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R W FRASER

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Nedlands, Western Australia 6009

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Introduction

The contribution of Newbery and Stiglitz (1981) to the welfare evaluation of commodity price stabilisation schemes has provided important theoretical and empirical insights into the desirability or not of this sort of government intervention.[1] For Newbery and Stiglitz: "Our main conclusion is that price stabilisation schemes have limited efficacy in stabilising the real spendable income of producing countries and that most of the other benefits associated with the schemes are transfer benefits which, in many cases, seem to benefit the consuming countries at the expense of the producing countries."[2] One assumption which underpins their analysis is that regarding producer response to price stabilisation. Specifically, in order to "throw some immediate light on the central issue of the benefits of price stabilisation" they assume "there is no response to price stabilisation" on the part of risk averse producers.[3] However, in subsequent theoretical analysis (Chapters 21 and 22), Newbery and Stiglitz show that by allowing for the supply response of producers to price stabilisation (the "long run") it is possible not just for "the long-run equilibrium welfare gains [to be] smaller or greater than the impact welfare gains" but even for "the long-run impact to be a reversal of the short-run impact".[4] In view of these theoretical conclusions it would not seem unreasonable to view the welfare evaluation of price stabilisation schemes as being incomplete until the effects of supply responses are accounted for.

The aim of this paper is to present a method for evaluating the supply response by individual producers to a price stabilisation scheme. In so doing, this method provides a basis for a more complete welfare assessment of such schemes. Supply response to the Australian Wool Corporation's Reserve Price Scheme is used as an example.

The structure of the paper is as follows. Section 1 sets out the theoretical model of producer behaviour. Section 2 presents a method for incorporating the effects of a price stabilisation scheme into this model. Section 3 explains the information requirements of the model and the procedures for satisfying these requirements. Section 4 presents and discusses the example results. Section 5 concludes the paper.

Section 1: The Model

The model of producer behaviour used in this paper is developed in Fraser (1984, 1986). It assumes that the only input to production is the farmer's own labour, ℓ, and that a single output is produced which is subject to multiplicative risk:

$$x = \theta f(\ell)$$

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1. Mention should also be made of the valuable review of Newbery and Stiglitz (1981) by Kanbur (1984).
2. Newbery and Stiglitz (1981) pp.39-40.
3. Newbery and Stiglitz (1981) p.92--original italics. Gilbert (1985) also excludes supply responses in his evaluation.
4. Newbery and Stiglitz (1981) p.312 and p.329 respectively.

$$x = \theta f(\ell)$$

where:

$f(\ell)$ = planned output $[f'(\ell) > 0,\ f''(\ell) < 0]$

θ = multiplicative risk term $[E(\theta) = 1]$

x = uncertain actual output $[E(x) = \bar{x} = f(\ell)]$.

With price also uncertain, the producer's random income (y) is thus given by:

$$y = px$$

where:

p = uncertain price $[E(p) = \bar{p}]$

It is further assumed that the producer's utility is (additively) separable in income and leisure so that his objective is to maximise by choice of labour input:

$$E[U(px)] - w\ell \qquad (1)$$

where:

w = marginal disutility of labour

$U(px)$ = utility of random income $(U' > 0\ ,\ U'' \leq 0)$.

It is shown in Fraser (1984) that using a Taylor series expansion (1) may be approximated by:

$$U(\bar{p}\bar{x}) + 0.5U''(\bar{p}\bar{x})\bar{x}^2(\sigma_p^2 + \sigma_\theta^2\bar{p}^2) - \sigma_{p\theta}\bar{x}\ U'(\bar{p}\bar{x})(R-1) - w\ell \qquad (2)$$

where:

σ_p^2 = variance of p

σ_θ^2 = variance of θ

$\sigma_{p\theta}$ = covariance of p,θ

$R = -U''(px)px/U'(px)$ = the producer's coefficient of relative risk aversion (evaluated at $\bar{p},\bar{x}$).

Note from (2) that whether a covariance of a given sign contributes positively or negatively to utility depends on whether R exceeds or is less than unity.

It is shown in Fraser (1986) that differentiating (1) with respect to ℓ gives the producer's first order condition as:

$$E[U'(px)p\theta]f'(\ell) = w \qquad (3)$$

which, using a Taylor series expansion, may be approximated by:

$$U'(\bar{p}\bar{x})\left[\bar{p}+0.5(\sigma_p^2/\bar{p}+\sigma_\theta^2\bar{p})[R(R-1)-\bar{p}\bar{x}R']+\sigma_{p\theta}[(R-1)^2-\bar{p}\bar{x}R']\right]f'(\ell) = w \ . \qquad (4)$$

Section 2: Price Stabilisation

The type of price stabilisation scheme analysed here is that which involves the use of a buffer stock. In this case, an authority is established to buy stock at times of unusually low prices, and to sell this stock at times of unusually high prices. More specifically, on the basis of an expected price, a "floor price" is determined at which the authority will enter the market to buy, and a "ceiling price" at which the authority will sell. In this way, the overall variation of price is reduced. Note that the average level of price may also be altered by the operation of such a scheme. In particular, if the ceiling price is set relatively further from the expected level than is the floor price, then the average price will be raised by a process of stock accumulation (and vice versa).

In order to incorporate the precise impact of a price stabilisation scheme into the model of Section 1, specific formulae for characterising this impact are needed. Unfortunately, the derivation of these formulae is not a simple procedure. For this reason, the analysis was confined to the case where price and output were assumed to be initially jointly normally distributed. A formal derivation of the formulae listed below is contained in the Appendix.

$$E(p_s) = F(p_f)p_f + [1-F(p_c)]p_c + [F(p_c)-F(p_f)]\epsilon_2 \tag{5}$$

$$\begin{aligned} Var(p_s) = {} & [F(p_c)-F(p_f)]\sigma_2^2 + F(p_f)[p_f-E(p_s)]^2 \\ & + [1-F(p_c)][p_c-E(p_s)]^2 \\ & + [F(p_c) - F(p_f)][\epsilon_2-E(p_s)]^2 \end{aligned} \tag{6}$$

$$\begin{aligned} Cov(p_s,x) = {} & \rho(\sigma_x/\sigma_p)\sigma_2^2[F(p_c)-F(p_f)] + \rho\sigma_x Z(p_f)(\epsilon_2-p_f) \\ & +\rho\sigma_x Z(p_c)(p_c-\epsilon_2) \end{aligned} \tag{7}$$

where:

p_f = floor price

p_c = ceiling price

$Z(p_f) = (1/\sqrt{2\pi})\exp\left[-0.5[(p_f-\bar{p})/\sigma_p]^2\right]$

$Z(p_c) = (1/\sqrt{2\pi})\exp\left[-0.5[(p_c-\bar{p})/\sigma_p]^2\right]$

$F(p_f)$ = cumulative probability of $p \leq p_f$

$F(p_c)$ = cumulative probability of $p \leq p_c$

$E(p_s)$ = expected stabilised price

$\epsilon_2 = \bar{p} + \sigma_p[Z(p_f) - Z(p_c)]/[F(p_c)-F(p_f)]$

$Var(p_s)$ = variance of the stabilised price

$Cov(p_s,x)$ = covariance of the stabilised price with output.

ρ = correlation coefficient of the underlying joint normal distribution

$$\sigma_2^2 = \sigma_p^2\left\{1 + \left(\left[\frac{p_f-\bar{p}}{\sigma_p}\right]Z(p_f) - \left[\frac{p_c-\bar{p}}{\sigma_p}\right]Z(p_c)\right)/\left(F(p_c-F(p_f)\right)\right.$$

$$\left. - \left([Z(p_f) - Z(p_c)]/[F(p_c)-F(p_f)]\right)^2\right\}$$

Note that the formula for $E(p_s)$ implicitly assumes that any differences in the elasticity of demand at times of buying and selling are manifested as stock adjustments. Nevertheless, the formula could be altered to take account of the mean price impact of such differences. Note also that if:

$$\bar{p} - p_f = p_c - \bar{p}$$

then the formulae simplify to:

$$E(p_s) = \bar{p} \tag{5'}$$

$$Var(p_s) = [F(p_c)-F(p_f)]\sigma_2^2 + 2F(p_f)(p_f-\bar{p})^2 \tag{6'}$$

$$Cov(p_s,x) = \rho[(\sigma_x/\sigma_p)\sigma_2^2[F(p_c)-F(p_f)] + \rho\sigma_x Z(p_f)(p_c-p_f) \tag{7'}$$

Section 3: Information Requirements

In order to be able to use (4) to evaluate a producer's supply response to price stabilisation, three broad types of information are required:

(i) a specification of the producer's risk aversion as characterised by his utility function;

(ii) a specification of the producer's initial economic circumstances. This is taken in what follows to comprise[5]:

(a) $f(\ell)$

(b) σ_p^2

(c) σ_θ^2

(d) $\sigma_{p\theta}$

(e) $\bar{p}$

5. The assumption that w is constant for a producer means that information about its value will not be required. See also note 6.

(iii) a specification of the floor price and ceiling at which the price stabilisation scheme operates--which in this case is taken to refer to the Australian Wool Corporation's Reserve Price Scheme.

It is assumed that the producer's attitude to income risk can be adequately represented by the constant relative risk aversion function:

$$U(px) = px^{(1-R)}/(1-R)$$

It should be noted that this assumption simplifies (4) by eliminating the terms related to whether R is increasing or decreasing (R'). In what follows, a range of values of R consistent with empirical estimates is considered (see Newbery and Stiglitz 1981, Chap.7).

The specification of the producer's initial economic circumstances requires a mixture of assumptions and actual industry data. The already-simplified relationship between the producer's labour input and his output [f(ℓ)] requires further simplification to a precise functional form. In what follows it is assumed that this form is given by:

$$\bar{x} = \ell^{m}$$

where it is also assumed that m lies in the range 0.5 to unity, and ℓ is given a positioning value equal to unity ($\bar{x}$ = 1).[6] The producer's information about the relative size of σ_p^2, σ_θ^2 and $\sigma_{p\theta}$ is based on actual industry data with the additional main assumption that the producer has rational expectations (ie, his beliefs about σ_p^2, σ_θ^2 and $\sigma_{p\theta}$ are correct). With the Australian Wool Corporation's Reserve Price Scheme to be used as the example price stabilisation scheme, suitable details of the breakdown of the overall income variation in the Australian wool industry are provided in Harris et al (1974). Using this breakdown, which is based on the following approximation[7]:

$$\sigma_y^2 = \bar{x}^2\sigma_p^2 + \bar{p}^2\sigma_x^2 + 2\bar{p}\bar{x}\ \sigma_{px}$$

and setting a positioning value for income variability of[8]:

$$\sigma_y^2 = 10$$

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6. Note that for $\bar{x}$ = 1 to satisfy (2) over a range of values of R, the value of w must be (precisely) inversely related to the value of R. However, as the results are calculated in percentage change terms, this additional assumption concerning w is not felt to be particularly restrictive.
7. See Harris et al (1974) pp.304-305.
8. Recalling note 6, σ_y^2 would also vary over a range of values of R but for the setting of $\bar{x}$ = 1 for all producers.

gives:

$$\bar{x}^2 \sigma_p^2 = 12.4$$

$$\bar{p}^2 \sigma_x^2 = 0.8$$

$$2\bar{p}\bar{x} \sigma_{px} = -3.2$$

which, recalling that initially:

$$\ell = 1$$

gives:

$$\sigma_p^2 = 12.4 \ .$$

However, further specification of this breakdown requires an initial setting of $\bar{p}$. A positioning value of:

$$\bar{p} = 13$$

was chosen with a view to establishing an initial coefficient of variation (CV) of each of the random variables which corresponded closely to the actual industry values calculated by Harris et al (1974 p.302). With this initial setting:

$$\sigma_x^2 = \sigma_\theta^2 = 0.005$$

$$\sigma_{px} = \sigma_{p\theta} = -0.123$$

so that (with actual industry values in parenthesis):

$$CV_p = 27.1\% \ (29.9\%)$$

$$CV_x = 7.1\% \ (7.5\%)$$

$$CV_y = 24.6\% \ (23.4\%)$$

Note also that these initial settings give:

$$E(y) = \bar{p}\bar{x} + \sigma_{px} = 12.877$$

as the initial value of expected income.

Finally, in what follows the price stabilisation scheme is considered to operate with a range of floor prices between 80 and 90 per cent of of the mean price, and with a symmetrically set ceiling price so that $E(p_s) = \bar{p}$.[9]

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9. By imposing a symmetrically set ceiling price, attention is focused solely on the price stabilising effects of the scheme. Nevertheless, the formula for $E(p_s)$ could readily be adjusted to take account of any "Transfer Benefit" (see Newbery and Stiglitz 1981, pp.266-267).

Section 4: Results and Discussion

On the basis of the information detailed in the previous section, the formulae for the impact of a stabilisation scheme on the producer's uncertain conditions given in Section 2 and the model of producer behaviour outlined in Section 1, it is possible to estimate the supply response of individual producers to the introduction of a price stabilisation scheme for a range of price floors (and ceilings) and attitudes to risk.

It must be emphasised at this point that the results to follow represent the impact on an individual producer's output of a price stabilisation scheme, and not an aggregate impact. For this reason, the results take no account of any price change which would itself follow a change in aggregate output (unless demand is perfectly elastic). Because such a price change would to some extent reverse the initial output response, the following results should be interpreted as (absolute) upper bound impacts. The issue of the aggregation of individual impacts, which is crucially bound up with the distribution of risk attitudes among individuals, and which is required to complete the welfare assessment of the price stabilisation scheme, will be discussed at the end of this section.

Turning now to the results, Table 1 gives the impact of the

Table 1: Impact of the Scheme on the Variance of Price and the Covariance of Price with Output

Floor Price (% of $\bar{p}$)	$Var(p_s)/\sigma_p^2$	$Cov(p_s,x)/\sigma_{px}$
80	0.342	0.539
85	0.219	0.420
90	0.110	0.288

scheme on the variance of price and the covariance of price with output for a range of price floors (with symmetrical ceilings). Clearly, the higher the price floor, the greater is the reduction in price variability and covariability. Equation (2) shows that for values of R less than unity both of these changes result in this case in a favourable welfare impact. In particular, it indicates that even a risk neutral producer would benefit from the introduction of a price stabilisation scheme because the associated reduction in the magnitude of the negative covariance of price with output increases expected income. By contrast, for values of R greater than unity, Equation (2) shows that the covariance effect is unfavourable to producer welfare. Nevertheless it seems unlikely that this effect would be strong enough to dominate the favourable impact of the reduction in the variance of prices.

These expectations are to a large extent confirmed in the results presented in Table 2, which gives estimates with m (labour productivity) set equal to 0.5 of supply responses for a range of price floors and individual attitudes to risk. For values of R less than unity, all responses are positive, reflecting the favourable welfare impact of the scheme. In addition, this impact is positively related to the size of the price floor as can be seen from the

Table 2: Estimates of Supply Response to Price Stabilisation (percentage change in output; m = 0.5)

Floor price	R						
(% of $\bar{p}$)	0	0.3	0.6	0.9	1.2	1.5	1.8
80	0.440	0.561	0.410	0.117	-0.253	-0.666	-1.098
85	0.554	0.678	0.489	0.139	-0.300	-0.788	-1.298
90	0.689	0.795	0.564	0.159	-0.342	-0.895	-1.473

increasing magnitude of responses down the table. This feature is consistent with the estimates of reduced variability and covariability presented in Table 1. Also consistent with the estimates in Table 1 is the larger magnitude of supply response for R = 0.3 than for R = 0. Specifically, while the risk neutral producer benefits from the reduced magnitude of the negative covariance, the risk averse producer also benefits from the reduction in price variability. In the case of R = 0.3, this additional favourable welfare impact is strong enough to result in a larger supply response than that for R = 0. Note, however, that for values of R > 0.3 (and less than unity), the tendency for increased risk aversion also to inhibit the willingness of a producer to respond to improved economic circumstances dominates the enhanced favourable welfare impact of the scheme and the positive supply responses diminish in magnitude. For values of R greater than unity, all responses are negative, once again representing a favourable welfare impact, and increase in magnitude both with the size of the price floor and with the size of R.[10]

A further point to note about these responses is that in all cases their magnitude represents approximately one per cent or less of initial output. The possibility that this unresponsiveness was due to relatively unproductive labour was examined by recalculating the estimates for m = 0.99 (ie, almost constant returns to labour). These responses are presented in Table 3. By comparison with those in

Table 3: Estimates of Supply Response to Price Stabilisation (percentage change in output; m = 0.99)

Floor price	R						
(% of $\bar{p}$)	0	0.3	0.6	0.9	1.2	1.5	1.8
80	54.431	2.373	1.078	0.245	-0.460	-1.100	-1.693
85	72.734	2.871	1.288	0.290	-0.546	-1.301	-2.001
90	95.620	3.357	1.482	0.332	-0.620	-1.475	-2.266

10. This can be seen by comparing equations (2) and (4), but see also Newbery and Stiglitz (1981, p.311) for a related discussion.

Table 2, they show a general increase in magnitude which is so marked at lower levels of risk aversion that the consequences of the additional favourable welfare impact of the scheme for risk averse producers can no longer be detected. Even though almost constant returns to labour may be considered an extreme example, these results clearly support the view that labour productivity can be an important determining factor of the size, but not the direction, of the supply response to price stabilisation.

Returning, finally, to the issue of the aggregate supply response to price stabilisation, it can be seen from the individual producer responses listed in Tables 2 and 3 that, even if all producers had the same (known) attitude to risk, determining the magnitude of the aggregate response still requires information on the distribution of total output among producers and on their individual labour productivity. However, the distribution of attitudes to risk among producers is unlikely to be of such a simple form, in which case the results in Tables 2 and 3 show the actual distribution will typically be a determinant of not only the magnitude but also the direction of the aggregate supply response. For example, consider the situation where all producers have the same labour productivity (given by m=0.5) and share of total output. With attitudes to risk distributed such that there are even numbers of producers at the indicated values of R between 0 and 1.8, price stabilisation with a floor price set at 90 per cent of $\bar{p}$ is estimated to <u>reduce</u> aggregate supply by 0.07 per cent; whereas with this even distribution but at values of R between 0.3 and 1.5, full price stabilisation is estimated to <u>increase</u> aggregate supply by 0.04 per cent.

While these information requirements are not inconsiderable, once an aggregate supply response to a price stabilisation scheme has been estimated, it can be combined with information on supply and demand elasticities to determine the equilibrium (mean) price impact, and consequently the complete welfare assessment, of the scheme. In this regard, it should be noted that the model in section 1 may also be used to estimate an aggregate supply elasticity contingent on the same information requirements outlined above. For example, using the initial data settings of section 3 but imposing a one per cent increase in mean price gives the range of supply responses listed in Table 4a. And making the same assumptions

<u>Table 4a</u>: Supply Responses to an Increase in Expected Price of One Per Cent

($\% \Delta \bar{p} = 1$)

	R						
	0	0.3	0.6	0.9	1.2	1.5	1.8
m = 0.5	1.007	0.541	0.250	0.052	-0.091	-0.199	-0.284
m = 0.99	170.321	2.338	0.686	0.117	-0.178	-0.362	-0.490

Table 4b: Examples of Estimated Aggregate Supply Elasticities

	$0 \leq R \leq 1.8$	$0.3 \leq R \leq 1.5$
m = 0.5	0.182	0.111
m = 0.99	24.633	0.520

as above about the labour productivity, output distribution and risk attitudes of producers (ie, identical output and productivity and an even distribution of producers at the indicated values of R) gives the range of estimated aggregate supply elasticities listed in Table 4b. It is clear from this table that both the levels of labour productivity and the distribution of risk attitudes among producers are important determinants of the magnitude of the aggregate supply elasticity.

Section 5: Conclusion

The intention of this paper has been to present a method for evaluating the supply response of individual producers to a price stabilisation scheme. This method required the development of precise formulae to take account of the impact of price stabilisation on the producer's uncertain conditions. The Australian Wool Corporation's Reserve Price Scheme was taken as a specific example of price stabilisation in practice. Individual supply responses indicated a favourable welfare impact of the scheme. Nevertheless, both the magnitude and the direction of individual responses were shown to vary depending on the level of the price stabilisation and the degree of risk aversion of the producer. The productivity of labour was also shown to determine the magnitude of the individual producer's supply response.

Appendix

Double Winsorising a Normal Distribution

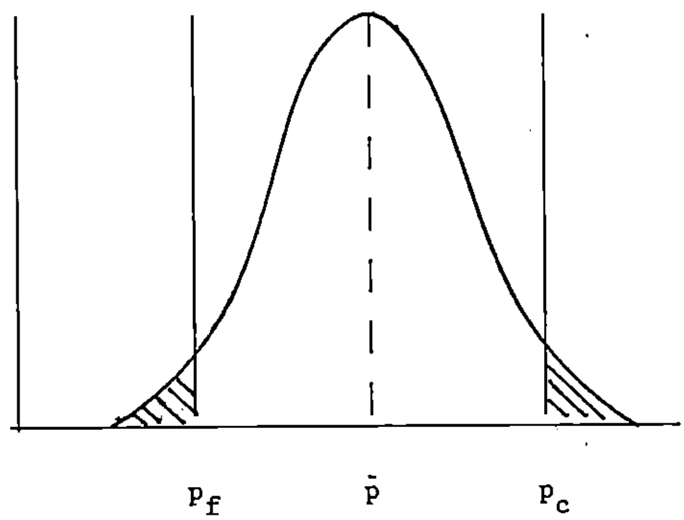

Figure 1: Winsorising a Normal Distribution

Let p_f be the lower and p_c the upper point of Winsorising, $\bar{p}$ the original mean and σ_p^2 the original variance of a normal distribution.

Then this is equivalent to mixing three distributions in the proportions: $F(p_f)$, $[F(p_c)-F(p_f)]$ and $[1-F(p_c)]$ where:

for $p \le p_f$: $\epsilon_1 = p_f \quad \sigma_1^2 = 0$

for $p_f < p < p_c$: $\epsilon_2 = E(p|p_f < p < p_c)$

$$\sigma_2^2 = Var(p|p_f < p < p_c)$$

for $p \ge p_f$: $\epsilon_3 = p_c \quad \sigma_1^2 = 0$

The second of these is a double truncated normal distribution where:

(1) $$\epsilon_2 = \bar{p} + \left[\left[Z(p_f) - Z(p_c)\right] / \left[F(p_c)-F(p_f)\right]\right]\sigma_p$$

(1) $$\epsilon_2 = \bar{p} + \left[\left(Z(p_f) - Z(p_c)\right)/\left(F(p_c)-F(p_f)\right)\right]\sigma_p$$

Note: If $\bar{p} - p_f = p_c - \bar{p}$, then $\epsilon_2 = \bar{p}$.

(2) $$\sigma^2 = \sigma_p^2\left\{1 + \left(\left[\frac{p_f - \bar{p}}{\sigma_p}\right]Z(p_f) - \left[\frac{p_c - \bar{p}}{\sigma_p}\right]Z(p_c)\right)/\left(F(p_c)-F(p_f)\right) - \left([Z(p_f)-Z(p_c)]/[F(p_c)-F(p_f)]\right)^2\right\}$$

Note: If $\bar{p} - p_f = p_c - \bar{p}$ then:

$$\sigma_2^2 = \sigma_p^2\left(1 - 2[(p_c-\bar{p})/\sigma_p]Z(p_f)/[1 - 2F(p_f)]\right)$$

(see Johnson and Kotz 1970, pp.81-83).

Given the following formulae for a mixture (p_s):

$$E(p_s) = \sum_{i=1}^{k} p_i \epsilon_i$$

$$Var(p_s) = \sum_{i=1}^{k} p_i \sigma_i^2 + \sum_{i=1}^{k} p_i(\epsilon_i-\bar{\epsilon})^2$$

where:

$$\bar{\epsilon} = \sum_{i=1}^{k} p_i \epsilon_i = E(p_s)$$

For k = 3 and the above information:

(3) $$E(p_s) = F(p_f)p_f + [1-F(p_c)]p_c + [F(p_c)-F(p_f)]\epsilon_2$$

Note: If $\bar{p} - p_f = p_c - \bar{p}$, then $E(p_s) = \bar{p}$.

(4) $$Var(p_s) = [F(p_c)-F(p_f)]\sigma_2^2 + F(p_f)[p_f-E(p_s)]^2$$

$$+ [1-F(p_c)][p_c-E(p_s)]^2 + [F(p_c)-F(p_f)][\epsilon_2-E(p_s)]^2$$

Note: If $\bar{p} - p_f = p_c - \bar{p}$, then:

$$Var(p_s) = [F(p_c)-F(p_f)]\sigma_2^2 + 2F(p_f)(p_f-\bar{p})^2$$

(see Johnson and Leone 1964, p.129).

To assess the impact of double Winsorising on the covariance between p_s and some other normally distributed variable v, let ρ be the correlation coefficient and $\rho\sigma_p\sigma_v$ the initial covariance. Then:

$$Cov(p_s,v) = \int_{-\infty}^{p_f}\int_{-\infty}^{\infty} [p_f-E(p_s)][v-E(v)]f_{pv}dpdv \quad (a)$$

$$+ \int_{p_f}^{p_c}\int_{-\infty}^{\infty} [p-E(p_s)][v-E(v)]f_{pv}dpdv \quad (b)$$

$$+ \int_{p_c}^{\infty}\int_{-\infty}^{\infty} [p_c-E(p_s)][v-E(v)]f_{pv}dpdv \quad (c)$$

Consider term (a):

$$= [p_f - E(p_s)]F(p_f)[\bar{v}_f - E(v)]$$

where:

$$\bar{v}_f = E(v) + \rho(\sigma_v/\sigma_p)[-\sigma_p Z(p_f)/F(p_f)]$$

(see Mood, Graybill and Boes 1974, p.167; Maddala 1983, p.367).

Consider term **(c)**:

$$= [p_c - E(p_s)][1 - F(p_c)][\bar{v}_c - E(v)]$$

where:

$$\bar{v}_c = E(v) + \rho(\sigma_v/\sigma_p)\left[\sigma_p Z(p_c)/[1 - F(p_c)]\right]$$

Consider term **(b)**:

$$= \int_{p_f}^{p_c} \int_{-\infty}^{\infty} (p - \epsilon_2)[v - E(v)] f_{pv} dp dv$$

$$+ \int_{p_f}^{p_c} \int_{-\infty}^{\infty} [\epsilon_2 - E(p_s)][v - E(v)] f_{pv} dp dv$$

$$= Cov(p_s, v | p_f < p < p_c)[F(p_c) - F(p_f)]$$

$$+ [\epsilon_2 - E(p_s)][F(p_c) - F(p_f)][\bar{v} - E(v)]$$

where:

$$\bar{v} = E(v) + \rho(\sigma_v/\sigma_p)\left(\sigma_p [Z(p_f) - Z(p_c)]/[F(p_c) - F(p_f)]\right)$$

Note: $f(p_s) = f(p)/[F(p_c) - F(p_f)]$

Note also that:

$$\text{Cov}(p_s, v \mid p_f < p < p_c) = \rho(\sigma_v/\sigma_p)\sigma_2^2$$

(see Johnson and Kotz 1972, p.112).

Collecting and simplifying terms (a), (b) and (c):

$$(5)\ \text{Cov}(p_s, v) = \rho(\sigma_v/\sigma_p)\sigma_2^2[F(p_c)-F(p_f)] + [\epsilon_2 - E(p_s)]\rho\sigma_v[Z(p_f)-Z(p_c)]$$

$$- [p_f - E(p_s)]\rho\sigma_v Z(p_f) + [p_c - E(p_s)]\rho\sigma_v Z(p_c)$$

$$= \rho(\sigma_v/\sigma_p)\sigma_2^2[F(p_c)-F(p_f)] + \rho\sigma_v Z(p_f)(\epsilon_2 - p_f)$$

$$+ \rho\sigma_v Z(p_c)(p_c - \epsilon_2)$$

Note: If $\bar{p} - p_f = p_c - \bar{p}$, then:

$$\text{Cov}(p_s, v) = \rho(\sigma_v/\sigma_p)\sigma_2^2[F(p_c)-F(p_f)] + \rho\sigma_v Z(p_f)(p_c - p_f)$$

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