11-3-2021

Stable Allocations of Vaccines in a Political Economy

Zephirin Nganmeni

Roland Pongou

Bertrand Tchantcho

Jean-Baptise Tondji
The University of Texas Rio Grande Valley

Recommended Citation

Nganmeni, Zephirin and Pongou, Roland and Tchantcho, Bertrand and Tondji, Jean-Baptiste, Stable Allocations of Vaccines in a Political Economy (November 3, 2021). Available at SSRN: https://ssrn.com/abstract=3955958 or http://dx.doi.org/10.2139/ssrn.3955958

Stable Allocations of Vaccines in a Political Economy

Zéphirin Nganmeni Roland Pongou Bertrand Tchantcho Jean-Baptiste Tondji*

November 3, 2021

Abstract

We develop a theory that addresses the problem of the existence of stable vaccine allocations in a political economy. These are allocation policies that a political leader can enforce without losing their popularity. Our analysis distinguishes between contexts where vaccination has positive externalities and contexts where it does not. We show that a stable allocation may not exist if vaccine supply is sufficiently low relative to the number of individuals eligible to receive a dose. We then fully characterize the minimum number of vaccine doses that guarantees the existence of a stable vaccine allocation, regardless of society's preference heterogeneity level. Minimum dose number depends only on a society's influence structure or voting rule. When individuals have unequal voting rights, stable allocations favor those with greater voting power. We generalize our main characterization result to economies where spatial proximity between individuals varies and preferences are unselfish due to positive vaccine externalities. Applying the theory, we find that a political leader can enforce stable vaccine allocation policies that are minority-inclusive only when the supply of vaccines is sufficiently high.

Keywords: Vaccine allocation game, influence structure, leader popularity, (un)selfish preferences, spatial proximity, externalities, minority and inclusion.

JEL Classification: C70, D81, E61, H12, I18.

*Nganmeni: Université Paris 8, zephirin.nganmeni@univ-paris8.fr; Pongou: University of Ottawa, rpongou@uottawa.ca; Tchantcho: University of Yaounde I, Advanced Teachers' Training College, btchantcho@yahoo.fr; Tondji: The University of Texas Rio Grande Valley, jeanbaptiste.tondji@utrgv.edu.
Special thanks are due to Andrew Dormer for excellent research assistance.

1 Introduction

We develop a theory that addresses the problem of the existence of stable vaccine allocations in a political economy. These are allocation policies that a political leader can enforce without losing their popularity. In our analysis, we distinguish between contexts where vaccination has positive externalities on the unvaccinated and contexts where it does not. When vaccination only benefits the vaccinated individual, agents have selfish preferences; and when it has positive externalities, agents have unselfish preferences.

Our analysis is timely and fitting, given the current COVID-19 crisis. Indeed, recent political events have demonstrated how the popularity of political leaders can depend on how well they manage major global crises such as pandemics. These crises can both reinforce and undermine the popularity of the status quo (or incumbent policy) depending on how a leader performs and how the electorate perceives this performance. Decisions made by political leaders or institutions in response to a major crisis can affect political support in two ways. First, major crises can rally the electorate around the status quo when they perceive that the decisions made in the crisis by incumbent governments bring some relief (Healy & Malhotra, 2009; Bechtel & Hainmueller, 2011; Bol et al., 2021). Second, when handled poorly, major crises can reduce support for incumbent leaders, shifting the electorate's preferences towards political rivals in democratic societies and fostering regime change in authoritarian ones (Aidt & Leon, 2016; Ruiz-Rufino & Alonso, 2017; Baccini et al., 2021). These mechanisms of retrospective performance evaluation induce accountability among leaders, as the electorate tends to reward leaders for making good leadership decisions and punishes them for making bad ones. The study by Herrera et al. (2020) which analyses government responses to the coronavirus disease 2019 (COVID-19) pandemic, is consistent with this view. It shows that, when experiencing increasing severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2)[1] infections and deaths, governments that prioritized health over short-term economic gains obtained higher approval rates than those that failed to do so. Along these lines, Baccini et al. (2021) and several viewpoints from news outlets, including, Acosta & Stracqualursi (2021), Oliphant (2021), and McEvoy (2021), have partly attributed Donald Trump's reelection loss to his administration's mismanagement of COVID-19 pandemic. On the same line, according to

[1]SARS-CoV-2 is the virus that causes COVID-19.

BALLOTPEDIA (2021), the COVID-19 pandemic has contributed to the recall election seeking to remove California Governor Gavin Newsom on September 14, 2021.[2] Similarly, faced with plunging approval ratings over the summer amid public dissatisfaction with how his administration handled the pandemic (especially the vaccination program) and the 2020 Tokyo Olympics, Yoshihide Suga, Japan's Prime Minister, said he would not seek re-election after just a year in office (Rich, 2021).

Given these realities, it is essential to investigate the conditions under which political leaders and governments have the ability to implement pandemic-containing policies that mitigate adverse short-term effects. We address this broad question in the specific context of the allocation of vaccines, which can help reverse the spread of a pandemic. How can a leader allocate a limited supply of vaccine doses in a heterogeneous society without risking their popularity or their chances of re-election? We address this question through the lens of an allocation model that we describe as follows.

A pandemic disease caused by a virus is affecting the livelihood of residents in a society which consists of a finite set of individuals or agents, $N = \{1, 2, ..., n\}$, with $n \geq 3$. Thanks to advanced technology in medical science and financial government investments, a safe and effective vaccine against the virus has been developed and is approved for emergency use. However, the number of available doses, μ, is limited: $\mu < n$, where μ is a non-zero integer. Given N and μ, an allocation of the μ vaccine doses to individuals is any $S \subseteq N$ such $|S| \leq \mu$, where $|S|$ denotes the cardinality of S. We denote by $N(\mu)$ the set of allocations of μ vaccine doses. Each individual $i \in N$ has a preference relation, which we denote by $\precsim_i$, over the set $N(\mu)$ which is a *weak ordering* (i.e., complete and transitive). In the society, an *influence structure* (or *voting rule*) $\mathcal{I}$ collects *influential* coalitions—groups of individuals that have the power or ability to rule out any proposed allocation in $N(\mu)$ regardless of the preferences of agents outside of these coalitions—. We refer to the pair $E = (N, \mathcal{I})$ as a *political economy*, and the couple (E, μ) as a *vaccine allocation problem*. For any preference profile $(\precsim_i)$, we consider the tuple $G = \langle (E, \mu), (\precsim_i) \rangle$ as a *vaccine allocation game*. Given these concepts, the goal of a social planner or political leader is to design a vaccine allocation solution that only proposes *stable outcomes*, i.e., allocations that would not be successfully challenged by

[2]The Governor Gavin Newsom survived the recall petition as a majority of California voters say "No" by allowing him to stay in office until at least 2023.

influential coalitions in the society.

With this model, we examine the following questions:

1. Under which conditions on $\mathcal{I}$ and μ does a stable vaccine allocation exist?

2. What is the minimal number of vaccine doses that guarantees the existence of a stable allocation?

3. How does voting power affect priority in access to a vaccine?

We exploit a classical equilibrium solution (Black, 1948; Gillies, 1959; Serrano, 1995) in the blocking approach (Dutta & Vohra, 2017) to investigate these questions in two different environments.[3] In the first environment, agents are selfish, and care only about maximizing their own utility. In the second environment, agents can be described as having "other regarding preferences", meaning that agents have preferences regarding other agents' material payoffs and may or may not have preferences regarding their own material payoffs. In this case we will assume that an agent prefers an allocation where their closest neighbor receives a vaccine dose (at the expense of a more distant neighbor) to one in which the converse happens. We will also assume that agents have no preferences regarding their material payoffs, meaning that agents will prefer allocations where close neighbors are prioritized over distant neighbors even if such allocations result in the agents themselves receiving no doses. While the assumptions in this second environment may be unrealistic, a less absolute form of this environment is highly prevalent in real life, as individuals have preferences that are determined by their ethnic, gender, and cultural proximity to others; see, for example, Rabin (1993), Fehr & Schmidt (1999), Dufwenberg et al. (2011), Dimick et al. (2018), Bosi et al. (2021), and an excellent survey by Kagel & Roth (2020). We exploit insights from this environment to derive implications for inclusive allocation policies in a society that features minority groups.

Under the assumption that each agent behaves selfishly in their attempt to obtain a vaccine dose[4], Theorem 1 shows that the number of vaccine doses and the society's influ-

[3] According to this equilibrium concept, a policy X is an equilibrium (or is stable) if there does not exist another policy Y that is preferred to X by an influential coalition under the prevailing influence structure (or voting rule). It follows that this solution also provides a measure of *popularity* since a policy that is not an equilibrium outcome is preferred less compared to another policy and is therefore unpopular.

[4] Recent studies (see, for instance, Bleier et al. (2021), Singanayagam et al. (2021), and Schiavone et al. (2021)) showing that adenovirus and mRNA candidate vaccines demonstrated persistent SARS-Cov2

ence structure are critical for the existence of a stable vaccine allocation. In Theorem 2, we introduce a new notion of a stability index, which corresponds to the minimum number of vaccine doses which guarantees the existence of a stable vaccine allocation. This index provides a structural characterization of the set of stable allocations, as it only depends on the prevailing influence structure. A corollary of these findings is that, from a vaccine allocation problem for which the number of vaccine doses available corresponds to the stability index, we can deduce all the stable allocations for any greater number of vaccine doses. As an application of our theory, Proposition 1 derives a closed-form expression of the stability index for vaccine allocations in the class of symmetrically weighted political economies.

Given the limited supply of vaccine doses, we also address the question of how individual influence (or a priori voting rights) in a political economy could affect priority in access to a vaccine. Different approaches to the measurement of influence in voting have been proposed in the literature.[5] A classical measure is the so-called influence relation. An agent i is said to have more influence than another agent j if whenever j is replaced by i in an influential (or a winning) coalition, the resulting coalition remains influential. The influence relation, like other classical voting power measures, is therefore a measure of "a priori" voting power. We know very little about whether this measure translate into the ability to bring about a social outcome that maximizes individual preferences (Pongou & Tchantcho, 2021). In Proposition 2, we show that, if agent i is at least as influential as j, then, i can replace j in any stable allocation without compromising the allocation's stability. In other words, more influential individuals have greater access to vaccine. For robustness, we use another measure of a priori influence—partner-dependence—to confirm this relationship. An agent k is partner-dependent of another agent i when any influential coalition containing k also contains i. In Proposition 3, we show that when the number of vaccine doses available is the stability index (or the minimum number that guarantees

virus in nasal swabs despite preventing COVID-19 disease justify selfish preferences for these vaccines. In simple words, a fully COVID-19 vaccinated individual can be infected from SARS-Cov2 and can transmit the virus. Thus, currently, the COVID-19 vaccination mainly has private benefits because I am only protecting myself from becoming sick if I am vaccinated.

[5]For a general discussion on voting power, we refer the reader to Shapley & Shubik (1954), Isbell (1958), Taylor et al. (1999), Tchantcho et al. (2008), Pongou & Tchantcho (2021) and Freixas & Pons (2021), among many others.

existence of a stable allocation), a stable allocation that contains agent i cannot contain another agent k which is partner-dependent of i. Propositions 2 and 3 imply that individuals that have more a priori voting power have higher priority access to the vaccine.

We extend the analysis to societies characterized by varying spatial proximity between individuals who have in-group preferences. Such preferences could be explained by altruism towards in-group individuals, or by the fact that certain vaccines have positive externalities; that is, individuals' vulnerability to infection decreases more if their closer neighbors are vaccinated. In these economies, preferences are unselfish due to positive vaccine externalities. Networks offer an alternative approach to representing the relationships between agents. We therefore add a spatial dimension to the vaccine allocation game. Spatial preferences come from the positions of agents in the metric space used to represent and analyze proximity between agents. In this environment, Theorem 3 characterizes the set of stable allocations. In line with our analysis for selfish societies, we provide in Corollary 1, a spatial stability index, which corresponds to the minimum number of vaccine doses which guarantees the existence of a stable vaccine allocation in a spatial vaccine allocation problem. In a practical application of our theory, we consider a network polarized community consisting of a majority and a minority group. We find that stable policies exist, but they may not be minority-inclusive in the sense that they may not allocate a vaccine dose to any agent in the minority group. Unless the supply of vaccine doses exceeds a certain threshold, stable allocations only allocate vaccines to members of the majority group. Additionally, inclusive policies pursued in such environments may not be stable. The central insight from this application is that, unless they can secure a sufficiently large number of vaccine doses, a social planner who desires to be fair while preserving their popularity will be unable to find solutions that satisfy both of these goals.

Contributions to the Related Literature. To our best knowledge, our analysis proposes the first model of resource allocation in a political economy in which the social planner's goal is to preserve their popularity. We provide necessary and sufficient conditions under which the planner's problem can be solved. While our analysis focuses on vaccine allocation, our results extend to the allocation of any scarce good in a political economy context. However, our application to vaccines is timely, given the current COVID-19 pandemic. In this application, we distinguish situations in which vaccines provide only private benefits, therefore justifying selfish preferences, and situations where vaccines carry positive externalities. We also address the question of how a priori voting rights affect access to vaccine. All of our results are original.

We view our work as contributing to several literatures. We add to the literature on the allocation of discrete goods. Several approaches to distributing such resources have been considered. The random solution is one such approach. Applied to the vaccine problem, it consists of randomly choosing μ individuals in N to receive a vaccine dose. This approach and its variations (Price, 1958; Dahl, 1970; Mueller et al., 1972) do not consider structural and individual factors that could be essential for a planner to achieve a stable allocation of resources. Another literature, that is more recent, addresses the problem of optimally allocating vaccines during a pandemic to satisfy a set of other objectives related to demographics and occupations. Some studies suggest using vaccine efficacy (Kirwin et al., 2021; Matrajt et al., 2021; Singanayagam et al., 2021), cost-benefit analyses by coupling epidemiological and economic models (Mylius et al., 2008; Medlock & Galvani, 2009; Matrajt et al., 2013; Duijzer et al., 2018; Rao & Brandeau, 2021), mechanism design (Westerink-Duijzer et al., 2020; Xue & Ouellette, 2020; Akbarpour et al., 2021; Pathak et al., 2021; Castillo et al., 2021), and ethics (Yi & Marathe, 2015; Wu et al., 2020; Emanuel et al., 2020; Nichol & Mermin-Bunnell, 2021; Pathak et al., 2021). These studies show that the nature of epidemics, human characteristics, and market conditions are essential in implementing optimal vaccine allocations. We add to this literature the notion that leader popularity matters in a political setting where a leader is evaluated based on how well they manage a pandemic. Our main contribution highlights that the composition of a society in terms of the distribution of political influence ($\mathcal{I}$) and technology (supplying μ) are valuable tools that can help political leaders enhance their popularity from implementing stable vaccine allocations.

Our study is also related to the literature that uses tools from voting environments to solve allocation problems. Examples of such problems include, but are not limited to, the apportionment and proportional representation problems (Johnston, 1983; Balinski & Laraki, 2007; Florek, 2012; Brill et al., 2018; Jones et al., 2020), and the claim problems (Ju et al., 2007; Flores-Szwagrzak, 2015). Classical voting models that use the core (Gillies, 1959) as a solution concept do not account for the patterns of relationships between voters. Our modeling of unselfish societies does. In this respect, we are closer to the coalition structure model introduced by Aumann & Dreze (1974). We however differ from these studies in our scope, analysis and empirical implications.

The unselfish society model that we propose is inspired by the spatial framework. In this approach, we assume that each agent $i \in N$ is associated with an (ideal) point q^i in a Euclidean space of dimension $m \geq 1$. The Euclidean preference assumption is that agents are more attracted to allocations near their ideal points. Several studies are related to the core and its variants in the spatial framework. We can quote, inter alia, the usual spatial core (Plott, 1967; Davis et al., 1972), the epsilon-core (Shubik & Wooders, 1983; Eban & Stephen, 1990; Tovey, 2010, 1991; Bräuninger, 2007), the heart (Schofield, 1995), the soul (Austen-Smith, 1996), the delta-core (M. Martin & Tovey, 2021), the minmax or Simpson-Kramer point (Kramer, 1977), the strong point or Copeland winner (Owen, 1990), the yolk (McKelvey, 1986; Ferejohn et al., 1984), and the finagle point (Wuffle et al., 1989). Again, while our work shares some features of these models, our main research question is different. In a spatial framework, we characterize the set of stable vaccine allocations and derive implications for how the supply of vaccine doses can affect both the stability and inclusiveness of possible vaccine allocations.

We also contribute to a recent literature on how a priori voting rights affect the ability to induce social outcomes that maximize own preferences. There is a large and growing literature on the measurement of a priori voting power (Shapley & Shubik, 1954; Isbell, 1958; Banzhaf III, 1964; Taylor et al., 1999; Lambo & Moulen, 2002; Tchantcho et al., 2008; Kurz et al., 2017, 2021; Freixas & Pons, 2021). This literature does not address the question of whether a priori power reflects the ability to affect social outcomes. This question has been addressed in a few studies (Diffo Lambo & Moulen, 2000; Lambo et al., 2012; Pongou & Tchantcho, 2021). While our study partly addresses a similar question, we note that our setting is different.

The rest of this study is organized as follows. In Section 2, we formalize the general problem of vaccine allocation. Section 3 addresses the problem of the existence and characterization of stable vaccine allocations when agents are selfish. Section 4 addresses the same problem when agents are unselfish (or organized into distinct groups and have in-group preferences). Section 5 offers concluding remarks.

2 General Framework of Vaccine Allocation Games

To formalize the notion of a vaccine allocation game, we need several intermediate concepts and assumptions.

2.1 Allocations and Preferences

For any non-zero integer n, let $N = \{1, ..., n\}$ be a finite set of agents. A coalition is any nonempty subset of agents; 2^N refers to the set of coalitions and $|S|$ refers to the cardinality of any coalition S. There are μ available doses of a vaccine, where μ is a non-zero integer. Given N and μ, an **allocation** of the μ doses to agents is any $S \subseteq N$ such $|S| \leq \mu$, where $|S|$ denotes the cardinality of S. Therefore, we denote $N(\mu) = \{S \subseteq N : |S| \leq \mu\}$ the set of allocations. Each agent has a preference relation over the set $N(\mu)$ which is a **weak ordering** over $N(\mu)$. We denote agent i's preferences by $\precsim_i$, the relation $\prec_i$ denotes the strict portion of $\precsim_i$, and the relation $\sim_i$ denotes the indifference portion of $\precsim_i$. We denote by $(\precsim_i)$ a preference profile over $N(\mu)$. In this first part of our model, which is more general, we do not impose any additional assumptions on agents' preferences. For example, it is not assumed that all agents desire to receive a vaccine dose; it might happen that an agent i prefers not being vaccinated over being vaccinated and another agent j prefers being vaccinated over being unvaccinated. Next, we introduce the concept of an influence structure.

2.2 Influence Structures

Generally, governments and citizens face different challenges during a pandemic than they do in regular times. Producing a vaccine that may reduce adverse effects on health and economic conditions is a rare event in many societies. Those who succeed in developing a new vaccine face an allocation problem at the onset of its production. This problem generally arises from the fact that the social planner (or decision-makers) and citizens eligible to receive vaccines have misaligned preferences. Generally, political leaders avoid implementing policies that may result in them becoming *unpopular* with citizens, significant donors, and other interest groups.[6] Since political leaders care about maximizing the

[6] For more information on how interests groups influence policymakers' decisions, we refer the reader to Dellis & Oak (2019); also, a recent study by Dellis (2021) describes the literature on legislative

power wielded by themselves and their political party, democratic leaders avoid becoming unpopular as it increases the probability of an electoral loss for the leader or their party. Though weaker in autocratic societies, the incentive to prevent unpopularity still exists. Although leaders in autocratic societies don't have to face electoral backlash, unpopular policies still increase the risk of civil wars, revolutions, and other conflicts which may result in government instability or leaders' death.

We will refer to any group able to influence the popularity of a leader (and therefore the vaccine allocation) as an *influential coalition*. When developing an allocation strategy, any social planner must design a vaccine allocation solution that only proposes *stable outcomes*, i.e., allocations that will not be successfully challenged by *any influential coalition*.[7] We denote the set of influential coalitions by $\mathcal{I}$. Throughout the study, we call $\mathcal{I}$, the influence structure. Each coalition $S \in \mathcal{I}$ has the ability to veto out any proposed allocation in $N(\mu)$ regardless of the preferences of agents outside of S. Non-influential coalitions (i.e., all $S \notin \mathcal{I}$) do not have this veto power. We will assume that $\mathcal{I}$ satisfies the following two conditions:

(1) $\varnothing \notin \mathcal{I}$ and $\mathcal{I} \neq \varnothing$

(2) for all coalitions S and T, if $S \in \mathcal{I}$ and $S \subseteq T$, then $T \in \mathcal{I}$.

Both assumptions (1) and (2) are natural. According to assumption (1), an empty coalition can't influence the allocation of vaccines. Assumption (1) also states that at least one influential coalition exists. Assumption (2) is a monotonicity condition which stipulates that the addition of new agents to an influential coalition yields another influential coalition. The two assumptions together imply that the entire community, N, is an influential coalition.

An influence structure is said to be **weighted** if the following two conditions hold.

- There exist a real number q called quota, with $0 < q \leq n$.
- For all $i \in N$, there exists a non-negative weight α_i such that for all subset S of N, $S \in \mathcal{I}$ if and only if $\sum\limits_{i \in S} \alpha_i \geq q$.

informational lobbying.

[7] In other words, a stable vaccine allocation is one that is not unpopular.

In this case, we denote $\mathcal{I} \equiv \mathcal{I}(q) = [q; \alpha_1, ..., \alpha_n]$. A weighted influence structure is **symmetrical** if all agents have the same weight, which by normalization is $\alpha_i = 1$, for each $i \in N$. We introduce below a formal definition of a political economy, a vaccine allocation problem, and a vaccine allocation game.

2.3 Political Economies and Vaccine Allocation Games

Throughout the study, a *political economy* is a pair $E = (N, \mathcal{I})$, and the couple $V = (E, \mu)$ is referred to as a *vaccine allocation problem*. A *vaccine allocation game*, which we denote G, is any pair $G = \langle V, (\precsim_i) \rangle$, where $(\precsim_i)$ is a preference profile over $N(\mu)$. Sometimes, we will also write a vaccine allocation game as $G = \langle N, \mathcal{I}, \mu, (\precsim_i) \rangle$. We denote by $\mathbb{V}$ the set of all vaccine allocation games. Next, we provide the definition of a *stable* allocation of vaccines.

2.4 Stable Allocations of Vaccines

Let $V = (E, \mu)$ be a vaccine allocation problem, X and Y be two allocations in $N(\mu)$. In the vaccine allocation game $G = \langle V, (\precsim_i) \rangle$, we denote by $P(X, Y, (\precsim_i)) := \{i \in N \backslash X : X \prec_i Y\}$, the set of agents who prefer allocation Y to X. The following definition introduces the stability concept in our model of vaccine allocation.

Definition 1. Let $G = \langle N, \mathcal{I}, \mu, (\precsim_i) \rangle$ be a vaccine allocation game, and X and Y be two allocations in $N(\mu)$.

1. Y *challenges* X (i.e., Y is **more popular** than X) if $P(X, Y, (\precsim_i))$ is an influential coalition, i.e, $P(X, Y, (\precsim_i)) \in \mathcal{I}$.

2. X is *challenged* (i.e., is unpopular) if, there is an allocation Y that challenges X.

3. A *stable* allocation is an allocation that is not challenged.

4. The **core** of the vaccine allocation game, denoted $C(G)$, consists of all stable allocations.[8]

In the following examples, we illustrate the notion of the core of a vaccine allocation game.

[8] In other words, the core of a vaccine allocation game is the set of all vaccine allocations that are not unpopular. Leaders who implement allocation policies that do not belong to the core will lose their popularity, if judged only by how well they perform on the vaccine allocation problem.

Example 1. Consider a set of five individuals, $N = \{1, 2, 3, 4, 5\}$, working for three different firms denoted A, B and C. These firms offer different services, and interactions between individuals and firms are described as follows: $A = \{1, 2, 3\}$; $B = \{2, 4\}$; and $C = \{3, 5\}$ (also, see Figure 1). In other words, individual 1 only works for firm A, and individual 2 works for two firms, A and B.

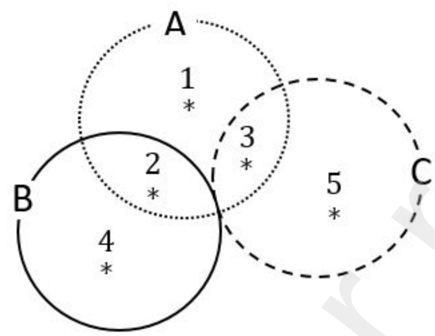

Figure 1: Representation of firms and services in the community

Assume that the supply of vaccine is limited, and therefore, not all the individuals in a firm can be vaccinated in the same period. How can a social planner solve the vaccine allocation problem? One approach which is widely used in the real-world consists of having an appropriate institution with the support of independent experts analyze the severity of the pandemic in different communities, then use this information to recommending vaccine priority.[9] This method of prioritization deals with the problem of vaccine scarcity while following some basic ethical principles such as: maximizing benefits and minimizing harms, mitigating health inequities, promoting fair access to the vaccine, and allowing for transparency in the allocation process. Assuming that agents in a group have some set of common characteristics which can used for analysis by institutions, an example of this prioritization policy could include agents in group A being offered vaccines first (because the nature of firm A services exposes its workers to a higher infection risk), followed by firm B, and then by firm C. It is essential to point out that the recommended priority

[9]For some recent proposals on vaccine prioritization, see, for instance, Persad et al. (2020), Pathak et al. (2021), Pollard & Bijker (2021), Han et al. (2021), World Health Organization (2021), Akbarpour et al. (2021), and the references therein. Currently, several countries are recommending eligible groups of individuals to a third COVID-19 vaccine shot (see, for example, Furlong & Deutsh (2021) and Anne Arundel County Department of Health (2021)).

decision is often made through bargaining and deliberation. In this context, individuals or firms can use their personal or economic power in the community to influence the allocation of the limited vaccine doses. It follows then that a society's influence structure will play a role in determining the set of agents who are likely to receive the vaccine first.

Following this discussion, we assume that a coalition is influential if it consists of more than half of the size of a firm. For instance, $\{1,2\}$ (more than half of the size of firm A) is influential while $\{1,5\}$ is not. In general, it follows that $\mathcal{I}$ contains coalitions S such that $|S \cap \{1,2,3\}| \geq 2$ or $\{2,4\} \subseteq S$ or $\{3,5\} \subseteq S$. Remark that any influential coalition contains at least one of the two individuals working for two different firms, 2 and 3, which implies that these individuals have more power to influence the allocation of vaccines. We consider the preference profile $(\precsim_i)$, such that for all allocations X and Y:

$$\forall i \in N \backslash \{3\}, \begin{cases} X \prec_i Y, \text{ if } i \in Y \backslash X \\ Y \prec_i X, \text{ if } i \in X \backslash Y \\ X \sim_i Y, \text{ otherwise} \end{cases} \text{ and for } i = 3, \begin{cases} X \prec_i Y, \text{ if } i \in X \backslash Y \\ Y \prec_i X, \text{ if } i \in Y \backslash X \\ X \sim_i Y, \text{ otherwise} \end{cases}$$

According to these preferences, individuals $1, 2, 4$ and 5 have a positive attitude towards vaccine since they would like to be vaccinated, unlike individual 3 who has a negative attitude towards vaccination. Let $(\precsim_i)$ be the preference profile given above, and let $E = (N, \mathcal{I})$ be a political economy. This situation can formally be analyzed using the vaccine allocation game $G = \langle E, \mu, (\precsim_i) \rangle$, where $\mu \leq 5$. We provide the set $C(E,\mu)$ of stable vaccine allocations for different values of μ.

- $C(E,1) = \{\{i\} : i \in N \backslash \{3\}\}$;
- $C(E,2) = \{\{2\}, \{1,2\}, \{1,4\}, \{2,4\}, \{2,5\}\}$;
- $C(E,3) = C(E,2) \cup \{\{1,2,4\}, \{1,2,5\}, \{1,4,5\}, \{2,4,5\}\}$;
- $C(E,4) = C(E,3) \cup \{\{1,2,4,5\}, \{1,2,3,5\}\}$; and
- $C(E,5) = C(E,3) \cup \{\{1,2,3,4,5\}\}$.

We note that when $\mu \leq 3$, agent 3 does not receive a vaccine dose in any stable allocation. Indeed, if agent 3 receives a dose of vaccine in an allocation X, then at least one agent j of $\{1,2,5\}$ must not receive a dose of vaccine. There is an allocation Y that challenges X via the influential structure $\{j,3\}$; consider any allocation Y that contains j but does not contain 3. Note also that when $\mu = 5$, the stable allocation $\{1,2,3,5\}$ reserves a vaccine dose for the hesitant agent 3 at the expense of agent 4 who, unlike 3, would like to be vaccinated.

Example 2. Consider a vaccine allocation game $G = \langle N, \mathcal{I}(2), \mu, (\precsim_i)\rangle$, where $N = \{1,2,3,4\}$, $\mathcal{I}(2) = [2;1,1,1,1]$ is a weighted symmetrical influence structure, $\mu = 2$, and $(\precsim_i)$ are the agents' preferences. If all the agents have a positive attitude towards vaccine, we show that the core $C(G)$ is empty, that is, $C(G) = \varnothing$. If only agent 4 has a negative attitude towards vaccine, then, $C(G) = \{\{1,2\},\{1,3\},\{2,3\}\}$.

Example 3. Consider a political economy $E = (N, \mathcal{I}(3))$, where $N = \{1,...,6\}$, and $\mathcal{I}(3) = [3;1,1,1,1,1,1]$. We assume that agent $i \in N$ has an *ideal* position denoted, q^i, in the usual Euclidean plane, with $q^i \neq q^j$ when $i \neq j$; see Figure 2. We derive agents' preference profile $(\precsim_i)$ from their *ideal* positions so that for all allocations X and Y, we have $X \precsim_i Y$ if $\min_{k\in Y}\left\{d\left(q^i,q^k\right)\right\} \leq \min_{l\in X}\left\{d\left(q^i,q^l\right)\right\}$, where d is the usual Euclidean distance.

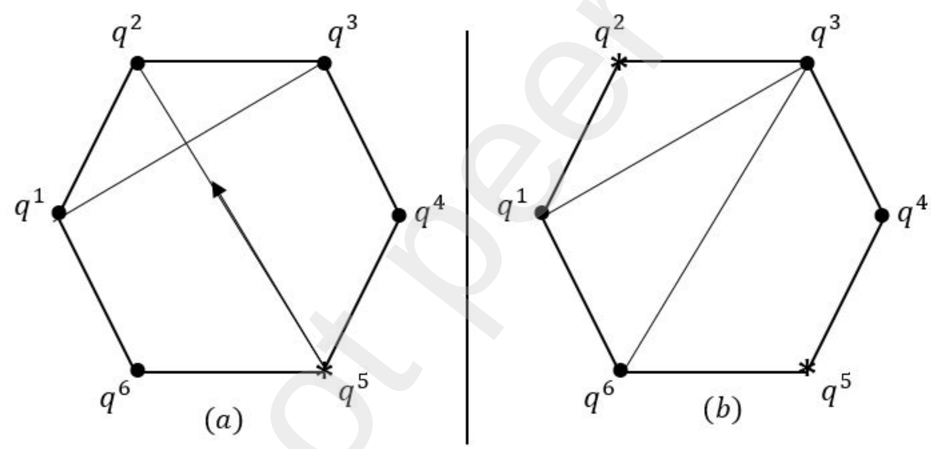

Figure 2: Illustration of an unselfish community in a spatial setting. The positions of agents in both Figures 2-(a) and 2-(b) form a regular hexagon.

- For $\mu = 1$, any allocation in the vaccine allocation game $G = (E, \mu, (\precsim_i))$ can be challenged, yielding an empty core. For instance, the allocation $X = \{5\}$ is challenged by $Y = \{2\}$ via $\{1,2,3\}$; see Figure 2-(a). Indeed, for any $i \in \{1,2,3\}$, $d(q^i,q^2) < d(q^i,q^5)$.
- For $\mu = 2$, the set of stable allocations in $G = (E, \mu, (\precsim_i))$ becomes non-empty. For instance, we show that $X = \{2,5\} \in C(G)$. If we consider an influential coalition S, it is not possible to find an allocation Y that collectively improves the preferences of the members of S with respect to X; see Figure 2-(b).

Though a stable vaccine allocation always exists in Example 1, this is not necessarily the case in Example 2 or Example 3. A natural question therefore arises: What conditions guarantee the existence of a stable vaccine allocation? This question is pertinent because if a stable vaccine allocation does not exist in a political economy, then any chosen allocation policy will be unpopular. Throughout the remaining of the study, we assume that agents have positive attitude towards vaccines. We address this issue in Sections 3 and 4. In Section 3, we assume that agents have selfish preferences in the sense that being vaccinated brings more utility to an agent. In this environment, agents only care about themselves. Agents' preferences in Example 2 illustrate such a selfish environment. In Section 4, we assume instead that agents are unselfish, i.e., they also care about others who are closer to them. We illustrate such a society using a spatial model in Example 4.

3 Stable Vaccine Allocations in Selfish Societies

In this section, we analyze the existence of stable vaccine allocations in an environment where agents are *selfish*. Selfish preferences in the context of vaccination are generally widespread in the early production stage of vaccines during pandemics. These preferences often induce social planners or policymakers to implement conservative and protectionist policies in the production and distribution of vaccines. This phenomenon is known as *vaccine nationalism* (see, for example, Bollyky & Bown (2020), Eaton (2021), and Katz et al. (2021)), a common situation where powerful countries deploy enough resources to secure vaccines and therapeutics at the expense of less-wealthy countries. Most people will not support a social planner who supplies vaccine doses to another country when the pandemic is not yet contained at home. Therefore, we might expect leaders to face tenuous challenges in allocating a limited supply of vaccines in selfish democratic communities where politicians seek popularity and re-election.

The main objective of each agent is to be among the first to receive a vaccine dose. Consequently, an agent prefers an allocation X to an allocation Y if X provides a vaccine dose to that agent and Y does not. An agent is indifferent between X and Y if they receive a vaccine dose in both allocations, or if neither allocation gives them a dose. The formal definition is below.

Definition 2. Let $V = (E, \mu)$ be a vaccine allocation problem, X and Y be two allocations in $N(\mu)$. Agent i's preferences $\precsim_i$ over $N(\mu)$ are *selfish* if the following hold.

1. Agent i prefers Y over X (or $X \prec_i Y$) if $i \in Y \backslash X$.

2. Agent i is indifferent between X and Y (or $X \sim_i Y$) if $i \in (X \cap Y)$ or $i \notin (X \cup Y)$.

3. Agent i weakly prefers Y over X (or $X \precsim_i Y$), if $X \prec_i Y$ or $X \sim_i Y$.

Without loss of generality, we assume that $\mu \leq n$. It is straightforward that, if a stable allocation X is such as $|X| < \mu$, then, by allocating the remainder of vaccine doses available to the agents chosen in any way in $N \backslash X$, the new induced allocation remains stable. In addition, the agents have a positive attitude towards vaccines by assumption. Therefore, we only consider efficient allocations i.e., $N(\mu) = \{S \subseteq N : |S| = \mu\}$.

3.1 Characterization of Stable Vaccine Allocations in Selfish Societies: Minimum Number of Vaccine Doses

We will now tackle the existence problem for stable allocations with a given number of vaccine doses. We already know from Example 2 that a stable allocation may not exist in a society where citizens are selfish. Our goal is to determine conditions under which a stable allocation always exists in such societies. In particular, we determine the minimum number of vaccine doses that guarantees the existence of a stable allocation. This is equivalent to finding a threshold number μ^* such that for any $\mu \geq \mu^*$, we have a non-empty core for a corresponding vaccine allocation game $G = \langle E, \mu, (\precsim_i) \rangle$. Such a minimal integer is justified given that the scarcity of a resource increases with its cost. Therefore, social planners can approve of a vaccine during a pandemic when the suppliers guarantee the production of the minimum supply of doses μ^*. Notice that the latter decision might depend on several characteristics of the society, including the preferences and responsibilities of the planners towards their constituents and special interest groups. In Example 1, we can show that $\mu^* = 1$. More generally, we will prove the existence of such a number μ^* in any given political economy $E = (N, \mathcal{I})$.

Let $\mathbb{N}^*$ be the set of non-zero integers. For any political economy $E = (N, \mathcal{I})$, let

$$\mathcal{H}_E = \{k \in \mathbb{N}^* : \forall t \geq k, \exists T \in N(t), \forall S \in \mathcal{I}, \text{if } |S| \leq t, \text{ then, } S \cap T \neq \varnothing\},$$

where $N(t) = \{S \subseteq N : |S| = t\}$. Obviously, $\mathcal{H}_E \neq \varnothing$ since $|N| = n \in \mathcal{H}_E$. We define the stability index as follows.

Definition 3. The stability index of a political economy $E = (N, \mathcal{I})$, denoted $\mu^*(E)$, is defined as $\mu^*(E) := \min(\mathcal{H}_E)$.

We shall prove that the stability index is essential to the determination of the stability of vaccine allocations in a selfish community. Before deriving the stability index, we first prove the following result.

Theorem 1. *Let $G = \langle N, \mathcal{I}, \mu, (\precsim_i) \rangle$ be a vaccine allocation game, and let $X \in N(\mu)$ be an allocation. Assume that each agent i's preferences $\precsim_i$ over $N(\mu)$ are selfish. Then :*

$X \in C(G)$ if and only if for any $S \in \mathcal{I}$, if $|S| \leq \mu$ then, $X \cap S \neq \varnothing$.

Proof. We proceed by double implications.

$\Rightarrow$) Let $X \in C(G)$ and let us show that for any $S \in \mathcal{I}$, if $|S| \leq \mu$ then, $X \cap S \neq \varnothing$. For this purpose, consider such a coalition S, and assume that $|S| \leq \mu$ and $X \cap S \neq \varnothing$. Let T be a subset of $\mu - |S|$ elements of $N \backslash S$, and consider the allocation $Y = S \cup T$. It is obvious that $S \subseteq P(X, Y)$; since S is an influential coalition, it follows that Y challenges X via S, which contradicts the assumption that X is a stable allocation. We have shown that for any $S \in \mathcal{I}$, if $|S| \leq \mu$, then $X \cap S \neq \varnothing$.

$\Leftarrow$) Assume that for any $S \in \mathcal{I}$, if $|S| \leq \mu$, then $X \cap S \neq \varnothing$. We need to show that X cannot be challenged in the allocation game G. Assume that X is challenged by another allocation $Y \in N(\mu)$. It follows that $P(X, Y) \in \mathcal{I}$. Naturally, we have two cases, either $|P(X, Y)| > \mu$ or $|P(X, Y)| \leq \mu$. If $|P(X, Y)| > \mu$, then according to the definition, it is not possible that X is challenged by Y, because $Y \in N(\mu)$ implies $|Y| = \mu$. Therefore, $|P(X, Y)| \leq \mu$, and by assumption, $X \cap P(X, Y) \neq \varnothing$, which is impossible. In fact, let $i \in X \cap P(X, Y)$. Then, agent i prefers X to Y, and Y to X, i.e, $X \prec_i Y \prec_i X$ or $X \prec_i X$ (since the preference $\prec_i$ is transitive), which is a contradiction. Hence, allocation $X \in C(G)$. □

Theorem 1 states that an allocation is in the core if it contains at least one member of each influential coalition whose size does not exceed μ. Overall, this result conveys a key message: any allocation of vaccine to μ agents in the community cannot be challenged so long as memberships in influential coalitions are costly. What we mean by cost is the capacity to gather a large number of agents in order to form an influential coalition. In fact, it is implied from the statements in Theorem 1, that, if the influence structure of the society is such that only coalitions which consist of more than μ agents can impact the allocation process of vaccines (i.e., $S \in \mathcal{I}$ if and only if $|S| > \mu$), then any allocation in $N(\mu)$ is stable: $C(G) = N(\mu)$. This situation happens not because of agents' unwillingness to challenge allocations, but simply because successful challenges require a significant degree of adhesion from members in the society. We have stable allocations because of the limited number of resources that prevents bargaining. This is similar to the concept of "abstention due to alienation;" see Zipp (1985), Plane & Gershtenson (2004), and Adams et al. (2006).

Definition 4. A vaccine allocation game $\langle N, \mathcal{I}, \mu, (\precsim_i)\rangle$ has a *stable horizon* if for any integer $\overline{\mu} \geq \mu$, the game $\langle N, \mathcal{I}, \overline{\mu}, (\precsim_i)\rangle$ has a non-empty core.

The main result of this section follows.

Theorem 2. *Let $E = (N, \mathcal{I})$ be a political economy. For any integer μ and selfish preference profile $(\precsim_i)$, the vaccine allocation game $G = \langle E, \mu, (\precsim_i)\rangle$ has a stable horizon if and only if $\mu \geq \mu^*(E)$.*

Proof. $\Rightarrow$) Assume that $G = \langle E, \mu, (\precsim_i)\rangle$ has a stable horizon, we must show that $\mu \geq \mu^*(E)$. Since $\mu^*(E)$ is the smallest element of $\mathcal{H}_E$, it is sufficient to show that $\mu \in \mathcal{H}_E$. For this purpose, let $t \geq \mu$, by assumption, G has a stable horizon thus, the core of the vaccine allocation game $G^t = \langle E, t, (\precsim_i)\rangle$ is non-empty. Let $X \in C(G^t) \subseteq N(t)$, Theorem 1 ensures that: for any $S \in \mathcal{I}$, if $|S| \leq t$ then, $S \cap X \neq \varnothing$. It follows that $\mu \in \mathcal{H}_E$ and therefore $\mu \geq \mu^*(E)$.

$\Leftarrow$) Conversely, assume that $\mu \geq \mu^*(E)$ and show that $G = \langle E, \mu, (\precsim_i)\rangle$ has a stable horizon. Let $\overline{\mu} \geq \mu$, we have $\overline{\mu} \geq \mu^*(E)$ and by definition of $\mathcal{H}_E$, there is $T \in N(\overline{\mu})$ such that for any $S \in \mathcal{I}$, if $|S| \leq \overline{\mu}$, then, $S \cap T \neq \varnothing$. According to Theorem 1, T is a stable allocation for the vaccine allocation game $G^{\overline{\mu}} = \langle E, \overline{\mu}, (\precsim_i)\rangle$, i.e., $T \in C(G^{\overline{\mu}})$. The latter completes the proof. $\square$

When there is no ambiguity, for any political economy $E = (N, \mathcal{I})$, and any selfish preference profile $(\precsim_i)$, we denote by $G^* = \langle E, \mu^*, (\precsim_i)\rangle$, with $\mu^* = \mu^*(E)$, the vaccine allocation game with a stable horizon that requires the smallest number of vaccine doses. In Proposition 1, we provide the stability index for the class of symmetrically weighted political economies. Precisely, we show that the stability index for a q-weighted symmetric political economy $E = (N, \mathcal{I}(q))$ is given by $\mu^*(E) = n-q+1$ if $q \leq \frac{n}{2}$, and by $\mu^*(E) = 1$ if $q > \frac{n}{2}$. A corollary of this finding is that when the quota exceeds half of the total number of agents (simple majority rule), then all the vaccine allocations are stable for any number of doses.

Proposition 1. *Let $E = (N, \mathcal{I}(q))$ be a symmetric weighted political economy with quota $q < n$. Then, the stability index of E is given by:* $\begin{cases} \mu^*(E) = n - q + 1 \text{ if } q \leq \frac{n}{2} \\ \mu^*(E) = 1 \text{ if } q > \frac{n}{2} \end{cases}$.

Proof. We differentiate two cases.

- *Case 1*: $q \leq \frac{n}{2}$. If $\mu^* \neq n - q + 1$, then we have either $\mu^* < n - q + 1$ or $\mu^* > n - q + 1$. Assume that $\mu^* < n - q + 1$ and let $t = n - q \geq \mu^*$. By assumption, there exists $T \in N(t)$ such that for any $S \in \mathcal{I}(q)$, if $|S| \leq t$, then $S \cap T \neq \varnothing$. For such a coalition T, we have $|N \backslash T| = q$. Moreover, $q \leq \frac{n}{2}$ is equivalent to $q \leq n - q = t$. Given that $\mathcal{I}(q)$ is a weighted influence structure with quota q, it follows that $S = N \backslash T$ is an influential coalition such that $|S| \leq t$ and $S \cap T = \varnothing$. This is in contradiction with the initial assumption that $\mu^* < n - q + 1$. Thus, we can conclude that $\mu^* \geq n - q + 1$. To show that $\mu^* \leq n - q + 1$, we consider the coalition $T = \{i \in N : i \leq n - q + 1\}$. It holds that $|T| = n - q + 1$ and $|N \backslash T| = q - 1$. Moreover, any influential coalition S contains at least q agents, and consequently, S cannot be contained in $N \backslash T$. Thus $S \cap T \neq \varnothing$, which proves that $\mu^* \leq n - q + 1$. We can conclude that $\mu^* = n - q + 1$.

- *Case 2*: $q > \frac{n}{2}$. To show that $\mu^* = 1$, it is sufficient to show that for any $t \geq 1$, there is $T \in N(t)$ such that for any $S \in \mathcal{I}$, if $|S| \leq t$, then $S \cap T \neq \varnothing$. Indeed, if $t \leq \frac{n}{2}$, then there is no coalition $S \in \mathcal{I}(q)$ such that $|S| \leq t$. If $t > \frac{n}{2}$, just take $T = \{i \in N : i \leq t\}$. Since $|S| = t > \frac{n}{2}$, it follows that $|N \backslash T| \leq \frac{n}{2} < q$ and therefore, no influential coalition can be included in $N \backslash T$, i.e., any influential coalition intersects T. This is especially true for $S \in \mathcal{I}(q)$ such that $|S| \leq t$.

□

3.2 Stable Vaccine Allocations and Individual Influence in a Political Economy

In this section, we ask how, in circumstances of scarce resources, individual influence (or voting power) in a political economy affects priority in access to vaccines. For simplicity, given that preferences are fixed, we will represent the core of the vaccine allocation game $G = \langle N, \mu, \mathcal{I}, (\precsim_i) \rangle$ by $C(G) = C(E, \mu)$, where $E = (N, \mathcal{I})$. Consider the following example, which illustrates the problem we aim to solve in this section.

Example 4. The allocation problem studied in Example 4 is similar to that in Example 1, with the exception that agents now have selfish preferences. We recall that $N = \{1, 2, 3, 4, 5\}$ and $\mathcal{I} = \{S \subseteq N : \{2, 4\} \subseteq S$ or $\{3, 5\} \subseteq S$ or $|S \cap \{1, 2, 3\}| \geq 2\}$.

• If $\mu = 1$ then, any allocation is stable and $C(E, 1) = \{\{i\} : i \in N\}$. Indeed, we need at least two individuals from the same firm to form an influential coalition to challenge an allocation. Since there is only one resource unit, any individual can get it. In fact, nobody has an incentive to cooperate with another individual from the same firm to challenge an allocation since doing so can only increase the utility of one member of the coalition, leaving the other member's utility unchanged.

• If $\mu = 2$, we show that $X = \{2, 3\}$ is the unique stable allocation. Each of individuals 2 and 3 belong to two firms, making them more influential than individuals 1, 4, and 5. The priority given to them in terms of accessing the vaccine is therefore in line with the influence structure of the society.

• If $\mu = 3$, we show that $C(E, 3) = \{\{1, 2, 3\}, \{2, 3, 4\}, \{2, 3, 5\}\}$.

• If $\mu = 4$, $C(E, 4) = \{\{1, 2, 3, 4\}, \{2, 3, 4, 5\}, \{1, 2, 3, 5\}\}$.

• If $\mu = 5$, the unique stable allocation is the entire community, i.e., $C(E, 5) = \{N\}$.

It follows from this example that stable allocations depend on preference structure. We note, for example that when the number of doses is at least equal to 2, both individuals 2 and 3 receive a vaccine dose; this is not the case in Example 1 where 3 is strongly opposed to vaccination.

We also note that when the number of vaccine doses is at least equal to 2, individuals 2 and 3 are given a vaccine dose in all stable allocations. However, there exist stable allocations where other individuals are not given any doses. This observation reveals that individuals 2 and 3 are more influential (or have higher voting power) than the other individuals (see, for example, Taylor et al. (1999), Tchantcho et al. (2008), Pongou & Tchantcho (2021) and Freixas & Pons (2021)). We will generalize this insight below. First, we need to introduce the notions of individual influence and partner-dependence in a political economy.

Definition 5. Let $E = (N, \mathcal{I})$ be a political economy, and let i, j and k be three agents.

1. Agent i is at least as influential as agent j in E when, for any $S \subseteq N \setminus \{i, j\}$, if $S \cup \{j\} \in \mathcal{I}$ then, $S \cup \{i\} \in \mathcal{I}$.
2. Agent k is partner-dependent of agent i when for any $S \in \mathcal{I}$, if $k \in S$ then, $i \in S$.

An individual i is at least as influential as another individual j if whenever j can turn a non-influential coalition S (not containing i and j) into an influential coalition by joining it, i can turn a non-influential coalition into an influential one by doing the same. An individual k is partner-dependent of another individual i if, whenever k belongs to an influential coalition S, i also belongs to S. The notion of partner-dependence reflects a certain correlation in the membership of influential coalitions. The dependent agent cannot belong to an influential coalition without their partners. Note that if k is partner-dependent of agent i, then i is at least as influential as agent k, but the converse is not true,

To illustrate these notions, consider the influence structure in Example 1. Agent 3 is at least as influential as agents 1 and 5; likewise, agent 2 is at least as influential as agents 1 and 4. Note that the converse is not true, as agents 1 and 5 are not as influential as agent 3 and agents 1 and 4 are not as influential as agent 2. Also note that, agent 4 is partner-dependent of agent 2 and, agent 5 is partner-dependent of agent 3.

For any agent $i \in N$, denote by $C^i(G) = \{X \in C(E, \mu) : i \in X\}$ the set of core allocations in the game $G = \langle E, \mu \rangle$ that contain i. The following result shows that the core is consistent with the structure of individual influence among agents in a game.

Proposition 2. *Let $G = \langle N, \mu, \mathcal{I} \rangle$ be a vaccine allocation game and $X \in N(\mu)$ be an allocation. Let $i, j \in N$ be such that i is at least as influential as j. The following statements hold.*

1. *For any $X \in C(G)$, if $j \in X$ and $i \notin X$ then, $(X \setminus \{j\}) \cup \{i\} \in C(G)$.*

2. $|C^j(G)| \leq |C^i(G)|$. *In particular, if for any $X \in C(G)$, we have $i \notin X$, then for all $X \in C(G)$, we also have $j \notin X$.*

Proof. 1. Assume that i is at least as influential as j and consider an allocation X such that $j \in X$ and $i \notin X$. We need to show that $Z = (X \backslash \{j\}) \cup \{i\}$ is a stable allocation. It is sufficient to show that no allocation challenges Z. Let Y be an allocation. Since X is stable, Y does not challenge X, that is the set $P(X,Y) := N \backslash X$ is not an influential coalition. By definition, $P(Z,Y) := N \backslash ((X \backslash \{j\}) \cup \{i\}) = N \backslash (X \cup \{i\}) \cup \{j\}$. Let $S = N \backslash (X \cup \{i\})$. It is straightforward that $S \subseteq N \backslash \{i,j\}$ and $S \cup \{i\} = N \backslash X = P(X,Y)$ is not an influential coalition. Since i is at least as influential as j, $S \cup \{j\} = N \backslash (X \cup \{i\}) \cup \{j\} = P(Z,Y)$ cannot be an influential coalition. We can conclude that there is no allocation Y that can challenge Z, and so Z is a stable allocation.

2. According to the first point, the correspondence f which maps any element X of $C^j(G)$ to the element $f(X)$ of $C^i(G)$, with $f(X) = (X \backslash \{j\}) \cup \{i\}$, is well defined. Moreover, f is an injective correspondence, thus $|C^j(G)| \leq |C^i(G)|$. The particular situation for which no stable allocation contains i corresponds to $|C^i(G)| = 0$. In this case, we also have $|C^j(G)| = 0$, which implies that no stable allocation contains j. □

The first statement in Proposition 2 stipulates that the core is consistent with the influence relation among agents in any vaccine allocation game. That is, if there exists a stable allocation that contains an agent j, then there also exists a stable allocation that contains any agent i that is at least as influential as j. The second statement in Proposition 2 deduces naturally from the first. In fact, the second statement says that, given two agents i and j, if agent i is at least as influential as agent j then, the core allocations which contain agent i are at least as numerous as those which contain agent j. In other words, the likelihood that an agent gets priority in access to a vaccine is higher if that agent is more influential.

Proposition 3 examines the case where agents have some partner-dependence relationship in a political economy and yields a result similar to that of Proposition 2.

Proposition 3. *Let $E = (N, \mathcal{I})$ be a political economy and $X \in N(\mu^*)$ be an allocation. Let i and $k \in N$ be two agents, with agent k being a partner-dependent of agent i. If X belongs to the core of the vaccine allocation game $G^* = \langle E, \mu^*, (\precsim_i) \rangle$ and $i \in X$, then $k \notin X$.*

Proof. By assumption, μ^* is the smallest strictly positive integer such that for any $\mu \geq \mu^*$, we have $C(E, \mu) \neq \varnothing$. By contradiction, consider $X \in C(G^*)$ such that $\{i, k\} \subseteq X$. It is clear that $\mu \geq 2$. Let $\mu = \mu^* - 1 \geq 1$ and $Y = X \setminus \{k\}$. For any $S \in \mathcal{I}$, we have by assumption $S \cap X \neq \varnothing$. For such a coalition S, if $i \in S$, then $i \in S \cap (X \setminus \{k\})$, that is, $S \cap Y \neq \varnothing$. Otherwise, if $i \notin S$, then $k \notin S$; but $S \cap X \neq \varnothing$, it follows that $S \cap (X \setminus \{k\}) \supseteq S \cap (X \setminus \{i, k\}) \neq \varnothing$, that is, $S \cap Y \neq \varnothing$. For $\mu = \mu^* - 1 < \mu^*$, we have $Y \in C(E, \mu)$ and by assumption, for any $\mu \geq \mu^*$, we have $C(E, \mu) \neq \varnothing$. We can deduce that for any $\mu \geq \mu^* - 1$, we have $C(E, \mu) \neq \varnothing$, which contradicts the assumption that μ^* is minimal. $\square$

In this section, we have assumed that agents only care about themselves in a vaccine allocation game. This consideration abstracts any information about the potential connections or links that may exist between agents. This assumption may in fact be unrealistic, as individuals that belong to the same group, whether defined by race, profession, or household, may have other-regarding preferences towards members of their own group for several reasons. One reason this may be the case is that members of the same group are more likely to physically interact with each other than with those outside of the group. Therefore, if a member of an individual's group is vaccinated, it decreases that individual's vulnerability to infection, even if they are not personally vaccinated. While this is also true for those outside of an individual's group, the individual benefits more from the vaccination of group members, with whom they frequently interact, than they do from vaccinating outsiders, with whom they have little to no interactions. Therefore, if an individual has to choose between two vaccine allocations X and Y to which they don't belong, that individual will choose the allocation that gives more vaccine doses to individuals closer to them. Moreover, two individuals may have preferences that are closer if they belong to the same group than if they belong to different groups. In the next section, we extend the analysis of stable allocations in a framework where agents may have some degree of direct or indirect connections in the community. In order to simulate these connections, we add a spatial dimension to a vaccine allocation game using concepts from Euclidean geometry. This spatial dimension captures the relationship that individuals may have with each other.

4 Stable Vaccine Allocations in Unselfish Societies

In the previous section we assumed that given an allocation X, an agent i is either satisfied ($i \in X$) or not ($i \notin X$). However, in practice, it might happen that even if agent i does not receive the vaccine, they prefer that agent j who is "closer" to them receives it than another agent k who is "further" from them. This is the case, for example, when vaccines have positive externalities in the sense that an unvaccinated individual is less vulnerable to infection when of their neighbors are vaccinated.

In order to capture agents' consideration of "closeness" in their preferences, we add a spatial dimension to the definition of a vaccine allocation game. We assume that each agent $i \in N$ is associated with a unique (ideal) point q^i in the metric space $\mathbb{R}^m$ of dimension $m \geq 1$. We denote by $\{q^i\}_{i\in N}$ or $\{q^i\}$ a profile of agents' ideal points. The metric space is endowed with a distance d. For simplicity, we assume that for all distinct $i, j \in N$, $q^i \neq q^j$. Therefore, agents in N are equivalent to their ideal points in $\{q^i\}_{i\in N}$, i.e., for any allocation $S \in N(\mu)$, with $|S| = \mu$, we can write $S = \{S^1, ..., S^\mu\}$, a subset of $\{q^i\}_{i\in N}$. A **spatial political economy**, which we denote E^d, is defined as the triple $E^d = (N, \mathcal{I}, \{q^i\})$. Given a spatial political economy E^d, the pair $V^d = (E^d, \mu)$ defines a **spatial vaccine allocation problem**. We denote by $\widetilde{\mathbb{V}}$ the domain of spatial vaccine allocation problems. In an unselfish society, agents have *unselfish* spatial preferences defined as follows.

Definition 6. Let $V^d = (E^d, \mu)$ be a spatial vaccine allocation problem, and X and Y be two allocations in $N(\mu)$. Agent i's preferences over $N(\mu)$, which we denote $\precsim_i^d$, are *unselfish* if the following hold.

1. Agent i prefers Y over X, denoted as $X \prec_i^d Y$, if there exists an element $j \in Y$ such that for all $k \in X$, $d(q^i, q^j) < d\left(q^i, q^k\right)$[10];

2. Agent i is indifferent between X and Y, denoted as $X \sim_i^d Y$, if neither $X \prec_i^d Y$ nor $Y \prec_i^d X$.

3. Agent i weakly prefers Y over X, denoted as $X \precsim_i^d Y$, if $X \prec_i^d Y$ or $X \sim_i^d Y$.

We define a **spatial vaccine allocation game**, which we denote G^d, as an array $G^d = \langle N, \mathcal{I}, \{q^i\}, \mu, (\precsim_i^d)\rangle$ or simply $G^d = \langle V^d, (\precsim_i^d)\rangle$. As in the previous sections, since

[10] We can also write this inequality as $d(i, Y) < d(i, X)$, where $d(i, Z)$ is the distance of agent i to an allocation Z, with $d(i, Z) := \min\{d\left(q^i, q^j\right) : j \in Z\}$.

preferences are fixed, we can represent the set of stable allocations of the spatial vaccine allocation game G^d simply by $C(G^d) \equiv C(E^d, \mu)$. In the following example, we determine the set of stable vaccine allocations in a simple spatial political economy.

Example 5. Consider a small community which consists of four juniors and three seniors. The community is affected by a deadly respiratory virus which creates a pandemic. Relative to juniors, seniors are more likely to get very sick from the disease and are more likely to need hospitalization, intensive care, or a ventilator to help them breathe. Additionally, relative to juniors, seniors have a higher probability of being killed by the disease if they catch it. Other factors, such as having certain underlying medical conditions also increase the likelihood of dying or becoming very ill if infected by the virus. In the community, most of these underlying medical conditions are concentrated among seniors.[11] With intensive investment in science and medical technology, a vaccine is discovered and it is shown to be highly effective against the adverse effects caused by the virus. However, not everyone can get a shot at the same time. The structure of the community is such that potential influential coalitions fall into one of the following categories: (1) a group of agents that includes at least two individuals in which there is at least one senior; or (2) a group of agents that includes at least four juniors. Such an influential coalition can challenge any allocation of vaccines that does not satisfy some specific interests. Let $J = \{1, 2, 3, 4\}$ denote the group of juniors and $S = \{5, 6, 7\}$ denote the group of seniors, so that $N = J \cup S$. We assume that the number of vaccine doses μ is less than 7, the cardinality of N.

Consider the two different spatial configurations given in Figure 3. Each of the two configurations reflects a certain proximity which induces an unselfish community.

The situation can be represented in both configurations by a vaccine allocation game whose underlying political economy $(N, \mathcal{I}(4))$ is such that $\mathcal{I}$ corresponds to the weighted influence structure, defined by $\mathcal{I}(4) = [4; 1, 1, 1, 1, 3, 3, 3]$. Notice that the weight of each senior is three times that of a junior. Such a difference might reflect a certain priority given to seniors due to the risk severity of the virus among seniors. Within a population with a fixed cardinality $n = N$, the number of influential coalitions increases with the proportion of seniors. In other words, in an aging population, there is more tension for access to the vaccine. For instance, in African countries where the population

[11] A typical real-world example of such a scenario is COVID-19.

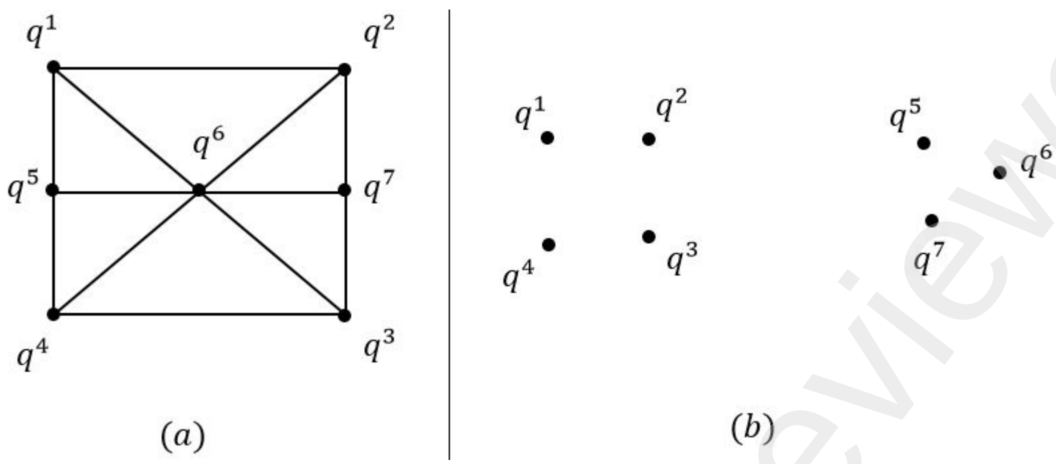

Figure 3: Representation of agents' relationships in an unselfish community

is younger, pressures for access to the vaccine against COVID-19 are less high than in Western countries with an aging population. Social planners can also consider separately or simultaneously other factors in the allocation of a limited number of vaccines. These factors include occupational exposure to disease and the epidemiological parameters such as transmission rate, effective reproductive numbers, and immunity threshold.

In this model, even if an agent i is in none of the two allocations X and Y, they can still support a challenge of X for Y, simply because agent i's ideal might be closer to Y than to X. For instance, when $\mu = 1$, the configuration of relationships given in Figure 2-(a) induces an empty set of stable vaccine allocations. If the allocation $\{1\}$ is proposed, then the influential coalition $\{2, 6\}$ prefers the allocation $\{6\}$; if $\{6\}$ is proposed, then the influential coalition $\{1, 5\}$ prefers $\{5\}$; if $\{5\}$ is proposed, then the influential coalition $\{2, 6\}$ prefers $\{6\}$, etcetera. Similarly, we can show that for $\mu = 2$, there is no stable vaccine allocations.

$\mu = 3$ is the lowest value of μ for which the core is not empty for the configuration in Figure 3-(a). Specifically, we show that $X = \{5, 6, 7\}$ is the only stable allocation in Figure 3-(a). Indeed, the only influential coalition that would have challenged X is $S = \{1, 2, 3, 4\}$, but there is no allocation that is collectively better than X for members of S. However, in the second configuration (Figure 3-(b)), several alternative allocations to X are available to members of S such that there is no stable vaccine allocations. This example shows that the social structure of the community affects the existence of a stable allocation of vaccines.

4.1 Characterization of Stable Allocations in Unselfish Societies

In what follows, we provide a characterization of the core for any vaccine allocation game $V^d \in \widetilde{\mathbb{V}}$. To do this, we need to introduce additional concepts and notations. Let $\mathcal{I}^-(\mu) = \{S \in \mathcal{I} : |S| \leq \mu\}$ denote the set of minimal influential coalitions, and $\mathcal{I}^+(\mu) = \mathcal{I} \backslash \mathcal{I}^-(\mu) = \{S \in \mathcal{I} : |S| > \mu\}$, the complementary set of $\mathcal{I}^-(\mu)$ in the influence structure $\mathcal{I}$. When there is no ambiguity, we use the notations $\mathcal{I}^-$ and $\mathcal{I}^+$ for $\mathcal{I}^-(\mu)$ and $\mathcal{I}^+(\nu)$, respectively. The set $\mathcal{I}^+$ consists of majority influential coalitions. Giving a majority influential coalition $A \in \mathcal{I}^+$, and an allocation $B \in N(\mu)$, we introduce the set $\Lambda_{A,B}$ defined as follows: $\Lambda_{A,B} = A \cup \{j \in N : \exists i \in A / d(q^i, q^j) \leq \alpha_B^i\}$, where $\alpha_B^i = \min_{k \in B} \{d(q^i, q^k)\}$. Lastly, we consider $\overline{\mathcal{I}} = \mathcal{I}^- \cup \{\Lambda_{A,B} : A \in \mathcal{I}^+, B \in N(\mu)\}$, $\overline{\mathcal{I}}^- = \{S \in \overline{\mathcal{I}} : |S| \leq \mu\}$ and $\overline{\mathcal{I}}^+ = \{S \in \overline{\mathcal{I}} : |S| > \mu\}$.

To illustrate the set $\Lambda_{A,B}$, let $N = \{1, ..., 7\}$ and the ideal points of Figure 3-(a) in Example 5. Consider the symmetric weighted influence structure $\mathcal{I}(4) = [4; 1, 1, 1, 1, 1, 1, 1]$. For $\mu = 1$, we have $\mathcal{I}(4)^- = \varnothing$, because any influential coalition contains more than one agent. It follows that $\mathcal{I}(4)^+ = \mathcal{I}(4)$. Furthermore, $N(\mu) = \{\{k\} : k \in N\}$.

- Consider for instance the winning coalition $A = \{1, 2, 3, 4\} \in \mathcal{I}(4)$ and the coalition $B = \{6\}$. We have $\alpha_B^i = d(q^i, q^6)$, and $N \backslash A = \{5, 6, 7\}$. By definition, $\Lambda_{A,B} = \{1, 2, 3, 4\} \cup \{j \in N : \exists i \in A / d(q^i, q^j) = d(q^i, q^6)\}$. For $j = 5$, we have $d(q^1, q^5) < d(q^1, q^6)$, and $1 \in A$; similarly, for $j = 6$, we have $d(q^1, q^6) = d(q^1, q^6)$, and $1 \in A$; and for $j = 7$, we have $d(q^2, q^7) < d(q^2, q^6)$, and $2 \in A$. Therefore, $\Lambda_{A,B} = N$.

- If $A = \{1, 4, 5, 6\} \in \mathcal{I}(4)$ and $B = \{6\}$ then, for any $j \in N \backslash A = \{2, 3, 7\}$ and for any $i \in A$, we have $d(q^i, q^6) < d(q^i, q^j)$, i.e., $j \notin \Lambda_{A,B}$. We deduce that $\Lambda_{A,B} = A$.

- Now, consider any influential coalition $A \in \mathcal{I}(4)$, and the coalition $B = \{k\}$, where $k \neq 6$. It is straightforward to show that there is $i \in A$ such that $d(q^i, q^6) \leq d(q^i, q^k) = \alpha_B^i$, i.e., 6 belongs to any $\Lambda_{A,B}$. It follows that $\{6\}$ is a stable vaccine allocation when $\mu = 1$. In fact, we show that $\{6\}$ is the only stable vaccine allocation.

Theorem 3 provides a complete characterisation of the set of stable vaccine allocations in a spatial model.

Theorem 3. *For all* $V^d = (E^d, \mu) \in \widetilde{\mathbb{V}}$, $C(V^d) = \left\{X \in N(\mu) : \forall S \in \overline{\mathcal{I}}, \{q^i\}_{i\in S} \cap X \neq \varnothing\right\}$.

Proof. $\subseteq$) Let $X = \{X^1, .., X^\mu\} \in N(\mu)$, we need to show that: if $X \in C\left(V^d\right)$, then for any $S \in \overline{\mathcal{I}}$ we have $\{q^i\}_{i\in S} \cap X \neq \varnothing$. By contraposition, it is sufficient to show that, if there is $S \in \overline{\mathcal{I}}$ such that $\{q^i\}_{i\in S} \cap X = \varnothing$, then $X \notin C\left(V^d\right)$. For these purposes, consider a coalition $S \in \overline{\mathcal{I}} = \overline{\mathcal{I}}^- \cup \left\{\Lambda_{A,B} : A \in \overline{\mathcal{I}}^+, B \subseteq N, |B| = \mu\right\}$. If $S \in \overline{\mathcal{I}}^-$ then, let $Y = \{Y^1, .., Y^\mu\} \in N(\mu)$ such that $\{q^i\}_{i\in S} \subseteq Y$. It is always possible to choose such a Y because by definition, $S \in \overline{\mathcal{I}}^-$ means that $|S| \leq \mu$. Since $\{q^i\}_{i\in S} \cap X = \varnothing$ and $\{q^i\}_{i\in S} \subseteq Y$, it is clear that any individual $i \in S$ prefers $q^i \in Y$ to any $X^k \in X$. It follows that X is challenged by Y because $S \in \overline{\mathcal{I}}$, therefore $X \notin C\left(V^d\right)$.

If $S \notin \overline{\mathcal{I}}^-$ then, $S \in \left\{\Lambda_{A,B} : A \in \overline{\mathcal{I}}^+, B \subseteq N, |B| = \mu\right\}$, that is there are $A \in \overline{\mathcal{I}}^+, B \subseteq N$ such that $|B| = \mu$ and $S = \Lambda_{A,B} = \{j \in N : \exists i \in A, d\left(q^i, q^j\right) \leq \alpha^i_B\}$, where for any $i \in A$, $\alpha^i_B = \min_{k\in B} \left\{d\left(q^i, q^k\right)\right\}$. Let $Y = \{Y^1, .., Y^\mu\} \in N(\mu)$ such that the components of Y are the points of $\{q^i\}_{i\in B}$. It should be noted that it is always possible to choose such a Y because by definition $|B| = \mu$. For any $i \in A$, let us choose $l^i \in B$ such that $\alpha^i_B = \min_{k\in B} \left\{d\left(q^i, q^k\right)\right\} = d\left(q^i, q^{l^i}\right)$. The existence of l^i is certain because, B is a nonempty and finite set. It should be noted that if $i \in B$ then, $l^i = i$ and $\alpha^i_B = 0$. For any $k \in \{1, .., \mu\}$, there is $j^k \in N$ such that $X^k = q^{j^k}$, where $j^k \notin S$ because $\{q^i\}_{i\in S} \cap X = \varnothing$. We know that $j^k \notin S$ implies $j^k \notin \{j \in N : \exists i \in A, d\left(q^i, q^j\right) \leq \alpha^i_B\}$, meaning, for any $i \in A, d\left(q^i, q^{j^k}\right) > \alpha^i_B \geq d\left(q^i, q^{l^i}\right)$. It follows that any individual $i \in A$ prefers $q^{l^i} \in Y$ to any $X^k \in X$, that is, X is challenged by Y via $A \in \overline{\mathcal{I}}^+ \subseteq \overline{\mathcal{I}}$, therefore $X \notin C\left(V^d\right)$.

$\supseteq$) Let $X = \{X^1, .., X^\mu\} \in N(\mu)$ such that for any $S \in \overline{\mathcal{I}}$ we have $\{q^i\}_{i\in S} \cap X \neq \varnothing$, we need to show that $X \in C\left(V^d\right)$, or equivalently, we must show that X is not challenged. Consider $Y = \{Y^1, .., Y^\mu\} \in N(\mu)$ and a winning coalition $T \in \overline{\mathcal{I}}$: Can Y dominate X via T? By assumption, for any $S \in \overline{\mathcal{I}} = \overline{\mathcal{I}}^- \cup \left\{\Lambda_{A,B} : A \in \overline{\mathcal{I}}^+, B \subseteq N, |B| = \mu\right\}$, we have $\{q^i\}_{i\in S} \cap X \neq \varnothing$. Thus, if $T \in \overline{\mathcal{I}}^-$ then $\{q^i\}_{i\in T} \cap X \neq \varnothing$. Let $j \in T$ such that there is $t \in \{1, .., \mu\}$, $q^j = X^t$, there is no $k \in \{1, .., \mu\}$ such that individual i prefers Y^k to X^t because $d\left(q^i, X^t\right) = d\left(q^i, q^i\right) = 0$. In this case, Y can't challenge X via T.

Assume that $T \notin \overline{\mathcal{I}}^-$ then, $T \in \overline{\mathcal{I}}^+$: let $B = \left\{k : q^k \in Y\right\}$, by definition $S = \Lambda_{T,B} \in \overline{\mathcal{I}}$ and by assumption, $\{q^i\}_{i\in S} \cap X \neq \varnothing$. Remember that $S = \Lambda_{T,B} =$

$\{j \in N : \exists i \in T, d(q^i, q^j) \leq \alpha_B^i\}$, where for any $i \in T$, $\alpha_B^i = \min_{k \in B} \{d(q^i, q^k)\}$. Consider $j \in N$ such that: (1) there is $t \in \{1, .., \mu\}$, $q^j = X^t$, and (2) there is $i \in T$, $d(q^i, q^j) \leq \alpha_B^i = \min_{k \in B} \{d(q^i, q^k)\}$. There is no k such that individual i prefers Y^k to X^t because: for any k, we have $d(q^i, X^t) = d(q^i, q^j) \leq \alpha_B^i \leq d(q^i, q^k)$. And then, Y can't challenge X via T. $\square$

In Section 3, we determine the minimum number of vaccine doses that guarantees the existence of a stable allocation in a selfish community. Theorem 3 provides a complete characterization of the core of a vaccine allocation game in a spatial model with unselfish preferences. The core may be empty as described in Example 3 in Section 2.4. We want to provide an investigation similar to the one in Section 3 on the effects of vaccine supply on the non-emptiness of the core in an unselfish society. We need additional notation. For a given spatial political economy $E^d = (N, \mathcal{I}, \{q^i\})$, let $\mathcal{H}_{E^d}$ be the set of non-zero integers k such that for any integer $t \geq k$ there is $S \in N(t)$ such that for any influential coalition $A \in \mathcal{I}$: if $|A| \leq t$, then $S \cap A \neq \varnothing$, otherwise, for any $B \in N(t)$, $S \cap \Lambda_{A,B} \neq \varnothing$.

Definition 7. The spatial stability index of a spatial political economy E^d, denoted $\mu^*(E^d)$, is defined as follows : $\mu^*(E^d) := min(\mathcal{H}_{E^d})$.

The notion of a vaccine allocation game with a stable horizon remains the same as in Definition 4 in Section 3.1. We have the following result.

Corollary 1. *Let $E^d = (N, \mathcal{I}, \{q^i\})$ be a spatial political economy. For any integer μ and unselfish preference profile $(\precsim_i^d)$, the spatial vaccine allocation game $G^d = \langle E^d, \mu, (\precsim_i^d)\rangle$ has a stable horizon if and only if $\mu \geq \mu^*(E^d)$.*

Proof. The proof of this corollary follows directly from Theorem 3. $\square$

For several reasons, including vaccine attributes, misinformation on public attitudes towards vaccination (Kreps et al., 2021; Brewer et al., 2017), online and offline misinformation surrounding the safety and effectiveness of vaccines (Loomba et al., 2021; Fridman et al., 2021), and culture (Schmelz & Bowles, 2021), the social planner may receive incomplete information about agents' ideal points. Theorem 4 shows that the set of stable vaccine allocations is robust to *small* perturbations of agents' preferences. In other words, if the difference between agents' perceived ideal points from the planner's viewpoint and agents' *true* ideal points is small (or does not exceed a certain threshold), then the allocation of vaccine will not jeopardize the planner's popularity.

Theorem 4. *Apart from a negligible set of configurations $\{q^i\}$, given $V_1^d = \langle N, \mu, \mathcal{I}, \{q^i\}\rangle \in \widetilde{\mathbb{V}}$, there exists a positive real number γ such that for all $V_2^d = \langle N, \mu, \mathcal{I}, \{p^i\}\rangle \in \widetilde{\mathbb{V}}$, if for any agent $i \in N$, $d(p^i, q^i) \leq \gamma$, then $C(V_1^d) = C(V_2^d)$.*

Proof. For purpose of clarity, we provide a detailed proof of Theorem 4 for two-dimensional games. One can naturally extend the arguments of the proof to any dimension. We proceed in three steps.

Step 1: Let us specify the set of configurations of the ideal points to be discarded. These are configurations such that three ideal points are aligned and configurations are such that one ideal point is located equidistant from two other distinct ideal points. It must be shown that these configurations are negligible in the sense of measurement. For this purpose, let us assume that the ideal points are drawn at random on a domain D of non-zero finite measure. Randomly drawing three distinct positions such that one is located at an equal distance from two others is equivalent to randomly drawing three distinct points X, Y and Z such that Z belongs to the perpendicular bisector of the segment $[X, Y]$. This probabilistic event can be translated into two events: (A_1) randomly choose two distinct positions X, Y; and (A_2) randomly draw Z on the perpendicular bisector of the segment $[X, Y]$. It is clear that $P(A_1) > 0$, the conditional probability rule results in: $P(A_1 \cap A_2) = P(A_1) \times P(A_2/A_1)$. Moreover A_2/A_1 is the probability of drawing Z on the perpendicular bisector of the segment $[X, Y]$ knowing X and Y, which is zero. To denote configurations which are such that no ideal point is on the perpendicular bisector of the segment formed by two other ideal points, we use the expression λ-almost any configuration.

Step 2: By assumption, for λ-almost any configuration, we have: $d\left(q^i, q^j\right) \neq d\left(q^i, q^k\right)$ i.e., $d\left(q^i, q^j\right) < d\left(q^i, q^k\right)$ or $d\left(q^i, q^k\right) < d\left(q^i, q^j\right)$. Let us show that there is a strictly positive real number ρ such that: given three distinct individuals i, j and k, such that $p^i \in B\left(q^i, \rho\right)$, $p^j \in B\left(q^j, \rho\right)$ and $p^k \in B\left(q^k, \rho\right)$, we have $d\left(q^i, q^j\right) < d\left(q^i, q^k\right)$ if and only if $d\left(p^i, p^j\right) < d\left(p^i, p^k\right)$. Assume that $d\left(q^i, q^j\right) < d\left(q^i, q^k\right)$, then for any $i \in N$, let i_1 be the agents of $N \backslash \{i\}$ whose ideal point is closer to that of i and i_2 be the agents of $N \backslash \{i, i_1\}$ whose ideal point is closer to that of i. By assumption, an ideal point is not located at the same distance from two others, therefore, $0 < d\left(q^i, q^{i_1}\right) < d\left(q^i, q^{i_2}\right)$. Let

$\rho_i = \frac{d\left(q^i, q^{i_2}\right) - d\left(q^i, q^{i_1}\right)}{4}$ and $\rho = \min\{\rho_i : i \in N\}$.

Given three distinct individuals i, j and k, such that $d\left(q^i, q^j\right) < d\left(q^i, q^k\right)$, if $p^i \in B\left(q^i, \rho\right)$, $p^j \in B\left(q^j, \rho\right)$ and $p^k \in B\left(q^k, \rho\right)$, then the most distant points p^i and p^j that can be obtained are as such: $d\left(p^i, p^j\right) = d\left(q^i, q^j\right) + 2\rho$. Likewise, the closest points p^i and p^k that can be obtained are as such: $d\left(p^i, p^k\right) = d\left(q^i, q^k\right) - 2\rho$. It follows that $d\left(p^i, p^k\right) - d\left(p^i, p^j\right) \geq d\left(q^i, q^k\right) - d\left(q^i, q^j\right) - 4\rho$. By construction, we have $\rho \leq \frac{d\left(q^i, q^k\right) - d\left(q^i, q^j\right)}{4}$ thus, $-4\rho \geq d\left(q^i, q^j\right) - d\left(q^i, q^k\right)$. We finally get $d\left(p^i, p^k\right) - d\left(p^i, p^j\right) \geq d\left(q^i, q^k\right) - d\left(q^i, q^j\right) > 0$, i.e., $d\left(p^i, p^j\right) \leq d\left(p^i, p^k\right)$.

In the same way, we can show that if $d\left(q^i, q^k\right) < d\left(q^i, q^j\right)$, then $d\left(p^i, p^k\right) < d\left(p^i, p^j\right)$.

Step 3: Now let us show that for a given vaccine allocation game $V_2^d = \left(N, \overline{\mathcal{I}}, \{p^i\}_i, \mu\right)$, if for any $i \in N$, $d(p^i, q^i) \leq \rho$, then $C\left(V_1^d\right) = C\left(V_2^d\right)$. It is sufficient to show that for any $S \in \overline{\mathcal{I}}$, for all $X = \{X^1, .., X^\mu\}$, $Y = \{Y^1, .., Y^\mu\}$, $\overline{X} = \left\{\overline{X}^1, .., \overline{X}^\mu\right\}$ and $\overline{Y} = \left\{\overline{Y}^1, .., \overline{Y}^\mu\right\}$ such that for any $k \in \{1, ..., \mu\}$, $\overline{X}^k \in B\left(X^k, \rho\right)$ and $\overline{Y}^k \in B\left(Y^k, \rho\right)$, we have: X is challenged by Y via S for V_1^d if and only if $\overline{X}$ is challenged by $\overline{Y}$ via S for game V_2^d. By definition, X is challenged by Y if and only if for any $i \in S$, $\min_{Z \in Y}\{d\left(q^i, Z\right)\} < \min_{Z \in X}\{d\left(q^i, Z\right)\}$. From Step 2, for all three distinct individuals i, j and k, we have $d\left(q^i, q^j\right) < d\left(q^i, q^k\right)$ if and only if $d\left(p^i, p^j\right) < d\left(p^i, p^k\right)$. It follows that, for any $i \in S$, $\min_{Z \in Y}\{d\left(q^i, Z\right)\} < \min_{Z \in X}\{d\left(q^i, Z\right)\}$ if and only if $\min_{Z \in \overline{Y}}\{d\left(p^i, Z\right)\} < \min_{Z \in \overline{X}}\{d\left(p^i, Z\right)\}$, i.e., X is challenged by Y via S for V_1^d if and only if $\overline{X}$ is challenged by $\overline{Y}$ via S for game V_2^d. The combination of the three steps allows the proof to be concluded with $\gamma = \rho$. $\square$

4.2 An Application to Minority Inclusion

In the spatial model, it is essential to know the coordinates of the ideal points in the considered space. However, in some circumstances, the social planner may not completely infer agents' ideal points. It is also necessary to account for the cost of the investigations to obtain the information on the agents. For this reason, we must consider alternative approaches to capturing the distance between agents. Networks provide a way of addressing this concern. Networks model connections (direct or indirect) that may exist between agents. We can use these relationships to evaluate how close an agent is to others. Factors such as gender, occupation, political ideology, beliefs, or ethnicity induce relationships between agents. In the specific case discussed in this application, agents are the nodes of a weighted undirected network structure. Each vertex between two agents is associated with a weight corresponding to the distance between the two agents connected by this vertex. If there is no vertex between two agents, then they are considered to be infinitely distant. In a network, a connected component is a maximum subset of nodes. Any two nodes in a component are connected by a path, i.e., a series of vertices between nodes in the considered subset.

To illustrate, consider an undirected weighted network with n nodes numbered from 1 to n. The nodes are the agents involved in a spatial vaccine allocation problem $V^A = (N, \mu, \mathcal{I}, A)$ such that $\mathcal{I}$ is the weighted influence structure defined by $\mathcal{I} \equiv \mathcal{I}(\lceil \frac{n}{2} \rceil) = [\lceil \frac{n}{2} \rceil ; 1, ..., 1]$, and A is a network structure that describes connections between agents. The quota, $q = \lceil \frac{n}{2} \rceil$, is the smallest integer which strictly exceeds the real number $\frac{n}{2}$; that is a simple majority influence structure. Note that the network A replaces the agents' ideal points profile in a spatial vaccine allocation problem.

To describe the network A, we assume that the set of agents N is partitioned into two subsets denoted N_1 and N_2. If N_1 contains n_1 agents and N_2 contains n_2 agents, then $n_1 + n_2 = n$. Each of the two subsets N_1 and N_2 corresponds to a connected component. Figure 4 displays a specific network A, involving 11 agents such that $N_1 = \{1, ..., 8\}$ and $N_2 = \{9, 10, 11\}$. For practical reasons, we assume that each vertex has a measure of 1, and the absence of a link is assimilated to an infinite distance. Thus, the preferences of the agents are such that, agents prefer themselves first, then the members of their network depending on the length of the connection path, and finally the other agents. Assuming that the total number n of agents is odd, it follows that one of the two groups is an influential coalition (majority) and the other is not (minority). Without loss of generality, assume that N_1 is an influential coalition and as a consequence, N_2 is not. For a spatial vaccine allocation problem $V^A = (N, \mu, \mathcal{I}, A)$, how does the number of vaccine doses μ affect minority inclusion in core allocations of these doses?

It is straightforward that, if $\mu \geq q$, then any vaccine allocation is stable. Assume that $\mu < q$. For the configuration considered in Figure 4, if there is only one vaccine dose, then $\{1\}$ is the only stable allocation. Indeed, if the single dose is given to agent 7 for instance or to an agent chosen in N_2, then, the members of the influential coalition $S = \{1, ..., 6\}$ will propose the allocation $\{1\}$ to challenge the proposed allocation. With the same reasoning, we show that if $\mu = 2$, then, the stable allocations are in the form

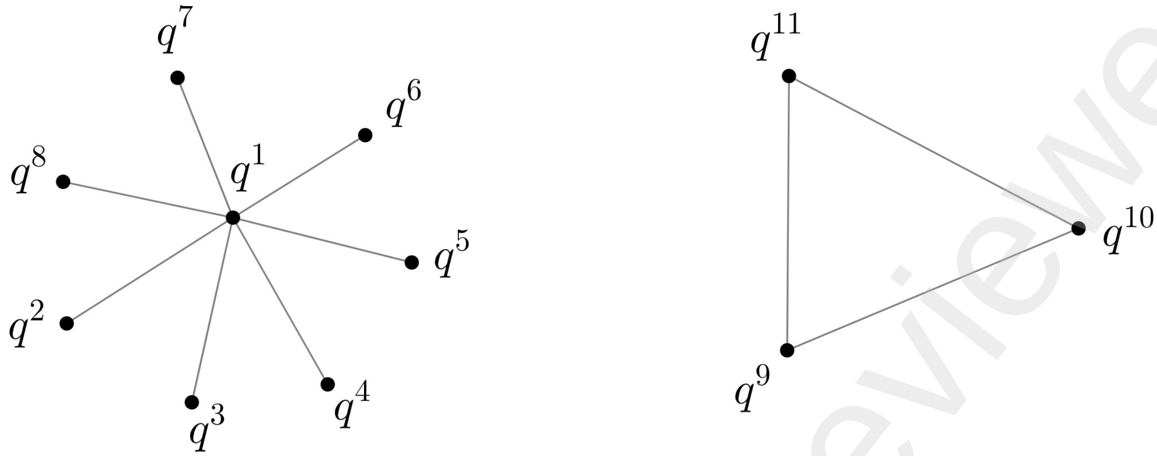

Figure 4: Illustration of an unselfish community on the network structure A

$\{1, k\}$ where $k \in \{2, ..., 8\}$. Remark that these allocations are not minority-inclusive as they do not reserve any vaccine dose for a minority individual. Similarly, if $\mu = 3$, no vaccine dose will go to a minority individual in any stable allocation. Stable allocations that allocate vaccine doses to minority individuals exist only for μ at least equal to 4.

In fact, it holds in this example that the analysis of the existence of a stable allocation in $V^A = (N, \mu, \mathcal{I}, A)$ is reduced to the vaccine allocation problem $\overline{V^A} = \left(N_2, \mu, \overline{\mathcal{I}}(\lceil \frac{n}{2} \rceil), A\right)$, where $\overline{\mathcal{I}}(\lceil \frac{n}{2} \rceil)$ is the weighted influence structure defined on N_2, with the same quota $q = \lceil \frac{n}{2} \rceil$ as the influence structure $\mathcal{I}(\lceil \frac{n}{2} \rceil)$. In this case, there is a better chance of obtaining a stable allocation since everything happens as if the members of N_1 were excluded but without changing the quota. In other words, the stability index of $\overline{V^A}$ cannot exceed that of V^A, that is $\mu^* \left(\overline{V^A}\right) \leq \mu^* \left(V^A\right)$. For $\mu = \mu^* \left(\overline{V^A}\right)$, there is at least one stable allocation but no member of the influential coalition N_1 belongs to a stable allocation. For $\mu > \mu^* \left(\overline{V^A}\right)$, in a stable allocation, up to $\mu - \mu^* \left(\overline{V^A}\right)$ doses of vaccine can be released for members of N_1. This shows that inclusive policies are possible only if the number of vaccine doses exceeds the threshold $\mu = \mu^* \left(\overline{V^A}\right)$.

Finally, if the members of N_1 are pairwise connected, then the set N_1 can be compared to a selfish community. In that case, $\mu^* \left(\overline{V^A}\right) = n_1 - q + 1$.

The application that we illustrate in Section 4 discusses the non-emptiness of the core and inclusion in a polarized and unselfish society as a function of the supply of vaccine doses μ. Though the findings in this application offer a glimpse of the issue, a formal analysis of the problem is an avenue for future research. Section 4.2 also opens a discussion on whether a political leader can implement a stable and inclusive vaccine allocation in a society organized around distinct groups where individuals have in-group preferences. The application shows that their ability to enforce such popular and inclusive allocations in a majoritarian democratic institution may depend on the supply of vaccines and, hence, on society's technological capability to produce vaccine doses.

5 Concluding Remarks

In this study, we developed a theory that addresses the problem of the existence of stable vaccine allocations in a political economy. Stable allocations are defined as feasible allocations that a social planner can implement without losing their popularity. We distinguished between contexts where vaccination has positive externalities on the unvaccinated and contexts where it does not. When vaccination does not have any positive externalities in that it only benefits the vaccinated individual, agents have selfish preferences; and when it has positive externalities, agents have unselfish preferences.

We have seen that when the supply of vaccine doses is sufficiently low relative to the number of individuals eligible to receive a dose, a stable allocation may not exist. In such a situation, any policy implemented by the social planner will be unpopular. The absence of a stable vaccine allocation is caused by highly divergent preferences. This observation motivated us to investigate "structural" conditions under which a stable allocation policy always exists. The identification of structural conditions is useful because such conditions do not depend on preferences and can therefore inform policy design. Assuming that individuals have selfish preferences, we characterized the minimum number of vaccine doses that guarantees the existence of a stable policy regardless of the level of preference heterogeneity. This number only depends on the society's influence structure.

We also studied some properties of stable vaccine allocations under selfish preferences. In particular, we investigated the relationship between voting rights and priority in access to vaccine. We found that when individuals have unequal voting rights, stable allocations favor those with greater voting power. In other words, if there exists a stable allocation that gives an individual a vaccine dose, there must also exist a stable allocation that gives any individual that possesses more voting power a vaccine dose. A direct implication is that the likelihood of an individual having priority in access to vaccines increases with the amount of voting power the individual wields.

When agents have unselfish preferences and the society is such that proximity between individuals varies, we also characterized the minimum number of vaccine doses that guarantees the existence of a stable vaccine allocation policy. This number depends both on the society's influence structure and the patterns of proximity between individuals. Unselfish preferences can be explained by the fact that a person's vulnerability to infection decreases if their close neighbors are less vulnerable or are vaccinated. In an application, we explore one implication of such preferences for minority inclusion in the design of stable vaccine allocations. We found that while a stable vaccine allocation policy may exist, it may not be minority-inclusive, especially if the number of vaccine doses is very small. Also, in such situations, minority-inclusive policies may not be stable, creating a dilemma for the social planner. These findings open discussions into the circumstances under which a social planner can feasibly design a vaccine allocation policy that is both stable and inclusive in a fragmented society.

Finally, we would like to emphasize that while our analysis focuses on vaccine allocation, our results extend to the allocation of any scarce good in a political economy context.

References

Acosta, J. & Stracqualursi, V. (2021). Trump pollster says Covid-19, not voter fraud, to blame for reelection loss. *Forbes*. Accessed: 2021-06-29.

Adams, J., Dow, J., & Merrill, S. (2006). The political consequences of alienation-based and indifference-based voter abstention: Applications to presidential elections. *Political Behavior*, 28, 65–86.

Aidt, T. S. & Leon, G. (2016). The democratic window of opportunity: Evidence from riots in sub-saharan africa. *Journal of Conflict Resolution*, 60(4), 694–717.

Akbarpour, M., Budish, E. B., Dworczak, P., & Kominers, S. D. (2021). An economic framework for vaccine prioritization. *Available at SSRN 3846931*.

Anne Arundel County Department of Health (2021). *COVID Booster for Eligible Groups*.

Technical report, Anne Arundel County Department of Health, Anne Arundel County Government. Accessed: 2021-9-28.

Aumann, R. & Dreze, J. (1974). Cooperative games with coalition structures. *International Journal of Game Theory*, 3, 217–237.

Austen-Smith, D. (1996). Refinements of the heart. In *Collective decision-making: social choice and political economy* (pp. 221–236). Springer.

Baccini, L., Brodeur, A., & Weymouth, S. (2021). The COVID-19 pandemic and the 2020 US presidential election. *Journal of Population Economics*, 34(2), 739–767.

Balinski, M. & Laraki, R. (2007). A theory of measuring, electing and ranking. *Proceedings of the National Academy of Sciences*, 104, 8720–8725.

BALLOTPEDIA (2021). Gavin Newsom recall, Governor of California (2019-2021). *BALLOTPEDIA*. Accessed: 2021-9-14.

Banzhaf III, J. F. (1964). Weighted voting doesn't work: A mathematical analysis. *Rutgers L. Rev.*, 19, 317.

Bechtel, M. M. & Hainmueller, J. (2011). How lasting is voter gratitude? an analysis of the short-and long-term electoral returns to beneficial policy. *American Journal of Political Science*, 55(4), 852–868.

Black, D. (1948). The median voter theorem. *Journal of Political Economy*, 56(1), 23–34.

Bleier, B. S., Ramanathan Jr, M., & Lane, A. P. (2021). Covid-19 vaccines may not prevent nasal sars-cov-2 infection and asymptomatic transmission. *Otolaryngology–Head and Neck Surgery*, 164(2), 305–307.

Bol, D., Giani, M., Blais, A., & Loewen, P. J. (2021). The effect of COVID-19 lockdowns on political support: some good news for democracy? *European Journal of Political Research*, 60(2), 497–505.

Bollyky, T. J. & Bown, C. P. (2020). The tragedy of vaccine nationalism: only cooperation can end the pandemic. *Foreign Aff.*, 99, 96.

Bosi, S., Camacho, C., & Desmarchelier, D. (2021). Optimal lockdown in altruistic economies. *Journal of Mathematical Economics*, 93, 102488.

Bräuninger, T. (2007). Stability in spatial voting games with restricted preference maximizing. *Journal of Theoretical Politics*, 19, 173–191.

Brewer, N. T., Chapman, G. B., Rothman, A. J., Leask, J., & Kempe, A. (2017). Increasing vaccination: putting psychological science into action. *Psychological Science in the Public Interest*, 18(3), 149–207.

Brill, M., Laslier, J.-F., & Skowron, P. (2018). Multiwinner approval rules as apportionment methods. *Journal of Theoretical Politics*, 30(3), 358–382.

Castillo, J. C., Ahuja, A., Athey, S., Baker, A., Budish, E., Chipty, T., Glennerster, R., Kominers, S. D., Kremer, M., Larson, G., et al. (2021). Market design to accelerate COVID-19 vaccine supply. *Science*, 371(6534), 1107–1109.

Dahl, R. (1970). *After the Revolution*. New Haven: Yale University Press.

Davis, O., DeGroot, M., & Hinich, M. (1972). Social preference orderings and majority rule. *Econometrica*, 40, 147–157.

Dellis, A. (2021). Legislative informational lobbying. *Available at SSRN 3686620*.

Dellis, A. & Oak, M. (2019). Informational lobbying and pareto-improving agenda constraint. *The Journal of Law, Economics, and Organization*, 35(3), 579–618.

Diffo Lambo, L. & Moulen, J. (2000). What power is measured in a voting game? *Mathematics & Social Sciences*, (152).

Dimick, M., Rueda, D., & Stegmueller, D. (2018). Models of other-regarding preferences, inequality, and redistribution. *Annual Review of Political Science*, 21, 441–460.

Dufwenberg, M., Heidhues, P., Kirchsteiger, G., Riedel, F., & Sobel, J. (2011). Other-regarding preferences in general equilibrium. *The Review of Economic Studies*, 78(2), 613–639.

Duijzer, L. E., van Jaarsveld, W. L., Wallinga, J., & Dekker, R. (2018). Dose-optimal vaccine allocation over multiple populations. *Production and Operations Management*, 27(1), 143–159.

Dutta, B. & Vohra, R. (2017). Rational expectations and farsighted stability. *Theoretical Economics*, 12(3), 1191–1227.

Eaton, L. (2021). Covid-19: WHO warns against "vaccine nationalism" or face further virus mutations. *BMJ*, 372(292).

Eban, G. & Stephen, W. (1990). Predicting committee behavior in majority rule voting experiments. *RAND Journal of Economics*, 21, 293–313.

Emanuel, E. J., Persad, G., Kern, A., Buchanan, A., Fabre, C., Halliday, D., Heath, J., Herzog, L., Leland, R., Lemango, E. T., et al. (2020). An ethical framework for global vaccine allocation. *Science*, 369(6509), 1309–1312.

Fehr, E. & Schmidt, K. M. (1999). A theory of fairness, competition, and cooperation. *The Quarterly Journal of Economics*, 114(3), 817–868.

Ferejohn, J., McKelvey, R., & Packel, E. (1984). Limiting distributions for continuous state Markov models. *Social Choice and Welfare*, 1, 45–67.

Florek, J. (2012). A numerical method to determine a degressive proportional distribution of seats in the european parliament. *Mathematical Social Sciences*, 63, 121–129. Around the Cambridge Compromise: Apportionment in Theory and Practice.

Flores-Szwagrzak, K. (2015). Priority classes and weighted constrained equal awards rules for the claims problem. *Journal of Economic Theory*, 160, 36–55.

Freixas, J. & Pons, M. (2021). A critical analysis on the notion of power. *Annals of Operations Research*, (pp. 1–23).

Fridman, A., Gershon, R., & Gneezy, A. (2021). COVID-19 and vaccine hesitancy: A longitudinal study. *PloS One*, 16(4), e0250123.

Furlong, A. & Deutsh, J. (2021). A country-by-country guide to coronavirus vaccine booster plans. *POLITICO*. Accessed: 2021-9-28.

Gillies, D. (1959). Solutions to general non-zero-sum games. *In A.W. Tucker and R.D. Luce (eds.): Contributions to the Theory of Games IV. Princeton University Press*, (pp. 47–85).

Han, S., Cai, J., Yang, J., Zhang, J., Wu, Q., Zheng, W., Shi, H., Ajelli, M., Zhou, X.-H., & Yu, H. (2021). Time-varying optimization of covid-19 vaccine prioritization in the context of limited vaccination capacity. *Nature communications*, 12(1), 1–10.

Healy, A. & Malhotra, N. (2009). Myopic voters and natural disaster policy. *American Political Science Review*, (pp. 387–406).

Herrera, H., Ordoñez, G., Konradt, M., & Trebesch, C. (2020). Corona politics: The cost of mismanaging pandemics. *Covid Economics*, (50), 3–32.

Isbell, J. R. (1958). A class of simple games. *Duke Mathematical Journal*, 25(3), 423–439.

Johnston, R. (1983). Proportional representation and fair representation in the European Parliament. *Area*, 15, 347–353.

Jones, M. A., McCune, D., & Wilson, J. M. (2020). New quota-based apportionment methods: The allocation of delegates in the republican presidential primary. *Mathematical Social Sciences*, 108, 122–137.

Ju, B.-G., Miyagawa, E., & Sakai, T. (2007). Non-manipulable division rules in claim problems and generalizations. *Journal of Economic Theory*, 132(1), 1–26.

Kagel, J. H. & Roth, A. E. (2020). *The Handbook of Experimental Economics, Volume 2*. Princeton University Press.

Katz, I. T., Weintraub, R., Bekker, L.-G., & Brandt, A. M. (2021). From vaccine nationalism to vaccine equity—finding a path forward. *New England Journal of Medicine*, 384(14), 1281–1283.

Kirwin, E., Rafferty, E., Harback, K., Round, J., & McCabe, C. (2021). A net benefit approach for the optimal allocation of a covid-19 vaccine. *Pharmacoeconomics*, (pp. 1–15).

Kramer, G. (1977). A dynamical model of political equilibrium. *Journal of Economic Theory*, 16, 310–334.

Kreps, S., Dasgupta, N., Brownstein, J. S., Hswen, Y., & Kriner, D. L. (2021). Public attitudes toward COVID-19 vaccination: The role of vaccine attributes, incentives, and misinformation. *npj Vaccines*, 6(1), 1–7.

Kurz, S., Maaser, N., & Napel, S. (2017). On the democratic weights of nations. *Journal of Political Economy*, 125(5), 1599–1634.

Kurz, S., Mayer, A., & Napel, S. (2021). Influence in weighted committees. *European Economic Review*, 132, 103634.

Lambo, L. D. & Moulen, J. (2002). Ordinal equivalence of power notions in voting games. *Theory and Decision*, 53(4), 313–325.

Lambo, L. D., Tchantcho, B., & Moulen, J. (2012). Comparing influence theories in voting games under locally generated measures of dissatisfaction. *International Journal of Game Theory*, 41(3), 719–731.

Loomba, S., de Figueiredo, A., Piatek, S. J., de Graaf, K., & Larson, H. J. (2021). Measuring the impact of COVID-19 vaccine misinformation on vaccination intent in the UK and USA. *Nature human behaviour*, 5(3), 337–348.

M. Martin, Z. N. & Tovey, C. (2021). Dominance in Spatial Voting with Imprecise Ideals: A New Characterization of the Yolk. *Mathematical Social Sciences*.

Matrajt, L., Eaton, J., Leung, T., Dimitrov, D., Schiffer, J. T., Swan, D. A., & Janes, H. (2021). Optimizing vaccine allocation for COVID-19 vaccines shows the potential role of single-dose vaccination. *Nature communications*, 12(1), 1–18.

Matrajt, L., Halloran, M. E., & Longini Jr, I. M. (2013). Optimal vaccine allocation for the early mitigation of pandemic influenza. *PLoS Computational Biology*, 9(3), e1002964.

McEvoy, J. (2021). Trump Campaign Autopsy Links Loss To Covid Response, Trustworthiness—Not Voter Fraud. *Forbes*. Accessed: 2021-06-29.

McKelvey, R. (1986). Covering, dominance, and institution free properties of social choice. *American Journal of Political Science*, 30, 283–314.

Medlock, J. & Galvani, A. P. (2009). Optimizing influenza vaccine distribution. *Science*, 325(5948), 1705–1708.

Mueller, D., Tollison, R., & Willett, T. (1972). Representative democracy via random selection. *Public Choice*, 12, 57–68.

Mylius, S. D., Hagenaars, T. J., Lugnér, A. K., & Wallinga, J. (2008). Optimal allocation of pandemic influenza vaccine depends on age, risk and timing. *Vaccine*, 26(29-30), 3742–3749.

Nichol, A. A. & Mermin-Bunnell, K. M. (2021). The ethics of COVID-19 vaccine distribution. *Journal of Public Health Policy*, (pp. 1–4).

Oliphant, J. (2021). U.S. election year shaped by pandemic and Trump's defiance. *Reuters*. Accessed: 2021-06-29.

Owen, G. (1990). Stable outcomes in spatial voting games. *Mathematical Social Sciences*, 19, 269–279.

Pathak, P. A., Sönmez, T., Ünver, M. U., & Yenmez, M. B. (2021). Fair allocation of vaccines, ventilators and antiviral treatments: leaving no ethical value behind in health care rationing. In *Proceedings of the 22nd ACM Conference on Economics and Computation* (pp. 785–786).

Persad, G., Peek, M. E., & Emanuel, E. J. (2020). Fairly prioritizing groups for access to covid-19 vaccines. *JAMA*, 324(16), 1601–1602.

Plane, D. & Gershtenson, J. (2004). Candidates' ideological locations, abstention, and turnout in u.s. midterm senate elections. *Political Behavior*, 26, 69–93.

Plott, C. (1967). A notion of equilibrium and its possibility under majority rule. *American Economic Review*, 57, 787–806.

Pollard, A. J. & Bijker, E. M. (2021). A guide to vaccinology: from basic principles to new developments. *Nature Reviews Immunology*, 21(2), 83–100.

Pongou, R. & Tchantcho, B. (2021). Round-robin political tournaments: Abstention, truthful equilibria, and effective power. *Games and Economic Behavior*, 130, 331–351.

Price, D. (1958). *Scientific Estate*. Oxford University Press.

Rabin, M. (1993). Incorporating fairness into game theory and economics. *The American Economic Review*, (pp. 1281–1302).

Rao, I. J. & Brandeau, M. L. (2021). Optimal allocation of limited vaccine to control an infectious disease: Simple analytical conditions. *Mathematical Biosciences*, 337, 108621.

Rich, M. (2021). Japan's Prime Minister Will Step Aside After Just a Year in Office. *The New York Times*. Accessed: 2021-9-08.

Ruiz-Rufino, R. & Alonso, S. (2017). Democracy without choice: Citizens' perceptions of government autonomy during the eurozone crisis. *European Journal of Political Research*, 56(2), 320–345.

Schiavone, M., Gasperetti, A., Mitacchione, G., Viecca, M., & Forleo, G. B. (2021). Response to: Covid-19 re-infection. vaccinated individuals as a potential source of transmission. *European Journal of Clinical Investigation*, (pp. e13544–e13544).

Schmelz, K. & Bowles, S. (2021). Overcoming COVID-19 vaccination resistance when alternative policies affect the dynamics of conformism, social norms, and crowding out. *Proceedings of the National Academy of Sciences*, 118(25).

Schofield, N. (1995). Coalition politics: A formal model and empirical analysis. *Journal of Theoretical Politics*, 7, 245–281.

Serrano, R. (1995). A market to implement the core. *Journal of Economic Theory*, 67(1), 285–294.

Shapley, L. S. & Shubik, M. (1954). A method for evaluating the distribution of power in a committee system. *American Political Science Review*, 48(3), 787–792.

Shubik, M. & Wooders, M. (1983). Approximate cores of replica games and economies : Part II: Set-up costs and firm formation in coalition production economies. *Mathematical Social Sciences*, 6, 285–306.

Singanayagam, A., Hakki, S., Dunning, J., Madon, K. J., Crone, M., Koycheva, A., Derqui-Fernandez, N., Barnett, J. L., Whitfield, M. G., Varro, R., et al. (2021). Community transmission and viral load kinetics of SARS-CoV-2 Delta (B. 1.617. 2) variant in vaccinated and unvaccinated individuals. *Available at SSRN: https://ssrn.com/abstract=3918287*.

Taylor, A. D., Zwicker, W. S., & Zwicker, W. (1999). *Simple games: Desirability relations, trading, pseudoweightings*. Princeton University Press.

Tchantcho, B., Lambo, L. D., Pongou, R., & Engoulou, B. M. (2008). Voters' power in voting games with abstention: Influence relation and ordinal equivalence of power theories. *Games and Economic Behavior*, 64(1), 335–350.

Tovey, C. (1991). Some foundations for empirical study in spatial euclidean social choice models. In W. Barnett (Ed.), *Political Economy: Institutions, Competition, and Representation* chapter 7. Cambridge University Press.

Tovey, C. (2010). The instability of instability of centered distributions. *Mathematical Social Sciences*, 59, 53–73.

Westerink-Duijzer, L. E., Schlicher, L. P., & Musegaas, M. (2020). Core allocations for cooperation problems in vaccination. *Production and Operations Management*, 29(7), 1720–1737.

World Health Organization (2021). *WHO SAGE roadmap for prioritizing uses of COVID-19 vaccines in the context of limited supply: an approach to inform planning and subsequent recommendations based on epidemiological setting and vaccine supply scenarios, first issued 20 October 2020, latest update 16 July 2021*. Technical report, World Health Organization.

Wu, J. H., John, S. D., & Adashi, E. Y. (2020). Allocating vaccines in a pandemic: the ethical dimension. *The American Journal of Medicine*, 133(11), 1241.

Wuffle, A., Feld, S., Owen, G., & Grofman, B. (1989). Finagle's law and the Finagle point: A new solution concept for two-candidate competition in spatial voting games without a core. *American Journal of Political Science*, 33, 348–375.

Xue, Q. C. & Ouellette, L. L. (2020). Innovation policy and the market for vaccines. *Journal of Law and the Biosciences*, 7(1), lsaa026.

Yi, M. & Marathe, A. (2015). Fairness versus efficiency of vaccine allocation strategies. *Value in Health*, 18(2), 278–283.

Zipp, J. (1985). Perceived representativess and voting: An assessment of the impact of "choices" vs. "echoes". *American Political Science Review*, 79, 50–61.