-0.0983	48.6869
10	1.5036	1.0201	0.0501	14.1857	-0.0724	48.3721
15	1.5879	1.1877	0.0280	13.9642	-0.0573	48.1449

where x is the source-detector distance for the medium in terms of mfp and b, the value of the exposure buildup factor at 1 mfp, K (E, x) is the dose multiplicative factor, and b, c, a, X_K and d are computed G-P fitting parameters which depends on the attenuating medium and source energy. The equation (4) represents the dependency of K on x; *a, c, d* and X_K are dependent upon

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Table 3: G-P fitting parameters for exposure buildup factors and equivalent atomic numbers for Holmium Titanate.

E (MeV)	Holmium Titanate					
	b	c	a	X_k	d	Z_{eq}
0.015	1.0036	0.4806	0.1375	15.8837	-0.0722	54.8357
0.02	1.0057	0.4088	0.1762	16.5083	-0.0622	55.4650
0.03	1.2565	0.4251	0.1963	13.8163	-0.0660	29.9850
0.04	1.3881	0.3290	0.2210	15.5150	-0.0885	30.3112
0.05	1.3644	0.3076	0.1624	12.7561	-0.1124	30.5883
0.06	2.4216	0.4864	0.0711	10.3089	-0.1006	53.1574
0.08	1.9521	0.1341	0.2497	16.0083	-0.0212	54.0915
0.1	1.4461	0.0445	0.6697	14.2439	-0.2166	54.6413
0.15	1.1994	0.2553	0.3434	13.7851	-0.1952	55.4176
0.2	1.2044	0.4578	0.1913	14.2652	-0.1024	55.8517
0.3	1.3160	0.5855	0.1289	13.8722	-0.0613	56.3876
0.4	1.4206	0.7160	0.0861	14.1208	-0.0502	56.7026
0.5	1.4894	0.8038	0.0598	14.0995	-0.0398	56.9151
0.6	1.5333	0.8646	0.0411	13.8087	-0.0304	57.0463
0.8	1.5859	0.9326	0.0233	13.6505	-0.0235	57.1653
1	1.5996	0.9724	0.0144	13.2345	-0.0224	57.2159
1.5	1.5416	1.0776	-0.0103	13.7136	-0.0101	56.1731
2	1.5545	1.0767	-0.0072	13.0902	-0.0154	53.3056
3	1.5280	1.0590	0.0034	12.9681	-0.0324	48.7426
4	1.4847	1.0299	0.0160	13.4005	-0.0440	46.4614
5	1.5065	0.9484	0.0460	13.6330	-0.0714	45.2662
6	1.4909	0.9251	0.0577	13.8539	-0.0812	44.5589
8	1.5297	0.8858	0.0791	14.1522	-0.0985	43.6622
10	1.5023	0.9755	0.0573	14.2054	-0.0773	43.1490
15	1.5751	1.1276	0.0340	14.1329	-0.0595	42.8001

attenuating medium and source energy, E. The Z_{eq} and G-P fitting parameters for the spin ice compounds are given in Table 1 to 4.

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Table 4: G-P fitting parameters for exposure buildup factors and equivalent atomic numbers for Holmium Stannate.

| E (MeV)	Holmium Stannate					
	b	c	a	X_k	d	Z_{eq}
0.015	1.0036	0.4789	0.1383	16.2521	-0.0751	54.6535
0.02	1.0054	0.4074	0.1773	16.3543	-0.0627	55.0724
0.03	3.1554	0.9560	0.1268	28.4026	-0.1868	39.2103
0.04	3.4713	0.3233	0.1125	21.9249	-0.0425	39.5539
0.05	2.8114	0.0840	-0.2352	12.0958	0.0271	39.7889
0.06	2.2176	0.5847	0.1097	12.3777	-0.1180	54.4411
0.08	1.9838	0.1598	0.1261	16.1313	0.0078	55.0371
0.1	1.4534	0.0428	0.6671	14.3096	-0.2013	55.3483
0.15	1.1981	0.2508	0.3475	13.7746	-0.1977	55.7821
0.2	1.2028	0.4566	0.1918	14.2553	-0.1027	56.0244
0.3	1.3169	0.5862	0.1287	13.8759	-0.0612	56.3170
0.4	1.4239	0.7181	0.0855	14.1214	-0.0499	56.4910
0.5	1.4945	0.8066	0.0591	14.0979	-0.0397	56.6030
0.6	1.5387	0.8680	0.0403	13.8189	-0.0302	56.6860
0.8	1.5910	0.9360	0.0225	13.6736	-0.0232	56.7650
1	1.6041	0.9754	0.0137	13.2449	-0.0220	56.8087
1.5	1.5417	1.0778	-0.0104	13.7159	-0.0101	56.1471
2	1.5540	1.0662	-0.0042	13.0869	-0.0177	54.4696
3	1.5287	1.0440	0.0083	13.1039	-0.0364	51.9010
4	1.4745	1.0294	0.0178	13.4565	-0.0463	50.5636
5	1.4907	0.9566	0.0460	13.6876	-0.0728	49.7889
6	1.4804	0.9321	0.0588	13.9125	-0.0839	49.3469
8	1.5258	0.9081	0.0766	14.1586	-0.0983	48.7986
10	1.5036	1.0210	0.0499	14.1853	-0.0723	48.4838
15	1.5881	1.1889	0.0278	13.9609	-0.0573	48.2556

3. RESULTS AND DISCUSSION

Variation of exposure buildup factor, EBF for the spin ice compounds with photon energy (0.015 to 15 MeV) for selected penetration depths 1, 5, 10, 20 and 40 mfp is shown in Fig. 1 (a-d). The EBF for the compounds are small in

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low photon energy region which can be explained by photon interaction cross section dependency on photon energy and atomic number for the elements. The Z_{eq} for a compound plays the similar role as Z for an element.

The EBF for the compounds are the minimum in low-energy due to dominance of photoelectric effect where interaction cross section is proportional to $Z_{eq}^{4-5}/E^{3.5}$. The large values of Z_{eq} of the compounds reduce the buildup factors. With increase in photon energy, EBF increases due to multiple scattering as Compton scattering dominates. The pair production takes over the Compton scattering process for photon energy equal or above 1.022 MeV. In the pair production, interaction cross section is proportional to Z_{eq}^2, so low-Z_{eq} shows lowest EBF in the pair production region. This type of high buildup factors for the elements, compounds and mixtures have been reported [21-24].

It is also noted that the compounds containing tin ($Dy_2Sn_2O_7$ and $Ho_2Sn_2O_7$) show peaks at photon energy 30 keV (see Fig.1 (b) and (d)). To explain this, we have plotted the EBF for tin [9] at different penetration depths (1, 5, 10, 20 and 40 mfp) in Fig.2. From Fig.2, it is observed that the sharp peak in EBF at 40 mfp for tin (2.88×10^{13}) is at energy 30 keV analogous to $Dy_2Sn_2O_7$ (3.34×10^7) and $Ho_2Sn_2O_7$ (3.29×10^7). The height of peaks in the spin ice compounds changes due to other elements. Therefore, it is concluded that peaks in EBF in low-energy are due to presence of the tin element.
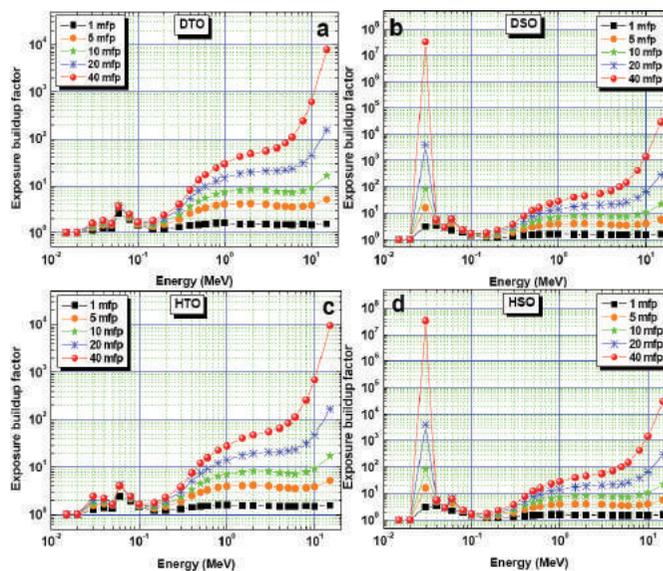
Figure 1 (a-d): Variation of exposure buildup factors for spin ice compounds with photon energy for penetration depths 1, 5, 10, 20 and 40 mfp.

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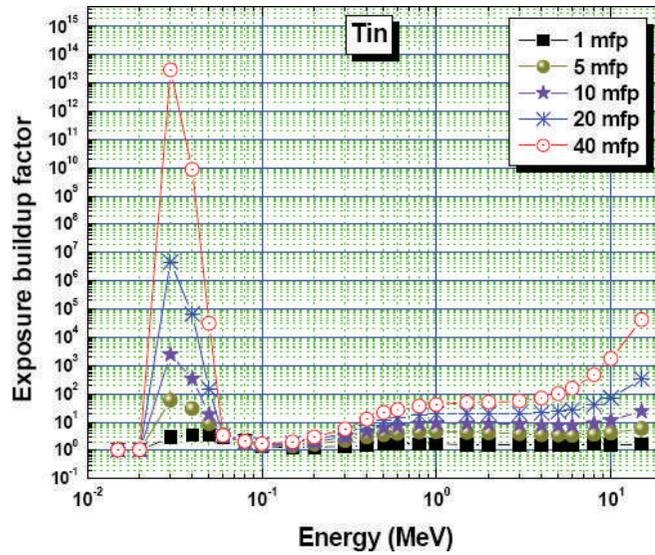
Figure 2: Exposure buildup factors for tin at penetration depth 1, 5, 10, 20 and 40 mfp.
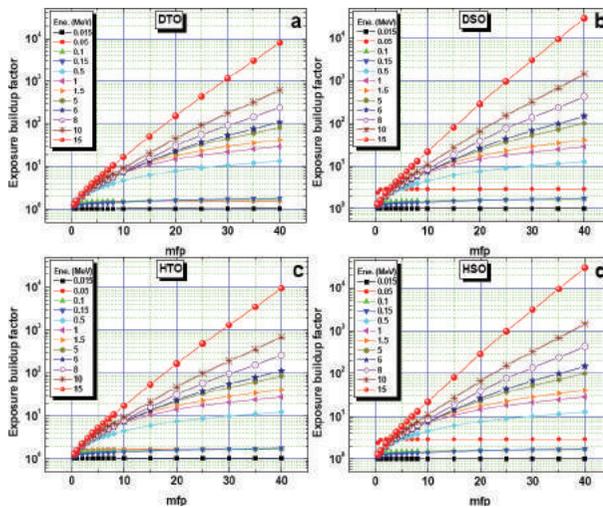
Figure 3: (a-d): Variation of exposure buildup factors for spin ice compounds with penetration depths for selected photon energies.

Variation of EBF for the spin ice compounds for penetration depth (0-40 mfp) is shown in Fig. 3 (a-d). It is to be noted that the EBF for the compounds increase with photon energy and penetration depth. However, in low-energies

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the EBF are found to be constant. The EBF for the selected compounds are found to be unity at 0.015 MeV photon energy. As photon energy increases, the EBF increases and become linear at high energy (15 MeV). The EBF for the compounds containing tin are found to be the largest.This type of behavior of buildup factor for superconductors has been reported [24].

4. CONCLUSIONS

In the present study, we studied exposure buildup factors for some spin ice compounds DysprosiumTitanate, Dysprosium Stannate, Holmium Titanate and Holmium Stannateuseful in nuclear engineering for photon energy range 0.015 to 15 MeV. The EBF for the compounds containing tin were found to be the largest as well as shown a peak at 30 keV photon energy.This study could be very useful for shielding evaluation of this and similar type of spin ice compounds.

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