Heavy-ion Fusion Cross Sections of ^{32}S on ^{90,96}Zr Targets Using Coulomb and Proximity Potential

K. P. SANTHOSH AND V. BOBBY JOSE

School of Pure and Applied Physics, Kannur University, Swami Anandatheertha Campus, Payyanur 670327, India.

Email: drkpsanthosh@gmail.com

Abstract The fusion excitation functions for the fusion of ^{32}S on ^{90,96}Zr have been calculated larger value, while using one-dimensional barrier penetration model, taking scattering potential as the sum of Coulomb and proximity potential and the calculated values are compared with experimental data with considerations to shape degrees of freedom. At and above the barrier the computed cross sections match well with the experimental data, whereas below the barrier, calculations with nuclear surface tension coefficient improved by Reisdorf in the proximity potential with considerations to shape degrees of freedom give an approximate fit. Reduced reaction cross sections for the systems ^{32}S on ^{90,96}Zr have also been described.

Keyword: Heavy-ion reactions, Sub-barrier fusion, Barrier penetration model.

1. INTRODUCTION

Many investigations, both experimental and theoretical, on heavy-ion fusion reactions in low energy range near and below the Coulomb barrier [1-10] have been an area of extensive studies for many years in Nuclear Physics. As the synthesis of superheavy elements (SHEs) is a very interesting problem and a hot topic nowadays, the observations in heavy-ion systems at and near the Coulomb barrier energies are quite important for the understanding of the complexity of collision processes at low energies. In the analysis of heavy-ion fusion reactions an internuclear interaction consisting of repulsive Coulomb and centrifugal potentials and attractive nuclear potential, which is a function

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of the distance between centers-of mass of the colliding nuclei plays a major role. The total potential attains a maximum value at a distance where the repulsive and attractive forces balance each other, referred to as Coulomb barrier and the energy of relative motion must overcome this barrier in order for the nuclei to be captured and fused.

Even though the simple one dimension barrier penetration model [1] explains the fusion reactions of heavy-ions above the barrier, the large enhancement in fusion cross-sections below the barrier in several orders of magnitude over those expected from the simple one dimension barrier penetration model can only be explained in terms of the coupling of relative motion to the internal degrees of freedom of the colliding nuclei, such as deformation [4, 11, 12], vibration [13–16], and nucleon transfer channels [17–20] or related to the gross features of nuclear matter such as neck formation [21, 22] between the two colliding nuclei.

In the present work, the fusion excitation functions for the fusion of ^{32}S on ^{90,96}Zr have been calculated using one-dimensional barrier penetration model, taking scattering potential as the sum of Coulomb and proximity potential [23] and the calculated values are compared with experimental data [24] with considerations to shape degrees of freedom. Reduced reaction cross sections for the systems ^{32}S+^{90}Zr and ^{32}S+^{96}Zr have been described, by using the usual reduction procedure of dividing the cross section by $\pi R_0^{\,2}$, where R_0 is the barrier radius and the division of energy by Coulomb barrier.

2. THEORY

2.1. The potential

Nuclear reactions are exclusively governed by the nucleus-nucleus potential and discovering a unique nuclear potential that describes the different reaction mechanisms is therefore a challenge for the last several years in Nuclear Physics.

It was shown that the nuclear potential can be written as a product of geometrical factor (proportional to the reduced radii of colliding nuclei) and a universal function so as to incorporate the role of different colliding nuclei in the geometrical factor. In this effort, the proximity potential of Blocki et al. [25] provides a simple formula for the nucleus-nucleus interaction energy as a function of the separation between the surfaces of the approaching nuclei. The formula is free of adjustable parameters and makes use of the measured values of the nuclear surface tension and surface diffuseness.

The interaction barrier for two colliding nuclei is given as:

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$$V = \frac{Z_1 Z_2 e^2}{r} + V_P(z) + \frac{\hbar^2 \ell(\ell+1)}{2\mu r^2} \tag{1}$$

where Z_1 and Z_2 are the atomic numbers of the projectile and the target, r is the distance between the centers of the projectile and the target, z is the distance between the near surfaces of the projectile and the target, ℓ is the angular momentum, μ is the reduced mass of the target and the projectile and $V_P(z)$ is the proximity potential given as:

$$V_P(z) = 4\pi\gamma b \frac{C_1 C_2}{C_1 + C_2} \phi\left(\frac{z}{b}\right) \tag{2}$$

with the nuclear surface tension coefficient,

$$\gamma = 0.9517[1 - 1.7826(N-Z)^2 / A^2] \tag{3}$$

ϕ, the universal proximity potential is given as:

$$\phi(\xi) = -4.41\exp(-\xi / 0.7176), \text{ for } \xi \geq 1.9475 \tag{4}$$

$$\phi(\xi) = -1.7817 + 0.9270\xi + 0.01696\xi^2 - 0.05148\xi^3, \text{ for } 0 \leq \xi \leq 1.9475 \tag{5}$$

$$\phi(\xi) = -1.7817 + 0.9270\xi + 0.0143\xi^2 - 0.09\xi^3, \text{ for } \xi \leq 0 \tag{6}$$

with $\xi = z/b$, where the width (diffuseness) of nuclear surface $b \approx 1$ and Siissmann Central radii C_i related to sharp radii R_i as $C_i = R_i - \frac{b^2}{R_i}$.

For R_i, we use the semi empirical formula in terms of the mass number A_i as:

$$R_i = 1.28 A_i^{1/3} - 0.76 + 0.8 A_i^{-1/3} \tag{7}$$

During the last three decades several attempts have been made to improve the proximity potential [26, 27]. In these works an improved version of nuclear surface tension co-efficient is presented by Reisdorf as:

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$$\gamma = 1.2496[1 - 2.3(N - Z)^2 / A^2] \tag{8}$$

The choice of the potential and its form to be adopted is one of the most challenging aspects, when one wants to compare the experimental fusion data with theory, both below and above the barrier. Among such potentials, proximity potential is well known for its simplicity and numerous applications in different fields. It is based on the proximity force theorem according to which the nuclear part of the interaction potential is a product of the geometrical factor depending on the mean curvature of the interaction surface and the universal function (depending on the separation distance) and is independent of the masses of the colliding nuclei.

2.2. The fusion cross section

To describe the fusion reactions at energies not too much above the barrier and at higher energies, the barrier penetration model developed by C. Y. Wong [1] has been widely used for the last four decades, which obviously explains the experimental result properly.

Following Thomas [28], Huizenga and Igo [29] and Rasmussen and Sugawara [30], Wong approximated the various barriers for different partial waves by inverted harmonic oscillator potentials of height E_ℓ and frequency ω_ℓ. For energy E, using the probability for the absorption of ℓth partial wave given by Hill-Wheeler formula [31], Wong arrived at the total cross section for the fusion of two nuclei by quantum mechanical penetration of simple one-dimensional potential barrier as:

$$\sigma = \frac{\pi}{k^2} \sum_\ell \frac{2\ell + 1}{1 + \exp[2\pi(E_\ell - E) / \hbar\omega_\ell]} \tag{9}$$

where $k = \sqrt{\dfrac{2\mu E}{\hbar^2}}$. Here $\hbar\omega_\ell$ is the curvature of the inverted parabola. Using some parameterizations in the region $\ell = 0$ and replacing the sum in Eq. (9) by an integral Wong gave the reaction cross section as:

$$\sigma = \frac{R_0^2 \hbar\omega_0}{2E} \ln \left\{ 1 + \exp \left[\frac{2\pi(E - E_0)}{\hbar\omega_0} \right] \right\} \tag{10}$$

For relatively large values of E, the above result reduces to the well-known formula:

$$\sigma = \pi R_0^2 \left[1 - \frac{E_0}{E}\right] \tag{11}$$

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For relatively small values of E, such that $E < E_0$:

$$\sigma = \frac{R_0^2 \hbar \omega_0}{2E} \exp\left[2\pi(E - E_0)/\hbar\omega_0\right] \tag{12}$$

Lefort and his collaborators [32] have shown that not a critical angular momentum but a critical distance of approach may be the relevant quantity limiting the complete fusion during a collision between two complex nuclei. In order to substantiate the finding of a critical distance of approach, it is necessary to check the linear dependence of σ on $1/E$ in the region of high energy. The value of critical distance was found to:

$$R_c = r_c(A_1^{1/3} + A_2^{1/3}), \, r_c = 1.0 \pm 0.07 \, fm \tag{13}$$

Gutbrod, Winn and Blann from their analysis of low energy data [33], obtain the fusion distance as:

$$R_B = r_B(A_1^{1/3} + A_2^{1/3}), \, r_B = 1.4 \, fm \tag{14}$$

that is 40% larger than the value of R_c and corresponds to the distance of the ions at the fusion barrier. In order to understand the difference between the two distances given by Eqs.(13) and (14), Glas and Mosel [34] set σ as:

$$\sigma = \pi \lambda^2 \sum_{\ell=0}^{\infty} (2\ell + 1)T_i P_i \tag{15}$$

where T_i is the penetration probability and $P_i = \begin{cases} 1, \ell \le \ell_c \\ 0, \ell > \ell_c \end{cases}$

Replacing the sum in Eq.(15) by an integration one obtains:

$$\sigma = \frac{\hbar\omega}{2} R_B^2 \frac{1}{E} \ln \left\{ \frac{1 + \exp\left[2\pi\{E - V(R_B)\}/\hbar\omega\right]}{1 + \exp[2\pi\{E - V(R_B) - (R_c/R_B)^2[E - V(R_c)]\}/\hbar\omega]} \right\} \tag{16}$$

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Below the barrier, the tunneling through the barrier has to occur in order to allow the fusion of the two nuclei and in terms of partial wave, the fusion cross section is given as:

$$\sigma = \frac{\pi}{k^2} \sum_{\ell=0}^{\ell=\ell_C} (2\ell+1)P \tag{17}$$

where $\ell_C = R_a\sqrt{2\mu(E_{CM} - V_{(R_a,\eta_{in},\ell=0)})}/\hbar$, R_a is the first turning point and η_{in} is the entrance channel asymmetry. Here, P is the WKB penetration probability given as:

$$P = \exp\left\{-\frac{2}{\hbar}\int_a^b \sqrt{2\mu(V-E)}dz\right\} \tag{18}$$

where a and b are the inner and outer turning points defined as $V(a)=V(b)=E$.

The Coulomb interaction between the two deformed and oriented nuclei [1] with higher multipole deformation included [35, 36] is given as,

$$V_C = \frac{Z_1 Z_2 e^2}{r} + 3Z_1 Z_2 e^2 \sum_{\lambda,i=1,2} \frac{1}{2\lambda+1} \frac{R_i^\lambda}{r^{\lambda+1}} Y_\lambda^{(0)}(\alpha_i)\left[\beta_{\lambda i} + \frac{4}{7}\beta_{\lambda i}^2 Y_\lambda^{(0)}(\alpha_i)\delta_{\lambda,2}\right] \tag{19}$$

$$R_i(\alpha_i) = R_{0i}\left[1 + \sum_\lambda \beta_{\lambda i} Y_\lambda^0(\alpha_i)\right] \tag{20}$$

where $R_{0i} = 1.28A_i^{1/3} - 0.76 + 0.8A_i^{-1/3}$.

Here α_i is the angle between the radius vector and symmetry axis of the ith nuclei.

2.3. The reduced reaction cross section

In order to compare the excitation functions of different reaction mechanisms induced by different projectiles on the same target nucleus, the procedure of eliminating the geometrical factors concerning different systems by 'reducing' the cross section and the centre-of-mass energy has extensively been used in recent years [24, 37, 38]. The normal procedure consists of the division of the cross section by πR_0^2, where R_0 is the barrier radius and the division of energy by Coulomb barrier E_0.

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3. RESULTS AND DISCUSSION

Interaction barrier for the fusion of ^{32}S on ^{90}Zr has been plotted in Fig.1, against the distance between the centers of the projectile and the target. The dotted lines in Fig.1 represent the interaction barrier calculated by using the usual nuclear surface tension co-efficient given by Eq.(3), denoted as γ-old and the dashed line represents the result while using the improved version of nuclear surface tension co-efficient given by Eq.(8) denoted as γ-new, without considering the shape degrees of freedom of the projectile and the target. The dash-dotted line and the solid line represent the barrier calculations using nuclear surface tension co-efficient given by Eq.(3) and Eq.(8) respectively with considerations to deformations, using Eq.(19) for $\ell = 0$. It should be noted that the barrier height E_0 decreases and the barrier radius R_0 shifts towards larger value, while considering the shape degrees of freedom for both the usual and the improved values of surface tension coefficients given by Eq.(3) and Eq.(8) respectively. In the both cases of with and without deformations the barrier height E_0 decreases and the barrier radius R_0 shifts towards larger value with improved value of the surface tension coefficient given by Eq. (8) than the usual value given by Eq.(3). Moreover, Eq. (8) gives deeper potential compared to Eq. (3).
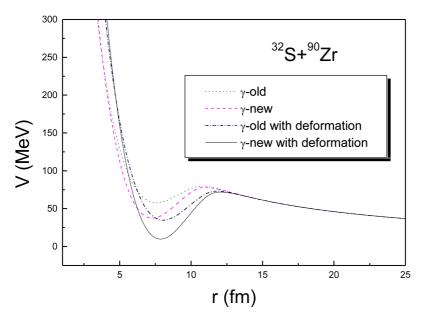

Figure 1: Scattering potential for the projectile ^{32}S on ^{90}Zr target consisting of repulsive Coulomb and centrifugal potentials and attractive nuclear potential.
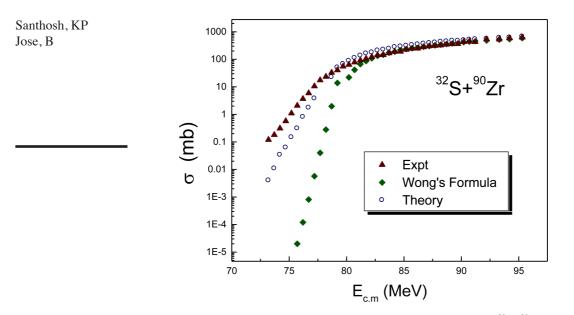
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Figure 2: Comparison of measured fusion excitation functions of ^{32}S+^{90}Zr reaction, while using usual nuclear surface tension co-efficient given by Eq. (3).

At, above and below the barrier, the total fusion cross-sections for the reactions of ^{32}S on ^{90,96}Zr have been calculated by using the values of barrier height E_0 and barrier radius R_0 taken from the respective figures corresponding to Fig.1 and using Eqs. (9) to (19). Below the barrier, the fusion process has been treated as a tunneling process and we have calculated the fusion cross sections using Eq. (17) and Eq. (19). While considering the shape degrees of freedom, the experimental deformation parameter for ^{32}S has been taken as $\beta_2 = 0.3120$, in all calculations. In Figs. 2 and 3 the calculated fusion cross sections are compared with the experimental data [24].

In Figs. 2 and 3, in the reactions of ^{32}S+^{90}Zr and ^{32}S+^{96}Zr, at and above the barrier, the fusion cross-sections (solid diamonds) computed using Wong's formula given by Eq. (9) and nuclear surface tension co-efficient given by Eq. (3) fit very well with the experimental data (solid up triangles), whereas below the barrier show some disagreement.

In the case of ^{32}S+^{90}Zr reaction, in Fig. 2, at and above the barrier the barrier the fusion cross-sections (open circles) computed using Eq. (16) and nuclear surface tension co-efficient given by Eq. (3) also show good fit with the experimental data. Below the barrier, we have considered the fusion process as a tunneling process and the cross sections calculated using Eqs. (3), (17) and (19) show some agreement with the experimental data. In the above
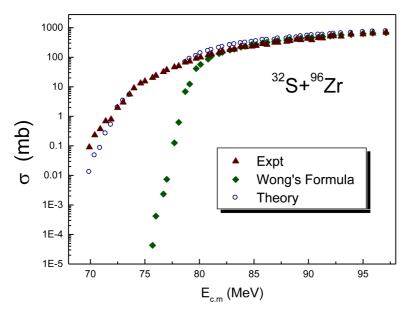
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Figure 3: Comparison of measured fusion excitation functions of ^{32}S+^{96}Zr reaction, while using improved version of nuclear surface tension co-efficient given by Eq. (8).
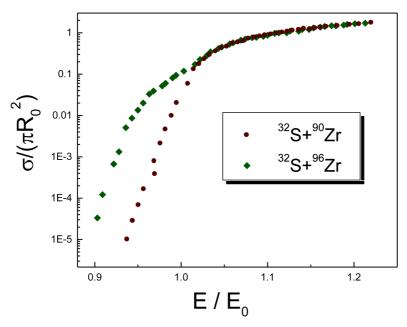

Figure 4: Reduced reaction cross sections for the systems consisting of ^{32}S on different targets ^{90}Zr and ^{96}Zr using the usual reduction procedure.

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calculation the experimental deformation parameter for ^{90}Zr has been taken as $\beta_2 = 0.0894$.

In Fig. 3, in the reaction of ^{32}S on ^{96}Zr, for getting a better result we have changed the value of the nuclear surface coefficient given by Eq. (3) by Eq. (8). It should be noted that the above the barrier, calculations using the nuclear surface tension co-efficient given by Eq. (8) along with Eq. (16) give good agreement with experimental data. Below the barrier, the computed fusion cross sections using Eq. (8) along with Eq. (17) and Eq. (19) give a comparatively better fit to the experimental results. The experimental deformation parameter for ^{96}Zr has been taken as $\beta_2 = 0.0800$. The enhancement in the measured cross sections reveals the importance of the inclusion of the couplings to the low-lying octupole vibrations in ^{32}S+^{90}Zr reaction and the inclusion of four sequential neutron transfer channels with the low-lying octupole vibrations in ^{32}S+^{96}Zr reaction.

Fig. 4 represents the reduced reaction cross sections for the systems ^{32}S+^{90}Zr and ^{32}S+^{96}Zr by the reduction procedure of dividing the cross section by πR_0^2 and the energy by E_0 . It should be noted that the reduced reaction cross section is larger for ^{32}S+^{96}Zr reaction than ^{32}S+^{90}Zr reaction.

4. CONCLUSIONS

At and above the barrier, the simple one dimension barrier penetration model developed by C. Y. Wong explains the fusion reactions of heavy ions very well, while using the scattering potential as the sum of Coulomb and proximity potentials. The enhancements in fusion cross sections below the Coulomb barrier orders of magnitude larger than the predictions of one dimension barrier penetration model reveals the important role played by nuclear structure of the colliding nuclei. Below the barrier larger deformations corresponds to large sub barrier enhancement of fusion cross sections.

Below the barrier the fusion process can be considered as a tunneling process and in the quantum mechanical tunneling of one dimension barrier penetration model the inclusion of nuclear deformation parameters in Coulomb and proximity potential model explains the nuclear fusion cross sections comparatively well. In the calculation of interaction barrier of deformed nuclei, the nuclear surface tension coefficient given by Reisdorf shows better results than the usual nuclear surface tension co-efficient of proximity potential. The reduced cross sections compare the different fusion reaction mechanisms induced by different targets with the same projectiles in the same figure.

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