Chapter

# Vortex Dynamics in Complex Fluids

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## Abstract

The present chapter provides an overview of vortex dynamics in complex fluids by taking examples of Taylor vortex flow. As complex fluids, non-Newtonian fluid is taken up. The effects of these complex fluids on the dynamic behavior of vortex flow fields are discussed. When a non-Newtonian shear flow is used in Taylor vortex flow, an anomalous flow instability is observed, which also affects heat and mass transfer characteristics. Hence, the effect of shear-thinning on vortex dynamics including heat transfer is mainly referred. This chapter also refers to the concept of new vortex dynamics for chemical process intensification technologies that apply these unique vortex dynamics in complex fluids in Conclusions.


**Keywords:** Taylor vortex flow, complex fluid, non-Newtonian fluid, heat transfer, process intensification

## 1. Introduction

Historically, innovative processes have been created using organized vortices. For example, in Japan, Kiyomasa Kato, a Sengoku daimyo (Japanese territorial lord in the Sengoku period) in Kumamoto Prefecture, made a canal (called "hanaguri canal") with a partition (baffle) having a semicircular hole at the bottom as shown in **Figure 1**. The flow velocity of the water flowing through the hole in the lower part of the partition increases due to the effect of the contraction of the flow, and a strong circulating vortex is formed in the water channel divided by the partition. By intensifying the flow in the canal, water can be supplied to about 95 ha of land in nine villages in the downstream without piling up volcanic ash or earth and sand, and the harvest has increased about three times. Based on this idea by Kiyomasa Kato, in order to solve the particle sedimentation problem in oscillatory baffled reactors (OBR) which is one of the hopeful process intensification techniques, our group [1] succeeded in preventing the particle sedimentation to the bottom of the reactor and obtaining extremely monodispersed particles in a calcium carbonate crystallization process by changing from a normal baffle with a hole in the center to a snout-type baffle as shown in **Figure 2**.

In addition, the function of vortex flow is not only to intensify the previously noticed transport phenomena such as mixing, heat transfer, and mass transfer, but also to have a new function that has not been previously noticed, such as classification and separation of particles. Ohmura et al. [2] found that particles with different

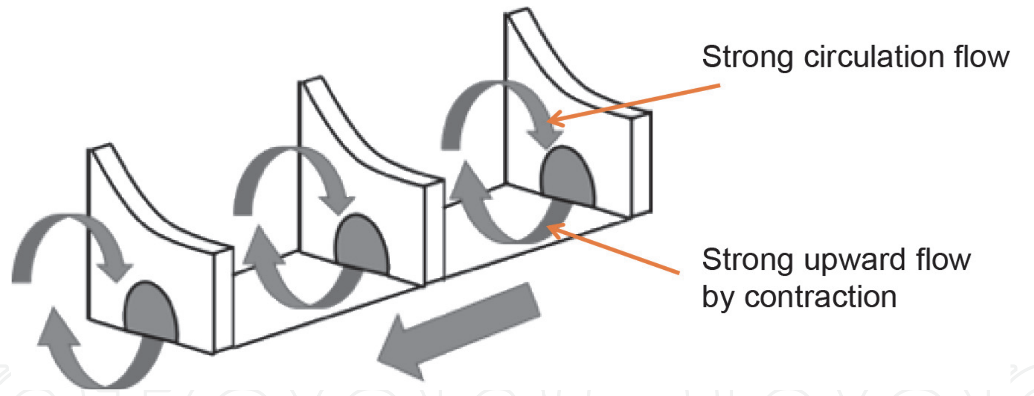


**Figure 1.**
*Schematic of Hanaguri Canal.*

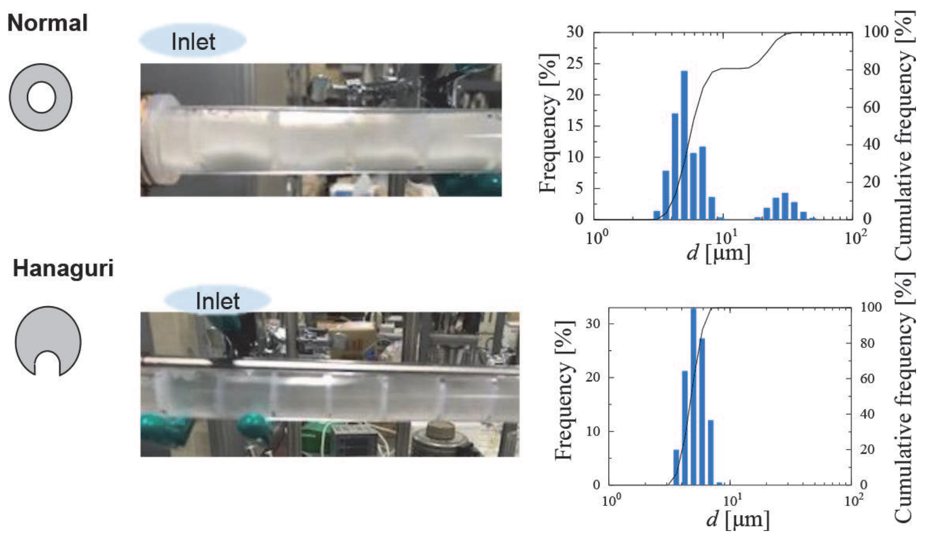


**Figure 2.**
*Comparison of performance of an oscillatory baffled crystallizer between using normal and Hanaguri-type baffles.*

sizes move on different streamlines within a Taylor cell and proposed that this could be applied to a particle classification device. Kim et al. [3] applied this idea to a continuous crystallizer and proposed a device for granulating particles of different sizes while classifying them. Wang et al. [4] also proposed a novel solid–liquid separation system that breaks the conventional stereotype of mixing equipment by applying the particle clustering phenomenon in isolated mixing regions in stirring tanks. In this way, vortices with a systematic structure have very attractive properties, such as solid accumulation, mixing and reaction enhancement, particle classification, and mass transport. If we can understand the characteristics of this organized vortex structure and manipulate it freely, we may be able to develop innovative chemical processes.

In many industrial processes, such as chemical, food, and mineral processes, the fluids handled are not only simple homogeneous Newtonian fluids, but also often complex fluids, such as non-Newtonian fluids, multi-phase fluids with highly dispersed phases, and viscoelastic fluids. Therefore, in order to apply the new "vortex dynamics" currently being constructed to process intensification technologies and implement it in society, it is necessary to develop the concept of new "vortex dynamics" from simple fluids to complex fluids. According to the abovementioned background, the present chapter provides an overview of vortex dynamics in complex fluids by taking examples of Taylor vortex flow.

## 2. Vortex dynamics with non-Newtonian fluids

A non-Newtonian fluid property causes a multiple fluid motion. These motions are quite interesting from fundamental and practical viewpoints. Especially, in vortex flow systems, fluid elements experience curved streamlines. In polymeric fluid systems, the polymer molecule chain does not line along curved stream lines, and consequently, hoop stress in a normal direction occurs. As a result, coupling normal stresses and curved streamlines causes elastic instabilities [5]. These instabilities are observed in various flows, e.g., Poiseuille flow [6], microchannel flow [7], and swirling flow [8]. Many polymeric fluids show not only viscoelastic behavior but also shear-thinning behavior. The shear-thinning property causes the viscosity distribution accompanied by the shear-rate distribution in the fluid system. Coelho and Pinho [9] showed that the shear-thinning affects the flow transition of vortex shedding in a cylinder flow. Ascanio et al. [10] reported that the mixing process of shear-thinning fluids under a time-periodic flow field is different from that of Newtonian fluid. Thus, vortex dynamics in non-Newtonian fluid systems is far from complete.

To investigate the effect of non-Newtonian property on vortex dynamics in more detail, many researchers have been utilizing Taylor–Couette flow, which is one of the most canonical flow systems in fluid mechanics, with non-Newtonian fluids [11–14]. Taylor–Couette flow is the flow between coaxial cylinders with the inner one rotating. This flow shows a cascade transition from laminar Couette flow to fully turbulent wavy vortex flow with the increase in circumferential Reynolds number ($Re$). When the value of $Re$ exceeds the critical $Re$ ($Re_{\mathrm{cr}}$), Taylor vortex flow firstly appears. As mentioned above, many researchers have been studied the Taylor–Couette flow with non-Newtonian fluids. For example, Muller et al. [11] and Larson et al. [12] revealed that the elastic instability occurs in Taylor–Couette flow and organized flow modes based on Deborah number ($De$), which the ratio of a characteristic relaxation time of the fluid to a characteristic residence time in the flow geometry [5]. **Figure 3** shows laminar Taylor–Couette flow with Newtonian (40 wt% glycerol aqueous solution) and viscoelastic fluid (0.75 wt% sodium polyacrylate aqueous solution).

The flow pattern was visualized by adding a small amount of Kalliroscope AQ-1000 flakes. As shown in **Figure 3**, the cellular structure of Taylor vortices seems to be complicated in the viscoelastic fluid even at the relatively low $Re$. The detailed mechanism is found in their papers [11–14]. Other interesting point is an

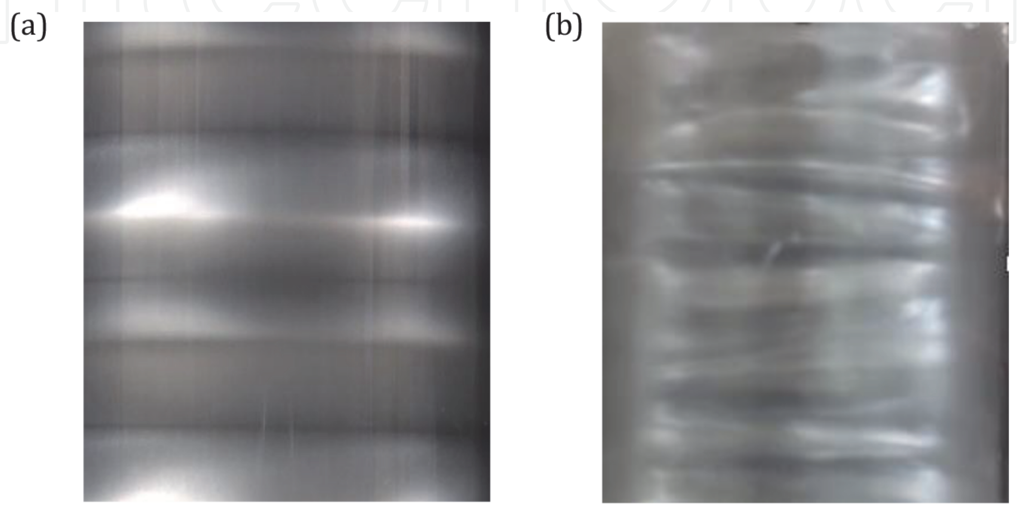


**Figure 3.**
*Flow visualization: (a) Newtonian fluid at* $\mathrm{Re} = 212$ *and (b) viscoelastic fluid at* $\mathrm{Re}_{eff} = 218$.

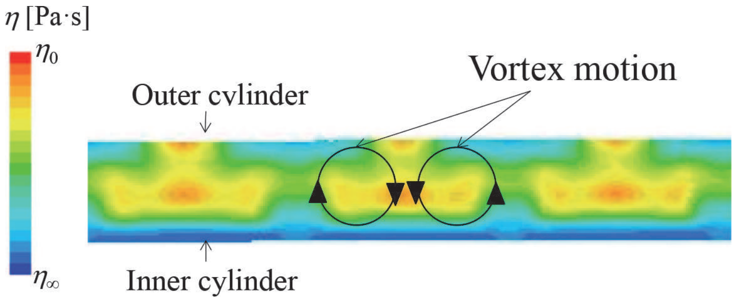


**Figure 4.**
*Viscosity distribution in the annular space obtained by numerical simulation [15]. The fluid was assumed to be a shear-thinning fluid.*

enlarged vortex structure by shear-thinning property. Escudier et al. [15] found that the cellular vortex is axially stretched and the vortex eye (the location of zero axial velocity in the vortex interior) is radially shifted toward the center body.

However, the first Taylor–Couette instability has not been fully understood yet in non-Newtonian fluid systems. One of the reasons is the discrepancy between $Re_{\mathrm{cr}}$ reported by several researchers for non-Newtonian fluids. Alibenyahia et al. [16] reviewed the discrepancy; Jastrebski et al. [17] reported $Re_{\mathrm{cr}}$ decreased with the shear-thinning property, on the other hand, Caton et al. [18] found the opposite tendency. Actually, this discrepancy is explained by the difference in how to define the effective Reynolds number, $Re_{\mathrm{eff}}$, in their papers. In non-Newtonian fluids, how to define $Re$ is quite complicated because the viscosity locally varies as shown in **Figure 4** [19]. Practically, $Re_{\mathrm{eff}}$ based on the effective viscosity in the system should be discussed. Several researchers have been trying to define more rational $Re_{\mathrm{eff}}$ in various flow systems, e.g., rising bubble flow in shear-thickening fluid [20], Rayleigh–Bénard convection with shear-thinning fluids [21], and non-Newtonian fluid flow past a circular cylinder [22].

We previously proposed a new definition of $Re_{\mathrm{eff}}$ based on the effective viscosity ($\eta_{\mathrm{eff}}$), which is obtained by numerical simulation. $\eta_{\mathrm{eff}}$ is calculated by averaging the locally distributed viscosity using a weight of dissipation function as follows [23]:

$$\eta_{\mathrm{eff}} = \sum_{i=1}^{N} \dot{\gamma}_i^2 \eta_i \Delta V_i / \sum_{i=1}^{N} \dot{\gamma}_i^2 \Delta V_i, \tag{1}$$

where $N$ is the total mesh number, $\eta_i$ [Pa·s] is the local viscosity, $\dot{\gamma}_i$ [1/s] is the local shear rate, and $\Delta V_i$ [m$^3$] is the local volume for each cell. It should be noted that $\eta_{\mathrm{eff}}$ is obtained using numerical simulation. The computational domain is shown in **Figure 5**. The governing equations are as follows:

$$\nabla \cdot \mathbf{u} = 0, \tag{2}$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\frac{\nabla p}{\rho} + \frac{1}{\rho} \nabla \cdot (2\eta \mathbf{D}) + \mathbf{g}, \tag{3}$$

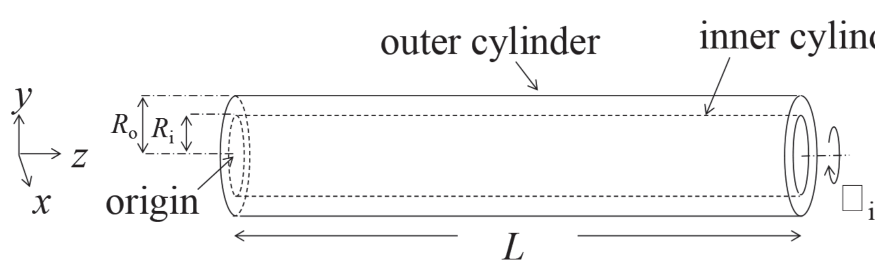


**Figure 5.**
*Computational domain [22].* $R_i$ *and* $R_o$ *are the radii of inner and outer cylinders, respectively.*

where **u** [m/s] is the velocity, $p$ [Pa] is the pressure, $\rho$ [kg/m$^3$] is the density, $\eta$ [Pa·s] is the viscosity depending on the shear rate, **D** (= ($\nabla\mathbf{u}$ + $\nabla\mathbf{u}^T$) / 2) [1/s] is the rate of deformation tensor, **g** [m/s$^2$] is the gravitational acceleration. The rheological property is characterized by Carreau model as follows [24]:

$$\eta = \eta_0\left[1 + (\beta \cdot \dot{\gamma})^2\right]^{(n-1)/2}, \tag{4}$$

where $\eta_0$ [Pa·s] is the zero shear-rate viscosity, $\dot{\gamma}$ [1/s] is the shear rate, $\beta$ [s] is the characteristic time, and $n$ [−] is the power index, which indicates the slope of decreasing viscosity with shear rate. In the case of $n < 1$, the fluid shows the shear-thinning behavior. The detailed information of numerical procedure is written in our paper [23].

**Figure 6** shows the critical value of $Re_{\text{eff}}$ for various shear-thinning fluids as a function of gap ratio $R_i$ / $R_o$. The theoretical $Re_{\text{cr}}$ for Newtonian fluids derived by Taylor [25] was denoted by the dashed line in **Figure 6**. It is found that the critical $Re_{\text{eff}}$ for shear-thinning fluids was in agreement with the theoretical value at $R_i$ / $R_o$ > 0.7. Thus, $Re_{\text{eff}}$ defined based on $\eta_{\text{eff}}$ by Eq. (1) is rational as a practical basis. The effect of shear-thinning property on the vortex structure is also interesting from the viewpoint of fluid dynamics. **Figure 7** shows the number of pairs of Taylor

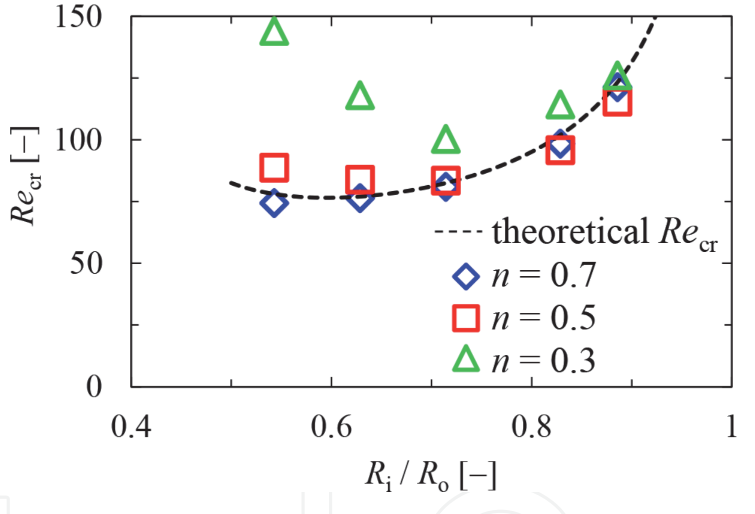


**Figure 6.**
*Re$_{cr}$ for various shear thinning [18].*

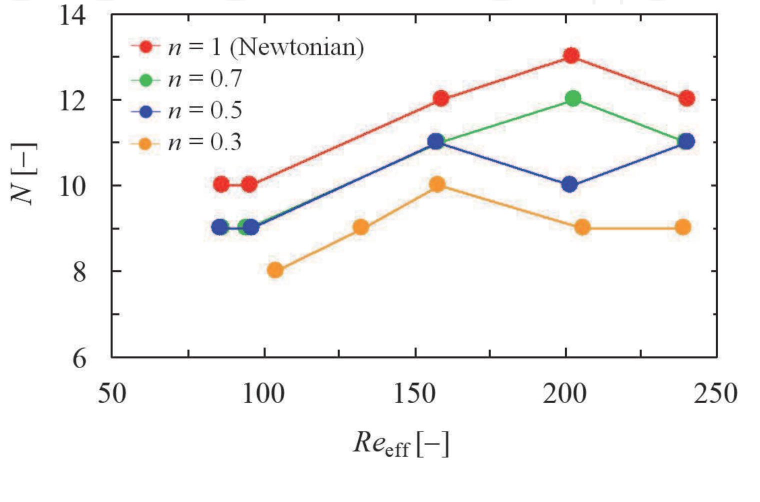


**Figure 7.**
*Variation in the number of pairs of Taylor vortices [23].*

cells, $N$, as a function of $Re_{\text{eff}}$ at the aspect ratio $\Gamma$ = 20 [26]. In all fluid systems, $N$ tended to increase with $Re_{\text{eff}}$. This tendency agrees with reports by other researchers [27]. Furthermore, the shear-thinning property seems to make Taylor cells large because $N$ decreases with the shear-thinning property at the same degree of $Re_{\text{eff}}$. This tendency was remarkable in the case of $n$ = 0.3. This means that the shear-thinning property axially enlarges Taylor cells. Although the detailed mechanism of enlarging Taylor cells is under consideration, it will be clarified by numerical simulation of development process of Taylor vortices.

We also introduce heat transfer characteristics of Taylor–Couette flow with shear-thinning fluids. In addition to Eqs. (2) and (3), energy equation was solved:

$$\frac{\partial}{\partial t}\left(\rho C_p\right) + \nabla \cdot \left(\rho C_{\mathrm{p}} T \mathbf{u}\right) = \nabla \cdot (\kappa \nabla T), \tag{5}$$

where $C_{\text{p}}$ [J/kg·K] is the specific heat capacity, $T$ [K] is the temperature, and $\kappa$ [J/m·s·K] is the thermal conductivity. **Figure 8** shows the axial variation in the local Nusselt number, $Nu_{\text{L}}$, at the surface of the outer cylinder at $Re_{\text{eff}}$ = 158 [26]. The $Nu_{\text{L}}$ at the surface of the outer cylinder was calculated as follows:

$$Nu_{\mathrm{L}} = \frac{2hd}{\kappa}, \tag{6}$$

where $h$ is a local heat transfer coefficient. As clearly shown in **Figure 6**, $Nu_{\text{L}}$ decreases with the increase in the shear-thinning property. This decrease is explained by increasing the thickness of velocity boundary layer for shear-thinning fluid systems (**Figure 9**). Generally speaking, it is said that the shear-thinning property improves heat transfer performance at same $Re$ [28, 29]. This is because the viscosity reduction by the shear-thinning property is not adequately reflected in $Re$ used in papers. In other words, the actual flow condition is underestimated in the case of shear-thinning fluids. Thus, the heat transfer performance is not accurately compared between Newtonian and shear-thinning fluids unless $Re_{\text{eff}}$ is used for representation of flow condition.

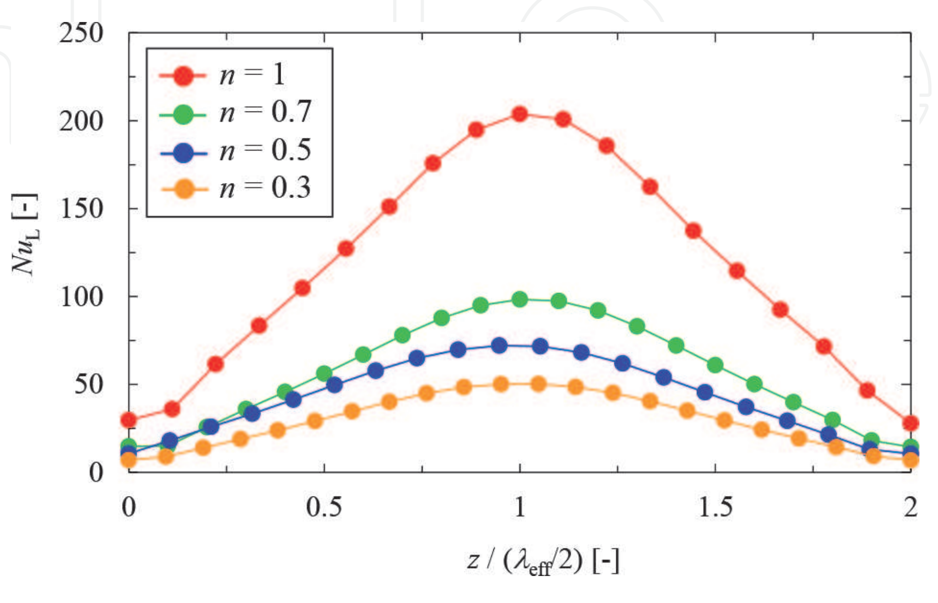


**Figure 8.**
*Axial variation in the local Nusselt number ($\mathrm{Nu}_L$) along the surface of the outer cylinder at $\mathrm{Re}_{eff}$ = 158 [23]. $\lambda_{eff}$ is the wavelength of Taylor cells.*

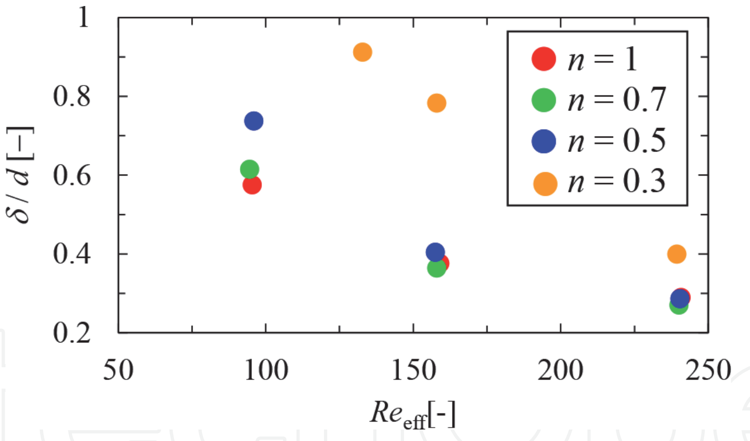


**Figure 9.**
*Dependence of the dimensionless thickness of the velocity boundary layer [23].*

## 3. Conclusions

In this section, we mainly refer the effect of shear-thinning on vortex dynamics including heat transfer. However, the viscoelastic property further complicates vortex dynamics as shown in **Figure 3**. In the future, vortex dynamics and transport phenomena in viscoelastic fluid systems should be investigated in more detail. In this case, it is considered to be important to construct a mathematical model by multi-scale analysis focusing on the interaction among scales of microstructure (molecular structure of polymers, micelles, particles, etc.), mesostructure (entanglement of polymer, particle aggregation, etc.), and macrostructure (vortex flow) of complicated fluid. For example, when a polymer solution flows in a micro channel having a sharp contraction part, an unsteady vortex called viscoelastic turbulence is generated in a corner part of the contraction part at higher Weissenberg number [30]. When the scale of the microchannel becomes small, the scale of the flow can be compared with the scale of the polymer. Since the influence of the elasticity derived from the deformation of the polymer itself on the flow becomes large, there is a possibility that the dynamic characteristics of the vortex generated in the contraction part can be controlled by the channel shape. In order to construct a methodology of controlling the viscoelastic vortex, a multi-scale simulation combined with molecular dynamics and computational fluid dynamics may be important.

As this viscoelastic vortex example shows, the field in which the vortex occurs affects the characteristics of the vortex. In the case of a Taylor vortex flow system, for example, the structure and dynamic characteristics of the vortices largely depend on the surface properties. It has been reported that heat transfer is enhanced by processing regular unevenness in the circumferential direction on the outer cylinder surface [31]. In the case of conical Taylor vortex flow, our previous work [32] successfully reproduced the phenomenon that the vortices move upward spontaneously under specific conditions by numerical analysis, and it was found that mass transfer was enhanced in polymer fluid system. In this way, it is possible to control the characteristics of the vortex flow by a structurally organized (having low entropy or fractal) nonuniform field rather than simply a random (high-entropy) nonuniform field. Therefore, in order to systematize a new vortex dynamics for freely manipulating vortices, it is necessary to quantitatively express the heterogeneity by introducing the concept of entropy and fractal and to clarify the relationship between the structure of the field and the characteristics of vortices.

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## Conflict of interest

The authors declare no conflict of interest.

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