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Authors	þÿSantana-Ros, T.; Bartczak, P.; MichaBowski, T.; Tang
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Testing the inversion of asteroids' *Gaia* photometry combined with ground-based observations

T. Santana-Ros,[1]* P. Bartczak,[1] T. Michałowski,[1] P. Tanga[2] and A. Cellino[3]

[1] *Astronomical Observatory Institute, Faculty of Physics, Adam Mickiewicz University, Słoneczna 36, PL-60-286 Poznań, Poland*
[2] *Laboratoire Cassiopée, Observatoire de la Côte d'Azur, BP 4229, F-06304 Nice Cedex 04, France*
[3] *INAF – Osservatorio Astronomico di Torino, strada Osservatorio 20, I-10025 Pino Torinese (TO), Italy*

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ABSTRACT

We investigated the reliability of the genetic algorithm which will be used to invert the photometric measurements of asteroids collected by the European Space Agency *Gaia* mission. To do that, we performed several sets of simulations for 10 000 asteroids having different spin axis orientations, rotational periods and shapes. The observational epochs used for each simulation were extracted from the *Gaia* mission simulator developed at the Observatoire de la Côte d'Azur, while the brightness was generated using a Z-buffer standard graphic method. We also explored the influence on the inversion results of contaminating the data set with Gaussian noise with different σ values. The research enabled us to determine a correlation between the reliability of the inversion method and the asteroid's pole latitude. In particular, the results are biased for asteroids having quasi-spherical shapes and low pole latitudes. This effect is caused by the low light-curve amplitude observed under such circumstances, as the periodic signal can be lost in the photometric random noise when both values are comparable, causing the inversion to fail. Such bias might be taken into account when analysing the inversion results, not to mislead it with physical effects such as non-gravitational forces. Finally, we studied what impact on the inversion results has combining a full light curve and *Gaia* photometry collected simultaneously. Using this procedure we have shown that it is possible to reduce the number of wrong solutions for asteroids having less than 50 data points. The latter will be of special importance for planning ground-based observations of asteroids aiming to enhance the scientific impact of *Gaia* on Solar system science.

Key words: methods: numerical – techniques: photometric – minor planets, asteroids: general.

1 INTRODUCTION

The potential of the sparse photometric data to provide physical information about asteroids has been extensively proved by several authors (Cellino et al. 2006; Durech et al. 2007). Generally, the inversion methods used to derive information about the physical properties of asteroids are taking profit of the fact that a simplified version of the asteroids' real shape (triaxial ellipsoid, convex representation) is, in the majority of cases, good enough to describe the asteroid brightness variation due to its rotation for a given period. If the observations are spread over a variety of aspect angles, it is then possible to derive the direction of the asteroid spin axis.

The main challenge to be solved when inverting sparse data is the correct determination of the rotation period. One possible approach to solve this issue is to fit an asteroid spin and shape on a given period interval (Kaasalainen 2004). Using a convex representation of the asteroid's body shape, some authors have successfully solved the inversion problem for a couple of hundreds of asteroids (Durech et al. 2009; Hanus et al. 2013). If any *dense* light curve is available for the object, the interval is reduced to some range around the observed period, saving a lot of computational time and increasing the solution reliability. But, unfortunately, obtaining full light curves of asteroids is a highly time consuming task, thus such observations are actually available only for ∼5000 asteroids (stored in the Minor Planet Lightcurve Database[1]). It is estimated that European Space Agency (ESA) *Gaia* mission will produce photometric measurements for more than 300 000 asteroids, which means that for the majority of inversion trials the period scanning shall be extended

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* E-mail: tonsan@amu.edu.pl

[1] http://www.minorplanetcenter.net/ (Marsden, Green & Williams 1994)

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to almost all the possible period values, namely from 2 h to several days (Eyer & Mignard 2005).

Unlike classical asteroid photometry, *Gaia* will not obtain full light curves, but sparse, single photometric data spread over five years. The number of detections depends on the orbits of the objects, being the average around 60–70 snapshots for main-belt asteroids. These data will cover a wide range of observational circumstances, and in particular wide ranges of ecliptic longitudes, resulting in a good coverage of aspect angle variation. In terms of observational cadence, these measurements will be similar to the data stored in the Minor Planet Lightcurve Database. None the less, *Gaia* snapshots will be photometrically 10 times more accurate, and what is more important, homogeneous, in the sense that they will be measured by a single detector, and not by different telescopes. To put it in other words, these sparse data can be considered as the single points of a time-extended light curve, describing the photometric variation of the asteroids not over a single rotation period, but over five years, characterized by a continuous change of the observing circumstances. Actually, the inversion problem related with deriving physical parameters of asteroids from such measurements has become a topical issue, since not only *Gaia*, but also new ground-based survey telescopes such as the Large Synoptic Survey Telescope (LSST) will produce this kind of data.

The inversion technique specifically developed to invert the *Gaia* sparse data for asteroids (Cellino et al. 2006) is based on a genetic algorithm, where the solution of the inversion problem is characterized by the best fit of a set of parameters that have been obtained by means of several random variations during a genetic mutation process. This solution should mitigate the risk of falling in secondary minima of the system and its capability to derive the *correct* inversion solution has been shown in some experiments with *Gaia* simulated observations and also with real data collected during the ESA *Hipparcos* mission (see for instance Cellino et al. 2009 or Carbognani et al. 2012). On the other hand, adding existing ground-based observations for a given asteroid is not speeding up the performance of this method (in fact the inversion becomes slower with greater number of measurements) and whether such observations can improve the method performance or not is a topic that needs to be studied.

In this paper, we make a more general and detailed reassessment of the expected performances of the *Gaia* inversion algorithm. To do that, we fed the algorithm with simulations for tens of thousands of asteroids with different spin axis orientations, different rotational periods and random shapes. Such work is necessary to correctly analyse the results generated with the *Gaia* inversion algorithm at the end of the mission, when asteroids' photometric observations will be released.

Now that all the parameters of the *Gaia* scanning law are fixed, we are able to predict exactly the observation sequence for Solar system objects. It means that we can plan to observe from the ground at the same time as *Gaia* does. For example, we can very easily add a full rotational (dense) light curve around (or very close to) an isolated observation by *Gaia*. The link between the two data sets would then be very strong, as a single *Gaia* measurement provides a very precise absolute magnitude that can be used to calibrate the ground-based light curve. The question is: How many such light curves per object we need to obtain a substantial improvement of the inversion? Maybe a single one? Or more? Therefore, this work is thought to address such questions and lay the foundations for a collaboration involving coordinated observations from the ground.

2 CONTROL TEST WITH TRIAXIAL ELLIPSOIDS AND 'GEOMETRIC' SCATTERING LAW

The first test was performed to detect any systematic divergence between our simulated asteroids' magnitude and the magnitude generated by the *Gaia* inversion algorithm. Since the magnitude–phase relationship is essentially linear for the typical range of phase angles covered by *Gaia* (Zappalà et al. 1990), the inversion algorithm includes a linear parameter to describe this effect. Thus we have not implemented any light-scattering model in our simulations, but we have considered the geometrical phases (Lindegren 1977). For this test, we simulated *Gaia*-like observations for 10 359 triaxial ellipsoid shapes. This amount of objects is not a random choice, but the result of generating a set of asteroids having their spin axis directions uniformly distributed. The procedure followed to generate such uniform distribution start defining an initial mesh, consisting of eight unit vectors with respect to a common origin, each of those being the vertex of a cube. Then we recursively subdivide the surface with the Catmull–Clark subdivision method (Catmull & Clark 1978), which smooths the initial mesh surface by dividing the surface's polygons into smaller ones. After seven iterations we obtain a mesh with 10 359 vertices, each of those being the spin axis orientation of a given simulated asteroid.

In order to generate the observational epochs for each object, we have used the *Gaia* mission simulator developed by F. Mignard and P. Tanga at the Observatoire de la Côte d'Azur (OCA), with a sample of asteroids having typical main-belt orbits. The population period distribution is shown in Fig. 1 and was generated following a Maxwellian distribution like the one described in Pravec, Harris & Michałowski (2002). The disc-integrated photometry simulations have been generated using a Z-buffer standard graphic method described by Catmull (1974). Z-buffering works by testing pixel depth. The z-value of any new point to be written into the buffer is compared with the z-value of the point already there. If the new point is behind, it is discarded, whereas if it is in front, it replaces the old value. We note that simpler and more efficient methods exist to generate the brightness of a triaxial ellipsoid, but the main objective of this test was to ensure that the algorithm used to generate the photometric simulations for further tests with more elaborate shape representations was well performing and we were not adding any bias in our analysis. The resulting distribution of magnitudes (i.e.

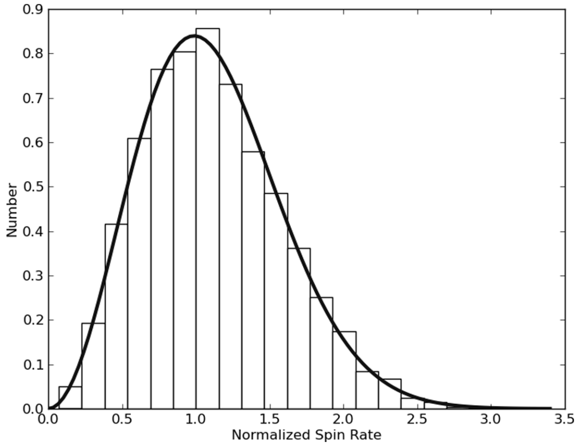

Figure 1. Histogram of $f/\langle f\rangle$ for the asteroid population generated in our simulations.

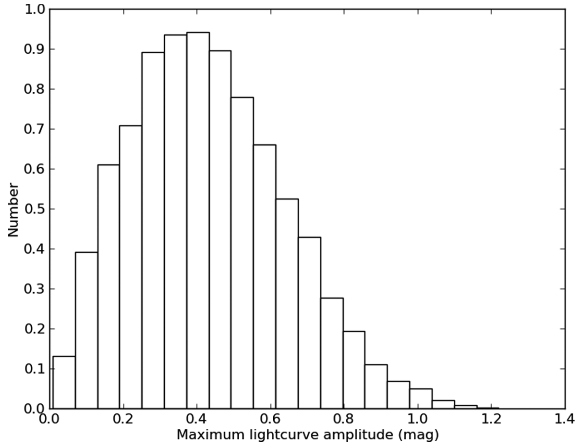

Figure 2. Histogram of maximum light-curve amplitude distribution for the asteroid population generated in our simulations.

the maximum light-curve amplitude for each asteroid) is shown in Fig. 2.

2.1 Test results overview: rotational period, spin axis orientation and overall shape

The inversion run was executed using the Poznań's observatory cluster which consist of 27 workstations equipped with a six-core AMD processors (3 GHz), and the outcome was obtained after one full day of computations. In terms of pole determination, the results were positive, as the inversion algorithm found the correct pole (within 5° of the true value) and shape (within 5 per cent of the true axis ratio) for more than 99 per cent of inversion runs. A few results presented an error in the pole determination, that increased as a function of the pole latitude. This situation can be interpreted as being caused by the double-pole ambiguity of derived spin states of asteroids orbiting close to the plane of the ecliptic. Thus this result is not an intrinsic problem of the method used, but a well-known limitation of the inversion techniques (Michałowski 1993).

The correct rotational period was also found for the majority of inversion attempts. In particular, for 10 319 of the 10 359 runs the correct value was found with an accuracy better than 0.01 per cent. The original orientation of the spin axis (ecliptic longitude λ and ecliptic latitude β) for the few attempts with a wrong period determination is shown in Fig. 3. The majority of the wrong solutions are confined to pole latitudes with $|\beta| < 30°$, and there was no wrong solution for pole latitudes higher than 60°. As will be shown later, it is a standard behaviour of the method to better perform for asteroids having high pole latitudes. A similar behaviour has been found in the past for other inversion techniques (e.g. in Hanus et al. 2011). As for our case, this effect can be explained by understanding one of the major advantages (paradoxically) of *Gaia*'s observations: its capability of seeing asteroids in a wide range of ecliptic longitudes in such a *short* period of time (the operational mission phase is planned to last for five years). For instance, if we consider a main-belt asteroid having a pole with $\beta \sim 0°$, the aspect angle (i.e. orientation of the object's spin axis with respect to the direction of sight of the observer) will be very low for two out of five apparitions observed by *Gaia* (see fig. 2 in Cellino et al. 2006). For such apparitions the asteroid's light curve is presenting almost no amplitude, resulting in the loss of information about the spin period signal in the sparse-in-time measurements obtained under such circumstances. If the observational sequence for such kind of objects is unluckily distributed, and the majority of measurements are collected under such geometries, the period search would become very sensitive to any asymmetries in the light curve, arising for example from an irregular shape. This could cause the genetic algorithm to find alternative solutions, resulting in a warning flag (refusing to generate a solution) or, in the worst case, it could lead to a wrong inversion solution. On the other hand, asteroids with high pole latitudes are always going to be observed under aspect angles far from zero (see fig. 1 in Cellino et al. 2006), thus in such cases, each measurement is bearing valuable information about the spin period. This is because, in such geometries, the instigator of the main periodical signal will be the asteroid axis with the greatest angular momentum (thus the longest). Consequently, the light-curve amplitude will be near its maximum value and the signals due to any shape irregularity would play a secondary role. Nevertheless, it should be highlighted that the effect of low pole latitude plays in opposite ways for pole determination and for spin period determination. In particular, asteroids having low pole latitudes are ideal for any inversion technique for deriving the pole, due to the high variation in light-curve amplitude for different ecliptic longitudes.

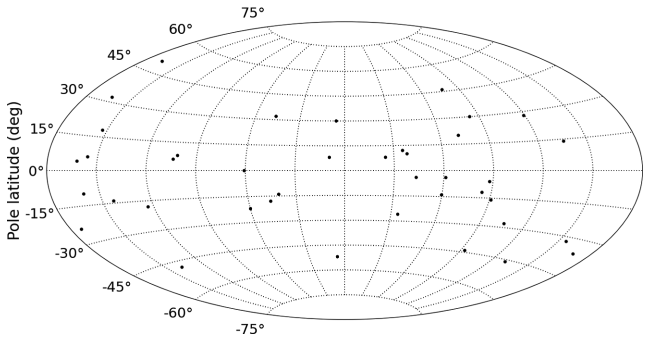

Figure 3. Result of the period determination. The black points represent the original pole orientation for the runs which period determination had an error greater than 0.01 per cent. The number of wrong period determination was less than 0.4 per cent of the 10 359 asteroids used in this test.

Concerning other results, we also studied the ellipsoid axis ratio *b*/*a* which describes the elongation of the body. This parameter was found with an accuracy better than 5 per cent for more than the 98 per cent of inversion attempts. In this case, no correlation with the pole latitude can be observed. On the other hand, in the case of the *c*/*a* axis ratio determination, there is a clear relation between its error and the pole latitude. In particular, the *c*/*a* axis ratio was found with an accuracy better than 5 per cent for 95 per cent of the attempts, and almost the totality of the problematic cases – meaning solutions with an accuracy worse than 5 per cent – were found for asteroids with extreme values of sin β. This result is not surprising and can be easily explained in terms of observational geometry, as for objects having high pole latitudes and orbits close to the ecliptic, the observations bring no or little information on the c axis.

Finally, it is worth mentioning that the presence of a small number of wrong inversion solutions is, in any case, unavoidable, due to the way the genetic approach works. There is always the possibility that several genetic inversion attempts for the same object will not be sufficient to catch the right solution. Increasing the number of attempts per object would certainly improve further the performances, but this is not feasible in the *Gaia* scenario due to CPU time constraints, as the inversion algorithm will have to process a number of the order of half a million of asteroids (in other words, two months of computations in our observatory cluster).

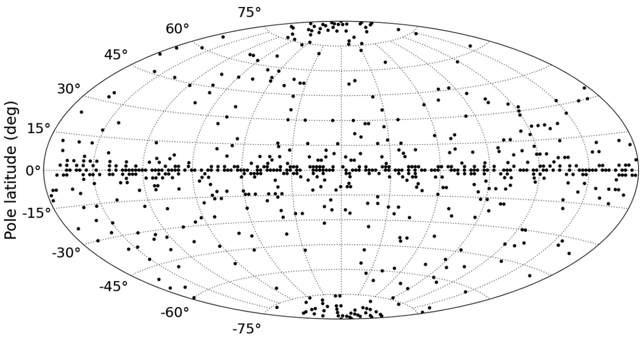

Figure 4. Distribution of the warnings received from the inversion algorithm.

2.2 Results control system

Since *dense* light curves are only available for a minority of the asteroids observed by *Gaia*, we would not be able to infer if the inversion results are providing real information of the asteroid's physical parameters or, in contrast, they are the result of an inversion artefact. In order to tackle this problem, the inversion algorithm developed for the *Gaia* data analysis pipeline, is including a *warning* criterion to select the acceptable results. In particular, a *warning* flag will be generated for those cases where the best fit is close to the second-best one, but their inversion solution is substantially different. It is still under discussion if these solutions shall be included in the final catalogue marked with a flag or, instead, they shall remain unpublished. The distribution of such cases can be seen in Fig. 4. We received a total of 660 warnings, being 59 per cent of them from asteroids having a pole latitude between $-15^\circ < \beta < 15^\circ$. This shall be taken into account when analysing the *Gaia* inversion statistics, as we expect them to show a lack of asteroids with low pole latitudes. Otherwise, one could tend to mislead this effect with some physical effects, such as non-gravitational forces.

2.3 Simulations contaminated with Gaussian noise

The good results obtained in the control test described above allow us to be confident with the methodology used, i.e. we are able to generate photometric simulations which are correctly inverted by the software algorithm that will be used for the analysis of *Gaia* asteroid photometry. However, we cannot expect the inversion of the real observations obtained by *Gaia* to have such a high reliability, as our simulations were generated under ideal assumptions (triaxial ellipsoid, geometric scattering law, no tumbling or binary asteroids, etc.). Obviously this is not the situation we are going to face when analysing the *Gaia* photometry. *Gaia* photometric accuracy for each single transit will be of the order of 0.01 mag for objects as faint as $V = 18.5$. Thus in the majority of cases the method systematic errors (coming, for instance, from the ellipsoid shape approximation or the scattering law used) will be of greater concern than the errors arising from the photometric accuracy. Thus, we contaminated our photometric simulations with Gaussian noise with different values of σ, and we repeated the inversion process for each case. The results distribution is shown in Fig. 5, and two different biases can be observed: (1) population bias, (2) inversion reliability bias. The first one is connected with the warnings obtained from the results control system described above. The number of rejected solutions is not homogeneously distributed, as the majority of them are concentrated around asteroids having low pole latitude. The second bias is affecting the reliability of the obtained results. For $\sigma \geq 0.03$ the results reliability is becoming proportional to the asteroid's pole latitude, being worse for the low pole latitudes. Worth noting that the inversion solutions studied are the ones accepted by the algorithm's warning system, thus the first and the second bias are superimposed.

3 REALISTIC TEST USING RANDOM NON-CONVEX SHAPES

Once we felt fully confident with our simulation-inversion procedure and after studying the methodological bias, we proceeded with a more demanding test. In order to recreate as close as possible the kind of data which will feed the *Gaia* inversion algorithm we generated a set of 10 359 random non-convex shapes using Gaussian spheres (Muinonen 1998). An example of a random shape generated with our procedure can be seen in Fig. 6. The spin axis and the rotational period for each object were chosen following the same manner as in the previous tests described above. Next we generated the brightness using the Z-buffer standard graphic method. This method is including the phase angle effects, but unlike the first tests with ellipsoids, this is also including the shadowing effects produced by the irregular surface structures. Consequently, the light curves generated by these non-convex shapes presented complex features such as multiple minima and maxima, which cannot be recreated with a simple ellipsoid model. Moreover, once the brightness was simulated, we contaminated the set with a Gaussian noise with $\sigma = 0.03$. As stated before, this is three times the photometric accuracy expected for each single detection made by *Gaia*. But this might allow us to be on the safe side and would cover unexpected methodological errors, for instance, those connected with the scattering properties. Thus, the main question to be answered is whether the *Gaia* inversion algorithm will be able to deal with such complex scenario.

3.1 Results overview

The answer to the previous question happened to be very optimistic: 65 per cent of the 10 359 asteroids were accepted by the inversion software and the correct solution was found for 83 per cent of them, or to be more precise, 6754 inversion results were generated, from which 5632 were correct and 1122 incorrect. If we interpret this results in terms of the expected performance of the *Gaia* mission for Solar system objects, this would allow us to derive the poles, sidereal periods and axial ratios for several thousand asteroids. None the less, we could find some correlations between the majority of wrong inversions and certain parameters: the number of observations, the asteroid shape and the object's spin axis. Understanding such dependences would allow us to refuse some of the wrong results or even correct them.

3.2 Influence of the number of measurements

It is of common sense to consider that the more data points the inversion attempt has the better the result will be. And this is, in fact, what our results are confirming. The histogram in Fig. 7 is showing the percentage of correct solutions found as a function of number of *Gaia* measurements and the asteroid pole latitude. The majority of regions having 70 measurements or more were found to be above the average of correct solutions. However, for lower regions, we found that the results are acceptable for asteroids having high pole latitudes, but worse than the average for asteroids having low

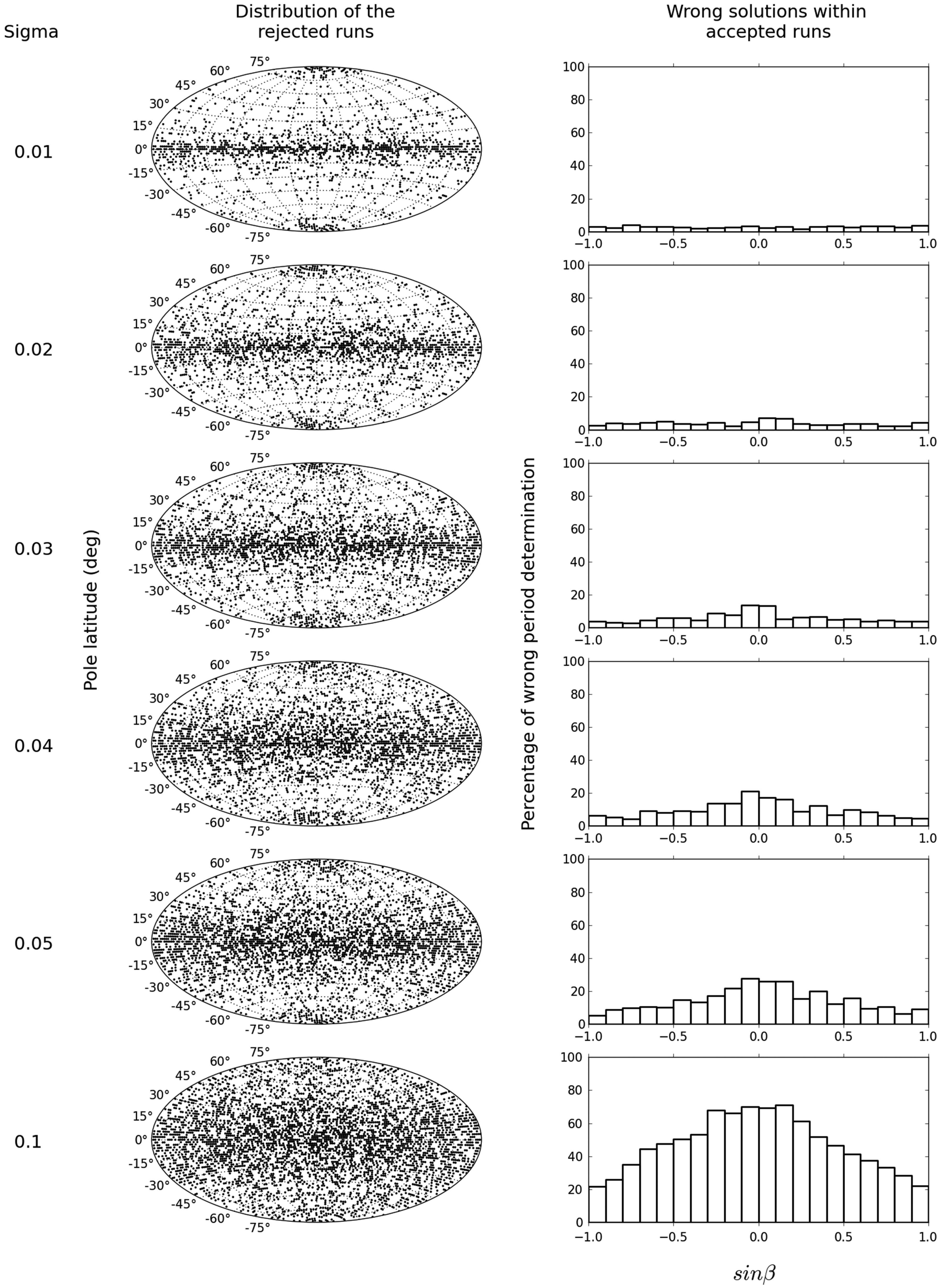

Figure 5. Distribution of the inversion results for each σ value for the noise. The projections on the left show the inversion runs for which the solution was not accepted by the algorithm. The histograms on the right show (in per cent) the distribution of the wrong solutions within the accepted runs as a function of the asteroids' pole latitude.

pole latitudes. The range of latitudes around zero for which the inversion is presenting a lower reliability is getting wider the smaller the number of measurements is. Still, the good news is that, on the average, main-belt asteroids will be detected on the *Gaia* focal plane for a number of times between 60 and 70 during the five-year operational lifetime of the mission (Mignard et al. 2007). Therefore, the number of *problematic* asteroids for which *Gaia* photometric inversion might produce wrong solutions will represent a small part (although

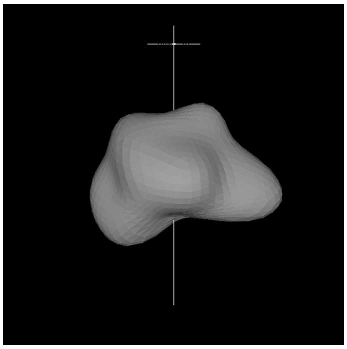

Figure 6. Example of a random non-convex shape used for generating the photometric simulations.

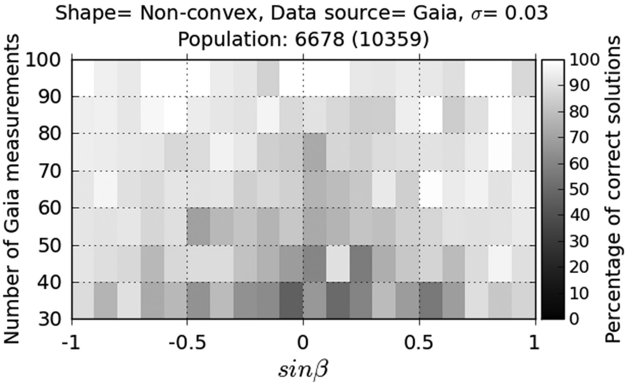

Figure 7. Histogram showing the results obtained for the inversion of the simulated set of irregular body shapes. The percentage of correct solutions is plotted as a function of the number of *Gaia* detections for each bin of asteroid's pole latitude. The population number is indicating the amount of generated solutions and the total of inversion runs executed (in brackets).

still several hundreds) of the hundreds of thousands of asteroids observed.

3.3 Influence of the asteroid shape

The *Gaia* inversion algorithm is assuming the asteroids to have the shapes of triaxial ellipsoids. This approach was chosen mainly due to two reasons: (1) it seeks to minimize the CPU time required, (2) there was a need to produce an automated, standard procedure for working on such large amount of data in unattended runs. Although this approximation would seem inaccurate at a first glance, the results are showing that, despite its simplicity, this approach is sufficient to fit the data in the great majority of cases. Certainly, the shape solution provided by the algorithm is only a first-order approximation of the asteroid's shape and might provide only a general idea of the body elongation.

In order to assess the goodness of the inversion solution we calculated the principal moment of inertia for each random shape, and we determined the triaxial ellipsoid with an equivalent moment of inertia. This operation enables us to obtain an indicator of the elongation of any irregular shape, as the asteroids have been simulated in a relaxed state (i.e. with the rotation axis coincident with its principal axis of rotation). Finally, we classified our set of random shapes into three groups, according to the value of the equivalent *b*/*a* axis ratio calculated. The results are presented in Fig. 8. As we could expect, the worst results are obtained for the quasi-spherical bodies and, in particular, for those having a low pole latitude. Such population is presenting the highest ratio of wrong solutions (around 30 per cent on the average) but also one out of two solutions is refused with a warning flag. These results are in agreement with the first tests presented above and can be explained using the same scheme (see Section 2.1 for more details). It is worth pointing out that, by definition, an ideally spherical object cannot be inverted, since the magnitude becomes dependent on the phase angle only. When *b*/*a* ratio approaches 1, the light-curve amplitude remains always very small, and the inversion algorithm will find a large number of equivalent solutions (in terms of residuals) characterized by a large variety of possible poles.

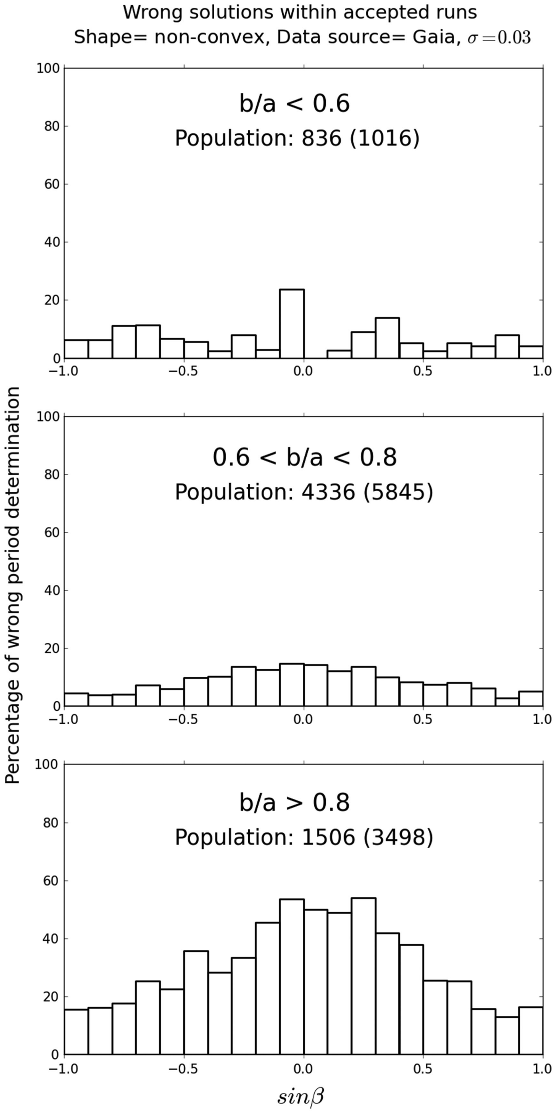

Figure 8. Histograms showing the inversion results obtained for three different groups of asteroids as a function of their equivalent *b*/*a* axis ratio (see text). The population numbers are indicating the amount of generated solutions and the total of inversion runs executed (in brackets).

4 *GAIA* PHOTOMETRY COMBINED WITH GROUND-BASED OBSERVATIONS

Combining *Gaia* observations of asteroids with ground-based light curves becomes straightforward when both observations are taken simultaneously. In contrast, if the light curve obtained from the ground does not include the epoch of observation by *Gaia*, there may be problems to link the *Gaia* observation to the rotational phase, and to calibrate the magnitudes of the ground-based data, especially in cases when the light curve is complex and the period resulting from the light curve is uncertain.

With the aim of supporting an observational campaign, it would be a good idea to publish the *Gaia* observation sequence for Solar system objects, allowing the observers to obtain a light curve of a certain asteroid at the same time as *Gaia* is collecting a very precise photometric measurement. Later on, it will be possible to calibrate the ground-based observation (even if it is relative photometry) with the *Gaia* absolute magnitude, and proceed with the inversion process normally. Formally, the only difference between data sources will appear during the preparation of the input file containing the photometric error associated with each observational instrument and the position vectors of the observer.

In order to study the impact of adding ground-based observations to *Gaia* data, we have simulated a full light curve with 60 point measurements for the asteroids non-convex shapes described in Section 3. The particular geometry of the scan movement of *Gaia* telescopes, which never point on the Sun nor its opposition, results in observations taken at relatively high phase angles. For instance, considering the *Gaia* observations of a main-belt asteroid, the measurement with the lowest phase angle will be usually above 10°. As the asteroid's magnitude becomes fainter for increasing phase angles, we selected the date of the *Gaia*'s measurement with the lowest phase angle as an epoch to generate the light curve. This choice was taking into account that ground-based supporting observations of asteroids will be probably done by small or mid-sized telescopes, and moreover, we are not interested in the projecting shadows appearing at higher phase angles. Finally, we contaminated all the simulated light curves with Gaussian noise ($\sigma = 0.03$). It should be pointed out that the results presented here are limited to the specific choice made when adding dense light curves and they could be different for observations obtained around opposition, more than one curve, etc.

4.1 Impact on the results as a function of the asteroid shape

After combining the simulated *Gaia* photometry and the full light curve, we use the resulting data set to feed the inversion algorithm. The results resemble the ones obtained for the *Gaia* data alone, i.e. a good overall result, albeit worse reliability for nearly spherical bodies. In Fig. 9, we show the percentage of wrong solutions as a function of the asteroid's pole latitude and the equivalent b/a ratio, as well as the improvement of combining both data sets. It resulted that, with the exception of nearly spherical objects, the improvement was almost negligible. It is worth noting that the relative improvement on the results is not caused by an increment of the correct inversions, but the reduction of the accepted wrong solutions.

4.2 Impact on the results as a function of the number of measurements

If we analyse the results as a function of the amount of points collected by *Gaia*, the improvement can be clearly appreciated for asteroids having less than 50 detections. The results of the inversion run and the corresponding improvement are shown in Fig. 10. For the majority of asteroids for which supporting ground-based observations will be planned, we would not have any a priori information of the physical parameters. In contrast, the total number of observations for each asteroid can be already calculated as all the parameters of the *Gaia* scanning law are already fixed and the mission has started its science phase. Considering that it is not feasible to obtain one light curve for each of the ∼300 000 asteroids observed by *Gaia*, it would be necessary to draw up an observational plan, thus we conclude that, for the purposes, the number of expected *Gaia* measurements can be a good selection criterion.

4.3 Discussion of the results

The results presented above could seem counter-intuitive. In particular, one could argue how it is possible that the addition of a full light curve does not generally improve the reliability of the method. In order to understand the situation, we should clarify some points:

(i) *The actual version of the inversion algorithm treats equally each single measurement.*

The goodness of the inversion solution is estimated on the basis of the fit between computed and observed single measurements. In the case of asteroids with abundant *Gaia* observations (for instance, more than 80 points), a single light curve will have a discreet influence on the inversion result, especially for light curves with low amplitude (Marciniak & Michałowski 2010). This situation can be faced by increasing the weight of the ground-based light curve to the detriment of the *Gaia* data.

(ii) *The additional light curves were blindly simulated in terms of asteroid's aspect angle.*

Observations obtained at high aspect angles represents the best-case scenario when deriving the rotation period, as asteroid light curve is then at its maximum amplitude. However, it would require us to know in advance the spin axis orientation of the given asteroid so to predict the appropriate observational epoch. For the great majority of asteroids observed by *Gaia* this is not feasible, as we do not know their rotational states. For this reason, the only selection criterion used when generating the additional light curves was the asteroid's phase angle (see the beginning of the section), thus some of the light curves present low amplitudes. Under this particular situation, the inversion fit's residual is very low no matter the rotation period, and so the impact on the inversion results is negligible.

(iii) *The triaxial ellipsoid assumption of the inversion algorithm might have not well-behaved cases.*

While it has been proven to work well for the majority of cases, the triaxial ellipsoid assumption might cause the inversion to fail under tough cases. For instance, very irregular shapes can generate light curves with multiple extrema, which cannot be inverted using a triaxial ellipsoid assumption, no matter how many complementary data are used.

5 CONCLUSIONS

We have tested the *Gaia* inversion algorithm fed with realistic simulations of asteroids and the results have been encouraging. The number of correct inversions remains above 80 per cent even under severe scenarios with photometric errors of 0.03 mag. Moreover, we have detected the most problematic scenarios for the method: (1) asteroids having a quasi-spherical shape, (2) asteroids with low pole latitudes and (3) asteroids with less than 50 data points. These

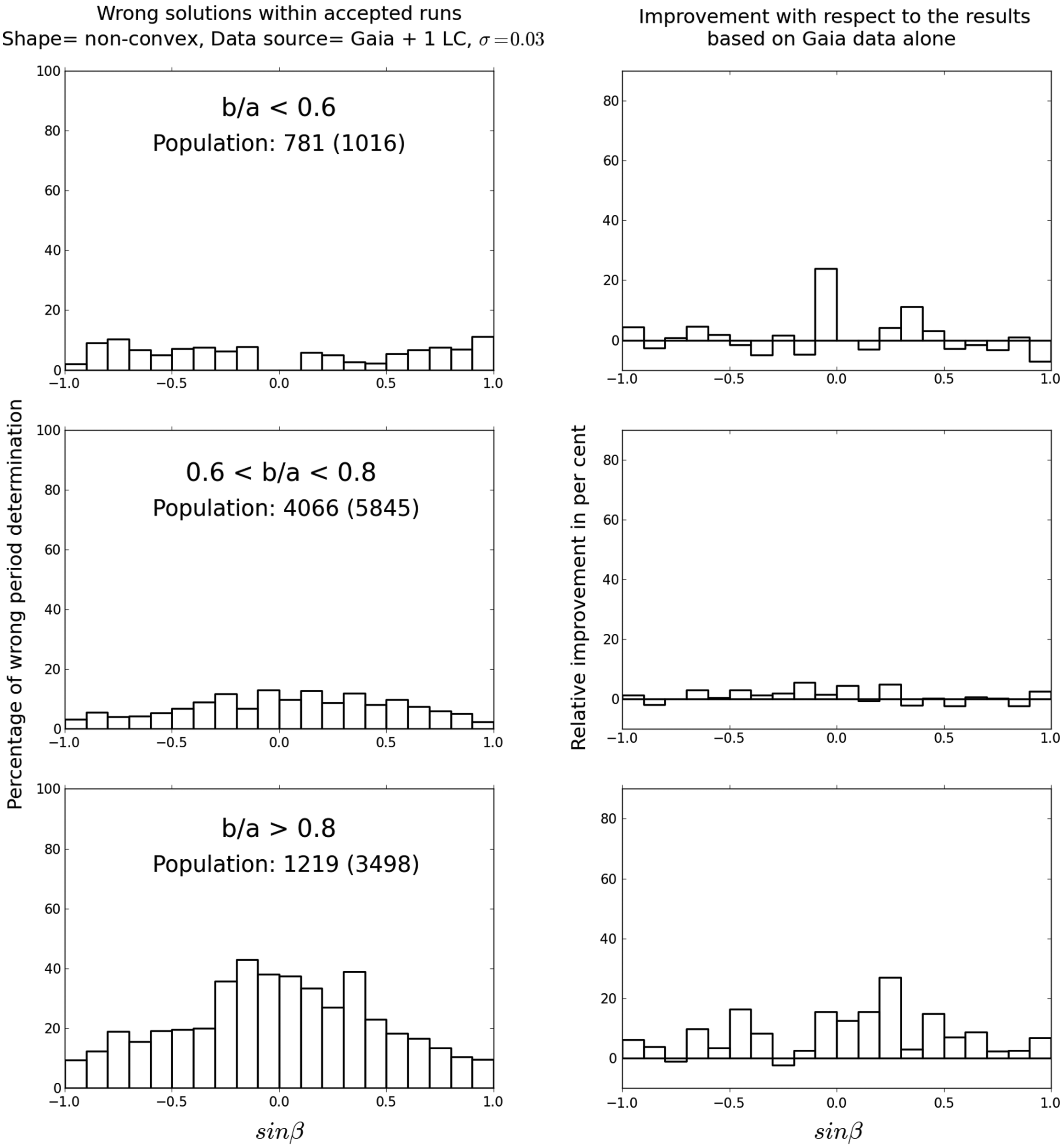

Figure 9. Histograms on the left show the inversion results obtained for a data set combining *Gaia* photometry and one full light curve. The results are divided into three groups as a function of the asteroids' equivalent *b/a* axis ratio and are plotted as a function of the asteroids' pole latitude. Histograms on the right show the relative improvement comparing with the inversion results obtained for *Gaia* data alone.

biases have to be considered before sketching physical interpretations of the future inversion results. Otherwise, the detected bias could be erroneously mislead with physical effects, like an over-interpretation of the Yarkovsky - O'Keefe - Radzievskii - Paddack effect resulting from the loss of many cases having poles far from perpendicular to the orbital plane. We have shown that it is possible to reduce the number of wrong solutions by adding a single light curve to *Gaia*'s measurements. Thus this pre-selection method can be used to coordinate an observational campaign with the aim of enhancing the *Gaia* Solar system science output. It is also of utmost importance to develop strategies for collaboration with ground-based optical surveys that will produce in the near future sparse photometric measurements similar to the ones produced by *Gaia* in terms of quantity and quality. The most outstanding project for the next decade is LSST, which, up to some extent, could be understood as a ground extension of the *Gaia* mission. In this sense, the inversion algorithm used in this paper can be easily adapted to process and combine data from other surveys. Such collaboration shall even greatly boost our statistical picture of physical parameters from Solar system objects and would allow us to derive shapes and spin states of asteroids far beyond the main belt.

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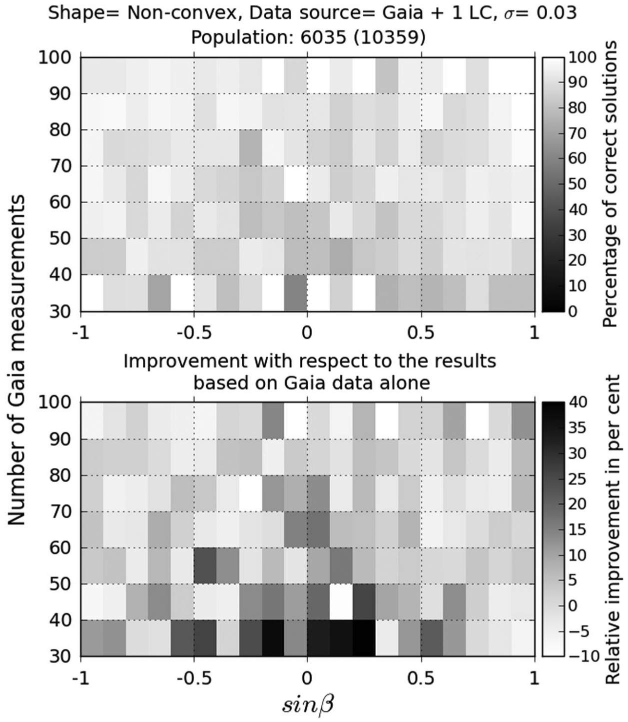

Figure 10. The histogram on the top shows the results obtained for the combined data set. The percentage of correct solutions is plotted as a function of the number of *Gaia* detections for each bin of asteroids' pole latitude. The population number is indicating the amount of generated solutions and the total of inversion runs executed (in brackets). The histogram on the bottom shows the relative improvement compared to the inversion results obtained for *Gaia* data alone.

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