

COST OPTIMIZATION OF TWO-STAGE HELICAL GEARBOXES WITH SECOND STAGE DOUBLE GEAR-SETS

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Abstract

In practice, the cost of a gearbox plays a very important role in the trade. Therefore, reducing the cost of gearboxes is an important task not only when manufacturing the gearboxes but also when designing them. In order to reduce the cost of a gearbox, there are many solutions in which determining the optimal partial gear ratios of a gearbox is an effective measure. This is because it not only the size, the mass but also the cost of a gearbox depends greatly on the partial gear ratios. This work presents a method for calculating the cost function of two-stage helical gearboxes with second-stage double gear-sets based on the mass of the components that construct the gearbox. The cost objective function is minimized to achieve the optimal transmission ratios. Furthermore, screening experiments are carried out with nine important input parameters that have significant effects on the optimum transmission ratio of the second stage. These parameters are the total gearbox ratio, the coefficient of wheel face width of the first stage, coefficient of wheel face width of the second stage, the allowable contact stress of the first stage, the allowable contact stress of the second stage, the output torque, the cost of gearbox housing, the cost of gears, and the shaft cost. The experimental results of were analysed by using the Analysis of Variance (ANOVA) method with the help of Minitab 19 software. The results demonstrate that the effective weight of the input parameters and their interactions on the output response was investigated. Also, a regression model for computing

the optimal transmission ratio of the second stage was proposed. This brings significance not only in the design process but also in manufacturing since the gearbox cost can decrease.

Keywords: cost optimization, optimization design, gear ratios, optimal gear ratios, helical gearbox.

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1. Introduction

Among many drive systems used in industries such as mechanical drive systems, electric drive systems, pneumatic drives, hydraulic drives, etc., mechanical drive systems are used most widely. This is because it has simple structure, easy design and manufacture, stable working, high reliability and low cost. A mechanical drive system is responsible for reducing the rotational speed and increasing the torque from the motor shaft to the shaft of the working machine. Usually, it is composed of an electric motor, a gearbox, an external transmission drive (either a V-belt or a chain drive) and a coupling of which the gearbox is the main part. Therefore, the optimal gearbox design is a very important problem when designing a mechanical drive system. In the optimal design of a gearbox, determining the optimal gear ratios is a decisive problem because the gear ratios is the factor that has the greatest influence on the size, the volume, the mass as well as the cost of the gearbox. **Fig. 1** [1] is an example for illustration of the importance of determining the optimum gear ratios for a two-stage helical gearbox. In this figure, the relationship between the mass of all gears in the gearbox and the gear ratio of the second stage u_2 is described. It is easy to know that the gear mass depends greatly on the gear ratio u_2 . When u_2 is equal to 2 (the optimal value of u_2), the mass of the gear is only approximately 178 kg while it will be about 275 kg when $u_2 = 6$. Therefore, determining the optimum gear ratios for gearboxes is a topic that attracts the attention of many researchers.

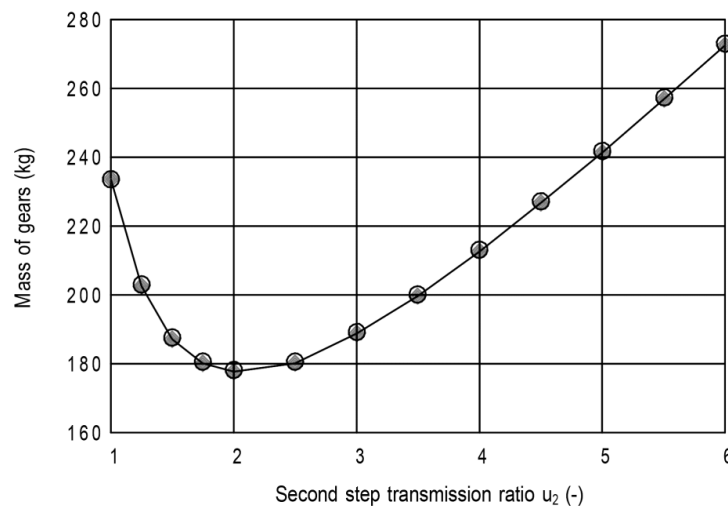


Fig. 1. Relation between gear mass and gear ratio of second stage [1]

Up to now, the optimal gear ratio of a gearbox has been determined using various methods such as using graphs [2, 3], using «practical formulas» [4] which have been found based on the results from gearbox companies, or using optimal models [5, 6]. In addition, the problem of calculating the optimal gear ratio has been performed for different types of gearboxes such as helical gear boxes [6–8], bevel gear boxes [9–11], worm gearboxes [12–15]. Besides, the optimal gear ratios have been determined for two-stage [7, 8, 14], three-stage [6, 10, 11], and four-stage gearboxes [16, 17]. Also, different objectives have also been selected when solving this problem such as the minimum mass of gears [16, 18], minimum mass of gearbox [19], the minimum gearbox cross-section section [8, 10, 14], the minimum gearbox length [17, 20], or minimum gearbox cost [9, 21].

Regarding two-stage helical gearboxes, there have been several studies for determining the optimal gear ratios for two-stage helical gearboxes with second stage double gear-sets. However,

up to now, there has been no research to determine the optimum gear ratios to achieve the minimum gearbox cost for this type of gearboxes.

From previous researches, it can realize that there have been many studies in determination of optimum gear ratios of different types of gearboxes. However, until now, the cost objective function has not been found for design of two-stage helical gearboxes with second stage double gear-sets. In this study, a method for calculating the optimum gear ratios of two-stage helical gearboxes with second stage double gear-sets is presented with novelties as the following:

- 1) a general cost object function was investigated;
- 2) significant effects of the main design parameters and their interactions on the gear ratios were evaluated;
- 3) a regression model was found to simply compute the optimal gear ratios by the main design parameters.

2. Materials and methods

2. 1. Optimization problem

In general, the cost of gearboxes depends on the cost of main components that construct gearboxes, such as the cost of gears, shafts, rolling bearings, and gearbox housing. Moreover, the cost of these components is evaluated by used materials, manufacturing methods, heat treatment, operators, etc. According to commerce, the price of each component can be calculated by unit price per kilogram. Therefore, the cost function of a gearbox denoted by C_{gb} , can be computed as:

$$C_{cg} = \sum_{i=1}^n c_i m_i, \quad (1)$$

where c_i and m_i are the cost per kilogram and mass of each component, respectively. In this work, three main components that powerfully affect the cost of gearboxes, such as the cost of gears, the shaft cost, and the gearbox housing cost are taken into consideration. The rolling bearing is ignored since this component is standard and is selected depending on the diameter calculation of the shafts in the design process. The gear cost, the shaft cost, and the gearbox housing cost per kilogram (USD/kg) are denoted by c_g , c_s and c_{gh} , respectively. Similarly, the gear mass, the shaft mass, and the gearbox housing mass (kg) are denoted by m_g , m_s and m_{gh} respectively. Therefore, (1) can be rewritten as:

$$C_{cg} = c_g m_g + c_s m_s + c_{gh} m_{gh}. \quad (2)$$

As shown in (2), the masses of gears, the shafts, and the gearbox housing are considered for computing the gearbox mass in order to determine the gearbox cost. **Fig. 2** presents the diagram of two-stage helical gearboxes with second stage double gear-set for calculating the mass of the main components.

For the two-stage helical gearboxes with the second stage double gear-set as shown in **Fig. 2**, the first stage has a couple gear-set and the second stage has two similar couple gear-sets. The total mass of gears is therefore computed by:

$$m_g = m_{g1} + 2m_{g2}, \quad (3)$$

where m_{g1} and m_{g2} are the gear mass of the first and the second stages of the gearbox (kg). The mass of each gear can be calculated by multiplying the weight density and the volume of gear. Therefore, the gear mass of the first and the second stages, denoted by m_{g1} and m_{g2} , can be respectively expressed as:

$$m_{g1} = \rho_g \cdot (\pi \cdot e_1 \cdot d_{w11}^2 \cdot b_{w1} / 4 + \pi \cdot e_2 \cdot d_{w21}^2 \cdot b_{w1} / 4), \quad (4)$$

$$m_{g2} = \rho_g \cdot (\pi \cdot e_1 \cdot d_{w12}^2 \cdot b_{w2} / 4 + \pi \cdot e_2 \cdot d_{w22}^2 \cdot b_{w2} / 4). \quad (5)$$

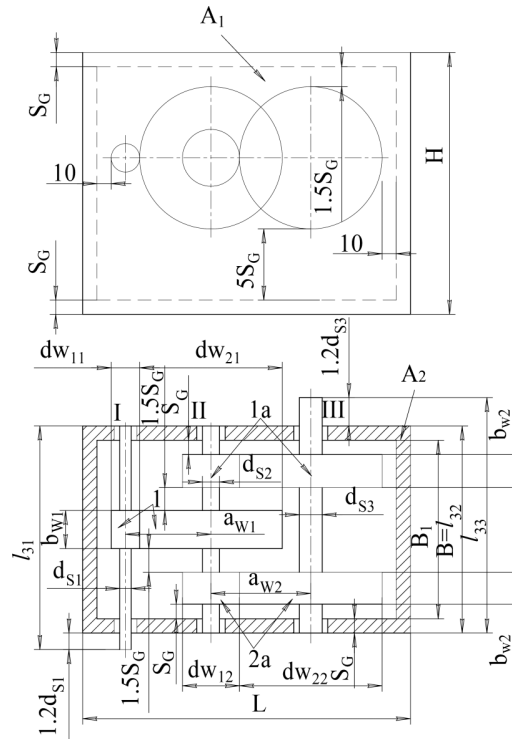


Fig. 2. Diagram of two-stage helical gearboxes with second stage double gear-set

In (4), (5), ρ_g is the weight density of gear material, $\rho_g = 7.82 \text{ (kg/m}^3\text{)}$ when the material is steel [22]; e_1 and e_2 are respectively the volume coefficients of the drive and driven gears, $e_1 = 1$, $e_2 = 0.6$ [3]; b_{w1} and b_{w2} are respectively the gear width of the first and second stages (mm). They can be computed as [3]:

$$b_{w1} = X_{ba1} \cdot a_{w1}, \quad (6)$$

$$b_{w2} = X_{ba2} \cdot a_{w2}, \quad (7)$$

in which, X_{ba1} and X_{ba2} are the coefficients of wheel face width of the first and second stages, respectively; a_{w1} and a_{w2} are the center distances of the first and second stages. They can be calculated by [3]:

$$a_{w1} = k_a (u_1 + 1) \sqrt[3]{T_{11} k_{H\beta 1} / ([\sigma_{H1}]^2 u_1 X_{ba1})}, \quad (8)$$

$$a_{w2} = k_a (u_2 + 1) \sqrt[3]{T_{12} k_{H\beta 2} / ([\sigma_{H2}]^2 u_2 X_{ba2})}. \quad (9)$$

All the parameters in (8), (9) are described as the following:

- k_a is the material coefficient, $k_a = 43$ for steel material;
 - $k_{H\beta 1}$ and $k_{H\beta 2}$ are respectively the contacting load ratios for pitting resistance; for the first stage of a two-stage helical reducer with second stage double gear-sets $k_{H\beta 1} = 1.0 \div 1.06$ [3]; for the second stage, $k_{H\beta 1} = 1.02 \div 1.28$ [3]. Therefore, $k_{H\beta 1}$ and $k_{H\beta 2}$ can be respectively chosen as $k_{H\beta 1} = 1.03$ and $k_{H\beta 2} = 1.15$;
 - $[\sigma_{H1}]$ and $[\sigma_{H2}]$ are the allowable contact stress of the first and second stages (MPa), respectively;
 - u_1 and u_2 are respectively the transmission ratios of the first and second stages;
 - T_{11} and T_{12} are respectively the torque on the first and second shafts of the gearbox (Nmm).
- They can be computed as:

$$T_{11} = T_{out} / (u_t \cdot \eta_{hg}^2 \cdot \eta_{be}^3), \quad (10)$$

$$T_{12} = T_{out} / (2u_t \cdot \eta_{hg} \cdot \eta_{be}^3), \quad (11)$$

where u_t is the total ratio of the gearbox; η_{hg} is the efficiency of helical gear set $\eta_{hg} = 0.96 \div 0.98$ [3]; η_{be} is the efficiency of a rolling bearing pair $\eta_{be} = 0.99 \div 0.995$ [3].

For using in the same way as the gear mass calculation, the total mass of all shafts of the gearbox as shown in **Fig. 2** can be computed as:

$$m_s = m_{s1} + m_{s2} + m_{s3}, \quad (12)$$

where m_{s1} , m_{s2} , and m_{s3} are respectively the mass of shaft 1, 2 and 3 of the gearbox (kg). They can be expressed as:

$$m_{s1} = \rho_s \cdot \pi \cdot d_{s1}^2 \cdot l_{s1} / 4, \quad (13)$$

$$m_{s2} = \rho_s \cdot \pi \cdot d_{s2}^2 \cdot l_{s2} / 4, \quad (14)$$

$$m_{s3} = \rho_s \cdot \pi \cdot d_{s3}^2 \cdot l_{s3} / 4. \quad (15)$$

In (13)–(15), ρ_s is the weight density of shaft material (kg/m^3); l_{s1} , l_{s2} , and l_{s3} are respectively the length of shaft 1, 2 and 3 of the gearbox as shown in **Fig. 2**. They can be written as:

$$l_{s1} = B + 1.2 \cdot d_{s1}, \quad (16)$$

$$l_{s2} = B, \quad (17)$$

$$l_{s3} = B + 1.2 \cdot d_{s3}. \quad (18)$$

In (16)–(18), preliminary diameters of the first, second and third shafts i.e., d_{s1} , d_{s2} , and d_{s3} are described as:

$$d_{s1} = [T_{11} / (0.2 \cdot [\tau])]^{1/3}, \quad (19)$$

$$d_{s2} = [T_{12} / (0.2 \cdot [\tau])]^{1/3}, \quad (20)$$

$$d_{s3} = [T_{13} / (0.2 \cdot [\tau])]^{1/3}, \quad (21)$$

where $[\tau]$ is the allowable shear stress (MPa), $[\tau] = 15 \div 20$ MPa [23]; it can be chosen as $[\tau] = 17$ MPa; T_{11} and T_{12} are respectively presented in (10), (11); T_{13} can be expressed as:

$$T_{13} = T_{out} / (u_t \cdot \eta_{be}). \quad (22)$$

The mass of gearbox housing can be calculated as the similar procedure:

$$m_{gh} = \rho_{gh} \cdot V_{gh}. \quad (23)$$

In here, ρ_{gh} is the weight density of gearbox housing material made by cast iron materials, $\rho_{gh} = 7.2 \text{ kg}/\text{m}^3$ for the cast iron material [22]; V_{gh} is the volume of the gearbox housing (m^3). For calculating V_{gh} , the volume gearbox housing can be divided into three parts i.e., V_{A1} is the volume of the front and back sides of the gearbox; V_{A2} is the volume of the left and right sides, and V_b is the volume of the top and bottom sides. Therefore, V_{gh} can be written as.

In which, V_{A1} , V_{A2} and V_b can be described as:

$$\begin{aligned} V_{A1} &= L \cdot H \cdot S_G, \quad V_{A2} = B_1 \cdot H \cdot S_G = (B - 2S_G) \cdot H \cdot S_G, \\ V_b &= L \cdot B \cdot S_G. \end{aligned} \quad (24)$$

In (24), L , H , S_G and B can be calculated as [23]:

$$L = (d_{w11} + d_{w21} / 2 + d_{w12} / 2 + d_{w22} + 22.5) / 0.975, \quad (25)$$

$$H = \max(d_{w21}, d_{w22}) + 6.5 \cdot S_G, \quad (26)$$

$$S_G = 0.005 \cdot L + 4.5, \quad (27)$$

$$B = b_{w1} + 2 \cdot b_{w2} + 7 \cdot S_G. \quad (28)$$

The parameters d_{w11} , d_{w21} , d_{w12} , d_{w22} in (25) and (26) are the gear pitch diameters of the first and second stages. They can be expressed as [3]:

$$\begin{aligned} d_{w11} &= 2 \cdot a_{w1} / (u_1 + 1), \\ d_{w21} &= 2 \cdot a_{w1} \cdot u_1 / (u_1 + 1), \\ d_{w12} &= 2 \cdot a_{w2} / (u_2 + 1), \\ d_{w22} &= 2 \cdot a_{w2} \cdot u_2 / (u_2 + 1). \end{aligned} \quad (29)$$

From the above procedure of the gearbox cost, it can be observed that all calculation parameters of the gearbox cost depend on the transmission ratio of the two stages in the gearbox, i.e., u_1 , and u_2 . In design process, a designer would choose reasonable transmission ratios for each type of gearbox in order to obtain the lowest gearbox cost. Therefore, the objective function as shown in eq. (2) must be minimized as follows:

$$\text{Minimize } C_{gb}, \quad (30)$$

where C_{gb} can be expressed as a function depending on the transmission ratios of the two stages, i.e., u_1 and u_2 in the gearbox:

$$C_{gb} = c_{gb}(u_1, u_2). \quad (31)$$

Subject to the following constraints:

$$1 \leq u_1 \leq 9 \quad \text{and} \quad 1 \leq u_2 \leq 9. \quad (32)$$

Besides, the total ratio of the gearbox u_t has the relation with the transmission ratio of the two stages by:

$$u_t = u_1 \cdot u_2. \quad (33)$$

As it is known that u_t should be used in the interval [5, 45], from (33), the objective function in (31) can be rewritten as:

$$\text{Minimize } C_{gb} = C_{gb}(u_2). \quad (34)$$

2. 2. Experimental procedures

(34) was solved by a computer program to obtain the optimal transmission ratios of the two stages. In addition, a screening experiment was conducted to illustrate the effect of input parameters on the optimal transmission ratio u_2 . As the procedure in Sec. 2, the input parameters including the total gearbox ratio u_t , the coefficients of wheel face width of the first and second stages X_{ba1} , X_{ba2} , the allowable contact stress of the first and second stages AS_1 , AS_2 , the output torque T_{out} , the cost of gearbox housing c_{gh} , the cost of gears c_g , and the cost of shaft c_s were considered to design the experiment. The nine input parameters and their value range are presented in **Table 1**. Normally, the full factorial design is 29 to denote the number of test runs in the simulation experiment; however, using Minitab 19 for the conducting experiments, the total of test runs with 1/4 fraction is 29-2, i.e., 128 test runs. The factorial design of the input parameters and the output respond u_2 is illustrated in **Table 2**.

Table 1

Input parameters and their value range

Factor	Code	Unit	Low	High
Total gearbox ratio	u_t	–	5	45
Coefficient of wheel face width of stage 1	X_{ba1}	–	0.3	0.35
Coefficient of wheel face width of stage 2	X_{ba2}	–	0.35	0.4
Allowable contact stress of stage 1	AS_1	MPa	350	420
Allowable contact stress of stage 2	AS_2	MPa	350	420
Output torque	T_{out}	Nmm	100	10000
Cost of gearbox housing	c_{gh}	USD/kg	1	5
Cost of gears	c_g	USD/kg	2	9
Cost of shafts	c_s	USD/kg	1.5	5

Table 2

Factorial design of the input parameters and output response

RunOrder	u_t	X_{ba1}	X_{ba2}	AS_1	AS_2	T_{out}	C_{gh}	C_g	C_s	u_2
1	5	0.3	0.4	350	420	10000	5	9	1.5	4.07
2	45	0.3	0.35	350	420	10000	1	2	1.5	8.49
3	5	0.35	0.35	350	420	10000	1	9	5	4.46
4	5	0.3	0.4	350	350	10000	5	9	5	4.00
5	5	0.35	0.4	420	420	100	1	2	5	4.85
6	5	0.35	0.35	420	350	100	1	9	5	13.97
...
127	45	0.3	0.35	420	350	100	5	9	5	6.41
128	45	0.35	0.4	350	420	10000	5	2	5	8.41

3. Results and discussions

After conducting the screening experiments, the results are presented as the following sections.

3. 1. Effect of input parameters on optimum transmission ratio

Based on the screening experiment as discussed in section 3, the main effects of each factor on the optimal transmission ratio u_2 are presented in **Fig. 2**. From the observation of the slope angles of all straight lines in this figure, the total gearbox ratio u_t has the largest influence on the optimum transmission ratio u_2 . Besides, the effect of these parameters on the optimum transmission ratio is much smaller than the influence of the total gearbox ratio. It is also seen from the lines of X_{ba1} , X_{ba2} , and c_g that the coefficients of wheel face width of the first and second stages X_{ba1} , X_{ba2} , and the cost of gears c_g have much less significant effect on the optimum transmission ratio.

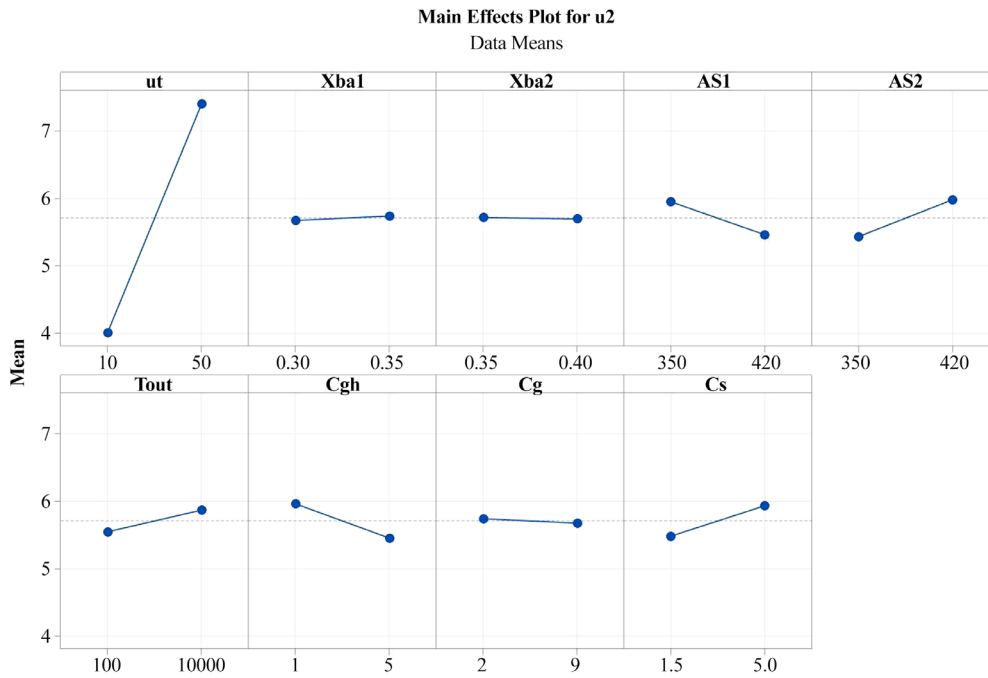


Fig. 3. Effects of the input parameters on the transmission ratio u_2

The influence of the input parameters can be seen more clearly in **Fig. 4**, which shows the Pareto chart of the standardized effects. For the respond model, the parameters are statistically significant at the 0.05 level. The magnitude of the effect of the input parameters on the optimal transmission ratio is arranged from the lowest value to the highest value. The strongest influence on the optimum transmission ratio u_2 comes from the total gearbox ratio u_t (factor *A* in **Fig. 4**). The effect is gradually reduced in the sequence of the remaining parameters i.e., the allowable contact stress of the second stages AS_2 (factor *E*), the cost of gearbox housing c_{gh} (factor *G*), the allowable contact stress of the first stages AS_1 (factor *D*), the cost of shaft c_s (factor *J*), the output torque T_{out} (factor *F*), the coefficient of wheel face width of the first stage X_{ba1} (factor *B*), the cost of gears c_g (factor *H*), the coefficient of wheel face width of the second stage X_{ba2} (factor *C*).

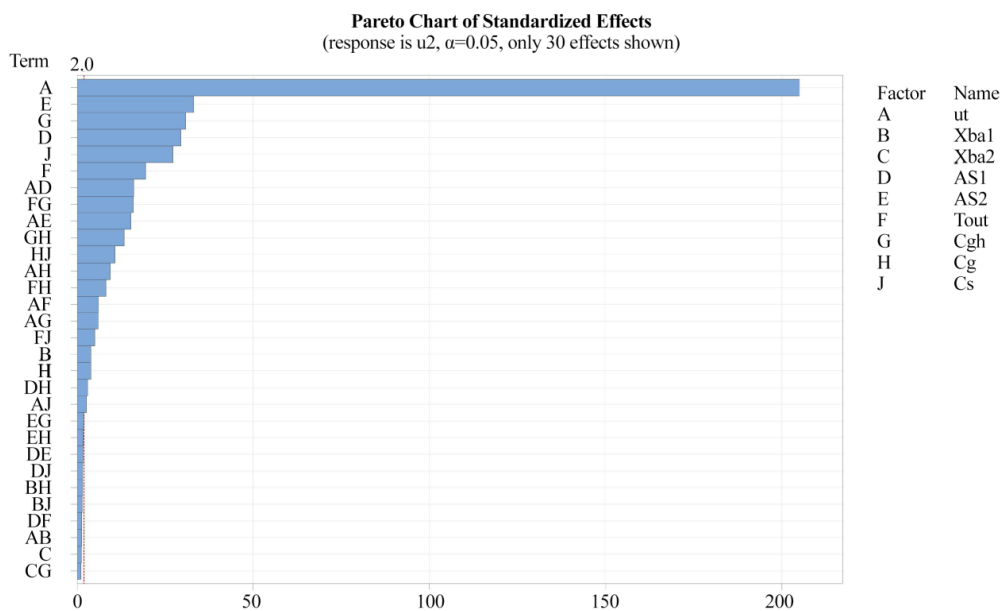


Fig. 4. Magnitude of the effect of the input parameters on the optimal transmission ratio u_2

In order to examine the increase and decrease of the optimum transmission ratio u_2 , **Fig. 5** shows the normal plot of the standardized effects. As shown in the **Fig. 5**, the positive influences of the four input parameters on the output respond, i.e., the total gearbox ratio u_t (factor A), the allowable contact stress of the second stages AS_2 (factor E), the cost of shaft c_s (factor J), and the output torque T_{out} (factor F) increase the optimal transmission ratio u_2 when these parameters are varied from the low value to high value.

However, the negative effects of the two factors, i.e., the allowable contact stress of the first stages AS_1 (factor D) and the cost of gearbox housing c_{gh} (factor G) reduce the optimum transmission ratio u_2 .

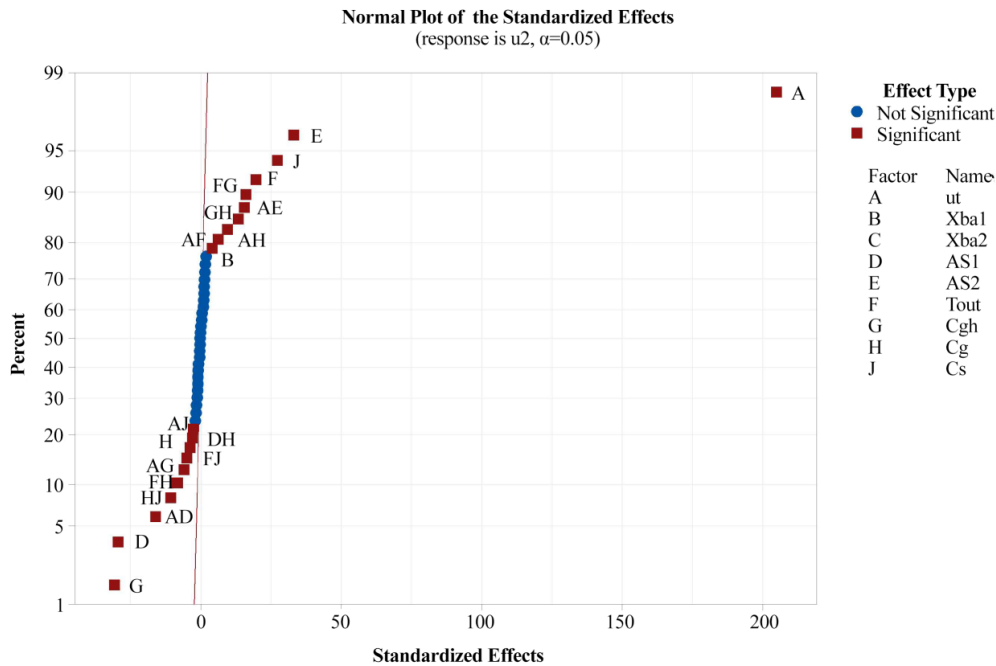


Fig. 5. Distribution of the standardized effects of the input parameters on u_2

3. 2. Regression analysis

The regression analysis of the input parameters and their interactions with the significant effects on the optimum transmission ratio is displayed in **Table 3**. As seen in the second column of this table, the positive and negative values show more clearly the increasing and decreasing influences of the input parameters and their interactions. In addition, the magnitudes of these factors describe the effective weight on the optimum transmission ratio, e.g., u_t has the largest effect on u_2 . Ignoring the factors that have the insignificant effects with P-Values smaller than the statistically significant value, i.e., $\alpha = 0.05$, brings the weighty effect to the output respond. This illustrates the reliability of the factors importantly affecting the optimum transmission ratio.

Furthermore, the coefficients of the input parameters and their interactions are also presented in the third column of **Table 3**. In order to simply compute the optimum transmission ratio of the second stage, the regression equation can be expressed as the following model:

$$\begin{aligned}
 u_2 = & 1.964 + 0.08476u_t + 1.329X_{ba1} - 0.001078AS_1 + 0.004015AS_2 + \\
 & + 0.000017T_{out} - 0.1576C_{gh} + 0.0641C_g + 0.2497C_s - 0.000189u_t \cdot AS_1 + \\
 & + 0.000182u_t \cdot AS_2 + 0.000001u_t \cdot T_{out} - 0.001236u_t \cdot C_{gh} + 0.001109u_t \cdot C_g - \\
 & - 0.000638u_t \cdot C_s - 0.000210AS_1 \cdot C_g - 0.000241AS_2 \cdot C_{gh} + 0.000013T_{out} \cdot C_{gh} - \\
 & - 0.000004T_{out} \cdot C_g - 0.000005T_{out} \cdot C_s + 0.01577C_{gh} \cdot C_g - 0.01465C_g \cdot C_s.
 \end{aligned} \tag{35}$$

The correlation coefficients as shown in **Table 4** i.e., $R^2=0.9977$, $\text{adj-}R^2=0.9972$ and $\text{pred-}R^2=0.9966$ are almost approximate to 1. For this, the proposed model in (35) presents a good fit to the experimental data. Therefore, based on the model, the optimum transmission ratio of the second stage u_2 is obtained. According to (33), the optimum transmission ratio of the first stage is also determined.

Table 3
Regression analysis for input parameters and their interactions

Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant	–	5.70803	0.00839	680.59	0.000	–
u_t	3.38441	1.69221	0.00839	201.77	0.000	1.00
X_{ba1}	0.06644	0.03322	0.00839	3.96	0.000	1.00
AS_1	–0.48779	–0.24389	0.00839	–29.08	0.000	1.00
AS_2	0.54807	0.27403	0.00839	32.67	0.000	1.00
T_{out}	0.32183	0.16092	0.00839	19.19	0.000	1.00
c_{gh}	–0.50865	–0.25432	0.00839	–30.32	0.000	1.00
c_g	–0.06607	–0.03304	0.00839	–3.94	0.000	1.00
c_s	0.44994	0.22497	0.00839	26.82	0.000	1.00
$u_t \cdot AS_1$	–0.26515	–0.13257	0.00839	–15.81	0.000	1.00
$u_t \cdot AS_2$	0.25410	0.12705	0.00839	15.15	0.000	1.00
$u_t \cdot T_{out}$	0.10060	0.05030	0.00839	6.00	0.000	1.00
$u_t \cdot c_{gh}$	–0.09889	–0.04944	0.00839	–5.90	0.000	1.00
$u_t \cdot c_g$	0.15526	0.07763	0.00839	9.26	0.000	1.00
$u_t \cdot c_s$	–0.04468	–0.02234	0.00839	–2.66	0.009	1.00
$AS_1 \cdot c_g$	–0.05146	–0.02573	0.00839	–3.07	0.003	1.00
$AS_2 \cdot c_{gh}$	–0.03372	–0.01686	0.00839	–2.01	0.047	1.00
$T_{out} \cdot c_{gh}$	0.26418	0.13209	0.00839	15.75	0.000	1.00
$T_{out} \cdot c_g$	–0.13761	–0.06880	0.00839	–8.20	0.000	1.00
$T_{out} \cdot c_s$	–0.08454	–0.04227	0.00839	–5.04	0.000	1.00
$c_{gh} \cdot c_g$	0.22071	0.11036	0.00839	13.16	0.000	1.00
$c_g \cdot c_s$	–0.17942	–0.08971	0.00839	–10.70	0.000	1.00

Table 4
Summary of model for u_2

S	R-sq	R-sq(adj)	R-sq(pred)
0.0948864	99.77 %	99.72 %	99.66 %

In order to evaluate the reliability of the regression model more carefully as shown in (35), the normal probability and versus order plots are depicted in **Fig. 6**. It can be observed from **Fig. 6, a** that the normal plot of the residuals approximately follows a straight line. Furthermore, in **Fig. 6, b** the residuals are randomly distributed on both sides of the horizontal axis. These results demonstrate that the regression model is consistent with the experiment data.

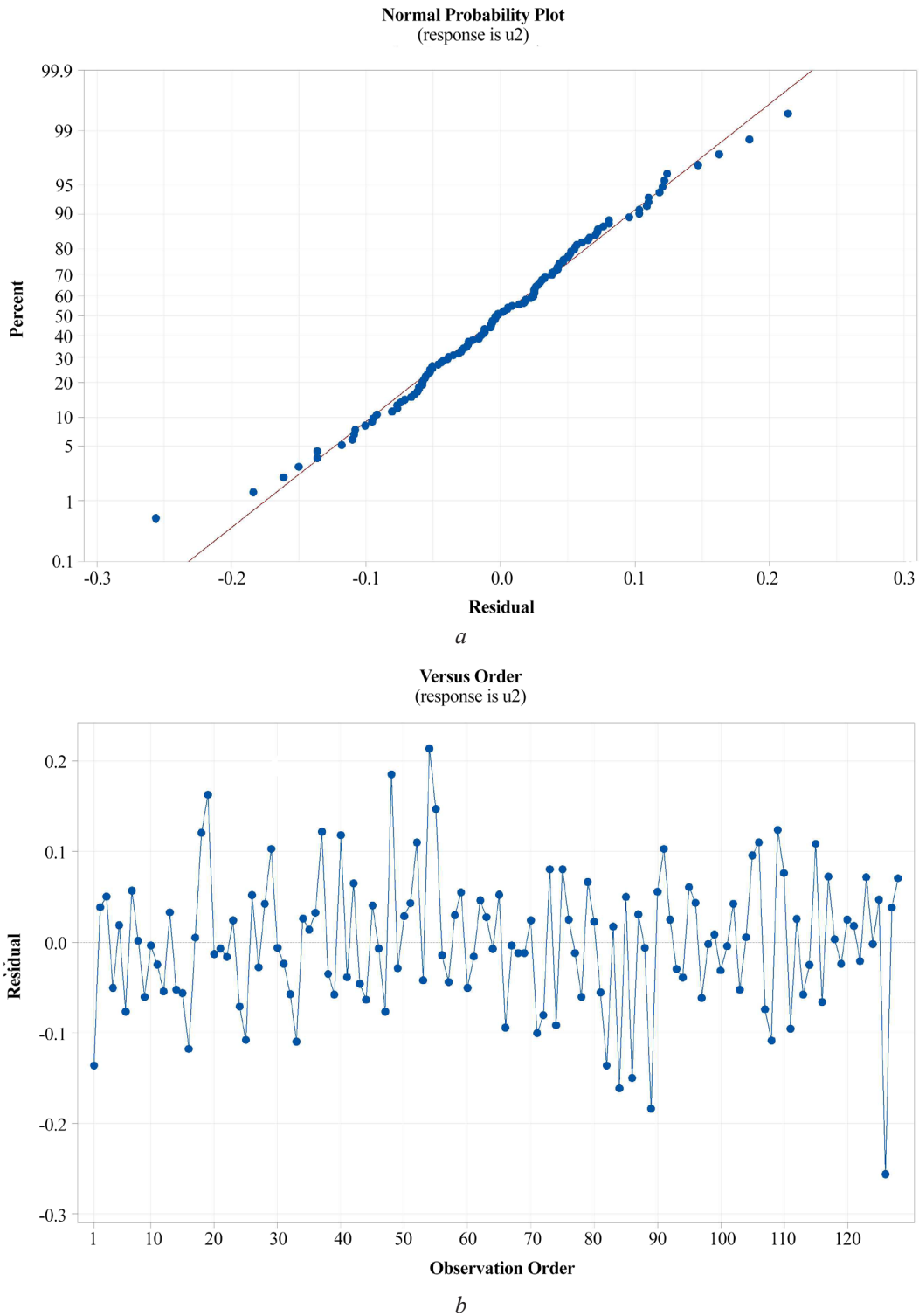


Fig. 6. Residuals versus fits: *a* – Normal Probability, *b* – Versus order

4. Conclusions

In this paper, a method for cost analysis of a two-stage helical gearbox with second stage double gear-set was proposed. The cost function was minimized in order to get the optimum transmission ratio of the stages. The Minitab 19 was used to design the experiment and analysis the results. The influence of nine input parameters including the total gearbox ratio u_t , the coefficient of wheel face width of the first and second stages X_{ba1} , X_{ba2} , the allowable contact stress of

the first and second stages AS_1 , AS_2 , the output torque T_{out} , the cost of gearbox housing c_{gh} , gears c_g , and shaft c_s on the optimum gear ratio was investigated. The results from the experiments clearly showed the significant influences of the input parameters and their interactions on the optimal transmission ratio u_2 . It was found that the largest influence on u_2 belongs to the total gearbox ratio u_f ; followed by the influence of AS_2 , c_{gh} , AS_1 , c_s , T_{out} , X_{ba1} , c_g , and X_{ba2} . Additionally, based on the regression and variance analysis, the mathematical model was achieved to calculate the optimum transmission ratio u_2 . This model was well evaluated by the correlation coefficients and residual versus fits. This is greatly significant in design and manufacturing. However, the formula for determining the optimal gear ratio still does not take into account the cost of the bearings. That is also the direction of future research for scientists.

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