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Nano Continuous Mappings via Nano $\mathcal{M}$ Open Sets

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Abstract

Nano $\mathcal{M}$ open sets aress a union of nano θ semi open sets and nano δ pre open sets. The properties of nano $\mathcal{M}$ open sets with their interior and closure operators are discussed in a previous paper. In this paper, nano $\mathcal{M}$-continuous and nano $\mathcal{M}$-irresolute functions are introduced in a nano topological spaces along with their continuous and irresolute mappings. Also, nano $\mathcal{M}$-open and nano $\mathcal{M}$-closed functions are introduced and compared with their near open and closed mappings in a nano topological spaces. Further, nano $\mathcal{M}$ homeomorphisms are also discussed in nano topological spaces. Also, we discuss nano e-Cts, nano e-Irr, nano eo and nano ec functions and nano $eHom$ in a nano topological space. Some of their properties are also well discussed.

Keywords: Nano $\mathcal{M}$-o set; Nano $\mathcal{M}$-c set; Nano $\mathcal{M}$-Cts; Nano $\mathcal{M}$-Irr; Nano $\mathcal{M}of$; Nano $\mathcal{M}cf$; Nano $\mathcal{M}Hom$

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1. Introduction and Preliminaries

Lellis Thivagar and Richard (2013) introduced the notion of Nano topology (briefly, $\mathfrak{N}\mathfrak{T}$) by using theory approximations and boundary region of a subset of an universe in terms of an equivalence relation on it and also defined Nano closed (briefly, $\mathfrak{N}c$) sets, Nano-interior (briefly, $\mathfrak{N}int$) and Nano-closure (briefly, $\mathfrak{N}cl$) in a nano topological spaces (briefly, $\mathfrak{N}ts$). Richard (2016) discussed some weak forms of $\mathfrak{N}o$ sets and $\mathfrak{N}\theta$ open (briefly, $\mathfrak{N}\theta o$) sets. Some generalizations of almost contra-super-continuity were made by Ekici (2007).

The notion of e-open sets in topological spaces was introduced by Ekici (2008c), who studied some of their properties. Also, a-open sets, A^*-sets and decompositions of continuity, super-continuity Ekici (2008b) and new forms of contra-continuity were studied by Ekici (2008a). The new sets, called e^*-open sets and $(D, S)^*$-sets, were introduced by Ekici (2009).

El-Maghrabi and Al-Juhani (2011) initroduced the notion of M-open sets in topological spaces, and they studied some of their properties. The class of sets, namely M-open sets, are playing more important roles in topological spaces because of their applications in various fields of Mathematics and other real fields. By these motivations, we present the concept of nano M-open sets (Padma et al. (2019)) and study their properties and applications in nano topological space. The purpose of this paper is to discuss nano $\mathcal{M}$-Cts, nano $\mathcal{M}$-Irr, nano $\mathcal{M}o$ and nano $\mathcal{M}c$ functions and nano $\mathcal{M}Hom$ by using the sets nano $\mathcal{M}$ (respectively, e) open sets.

The definitions and properties needed in this paper are shown in Bhuvaneswari et al. (2016), Lellis Thivagar and Richard (2013), Lellis Thivagar and Richard (2013), Padma et al. (2019), Pankajam and Kavitha (2017), Revathy and Gnanambal (2015), Richard (2016), and Sujatha and Angayarkanni (2019).

Throughout this paper, $(U, \tau_R(X))$ is a $\mathfrak{N}ts$ with respect to X where $X \subseteq U,\ R$ is an equivalence relation on U. Then, U/R denotes the family of equivalence classes of U by R. All other undefined notions are from Lashin and Medhat (2015), Lellis Thivagar and Richard (2013), and Pawlak (2016).

2. Nano $\mathcal{M}$ continuous functions

Definition 2.1.

A function $h\colon (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ is said to be Nano $\mathcal{M}$ (respectively, δ, δ-pre, δ-semi and e) continuous (briefly, $\mathfrak{N}\mathcal{M}$ (respectively, $\mathfrak{N}\delta$, $\mathfrak{N}\delta\mathcal{P}$, $\mathfrak{N}\delta\mathcal{S}$ and $\mathfrak{N}e$) Cts), if for each $\mathfrak{N}c$ set K of V_1, the set $h^{-1}(K)$ is $\mathfrak{N}\mathcal{M}c$ (respectively, $\mathfrak{N}\delta c$, $\mathfrak{N}\delta\mathcal{P}c$, $\mathfrak{N}\delta\mathcal{S}c$ and $\mathfrak{N}ec$) set of U_1.

Theorem 2.1.

Let $h\colon (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ be a mapping. Then,

(i) Every $\mathfrak{N}\delta$ Cts is $\mathfrak{N}$ Cts.

(ii) Every $\mathfrak{N}$ Cts is $\mathfrak{N}\delta\mathcal{P}$ Cts.
(iii) Every $\mathfrak{N}\delta$ Cts is $\mathfrak{N}\delta\mathcal{S}$ Cts.
(iv) Every $\mathfrak{N}\theta$ Cts is $\mathfrak{N}\delta$ Cts.
(v) Every $\mathfrak{N}\theta\mathcal{S}$ Cts is $\mathfrak{N}\mathcal{M}$ Cts.
(vi) Every $\mathfrak{N}\theta$ Cts is $\mathfrak{N}\theta\mathcal{S}$ Cts.
(vii) Every $\mathfrak{N}\theta$ Cts is $\mathfrak{N}$ Cts.
(viii) Every $\mathfrak{N}\delta\mathcal{P}$ Cts is $\mathfrak{N}\mathcal{M}$ Cts.
(ix) Every $\mathfrak{N}\delta\mathcal{P}$ Cts is $\mathfrak{N}e$ Cts.
(x) Every $\mathfrak{N}\mathcal{M}$ Cts is $\mathfrak{N}e$ Cts.
(xi) Every $\mathfrak{N}\delta\mathcal{S}$ Cts is $\mathfrak{N}e$ Cts.

Proof:

(i) Let $h\colon (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ be $\mathfrak{N}\delta$ Cts and L is a $\mathfrak{N}c$ set in V_1. Then, $h^{-1}(L)$ is $\mathfrak{N}\delta c$ in U_1. Since every $\mathfrak{N}\delta c$ set is $\mathfrak{N}c$, $h^{-1}(L)$ is $\mathfrak{N}c$ set in U_1. Therefore, h is $\mathfrak{N}$ Cts.

(ii) Let $h\colon (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ be $\mathfrak{N}$ Cts and L is a $\mathfrak{N}c$ set in V_1. Then, $h^{-1}(L)$ is $\mathfrak{N}c$ in U_1. Since every $\mathfrak{N}c$ set is $\mathfrak{N}\delta\mathcal{P}c$, $h^{-1}(L)$ is $\mathfrak{N}\delta\mathcal{P}c$ set in U_1. Therefore, h is $\mathfrak{N}\delta\mathcal{P}$ Cts.

(iii) Let $h\colon (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ be $\mathfrak{N}\delta$ Cts and L is a $\mathfrak{N}c$ set in V_1. Then, $h^{-1}(L)$ is $\mathfrak{N}\delta c$ in U_1. Since every $\mathfrak{N}\delta c$ set is $\mathfrak{N}\delta\mathcal{S}c$, $h^{-1}(L)$ is $\mathfrak{N}\delta\mathcal{S}c$ set in U_1. Therefore, h is $\mathfrak{N}\delta\mathcal{S}$ Cts.

(iv) Let $h\colon (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ be $\mathfrak{N}\theta$ Cts and L is a $\mathfrak{N}c$ set in V_1. Then, $h^{-1}(L)$ is $\mathfrak{N}\theta c$ in U_1. Since every $\mathfrak{N}\theta c$ set is $\mathfrak{N}\delta c$, $h^{-1}(L)$ is $\mathfrak{N}\delta c$ set in U_1. Therefore, h is $\mathfrak{N}\delta$ Cts.

(v) Let $h\colon (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ be $\mathfrak{N}\theta\mathcal{S}$ Cts and L is a $\mathfrak{N}c$ set in V_1. Then, $h^{-1}(L)$ is $\mathfrak{N}\theta\mathcal{S}c$ in U_1. Since every $\mathfrak{N}\theta\mathcal{S}c$ set is $\mathfrak{N}\mathcal{M}c$, $h^{-1}(L)$ is $\mathfrak{N}\mathcal{M}c$ set in U_1. Therefore, h is $\mathfrak{N}\mathcal{M}$ Cts.

(vi) Let $h\colon (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ be $\mathfrak{N}\theta$ Cts and L is a $\mathfrak{N}c$ set in V_1. Then, $h^{-1}(L)$ is $\mathfrak{N}\theta c$ in U_1. Since every $\mathfrak{N}\theta c$ set is $\mathfrak{N}\theta\mathcal{S}c$, $h^{-1}(L)$ is $\mathfrak{N}\theta\mathcal{S}c$ set in U_1. Therefore, h is $\mathfrak{N}\theta\mathcal{S}$ Cts.

(vii) Let $h\colon (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ be $\mathfrak{N}\theta$ Cts and L is a $\mathfrak{N}c$ set in V_1. Then, $h^{-1}(L)$ is $\mathfrak{N}\theta c$ in U_1. Since every $\mathfrak{N}\theta c$ set is $\mathfrak{N}c$, $h^{-1}(L)$ is $\mathfrak{N}c$ set in U_1. Therefore, h is $\mathfrak{N}$ Cts.

(viii) Let $h\colon (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ be $\mathfrak{N}\delta\mathcal{P}$ Cts and L is a $\mathfrak{N}c$ set in V_1. Then, $h^{-1}(L)$ is $\mathfrak{N}\delta\mathcal{P}c$ in U_1. Since every $\mathfrak{N}\delta\mathcal{P}c$ set is $\mathfrak{N}\mathcal{M}c$, $h^{-1}(L)$ is $\mathfrak{N}\mathcal{M}c$ set in U_1. Therefore, h is $\mathfrak{N}\mathcal{M}$ Cts.

(ix) Let $h\colon (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ be $\mathfrak{N}\delta\mathcal{P}$ Cts and L is a $\mathfrak{N}c$ set in V_1. Then, $h^{-1}(L)$ is $\mathfrak{N}\delta\mathcal{P}c$ in U_1. Since every $\mathfrak{N}\delta\mathcal{P}c$ set is $\mathfrak{N}ec$, $h^{-1}(L)$ is $\mathfrak{N}ec$ set in U_1. Therefore, h is $\mathfrak{N}e$ Cts.

(x) Let $h\colon (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ be $\mathfrak{N}\mathcal{M}$ Cts and L is a $\mathfrak{N}c$ set in V_1. Then, $h^{-1}(L)$ is $\mathfrak{N}\mathcal{M}c$ in U_1. Since every $\mathfrak{N}\mathcal{M}c$ set is $\mathfrak{N}ec$, $h^{-1}(L)$ is $\mathfrak{N}ec$ set in U_1. Therefore, h is $\mathfrak{N}e$ Cts.

(xi) Let $h\colon (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ be $\mathfrak{N}\delta\mathcal{S}$ Cts and L is a $\mathfrak{N}c$ set in V_1. Then, $h^{-1}(L)$ is $\mathfrak{N}\delta\mathcal{S}c$ in U_1. Since every $\mathfrak{N}\delta\mathcal{S}c$ set is $\mathfrak{N}ec$, $h^{-1}(L)$ is $\mathfrak{N}ec$ set in U_1. Therefore, h is $\mathfrak{N}e$ Cts. ■

The converse of Theorem 2.1 need not be true by the following examples.

Example 2.1.

Let $U_1 = \{L_a, L_b, L_c, L_d\}$ with $U_1/R = \{\{L_a, L_b\}, \{L_c, L_d\}\}$, $P = \{L_a, L_b\}$, $\tau_R(P) = \{U_1, \phi, \{L_a, L_b\}\}$. Define the identity map $h : U_1 \to U_1$ which is $\mathfrak{N}\ Cts$ but not $\mathfrak{N}\delta\ Cts$, and the set $h^{-1}(\{L_a, L_b\}) = \{L_a, L_b\}$ which is $\mathfrak{N}o$ but not $\mathfrak{N}\delta o$ in U_1.

Example 2.2.

Let $U_1 = V_1 = \{M_a, M_b, M_c, M_d, M_e\}$ with $U_1/R = \{\{M_c\}, \{M_a, M_b\}, \{M_d, M_e\}\}$, $P = \{M_a, M_c\}$, $\tau_R(P) = \{U_1, \phi, \{M_c\}, \{M_a, M_b\}, \{M_a, M_b, M_c\}\}$ and $V_1/R' = \{\{M_e\}, \{M_a, M_b\}, \{M_c, M_d\}\}$, $Q = \{M_c, M_e\}$, $\tau_{R'}(Q) = \{V_1, \phi, \{M_e\}, \{M_c, M_d\}, \{M_c, M_d, M_e\}\}$ Then, the mapping $h : (U_1, \tau_R(P)) \to (V_1, \tau_{R'}(Q))$ is defined by

(i) $h(M_a) = M_d,\ h(M_b) = M_e,\ h(M_c) = M_c,\ h(M_d) = M_a$ and $h(M_e) = M_b$ is $\mathfrak{N}\delta\mathcal{P}\ Cts$ but not $\mathfrak{N}\ Cts$, the set $\{M_e\}$ is $\mathfrak{N}o$ in V_1 but $h^{-1}(\{M_e\}) = \{M_b\}$ is not $\mathfrak{N}o$ in U_1.
(ii) $h(M_a) = M_c,\ h(M_b) = h(M_e) = M_d,\ h(M_c) = M_e$ and $h(M_d) = M_a$ is $\mathfrak{N}\delta\mathcal{S}\ Cts$ but not $\mathfrak{N}\delta\ Cts$, the set $\{M_c, M_d\}$ is $\mathfrak{N}o$ in V_1 but $h^{-1}(\{M_c, M_d\}) = \{M_a, M_b, M_e\}$ is not $\mathfrak{N}\delta o$ in U_1.
(iii) $h(M_a) = M_c,\ h(M_b) = M_d,\ h(M_c) = M_e,\ h(M_d) = M_a$ and $h(M_e) = M_b$ is $\mathfrak{N}\delta\ Cts$ but not $\mathfrak{N}\theta\ Cts$, the set $\{M_e\}$ is $\mathfrak{N}o$ in V_1 but $h^{-1}(\{M_e\}) = \{M_c\}$ is not $\mathfrak{N}\theta o$ in U_1.
(iv) $h(M_a) = M_e,\ h(M_b) = M_d,\ h(M_c) = M_c,\ h(M_d) = M_b$ and $h(M_e) = M_a$ is $\mathfrak{N}\mathcal{M}\ Cts$ but not $\mathfrak{N}\theta\mathcal{S}\ Cts$, the set $\{M_e\}$ is $\mathfrak{N}o$ in V_1 but $h^{-1}(\{M_e\}) = \{M_a\}$ is not $\mathfrak{N}\theta\mathcal{S}o$ in U_1.
(v) $h(M_a) = M_c,\ h(M_b) = M_d,\ h(M_c) = M_e,\ h(M_d) = M_a$ and $h(M_e) = M_b$ is $\mathfrak{N}\ Cts$ but not $\mathfrak{N}\theta\ Cts$, the set $\{M_e\}$ is $\mathfrak{N}o$ in V_1 but $h^{-1}(\{M_e\}) = \{M_c\}$ is not $\mathfrak{N}\theta o$ in U_1.

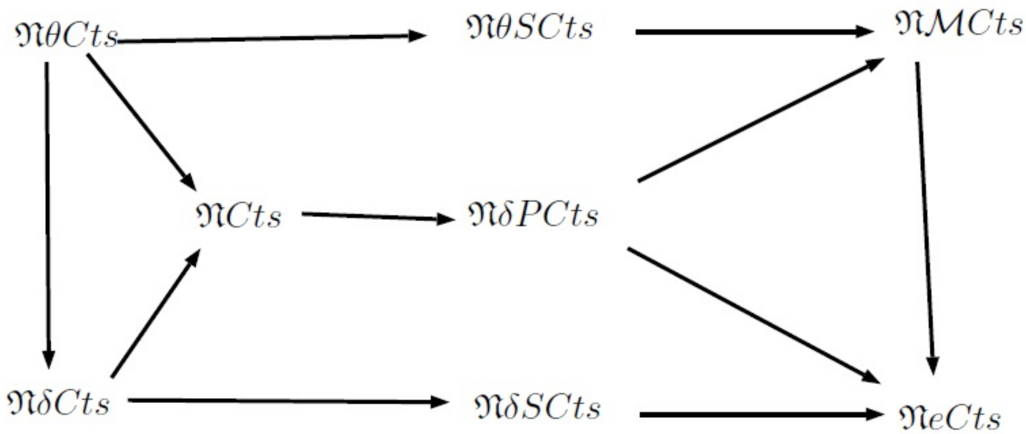
Example 2.3.

Let $U_1 = V_1 = W_1 = W_1' = \{M_a, M_b, M_c, M_d, M_e\}$ with $U_1/R = \{\{M_c\}, \{M_a, M_b\}, \{M_d, M_e\}\}$, $P = \{M_a, M_c\}$, $\tau_R(P) = \{U_1, \phi, \{M_c\}, \{M_a, M_b\}, \{M_a, M_b, M_c\}\}$; $V_1/R' = \{\{M_a\}, \{M_b\}, \{M_c, M_d, M_e\}\}$, $Q = \{M_c, M_d, M_e\}$, $\tau_{R'}(Q) = \{V_1, \phi, \{M_c, M_d, M_e\}\}$; $W_1/R'' = \{\{M_c\}, \{M_e\}, \{M_a, M_b, M_d\}\}$, $S = \{M_a, M_b, M_d\}$, $\tau_{R''}(S) = \{W_1, \phi, \{M_a, M_b, M_d\}\}$ and $W_1'/R''' = \{\{M_b\}, \{M_e\}, \{M_a, M_c, M_d\}\}$, $S' = \{M_a, M_c, M_d\}$ and $\tau_{R'''}(S') = \{W_1', \phi, \{M_a, M_c, M_d\}\}$. Then, the identity mapping

(i) $h : (U_1, \tau_R(P)) \to (V_1, \tau_{R'}(Q))$ is $\mathfrak{N}\theta\mathcal{S}\ Cts$ but not $\mathfrak{N}\theta\ Cts$, the set $\{M_c, M_d, M_e\}$ is $\mathfrak{N}o$ in V_1 but $h^{-1}(\{M_c, M_d, M_e\}) = \{M_c, M_d, M_e\}$ is not $\mathfrak{N}\theta o$ in U_1.
(ii) $h : (U_1, \tau_R(P)) \to (V_1, \tau_{R'}(Q))$ is $\mathfrak{N}\mathcal{M}\ Cts$ but not $\mathfrak{N}\delta\mathcal{P}\ Cts$, the set $\{M_c, M_d, M_e\}$ is $\mathfrak{N}o$ in V_1 but $h^{-1}(\{M_c, M_d, M_e\}) = \{M_c, M_d, M_e\}$ is not $\mathfrak{N}\delta\mathcal{P}o$ in U_1.
(iii) $h : (U_1, \tau_R(P)) \to (V_1, \tau_{R'}(Q))$ is $\mathfrak{N}e\ Cts$ but not $\mathfrak{N}\delta\mathcal{P}\ Cts$, the set $\{M_c, M_d, M_e\}$ is $\mathfrak{N}o$ in V_1 but $h^{-1}(\{M_c, M_d, M_e\}) = \{M_c, M_d, M_e\}$ is not $\mathfrak{N}\delta\mathcal{P}o$ in U_1.
(iv) $g : (U_1, \tau_R(P)) \to (W_1, \tau_{R''}(S))$ is $\mathfrak{N}e\ Cts$ but not $\mathfrak{N}\mathcal{M}\ Cts$, the set $\{M_a, M_b, M_d\}$ is $\mathfrak{N}o$ in W_1 but $g^{-1}(\{M_a, M_b, M_d\}) = \{M_a, M_b, M_d\}$ is not $\mathfrak{N}\mathcal{M}o$ in U_1.

(v) $h: (U_1, \tau_R(P)) \to (W_1^{'}, \tau_{R'''}(S^{'}))$ is $\mathfrak{N}e\ Cts$ but not $\mathfrak{N}\delta\mathcal{S}\ Cts$, the set $\{M_a, M_c, M_d\}$ is $\mathfrak{N}o$ in $W_1^{'}$ but $h^{-1}(\{M_a, M_c, M_d\}) = \{M_a, M_c, M_d\}$ is not $\mathfrak{N}\delta\mathcal{S}o$ in U_1.

From the above discussions, the following implications hold for any set in $\mathfrak{N}ts$.



$K \to L$ denotes K implies L, but not conversely

Theorem 2.2.

A function $h\colon (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ is $\mathfrak{N}\mathcal{M}\ Cts$ if and only if the inverse image of every $\mathfrak{N}o$ set in V_1 is $\mathfrak{N}\mathcal{M}o$ in U_1.

Proof:

Let h be $\mathfrak{N}\mathcal{M}\ Cts$ and O is $\mathfrak{N}o$ in V_1. That is, $V_1 - O$ is $\mathfrak{N}c$ in V_1. Since h is $\mathfrak{N}\mathcal{M}\ Cts$, $h^{-1}(V_1 - O)$ is $\mathfrak{N}\mathcal{M}c$ in U_1. That is, $U_1 - h^{-1}(O)$ is $\mathfrak{N}\mathcal{M}c$ in U_1. Therefore, $h^{-1}(O)$ is $\mathfrak{N}\mathcal{M}o$ in U_1.

Conversely, let the inverse image of every $\mathfrak{N}o$ set be $\mathfrak{N}\mathcal{M}o$ set. Let C be $\mathfrak{N}c$ in V_1. Then, $V_1 - C$ is $\mathfrak{N}o$ in V_1. Then, $h^{-1}(V_1 - C)$ is $\mathfrak{N}\mathcal{M}o$ in U_1. That is $U_1 - h^{-1}(C)$ is $\mathfrak{N}\mathcal{M}o$ in U_1. Therefore, $h^{-1}(C)$ is $\mathfrak{N}\mathcal{M}c$ in U_1. Thus, the inverse image of every $\mathfrak{N}c$ set in V_1 is $\mathfrak{N}\mathcal{M}c$ in U_1. That is, h is $\mathfrak{N}\mathcal{M}\ Cts$ on U_1.

The maps $\mathfrak{N}\delta\ Cts$, $\mathfrak{N}\delta\mathcal{P}\ Cts$, $\mathfrak{N}\delta\mathcal{S}\ Cts$ and $\mathfrak{N}e\ Cts$ satisfy the Theorem 2.2 for their respective open sets. ■

Theorem 2.3.

A function $h\colon (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ is $\mathfrak{N}\mathcal{M}\ Cts$ if and only if $h(\mathfrak{N}\mathcal{M}cl(K)) \subseteq \mathfrak{N}cl(h(K))$ for every subset K of U_1.

Proof:

Let h be $\mathfrak{N}\mathcal{M}\ Cts$ and $K \subseteq U_1$. Then, $h(K) \subseteq V_1$. Since h be $\mathfrak{N}\mathcal{M}\ Cts$ and $\mathfrak{N}cl(h(K))$ is $\mathfrak{N}c$ in V_1, $h^{-1}(\mathfrak{N}cl(h(K)))$ is $\mathfrak{N}\mathcal{M}c$ in U_1. Since $h(K) \subseteq \mathfrak{N}cl(h(K))$, $h^{-1}(h(K)) \subseteq h^{-1}(\mathfrak{N}cl(h(K)))$, then $K \subseteq h^{-1}(\mathfrak{N}cl(h(K)))$. $\mathfrak{N}\mathcal{M}cl(K) \subseteq \mathfrak{N}\mathcal{M}cl[h^{-1}(Nclh(K))] = h^{-1}(\mathfrak{N}cl(h(K)))$. Thus, $\mathfrak{N}\mathcal{M}cl(K) \subseteq h^{-1}(\mathfrak{N}cl(h(K)))$. Therefore, $h(\mathfrak{N}\mathcal{M}cl(K)) \subseteq \mathfrak{N}cl(h(K))$ for every subset K of

U_1.

Conversely, let $h(\mathfrak{N}\mathcal{M}cl(K)) \subseteq \mathfrak{N}cl(h(K))$ for every subset K of U_1. If C is $\mathfrak{N}c$ in V_1 and since $h^{-1}(C) \subseteq U_1$, $h(\mathfrak{N}\mathcal{M}cl(h^{-1}(C))) \subseteq \mathfrak{N}cl(h(h^{-1}(C))) = \mathfrak{N}cl(C) = C$. That is, $h(\mathfrak{N}\mathcal{M}cl(h^{-1}(C))) \subseteq C$. Thus, $\mathfrak{N}\mathcal{M}cl(h^{-1}(C)) \subseteq h^{-1}(C)$. But $h^{-1}(C) \subseteq \mathfrak{N}\mathcal{M}cl(h^{-1}(C))$. Hence, $\mathfrak{N}\mathcal{M}cl(h^{-1}(C)) = h^{-1}(C)$. Therefore, $h^{-1}(C)$ is $\mathfrak{N}\mathcal{M}c$ in U_1, for every $\mathfrak{N}c$ set C in V_1. Thus h is $\mathfrak{N}\mathcal{M}$ Cts. ■

Remark 2.1.

A function $h\colon (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ is $\mathfrak{N}\mathcal{M}$ Cts. Then, $h(\mathfrak{N}\mathcal{M}cl(K))$ is not necessarily equal to $\mathfrak{N}cl(h(K))$ where $K \subseteq U_1$. It is shown in the following examples.

Example 2.4.

In Example 2.3, $h : (U_1, \tau_R(P)) \to (V_1, \tau_{R'}(Q))$ is $\mathfrak{N}\mathcal{M}$ Cts. Let $A = \{M_a\} \subset U_1$. Then, $\mathfrak{N}\mathcal{M}cl(A) = h(\mathfrak{N}\mathcal{M}cl(\{M_a\})) = h(\{M_a\}) = \{M_a\}$. But $\mathfrak{N}clh(A) = \mathfrak{N}cl(\{M_a\}) = \{M_a, M_b\}$. Thus $h(\mathfrak{N}\mathcal{M}cl(A)) \neq \mathfrak{N}cl(h(A))$, even though h is $\mathfrak{N}\mathcal{M}$ cts. That is equality does not hold.

Theorem 2.4.

A function $h\colon (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ is $\mathfrak{N}\mathcal{M}$ Cts if and only if $\mathfrak{N}\mathcal{M}cl(h^{-1}(L_1)) \subseteq h^{-1}(\mathfrak{N}cl(L_1))$ for every subset L_1 of V_1.

Proof:

If h is $\mathfrak{N}\mathcal{M}$ Cts and $L_1 \subseteq V_1$. $\mathfrak{N}cl(L_1)$ is $\mathfrak{N}c$ in V_1, and hence, $h^{-1}(\mathfrak{N}cl(L_1))$ is $\mathfrak{N}\mathcal{M}c$ in U_1. Therefore, $\mathfrak{N}\mathcal{M}cl(h^{-1}(\mathfrak{N}\ cl(L_1))) = h^{-1}(\mathfrak{N}cl(L_1))$. Since $L_1 \subseteq \mathfrak{N}cl(L_1)$, $h^{-1}(L_1) \subset h^{-1}(\mathfrak{N}cl(L_1))$. Therefore, $\mathfrak{N}\mathcal{M}cl(h^{-1}(L_1)) \subset \mathfrak{N}\mathcal{M}cl(h^{-1}(\mathfrak{N}cl(L_1))) = h^{-1}(\mathfrak{N}cl(L_1))$. That is, $\mathfrak{N}\mathcal{M}cl(h^{-1}(L_1)) \subseteq h^{-1}(\mathfrak{N}cl(L_1))$.

Conversely, let $\mathfrak{N}\mathcal{M}cl(h^{-1}(L_1)) \subseteq h^{-1}(\mathfrak{N}cl(L_1))$ for every subset L_1 of V_1. If L_1 is $\mathfrak{N}c$ in V_1, then $\mathfrak{N}cl(L_1) = L_1$. By assumption, $\mathfrak{N}\mathcal{M}cl(h^{-1}(L_1)) \subseteq h^{-1}(\mathfrak{N}cl(L_1)) = h^{-1}(L_1)$. Thus, $\mathfrak{N}\mathcal{M}cl(h^{-1}(L_1)) \subseteq h^{-1}(L_1)$. But $h^{-1}(L_1) \subseteq \mathfrak{N}\mathcal{M}cl(h^{-1}(L_1))$. Therefore, $\mathfrak{N}\mathcal{M}cl(h^{-1}(L_1)) = h^{-1}(L_1)$. Hence, $h^{-1}(L_1)$ is $\mathfrak{N}\mathcal{M}c$ in U_1, for every $\mathfrak{N}c$ set L_1 in V_1. Therefore, h is $\mathfrak{N}\mathcal{M}$ Cts on U_1.

The maps $\mathfrak{N}\delta$ Cts, $\mathfrak{N}\delta\mathcal{P}$ Cts $\mathfrak{N}\delta\mathcal{S}$ Cts and $\mathfrak{N}e$ Cts satisfy the Theorems 2.3 and 2.4 for their respective closures. ■

Remark 2.2.

A function $h\colon (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ is $\mathfrak{N}\mathcal{M}$ Cts. Then, $\mathfrak{N}\mathcal{M}cl(h^{-1}(L))$ is not necessarily equal to $h^{-1}(\mathfrak{N}cl(L))$ where $L \subseteq V_1$. It is shown in the following examples.

Example 2.5.

In Example 2.3, $h : (U_1, \tau_R(P)) \to (V_1, \tau_{R'}(Q))$ is $\mathfrak{N}\mathcal{M}$ Cts. Let $B = \{M_a\} \subset V_1$.

Then, $\mathfrak{N}\mathcal{M}clh^{-1}(B) = \mathfrak{N}\mathcal{M}clh^{-1}(\{M_a\}) = \mathfrak{N}\mathcal{M}cl(\{M_a\}) = \{M_a\}$. But $h^{-1}(\mathfrak{N}cl(B)) = h^{-1}(\mathfrak{N}cl(\{M_a\})) = h^{-1}(\{M_a, M_b\}) = \{M_a, M_b\}$. Thus, $\mathfrak{N}\mathcal{M}cl(h^{-1}(B)) \neq h^{-1}(\mathfrak{N}cl(B))$, even though h is $\mathfrak{N}\mathcal{M}$ cts. That is, equality does not hold.

Theorem 2.5.

A function $h\colon (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ is $\mathfrak{N}\mathcal{M}$ Cts if and only if $h^{-1}(\mathfrak{N}int(K_1)) \subseteq \mathfrak{N}\mathcal{M}int(h^{-1}(K_1))$ for every subset K_1 of V_1.

Proof:

If h is $\mathfrak{N}\mathcal{M}$ Cts and $K_1 \subseteq V_1$. $\mathfrak{N}int(K_1)$ is $\mathfrak{N}o$ in V_1, and hence, $h^{-1}(\mathfrak{N}int(K_1))$ is $\mathfrak{N}\mathcal{M}o$ in U_1. Therefore, $\mathfrak{N}\mathcal{M}int$ $(h^{-1}(\mathfrak{N}int(K_1))) = h^{-1}(\mathfrak{N}int(K_1))$. Also, $\mathfrak{N}int(K_1) \subseteq K_1$, implies that $h^{-1}(\mathfrak{N}int(K_1)) \subseteq h^{-1}(K_1)$. Therefore, $\mathfrak{N}\mathcal{M}int(h^{-1}(\mathfrak{N}int(K_1))) \subseteq \mathfrak{N}\mathcal{M}int(h^{-1}(K_1))$. That is, $h^{-1}(\mathfrak{N}int(K_1)) \subseteq \mathfrak{N}\mathcal{M}int(h^{-1}(K_1))$.

Conversely, let $h^{-1}(\mathfrak{N}int(K_1)) \subseteq \mathfrak{N}\mathcal{M}int(h^{-1}(K_1))$ for every subset K_1 of V_1. If K_1 is $\mathfrak{N}o$ in V_1, then $\mathfrak{N}int(K_1) = K_1$. By assumption, $h^{-1}(\mathfrak{N}int(K_1)) \subseteq \mathfrak{N}\mathcal{M}int(h^{-1}(K_1))$. Thus, $h^{-1}(K_1) \subseteq \mathfrak{N}\mathcal{M}int(h^{-1}(K_1))$. But $\mathfrak{N}\mathcal{M}int(h^{-1}(K_1)) \subseteq h^{-1}(K_1)$. Therefore, $\mathfrak{N}\mathcal{M}int(h^{-1}(K_1)) = h^{-1}(K_1)$. That is, $h^{-1}(K_1)$ is $\mathfrak{N}\mathcal{M}o$ in U_1, for every $\mathfrak{N}o$ set K_1 in V_1. Therefore, h is $\mathfrak{N}\mathcal{M}$ Cts on U_1. ∎

Remark 2.3.

A function $h\colon (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ is $\mathfrak{N}\mathcal{M}$ Cts. Then $h^{-1}(\mathfrak{N}int(L_1))$ is not necessarily equal to $\mathfrak{N}\mathcal{M}int(h^{-1}(L_1))$ where $L_1 \subseteq V_1$. It is shown in the following examples.

Example 2.6.

In Example 2.3, $h : (U_1, \tau_R(P)) \to (V_1, \tau_{R'}(Q))$ is $\mathfrak{N}\mathcal{M}$ Cts. Let $B = \{M_c\} \subset V_1$. Then, $\mathfrak{N}\mathcal{M}int(h^{-1}(B)) = \mathfrak{N}\mathcal{M}inth^{-1}(\{M_c\}) = \mathfrak{N}\mathcal{M}int(\{M_c\}) = \{M_c\}$. But $h^{-1}(\mathfrak{N}int(B)) = h^{-1}(\mathfrak{N}int(\{M_c\})) = h^{-1}(\{\phi\}) = \phi$. Thus, $\mathfrak{N}\mathcal{M}int(h^{-1}(B)) \neq h^{-1}(\mathfrak{N}int(B))$, even though h is $\mathfrak{N}\mathcal{M}$ cts. That is, equality does not hold.

Theorem 2.6.

In a $\mathfrak{N}ts$ $(U_1, \tau_R(P))$, if the collection of $\mathfrak{N}\mathcal{M}O(U_1, P)$ is $\mathfrak{N}c$ under arbitrary union and let $h\colon (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ be a function. Then, the function h is $\mathfrak{N}\mathcal{M}$ Cts if and only if for each $x \in U_1$ and each $\mathfrak{N}o$ set O in V_1 with $h(x) \in O$ $\exists$ $\mathfrak{N}\mathcal{M}o$ set G in $U_1 \ni x \in G$ & $h(G) \subset O$.

Proof:

Let $x \in U_1$ and O be a $\mathfrak{N}o$ set in V_1 with $h(x) \in O$, then $x \in h^{-1}(O)$. Since h is $\mathfrak{N}\mathcal{M}$ Cts, $h^{-1}(O)$ is a $\mathfrak{N}\mathcal{M}o$ set in U_1. Put $G = h^{-1}(O)$. Then, $x \in G$ and $h(G) = h(h^{-1}(O)) \subset O$.

Conversely, let $x \in U_1$ and O be a $\mathfrak{N}o$ set in V_1 containing $h(x)$. By hypothesis, there exists a $\mathfrak{N}\mathcal{M}o$ set G_x in $U_1 \ni x \in G_x$ and $h(G_x) \subset O$. This implies $x \in G_x \subset h^{-1}(O)$, which implies

$h^{-1}(O)$ is $\mathfrak{N}\mathcal{M}Nbd(x)$. Since x is arbitrary, $h^{-1}(O)$ is $\mathfrak{N}\mathcal{M}Nbd$ of each its points. Which implies $h^{-1}(O)$ is a $\mathfrak{N}\mathcal{M}o$ set in U_1. Therefore, h is $\mathfrak{N}\mathcal{M}\ Cts$. ■

Theorem 2.7.

In a $\mathfrak{N}ts$ $(U_1, \tau_R(P))$, if the collection of $\mathfrak{N}\mathcal{M}O(U_1, X)$ is $\mathfrak{N}c$ under arbitrary union and let $h\colon (U_1, \tau_R(P)) \rightarrow (V_1, \sigma_{R'}(Q))$ be a function. Then, the function h is $\mathfrak{N}\mathcal{M}\ Cts$ if and only if $\forall\ x \in U_1$, the inverse of every $\mathfrak{N}Nbd$ of $h(x)$ is $\mathfrak{N}\mathcal{M}Nbd(x)$.

Proof:

Let $x \in U_1$ and H be a $\mathfrak{N}Nbd$ of $h(x)$. There exists a $\mathfrak{N}o$ set O in $V_1 \ni h(x) \in O \subset H$, and hence, $x \in h^{-1}(O) \subset h^{-1}(H)$. Since h is $\mathfrak{N}\mathcal{M}\ Cts$ and $h^{-1}(O)$ is $\mathfrak{N}\mathcal{M}o$ set in U_1, therefore, $h^{-1}(H)$ is $\mathfrak{N}\mathcal{M}Nbd(x)$.

Conversely, let $x \in U_1$ and O be a $\mathfrak{N}o$ set in V_1 containing $h(x)$. This implies O is $\mathfrak{N}Nbd$ of $h(x)$. By hypothesis, $h^{-1}(O)$ is $\mathfrak{N}\mathcal{M}Nbd(x)$. Since x is arbitrary, $h^{-1}(O)$ is $\mathfrak{N}\mathcal{M}Nbd$ of each of its point. Hence, $h^{-1}(O)$ is a $\mathfrak{N}\mathcal{M}o$ set in U_1. Therefore, h is $\mathfrak{N}\mathcal{M}\ Cts$.

The maps $\mathfrak{N}\delta\ Cts$, $\mathfrak{N}\delta\mathcal{P}\ Cts$, $\mathfrak{N}\delta\mathcal{S}\ Cts$ and $\mathfrak{N}e\ Cts$ satisfy the Theorems 2.6 and 2.7 for their respective family of open sets. ■

Remark 2.4.

The composition of two $\mathfrak{N}\mathcal{M}\ Cts$ functions need not be $\mathfrak{N}\mathcal{M}\ Cts$ as seen from the following example.

Example 2.7.

Let $U_1 = V_1 = W_1 = \{L_a, L_b, L_c, L_d, L_e\}$ with $U_1/R = \{\{L_c\}, \{L_a, L_b\}, \{L_d, L_e\}\}$, $P = \{L_a, L_c\}$, $\tau_R(P) = \{U_1, \phi, \{L_c\}, \{L_a, L_b\}, \{L_a, L_b, L_c\}\}$ and $V_1/R^{'} = \{\{L_e\}, \{L_a, L_b\}, \{L_c, L_d\}\}$, $Y = \{L_a, L_c, L_d\}$, $\sigma_{R'}(Q) = \{V_1, \phi, \{L_a, L_b\}, \{L_c, L_d\}, \{L_a, L_b, L_c, L_d\}\}$. Then, the identity mappings $h : (U_1, \tau_R(P)) \rightarrow (V_1, \sigma_{R'}(Q))$ and $g : (V_1, \sigma_{R'}(Q)) \rightarrow (W_1, \sigma_{R'}(Q))$ are $\mathfrak{N}\mathcal{M}\ Cts$ but the composition $g \circ h$ is not $\mathfrak{N}\mathcal{M}\ Cts$. The set $\{L_c, L_d\}$ is $\mathfrak{N}o$ in V_1 but $(g \circ h)^{-1}(\{L_c, L_d\}) = \{L_c, L_d\}$ is not $\mathfrak{N}\mathcal{M}o$ in U_1.

Theorem 2.8.

Let $h\colon (U_1, \tau_R(P)) \rightarrow (V_1, \sigma_{R'}(Q))$ and $g\colon (V_1, \sigma_{R'}(Q)) \rightarrow (W_1, \mu_{R''}(R))$ be any two functions. If h is a $\mathfrak{N}\mathcal{M}\ Cts$ and g is $\mathfrak{N}\ Cts$ function, then $g \circ h$ is $\mathfrak{N}\mathcal{M}\ Cts$.

Proof:

Let C be any $\mathfrak{N}c$ set in W_1. As g is $\mathfrak{N}\ Cts$, $g^{-1}(C)$ is $\mathfrak{N}c$ in V_1. Since h is $\mathfrak{N}\mathcal{M}\ Cts$, implies $h^{-1}(g^{-1}(C)) = (g \circ h)^{-1}(C)$ is $\mathfrak{N}\mathcal{M}c$ in U_1. Therefore, $g \circ h$ is $\mathfrak{N}\mathcal{M}\ Cts$. ■

3. Nano $\mathcal{M}$ Irresolute Functions

Definition 3.1.

A function $h : (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ is called Nano $\mathcal{M}$ (respectively, θ, δ, θ semi, δ pre, δ semi & e) irresolute (briefly, $\mathfrak{N}\mathcal{M}Irr$ (resp. $\mathfrak{N}\theta Irr$, $\mathfrak{N}\delta Irr$, $\mathfrak{N}\theta\mathcal{S}Irr$, $\mathfrak{N}\delta\mathcal{P}Irr$, $\mathfrak{N}\delta\mathcal{S}Irr$ and $\mathfrak{N}eIrr$)) function, if for each $\mathfrak{N}\mathcal{M}c$ (respectively, $\mathfrak{N}\theta c$, $\mathfrak{N}\delta c$, $\mathfrak{N}\theta\mathcal{S}c$, $\mathfrak{N}\delta\mathcal{P}c$, $\mathfrak{N}\delta\mathcal{S}c$ and $\mathfrak{N}\ ec$) subset K of V_1, the set $h^{-1}(K)$ is $\mathfrak{N}\mathcal{M}c$ (respectively, $\mathfrak{N}\theta c$, $\mathfrak{N}\delta c$, $\mathfrak{N}\theta\mathcal{S}c$, $\mathfrak{N}\delta\mathcal{P}c$, $\mathfrak{N}\delta\mathcal{S}c$ and $\mathfrak{N}\ ec$) subset of U_1.

Theorem 3.1.

A function h: $(U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ is called

(i) $\mathfrak{N}Irr$, then h is $\mathfrak{N}\mathcal{S}\ Cts$.
(ii) $\mathfrak{N}\delta\mathcal{P}Irr$, then h is $\mathfrak{N}\delta\mathcal{P}\ Cts$.
(iii) $\mathfrak{N}\mathcal{M}Irr$, then h is $\mathfrak{N}\mathcal{M}\ Cts$.
(iv) $\mathfrak{N}\delta\mathcal{S}Irr$, then h is $\mathfrak{N}\delta\mathcal{S}\ Cts$.

Proof:

(i) Let C be $\mathfrak{N}c$ in V_1. Then C is $\mathfrak{N}\mathcal{S}c$ in V_1, since every $\mathfrak{N}c$ set is $\mathfrak{N}\mathcal{S}c$. By hypothesis, $h^{-1}(C)$ is $\mathfrak{N}\mathcal{S}c$. Therefore, h is $\mathfrak{N}\mathcal{S}\ Cts$.

(ii) Let C be $\mathfrak{N}c$ in V_1. Then C is $\mathfrak{N}\delta\mathcal{P}c$ in V_1, since every $\mathfrak{N}c$ set is $\mathfrak{N}\delta\mathcal{P}c$. By hypothesis, $h^{-1}(C)$ is $\mathfrak{N}\delta\mathcal{P}c$. Therefore, h is $\mathfrak{N}\delta\mathcal{P}\ Cts$.

(iii) Let C be $\mathfrak{N}c$ in V_1. Then C is $\mathfrak{N}\mathcal{M}c$ in V_1, since every $\mathfrak{N}c$ set is $\mathfrak{N}\mathcal{M}c$. By hypothesis, $h^{-1}(C)$ is $\mathfrak{N}\mathcal{M}c$. Therefore, h is $\mathfrak{N}\mathcal{M}\ Cts$.

(iv) Let C be $\mathfrak{N}c$ in V_1. Then C is $\mathfrak{N}\delta\mathcal{S}c$ in V_1, since every $\mathfrak{N}c$ set is $\mathfrak{N}\delta\mathcal{S}c$. By hypothesis, $h^{-1}(C)$ is $\mathfrak{N}\delta\mathcal{S}c$. Therefore, h is $\mathfrak{N}\delta\mathcal{S}\ Cts$. ■

Remark 3.1.

The converse of the above theorem need not be true as shown in the following example.

Example 3.1.

Let $U_1 = V_1 = \{L_a, L_b, L_c, L_d, L_e\}$ with $U_1/R = \{\{L_c\}, \{L_a, L_b\}, \{L_d, L_e\}\}$, $P = \{L_a, L_c\}$. Then, $\tau_R(P) = \{U_1, \phi, \{L_c\}, \{L_a, L_b\}, \{L_a, L_b, L_c\}\}$ and $V_1/R' = \{\{L_e\}, \{L_a, L_b\}, \{L_c, L_d\}\}$, $Y = \{L_a, L_c, L_d\}$. Then, $\sigma_{R'}(Q) = \{V_1, \phi, \{L_a, L_b\}, \{L_c, L_d\}, \{L_a, L_b, L_c, L_d\}\}$. Define $h : (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ as $h(L_a) = L_a,\ h(L_b) = L_b,\ h(L_c) = L_c,\ h(L_d) = L_e$ and $h(L_e) = L_e$. Then, h is $\mathfrak{N}\mathcal{M}\ Cts$, but h is not $\mathfrak{N}\mathcal{M}Irr$, since $h^{-1}(\{L_b, L_d, L_e\}) = \{L_b, L_d, L_e\}$ which is not $\mathfrak{N}\mathcal{M}o$ (respectively, not $\mathfrak{N}\delta\mathcal{P}o$) in U_1 whereas $\{L_b, L_d, L_e\}$ is $\mathfrak{N}\mathcal{M}o$ (respectively, $\mathfrak{N}\delta\mathcal{P}o$ in V_1.

Example 3.2.

Let $U_1 = V_1 = \{L_a, L_b, L_c, L_d, L_e\}$ with $U_1/R = \{\{L_e\}, \{L_a, L_b\}, \{L_c, L_d\}\}$, $P = \{L_a, L_c, L_d\}$. Then, $\tau_R(P) = \{U_1, \phi, \{L_a, L_b\}, \{L_c, L_d\}, \{L_a, L_b, L_c, L_d\}\}$. $V_1/R' = \{\{L_c\}, \{L_a, L_b\}, \{L_d, L_e\}\}$, $Q = \{L_a, L_c\}$. Then, $\sigma_{R'}(Q) = \{V_1, \phi, \{L_c\}, \{L_a, L_b\}, \{L_a, L_b, L_c\}\}$, Define $h : (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ as $h(L_a) = L_a,\ h(L_b) = L_b,\ h(L_c) = L_d,\ h(L_d) = L_e$ and $h(L_e) = L_e$. Then, h is $\mathfrak{N}\delta\mathcal{S}\ Cts$, but h is not $\mathfrak{N}\delta\mathcal{S}Irr$, since $h^{-1}(\{L_c, L_e\}) = \{L_d, L_e\}$ which is not $\mathfrak{N}\delta\mathcal{S}o$ in U_1 whereas $\{L_d, L_e\}$ is $\mathfrak{N}\delta\mathcal{S}o$ in V_1.

Example 3.3.

In Example 3.2, h is $\mathfrak{N}$-Cts, but h is not $\mathfrak{N}Irr$, since $h^{-1}(\{L_c, L_d\}) = \{L_c\}$ which is not $\mathfrak{N}\delta\mathcal{S}o$ in U_1 whereas $\{L_c, L_d\}$ is $\mathfrak{N}\delta\mathcal{S}o$ in V_1.

Theorem 3.2.

A function $h : (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ is called $\mathfrak{N}\mathcal{M}Irr$ (respectively, $\mathfrak{N}eIrr$) if and only if for every $\mathfrak{N}\mathcal{M}o$ (respectively, $\mathfrak{N}eo$) set K in V_1, $h^{-1}(K)$ is $\mathfrak{N}\mathcal{M}o$ (respectively, $\mathfrak{N}eo$) in U_1.

Proof:

This follows from the fact that the complement of $\mathfrak{N}\mathcal{M}o$ (respectively, $\mathfrak{N}eo$) set is $\mathfrak{N}\mathcal{M}c$ (respectively, $\mathfrak{N}ec$) and vice versa. ■

Theorem 3.3.

If $h : (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ and $g : (V_1, \sigma_{R'}(Q)) \to (W_1, \mu_{R''}(S))$ are both $\mathfrak{N}\mathcal{M}Irr$, then $g \circ h : (U_1 : \tau_R(P)) \to (W_1, \mu_{R''}(S))$ is $\mathfrak{N}\mathcal{M}Irr$.

Proof:

Let K be $\mathfrak{N}\mathcal{M}o$ in W_1. Then, $g^{-1}(K)$ is $\mathfrak{N}\mathcal{M}o$ in V_1, since g is $\mathfrak{N}\mathcal{M}Irr$ & $h^{-1}(g^{-1}(K)) = (g \circ h)^{-1}(K)$ is $\mathfrak{N}\mathcal{M}o$ in U_1, since h is $\mathfrak{N}\mathcal{M}Irr$. Hence $g \circ h$ is $\mathfrak{N}\mathcal{M}Irr$. ■

The maps $\mathfrak{N}\delta Irr$, $\mathfrak{N}\delta\mathcal{P}Irr$, $\mathfrak{N}\delta\mathcal{S}Irr$ and $\mathfrak{N}eIrr$ satisfy the Theorem 3.3 for their respective open sets.

Theorem 3.4.

(i) If $h : (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ is $\mathfrak{N}\mathcal{M}Irr$ and $g : (V_1, \sigma_{R'}(Q)) \to (W_1, \mu_{R''}(S))$ is $\mathfrak{N}\mathcal{M}$ Cts, then $g \circ h : (U_1, \tau_R(P)) \to (W_1, \mu_{R''}(S))$ is $\mathfrak{N}\mathcal{M}\ Cts$.

(ii) If $h : (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ is $\mathfrak{N}\mathcal{M}\ Cts$ and $g : (V_1, \sigma_{R'}(Q)) \to (W_1, \mu_{R''}(S))$ is $\mathfrak{N}$ Cts, then $g \circ h : (U_1, \tau_R(P)) \to (W_1, \mu_{R''}(S))$ is $\mathfrak{N}\mathcal{M}\ Cts$.

Proof:

(i) Let K be $\mathfrak{N}o$ in W_1. Then, $g^{-1}(K)$ is $\mathfrak{N}\mathcal{M}o$ in V_1, since g is $\mathfrak{N}\mathcal{M}Cts$ & $h^{-1}(g^{-1}(K)) = (g \circ h)^{-1}(K)$ is $\mathfrak{N}\mathcal{M}o$ in U_1, since h is $\mathfrak{N}\mathcal{M}Irr$. Hence $g \circ h$ is $\mathfrak{N}\mathcal{M}Cts$.

(ii) Let K be $\mathfrak{N}o$ in W_1. Then, $g^{-1}(K)$ is $\mathfrak{N}o$ in V_1, since g is $\mathfrak{N}Cts$ & $h^{-1}(g^{-1}(K)) = (g \circ h)^{-1}(K)$ is $\mathfrak{N}\mathcal{M}o$ in U_1, since h is $\mathfrak{N}\mathcal{M}Cts$. Hence $g \circ h$ is $\mathfrak{N}\mathcal{M}Cts$. ∎

The other respective functions satisfy Theorem 3.4 for their respective open sets.

4. Nano $\mathcal{M}$ closed functions

Definition 4.1.

A function $h\colon (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ is said to be Nano $\mathcal{M}$ closed (respectively, θ closed, δ closed, θ semi closed, δ pre closed, δ semi closed and e closed) function (briefly, $\mathfrak{N}\mathcal{M}cf$ (respectively, $\mathfrak{N}\theta cf$, $\mathfrak{N}\delta cf$, $\mathfrak{N}\theta\mathcal{S}cf$, $\mathfrak{N}\delta\mathcal{P}cf$, $\mathfrak{N}\delta\mathcal{S}cf$ and $\mathfrak{N}ecf$)) if the direct image $h(K)$ is $\mathfrak{N}\mathcal{M}c$ (respectively, $\mathfrak{N}\theta c$, $\mathfrak{N}\delta c$, $\mathfrak{N}\theta\mathcal{S}c$, $\mathfrak{N}\delta\mathcal{P}c$, $\mathfrak{N}\delta\mathcal{S}c$ and $\mathfrak{N}ec$) set in V_1 whenever K is $\mathfrak{N}c$ in U_1.

Definition 4.2.

A function $h\colon (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ is said to be Nano $\mathcal{M}$ open (respectively, θ open, δ open, θ semi open, δ pre open, δ semi open and e open) function (briefly, $\mathfrak{N}\mathcal{M}of$ (respectively, $\mathfrak{N}\theta of$, $\mathfrak{N}\delta of$, $\mathfrak{N}\theta\mathcal{S}of$, $\mathfrak{N}\delta\mathcal{P}of$, $\mathfrak{N}\delta\mathcal{S}of$ and $\mathfrak{N}eof$)) if the direct image $h(K)$ is $\mathfrak{N}\mathcal{M}o$ (respectively, $\mathfrak{N}\theta o$, $\mathfrak{N}\delta o$, $\mathfrak{N}\theta\mathcal{S}o$, $\mathfrak{N}\delta\mathcal{P}o$, $\mathfrak{N}\delta\mathcal{S}o$ and $\mathfrak{N}eo$) set in V_1 whenever K is $\mathfrak{N}o$ in U_1.

Theorem 4.1.

A function $h\colon (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$,

(i) Every $\mathfrak{N}\theta cf$ is $\mathfrak{N}cf$.
(ii) Every $\mathfrak{N}\theta cf$ is $\mathfrak{N}\delta cf$.
(iii) Every $\mathfrak{N}\delta cf$ is $\mathfrak{N}cf$.
(iv) Every $\mathfrak{N}\theta cf$ is $\mathfrak{N}\theta\mathcal{S}cf$.
(v) Every $\mathfrak{N}cf$ is $\mathfrak{N}\delta\mathcal{P}cf$.
(vi) Every $\mathfrak{N}\delta cf$ is $\mathfrak{N}\delta\mathcal{S}cf$.
(vii) Every $\mathfrak{N}\theta\mathcal{S}cf$ is $\mathfrak{N}\mathcal{M}cf$.
(viii) Every $\mathfrak{N}\delta\mathcal{P}cf$ is $\mathfrak{N}\mathcal{M}cf$.
(ix) Every $\mathfrak{N}\delta\mathcal{P}cf$ is $\mathfrak{N}ecf$.
(x) Every $\mathfrak{N}\delta\mathcal{S}cf$ is $\mathfrak{N}ecf$.
(xi) Every $\mathfrak{N}\mathcal{M}cf$ is $\mathfrak{N}ecf$.

Proof:

(i) Let $h\colon (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ be $\mathfrak{N}\theta cf$ and L is a $\mathfrak{N}c$ set in U_1. Then, $h(L)$ is $\mathfrak{N}\theta c$ in V_1. Since every $\mathfrak{N}\theta c$ set is $\mathfrak{N}c$, $h(L)$ is $\mathfrak{N}c$ set in V_1. Therefore, h is $\mathfrak{N}cf$.

(ii) Let $h\colon (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ be $\mathfrak{N}\theta cf$ and L is a $\mathfrak{N}c$ set in U_1. Then, $h(L)$ is $\mathfrak{N}\theta c$ in V_1. Since every $\mathfrak{N}\theta c$ set is $\mathfrak{N}\delta c$, $h(L)$ is $\mathfrak{N}\delta c$ set in V_1. Therefore, h is $\mathfrak{N}\delta cf$.

(iii) Let $h\colon (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ be $\mathfrak{N}\delta cf$ and L is a $\mathfrak{N}c$ set in U_1. Then, $h(L)$ is $\mathfrak{N}\delta c$ in V_1. Since every $\mathfrak{N}\delta c$ set is $\mathfrak{N}c$, $h(L)$ is $\mathfrak{N}c$ set in V_1. Therefore, h is $\mathfrak{N}cf$.

(vi) Let $h\colon (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ be $\mathfrak{N}\theta cf$ and L is a $\mathfrak{N}c$ set in U_1. Then, $h(L)$ is $\mathfrak{N}\theta c$ in V_1. Since every $\mathfrak{N}\theta c$ set is $\mathfrak{N}\theta\mathcal{S}c$, $h(L)$ is $\mathfrak{N}\theta\mathcal{S}c$ set in V_1. Therefore, h is $\mathfrak{N}\theta\mathcal{S}cf$.

(v) Let $h\colon (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ be $\mathfrak{N}cf$ and L is a $\mathfrak{N}c$ set in V_1. Then, $h(L)$ is $\mathfrak{N}c$ in U_1. Since every $\mathfrak{N}c$ set is $\mathfrak{N}\delta\mathcal{P}c$, $h(L)$ is $\mathfrak{N}\delta\mathcal{P}c$ set in U_1. Therefore, h is $\mathfrak{N}\delta\mathcal{P}cf$.

(vi) Let $h\colon (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ be $\mathfrak{N}\delta cf$ and L is a $\mathfrak{N}c$ set in U_1. Then, $h(L)$ is $\mathfrak{N}\delta c$ in V_1. Since every $\mathfrak{N}\delta c$ set is $\mathfrak{N}\delta\mathcal{S}c$, $h(L)$ is $\mathfrak{N}\delta\mathcal{S}c$ set in V_1. Therefore, h is $\mathfrak{N}\delta\mathcal{S}cf$.

(vii) Let $h\colon (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ be $\mathfrak{N}\theta\mathcal{S}cf$ and L is a $\mathfrak{N}c$ set in U_1. Then, $h(L)$ is $\mathfrak{N}\theta\mathcal{S}c$ in V_1. Since every $\mathfrak{N}\theta\mathcal{S}c$ set is $\mathfrak{N}\mathcal{M}c$, $h(L)$ is $\mathfrak{N}\mathcal{M}c$ set in V_1. Therefore, h is $\mathfrak{N}\mathcal{M}cf$.

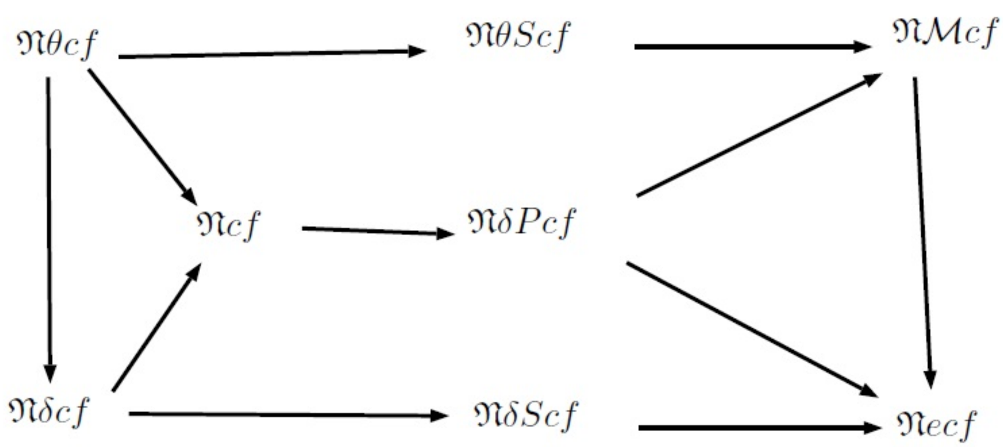
(viii) Let $h\colon (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ be $\mathfrak{N}\delta\mathcal{P}cf$ and L is a $\mathfrak{N}c$ set in U_1. Then, $h(L)$ is $\mathfrak{N}\delta\mathcal{P}c$ in V_1. Since every $\mathfrak{N}\delta\mathcal{P}c$ set is $\mathfrak{N}\mathcal{M}c$, $h(L)$ is $\mathfrak{N}\mathcal{M}c$ set in V_1. Therefore, h is $\mathfrak{N}\mathcal{M}cf$.

(ix) Let $h\colon (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ be $\mathfrak{N}\delta\mathcal{P}cf$ and L is a $\mathfrak{N}c$ set in U_1. Then, $h(L)$ is $\mathfrak{N}\delta\mathcal{P}c$ in V_1. Since every $\mathfrak{N}\delta\mathcal{P}c$ set is $\mathfrak{N}ec$, $h(L)$ is $\mathfrak{N}ec$ set in V_1. Therefore, h is $\mathfrak{N}ecf$.

(x) Let $h\colon (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ be $\mathfrak{N}\delta\mathcal{S}cf$ and L is a $\mathfrak{N}c$ set in U_1. Then, $h(L)$ is $\mathfrak{N}\delta\mathcal{S}c$ in V_1. Since every $\mathfrak{N}\delta\mathcal{S}c$ set is $\mathfrak{N}ec$, $h(L)$ is $\mathfrak{N}ec$ set in V_1. Therefore, h is $\mathfrak{N}ecf$.

(xi) Let $h\colon (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ be $\mathfrak{N}\mathcal{M}cf$ and L is a $\mathfrak{N}c$ set in U_1. Then, $h(L)$ is $\mathfrak{N}\mathcal{M}c$ in V_1. Since every $\mathfrak{N}\mathcal{M}c$ set is $\mathfrak{N}ec$, $h(L)$ is $\mathfrak{N}ec$ set in V_1. Therefore, h is $\mathfrak{N}ecf$. ∎

From the above discussions, the following implications are hold for any set in $\mathfrak{N}ts$.



Note: $K \to L$ denotes K implies L, but not conversely

Example 4.1.

Let $U_1 = V_1 = \{L_a, L_b, L_c, L_d\}$ with $U_1/R = \{\{L_a, L_b\}, \{L_c, L_d\}\}$, $P = \{L_a, L_b\}$, $\tau_R(P) = \{U_1, \phi, \{L_a, L_b\}\}$. Define the identity map $h : U_1 \to V_1$ is $\mathfrak{N}cf$ but not $\mathfrak{N}\delta cf$. The set $\{L_c, L_d\}$ is $\mathfrak{N}c$ in U_1 but $h(\{L_c, L_d\}) = \{L_c, L_d\}$ which is not $\mathfrak{N}\delta c$ in V_1.

Example 4.2.

Let $U_1 = V_1 = \{M_a, M_b, M_c, M_d, M_e\}$ with $U_1/R = \{\{M_e\}, \{M_a, M_b\}, \{M_c, M_d\}\}$, $P = \{M_c, M_e\}$, $\tau_R(P) = \{U_1, \phi, \{M_e\}, \{M_c, M_d\}, \{M_c, M_d, M_e\}\}$ and $V_1/R' = \{\{M_c\}, \{M_a, M_b\}, \{M_d, M_e\}\}$, $Q = \{M_a, M_c\}$, $\tau_{R'}(Q) = \{V_1, \phi, \{M_c\}, \{M_a, M_b\}, \{M_a, M_b, M_c\}\}$. Then, the mapping $h : (U_1, \tau_R(P)) \to (V_1, \tau_{R'}(Q))$ is defined by

(i) $h(M_a) = M_c,\ h(M_b) = M_d,\ h(M_c) = M_e,\ h(M_d) = M_a$ and $h(M_e) = M_b$ is $\mathfrak{N}cf$ (respectively, $\mathfrak{N}\delta cf$) but not $\mathfrak{N}\theta cf$ (respectively, $\mathfrak{N}\theta cf$). The set $\{M_a, M_b\}$ is $\mathfrak{N}c$ in U_1 but $h(\{M_a, M_b\}) = \{M_c, M_d\}$ is not $\mathfrak{N}\theta c$ in V_1.

(ii) $h(M_a) = M_d,\ h(M_b) = M_e,\ h(M_c) = M_c,\ h(M_d) = M_a$ and $h(M_e) = M_b$ is $\mathfrak{N}\delta\mathcal{P}cf$ but not $\mathfrak{N}cf$. The set $\{M_a, M_b, M_e\}$ is $\mathfrak{N}c$ in U_1 but $h(\{M_a, M_b, M_e\}) = \{M_b, M_d, M_e\}$ is not $\mathfrak{N}c$ in V_1.

(iii) $h(M_a) = M_c,\ h(M_b) = h(M_e) = M_d,\ h(M_c) = M_e$ and $h(M_d) = M_a$ is $\mathfrak{N}\delta\mathcal{S}cf$ but not $\mathfrak{N}\delta cf$. The set $\{M_a, M_b\}$ is $\mathfrak{N}c$ in U_1 but $h(\{M_a, M_b\}) = \{M_c, M_d\}$ is not $\mathfrak{N}\delta c$ in V_1.

(iv) $h(M_a) = M_e,\ h(M_b) = M_d,\ h(M_c) = M_c,\ h(M_d) = M_b$ and $h(M_e) = M_a$ is $\mathfrak{N}\mathcal{M}cf$ but not $\mathfrak{N}\theta\mathcal{S}cf$. The set $\{M_a, M_b\}$ is $\mathfrak{N}c$ in U_1 but $h(\{M_a, M_b\}) = \{M_d, M_e\}$ is not $\mathfrak{N}\theta\mathcal{S}c$ in V_1.

Example 4.3.

Let $U_1 = V_1 = W_1 = W_1^{'} = \{M_a, M_b, M_c, M_d, M_e\}$ with $U_1/R = \{\{M_a\}, \{M_b\}, \{M_c, M_d, M_e\}\}$, $P = \{M_c, M_d, M_e\}$, $\tau_R(P) = \{U_1, \phi, \{M_c, M_d, M_e\}\}$; $V_1/R^{'} = \{\{M_c\}, \{M_a, M_b\}, \{M_d, M_e\}\}$, $Q = \{M_a,\ M_c\}$, $\tau_{R'}(Q) = \{V_1, \phi, \{M_c\}, \{M_a, M_b\}, \{M_a, M_b, M_c\}\}$; $W_1/R^{''} = \{\{M_c\}, \{M_e\}, \{M_a, M_b, M_d\}\}$, $S = \{M_a, M_b, M_d\}$, $\tau_{R''}(S) = \{W_1, \phi, \{M_a, M_b, M_d\}\}$ and $W_1^{'}/R^{'''} = \{\{M_b\}, \{M_e\}, \{M_a, M_c, M_d\}\}$ $S^{'} = \{M_a, M_c, M_d\}$, $\tau_{R'''}(Z^{'}) = \{U_1, \phi, \{M_a, M_c, M_d\}\}$. Then, the identity mappings

(i) $h : (U_1, \tau_R(P)) \to (V_1, \tau_{R'}(Q))$ is $\mathfrak{N}\theta\mathcal{S}cf$ but not $\mathfrak{N}\theta cf$. The set $\{M_a, M_b\}$ is $\mathfrak{N}c$ in U_1 but $h(\{M_a, M_b\}) = \{M_a, M_b\}$ is not $\mathfrak{N}\theta c$ in V_1.

(ii) $h : (U_1, \tau_R(P)) \to (V_1, \tau_{R'}(Q))$ is $\mathfrak{N}\mathcal{M}cf$ but not $\mathfrak{N}\delta\mathcal{P}cf$. The set $\{M_a, M_b\}$ is $\mathfrak{N}c$ in U_1 but $h(\{M_a, M_b\}) = \{M_a, M_b\}$ is not $\mathfrak{N}\delta\mathcal{P}c$ in V_1.

(iii) $h : (U_1, \tau_R(P)) \to (V_1, \tau_{R'}(Q))$ is $\mathfrak{N}ecf$ but not $\mathfrak{N}\delta\mathcal{P}cf$. The set $\{M_a, M_b\}$ is $\mathfrak{N}c$ in U_1 but $h(\{M_a, M_b\}) = \{M_a, M_b\}$ is not $\mathfrak{N}\delta\mathcal{P}c$ in V_1.

(iv) $g : (W_1^{'}, \tau_{R'''}(S^{'})) \to (V_1, \tau_{R'}(Q))$ is $\mathfrak{N}ecf$ but not $\mathfrak{N}\delta\mathcal{S}cf$. The set $\{M_b, M_e\}$ is $\mathfrak{N}c$ in $W_1^{'}$ but $g(\{M_b, M_e\}) = \{M_b, M_e\}$ is not $\mathfrak{N}\delta\mathcal{S}c$ in V_1.

(v) $h : (W_1, \tau_{R''}(S)) \to (V_1, \tau_{R'}(Q))$ is $\mathfrak{N}ecf$ but not $\mathfrak{N}\mathcal{M}cf$. The set $\{M_c, M_e\}$ is $\mathfrak{N}c$ in W_1 but $h(\{M_c, M_e\}) = \{M_c, M_e\}$ is not $\mathfrak{N}\mathcal{M}c$ in V_1.

Theorem 4.2.

A function $h\colon (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ is $\mathfrak{N}\mathcal{M}c$ if and only if $h(K)$ is $\mathfrak{N}\mathcal{M}o$ in V_1 for every $\mathfrak{N}o$ set K in U_1.

Proof:

Suppose $h\colon (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ is $\mathfrak{N}\mathcal{M}cf$ and O is a $\mathfrak{N}o$ set in U_1. Then, $U_1 - O$ is $\mathfrak{N}c$ in U_1. By hypothesis $h(U_1 - O) = V_1 - h(O)$ is a $\mathfrak{N}\mathcal{M}c$ set in V_1, and hence, $h(O)$ is $\mathfrak{N}\mathcal{M}o$ in V_1.

Conversely, if C is $\mathfrak{N}c$ set in U_1, then $U_1 - C$ is a $\mathfrak{N}o$ set in U_1. By hypothesis $h(U_1 - C) = V_1 - h(C)$ is $\mathfrak{N}\mathcal{M}o$ set in V_1, implies $h(C)$ is $\mathfrak{N}\mathcal{M}c$ in V_1. Therefore, h is $\mathfrak{N}\mathcal{M}cf$. ■

Theorem 4.3.

A function $h\colon (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ is a $\mathfrak{N}\mathcal{M}cf$ if and only if $\mathfrak{N}\mathcal{M}cl(h(K)) \subseteq h(\mathfrak{N}cl(K))$ for every subset K of U_1.

Proof:

Suppose h is $\mathfrak{N}\mathcal{M}c$ and $K \subseteq U_1$. Then, $h(\mathfrak{N}cl(K))$ is $\mathfrak{N}\mathcal{M}c$ in V_1. Since $h(K) \subseteq h(\mathfrak{N}cl(K))$, we get $\mathfrak{N}\mathcal{M}clh(K) \subseteq \mathfrak{N}\mathcal{M}clh(\mathfrak{N}cl(K)) = h(\mathfrak{N}cl(K))$. Hence, $\mathfrak{N}\mathcal{M}cl(h(K)) \subseteq h(\mathfrak{N}cl(K))$.

Conversely, let C is any $\mathfrak{N}c$ set in U_1. Then, $\mathfrak{N}cl(C) = C$. Therefore, $h(C) = h(\mathfrak{N}cl(C))$. By hypothesis $\mathfrak{N}\mathcal{M}clh(C) \subseteq h(\mathfrak{N}cl(C)) = h(C)$, which implies $\mathfrak{N}\mathcal{M}clh(C) \subseteq h(C)$. But $h(C) \subseteq \mathfrak{N}\mathcal{M}clh(C)$ is always true. This shows $\mathfrak{N}\mathcal{M}clh(C) = h(C)$. Therefore, $h(C)$ is $\mathfrak{N}\mathcal{M}c$ in V_1 and hence h is $\mathfrak{N}\mathcal{M}c$. ■

Theorem 4.4.

Let $h\colon (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ be a function and $\mathfrak{N}\mathcal{M}O(U_1, P)$ is closed under arbitrary union. The following statements are equivalent:

(i) h is $\mathfrak{N}\mathcal{M}of$.
(ii) For each subset K of U_1, $h(\mathfrak{N}int(K)) \subseteq \mathfrak{N}\mathcal{M}int(h(K))$.
(iii) For each $x \in U_1$, the image of every $\mathfrak{N}Nbd$ of x is $\mathfrak{N}\mathcal{M}Nbd$ of $h(x)$

Proof:

(i) ⇒ (ii): Suppose (i) holds and $K \subseteq U_1$. Then, $\mathfrak{N}int(K)$ is $\mathfrak{N}o$ set in U_1. By (i), $h(\mathfrak{N}int(K))$ is a $\mathfrak{N}\mathcal{M}o$ set in V_1. Therefore, $\mathfrak{N}\mathcal{M}int(h(\mathfrak{N}int(K))) = h(\mathfrak{N}int(K))$. Since $h(\mathfrak{N}int(K)) \subseteq h(K)$, implies $\mathfrak{N}\mathcal{M}int(h(\mathfrak{N}int(K))) \subseteq \mathfrak{N}\mathcal{M}int(h(K))$. That is $h(\mathfrak{N}int(K)) \subseteq \mathfrak{N}\mathcal{M}int(h(K))$.

(ii) ⇒ (iii): Suppose (ii) holds. Let $x \in U_1$ and X be an arbitrary $\mathfrak{N}Nbd$ of x in U_1. Then, $\exists$ $\mathfrak{N}o$ set G in $U_1 \ni x \in G \subset X$. By (ii), $h(G) = h(\mathfrak{N}int(G)) \subseteq \mathfrak{N}\mathcal{M}int(h(G))$. But $\mathfrak{N}\mathcal{M}int(h(G)) \subseteq h(G)$ is always true. Therefore, $h(G) = \mathfrak{N}\mathcal{M}int(h(G))$, and hence, $h(G)$ is $\mathfrak{N}\mathcal{M}o$ set in V_1. Further $h(x) \in h(G) \subset h(X)$, this implies, $h(X)$ is $\mathfrak{N}\mathcal{M}Nbd$ of $h(x)$ in V_1. Hence (iii) holds.

(iii) ⇒ (i): Suppose (iii) holds. Let G be any $\mathfrak{N}o$ set in U_1 and $x \in G$ then $y = h(x) \in h(G)$. By (iii), $\forall\, y \in h(G)$, $\exists$ $\mathfrak{N}\mathcal{M}Nbd$ K_y of y in V_1. Since K_y is $\mathfrak{N}\mathcal{M}Nbd$ of y, $\exists$ $\mathfrak{N}\mathcal{M}o$ set H_y in $V_1 \ni y \in H_y \subset K_y$. Therefore, $h(G) = \cup\{H_y : y \in h(G)\}$, which is union of $\mathfrak{N}\mathcal{M}o$ sets, and hence, $h(G)$ is $\mathfrak{N}\mathcal{M}o$ in V_1. Therefore, h is $\mathfrak{N}\mathcal{M}of$. ■

Theorem 4.5.

A function $h\colon (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ is $\mathfrak{N}\mathcal{M}c$ if and only if for each subset S of V_1 and $\forall$ $\mathfrak{N}o$ set G in U_1 containing $h^{-1}(S)$, there exists a $\mathfrak{N}\mathcal{M}o$ set H of $V_1 \ni S \subseteq H$ and $h^{-1}(H) \subseteq G$.

Proof:

Let $S \subseteq V_1$ be a $\mathfrak{N}o$ subset of U_1 containing $h^{-1}(S)$. Let h is a $\mathfrak{N}\mathcal{M}cf$ and $U_1 - G$ is $\mathfrak{N}c$ in U_1, therefore, $h(U_1 - G)$ is a $\mathfrak{N}\mathcal{M}c$ set in V_1. Then, take $H = V_1 - h(U_1 - G)$ implies $H = h(G)$ where H is $\mathfrak{N}\mathcal{M}o$ set in V_1. Since $h^{-1}(S) \subseteq G$, $S \subseteq h(G)$, $S \subseteq H$. Therefore, $h(U_1 - G) = V_1 - H \Rightarrow h(U_1 - G) \subseteq V_1 - S$ and $h^{-1}(H) \subseteq h^{-1}(V_1 - h(U_1 - G)) \subseteq U_1 - (U_1 - G) = G$. Thus, H is $\mathfrak{N}\mathcal{M}o$ set in V_1 such that $S \subseteq H$ and $h^{-1}(H) \subseteq G$.

Conversely, let G be a $\mathfrak{N}c$ set in U_1. Then $U_1 - G$ is a $\mathfrak{N}o$ set in U_1. Take $S = V_1 - h(G)$ to be a subset of V_1, $h^{-1}(S) = h^{-1}(V_1 - h(G)) \subseteq U_1 - G$. By hypothesis, there is a $\mathfrak{N}\mathcal{M}o$ set H of $V_1 \ni V_1 - h(G) \subseteq H$ & $h^{-1}(H) \subseteq U_1 - G$. Therefore, $V_1 - H \subseteq h(G) \subseteq h(U_1 - h^{-1}(H)) \subseteq V_1 - H$, that is, $h(G) = V_1 - H$. Since H is $\mathfrak{N}\mathcal{M}o$ set in V_1 and so $h(G)$ is $\mathfrak{N}\mathcal{M}c$ in V_1. Hence, h is $\mathfrak{N}\mathcal{M}cf$. ∎

Theorem 4.6.

If $h\colon (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ is a $\mathfrak{N}\mathcal{M}cf$, then for each $\mathfrak{N}c$ set K of V_1 and each $\mathfrak{N}o$ set G of U_1 containing $h^{-1}(K)$, there exists $H \in \mathfrak{N}\mathcal{M}O(V_1, Q)$ containing K such that $h^{-1}(H) \subseteq G$.

Proof:

Suppose h is $\mathfrak{N}\mathcal{M}cf$. Let K be any $\mathfrak{N}c$ set of V_1 and G is a $\mathfrak{N}o$ set in U_1 containing $h^{-1}(K)$. By Theorem 4.5, $\exists$ $\mathfrak{N}\mathcal{M}o$ set F of $V_1 \ni K \subseteq F$ and $h^{-1}(F) \subseteq G$. Since K is $\mathfrak{N}c$ and F is a $\mathfrak{N}\mathcal{M}o$ set containing K, then $K \subseteq \mathfrak{N}\mathcal{M}int(F)$. Put $H = \mathfrak{N}\mathcal{M}int(F)$. Then $K \subseteq H \in \mathfrak{N}\mathcal{M}O(V_1, Q)$ and $h^{-1}(H) \subseteq G$. ∎

Theorem 4.7.

Let $h\colon (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ and $g\colon (V_1, \tau_{R'}(Q)) \to (W_1, \mu_{R''}(R))$ be any two functions. Then, $g \circ h\colon (U_1, \tau_R(P)) \to (W_1, \sigma_{R''}(R))$ is a $\mathfrak{N}\mathcal{M}cf$ if h is $\mathfrak{N}c$ and g is a $\mathfrak{N}\mathcal{M}cf$.

Proof:

Suppose F is a $\mathfrak{N}c$ set in U_1. Since h is a $\mathfrak{N}cf$, $h(F)$ is a $\mathfrak{N}c$ set in V_1. Now g is a $\mathfrak{N}\mathcal{M}cf$, implies $g(h(F)) = (g \circ h)(F)$ is a $\mathfrak{N}\mathcal{M}c$ set in W_1. Hence $g \circ h$ is a $\mathfrak{N}\mathcal{M}cf$. ∎

Theorem 4.8.

Let $h\colon (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ and $g\colon (V_1, \tau_{R'}(Q)) \to (W_1, \mu_{R''}(R))$ be any two functions such that $g \circ h\colon (U_1, \tau_R(P)) \to (W_1, \sigma_{R''}(R))$ be a $\mathfrak{N}\mathcal{M}cf$. Then, the following results hold.

(i) If h is $\mathfrak{N}$-Cts surjection, then g is a $\mathfrak{N}\mathcal{M}cf$.
(ii) If g is $\mathfrak{N}\mathcal{M}Irr$ and injective, then h is a $\mathfrak{N}\mathcal{M}cf$.

Proof:

(i) Suppose F_1 is a $\mathfrak{N}c$ set in V_1. Since h is a $\mathfrak{N}\ Cts$ function, $h^{-1}(F_1)$ is a $\mathfrak{N}c$ set in U_1. Therefore, $(g \circ h)(h^{-1}(F_1)) = g(F_1)$ is a $\mathfrak{N}\mathcal{M}c$ set in W_1. Hence, g is a $\mathfrak{N}\mathcal{M}cf$.

(ii) Suppose F_1 is $\mathfrak{N}c$ set in U_1. Then, $(g \circ h)(F_1)$ is a $\mathfrak{N}\mathcal{M}c$ set in W_1. Since g is a $\mathfrak{N}\mathcal{M}Irr$ function, this implies $g^{-1}((g \circ h)(F_1)) = h(F_1)$ is a $\mathfrak{N}\mathcal{M}c$ set in V_1. Hence, h is a $\mathfrak{N}\mathcal{M}cf$. ■

5. Nano $\mathcal{M}$ Homeomorphisms

Definition 5.1.

Let $(U_1, \tau_R(P))$ and $(V_1, \sigma_{R'}(Q))$ be $\mathfrak{N}ts$ and let $h\colon (U_1, \tau_R(P)) \to (V_1, \sigma_{R''}(Q))$ be a bijective function. If both the function h and the inverse function h^{-1} are nano $\mathcal{M}$ (respectively, θ, δ, θ semi, δ pre, δ semi and e) Cts (briefly, $\mathfrak{N}\mathcal{M}$ (respectively, $\mathfrak{N}\theta$, $\mathfrak{N}\delta$, $\mathfrak{N}\theta\mathcal{S}$, $\mathfrak{N}\delta\mathcal{P}$, $\mathfrak{N}\delta\mathcal{S}$ and $\mathfrak{N}e$) Cts), then h is called $\mathfrak{N}\mathcal{M}$ (respectively, $\mathfrak{N}\theta$, $\mathfrak{N}\delta$, $\mathfrak{N}\theta\mathcal{S}$, $\mathfrak{N}\delta\mathcal{P}$, $\mathfrak{N}\delta\mathcal{S}$ and $\mathfrak{N}e$) homeomorphism (briefly, $\mathfrak{N}\mathcal{M}$ (respectively, $\mathfrak{N}\theta$, $\mathfrak{N}\delta$, $\mathfrak{N}\theta\mathcal{S}$, $\mathfrak{N}\delta\mathcal{P}$, $\mathfrak{N}\delta\mathcal{S}$ and $\mathfrak{N}e$) Hom). Equivalently, if h both $\mathfrak{N}\mathcal{M}$ (respectively, $\mathfrak{N}\theta$, $\mathfrak{N}\delta$, $\mathfrak{N}\theta\mathcal{S}$, $\mathfrak{N}\delta\mathcal{P}$, $\mathfrak{N}\delta\mathcal{S}$ and $\mathfrak{N}e$) Cts and $\mathfrak{N}\mathcal{M}o$ (respectively, $\mathfrak{N}\theta o$, $\mathfrak{N}\delta o$, $\mathfrak{N}\theta\mathcal{S}o$, $\mathfrak{N}\delta\mathcal{P}o$, $\mathfrak{N}\delta\mathcal{S}o$ and $\mathfrak{N}eo$) then h is called $\mathfrak{N}\mathcal{M}$ (respectively, $\mathfrak{N}\theta$, $\mathfrak{N}\delta$, $\mathfrak{N}\theta\mathcal{S}$, $\mathfrak{N}\delta\mathcal{P}$, $\mathfrak{N}\delta\mathcal{S}$ and $\mathfrak{N}e)Hom$.

The family of all $\mathfrak{N}\mathcal{M}Hom$'s in U_1 is denoted by $\mathfrak{N}\mathcal{M}H(U_1, P)$.

Theorem 5.1.

Let $h : (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$,

(i) Every $\mathfrak{N}\theta Hom$ is $\mathfrak{N}Hom$.
(ii) Every $\mathfrak{N}\theta Hom$ is $\mathfrak{N}\delta Hom$.
(iii) Every $\mathfrak{N}\delta Hom$ is $\mathfrak{N}Hom$.
(iv) Every $\mathfrak{N}Hom$ is $\mathfrak{N}\delta\mathcal{P}Hom$.
(v) Every $\mathfrak{N}\theta\mathcal{S}Hom$ is $\mathfrak{N}\mathcal{M}Hom$.
(vi) Every $\mathfrak{N}\delta\mathcal{P}Hom$ is $\mathfrak{N}\mathcal{M}Hom$.
(vii) Every $\mathfrak{N}\delta\mathcal{P}Hom$ is $\mathfrak{N}eHom$.
(viii) Every $\mathfrak{N}\delta\mathcal{S}Hom$ is $\mathfrak{N}eHom$.
(ix) Every $\mathfrak{N}\mathcal{M}Hom$ is $\mathfrak{N}eHom$.

but not conversely.

Proof:

(i) Let $h : (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ be $\mathfrak{N}\theta Hom$. Then, h and h^{-1} are $\mathfrak{N}\theta\ Cts$ and h is bijection. Since every $\mathfrak{N}\theta\ Cts$ function is $\mathfrak{N}\ Cts$, we have h and h^{-1} are $\mathfrak{N}\ Cts$. Therefore, h is $\mathfrak{N}Hom$.

(ii) Let $h : (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ be $\mathfrak{N}\theta Hom$. Then, h and h^{-1} are $\mathfrak{N}\theta\ Cts$ and h is bijection. Since every $\mathfrak{N}\theta\ Cts$ function is $\mathfrak{N}\delta\ Cts$, we have h and h^{-1} are $\mathfrak{N}\delta\ Cts$. Therefore, h is $\mathfrak{N}\delta Hom$.

(iii) Let $h : (U_1, \tau_R(P)) \rightarrow (V_1, \sigma_{R'}(Q))$ be $\mathfrak{N}\delta Hom$. Then, h and h^{-1} are $\mathfrak{N}\delta\ Cts$ and h is bijection. Since every $\mathfrak{N}\delta\ Cts$ function is $\mathfrak{N}\ Cts$, we have h and h^{-1} are $\mathfrak{N}\ Cts$. Therefore, h is $\mathfrak{N}Hom$.

(iv) Let $h : (U_1, \tau_R(P)) \rightarrow (V_1, \sigma_{R'}(Q))$ be $\mathfrak{N}Hom$. Then, h and h^{-1} are $\mathfrak{N}\ Cts$ and h is bijection. Since every $\mathfrak{N}\ Cts$ function is $\mathfrak{N}\delta\mathcal{P}\ Cts$, we have h and h^{-1} are $\mathfrak{N}\delta\mathcal{P}\ Cts$. Therefore, h is $\mathfrak{N}\delta\mathcal{P}Hom$.

(v) Let $h : (U_1, \tau_R(P)) \rightarrow (V_1, \sigma_{R'}(Q))$ be $\mathfrak{N}\theta\mathcal{S}Hom$. Then, h and h^{-1} are $\mathfrak{N}\theta\mathcal{S}\ Cts$ and h is bijection. Since every $\mathfrak{N}\theta\mathcal{S}\ Cts$ function is $\mathfrak{N}\mathcal{M}\ Cts$, we have h and h^{-1} are $\mathfrak{N}\mathcal{M}\ Cts$. Therefore, h is $\mathfrak{N}\mathcal{M}Hom$.

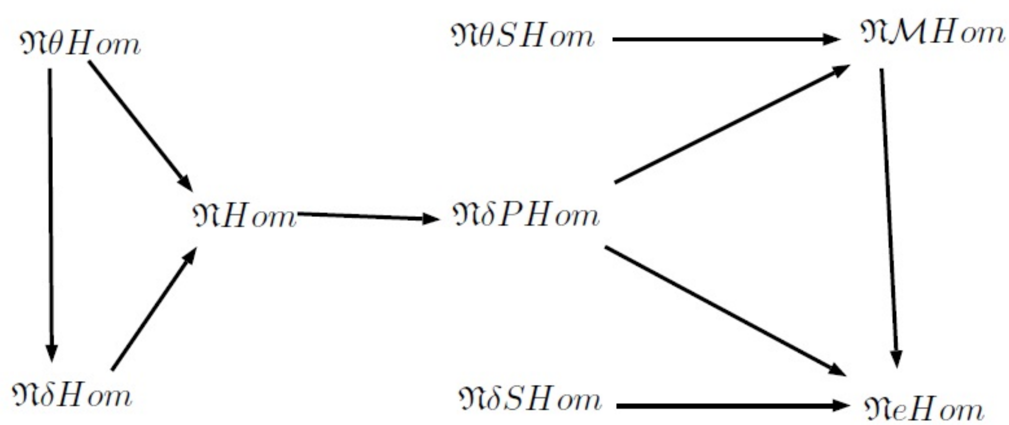
(vi) Let $h : (U_1, \tau_R(P)) \rightarrow (V_1, \sigma_{R'}(Q))$ be $\mathfrak{N}\delta\mathcal{P}Hom$. Then, h and h^{-1} are $\mathfrak{N}\delta\mathcal{P}\ Cts$ and h is bijection. Since every $\mathfrak{N}\delta\mathcal{P}\ Cts$ function is $\mathfrak{N}\mathcal{M}\ Cts$, we have h and h^{-1} are $\mathfrak{N}\mathcal{M}\ Cts$. Therefore, h is $\mathfrak{N}\mathcal{M}Hom$.

(vii) Let $h : (U_1, \tau_R(P)) \rightarrow (V_1, \sigma_{R'}(Q))$ be $\mathfrak{N}\delta\mathcal{P}Hom$. Then, h and h^{-1} are $\mathfrak{N}\delta\mathcal{P}\ Cts$ and h is bijection. Since every $\mathfrak{N}\delta\mathcal{P}\ Cts$ function is $\mathfrak{N}e\ Cts$, we have h and h^{-1} are $\mathfrak{N}e\ Cts$. Therefore, h is $\mathfrak{N}eHom$.

(viii) Let $h : (U_1, \tau_R(P)) \rightarrow (V_1, \sigma_{R'}(Q))$ be $\mathfrak{N}\delta\mathcal{S}Hom$. Then, h and h^{-1} are $\mathfrak{N}\delta\mathcal{S}\ Cts$ and h is bijection. Since every $\mathfrak{N}\delta\mathcal{S}\ Cts$ function is $\mathfrak{N}e\ Cts$, we have h and h^{-1} are $\mathfrak{N}e\ Cts$. Therefore, h is $\mathfrak{N}eHom$.

(ix) Let $h : (U_1, \tau_R(P)) \rightarrow (V_1, \sigma_{R'}(Q))$ be $\mathfrak{N}\mathcal{M}Hom$. Then, h and h^{-1} are $\mathfrak{N}\mathcal{M}\ Cts$ and h is bijection. Since every $\mathfrak{N}\mathcal{M}\ Cts$ function is $\mathfrak{N}e\ Cts$, we have h and h^{-1} are $\mathfrak{N}e\ Cts$. Therefore, h is $\mathfrak{N}eHom$. ■

From the above discussions, the following implications hold for any set in $\mathfrak{N}ts$.



Note: $K \rightarrow L$ denotes K implies L, but not conversely.

Example 5.1.

Let $U_1 = V_1 = \{M_a, M_b, M_c, M_d, M_e\}$ with $U_1/R = \{\{M_c\}, \{M_a, M_b\}, \{M_d, M_e\}\}$ and $X = \{M_a, M_c\}$. Then, $\tau_R(X) = \{U_1, \phi, \{M_c\}, \{M_a, M_b\}, \{M_a, M_b, M_c\}\}$. Then, the identity map $h : (U_1, \tau_R(X)) \to (V_1, \tau_R(X))$ is $\mathfrak{N}Hom$ (respectively, $\mathfrak{N}\delta Hom$, $\mathfrak{N}\mathcal{M}Hom$), but h is not $\mathfrak{N}\theta Hom$ (respectively, $\mathfrak{N}\theta Hom$, $\mathfrak{N}\theta\mathcal{S}Hom$), since

(i) $h^{-1}(\{M_c\}) = \{M_c\}$ which is not $\mathfrak{N}\theta o$ (respectively, $\mathfrak{N}\theta o$) in U_1 whereas $\{M_c\}$ is $\mathfrak{N}o$ (respectively, $\mathfrak{N}o$) in V_1.
(ii) $h^{-1}(\{M_a, M_b\}) = \{M_a, M_b\}$ which is not $\mathfrak{N}\theta\mathcal{S}o$ in U_1 whereas $\{M_a, M_b\}$ is $\mathfrak{N}o$ in V_1.

Example 5.2.

Let $U_1 = V_1 = \{M_a, M_b, M_c, M_d, M_e\}$ with $U_1/R = \{\{M_a\}, \{M_b\}, \{M_c, M_d, M_e\}\}$ and $X = \{M_c, M_d, M_e\}$. Then, $\tau_R(X) = \{U_1, \phi, \{M_c, M_d, M_e\}\}$. Then, the identity map $h : (U_1, \tau_R(X)) \to (V_1, \tau_R(X))$ is $\mathfrak{N}Hom$, but h is not $\mathfrak{N}\delta Hom$, since $h^{-1}(\{M_c, M_d, M_e\}) = \{M_c, M_d, M_e\}$ which is not $\mathfrak{N}\delta o$ in U_1 whereas $\{M_c, M_d, M_e\}$ is $\mathfrak{N}o$ in V_1.

Example 5.3.

Let $U_1 = V_1 = \{M_a, M_b, M_c, M_d, M_e\}$ with $U_1/R = \{\{M_a\}, \{M_b\}, \{M_c, M_d, M_e\}\}$ and $X = \{M_a, M_c, M_d\}$. Then, $\tau_R(X) = \{U_1, \phi, \{M_a\}, \{M_c, M_d, M_e\}, \{M_a, M_c, M_d, M_e\}\}$, $V_1/R' = \{\{M_e\}, \{M_a, M_b\}, \{M_c, M_d\}\}$ and $Y = \{M_a, M_c\}$. Then, $\sigma_{R'}(Y) = \{V_1, \phi, \{M_c\}, \{M_a, M_b\}, \{M_a, M_b, M_c\}\}$. Define $h : (U_1, \tau_R(X)) \to (V_1, \sigma_{R'}(Y))$ as $h(M_a) = M_a$, $h(M_b) = M_d$, $h(M_c) = M_c$, $h(M_d) = M_b$ and $h(M_e) = M_e$. Then, h is $\mathfrak{N}\delta\mathcal{P}Hom$, but h is not $\mathfrak{N}Hom$, since $h^{-1}(\{M_c\}) = \{M_c\}$ which is not $\mathfrak{N}o$ in U_1 whereas $\{M_c\}$ is $\mathfrak{N}o$ in V_1.

Example 5.4.

Let $U_1 = V_1 = \{M_a, M_b, M_c, M_d, M_e\}$ with $U_1/R = \{\{M_a\}, \{M_b\}, \{M_c, M_d, M_e\}\}$ and $X = \{M_a, M_c, M_d\}$. Then, $\tau_R(X) = \{U_1, \phi, \{M_a\}, \{M_c, M_d, M_e\}, \{M_a, M_c, M_d, M_e\}\}$, $V_1/R' = \{\{M_c\}, \{M_a, M_b\}, \{M_d, M_e\}\}$ and $Y = \{M_a, M_c\}$. Then, $\sigma_{R'}(Y) = \{V_1, \phi, \{M_c\}, \{M_a, M_b\}, \{M_a, M_b, M_c\}\}$. Then, the identity map $h : (U_1, \tau_R(X)) \to (V_1, \sigma_{R'}(Y))$ is $\mathfrak{N}\mathcal{M}Hom$, but h is not $\mathfrak{N}\delta\mathcal{P}Hom$, since $h^{-1}(\{M_a, M_b\}) = \{M_a, M_b\}$ which is not $\mathfrak{N}\delta\mathcal{P}o$ in U_1 whereas $\{M_a, M_b\}$ is $\mathfrak{N}o$ in V_1.

Example 5.5.

Let $U_1 = V_1 = \{L_a, L_b, L_c, L_d, L_e\}$ with $U_1/R = \{\{L_c\}, \{L_a, L_b\}, \{L_d, L_e\}\}$ and $X = \{L_a, L_c\}$. Then, $\tau_R(X) = \{U_1, \phi, \{L_c\}, \{L_a, L_b\}, \{L_a, L_b, L_c\}\}$, $V_1/R' = \{\{L_e\}, \{L_a, L_b\}, \{L_c, L_d\}\}$ and $Y = \{L_a, L_c, L_d\}$. Then, $\sigma_{R'}(Y) = \{V_1, \phi, \{L_a, L_b\}, \{L_c, L_d\}, \{L_a, L_b, L_c, L_d\}\}$. Then,

(i) the identity map $h : (U_1, \tau_R(X)) \to (V_1, \sigma_{R'}(Y))$ is $\mathfrak{N}eHom$, but h is not $\mathfrak{N}\delta\mathcal{S}Hom$, since $h(\{L_c\}) = \{L_c\}$ which is not $\mathfrak{N}\delta\mathcal{S}o$ in V_1 whereas $\{L_c\}$ is $\mathfrak{N}o$ in U_1.
(ii) the identity map $h : (V_1, \sigma_{R'}(Y)) \to (U_1, \tau_R(X))$ is $\mathfrak{N}eHom$, but h is not $\mathfrak{N}\delta\mathcal{P}Hom$, since $h(\{L_c, L_d\}) = \{L_c, L_d\}$ which is not $\mathfrak{N}\delta\mathcal{P}o$ in V_1 whereas $\{L_c, L_d\}$ is $\mathfrak{N}o$ in U_1.

Example 5.6.

Let $U_1 = V_1 = \{L_a, L_b, L_c, L_d, L_e\}$ with $U_1/R = \{\{L_c\}, \{L_a, L_b\}, \{L_d, L_e\}\}$ and $X = \{L_a, L_c\}$. Then, $\tau_R(X) = \{U_1, \phi, \{L_c\}, \{L_a, L_b\}, \{L_a, L_b, L_c\}\}$, $V_1/R' = \{\{L_e\}, \{L_a, L_b\}, \{L_c, L_d\}\}$ and $Y = \{L_a, L_c, L_d\}$. Then, $\sigma_{R'}(Y) = \{V_1, \phi, \{L_a, L_b\}, \{L_c, L_d\}, \{L_a, L_b, L_c, L_d\}\}$. Define $h : (U_1, \tau_R(X)) \to (V_1, \sigma_{R'}(Y))$ as $h(L_a) = L_c$, $h(L_b) = L_d$, $h(L_c) = L_a$, $h(L_d) = L_b$ and $h(L_e) = L_e$. Then, h is $\mathfrak{N}eHom$, but h is not $\mathfrak{N}\mathcal{M}Hom$, since $h^{-1}(\{L_a, L_b\}) = \{L_c, L_d\}$ which is not $\mathfrak{N}\mathcal{M}o$ in U_1 whereas $\{L_a, L_b\}$ is $\mathfrak{N}o$ in V_1.

Theorem 5.2.

For any bijection $h\colon (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q)$ the following statements are equivalent:

(i) Inverse of h is $\mathfrak{N}\mathcal{M}Cts$.

(ii) h is a $\mathfrak{N}\mathcal{M}of$.

(iii) h is a $\mathfrak{N}\mathcal{M}cf$

Proof:

(i) $\Rightarrow$ (ii): Suppose G_1 is a $\mathfrak{N}o$ set in U_1. Then by (i), $(h^{-1})^{-1}(G_1) = h(G_1)$ is a $\mathfrak{N}\mathcal{M}o$ set in V_1, and hence, h is a $\mathfrak{N}\mathcal{M}of$.

(ii) $\Rightarrow$ (iii): Suppose F_1 is $\mathfrak{N}c$ in U_1. Then $U_1 - F_1$ is $\mathfrak{N}o$ in U_1. By (ii), $h(U_1 - F_1) = V_1 - h(F_1)$ is a $\mathfrak{N}\mathcal{M}o$ set in V_1 which implies $h(F_1)$ is a $\mathfrak{N}\mathcal{M}c$ set in V_1. Therefore, h is $\mathfrak{N}\mathcal{M}cf$.

(iii) $\Rightarrow$ (i): Let F_1 be a $\mathfrak{N}c$ set in U_1. By (iii), $h(F_1) = (h^{-1})^{-1}(F_1)$ is a $\mathfrak{N}\mathcal{M}c$ set in V_1, and hence, the inverse of h is a $\mathfrak{N}\mathcal{M}Cts$ function. ∎

Theorem 5.3.

If $h\colon (U_1, \tau_R(P)) \to (V_1, \sigma_{R'}(Q))$ is bijective and $\mathfrak{N}\mathcal{M}Cts$, then the following statements are equivalent:

(i) h is $\mathfrak{N}\mathcal{M}o$.

(ii) h is a $\mathfrak{N}\mathcal{M}Hom$.

(iii) h is a $\mathfrak{N}\mathcal{M}c$

Proof:

(i) $\Rightarrow$ (ii): By the assumption h is bijective, $\mathfrak{N}\mathcal{M}Cts$ and $\mathfrak{N}\mathcal{M}o$. Then, by definition, h is $\mathfrak{N}\mathcal{M}Hom$.

(ii) $\Rightarrow$ (iii): By the assumption h is bijective and $\mathfrak{N}\mathcal{M}o$. Then, by Theorem 5.2, h is $\mathfrak{N}\mathcal{M}c$.

(iii) $\Rightarrow$ (i): By the assumption h is bijective and $\mathfrak{N}\mathcal{M}c$. Then, by Theorem 5.2, h is $\mathfrak{N}\mathcal{M}o$. ∎

6. Conclusion

In this paper, we have studied many interesting notions on various forms of nano $\mathcal{M}$ open sets such as nano $\mathcal{M}$-continuous and nano $\mathcal{M}$-irresolute functions in a nano topological spaces along with their continuous and irresolute mappings. Also discussed were nano $\mathcal{M}$-open and nano $\mathcal{M}$-closed functions, and these were compared with their near open and closed mappings in a nano topological spaces. Finally, we discussed nano $\mathcal{M}$ homeomorphisms in nano topological spaces and studied some of their properties. In future work, nano $\mathcal{M}$ open sets can be applied in an application field of real-life experience.

Zadeh (1965) introduced the concept of a fuzzy set (FS) to the world. In FS theory, the membership value of each element in a set is specified by a real number from the closed interval of $[0, 1]$. Later, Atanassov (1989) defined the notion of an intuitionistic fuzzy set (IFS) as an extension of FS. In IFS theory, the elements are assumed to posses both membership and non-membership values with the condition that their sum does not exceed unity. Also, Atanassov (1989) established some properties of IFS.

Lellis Thivagar and Richard (2013) introduced the notion of Nano topology (briefly, $\mathfrak{N}\mathfrak{T}$) by using theory approximations and boundary region of a subset of an universe in terms of an equivalence relation on it and also defined Nano closed (briefly, $\mathfrak{N}c$) sets, Nano-interior (briefly, $\mathfrak{N}int$) and Nano-closure (briefly, $\mathfrak{N}cl$) in a nano topological spaces (briefly, $\mathfrak{N}ts$).

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