КРАТКИЕ ВЫВОДЫ

1. Применение балочно-вантовой системы для усиления деревянных балок чердачного перекрытия позволило при минимальных затратах повысить их несущую способность более чем в четыре раза, что обеспечивает достаточную их надежность при снижении рабочего поперечного сечения по высоте и ширине на 35÷40%.
2. Особо следует отметить простоту монтажа элементов пространственного блока покрытия и функциональную увязку несущих конструкций стропильной системы с вписанной в нее балочно-вантовой системой усиления деревянных балок чердачного перекрытия.
3. Стоимость усиленного чердачного перекрытия снижена более чем на 45% в сравнении со стоимостью замены новым конструктивным решением.
4. Конструктивная схема пространственного блока покрытия с балочно-вантовой системой усиления внедрена в проект капитального ремонта кровли цеха. Это позволило получить натурную модель для проведения экспериментальных исследований. Результаты этих исследований будут использованы при разработке рекомендации по эффективному применению балочно-вантовых систем для усиления несущих конструкций чердачных перекрытий.

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EVALUATION OF SUSPENSION CABLE NONLINEAR DISPLACEMENTS

ABSTRACT

Many complex engineering structures contain specific behavior load bearing members – cables, very effective to resist tensile forces. Specific behavior is described by the small cable resistance capability to bending and compression, which actually approaches to zero. Therefore the cable responses to loading in nonlinear way, adapting to new equilibrium form for each loading increment (change). One can group all nonlinearities met in cable analysis in respect of two aspects: those, caused by straining, and those caused by structure adaptation to new equilibrium form. The displacement components, induced by the second aspect reason, are named as kinematical ones. The investigation is assigned to kinematical displacement evaluation methods. The simplified (standard) displacement evaluation methods, applied currently in engineering practice are not sufficient accurate. The more exact methods in terms of analytical relations to identify cable kinematical displacements of the suspension cable are under consideration. An analysis of kinematical displacements estimated by standard (engineering) and proposed method, including the obtained errors causalities is provided. The possible engineering tools to reduce kinematical displacements and to stabilize suspension cable shape are presented on the basis of proposed displacement evaluation method.

1. INTRODUCTION

Many engineering complex structures, employed to cover large span distances or areas, usually contain cables (cable networks). Bridges, roofing systems of stadiums, masts, other previously and recently erected structures contain the main loading bearing element – flexible cable or system of flexible cables [1-12]. The exceptional peculiarity of the member is the large displacements caused by asymmetric loading [2-4,6,10-11]. The large displacements are conditioned by kinematic displacements, resulted by cable adaptation process to new equilibrium form, induced by complementary loading in terms of asymmetrically distributed and/or concentrated loads. Recently the identification of kinematical displacements is widely investigated [2,4,6,10-12]. But one must note that most researchers applied the simplified engineering methods to identify kinematical displacements in order to calculate the total displacements. The latter methods are based on superposition principle, when splitting the actual loads to symmetric and asymmetric loads. The suspension cables are the elements of dominantly geometrically nonlinear behavior. Thus, one must provide the exact analysis of relation of kinematical displacements versus subjected loading in order to ensure reliable evaluation of internal forces and displacement fields. Such the analysis should enable not only to fix the application bounds of simplified engineering methods, but also should serve as a good basis to find the effective tools when stabilizing primary form of cable or cable network.

The current investigation is devoted to creation of improved (more accurate vs engineering) method to identify cable kinematic displacements, both for vertical and horizontal its components. The error, resulted from application of engineering methods, analysis is to be provided and eventual engineering tools to stabilize the cable primary equilibrium form are to be presented.

2. SUSPENSION CABLE VERTICAL DISPLACEMENT EVALUATION

The parabola primary shape cable (corresponding to uniformly distributed e.g. of cable self weight loading) is subjected by asymmetrically complementary loading (see Fig.1). The relation of vertical displacement vs complementary loading is under investigation.

The pure kinematic displacements from total ones are separated by absolutely reducing the straining caused (elastic) displacements via taking the cable axial stiffness to be $EA \to \infty$. Then the elastic displacements are equal to zero ($\omega_{el} = 0$) and the total displacements coincide with the kinematic ones.

The shape of the cable, subjected to the uniformly distributed load q, satisfies the quadratic parabola law:

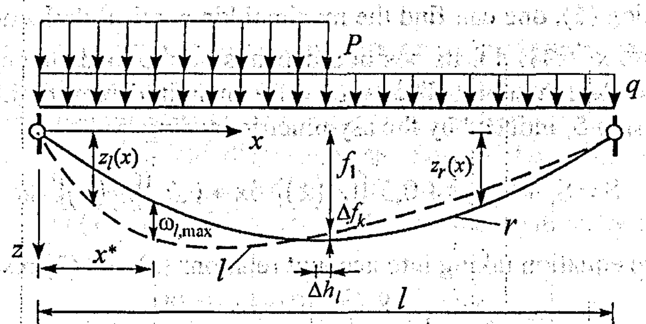

Figure 1 – Deformed shape of cable, subjected by complementary asymmetric loading

$$z(x) = f_0\left[\left(\frac{4x}{l}\right) - \left(\frac{4x^2}{l^2}\right)\right], \tag{1}$$

where f_0 is the cable sag.

The cable, subjected to asymmetric load p, changes its primary shape (see dashed line in Fig. 1). Divide the cable to the part (left), subjected by asymmetric loading p, and the part free of loading. Then the axial cable curvature equations for left and right parts are as follow:

$$z_l(x) = f_1\left[\left(\frac{4x}{l} - \frac{4x^2}{l^2}\right) + \frac{\gamma}{2}\left(\frac{6x}{l} - \frac{6x^2}{l^2}\right)\right] \Big/ \left(1 + \frac{\gamma}{2}\right), \tag{2}$$

for $x \le l/2$, and

$$z_r(x) = f_1\left[\left(\frac{4x}{l} - \frac{4x^2}{l^2}\right) + \frac{\gamma}{2}\left(\frac{2x}{l} - 2\right)\right] \Big/ \left(1 + \frac{\gamma}{2}\right), \tag{3}$$

for $l/2 \le x \le l$.

The above formulae contain the following values:

γ = p/q is the ratio of symmetrical and symmetrical loads intensities; $f_1 = f_0 \pm \Delta f_k$ is the vertical displacement of loaded cable at middle span x = l/2; Δf_k is the kinematical vertical displacement of loaded cable at middle span x = l/2.

The maximal deflection z(x) = $z_{max}(x^*)$ one can find only after when its coordinate $x = x^*$ (see Fig.1.) is fixed. The required coordinate is obtained by equating the cable left part expression (2) first derivative to zero. After some transformations one can find

$$x^* = \frac{1}{4} \cdot \frac{(2+3\gamma/2)}{(1+\gamma)}. \tag{4}$$

One must note, that the maximal deflection $z(x) = z_{max}(x^*)$ location point depends on ratio γ, and its magnitude is in direct proportionality to maximal middle span deflection f_1. When γ varies from 1 to 10, the coordinate x^* changes insignificantly, varying in bounds $x^* = 0.4371 \div 0.3861$. Thus, one can find, that the maximal cable deflection position $z_{max}(x^*)$ deviation is small enough from the cable middle span.

Engineering design process requires to check the stiffness requirements in terms of displacement limitations. Therefore one must possess the relations of vertical kinematical displacements. The latter can by calculated by

$$\omega_1(x) = z_{11}(x) - z_{10}(x). \tag{5}$$

The maximal kinematical vertical displacement $\omega_1(x) = \omega_{1,max}(x^{**})$ can be obtained, having fixed its position

$$x^{**} = \frac{1}{4} \frac{[f_0\gamma/2 + \Delta f_k(2+3\gamma/2)]}{[f_0\gamma/2 + \Delta f_k(1+\gamma)]}. \tag{6}$$

Analyzing the equation (6), one can find the maximal kinematical deflection point to be outside of the first cable span quarter ($0 \le x^{**} \le 1/4$). Its position depends on the loads intensities ratio γ and the middle span kinematical deflection Δf_k magnitude The latter value magnitude can be identified, having determined the suspension cable extension S, induced by the asymmetric loading p:

$$S = S_l + S_r \cong 1 + 0{,}5 \int_0^{l/z} [z_l'(x)]^2 dx + 0{,}5 \int_{l/2}^{1} [z_r'(x)]^2 dx. \tag{7}$$

Solution of the above equation taking into account relations (2) and (3) results

$$S_l = 1 + \frac{8}{3}\frac{f_1^2}{1}\frac{[1+\gamma+\gamma^2/4]}{[1+\gamma+5\gamma^2/16]}. \tag{8}$$

By employing the S_l expression, one can obtain the equation for middle span deflection Δf_k:

$$\Delta f_k = f_0\left(\sqrt{\psi} - 1\right), \tag{9}$$

where parameter ψ is obtained by

$$\psi = \frac{1+\gamma+\gamma^2/4}{1+\gamma+5\gamma^2/16}. \tag{10}$$

The analysis of the relation (10), shows that the $\psi < 1.0$ results the negative Δf_k magnitude. Thus, the middle span point of cable, subjected by asymmetric loading, is lifted up, proportionally to the ratio γ. For instance, when $\gamma = 0{,}5$, $\Delta f_k = -0{,}005\ f_0$; when $\gamma = 2$, $\Delta f_k = -0.03\ f_0$; when $\gamma = 3$, $\Delta f_k = -0.042\ f_0$.

The vertical kinematical displacements of the loaded cable part are determined by

$$\omega_1(x) = f_0\left[\left(\frac{4x}{l} - \frac{4x^2}{l^2}\right)\left(\frac{1}{\xi} - 1\right) + \frac{\gamma}{2\xi}\left(\frac{6x}{l} - \frac{6x^2}{l^2}\right)\right], \tag{11}$$

$$\xi = \sqrt{1+\gamma+5\gamma^2/16}. \tag{12}$$

The formula (11) is convenient for application as kinematical displacements are expressed only via cable primary maximal coordinate f_0 and ratio γ. It is obvious that at point $x = l/2$ kinematical vertical displacement is $\omega_1(x) = \Delta f_k$.

Transforming equation (6) one can obtain:

$$x^{**} = \frac{1}{4} \frac{(2+3\gamma/2-2\xi)}{(1+\gamma-\xi)}. \tag{13}$$

The formula (13) is simpler than formula (6), as maximal vertical displacement relation $\omega_1(x) = \omega_{1,max}(x^{**})$, excludes the cable middle span deflection Δf_k item. Analyzing the expression (13), one can find the maximal kinematical displacement location point of the loaded left part will be always inside of the first cable span quarter, i.e. $x^{**} < l/4$, versus as is adopted by engineering method authors [2,6,7,10,11,13]. Furthermore, this displacement location point always will be back away from the left support by the distance, being less than l/4. This distance decreases by increasing the ratio γ. When the ratio of asymmetric and symmetric loads intensities $\gamma = 1$, $x^{**} = 0{,}957l/4$; when $\gamma = 2$, $x^{**} = 0{,}934l/4$; and when $\gamma = 3$, $x^{**} = 0{,}921l/4$.

Aiming to obtain the maximal kinematical displacements of the cable left part, loaded to asymmetrical load p, provide transformations of relation (11), taking into account relation (13). After certain transformations it reads:

$$\omega_{l,max} = f_0\left[\left(2u - u^2\right)\left(\frac{1}{\xi} - 1\right) + \frac{\gamma}{2\xi}\left(3u - 2u^2\right)\right], \quad (14)$$

$$u = \frac{1 - \xi + 3\gamma/4}{1 - \xi + \gamma}. \quad (15)$$

The expression (14) is more convenient as it does not requires to identify the maximal kinematical displacement position, defined by formula (13).

3. SUSPENSION CABLE HORIZONTAL DISPLACEMENT EVALUATION

Horizontal kinematic displacement is the component of the total kinematic displacement. The horizontal kinematic displacement, induced by complementary loading are directed to the loaded part of the cable.

Analyzing the horizontal kinematical displacements. Divide the cable to the left part, loaded by asymmetrically distributed load p, and the right part, free of loading.

Combining the expressions (1), (2) and (7), one can derive the relation for the left part horizontal middle span displacement:

$$\Delta h_l = \frac{4}{3l}\left[f_1^2\frac{\left(1 + 5\gamma/4 + 7\gamma^2/16\right)}{\left(1 + \gamma/2\right)^2} - f_0^2\right]. \quad (16)$$

Analyzing the formula (16), one can find the horizontal kinematical displacement to be in direct proportion on the loads intensities ratio γ and the cable sag f_0. The horizontal kinematical displacements are of the same order as the vertical displacement magnitude in the middle span Δf_k.

The left cable part horizontal kinematical displacement is determined by the relation:

$$\Delta h_r = \Delta S_{kl} = \frac{4}{3r}\left[f_1^2\frac{\left(1 + 3\gamma/4 + 3\gamma^2/16\right)}{\left(1 + \gamma/2\right)^2} - f_0^2\right]. \quad (17)$$

One must note that kinematical horizontal displacements are absolutely identical magnitudes, i.e. $\Delta h_l = \Delta h_r$.

The horizontal displacements of the other cable points can be determined analogously, having compared the cable lengths before loading and after loading by asymmetric loading.

Analyzing expressions of vertical and horizontal kinematical expressions, one can find them to be related ones. Thus, aiming to reduce vertical displacements, one must constrain the cable ability to deform horizontally. When constraining the cable horizontal displacements via the horizontal link, introduced in the middle span, the magnitudes of vertical displacements approach to zero. This constructional tool (realized e.g. by connecting the cable with stiffness beam) is applied in suspension cable bridges in order to stabilize its primary equilibrium form [7]. The analogous constructional decisions are applied for various suspension cable and complex structures [2,5,13].

3. NUMERICAL SIMULATIONS

To illustrate the proposed method of kinematical displacement evaluation and to provide error, obtained when applying the approximate engineering methods, the numerical simulations are provided.

The 100 meters span flexible suspension cable was subjected by symmetrically and asymmetrically distributed loads q and p, respectively (see Fig. 1). The displacement analysis is provided in respect of the sag f_0 and the ratio of symmetric and asymmetric loads intensities γ variations.

Analyzing vertical kinematical displacements. Numerical simulation cleared that the sag f_0 magnitude has direct influence to the cable vertical and horizontal displacements (increments of sag cause adequate increments of kinematic displacement components).

Taking the ratio $\gamma = 1$ and the primary coordinate to be $f_0 = 1/10 = 10\text{m}$, the induced vertical kinematical displacement is $\omega_{1,max} = 0{,}721\text{m}$. The vertical displacement of the cable, loaded under conditions $\gamma = 1$ and $f_0 = 1/5 = 20\text{m}$, doubled the displacement, comparing with the previous magnitude, and was $\omega_{1,max} = 1{,}442\text{m}$. One can conclude, that aiming to reduce kinematical displacements magnitudes under the constant loads intensities ratio γ, one must reduce the primary maximal coordinate f_0. The performed numerical experiment cleared that kinematical displacement values of loaded (left) part of the cable in absolute magnitudes of are less when compared with the remaining (right) part of the cable. This, looking to be paradoxical result is conditioned by the negative magnitude of the middle span displacement Δf_k. The analogous distribution of kinematical displacements was analyzed briefly in investigation [13]. The middle span kinematical displacement Δf_k also increases when one increases the γ magnitude. One obtained, that varying γ = 2 ÷ 5 for constant $f = l/10 = 10m$, the Δf_k varied respectively in the bounds - 0,136 m ÷ 0,769 m. For constant $f = l/5 = 20m$ and variable γ = 1 ÷ 10, the middle span vertical kinematical displacement magnitudes varied by - 0,272 m ÷ - 1,538 m.

The relation analysis of the ratio γ versus maximal vertical kinematical displacement $\omega_{1,max}$ shows it to be the nonlinear one. Varying γ by 1 ÷ 10 (in case of $f = l/5 = 20m$), the vertical displacement increments asymptotically reduce (see the Table). The latter result means that the suspension cable is sensitive to asymmetric loading per loading history. Analyzing the loading history in respect of the analogous increments of γ. In previous stages of loading the cable responded by the relatively larger vertical displacements, but the further increments of γ resulted to the respectively smaller displacement magnitudes.

Having performed numerical simulations, it was found, that engineering design methods, based on superposition principle, lead to the errors when estimating magnitudes of vertical displacements. The errors do not depend on primary maximal coordinate magnitudes, but are sensitive to the ratio γ magnitude. When γ = 1, the error approximately equals 15.5%, when γ = 5, the error is 42%, and when γ = 10, the error is 52% (see Fig.2).

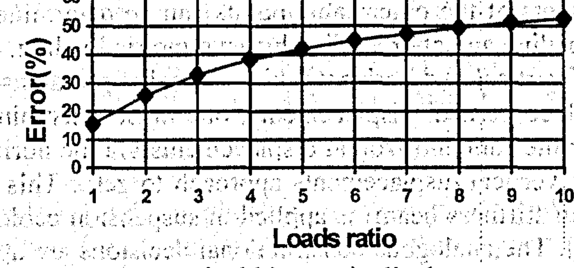

Figure 2 – Vertical kinematic displacement vs γ

It is obvious, that aiming to estimate suspension cable kinematical and, in general, total displacements more accurately, one can apply the engineering methods only in case when $\gamma \le 1$ [2,6,10,11]. It is proved, that varying x^{**}, the maximal kinematical displacement magnitude $\omega_{l,max}$ variation is small. Assuming the maximal vertical displacement to be approximately $x^{**} = l/4$, the kinematical displacements, calculated by (11) and (12), result insignificant errors up to 2% for whole γ variation range under investigation.

4. CONCLUSIONS

1. The performed numerical simulations applying the proposed analytical method resulted the following conclusions:
2. The loaded cable part vertical displacements are less the ones of unloaded part (in absolute magnitudes);
3. An increase of the loads intensities ratio γ results the nonlinear increase (reduce) of the maximal vertical kinematical displacement $\omega_{l,\max}$.

Table 1 – Vertical maximal kinematic displacements

Loads ratio γ	Displacements by expression (14) $\omega_{l,\max}$ in m	Displacements by engineering methods $\omega_{l,\max}$ in m
1	2	3
1	1.442	1.766
2	1.998	2.500
3	2.258	3.000
4	2.416	3.334
5	2.518	3.572
6	2.588	3.750
7	2.640	3.888
8	2.680	4.000
9	2.710	4.090
10	2.736	4.166

4. Vertical kinematic displacement handling possibilities can be realized via the following means: a reduction of the cable sag; an increase of the symmetric load magnitude; by introducing artificial constraints of the cable to restrict cable horizontal displacements.
5. The provided numerical simulations cleared that the error, obtained when calculating kinematical displacements by engineering methods, in case of $\gamma > 1$ out measures 16%. Thus, the engineering methods of displacement evaluation application is restricted for loading cases, when $\gamma \le 1$.

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