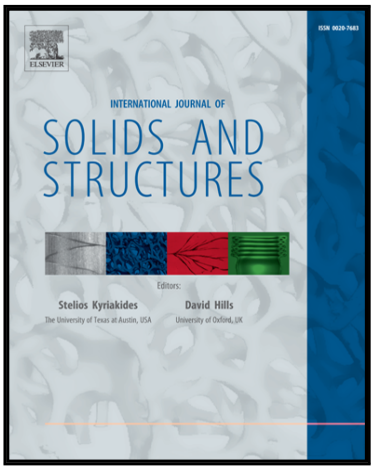

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Methods to solve half-plane partial slip contact problems

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Abstract

There exists a family of methods for finding the extent of partial slip in contact problems between elastically similar bodies, capable of idealisation by half-planes. Closed form solutions are given to problems subject to a constant normal load and subsequent application of either an increasing shear force or differential bulk tension parallel with the surface. The starting point may be either a sliding contact (as is customary), or a fully adhered contact. The corrections required to impose a point-wise interpretation of Coulomb's law of friction may be either in the form of a shear traction distribution or as dislocation arrays. The latter, when applied to the fully adhered contact, has the merit of automatically preserving the locked-in relative surface strains.

1. Introduction

Contact problems involving components in partial slip are of interest because the relative surface movement causes damage. This accelerates the nucleation of cracks which are capable of propagating through the component leading to failure. The time to nucleate a crack may be estimated from the peak energy dissipated at the interface over a load cycle [1]. As well as causing damage, frictional slip also provides damping. This damping is often desirable as it limits the amplitude of vibrations.

Considerable progress has been made in the study of contact problems in partial slip in which the bodies pressed together may be idealised as half-planes. Early considerations related to the Hertzian contact [2], and invariably considered cases where the normal load was applied first and held constant. A monotonically increasing shearing force, less than that needed to cause sliding, was subsequently applied. This problem was first studied by Cattaneo [3], and re-examined ten years or so later by Mindlin [4]. All solutions, to date, start off with a shear traction distribution which corresponds to sliding. A corrective shear traction distribution is then applied over the central part of the contact so as to make the surface strains in the two bodies equal, and hence achieve a condition of stick. Cattaneo and Mindlin noticed that, for a Hertzian contact, the corrective traction was similar to that induced by the sliding contact but scaled, and Ciavarella [5] and Jäger [6] both noted that this similarity argument would apply whatever the contacting profile. Although it was not solved this way at the time, these solutions can be found using an integral equation formulation, in which the kernel represents the effect of a shear traction on the half-plane surfaces. This approach may be extended to problems where the surface shear traction is induced by differential bulk tension, and where, again, the basic solution is one where, at all points, the interface is in a state of slip (but the direction of slip is opposite at one end of the contact from the other) [7, 8, 9].

This is not the only way to formulate the problem; Table A shows four different solution methods. The numbers given in parenthesis shows the solution form summarised in Table A. We can start from a solution that satisfies the sliding conditions (1,3). This means the corrective solution must re-establish stick over a central region of the contact. Alternatively,

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we can start from a solution that satisfies full stick (2,4). The corrective solution must now permit slip at the edges of the contact. The correction to these initial solutions may be in the form of a traction (1,2) or displacements (3,4).

The disadvantages with methods (2,3) are that the correction required (a) is over the entire contact patch and (b) it must be carried out in two stages. On the other hand, method (4) is particularly useful for complex problems that involve a locked in displacement difference in the stick region. A further advantage is that the primary output is the slip displacement - a key ingredient in finding the energy dissipated.

2. The Ciavarella-Jäger method (1)

This procedure was independently published by Ciavarella [5] and Jäger [6], and represents a generalisation of the results of Mindlin and Cattaneo for bodies with an arbitrary convex profile. Figure 1a shows two symmetrical, undeformed elastic bodies, made of the same material and having plane strain elastic modulus $E^* = \frac{E}{2(1-\nu^2)}$, on the point of being brought into contact, and with a gap function $h_0(x)$. The integral equation defining the contact pressure distribution, $p(x)$, as the contact is formed, is

$$E^* \frac{dh_0}{dx} = \frac{2}{\pi} \int_{-a}^{a} \frac{p(\xi)\, d\xi}{\xi - x} \qquad |x| \leq a, \tag{1}$$

where a is the contact half-width and, from normal equilibrium, the corresponding normal load, P is

$$P = \int_{-a}^{a} p(x)\, dx. \tag{2}$$

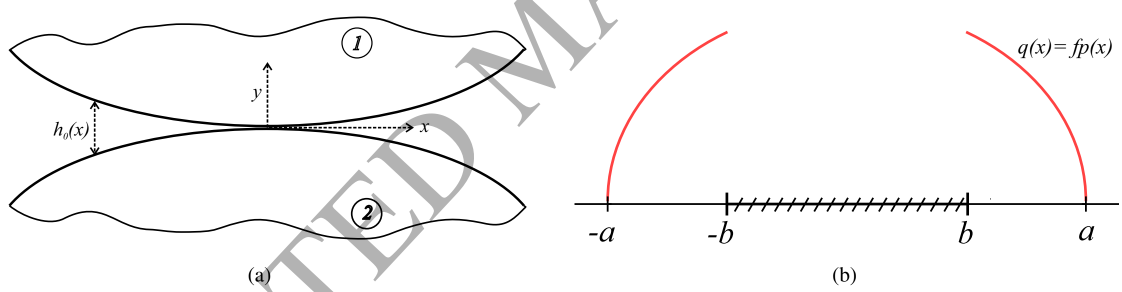

Figure 1: (a) Convex contact; and (b) Conditions for partial slip

Equal surface strains will develop in each body when a normal load is applied, but no shear traction will arise because $\epsilon_{xx1}(x) = \epsilon_{xx2}(x)$. A monotonically increasing shear force causes slip at the edges of the contact (as the contact pressure goes smoothly to zero and is unable to sustain sufficient shearing traction to preserve stick) while the central part of the contact ($|x| < b$) remains stuck. Figure 1b shows the shear traction in the slip zone is equal to the sliding solution and any pre-existing strains are preserved in the stick zone. We write down the shear traction as the sum of the sliding distribution, $f p(x)$, and a corrective term, $q_c(x)$ over the stick interval. The integral giving the relationship between the *difference* in surface strains, $\frac{du}{dx} = \epsilon_{xx1}(x) - \epsilon_{xx2}(x)$ and the shear traction present is

$$E^* \frac{du}{dx} = \frac{2f}{\pi} \int_{-a}^{a} \frac{p(\xi)\, d\xi}{\xi - x} + \frac{2}{\pi} \int_{-b}^{b} \frac{q_c(\xi) d\xi}{\xi - x}. \tag{3}$$

To turn this into an integral equation, we impose the requirement that the strain difference is preserved (at zero) in the stick zones, i.e.

$$\frac{du}{dx} = 0 \qquad |x| \leq b, \tag{4}$$

and, as $a \geq b$, we see that this means

$$f E^* \frac{dh_0}{dx} = -\frac{2}{\pi} \int_{-b}^{b} \frac{q_c(\xi) d\xi}{\xi - x} \quad |x| \leq b. \tag{5}$$

A comparison of Equations 1 and 5 shows that the corrective shear traction, $q_c(x) = -f p_c(x)$ may be obtained by solving the *normal* load contact equation, but at a reduced load, P_c. If the actual shear force exerted at this point is Q, tangential equilibrium requires that, $Q = f(P - P_c)$, giving the value of P_c.

2.1. Application to a Hertzian contact

In a Hertzian contact with a relative radius of curvature between the contacting bodies R, $dh_0/dx = -2x/R$. If this is substituted into Equation 1 and a bounded-both-ends solution sought the resulting pressure distribution is elliptical and given by

$$p(x) = p_0 \sqrt{1 - \left(\frac{x}{a}\right)^2} \quad |x| \leq a, \tag{6}$$

where the peak contact pressure, p_0, is given by $p_0 = \frac{2P}{\pi a} = \sqrt{\frac{PE^*}{\pi R}}$. Normal equilibrium gives the half-width of the contact as $a = \sqrt{\frac{4PR}{\pi E^*}}$. With the normal contact problem now fully solved we may turn our attention to the corrective term to be used to establish the stick region. The equation for the normal pressure may be used but with P replaced by P_c and the half-width of the contact, a replaced by the half-width of the stick zone, b. Explicitly, the shear traction distribution in partial slip is given by

$$\frac{q(x)}{f} = \sqrt{\frac{PE^*}{\pi R}} \left[\sqrt{1 - \left(\frac{x}{a}\right)^2} - \sqrt{1 - \frac{Q}{fP}} \sqrt{1 - \left(\frac{x}{b}\right)^2} \right], \tag{7}$$

where the second term is included only if $|x| \leq b$ and,

$$\frac{b}{a} = \sqrt{1 - \frac{Q}{fP}}. \tag{8}$$

3. Dislocation correction to the fully adhered solution (4)

An alternative approach is to take the solution for a fully adhered contact subject to shear, and then to permit slip to develop in the regions where the frictional stick condition, $|q(x)| < f p(x)$, is violated. For *all* adhered half-plane contacts, the shear traction distribution, for a pre-formed contact of half-width a, due to the subsequent application of a shear force, Q is given by

$$q_{st}(x) = \frac{Q}{\pi \sqrt{a^2 - x^2}} \quad |x| \leq a, \tag{9}$$

and is independent of geometry. The fully stuck shear traction is found from knowing the relative strain difference between the two bodies must be zero [10]. The appropriate strain nucleus, needed to permit slip, is a glide dislocation, $b_x(x)$, located on the interface between two half-planes, adhered over the interval $[-a\ a]$, $(|x| \leq a)$, but where the surfaces of the half-planes are traction-free outside this interval. The shear traction distribution, due to a single dislocation, b_x at the point c (Appendix A), is

$$q(x) = \frac{E^* b_x(c)}{2\pi \sqrt{a^2 - x^2}} \frac{\sqrt{a^2 - c^2}}{c - x} \quad |x| \leq a. \tag{10}$$

For the time being we assume that the stick region is located at $-b_2 \leq x \leq b_1$, so dislocations must be distributed over the two slip zones, and hence the corrective shear traction distribution, $q_c(x)$ (not the same as that defined in Section 2) is given by

$$q_c(x) = \frac{E^*}{2\pi\sqrt{a^2-x^2}}\left[\int_{b_1}^{a} \frac{\sqrt{a^2-\xi^2}B_x(\xi)}{\xi - x}d\xi + \int_{-a}^{-b_2} \frac{\sqrt{a^2-\xi^2}B_x(\xi)}{\xi - x}d\xi\right], \tag{11}$$

where $B_x(x) = \frac{db_x}{dx}$ is the dislocation density. The integral equation defining the dislocation density is given by

$$\frac{E^*}{2\pi\sqrt{a^2-x^2}}\left[\int_{b_1}^{a} \frac{\sqrt{a^2-\xi^2}B_x(\xi)}{\xi - x}d\xi + \int_{-a}^{-b_2} \frac{\sqrt{a^2-\xi^2}B_x(\xi)}{\xi - x}d\xi\right] = fp(x) - \frac{Q}{\pi\sqrt{a^2-x^2}} \quad -a \leq x \leq -b_2,\ b_1 \leq x \leq a, \tag{12}$$

where, as required, the range of the integral is the same as the range over which the right hand side is imposed. When the bodies are symmetrical about $x = 0$, so that the contact pressure is also symmetrical, and when the shear tractions are induced by a shear force alone, the slip displacements themselves constitute an even function of x and hence $b_1 = b_2 = b$ so that the dislocation density is odd. Therefore, $B_x(x) = -B_x(-x)$, giving

$$q_c(x) = \frac{E^*}{\pi\sqrt{a^2-x^2}}\int_{b}^{a}\left[\frac{\sqrt{a^2-\xi^2}B_x(\xi)\xi}{x+\xi}\,\frac{1}{\xi - x}\right]d\xi \quad b \leq |x| \leq a, \tag{13}$$

when

$$q_c(x) = fp(x) - \frac{Q}{\pi\sqrt{a^2-x^2}}. \tag{14}$$

This is best illustrated by reference to an example geometry, and here we use a Hertzian contact.

3.1. Application to a Hertzian contact

We have already shown, in §2.1, that for this problem the contact pressure distribution is given by Equation 6 where a, P are related by the contact law, and hence Equation 13 becomes,

$$\int_{b}^{a}\left[\frac{\sqrt{a^2-\xi^2}B_x(\xi)\xi}{x^2-\xi^2}\right]d\xi = \frac{Q}{E^*} - \frac{2Pf(a^2-x^2)}{a^2E^*} \quad b \leq x \leq a. \tag{15}$$

The kernel does not have a standard form, so we use the transformation

$$g = a^2 \quad e = b^2 \quad y = x^2 \quad \eta = \xi^2 \tag{16}$$

so that

$$\int_{e}^{g}\left[\frac{\sqrt{g-\eta}B_x(\eta)}{y-\eta}\right]d\eta = \frac{Q}{E^*} - \frac{2Pf}{E^*}\left(1 - \frac{y}{g}\right) \quad e \leq y \leq g. \tag{17}$$

A bounded-both-ends solution is sought using the standard Riemann-Hilbert procedure [11]. The side condition,

$$\int_{-a}^{a}\left[\frac{Q}{E^*} - \frac{2Pf}{E^*}\left(1 - \frac{s}{g}\right)\right]\frac{ds}{\sqrt{a^2-s^2}} = 0, \tag{18}$$

provides the size of the stick zone (Equation 8 but revealed by this different route) and the inversion gives

$$B_x(x) = \frac{2fp_0}{E^*}\frac{x}{a}\sqrt{1-(b/x)^2} \qquad b \leq |x| \leq a. \tag{19}$$

A consistent slip displacement gradient (the same as the dislocation density) and stick interval are now known. Note that $B_x(\pm a)$ is finite.

3.2. Pure bulk tension

We restrict our attention to the case where the contacting bodies are symmetrical, but when the shear traction is excited by bulk tension alone, again imposed after the contact has been formed. The shear traction when the contact is fully stuck is given by

$$q_{st}(x) = \frac{\sigma x}{4\sqrt{a^2 - x^2}} \quad |x| \leq a, \tag{20}$$

and is again independent of the contact geometry. The kernel for the dislocations is identical to the shear case. However, the displacements are now an odd function of x and so the dislocation density is an even function, $B_x(x) = B_x(-x)$. As before, the slip regions are of equal size, so that

$$\frac{xE^*}{\pi\sqrt{a^2 - x^2}} \int_b^a \left[\frac{\sqrt{a^2 - \xi^2} B_x(\xi)}{x + \xi} \frac{1}{\xi - x}\right] d\xi = sgn(x) f p(x) - \frac{\sigma x}{4\sqrt{a^2 - x^2}} \quad b \leq |x| \leq a. \tag{21}$$

The associated side condition,

$$\int_{-a}^{a} \left[\frac{\sigma s}{E^*} - \frac{2Pf sign(s)}{E^*}\left(1 - \frac{s}{g}\right)\right] \frac{ds}{\sqrt{a^2 - s^2}} = 0, \tag{22}$$

gives the size of the stick zone, which for a Hertzian contact is found to be,

$$K\left(\sqrt{1 - \left(\frac{b}{a}\right)^2}\right) - E\left(\sqrt{1 - \left(\frac{b}{a}\right)^2}\right) = \frac{\sigma\pi}{8fp_0}. \tag{23}$$

The slip displacement cannot be written in closed form.

4. Traction correction to the fully adhered solution (2)

A further method, with little physical interpretation, is to take the fully stuck solution, and to introduce a corrective shear traction $q_c(x)$ in the slip region whilst ensuring that the displacements in the stick zone remain zero. The correction must be carried out in two steps and over the entire contact patch. First, a corrective shear traction is imposed on the slip regions, which for a symmetrical P load is given by

$$q_{c1}(x) = f p(x) - \frac{Q}{\pi\sqrt{a^2 - x^2}} \quad b \leq |x| \leq a. \tag{24}$$

This has the consequence of developing displacements in the stick zone, which means a further corrective traction is required in the stick zone to return the displacements to zero in the stick zone. As the boundary is not fixed the correction is non singular and hence requires a different kernel. This is derived in Appendix B. The condition to be imposed is

$$\frac{du}{dx} = \frac{2}{\pi E^*} \int_{-b}^{b} \frac{q_{c2}(\xi) d\xi}{\xi - x} + \left.\frac{du}{dx}\right|_{c1} = 0 \quad |x| \leq b, \tag{25}$$

Which may be inverted to find the corrective traction in the stick zone.

5. Dislocation correction to the sliding solution (3)

Finally, suppose that we have formed a contact by pressing the two bodies together. Suppose, further, that a shear force sufficient to cause sliding was subsequently imposed, so that the shear traction distribution present is everywhere equal to its limiting value. It would follow that, at incipient sliding (that is, without rigid body motion having actually taken place) the relative slip displacement between corresponding surface particles, $u_{sl}(x)$, is given by

$$\left.\frac{du}{dx}\right|_{sl} = \frac{2f}{\pi E^*}\int_{-a}^{a}\frac{p(\xi)d\xi}{\xi - x}. \tag{26}$$

If we now think of going back to our original problem of a monotonically increasing shear force up to some value less than that needed to cause sliding, relative slip needs to be preserved at its original (zero) value within the stick interval. This may be done with the imposition of an equal and opposite correction term $\left.\frac{du}{dx}\right|_{c1}$. Such that

$$\left.\frac{du}{dx}\right|_{sl} + \left.\frac{du}{dx}\right|_{c1} = 0 \qquad |x| \leq b. \tag{27}$$

Thus, the corrective slip displacement gradient, which is otherwise the density of an array of glide dislocations within the stick interval, is simply equal and opposite to the slip displacement produced by the sliding shear traction, over the stick region. We use the solution for the shear traction developed by a glide dislocation which is again bounded at each end. The derivation is given in Appendix B. We know that the displacement must be zero in the stick zone and hence we require

$$B_x(x) = -\left.\frac{du}{dx}\right|_{sl} \quad |x| \leq b. \tag{28}$$

The effect of the glide dislocations inserted in the stick zone is to induce tractions in the slip zone, which may be found from an integral such as that given in equation (3), but now the condition $q(x) = f\,p(x)$ is violated in the slip zone. Further glide dislocations are introduced in the slip zone to restore the shear traction in the slip zone.

6. Conclusion

A new family of methods for solving partial-slip half-plane contact problems gives us more options when tackling half-plane partial slip problems than available hitherto (see Table A). Two of the procedures - applying a corrective shear to a sliding solution (1) the classical approach, and applying a corrective displacement to the adhered solution (4) are very attractive. Two further procedures, were also developed viz. imposing a corrective shear on the adhered solution (2), and applying a corrective displacement to the sliding solution (3). The method of superimposing a corrective strain (the dislocation density) on the fully stuck solution is attractive when we look at complicated loading trajectories because: (a) it automatically preserves locked in displacements and (b) it gives the displacements in the slip zone immediately, which can be used to calculate the energy dissipated.

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Appendix A. Solution for a glide dislocation on interface between partially bonded half-planes

Two elastically similar half planes are bonded together over the line segment $-a \leq x \leq a$, but devoid of surface tractions outside this interval ($|x| \leq a$). Suppose that the half-planes are bonded with a relative shift between corresponding points on the surfaces of the half-planes, $u(x) = u_1(x) - u_2(x)$. The corresponding interface shearing traction present, $q(x)$ is given by

$$\frac{du}{dx} = \frac{2}{\pi E^*} \int_{-a}^{a} \frac{q(\xi)d\xi}{\xi - x} \tag{A.1}$$

and the inversion of this equation when the end points are fixed and (hence the singular solution), is

$$q(x) = \frac{1}{\pi\sqrt{a^2 - x^2}} \left[C + \frac{E^*}{2} \int_{-a}^{a} \frac{\sqrt{a^2 - \xi^2} u'(\xi) d\xi}{(\xi - x)} \right] \quad |x| \leq a. \tag{A.2}$$

Suppose that we set the slip displacement gradient to be zero everywhere except at the point $\xi = c$ where we insert a dislocation of Burgers vector b_x, so that $u'(x) = b_x(c)\delta(x - c)$ where $\delta(\bullet)$ is Dirac's delta function, and hence

$$q(x) = \frac{E^* b_x}{2\pi\sqrt{a^2 - x^2}} \int_{-a}^{a} \frac{\sqrt{a^2 - \xi^2}\delta(\xi - c)d\xi}{(\xi - x)} = \frac{E^* b_x(c)}{2\pi\sqrt{a^2 - x^2}} \frac{\sqrt{a^2 - c^2}}{c - x} \quad |x| \leq a \tag{A.3}$$

where the rigid body term, C, has been dropped. A distribution of dislocations, $B_x(x) = \frac{db_x}{dx}$, therefore gives rise to a shear traction distribution

$$q(x) = \frac{E^*}{2\pi\sqrt{a^2 - x^2}} \int_{-a}^{a} \frac{\sqrt{a^2 - \xi^2} B_x(\xi)\, d\xi}{(\xi - x)} \quad |x| \leq a \tag{A.4}$$

Appendix B. Bounded distribution of glide dislocations

A slightly different solution arises when a distribution of glide dislocations is needed which does not extend to the ends of the bonded section. We start, again, with

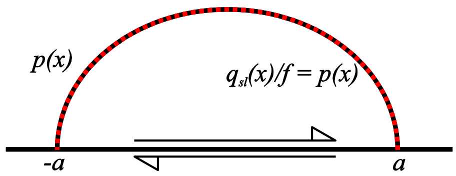
$$\frac{du}{dx} = \frac{2}{\pi E^*} \int_{-b}^{b} \frac{q(\xi)d\xi}{\xi - x} \tag{B.1}$$

but now require a solution for the shear stress where the distribution is bounded at both ends. This is appropriate when the edges of the stick zone are not pre-determined, and find

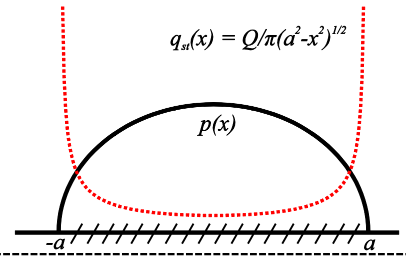
$$q(x) = \frac{E^*\sqrt{b^2 - x^2}}{2\pi} \int_{-b}^{b} \frac{u'(\xi)d\xi}{\sqrt{b^2 - \xi^2}(\xi - x)} = \frac{E^*\sqrt{b^2 - x^2}}{2\pi} \int_{-b}^{b} \frac{B_x(\xi)d\xi}{\sqrt{b^2 - \xi^2}(\xi - x)}. \tag{B.2}$$

Where a distribution of dislocations, $B_x(x) = \frac{du}{dx}$, gives rise to a shear traction distribution along with the consistency condition, which we know will be automatically satisfied as $u'(x)$ is an odd function.

Sliding

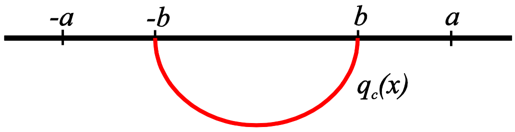


Fully stuck



Corrective tractions

1



The displacement must be zero in the stick zone. A corrective traction is required only in the stick zone as the tractions in the slip zone are already correct.

$$\left.\frac{du}{dx}\right|_{sl} + \left.\frac{du}{dx}\right|_{c} = 0; \quad |x| \leq b$$

The displacements due to the sliding solution may be found from:

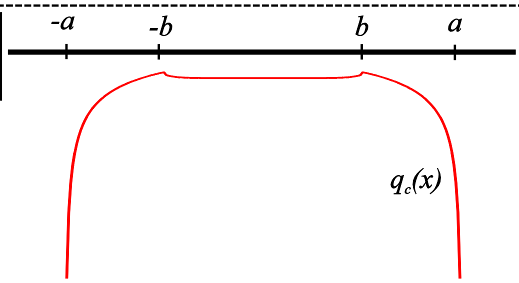
$$\left.\frac{du}{dx}\right|_{sl} = \frac{2f}{\pi E^*} \int_{-a}^{a} \frac{p(\xi)d\xi}{\xi - x}$$

The corrective term produces the displacement

$$\left.\frac{du}{dx}\right|_{c} = \frac{2}{\pi E^*} \int_{-b}^{b} \frac{q_c(\xi)d\xi}{\xi - x}$$

It is found that $q_c(x) = -f\,p_c(x)$, a scaled normal traction

2



A corrective traction is required in the slip zone, so $q(x) = fp(x)$.

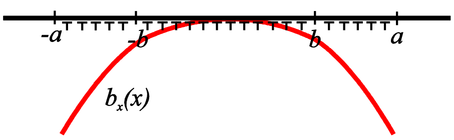
$$q_{c1} = fp(x) - \frac{Q}{\pi\sqrt{a^2 - x^2}}; \quad b \leq |x| \leq a$$

This will cause a non zero displacement in the stick zone. A corrective traction is required in the stick zone to ensure the displacement is zero in the stick zone.

$$\frac{du}{dx} = \frac{2}{\pi E^*}\left(\int_{-b}^{b} \frac{fp(\xi)d\xi}{\xi - x} + \int_{-b}^{b} \frac{q_{c2}(\xi)d\xi}{\xi - x}\right) = 0; \quad |x| \leq b$$

Corrective displacements (dislocations)

3



Find the displacements in the stick zone due to the ***sliding solution***. Dislocations are introduced to bring the surface displacement to zero in the stick region

$$B_x(x) = -\left.\frac{du}{dx}\right|_{sl}; \quad |x| \leq b$$

The tractions due to these dislocations are given by

$$q_{c1}(x) = \frac{E^*\sqrt{b^2 - x^2}}{2\pi} \int_{-b}^{b} \frac{B_x(\xi)d\xi}{\sqrt{b^2 - \xi^2}(\xi - x)}$$

A corrective displacement in the slip zone is required such that

$$q_{c1}(x) + q_{c2}(x) = 0; \quad b \leq |x| \leq a$$

4

The tractions in the slip zone must be corrected whilst keeping the stick zone displacement at zero

$$q_{st} + q_c = fp(x) \quad b \leq |x| \leq a$$

Dislocations are introduced in the slip zones which gives rise to a corrective traction

$$q_c(x) = \frac{E^*}{2\pi\sqrt{b^2 - x^2}} \int_{-a}^{-b} + \int_{b}^{a} \frac{\sqrt{a^2 - \xi^2}B_x(\xi)d\xi}{\xi - x}$$

Table A: Solution techniques for a pure shear half-plane partial slip problem