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Effect of Rotation on Plane Waves of Generalized Magneto-Thermoelastic Medium with Voids under Thermal Loading Due to Laser Pulse

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Abstract

The investigation in this paper deals with the rotation of the magneto-thermoelastic solid and with voids subjected to thermal loading due to laser pulse. The problem is studied in the context of three theories of generalized magneto thermoelasticity: Lord-Schulman (L-S), Green-Lindsay (G-L) and the coupled theory (CD) with the effect of rotation, magnetic field, thermal loading and voids. The methodology applied here is using the normal mode analysis to solve the physical problem to obtain the exact expressions for the displacement components, the stresses, the temperature, and the change in the volume fraction field have been shown graphically by comparison between three theories, in the presence and the absence of rotation, magnetic field and for two different values of time on thermoelastic material in the presence of voids.

Keywords: Rotation; Magnetic field; Voids; Laser pulse; Coupled theory; Lord-Shulman; Green-Lindsay; Thermoelasticity

MSC 2010 No.: 74A45, 93A30

1. Introduction

Thermoelasticity theories, which admit a finite speed for thermal signals, have received a lot of attention for the past four decades. In contrast to the coupled thermoelasticity theory based on a parabolic heat equation Biot (1956), which predicts an infinite speed of the propagation of heat, these theories involve a hyperbolic heat equation and are referred to as generalized thermoelasticity theories. The first generalization, for isotropic bodies, is due to the Lord and Shulman (1967) who obtained a wave-type heat equation by postulating a new law of heat conduction to replace the classical Fourier's law. The second generalization is known as the theory of thermoelasticity with two relaxation times, or the theory of temperature-rate-dependent thermoelasticity and was proposed by the Green and Lindsay (1972). It is based on a form of the entropy inequality proposed by Green and Laws (1972). Some researches in the past have investigated different problems of rotating media. In a paper by Schoenberg and Censor (1973), the propagation of plane harmonic waves in a rotating elastic medium without a thermal field has been studied. It was shown there that the rotation causes the elastic medium to be depressive and anisotropic. Many authors [Chand et al. (1990), Othman (2005), Othman and Singh (2007), Samia et al. (2017)] studied the effect of rotation on elastic waves.

These problems are based on the more realistic elastic model since earth, the moon and other planets have angular velocity. Othman [(2004), (2005), (2010)] used the normal mode analysis to study the effect of rotation on plane waves in generalized thermoelasticity with one and two relaxation times. The theory of magneto-thermo-elasticity is concerned with the interacting effects of applied magnetic field on the elastic and thermoelastic deformations of a solid body. This theory has aroused much interest in many industrial appliances, particularly in nuclear devices, where, there exists a primary magnetic field; various investigations are to be carried out by considering the interaction between magnetic, thermal and strain fields. Analyses of such problems also influence various applications in biomedical engineering as well as in different geomagnetic studies.

The development of the interaction of electromagnetic field, the thermal field and the elastic field is available in many works such as Choudhuri and Debnath (1985), Abbas and Youssef (2009), Abd-Alla (2000), Othman (2005), Othman et al. (2018), Bhatti et al. [(2016a, b, c), (2017)], Ellahi et al. [(2016), (2017)], Hassan et al. (2017), Majeed et al. (2016), and Zeeshan et al. (2016). Problems related to magneto-thermoelasticity with thermal relaxation times have been investigated by Othman et al. (2009). The theory of linear elastic materials with voids is one of the most important generalizations of the classical theory of elasticity. This theory is useful for investigating various types of geological and biological materials for which the elastic theory is inadequate. This theory is concerned with elastic materials consisting of a distribution of small porous (voids), in which the void volume is included among the kinematics variables and in the case of vanishing this volume it reduces to the classical theory of elasticity.

A theory of thermoelastic materials with voids and without energy dissipation is developed by Cicco and Diaco (2002). Puri and Cowin (1985) studied the behavior of plane waves in a linear elastic material with voids. Iesan (1986) presented a linear theory for thermoelastic material with voids. He derived the basic equations and proved the uniqueness of the solution, reciprocity relation and variation characterization of the solution in the dynamical theory. Nunziato and Cowin (1979) studied a nonlinear theory of elastic materials with voids. Chandrasekharaiah (1987) investigated the plane waves in a rotating elastic solid with voids. The domain of influence theorem in the linear theory of elastic materials with voids was discussed by Dhaliwal and Wang (1994). Well-posedness theorems for linear elastic materials with voids studied by Scarpetta (1995). The study of dynamic response of an isotropic generalized thermoelastic solid with additional parameters is helpful in solving many practical problems. Initial stresses are developed in the medium due to many reasons, resulting from difference of temperature, the earth is supposed to be under high initial stresses. It is therefore of great interest to study the effect of these stresses on the mechanical and thermal state of the medium.

The so-called ultra-short lasers are those with pulse duration ranging from nanoseconds to femto-seconds. In the case of ultra-short-pulsed laser heating, the high-intensity energy flux and ultra-short duration laser beam have introduced situations where, very large thermal gradients or an ultra-high heating rate may exist on the boundaries by Sun et al. (2008). Researchers have proposed several models to describe the mechanism of heat conduction during short-pulse laser heating. It has been found that usually the micro-scopic two-step models, that is, parabolic and hyperbolic are useful for modification material as thin films. When a metal film is heated by a laser pulse, a thermoelastic wave is generated due to thermal expansion near the surface. Wang and Xu (2002) have studied the stress wave induced by pico and femto-second laser pulses in a semi-infinite metal by expressing the laser pulse energy as a Fourier series. Othman and Eraki (2018) studied the effect of gravity on generalized thermoelastic diffusion due to laser pulse using dual-phase-lag model.

The present article is proposed to determine the components of displacement, the stresses, the temperature distribution, and the volume fraction field in a homogenous, linear, isotropic, thermoelastic solid with voids in the case of the absence and presence of the rotation, the magnetic field and two values of time. The model is illustrated in the context of (L-S), (G-L) and (CD) theories. The normal mode analysis is used to obtain the exact expressions for physical quantities. The distributions of the field quantities are represented graphically.

2. Formulation of the problem and basic equations

Consider a homogeneous, linear, isotropic, thermoelastic material with voids with a half space $(x \geq 0)$, the rectangular Cartesian coordinate system (x, y, z) having originated on the surface $z = 0$, for two dimensional problem assume the dynamic displacement vector as $\boldsymbol{u} = (u, v, 0)$. The elastic material is rotating uniformly with an angular velocity $\boldsymbol{\Omega} = \Omega\,\mathbf{n}$ since $\mathbf{n}$ is a unit vector representing the direction of the axis of rotation. All quantities considered will be a function of the time variable t, and of the coordinates x and y. The displacement equation of motion in a rotating frame has two additional terms, according to Schoenberg and Censor (1973) the centripetal acceleration, $\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \boldsymbol{u})$ due to the time varying motion only and the Coriolis acceleration $2\boldsymbol{\Omega} \times \dot{\boldsymbol{u}}$, where, $\boldsymbol{\Omega} = (0, 0, \Omega)$. These terms, do not appear in non-rotating media. A uniform magnetic field $\boldsymbol{H} = (0, 0, H_0)$ is acting towards the associative direction of z - axis. Due to the application of this magnetic field, there results an induced magnetic field $\boldsymbol{h}$ and an induced electric field $\boldsymbol{E}$, assume that both $\boldsymbol{h}$ and $\boldsymbol{E}$ are small in magnitude in accordance either the assumptions of the linear theory of thermoelasticity. In these equations a dot denotes differentiation with respect to time, while a comma denotes material derivatives. The initial magnetic field vector $\boldsymbol{H}$ is given by $H_x = 0,\ H_y = 0,\ H_x = 0,\ H_z = H_0$.

The electric field intensity vector is normal to both the magnetic intensity and the displacement vector. Therefore $\boldsymbol{E}$ have the components $E_x = E_1, E_y = E_2,\ E_z = 0$.

The simplified linear equations of electrodynamics for a slowly moving perfectly conducting medium given by Maxwell's equations

$$\boldsymbol{J} = \operatorname{curl} \boldsymbol{h} - \varepsilon_0 \dot{\boldsymbol{E}}, \tag{1}$$

$$\boldsymbol{E} = -\mu_0 \left(\dot{\boldsymbol{u}} \times \boldsymbol{H} \right), \tag{2}$$

$$\operatorname{curl} \boldsymbol{E} = -\mu_0 \dot{\boldsymbol{h}}, \tag{3}$$

$$\operatorname{div} \boldsymbol{h} = 0. \tag{4}$$

From Equations (1)-(4) one can obtain

$$E_1 = -\mu_0 H_0 \dot{v}, \quad E_2 = \mu_0 H_0 \dot{u},$$

$$E_3 = 0, \quad h = -H_0 \left(0, 0, e\right),$$

$$J_1 = -H_0 \frac{\partial e}{\partial y} + \varepsilon_0 \mu_0 H_0 \frac{\partial^2 v}{\partial t^2},$$

$$J_2 = H_0 \frac{\partial e}{\partial x} - \varepsilon_0 \mu_0 H_0 \frac{\partial^2 u}{\partial t^2}, \qquad J_3 = 0.$$

The system of governing equations of an effect of rotation on a generalized magneto-thermoelasticity medium with voids under thermal loading due to laser pulse with three theories (L-S), (G-L), (CD) and without body forces, as

$$\mu \nabla^2 \boldsymbol{u} + \left(\lambda + \mu\right) \nabla \left(\nabla . \boldsymbol{u}\right) + b \nabla \phi - \beta \left(\nabla T + \upsilon_0 \nabla \dot{T}\right) + \boldsymbol{F} = \rho \left[\ddot{\boldsymbol{u}} + \boldsymbol{\Omega} \times \left(\boldsymbol{\Omega} \times \boldsymbol{u}\right) + 2 \boldsymbol{\Omega} \times \dot{\boldsymbol{u}} \right], \tag{5}$$

$$\alpha \nabla^2 \phi - b e - \xi \phi - \omega_0 \dot{\phi} + m \left(T + \upsilon_0 \frac{\partial T}{\partial t} \right) = \rho \chi \ddot{\phi}, \tag{6}$$

$$K \nabla^2 T = \rho C_E \left(\frac{\partial T}{\partial t} + \tau_0 \frac{\partial^2 T}{\partial t^2} \right) + \beta T_0 \left(\frac{\partial e}{\partial t} + n_0 \tau_0 \frac{\partial^2 e}{\partial t^2} \right) + m T_0 \left(1 + n_0 \tau_0 \frac{\partial}{\partial t} \right) \phi - \rho \left(Q + n_0 \tau_0 \frac{\partial Q}{\partial t} \right), \tag{7}$$

$$\sigma_{ij} = \lambda \left(\nabla . \boldsymbol{u}\right) \delta_{ij} + 2 \mu e_{ij} + b \phi \delta_{ij} - \beta \left(T + \upsilon_0 \frac{\partial T}{\partial t} \right) \delta_{ij}, \quad i, j = 1, 2 \tag{8}$$

$$e_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_i} + \frac{\partial u_j}{\partial x_j} \right), \ i, j = 1, 2. \tag{9}$$

Since $\boldsymbol{F}$ is the Lorentz force and given by $\boldsymbol{F} = \mu_0 \left(\boldsymbol{J} \times \boldsymbol{H} \right)$.

The components of the Lorentz force will be

$$F_1 = \mu_0 H_0^2 \frac{\partial e}{\partial x} - \varepsilon_0 \mu_0^2 H_0^2 \frac{\partial u^2}{\partial t^2}, \quad F_2 = \mu_0 H_0^2 \frac{\partial e}{\partial y} - \varepsilon_0 \mu_0^2 H_0^2 \frac{\partial v^2}{\partial t^2}, \quad F_3 = 0, \tag{10}$$

where, σ_{ij} are the components of stress tensor, e_{ij} are the components of strain, λ, μ are the Lamé constants, $\beta = (3\lambda + 2\mu)\alpha_t$ such that α_t is the coefficient of thermal expansion, δ_{ij} is the Kronecker delta, $i, j = x, y$, p is the initial stress, $\alpha, b, \xi, \omega_0, m, \chi$ are the material constants due to the presence of voids, ρ is the density, C_E is the specific heat at constant strain, n_0 is a parameter, τ_0, υ_0 are the thermal relaxation times, K is the thermal conductivity, T_0 is the reference temperature is chosen so that $|(T - T_0)/T_0| \ll 1$, ϕ is the change in the volume fraction field, and Q is the heat input of the laser pulse.

Equations (5)-(8) are the field equations of the generalized linear magneto-thermo-elasticity for a rotating media, applicable to the coupled theory, four generalizations, as follows:

1. The coupled (CD) theory, when

$$n_0 = 0, \quad \tau_0 = \upsilon_0 = 0.$$

2. Lord-Shulman (L-S) theory, when

$$n_0 = 1, \quad \upsilon_0 = 0, \; \tau_0 > 0.$$

3. Green-Lindsay (G-L) theory, when

$$n_0 = 0, \qquad \upsilon_0 \geq \tau_0 > 0.$$

The plate surface is illuminated by a laser pulse given by the heat input

$$Q = \frac{I_0 \gamma t}{2\pi r^2 t_0^2} \exp\left(\frac{-x^2}{r^2} - \frac{t}{t_0} - \gamma y \right), \tag{11}$$

where, I_0 is the energy absorbed, t_0 is the pulse rise time r is the beam radius and as a function of the depth y.

The components of stress tensor are

$$\sigma_{xx} = \lambda \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + 2\mu \frac{\partial u}{\partial x} + b\phi - \beta \left(T + \upsilon_0 \frac{\partial T}{\partial t} \right), \tag{12}$$

$$\sigma_{yy} = \lambda\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+2\mu\frac{\partial v}{\partial y}+b\phi-\beta\left(T+\upsilon_0\frac{\partial T}{\partial t}\right), \tag{13}$$

$$\sigma_{xy} = \mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right). \tag{14}$$

From Equations (10), (12)-(14) Equation (5) has the form

$$\mu\,\nabla^2 u+(\lambda+\mu)\frac{\partial e}{\partial x}+b\frac{\partial \phi}{\partial x}-\beta\left(\frac{\partial T}{\partial x}+\upsilon_0\frac{\partial^2 T}{\partial t\,\partial x}\right)+\mu_0 H_0^2\frac{\partial e}{\partial x}-\varepsilon_0\mu_0^2 H_0^2\frac{\partial^2 u}{\partial t^2}=\rho\left(\frac{\partial^2 u}{\partial t^2}-\Omega^2 u-2\Omega\frac{\partial v}{\partial t}\right), \tag{15}$$

$$\mu\,\nabla^2 v+(\lambda+\mu)\frac{\partial e}{\partial y}+b\frac{\partial \phi}{\partial y}-\beta\left(\frac{\partial T}{\partial y}+\upsilon_0\frac{\partial^2 T}{\partial t\,\partial y}\right)+\mu_0 H_0^2\frac{\partial e}{\partial y}-\varepsilon_0\mu_0^2 H_0^2\frac{\partial^2 v}{\partial t^2}=\rho\left(\frac{\partial^2 v}{\partial t^2}-\Omega^2 v-2\Omega\frac{\partial u}{\partial t}\right). \tag{16}$$

For simplification, the following non-dimensional variables are used:

$$(x', y') = \frac{\omega_1^*}{c_0}(x, y),\ (u', v') = \frac{\omega_1^*}{c_1}(u, v),\ \sigma'_{ij} = \frac{\sigma_{ij}}{\mu}\sigma_{ij},\ \phi' = \frac{\omega_1^*\chi_0}{c_1^2}\phi,\ T' = \frac{T}{T_0},\ t' = \omega_1^* t,$$

$$\Omega' = \frac{\Omega}{\omega_1^*},\ c_1^2 = \frac{\lambda+2\mu}{\rho},\ \omega_1^* = \frac{\rho C_E c_1^2}{k},\ \upsilon'_0 = \omega_1^*\upsilon_0,\ \tau_0' = \omega_1^*\tau_0,\ Q' = \frac{Q}{\omega_1^* T_0 C_E}. \tag{17}$$

From Equation (17) in Equations (15), (16), (6), (7) we get

$$\nabla^2 u + A_1\frac{\partial e}{\partial x}+A_2\frac{\partial \phi}{\partial x}-A_3\left(\frac{\partial T}{\partial x}+\upsilon_0\frac{\partial^2 T}{\partial t\,\partial x}\right) - A_4\frac{\partial^2 u}{\partial t^2}=A_5\left(-\Omega^2 u-2\Omega\frac{\partial v}{\partial t}\right), \tag{18}$$

$$\nabla^2 v + A_1\frac{\partial e}{\partial y}+A_2\frac{\partial \phi}{\partial y}-A_3\left(\frac{\partial T}{\partial y}+\upsilon_0\frac{\partial^2 T}{\partial t\,\partial y}\right) - A_4\frac{\partial^2 v}{\partial t^2}=A_5\left(-\Omega^2 v+2\Omega\frac{\partial u}{\partial t}\right), \tag{19}$$

$$\nabla^2\phi - A_6 e - A_7\phi - A_8\frac{\partial \phi}{\partial t}+A_9\left(T+\upsilon_0\frac{\partial T}{\partial t}\right)=A_{10}\frac{\partial^2\phi}{\partial t^2}, \tag{20}$$

$$\varepsilon_1\nabla^2 T - A_{11}\left(\frac{\partial\phi}{\partial t}+\mathrm{n}_0\tau_0\frac{\partial^2\phi}{\partial t^2}\right)-\left(\frac{\partial T}{\partial t}+\tau_0\frac{\partial^2 T}{\partial t^2}\right)-\varepsilon_2\left(\frac{\partial e}{\partial t}+\mathrm{n}_0\tau_0\frac{\partial^2 e}{\partial t^2}\right)=-\left(Q_0+\mathrm{n}_0\tau_0\frac{\partial Q_0}{\partial t}\right). \tag{21}$$

Also, the constitutive Equations (12)-(14) reduces to

$$\sigma_{xx} = A_{12}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+2\frac{\partial u}{\partial x}+A_2\phi-A_3\left(T+\upsilon_0\frac{\partial T}{\partial t}\right), \tag{22}$$

$$\sigma_{yy} = A_{12}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+2\frac{\partial v}{\partial y}+A_2\phi-A_3\left(T+\upsilon_0\frac{\partial T}{\partial t}\right), \tag{23}$$

$$\sigma_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right). \tag{24}$$

We define displacement potential ψ_1 and the vector potential $\boldsymbol{\psi}_2$ which relate to displacement components u and v as,

$$u = \frac{\partial \psi_1}{\partial x} + \frac{\partial \psi_2}{\partial y}, \quad v = \frac{\partial \psi_1}{\partial y} - \frac{\partial \psi_2}{\partial x}, \quad e = \nabla^2 \psi_1 \text{ and } \left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) = \nabla^2 \psi_2. \tag{25}$$

By substituting from Equation (25) in Equations (18)-(21), this yields

$$(1+A_1)\nabla^2\psi_1 + A_2\phi - A_3\left(T + \upsilon_0 \frac{\partial T}{\partial t}\right) - 2A_5\Omega\frac{\partial}{\partial t}\psi_2 = \left(A_4\frac{\partial^2}{\partial t^2} - A_5\Omega^2\right)\psi_1, \tag{26}$$

$$\left(\nabla^2 - A_4\frac{\partial^2}{\partial t^2} + A_5\Omega^2\right)\psi_2 + 2A_5\Omega\frac{\partial}{\partial t}\psi_1 = 0, \tag{27}$$

$$-A_6\nabla^2\psi_1 + \left(\nabla^2 - A_7 - A_8\frac{\partial}{\partial t} - A_{10}\frac{\partial^2}{\partial t^2}\right)\phi + A_9\left(T + \upsilon_0\frac{\partial T}{\partial t}\right) = 0, \tag{28}$$

$$-\varepsilon_2\left(\frac{\partial}{\partial t} + \mathrm{n}_0\tau_0\frac{\partial^2}{\partial t^2}\right)\nabla^2\psi_1 - A_{11}\left(\frac{\partial \phi}{\partial t} + \mathrm{n}_0\tau_0\frac{\partial^2\phi}{\partial t^2}\right) + \varepsilon_1\nabla^2 T - \left(\frac{\partial T}{\partial t} + \tau_0\frac{\partial^2 T}{\partial t^2}\right) = -\left(\mathrm{Q} + \mathrm{n}_0\tau_0\frac{\partial \mathrm{Q}}{\partial t}\right). \tag{29}$$

3. Normal mode analysis

The solution of the considered physical variable can be decomposed in terms of normal modes as the following form

$$\left[u, v, T, \psi_1, \psi_2, \phi, \sigma_{ij}\right](x, y, t) = \left[u^*, v^*, T^*, \psi_1^*, \psi_2^*, \phi^*, \sigma_{ij}^*\right](y)\exp\left[i\left(\omega t + a x\right)\right], \tag{30}$$

where, $\left[u^*, v^*, T^*, \psi_1^*, \psi_2^*, \phi^*, \sigma_{ij}^*\right](y)$ are the amplitudes of the functions $\left[u, v, T, \psi_1, \psi_2, \phi, \sigma_{ij}\right]$, ω is the complex time constant, $\mathrm{i} = \sqrt{-1}$ and a is the wave number in *x*-direction.
Using Equation (30) into Equations (26)-(29), then we have,

$$\left(\mathrm{D}^2 - S_2\right)\psi_1^* - S_3\psi_2^* + S_4\phi^* - S_5 T^* = 0, \tag{31}$$

$$\mathrm{S}_6\psi_1^* + \left(\mathrm{D}^2 - S_7\right)\psi_2^* = 0, \tag{32}$$

$$\left(-A_6\mathrm{D}^2 + S_8\right)\psi_1^* + \left(\mathrm{D}^2 - S_9\right)\phi^* + S_{10}T^* = 0, \tag{33}$$

$$S_{11}\left(\mathrm{D}^2 - a^2\right)\psi_1^* - S_{12}\phi^* + \left(\mathrm{D}^2 - S_{13}\right)T^* = -Q_0' f\,(x,t)\,\mathrm{e}^{-\gamma y}. \tag{34}$$

Eliminating ψ_2^*, ϕ^* and T^* among Equations (31)-(34), we get the following differential equation satisfied by ψ_1^* can be obtained

$$\left[\mathrm{D}^8 - A\,\mathrm{D}^6 + B\,\mathrm{D}^4 - C\,\mathrm{D}^2 + F\right]\psi_1^*(\mathrm{y}) = -\,l_1 Q_0' f\,(x,t)\exp(-\gamma y). \tag{35}$$

In a similar manner, it can be shown that ψ_2^*, ϕ^* and T^* satisfy the following equations

$$\left[\mathrm{D}^8 - A\,\mathrm{D}^6 + B\,\mathrm{D}^4 - C\,\mathrm{D}^2 + G\right]\psi_2^*(\mathrm{y}) = -\,l_2 Q_0' f\,(x,t)\exp(-\gamma y), \tag{36}$$

$$\left[\mathrm{D}^8 - A\,\mathrm{D}^6 + B\,\mathrm{D}^4 - C\,\mathrm{D}^2 + G\right]\phi^*(\mathrm{y}) = -\,l_3 Q_0' f\,(x,t)\exp(-\gamma y), \tag{37}$$

$$\left[\mathrm{D}^8 - A\mathrm{D}^6 + B\,\mathrm{D}^4 - C\,\mathrm{D}^2 + G\right]T^*(\mathrm{y}) = -\,l_4 Q_0' f\,(x,t)\exp(-\gamma y). \tag{38}$$

Equation (35) can be factored as

$$\left(\mathrm{D}^2 - k_1^2\right)\left(\mathrm{D}^2 - k_2^2\right)\left(\mathrm{D}^2 - k_3^2\right)\left(\mathrm{D}^2 - k_4^2\right)\psi_1^* = -\,\ell_1 Q_0' f\,(x,t)\,e^{-\gamma y}, \tag{39}$$

where, $k_n^2, (n = 1,2,3,4)$ are the roots of the characteristic equation of Equation (39).
The general solutions of Equations (36)-(38), bound as $y \to \infty$, are given by:

$$\psi_1 = \sum_{n=1}^{4} M_n\, e^{(-k_n y + i\omega t + i a x)} + N_1 \ell_1 Q_0' f_1(x,t) e^{-\gamma y}, \tag{40}$$

$$\psi_2 = \sum_{n=1}^{4} H_{1n} M_n\, e^{(-k_n y + i\omega t + i a x)} + N_1 \ell_2 Q_0' f_1(x,t) e^{-\gamma y}, \tag{41}$$

$$\phi = \sum_{n=1}^{4} H_{2n} M_n\, e^{(-k_n y + i\omega t + i a x)} + N_1 \ell_3 Q_0' f_1(x,t) e^{-\gamma y}, \tag{42}$$

$$T = \sum_{n=1}^{4} H_{3n} M_n\, e^{(-k_n y + i\omega t + i a x)} + N_1 \ell_4 Q_0' f_1(x,t) e^{-\gamma y}, \tag{43}$$

where, $M_n, (n = 1,2,3,4)$ are some coefficients.

To obtain the components of the displacement vector, substituting from Equations (40) and (41) in Equation (25), then

$$u = \sum_{n=1}^{4} V_{1n} M_n\, e^{(-k_n y + i\omega t + i a x)} - \left(\gamma \ell_2 + \frac{2x\,\ell_1}{r^2}\right) N_1 Q_0' f_1(x,t) e^{-\gamma y}, \tag{44}$$

$$v = -\sum_{n=1}^{4} V_{2n} M_n\, e^{(-k_n y + i\omega t + i a x)} + [\frac{2x\,\ell_2}{r^2} - \gamma \ell_1] N_1 Q_0' f_1(x,t) e^{-\gamma y}. \tag{45}$$

Substituting from Equations (40)-(45) in Equations (22)-(24), respectively, the stress components and the chemical potential can be written as follows:

$$\sigma_{xx} = \sum_{n=1}^{4} H_{4n} M_n e^{(-k_n y + i\omega t + iax)} + N_1 N_2 Q_0' f_1(x,t) e^{-\gamma y} - N_1 N_3 Q_0' f_2(x,t) e^{-\gamma y}, \tag{46}$$

$$\sigma_{yy} = \sum_{n=1}^{4} H_{5n} M_n e^{(-k_n y + i\omega t + iax)} + N_1 N_4 Q_0' f_1(x,t) e^{-\gamma y} - N_1 N_3 Q_0' f_2(x,t) e^{-\gamma y}, \tag{47}$$

$$\sigma_{xy} = -\sum_{n=1}^{4} H_{6n} M_n e^{(-k_n y + i\omega t + iax)} + N_1 N_5 Q_0' f_1(x,t) e^{-\gamma y}. \tag{48}$$

4. Boundary conditions

In this section, we need to consider the boundary conditions at $y = 0$, in order to determine the parameter $M_n, (n = 1, 2, 3, 4)$.

(1) The mechanical boundary condition

$$\sigma_{yy} = -P_1 e^{i(\omega t + ax)}, \qquad \sigma_{xy} = 0, \qquad \frac{\partial \phi}{\partial y} = 0. \tag{49}$$

(2) The thermal boundary condition that the surface of the half-space is subjected to

$$T = P_2 e^{i(\omega t + ax)}, \tag{50}$$

where P_1 is the magnitude of the applied force in of the half-space and P_2 is the applied constant temperature to the boundary.

Using the expressions of the variables into the above boundary conditions (49), (50), we obtain

$$\sum_{n=1}^{4} H_{5n} M_n = -P_1, \tag{51}$$

$$\sum_{n=1}^{4} H_{6n} M_n = 0, \tag{52}$$

$$\sum_{n=1}^{4} -k_n H_{2n} M_n = 0, \tag{53}$$

$$\sum_{n=1}^{4} H_{3n} M_n = P_2. \tag{54}$$

Invoking boundary conditions (51)-(54) at the surface $y = 0$ of the plate, we obtain a system of four equations. After applying the inverse of matrix method, we get the values of the four constants $M_n, (n = 1, 2, 3, 4)$.

5. Numerical results and discussions

The copper material was chosen for purposes of numerical evaluations and the constants of the problem were taken as follows:

$$\lambda = 7.76\times 10^{10}\ N.m^{-2},\ \ \mu = 3.86\times 10^{10}\ kg.m^{-1}.s^{-2},$$
$$K = 386\, w.m^{-1}.k^{-1},\quad \alpha_t = 1.78\times 10^{-5}\, k^{-1},$$

$$\rho = 8954\ kg.m^{-3},\ \ C_E = 383.1\ J.kg^{-1}.k^{-1},\ T_0 = 293\, K,$$
$$\beta = 2.68\times 10^{6} N/m^2 \deg,\ \ \omega_1^* = 3.58\times 10^{11}/s.$$

The voids parameters are

$$\chi = 1.753\times 10^{-15} m^2,\qquad \xi = 1.475\times 10^{10} N/m^2,$$
$$b = 1.13849\times 10^{10} N/m^2,\qquad \alpha = 3.688\times 10^{-5} N,$$
$$m = 2\times 10^{6} N/m^2 deg,\quad \omega_0 = 0.0787\times 10^{-3} N/m^2 s.$$

The laser pulse parameters are

$$I_0 = 10^5,\quad r = 1\times 10^2\ \mu m,\quad \gamma = 1\times 10^5\ m^{-1},\quad t_0 = 8\, nan.\sec.$$

The Magnetic field parameters, were

$$H_0 = 10^8,\quad \mu_0 = 4\pi\times 10^{-7}\ H/M,\quad \varepsilon_0 = 10^{-9}/(36\pi)\ F/M.$$

The comparisons were carried out for

$$x = 140,\ t = 0.1,\ \omega = \zeta_0 + \mathrm{i}\zeta_1,\ \zeta_0 = 1,\quad \zeta_1 = -4,$$
$$p_1 = 0.1,\ \ p_2 = 1,\ \tau_0 = 0.05,\ \upsilon_0 = 0.5,$$
$$a = 1.25,\quad \Omega = 0.5,\quad 0 \le y \le 3.$$

The above numerical technique, was used for the distribution of the real part of the displacement component u, the temperature T, the stress components σ_{yy}, σ_{xy} and the change in the volume fraction field ϕ with distance y, in the three theories, in the following cases:

(i) With and without rotation effect are shown graphically in Figures 1-5 in the case of two different values of rotation [$\Omega = 0.5, 0,$ with $H_0 = 10^8,\ t = 0.1$].

(ii) With and without the magnetic field [$H_0 = 10^8, 0$ with $\Omega = 0.5, t = 0.1$], are shown graphically in Figures 6-10.

(iii) With two different values of time [$t = 0.05\times10^{-13},\ 0.08$] are shown graphically in Figures 11-15.

All the considered variables depend not only on the variables t, x and y, but also depend on the thermal relaxation times τ_0 and υ_0, the results are shown in Figures 1-15.

Figure 1 shows that the distribution of the displacement component u for $\Omega = 0.5, 0,$ in the context of the three theories, decreases with the increase of the rotation in the range $0 \le y \le 1.25$, while it increases with the increase of the rotation in the range $1.25 < y$.

Figure 2 explains that the distribution of the temperature T for $\Omega = 0.5, 0,$ increases with the increase of the rotation in the context of the three theories, and converges to zero as y increasing.

Figure 3 expresses that the distribution of the change in the volume fraction field ϕ in the context of the three theories for $\Omega = 0.5, 0,$ increases with the decrease of the rotation in the range $0 \le y \le 0.75,$ while it increases with the increase of the rotation in the range $0.75 < y$.

Figure 4 displays that the distribution of the stress component σ_{yy} for $\Omega = 0.5, 0,$ increases with the increase of the rotation and satisfies the boundary condition at $y = 0.$

Figure 5 exhibits that the distribution of the stress tensor components σ_{xy} for $\Omega = 0.5, 0,$ increases with the decrease of the rotation and satisfies the boundary condition at $y = 0,$ then converges to zero as y increasing.

Figure 6 shows that the distribution of the displacement component u for $t = 0.08,$ $0.05\times10^{-13},$ in the context of the three theories; decreases with the increase of the time for $y > 0.$

Figure 7 explains that the distribution of the temperature T for $t = 0.08,\ 0.05\times10^{-13},$ in the context of the three theories, increases with the increase of the time values for $y > 0.$

Figure 8 displays that the distribution of change in the volume fraction field ϕ in the context of the three theories, for $t = 0.08,\ 0.05\times10^{-13},$ increases with the increase of the time t for $y > 0$, in the context of three theories.

Figures 9 demonstrate that the distribution of the stress component σ_{yy} in the context of the three theories, begins from a negative value for $t = 0.08,\ 0.05\times10^{-13}.$ It noticed that the distribution of the stress component σ_{yy} was increased with the increase of the time t for $y > 0$.

Figures 10 show that the distribution of the stress component σ_{xy} with distance y has been shown. In the context of the three theories, the values of σ_{xy} increase in the range $0 \le y \le 0.3,$ then decreases in the range $0.3 \le y \le 3,$ and also move in wave propagation for $t = 0.08,\ 0.05 \times 10^{-13}.$

Figure 11 exhibits that the distribution of the displacement component u for $H_0 = 0{,}10^8,$ in the context of the three theories, decreases with the increase of the magnetic field in the range $0 \le y \le 1.27$, while it increases with the increase of the magnetic field in the range $1.27 < y$.

Figure 12 demonstrates that the distribution of the thermodynamic temperature T begins from a positive value for $H_0 = 0{,}10^8,$ in the context of the three theories, T decrease in the range $0 \le y \le 3.$ We also notice that the magnetic field has no great effect on the distribution of the conductive temperature T.

Figure 13 explains the distribution of change in the volume fraction field ϕ in the context of the three theories, for $H_0 = 0{,}10^8,$ is decreasing with the increase of the magnetic field value for $y > 0,$ while the magnetic field has a great effect on the distribution of $\phi.$

Figure 14 exhibits that the distribution of the stress component σ_{yy} always begin from negative values, and satisfies the boundary condition at $y = 0.$ In the context of the three theories for $H_0 = 0{,}10^8,$ it observed that the distributions of σ_{yy} is directly proportional to the magnetic field while they increasing with the increase of the magnetic field for $y > 0.$

Figure 15 shows the distribution of the stress component σ_{xy} and demonstrates that it reaches a zero value and satisfies the boundary conditions at $y = 0.$ In the context of the three theories, for $H_0 = 0{,}10^8,$ it observed that the distribution of the stress component σ_{xy} is increasing with the increase of the magnetic field value for $y > 0.$

6. Conclusion

By comparing the figures that were obtained for the three thermoelastic theories, important phenomena are observed that the values of all physical quantities converge to zero with increasing distance y, and all functions are continuous. The magnetic field, and
the time for thermoelastic material with voids under thermal loading due to laser pulse is an interesting problem of mechanics. The presence and absence of rotation and magnetic field effect in the current model is of significance. The normal mode analysis technique has

been used which is applicable to a wide range of problems in thermoelasticity. Finally, it deduced that the deformation of a body depends on the nature of the applied forces and thermal loading due to laser pulse as well as the type of boundary conditions.

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APPENDIX

$$A_1 = \frac{\lambda+\mu}{\mu},\ A_2 = \frac{bc_1^2}{\mu\omega_1^{*2}\chi},\ A_3 = \frac{\beta T_0}{\mu},\ A_4 = \frac{\mu_0 H_0^2 \varepsilon_0 c_1^2 + \rho c_1^2}{\mu},$$

$$A_5 = \frac{\rho c_1^2}{\mu},\ A_6 = \frac{b\chi}{\alpha},\ A_7 = \frac{\xi c_1^2}{\alpha\,\omega_1^{*2}}\quad A_8 = \frac{\omega_0 c_1^2}{\alpha\,\omega_1^{*}},$$

$$A_9 = \frac{mT_0\chi}{\alpha},\ A_{10} = \frac{\rho c_1^2\chi}{\alpha},\ A_{11} = \frac{mc_1^2}{\rho C_E\,\omega_1^{*2}\chi},\ A_{12} = \frac{\lambda}{\mu},$$

$$\varepsilon_1 = \frac{K\,\omega_1^{*}}{\rho c_1^2 C_E},\quad \varepsilon_2 = \frac{\beta}{\rho C_E},\ S_1 = 1 + A_1,\ S_2 = \frac{S_1 a^2 - A_4\omega^2 - A_5\Omega^2}{S_1},$$

$$S_3 = \frac{2\mathrm{i}A_5\omega\Omega}{S_1},\ S_4 = \frac{A_2}{S_1},\ S_5 = \frac{A_3(1+\mathrm{i}\upsilon_0\omega)}{S_1},\ S_6 = 2\mathrm{i}A_5\omega\Omega,\ S_7 = a^2 - i\,\omega A_4 - A_5\Omega^2,$$

$$S_8 = A_6 a^2,\ S_9 = a^2 + A_7 + \mathrm{i}\omega A_8 - A_{10}\omega^2,\ S_{10} = A_9\left(1 + i\,\upsilon_0\omega\right),$$

$$S_{11} = \frac{-\varepsilon_2\left(\mathrm{i}\omega - n_0\tau_0\omega^2\right)}{\varepsilon_1},\ S_{12} = \frac{A_{11}\left(\mathrm{i}\omega - n_0\tau_0\omega^2\right)}{\varepsilon_1},\quad S_{13} = \frac{\varepsilon_1 a^2 + \mathrm{i}\omega - \tau_0\omega^2}{\varepsilon_1},$$

$$f\left(x,t\right) = \left[t + n_0\tau_0\left(1 - \frac{t}{t_0}\right)\right]\exp\left(\frac{-t}{t_0} - i\,\omega t - i\,ax - \frac{x^2}{r^2}\right),$$

$$Q_0^{'} = \frac{\mathrm{Q}_0}{\varepsilon_1},\quad Q_0 = \frac{I_0\gamma}{2\pi r^2 t_0^2},\ \mathrm{D} = \frac{\mathrm{d}}{\mathrm{dy}}.$$

$$A = S_{13} + S_9 + S_7 + S_2 - A_6S_4 - S_5S_{11},$$

$$B = S_9S_{13} + S_{10}S_{12} + S_7S_{13} + S_7S_9 + S_2S_{13} + S_2S_9 + S_2S_7 + S_6S_3 - A_6S_4S_{13} - S_4S_8 + S_4S_{10}S_{11} - A_6S_4S_7 \\ - A_6S_5S_{12} - S_5S_{11}a^2 - S_5S_9S_{11} - S_5S_7S_{11},$$

$$E = S_7S_9S_{13} + S_7S_{10}S_{12} + S_2S_9S_{13} + S_2S_{10}S_{12} + S_2S_7S_{13} + S_2S_7S_9 + S_3S_6S_{13} + S_3S_6S_9 \\ - S_4S_8S_{13} + S_4S_{10}S_{11}a^2 - A_6S_4S_7S_{13} - S_4S_7S_8 + S_4S_7S_{10}S_{11} - S_5S_8S_{12}^2 \\ - S_5S_9S_{11}a^2 - A_6S_5S_7S_{12} - S_5S_7S_{11}a^2 - S_5S_7S_9S_{11},$$

$$G = S_2S_7S_9S_{13} + S_2S_7S_{10}S_{12} + S_3S_6S_9S_{13} + S_3S_6S_{10}S_{12} - S_4S_7S_8S_{13} \\ + S_4S_7S_{10}S_{11}a^2 - S_5S_7S_8S_{12} - S_5S_7S_9S_{11}a^2,$$

$$l_1 = S_5\gamma^4 - \left(S_5S_7 + S_5S_8 + S_7S_8 - S_4S_9\right)\gamma^2 + \left(S_5S_7S_8 - S_4S_7S_9\right),$$
$$l_2 = -S_5S_6\gamma^2 + \left(S_5S_6S_8 - S_4S_6S_9\right),$$

$$l_3 = \left(A_5S_5 - S_9\right)\gamma^4 + \left(S_2S_9 + S_7S_9 - A_5S_5a^2 - A_5S_5S_7\right)\gamma^2 - \left(S_2S_7S_9 + S_3S_6S_9 + A_5S_5S_7a^2\right),$$

$$l_4 = \gamma^6 - \left(S_2 + S_7 + S_8 - A_5S_4\right)\gamma^4 + \left(S_7S_8 + S_2S_7 + S_2S_8 + S_3S_6 - A_5S_4S_7 - A_5S_4a^2\right)\gamma^2 \\ - \left(S_2S_7S_8 + S_3S_6S_8 - A_5S_4S_7a^2\right).$$

$$V_{1n} = \mathrm{i}a - k_nH_{1n}, \quad V_{2n} = k_n + \mathrm{i}aH_{1n}, \quad H_{1n} = \frac{-\mathrm{S}_6}{\left(k_n^2 - S_7\right)},$$

$$H_{2n} = \frac{-\left[S_{10}\left(k_n^2 - S_2\right) - S_3S_{10}H_{1\mathrm{n}} + S_5\left(-\mathrm{A}_6k_n^2 + S_8\right)\right]}{\left[S_5\left(k_n^2 - S_9\right) + S_4S_{10}\right]},$$

$$H_{3\mathrm{n}} = \frac{-A_6k_n^2 + S_8 + \left(k_n^2 - \mathrm{S}_9\right)H_{2\mathrm{n}}}{S_{10}},$$

$$H_{4n} = A_{12}\left(\mathrm{i}aV_{1n} + k_nV_{2n}\right) + 2\mathrm{i}aV_{1n} + A_{13}H_{2n} - A_{14}H_{3n}\left(1 + \mathrm{i}\upsilon_0\omega\right),$$

$$H_{5n} = A_{12}\left(\mathrm{i}aV_{1n} + k_nV_{2n}\right) + 2k_nV_{2n} + A_{13}H_{2n} - A_{14}H_{3n}\left(1 + \mathrm{i}\upsilon_0\omega\right), \qquad H_{6n} = \left(k_nV_{1n} + \mathrm{i}aV_{2n}\right),$$

$$f_1(x,t) = f_1^*e^{(\omega t + ikx)}, \quad f_2(x,t) = \upsilon_0\left\{\left(1 - \frac{n_0\tau_0}{t_0}\right) - \frac{1}{t_0}\left[t + n_0\tau_0(1 - \frac{t}{t_0})\right]\right\}e^{\left(-\frac{x^2}{r^2} - \frac{t}{t_0}\right)},$$

$$N_1 = \frac{-1}{\gamma^8 - A\gamma^6 + B\gamma^4 - E\gamma^2 + G},$$

$$N_2 = A_{12}\left[\frac{-2\ell_1}{r^2} - \left(\frac{2x\,\ell_1}{r^2} + \gamma\ell_2\right)\left(\frac{-2x}{r^2}\right) - \gamma\left(\frac{2x\,\ell_2}{r^2} - \gamma\ell_1\right)\right] - \frac{4\ell_1}{r^2} - \left(\frac{4x\,\ell_1}{r^2} + 2\gamma\ell_2\right)\left(\frac{-2x}{r^2}\right) + A_2\ell_3,$$

$$N_3 = A_3\ell_4,$$

$$N_4 = A_{12}\left[\frac{-2\ell_1}{r^2} - \left(\frac{2x\,\ell_1}{r^2} + \gamma\ell_2\right)\left(\frac{-2x}{r^2}\right) - \gamma\left(\frac{2x\,\ell_2}{r^2} - \gamma\ell_1\right)\right] - 2\gamma\left(\frac{2x\,\ell_2}{r^2} - \gamma\ell_1\right) + A_2\ell_3,$$

$$N_5 = \gamma\left(\frac{2x\,\ell_1}{r^2} + \gamma\ell_2\right) - \gamma\left(\frac{2x\,\ell_2}{r^2} - \gamma\ell_1\right).$$

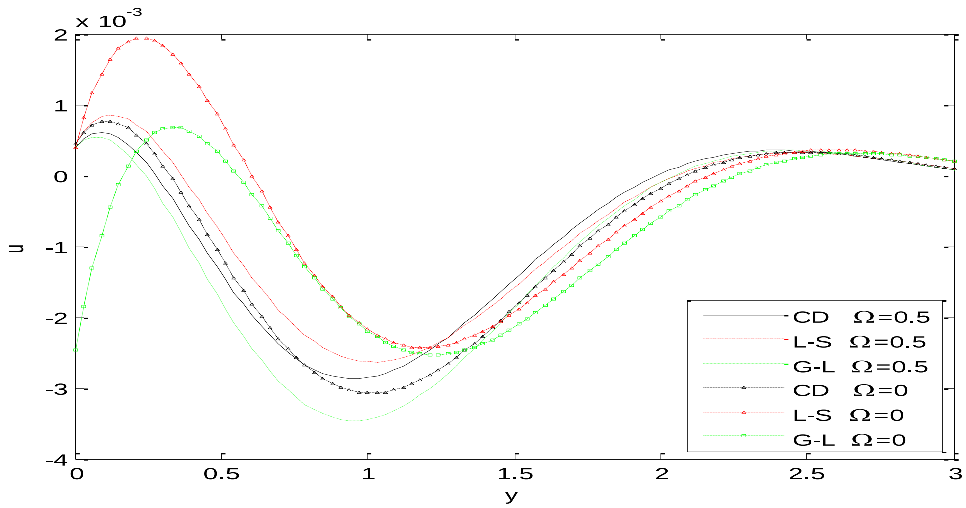

Figure 1. Horizontal displacement distribution u in the absence and presence of rotation

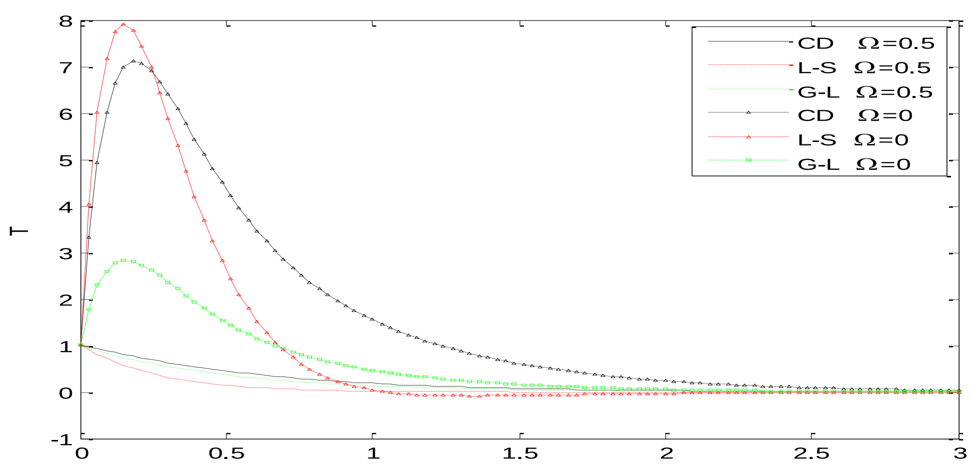

Figure 2. The temperature distribution T in the absence and presence of rotation

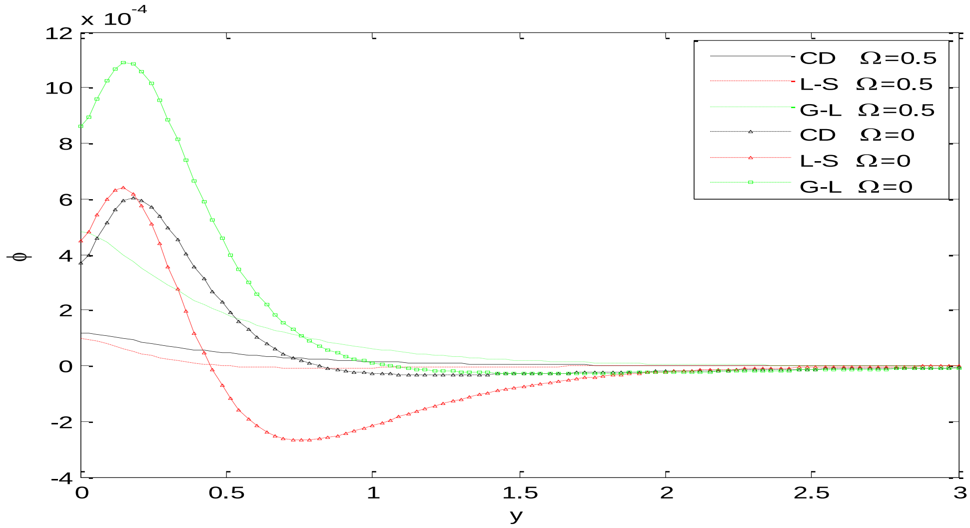

Figure 3. The displacement of volume fraction field ϕ in the absence and presence of rotation

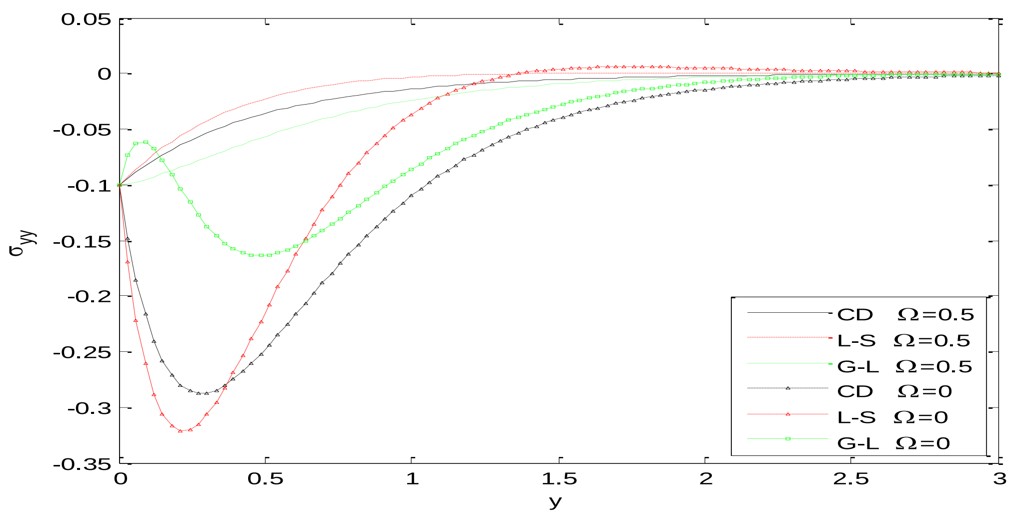

Figure 4. Distribution of stress component σ_{yy} in the absence and presence of rotation

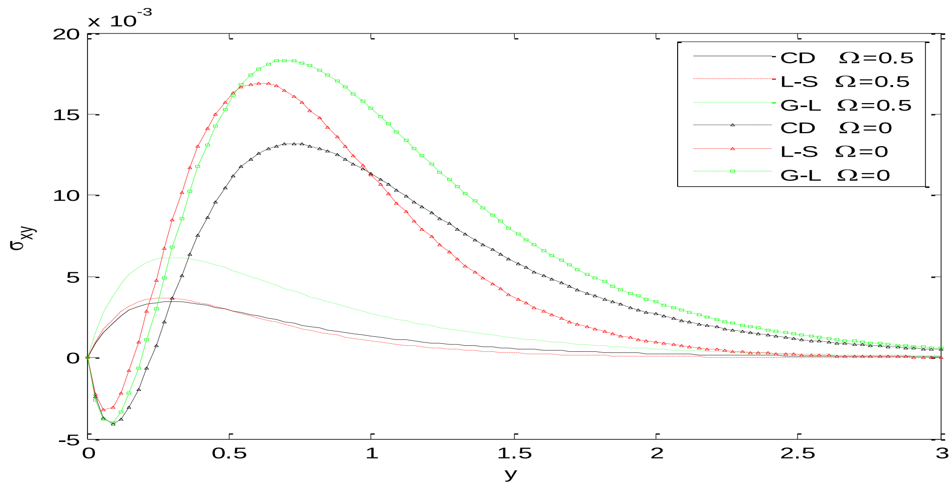

Figure 5. Distribution of stress component σ_{xy} in the absence and presence of rotation

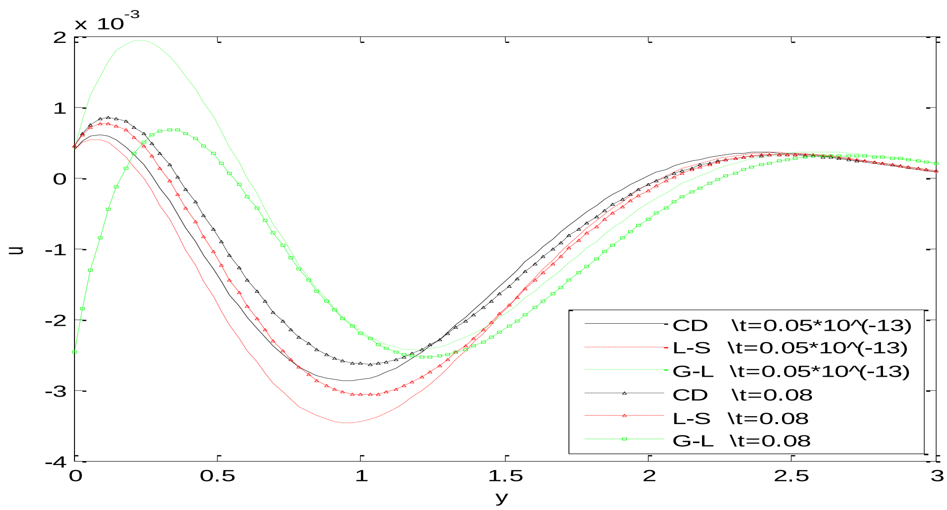

Figure 6. Distribution of the displacement component u with two values of time

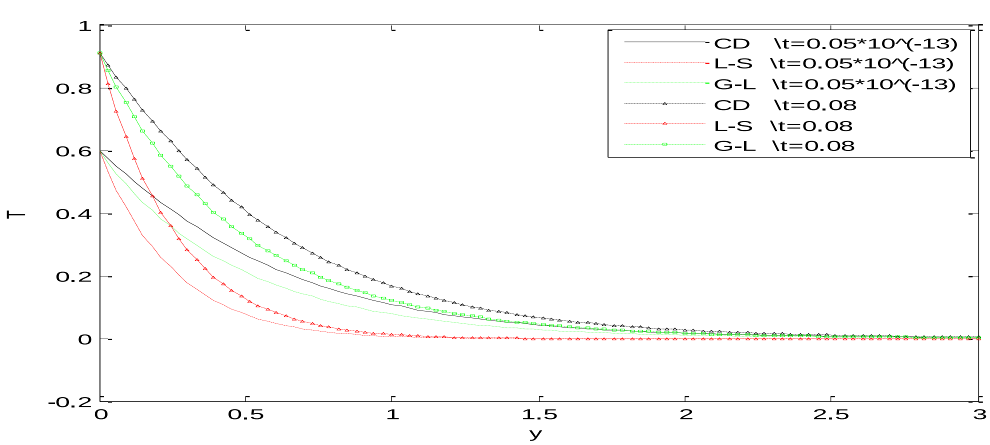

Figure 7. The temperature distribution T with two values of time

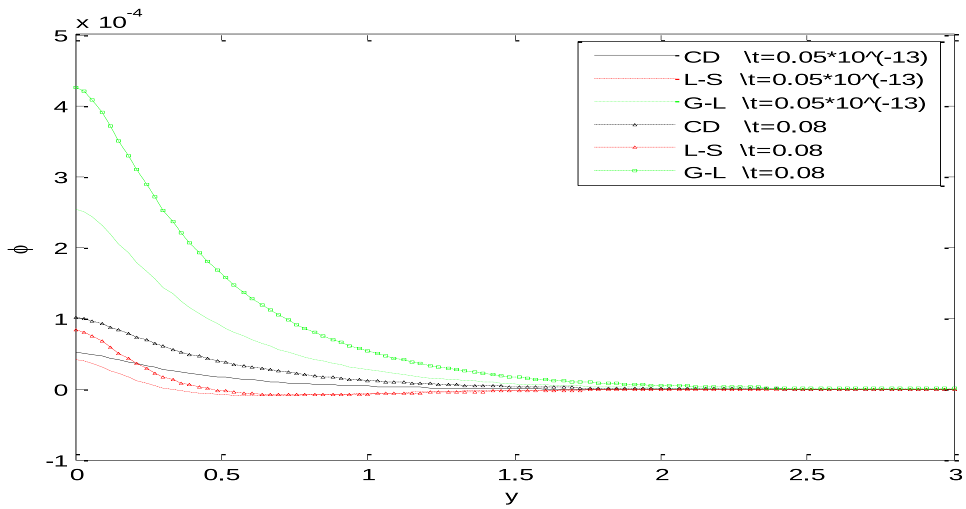

Figure 8. The change in volume fraction field distribution ϕ with two values of time

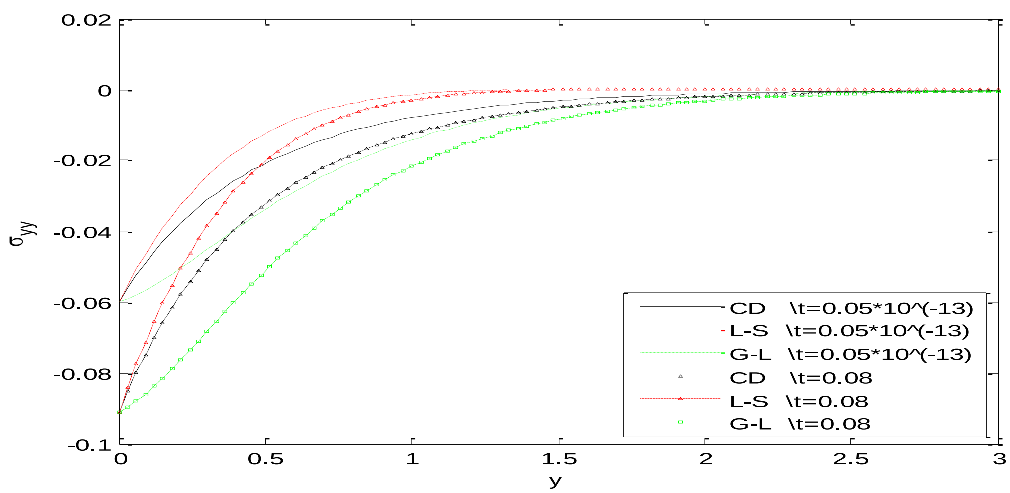

Figure 9. The distribution of the stress component σ_{yy} with two values of time

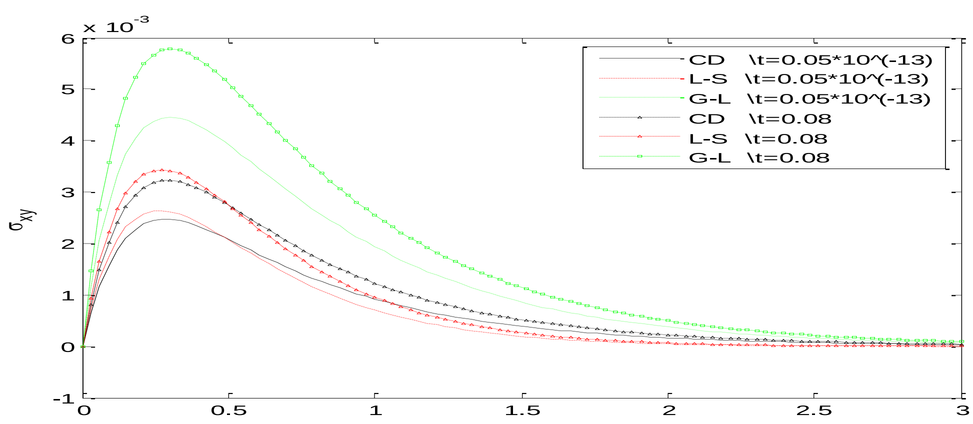

Figure 10. Distribution of the stress component σ_{xy} with two values of time

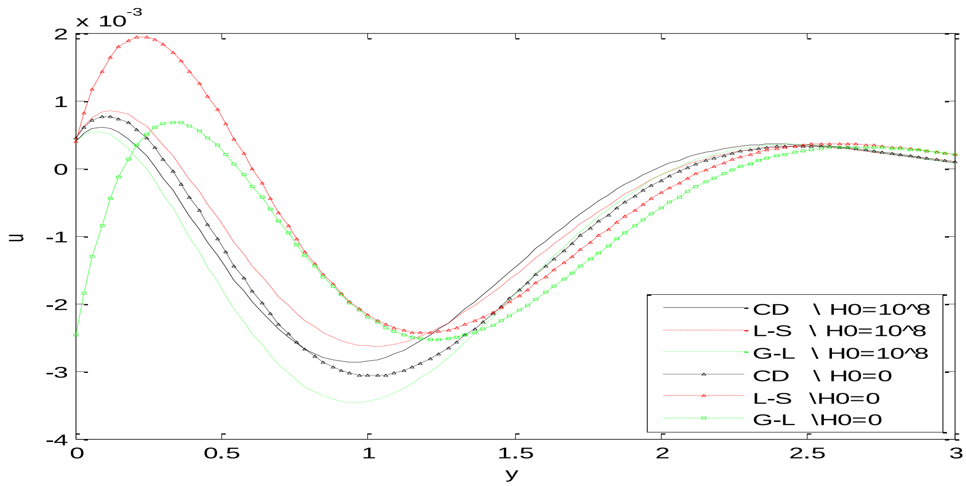

Figure 11. The horizontal displacement distribution u in the absence and presence of magnetic field

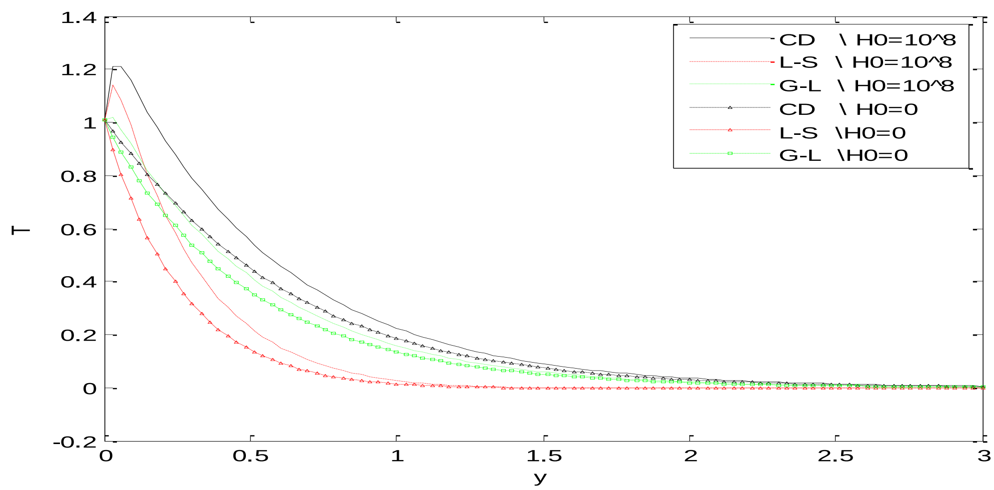

Figure 12. The temperature distribution T in the absence and presence of magnetic field

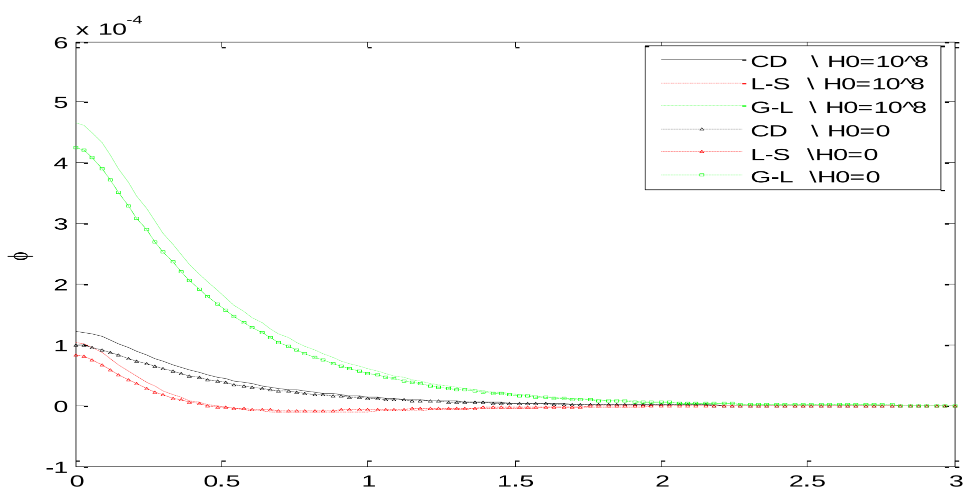

Figure 13. The change in volume fraction field distribution ϕ in the absence and presence of magnetic field

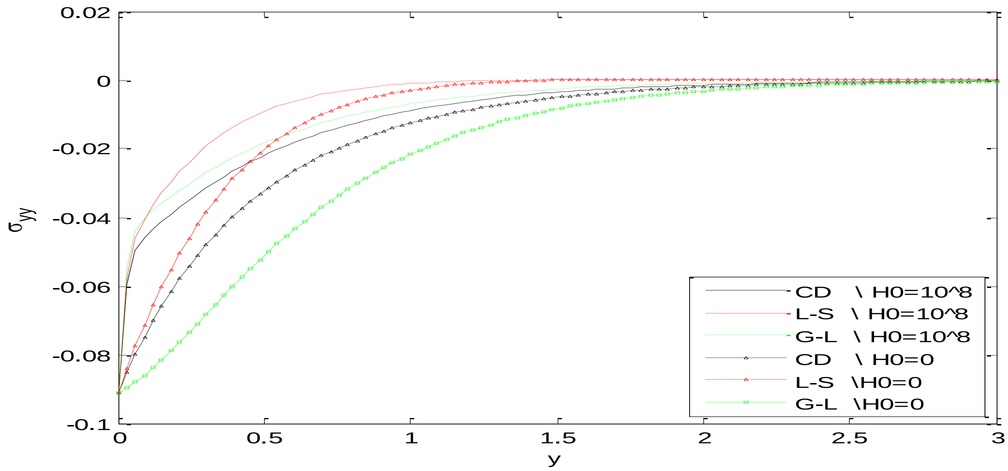

Figure 14. The distribution of the stress component σ_{yy} in the absence and presence of magnetic field

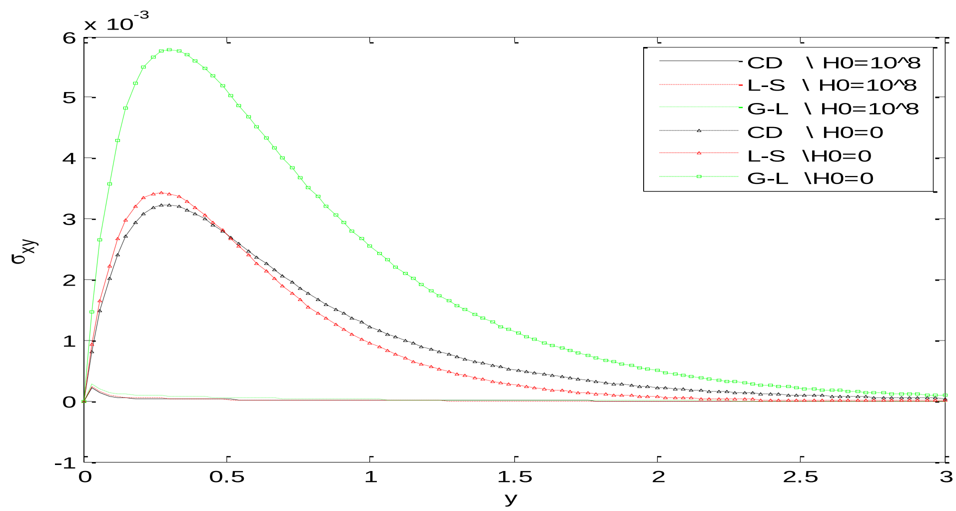

Figure 15. Distribution of stress component σ_{xy} in the absence and presence of magnetic field