The accretion rate and minimum spin period of accreting pulsars

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ABSTRACT

We consider combined rotational, magnetic, and thermal evolution of the neutron star during the accretion phase in a binary system. A rapid accretion-driven decay of the magnetic field decreases substantially the efficiency of angular momentum transfer. As a result, the neutron star cannot spin-up to periods shorter than some limiting value even if accretion is very long and accretion rate is high. The proposed mechanism can explain a discrepancy between the shortest detected period and minimal possible spin period of neutron stars.

Key words: magnetic fields – stars: neutron – pulsars: general – stars: rotation.

1 INTRODUCTION

According to the generally accepted point of view (Bhattacharya & van den Heuvel 1991), the population of radio millisecond pulsars originates from low-mass X-ray binary systems (LMXBs). The re-cycling scenario suggests that a neutron star born in a binary system with more or less typical parameters can accrete the gas stripped by the companion until its magnetic field and spin period will fit the so-called millisecond pulsar box in the *B–P* (magnetic field–period) plane. The characteristic values of the magnetic field and period for millisecond pulsars are $B \sim 10^8$–10^9 G and $P \sim 1$–10 ms, respectively, and these values can be reached as a result of evolution in a binary system. The evolution of a neutron star in a low-mass binary is extremely long and complex, because even the main-sequence lifetime of a low-mass companion exceeds 10^9 yr. In accordance with the standard scenario (Pringle & Rees 1972; Illarionov & Sunyaev 1975) the neutron star in a close binary passes throughout several evolutionary phases.

(i) The initial phase, in which the pressure of the pulsar radiation keeps the wind plasma of a companion away from the neutron star magnetosphere. The magnetic, thermal, and rotational evolution do not differ from those of an isolated star.

(ii) The propeller phase, in which the radiation pressure reduced by the spin-down and field decay cannot prevent the wind plasma from interaction with the magnetosphere. However, rotation is still sufficiently fast to eject the wind plasma by a propeller effect.

(iii) The wind accretion phase, in which the wind plasma falls down on to the surface of a neutron star and nuclear burning of the accreted material heats the star and, due to this, accelerates the field decay.

(iv) The enhanced accretion phase, which starts when the companion leaves the main sequence and fills its Roche lobe. The Roche lobe overflow drastically increases accretion on to the neutron star. During this phase, a vigorous mass transfer heats the neutron star interiors to a very high temperature $(1–3) \times 10^8$ K and accelerates essentially the field decay. The accretion torque spins up the neutron star to a short period.

The pulsars processed in the above transformations can have finally the parameters close to those of millisecond pulsars (Urpin, Geppert & Konenkov 1998a,c) even if they had initially the typical for pulsars magnetic fields and periods. Certainly, the neutron star experiences the most dramatic changes in the course of the phase IV, and this phase determines the final state of the pulsar. During the enhanced accretion phase, the neutron star initially spins up very rapidly until it approaches the so-called spin-up line corresponding to the accretion rate. The spin-up line in the *B–P* plane is determined by corotation at the Alfvén radius. It was suggested by Bhattacharya & Srinivasan (1991) that, during the further evolution, a balance between spin-up and the rate of field decay is reached, so that the neutron star slides down the corresponding spin-up line with a rate that is determined by the field decay.

However, the behaviour of a neutron star during the enhanced accretion phase can be much more complicated if one takes into account the combined magnetic, thermal, and rotational evolution. Because of a high internal temperature, the magnetic field decay during the accretion phase can be very fast such as mass transfer is not able to provide a sufficient amount of the angular momentum to maintain a balance at the spin-up line. Therefore, a significant departure from the spin-up line can be expected during the accretion phase. In this paper, we will show that these departures occur at both low and high accretion rates and, owing to them, the minimum period exists that can limit spin-up of the accreting neutron star. The present study is addressed mainly the neutron stars in LMXBs.

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However, the same qualitative behaviour is typical for spin-up of accreting neutron stars in other types of binary systems as well. This concerns particularly high-mass X-ray binaries where the accretion rate is very high and field decay is fast (Urpin, Konenkov & Geppert 1998b). In such systems, the angular momentum transfer can also be substantially influenced by the magnetic field decay and thermal evolution. The considered mechanism of spin-up can be the key issue in understanding the origin of various classes of accreting pulsars.

2 BASIC EQUATIONS

Consider the evolution of a neutron star during the enhanced accretion phase that begins at the end of the main-sequence life of a companion. The magnetic, thermal, and spin evolution of the neutron star may be essentially affected by such accretion. We assume that the magnetic field is maintained by electric currents in the crust. Such magnetic configuration can be generated, for example, by turbulent dynamo during first minutes of the neutron star life (Bonanno, Urpin & Belvedere 2005, 2006). The evolution of the crustal field is determined by the conductive properties of the crust and material motion throughout it. In a very strong magnetic field, the ohmic dissipation can be accompanied also by non-dissipative Hall currents. These currents affect the decay of the magnetic field indirectly, coupling different modes and generating magnetic features with a smaller length-scale than the background magnetic field (Naito & Kojima 1994). Numerical simulations (Hollerbach & Rüdiger 2002, 2004) indicate that some acceleration of the decay of a large-scale field (for instance, dipole) can occur if the Hall parameter is very large but the effect is not significant for typical pulsar fields. Besides, this effect turns to be sensitive to the initial magnetic geometry. For example, the presence of both the toroidal and dipole field components decreases the rate of dissipation caused by the Hall effect, and dissipation proceeds approximately on the ohmic time-scale (Hollerbach & Rüdiger 2002, 2004). Note also that the Hall currents are important only for a very strong magnetic field, but, at the beginning of enhanced accretion, the field of a neutron star is usually weaker than the 'standard' pulsar field (Urpin et al. 1998a,c). Therefore, we will neglect the Hall currents. Then, the induction equation in the crust reads

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\frac{c^2}{4\pi}\nabla \times \left(\frac{1}{\sigma}\nabla \times \boldsymbol{B}\right) + \nabla \times (\boldsymbol{v} \times \boldsymbol{B}), \tag{1}$$

where σ is the conductivity and $\boldsymbol{v}$ is the velocity of crustal matter. The velocity $\boldsymbol{v}$ is caused by the flux of the accreted matter and is non-zero only during the periods when the neutron star undergoes accretion. This flux carries out the magnetic field into the deep crustal layers. Assuming spherical symmetry of the material flow, the velocity in the negative radial direction can be written as

$$v = \frac{\dot{M}}{4\pi r^2 \rho}, \tag{2}$$

where $\dot{M}$ is the accretion rate and ρ is the density. We consider the evolution of an axisymmetric magnetic field following Wendell, van Horn & Sargent (1987). Introducing the vector potential $\boldsymbol{A} = (0, 0, A_\varphi)$, where $A_\varphi = S(r, \theta, t)/r$ and (r, θ, φ) are the spherical coordinates, we obtain the equation for S from equation (1). The function S can be separated in r and θ in the form

$$S = \sum_{l=1}^{\infty} s_1(r, t) P_1^1(\cos\theta), \tag{3}$$

where $P_1^1(\cos\theta)$ is the associated Legendre polynomial with the index 1. Using the properties of Legendre polynomials, we have

$$\frac{\partial s_1}{\partial t} = v\frac{\partial s_1}{\partial r} + \frac{c^2}{4\pi\sigma}\left[\frac{\partial^2 s_1}{\partial r^2} - \frac{l(l+1)}{r^2}s_1\right]. \tag{4}$$

In the case of a dipole field ($l = 1$), the function $s_1(r, t)$ can be related to the surface magnetic field at the pole, $B_{\rm p}(t)$, by $B_{\rm p}(t) = 2s_1(R, t)/R^2$, where R is the stellar radius (Urpin, Chanmugam & Sang 1994). Since equation (5) is linear, we can normalize s_1 in such a way that $s_1(R, 0) = 1$. Then, the ratio $B_{\rm p}(t)/B_{\rm p}(0)$ is given by $s_1(R, t)$.

Continuity of the magnetic field at the stellar surface $r = R$ yields the boundary condition for equation (5)

$$\frac{\partial s_1}{\partial r} + \frac{l}{R}s_1 = 0. \tag{5}$$

In this paper, we consider the case of a dipolar field with $l = 1$.

Since the crustal conductivity depends on the temperature T, accretion must influence the field decay by changing both T and inward directed flux of matter. Therefore, the magnetic evolution of a neutron star in a binary turns out to be strongly coupled to its thermal evolution. The thermal evolution is determined mainly by pycnonuclear reactions and, due to them, the crust is heated by nuclear burning of the accreted material. The enhanced accretion heats the neutron star to a high temperature $\sim(1–3) \times 10^8$ K that leads to a rapid field decay. The thermal structure of the star with pycnonuclear reactions has been studied by a number of authors (Brown & Bildsten 1998; Haensel & Zdunik 2008, and references therein). In LMXBs, accretion due to Roche lobe overflow can last as long as 10^6–10^8 yr and the magnetic field can be drastically reduced in the course of this phase.

Our model is in contrast to the widely accepted assumption of a proportionality between to the amount of accreted material and the field decay rate at any time along the spin-up line (Shibazaki et al. 1989). This simple model is obviously inconsistent from a theoretical point of view because the decay is determined not only by the duration of accretion (or, equivalently, by the total amount of accreted mass) but also by the conductivity of the crust. The latter is determined by the temperature which is dependent on the accretion rate. As a result, a consistent description of the field decay must take into account both the duration of accretion (or total amount of accreted mass) and accretion rate. A detailed study of the accretion-driven field decay (Urpin & Geppert 1995) reveals that, generally, the decay is not proportional to the amount of accreted mass at any accretion rate. Departures from this simple relation are essential even at a low accretion rate but the dependence becomes particularly complicated if the accretion rate is high, $\geq 10^{-10}$ M$_\odot$ yr^{-1} (see e.g. fig. 5 of the paper by Urpin & Geppert 1995). It is worth noting that a simple model with a field decay proportional to the amount of accreted material is not in agreement with observational data as well. This point is discussed in detail by Wijers (1997). In the model developed in the present study, the magnetic field decay is determined by both the accretion rate and duration of accretion.

The rotational evolution is entirely determined by accretion as well because the mass flow carries a large amount of the angular momentum to the neutron star. It is widely believed that the mass flow interacts with the magnetosphere at the so-called Alfvén radius, $R_{\rm A}$, which is determined by a balance of the magnetic pressure and the dynamical pressure of plasma,

$$R_{\rm A} = \left(\frac{2R^6 B_{\rm p}^2}{4\dot{M}\sqrt{GM}}\right)^{2/7}, \tag{6}$$

where M is the neutron star mass and G is the gravitational constant. If the neutron star rotates rapidly and its angular velocity is larger than the Keplerian angular velocity at the Alfvén radius, $\Omega > \Omega_K(R_A) = (GM/R_A^3)^{1/2}$, plasma penetrating to the Alfvén radius should be provided some portion of the angular momentum from the rapidly rotating magnetosphere and, as a result, this plasma must be expelled. On the contrary, matter carrying the angular momentum can fall down on to the neutron star if its rotation is relatively slow and $\Omega < \Omega_K(R_A)$. Due to this angular momentum, the neutron star spins up to a shorter period. However, the star cannot spin-up to a period shorter than the Keplerian period at the Alfvén radius because otherwise it will work as a propeller and expels plasma instead of accreting it. It has been argued by Bhattacharya & Srinivasan (1991) that a balance should be reached in spin-up and the rate of the field decay, thus the neutron star slides down the spin-up line with the corresponding accretion rate.

Since the interaction with the magnetosphere occurs at the Alfvén radius, the angular momentum carried by the accreted plasma can be characterized by its Keplerian value at R_A, $\Omega_K(R_A)R_A^2$, multiplied by some 'efficiency' factor ξ, $\xi < 1$. This factor depends on the geometry of accretion flow. If the accreted matter forms an accretion disc around the neutron star, then $\xi \sim 1$. If accretion is close to spherical one, then ξ is likely much smaller. The rate of the angular momentum transfer to the neutron star, $\dot{J}$, can be estimated as $\sim \xi\dot{M}(\Omega_K(R_A)R_A^2)$, The corresponding spin-up rate is

$$\dot{P} \sim -\frac{P^2\dot{J}}{2\pi I} \approx -\xi\beta P^2 B_p^{2/7}\dot{M}_G^{6/7}, \tag{7}$$

where $\beta = (GMR^2/4)^{3/7}/\pi I$, I is the moment of inertia of the neutron star. The star spins up until the angular velocity of the neutron star Ω becomes comparable to $\Omega_K(R_A)$. This condition determines the critical period, P_{eq}, which distinguishes the accretion and propeller phases. We have for P_{eq}

$$P_{eq} \approx \frac{12.4 B_{12}^{6/7} R_6^{18/7}}{\dot{M}_{-10}^{3/7} M_{1.4}^{5/7}} \text{ s}, \tag{8}$$

where $B_{12} = B_p/10^{12}$ G, $\dot{M}_{-10} = \dot{M}/10^{-10}\,M_\odot$ yr^{-1}, $M_{1.4} = M/1.4\,M_\odot$, and $R_6 = R/10$ km. Equation (8) determines the so-called spin-up line in the B–P plane. Likely, the accreted matter forms the Keplerian disc during this phase. Therefore, spin-up of the neutron star is driven by equation (7) with $\xi \approx 1$. At the beginning of the accretion phase, the neutron star approaches the spin-up line corresponding to an enhanced accretion rate and then it slides down this line. Evolution on the spin-up line can last until either the accretion regime is changed or the magnetic field becomes too weak to maintain a balance in spin-up and field decay. In the latter case, the neutron star must leave the spin-up line and evolve in a very particular way. We assume that the spin period of a neutron star follows equation (7) if $P \geq P_{eq}$. However, as it was discussed above, we have to suppose $P = P_{eq}$ if equation (7) leads to $P < P_{eq}$ because accretion must stop at such rotation and the neutron star begins to work as a propeller if $P < P_{eq}$, As a result of accretion, we obtain a rapidly rotating neutron star with a low magnetic field that can manifest itself as a radio pulsar after accretion is exhausted.

3 NUMERICAL RESULTS

The thermal evolution of a neutron star is considered by making use of the public code developed by Page, NSCOOL[1] in which the magnetothermal evolution has been taken into account by coupling the induction equation with the thermal equation via the Joule heating as discussed by Bonanno et al. (2014). In particular the induction equation has been solved via an implicit scheme and the electron conductivity has been calculated with the approach described (Potekhin 1999).[2] Actual calculations have been performed for a neutron star of $M = 1.4\,M_\odot$ based on the APR equation of state (EOS; Akmal, Pandharipande & Ravenhall 1998). The radius and the thickness of the crust for this star are 11.5 km and ≈ 1.0 km, respectively. For the chemical composition of the crust, we use the same so-called accreted matter model by Haensel & Zdunik (2008). In addition we have also considered models of $M = 1.4\,M_\odot$ and $M = 1.6\,M_\odot$ obtained with a stiffer EOS where the baryonic matter is calculated using a field-theoretical models at the mean field level (Glendenning & Moszkowski 1991). In particular, the extension of the crust for $M = 1.4\,M_\odot$ for this EOS is about 2.1 km with a radius of 13.8 km, at variance with the APR EOS case.

The impurity parameter, Q, is assumed to be constant throughout the crust and equal to 0.001. The spin period at the beginning of the enhanced accretion phase is assumed to be $P_0 = 100$ s in all runs. Note, however, that the results are not sensitive to these values. The magnetorotation evolution of the neutron star depends on the field strength and its distribution in the crust. Our models at the beginning of the accretion phase are produced according to the framework discussed in Urpin et al. (1998a) for wind accretion rate 10^{-14} M$_\odot$ yr^{-1}. The magnetic field at the neutron star birth is assumed to be confined to the outer layers of the crust with densities $\rho \leq \rho_0 10^{13}$ g cm^{-3} but this choice does not influence our main conclusions. Calculations are made for two values of the polar magnetic field at the neutron star birth, $B_p(0) = 3 \times 10^{13}$ and 6×10^{12} G. This magnetic field processed in the course of phases I, II, and III then was used as the initial magnetic configuration for the phase IV. Our calculations of phase IV stop when the magnetic field of a neutron star approaches the value $B_p = 10^8$ G because, at the present, it is difficult to detect pulsars with $B_p < 10^8$ G.

In Fig. 1, we plot the evolutionary tracks of a neutron star for three different accretion rates during the enhanced accretion phase, $\dot{M} = 2.2 \times 10^{-9}$ (solid line), 4×10^{-9} (dashed), and 5×10^{-10} M$_\odot$ yr^{-1} (dot–dashed). The critical temperature for the neutron $^3P_2$ gap is 10^9 K and the magnetic field at the beginning of enhanced accretion is 8×10^{11} G as it follows from our choice of the initial condition for phase IV. After accretion starts, the neutron star moves rapidly to the corresponding spin-up line and after 9.8×10^5 yr reaches it. In its further evolution, the star slides down the spin-up line until the balance is maintained between the spin-up and rate of the field decay. However, this balance cannot be maintained for a long time because the magnetic field will eventually decay and become too weak for that. Due to the field decay, the Alfvén radius becomes small where the accretion flow interacts with a magnetosphere and, therefore, the amount of angular momentum transferred to the neutron star is small as well. As a result, the neutron star begins to depart from the spin-up line. This behaviour is typical for all considered models and can be clarified by equation (7). The rate of angular
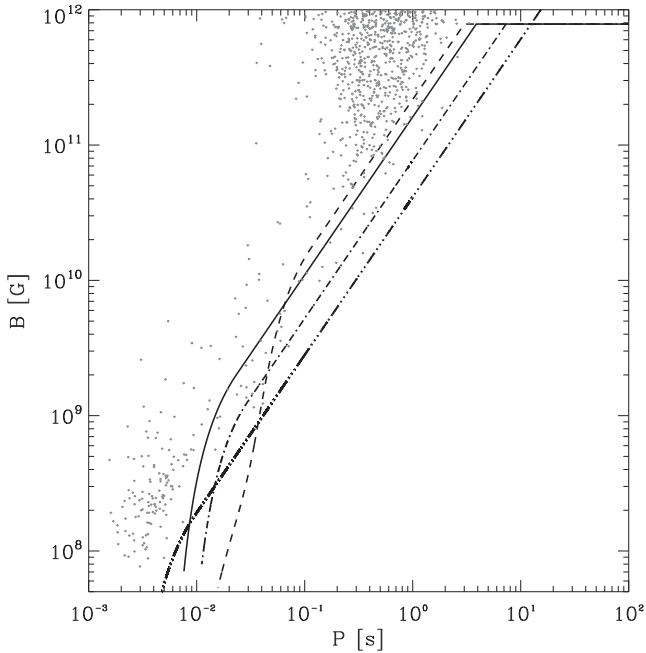
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[1] http://www.astroscu.unam.mx/neutrones/NSCool/
[2] http://ioffe.ru/astro/conduct

Figure 1. Evolutionary tracks of pulsars in the B–P plane for few accretion rates, $\dot{M} = 2.2 \times 10^{-9}$ (solid), 4×10^{-9} (dashed), and 5×10^{-10} (dash–dotted) $M_\odot$ yr^{-1}. The magnetic field at the beginning of enhanced accretion is 8×10^{11} G. The long-dashed represents the same model of the solid line but for a stiffer EOS (mean-field) for 1.4 $M_\odot$.

momentum transfer is proportional to $B_p^{2/7}$ and, hence, it decreases with decreasing B_p. On the contrary, the rate of field decay does not depend on the magnetic field and, at a certain moment, the rate of angular momentum transfer should become smaller than the rate of field decay. Starting from this moment, the star cannot slide down the spin-up line anymore. Departures from the spin-up line become significant, however, at different P for different accretion rates. A further evolution leads to a formation of the pulsar with low magnetic field and relatively long spin period. We define the minimum period, P_m, that the star can reach at a given accretion rate as the period that a neutron star has at $B_p = 10^8$ G, and we stop calculations at that time. For models considered in Fig. 1, P_m is equal to 6.5 ms for the accretion rate of 2.2×10^{-9} $M_\odot$ yr^{-1} (solid line in Fig. 1). The minimum period is determined by different mechanisms of field decay at different accretion rates. At a low accretion rate, $\dot{M} < 2.2 \times 10^{-9}$ $M_\odot$ yr^{-1}, a decrease of the magnetic field is determined mainly by the ohmic dissipation (the second term on the r.h.s. of equation 4). In this case, the minimum period P_m decreases if the accretion rate increases. At a high accretion rate, $\dot{M} > 2.2 \times 10^{-9}$ $M_\odot$ yr^{-1}, the advective term in equation (4) (the first term on the r.h.s.) is greater than the dissipative one, and the accretion flow pushes the magnetic field into deep crustal layers. This submergence of the magnetic field leads to its rapid dissipation since we assume that the neutron star core is superconductive and the field cannot penetrate into the core. Therefore, the narrow transition layer is formed near the crust–core boundary and the field decays rapidly in this layer because of its relatively small thickness. The field decay is extremely fast in this case and its rate exceeds the rate of angular momentum transfer even if the magnetic field is relatively high. An increase of the accretion rate leads to more and more rapid decay of the field and, therefore, departures from the spin-up line can manifest itself even at higher magnetic fields. As a result, the minimum period increases with the increasing accretion
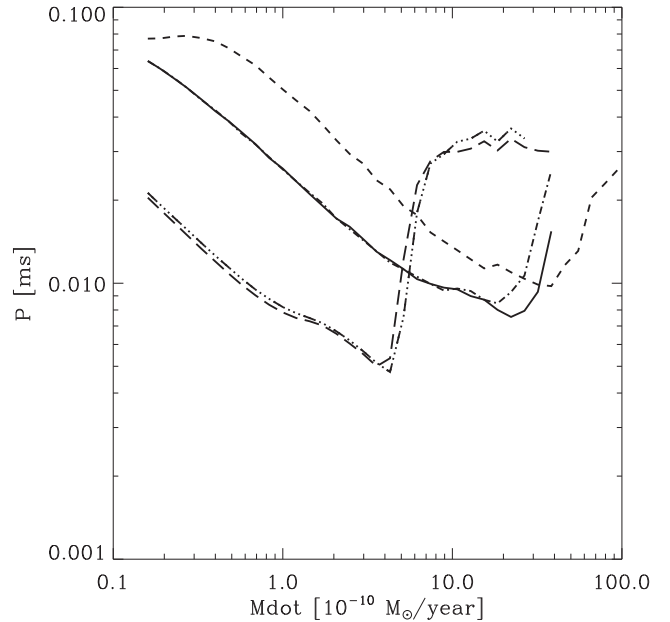
Figure 2. The minimum period, P_m that the neutron star can reach by accretion as a function of the accretion rate. The solid line corresponds to a neutron star with APS EOS and the magnetic field at the beginning of enhanced accretion 8×10^{11} G and a neutron $^3P_2$ gap with $T_c = 10^9$K. The dot–dashed line is the same model of the solid line but with $T_c = 10^{10}$ K. The dashed line represents the same model of the dot–dashed but for the initial field of 1.6×10^{11} G. The long-dashed and triple-dot–dashed lines represent the same model of the solid line but for a stiffer EOS (mean-field) for 1.4 $M_\odot$ and 1.6 $M_\odot$ solar masses, respectively.

rate. Hence, the accretion rate exists at which the neutron star slides down the spin-up line during the longest time and during this sliding down, the neutron star reaches the minimal value of minimum spin periods. If the accretion rate is higher or low than this value, the neutron star span up by accretion will rotate slower. For the model with the APR EOS, represented in Fig. 1, the minimal value of P_m is $\approx$6.5 ms and the corresponding accretion rate is $\approx$2 $\times$ 10^{-9} $M_\odot$ yr^{-1}. Note that the results are qualitatively the same for the neutron star models based on other EOS but the minimum periods have other values.

In Fig. 2, we plot the minimum period defined above, P_m, as a function of the accretion rate for the neutron star models based on the APR EOS and mean-field EOS. In particular, the solid curve represents a neutron star with a critical temperature for the $^3P_2$ gap of 10^9 K [model 'a' in Page et al. (2004), see also Elgarøy et al. (1996)], and the magnetic field of 8×10^{11} G when accretion starts. The dot–dashed line represents the same model of the solid line but with $T_c = 10^{10}$ K, (gap model 'c'). The dashed line instead corresponds to the same model of the dot–dashed line, but with a smaller magnetic field at the beginning of the phase IV, $B_p(0) = 1.6 \times 10^{11}$. On the other hand, the long-dashed and triple-dot–dashed line represent the same model of the solid line but for a stiffer EOS (mean-field) for 1.4 $M_\odot$ and 1.6 $M_\odot$ solar masses, respectively. In this latter case the minimum period is as low as 4 ms.

By comparing the dot–dashed line and the dashed line, we notice that the final spin period clearly depends on the initial field strength being higher for a smaller magnetic field, as it is expected. On the other hand by comparing the solid line and the dot–dashed line we observe that the spin period does not depend on the superconducting gap at the beginning of the accretion phase if the accretion rate is not

very high, $\dot{M} \leq 2.2 \times 10^{-9}$ M$_\odot$ yr^{-1}. However, this is not the case if accretion is heavy and $\dot{M} \geq 2.2 \times 10^{-9}$ M$_\odot$ yr^{-1} the resulting minimal period is longer for higher T_c.

We can conclude that the minimal spin period is essentially determined by the physical property of the neutron star matter. In particular, we argue that P_m depends on the EOS and can be significantly shorter than ≈ 7 ms if an EOS stiffer than the APR one is used (Urpin et al. 1998a,c). For example, the minimal spin period is ~ 4 ms for the EOS by Glendenning & Moszkowski (1991). Therefore, measurements of the minimal spin period of neutron stars processed in low-mass binaries can provide information regarding the EOS. Our study shows that stiff EOSs are in better agreement with observational data.

4 DISCUSSION

We have considered a combined magnetic, thermal, and rotational evolution of the accreting neutron star based on different EOSs. It turns out that the accreting neutron star cannot spin-up to a very short period even if the accretion rate is high and accretion lasts sufficiently long time. The reason for this is a strong coupling between the rotational, magnetic, and thermal evolution during the accretion phase. At a given accretion rate, there exists the minimum period, P_m, that the neutron star cannot overcome. This limit is caused by the fact that accretion-driven decay of the magnetic field leads to a rapid decrease of the angular momentum transfer. At some stage, the rate of the angular momentum transfer becomes so low that the neutron star cannot maintain a balance with the rate of field decay in order to slide down the spin-up line. Since the field decay is faster, the star leaves the spin-up line and evolves into the region of low-magnetic and relatively long-periodic pulsars in the B–P plane.

The point where a neutron star begins to depart from the spin-up line can be qualitatively estimated by comparing the rate of angular momentum transfer with the rate of field decay. The inverse time-scale of spin-up is given by equation (7), $\dot{P}/P \sim -Pj/2\pi I$. Since $j \sim \dot{M}\Omega_K(R_A)R_A^2 \sim \dot{M}\sqrt{GM}R_A^{1/2}$ during the enhanced accretion phase, we obtain

$$\frac{\dot{P}}{P} \sim -\frac{\dot{M}P}{2\pi I}\sqrt{GM}R_A^{1/2} \sim -\frac{\Omega_K(R_A)}{\Omega} \cdot \frac{\dot{M}R_A^2}{I}. \qquad (9)$$

On the other hand, the inverse time-scale of the magnetic field decay can be estimated as $\dot{B}/B \sim -1/\Delta t$ where Δt is the time from the beginning of the enhanced accretion (see e.g. Urpin & Geppert 1995). The field decay is faster than spin-up if $\dot{B}/B > \dot{P}/P$, or

$$\Delta M R_A^2 > I(\Omega/\Omega_K(R_A)), \qquad (10)$$

where $\Delta M = \dot{M}\Delta t$ is the accreted mass. Generally, departures from the spin-up line depend on the magnetic field and the mass and radius of a neutron star and, hence, on the EOS of nuclear matter. Neutron stars with a soft EOS (with smaller R at given M) need to accrete a smaller mass to leave the spin-up line. On the contrary, stars with a stiff EOS (greater R at given M) leave this line only if ΔM is sufficiently large.

In this paper, we define P_m as the period reached by a neutron star when its magnetic field becomes equal to 10^8 G. This definition is rather arbitrary and we introduce it mainly in order to stop calculations when the rotational evolution becomes very slow. In fact, the spin period of a neutron star continues to decrease slowly after it reaches the value P_m. However, this decrease turns out to be very slow and it becomes slower during the further evolution. For example, P_m is equal ~ 6 ms if we stop calculations when the mag-

netic field reaches the value 10^7 G in comparison with $P_m \approx 7$ ms obtained for $B = 10^8$ G in Fig. 2. Besides, detection of the neutron stars with $B < 10^8$ G is a very complicated problem for the present telescopes. Therefore, we stop calculations when $B < 10^8$ G and restrict our discussion by the properties of neutron stars that can be observed at the present.

The main reason for this relatively high minimal period of accreting pulsars is the rapid decrease of the magnetic field which leads to a low value of R_A. Transfer of the angular momentum becomes too slow at small R_A. If the accretion flow forms the Keplerian disc around a neutron star, the rate of angular momentum transfer can be estimated as $\dot{M}\Omega_K(R_A)R_A^2$. Note that this estimate is valid only if $R_A \gg R$. For the considered accretion rate, this condition is satisfied if the magnetic field of a neutron star is greater than $\sim 10^7$ G. If the magnetic field is very weak ($B_p < 10^7$ G), then $R_A \leq R$ and the rate of angular momentum transfer is of the order of $\dot{M}\Omega_K(R)R^2$. Since $\Omega_K(R) \propto R^{-3/2}$, the angular momentum transport becomes very slow in this case. Despite the inner disc radius cannot be smaller than the radius of the inner marginally stable orbit, the rate of angular momentum transport turns out to be low even if the accretion disc is extended to this orbit. Therefore, the further spin evolution of a neutron star is very slow and, in order to reach short periods, enhanced accretion should last extremely long time.

As it was noted, the minimum period, P_m, is determined by the accretion rate and EOS of nuclear matter. However, this minimum period cannot be very short even if the accretion rate is high. A high accretion rate leads to a very rapid decrease of the magnetic field. In this case, the magnetic evolution in the crust is dominated by advection: the accretion flow pushes the magnetic field into the deep layers where the field dissipates rapidly in the narrow transition region near the superconductive core. A fast submergence of the magnetic field leads to a rapid decrease of the rate of the angular momentum transfer and substantial departures from the spin-up line. Due to this, the neutron star does not spend much time, sliding down the spin-up line, and can spin-up only to relatively long periods. As a result, there exists the accretion rate at which P_m reaches the minimal value for a given EOS. The minimal value of P_m for the considered model with the APR EOS is ≈ 7 ms and the neutron star reaches it if the accretion rate is $\approx 2 \times 10^{-9}$ M$_\odot$ yr^{-1}. If the accretion rate is higher or lower than this value, the neutron star can spin-up only to a longer period. This limit of the spin period is determined by the EOS and can differ from the value ~ 7 for other EOS. Note that the limit is smaller for stiff EOSs and greater for soft EOSs. For example, the minimal period for a neutron star with the EOS by Glendenning & Moszkowski (1991) is ~ 4 ms and this period can be reached if the accretion rate is $\sim 4 \times 10^{-10}$ M$_\odot$ yr^{-1}. Note that, for a stiffer EOS, the neutron star reaches the minimal period at lower accretion rate.

The shortest spin period detected up to date for PSR J1748-244 is 1.4 ms (Hessels et al. 2006). The minimal spin periods attainable for neutron stars and determined by balance of the gravity and centrifugal forces are well below this value for most of the proposed EOSs of a nuclear matter. A discrepancy between the minimal possible spin period of neutron stars and the shortest detected period can be caused by the mechanism considered in this paper. The calculated values of a minimal period, ~ 7 ms and ~ 4 ms, clearly point out that the EOS should be stiffer than that by Akmal et al. (1998) and, likely, even stiffer than that by Glendenning & Moszkowski (1991).

Our calculations show that formation of shortly periodic pulsars (like millisecond pulsars) is possible only if the accretion rate belongs to some not very wide range around $\dot{M} \sim 2 \times 10^{-9}$ M$_\odot$ yr^{-1}

for APS EOS. If the accretion rate is lower than this value, a pulsar in the binary system has no chance to become a millisecond one even if accretion lasts very long. Note that some observational indications (for instance, a distribution of pulsars in the *B–P* plane) also point out on the existence of the minimal accretion rate that can lead to a formation of millisecond pulsars (Pan, Wang & Zhang 2013).

REFERENCES

Akmal A., Pandharipande V. R., Ravenhall D. G., 1998, Phys. Rev. C, 58, 1804
Bhattacharya D., Srinivasan G., 1991, in Ventura J., Pines D., eds, Neutron Stars: Theory and Observations. Kluwer, Dordrecht, p. 219
Bhattacharya D., van den Heuvel E. P. J., 1991, Phys. Rep., 203, 1
Bonanno A., Urpin V., Belvedere G., 2005, A&A, 440, 199
Bonanno A., Urpin V., Belvedere G., 2006, A&A, 451, 1049
Bonanno A., Baldo M., Burgio G. F., Urpin V., 2014, A&A, 561, L5
Brown E. F., Bildsten L., 1998, ApJ, 496, 915
Elgarøy Ø., Engvik L., Hjorth-Jensen M., Osnes E., 1996, Nucl. Phys. A, 607, 425
Glendenning N. K., Moszkowski S. A., 1991, Phys. Rev. Lett., 67, 2414
Haensel P., Zdunik J. L., 2008, A&A, 480, 459
Hessels J. W. T., Ransom S. M., Stairs I. H., Freire P. C. C., Kaspi V. M., Camilo F., 2006, Science, 311, 1901
Hollerbach R., Rüdiger G., 2002, MNRAS, 337, 216
Hollerbach R., Rüdiger G., 2004, MNRAS, 347, 1273
Illarionov A. F., Sunyaev R. A., 1975, A&A, 39, 185
Naito T., Kojima Y., 1994, MNRAS, 266, 597
Page D., Lattimer J. M., Prakash M., Steiner A. W., 2004, ApJS, 155, 623
Pan Y. Y., Wang N., Zhang C. M., 2013, Ap&SS, 346, 119
Potekhin A. Y., 1999, A&A, 351, 787
Pringle J. E., Rees M. J., 1972, A&A, 21, 1
Shibazaki N., Murakami T., Shaham J., Nomoto K., 1989, Nature, 342, 656
Urpin V., Geppert U., 1995, MNRAS, 275, 1117
Urpin V. A., Chanmugam G., Sang Y., 1994, ApJ, 433, 780
Urpin V., Geppert U., Konenkov D., 1998a, MNRAS, 295, 907
Urpin V., Konenkov D., Geppert U., 1998b, MNRAS, 299, 73
Urpin V., Geppert U., Konenkov D., 1998c, A&A, 331, 244
Wendell C. E., van Horn H. M., Sargent D., 1987, ApJ, 313, 284
Wijers R. A. M. J., 1997, MNRAS, 287, 697

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