Does the Weak Trace Show the Past of a Quantum Particle in an Unperturbed System?

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We investigate the weak trace method for determining the path of a quantum particle in an unperturbed system. Specifically, looking at nested interferometer experiments, when internal interferometers are tuned to destructive interference, we show that the weak trace method gives misleading results. This is because the methods used experimentally to obtain the weak value of the position operator necessarily perturb the system, hence, in some cases the assumption that weak coupling being equivalent to no coupling is incorrect. Experiments performed that are claimed to support the interpretation simply show the effects of this coupling acting as measurement, rather than tapping into the underlying reality of what happens in a quantum system when no-one is looking.

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I. INTRODUCTION

According to the weak trace approach to particle presence, based on weak measurement and post-selection [1], non-zero weak values of the spatial projection operator at a given location indicate where a particle has been [33]. Vaidman presents this as an approach to considering where a quantum object has been, that fully conveys quantum aspects [7, 36]. Based on the two-state vector approximation, it allows discontinuous particle trajectories, showing how seemingly disconnected events can affect one another, in ways which explain previously arcane-seeming experimental results.

Despite these benefits, the weak trace approach has been highly controversial [12, 16, 18, 34, 38, 40]. In this paper, we first show how to obtain the weak value of an operator (Section II), then give a case where this weak value for the spatial projection operator is unexpectedly non-zero (Section III). We then raise two key problems with the weak trace approach.

The first of these problems, which we present in Section IV, is that even weak measurements disturb a system, so any approach relying on such a perturbation to determine the location of a quantum particle will only describe this disturbed system, rather than the actual undisturbed state we care about. We highlight this using the case of a balanced interferometer tuned to have destructive interference (i.e. no light exiting at its dark port) where such a perturbation has dramatic effects - the unperturbed state is vacuum, while the perturbed state has light present. While the measurement effect can be made arbitrarily small, this is not the same as removing it entirely - a fact that has been neglected in previous analyses, and which needs to be made more explicit.

The second problem, which we raise in Section V, is that, even if we assume there is no disturbance, there is no reason to associate the weak value of the spatial projection operator with the classical idea of 'particle presence', especially if it has features which go against the classical ideas associated with a particle being present (i.e. a particle having a single, continuous path).

II. TWO-STATE VECTOR FORMALISM AND WEAK VALUES

Aharonov et al rediscovered the Two-State Vector Formalism (TSVF; originally given by Watanabe as the Double Inferential-state Vector Formalism [44]), which implies that we need to consider both the backwards and forwards-evolving quantum states, rather than just the forward (as standard quantum mechanics does) [2]. Extrapolation from the present to the past (retrodiction) can be as useful as extrapolation from the present to the future (prediction).

This TSVF can be formalised as the effect on some operator $\hat{O}$ of the forwards travelling initial state $|\psi_i\rangle$ and the backwards travelling final state $\langle\psi_f|$ as

$$\langle\psi_f|\,\hat{O}\,|\psi_i\rangle \tag{1}$$

where $\langle\psi_f|\,|\psi_i\rangle$ is referred to as the Two-State Vector.

This time symmetric formalism was developed by Aharonov et al into the weak value O_w of an operator $\hat{O}$, where

$$O_w = \langle\hat{O}\rangle_w = \frac{\langle\psi_f|\,\hat{O}\,|\psi_i\rangle}{\langle\psi_f|\psi_i\rangle} \tag{2}$$

As this weak value increases the closer $|\psi_i\rangle$ and $|\psi_f\rangle$ are to being orthogonal, this is a useful tool for metrology, amplifying signals from delicate results so they can be observed experimentally, at the expense of postselection meaning this only succeeds rarely [9, 15, 17]. However, weak values also lead to a range of paradoxes [4].

We first derive the formula we give above for the weak value. To do this, we first couple our initial system $|\psi_i\rangle$ to our pointer, $|\phi(x)\rangle$, then weakly measure this with the

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probe Hamiltonian $\hat{H} = \hat{O} \otimes \hat{P}_d$, to get the state

$$|\psi_w\rangle = e^{-i(\hat{O}\otimes\hat{P}_d)T/\hbar} |\psi_i\rangle \otimes |\phi(x)\rangle \quad (3)$$

where T is the time of the interaction between state and probe, and $\hat{P}_d$ is the momentum of that pointer.

We then strongly measure this weak-measured state $|\psi_w\rangle$ with the operator

$$\hat{F}_1 = |\psi_f\rangle\langle\psi_f| \otimes \hat{I}_d \quad (4)$$

Assuming $\hat{P}_d$ is distributed around 0 with low variance (so $\hat{X}_d$, the position of the pointer, has high variance), we can make the assumption that

$$\begin{aligned} e^{(\hat{O}\otimes\hat{P}_d)} &= \sum_{k=0}^{\infty} (\hat{O}\otimes\hat{P}_d)^k/k! \\ &= \mathbb{1} + (\hat{O}\otimes\hat{P}_d) + \frac{(\hat{O}\otimes\hat{P}_d)^2}{2} + ... \\ &\approx \mathbb{1} + (\hat{O}\otimes\hat{P}_d) \end{aligned} \quad (5)$$

and so that this strong measurement gives the result

$$\begin{aligned} & |\psi_f\rangle\langle\psi_f| \, e^{-i(\hat{O}\otimes\hat{P}_d)T/\hbar} |\psi_i\rangle \otimes |\phi(x)\rangle \\ & \approx |\psi_f\rangle\langle\psi_f| \, (\mathbb{1} - i(\hat{O}\otimes\hat{P}_d)T/\hbar) \, |\psi_i\rangle \otimes |\phi(x)\rangle \\ & = |\psi_f\rangle \otimes \langle\psi_f|\psi_i\rangle(\mathbb{1} - iO_w\hat{P}_d T/\hbar) \, |\phi(x)\rangle \\ & \approx |\psi_f\rangle \otimes \langle\psi_f|\psi_i\rangle e^{-iO_w\hat{P}_d T/\hbar} |\phi(x)\rangle \end{aligned} \quad (6)$$

This means the pointer state $|\phi(x)\rangle$, acting as our "readout needle", shifts ($x \to (x-a)$ for some shift a), due to the application of the effective operator $(\mathbb{1} - O_w\hat{P}_d T/\hbar)$. a is distributed over a wide range of values for many repeats, but the distribution of "a"s will be a Gaussian centred on the weak value O_w. Peculiarly, this weak value O_w can be very far from any of the eigenvalues of $\hat{O}$, or even imaginary [1, 10, 31] - which is odd given it appears in exactly the place in the equation that an eigenvalue of $\hat{O}$ would for standard Von Neumann measurement (i.e. if $\hat{X}_d$ had no variance). This led to the belief that the weak value O_w represented some fundamental value of the operator $\hat{O}$ **between** measurements.

Vaidman, amongst others, took this to mean that, when the weak value of the spatial projection operator along a certain path (which, for strong measurements, has eigenvalues determining if a quantum particle is present) was non-zero for a given $|\psi_i\rangle$ and $\langle\psi_f|$, the quantum particle described initially by $|\psi_i\rangle$ and finally by $\langle\psi_f|$ had been on that path, and left a weak trace along it [33].

Due to their interrelatedness, this approach can be represented easily in a graphical format through the TSVF: asserting a quantum particle is present wherever its forward-evolving state (possible paths the particle could have travelled via from its original position) and backwards-evolving state (possible locations the particle could have come from to reach its final position) visibly overlap (as shown in Fig.1).

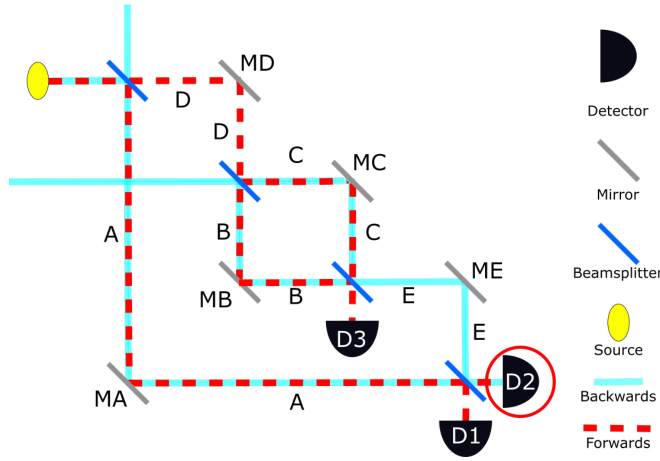

FIG. 1. Nested-interferometer model, used by supporters of the weak trace approach to discuss cases where the Two-State Vector Formalism contradicted "common-sense" continuous-trajectory approaches [33]. We describe this in more detail in Section III.

III. VAIDMAN'S NESTED-INTERFEROMETER

The nested-interferometer model, which we show in Fig.1, was pointed out as a case where the Two-State Vector Formalism contradicted "common-sense" continuous-trajectory approaches [33]. The inner interferometer is balanced such that a photon entering from arm D exits to detector $D3$ - consequently, the outer interferometer is unbalanced, so there is an equal probability of a photon introduced from the source ending in $D1$ or $D2$. Considering when the photon ends at $D2$, common sense would tell you it must have travelled via path A, as had it travelled via D, into the inner interferometer, there is no way it could exit onto path E and so reach $D2$. However, supporters of the weak trace approach claim, while it never travelled paths D or E, the photon, as well as being on path A, travelled along paths B and C. He bases his rationale for this on the Two-State Vector Formalism.

As a test of this, Danan et al placed oscillating mirrors at MA, MB, MC, MD and ME, each oscillating at a different frequency, and sent weak coherent pulses through the arrangement, to probe where, supposedly, a photon had been, by looking for the oscillations in the Fourier transform of the position of the centre of the beam detected at $D2$ [7][1]. They showed only the oscillations from MA, MB and MC, which they claim shows photons never travel by paths D or E, and proves Vaidman's hypothesis.

[1] Zhou et al repeated the experiment and obtained the same results [47].

However, it is debatable if this is a weak value experiment, or if it shows the validity of the weak trace approach, as we discuss below.

IV. JUST HOW WEAK IS A WEAK MEASUREMENT?

A. Position-related perturbations

Using the process we describe in Section II leads to a small shift in the position X_d of our pointer system $|\phi(x)\rangle$ of

$$\delta = \epsilon \Delta X_d \tag{7}$$

where $\epsilon << 1$, and ΔX_d is the initial width of the pointer position's Gaussian distribution before the coupling. Performing the measurement on a pre-and-postselected ensemble of N particles will allow the measurement of the shift to precision $\Delta X_d/\sqrt{N}$, so supporters of the weak trace approach claim, as long as $N > 1/\epsilon^2$, the presence of the particle will be revealed. However, this is still a coupling— so long as $\epsilon \neq 0$, the system still involves measurement.

This comes down to a key issue with weak measurement as a whole - that weak interaction never goes to non-interaction. The relationship between the amplitude and the intensity (and so energy, and so particle presence) is polynomial - therefore, while reduction of amplitude in coupling/transmission may polynomially reduce the intensity (and so chance of a quantum particle coupling/being transmitted), this is not the same as exponential reduction in this. This means, so long as we are taking a non-zero coupling/transmission amplitude, as all weak value experiments do, we cannot claim we are looking at the case when any interaction/measurement goes to zero.

Supporters of the weak trace approach rationalise this by saying "All particles have some nonvanishing interaction with the environment and they leave some trace" [33]. Further, Peleg and Vaidman say, "Note that in a hypothetical world with vanishing interaction of the photon with the environment, Vaidman's definition is not applicable, but in the real world there is always some nonvanishing local interaction. Unquestionably, there is an unavoidable interaction of the photon with the mirrors and beam splitters of the interferometer." [19].

However, this ignores two key factors. Firstly, due to their innate resistance from decoherence, it is very debatable as to whether there in fact **is** an unavoidable interaction between the photon and the mirrors/beamsplitters when that photon manages to arrive at the detector. Secondly, even if there were such an interaction on an individual photon-by-photon level, this would be nowhere near the same order of magnitude as the weak measurements appearing from paths D and E.

The experiment in Section III requires we preserve the interference pattern on the beamsplitters, to ensure no photons travel along paths D or E. Danan and Vaidman both claim we know this to be the case when we observe no trace from MD or ME on the output at $D2$, and can be double-checked by ensuring the intensity detected at $D3$ remains constant.

However, this ignores the difference in scale between the effects of oscillations on the beam in the upper path when it is combined (say that caused by MD or ME), and the interference-altering effects of oscillations when the beam is split (say, that caused by MB and MC) [16]. By definition, the raw oscillations have to be small, in order for the perturbation to count as a weak measurement, but small compared to the beam diameter at $D2$ ($\mathcal{O}(10^{-3}m)$) is very different to small compared to the wavelength of the photon used ($\mathcal{O}(10^{-6}m)$ (hence the advantages of homodyne detection). Therefore, the trace from the path mirrors MD and ME will obviously not show up when the spectrum is scaled to the order of the oscillations from MB and MC - but this doesn't mean photons didn't travel along paths D or E.

To quantify this, Danan says the oscillating mirrors each caused an angular change of $300nrad$. While they do not give the dimensions of the inner interferometer, we can assume it is of the order of centimetres, meaning the change in position on the recombining beamsplitter in the inner interferometer through each round is roughly $10^{-8}m$. Given the wavelength of light used was $785nm$, this is a comparatively large change in interference, allowing 1/40 of the light going into the inner interferometer via D to exit via E. Were this the case, one would expect this to show up on detector $D3$ - sadly, Danan et al do not show us the detected spectrum from $D3$; the closest they give us is a derived quantity from it for stability analysis, in their Supplemental III [8].

However, by showing that classical electrodynamics (through Maxwell's equations) would also lead us to expect to see to first order oscillations from MA, MB and MC, but not from MD or ME, if light went via D, through the inner interferometer, and back out through E, Danan et al make our point for us - their experiment gives us no reason to believe light can discontinuously "hop" into the inner interferometer without travelling through arms D and E. Saldanha, and Potŏchek and Ferenczi, both give the same analysis of Danan et al's results being obtainable using classical optics, with light slipping through when the oscillations at MB and MC unbalance the interferometer [20, 21].

This analysis is reinforced by that of Salih [22], Wieśniak [46], and Svensson [29, 30], who all independently show that, if MB and MC oscillate at the same rate, their respective traces on $D2$ disappear, as the interferometer is kept balanced. Similarly, Alonso and Jordan show that adding a Dove prism to the set-up along path C, while still balancing interferometer CD, allows the oscillations at MD to be seen, yet again proving the visibility of MB and MC but not MD and ME to be the result of differences in scale of oscillation effect, not due to light never passing along D or E [6].

Supporters of the weak trace approach attempt to say that this invalidates the experiment, by conceding that the Danan et al experiment isn't a direct measurement of the trace left by the photon [42]. However, this makes our point for us, that the experiment isn't a real measurement of weak value, and so it contributes little to confirming or denying the weak trace at a given point.

B. Non-position based versions of Danan et al's experiment

A natural response to this would be to alter Danan et al's experiment to avoid the position-related change in interference pattern, which the above analysis shows is a key loophole in the weak trace's conclusions. To do this, we change the pointer system (to which we weakly couple) from the beam position to a different degree of freedom. The polarisation of the beam presents a natural choice for this.

We first define the Bloch sphere for polarisation (or Poincaré sphere) such that the poles are $|H\rangle$ and $|V\rangle$, and the rotation

$$\hat{\mathbf{R}}_y(\theta) = \begin{pmatrix} \cos\left(\frac{\theta}{2}\right) & -\sin\left(\frac{\theta}{2}\right) \\ \sin\left(\frac{\theta}{2}\right) & \cos\left(\frac{\theta}{2}\right) \end{pmatrix} = e^{-i\theta\hat{\sigma}_y/2} \tag{8}$$

for dummy variable θ, and Pauli matrices $\hat{\sigma}_{x,y,z}$.

We take Danan et al's apparatus, but replace the oscillating mirrors with standard mirrors each paired with a variable polarisation rotator, designed to implement a $\hat{\mathbf{R}}_y(\theta)$ rotation of variable θ. By inserting a weak coherent light beam with polarisation $|H\rangle$, and having each rotator oscillate the applied rotation between the same magnitudes (e.g. $\pm 1°$) but at a different frequency, we would see the same results as Danan et al - the Fourier transform of the polarisation detected at $D2$ would show a massive peak at the frequency associated with path A, large peaks at the frequencies associated with paths B and C, and far smaller peaks at the frequencies associated with paths D and E. [2]

However, here, the difference in scale between the peaks at B and C and those at D and E is much more obviously due to the polarisation rotations making the beams at B and C distinct, and so stopping them from interfering with one another. This allows light to leak through from the inner interferometer to $D2$ - light carrying the oscillating tags from B, C, D and E. While the overall change in light intensity this light adds may be small, it will form the majority of the change in intensity observed, and so will be magnified heavily in the Fourier transform of the output polarisation with respect to time.

[2] To make this even closer to experiment given by Danan at al, the initial polarisation state of the input beam can be made a mixture, in a Gaussian distribution around $|H\rangle$ with a standard deviation far larger than the choice of maximum magnitude angle of the oscillations (e.g. $5°$).

C. General roof that weak trace experiments perturb a system

In both of the above cases, we see that the application of a weak measurement affects the interference pattern stopping light from traversing the inner interferometer, and so is responsible for the peculiar effects observed. We now attempt to generalise this, to show that any experiment of this sort will always cause interference breakdown. We do this by first looking at Englert's Visibility-Distinguishability Inequality [11],

$$\mathcal{D}^2 + \mathcal{V}^2 \leq 1 \tag{9}$$

This inequality shows that, with light travelling a two-path interferometer, knowledge about the distinguishability (or identifiability of which path the light travelled) $\mathcal{D}$, and fringe-visibility $\mathcal{V}$ at the output of the interferometer are negatively related. This is the case even when the inequality is saturated (as occurs when using a pure state). Therefore, doing anything which would increase the distinguishability between the two paths (such as placing different tags on B and C) will affect the interference pattern at the BCE beamsplitter. Given perfect interference is required to ensure all light that enters the inner interferometer from D exits into $D3$, anything which allows distinguishability between paths B and C will allow light to leak through onto E, and so a trace to be shown in $D2$. However, in all these cases, it will never be showing what would happen in an unperturbed system.

Given this breakdown in interference is due to distinguishability between B and C, we are left wondering what happens if we look at the weak trace of B and C together (where the two paths are indistinguishable), rather than separately - a case we look at in Section V B.

V. WHY SHOULD THE WEAK TRACE TELL US THE PAST OF A QUANTUM PARTICLE?

A. What does presence mean?

To try and work out if the weak trace should tell us the past location of a quantum particle, we must first work out what it means for a particle to be present at a certain place, at a certain point in time. Intuitively, we can say that an object being present at some location means:

1. It is only at that location (i.e. it can not be at both that location and another location simultaneously);

2. It has travelled to that location via a continuous

path, and will travel from that location via a continuous path; and,

3. It can affect/be affected by other objects/fields local to that location.

A fourth condition we can give for a particle being present at a location is that it leaves a weak trace at that location. This can be seen from the fact that, whenever a particle satisfies the above three conditions, it also satisfies this condition. However, this makes it a necessary condition for particle presence, rather than a sufficient condition - we have shown above cases where a particle has a non-zero weak trace at a location, but has no continuous trajectory (here continuous path with non-zero weak trace) to/from this location, and where we cannot say for certain that the particle would affect/be affected by other objects/fields local to that location (due to any such effect causing a breakdown in interference).

B. Incoherence of weak trace approach

Using the two-state vector formalism and weak value tools from Section II, we can quantitatively analyse the nested interferometer set-up in Section III. We first define the forwards-travelling initial vector and backwards-travelling final vector as

$$\begin{aligned} |\psi_i\rangle &= \frac{\sqrt{2}\,|A\rangle + |B\rangle + |C\rangle}{2} \\ \langle\psi_f| &= \frac{\sqrt{2}\,\langle A| + \langle B| - \langle C|}{2} \end{aligned} \tag{10}$$

with respect to the which path they evolve to/from respectfully. Using these, and defining the spatial projection operator for arm A as $\hat{P}_A = |A\rangle\langle A|$ (similarly for B and C), we get the weak values Vaidman does in [33]:

$$\begin{aligned} \langle\hat{P}_A\rangle_w &= \frac{\langle\psi_f|\,|A\rangle\langle A|\,|\psi_i\rangle}{\langle\psi_f|\psi_i\rangle} = 1 \\ \langle\hat{P}_B\rangle_w &= \frac{\langle\psi_f|\,|B\rangle\langle B|\,|\psi_i\rangle}{\langle\psi_f|\psi_i\rangle} = \frac{1}{2} \\ \langle\hat{P}_C\rangle_w &= \frac{\langle\psi_f|\,|C\rangle\langle C|\,|\psi_i\rangle}{\langle\psi_f|\psi_i\rangle} = -\frac{1}{2} \end{aligned} \tag{11}$$

However, if we want to see if the particle was present in the inner interferometer as a whole (as in either arm B or C), we define $\hat{P}_{BC} = |BC\rangle\langle BC| = |B\rangle\langle B| + |C\rangle\langle C|$ (as we are allowed to do in standard quantum mechanics, given its linearity), and instead find

$$\begin{aligned} \langle\hat{P}_{BC}\rangle_w &= \frac{\langle\psi_f|\,(|B\rangle\langle B| + |C\rangle\langle C|)\,|\psi_i\rangle}{\langle\psi_f|\psi_i\rangle} \\ &= \frac{1}{2} - \frac{1}{2} = 0 \end{aligned} \tag{12}$$

If we assumed a non-zero weak trace implies particle presence, this would mean the photon was never in the inner interferometer (made up of arms B or C) overall[3]. This seems incoherent, given, taken separately, it claims the particle was in arm B, and was in arm C[4]. This illustrates the importance of the sign of the weak values, which the weak trace approach, and the graphical form of the TSVF (as in Fig.1), neglect.

Further, given what we said in Section IV C, in some senses $\langle\hat{P}_{BC}\rangle_w$ is a far better measure of what light, if any, that reaches $D2$ was ever in the inner interferometer. This is as measuring $\langle\hat{P}_{BC}\rangle_w$ doesn't cause distinguishability between paths B and C in the inner interferometer, and so doesn't affect the interference pattern required to output all inner interferometer light into $D3$. $\langle\hat{P}_{BC}\rangle_w$ being 0 provides at least some element of support for a 'common-sense' trajectory (i.e. light only travelling via A to reach $D2$) in an unperturbed system (even though the true situation may be more complicated, as we discuss in Section V D). This could be demonstrated experimentally (e.g. using polarisation, as we talk about in Section IV B.

C. Weak trace without detection

The nested-interferometer set-up in Section III can be generalised by adjusting the beamsplitters, with the $A : D$ and $A : E$ beamsplitter set to $\alpha : (\beta + \gamma)$ and its adjoint respectively, and the $D \to B : C$ and $B : C \to E$ beamsplitter set to $\beta : \gamma$ and its adjoint respectively, where $\alpha^2 + \beta^2 + \gamma^2 = 1$. This gives initial and final states of

$$\begin{aligned} |\psi_i\rangle &= \alpha\,|A\rangle + \beta\,|B\rangle + \gamma\,|C\rangle \\ \langle\psi_f| &= \alpha\,\langle A| + \gamma\,\langle B| - \beta\,\langle C| \end{aligned} \tag{13}$$

and so the weak values of the spatial projection operators are

$$\begin{aligned} \langle\hat{P}_A\rangle_w &= \frac{\langle\psi_f|\,|A\rangle\langle A|\,|\psi_i\rangle}{\langle\psi_f|\psi_i\rangle} = \frac{\alpha^2}{\alpha^2} = 1 \\ \langle\hat{P}_B\rangle_w &= \frac{\langle\psi_f|\,|B\rangle\langle B|\,|\psi_i\rangle}{\langle\psi_f|\psi_i\rangle} = \frac{\beta\gamma}{\alpha^2} \\ \langle\hat{P}_C\rangle_w &= \frac{\langle\psi_f|\,|C\rangle\langle C|\,|\psi_i\rangle}{\langle\psi_f|\psi_i\rangle} = \frac{-\beta\gamma}{\alpha^2} \end{aligned} \tag{14}$$

When $\alpha \to 0$, given how the inner interferometer is balanced, this should lead to there being no chance of the

[3] Vaidman explicitly says that weak values obey the sum rule, and so allows us to say $\langle\hat{P}_{BC}\rangle_w$ must equal $\langle\hat{P}_B\rangle_w + \langle\hat{P}_C\rangle_w$ [32].

[4] Aharonov et all consider a similar scenario in their three-box experiment, and also discuss this idea of negative weak value in the equivalent of arm C cancelling the positive weak value in the equivalent of arm B [3].

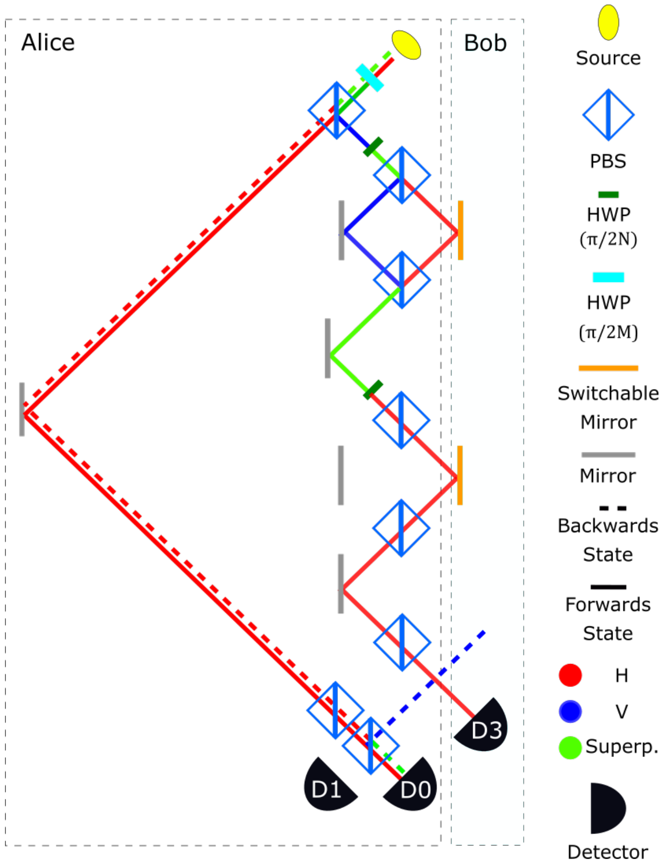

FIG. 2. Two-State Vector Formalism analysis of Salih et al's polarisation-based single-outer-cycle protocol for counterfactual communication. Note, here, we consider the polarisation, given it determines direction of travel through the polarising beamsplitters (PBSs). The forwards- and backwards-travelling states do not overlap anywhere on the inner interferometer chain in this case (when there is a detection at $D0$), meaning, by the weak trace approach, the particle detected at $D0$ was never at Bob. This means Bob's ability to communicate with Alice (him turning off/on his switchable mirrors to determine whether the photon goes to $D1$ or $D0$ respectively) is not explained by the weak trace approach any more than it is by standard quantum mechanics.

photon arriving in $D2$ - however, the TSVF still gives a trace at B and C, with the weak value of the projection operator at each going to $\pm\infty$ respectively. This proves that a trace at B or C needn't be associated with a detection at $D2$, even when we use a $|\psi_f\rangle$ tracing the state back from $D2$ - and so a weak trace at B or C needn't prove the photon detected at $D2$ was at B or C.

D. Why even use the weak trace approach?

An argument advanced by supporters of the weak trace approach is that it provides more information about the underlying state of the system than standard quantum mechanics. They claim that an issue with Wheeler's "common-sense" approach to particle trajectories (e.g. in his delayed-choice eraser) [45], was that it didn't tell us anything about the underlying mechanisms at work, just the final result—that it was entirely operational [33]. However, we turn this criticism back on these supporters - the weak trace approach doesn't tell us anything about the underlying system either, beyond standard quantum mechanics.

To qualify this, a key motivation behind the weak trace approach was to explain when the results of interference are affected by changes to disconnected regions, such as Wheeler's Delayed Choice, or Salih et al's Counterfactual Communication protocol [13, 27] and related effects [14, 23–26], and to show they aren't as "spooky" as they appear [35, 37, 39, 41]. However, Salih et al have shown that, both theoretically and experimentally, there is no weak trace by Vaidman's criterion at Bob when Alice receives the quantum particle (see Fig.2) [28]. Aharonov and Vaidman have also given an altered protocol for counterfactual communication where there is also no weak trace at Bob [5, 43]. These both show that the weak trace doesn't capture the cause of this phenomena.

While possibly hinting at some necessity of time-symmetry to quantum processes (through the TSVF), the weak trace doesn't suggest by itself any new mechanisms or elements of reality [32], above and beyond standard quantum theory, which would explain the causes of counterintuitive quantum effects when there is a weak trace. All it does is posit that, in some way, the quantum particle was present wherever these traces exist. Therefore, all it does is add a (potentially flawed) label rather than contributing anything testable to our ontology. This label confuses matters by oversimplifying a complex concept: what it means for a specific particle to be sequentially "present" at a two specific places in quantum field theory.

VI. CONCLUSION

We have shown that not only does the weak trace approach to particle presence give incoherent results (claiming that a particle can be in B, or in C, but not in B or C), but that it rests on faulty assumptions about weak coupling being equivalent to no coupling. Experimental "evidence" for the approach simply shows the effects of this coupling acting as measurement - rather than tapping into the underlying reality of what happens in quantum systems when no-one is looking.

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